In [1]:
```
import numpy as np
import matplotlib.pyplot as plt
import scipy.stats as ss
```

EM parameter estimator

A, B, C three coins.

A head ->B A tail ->C

B head ->1 B tail ->0

C head ->1 C tail ->0

A is a hidden variable or called latent variable

$P(y|\theta) = \sum_z P(y, z|\theta) = \sum_z P(z|\theta)P(y|z, \theta)$

In [2]:
```
# Three coins model

pi_i0 = 0.4
p_i0 = 0.6
q_i0 = 0.7

seq = [1,1,0,1,0,0,1,0,1,1]
seq_vector = np.array(seq)
'''

theta_0 = {pi_0, p_0, q_0}

P (y|theta)
= pi * p^y * (1-p)**(1-y) + (1-pi) * q**y * (1-q)**(1-y)


'''
```

Out[2]:
```
'\n\ntheta_0 = {pi_0, p_0, q_0}\n\nP (y|theta)\n= pi * p^y * (1-p)**(1-y) + (1-pi) * q**y * (1-q)**(1-y)\n\n
\n'
```

In [3]:
```
# calculate the P(A), based on the parameters assumed, E steps

def ComputeHiddenP(pi, p, q, y):
    '''
    pi, P(A) latent variable probability
    p,  P(B)
    q,  P(C)
    y,  coin sequence

    return mu, P(B) based on the assumed parameters
    '''
    mu = []
    for y_i in y:
        mu_i = (pi * p**y_i * (1-p)**(1-y_i))/(pi * p**y_i * \
                (1-p)**(1-y_i) + (1-pi) * q**y_i * (1-q)**(1-y_i))
        mu.append(mu_i)
    return mu
```

In [4]:
```
def UpdateParameters(mu, y):
    '''
    mu, numpy array

    '''
    seq_vector = np.array(y)
    pi = np.sum(mu)/len(mu)
    p_i = (np.dot(mu,seq_vector.transpose()))/(np.sum(mu))
    q_i = (np.dot(1 - np.array(mu),seq_vector.transpose()))/(np.sum(1 - np.array(mu)))
    return pi, p_i, q_i
```

In [6]:
```
# initial
pi_0, p_0, q_0 = 0.4, 0.6, 0.7


# iter 1
mu_1 = np.array(ComputeHiddenP(pi_0,p_0,q_0, seq))
pi_1, p_1, q_1 = UpdateParameters(mu_1, seq_vector)

# iter 2
mu_2 = np.array(ComputeHiddenP(pi_1, p_1, q_1, seq))
pi_2, p_2, q_2 = UpdateParameters(mu_2, seq_vector)

# iter 3

mu_3 = np.array(ComputeHiddenP(pi_2, p_2, q_2, seq))
pi_3, p_3, q_3 = UpdateParameters(mu_3, seq_vector)

print(pi_3, p_3, q_3)
```

```
0.40641711229946526 0.5368421052631578 0.6432432432432432
```

Gaussian Mixture Model

Clustering the following sequence -67, -48, 6, 8, 14, 16, 23, 24, 28, 29, 41, 49, 56, 60, 75

Estimate the parameters for 2 Gaussian distributions

mu_1, sigma_1, mu_2, sigma_2, alpha_k1, alpha_k2 alpha_k1 + alpha_k2 = 1

In [7]:
```
# Gaussian probability density calculating
def GaussianFunction(x, mean, std):
    '''
    x : array
    mean : array
    std : array
    '''
    return (1/(np.sqrt(2 * np.pi)* std)) * np.exp(-1 * (x - mean)* (x - mean)/(2 * std**2))
```

In [8]:
```
# Complete data log likelihood function, basically this is the target function which need to be optimized


def LogLikelihood(x, mean, std, alpha, zeta):
    loglikelihood = 0
    for k in range(len(alpha)):
            loglikelihood += np.sum(zeta[k]) + zeta[k] * \
            (np.log(1/(np.sqrt(2*np.pi))) - np.log(std[k]) - (1/(2*(std[k]**2))*(x - mean[k])**2))
    return np.sum(loglikelihood)
```

In [9]:
```
# M step function

def maximizationstep(x, k, mean, std, zeta):
    mu = []
    sigma = []
    alpha = []

    for i in range(0,k):

        mu_new = np.dot(zeta[i], x)/np.sum(zeta[i])

        sigma_new = np.sqrt((np.dot(zeta[i],((x - mu_new)**2)))/(np.sum(zeta[i])))

        alpha_new = np.sum(zeta[i])/len(zeta[i])

        mu.append(mu_new)
        sigma.append(sigma_new)
        alpha.append(alpha_new)


    return mu, sigma, alpha
```

In [11]:
```
# the observations need to be clustered
y = np.array([-67, -48, 6, 8, 14, 16, 23, 24, 28, 29, 41, 49, 56, 60, 75])
y = (y - np.min(y))/(np.max(y) - np.min(y))
# inital settings: numbers of iterations, inital alpha, mu, sigma
iters = 20
alpha_k1 = 0.5
mu_1, sigma_1 =  0, 1
mu_2, sigma_2 =  1, 1
loglikelihood = 0
mu_current = np.array([mu_1, mu_2])
sigma_current = np.array([sigma_1, sigma_2])
alpha_current = [alpha_k1, 1 - alpha_k1]

# Training Process
for i in range(0, iters):
    # E step
    mu = mu_current
    sigma = sigma_current
    alpha = alpha_current

    print('iteration', str(i) ,'finished, ',  'mu is ', mu_current, '| Sigma is ' ,sigma_current ,'| Alpha is
          'Loglikelihood is ', loglikelihood)
    print('\n')


    zeta = []

    zeta_k1 = alpha[0] * GaussianFunction(y, mu[0], sigma[0])
    zeta_k2 = alpha[1] * GaussianFunction(y, mu[1], sigma[1])

    zeta_sum = zeta_k1 + zeta_k2

    zeta.append(zeta_k1/zeta_sum)
    zeta.append(zeta_k2/zeta_sum)

    # M step
    mu_current, sigma_current, alpha_current = maximizationstep(y, 2, mu, sigma, zeta)
    loglikelihood = LogLikelihood(y, mu_current, sigma_current, alpha_current, zeta)
```

```
iteration 0 finished,  mu is  [0 1] | Sigma is  [1 1] | Alpha is  [0.5, 0.5] Loglikelihood is  0


iteration 1 finished,  mu is  [0.5847415939776828, 0.6498941579895441] | Sigma is  [0.27122608317076197,
0.23911814936734863] | Alpha is  [0.4703626351970613, 0.5296373648029388] Loglikelihood is  224.288934759
06023


iteration 2 finished,  mu is  [0.5694713494116488, 0.6632546131198166] | Sigma is  [0.28783163458167954,
0.21649666085639077] | Alpha is  [0.4692285711624703, 0.5307714288375296] Loglikelihood is  224.664075766
7338


iteration 3 finished,  mu is  [0.5414885289077966, 0.6861214376575723] | Sigma is  [0.3050950906913587,
0.18146596129354212] | Alpha is  [0.46236096573419927, 0.5376390342658008] Loglikelihood is  225.71289767
388615


iteration 4 finished,  mu is  [0.5107693696061694, 0.7038809221667734] | Sigma is  [0.31804028875351986,
0.1487663229030275] | Alpha is  [0.4382549606872101, 0.5617450393127899] Loglikelihood is  227.3017992920
508


iteration 5 finished,  mu is  [0.4885068026421883, 0.7073570516556931] | Sigma is  [0.32616628415882976,
0.13930401179876242] | Alpha is  [0.4025959566496822, 0.5974040433503177] Loglikelihood is  228.144751514
36017


iteration 6 finished,  mu is  [0.46967691980101345, 0.7071547114536995] | Sigma is  [0.3317046990302629,
0.13840822690364857] | Alpha is  [0.3701646564542043, 0.6298353435457957] Loglikelihood is  228.526071346
79351


iteration 7 finished,  mu is  [0.4500728501746608 7, 0.707078909536988] | Sigma is  [0.33493688576206254,
0.13915172088887784] | Alpha is  [0.3417432392988264, 0.6582567607011736] Loglikelihood is  228.796086017
2146


iteration 8 finished,  mu is  [0.42914274839082484, 0.7074442797104842] | Sigma is  [0.33586300376243455,
0.140203698744861] | Alpha is  [0.3169061018140875, 0.6830938981859125] Loglikelihood is  229.03303504049
57
```

```
iteration 9 finished,  mu is  [0.40725207356831006, 0.7080502068637416] | Sigma is  [0.33460910911294023,
0.14125803270888151] | Alpha is  [0.29521918769836486, 0.7047808123016351] Loglikelihood is  229.25458392
131196


iteration 10 finished,  mu is  [0.3848471820660406, 0.7087092315317869] | Sigma is  [0.3313827646484411,
0.14221658007788338] | Alpha is  [0.2762299731885349, 0.7237700268114651] Loglikelihood is  229.466945308
31414


iteration 11 finished,  mu is  [0.3622230870187485, 0.7093118380131496] | Sigma is  [0.32638021431938663,
0.14304727569487036] | Alpha is  [0.25948121759648296, 0.7405187824035168] Loglikelihood is  229.67398009
16516


iteration 12 finished,  mu is  [0.3394608331041483, 0.7098068183974465] | Sigma is  [0.3197143697885499,
0.14374848743844632] | Alpha is  [0.24452267798893157, 0.7554773220110685] Loglikelihood is  229.87933867
047133


iteration 13 finished,  mu is  [0.31642464973714557, 0.710172541154801] | Sigma is  [0.3113664244942726,
0.14433133227999181] | Alpha is  [0.23091860768157554, 0.7690813923184244] Loglikelihood is  230.08742030
724426


iteration 14 finished,  mu is  [0.2927739690943331, 0.710394502474319] | Sigma is  [0.30114378659772667,
0.14480912060416654] | Alpha is  [0.2182499874839903, 0.7817500125160097] Loglikelihood is  230.304058280
57653


iteration 15 finished,  mu is  [0.2679654131642797, 0.7104494736115545] | Sigma is  [0.28862102587014593,
0.14519159759822378] | Alpha is  [0.20611058221687514, 0.7938894177831249] Loglikelihood is  230.53728182
26388


iteration 16 finished,  mu is  [0.2412345873297419, 0.7102922867548611] | Sigma is  [0.2730352595915523,
0.14548213874699154] | Alpha is  [0.1940986377056912, 0.8059013622943088] Loglikelihood is  230.798536331
14797


iteration 17 finished,  mu is  [0.21156256341611715, 0.7098417883279865] | Sigma is  [0.2530842163784040
6, 0.14567753337168687] | Alpha is  [0.18181163794844116, 0.8181883620515589] Loglikelihood is  231.10502
70294528


iteration 18 finished,  mu is  [0.1776692869427136, 0.7089694149975849] | Sigma is  [0.22648141574709163,
0.14577424758635638] | Alpha is  [0.16886987969488876, 0.8311301203051114] Loglikelihood is  231.48529310
397436


iteration 19 finished,  mu is  [0.13830638903306675, 0.7075243862878333] | Sigma is  [0.1887904012059458
4, 0.1457941206361389] | Alpha is  [0.15508216609891157, 0.8449178339010884] Loglikelihood is  231.998122
79254928
```

In [12]:

```
# Using the parameters we found calculates the probability for each observations
predict_0 = GaussianFunction(y, mu_current[0], sigma_current[0]) * alpha[0]
predict_1 = GaussianFunction(y, mu_current[1], sigma_current[1]) * alpha[1]
```

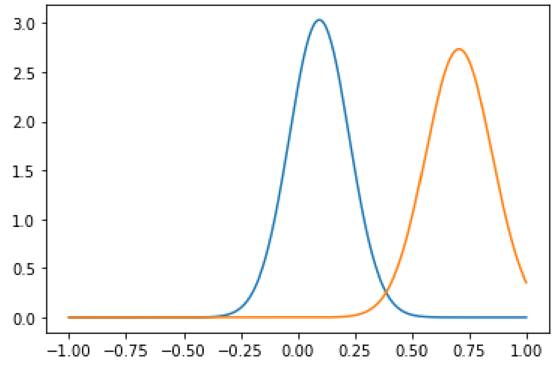
In [13]:

```
# label the observations based on their probabilities
label = []
for i in range(len(predict_0)):
    if predict_0[i] > predict_1[i]:
        label.append(1)
    else:
        label.append(0)
label
```

Out[13]:

```
[1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
```

In [14]:
```
# visualize two Gaussian distributions,
x  = np.linspace(-1, 1, 5000)
y_pdf1 = ss.norm.pdf(x, mu_current[0], sigma_current[0])
y_pdf2 = ss.norm.pdf(x, mu_current[1], sigma_current[1])
plt.plot(x, y_pdf1, label='pdf')
plt.plot(x, y_pdf2, label='pdf')
plt.show()
```



In [15]:
```
# visulize the clustering results
x = np.arange(0,15,1)
group = label
plt.scatter(x,y,c=group,label=label)
plt.plot()
```

Out[15]: []