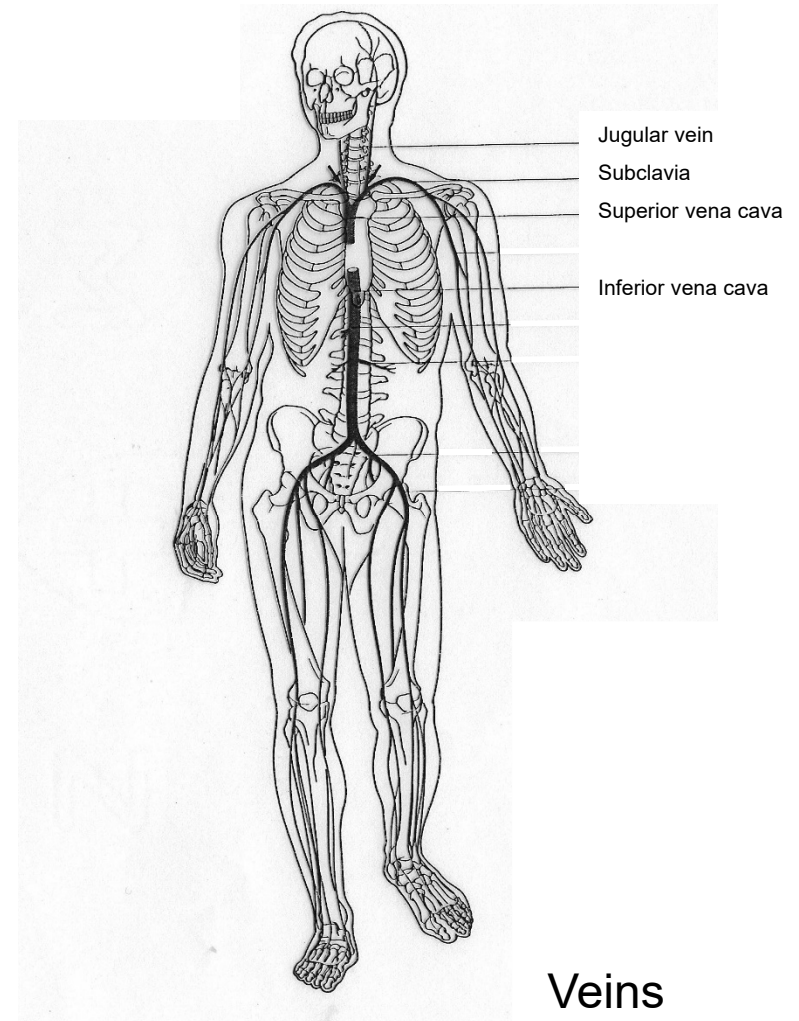
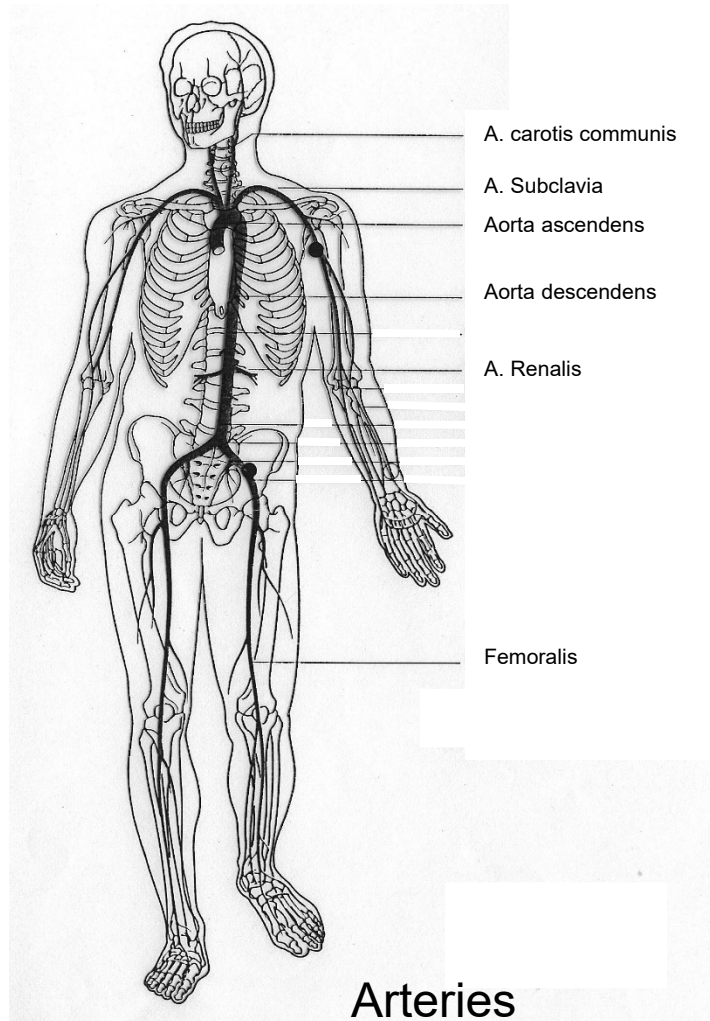


# Arteries, Veins, Circulation



**Key physical quantity: blood flow**

# Christian Doppler

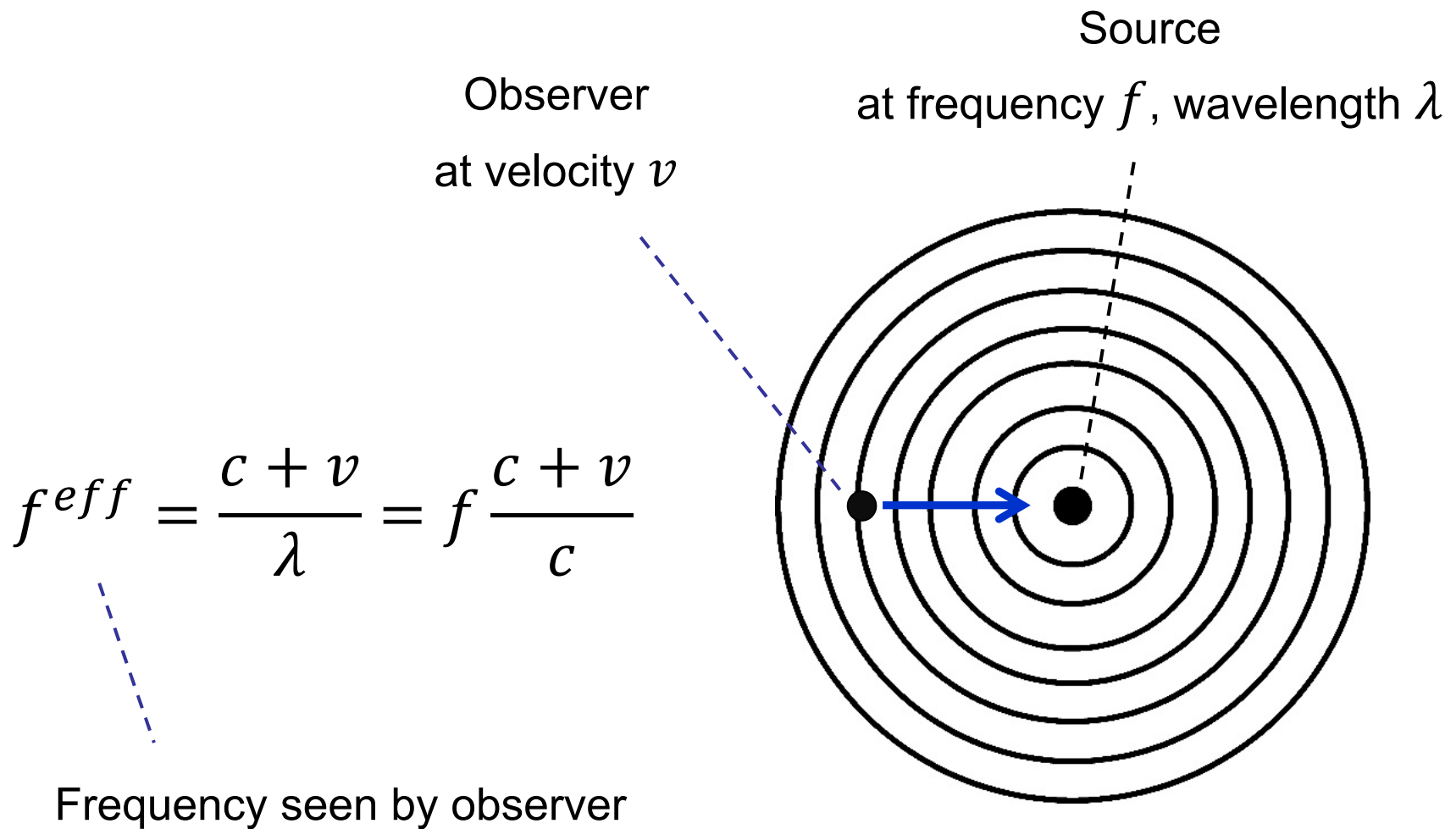


Christian Doppler (1803-1853), Austrian mathematician and physicist

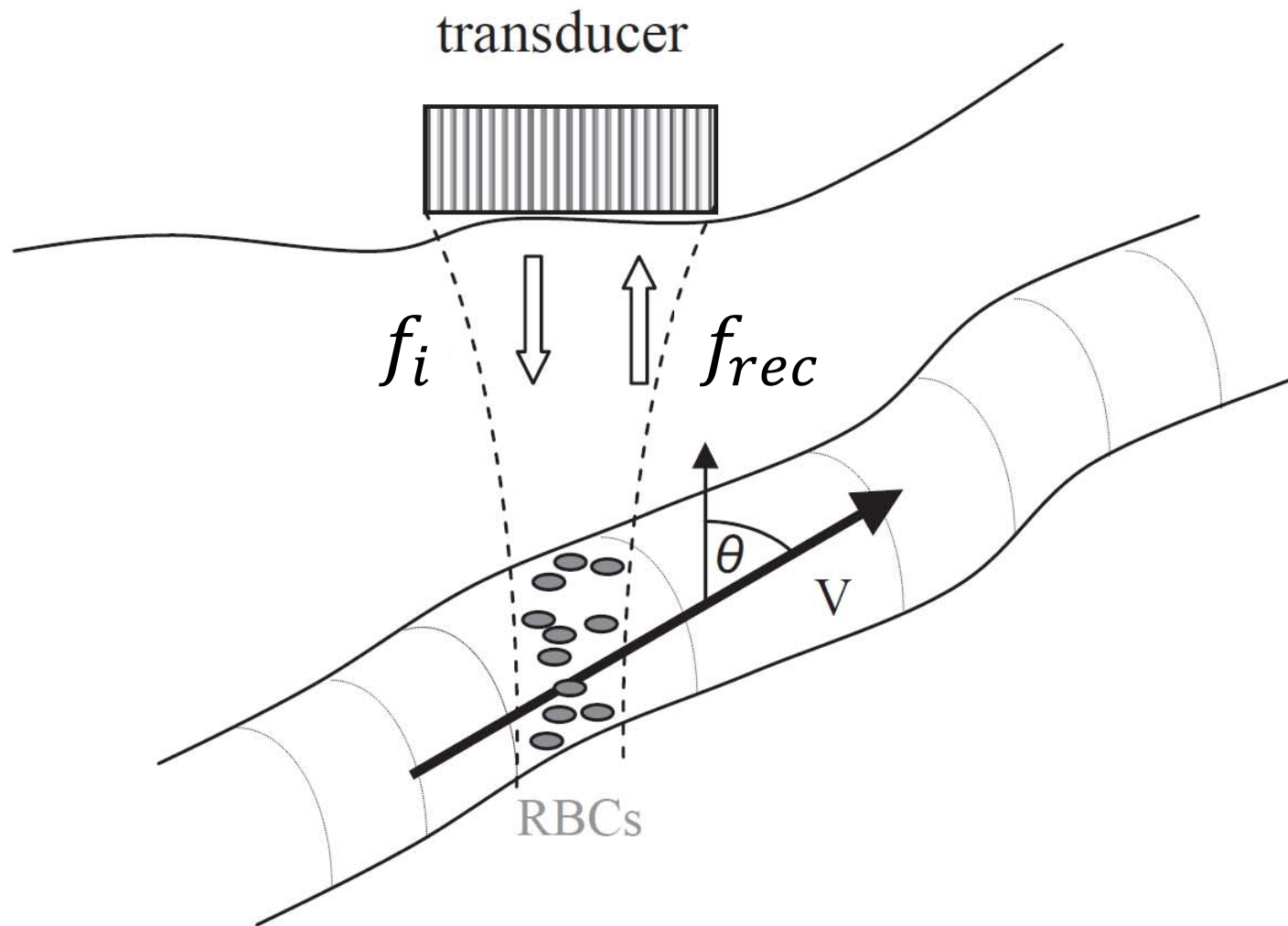
Postulated the frequency shift of the light of binary stars

1845: Acoustic verification using the Amsterdam – Utrecht railway

# Doppler Effect



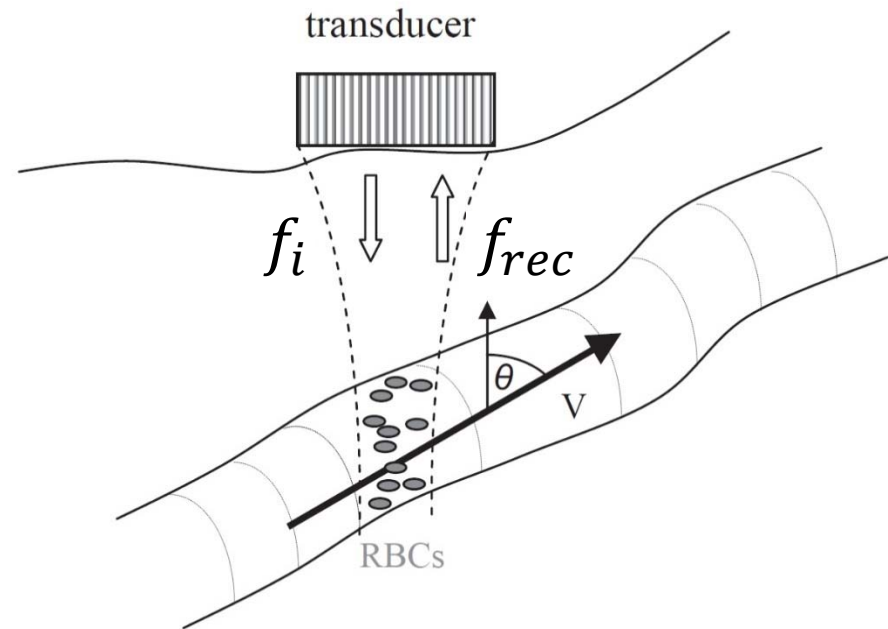
# Doppler Effect



# Doppler Effect

$$f_i^{eff} = \frac{c + v \cos \theta}{\lambda}$$

$$= \frac{f_i (c + v \cos \theta)}{c}$$



$$f_{rec} = \frac{f_i^{eff} (c + v \cos \theta)}{c} = \frac{f_i (c + v \cos \theta)^2}{c^2}$$

$$= f_i + \frac{2 f_i v \cos \theta}{c} + \frac{f_i v^2 \cos^2 \theta}{c^2}$$

negligible for  
 $v \ll c$

$$f_D = f_i - f_{rec} \approx \frac{2 f_i v \cos \theta}{c}$$

Doppler shift

# Doppler Effect

$$f_D = \frac{2 f_i v \cos \theta}{c} \quad \Rightarrow \quad v = \frac{c f_D}{2 f_i \cos \theta} \quad \text{blood velocity}$$

Example:

$$f_i = 5 \text{ MHz}, \quad \theta = 45^\circ, \quad v = 50 \text{ cm/s} \quad f_D = 2.3 \text{ kHz} \quad (< 0.05 \%)$$

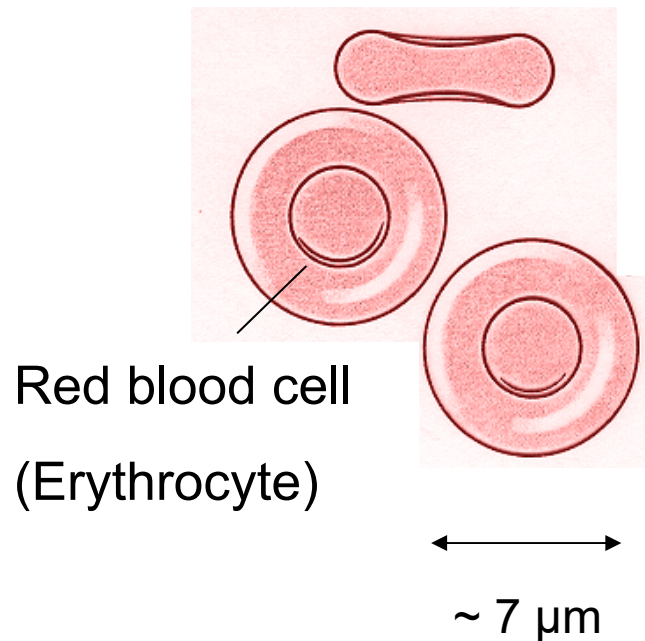
Doppler shifts are very small !

→ use high frequency, small wavelength

Must know angle  $\theta$ , usually from additional B-mode scan

$\theta$  must not be too close to  $90^\circ$  to solve robustly for  $v$

# Scattering of Ultrasound by Red Blood Cells



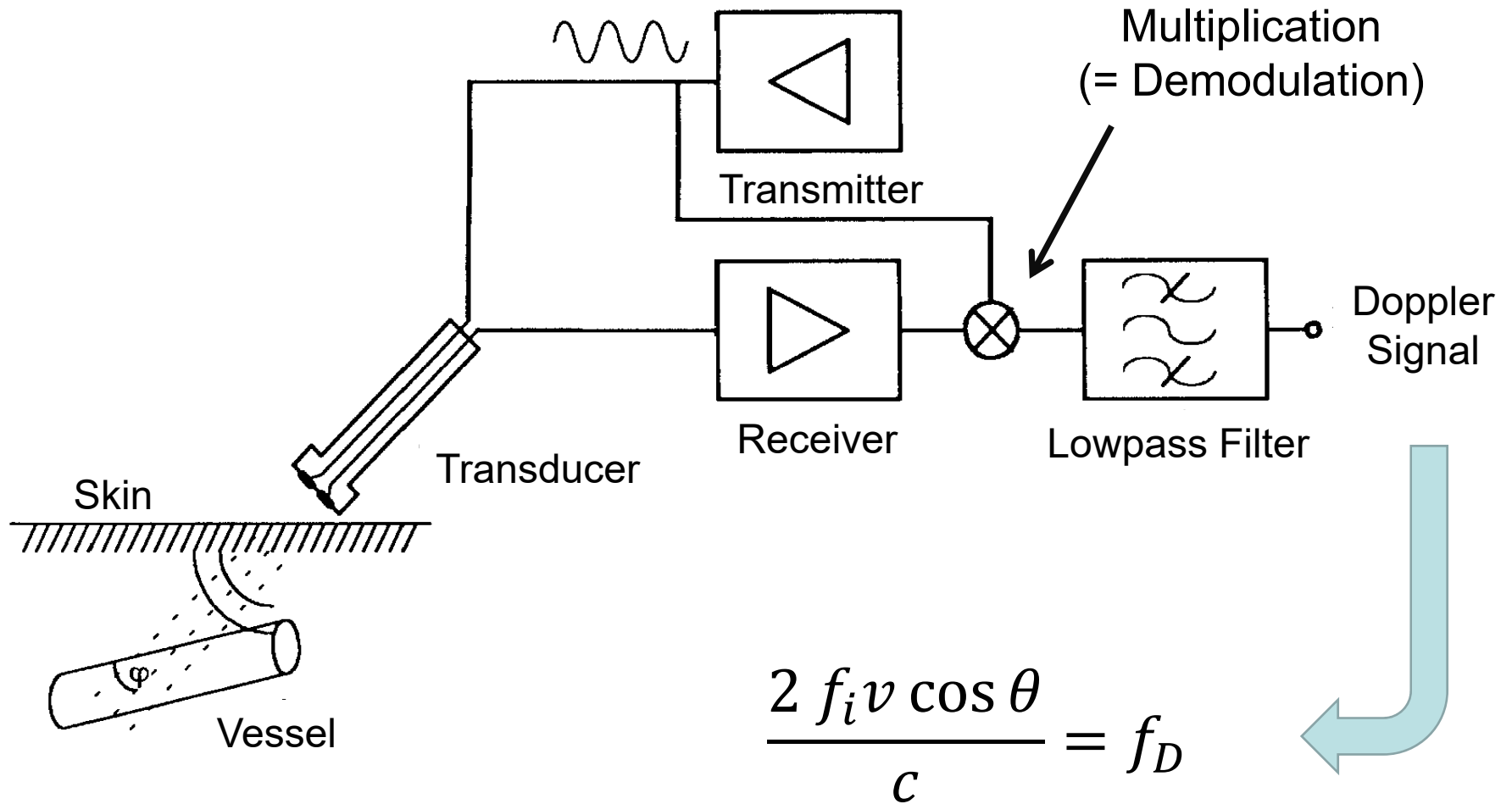
Ultrasound wavelength:  
100  $\mu\text{m}$  – 1 mm

Cells much smaller than wavelength

→ Rayleigh scattering:  $\sigma_s \propto \frac{1}{\lambda^4}$

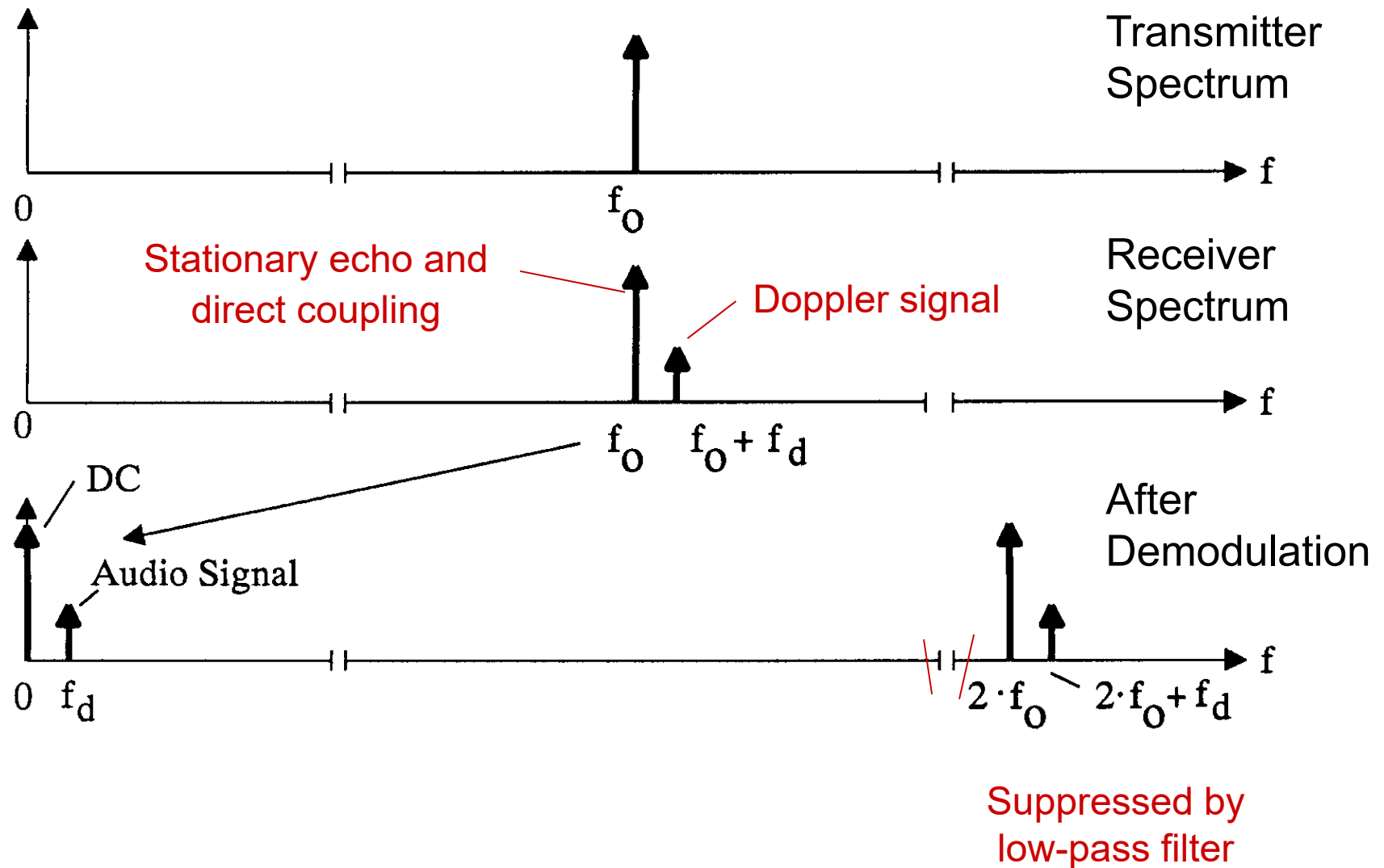
→ use high frequency, small  $\lambda$   
typical: 5 MHz

# Continuous-wave (cw) Doppler

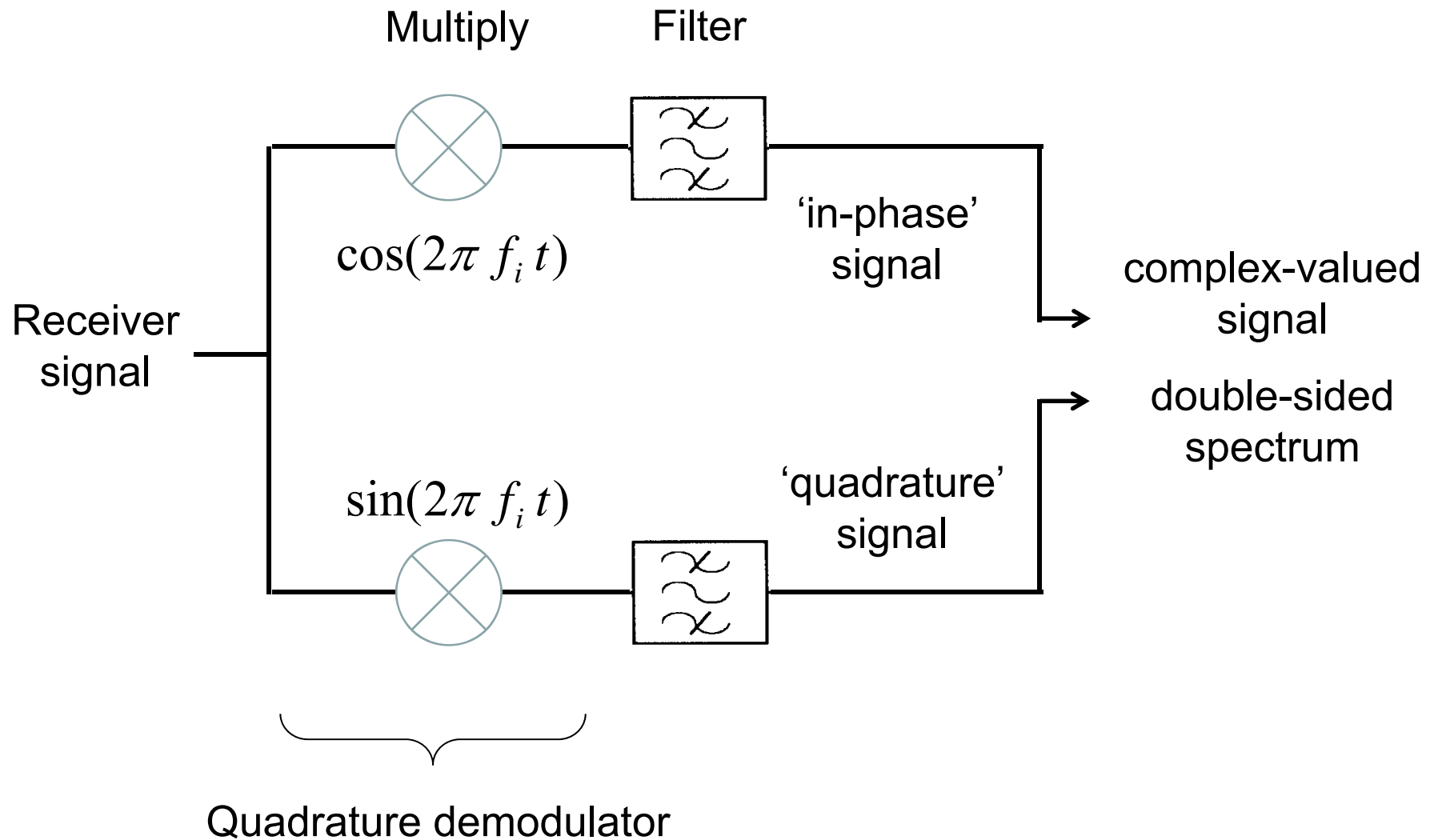




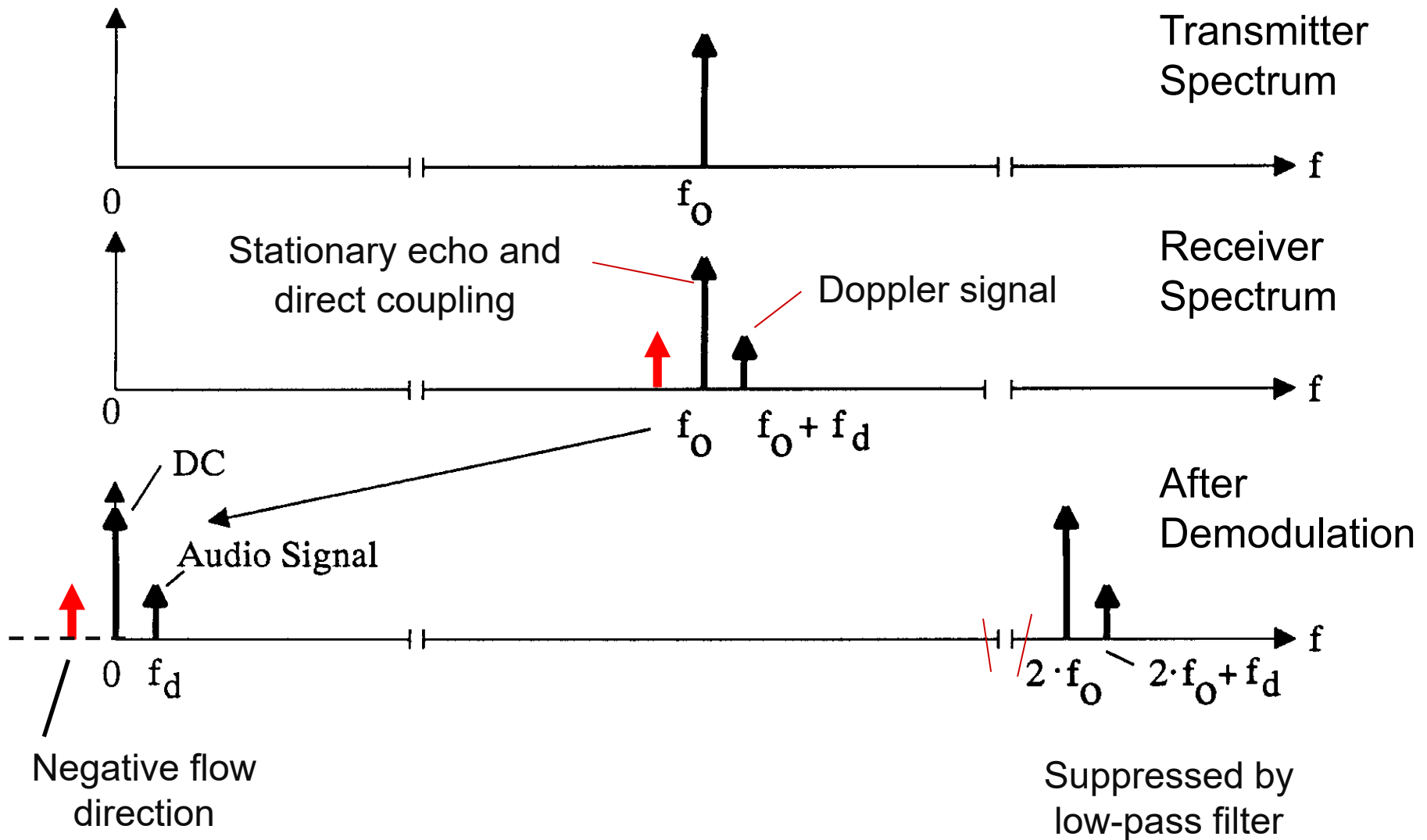
# Doppler Spectrum and Demodulation



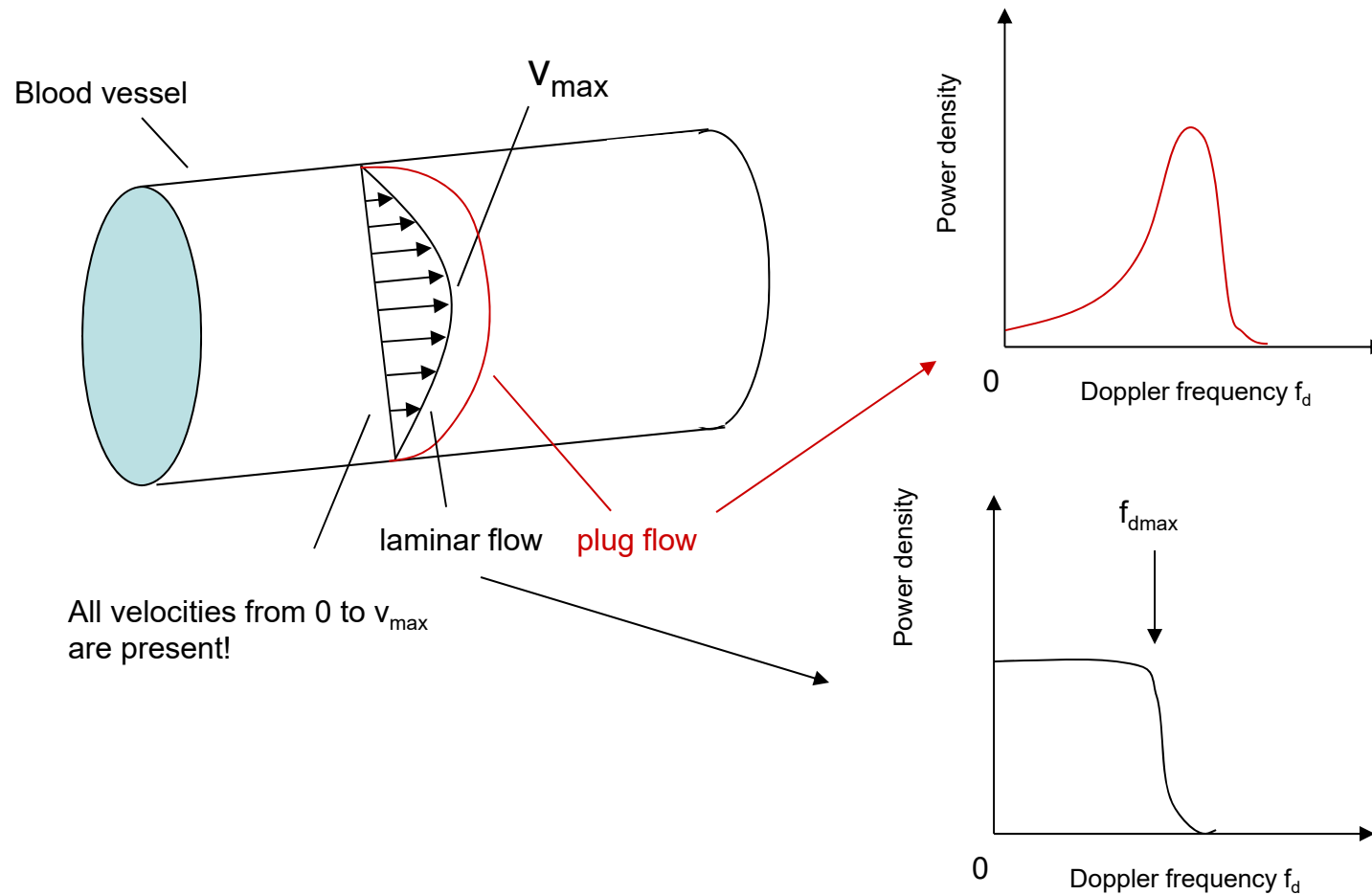
# Detection of Flow Direction



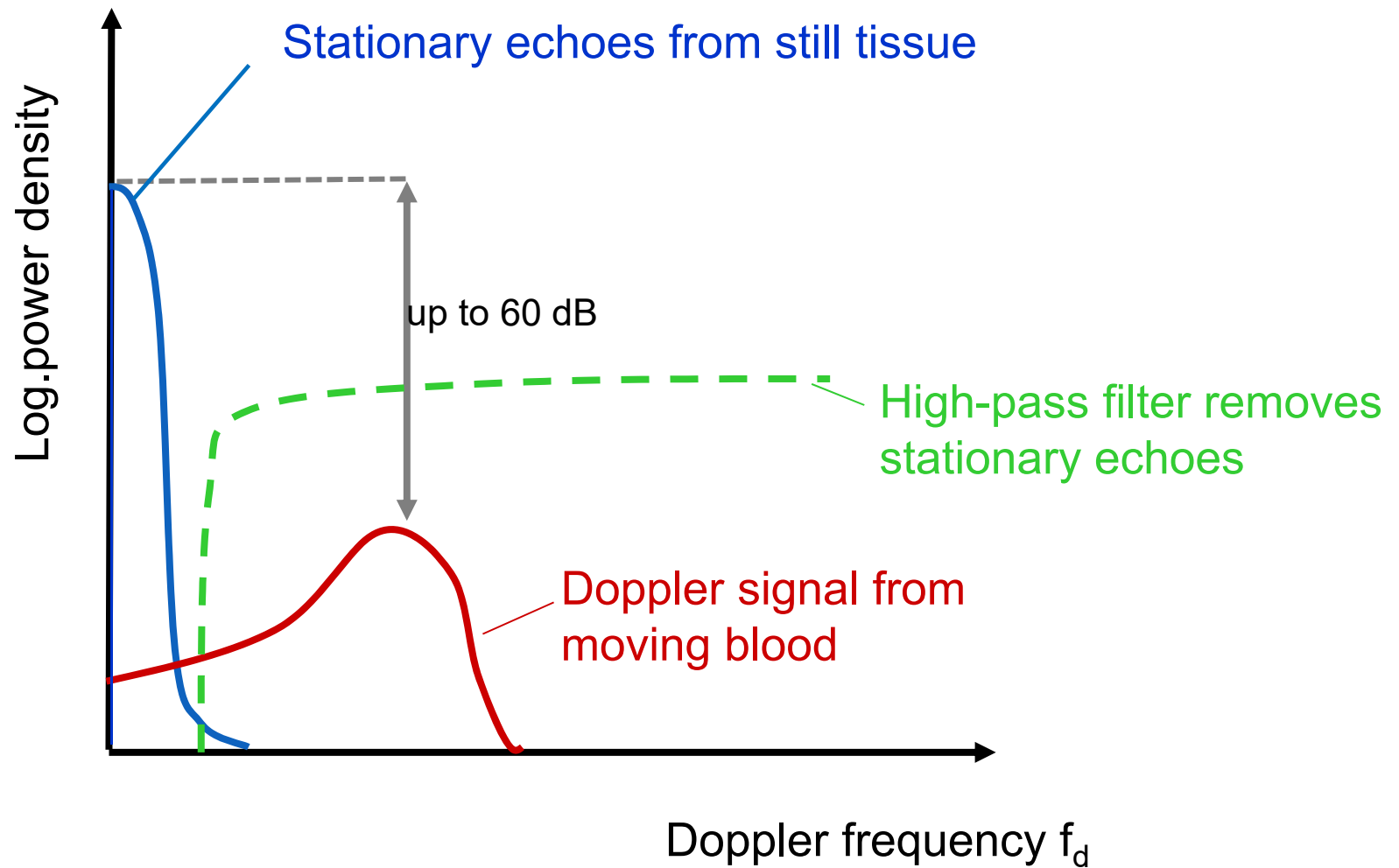
# Doppler Spectrum and Demodulation



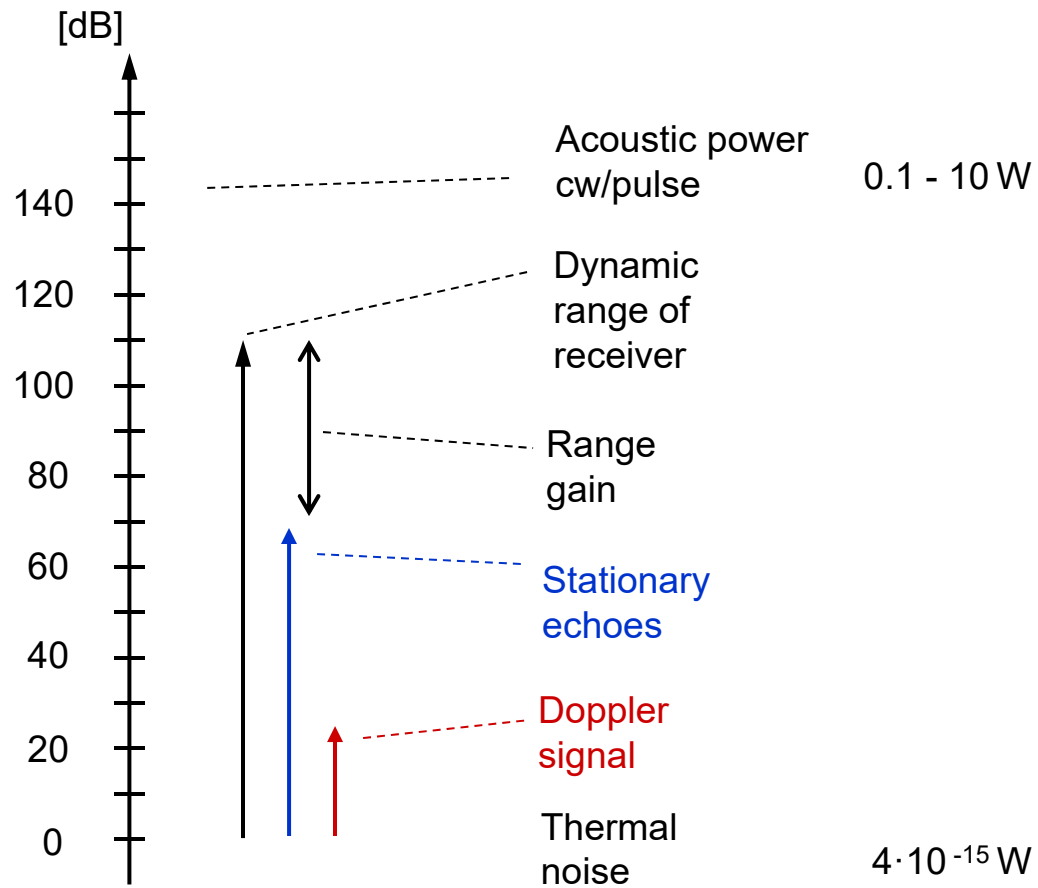
# Flow Profile, Doppler Spectrum



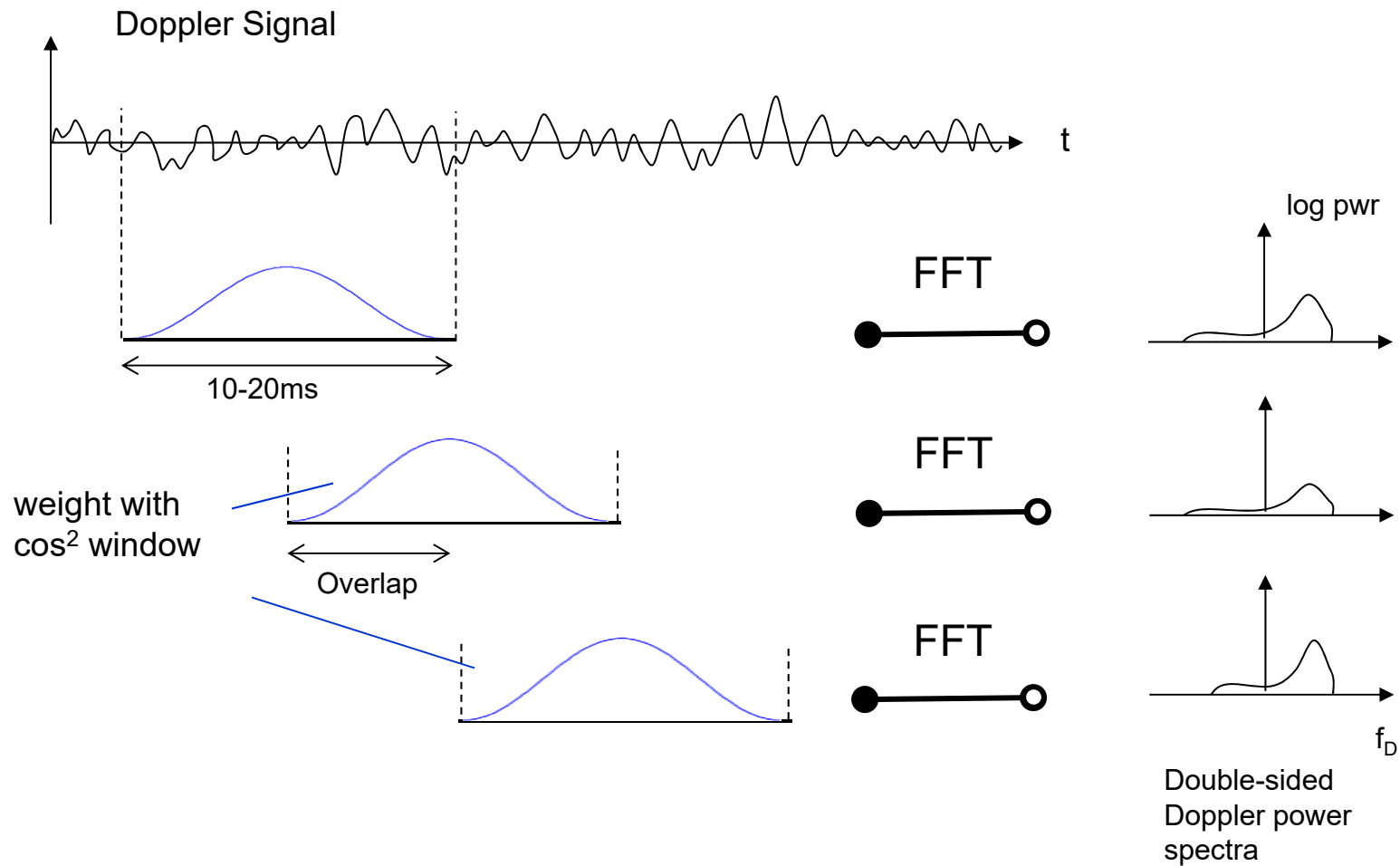
# Real Doppler Spectrum with Stationary Echoes



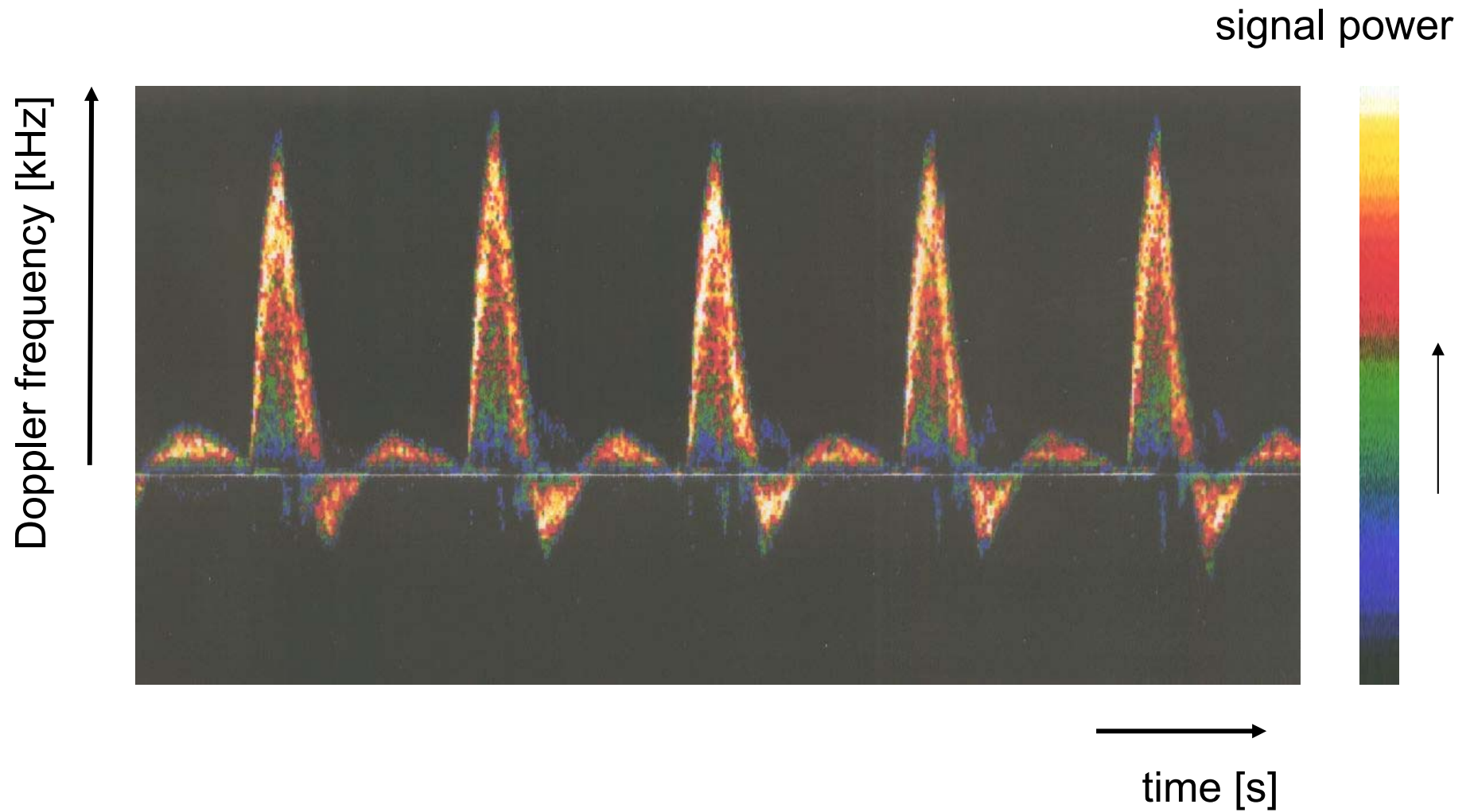
# Dynamic Range



# Time-Resolved Frequency Analysis



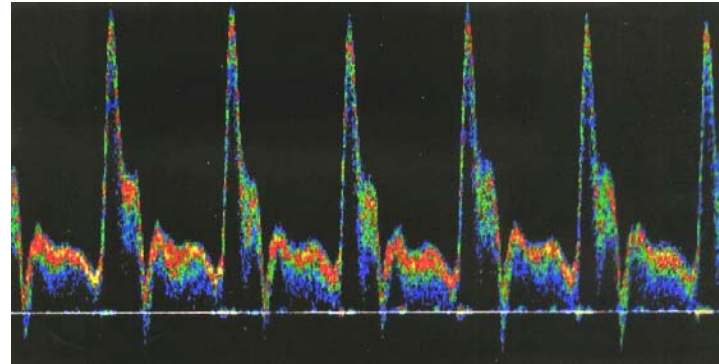
# Sonogram: Arteria Femoralis



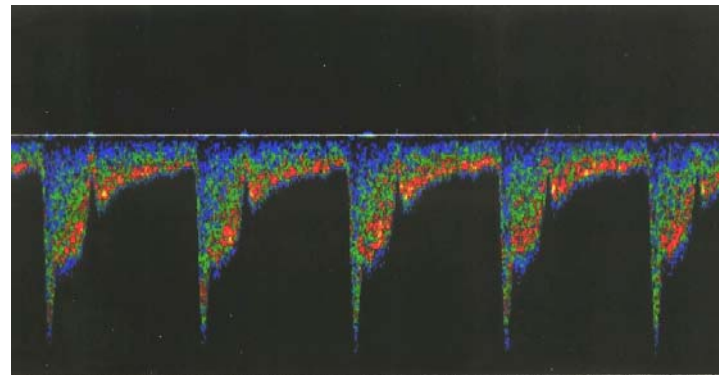


# Example: Carotis Bifurcation

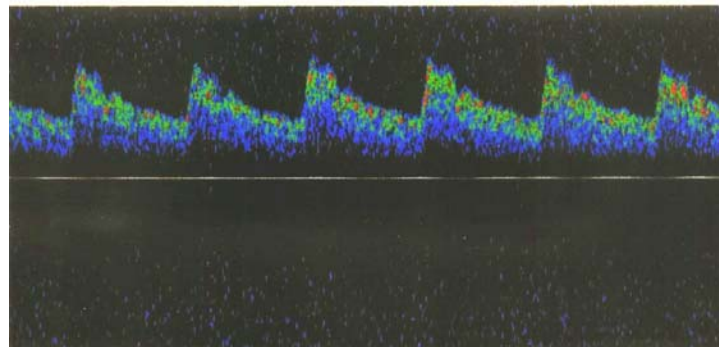
Carotis communis



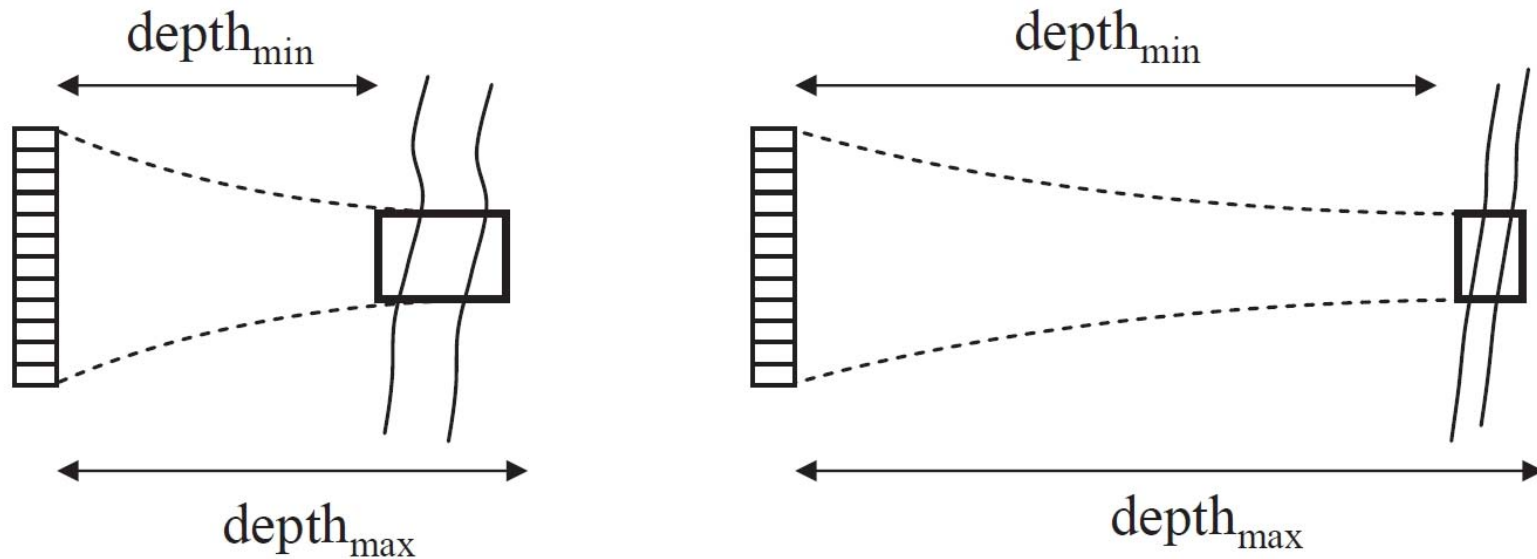
Carotis externa



Carotis interna



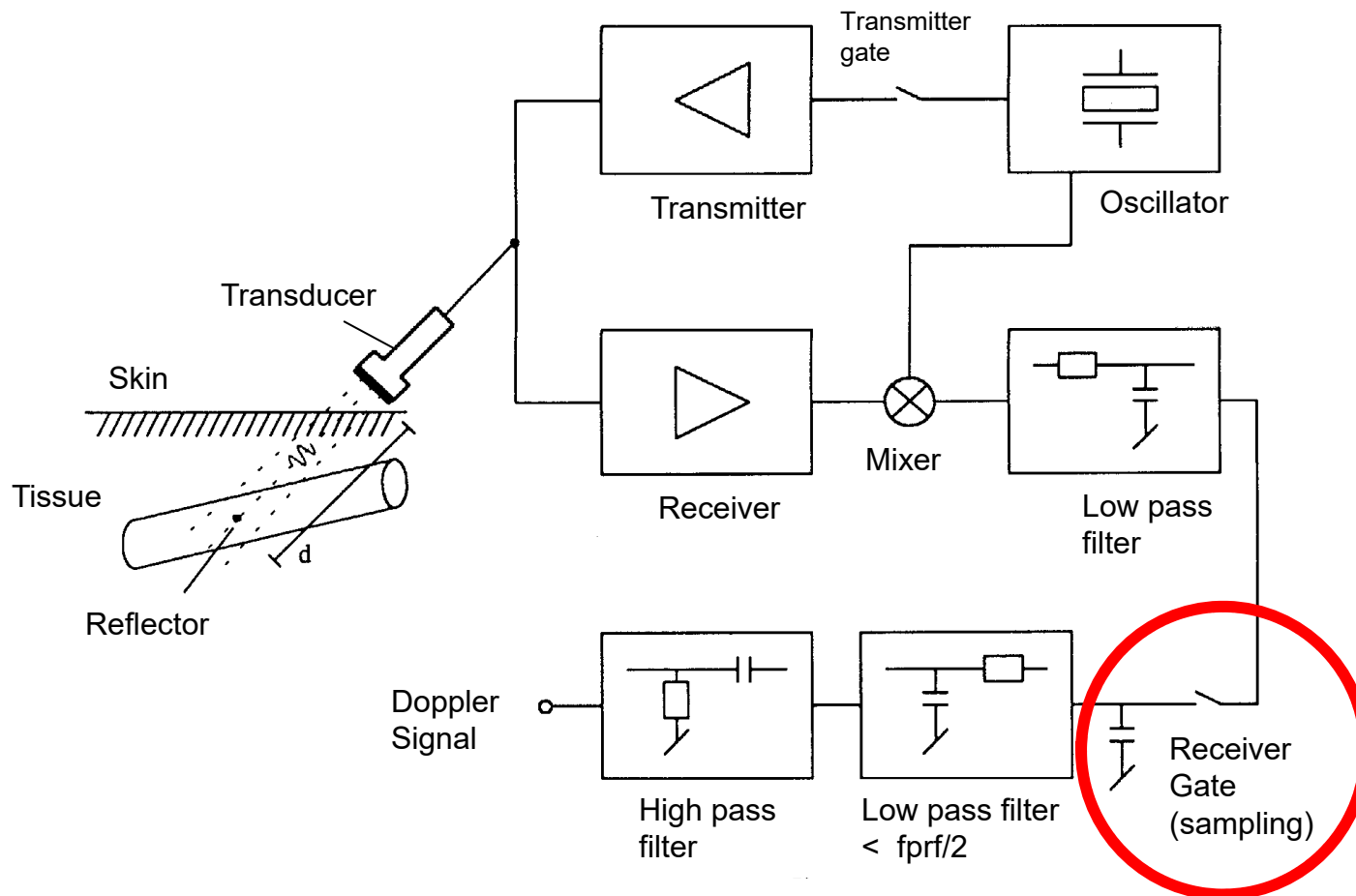
# Spatial Resolution: Pulsed Doppler



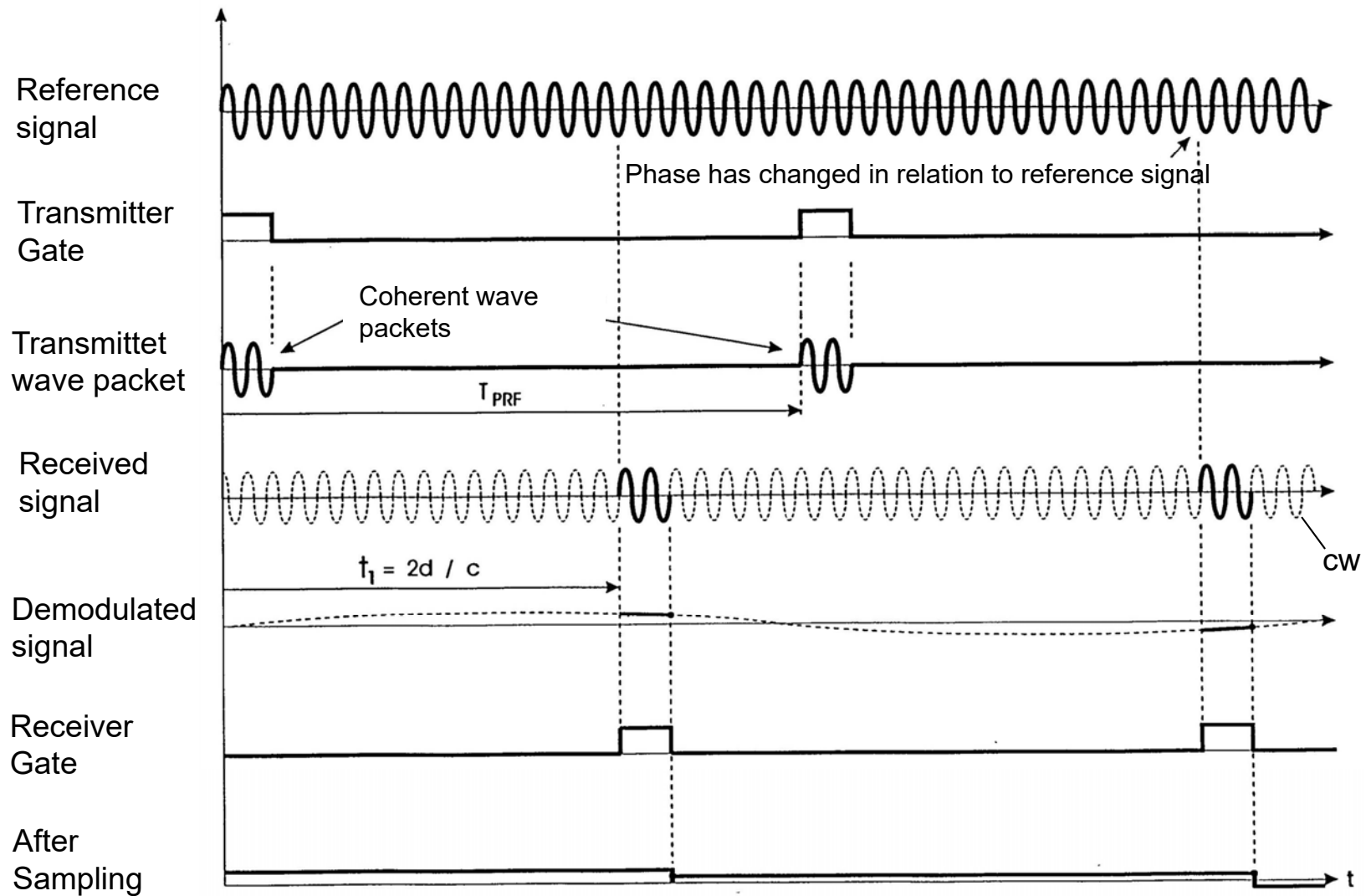
Focus beam to volume of interest

Choose gate timing for depth-selectivity

# Pulsed Doppler



# Time Domain



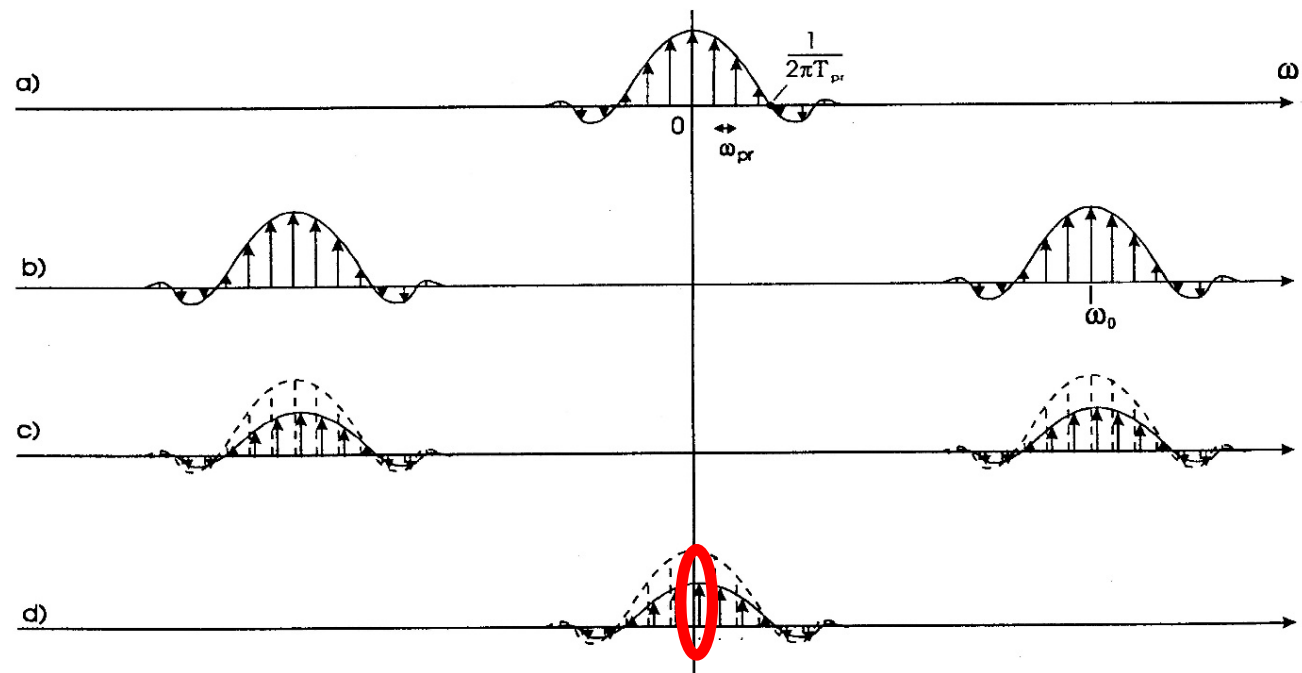
# Frequency Domain

Transmitter  
Gate

Transmitted  
wave package

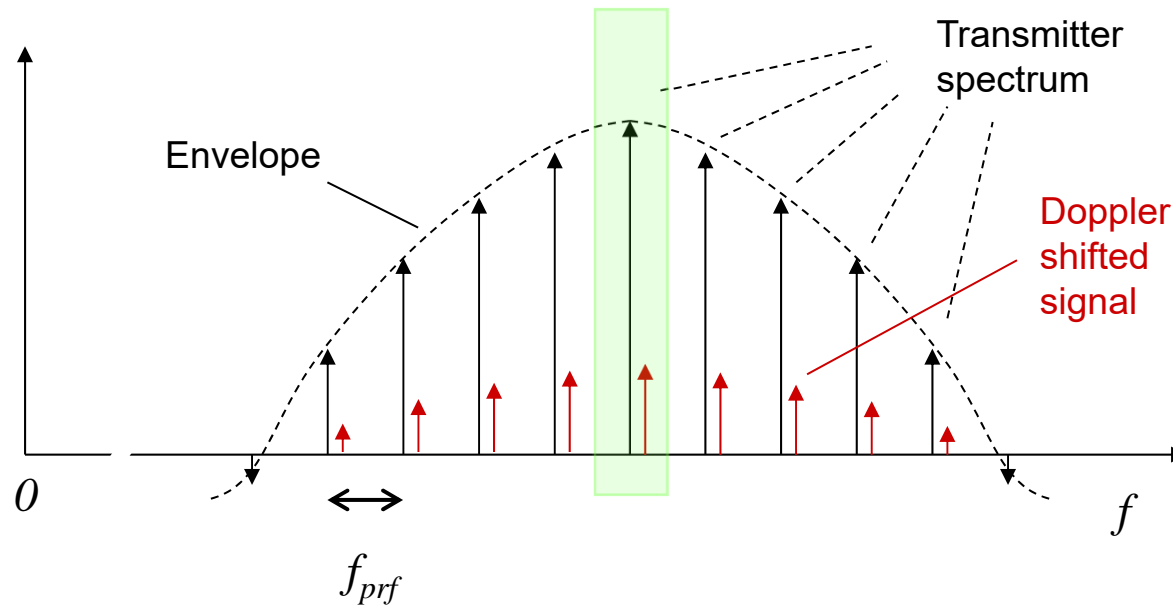
Received  
signal

Demodulated  
signal



Select main peak by  
low-pass filter

# Nyquist Limit



Nyquist limit (sampling theorem):  $-\frac{1}{2} f_{prf} < f_D < \frac{1}{2} f_{prf}$

## Pulsed Doppler mode

- permits spatially selective velocity measurement
- at limited velocity range