Biomedical Imaging

Magnetic Resonance Imaging

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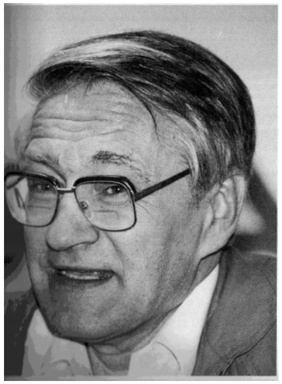
Magnetic Resonance Imaging

- **Probe:** radiofrequency waves
- **Wavelength:** **10 – 50 cm**
- **Matter interaction:** nuclear spin transitions

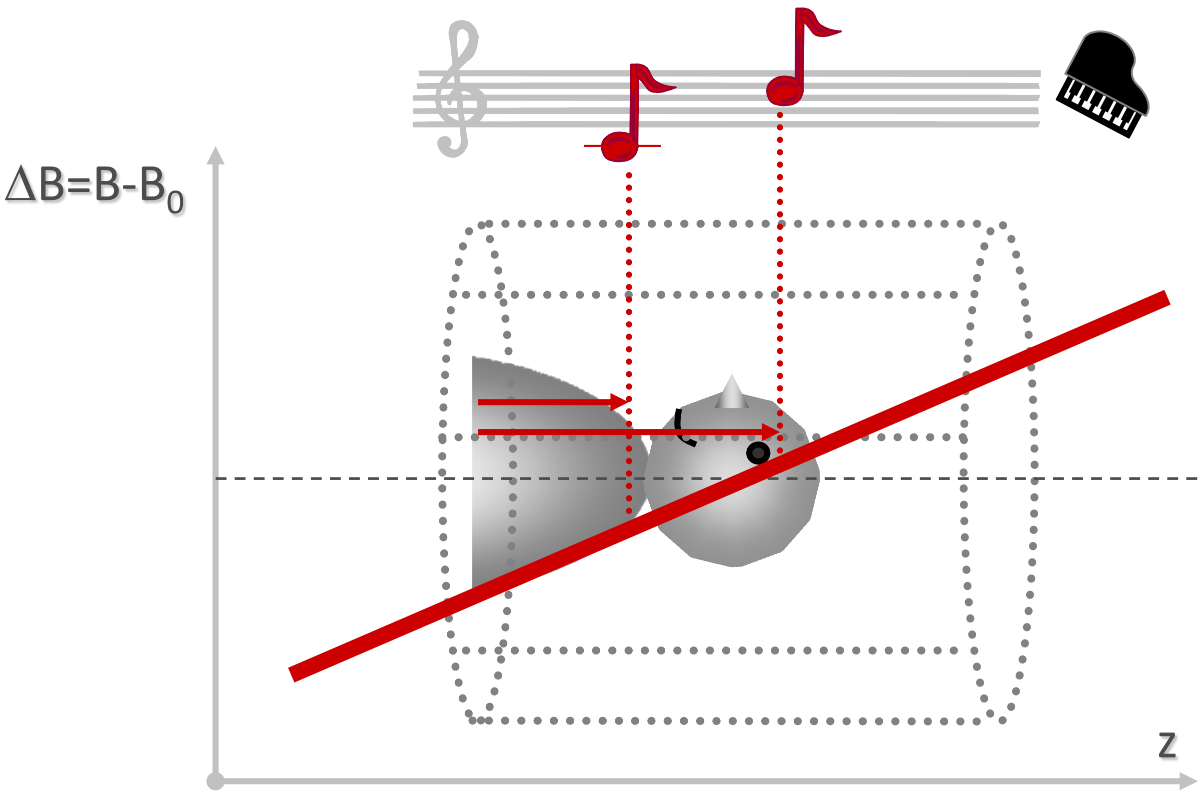


The radio-frequency window, on the other hand, was not exploited until 1972. This is not astonishing considering the achievable resolution, which is usually limited by the wavelength of the applied radiation through the uncertainty relation. The maximum radio-frequency useful for imaging is about 100 MHz, leading to a resolution of 3 m which is not sufficient even for imaging elephants.

The crucial idea, as first proposed by Lauterbur (10.1, 10.2), is to utilize a magnetic field *gradient* to disperse the NMR resonance fre-

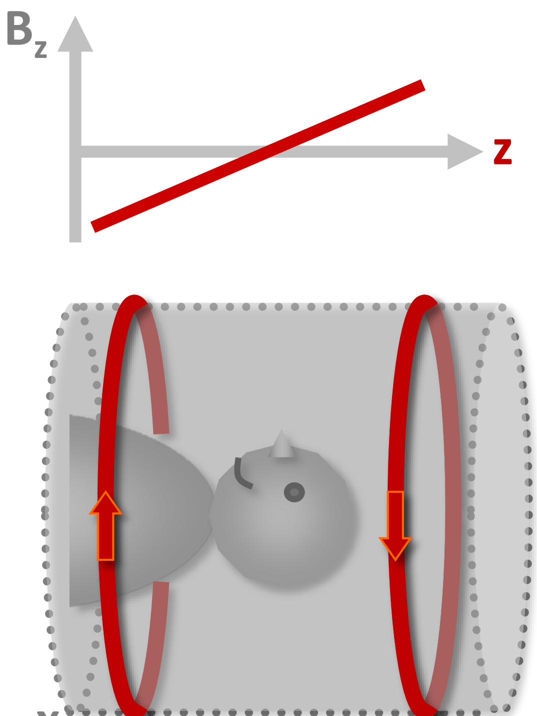
R.R. Ernst et al. (1987)

Position → Frequency

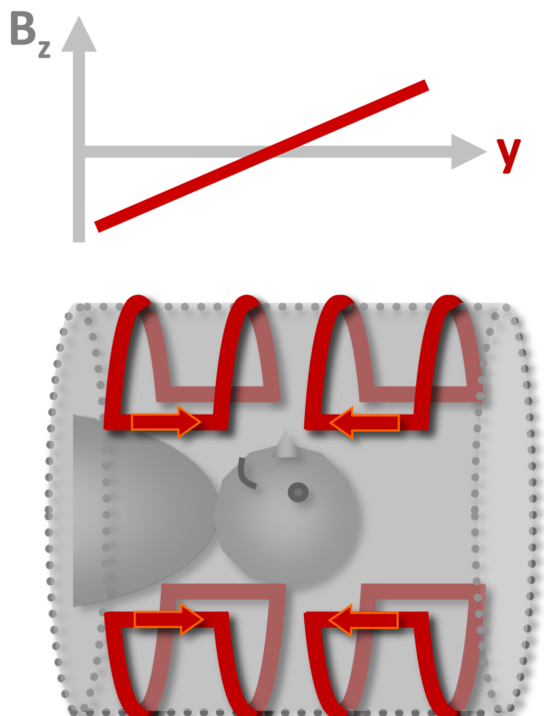


$$\omega(z) = \gamma\left(B_0 + \frac{\partial B_z}{\partial z} z\right)$$

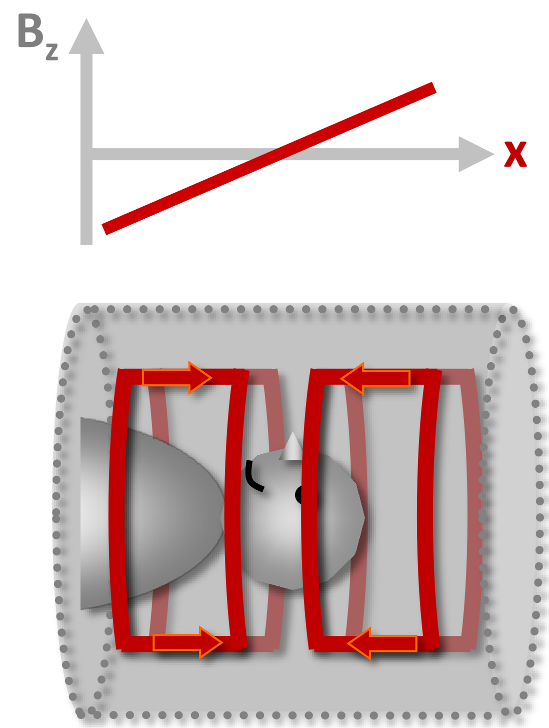
Magnetic gradient fields



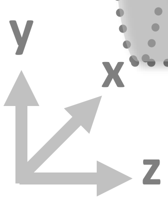
“Maxwell” pair



“Golay” pair



“Golay” pair



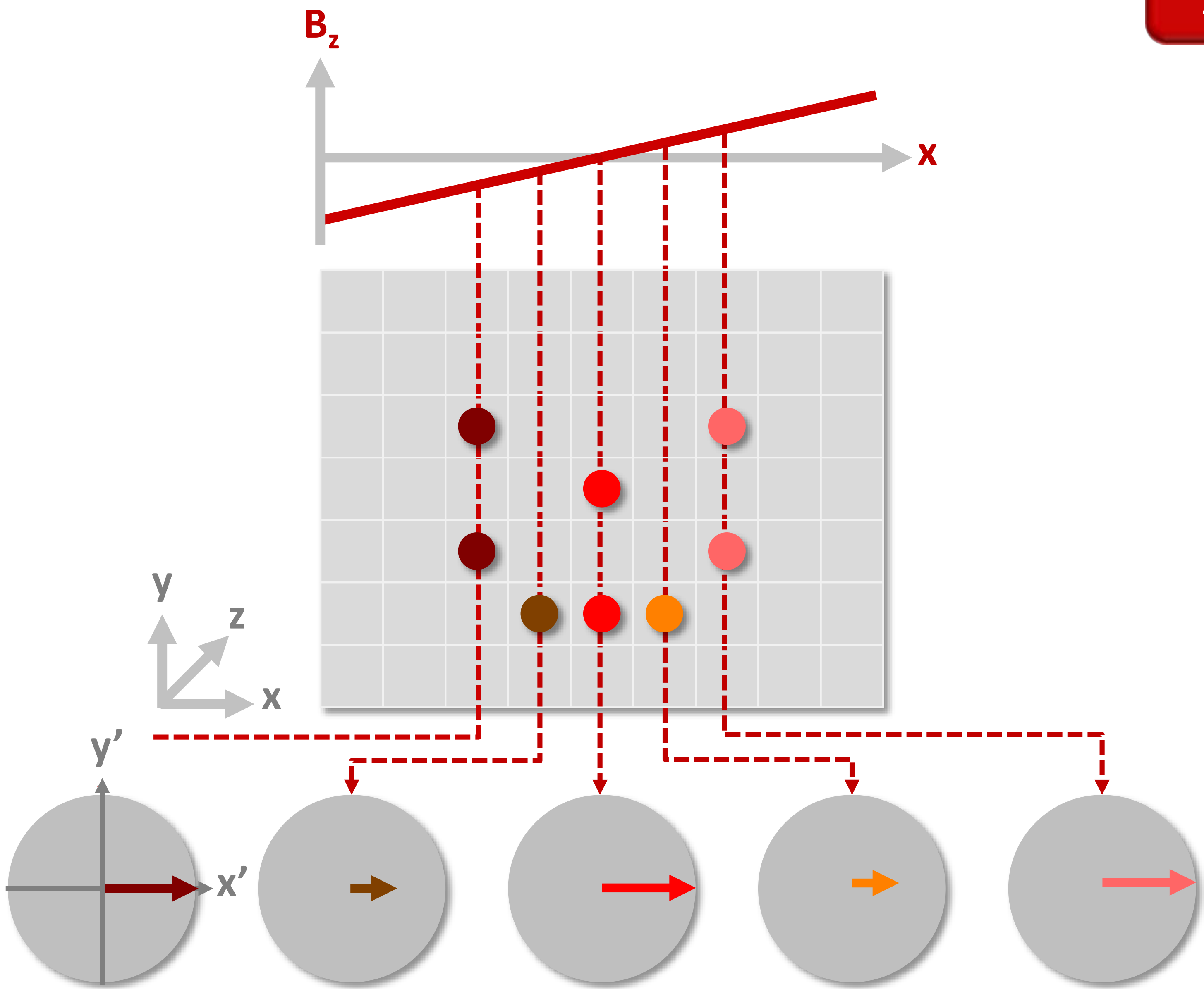
$$G_z = \frac{\partial B_z}{\partial z}$$

$$G_y = \frac{\partial B_z}{\partial y}$$

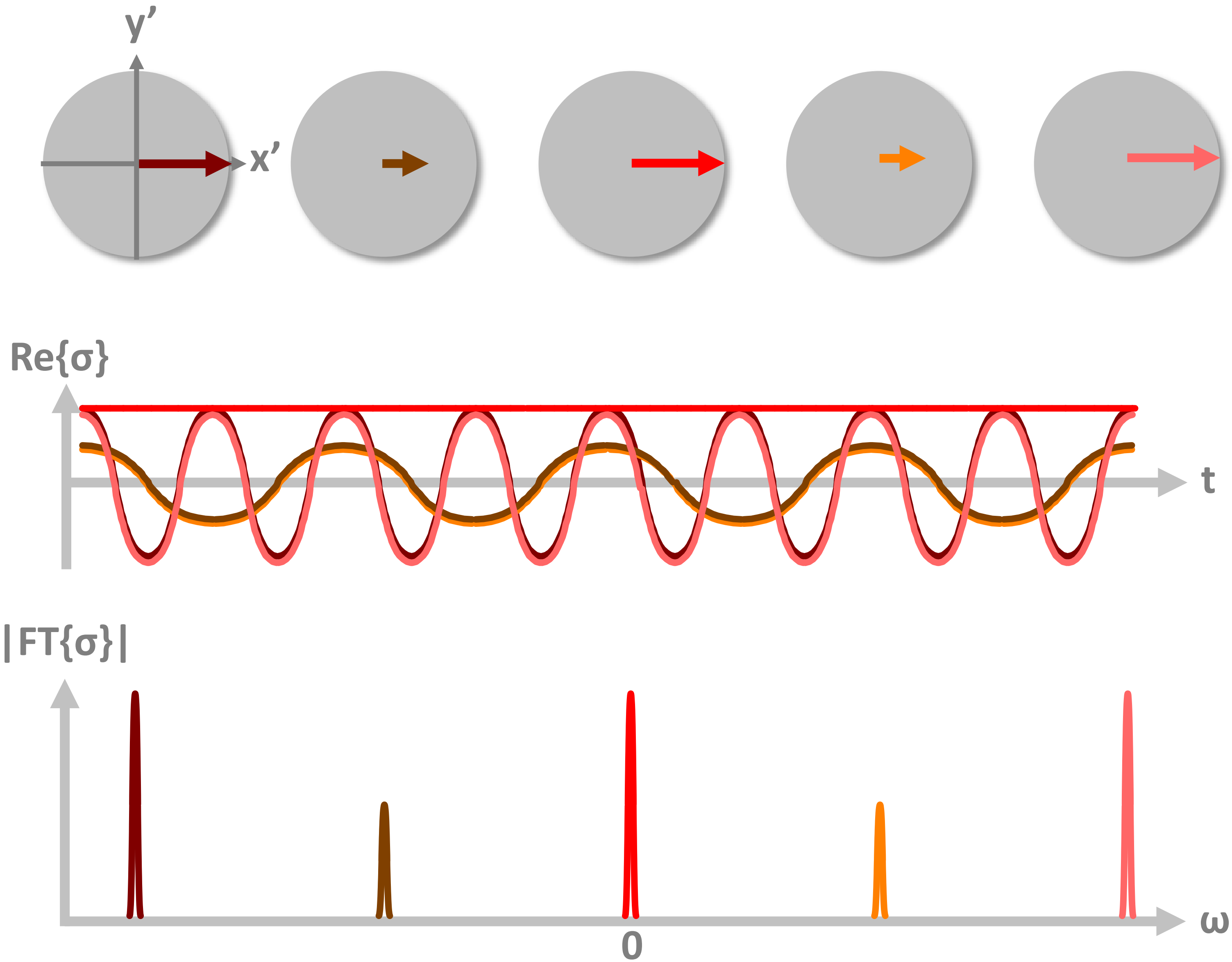
$$G_x = \frac{\partial B_z}{\partial x}$$

Frequency encoding

Chapter 5.9.3

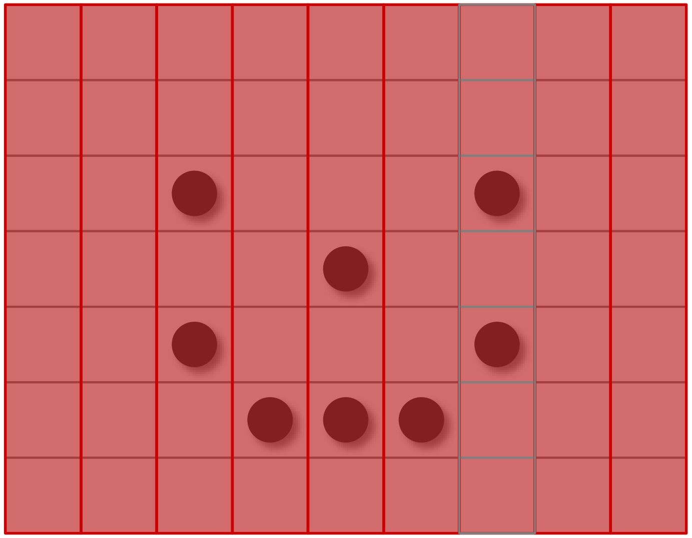


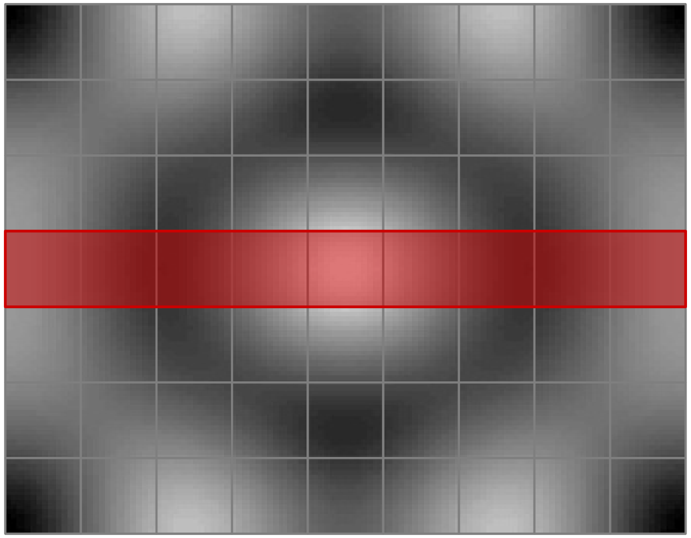
Frequency encoding

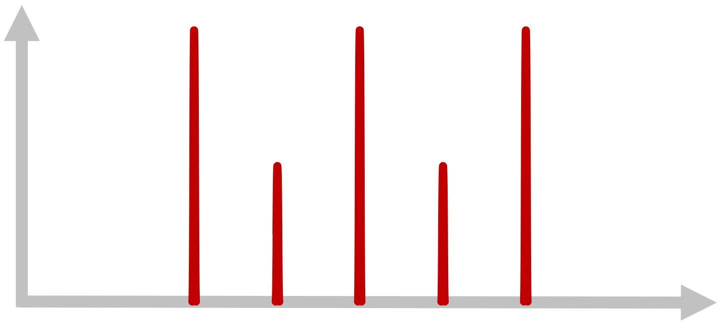


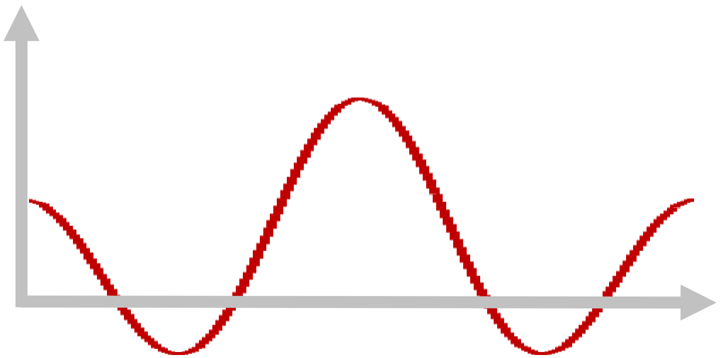
Object and k-space

Chapter 5.10



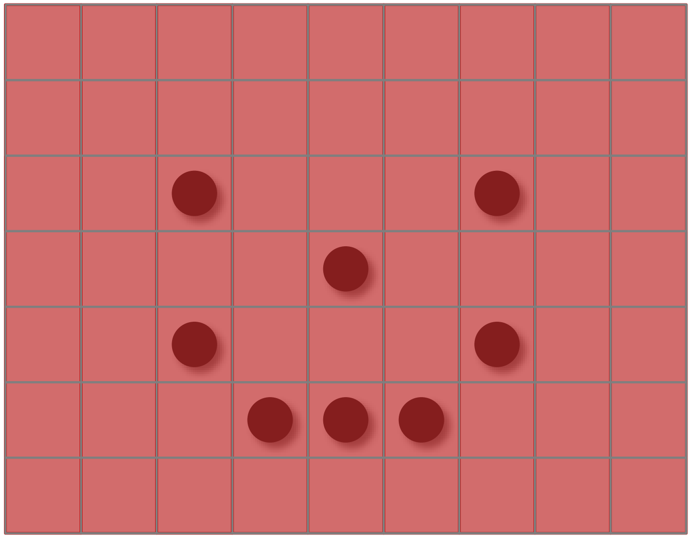




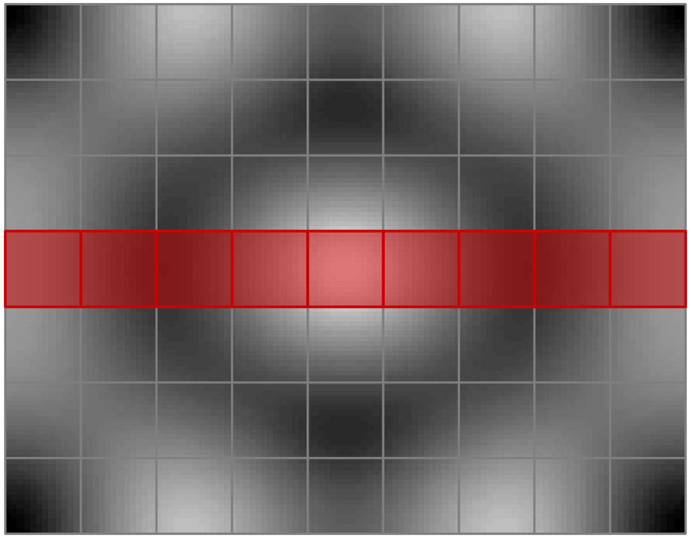


Object and k-space

Object space



K-space



Δx

Δk_x

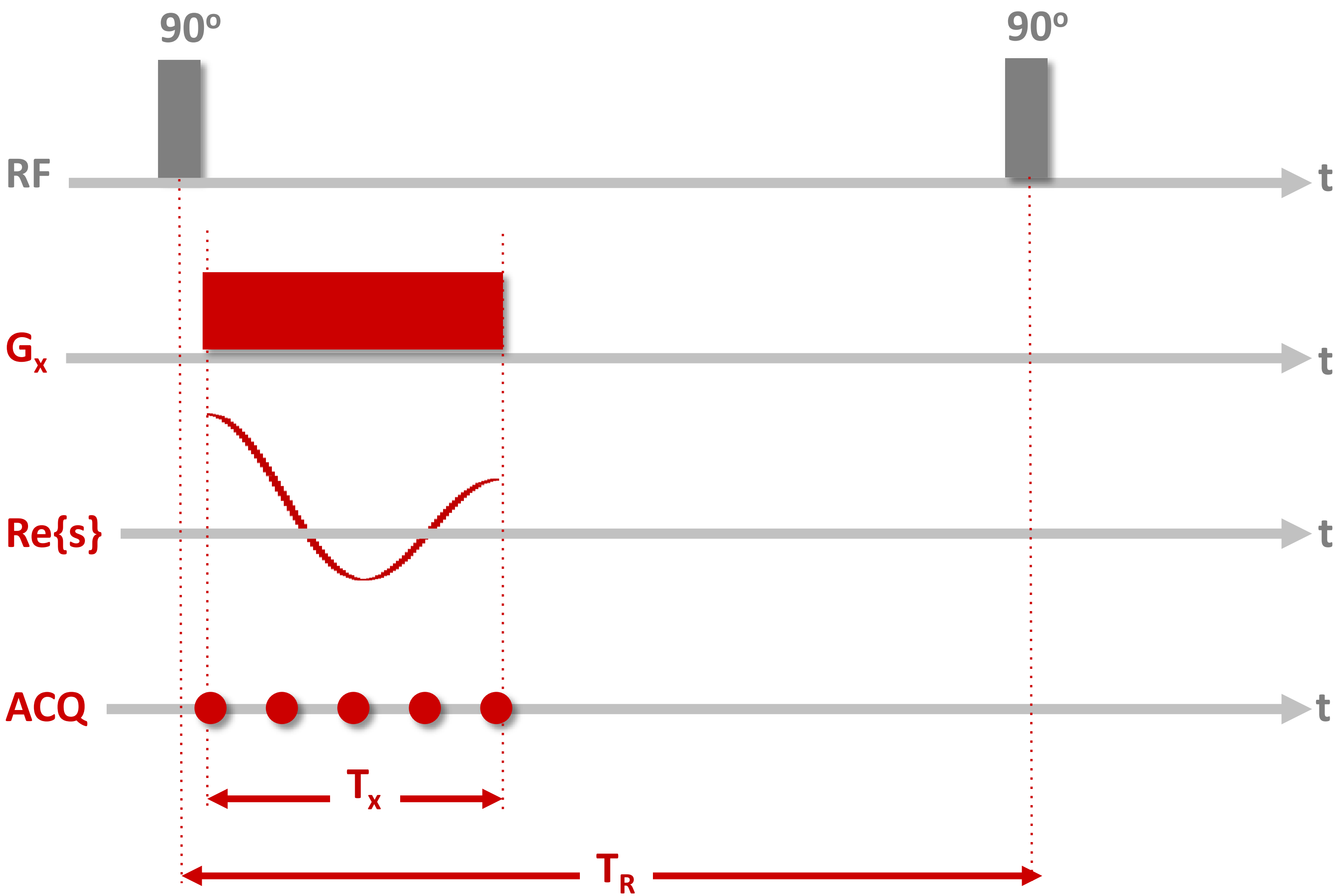
FOV_x
(field-of-view)

BW_x
(bandwidth)

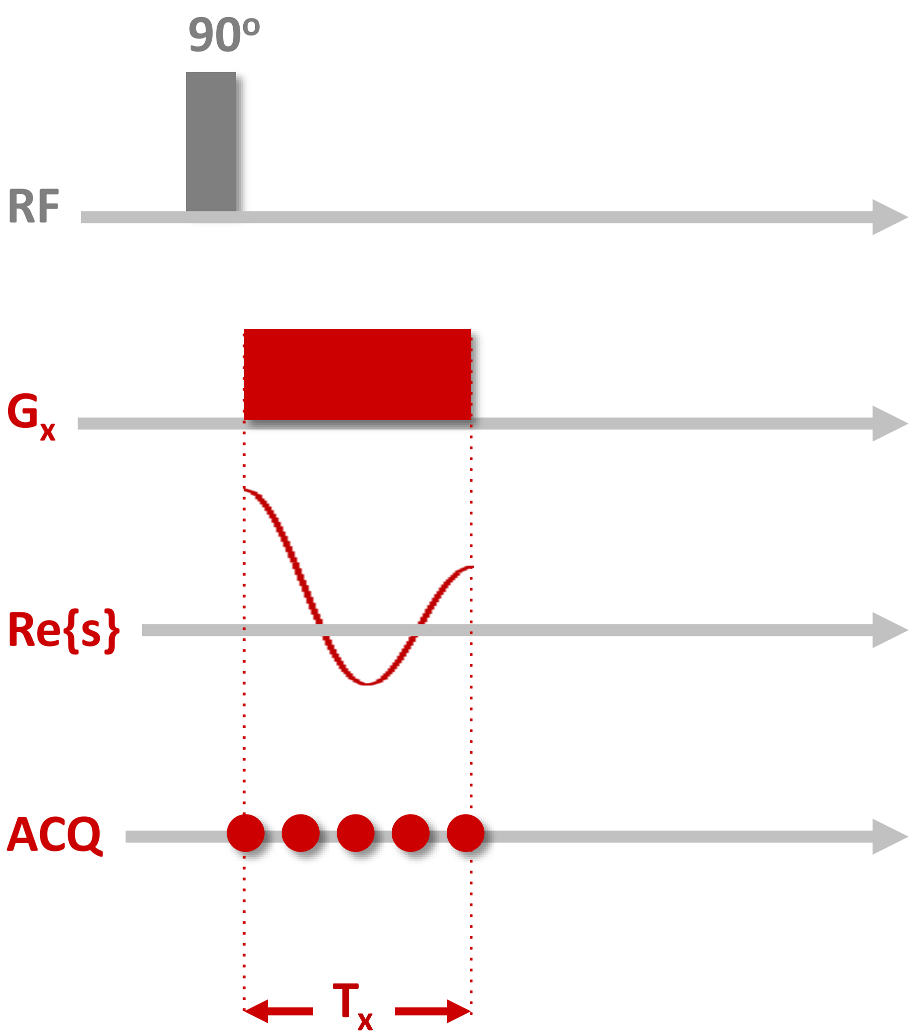
$$\Delta x = \frac{\mathrm{FOV}_x}{N_x}$$

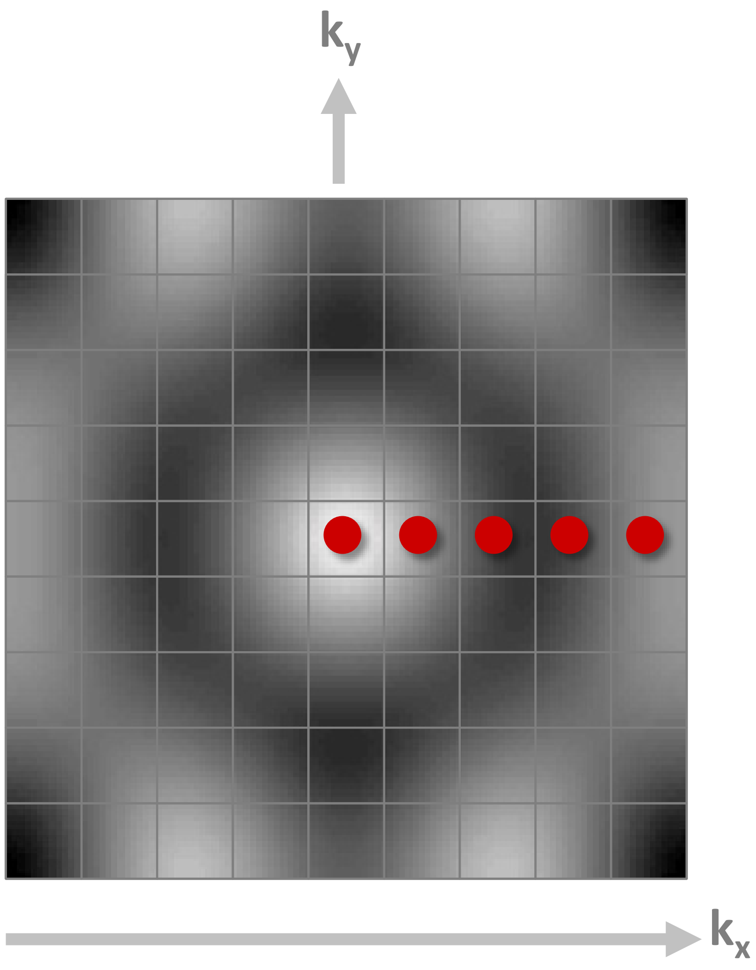
$$\Delta k_x = \frac{\mathrm{BW}_x}{N_x} = \frac{2\pi}{\mathrm{FOV}_x}$$

Frequency encoding experiment

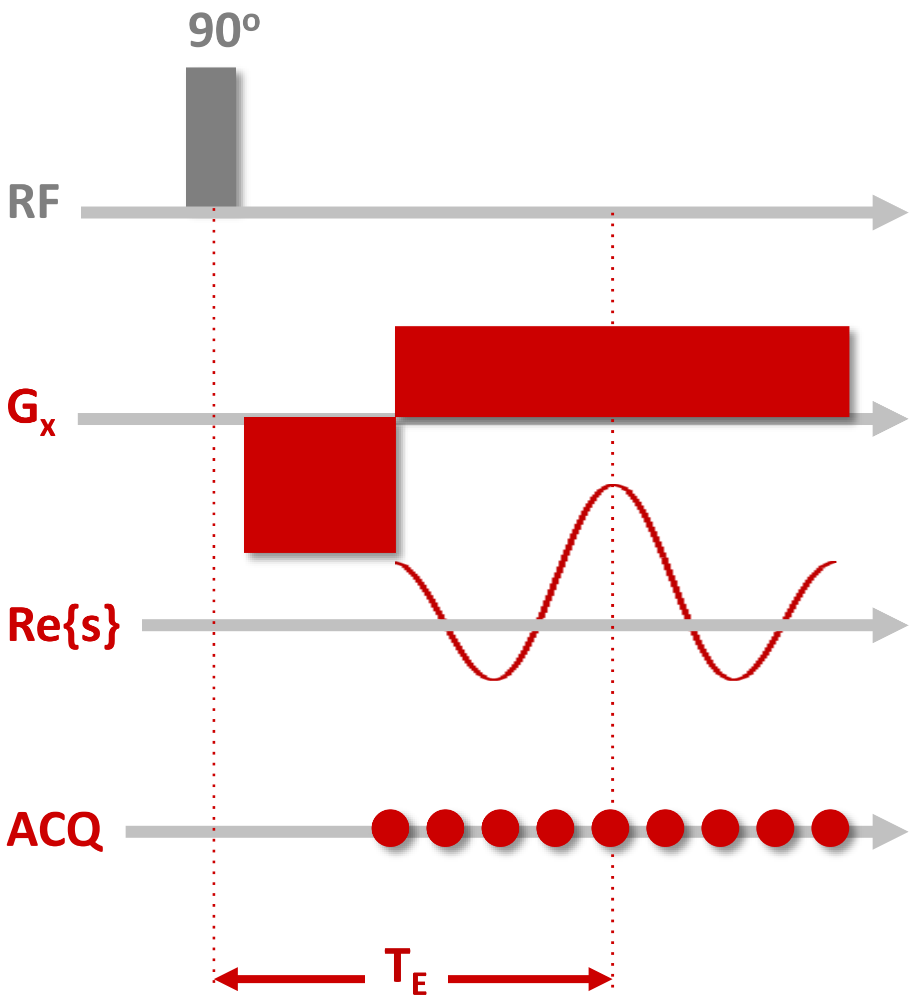


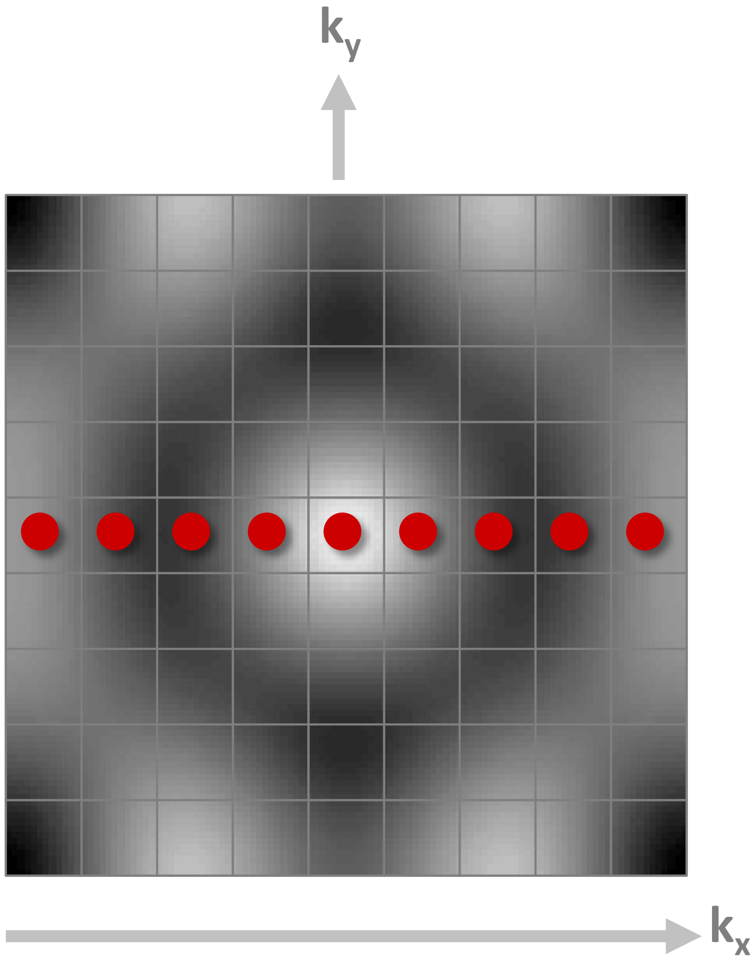
Frequency encoding experiment



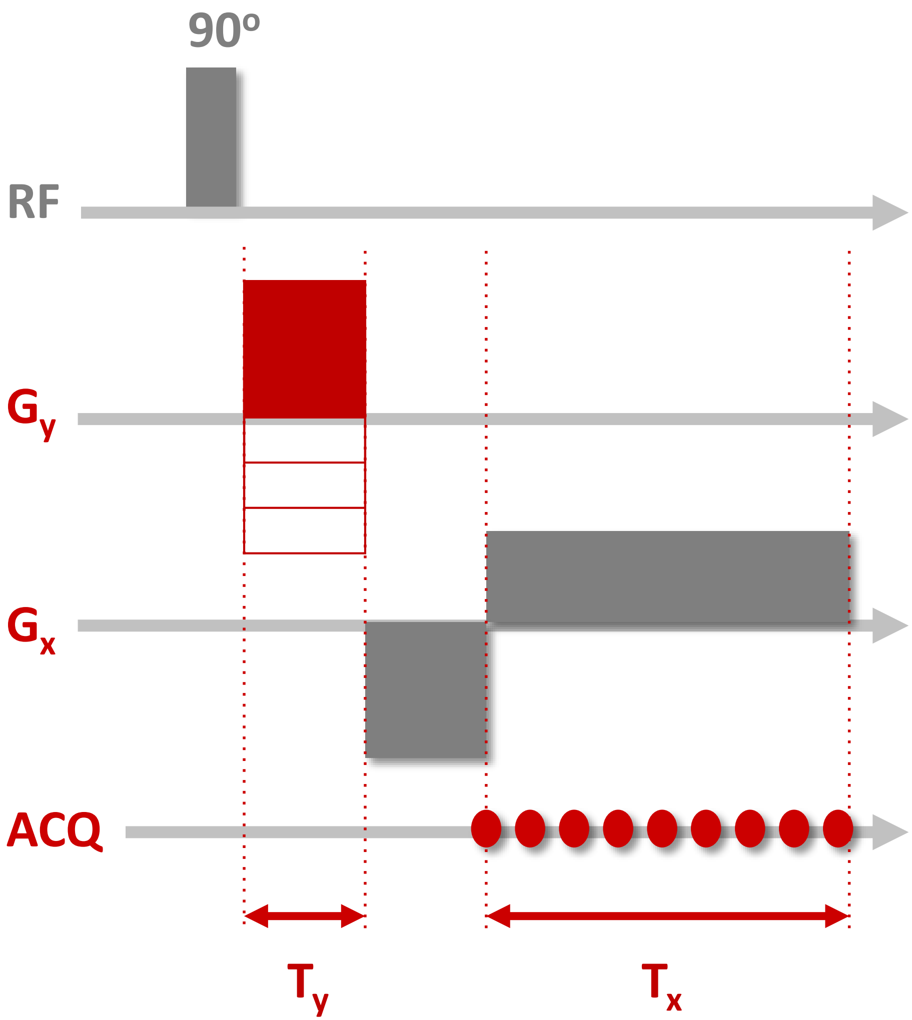


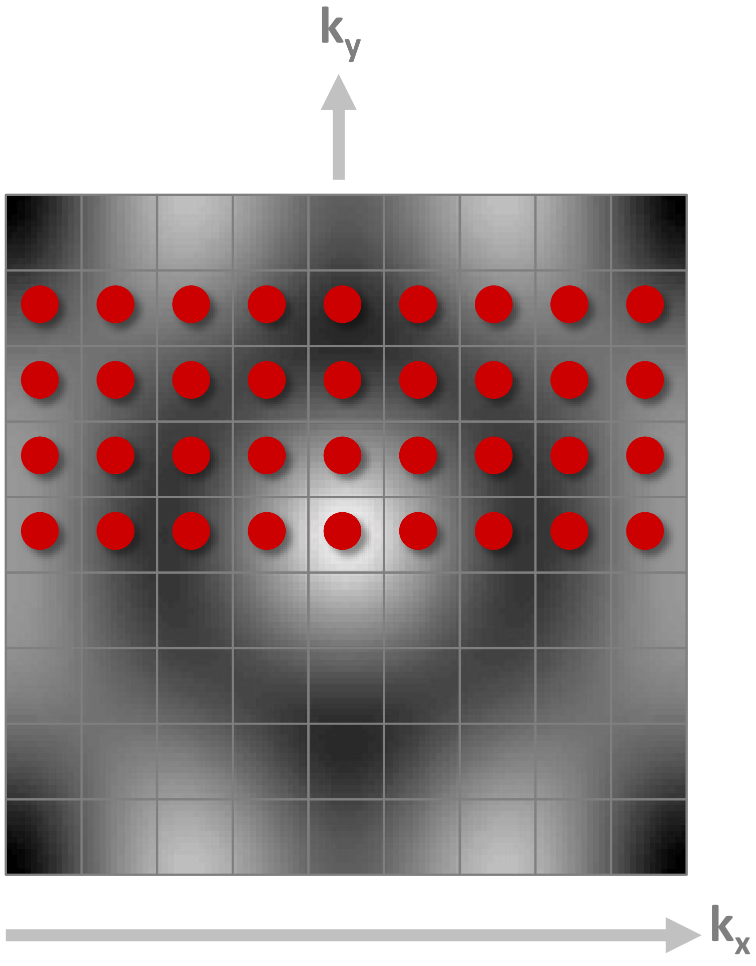
Gradient echo encoding experiment





Gradient echo encoding experiment





Gradient Encoding

Gradient in x direction: $$B(x) = B_0 + G\,x$$

Gradients in all directions: $$B(x,y,z) = B_0 + G_x\,x + G_y\,y + G_z\,z$$

$$\boldsymbol{r} = (x,y,z)^T,\ \boldsymbol{G} = (G_x, G_y, G_z)^T$$

$$B(\boldsymbol{r}) = B_0 + \boldsymbol{G}\cdot\boldsymbol{r}$$

Frequency at position ***r***, time *t*: $$\omega(\boldsymbol{r},t) = \gamma\,B(\boldsymbol{r},t) = \gamma\,B_0 + \gamma\,\boldsymbol{G}(t)\cdot\boldsymbol{r}$$

Phase of signal from position ***r***: $$\varphi(\boldsymbol{r},t) = \gamma\,B_0\,t + \gamma\left(\int_0^t \boldsymbol{G}(\tau)\,d\tau\right)\cdot\boldsymbol{r}$$

Gradient Encoding

Phase of signal from position $\boldsymbol{r}$:

$$\varphi(\boldsymbol{r},t) = \gamma B_0 t + \underbrace{\left(\gamma \int_0^t \boldsymbol{G}(\tau) d\tau \right)} \cdot \boldsymbol{r}$$

$$\varphi(\boldsymbol{r},t) = \omega_0 t + \boldsymbol{k}(t) \cdot \boldsymbol{r}$$

Signal from position $\boldsymbol{r}$:

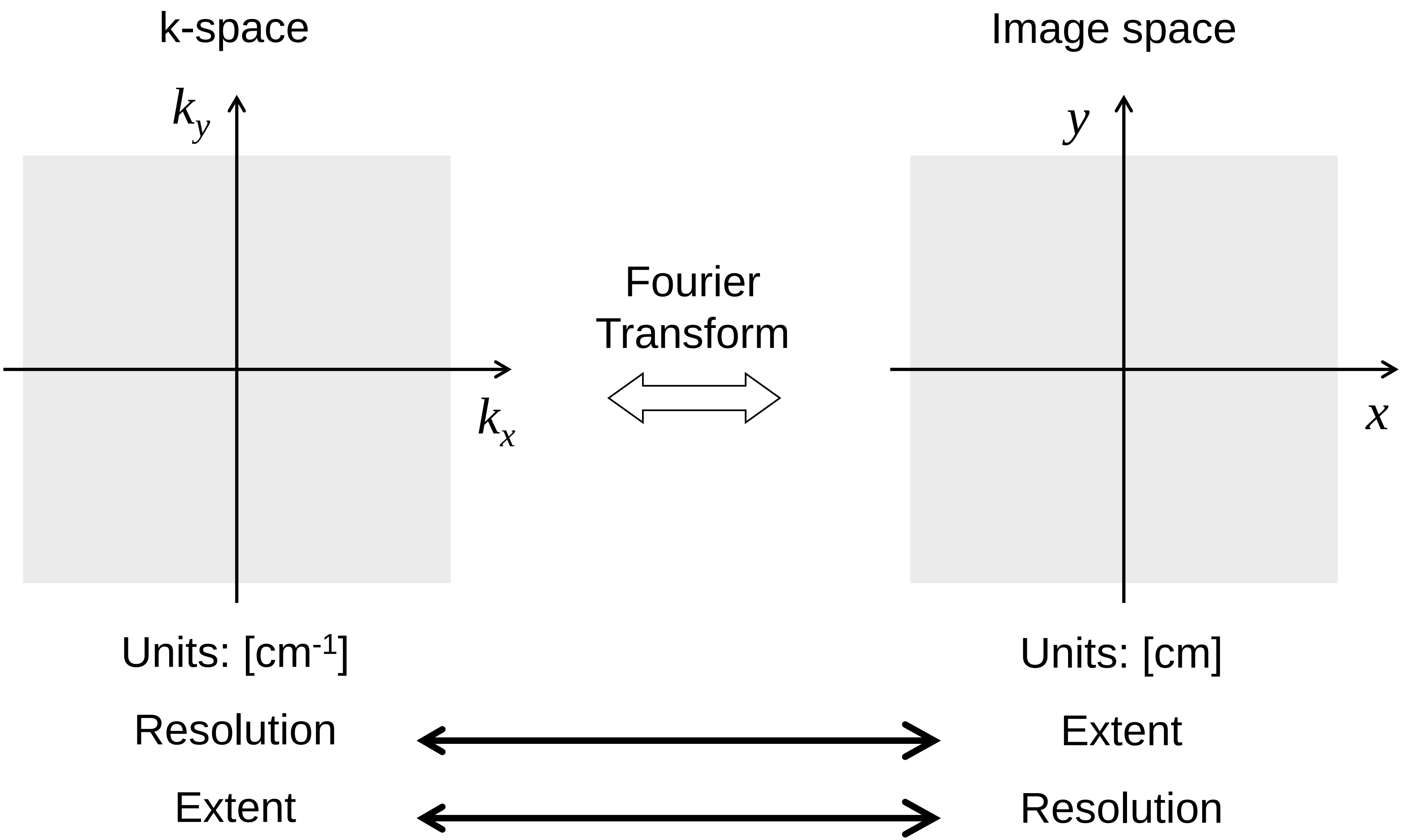
$$s(\boldsymbol{r},t) = \rho(\boldsymbol{r}) e^{j\omega_0 t} e^{j\boldsymbol{k}(t)\cdot\boldsymbol{r}}$$

Signal from entire object:

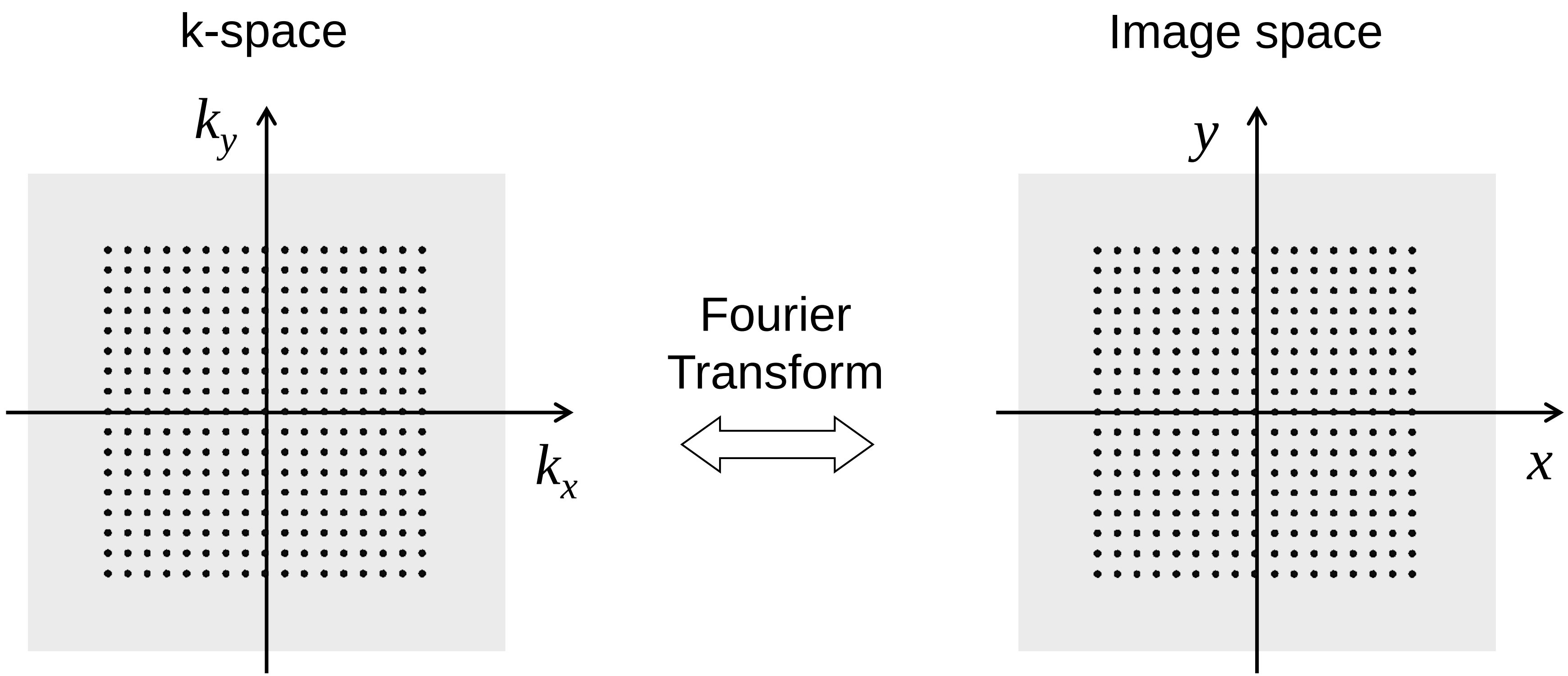
$$S(t) = e^{j\omega_0 t} \int_{obj} \rho(\boldsymbol{r})\, e^{j\boldsymbol{k}(t)\cdot\boldsymbol{r}} d^3 r$$

$$S(\boldsymbol{k}) = e^{j\omega_0 t} \underbrace{\int_{obj} \rho(\boldsymbol{r})\, e^{j\boldsymbol{k}\cdot\boldsymbol{r}} d^3 r}_{\text{Fourier Transform of } \rho(\boldsymbol{r})}$$

Fourier Imaging



Fourier Imaging

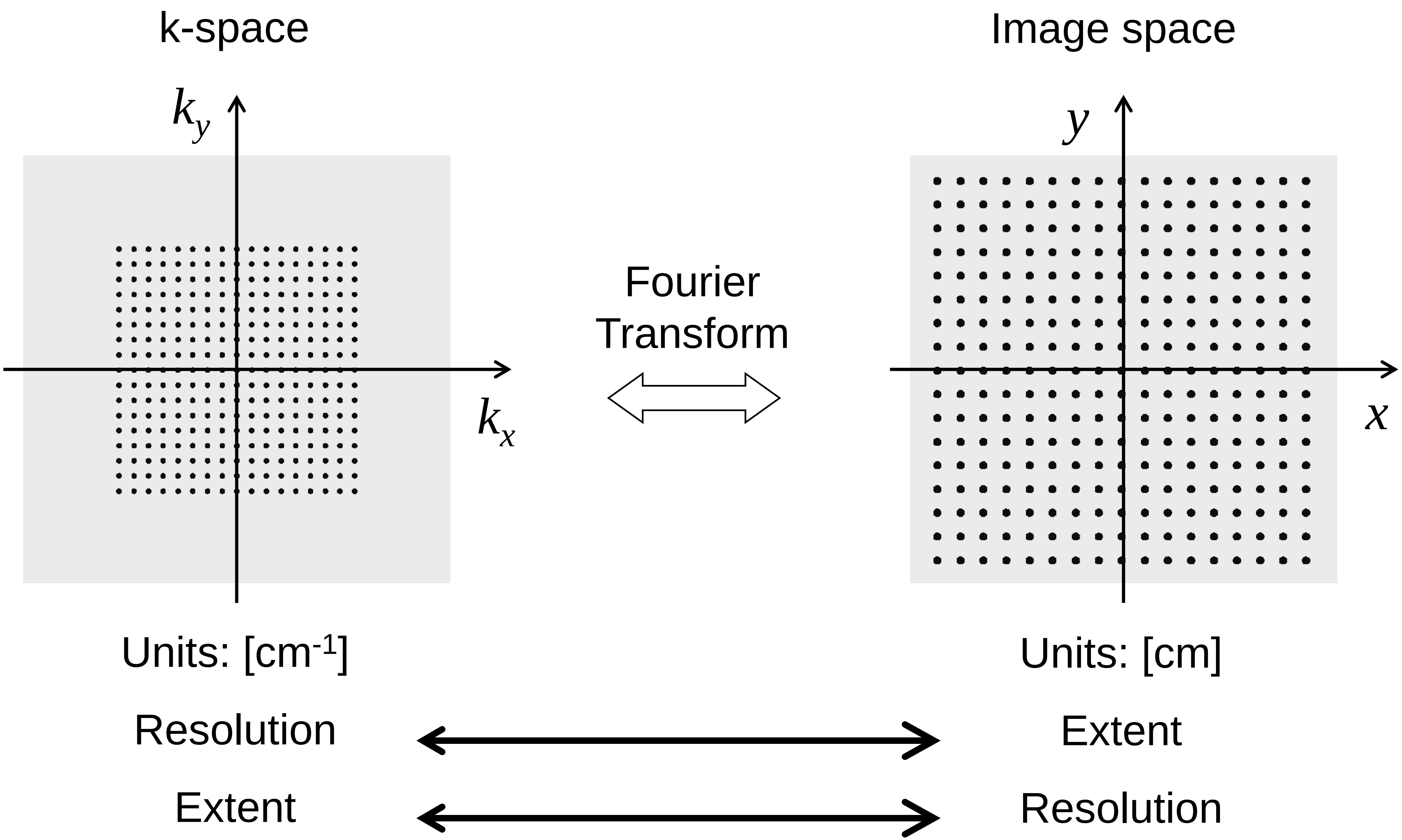


Units: [cm^{-1}] Units: [cm]

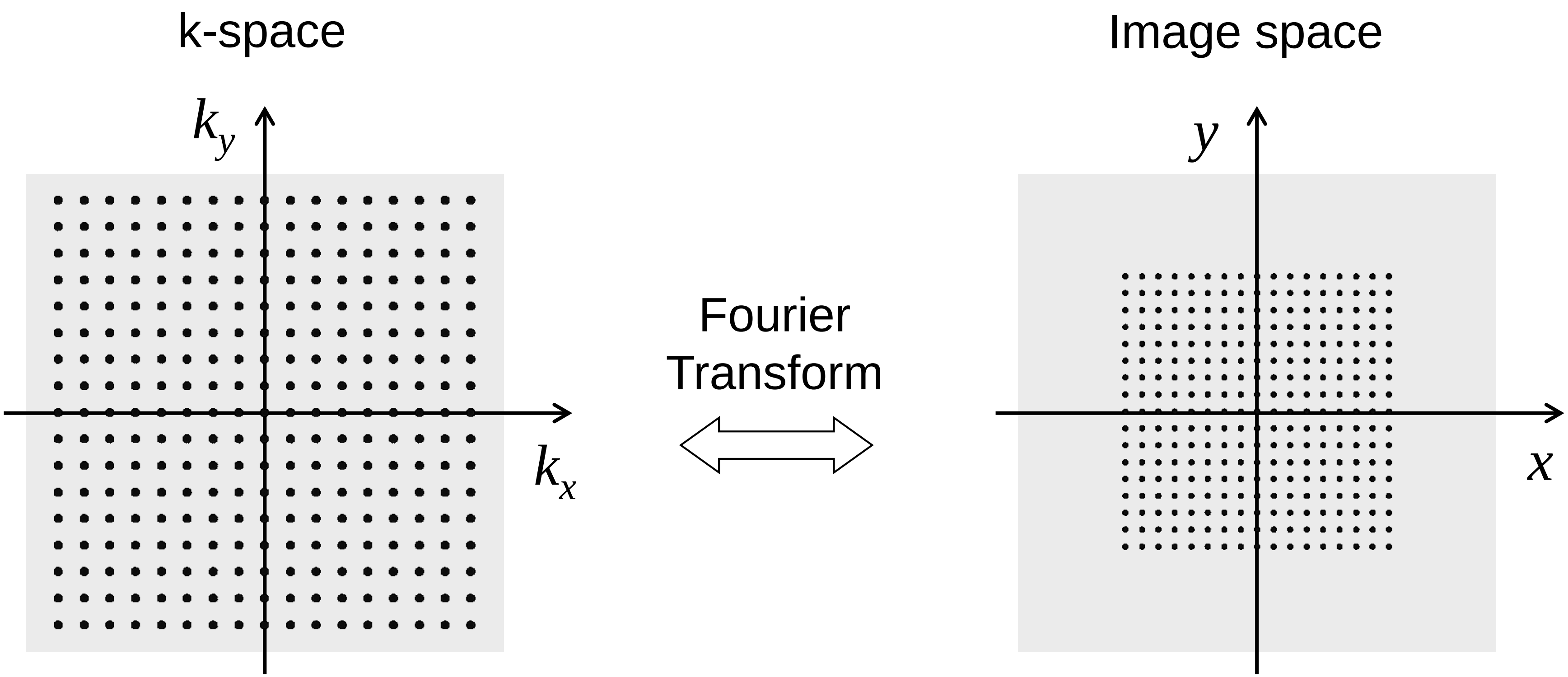
Resolution ⟷ Extent

Extent ⟷ Resolution

Fourier Imaging

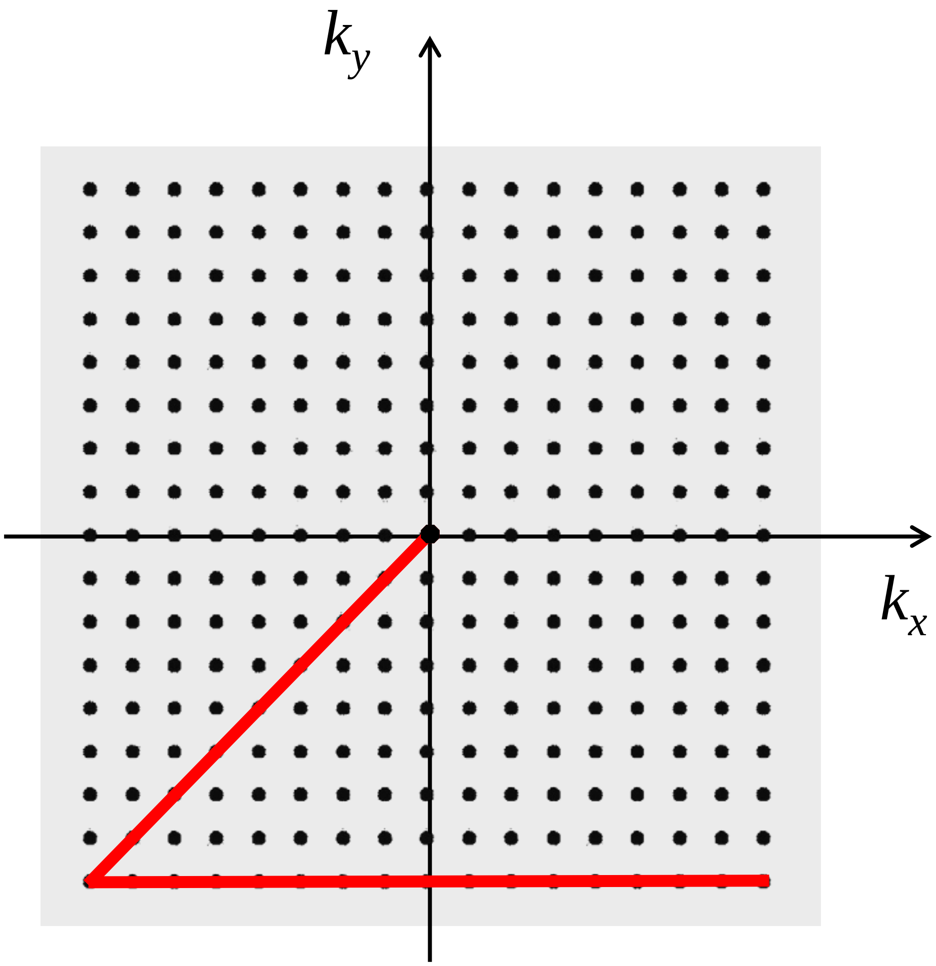


Fourier Imaging



k-space		Image space
Units: [cm^{-1}]		Units: [cm]
Resolution	⟷	Extent
Extent	⟷	Resolution

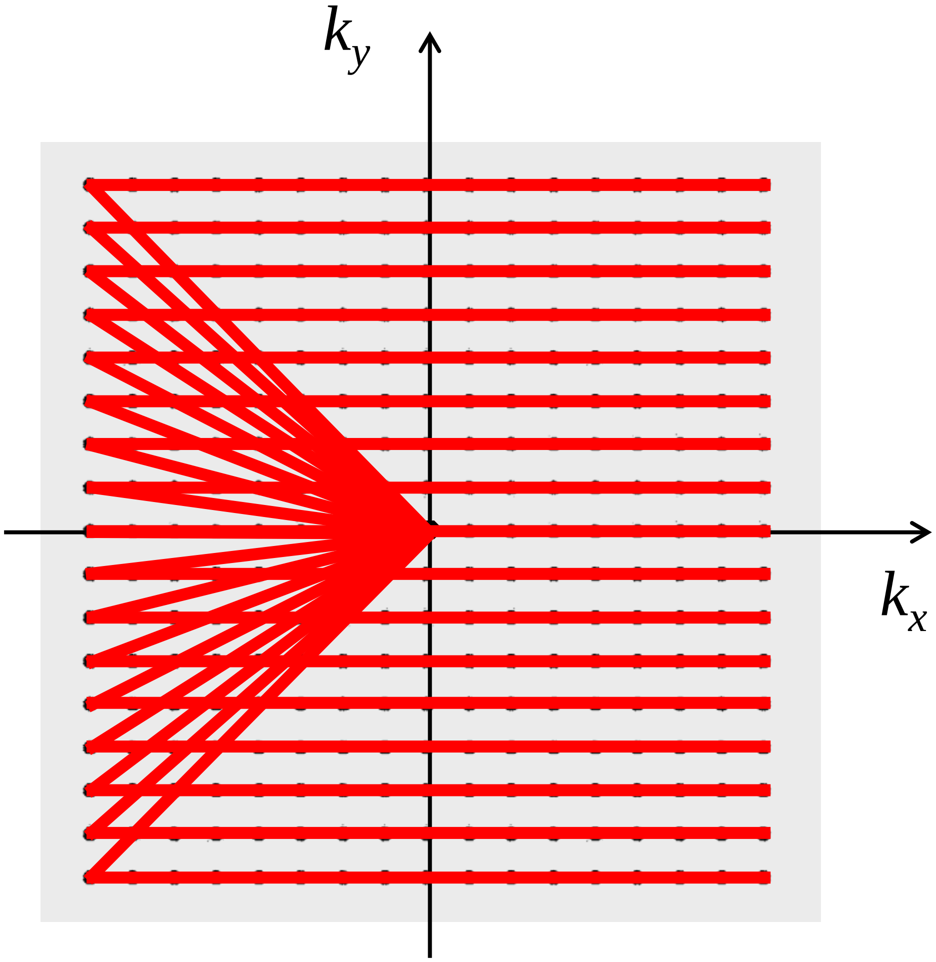
Fourier Imaging



$$\boldsymbol{k}(t) = \gamma \int_0^t \boldsymbol{G}(\tau)\,d\tau$$

$$\dot{\boldsymbol{k}} = \gamma \boldsymbol{G}(t)$$

Fourier Imaging



$$\boldsymbol{k}(t) = \gamma \int_0^t \boldsymbol{G}(\tau)\,d\tau$$

$$\dot{\boldsymbol{k}} = \gamma \boldsymbol{G}(t)$$

After excitation: $\boldsymbol{k} = \boldsymbol{0}$

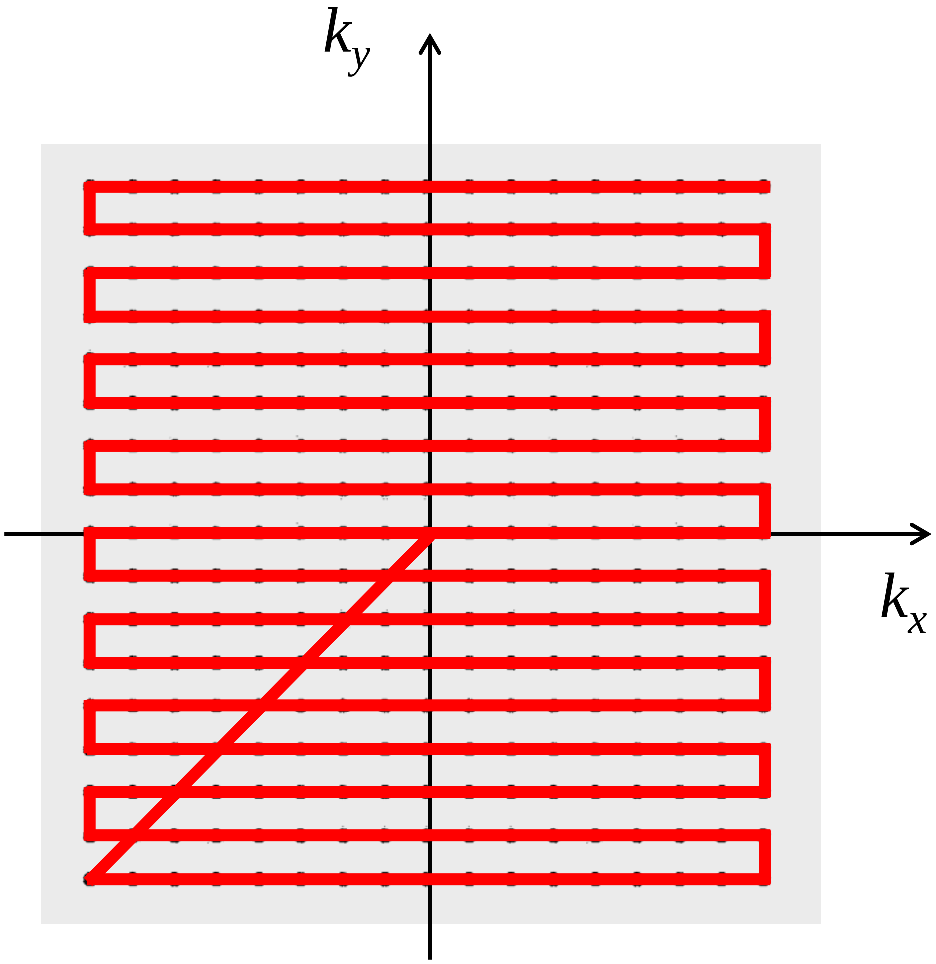
To bottom left with

$$\boldsymbol{G} = (-1,-1,0)$$

Then right with

$$\boldsymbol{G} = (+1,0,0)$$

Fourier Imaging



$$\boldsymbol{k}(t) = \gamma \int_0^t \boldsymbol{G}(\tau)d\tau$$

$$\dot{\boldsymbol{k}} = \gamma \boldsymbol{G}(t)$$

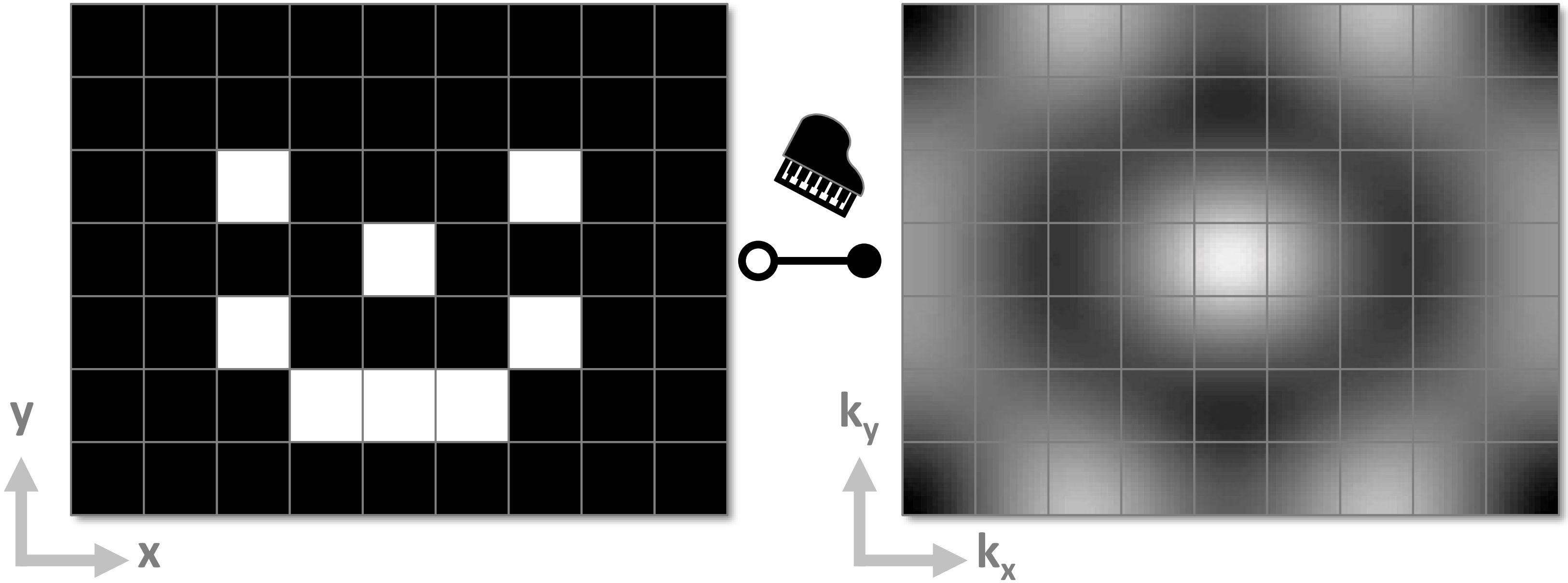
After excitation: $\boldsymbol{k} = \boldsymbol{0}$

Any other way of traversing k-space

E.g., Meandering (Echo-planar imaging, EPI)

Signal encoding

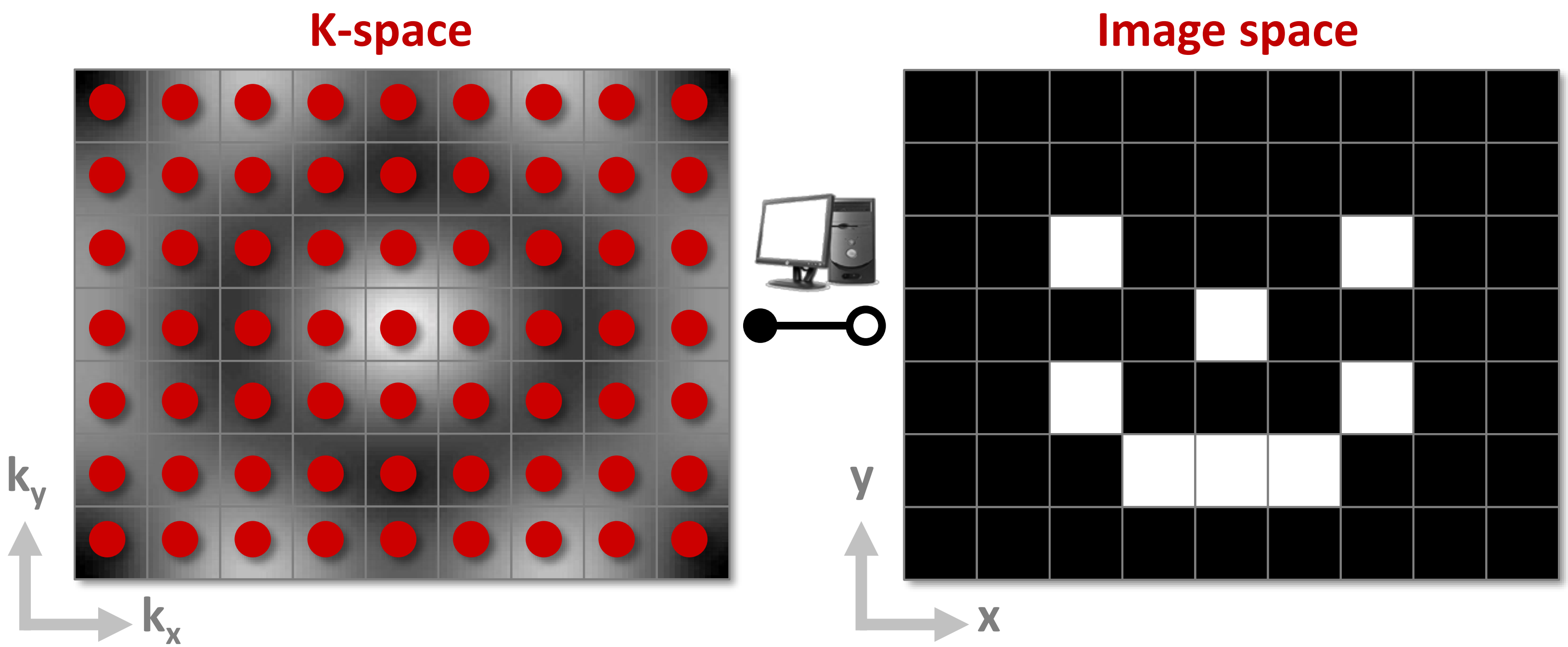
Object space **K-space**



$$s\left(k_{xp},k_{yq}\right) \propto \sum_{i}\sum_{j}\rho\left(x_i,y_j\right)e^{j\left(k_{xp}x_i+k_{yq}y_j\right)}$$

= Fourier Transform

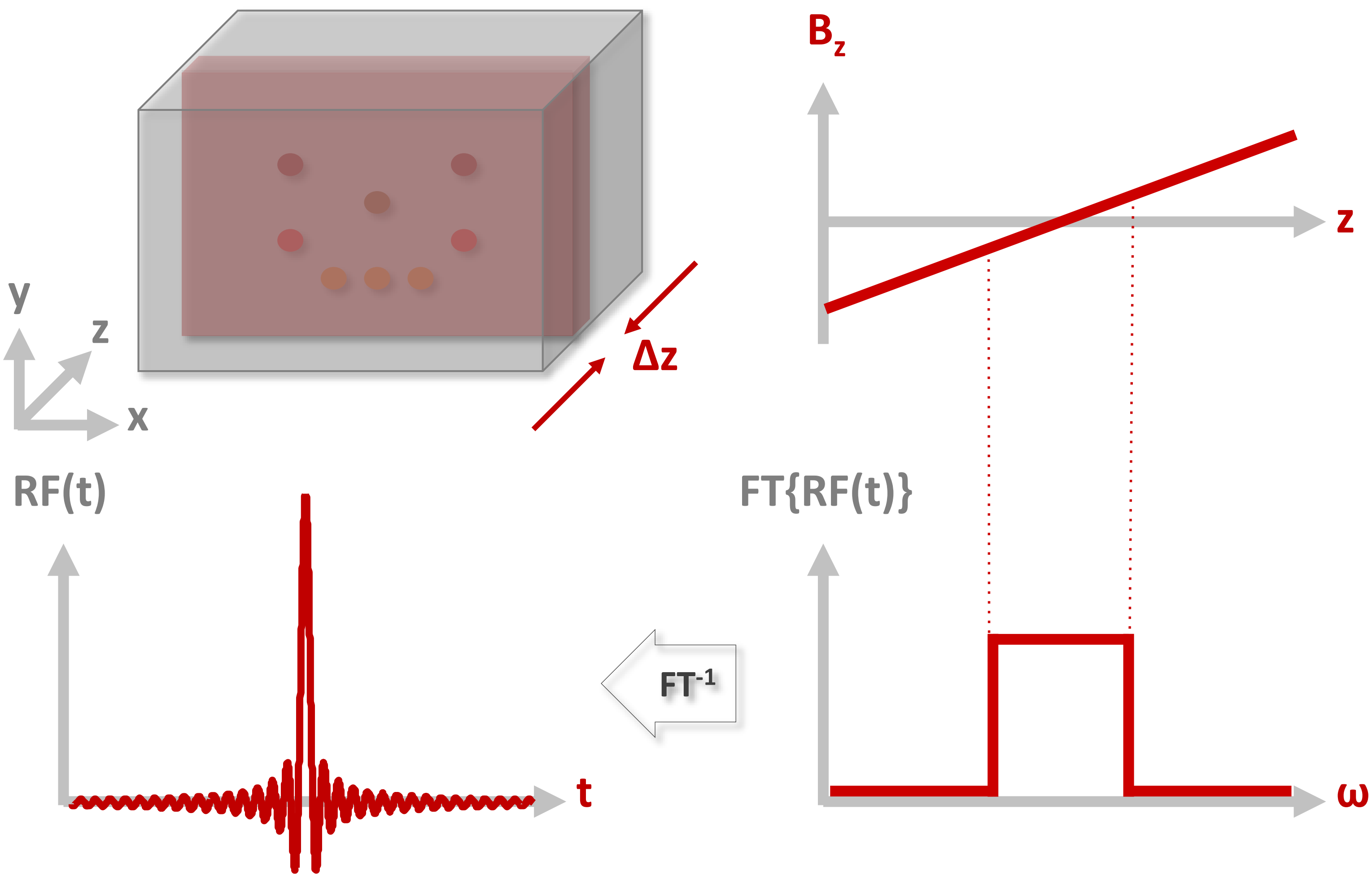
Image reconstruction



$$i(x_i, y_j) = \sum_p \sum_q s(k_{xp}, k_{yq}) e^{-j(k_{xp}x_i + k_{yq}y_j)}$$

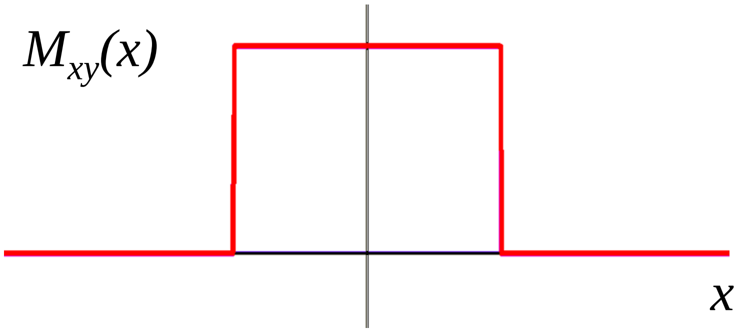
= Inverse Fourier Transform

Slice selection

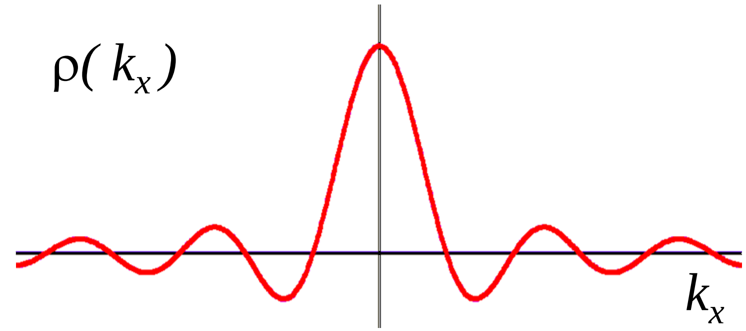


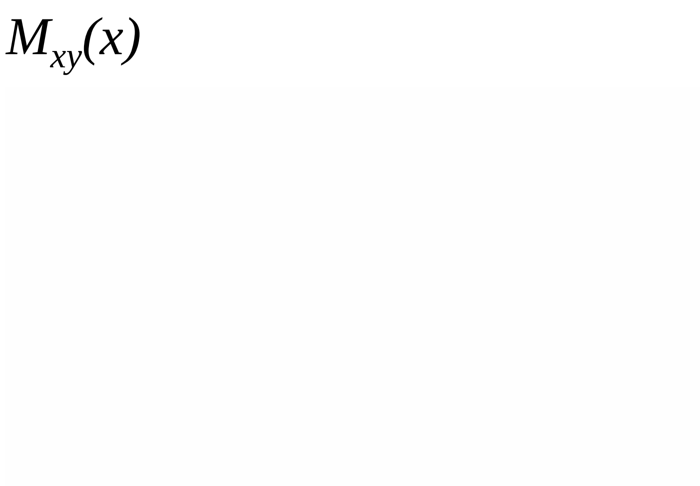
Slice Selection

Ideal slice profile: *Rectangle*

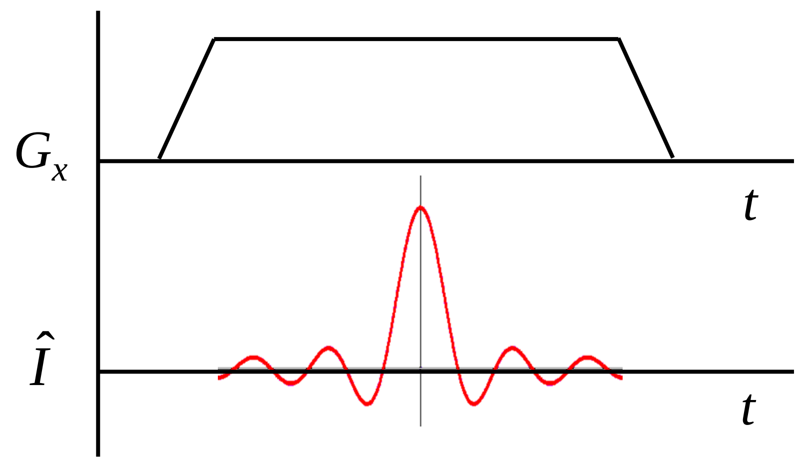


Ideal waveform: *Sinc*

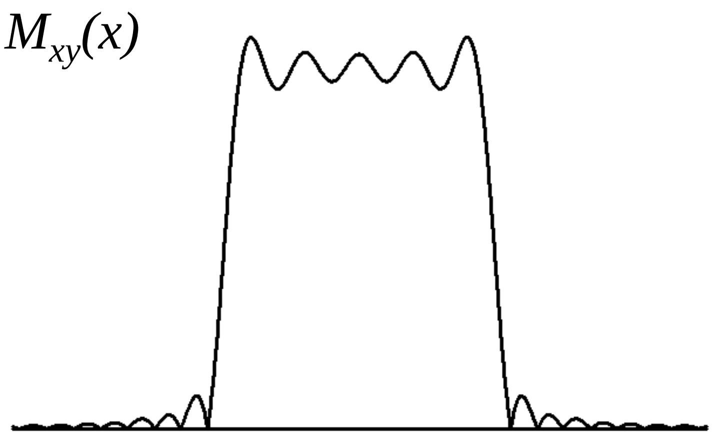




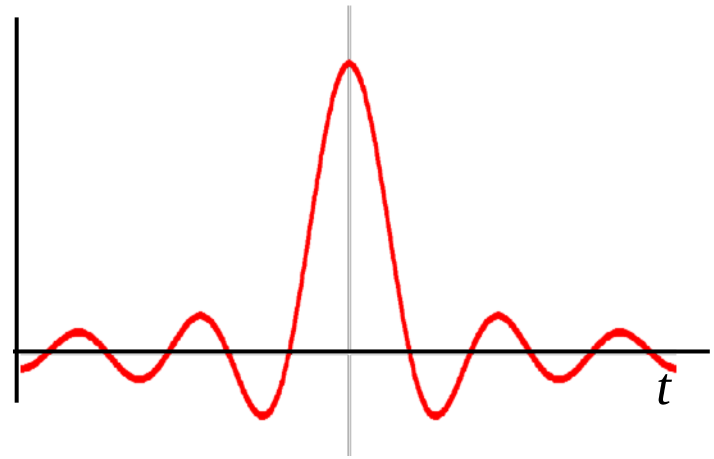
Implementation:



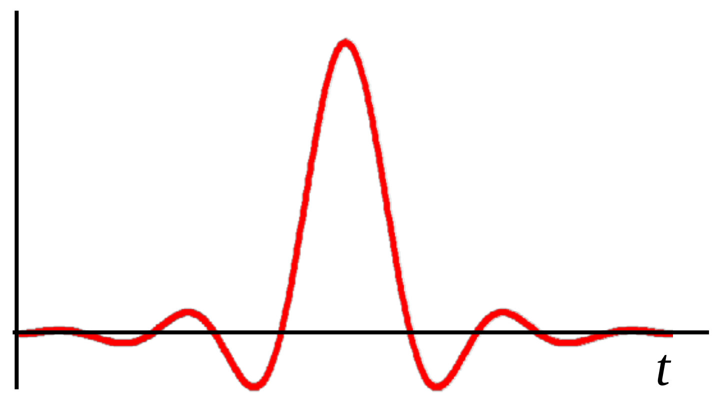
Slice Selection



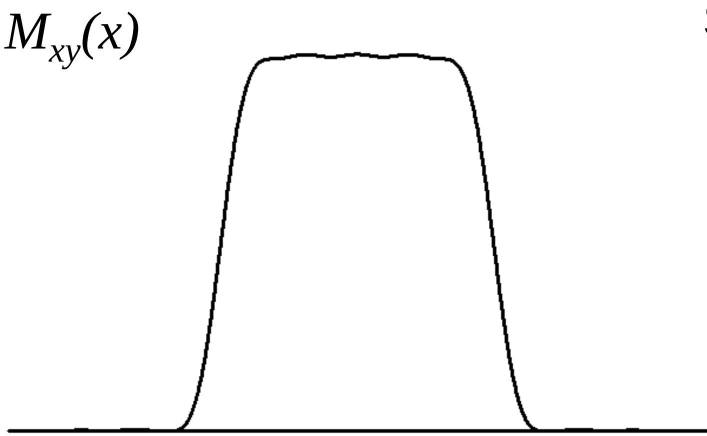
Plain Sinc



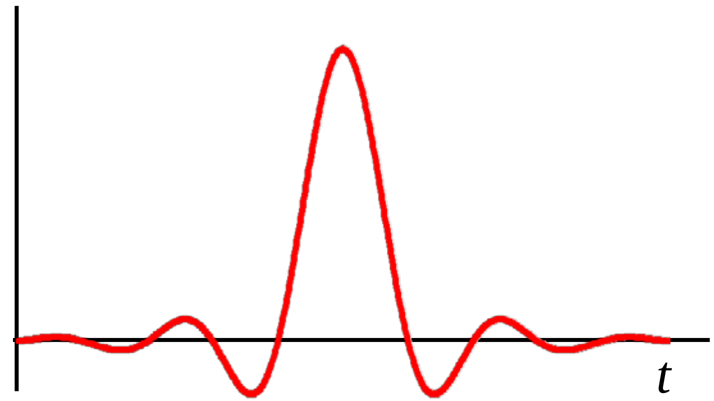
Sinc x Gauss



Slice Selection

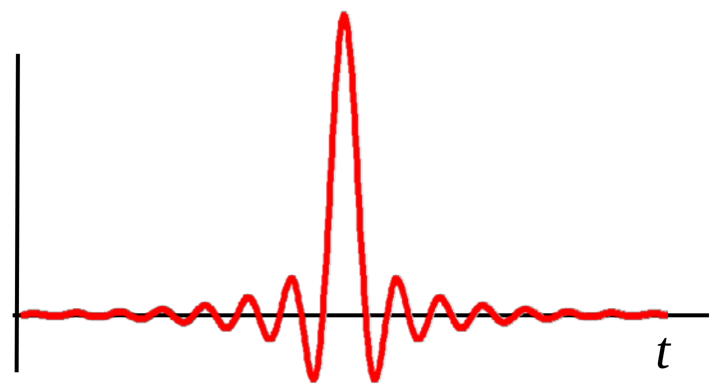


Sinc x Gauss

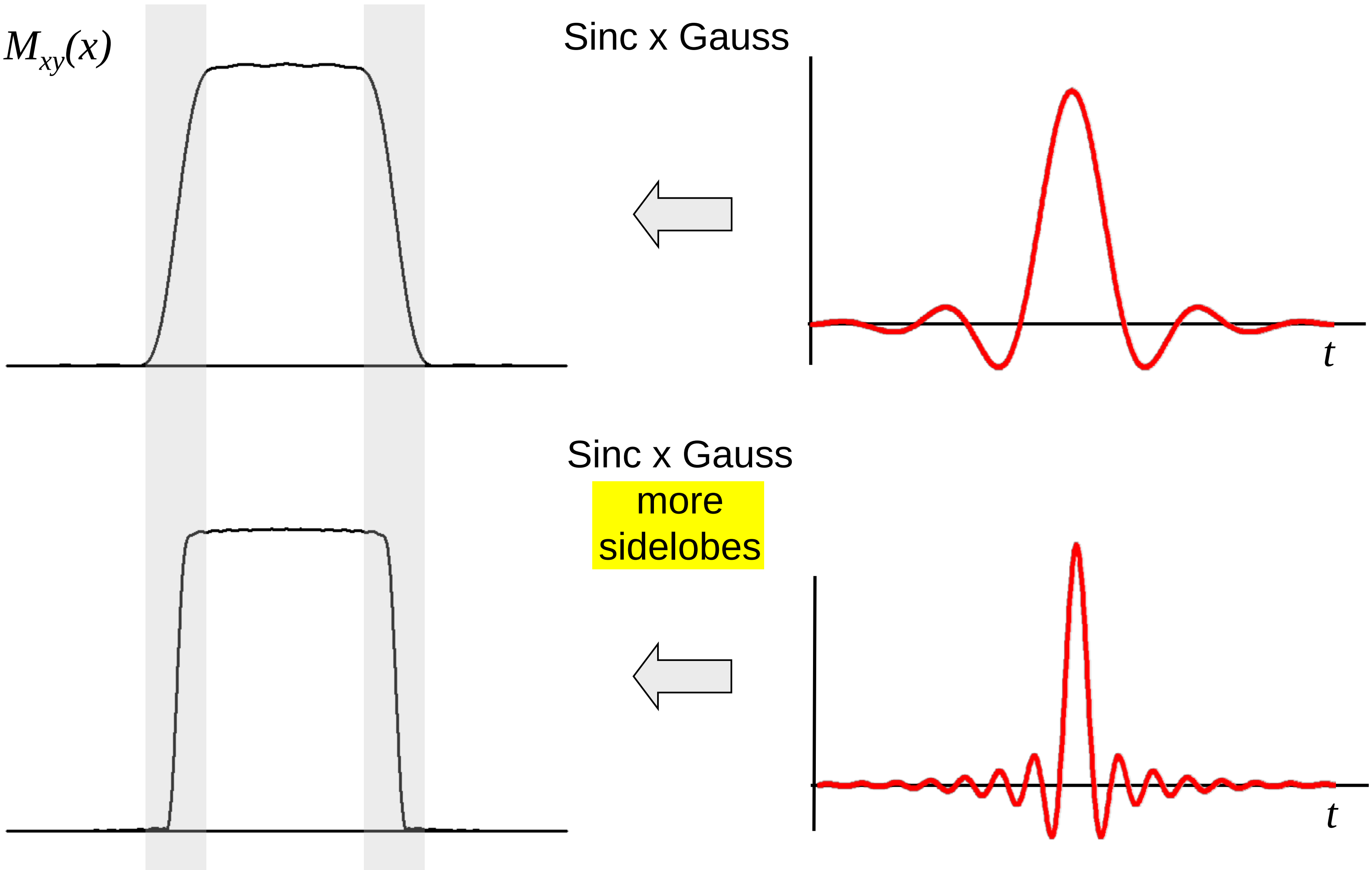


Sinc x Gauss

more sidelobes



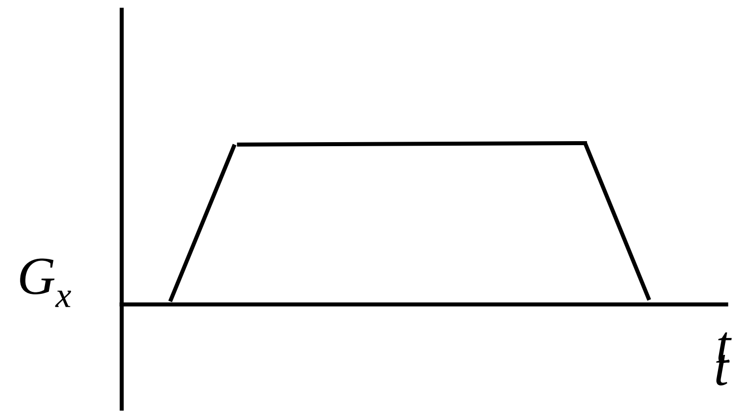
Slice Selection

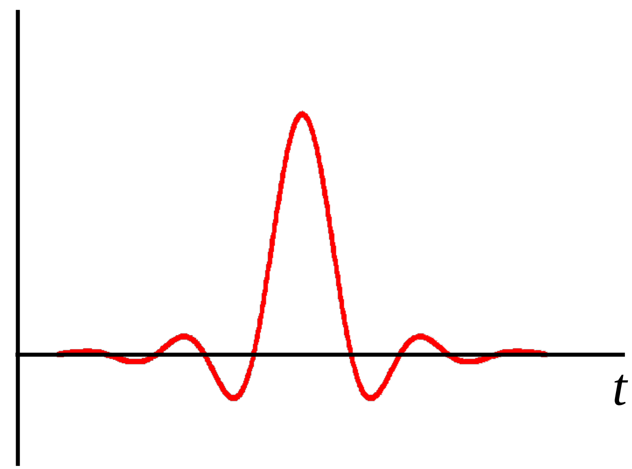


Slice Selection

Sinc x Gauss

M_x M_y $\left|M_{xy}\right|$

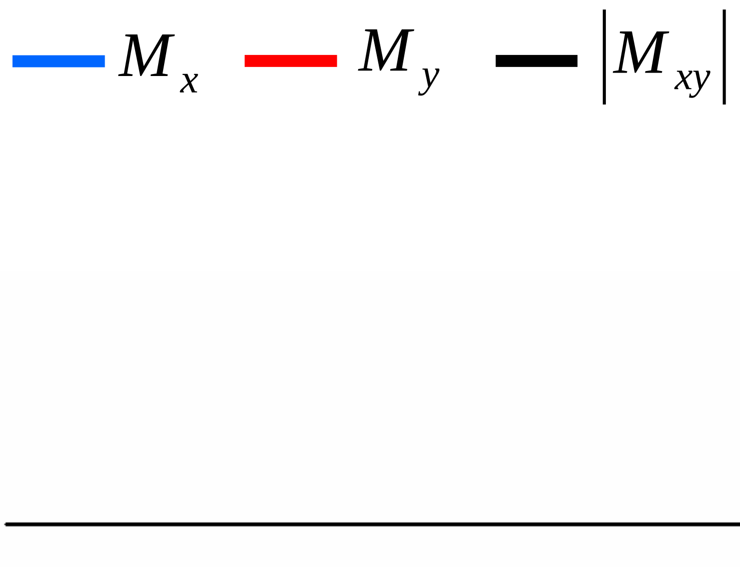


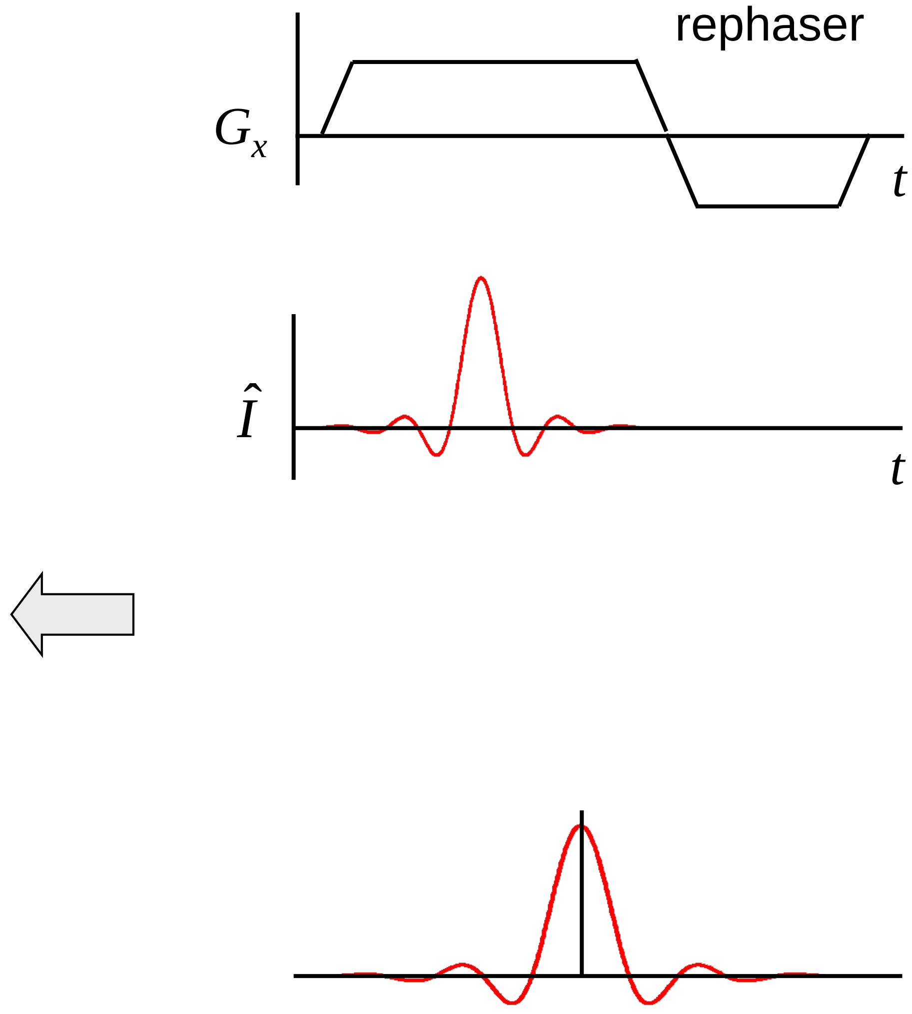


Linear phase ! Shifted from center !

Fourier Pulse Design

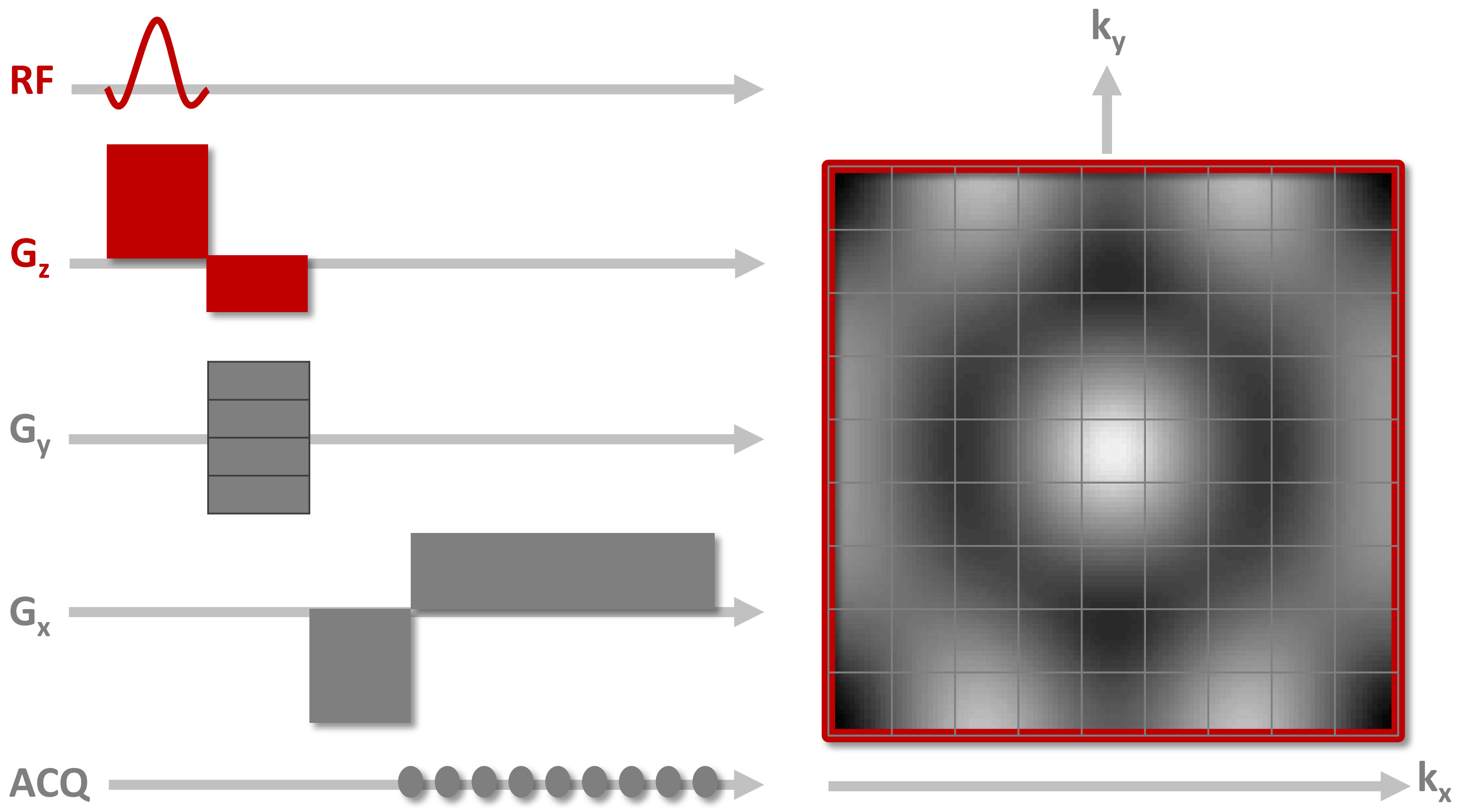
Sinc x Gauss



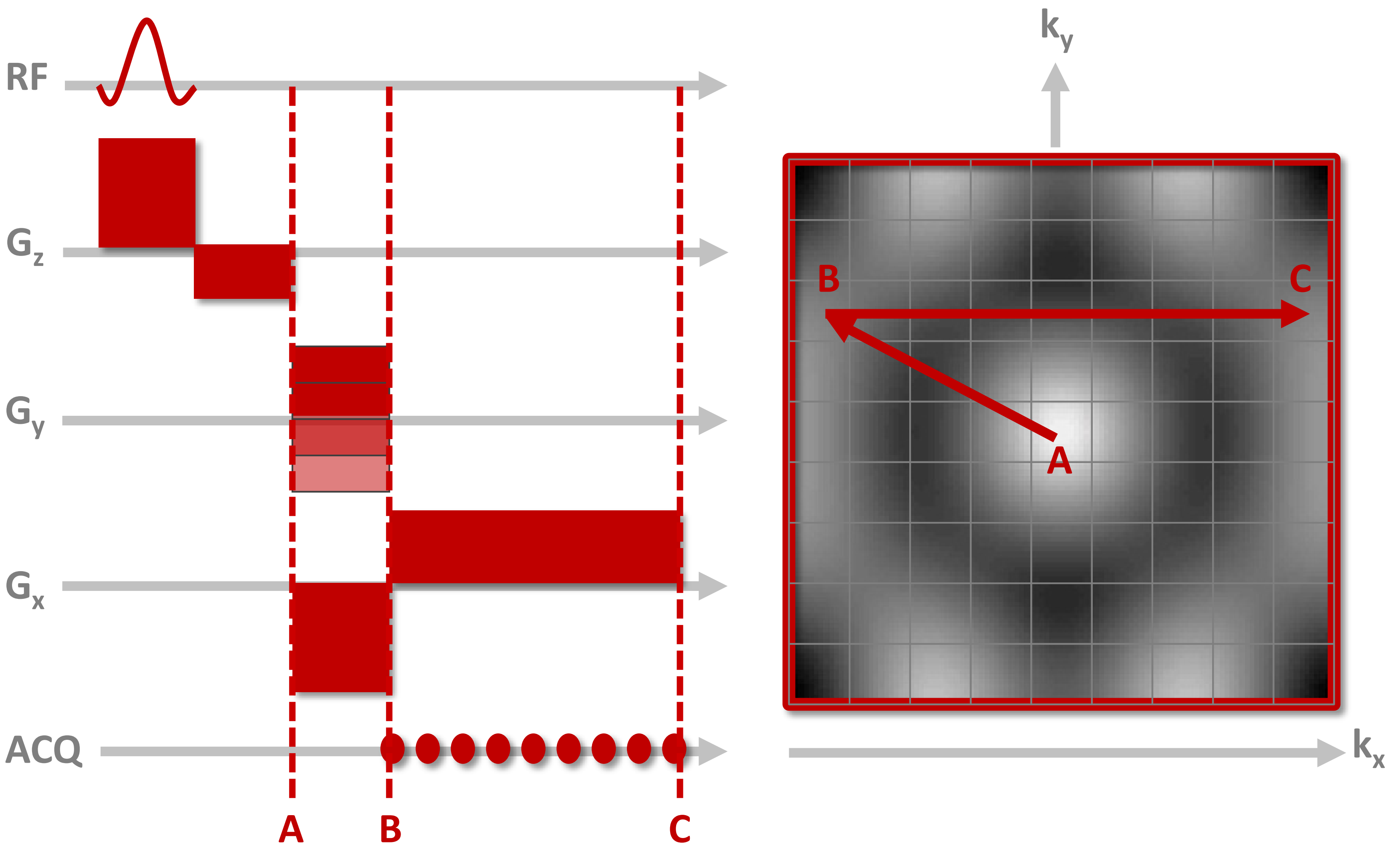


Gradient echo encoding experiment

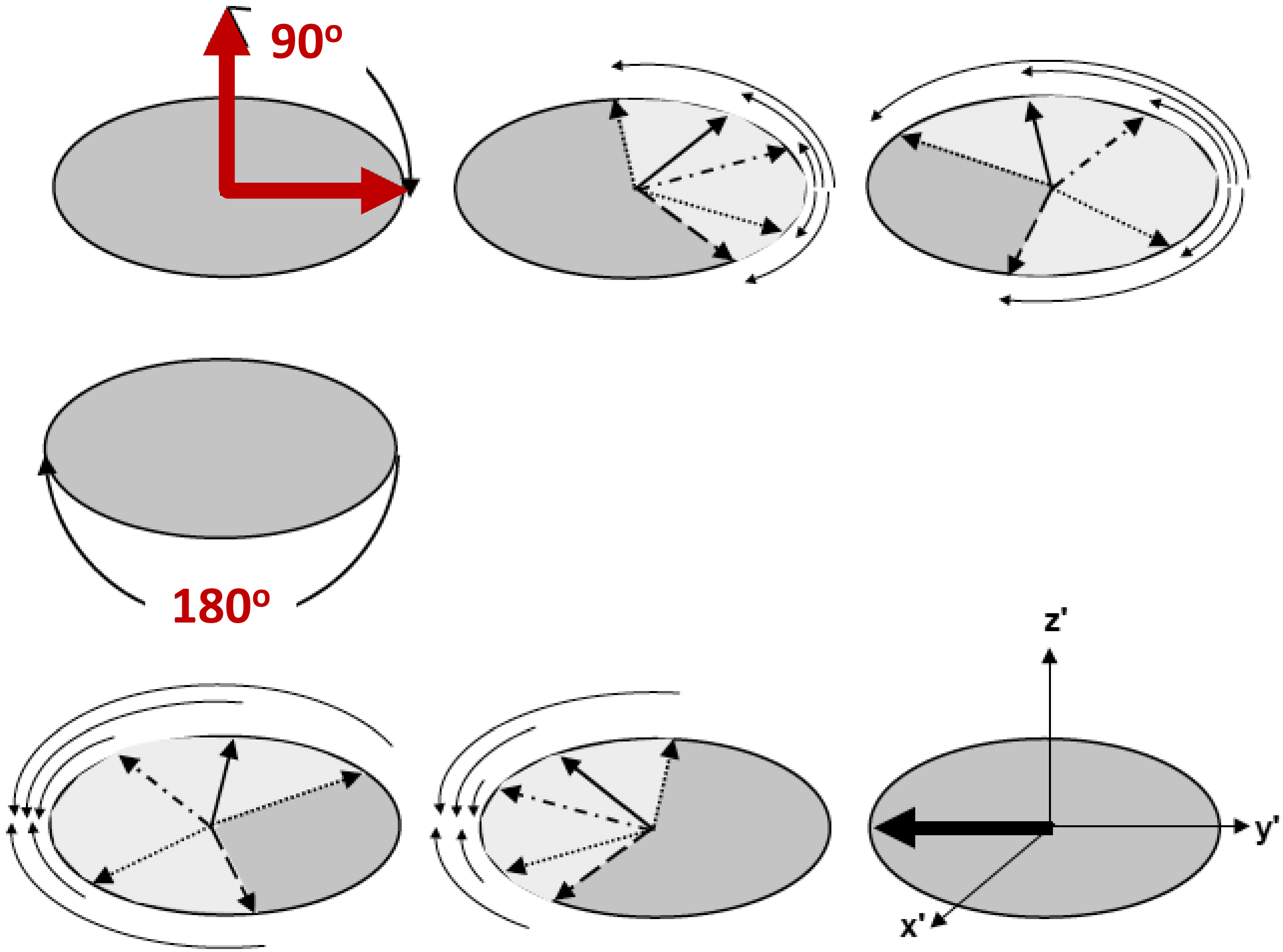
Chapter 5.12



Gradient echo encoding experiment

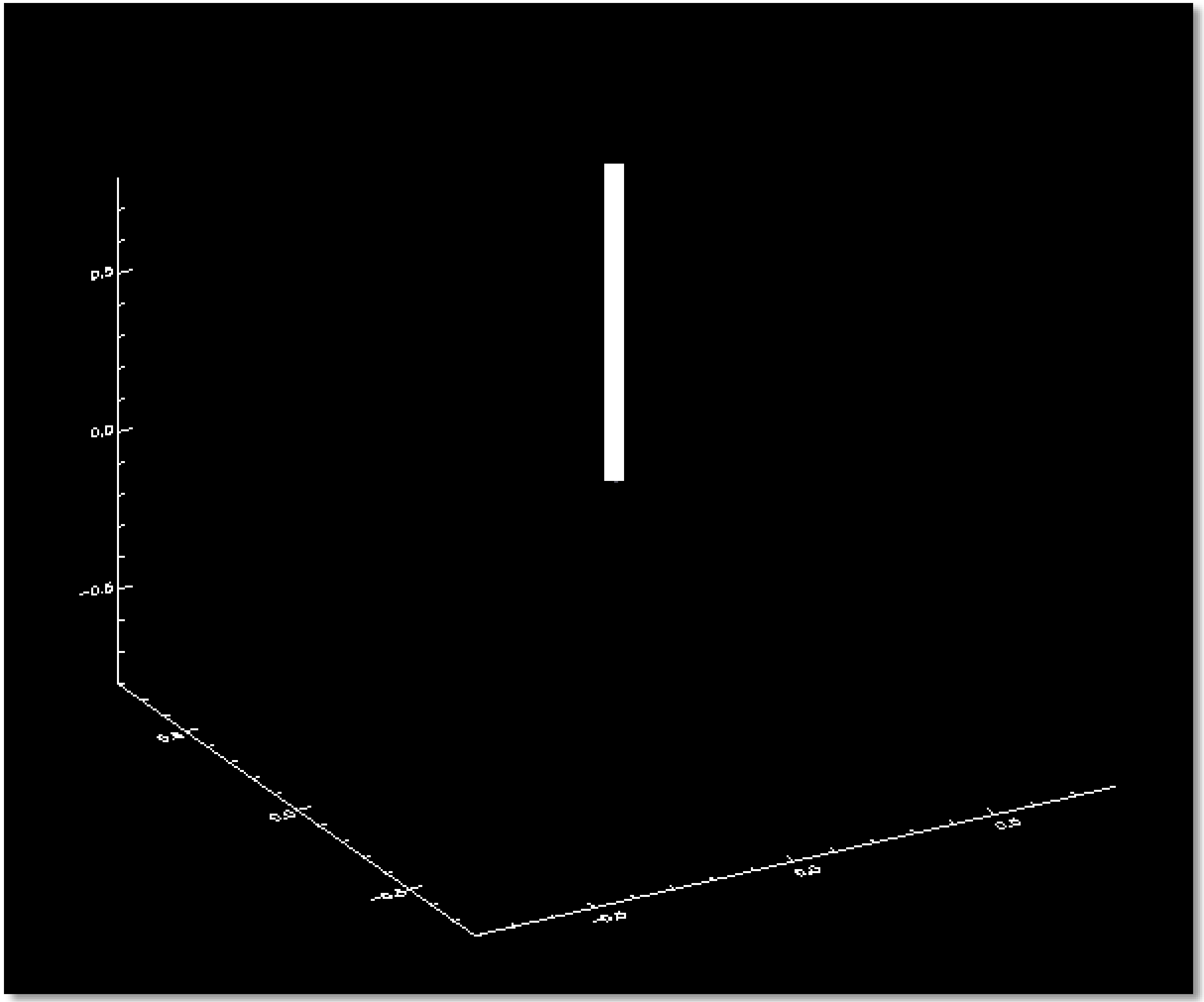


Microscopic B_0 variation and spin echos

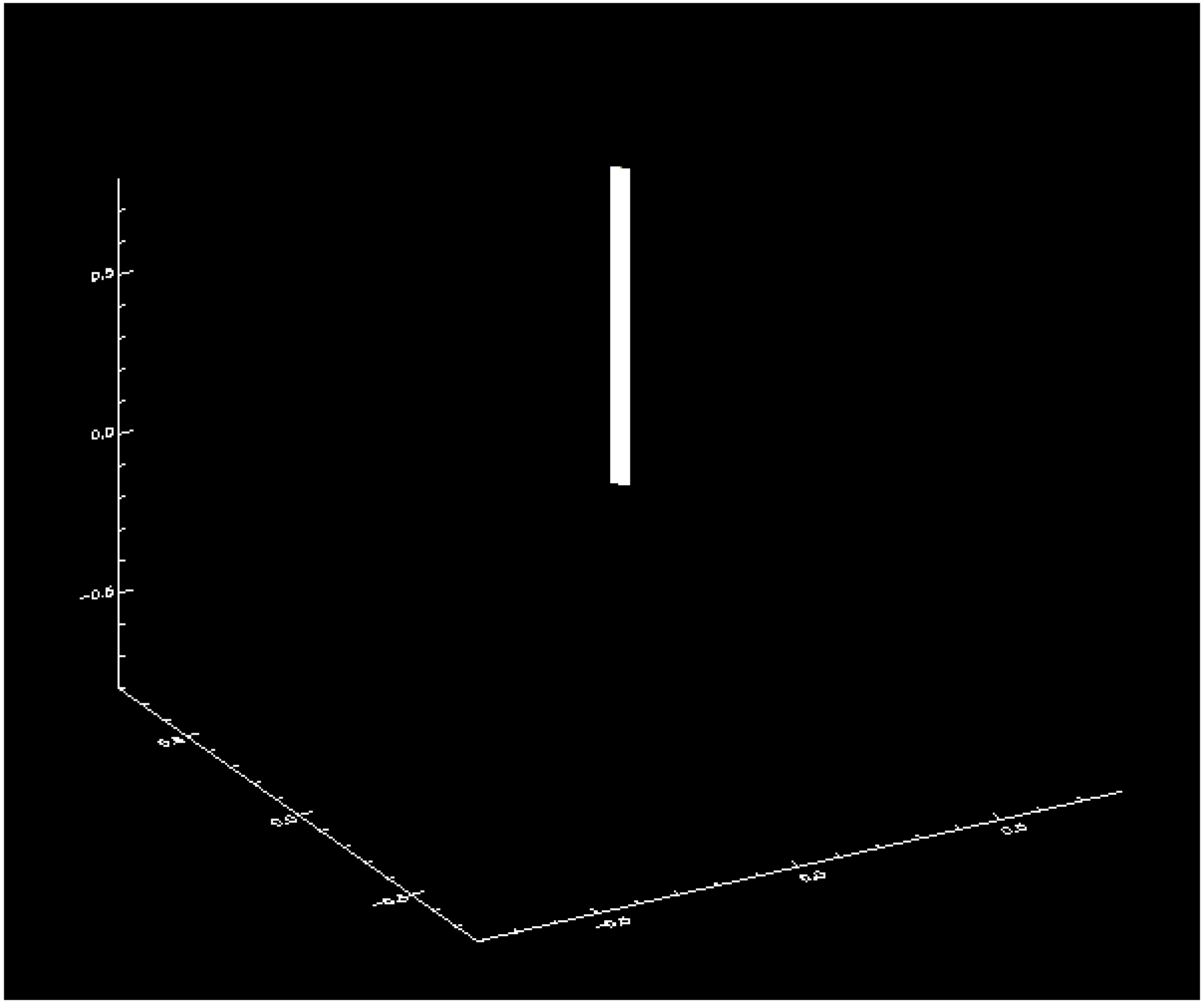


Spin echo formation

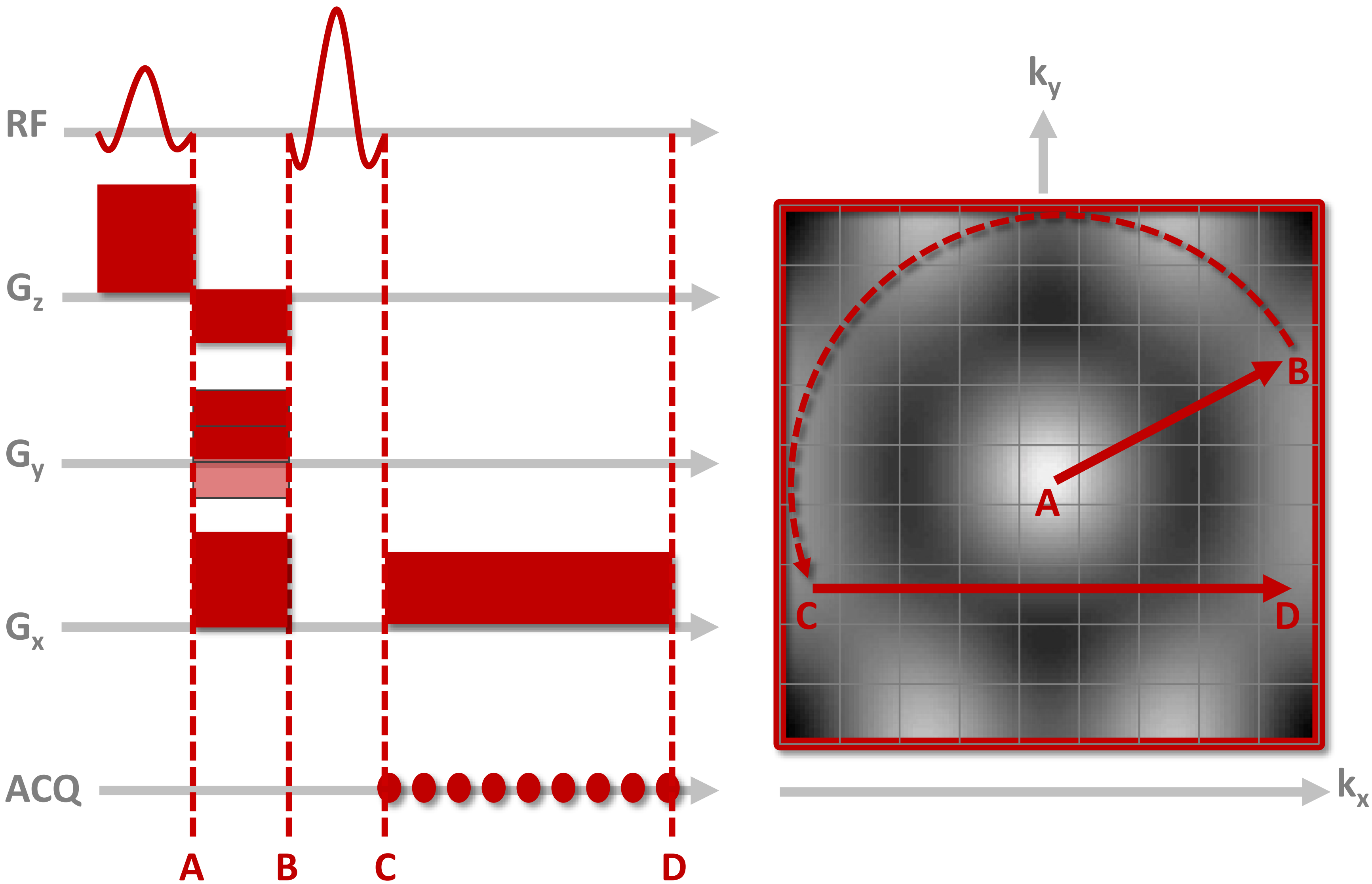
Ideal



Real

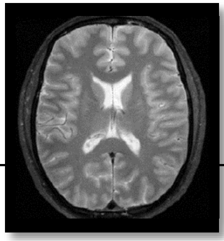


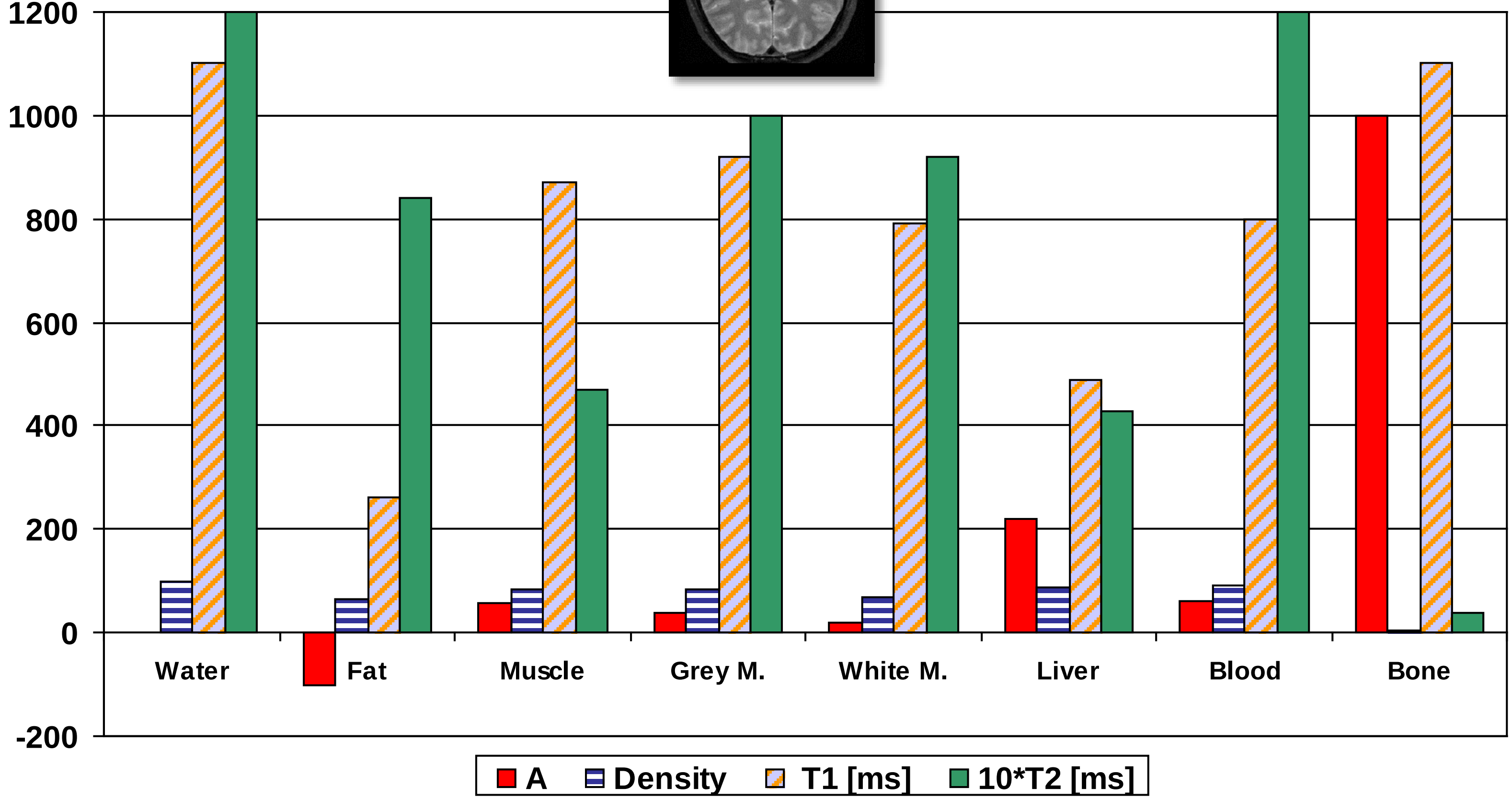
Spin echo encoding experiment with slice selection



Tissue properties

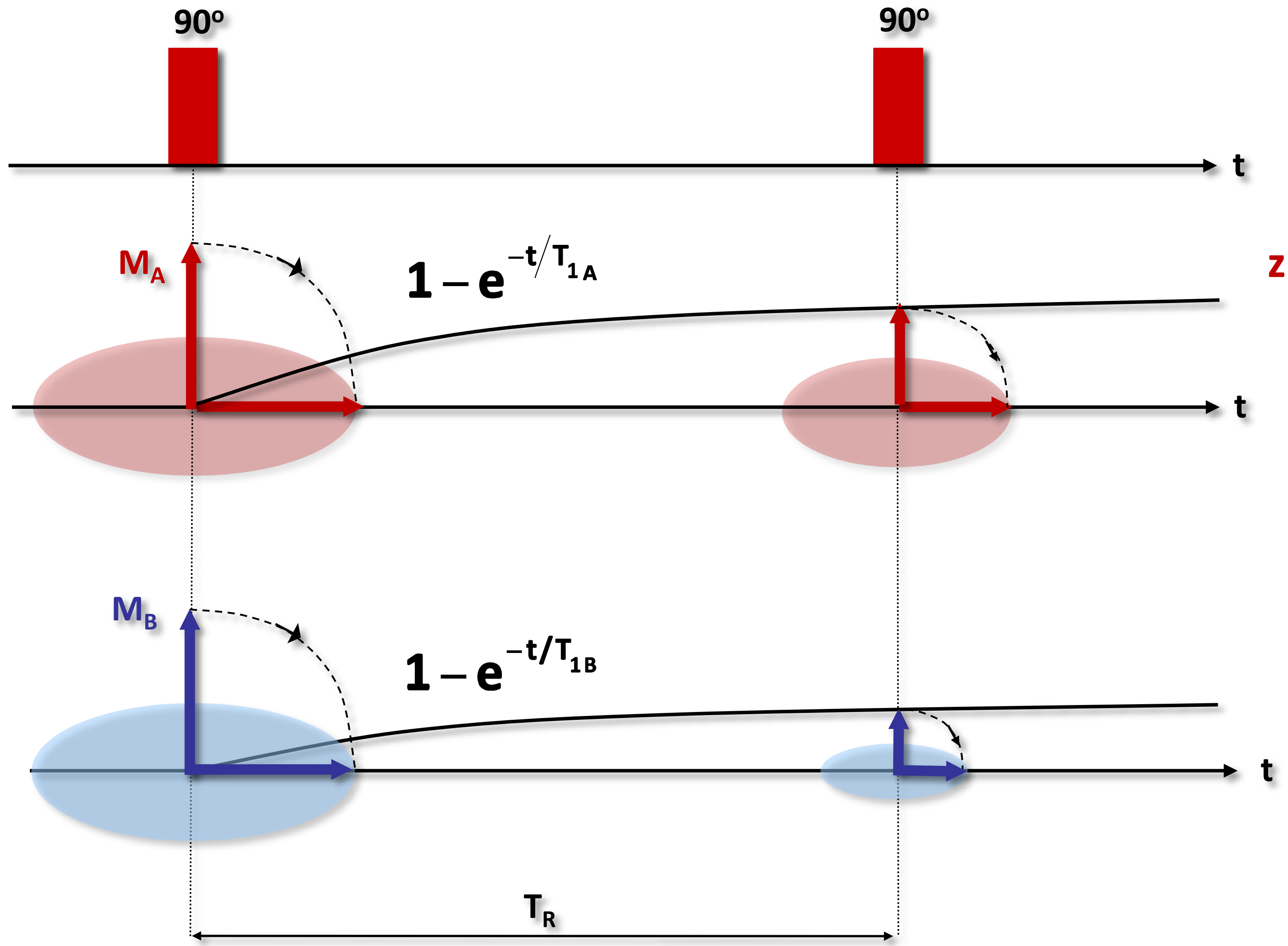
Chapter 5.13





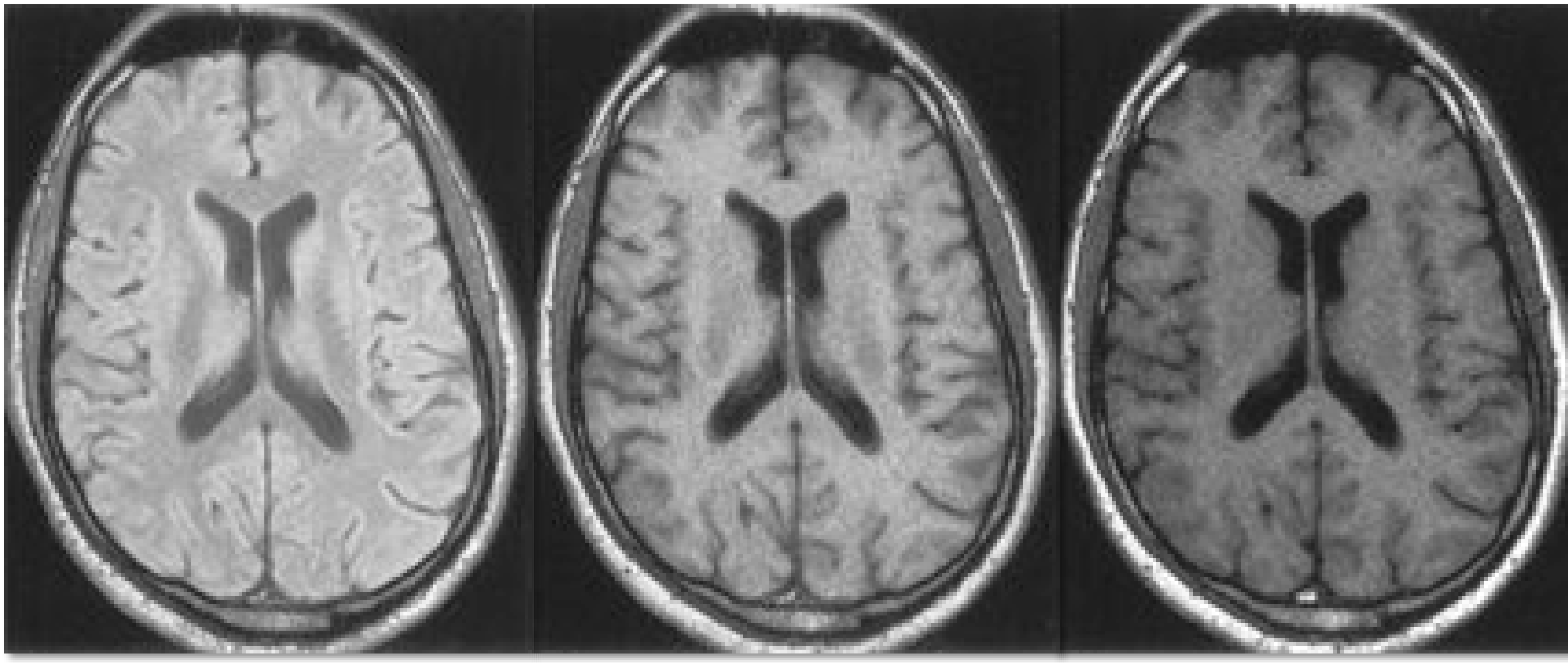
Saturation method

Chapter 5.12



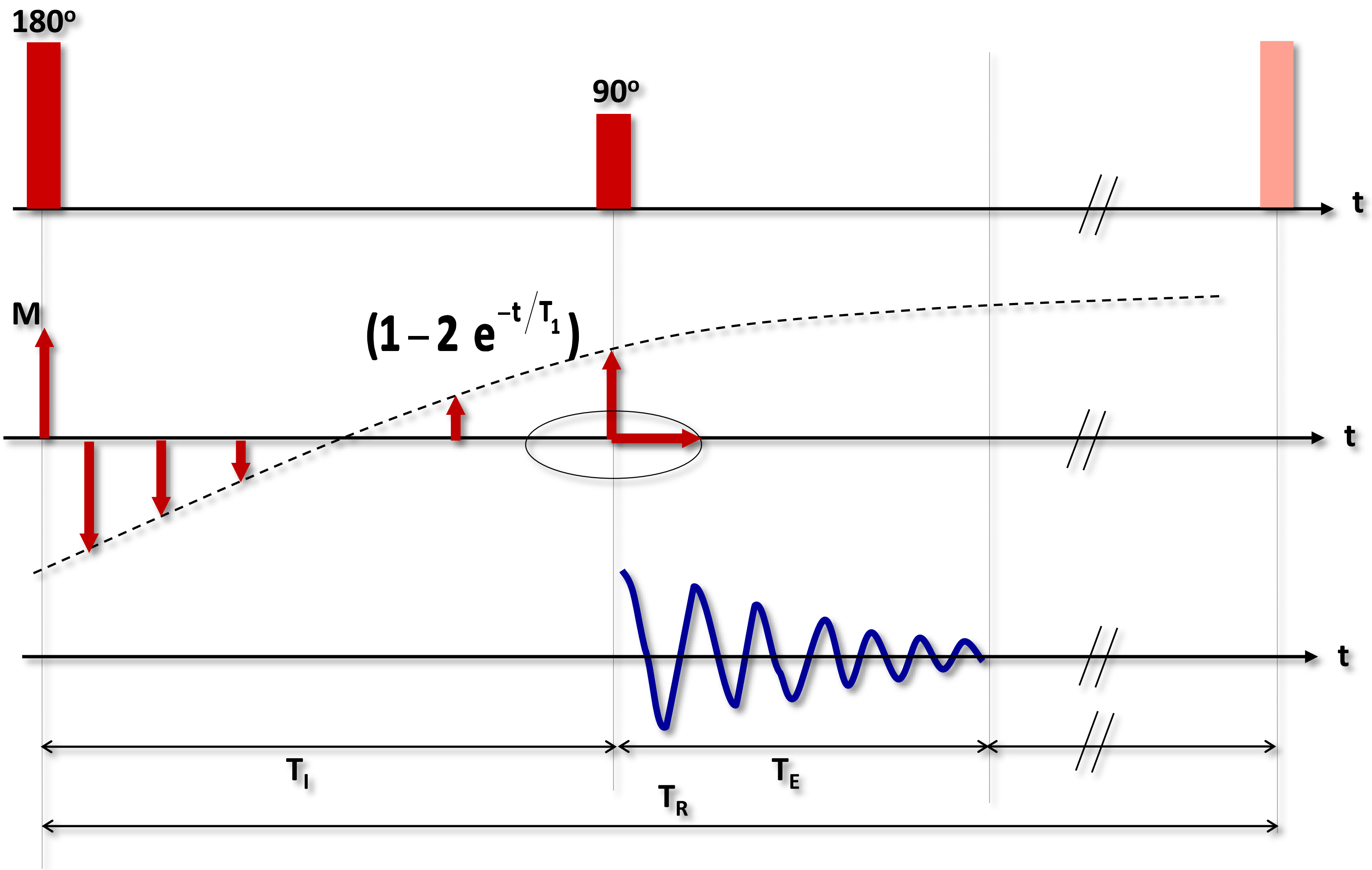
Saturation method

Density weighted **T_1 weighted**

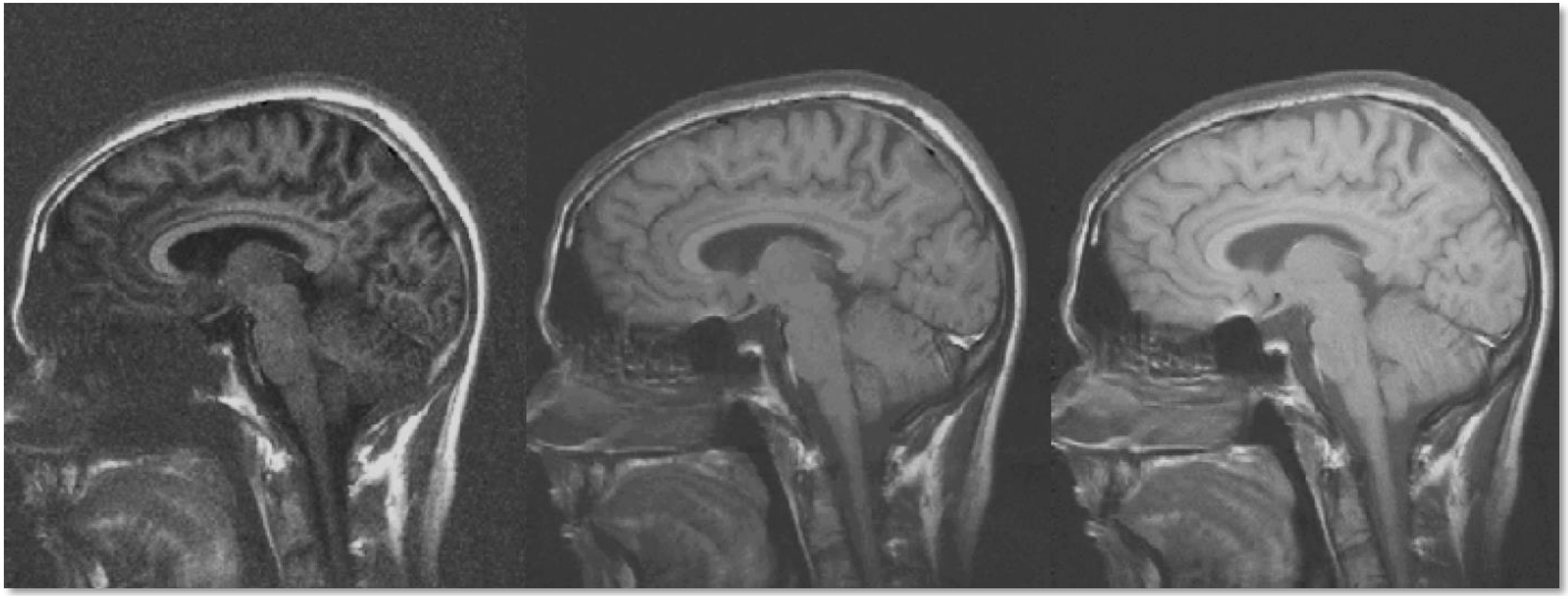


T_{Rec} = 2000ms T_{Rec} = 800ms T_{Rec} = 400ms

Inversion recovery method

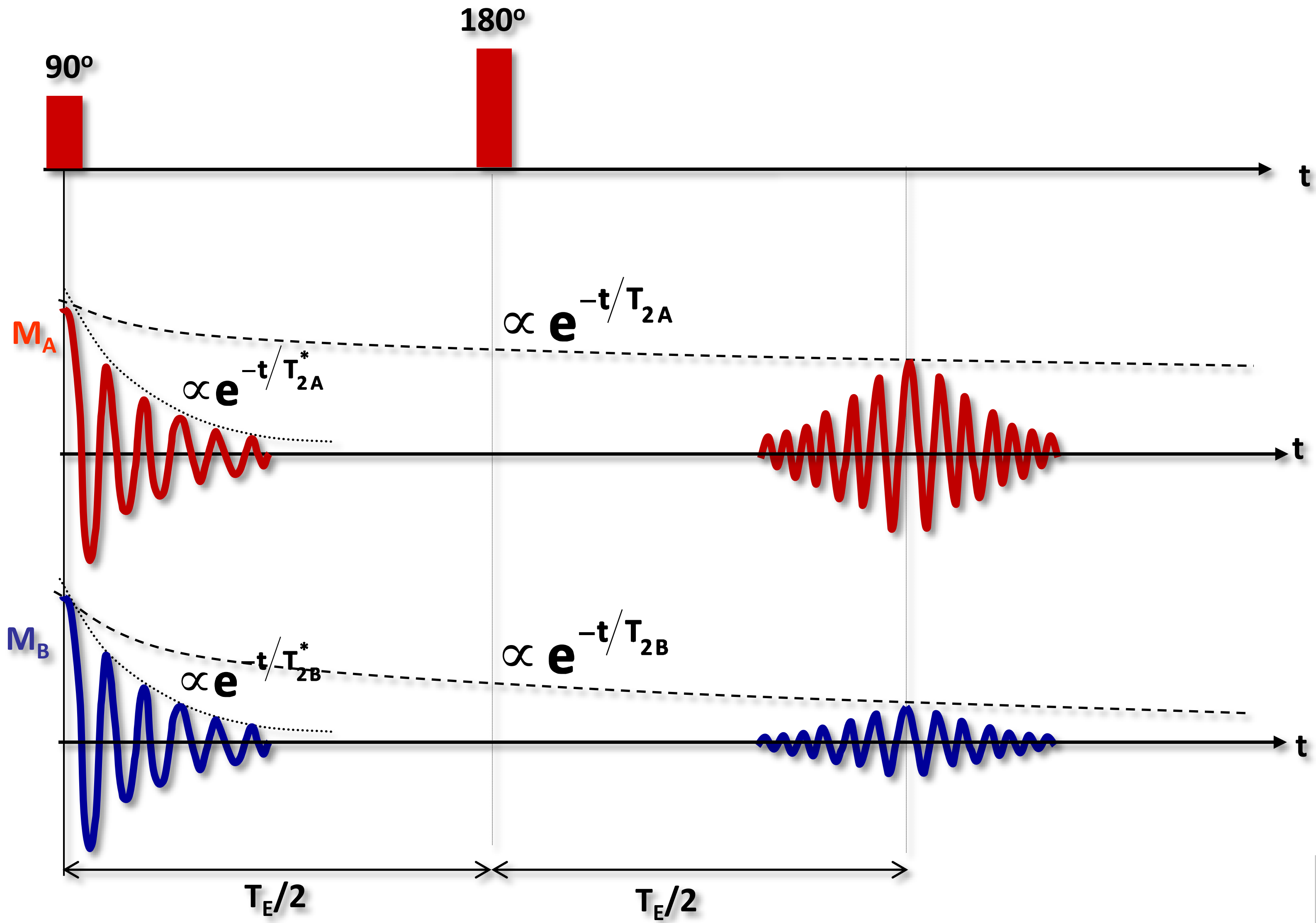


Inversion recovery (T_R=0.5, 1.5T)



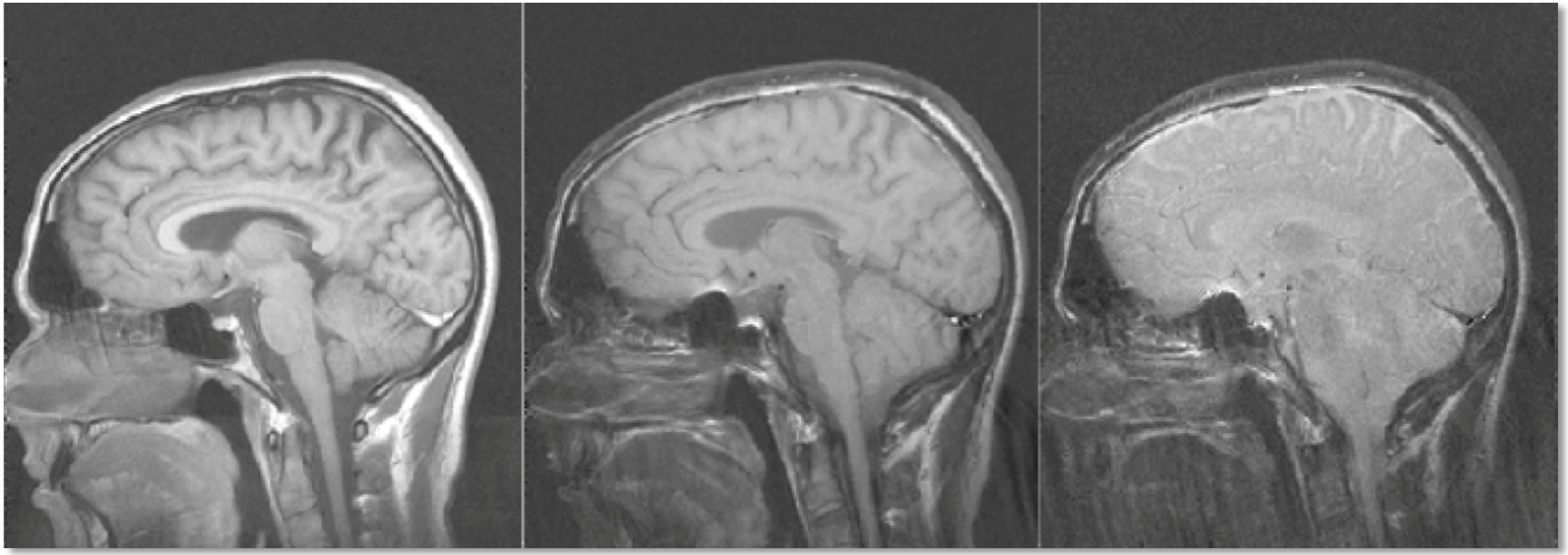
T_I = 200ms T_I = 300ms T_I = 400ms

Spin-echo method



Spin-echo method

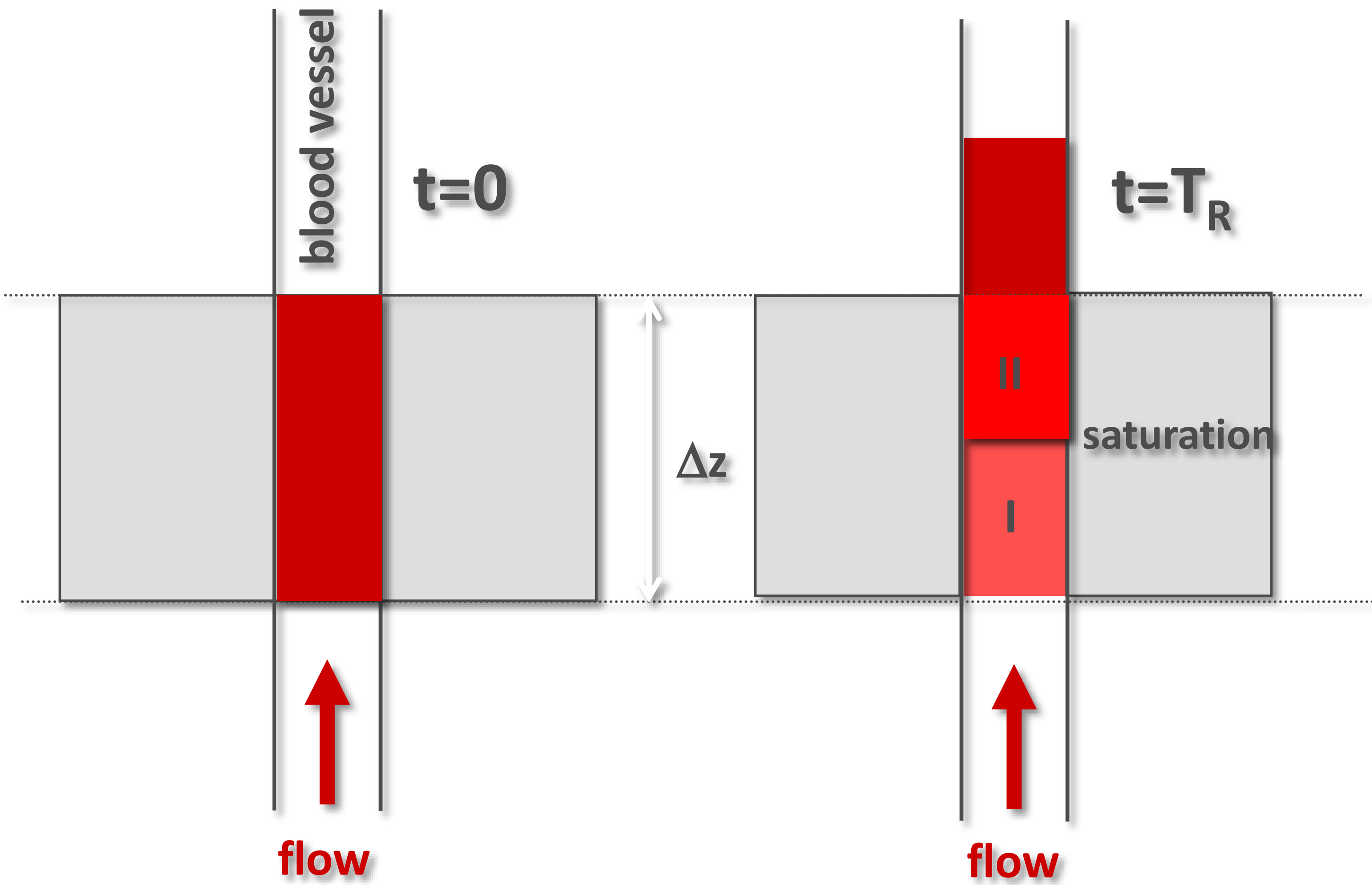
Spin-echo (T_R=0.5, 1.5T)



T_E = 8ms | T_E = 50ms | T_E = 100ms

Inflow contrast

Chapter 5.17



Inflow contrast in brain