GTES: An Optimized Game-Theoretic Demand-Side Management Scheme for Smart Grid

Zubair Md. Fadlullah, *Member, IEEE*, Duong Minh Quan, *Student Member, IEEE*, Nei Kato, *Fellow, IEEE*, and Ivan Stojmenovic, *Fellow, IEEE*

***Abstract*—Demand-side management in smart grids has emerged as a hot topic for optimizing energy consumption. In conventional research works, energy consumption is optimized from the perspective of either the users or the power company. In this paper, we investigate how energy consumption may be optimized by taking into consideration the interaction between both parties. We propose a new energy price model as a function of total energy consumption. Also, we propose a new objective function, which optimizes the difference between the value and cost of energy. The power supplier pulls consumers in a round-robin fashion and provides them with energy price parameter and current consumption summary vector. Each user then optimizes his own schedule and reports it to the supplier, which, in turn, updates its energy price parameter before pulling the next consumers. This interaction between the power company and its consumers is modeled through a two-step centralized game, based on which we propose our game-theoretic energy schedule (GTES) method. The objective of our GTES method is to reduce the peak-to-average power ratio by optimizing the users' energy schedules. The performance of the GTES approach is evaluated through computer-based simulations.**

***Index Terms*—Energy optimization, game theory, real-time pricing, smart grid.**

I. Introduction

RECENTLY, smart grid has emerged as a hot research topic and has attracted government, industry, and academia alike [1]–[4]. For the successful deployment of the smart grid, demand-side management or demand response [5]–[7] is crucial. Demand-side management refers to the planning and implementation of the electric utility activities, designed to influence the customers' consumption of electricity in such a fashion that produces desired changes in the shape of loads of the utility company. While demand-side management aims at producing a change in the load shape, it needs to balance the requirement of the utility provider (i.e., the power company) and that of the customers.

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Z. M. Fadlullah, D. M. Quan, and N. Kato are with the Graduate School of Information Sciences, Tohoku University, Sendai 980-8579, Japan (e-mail: zubair@it.ecei.tohoku.ac.jp; minhquan@it.ecei.tohoku.ac.jp; kato@it.ecei.tohoku.ac.jp).

I. Stojmenovic is with University of Ottawa, Ottawa, ON K1N 6N5 Canada (e-mail: ivan@site.uottawa.ca).

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Traditionally, the demand-side management technique may either shift or reduce the energy consumption. Shifting the energy consumption can effectively mitigate the aggregate energy load during the peak hours (which is the main reason for power outage and load-shedding events). In this vein, the ratio of the highest peak time of energy consumption to the average consumption in a whole day, referred to as the peak-to-average power ratio (PAR), is used to measure the imbalance in load shape of daily energy consumption [8]. By shifting energy consumption from the peak hours to off-peak hours, it is possible to reduce the PAR. Another demand-side management technique is to reduce the energy consumption by encouraging energy-aware consumption patterns [9].

Recent research studies and initiatives indicated that dynamic pricing is an efficient way to implement demand-side management in a smart grid. As its name implies, in dynamic pricing, the cost of energy varies dynamically over time. The energy price may change depending on the wholesale market (to reflect the fluctuation of energy price) or the energy consumption level. By considering an adequate dynamic-pricing plan as per the desired objective, smart grid customers may be provided with incentives for participating in such collaborative scheduling in return of monetary incentive. For example, they may receive a reduction of electricity bills or an amount of money for their contributed curtailment of power usage, while the utility company is significantly benefited from the apparently small contribution from individual users. In the survey conducted in [10], approximately 90% of the customers in a smart grid initiative demonstrated money saving as the prime reason for their participation. Therefore, our work also considers monetary incentive as the principal motivation for the smart grid users.

Scheduling energy consumption for all home appliances requires considering numerous parameters and constraints. Therefore, optimizing energy consumption for several users at the same time is difficult in terms of computation, running time, and even convergence guarantee of the optimization algorithm. Since direct connections among the users are not desired as redundant communication is unnecessary and might invite security problems, the smart grid users should only be able to communicate with the control center of the utility company. In our paper, we consider a practical smart grid architecture, whereby a two-step game-based approach [11] for a centralized optimized energy consumption schedule is proposed. Our proposed game-based approach, referred to as game-theoretic energy schedule (GTES) method, aims at reducing the system PAR by optimizing energy schedules of the users. In our proposal, we consider different objectives and conduct an extensive analysis on users' preferences about their perceived value of energy. In addition, the interactions between the power supplier and consumers are also considered.

The remainder of this paper is structured as follows. Section II surveys relevant research work. Section III presents the existing smart grid energy consumption model. Our considered modifications to the existing model are described and proposed in Section IV. Section IV also presents our proposed GTES method. The performance of the proposed algorithm is evaluated in Section V. Finally, Section VI concludes this paper.

II. Related Work

The work in [10] analyzes case studies of dynamic-pricing programs, which were offered by electric companies in a pilot smart grid project carried out in the U.S. Smart meters assigned to the participants of the project were used to study their energy consumption patterns under various dynamic-pricing schemes, e.g., real-time pricing, critical peak pricing, and critical peak rebates. Based on the energy price, the participants had to manually plan their schedules. The power company announced updated prices according to the forecast of market fluctuation and system peak load. Even though the project contributed to PAR reduction improvement, it did not consider any user-side action, which could further improve the performance of the adopted pricing schemes.

Also, the survey and analysis conducted in [10] indicated monetary incentive to be the prime motivation of the participants of smart grid projects. Therefore, the attitude of customers toward the energy price is important. In addition, it was suggested that the energy-pricing performance could be significantly enhanced by using automated devices, e.g., smart thermostat to automatically reduce consumption of air conditioning and central heating during peak periods. Such an idea of automatic residential energy consumption schedule is also explored by Mohsenian-Rad *et al.* in their work in [12] by assuming that the advantages of real-time pricing are currently limited due to the following: 1) the lack of efficient automation systems in the building and 2) the users' difficulty to adjust with dynamic prices. Also, the work presented a framework to predict the fluctuated energy price. The idea clearly indicates the potential of exploiting smart pricing in the smart grid.

Samadi *et al.* [9] introduced the use of a utility function in an energy schedule optimization whereby the energy generation capacity over time is limited. By using the utility function, the power company can acquire the user preferences for energy use and can accordingly set an adequate price to limit the power consumption. Furthermore, Samadi *et al.* introduced a different energy-pricing scheme for different users [9]. However, this raised a psychological unfairness issue among the users. As a remedy to this unfairness issue, Tarasak [13] added "load uncertainty" to the pricing scheme.

The energy consumption scheduling problem was viewed from a game-theoretic perspective in the work in [14]. In this paper, an energy consumption schedule game was constructed for reducing PAR by shifting energy use. By using a distributed algorithm, each user plays the game by connecting with others and reviewing the schedules until an "equilibrium" is reached whereby all the players are satisfied with their *status quo*. However, the work raises the following issues. First, in the smart grid, the users should not communicate with one another as it raises privacy problems. Second, the schedule needs to be planned ahead of time (e.g., for the next 24 h and so on). To address these issues, a centralized control may offer a more effective solution.

A centralized scheme for optimally scheduling energy use is introduced in the work in [8]. The scheme in [8] is based upon a social welfare function in terms of the users' preference toward energy consumption, which is the difference of the utility and cost of the energy. However, analysis on this approach is basic, particularly as it deals with two different units (i.e., utility of energy and cost of energy). Also, the work in [8] did not demonstrate how to select an appropriate utility function and how to take into account monetary incentive. Another centralized scheme exists in literature for capturing game-based interactions among customers, retailers, and energy companies in the smart grid [15]. However, that work does not focus upon load balancing or energy consumption scheduling in the power grid. In the next section, we present an existing system model of smart grid for power, energy cost, and load control modeling [14].

III. Existing System Model

We employ the smart grid infrastructure as the basis for our system model, as shown in Fig. 1. For further details of the smart grid infrastructure, interested readers are referred to [16] and [17]. In our model, we consider a scheme with one energy supplier (i.e., the power company) and multiple consumers (i.e., users). The consumers are equipped with smart meters, each of which is assumed to have the capability of scheduling the energy consumption of the respective consumer residence. The smart grid users are connected to the power company's control centers. The bidirectional communication between the center and its consumers is possible through the smart meters. We assume that the smart meters are able to monitor and collect all the data of electrical appliances plugged into the grid. The smart meters also have the ability to turn on/off and choose the level of energy consumption for these appliances if necessary. In addition, the smart meters are capable of informing the power company or the supplier about users' energy consumption schedules.

The existing smart grid model comprises three aspects, namely, power system modeling, appropriate energy cost function modeling, and load control on consumer-end modeling. These are described in the following discussion.

1) Power System Modeling: The work in [14] constructs the following power system model. The model assumes a set of consumers in the considered smart grid obtaining electric power from the power company (e.g., as shown in Fig. 1) as $\mathcal{N}$. Assume that the number of consumers is $N \triangleq |\mathcal{N}|$. For every user, $n \in \mathcal{N}$, let l_n^h indicate the total load during hour $h \in \mathcal{H} \triangleq \{1, \ldots, H\}$, where $H = 24$. Then, let the daily load for the nth user be denoted by the energy consumption vector $\mathbf{l}_n \triangleq \{l_n^1, \ldots, l_n^H\}$. L_h, which represents the overall load (for

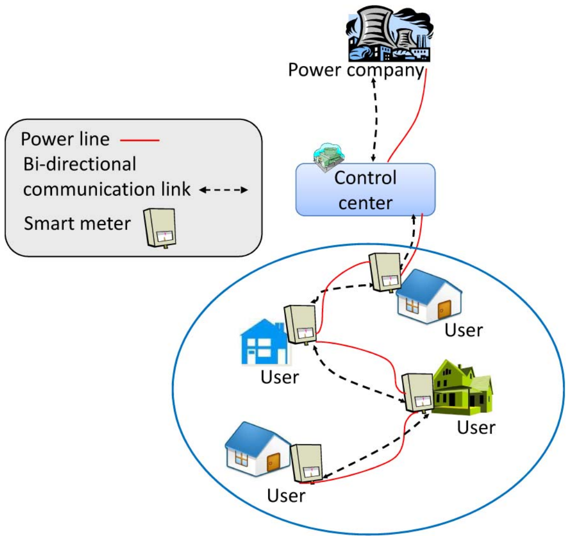

Fig. 1. Considered system architecture of the smart grid.

all the users) during every hour of a day (i.e., $h \in \mathcal{H}$), may be computed as follows:

$$L_h \triangleq \sum_{n \in \mathcal{N}} l_n^h. \tag{1}$$

The daily peak and average load levels are calculated as

$$L_{\text{peak}} = \max_{h \in \mathcal{H}} L_h \tag{2}$$

$$L_{\text{avg}} = \frac{1}{H} \sum_{h \in \mathcal{H}} L_h \tag{3}$$

respectively. Therefore, the peak-to-average ratio (PAR) in the load demand is given by

$$\textbf{PAR} = \frac{L_{\text{peak}}}{L_{\text{avg}}} = \frac{H \max_{h \in \mathcal{H}} L_h}{\sum_{h \in \mathcal{H}} L_h}. \tag{4}$$

2) Modeling an Appropriate Energy Cost Function: The hourly energy price varies with real time, proportional to the system energy consumption during that hour. In this way, the consumers will have incentives to refrain from using electricity at peak hours, resulting in a lower PAR. The cost function is an increasing function, i.e., the energy cost increases along with the total energy consumption L_h. For every $h \in \mathcal{H}$, we have $C_h(L_h^1) < C_h(L_h^2)\ \forall L_h^1 < L_h^2$. This assumption is made for the higher energy consumption to have more impact on the increase on the energy price. Also, the energy cost functions are assumed to be strictly convex. In other words, for every $h \in \mathcal{H}$, $C_h(\theta L_h^1 + (1-\theta)L_h^2) < \theta C_h(L_h^1) + (1-\theta)C_h(L_h^2)$. Here, L_h^1, L_h^2, and θ are real numbers such that $L_h^1, L_h^2 \geq 0$, and $0 < \theta < 1$.

3) Modeling Load Control on Consumer End: Let A_n indicate the set of residential electrical equipment (e.g., air conditioner, heater, kitchen appliances, television, fridge, and so forth) for the consumer $n \in \mathcal{N}$. For every appliance for the nth user, an energy consumption scheduling vector is constructed as follows:

$$x_{n,a} = \left[x_{n,a}^1, \ldots, x_{n,a}^H\right] \tag{5}$$

where the component $x_{n,a}^h$ denotes the corresponding 1-h energy consumption, which is scheduled for the appliance a by consumer n during hour h. Let l_n^h, L_h, and $L_{-n,h}$ denote the total energy consumed by the nth consumer, all consumers, and all users except the nth consumer. These can be computed as follows:

$$l_n^h = \sum_{a \in A_n} x_{n,a}^h, \quad h \in H \tag{6}$$

$$L_h = \sum_{n \in N} l_n^h, \quad h \in H \tag{7}$$

$$L_{-n,h} = L_h - l_n^h, \quad h \in H. \tag{8}$$

Then, at each consumer residence, the task of the smart meter is to determine the optimal schedule of energy consumption vector $x_{n,a}$ for all the appliances belonging to that consumer. The schedule for any appliance has several constraints, such as power consumption, minimum and maximum energy requirement to finish operation, starting time of the schedule, and stopping time of the schedule. The feasible set for energy consumption scheduling vector is defined to satisfy these conditions.

For each user $n \in N$ and each appliance $a \in A_n$, we denote the minimum daily energy consumption as $E_{n,a}^{\min}$ and the maximum daily energy consumption as $E_{n,a}^{\max}$. In order to shift and reduce energy consumption at the same time, there should be some bound on the energy consumption vector for all the appliances of the residence. The users also need to select the time interval $\mathcal{H}_{n,a}$ during which the appliances can be scheduled. Let the beginning and end time instants of this scheduling interval be denoted by $\alpha_{n,a} \in \mathcal{H}$ and $\beta_{n,a} \in \mathcal{H}$, respectively (i.e., $\alpha_{n,a} < \beta_{n,a}$). The scheduling interval must be equal to or longer than the normal time required for completing the operation for each appliance. For an appliance with schedulable operation, the scheduling interval will be more than the normal requirement time. On the other hand, for an appliance with nonschedulable operation, its scheduling interval is either a whole day with constant energy consumption (e.g., refrigerator) or equal to the normal requirement time in order to avoid further change to the plan.

The power level of each appliance $a \in A_n$ also needs to be constrained by the minimum standby power level $\gamma_{n,a}^{\min}$ and the maximum power level $\gamma_{n,a}^{\max}$. Then, it is clear that

$$\gamma_{n,a}^{\min} \leq x_{n,a}^h \leq \gamma_{n,a}^{\max} \quad \forall h \in \mathcal{H}_{n,a}. \tag{9}$$

$$x_{n,a}^h = 0 \quad \forall h \in \mathcal{H} \backslash \mathcal{H}_{n,a}. \tag{10}$$

Finally, the feasible energy consumption scheduling set corresponding to user n is defined as follows:

$$\chi_n = \Bigg\{ x_n | E_{n,a}^{\min} \leq \sum_{\alpha_{n,a}}^{\beta_{n,a}} x_{n,a}^h \leq E_{n,a}^{\max}, \gamma_{n,a}^{\min} \leq x_{n,a}^h \leq \gamma_{n,a}^{\max} \quad \forall h \in \mathcal{H}_{n,a} \quad x_{n,a}^h = 0 \quad \forall h \in \mathcal{H} \backslash \mathcal{H}_{n,a} \Bigg\}. \tag{11}$$

In the following section, to improve the assumptions of the existing model, we propose a number of modifications to it and also formulate an optimization problem.

IV. Proposed Strategies of Users and Power Companies, and Envisioned Game Theoretic Solution

In the following, we first propose the user's and power company's strategies, respectively, by making some modifications to the existing system model described in the earlier section. Based on these strategies, we then present our envisioned two-step game.

A. Strategy of the Users

In (11), the energy consumption schedule of all users is not optimized at once. Instead, each smart meter will optimize the schedule of its user according to that user's need. Clearly, the energy consumption schedule vector must belong to the feasible set defined in (11). The objective is to optimize the users' payoff. More precisely, it aims to maximize the users' benefit in consuming energy. The unit of the objective function is in terms of monetary units (e.g., in U.S. dollars) because we consider that the users are interested in monetary incentive or reward. The objective function is a function of energy consumption schedule vector to represent the users' payoffs by consuming that amount of electricity from the supplier. We propose adopting the following utility function:

$$\begin{aligned} W_i(x_1,\ldots,x_{24}) &= \text{Value of Energy} - \text{Cost of Energy} \\ &= V(X_i) - \sum_{h=1}^{24} x_h C_h(L_h) \end{aligned} \tag{12}$$

where $(x_1,\ldots,x_{24})$ denotes the energy consumption scheduling vector comprising the energy loads of user i during different hours in a day, $X_i = \sum_{h=1}^{24} x_h$ represents the total energy consumption, and $V(X_i)$ is the value of that amount of energy.

In the remainder of the section, we present an analysis of the value of energy function and cost of energy function.

1) Value of Energy Perceived by Users: Let $V(X_i)$ represent how much energy value is given by user i. However, each user in the power system is likely to have a different energy consumption pattern. Their energy consumption schedule will change based on their own parameters. Even if some users have the same energy consumption, their attitude toward energy value might still be different due to their characteristics and habits. Therefore, it is not easy to capture the response and energy demand of different users toward the same energy price. However, we can analytically model the users' preferences toward energy consumption by adopting the concept of utility function from microeconomics [18].

Utility describes the measurement of "usefulness" that an agent obtains from the available resources. It is the way that the agent values how much he can make the best use of the resources. According to utility theory, a legitimate utility function should satisfy the following three characteristics.

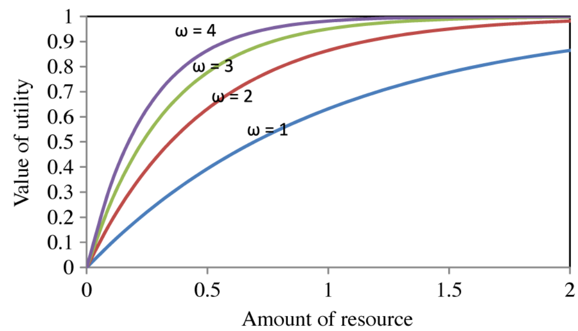

Fig. 2. Value of utility for different resources.

1) A utility function should be upper bounded:

$$u(x) \le M, \qquad M > 0. \tag{13}$$

2) The utility function should be an increasing function. More money or resource means a higher value of utility. This could be mathematically expressed as follows:

$$\frac{du(x)}{dx} > 0. \tag{14}$$

3) The utility function should be decreasing marginal or concave. Consider a user having 100 dollars. The benefit of gaining one more dollar will be smaller compared to the situation when the user has nothing. This is referred to as the decreasing marginal characteristic. This again can be expressed as

$$\frac{du^2(x)}{dx^2} < 0. \tag{15}$$

Functions satisfying the condition in (15) belong to the **concave functions** class.

There are several utility functions satisfying these characteristics, e.g., quadratic and exponential functions. According to the work in [19], the exponential function could give better model of the user's preference. Here, we use the exponential function as its value domain has been normalized between (0,1) [18]

$$u(X) = 1 - e^{-\omega X}. \tag{16}$$

A plot of utility functions is illustrated in Fig. 2, where ω is a parameter representing the users' tolerance toward energy consumption curtailment. As evident from the figure, a utility function with a relatively larger value of ω approaches the upper bound in a rather slow manner. The users with large values of ω are stricter with the energy curtailment, i.e., their utility values are lower than those of the other users even with the same amount of energy consumption. Since the value of ω is a private information of the users, energy optimization algorithms should not leak out this information.

Then, we define the value function of energy as follows:

$$V(X_i) = pX_{\max}u(X_i) = pX_{\max}(1 - e^{-\omega X_i}). \tag{17}$$

Here, $X_{\max}$ denotes the maximum amount of energy that the user can consume. p denotes the average price for a unit of energy that displays the value (in money) of acquiring that unit of energy, regardless of other elements such as time of day, level of energy consumption, the extra cost for peak hours, extra cost of delivery, and so forth.

The utility function that we choose has the maximum value of one. By scaling it up with the product of average energy price and the maximum amount of energy consumption that a user can consume, the maximum value of $V(X_i)$ is equal to the value of the maximum energy consumption. It implies that the user will value his consumption amount not greater than the average money he needs to pay to satisfy his maximum demand in monetary units.

2) Cost of Energy: In contrast with piecewise and quadratic linear functions described in [8], [14], the difference in energy price between peak hours and off-peak hours is significantly large. For instance, the PAR value of "three" indicates that the load during peak hours is three times the average load of the entire day. Then, the difference between the price of energy would be nine times. Due to tight schedules (especially during peak hours), this large difference would pose inconvenience to the customers.

Therefore, a proportional increase in the energy cost in accordance with the total load is crucial to encourage users' participation in balancing the PAR. In [10], the energy prices were chosen carefully by taking into consideration the users' reaction and by not allowing the differences between energy prices to exceed three times.

It is worth stressing that, in the first glance, we may intuitively consider that the more drastically the function changes, the better PAR reduction we might get. After all, the energy cost of the consumer is related with the imbalance of energy consumption distribution. However, the computation time increases significantly as the energy cost function varies drastically. Therefore, there is a trade-off for the selection of energy price. In our work, we propose the following energy price function:

$$C_h(L_h) = \alpha L_h \log(L_h + 1) \tag{18}$$

where α is referred to as the "price parameter." The supplier can manipulate this parameter to change the energy price of the whole day to control the energy consumption. The price difference between the peak and off-peak hours still remains unchanged. The logarithmic function in (18) gives a near linear shape as demonstrated in Fig. 3. The figure also illustrates the comparison between the proposed energy price function and the conventional quadratic price function.

By substituting (17) and (18) into (12), user i's objective function can be rewritten as

$$W_i(x_1, \ldots, x_{24}) = pX_{\max}\left(1 - e^{-\omega \sum_{h=1}^{24} x_h}\right) - \sum_{h=1}^{24} \alpha x_h L_h \log(L_h + 1). \tag{19}$$

B. Strategy of the Power Supplier

It is important to investigate the way the supplier decides the energy-pricing scheme. In the dynamic pricing, the energy

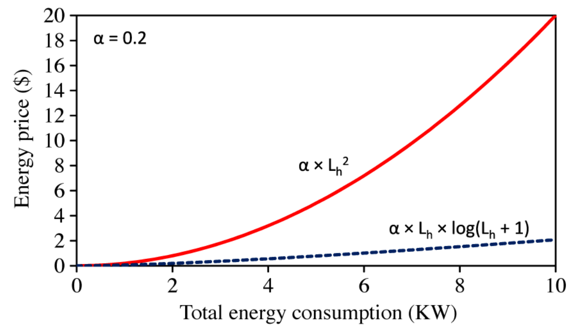

Fig. 3. Increase of energy price with the total energy consumption in case of the proposed energy price function and that in the quadratic function.

price changes according to various factors. In our proposed model, the energy price changes according to the total load during different hours. For our proposed price function, the energy price is proportional to $(L_h \log(L_h + 1))$. This property is exhibited by defining the energy price function as $(C_h(L_h) = \alpha L_h \log(L_h + 1))$, where L_h is the total load at hour h. Then, we have an energy price vector, denoted by $\mathcal{C}(\mathcal{L})$, for the entire day as follows:

$$\mathcal{C}(\mathcal{L}) = \alpha \cdot (L_1 \log(L_1+1) + \cdots + L_{24} \log(L_{24}+1)). \tag{20}$$

Therefore, the supplier can manipulate parameter α to influence the whole price vector and can impose, in turn, some constraints on the users' energy consumption. The users tend to curtail their consumption if they consider a substantial increase in the energy price. The supplier needs to consider this phenomenon. As a consequence, without any constraint for choosing an appropriate value of α, it is difficult to illustrate the effect of the users' dissatisfaction. Furthermore, it is also important to consider the real value of the energy cost for deciding the dynamic energy price. The supplier needs to consider the average energy price to obtain a close equivalent of the dynamic-pricing scheme. Next, we discuss the choice of parameter α from the supplier's side.

Let us consider a fixed-price scheme as the baseline. The energy price is fixed for every unit of energy regardless of the time of day or total energy consumption of the system. Indeed, this fixed-price scheme is adopted by the contemporary power grids. In case of Japan, Tokyo Electric Power Company [20] charges 17.87 yen for the first 120 kW and 22.86 yen for the next 180 kW for the light residential user with 20-A power line. Therefore, we assume that the supplier has the average price for the energy that it sells to its customers. This average price was referred to as parameter p earlier in the users' objective function in (17). When the supplier uses the dynamic-price scheme, we assume that the total cost that they charge for the whole system is equal to the fixed-price scheme with the same load. This can be represented by the following expression:

$$p\sum_{h=1}^{24} L_h = \sum_{h=1}^{24} L_h C_h(L_h) = \alpha \sum_{h=1}^{24} L_h^2 \log(L_h + 1). \tag{21}$$

Therefore, α can be evaluated as follows:

$$\alpha = \frac{p\sum_{h=1}^{24} L_h}{\sum_{h=1}^{24} L_h^2 log(L_h+1)}. \tag{22}$$

This is how the supplier may compute the energy price parameter α and also the energy price for different hours in a day. We investigate a little more about this price scheme to have some insight into the difference between the fixed-price and dynamic-price schemes.

C. Proposed GTES Algorithm

In this section, we propose a game-theoretic approach for optimizing energy consumption. We refer to our approach as GTES algorithm. The game is played between the power company and its consumers. The game aims at reducing the system PAR by optimizing energy schedules of the customers. The optimization process can be modeled as a two-stage game [11] as depicted in Algorithms 1 and 2.

1) The users will try to maximize their pay-offs by optimizing functions [shown in (19)] using IPM.
2) The supplier will then adjust the energy price parameter consistent with the user's energy consumption schedule according to (22).

Algorithm 1 Power company's (control center's) game.

Begin: Gather original schedule from all users}
  All users initialize their schedules from feasible sets
End
Calculate initial energy price parameter α according to (22)
Repeat
  Randomly choose user $n \in N$
  Signal user n to run **algorithm 2**
  Update the new schedule vector $\mathcal{L}_n$ from user n
  Update energy price parameter α according to (22)
Until No user wants to change schedule

Algorithm 2 User's game.

Begin: Receive signal from supplier
  Request α, vector $\mathcal{L}$
  User n optimizes schedule by solving the problem in (19) by using IPM
  Ifx_n changes compared to current schedule **Then**
    Inform the center of the new schedule vector $\mathcal{L}_n$
  End If
End

When the game reaches an equilibrium state, neither users nor supplier will change their strategies. At the same time, the system PAR and total energy consumption are reduced.

Algorithms 1 and 2 describe the moves made by the power company (typically the control center) and the consumers, respectively. At the start of the day (e.g., at 12 A.M.), when the control center will start the algorithm, it sends messages to all consumers to request their default energy consumption schedules. The users have to initialize energy consumption schedules, which satisfy all the earlier mentioned constraints. After receiving all the basic information from the users, the control center calculates the initial value for energy price parameter (α) and broadcasts it to all the users. Then, the loop in Algorithm 1 is executed until the algorithm converges. The supplier randomly selects a user to run the optimization in the loop. The selected user is not to be chosen again until all other users have executed the loop. The selected user receives a message with the total energy consumption schedule vector $\mathcal{L}_n$ of the whole system. Based on this information, together with the energy price parameter α, the user is able to optimize his objective function to find the best schedule for himself. Each smart meter is assumed to be equipped with the program to solve the local optimization problem using interior-point method (IPM) [21] for convex optimization as shown in Algorithm 2. Since IPM has a high degree of accuracy, each user makes the best possible response. If the user's schedule changes, his smart meter updates the new schedule and announces the new energy consumption schedule vector (i.e., the updated $\mathcal{L}_n$) to the center. The center therefore has to adjust the energy price parameter again based on the new system state.

The supplier center thus serves as a data-collecting entity. It receives update schedules from the users and provides them with general information: system energy consumption schedule vector $\mathcal{L}_n$. It has to update the energy price parameter α, but the update is also simple and does not require much computation. For users, solving the local optimization problem in IPM is fast and highly efficient. Also, the users do not have to reveal all the details about their schedules. In other words, reporting only the total energy consumption schedule vector $(\mathcal{L}_n)$ is sufficient for them. Normally, this type of information is already monitored by the supplier's control center. The knowledge of parameter ω, which represents the users' tolerance toward energy consumption curtailment, is not required, and this is not revealed to others as this is to remain private (as discussed earlier in Section IV-A1). In other words, in the proposed algorithm, the users do not have to worry about revealing the sensitive information to the control center.

V. Performance Evaluation

In this section, the performance of our proposed GTES approach is evaluated in terms of its energy consumption reduction in contrast with two other conventional methods. One of the conventional methods, referred to as centralized convex optimization (CCO) in the remainder of this section, gathers all parameters and constraints from every user and schedules energy consumption by applying convex optimization to find the schedules for all the users at once. The other conventional method, used for comparison, is the autonomous energy-theoretic optimization algorithm in [14] that we refer to as the energy consumption game (ECG) for ease of reference.

MATLAB [22] is used to construct our smart grid simulation environment with a supplier serving multiple customers. The

TABLE I
ENERGY APPLIANCES AND THEIR AVERAGE CONSUMPTION ON A DAILY BASIS

Appliances	Average Consumption per day (kw)
Clothes Dryer	2.47
Dishwasher	0.99
Lighting	3.29
Refrigerator	5.89
Washing Machine	0.28

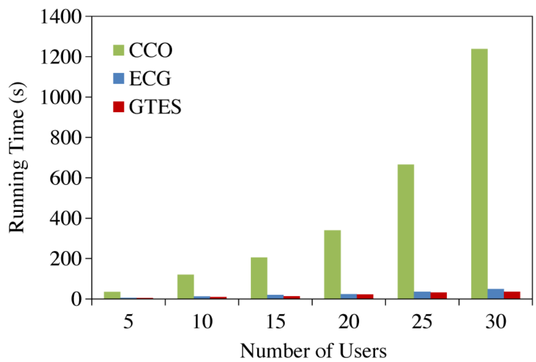

Fig. 4. Comparison of the running time between the proposed GTES and conventional CCO and ECG algorithms for various number of users.

number of customers is varied from 5 to 500. Each customer is supposed to be connected to the supplier via smart meter and is also assumed to have 10–15 schedulable appliances and 10–15 unschedulable appliances. The schedulable appliances include residential electrical devices having a flexible schedule such as washing machines, dish washers, and plug-in hybrid electric vehicles. On the other hand, unschedulable appliances (e.g., light bulbs/lamps, heaters, and so on) need to consume energy continuously, i.e., they require a fixed schedule. A list of some typical home electrical appliances is displayed in Table I based on [23]. These settings are randomly generated every time the simulation is conducted. Note that these settings remain the same for comparing the different methods in a specific situation.

First, we conduct simulations to compare the running time of the three methods with up to 30 users. Then, we evaluate the performance of the proposed method, also giving some comparison with the distributed game-theoretic ECG. Finally, we also include some comparisons between quadratic energy price function and our proposed price function.

The results depicted in Fig. 4 elucidate that the running time of CCO increases quite fast, almost at an exponential rate, while the two game-theoretic methods need significantly low completion time with increasing number of customers. Even with a relatively small number of users, the large number of parameters seriously affects the running time of convex optimization (i.e., CCO). In addition, it can hamper the convergence guarantee. In our conducted simulation, CCO often fails to converge when the number of consumers exceeds 30. In contrast, the number of appliances for each user in real life is supposed to be even higher. As a consequence, fully centralized control using CCO may not be practical. Figs. 5 and 6 illustrate more insight into this issue.

In Figs. 5 and 6, the number of iterations needed for each algorithm to converge and the average time per iteration are plotted, respectively. The running time in the previous analysis

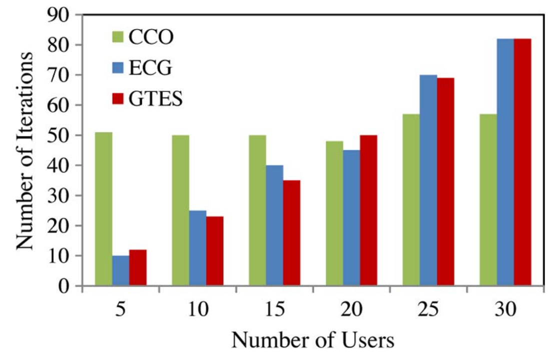

Fig. 5. Number of iterations until convergence in case of CCO, ECG, and the proposed GTES for different numbers of users.

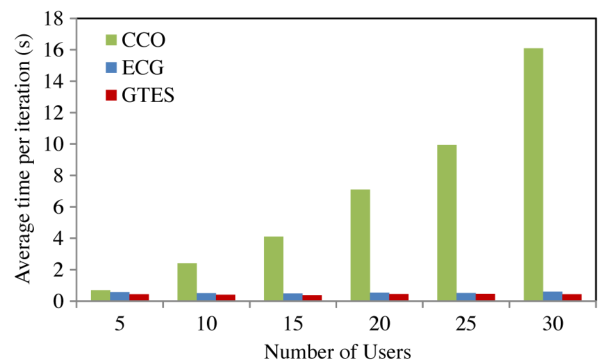

Fig. 6. Average time needed for each iteration in case of CCO, ECG, and the proposed GTES for different numbers of users.

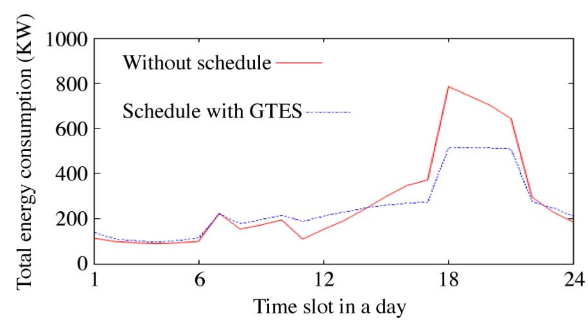

Fig. 7. Load shapes for the schemes without and with our proposed GTES scheme, respectively, illustrating how the total energy consumption becomes smooth with the proposal.

is the product of these two parameters. Fig. 5 demonstrates how the number of iterations required for CCO and both the game-theoretic algorithms (including our proposed GTES) increases gradually with the number of consumers. However, the average time needed for each iteration (as shown in Fig. 6) changes slowly for the game-theoretic algorithms, while it rises drastically in case of CCO. This is expected as the game-theoretic approaches only require solving the local optimization for each consumer. These results indicate that CCO is not scalable enough for a large number of consumers in contrast with its game-theoretic counterparts, i.e., ECG and GTES. As a consequence, in the remainder of this section, the performances of ECG and the proposed GTES are taken into consideration for comparison.

Figs. 7–9 demonstrate the load shape, convergence of system PAR, and supplier price, respectively, for the proposed GTES

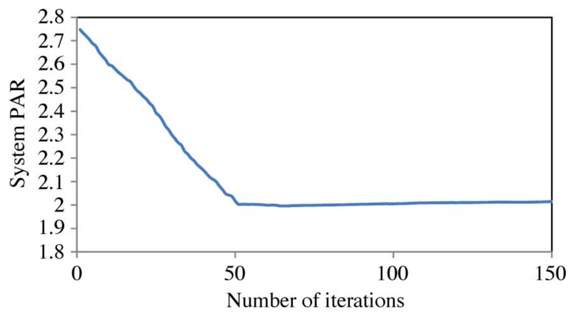

Fig. 8. Convergence of system PAR in case of the proposed GTES.

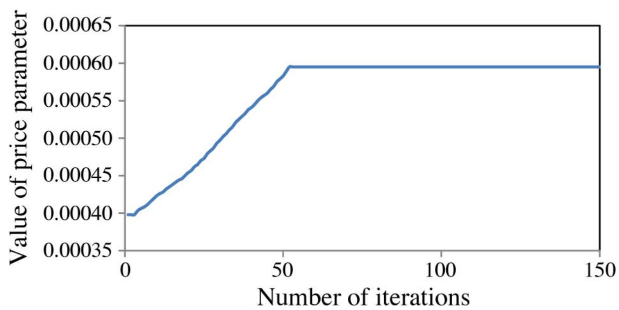

Fig. 9. Convergence of supplier price in case of the proposed GTES.

approach for a scenario of 50 users. Fig. 7 illustrates the system load shape for the nonscheduled scheme and that for the scheduling under the proposed GTES scheme over 24 h. It is evident that the energy consumption had been shifting and adjusting at the same time, resulting in a more balanced load shape and a lower total load. The system PAR changes from 2.258 to 1.837. In addition, the total system energy consumption also reduces from 3707.8 to 3447.7 kW. On the other hand, Fig. 8 shows an interesting observation where the convergence occurs significantly fast, and the system PAR drops drastically after the first round (when all the consumers had run the algorithm once). Then, the system PAR changes slowly and stabilizes after two to three rounds. This observation verifies that game-theoretic optimization would converge within $O(n)$ if each player (i.e., each of the consumers as well as the power company) follows the best move. Because, GTES uses IPM to solve the local problem for each user, all the users are able to find their optimal schedules, and therefore, the system approaches an equilibrium state quite rapidly. Fig. 9 illustrates the convergence of the energy price parameter α. As shown in the result in the figure, this parameter convergence also occurs significantly fast, similar to that on the user side. Thus, both the games on the user side and the supplier end converge to equilibrium.

Fig. 10 demonstrates the number of iterations required for the convergences of ECG and the proposed GTES. Because both of these methods are based on game theory, they converge very fast, and note that their convergence speed is proportional with the number of consumers. Additionally, as the number of consumers grows larger, the ratio between the number of iterations and the number of users decreases a little. This can be explained by the fact that the bigger the system becomes, the less effect is inflicted by changing a single consumer's schedule. Thus, it may be concluded that the game-theoretic optimization approaches have high convergence speeds, and they scale well with the increase in the number of consumers.

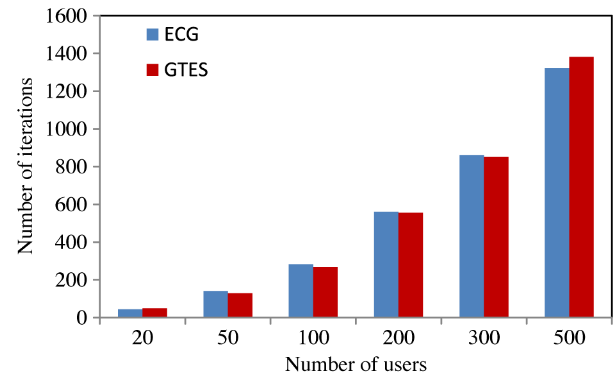

Fig. 10. Number of iterations required for ECG and the proposed GTES for a large number of users (up to 500).

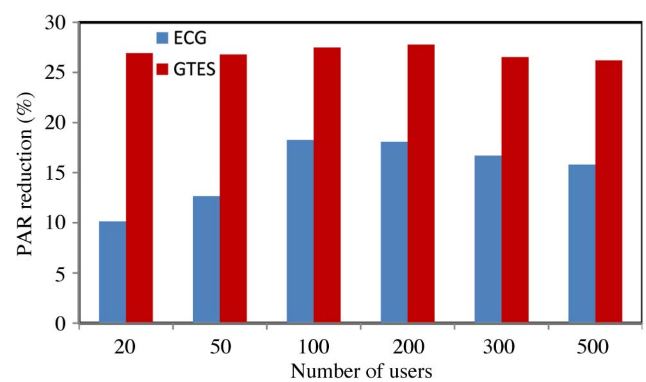

Fig. 11. PAR reduction in ECG and the proposed GTES for different numbers of users.

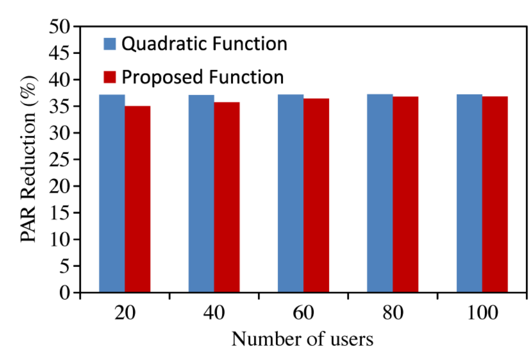

Fig. 12. Improvement of PAR due to the proposed price function in contrast with the conventional quadratic function.

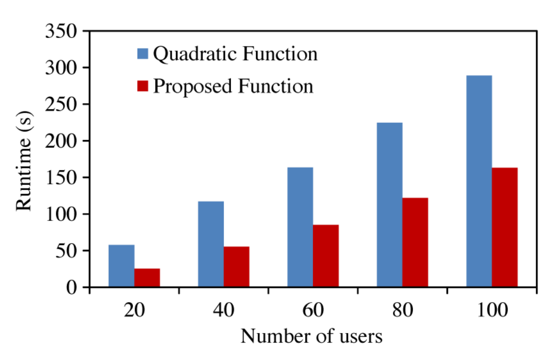

Fig. 13. Running time of the proposed price function and the conventional quadratic function.

Next, in Fig. 11, we show the comparison of the PAR reduction in ECG and that in case of the proposed GTES. Fig. 11 demonstrates that our proposed GTES method results in higher PAR reduction in contrast with ECG. This good performance of the proposed method can be attributed to its consideration of not only shifting energy consumption while scheduling but also its ability to adjust energy consumption levels during different hours.

Finally, we conduct a simulation to compare the differences between energy price functions. As we have mentioned in Section IV-A2, choosing between the more drastic quadratic function and our logarithmic-based function is a trade-off between the system performance and computation time. In order to compare these two functions, we use the game-theoretic algorithm with only one objective in ECG, namely, shifting energy consumption to reduce PAR. The consumers try to minimize their energy cost by shifting their consumption to "cheaper" hours. Clearly, unlike our proposed GTES method, the total energy consumption will be fixed in ECG, and the same holds for the energy price parameter. The results are depicted in Figs. 12 and 13. As the results demonstrate, the quadratic function displays a small improvement in PAR reduction in contrast with that in our energy price function, and the difference becomes more insignificant when the number of users increases. However, the running time of the quadratic function remains larger (in fact nearly double) compared to the proposed energy price function used in GTES. This result indicates that the performance of the proposed logarithmic-based price function is better than the existing energy price function.

VI. Conclusion

In this paper, we have considered a practical smart grid infrastructure, where the power company and its consumers are proposed to play their own games to optimize their energy schedules. In our proposed two-stage game in smart grid, the objective is to achieve a reduction in the system PAR. We apply a pricing scheme, whereby the energy price changes according to the whole system energy consumption during each hour so that all the consumers may have appropriate monetary incentives to follow the system. Since the number of parameters to be optimized is rather high, a fully centralized control by using convex optimization is not found to scale well with a high population of users. To overcome this, we proposed a game-theoretic centralized optimization scheme. The simulation results demonstrated that the proposed algorithm converged in $O(n)$ iterations and achieved good PAR reduction and energy consumption reduction. These results are desired from the power company's point of view, and provided that it has the knowledge of the consumers' energy schedules, the power company would be able to predict the total energy consumption at each hour in order to produce or estimate the necessary amount of energy (to purchase from the wholesale market). Also, every user has his electric cost reduced in an effective manner, and the values of the users' objective functions (which represent their pay-offs) are increased.

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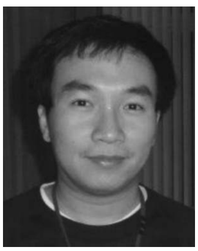
Zubair Md. Fadlullah (M'11) received the B.Sc. degree in computer science (with honors) from the Islamic University of Technology, Gazipur, Bangladesh, in 2003 and the M.S. and Ph.D. degrees from the Graduate School of Information Sciences (GSIS), Tohoku University, Sendai, Japan, in 2008 and 2011, respectively.

He is currently an Assistant Professor at GSIS. His research interests are in the areas of smart grid, network security, intrusion detection, game theory, and quality of security-service-provisioning mechanisms.

Dr. Fadlullah was a recipient of the prestigious Deans and Presidents awards from Tohoku University in March 2011 for his outstanding research contributions.



Duong Minh Quan (S'11) received the B.S. degree in information technology from the Graduate School of Information Sciences, Tohoku University, Sendai, Japan, in 2012.

His research interests include smart grid and game theory.

Nei Kato (M'03–SM'05–F'13) received the M.S. and Ph.D. degrees in information engineering from Tohoku University, Sendai, Japan, in 1988 and 1991, respectively.

Since 2003, he has been a Full Professor at the Graduate School of Information Sciences, Tohoku University, Sendai, Japan. He has published more than 300 papers in peer-reviewed journals and conference proceedings. He has been engaged in research on satellite communications, computer networking, wireless mobile communications, smart grid, image processing, and pattern recognition.

Prof. Kato is a distinguished lecturer of IEEE ComSoc and a fellow of the Institute of Electronics, Information and Communication Engineers (IEICE). He currently serves as the Vice Chair of the IEEE Ad Hoc & Sensor Networks Technical Committee. He was the recipient of the Minoru Ishida Foundation Research Encouragement Prize in 2003, the Distinguished Contributions to Satellite Communications Award from the Satellite and Space Communications Technical Committee of the IEEE Communications Society in 2005, the FUNAI Information Science Award in 2007, the TELCOM System Technology Award from the Foundation for Electrical Communications Diffusion in 2008, the IEICE Network System Research Award in 2009, the IEICE Satellite Communications Research Award in 2011, the KDDI Foundation Excellent Research Award in 2012, the IEICE Communications Society Distinguished Service Award in 2012, the IEEE GLOBECOM Best Paper Award (twice), the IEEE WCNC Best Paper Award, and the IEICE Communications Society Best Paper Award in 2012.

Ivan Stojmenovic (F'08) is a Full Professor at the University of Ottawa, Ottawa, ON, Canada. He published over 300 different papers and edited 7 books on wireless, ad hoc, sensor, and actuator networks, and applied algorithms with Wiley.

Dr. Stojmenovic is the Editor-in-Chief of the IEEE Transactions on Parallel and Distributed Systems (2010–2013) and the Founder and Editor-in-Chief of three journals. He is the Associate Editor-in-Chief of *Tsinghua Journal of Science and Technology*, a steering committee member of the IEEE Transactions on Emerging Topics in Computing, and the Editor of IEEE Network, IEEE Transactions on Cloud Computing, IEEE Transactions on Computers, *ACM Wireless Networks*, and some other journals. He is on Thomson Reuters list of Highly Cited Researchers (from 2013; less than 300 computer scientist) and has h-index 59, top h-index in Canada for mathematics and statistics, and more than 13 000 citations. He received five best paper awards and the Fast Breaking Paper by Thomson ISI ESI in October 2003. He received the Royal Society Research Merit Award, U.K. (2006), and the Humboldt Research Award, Germany (2012). He is Tsinghua 1000 Plan Distinguished Professor (2012–2015). He is a fellow of the IEEE Communications Society (class 2008) and has been a fellow of the Canadian Academy of Engineering since 2012 and a member of the Academia Europaea (The Academy of Europe) since 2012 (section: Informatics). He was an IEEE CS Distinguished Visitor in 2010–2011 and received the 2012 Distinguished Service Award from the IEEE ComSoc Communications Software TC. He received the Excellence in Research Award from the University of Ottawa in 2009. He chaired and/or organized more than 60 workshops and conferences and served in more than 200 program committees. He was the Program Cochair of IEEE PIMRC 2008, IEEE AINA-07, and IEEE MASS-04&07 and founded several workshop series. He is/was the Workshop Chair at IEEE ICDCS 2013, IEEE INFOCOM 2011, IEEE MASS-09, and ACM Mobihoc-07&08.