Coalitional Game Theory for Cooperative Micro-Grid Distribution Networks

Walid Saad[1], Zhu Han[2], and H. Vincent Poor[1]

[1] Electrical Engineering Department, Princeton University, Princeton, USA, Emails: {saad,poor}@princeton.edu

[2] Electrical and Computer Engineering Department, University of Houston, Houston, USA, Email: zhan2@mail.uh.edu

***Abstract*—Micro-grid distribution networks that use distributed energy sources are expected to lie at the heart of the emerging smart grid technology. While existing approaches have focused on control and communication aspects in micro-grids, this paper uses coalitional game theory to study novel cooperative strategies between the micro-grids of a distribution network. For this purpose, a coalitional game is formulated between a number of micro-grids (e.g., solar panels, wind turbines, PHEVs, etc.) that are, each, servicing a group of consumers (or an area) and that are connected to a macro-grid substation. For forming coalitions, an algorithm is proposed to allow the micro-grids to autonomously cooperate and self-organize into a partition composed of disjoint micro-grid coalitions. Each formed coalition consists of micro-grids that have a surplus of power to transfer or sell and of micro-grids that need to buy or acquire additional power to meet their demand. Within every coalition, the micro-grids coordinate the power transfer among themselves as well as with the macro-grid station, in a way to optimize a utility function that captures the total losses over the distribution power lines. Also, the proposed algorithm allows the micro-grids, in a distributed manner, to self-adapt to environmental changes such as variations in the power needs of the micro-grids. Simulation results show that the proposed algorithm yields a reduction in terms of the average power losses (over the distribution line) per micro-grid, reaching up to 31% improvement relative to the non-cooperative case.**

I. Introduction

The smart grid has emerged as a promising technology that can improve the efficiency, reliability, and robustness of power and energy grids [1]. The deployment of the smart grid is envisioned to revolutionize the way energy is consumed and traded between appliances, generators, and other devices [1]. One key component of the smart grid is the so-called *micro-grid distribution network* [2] which connects a variety of distributed energy resources such as solar cells, wind turbines, or other renewable energy sources to consumers or loads in different areas. Essentially, the micro-grid (MG) is a network of distributed generators that must be able to provide power for consumers both in coordination with the main power grid, i.e., the *macro-grid* and in an autonomous manner without reliance on the macro-grid [2].

The efficient introduction of MG networks faces numerous challenges on many fronts such as design, control, and implementation [2]. In [3], the authors investigate recent advances in channel modeling, network topology, and frequency allocation for designing a communications infrastructure suitable for smart grid and MG networks. The work in [4] presents a scheme based on wireless sensor networks for enabling a reliable communication between the energy sources of an MG. Beyond the communication infrastructure, a number of strategies for controlling the distributed MG elements are proposed in [5] allowing the MGs to operate, autonomously, without being connected to the medium voltage network. Existing literature has, thus, established a solid control and communication infrastructure to support MG distribution networks in practical power systems. As a result, there is an emerging need for exploiting this infrastructure by enabling the MGs to perform distributed decision making. In particular, it is of interest to enable the MGs, based on their power needs and characteristics, to make cooperative decisions, in order to exchange and trade power among themselves. For example, by cooperating, the MGs can alleviate the load on the macro-station, as well as reduce the costs, in terms power losses over the distribution lines, within the distribution network.

This material is based upon work supported by the DTRA under Grant No. HDTRA1-07-1-0037 and is also supported by NSF projects CNS-0910461, CNS-0953377, CNS-0905556, and ECCS-1028782.

The main contribution of this paper is to design novel cooperative strategies between the MGs (e.g., solar panels, wind farms, PHEVs, etc.) of a distribution network, allowing them to trade power, reduce the load on the main macro-grid, and minimize the losses of power over the distribution lines. In this regard, we formulate a coalitional game between the MGs and we propose a distributed algorithm for coalition formation. We show that, using the proposed algorithm, the MGs can autonomously cooperate and self-organize into a distribution network partition composed of disjoint coalitions. Each formed coalition consists of MGs that have a surplus of power to transfer or sell and of MGs that are in need of additional power to meet the demand of their consumers. Inside every coalition, the MGs coordinate the power transfer between one another, in a way to optimize a utility function representing the costs, in terms of power losses over the distribution lines. We also investigate how, using coalition formation, the MGs can self-adapt to environmental changes such as a change in the power requirements of the MGs. Results show that the proposed algorithm yields significant power savings compared to the scheme that relies on the macro-grid. Note that, although coalitional game theory has been applied in economics or communication networks, to the best of our knowledge, this paper is the first that uses coalitional games in the smart grid.

This paper is organized as follows: Section II presents the system model. In Section III, we formulate the coalitional game between the MGs and in Section IV, we propose a distributed algorithm for coalition formation. Simulation results are presented in Section V. Finally, conclusions are drawn in Section VI.

II. System Model

Consider a distribution network composed of a single substation, the *macro-station*, which is connected to the main macro-grid as well as to a network of N MGs (e.g., solar panels or farms, wind farms, PHEVs, etc.). Let $\mathcal{N}$ denote the set of all MGs. In a given time period, each MG $i \in \mathcal{N}$ is able to generate a total power G_i and is required to service a group of consumers having a demand D_i. For every MG, we define the real quantity $Q_i = G_i - D_i$ as the power surplus or need of i. The value of Q_i determines whether MG i has a surplus of power ($Q_i > 0$) to sell, needs to acquire power to meet its demand ($Q_i < 0$), or is able to meet its demand ($Q_i = 0$). Any MG $i \in \mathcal{N}$ having $Q_i > 0$ will be referred to as a "seller" while any MG $i \in \mathcal{N}$ with $Q_i < 0$ is referred to as a "buyer". In practical smart grid networks, the demand D_i is often considered as being random (e.g., see [6]) since it depends on many unpredictable factors such as consumption level, consumer behavior, among others.

978-1-61284-955-3/11/$26.00 ©2011 IEEE

Moreover, in order to meet the changing demand, the amount of power generated needs to ramp up very slowly and can also be random [6]. As a result, hereinafter, we assume that, for a given time period, the value of Q_i is a random variable with a certain observed distribution.

In the non-cooperative case, each MG $i \in \mathcal{N}$ exchanges (or acquires) the power Q_i with the main smart grid using the macro-station. This transfer of power between the macro-station and the MGs is, in general, done at a medium voltage U_0. In this paper, we restrict our attention to the power transfer *inside the distribution network* (the MGs and a single macro-station) and do not consider the power transfer between the macro-station and the transmission network (i.e., the main smart grid) which takes place over high voltage transmission lines. Any power transfer between an MG i and the macro-station is accompanied with a cost corresponding to the loss of power over the distribution lines inside the MG network. The transfer of power between an MG $i \in \mathcal{N},\ Q_i > 0$ and the macro-station, incurs a power loss P_{i0}^{loss} expressed by [7]

$$P_{i0}^{\text{loss}} = R_{i0} I_0^2 + \beta P_i(Q_i) = \frac{P_i(Q_i)^2 R_{i0}}{U_0^2} + \beta P_i(Q_i), \quad (1)$$

where R_{i0} is the resistance of the distribution line between MG i and the macro-station, β is the fraction of power lost in the transformer at the macro-station, and $I_0 = \frac{P_i(Q_i)}{U_0}$ is the current flowing over the distribution line when power is transferred between MG i and the macro-station. $P_i(Q_i)$ represents the power flowing between the macro-grid and MG i given by

$$P_i(Q_i) = \begin{cases} Q_i & \text{if } Q_i > 0, \\ L_i^* & \text{if } Q_i < 0, \\ 0 & \text{otherwise,} \end{cases} \quad (2)$$

where L_i^* is the total amount of power that needs to be generated (or available) to ensure that MG i receives $P_i^{\text{required}} = -Q_i,\ Q_i < 0$ and is given by a solution to the following equation [7]:

$$L_i = P_{i0}^{\text{loss}} + P_i^{\text{required}} = \frac{L_i^2 R_{i0}}{U_0^2} + \beta L_i - Q_i. \quad (3)$$

Depending on the values of the resistance R_{i0}, the required power Q_i, and the voltage U_0, the quadratic equation (3) can admit zero, one, or two solutions, for any given power Q_i. Whenever (3) admits two positive roots, we consider that the amount of power generated L_i^* is equal to the *smallest root* which, in fact, yields the smallest power losses over the distribution lines. Whenever (3) has no solution, we assume that the macro-grid, in this case, will generate the energy $L_i^* = \frac{(1-\beta)U_0^2}{2R_{i0}}$ which ensures a maximum received power to the MG (although, in this case, this power cannot meet the entire demand of MG i)[1]. For convenience, in the non-cooperative case, we consider a distribution network in which the macro-station can generate enough power to satisfy the required powers by all buyers in the network. Thus, denoting by $\mathcal{N}_b$ the set of all buyers in the network, we consider that the macro-station has enough power to provide $\sum_{i \in \mathcal{N}_b} L_i^*$, where L_i^* is the chosen solution to (3) for a buyer i (as already mentioned, if, for a buyer i, (3) has no roots, then $L_i^* = \frac{(1-\beta)U_0^2}{2R_{i0}}$).

As given by (1) and (2), the power loss over the distribution lines depends on several factors such as the distance between the MG and the macro-station (through the resistance), the transmission voltage U_0, and the power Q_i transferred (sent or received) between MG i and the macro-station. Thus, we define the non-cooperative payoff (utility) of any MG i as the total power loss over the distribution line due to the power transfer, as follows:

$$u(\{i\}) = -w_i P_{i0}^{\text{loss}}, \quad (4)$$

where w_i is the price paid by i per unit of power loss and the minus sign is inserted to turn the problem into a maximization.

[1]This solution is found by setting the first-order derivative of (3) to 0.

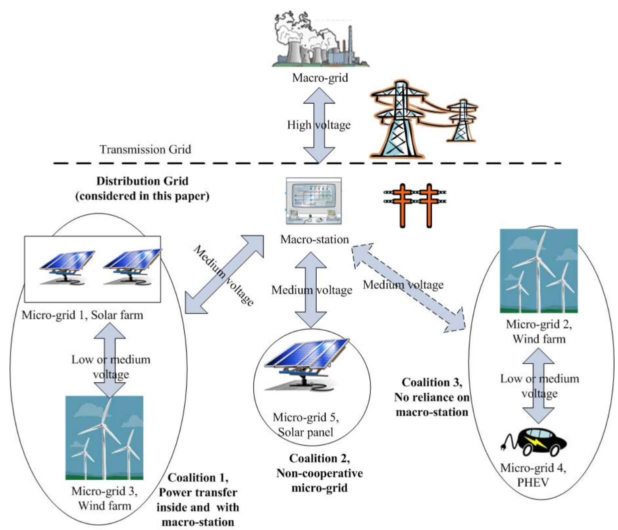

Fig. 1. An illustration of the studied system model with cooperative MGs.

Instead of relying on the macro-station, the MGs can decide to form cooperative groups, i.e., coalitions, so as to alleviate the load on the main smart grid and maximize their payoff in (4) by reducing the power losses. When a coalition $S \subseteq \mathcal{N}$ of MGs forms, the members of S can, depending on their need or surplus, transfer power locally with little reliance on the macro-station. By doing so, they can decrease the power losses over the distribution lines due, mainly, to two reasons: On one hand, whenever the MGs are located closer to each other than to the macro-station, exchanging power locally certainly yields reduced power losses over the distribution lines. On the other hand, by transferring power locally, the MGs can minimize the power flowing from or to the macro-station (and its corresponding power loss) by reducing the number of transmissions to or from the macro-station. Also, by cooperating, the MGs can avoid the power losses at the level of the macro-station's transformer. Therefore, depending on their location, their power need or surplus, the MGs have an incentive and a mutual benefit to cooperate and trade power locally within their given distribution network. An illustration of the proposed system model with cooperative coalitions is shown in Fig. 1 for a distribution network with 5 MGs. In this figure, each MG serves a group of consumers or area (not shown for figure clarity). In Fig. 1, we can see how some coalitions such as Coalition 1 can exchange energy locally as well as to the macro-station, while others, such as Coalition 3; can cooperate and transfer power with no reliance on the macro-station. In the next section, we introduce a coalitional game model suitable for studying the cooperative behavior of the MGs as in Fig. 1.

III. Cooperative MGs as a Coalitional Game

To formally study the cooperative behavior of the MGs, we use the framework of coalitional game theory [8], [9]. A coalitional game is defined as a pair $(\mathcal{N}, v)$ where $\mathcal{N}$ is the players' set and $v : 2^{\mathcal{N}} \rightarrow \mathbb{R}$ is a function that assigns for every coalition $S \subseteq \mathcal{N}$ a real number representing the total benefit achieved by S [8]. In order to formulate a coalitional game between the MGs in $\mathcal{N}$, we must define the value function $v(S)$ for any coalition $S \subseteq \mathcal{N}$. To do so, we need to investigate the operation of the MGs that belong to a given coalition S. We divide any coalition S of MGs into two groups: the group of sellers denoted by $S_s \subset S$ and the group of buyers which we denote by $S_b \subset S$, such that $S_s \cup S_b = S$. Hence, the members of S_s have a surplus of power and are willing to sell or transfer this surplus, i.e., $\forall i \in S_s, Q_i > 0$ while the members of S_b are in need of buying or receiving power, i.e., $\forall i \in S_b, Q_i < 0$. Note that, it is clear that, for any coalition $S \subset \mathcal{N}$ to form, it should have at least one seller and one buyer.

Subsequently, for any coalition $S = S_s \cup S_b$, we study the local power transfer between the sellers in S_s, the buyers in S_b and the macro-station. Although many approaches for assigning sellers to buyers can be used (e.g., those used in economics [8]), for ease of analysis of the coalitional game (which is the main focus of this paper), we propose a simple scheme for matching the buyers to the sellers that relies mainly on the preferences of the buyers inside the coalition. Given a coalition S with k buyers in the set $S_b \subset S$, assuming that the buyers are ordered in a certain sequence, i.e., $S_b = \{b_1, \dots, b_k\}$, the buyers can act sequentially, to obtain their needed power as follows:

1) Buyer $b_l \in S_b$ requests to acquire its needed energy from the seller $s_l \in S_s$ that will potentially yield the smallest power loss (for example, the closest seller to b_l).
   - If this seller s_l can ensure a received power of $-Q_{b_l}$ to buyer b_l, then, the buyer does not act further.
   - Otherwise, b_l buys as much power as possible from s_l and, then, tries to buy the rest of its power need from the seller in S_s having the next smallest power loss.
2) Buyer b_l repeats the above sequence until it has satisfied its power need Q_{b_l} or until no available sellers in S_s exist.

This process is repeated for all the buyers in S_b. If, during the exchange of power, a buyer b_l is unable to find a seller in S_s with available power and b_l still has a need for power, then, b_l will buy the rest from the macro-station. Similarly, if some of the sellers in S_s still have power to sell after the local transfer is complete, they will sell it to the macro-grid. In essence, a seller or a buyer, tries to transfer as much energy as possible locally inside the coalition, before using the macro-station. Note that, whenever an MG i, part of a coalition S, buys or sells all its power to the macro-station, without any local transfer inside S, this MG i will be excluded by the other members since it adds no extra benefit.

For any coalition S, let $\mathfrak{T}_S$ be the set of orderings over the buyers in S. Then, given an order $\Pi \in \mathfrak{T}_S$, the total losses over the distribution lines incurred by the power transfers to or from the members of S are given by:

$$u(S,\Pi) = -\left(\sum_{i \in S_s, j \in S_b} P_{ij}^{\text{loss}} + \sum_{i \in S_s} P_{i0}^{\text{loss}} + \sum_{j \in S_b} P_{j0}^{\text{loss}} \right), \quad (5)$$

where P_{i0}^{loss} and P_{j0}^{loss} are given by (1) and (2) and represent, respectively, the power losses during the distribution of power (if any) between the sellers and buyers of S and the macro-station.

P_{ij}^{loss} is the power lost in the distribution lines during the local power transfer, inside S, between a seller i and a buyer j which is also given by (1) and (2), with a few considerations. First, the local power transfer between any seller $i \in S_s$ and any buyer $j \in S_b$ inside S yields no transformer losses, i.e., $\beta = 0$. Second, any power transfer between any seller $i \in S_s$ and any buyer $j \in S_b$ inside S is done at a low-to-medium voltage U_1 which is, in general, smaller than U_0 [7]. Also, when computing P_{ij}^{loss}, one must account for the fact that every seller $i \in S_s$ has a maximum power surplus Q_i (prior to any power transfer), and, thus, it might not be able to provide the total power needed by a certain buyer. Thus, if the amount needed by any buyer $j \in S_b$ from a seller $i \in S_s$ exceeds the maximum surplus of the seller (or (3) yields no roots), then, buyer j will buy the minimum between the power $\frac{U_1^2}{2R_{ij}}$ (R_{ij} is the resistance between i and j) that maximizes (3) (maximum power that i can ensure to j) and the maximum available surplus (equal to Q_i if i did not sell any of its power yet) at seller i. Using (5), which represents the total power loss incurred by the different power transfers for S, we can define the value function for the MGs $(\mathcal{N}, v)$ coalitional game:

$$v(S) = \max_{\Pi \in \mathfrak{T}_S} u(S,\Pi). \quad (6)$$

The value function in (6) represents the maximum total utility generated by any coalition $S \subset \mathcal{N}$, i.e., the minimum cost paid for the power losses over the distribution lines incurred by any power transfer originating from or terminating at an MG in S. In a coalitional game, it is important to provide a rule that maps the value in (6) achieved by the coalition as a whole to a payoff vector $\boldsymbol{\phi}(S)$ where each element $\phi_i(S)$ is the payoff of player $i \in S$, i.e., the contribution of MG $i \in S$ to the total value $v(S)$. Since (6) represents a cost paid by the coalition, i.e., a certain amount of money, it can thus be divided in any arbitrary manner between the members of S which implies that we have a game with *transferable utility*. Although a number of fairness criteria (e.g., egalitarian fair, Shapley value, nucleolus, etc.) exist for the division of payoffs [9], due to space limitation, we will adopt a proportional fair payoff division which takes into account the non-cooperative utilities (losses) of the cooperating MGs. In this scheme, the extra benefits from cooperation are divided in weights according to the MGs' non-cooperative utilities. Thus, for an MG $i \in S$,

$$\phi_i = \alpha_i \left(v(S) - \sum_{j \in S} v(\{j\}) \right) + v(\{i\}), \quad (7)$$

where $\sum_{i \in S} \alpha_i = 1$ and within the coalition $\frac{\alpha_i}{\alpha_j} = \frac{v(\{i\})}{v(\{j\})}$. Note that, it can be easily seen from (6), that the grand coalition, i.e., a single coalition of all MGs, can *seldom* form. Instead, disjoint coalitions such as in Fig. 1 will emerge, and, thus, we have a *coalition formation game* [9] in which the objective is to build an algorithm for forming coalitions, as discussed next.

IV. Distributed MG Coalition Formation Algorithm

In order to devise an MG coalition formation algorithm, it will prove useful to define the following concept [10]:

Definition 1: Consider two collections of disjoint coalitions $\mathcal{C} = \{C_1, \dots, C_l\}$ and $\mathcal{K} = \{K_1, \dots, K_n\}$ formed out of the same players. For a collection $\mathcal{C} = \{C_1, \dots, C_l\}$, the payoff of a player j in a coalition $C_j \in \mathcal{C}$ is $\phi_j(\mathcal{C}) = \phi_j(C_j)$ where $\phi_j(C_j)$ is given by (7) for coalition C_j. Collection $\mathcal{C}$ is preferred over $\mathcal{K}$ by *Pareto order*, i.e., $\mathcal{C} \rhd \mathcal{K}$, if and only if

$$\mathcal{C} \rhd \mathcal{K} \Longleftrightarrow \{\phi_j(\mathcal{C}) \geq \phi_j(\mathcal{K}) \ \forall\ j \in \mathcal{C}, \mathcal{K}\}$$
$$\text{with } \textit{at least one strict inequality } (>) \text{ for a player } k. \quad (8)$$

The Pareto order implies that a group of players prefers to be partitioned by a collection $\mathcal{C}$ rather than by $\mathcal{K}$, if, at least one player, is able to improve its payoff when the structure changes from $\mathcal{K}$ to $\mathcal{C}$ without hurting any of the other players, i.e., without decreasing their payoffs. For coalition formation, we use two distributed rules named *merge* and *split* [10] defined as follows:

TABLE I
ONE ROUND OF THE PROPOSED MGS COOPERATION ALGORITHM.

Initial State

The network is partitioned by $\mathcal{S} = \{S_1, \ldots, S_k\}$ (initially $\mathcal{S}$ = $\mathcal{N}$ = $\{1, \ldots, N\}$ with non-cooperative MGs).

Stage 1 - Coalition Formation:

repeat

a) $\mathcal{M}$ = Merge($\mathcal{S}$); MG coalitions in $\mathcal{S}$ decide to merge as explained in Section IV.

b) $\mathcal{S}$ = Split($\mathcal{M}$); MG coalitions in $\mathcal{M}$ can take distributed split decisions using the Pareto order in (8).

until merge-and-split converges to a final partition $\mathcal{S}_{\text{final}}$.

Stage 2 - Cooperative MG Power Transfer:

a) Each coalition of MGs $S_i \in \mathcal{S}_{\text{final}}$ orders its seller in a way to minimize the losses (maximize (6)).

repeat, for every $S_i \in \mathcal{S}_{\text{final}}$

b) Sequentially, each buyer j in a coalition $S_i \in \mathcal{S}_{\text{final}}$ attempts to satisfy its power needs Q_j using the local sellers in S_i.

until no local power transfer is possible.

c) Within every coalition $S_i \in \mathcal{S}_{\text{final}}$, any seller or buyer, which still has power to transfer, can do so using the macro-station.

These two stages can be repeated periodically to adapt the partition to environmental changes (e.g., power need changes).

- **Merge -** Merge any set of coalitions $\{S_1, \ldots, S_l\}$ where $\{\bigcup_{j=1}^{l} S_j\} \triangleright \{S_1, \ldots, S_l\}$, hence, $\{S_1, \ldots, S_l\} \rightarrow \{\bigcup_{j=1}^{l} S_j\}$.
- **Split -** Split any coalition $\bigcup_{j=1}^{l} S_j$ where $\{S_1, \ldots, S_l\} \triangleright \{\bigcup_{j=1}^{l} S_j\}$, thus, $\{\bigcup_{j=1}^{l} S_j\} \rightarrow \{S_1, \ldots, S_l\}$.

By using merge, a group of MGs (or a group of MG' coalitions) can cooperate and form a single, larger coalition if this formation increases the payoff (reduces the power losses) of at least one of the MGs without decreasing the payoff of any of the other involved MGs. Hence, a merge decision by Pareto order ensures that all the involved MGs agree on its occurrence. In an analogous manner, a coalition can decide to split and divide itself into smaller coalitions if splitting is preferred by Pareto order.

For the MG cooperation game, we propose a coalition formation algorithm using merge and split and composed of two stages: coalition formation and cooperative power transfer. In this algorithm, first, each coalition (or individual MG) can obtain information on its environment (e.g., surrounding MGs, locations, etc.) using the communication infrastructure [3], [4]. Then, the coalition formation stage starts in which the merge process occurs as follows: Given a partition $\mathcal{S} = \{S_1, \ldots, S_k\}$, each coalition $S_i \in \mathcal{S}$ negotiates, in a pairwise manner, using the underlying communication structure (e.g., as in [3], [4]) with neighboring MGs to assess a potential merge. Using these pairwise negotiations, two coalitions can exchange some information on their members and capabilities so as to evaluate their projected payoff from a merge decision (e.g., by simulating the proposed seller-buyer mechanism) as per (6) and (7). The two coalitions will, subsequently, decide on whether to merge or not by using the Pareto order in (8). Whenever a merge decision occurs, a coalition can investigate the possibility of a split, through internal communication between its MGs. Clearly, a merge or split operation is a distributed decision that an MG (or a coalition of MGs) can make, individually and with no reliance on the main grid or other entities. After successive merge-and-split iterations, the network converges to a partition composed of disjoint coalitions and in which no coalition has any incentive to further merge or split (the partition is *merge-and-split proof*). The convergence of any merge-and-split iterations such as the proposed algorithm, is

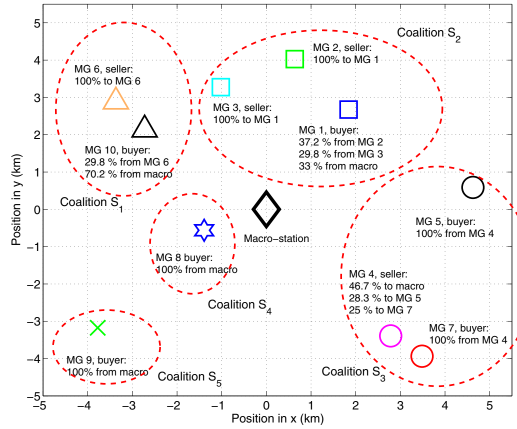

Fig. 2. Snapshot of an MG distribution network partition resulting from the proposed coalition formation algorithm with $N = 10$ MGs.

guaranteed as shown in [10]. Upon convergence, the MGs within each formed coalition will start the actual power transfer stage using the mechanism described in Section III which constitutes the last stage. Table I shows a summary of the proposed algorithm.

In addition to being *stable* or merge-and-split proof, the partition resulting from the proposed algorithm can also be a $\mathbb{D}_c$-stable partition in which no MG or coalition of MGs has an incentive to leave this partition using *any* operation (not just merge or split) [10]. When using the Pareto order, the $\mathbb{D}_c$-stable partition is not only stable but also has a Pareto optimal payoff distribution for the MGs. Although, for an MG network, a $\mathbb{D}_c$-stable partition may not exist (see [9] for a discussion on the existence of $\mathbb{D}_c$-stable partitions in networks which is omitted due to space limitation), our proposed algorithm is guaranteed reach it when it exists, i.e.,:

Lemma 1: In the studied $(\mathcal{N}, v)$ MGs coalitional game, the proposed coalition formation algorithm in Table I converges to the Pareto-optimal $\mathbb{D}_c$-stable partition, if such a partition exists. Otherwise, the final partition is merge-and-split proof.

This result is immediate from the fact that, as shown in [10], the $\mathbb{D}_c$-stable partition is a *unique* outcome of any algorithm based on merge-and-split iterations such as our algorithm in Table I.

Finally, the proposed algorithm in Table I also enables the MGs to take distributed decisions to adapt their network structure to environmental changes such as variations in the surplus or need of power due to changes in the demand or production of one or more MGs. To do so, the algorithm can be repeated, periodically, allowing the MGs to make new merge or split decisions so as to adapt to the dynamics of their environment.

V. SIMULATION RESULTS AND ANALYSIS

For our simulations, we set up a distribution network within a square of 10 km $\times$ 10 km with the macro-station located at the center and the MGs randomly deployed within this area. Analogously to [6], the power (surplus or need) Q_i of any MG i is assumed to be a Gaussian random variable with zero mean and a variance that is uniformly distributed between 100 and $100,000$, i.e., a standard deviation uniformly distributed between 10 MW and 316 MW. The resistance between any two nodes (MGs or macro-station) and the transformer loss are set, respectively, to typical values of $R = 0.2$ ohm per km and $\beta = 0.02$ [7]. The voltages U_0 and U_1 are set, respectively, to 50 kV and 22 kV which are practical values in a variety of smart grid distribution networks [7]. The price is set to $w_i = 1,\ \forall i \in \mathcal{N}$. All statistical results are averaged over a large number of runs with different random positions and variances for the MGs.

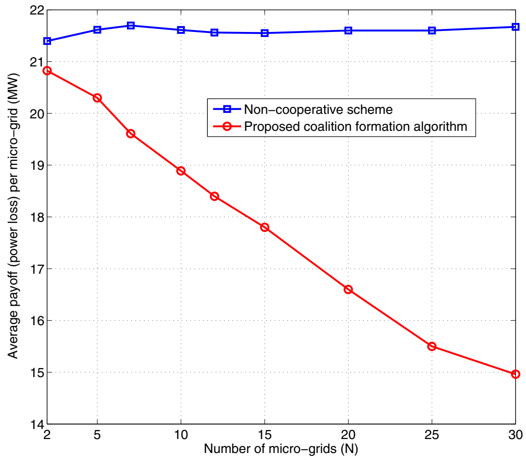

Fig. 3. Performance assessment in terms of the average power loss per MG resulting from the proposed algorithm as the number of MGs N varies.

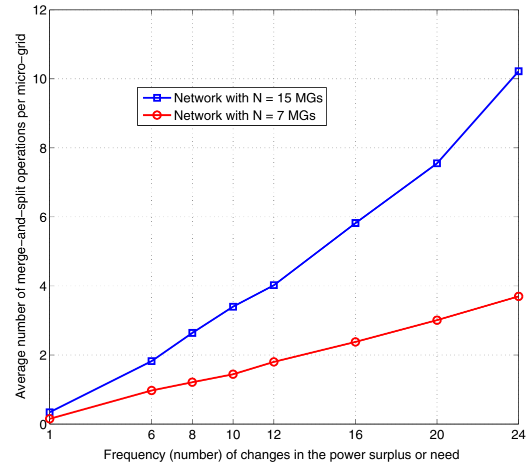

Fig. 4. Average number of merge-and-split operations per MG versus the frequency of changes in the power needs of the MGs over a period of 24 hours.

In Fig. 2, we show a snapshot of the network partition resulting from coalition formation in a distribution network with $N = 10$ randomly deployed MGs. In Fig. 2, we see that, based on their location and power needs, the MGs can cooperate and form coalitions. For example, by forming coalition $S_2 = \{1, 2, 3\}$, MGs $1, 2,$ and 3 can improve their payoffs from $\phi_1(\{1\}) = -8.65$, $\phi_2(\{2\}) = -2$, and $\phi_3(\{3\}) = -1.4$ to $\phi_1(S_2) = -4.3$, $\phi_2(S_2) = -1$, and $\phi_3(S_2) = -0.7$, respectively (about 50% of improvement per MG). Also, Fig. 2 shows how, inside each coalition, the MGs organize into sellers and buyers while distributing their power locally and with the macro-station, if needed. As an example, in the formed coalition $S_1 = \{4, 5, 7\}$, MG 4 is a seller, having a surplus power of 134.3 MW and, MGs 5 and 7 are two buyers that need, respectively, a power of 35.4 MW and 33.2 MW. During power transfer, first, MG 4 satisfies the power needs of the buyers inside S_1 by transferring 28.3% and 25% of its power, respectively, to MGs 5 and 7. The remaining 46.7% of the power surplus of MG 4 is transferred to the macro-station. Similar results can also be seen for the rest of the coalitions.

Fig. 3 shows the average power loss per MG resulting from the proposed coalition formation algorithm and the non-cooperative case as the number of MGs N in the network varies. Fig. 3 shows that, as the number of MGs N increases, the performance of the proposed algorithm improves as the resulting power loss per MG decreases while that of the non-cooperative scheme remains comparable. This is due to the fact that, for the proposed algorithm, as N increases, it becomes more likely for the MGs to find cooperating partners with which they can trade power, thereby decreasing the losses and increasing the advantage of coalition formation. In Fig. 3, we also observe that, compared to the non-cooperative case, the proposed algorithm has a significant performance advantage, in terms of average power loss per MG, which is increasing with N and reaching up to 31% of loss reduction (at $N = 30$ MGs) relative to the non-cooperative case.

In Fig. 4, we plot the average number of merge-and-split operations per MG versus the frequency of changes in the power needs of the MGs, over a period of 24 hours, for $N = 7$ MG and $N = 15$ MGs. Fig. 4 shows that, as the dynamics of the environment become faster, i.e., the frequency of changes increases, the MGs require a higher number of merge-and-split operations to adapt the network structure to this change. For example, while 2.7 merge-and-split operations are required when the power needs change 6 times every 24 hours (for $N = 15$ MGs), this number increases to 10.2 merge-and-split operations per MG when the power needs change roughly every hour. In addition, by comparing the average number of merge-and-split iterations between $N = 7$ MGs and $N = 15$ MGs in Fig. 4, we can observe that, as the network size grows, a larger number of operations is needed to self-adapt the structure to the change in the power requirements.

VI. Conclusions

In this paper, we have proposed novel cooperative strategies for MG distribution networks which are expected to be a key component in the emerging smart grid technology. The proposed approach, based on coalitional game theory, allows a number of MGs to cooperate and form coalitions, in order to minimize the costs incurred by the losses of power over the distribution lines. In this context, we have formulated a coalition formation game between the MGs and we have proposed an algorithm for forming the coalitions. The proposed algorithm allows the MGs to make distributed decisions on whether to form or break coalitions, while minimizing their utility function that accounts for the power loss costs. In addition, we have shown how the proposed algorithm enables the MGs to adjust their coalition formation decision so as to meet variations in their power surplus or needs. Results show that the proposed coalitional game solution yields a significant reduction in the power losses in the distribution network.

References

[1] ISO New England Inc., "Overview of the smart grid: Policies, initiatives and needs," Feb. 2009.

[2] N. Hatziargyriou, H. Sano, R. Iravani, and C. Marnay, "Microgrids: An overview of ongoing research development and demonstration projects," *IEEE Power and Energy Magazine*, vol. 27, pp. 78–94, Aug. 2007.

[3] S. Galli, A. Scaglione, and Z. Wang, "Power line communications and the smart grid," in *Proc. International Conference on Smart Grid Communications*, Gaithersburg, MD, USA, Oct. 2010.

[4] G. Bag, R. Majumder, and K. Kim, "Low cost wireless sensor network in distributed generation," in *Proc. International Conference on Smart Grid Communications*, Gaithersburg, MD, USA, Oct. 2010.

[5] J. A. P. Lopes, C. L. Moreira, and A. G. Madureira, "Defining control strategies for microgrids islanded operation," *IEEE Transactions on Power Systems*, vol. 21, no. 2, pp. 916–924, May 2006.

[6] H. Li and W. Zhang, "QoS routing in smart grid," in *Proc. IEEE Global Commun. Conf.*, Miami, FL, USA, Dec. 2010.

[7] J. Machowski, J. W. Bialek, and J. R. Bumby, *Power Systems Dynamics: Stability and Control*. New York, USA: Wiley, 2008.

[8] R. B. Myerson, *Game Theory, Analysis of Conflict*. Cambridge, MA, USA: Harvard University Press, Sep. 1991.

[9] W. Saad, Z. Han, M. Debbah, A. Hjørungnes, and T. Başar, "Coalitional game theory for communication networks: A tutorial," *IEEE Signal Processing Mag.*, vol. 26, no. 5, pp. 77–97, Sep. 2009.

[10] K. Apt and A. Witzel, "A generic approach to coalition formation," in *Proc. of the Int. Workshop on Computational Social Choice (COMSOC)*, Amsterdam, the Netherlands, Dec. 2006.