Biomedical Imaging FS 2019

MRI 1

Exercises

1. Equilibrium Magnetization

- Calculate the relative difference $\frac{\Delta n}{n}$ in up- and down-state spin population for the common case of
  - Protons (^1H)
  - Body temperature (310 K)
  - 3T field strength

According to Boltzmann statistics, the ratio between the two population numbers is

$$\frac{n_{down}}{n_{up}} = e^{-\frac{\Delta E}{k_B T}}$$

The energy gap is

$$\Delta E = \hbar \gamma B_0$$

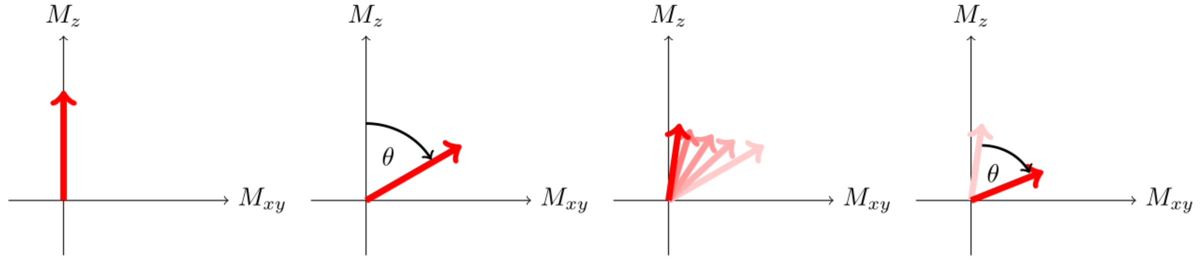
With $\gamma/2\pi = 42.576\ MHz/T$ for protons, $B_0 = 3\ T$, and $T = 310\ K$, one obtains

$$\frac{n_{down}}{n_{up}} = 0.999980227$$

$$\frac{\Delta n}{n} = \frac{n_{up} - n_{down}}{n_{up} + n_{down}} = \frac{n_{up}(1 - 0.999980227)}{n_{up}(1 + 0.999980227)} = 9.88659 \cdot 10^{-6}$$

So even at high field of multiple tesla the equilibrium magnetization amounts to just about 10^{-5} times the total of nuclear moments available.

2. Magnetization Dynamics



Initial state **First excitation** **Relaxation** **Next excitation …**

- Write a Matlab program that simulates and visualizes repeated on-resonance excitation of nuclear magnetization at one point
  - perform equal excitations of given flip angle θ at a given repetition time T_R
  - assume excitation to be an instantaneous rotation by θ. Excitation is on-resonance, so all rotations are about the same axis.
  - consider the fact that relaxation between excitations is incomplete
  - vary T_1, T_2 and the flip angle. Study the magnetization behaviour. What happens in the course of long pulse series?

For all parameter combinations with finite T1 and T2 the system converges to periodic magnetization dynamics, a so-called steady state. The magnitude of the steady-state magnetization depends strongly on the parameter configuration. Attempts to increase the signal yield by ever-larger flip angle result in saturation and magnetization loss.

- Repeated excitation leads into periodic magnetization dynamics. Assuming complete transverse relaxation per interval, which flip angle yields maximum transverse magnetization in the periodic regime? Calculate analytically for given T_1 and T_R.

Let M_z^s denote the z magnetization in the steady state just prior to the next excitation pulse. Assuming fully relaxed transverse magnetization, excitation with flip angle θ results in

$$M_z' = M_z^s \cos\theta$$

Subsequent relaxation during the pulse interval T_R results in

$$M_z'' = M_0 + (M_z' - M_0)e^{-\frac{T_R}{T_1}}$$

According to the steady-state assumption this magnetization must be equal to that present prior to the preceding excitation pulse:

$$M_z'' = M_z^s$$

and hence

$$M_z^s = M_0 + (M_z^s \cos\theta - M_0)e^{-\frac{T_R}{T_1}}$$

which rearranges to

$$M_z^s = M_0 \frac{1 - e^{-\frac{T_R}{T_1}}}{1 - \cos\theta\, e^{-\frac{T_R}{T_1}}}$$

The signal obtained in this steady state scales with the transverse magnetization

$$M_{xy} = M_z^s \sin\theta = M_0 \sin\theta \frac{1 - e^{-\frac{T_R}{T_1}}}{1 - \cos\theta\, e^{-\frac{T_R}{T_1}}}$$

The optimal flip angle is that which maximizes the expression on the right. To find it, differentiate with respect to θ and set the derivative to zero

$$\frac{dM_{xy}}{d\theta} = M_0 \left(1 - e^{-\frac{T_R}{T_1}}\right) \frac{\cos\theta \left(1 - \cos\theta\, e^{-\frac{T_R}{T_1}}\right) - \sin^2\theta\, e^{-\frac{T_R}{T_1}}}{\left(1 - \cos\theta\, e^{-\frac{T_R}{T_1}}\right)^2} = 0$$

$$\cos\theta \left(1 - \cos\theta\, e^{-\frac{T_R}{T_1}}\right) - \sin^2\theta\, e^{-\frac{T_R}{T_1}} = 0$$

$$\cos\theta - (\cos^2\theta + \sin^2\theta)\, e^{-\frac{T_R}{T_1}} = 0$$

$$\theta\ = \cos^{-1}\left(e^{-\frac{T_R}{T_1}}\right),$$

which is known as the *Ernst* angle.

- Verify your solution with your simulation code

E.g., for $T_1 = 200\ ms, T_R = 50\ ms$ the Ernst angle is 38.85 degrees. Using the simulation tool one can check that both larger and smaller angles indeed yield less transverse magnetization. (Make sure to enforce full transverse relaxation and perform sufficient iterations to reach the steady state).