

# Game Theory Methods in Microgrids

Petros Aristidou, Aris Dimeas, and Nikos Hatziargyriou

School of Electrical and Computer Engineering  
National Technical University of Athens  
Zografou Campus  
15780 Athens  
Greece

`el05614@mail.ntua.gr, {adimeas, nh}@power.ece.ntua.gr`

**Abstract.** Game theory is a branch of applied mathematics that is, most notably, used in economics as well as in engineering and other disciplines. Game theory attempts to mathematically capture behaviour in strategic situations, in which an individual's success in making choices depends on the choices of others. The microgrid encompasses a portion of an electric power distribution system that is located downstream of the distribution substation, and it includes a variety of DER units and different types of end users of electricity and/or heat. Microgrids promote the use of new technologies, under the general Smart Grids' umbrella, in order to achieve more efficient use of electric energy, better protection, improved control and provide services to the users. For the materialization of the infrastructure needed to implement this model, engineers have nominated technologies like smart agents, distributed computing, smart sensors and others, as well as, a solid and fast communication infrastructure. In this decentralized environment, multiple decision making participants interact, each striving to optimize its own objectives. Thus, a game theoretic approach is attempted to model and analyse the strategic situations arising from the interactions.

**Key words:** Microgrid, game theory, decision makers

## 1 Introduction

The Microgrid encompasses a portion of an electric power distribution system that is located downstream of the distribution substation, and it includes a variety of DG units and different types of end users of electricity and/or heat. DG units include both distributed generation (DG) and distributed storage (DS) units with different capacities and characteristics. The electrical connection point of the microgrid to the utility system, at the low-voltage bus of the substation transformer, constitutes the microgrid point of common coupling (PCC). The microgrid serves a variety of customers, e.g., residential buildings, commercial entities, and industrial parks. Depending on the type and depth of penetration of Distributed Generation (DG) units, load characteristics and power quality constraints, and market participation strategies, the required control and

operational strategies of a microgrid can be significantly, and even conceptually, different than those of the conventional power systems [7] [4] [8].

The model introduced here is based on the general Multi-agent Microgrid model. The economic operation of the Microgrid is undertaken by a company (ESCO/ Aggregator) which has the responsibility of meeting the energy demand of the consumers, buying energy from the distributed generation units and the wholesale markets. All of the distributed generator units and some of the consumers are equipped with Local Controllers (Smart Agents) capable of receiving signals from the ESCO/ Aggregator and alternating the distributed generation or the consumption respectively. The LCs aim in maximizing the revenue of the DG units or consumers, locally, by following a predefined action course, or a *strategy*. In other words, we have multiple decision makers interacting, with each striving to achieve its goal.

## 2 Game Theoretic Analysis

Game Theory is called upon to model and analyse situations where multiple decision makers interact, with each participant trying to achieve its objective. In this way, Game Theory attempts to mathematically capture behaviour in strategic situations. The primary notion deriving from the Game Theoretic analysis of a strategic situation is the Nash Equilibrium.

### 2.1 Nash Equilibrium

One way to motivate the definition of Nash equilibrium is to argue that if game theory is to provide a unique solution to the game theoretic problem then the solution must be a Nash equilibrium, in the following sense. Suppose that game theory makes a unique prediction about the strategy each player will choose. In order for this prediction to be correct, it is necessary that each player be willing to choose the strategy predicted by the theory. Thus, each player's predicted strategy must be that player's best response to the predicted strategies of the other players. Such prediction could be called *strategically stable* or *self-enforcing*, because no single player wants to deviate from his or her predicted strategy [12]. Such a prediction is called Nash equilibrium:

**Definition 1** *The strategies  $(s_1^*, s_2^*, \dots, s_i^*, \dots, s_n^*)$  are a Nash equilibrium if, for each player  $i$ ,  $s_i^*$  is player  $i$ 's best response to the strategies specified for the  $n - 1$  players,  $(s_1^*, s_2^*, \dots, s_{i-1}^*, s_{i+1}^*, \dots, s_n^*)$ :*

$$R_i(s_1^*, s_2^*, \dots, s_i^*, \dots, s_n^*) \geq R_i(s_1^*, s_2^*, \dots, s_i, \dots, s_n^*)$$

that is,  $s_i^*$  solves:

$$\max R_i(s_1^*, s_2^*, \dots, s_i, \dots, s_n^*)$$

where  $R_i$  denotes the revenue of  $i$  player as function of the strategies followed by every player.

## 2.2 Subgame Perfect Nash Equilibrium

A subgame perfect Nash equilibrium is a reinforcement of the Nash equilibrium notion for coping with dynamic games (that is games of more than one rounds). It's an equilibrium such, that players' strategies constitute a Nash equilibrium in every subgame of the original game. It may be found by **backward induction**, an iterative process for solving finite extensive form or sequential games. First, one determines the optimal strategy of the player who makes the last move of the game. Then, the optimal action of the next-to-last moving player is determined taking the last player's action as given. The process continues in this way backwards in time until all players' actions have been determined [11] [3].

## 2.3 Computing Nash Equilibria

The analysis of a strategic interaction lies with the problem of finding the Nash equilibrium of a given game. This notorious problem has been described as the "most fundamental computational problem" at the interface of computer science and game theory (Papadimitriou, 2001). Despite several decades of research into this problem it remains thorny; its precise computational complexity is unknown, and new algorithms have been relatively few and far between.

An equilibrium concept should be efficiently computable if it is to be taken seriously as a prediction of what participants will do. Because, if computing a particular kind of equilibrium is an intractable problem, of the kind that take lifetimes of the universe to solve on the world's fastest computers, it is ludicrous to expect that it can be arrived at in real life. This consideration suggests the following important question: *Is there an efficient algorithm for computing a Nash equilibrium?*

Although in many strategic interactions (games) the answer is negative, in some games Nash Equilibrium can be compute directly or approximated with the use of an heuristic method.<sup>1</sup>

## 2.4 Use of Nash Equilibrium solutions

Another question one may ask is the utility of the results arising from the game theoretic analysis of a situation. The results may be used in two ways: Descriptive or Prescriptive.

---

<sup>1</sup> A heuristic method is used to come to a solution rapidly that is hoped to be *close* to the best possible answer, or 'optimal solution'

**Descriptive** The first known use is to describe how individuals behave. Finding the equilibria of games we can predict how the actual participants will behave when confronted with situations analogous to the game being studied. In other words, game theoretic analysis gives us an insight to the strategic situations arising [5]. In the microgrid model we can investigate how the players will act in different situations. In this way we can predict technical or economical disadvantages of a model and decide whether to modify or discard it.

**Prescriptive or normative** On the other hand, game theory can be used not as a predictive tool for the behaviour of the participants, but as a suggestion for how individuals ought to behave. Since a Nash equilibrium of a game constitutes one's best response to the actions of the other players, playing a strategy that is part of a Nash equilibrium seems appropriate. Normative aspects of game theory may be sub-classified using various dimensions. One is whether we are advising a single player (or group of players) on how to act best in order to maximize pay-off to himself, if necessary at the expense of the other players; and the other is advising society as a whole (or a group of players) of reasonable ways of dividing pay-off among themselves [5].

The distinction between the descriptive and the normative modes is not as sharp as might appear, and often it is difficult to decide which of these two we are talking about. For example, when we use game or economic theory to analyse existing norms, is that descriptive or is it normative? We must also be aware that a given solution concept will often have both descriptive and normative interpretations, so that one will be talking about both aspects at the same time. Indeed, there is a sense in which the two aspects are almost tautologically the same [1].

### 3 Application to the Microgrid Model

An example of modelling and analysing a Microgrid model using game theory is presented below. We model the interaction between the LCs of consumers, DG units and the ESCO as a Game Theory game and analyse it using game theoretic tools and methods.

#### 3.1 Microgrid Model

The model used is shown in the schematic diagram 1. All Consumers and Distributed Generators are equipped with Smart Meters. With this technology, the application of a variable pricing scheme for the consumers and variable payment scheme for the distributed generation is possible. In other words, we have price-responsive demand and generation inside the Microgrid [2]. Moreover, the Smart Meters commute real-time measurements of the consumption of each individual consumer and the generation of each individual DG to the DNO/DSO. The ESCO/Aggregator is informed of these values from the DNO/DSO and can use this information to define the pricing in the Microgrid.

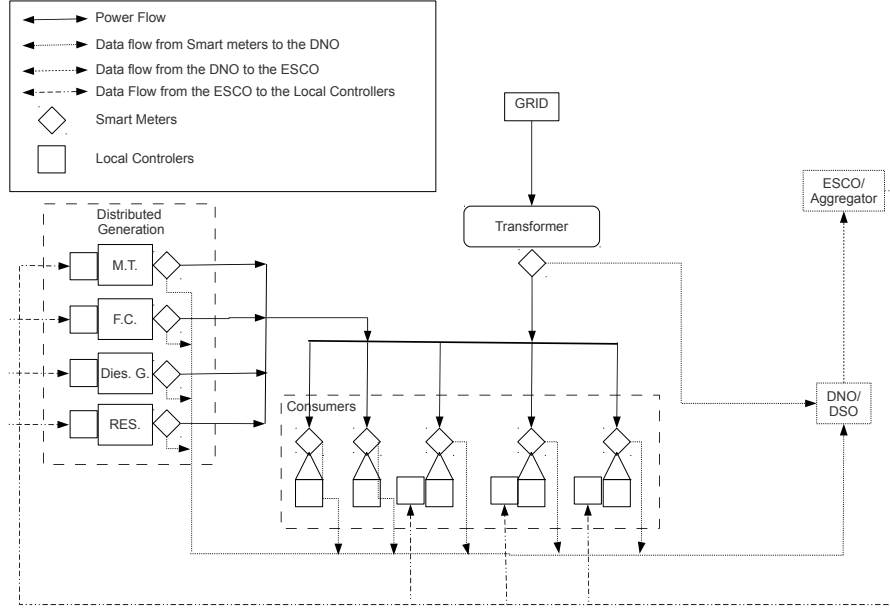


Fig. 1. Microgrid Model

### 3.2 Game Description

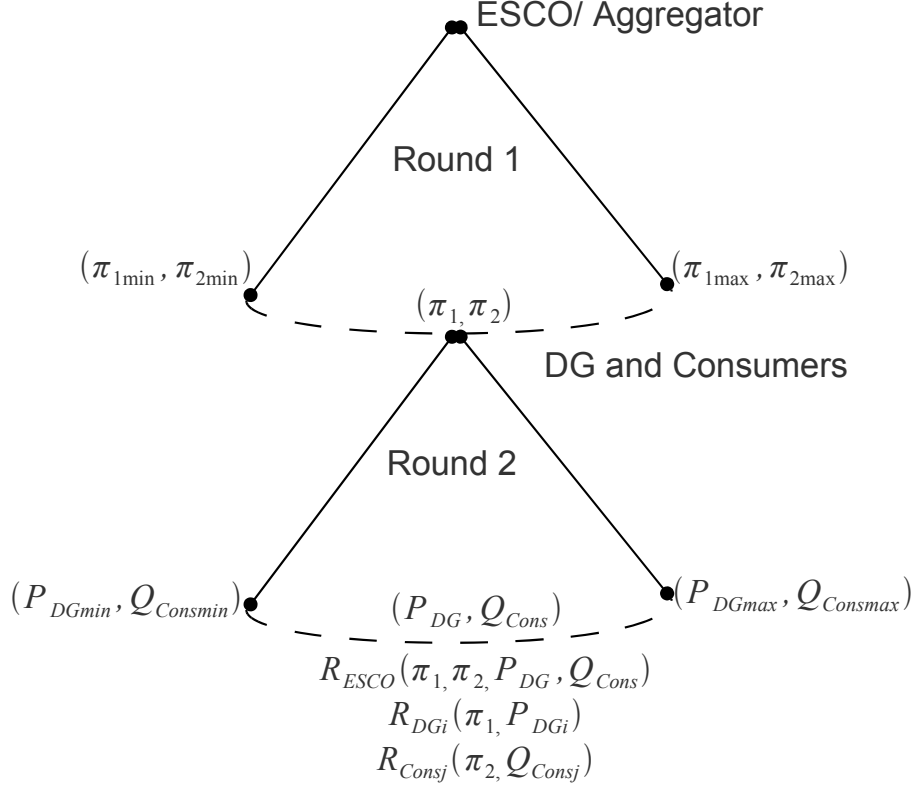
The players defined by the game consist of the ESCO/ Aggregator, the DG units and the consumers equipped with LCs. We assume that all players seek to maximize their revenue. The game is described as follows: The ESCO/ Aggregator plays first and chose its action, which consists of pair of values  $(\pi_1, \pi_2)$ , representing the variable pricing values. Following, each DG unit, having received the value  $\pi_1$ , acts on it modifying its generation  $P_{DG}$ . At the same time, each consumer, having received the value  $\pi_2$ , acts on it modifying its consumption  $Q_{LC}$  [10]. The game then ends and each player receives his revenue as a function of all the actions in the game:

$$R_{player}(\pi_1, \pi_2, P_{DG_1}, \dots, P_{DG_n}, Q_{LC_1}, \dots, Q_{LC_m})$$

### 3.3 Modelling as a Game

The Microgrid model can be examined under the class of games called "Dynamic Games of Perfect Information". Dynamic games are games with sequential move order, meaning, some players move or act only after having observed someone else's move or action. Perfect Information denotes that at every round of the game, the player whose turn is to chose an action knows the actions other players have chosen before him. The game formed by the Microgrid model can be of complete information, meaning that every players revenue function

is common knowledge, or incomplete information [6]. We can see the extensive form representation of the game in 2.



**Fig. 2.** Extensive form representation of the game

### 3.4 Game Analysis

As noted previously, the goal of each player is to maximize his revenue function. For doing so, the ESCO/Aggregator has to choose the best price values  $(\pi_1, \pi_2)$  from the strategy space of feasible values  $([\pi_{1min}, \pi_{1max}], [\pi_{2min}, \pi_{2max}])$ . Each DG unit chooses the best power generation  $P_{DG}(\pi_1)$  from the strategy space of feasible values  $[P_{DGmin}, P_{DGmax}]$ . Each consumer chooses the best consumption level  $Q_{LC}(\pi_2)$  from the strategy space of feasible values that is constructed according to the controllable loads available to the LC.

Using the *backwards-induction* method we begin the analysis from the second round of the game (schematics 2). DG units and consumers will face the following optimization problems:

$$\max\{R_{DG_i}(\pi_1)\} \quad \text{and} \quad \max\{R_{Consumer_i}(\pi_2)\}$$

respectively.

Assuming that for every value  $\pi_1$  the DG problem has a unique solution  $P_{DG}(\pi_1)$  and for every value  $\pi_2$  the consumer's problem has a unique solution  $Q_{LC}(\pi_2)$ . Then, at the first stage of the game, the ESCO/ Aggregator has to solve the problem:

$$\max\{R_{ESCO}(\pi_1, \pi_2, P_{DG}(\pi_1), Q_{LC}(\pi_2))\}$$

Assuming the ESCO/ Aggregator also has a unique solution, denoted  $\{\pi_1^*, \pi_2^*\}$ , then the solution  $\{\pi_1^*, \pi_2^*, P_{DG_1}^*, \dots, P_{DG_n}^*, Q_{LC_1}^*, \dots, Q_{LC_m}^*\}$  is called *backward-induction outcome* of the game. Although this kind of games have several Nash Equilibria, the only subgame-perfect Nash equilibrium is the equilibrium associated with backward-induction outcome [6].

## 4 Concluding Remarks

Situations of strategic interactions always occur when multiple decision makers, like smart agents, smart devices and local controllers, coexist within a system. Such decision makers are programmed to strive to optimize a local revenue function taking into consideration the system's state and the actions of others. The reasons for integrating these distributed decision makers (like smart agents) into the Smart Grids have been thoroughly presented in many papers [8]. Some of the reasons being the large amount of data needed to be processed otherwise, making the system more robust and reliable; and providing various services to the system's users. The strategic interaction of many decision makers in the Smart Grid, leads to the need of new ways of mathematically capturing and analysing the occurring situations.

The strategic interaction of distributed decision makers has been examined extensively by computer science researchers [9]. Game Theory has been proposed to provide the tools and methods for modelling and analysing such situations. Using the knowledge obtained from that research, we can customize the algorithms to solve and predict situations modelled with game theoretic tools in Microgrids and other Smart Grid models. The results from this analysis can be used to compare different variations of a model. We can compare the anticipated profit yielding from applying a certain technology with the estimated cost of implementing and integrating it. Moreover, we can investigate the impact that modifying the protocols governing the model has on them.

## References

1. R.J. Aumann, *What is game theory trying to accomplish*, Frontiers of economics **29** (1985), 28–99.

2. S. Braithwait, *Behavior modification*, Power and Energy Magazine, IEEE **8** (2010), no. 3, 36–45.
3. Camerer Colin, *Behavioral game theory: experiments in strategic interaction*, Russesll Sage Foundation, 2003.
4. A.L. Dimeas and N.D. Hatziargyriou, *Operation of a multiagent system for microgrid control*, IEEE Transactions on Power Systems **20** (2005), no. 3, 1447–1455.
5. John Eatwell, Murray Milgate, and Peter Newman (eds.), *The new palgrave: A dictionary of economics*, ch. 2, p. 46082, Palgrave Macmillan, 1987.
6. R. Gibbons, *Game theory for applied economists*, 1992.
7. F. Katiraei, R. Iravani, N. Hatziargyriou, and A. Dimeas, *MicroGrids Management, control and Operation Aspects of MicroGrids*, IEEE Power Energy Mag **6** (2008), no. 3, 54–65.
8. S.D.J. McArthur, E.M. Davidson, V.M. Catterson, A.L. Dimeas, N.D. Hatziargyriou, F. Ponci, and T. Funabashi, *Multi-agent systems for power engineering applicationsPart II: Technologies, standards, and tools for building multi-agent systems*, IEEE Transactions on Power Systems **22** (2007), no. 4, 1753–1759.
9. S. Parsons and M. Wooldridge, *Game theory and decision theory in multi-agent systems*, Autonomous Agents and Multi-Agent Systems **5** (2002), no. 3, 243–254.
10. Peter C. Reiss and Matthew W. White, *Household electricity demand*, (2002).
11. Mikhael Shor, *Dictionary of game theory terms*, *game theory.net*, online, 07 2010.
12. John von Neumann and Oskar Morgenstern, *Theory of games and economic behavior*, Princeton University Press, 1944.