Biomedical Imaging - Nuclear Imaging (II) (1)

Rem: $E = m \cdot c^2 \rightarrow E = 2 \cdot 9.1 \cdot 10^{-31}\,kg \cdot (3 \cdot 10^8 \frac{m}{s})^2$

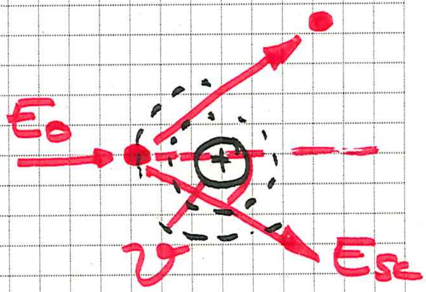
$E = 1.64 \cdot 10^{-13}\,J$

$E_\gamma = E / 1.6 \cdot 10^{-19} / 2 = \underline{512\,keV}$

Scattered coincidence

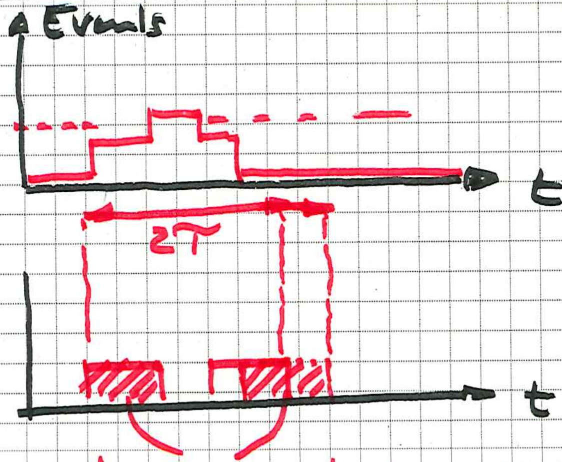
$$E_{sc} = \frac{m_e c^2}{\frac{m_e c^2}{E_0} + 1 - \cos\vartheta}$$

$m_e c^2$ – 511 keV



estimate scatter distribution

Random coincidence



'random' → discard

Dead time

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→ due to finite counting in detector

Technetium generator (SPECT)

$$\underbrace{{}^{99}_{42}Mo}_{N_1} \xrightarrow[k_1]{} \beta^- + \underbrace{{}^{99m}_{43}Tc}_{N_2} \xrightarrow[k_2]{} {}^{99}_{42}Tc + \gamma$$

$$\frac{dN_1}{dt} = -k_1 N_1 \qquad \frac{dN_2}{dt} = +k_1 N_1 - k_2 N_2$$

$$\underline{\underline{N_1 = N_0 e^{-k_1 t}}} \qquad \frac{dN_2}{dt} + k_2 N_2 = 0$$

$$N_2 = C \cdot e^{-k_2 t} \text{ (homogenous)}$$

$$\underline{N_2 = D \cdot e^{-k_1 t}} \text{ (particular)}$$

$$\rightarrow -k_1 \cdot D \cdot \cancel{e^{-k_1 t}} + k_2 \cdot D \cancel{e^{-k_1 t}} = k_1 \cdot N_0 \cdot \cancel{e^{-k_1 t}}$$

$$D \cdot (k_2 - k_1) = k_1 \cdot N_0 \quad \Rightarrow \quad \underline{D = \frac{k_1 \cdot N_0}{k_2 - k_1}}$$

$$\underline{\underline{N_2 = \frac{k_1 N_0}{k_2 - k_1}\left(e^{-k_1 t} - e^{-k_2 t}\right)}} \qquad \underline{C = -\frac{k_1 N_0}{k_2 - k_1}}$$

Cyclotron (PET)

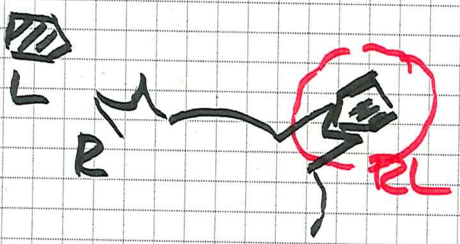
$${}^{18}_{8}O + {}^{1}_{1}P \longrightarrow {}^{18}_{9}F + {}^{1}_{0}n$$

$T_{1/2} \sim 110$ min

$$^{14}_{7}N + ^{1}_{1}p \longrightarrow ^{11}_{6}C + ^{4}_{2}He$$

$T_{1/2} \sim 20$ min

Quantitative PET



$$\frac{[RL]}{[L]} = -\frac{1}{kd}[RL] + \frac{[R_T]}{kd}$$

$[R_T]$ ... total receptor conc.

$\longrightarrow [R_T] \gg [RL]$

$\rightarrow \dfrac{[RL]}{[L]} = \dfrac{[R_T]}{kd}$

- Trace extraction from blood vessel

Flow $F \rightarrow 0 \;\rightarrow\; E = 1 - e^{-\frac{PS}{F}} \rightarrow 1$

$F \rightarrow \infty \;\rightarrow\; E \rightarrow 0$

- Distribution volume

$$V_D = \frac{M}{C} = \frac{1\,\mu g/kg \cdot 70\,kg}{1.4\,\mu g/l} = \underline{\underline{50\,l}}$$

• Compartment modelling

$$[L] + [R] \rightleftharpoons [LR]$$

$$\underbrace{\boxed{C_p}}_{\text{in blood}} \underset{k_2}{\overset{k_1}{\rightleftharpoons}} \underbrace{\boxed{C_f}}_{\text{unbound}} \underset{k_4}{\overset{k_3}{\rightleftharpoons}} \underbrace{\boxed{C_b}}_{\text{bound}}$$

$$\frac{dC_f}{dt} = +k_1 C_p(t) - k_2 C_f(t) - k_3 C_f(t) + k_4 C_b(t)$$

$$\frac{dC_b}{dt} = +k_3 C_f(t) - k_4 C_b(t)$$

PET signal : $C_t(t) = C_p(t) + C_f(t) + C_b(t)$

(assuming equal volume fraction per compartment)

Rem : $\underbrace{\int O(x)\, \delta(x-x')\, dx}_{\text{Truth, } \delta\text{-Dirac}} \xrightarrow{\text{Imaging}} \underbrace{\int O(x)\, LSF(x'-x)\, dx}_{\text{Image, LSF..}}$

Line-spread-funct

$$\rightarrow \underbrace{C_t(t)}_{\text{Output}} = \int \underbrace{C_p(t')}_{\text{Input}}\, \underbrace{h(t-t')}_{\text{Impulse response}}\, dt'$$

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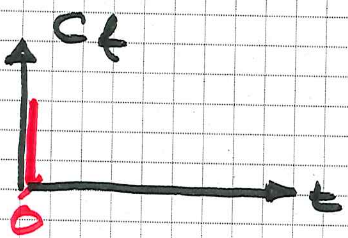
$$\boxed{C_p} \underset{k_2}{\overset{k_1}{\rightleftharpoons}} \boxed{C_f} \underset{k_4}{\overset{k_3}{\rightleftharpoons}} \boxed{C_b}$$

Impulse response: $C_f(t=0) = k_1$

$C_b(t=0) = 0$



$$\frac{dC_f}{dt} = -k_2 C_f(t) - k_3 C_f(t)$$

$$\frac{dC_b}{dt} = +k_3 C_f(t)$$

Solution yields impulse response for free and bound comp.

Example: energy metabolism

$$ATP \rightleftharpoons ADP + P_i + 31 \frac{kJ}{mol}$$

adenosin-triphosphat adenosin-diphosphat inorganic phosphor

→ ATP main energy source to cells