# Biomedical Imaging

## Exercise XCT #3 – Signal and Noise, Undersampling and Motion Artefacts

The objective of the exercise is to extend the CT simulator to take into consideration realistic experimental parameters including quantum noise. To assess image quality, the image signal-to-noise ratio (SNR) will be determined and manipulated by changing experimental parameters and filter settings. The impact of radial undersampling and the presence of object motion on image quality will be studied.

### Task 3.1

- Start Matlab and enter “XCT_EXERCISE3”; the simple thorax phantom as implemented in Task 1.3 is displayed.
- Open XCT_EXERCISE3.m in the editor and read the code lines and comments carefully.
- Consider an X-ray tube with following parameters:
  - *tube voltage:* *50 kV*
  - *anode current* *100 mA*
  - *anode material* *tungsten*
  - *relative X-ray yield* $10^{-15}*Z\ kV^{-1}$ *(Z: atomic number)*
  - *X-ray burst duration* *10 ms*
- Calculate the number of photons $N_0$ incident to the object during an X-ray burst.
```
e = 1.6e-19;
Ne = Ia*1e-3/e;
N0 = yield*Ua*Ne*dt;
```
- Compute the number of photons *N* that are transmitted through the object using Beer Lamberts law.
```
N = N0*exp(-phantom.sino(:,idx));
```
- Edit function *AddNoise()* to add quantum noise according to a Poisson distribution[1].
```
% Create Poisson noise
projection = poissrnd(projection,size(projection));
% Calculate projection
phantom.sino(:,idx) = -log(N/N0);
```
- Run the simulation and inspect projections and image.

### Task 3.2

- A Gaussian filter with variable full-width-at-half-maximum (FWHM) is to be implemented in function *ApplyGaussianFilter()* to reduce noise and make the image data more appealing.
- Calculate the required standard deviation *sigma* of the Gaussian function to obtain a required FWHM in the spatial frequency domain.
```
sigma = matrix/factor/(2*sqrt(2*log(2)));
```
- Implement a two-dimensional Gaussian to filter the data.
```
filter = exp(-(p.^2+q.^2)/(2*sigma^2));
```
- Run the simulation and inspect projections and images for different resolution reduction factors *resolfact*. How do resolution and noise relate to each other?

  SNR increases by the same factor by which resolution decreases.

[1] Use Matlab function poissrnd() to generate Poisson noise.

**Task 3.3**

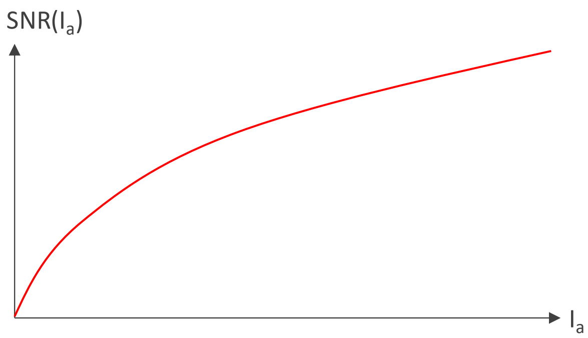
- Let's measure the signal-to-noise ratio (SNR) in the heart.
- Edit function *CalcSNR()* to identify the region of the heart. Compute mean and standard deviation of the image intensities inside the heart region[2].

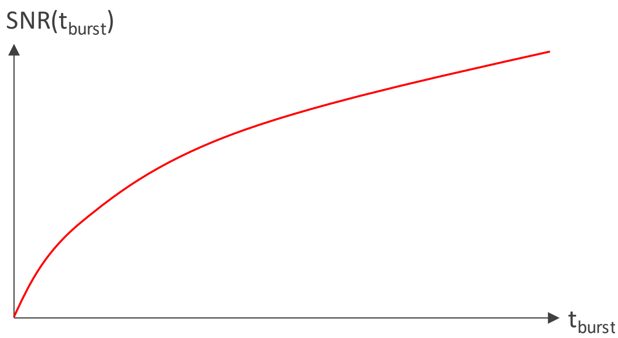
```
[x,y] = meshgrid(-fix(size(phantom.fbp,1)/2):+fix(size(phantom.fbp,1)/2));
Theta = phantom.ellipse(k,5)*pi/180;
X0 = [x(:)'-phantom.ellipse(k,1);y(:)'-phantom.ellipse(k,2)];
D = [1/(phantom.ellipse(k,3)-3) 0;0 1/(phantom.ellipse(k,4)-3)];
Q = [cos(theta) sin(theta); -sin(theta) cos(theta)];
equ = sum((D*Q*X0).^2);
i = find(equ<=1);
snr = mean(phantom.img(i))/std(phantom.img(i));
```

- How does SNR(heart) change if the tube current is doubled and tripled?

  SNR increases by factor sqrt(2) when doubling and sqrt(3) when tripling the tube current.
- How does SNR(heart) change if the burst duration is increased from 10 to 20 and 40 ms?

  SNR increases by factor sqrt (2) when changing burst duration from 10 to 20 ms and sqrt(4) when changing burst duration from 10 to 40 ms.
- Plot the relation between image SNR, tube current and burst duration in the graphs below.




**Task 3.4**

- Let the heart beat. Implement code to simulate sinusoidal contraction of the heart (at a burst duration of 40 ms the heart will perform a full cycle every 25 bursts = 1 heart beat/sec).

  `phantom.ellipse(8,4) = heart_axis+5*sin(2*pi*dt*idx);`
- Run the simulation and inspect sinogram and images. What is seen and why?

  Motion artefacts show up.
- To address the motion problem you may radially undersample i.e. projections are skipped and hence the overall imaging process is faster. Set undersampling factor *R* to 2, 4 and 8 and study the resulting images.
- Discuss the trade-off between imaging speed, motion artefacts and image SNR! How can the motion problem be dealt with in a clinical setting?

  Imaging is faster, motion artefacts decrease and image SNR decreases when undersampling is applied. Due to a violation of the sampling theorem, aliasing occurs in data reconstructed from undersampled acquisitions (pseudo-noise like structure). In a clinical setting, ECG gating can be used to acquire data only during a specific part of the cardiac cycle, typically during diastole.

[2] Use code from function CalcDiscretePhantom(); also, make sure that the mask to measure SNR is somewhat smaller than the actual heart to avoid inaccuracy of SNR calculation.

## Questions?

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