Let $y = -1$, then for all real numbers x,

$$f(x + f(-x)) = f(x)(1 - 1) = 0.$$

Let a be any real number, and let $y = a + f(-a)$. Since $f : \mathbb{R} \to \mathbb{R}$, y is a real number. If we then set $x = 1$, f must then satisfy

$$f(1 + f(a + f(-a))) = f(1)(1 + (a + f(-a))).$$

Since we've proven that $f(a + f(-a)) = 0$ for all real numbers a, we get

$$f(1 + f(a + f(-a))) = f(1) = f(1)(1 + a + f(-a)).$$

Thus, for all real numbers a,

$$\begin{aligned}
&(1 + a + f(-a)) = 1, \\
&a + f(-a) = 0, \\
&f(-a) = -a
\end{aligned}$$

It is well known that the only function satisfying $f(x) = x$ for all real numbers x is the identity function. Thus, the only function $f : \mathbb{R} \to \mathbb{R}$ satisfying the functional equation is $f(x) = x$.