LAB # 1

Modelling of mechanics of the actuators.

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Contents

1 Introduction

§1.1 Student information

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§1.2 Introduction

In this report, we will solve the following tasks:

Part1. **Mathematical modelling of two-mass mechanism**

Task 1.1. Design a model of the two-mass mechanism without any disturbances (load torques, frictions);

Task 1.2. Add backlash in a model of the two-mass mechanism;

Task 1.3. Add torque of viscous friction in a model of the two-mass mechanism;

Part2. **Mathematical modelling of DC-motor with two-mass mechanism**

Task 2.1. Modelling of the DC-motor with one-mass load;

Task 2.2. Modelling of the DC-motor with two-body mechanism.;

2 Notations

Table 2.1: Notation used in this report

Symbol	Definition
T_L	load torque
J	moment of inertia
ω	angular velocity
b	viscous friction
T_{s12}	spring torque between 1 and 2 objects
ω_{R1}	resonance frequency
V_a	Armature voltage
R_a	winding resistance
L_a	winding inductance
e_a	back-EMF induced by the rotation of the armature winding
k_e	back-EMF constant
k_T	torque constant

⋆ Other notations instructions will be given in the text.

3 Solution of problem

§3.1 Part1.

§3.1.i Task 1.1. Design a model of the two-mass mechanism without any disturbances (load torques, frictions)

Mathematic model of two-mass mechanism

Consider the two-mass system shown in Fig.3.1 below:

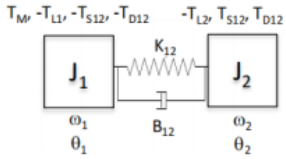


Fig. 3.1. Two-mass system

Analyzing the force of the 2-mass system to obtain a mathematical model of differential equations:

$$\begin{cases} J_1 \frac{d\omega_1}{dt} = T - T_{S12} - b_{12}\left(\omega_1 - \omega_2\right) - T_{L1} \\ \frac{dT_{s12}}{dt} = K_{12}\left(\omega_1 - \omega_2\right) \\ J_2 \frac{d\omega_2}{dt} = T_{s12} + b_{12}\left(\omega_1 - \omega_2\right) - T_{L2} \end{cases} \tag{3.1}$$

The simplified (3.1) model without considering any disturbances (load torques, frictions) is shown below (3.2):

$$\begin{cases} J_1 \frac{d\omega_1(t)}{dt} = T(t) - T_{S12}(t) \\ \frac{dT_{s12}(t)}{dt} = K_{12}\left(\omega_1(t) - \omega_2(t)\right) \\ J_2 \frac{d\omega_2(t)}{dt} = T_{s12}(t) \end{cases} \tag{3.2}$$

Mathematic model of the two-mass mechanism in Simulink

Model (3.2) in Simulink as shown in Fig.3.2

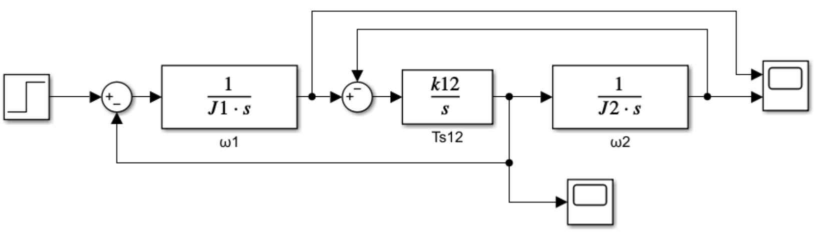
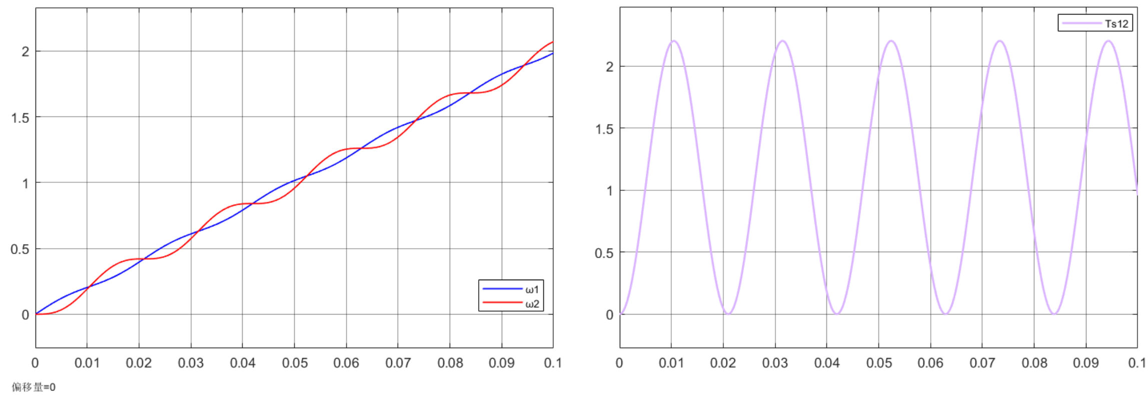


Fig. 3.2. Math model of the two-mass mechanism in Simulink

transient response

The transient response of ω_1(t), ω_2(t), T_{s12}(t) by the step reference signal T with value $0.1T_{\text{rated}}$ (at $T_{L1} = 0$, $T_{L2} = 0$).in the system obtained by the simulated system is shown in Fig.3.3:



(a) transient response of $\omega_1(t), \omega_2(t)$ **(b)** transient response of $T_{s12}(t)$

Fig. 3.3. Transient Response

Bode diagram

The Bode diagram of the system obtained by the simulation system is as follows Fig.3.5:

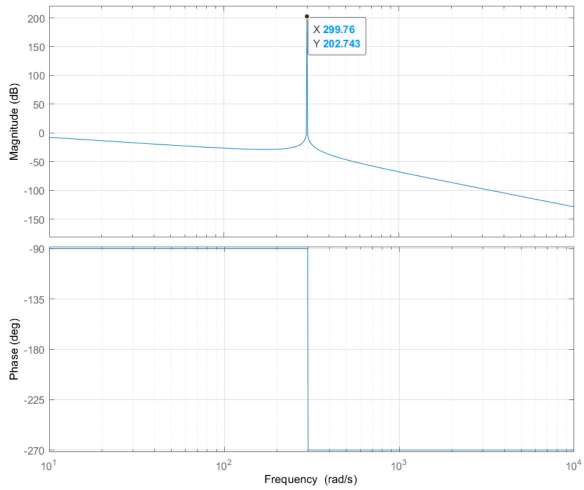


Fig. 3.4. Bode diagram of the two-mass mechanism

Calculated Parameters of Transient

1) **The resonance frequency:**

$$\omega_{R1} = \sqrt{k_{12}\frac{J_1 + J_2}{J_1 J_2}} = 299,7698c^{-1} \tag{3.3}$$

2) **the average angular acceleration:**

$$\varepsilon_{\text{av}} = \frac{T}{\mathrm{J}_1 + \mathrm{J}_2} = \frac{0.1 \times \mathrm{T}_{\text{rated}}}{J_1 + J_2} = 20\text{rad/s}^2 \tag{3.4}$$

3) **The magnitudes of bodies fluctuation:**

$$A_1 = \frac{J_2\varepsilon_{av}}{J_1\omega_{R1}} = 0,0201\text{rad/s}$$
$$A_2 = \frac{\varepsilon_{av}}{\omega_{R1}} = 0,0669\text{rad/s} \quad (3.5)$$

parameters of transient got by simulation

1) **The resonance frequency:**

From the Bode plot in Fig.3.4, it can be seen that the resonance frequency of the system is:

$$\omega_{R1}^{exp} = 299,76c^{-1} \quad (3.6)$$

2) **the average angular acceleration:**

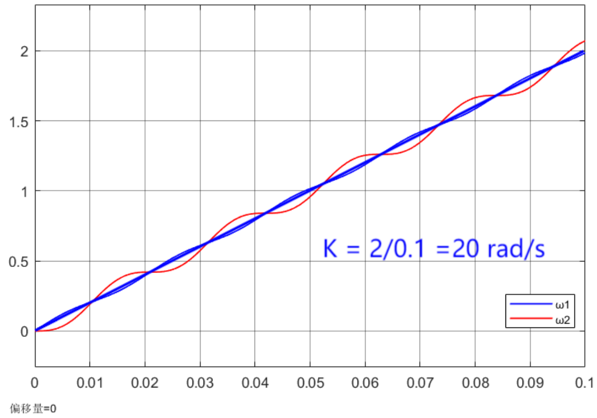


Fig. 3.5. transient response of $\omega_1(t), \omega_2(t)$

$$\varepsilon_{\text{av}}^{exp} = K = 20\text{rad/s}^2 \quad (3.7)$$

3) **The magnitudes of bodies fluctuation:**

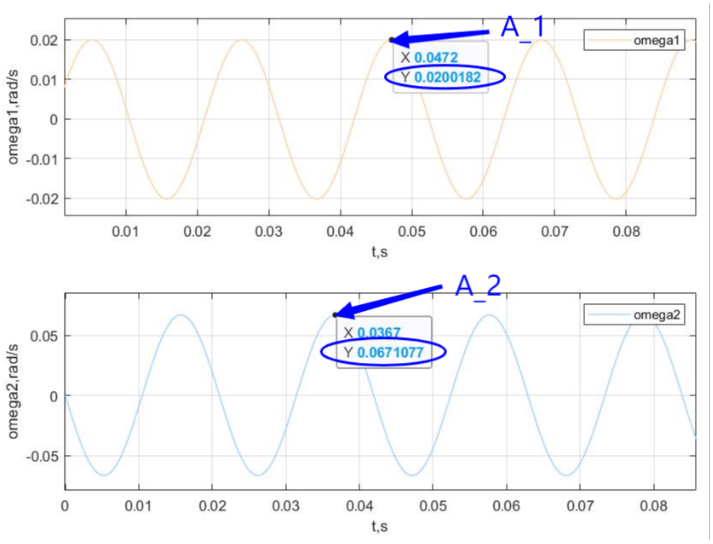


Fig. 3.6. The oscillation process of $\omega_1(t), \omega_2(t)$

$$
\begin{aligned}
A_1^{exp} &= 0.020\text{rad/s} \\
A_2^{exp} &= 0.067\text{rad/s}
\end{aligned}
\tag{3.8}
$$

Conclusion

Compare calculated parameters of transient and parameters got by simulation as shown in Table 3.1 below:

Table 3.1: Output value characteristic calculation

Parameter	calculated value	Experimental values	unit
ω_{R1}	299.7698	299.76	c^{-1}
ε_{av}	20	20	rad/s^2
A_1	0.0201	0.02002	rad/s
A_2	0.0669	0.0671	rad/s

- ⋆ It can be seen from Table 3.1 that the calculated parameters and experimental parameters are basically the same, indicating the accuracy of the model

§3.1.ii Task 1.2. Add backlash in a model of the two-mass mechanism

Mathematic model of the two-mass mechanism in Simulink

The model after adding the rebound to the model in Fig.3.2 is shown in Fig.3.7:

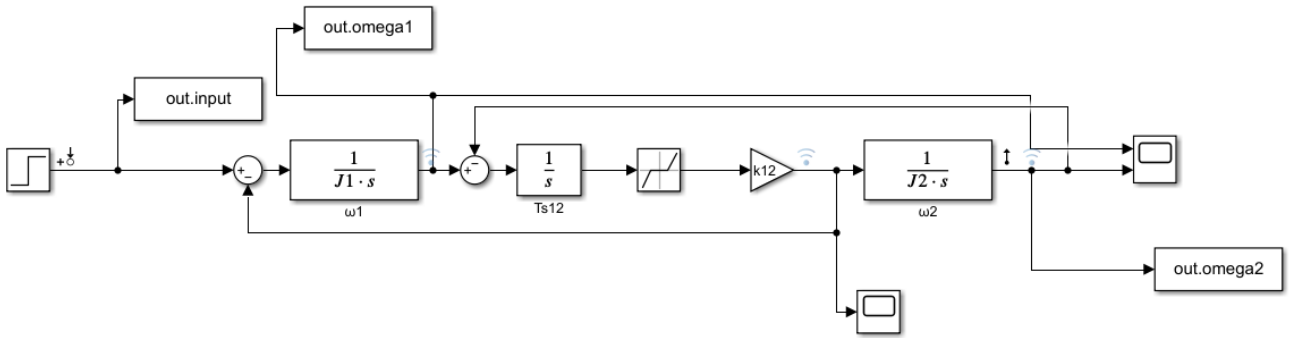
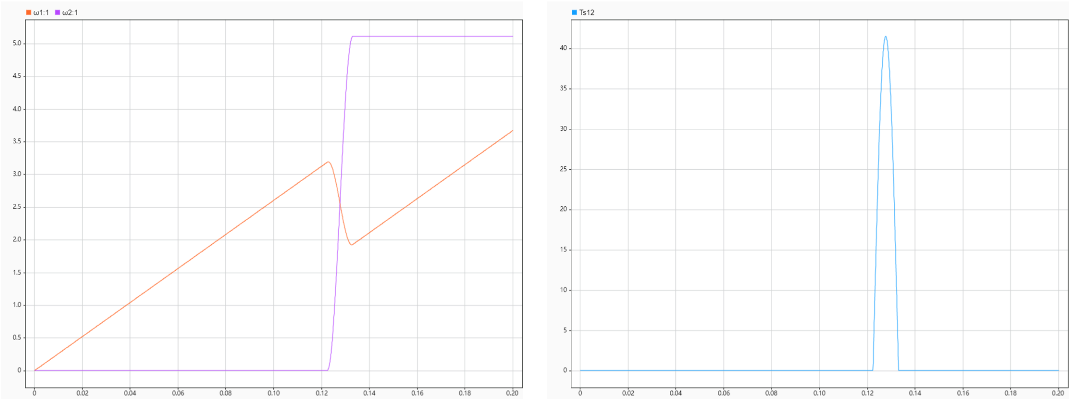


Fig. 3.7. Mathematic model of the two-mass mechanism(Add backlash) in Simulink

transient response

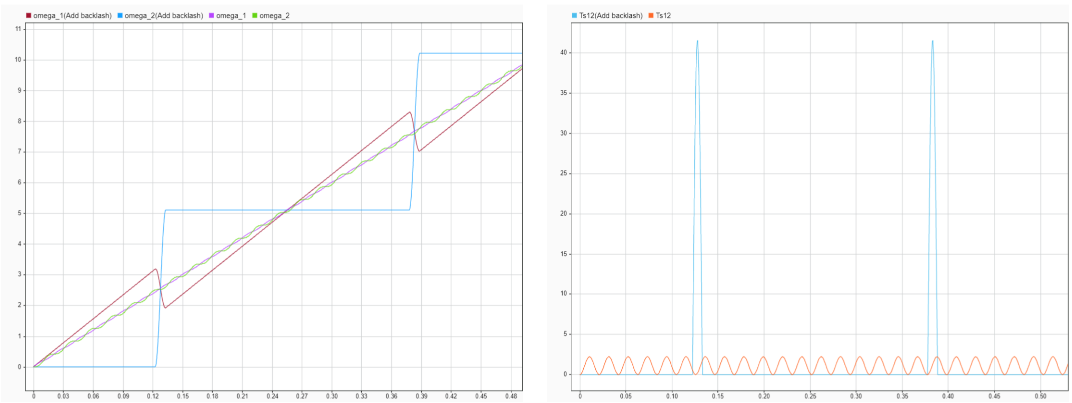
The transient response of ω_1(t), ω_2(t), T_{s12}(t) by the step reference signal T with value $0.1T_{\text{rated}}$ (at $T_{L1} = 0$, $T_{L2} = 0$,Add backlash).in the system obtained by the simulated system is shown in Fig.3.8:



(a) transient response of $\omega_1(t), \omega_2(t)$ **(b)** transient response of $T_{s12}(t)$

Fig. 3.8. Transient Response(Add backlash)

Compare T_{s12}(t), ω_1(t), ω_2(t) in mechanism without and with backlash in gearbox



(a) transient response of $\omega_1(t), \omega_2(t)$ **(b)** transient response of $T_{s12}(t)$

Fig. 3.9. Transient Response(Add backlash)

Conclusion

- ⋆ T_{s12}(t), ω_1(t), ω_2(t) of the model after adding the backlash presents a "dead zone" feature: 0 for a period of time, and concentrated output for the rest of the time.
- ⋆ The overall trend of the model after adding the rebound force does not change: the output signal ω_1(t), ω_2(t) changes by the same value after each cycle (about 0.13s)

§3.1.iii Task 1.3 Design a model of the two-mass mechanism with viscous frictions

Mathematic model of two-mass mechanism

The simplified (3.1) model with viscous frictions without considering any load torques(3.2):

$$\begin{cases} J_1 \frac{d\omega_1}{dt} = T - T_{S12} - b_{12}\left(\omega_1 - \omega_2\right) \\ \frac{dT_{s12}}{dt} = K_{12}\left(\omega_1 - \omega_2\right) \\ J_2 \frac{d\omega_2}{dt} = T_{s12} + b_{12}\left(\omega_1 - \omega_2\right) \end{cases} \tag{3.9}$$

Mathematic model of the two-mass mechanism in Simulink(with viscous frictions)

Model (3.2) in Simulink as shown in Fig.3.10

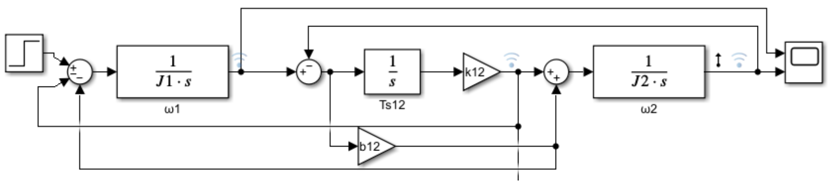


Fig. 3.10. Mathematic model of the two-mass mechanism in Simulink(with viscous frictions)

choose b considering that the oscillation damp in 5 periods.

We want to make the oscillation decay in 5 oscillation cycles, and calculate the viscous friction coefficient as follows:

$$\begin{aligned} a_v &\approx \frac{3\lambda_v \cdot \omega_{R1}}{2\pi} = \frac{3\omega_{R1}}{10\pi} \\ b_{12} &= \frac{2a_v J_1 J_2}{J_1 + J_2} = \frac{6\omega_{R1} J_1 J_2}{10\pi\left(J_1 + J_2\right)} = 2,4211 \end{aligned} \tag{3.10}$$

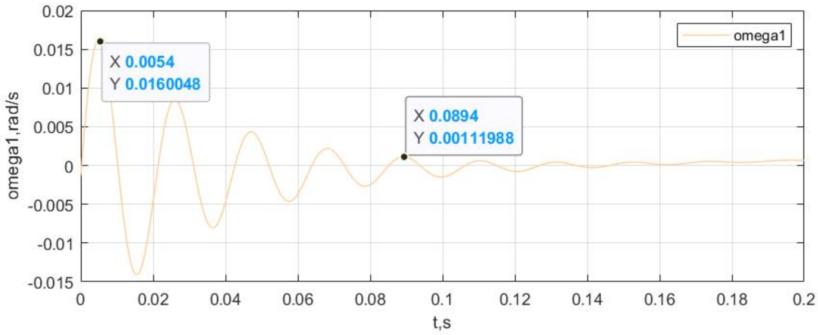
where:

λ_v – logarithmic decrement

a_v - attenuation coefficient

oscillation process

The output signal of the oscillation process of the system after adding the viscosity coefficient is shown in Fig.11:



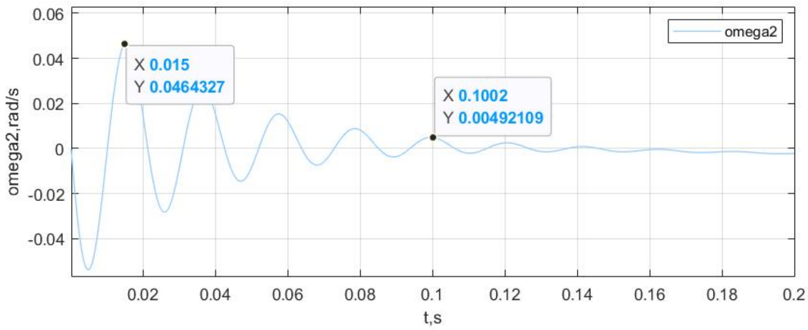


Fig. 3.11. oscillation process(with viscous frictions)

Conclusion

- ★ Adding viscous friction can make the oscillation of the system decay over a period of time

§3.2 Part2.

§3.2.i Task 2.1 Modelling of the DC-motor with one-mass load.

Mathematic model of the DC-motor with one-mass load.

Consider the Equivalent circuit of a DC motor shown in Fig.3.12 below:

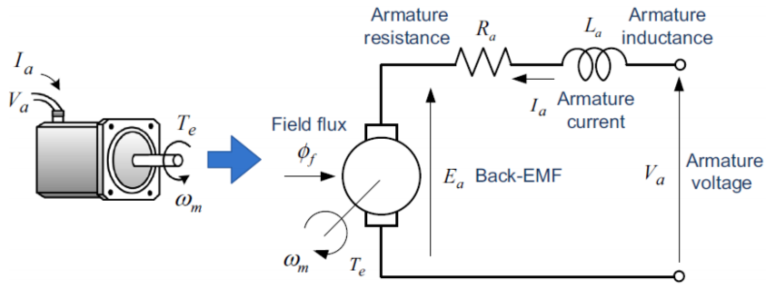


Fig. 3.12. Two-mass system

1. Topological equations (KVL) & Component equations

$$V_a = R_a i_a + L_a \frac{di_a}{dt} + e_a \tag{3.11}$$

2. Electromagnetic force

1) back-EMFe_a

$$e_a = k_e \phi_f \omega_m (\leftarrow e = Blv) \tag{3.12}$$

where i_a is winding current, L_a is winding inductance, R_a is winding resistance, e_a is back-EMF induced by the rotation of the armature winding in a magnetic field,k_e is the back-EMF constant (Vs/rad/W | b).

2) Equation of motion

$$T_e = J\frac{d\omega_m}{dt} + b\omega_m + T_L \tag{3.13}$$

where k_M is the torque constant (Nm/Wb/A). The numerical values of k_T and k_e constants are equal in SI units (the International System of Units), i.e., $k_T = k_e = k$.

3. One-Mass System Equations

$$J_\Sigma \frac{d\omega_1}{dt} = T - T_L \tag{3.14}$$

Combining (3.12), (3.13), (3.14) to get the mathematical model of a single-mass load DC motor:

$$\begin{cases} V_a = R_a i_a + L_a \frac{di_a}{dt} + e_a \\ T_e - T_{L1} = (J_1 + J_2)\frac{d\omega_1}{dt} \\ e_a = k_e \phi_f \omega_1 \\ k_T = k_e = k \\ T_e = k_T \cdot \phi_f \cdot i_a \end{cases} \tag{3.15}$$

Simplified (3.15) model for no load torque:

$$\begin{cases} V_a = R_a i_a + L_a \frac{di_a}{dt} + e_a \\ T_e = (J_1 + J_2)\frac{d\omega_1}{dt} \\ e_a = k_e \phi_f \omega_1 \\ k_T = k_e = k \\ T_e = k_T \cdot \phi_f \cdot i_a \end{cases} \tag{3.16}$$

Mathematic model of the DC-motor with one-mass load in Simulink

Using the (3.16) model to build a simulation model in Simulink is shown below:

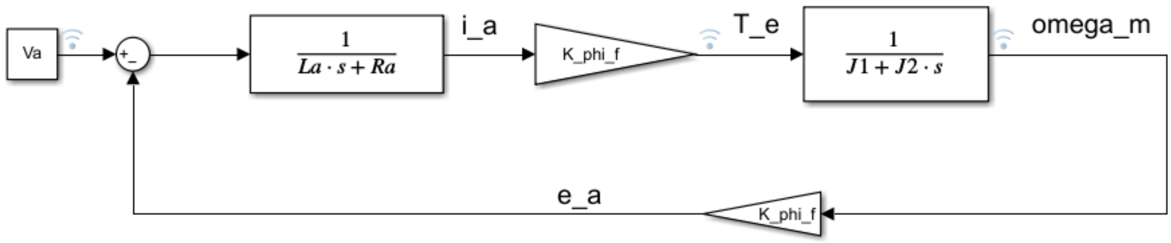


Fig. 3.13. Math model of the two-mass mechanism in Simulink

transient response

The transient response of ω_1(t), T_e by the step reference signal T with value $0.1T_{\text{rated}}$ (at $T_{L1} = 0$, $T_{L2} = 0$).in the system obtained by the simulated system is shown in Fig.3.14:

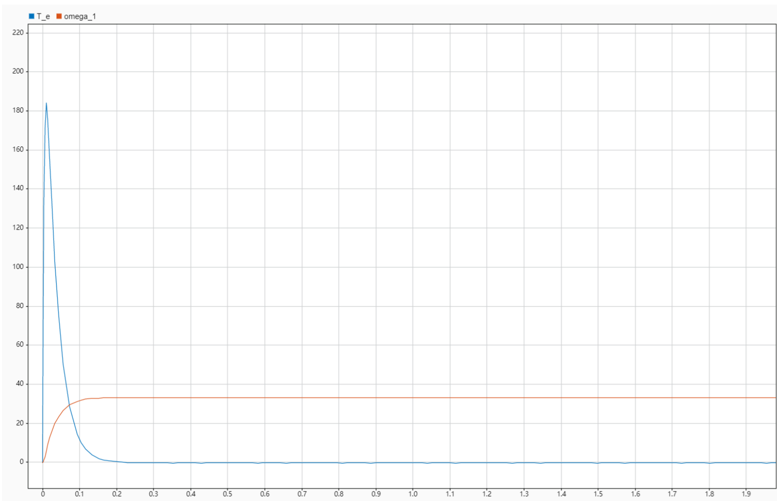


Fig. 3.14. transient response of the DC-motor with one-mass load in Simulink

§3.2.ii Task 2.2 Modelling of the DC-motor with two-body mechanism.

Mathematic model of the DC-motor with two-mass load.

Combining (3.1), (3.11), (3.12),(3.13) to get the mathematical model of a two-mass load DC motor:

$$\begin{cases} V_a = R_a i_a + L_a \frac{di_a}{dt} + e_a \\ e_a = k_e \phi_f \omega_1 \\ T_e = k_T \cdot \phi_f \cdot i_a \\ J_1 \frac{d\omega_1}{dt} = T_e - T_{S12} - T_{L1} - b_{12}(\omega_1 - \omega_2) \\ \frac{dT_{s12}}{dt} = K_{12}(\omega_1 - \omega_2) \\ J_2 \frac{d\omega_2}{dt} = T_{s12} - T_{L2} + b_{12}(\omega_1 - \omega_2) \\ k_T = k_e = k \end{cases} \tag{3.17}$$

Simplified (3.15) model for without any disturbances (load torques, frictions):

$$\begin{cases} V_a = R_a i_a + L_a \frac{di_a}{dt} + e_a \\ e_a = k_e \phi_f \omega_1 \\ T_e = k_T \cdot \phi_f \cdot i_a \\ J_1 \frac{d\omega_1}{dt} = T_e - T_{S12} \\ \frac{dT_{s13}}{dt} = K_{12}(\omega_1 - \omega_2) \\ J_2 \frac{d\omega_2}{dt} = T_{s12} \\ k_T = k_e = k \end{cases} \tag{3.18}$$

Mathematic model of the DC-motor with two-mass load in Simulink

Model (3.18) in Simulink as shown in Fig.3.15

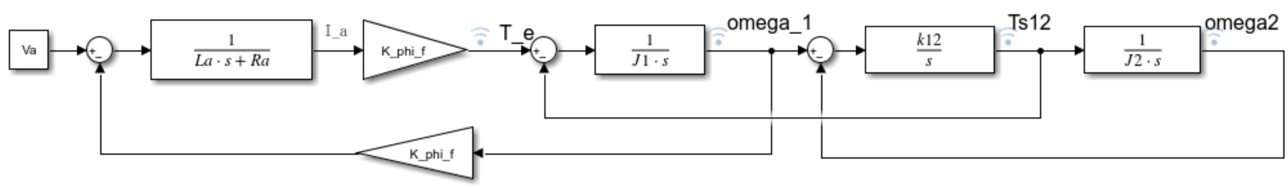


Fig. 3.15. Math model of the two-mass mechanism in Simulink

transient response

The transient response of $\omega_1(\mathrm{t}), \omega_2(\mathrm{t}), T_{\mathrm{s}12}(\mathrm{t}), M$ by the step reference signal T with value $0.1T_{\mathrm{rated}}$ (at $T_{L1} = 0$, $T_{L2} = 0$).in the system obtained by the simulated system is shown in Fig.3.16:

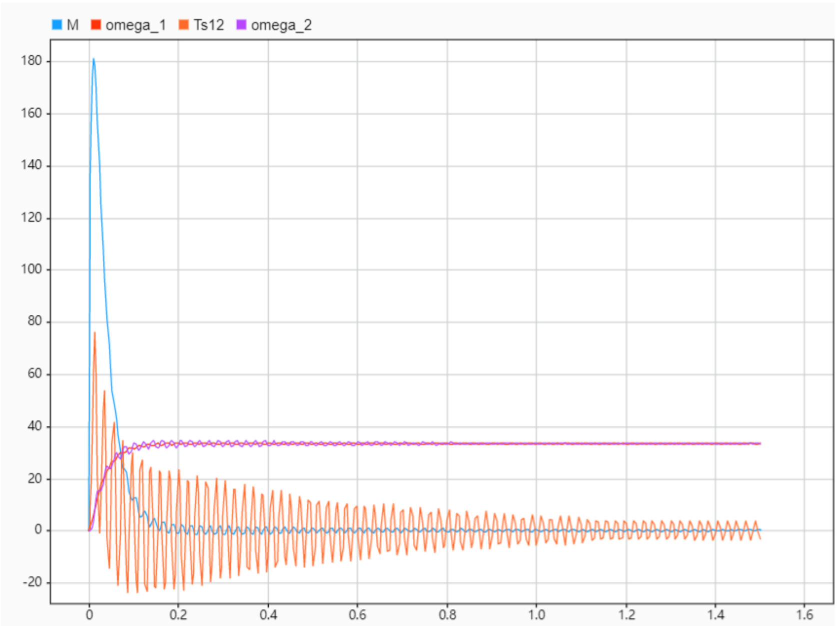


Fig. 3.16. transient response of the DC-motor with two-mass load in Simulink

I

Appendix

A Complete source code

§A.1 Parameter calculation

Problem 1

```
clear all;
J1 = 0.183;
J2 = 0.055;
k12 = 3800;
delta_phi = 0.39;
Va = 400;
Ra = 21.45;
Ta = 0.004;
La = Ta*Ra;
K_phi_f = 12;
T_rated = 47.7;
b12 = 0;
omega_r1 = sqrt(k12*(J1+J2)/(J1*J2))
epson_av = 0.1*T_rated/(J1+J2)
A1 = J2*epson_av/J1/omega_r1
A2 =epson_av/omega_r1
```