Biomedical Imaging FS 2019

Ultrasound 2

Please prepare solutions in pdf format and upload them on the Moodle platform (https://moodle-app2.let.ethz.ch/).

Exercise

This exercise is about a phased array and its operation. Consider a linear array of equidistant transducer elements (aligned in the y direction) sending ultrasound of 1.5 MHz into a water container that is adjacent in the x direction. The prepared Matlab code calculates the complex-valued amplitude of the resulting pressure wave in the x-y plane. It does so by summing up partial waves that emanate from the individual transducer elements. The array elements are assumed to be long in the z direction and of negligible width in the y direction such that the partial waves can be approximated as cylindrical.

Tasks

1. The number of array elements is initially set to just 1. Examine the resulting wave. If we view it as a limiting case of a single-transducer beam as discussed in the lecture, what are its NFB, lateral resolution, and broadening angle Θ?

   The wave generated by a single transducer of negligible width is cylindrical and thus not directed in any way. To view it in terms of beam characteristics consider the formulas given in the lecture:

$$NFB = \frac{r^2}{\lambda}$$

$$\text{best lateral resolution} = 2r$$

$$\theta = \arcsin\left(\frac{0.61\,\lambda}{r}\right)$$

   So the NFB is zero and the best lateral resolution is in fact very high, yet only up to the NFB and thus up to zero depth. In other words, a single narrow transducer offers high resolution but only at the very surface. Beyond the NFB, broadening limits lateral resolution. In the limiting case of r much less than λ, arcsin formula is apparently outside its meaningful range. From the wave's cylindrical nature it is clear that lateral resolution deteriorates rapidly in proportion to depth.

2. Increase the number of transducer elements at the preset pitch of lambda/2. What happens? How does the many-element case relate to what you expect from a single flat transducer?

   With increasing number of elements the array forms an ever narrower beam with sidelobes that likewise successively decrease in opening angle. For larger element counts the beam geometry matches that of a single flat transducer of the same width. Notably, at this point the phased-array is actually not phased yet since all elements transmit with the same phase. In this case, the array of negligible-width elements is a discrete approximation of a single, flat, finite-width transducer.

From here on, use 40 transducer elements.

3. Initially all transducers are driven in-phase (=0). Now alter the phasing to deflect the beam by 20°.

   To deflect the beam the array elements must be phased such that their partial waves are in-phase along a wave front orthogonal to the desired direction. To this end, each element's distance to the wavefront is compensated by a negative phase correction that makes up of for the distance. The translation of distance into phase difference is given by the wave number k, thus

$$\Delta\varphi = -\,k\;y\,\sin(\alpha),$$

   where y denotes the coordinate of the transducer element.

4. Provoke a grating lobe.

   Grating lobes can occur when the array's pitch is more than $\lambda/2$. At a deflection angle of 20° a prominent grating lobe is observed, e.g., for pitch = λ.

5. Manipulate the phasing to focus at a depth of 5 cm.

   Similar to deflection, focusing is equally achieved by phasing the array elements such that their partial waves have equal phase at the focus point. Simple trigonometry (Pythagoras) yields

$$\Delta\varphi = -\,k\,\sqrt{y^2 + d^2},$$

   where d is the focus distance long the x direction.

   An addition that is not part of the task, but interesting and fun: Simultaneous deflection <u>and</u> focusing can be achieved approximately by just adding up the phase corrections. A more accurate solution is obtained by considering the trigonometry of the combined case. Using the law of cosines (instead of Pythagoras) yields

$$\Delta\varphi = -k\sqrt{y^2 + \left(\frac{d}{\cos(\alpha)}\right)^2 + 2yd\frac{\sin(\alpha)}{\cos(\alpha)}}$$

6. As prepared, the code assumes unit amplitude for all transducer elements. Now vary the amplitude according to a Gaussian bell function along the array. Choose the width of the Gaussian such that its drops to about 10% for the outer elements.

   Trial and error yields a standard deviation of the bell function of $\sigma \approx 0.23$ normalized to the width of the whole array. Varying the relative amplitude of the transducer inputs according to such a Gaussian (so-called apodization) mitigates sidelobes at some expense in lateral resolution.

Questions?

Thomas Ulrich	(ulrich@biomed.ee.ethz.ch)
Franz Patzig	(patzig@biomed.ee.ethz.ch)
Jennifer Nussbaum	(nussbaum@biomed.ee.ethz.ch)
Samuel Bianchi	(bianchi@biomed.ee.ethz.ch)