The Mechanics of Identifying Pure Strategy Nash Equilibria

Finding the Nash equilibrium for any game involves two stages. *First*, we identify each player's optimal strategy in response to what the other players might do. This involves working through each player in turn and determining their optimal strategies. This is done for every combination of strategies by the other players. *Second*, a Nash equilibrium is identified when all players are playing their optimal strategies simultaneously.

Strictly speaking, the above methodology only identifies pure-strategy Nash equilibria. It does not identify mixed-strategy Nash equilibria. A pure-strategy equilibrium is where each player plays one specific strategy. A mixed-strategy equilibrium is where at least one player in the game randomizes over some or all of their pure strategies. This means that players place a probability distribution over alternative strategies. For example, players might decide to play each of two available pure strategies with a probability of 0.5, and never play any other strategy. A pure strategy is therefore a restricted mixed strategy with a probability of one given to the chosen strategy, and zero to all the others. The concept of mixed-strategy Nash equilibrium is discussed later in this section.

To illustrate the two-stage methodology for finding a (pure-strategy) Nash equilibrium we apply it to the prisoners' dilemma game. This is shown in Fig. 2.7.

		Prisoner 2	
		Confess	Don't confess
Prisoner 1	Confess	$\underline{-6}$,$\underline{-6}$	$\underline{0}$,−9
	Don't confess	−9,$\underline{0}$	−1,−1

Fig. 2.7 The Nash Equilibrium of the Prisoners' Dilemma Game

Stage One.
We first need to identify the optimal strategies for each prisoner, dependent upon what the other prisoner might do. If prisoner 1 expects prisoner 2 to confess, then prisoner 1's best strategy is also to confess (−6 is better than −9). This is shown in Fig. 2.7 by underlining this pay-off element for prisoner 1 in the cell corresponding to both prisoners confessing. If prisoner 1 expects prisoner 2 not to confess, then prisoner 1's best strategy is still to confess (this time 0 is better than −1). Again we show this by underlining this pay-off element for prisoner 1. The same analysis is undertaken for prisoner 2 and his best strategy pay-offs are underlined.

Stage Two.
Next we determine whether a Nash equilibrium exists by examining the occurrence of the previously identified optimal strategies. If all the pay-offs in a cell are underlined, then that cell corresponds to a Nash equilibrium. This is true by definition, since in a Nash equilibrium all players are playing their optimal strategy given that other players also play their optimal strategies. In the prisoners' dilemma game only one cell has all its elements underlined. This corresponds to both prisoners confessing, and so this is the unique Nash equilibrium for this game.