Biomedical Imaging FS 2019

Ultrasound 1

Exercises

1. Consider an ultrasound beam that hits the boundary between two materials perpendicularly. Using the information provided in the lecture slides, show that transmission and reflection at the boundary conserve total beam intensity.

   Let p_{inc}, p_{trans}, p_{ref} denote the pressure amplitudes of the incident, transmitted, and reflected beams, respectively. According to the lecture (for perpendicular incidence) they are related by

$$p_{ref} = p_{inc} \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

$$p_{trans} = p_{inc} \frac{2\, Z_2}{Z_2 + Z_1}$$

   The intensity of a beam is given by

$$I = \frac{1}{2}\, p\, u_z$$

   where u_z denotes particle velocity amplitude, which is equal to pressure divided by impedance so

$$I = \frac{1}{2} \frac{p^2}{Z}$$

   Now let's see if the sum of intensities of the transmitted and reflected beams is equal to that of the incident beam:

$$\frac{1}{2} \frac{p_{trans}^2}{Z_2} + \frac{1}{2} \frac{p_{ref}^2}{Z_1} = \frac{1}{2} \frac{p_{inc}^2}{Z_1} \quad ?$$

   Substitute pressures:

$$\frac{p_{inc}^2\, 4\, Z_2^2}{Z_2\, (Z_2 + Z_1)^2} + \frac{p_{inc}^2\, (Z_2 - Z_1)^2}{Z_1\, (Z_2 + Z_1)^2} = \frac{p_{inc}^2}{Z_1} \quad ?$$

$$\frac{4\, Z_1 Z_2}{(Z_2 + Z_1)^2} + \frac{(Z_2 - Z_1)^2}{(Z_2 + Z_1)^2} = 1 \quad ?$$

$$\frac{Z_2^2 + 2\, Z_1 Z_2 + Z_1^2}{(Z_2 + Z_1)^2} = 1 \quad !$$

By subtracting the first two equations one can also obtain a simple relationship that is independent of wave impedances:

$$p_{trans} - p_{ref} = p_{inc}$$

Some of you were tempted to work from this relationship. However from this equation alone one cannot derive conservation of intensity. To appreciate this, consider adding any Δp to both p_{trans} and p_{ref}. Then the difference equation will still hold while the sum of the intensities of the transmitted and reflected beams can assume any value.

2. Consider a flat transducer with a circular face (diameter = 3 mm) performing 1D ultrasound imaging of two layers of tissue (fat, muscle) as shown below, using pulses of frequency 3 MHz and duration 1 μs.

Due to Prof. Göksel standing in on short-notice, for some of the relevant material properties multiple values appear in the lecture materials. In these cases, either value is OK to use, of course. Please accept our apologies for the confusion.

a) Where is the near-field boundary (NFB)?

$$NFB = \frac{r^2}{\lambda_{fat}} = \frac{r^2 f}{c_{fat}} = \frac{(0.0015\, m)^2\, 3 \cdot 10^6 Hz}{1476\, m/s} = 0.457\, cm$$

So the NFB is still within the fat layer. Therefore it is correct to consider only the wavelength in fat to calculate it.

b) Let the maximum (i.e. on-axis) pulse intensity be 10 mW/cm^2 at the level of the NFB. What are the on-axis pressure and intensity levels entering the muscle layer?

At the level of the NFB the on-axis pressure amplitude is

$$p_{NFB} = \sqrt{2\, I\, Z_{fat}} = \sqrt{2 \cdot 100 \frac{W}{m^2} \cdot 1.36 \cdot 10^6\, kg/(m^2\, s)} = 16.49\, kPa$$

Until it reaches the interface the beam crosses another 0.543 cm of fat. The attenuation involved is

$$att = 0.543\, cm \cdot 0.5 \frac{dB}{cm\, MHz} \cdot 3\, MHz = 0.81\, dB$$

The attenuated pressure at 1cm depth is then

$$p_{1cm,fat} = 16.49\, kPa \cdot 10^{-\frac{0.81 dB}{20\, dB}} = 15.02\, kPa$$

The transmitted beam has pressure

$$p_{trans} = p_{1cm,fat} \frac{2\, Z_{muscle}}{Z_{muscle} + Z_{fat}} = 16.51\; kPa$$

and hence intensity

$$I_{trans} = \frac{1}{2} \frac{p_{trans}^2}{Z_{muscle}} = 82.1 \frac{W}{m^2} = 8.21 \frac{mW}{cm^2}$$

c) What is the peak particle velocity upon entrance of the pulse into the muscle layer?

$$u_{z,trans} = \frac{p_{trans}}{Z_{muscle}} = 9.95\; \frac{mm}{s}$$

d) At which depth in the muscle has the sound intensity dropped to 1 mW/cm^2 ?

(Assume 2 dB/cm/MHz of attenuation in muscle).

The additional attenuation in dB is

$$att = 10\; \log_{10}\left(\frac{1\; mW cm^2}{8.21\; mW\; cm^2}\right) dB = \; -9.14\; dB$$

which corresponds to

$$\Delta z = \frac{-9.14\; dB\; cm\; MHz}{2\; dB\; 3\; MHz} = 1.52\; cm$$

So the beam is down to 1 mW/cm^2 at a total depth of 2.52 cm.

Note that beam broadening beyond he NFB actually also reduces intensity a bit by causing the beam´s power to be spread across a somewhat larger area. This effect could but did not need to be accounted for.

e) What is the lateral resolution at that depth?

The broadening angle is

$$\theta = \arcsin\left(\frac{0.61\; \lambda}{r}\right) = 12.3°$$

So, starting at a width of 3 mm at the level of the NFB, the beams broadens to

$$\Delta x = 0.3\; cm + 2\; \cdot\; (2.52\; cm - 0.457\; cm) tan(\theta) = 1.2\; cm$$

Alternatively, one can work from the approximation $\Delta x = \lambda\, z/r$.

f) What is the axial resolution in fat and muscle?

$$\Delta z_{fat} = \frac{1\; \mu s\; \cdot c_{fat}}{2} = 0.74\; mm$$

$$\Delta z_{muscle} = \frac{1\ \mu s\ \cdot c_{muscle}}{2} = 0.78\ mm$$