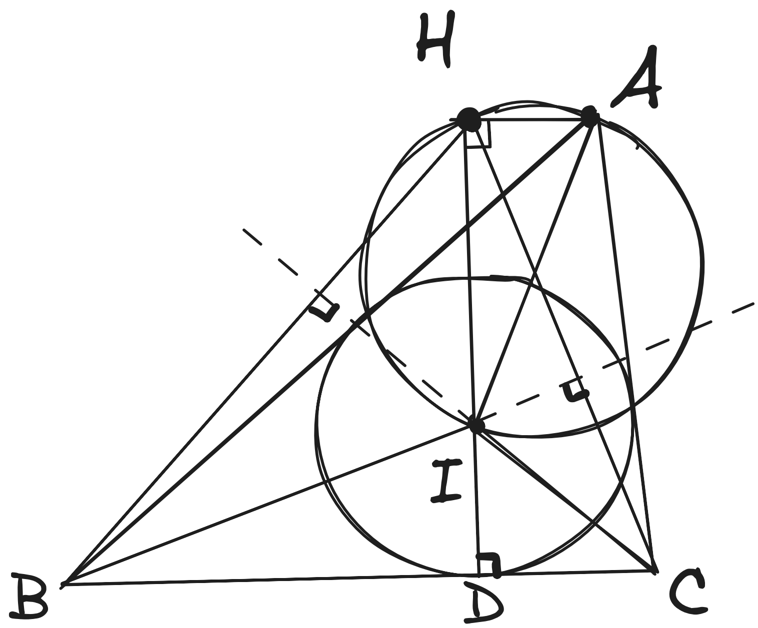
Part (a)

Let D be the foot of the altitude from I to BC.



First, we will prove that if AH is parallel to BC, then H lies on the circle. Since H is the orthocenter of $\triangle BIC$, H is perpendicular to BC which means $\angle BDH = 90^\circ$. Since AH is parallel to BC, $\angle AHI = \angle BDH = 90^\circ$. By a well-known theorem, if we know that AI is the diameter of a circle O and $\angle AHI = 90^\circ$, then point H is on circle O. Since we have proven $\angle AHI = 90^\circ$, H must lie on the circle.

Now we will prove that if H lies on the circle, then AH is parallel to BC. Since H is on the circle with diameter AI, $\angle AHI = \angle AHD = 90^\circ$. We also know that $\angle BDH = 90^\circ$. Because D is on BC, HD is perpendicular to both BC and AH. Thus, AH and BC must be parallel.

Part (b)

Let I be the incenter
Based on Eulers formula, $IT^2 = R^2 - 2Rr$

Part b)

Eulers formulas

d2 = R (R - 2r)

https://www.google.com/search?q=eulers+formula+circumcircle+incirc+le&rlz=1C1CHBD_enCA775CA775&oq=eulers+formula+circumcircle+incirc+le&gs_lcrp=EgZjaHJvbWUyBggAEEUYOTIJCAEQIRgKGKABMgkIAhAhGAoYoAEyCQgDECEYChigATIJCAQQIRgKGKABMgkIBRAhGAoYoAHSAQg1NTc0ajBqN6gCALACAA&sourceid=chrome&ie=UTF-8

https://math.stackexchange.com/questions/4324867/formula-for-distance-between-incenter-and-orthocenter

"IH^2 = 4R^2 + 4Rr + 3r^2 - p^2"