Lets assume that real constants p, q, r are currently fixed such that the system of equations yields the maximum number of solutions. Lets fix c to be any real number. Then, let $x = p - c$ where x is a fixed constant. From the first equation, we have

$$b = x - a^2.$$

Plugging this into the second equation, we get

$$a + (x - a^2)^2 + (c - q) = a^4 - (2x)a^2 + a + (c - q + x^2) = 0.$$

We know that c, q, x are all currently set to be a fixed value. Thus, $c - q + x^2$ is a fixed constant. Based on the Fundamental Theorem of Algebra, the above polynomial $a^4 - (2x)a^2 + a + (c - q + x^2) = 0$ has at most 4 real roots. This means that for any real number c and the optimal combination p, q, r, there at most 4 real values of a.

For every pair of solutions (a, c) value, there is exactly one corresponding b value. This is because $x = p - c$ and a^2 will both be a fixed number based on the values of a and c. Thus, there is only one value for $b = x - a^2$.

From the third equation, we get

$$c^2 = r - a - b = r - a - (p - c - a^2).$$

Rearranging, we have

$$c^2 - c + (p - r - a^2 + a) = 0.$$

We have proven that for every c, there are at most $4 \cdot 1 = 4$ pairs of solutions (a, b) that will satisfy the previous two equations. To satisfy the third and final equation, there are at most two possible c value we can choose from based on the Fundamental Theorem of Algebra. This is because for every corresponding c we pick, we have at most four real values of a satisfying the system. In the most optimal scenario, we predict and fix the four real values of a that will yield the maximum number of solutions. Then, the above equation becomes a polynomial of degree 2 as a, p, r are all fixed constant values. Therefore, there are at most two c values that go with every pair of solutions (a, b) for a total of $4 \cdot 2 = 8$ solutions (a, b, c).

However, this is just the best case scenario where we assumed all our solutions we found were real. Now, I will give an example of p, q, r that achieves a total of 8 solutions of real triplets (a, b, c). Setting $p = 2, q = 2, r = 2$, we yield the following 8 solutions:

$$
\begin{aligned}
&(2, -1, -1), \\
&(-1, 2, -1), \\
&(-1, -1, 2), \\
&(1, 0, 0), \\
&(0, 1, 0), \\
&(0, 0, 1), \\
&(\sqrt{3} - 1, \sqrt{3} - 1, \sqrt{3} - 1), \\
&(-\sqrt{3} + 1, -\sqrt{3} + 1, -\sqrt{3} + 1)
\end{aligned}
$$

Plugging each of these triplets (a, b, c) into the system of equations, as well as $(p, q, r) = (2, 2, 2)$, we find that they are all valid real solutions.