Biomedical Imaging

Exercise XCT #1 – X-ray Projection and Digital Subtraction Angiography

The purpose of the exercise is to understand and study X-ray planar projection imaging and digital subtraction angiography. To this end, a simple test and an analytical computer model of the human thorax is implemented using circles/ellipses assigned with realistic densities and mass attenuation coefficients. Line integrals will be computed to obtain X-ray projection data.

Task 1.1

- Start Matlab and enter "XCT_EXERCISE2"; a test object along with an intensity profile is displayed.
- Open XCT_EXERCISE2.m in the editor and read the code lines and comments carefully.
- Update density **ρ** and mass attenuation coefficients **μ/ρ** for anode voltages (U_a) of 50 and 150 keV (refer to Tables 2+4 at www.nist.gov/pml/data/xraycoef/).

	ρ (g/cm³)	**μ/ρ (cm²/g) (50 keV)**	**μ/ρ (cm²/g) (150 keV)**
blood	1.060	0.228	0.149
bone	1.920	0.424	0.148
lung	0.001	0.208	0.136
muscle	1.050	0.226	0.149

- Fill in code lines to compute linear attenuation coefficients.

```
mue_blood     = rho_blood.*mac_blood(:);
...
```

- How do linear attenuation coefficients **μ** vary as a function of anode voltage and why?

  At low energies, the photoelectric effect dominates over Compton scattering, while at higher X-ray energies the contribution from Compton scattering becomes more important.

- Ellipses are defined using center point $\mathbf{x_0,y_0}$, half axes **a,b** and tilt angle **θ** relative to the x-axis (*see Figure 1*) according to:

$$\left(\vec{x}-(x_0,y_0)^T\right)^T Q^T D^2 Q\left(\vec{x}-(x_0,y_0)^T\right) \leq 1 \text{ with} \qquad (1)$$

$$D=\begin{bmatrix} 1/a & 0 \\ 0 & 1/b \end{bmatrix}, Q=\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

- Function ***CalcDiscretePhantom*** demonstrates how to use the formalism of defining elliptic/circular areas.
- Modify the test object by changing position, size, shape and check the intensity profile display. Compute the sum along the vertical direction (per column) and compare the resulting intensity. What is seen and why?

```
DisplayData(sum(phantom.discrete(:,:),1)',[1,4,3]);
title('...
```

  Discontinuities can be observed in the projection, the projection is based on a discrete object.

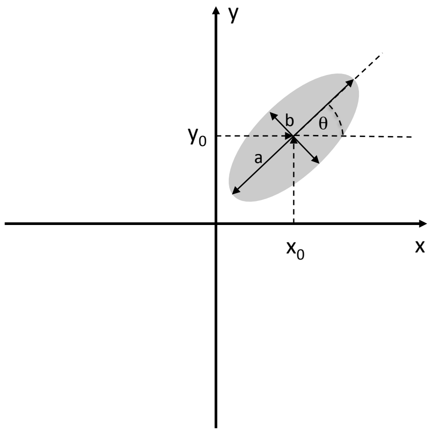

Figure 1: Definition of ellipse using center point (x_0,y_0), half axes (a,b) and tilt angle θ

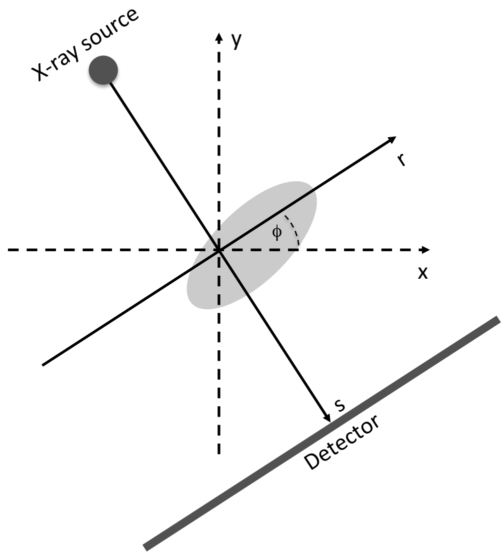

Figure 2: Definition of line integral L(r,s,ϕ). Location of X-ray source and detector are indicated.

Task 1.2

- Edit function ***CalcLineIntegral*** to calculate the projection at arbitrary angles ϕ. Consider that any point along **L(r,s,ϕ)** can be written as follows (*see Figure 2*):

$$\vec{x} = (x,y)^T = r\begin{pmatrix} \cos\varphi \\ \sin\varphi \end{pmatrix} + s\begin{pmatrix} \sin\varphi \\ -\cos\varphi \end{pmatrix} \quad (2)$$

  To compute the projection, points of **L(r,s,ϕ)** that intersect an ellipse have to be found. Since ellipses are convex there can only be one entry and one exit point; these two determine the integration borders.

  Implement the equation to compute the line integral or projection (proj) in function ***CalcLineIntegral.***

  proj = mue*abs(sp-sq);

- Modify the test object by changing position, size, shape and check the intensity profile display. How does the result compare to the projection computed using the sum in Task 2.1.

  Projection is now continuous instead of discrete.

Task 1.3

- Extend the test object to represent a simple model of the human thorax as shown below (*see Figure 3*). Consider that line integrals are additive. Accordingly, to generate the lung spaces on top of the thorax (muscle) the difference of linear attenuation coefficients of muscle and lung needs to be assigned to the lung space.

0	0	90	80	0	mue_muscle(idx)
0	0	70	60	0	mue_lung(idx)-mue_muscle(idx)
110	0	15	15	0	mue_muscle(idx)
110	0	5	5	0	mue_bone(idx)-mue_muscle(idx)
-110	0	15	15	0	mue_muscle(idx)
-110	0	5	5	0	mue_bone(idx)-mue_muscle(idx)
0	0	10	10	0	mue_blood(idx)-mue_lung(idx)
30	25	25	20	35	mue_muscle(idx)-mue_lung(idx)

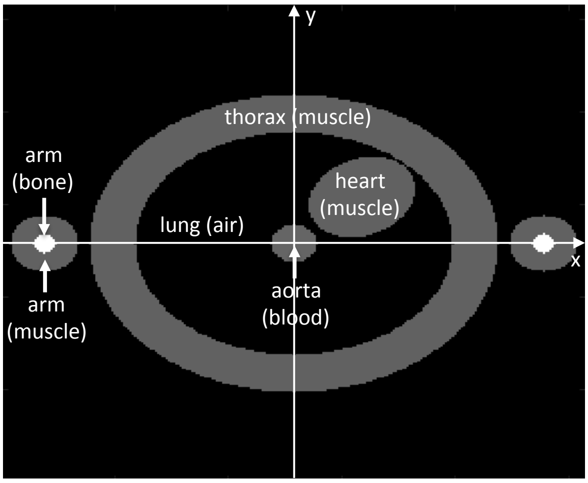

Figure 3: Example of a simple thorax phantom consisting of thorax muscle, left and right arm muscle and bone, lung air space, heart muscle and the aortic vessel filled with blood.

- Compute horizontal projections for anode voltages of 50 versus 150 keV. How does contrast between bone and muscle change and why?

  Since the probability of photoelectric interaction ~$\rho*Z^3/E^3$, there is far greater attenuation in bone than in tissue and excellent tissue contrast at low energies. As the probability for Compton scattering ~ρ/E, contrast drops for higher energies.

- Compute horizontal, vertical and angulated projections. Why is the aorta (blood) not seen well?

  Linear attenuation coefficients of blood, lung and muscle are very similar.

- Derive general equation of blood contrast based on Beer-Lamberts law for the DSA principle.

  $I = I_0 * \exp(-(\mu\_contrast\_agent-\mu\_blood)*d)$

- Implement Digital Subtraction Angiography (DSA) using two projections – one without and one upon intravascular administration of an iodine contrast agent (mass attenuation coefficient of blood with iodine contrast agent is doubled relative to the value of blood only).

  phantom.ellipse(7,6) = 2*mue_blood(idx);
  ...
  phantom_dsa = phantom.discrete-phantom_no_contrast;
  project_dsa = phantom.projection-project_no_contrast;

- Document the DSA result and explain why vessel contrast has improved?

  After subtraction of the images, all signal except signal originating from the contrast agent is cancelled out.

Questions?

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