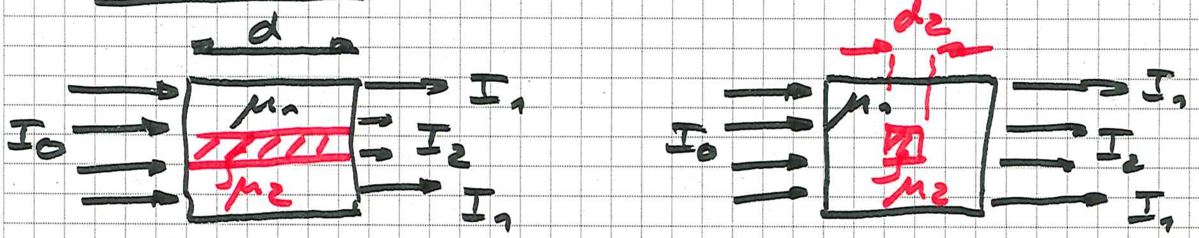
Biomedical Imaging – X-Ray Imaging (II) ①

- **Contrast C**



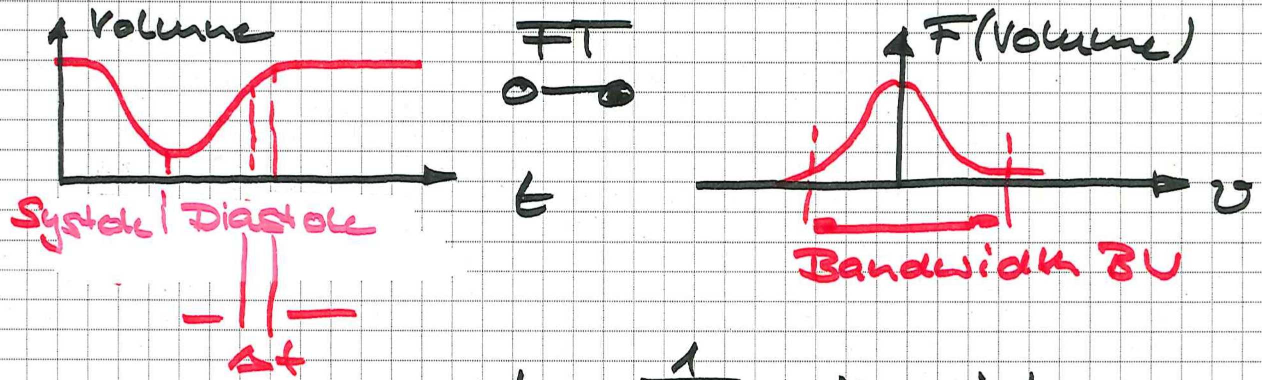
$$I_1 = I_0 e^{-\mu_1 d} \qquad I_1 = I_0 e^{-\mu_1 d}$$

$$I_2 = I_0 e^{-\mu_2 d} \qquad I_2 = I_0 e^{-\mu_1 (d-d_2)} e^{-\mu_2 d_2}$$

$$C \propto |\ln(I_1) - \ln(I_2)|$$

$$C \propto (\mu_2 - \mu_1) d \qquad C \propto (\mu_2 - \mu_1) d_2$$

- **Temporal resolution Δt**



$$\Delta t = \frac{1}{BW} \quad \text{Nyquist}$$

- **Detection – Photographic film**

$$AgBr \rightleftharpoons Ag^+ + Br^-$$

X-Ray photon

$$Br^- \longrightarrow Br + e^-$$

$$Ag^+ + e^- \longrightarrow \underbrace{Ag}_{\text{silver seeds washed out}}$$

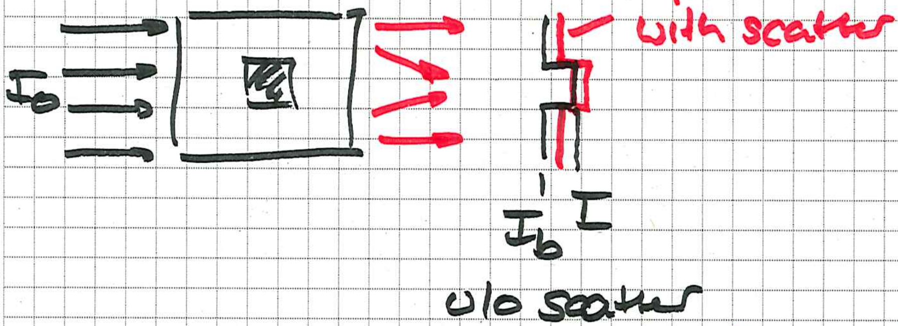
ETH zürich

(2)

→ efficiency of film only ~ 1%

↳ convert X-rays into light using fluorescent material

- **Contrast and Scatter**

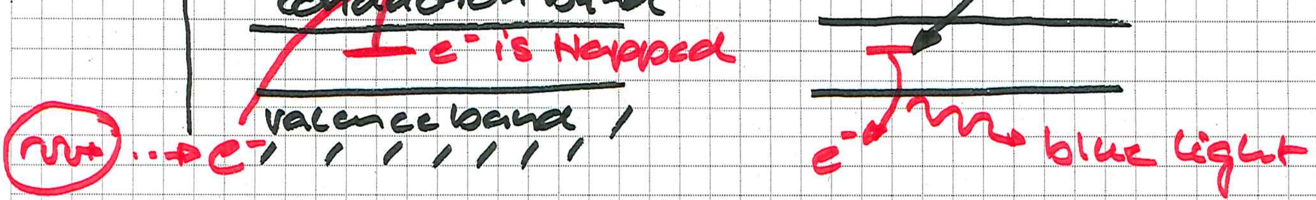


$$\underbrace{C = \frac{I - I_b}{I_b}}_{\text{w/o scatter}} \qquad \underbrace{C^s = \frac{(I+I_s) - (I_b + I_s)}{I_b + I_s}}_{\text{with scatter}}$$

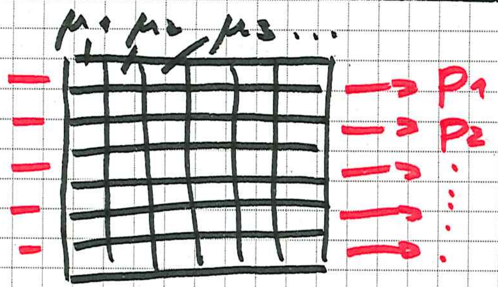
$$\rightarrow \quad C^s = C\,\frac{I_b}{I_b + I_s} = C\,\frac{1}{1 + I_s/I_b}$$

↳ scatter reduces contrast!

- **Detection – Luminescence**



CT reconstruction problem



Discretized object

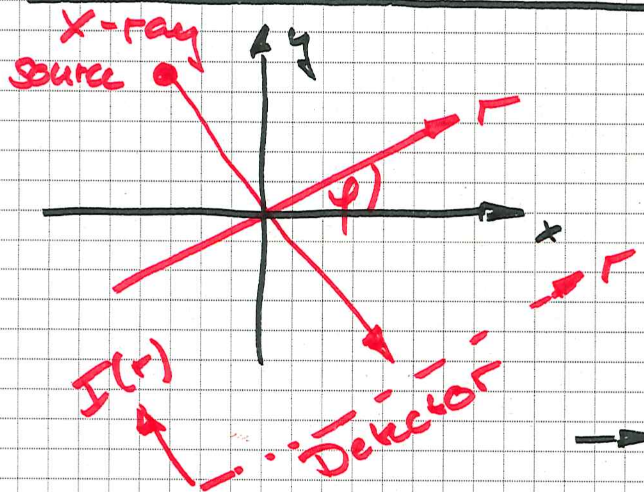
$$\begin{bmatrix} p_1 \\ p_2 \\ \vdots \end{bmatrix} = \underbrace{\begin{bmatrix} \lambda \lambda \lambda \ldots \\ \\ \\ \end{bmatrix}}_{} \cdot \underbrace{\begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \end{bmatrix}}_{}$$

$$\vec{p} = E \cdot \vec{\mu}$$

projections — lin. att. coefficients

$$\rightarrow E^T \vec{p} = E^T E \vec{\mu}$$

$$\rightarrow \vec{\mu} = (E^T E)^{-1} E^T \vec{p}$$



$$I(r) = I_0 e^{-\int \mu(r,s)\,ds}$$

Projection $P_\varphi(r)$:

$$P_\varphi(r) = \ln\left(\frac{I_0}{I(r)}\right)$$

$$\rightarrow \underline{P_\varphi(r) = \int \mu(r,s)\,ds}$$

$$P_\varphi(r) = \int \mu(r,s)\,ds$$

$$r = x\cos\varphi + y\sin\varphi$$

$$\downarrow$$

$$P_\varphi(r) = \iint \mu(x,y)\,\delta(x\cos\varphi + y\sin\varphi - r)\,dx\,dy$$

CT Fourier Transform reconstruction

$$P_\varphi(r) = \int \mu(r,s)\,ds$$

Forward FT

$$\mathcal{F}\{P_\varphi(r)\} = \int P_\varphi(r)\, e^{-jur}\, dr$$
$$= \iint \mu(r,s)\, e^{-jur}\, dr\, ds$$

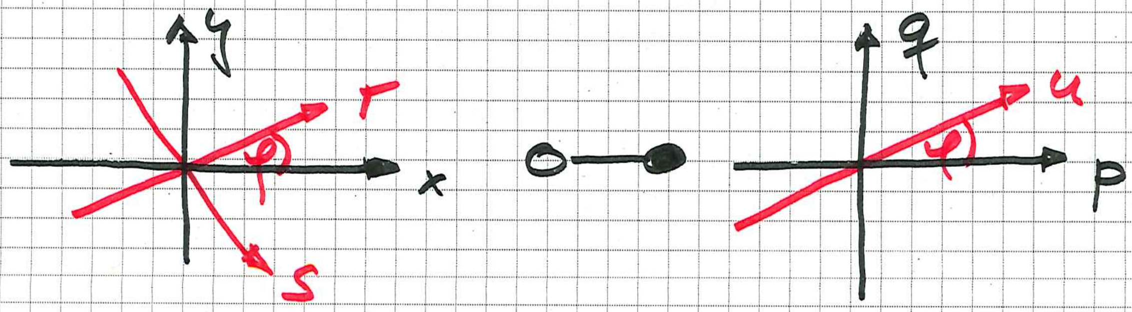
$r = x\cos\varphi + y\sin\varphi$

$$\mathcal{F}\{P_\varphi(r)\} = \iint \mu(x,y)\, e^{\overbrace{-jxu\cos\varphi}^{p}} \cdot e^{\overbrace{-jyu\sin\varphi}^{q}}\, dx\, dy$$

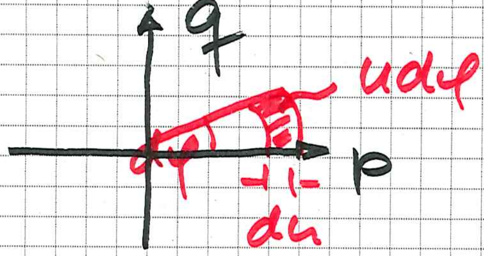
$$\mathcal{F}\{P_\varphi\} = \iint \underbrace{\mu(x,y)}_{\text{Wanted!}}\, e^{-jxp}\, e^{-jyq}\, dx\, dy$$

Inverse FT

$$\boxed{\mu(x,y) = \iint \mathcal{F}\{P_\varphi(p,q)\}\, e^{jxp}\, e^{jyq}\, dp\, dq}$$



→ Fourier-Slice theorem



$$dp\,dq = u\,du\,d\varphi$$

CT Filtered Backprojection (FBP)

$$\mu(x,y) = \int_0^{2\pi}\int \mathcal{F}\{P_\varphi(u)\}\, e^{jur}\, \underline{u\,du\,d\varphi}$$

$$\mu(x,y) = \int_0^{\pi}\int \underbrace{\mathcal{F}\{P_\varphi(u)\}}\, e^{jur}\, \underbrace{|u|}\, du\,d\varphi$$

$$P_\varphi(r) \circledast f(r)$$

→ Projection ↑ Filter

Convolution

$$\boxed{\mu(x,y) \leftarrow \int_0^{\pi} P_\varphi(r) \circledast f(r)\, d\varphi}$$

→ FBP more efficient than FT