11-7-2024 Testing Round 1
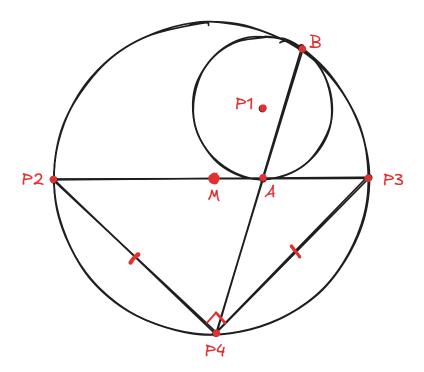
Problem 1: A millennium problem in ancient times is the chicken and rabbits problem. There are 54 heads and 152 legs. Let the number of rabbits be x. What is the sum of the digits of x?

Problem 2: To get ready, you must choose a colored hat, robe, staff. For each object, you can choose one of 3 colors. However, it must be that two of the colors of the objects must be the same, and the third color different. How many choosing's are there?

Problem 3: As you are headed to devil mountain, you and your 3 other demon hunter friends wants to pay for the carriage fee. All of you have exactly one of $\$1, \$2, \$3$ coin. Each of you wants to pitch in exactly one coin to the total carriage fee, which is $\$8$. How many ways are possible?

Problem 4: The entrance to the enchanted forest is protected by the demon's magic. There are 100 doors numbered 1 to 100 from left to right, all remain closed at the beginning. Toggling the state of the door means if it's open, then close it, and if it's closed, then open it. The procedure to finding the "safe" door is as follows: For the first round, toggle all the doors. For the second round, toggle every even door. For the third round, toggle every third door. You must follow this strategy in 100 rounds. More formally, for each round i from $1 \le i \le 100$, you must toggle every ith door. The doors that are open at the end of this process are "safe". What is the sum of the index of all the safe doors.

Problem 5: You come face to face with the demon. Since the demon is big, you and your friends must set up a precise spell (zheng) to defeat the monster. To cast the spell, all of you must work together and stand in fixed positions and cast individual spells precisely. The spell can be modelled as a geometric diagram. Let M be the monster, which stands in the middle of the circumcircle $(P_2P_3P_4)$, where your 3 friends stand. Also, $P_2P_3 = P_3P_4$ and $\angle P_2P_3P_4 = 90°$. You must then stand at P_1, the center of a circle tangent to line P_2P_3 at A and circle $(P_2P_3P_4)$ at B. By the scale of the monster, it is known that the radius of circle $(P_2P_3P_4)$ is 10 and P_4A is 13. You are tasked to cast a spell that goes from A to B. The length AB can be represented as $\frac{a}{b}$ in its lowest form. What is $a + b$?

(note: the circle may not be drawn to scale)