

# Distributed Game-theoretic Interactive Algorithm for Microgrid Optimization

Qiaoqiao Wang<sup>1</sup>, Jun Zeng<sup>1</sup>, Zhengang Wang<sup>1</sup>, Junfeng Liu<sup>2</sup>

1. School of Electric Power, South China University of Technology, Guangzhou 510640

E-mail: junzeng@scut.edu.cn

2. School of Automation Science and Engineering, South China University of Technology, Guangzhou 510640

E-mail: aujfliu@scut.edu.cn

**Abstract:** Considering the decentralization characteristic of microgrid, this paper presents a distributed game-theoretic interactive optimization algorithm. It aims at achieving distributed operation optimization for microgrid energy management system and supporting plug-and-play. On the basis of analysis of microgrid structure, game models are built for modular component units according to the three elements of a strategic game. In the meantime, considering satisfaction of load and cost of electricity, demand response is introduced and an iterative game-theoretic resource-storage-load optimization model is built. And a best-response mechanism is adopted to solve the Nash equilibrium. Finally, the feasibility and availability of the proposed algorithm are validated by simulation of a typical islanded microgrid.

**Key Words:** Microgrid, Distributed Operation Optimization, Potential Game, Demand Response

## 1 Introduction

Smart grid is flexible, clean, safe, economic, and friendly, and it is the direction for future power grid [1]. High penetration of distributed power sources (including energy storage), full use of distributed energy resource, and demand-side management are motivation of smart grid [2]. Microgrid is the best bridge between distributed energy resource and power grid, usually defined as a low-voltage distribution network/system containing distributed power sources, energy storage devices and controllable load, and it can operate in islanded mode or grid-connected mode [3]. Therefore, microgrid is regarded as a “building block” of smart grid. This paper focuses on the operation optimization of microgrid that is the core of microgrid energy management system (MEMS).

With the development of information technology and cyber physics system (CPS), more self-seeking distributed power sources and load are connected to microgrid, which further enhances the self-interest and intelligence of microgrid. Distribution and plug-and-play (PNP) are inevitable trend of microgrid. It requires flexible grid structure and good scalability for microgrid:

1) In the bottom layer, power electronics provides controllable interfaces to support PNP;

2) CPS links loose modular component units into an organic whole;

3) Distributed energy management and optimization provide coordinated control for system operation and PNP from the viewpoint of software.

This paper is conducted to present a distributed game-theoretic optimization algorithm from the point of view of energy management and optimization. Much work so

far has focused on centralized optimization of microgrid, for example, evolutionary algorithms [4] -[5], stochastic programming [6], and model predictive control method [7]. Obviously, it is vulnerable to failure of single point, and the high coupling of central processing facility reduces the popularity and scalability. It is hard to realize PNP. Researches about decentralized optimization are multi-agent system (MAS) [8] -[9]. Basically, MAS that is based on distributed information and computing algorithm is just an excellent technical solution, and it cannot fully express self-interests of distributed power sources and load. Thus there is a need to design a distributed optimization algorithm with self-interests considered.

Game theory is a theory of conflict and cooperation between intelligent decision makers, and it is successfully applied to microgrid [10] -[11]. Potential game is a special form of non-cooperative game, proposed by Monderer and Shapely in 1996. Potential game possesses distributed characteristics. On one hand, every player makes decision independently to seek for self-interest; on the other hand, overall utility of microgrid can be guaranteed at the same time as long as the potential function is set properly. In addition, potential game has finite improvement property (FIP), which means that the game must have a Nash equilibrium point. As a result, potential game is very suitable for distributed optimization of microgrid considering self-interest and intelligence.

This paper presents a distributed game-theoretic interactive algorithm for microgrid optimization. Section 2 introduces basic concepts of potential game. Section 3 is modeling for microgrid. Section 4 formulates a potential game model for microgrid and implements this algorithm.

\* This work is supported by National Natural Science Foundation (NNSF) of China under Grant 61573155 & Guangdong Science and Technology Planning Project under Grant 2016A030313508.

Section 5 is simulation and analysis of the proposed algorithm. Section 6 concludes this paper.

## 2 Game Theory and Potential Game

Game theory is a branch of applied mathematics, which can be divided into cooperative one and non-cooperative one. Non-cooperative game can be used to analyze the strategic decision-making processes of independent entities that have partially or totally conflicting interests, and it is widely used.

### 2.1 Basic Concepts

A strategic game consists of 3 elements, including players, strategies and payoff functions. For a strategic game  $\Gamma = \langle N, \{Y^i\}_{i \in N}, \{U_i\}_{i \in N} \rangle$ ,  $N = \{1, 2, \dots, n\}$  is the set of players,  $Y^i$  denotes the strategy set of player  $i$ , player  $i$  has the utility function  $U_i: Y \rightarrow R$  ( $Y = Y^1 \times Y^2 \times \dots \times Y^n$  is strategy profiles, and  $R$  is set of real numbers). If  $S$  is a subset of  $N$ ,  $-S$  denotes the complementary set of  $S$ , and  $Y^S$  denotes the Cartesian product  $\times_{i \in S} Y^i$ . For singleton set  $\{i\}$ ,  $Y^{-\{i\}}$  is denoted by  $Y^{-i}$ . A strategy profile  $y = (y^1, y^2, \dots, y^n) \in Y$  can be written as  $y = (y^i, y^{-i})$ ,  $y^i \in Y^i$ ,  $y^{-i} \in Y^{-i}$ .

Nash equilibrium is a significant concept of non-cooperative game. It demonstrates a state where no player can change its strategy to improve its utility with others' strategies fixed. And it is the steady state of the game.

### 2.2 Potential Game

Potential game is a special form of non-cooperative game. Definition and properties are given below.

**Definition<sup>[12]</sup>**: For a game  $\Gamma = \langle N, \{Y^i\}_{i \in N}, \{U_i\}_{i \in N} \rangle$ , there always exists function  $G: Y \rightarrow R$ , for  $\forall i \in N, \forall y^{-i} \in Y^{-i}, \forall x, z \in Y^i$ ,

$$U_i(x, y^{-i}) - U_i(z, y^{-i}) = G(x, y^{-i}) - G(z, y^{-i}) \quad (1)$$

Where,  $\Gamma$  is called an exact potential game, and function  $G$  is an exact potential for  $\Gamma$ .

**Property 1:** Let  $G$  be an ordinal potential for  $\Gamma$ , then the equilibrium set of  $\Gamma = \langle N, \{Y^i\}_{i \in N}, \{U_i\}_{i \in N} \rangle$ , coincides with the equilibrium set of  $\Gamma = \langle N, \{Y^i\}_{i \in N}, \{G\}_{i \in N} \rangle$ .

**Property 2:** Every finite ordinal potential game possesses a pure-strategy Nash equilibrium.

**Property 3:** Every finite ordinal potential game has the FIP.

## 3 Modeling for Microgrid

### 3.1 Structure of Microgrid

A typical microgrid consisting of photovoltaic array (PV), wind turbine (WT), diesel engine (DE), energy storage device (e.g. battery, BA), and load (LD) is depicted in Fig. 1. MG Manager receives dispatch of distribution management system (DMS) and realizes parallel and off-grid transformation via the point of common coupling (PCC). PV, WT, DE, BA, and LD respectively combine with corresponding sensors, controllers, smart switches and power converters (AC/DC or AC/AC) to form a controllable component unit. Local Manager (LM) is the core of this component unit. All LMs and MG Manager are connected through communication network to finish local decision making and collaborative optimization. It is noted that modular component units are distributed at different area and belong to different owners. Therefore, potential game with distributed properties is suitable for the distributed optimization problem of microgrid.

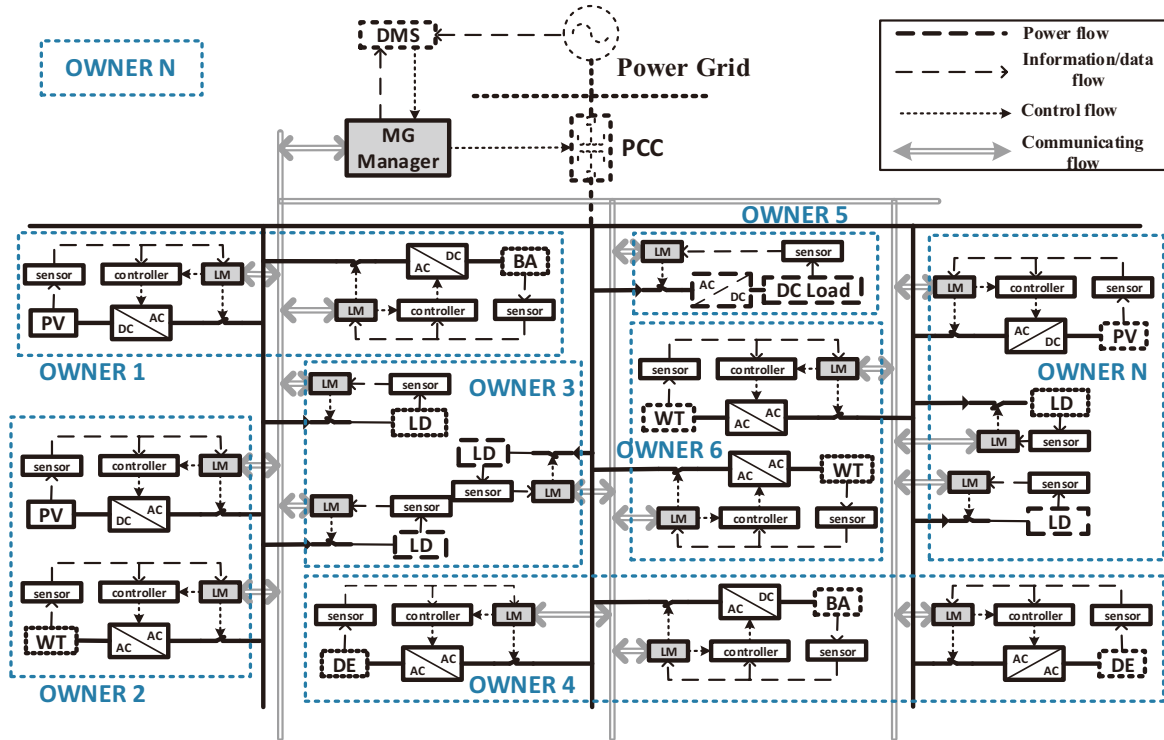


Fig. 1: The structure of microgrid

### 3.2 Component models

#### (1) PV

$$F_{pv} = \rho P_{pv} \Delta t - \gamma_{pv} P_{pv} \Delta t \quad (2)$$

$F_{pv}$  is the utility of PV,  $\rho$  denotes unit electric price, and  $\gamma_{pv}$  denotes unit maintenance cost of PV.

Maximum output power constraint is formulated as (3), and  $P_{pv\max}$  is the maximum output power of PV calculated according to a power prediction model.

$$0 \leq P_{pv} \leq P_{pv\max} \quad (3)$$

#### (2) WT

$$F_{wt} = \rho P_{wt} \Delta t - \gamma_{wt} P_{wt} \Delta t \quad (4)$$

$F_{wt}$  is the utility of WT, and  $\gamma_{wt}$  denotes unit maintenance cost of WT.

Maximum output power constraint is formulated as (5), and  $P_{wt\max}$  is the maximum output power of WT calculated according to a power prediction model.

$$0 \leq P_{wt} \leq P_{wt\max} \quad (5)$$

#### (3) DE

$$F_{de} = (\rho P_{de} - \rho_{oil} D_{oil} - \gamma_{de} P_{de}) \Delta t \quad (6)$$

$F_{de}$  is the utility of DE,  $\rho_{oil}$  denotes unit oil price,  $\gamma_{de}$  denotes unit maintenance cost of DE, and  $D_{oil}$  is the rate of oil consumption, a quadratic function about  $P_{de}$  ( $a$ ,  $b$  and  $c$  are parameters about oil consumption property of DE), as shown below.

$$D_{oil} = aP_{de}^2 + bP_{de} + c \quad (7)$$

Equation (8) indicates the mechanical output constraints, where  $P_{de\min}$  and  $P_{de\max}$  are mechanical lower boundary and upper boundary respectively. Equation (9) indicates the limits to ramp rate, where  $R_{up}$  and  $R_{down}$  are rising rate and declining rate respectively. Equation (10) indicates unit commitment constraint, where binary variables  $s_{on}^t$  and  $s_{off}^t$  ( $s_{on}^t + s_{off}^t = 1$ ) denote online and offline state respectively, and  $d_r$  and  $d_s$  are minimum running time and outage time respectively.

$$P_{de\min} \leq P_{de} \leq P_{de\max} \quad (8)$$

$$\begin{cases} P_{de,t} - P_{de,t-1} \leq \Delta t R_{up} \\ P_{de,t-1} - P_{de,t} \leq \Delta t R_{down} \end{cases} \quad (9)$$

$$\begin{cases} s_{on}^t + \sum_{i=1}^{d_r} s_{off}^{t+i} \leq 1 \\ s_{off}^t + \sum_{i=1}^{d_s} s_{on}^{t+i} \leq 1 \end{cases} \quad (10)$$

#### (4) BA

$$F_{ba} = \rho P_{ba} \Delta t - \gamma_{ba} |P_{ba}| \Delta t \quad (11)$$

$F_{ba}$  is the utility of BA,  $\gamma_{ba}$  denotes unit maintenance cost of BA,  $P_{ba} \geq 0$  for charging,  $P_{ba} < 0$  for discharging.

The state of charge (SOC) is used to model the battery as (12), where  $\eta$  is efficiency of battery, to be specific,  $\eta_c$  and  $\eta_d$  denote charging efficiency and discharging efficiency respectively.  $E_r$  is rated capacity, and  $\sigma$  is self-discharge rate.

$$\begin{cases} SOC(t) = SOC(t-1)(1-\sigma) - \frac{P_{ba} \Delta t}{E_r} \eta \\ \eta = \eta_c, \text{ if } P_{ba} \leq 0; \eta = 1/\eta_d, \text{ if } P_{ba} > 0 \end{cases} \quad (12)$$

The constraints of BA are formulated as (13)-(15).

$$SOC_{\min} \leq SOC \leq SOC_{\max} \quad (13)$$

$$-20\% E_r / \eta_c \leq P_{ba} \Delta t \leq 20\% E_r \eta_d \quad (14)$$

$$\begin{cases} P_{c\max} = \max(P_c, -(SOC_{\max} - SOC) E_r / \eta_c) \\ P_{d\max} = \min(P_d, (SOC - SOC_{\min}) E_r \eta_d) \\ P_c = -20\% E_r / \eta_c, P_d = 20\% E_r \eta_d \end{cases} \quad (15)$$

#### (5) LD

$$F_{ld} = -[(1-\omega) \rho P_{ld} \Delta t + \omega S(P_{ld}, L_{ld})] \quad (16)$$

$F_{ld}$  is the utility of load,  $S(P_{ld}, L_{ld})$  is satisfaction of load [10] as (17),  $\omega$  denotes the weight for satisfaction.

$$S(P_{ld}, L_{ld}) = -L_{ld} \left[ \left( \lambda \frac{P_{ld}}{L_{ld}} + \mu \right)^{-1} - (\lambda + \mu)^{-1} \right] \quad (17)$$

Where  $P_{ld}$  is actual demand of load, and  $L_{ld}$  is nominal demand of load ( $P_{ld} < 0$  &  $L_{ld} < 0$ );  $P_{ld} = L_{ld}$ ,  $S(P_{ld}, L_{ld}) = 0$ , reference of satisfaction;  $|P_{ld}| > |L_{ld}|$ ,  $S(P_{ld}, L_{ld}) < 0$ , more satisfied;  $|P_{ld}| < |L_{ld}|$ ,  $S(P_{ld}, L_{ld}) > 0$ , less satisfied;  $\partial S / \partial P_{ld} < 0$ ,  $\partial^2 S / \partial P_{ld}^2 > 0$ , improvement of satisfaction slows down with  $|P_{ld}|$  increasing constantly, but satisfaction falls down rapidly as  $|P_{ld}|$  decreases.

Lower and upper boundary of load is given in (18).

$$P_{ld\min} \leq P_{ld} \leq P_{ld\max} \quad (18)$$

## 4 Modeling of Potential Game and Implementation of the Proposed Algorithm

### 4.1 Modeling of Potential Game

#### (1) Identifying player $i$

$$i \in N, N = \{pv, wt, de, ba, ld\} \quad (19)$$

All power sources, energy storage devices, and load are identified as game players.  $N$  is the set of players and  $|N|$  denotes the number of players.

#### (2) Determining strategy set $Y^i$

Player's strategy is  $P_i \in Y^i$ ,  $i \in N$ , which means the output power of power sources, charging and discharging of energy storage devices, or demand of load. The positive values of  $P_i$  mean production and the negative values mean consumption. Strategy sets are formulated as (20).

$$\begin{cases} Y^{pv} = \{P_{pv} : 0 \leq P_{pv} \leq P_{pv\text{-max}}\} \\ Y^{wt} = \{P_{wt} : 0 \leq P_{wt} \leq P_{wt\text{-max}}\} \\ Y^{de} = \{P_{de} : eq.(8)(9)(10)\} \\ Y^{ba} = \{P_{ba} : P_{c\text{-max}} \leq P_{ba} \leq P_{d\text{-max}}\} \\ Y^{ld} = \{P_{ld} : P_{ld\text{-min}} \leq P_{ld} \leq P_{ld\text{-max}}\} \end{cases} \quad (20)$$

(3) Constructing utility function

$$\sum_j^{|N|} P_j = 0 \quad (21)$$

According to the global power balance constraint (21), a penalty function is defined as (22),

$$F_{pf}(y^i, y^{-i}) = (\sum_j^{|N|} P_j)^2 \quad (22)$$

Then utility function is constructed as (23), and  $\alpha$  is penalty factor.

$$U_i(y^i, y^{-i}) = F_i - \alpha F_{pf}(y^i, y^{-i}), i \in N \quad (23)$$

$$G(y) = G(y^i, y^{-i}) = \sum_j^{|N|} F_j - \alpha F_{pf}(y^i, y^{-i}) \quad (24)$$

There exists a potential function (24), for any player  $i$ , any strategies  $y^i, y^{i*} \in Y^i$ , and  $y^{-i} \in Y^{-i}$ , it satisfies (25)-(27).

$$\begin{aligned} \Delta U_i &= U_i(y^i, y^{-i}) - U_i(y^{i*}, y^{-i}) \\ &= (F_i - F_{i*}) - \alpha (F_{pf}(y^i, y^{-i}) - F_{pf}(y^{i*}, y^{-i})) \end{aligned} \quad (25)$$

$$\begin{aligned} \Delta G &= G(y^i, y^{-i}) - G(y^{i*}, y^{-i}) \\ &= (F_i + \sum_{j, j \neq i}^{|N|} F_j) - (F_{i*} + \sum_{j, j \neq i}^{|N|} F_j) \\ &\quad - \alpha (F_{pf}(y^i, y^{-i}) - F_{pf}(y^{i*}, y^{-i})) \\ &= (F_i - F_{i*}) - \alpha (F_{pf}(y^i, y^{-i}) - F_{pf}(y^{i*}, y^{-i})) \end{aligned} \quad (26)$$

$$\Delta G = \Delta U_i, i \in N \quad (27)$$

Then an exact potential game is built according to the definition of potential game.

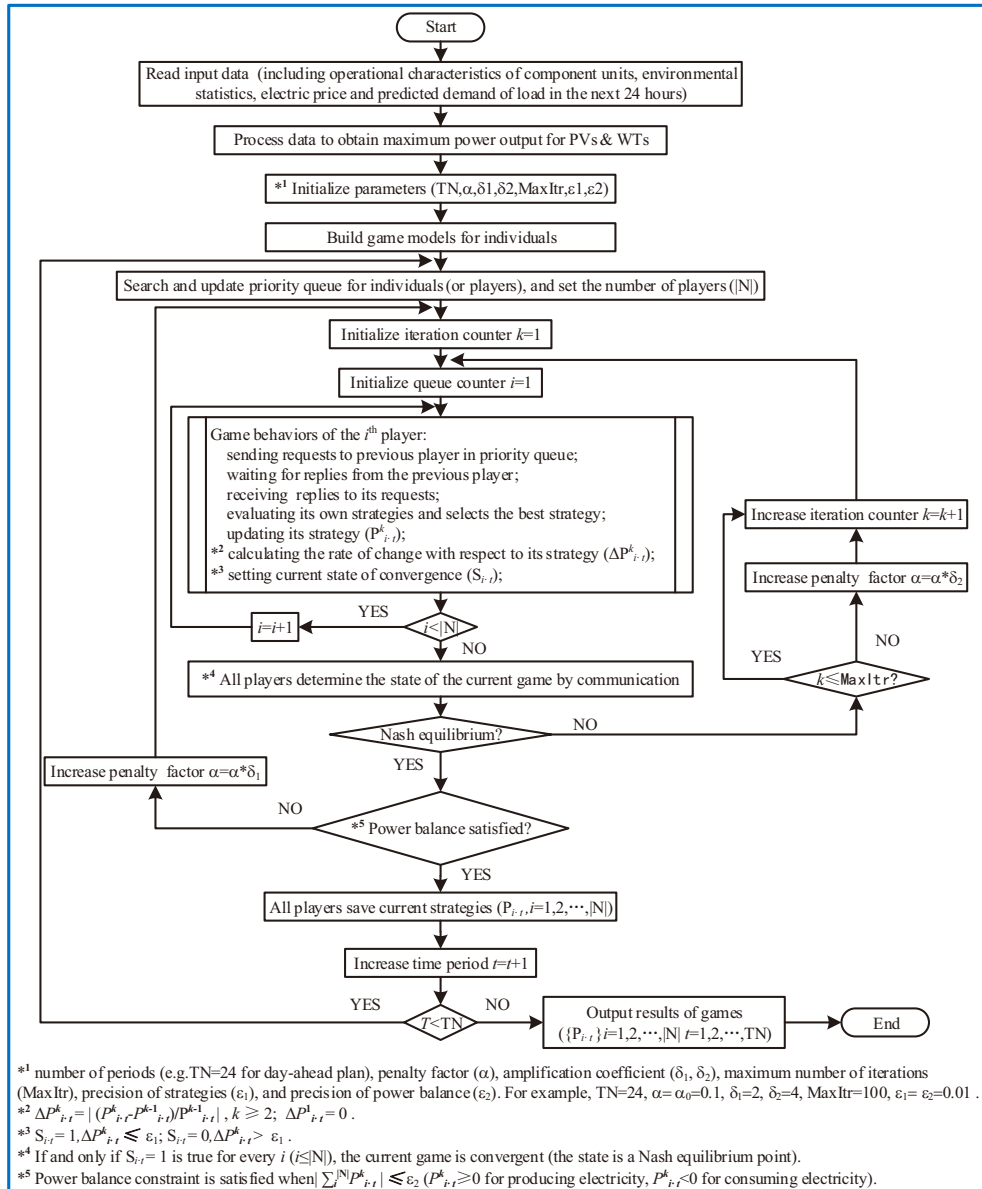


Fig. 2: Flowchart of the algorithm

## 4.2 Implementation and Analysis of the Proposed Algorithm

A day is divided into TN time periods, taking the day-ahead optimization of microgrid as example, TN=24. The optimization based on potential game is a distributed iterative process. There should be a priority to keep players updating strategies in order. And the priority queue is set from highest to lowest priority as follows: PV, WT, DE, BA & LD. A best-response mechanism is adopted here, that is, every player always chooses its best strategy in response to others' strategies. Implementation of the proposed algorithm is depicted in Fig. 2.

The distributed properties of the proposed algorithm are analyzed in detail as follows:

Distributed players are the basis of the proposed algorithm, and there is a one-to-one correspondence between each player and each component unit. Player contains all information of this component unit and is capable of local decision making.

From the viewpoint of energy, all component units are connected to power bus through power electronic interface to convert and transmit energy; from the viewpoint of information, all LMs are connected via communication network to transmit information.

The utility functions of players represent the utility of corresponding component units, and the potential function agrees with the interests of whole microgrid. Potential game has no limit to the number of players but the form of utility functions. Consequently, in the framework of potential game, join or exit of players does not affect the proposed game model.

In conclusion, players agree with component units, and energy agrees with information. Potential game is the bridge between them. The proposed algorithm has the following features: 1) local decision making, 2) dynamics, 3) openness, and 4) low coupling and easy to be extended. Therefore, it can support the distributed application of PNP from the viewpoint of algorithm.

## 5 Simulation and Analysis

A simulation is made regarding a typical islanded microgrid consisting of PVs, WTs, DEs, battery and load, by means of potential game on the platform of MATLAB. The prediction of RE (renewable energy that is sum of maximal output power of PVs and WTs) and nominal load are given in Fig. 3.

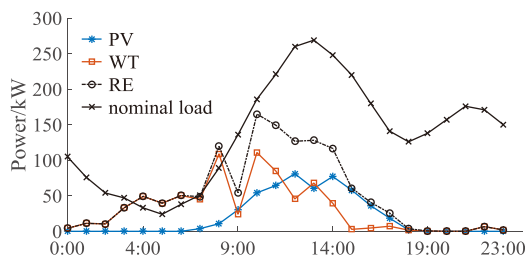


Fig. 3: Prediction of renewable energy and nominal load

Four typical conditions are considered here: 1) Sufficient RE relative to demand (3:00~6:00); 2) Insufficient RE relative to demand (9:00~18:00); 3) Little RE relative to demand (18:00~23:00), and 4) Fluctuation of RE: RE is

always fluctuant (7:00~10:00). Results of the game are shown in Fig. 4 – Fig. 8.

Fig. 4 shows that game output power of PVs and WTs equals maximal output. PVs and WTs operate at maximum power point.

Fig. 5 shows that DEs mainly start up when RE is insufficient or there is little RE (18:00~23:00); competition among DEs is obvious: DE. 1 starts up first and has the maximal output, DE.2 is superior to DE. 3 when required power from DEs is low.

Fig. 6 shows that battery discharges at first and maintains a low SOC.

Fig. 7 shows that actual demand of load changes a lot in response to variation of RE. And Fig. 8 shows the satisfaction of load. Average value of satisfaction is 0.0088. It is close to the reference of satisfaction.

From Fig. 4 to Fig. 8, the results show that the game is reasonable and valid. The characteristics can be summarized as follows:

- 1) RE is made full use of, PVs and WTs are running at the maximum power point;
- 2) The competition brought by potential game makes a more economic outcome for DEs;
- 3) The dependence on energy storage devices is alleviated, hence the capacity and quantity of battery can be cut down. And their life would be extended effectively;
- 4) The load is active and aggressive exchanging little loss of satisfaction for great benefits and improves the performances of power sources and energy storage devices simultaneously.

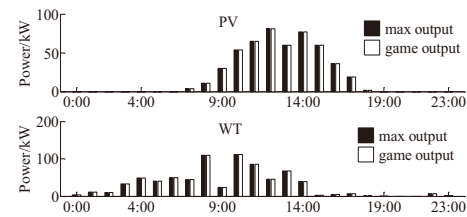


Fig. 4: Game results of PVs & WTs

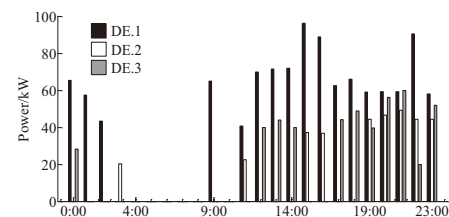


Fig. 5: Game results of DEs

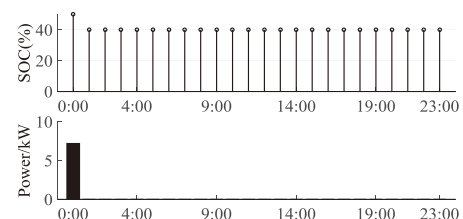


Fig. 6: Game results of BA



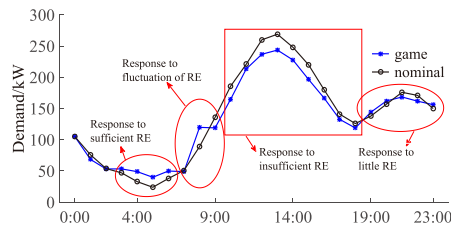


Fig. 7: Game results of load

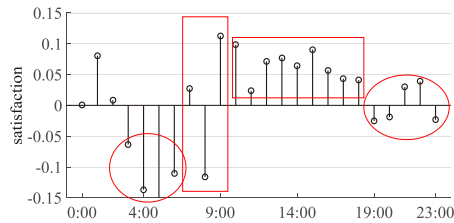


Fig. 8: satisfaction of load

## 6 Conclusion

This paper presents a resource-storage-load interactive optimization algorithm according to potential game, fully considering self-interests and intelligence of distributed power sources, energy storage devices, and load. It adopts an iterative method based on best response to find the Nash equilibrium of the game. Simulation regarding a typical islanded microgrid shows that individuals of microgrid are autonomous and independent, and participation of load improves the performance of distributed power sources and energy storage devices. Modular component units, as players, can join or exit the game without affecting convergence of the game, hence, it is a strong support for PNP. Results indicate feasibility and validity of the proposed algorithm.

## References

- [1] W. Zhang, Z. Liu, M. Wang, Research Status and Development Trend of Smart Grid. *Power System Technology*, 2009, 33(13):1-11.
- [2] Y. Yu, W. Luan, Smart Grid and Its Implementations. *Proceedings of the CSEE*, 2009, 29(34):1-8.
- [3] R. H. Lasseter, P. Paigi, Microgrid: a conceptual solution. *IEEE Power Electronics Specialists Conference*, 2004, 6:4285-4290.
- [4] Moghaddam, Amjad Anvari, A. Seifi, and T. Niknam. Multi-operation management of a typical micro-grids using Particle Swarm Optimization: A comparative study. *Renewable & Sustainable Energy Reviews*, 2012, 16(2):1268-1281.
- [5] Borhanazad, Hanieh, et al. Optimization of micro-grid system using MOPSO. *Renewable Energy*, 2014, 71(11):295-306.
- [6] Niknam, Taher, R. Azizipanah-Abarghooee, and M. R. Narimani. An efficient scenario-based stochastic programming framework for multi-objective optimal micro-grid operation. *Applied Energy*, 2012, 99(6):455-470.
- [7] Parisio, Alessandra, et al. Use of model predictive control for experimental microgrid optimization. *Applied Energy*, 2014, 115(4):37-46.
- [8] J. Zeng, et al. A multi-agent solution to energy management in hybrid renewable energy generation system. *Renewable Energy*, 2011, 36(5):1352-1363.
- [9] B. Zhao, et al. An MAS based energy management system for a stand-alone microgrid at high altitude. *Applied Energy*, 2015, 143:251-261.
- [10] Yang, Peng, G. Tang, and A. Nehorai. A game-theoretic approach for optimal time-of-use electricity pricing. *IEEE Trans. on Power Systems*, 2013, 28(2):884-892.
- [11] Li, Peng, J. Ma, and B. Zhao. Game theory method for multi-objective optimal Operation of Microgrid. *IEEE Power & Energy Society General Meeting*, 2015:1-5.
- [12] Monderer, Dov, and L. S. Shapley. Potential Games. *Games & Economic Behavior*, 1996, 14(1):124-143.