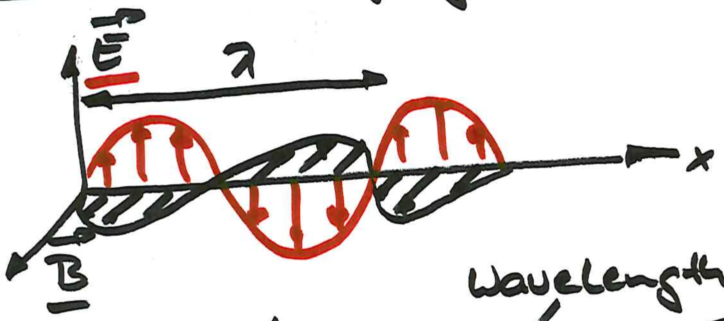
Biomedical Imaging XCT(I)

⑤



$$\frac{\partial^2 \vec{E}}{\partial t^2} - c^2 \vec{\nabla}^2 \vec{E} = 0$$

$$\frac{\partial^2 \vec{B}}{\partial t^2} - c^2 \vec{\nabla}^2 \vec{B} = 0$$

wavelength, Frequency

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = \lambda \cdot \nu$$

$$\boxed{E = h \cdot \nu = h \cdot \frac{c}{\lambda}}$$ Energy

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⑥ $\lambda_{x\text{-rays}} = 10\,pm - 10\,nm$

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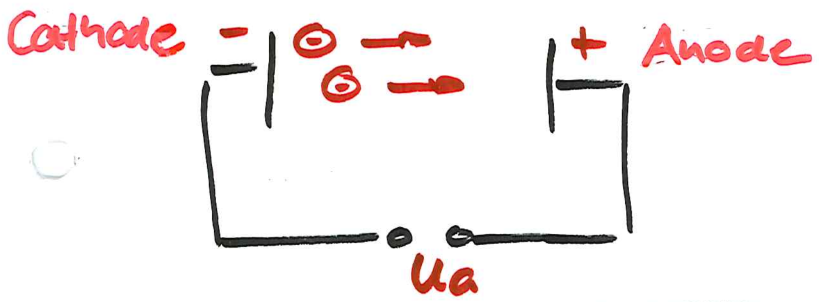
⑫ $E_{rad} = n \cdot E_{photon}$ (= single quantum energy)

$$h \cdot \nu = e \cdot \underline{U_a}$$

anode voltage

$$m \cdot c = p_{Photon} = \frac{h}{\lambda}$$



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⑬ given: $U_a = 100\,kV$ what's λ_{min} ?

$$h \cdot \frac{c}{\lambda} = e \cdot U \rightarrow \lambda = \frac{h \cdot c}{e \cdot U} = \frac{6.6 \cdot 10^{-34}\,Js \cdot 3 \cdot 10^8 \frac{m}{s}}{1.6 \cdot 10^{-19}\,C \cdot 1 \cdot 10^5\,V}$$

$$1\,[C] = 1\,[J/V]$$

$$\underline{\underline{\lambda_{min} = 12.5\,pm}}$$

$$E_{max} = h \cdot \frac{c}{\lambda_{min}} = \underline{\underline{1.6 \cdot 10^{-14}\,J}}$$

XCT(I)

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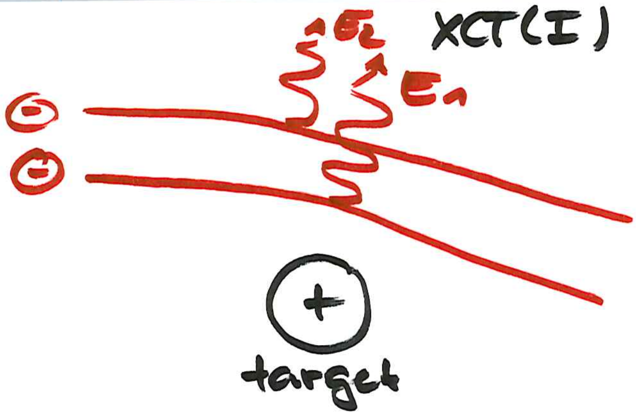
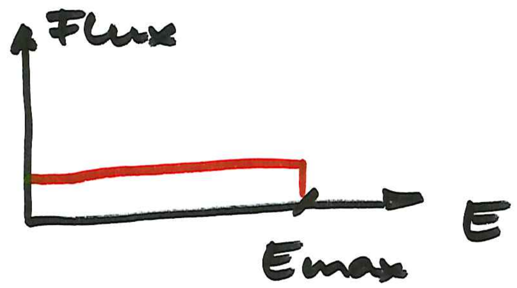
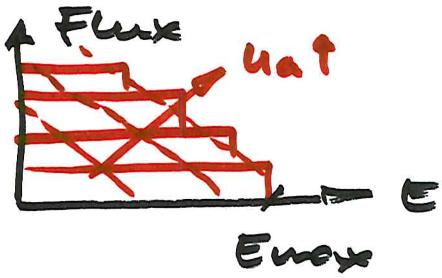


energy depends on distance e⁻ relation to target

$E_1 > E_2$





Very thin target

Thick target

$\boxed{\text{Bremsstrahlung}}$

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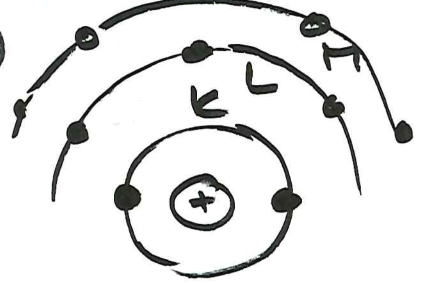

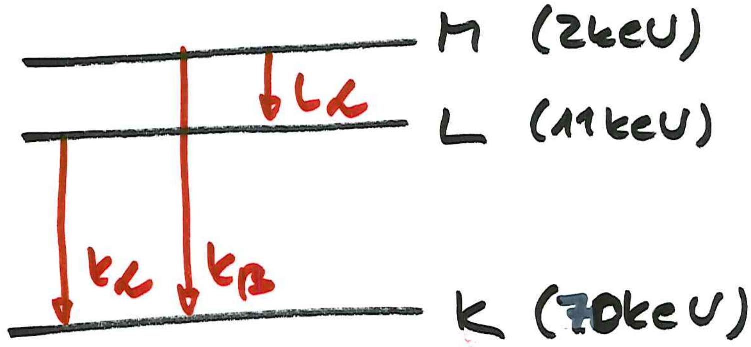

$E_{binding,K} = 13.6\,eV\, Z_{eff}^2 \qquad Z_{eff} = Z - 2$

e.g. tungsten $Z = 74$

$$\underbrace{\begin{array}{l} E_{K\alpha} = 59\,keV \\ E_{K\beta} = 68\,keV \end{array}}_{\text{not a function of Ua}}$$

$\boxed{\text{Characteristic radiation}}$

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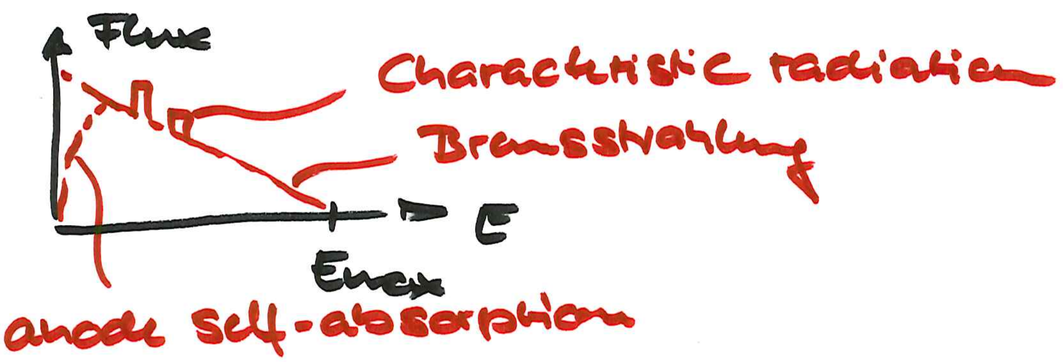

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$U_a = 100\,kV$
$I_a = 100\,mA$
$T = 2000\,K$

Power $P = U_a \cdot I_a = \underline{10\,kW}$

1% X-ray | 99% heat

Diffusion $\sim \Delta T$ | Radiation $\sim T^4$

$\lambda_{heat} = \frac{b}{T} = \underline{1.5\,\mu m}$

⇝ Infrared heat radiation

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$Z_{tissue} \ll Z_{anode}$

7.4 - 20 | 74 (e.g. tungsten)

$E_{photon} = E_k + \underbrace{\frac{1}{2} m_e v^2}_{\text{kinetic energy of } e^-_{photo}}$

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$\vec{p} - \vec{p}\,' = \vec{p}_e$

$E_p + E_e = E_{pe} + E_{p'}$

$$\boxed{\lambda_p - \lambda_{p'} = \frac{h}{m_e c}(1 - \cos\varphi)}$$

e.g. $\varphi = 30°$

⇝ $\lambda_p - \lambda_{p'} \approx 3\,pm$

⇝ Scattered photon mistaken as primary photons

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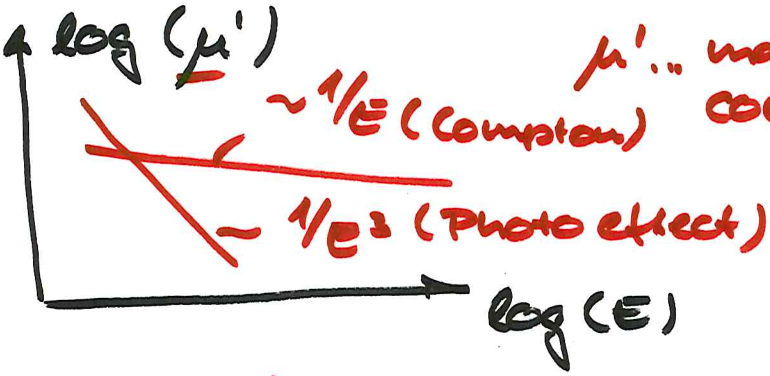


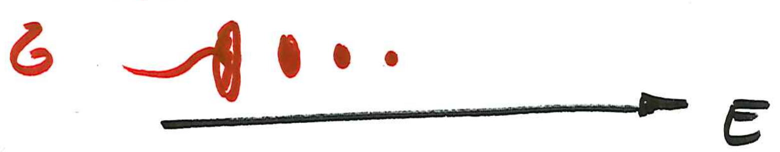
μ' .. mass absorption coefficient

$$\mu' = \sigma \cdot \frac{N_A}{A}$$

N_A - Avogadro number

A - atomic weight

σ - effective area



$\rightarrow \mu' \sim \frac{1}{E^\alpha}$

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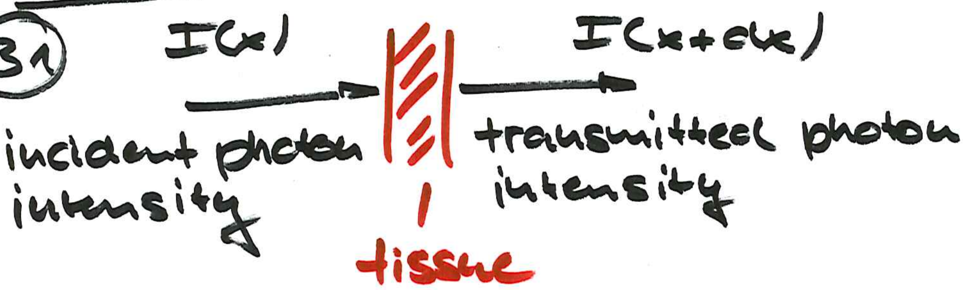

$$I(x+dx) - I(x) = -\mu I dx$$

$$\frac{dI}{I} = -\mu dx$$

$$\log|I| = -\mu x + C$$

$$\boxed{I = I_0 e^{-\mu x}} \rightarrow I_x = I_0 e^{-\int \mu(x) dx}$$

Beer-Lambert's law

XCT (I)

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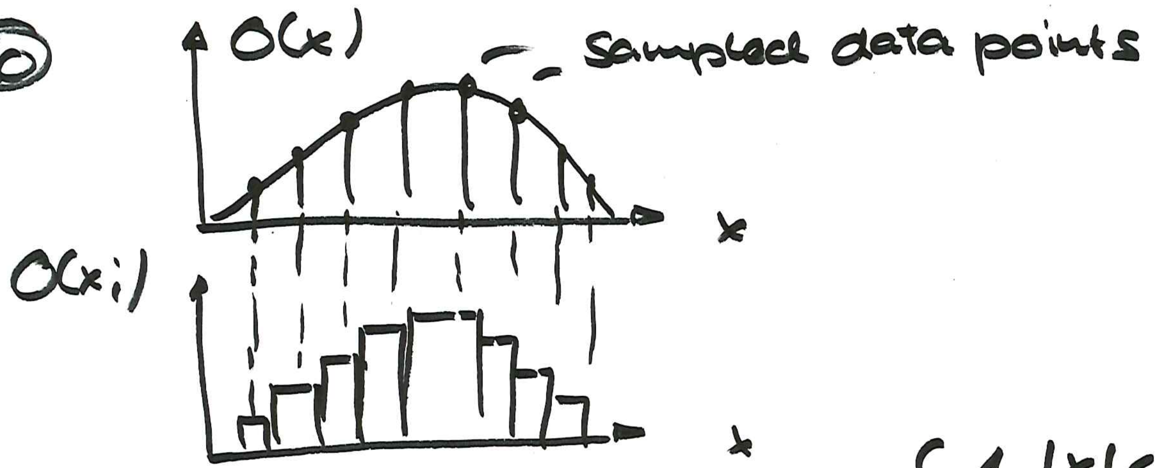


sample function $\quad \sqcap(x) = \begin{cases} 1 & |x| < 1/2 \\ 0 & \text{else} \end{cases}$

$\downarrow$

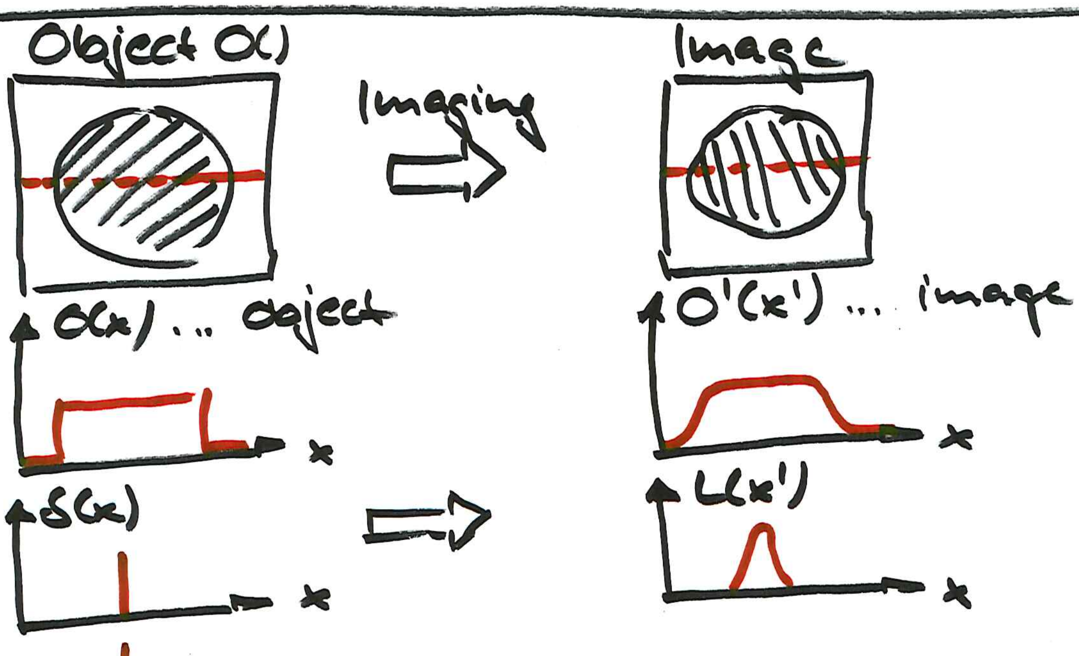
$$\delta_n(x) = n \sqcap(nx)$$

$\downarrow n \to \infty$

$$\int \delta(x)\, dx = 1$$

$$\int \underbrace{O(x)}_{\text{object}} \underbrace{\delta(x - x')}_{\text{sampling}} dx = \underbrace{O(x')}_{\text{object}}$$

$$O(x') \underbrace{*}_{\text{convolution}} \delta(x') = O(x')$$



Line is imaged as blurred line

XCT (I)

$$\int O(x)\,\underbrace{\delta(x-X)}_{\text{ideal}}\,dX \xRightarrow{\text{Imaging}} \int O(X)\,\underbrace{L(x'-X)}_{\text{real}}\,dX$$

→ line-spread function (LSF)

Let's decompose an object into it's harmonic content:

$$O(x) = \int \underbrace{a(k)\sin(kx)}_{\mathcal{J}_k(x)\,=\,a(k)\sin(kx)} + b(k)\cos(kx)\,dx$$

$$O(x) \Rightarrow O'(x') \rightsquigarrow \mathcal{J}_k(x) \Rightarrow \mathcal{J}'_k(x')$$

$$\hookrightarrow a(k)\int \delta(x-X)\sin(kX)\,dX \Rightarrow a(k)\int \underbrace{L(x'-X)}_{x''=x'-X}\sin(kX)\,dX$$

$$a(k)\int L(x'')\sin(kx'-kx'')\,dx''$$

$$\sin(kx'-kx'') = \sin(kx')\cos(kx'') - \cos(kx')\sin(kx'')$$

$$\mathcal{J}'_k(x') = a(k)\int L(x'')\left[\sin(kx')\cos(kx'') - \cos(kx')\sin(kx'')\right]dx''$$

$a_1(k)$ $\qquad$ $a_2(k)$

$$\mathcal{J}'_k(x') = a(k)\left[a_1(k)\sin(kx') - a_2(k)\cos(kx')\right]$$

$$\boxed{\mathcal{J}'_k(x') = a(k)\,\eta(k)\,\sin(kx'-\varphi)}$$

Modulation Transfer Function

k ← spatial frequency [1/m]