Practical Assignment № 1

Adaptive and Robust Control

Design of adaptive control systems for undisturbed objects

variant number : 16

Student Name： Xu Miao, Zhou Haojie

HDU Number： 19322103, 19322233

ITMO number : 293687, 293806

A.Problem statement

Consider the plant

$$\dot{x} = \theta x + u, \qquad (1.1)$$

where x is the the state variable (coincides with the output), u is the the control, θ is the unknown scalar parameter.

The objective is to design a control that will completely compensate for uncertainty θ and provide the following equality:

$$\lim_{t\to\infty}(x_m(t) - x(t)) = \lim_{t\to\infty}\varepsilon(t) = 0, \qquad (1.2)$$

where $\varepsilon = x_m - x$ is the control error, x_m is the output of the reference model

$$\dot{x}_m = -\lambda x_m + \lambda g, \qquad (1.3)$$

g is the reference signal, $\lambda > 0$ is the parameter responsible for the transient performance of closed-loop system after adaptation process. The reference model (1.3) defines the desired tracking performance of the closed-loop system.

B.Theoretical background

- **Non--adaptive Controller**

Let us consider the solution of the adaptive tracking problem, starting with the problem statement and ending with the analysis of the properties of the closed system.

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Problem solution. Under assumption that the parameter θ is known, in view of (1.1) and (1.3) we calculate the time-derivative of the tracking error ε:

$$\dot{\varepsilon} = \dot{x}_m - \dot{x} = -\lambda x_m + \lambda g - \theta x - u. \tag{1.4}$$

The tracking error exponentially tends to zero if the following equality holds:

$$\dot{\varepsilon} = -\lambda\varepsilon.$$

By equating the right-hand sides of the last equation with the model (1.4), we find the desired control law:

$$\begin{gathered} -\lambda x_m + \lambda g - \theta x - u = -\lambda\varepsilon = -\lambda x_m + \lambda x, \\ u = -\theta x - \lambda x + \lambda g. \end{gathered} \tag{1.5}$$

Thus, in the case if the parameter θ is known, the control law (1.5) provides exponential tracking of the plant output. However, this parameter is unknown, hence the control (1.5) is not implementable.

Now, we consider the case of unknown θ and being motivated by the structure (1.5) we design the adjustable controller by replacing θ by its estimate $\hat{\theta}$:

$$u = -\hat{\theta}x - \lambda x + \lambda g. \tag{1.6}$$

- **Adaptive Controller**

Onward, the solution of the problem is reduced to finding the function $\hat{\theta}(t)$, which will ensure the stability of the closed system and achievement of the objective (1.2). In order to find this function $\hat{\theta}(t)$ we derive the tracking error model and the parametric error model and, then apply the method of Lyapunov functions [1-3]. By substituting the last expression into the model (1.1) we get:

$$\dot{x} = \theta x - \hat{\theta}x - \lambda x + \lambda g = \tilde{\theta}x - \lambda x + \lambda g$$

where $\tilde{\theta} = \theta - \hat{\theta}$ is the parametric error.

Let us derive the tracking error model:

$$\dot{\varepsilon} = \dot{x}_m - \dot{x} = -\lambda x_m + \lambda g - \tilde{\theta}x + \lambda x - \lambda g = -\lambda\varepsilon - \tilde{\theta}x$$

or

$$\dot{\varepsilon} = -\lambda\varepsilon - \tilde{\theta}x. \tag{1.7}$$

We choose the Lyapunov function

$$V = \frac{1}{2}\varepsilon^2 + \frac{1}{2\gamma}\tilde{\theta}^2, \tag{1.8}$$

where $\gamma > 0$ is the parameter can calculate its time derivative in view of (1.7):

$$\dot{V} = \frac{1}{2}2\varepsilon\dot{\varepsilon} + \frac{1}{2\gamma}2\tilde{\theta}\dot{\tilde{\theta}} = \varepsilon(-\lambda\varepsilon - \tilde{\theta}x) - \frac{1}{\gamma}\dot{\theta}\dot{\hat{\theta}} = -\lambda\varepsilon^2 - \tilde{\theta}x\varepsilon - \tilde{\theta}\frac{1}{\gamma}\dot{\hat{\theta}}.$$

Selecting the law of parameter estimation as

$$\dot{\hat{\theta}} = -\gamma x\varepsilon \tag{1.9}$$

we have:

$$\dot{V} = -\lambda\varepsilon^2 < 0 \forall\varepsilon \neq 0$$

By taking integral from the $\dot{V}$ we get:

$$V(t) = V(0) - \lambda\int_0^t \varepsilon^2(\tau)d\tau$$

Since V is positive, while $\dot{V}$ is nonpositive, then V is bounded, and hence ε, x, $\tilde{\theta}$ are bounded. Therefore, it follows from the latter equality that ε is a squareintegrable bounded function. Now, applying the Barbalat's lemma [4] we have: $\varepsilon \to 0$ as $t \to \infty$. As a result, it is followed from (1.9) and the boundedness of x that $\dot{\hat{\theta}} \to 0$ as $t \to \infty$.

The algorithm (1.9), which provides the estimation of θ, is called as adaptation algorithm, while the parameter γ is called as adaptation gain.

Thus, the adaptive control algorithm ensuring the control objective consists of the controller (1.6) and the adaptation algorithm (1.9).

Summarizing the results presented we see that for any initial conditions $x(0), \hat{\theta}(0)$ and for any bounded signal g the control algorithm provides the following properties of the closed system:

C.1. boundedness of all signals;

C.2. asymptotic convergence of the tracking error ε to zero;

C.3. exponential convergence of ε to θ under the persistent excitation condition

$$\int_{t}^{t+T} x^2(\tau)d\tau > \alpha I > 0 \tag{1.10}$$

(a particular case of that condition is (3.14), which is almost always satisfied for scalar signals);

C.4. if the condition (1.10) holds, there exists an optimal γ, for which the rate of convergence of $\tilde{\theta}$ to zero is maximum. Small γ lead to slow aperiodic parameters convergence, while for large γ one can observe the oscillations of parameters tuning [5].

C. Experimental part

- **Experimental parameters（Group 16）**

Task No.	Plant parameter θ	Reference model parameter λ	Reference $g(t)$
16	7	1	$\cos t + 3$

From the previous theoretical background we can obtain the equation of state of the system as follows:

$$\dot{x} = \theta x + u, \tag{2.1}$$

where x is the the state variable (coincides with the output), u is the the control, θ is the unknown scalar parameter.

- **Non--adaptive Controller**

1. design the non-adaptive controller

From the previous theoretical background we can obtain the Non--adaptive Controller as follows:

$$\begin{aligned} \dot{x}_m &= -\lambda x_m + \lambda g \\ u &= -\theta x - \lambda x + \lambda g \\ \varepsilon &= x_m - x \end{aligned} \tag{2.2}$$

The simulink scheme is constructed from (2.1) and (2.2) as follows:

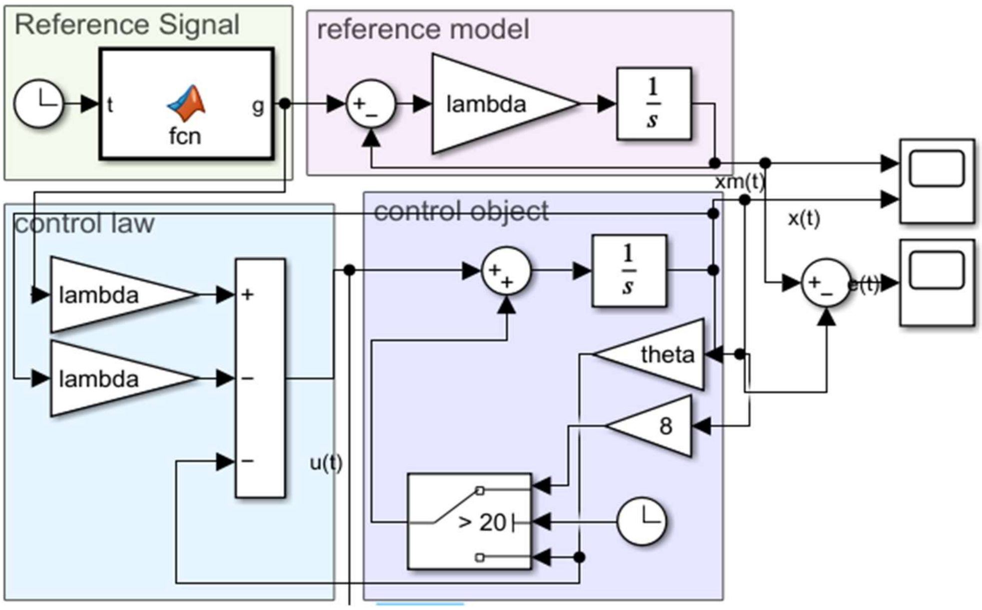

Figure 1： non-adaptive controller simulation scheme

2. Simulation

Make a simulation experiment, in which the plant parameter θ is increased rapidly (controller parameters are not changed)

we change the parameter θ as follows:

$$\theta : 7 \rightarrow 8(t = 20)$$

The simulation results are shown in the following figure:

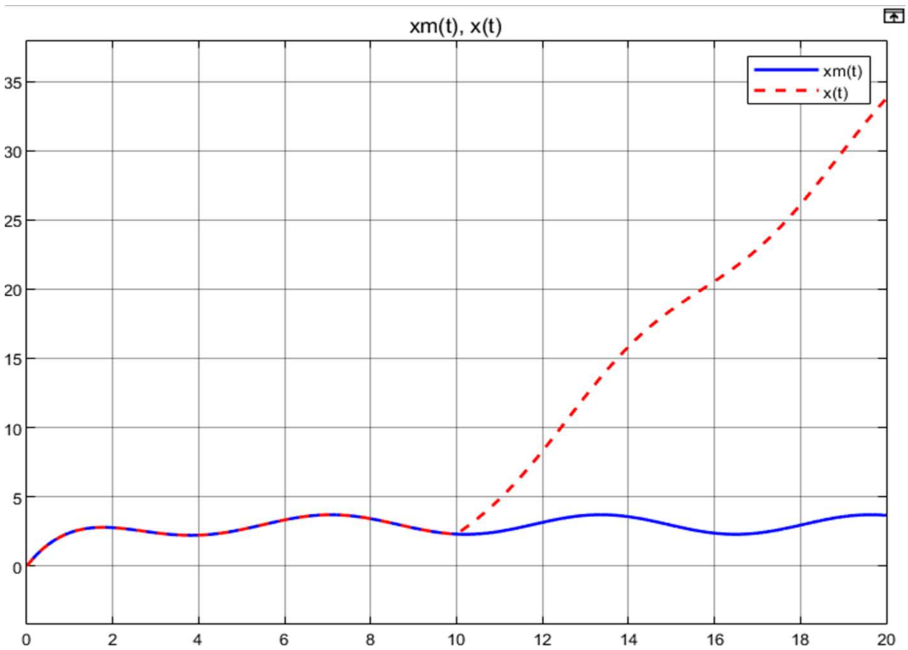

Figure 2： non-adaptive controller simulation result（$x(t), x_m(t)$）

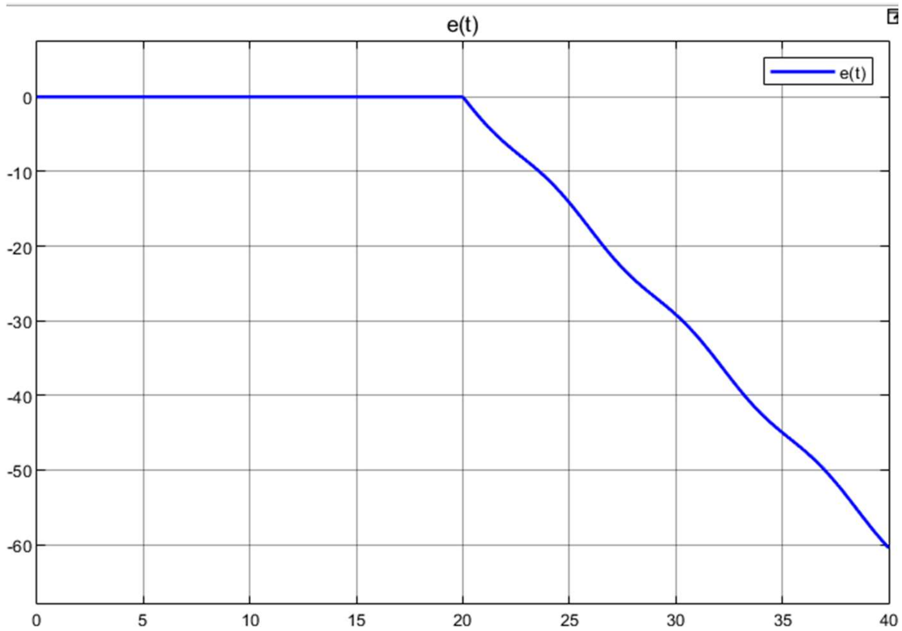

Figure 3： non-adaptive controller simulation result（$\epsilon(t)$）

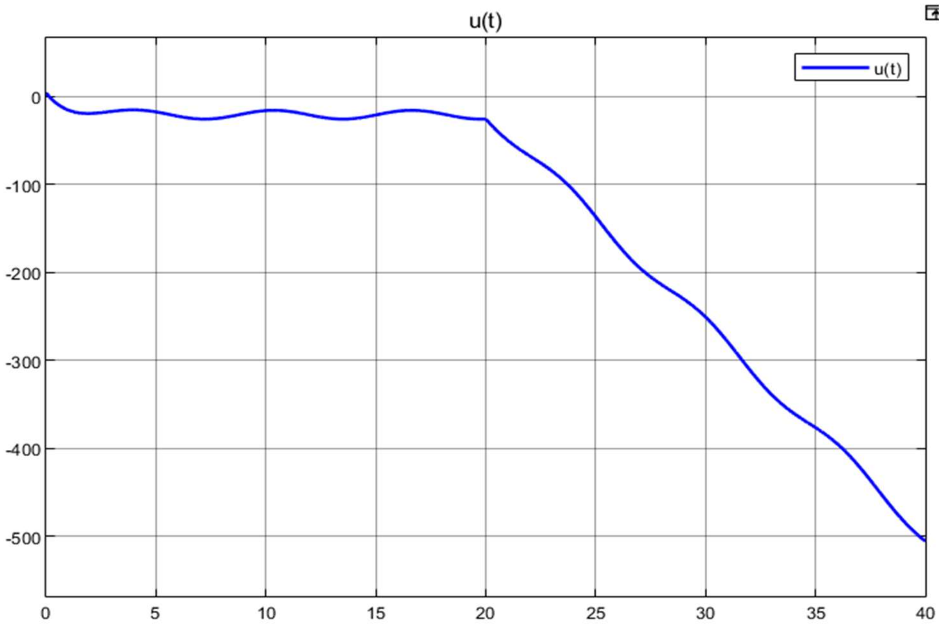

Figure 4： non-adaptive controller simulation result（$u(t)$）

3. Conclusion

It can be seen from Figures 2, 3, and 4 that the closed-loop system using the non-adaptive controller loses stability when the plant parameter θ is increased rapidly.($x(t), \epsilon(t), u(t) \to \infty$)

Adaptive Controller

1. design the Adaptive Controller

From the previous theoretical background we can obtain the Adaptive Controller as follows:

$$\begin{aligned} \dot{x}_m &= -\lambda x_m + \lambda g \\ \varepsilon &= x_m - x \\ \dot{\hat{\theta}} &= -\gamma x \varepsilon \\ u &= -\hat{\theta} x - \lambda x + \lambda g. \end{aligned} \tag{2.3}$$

The simulink scheme is constructed from (2.1) and (2.3) as follows:

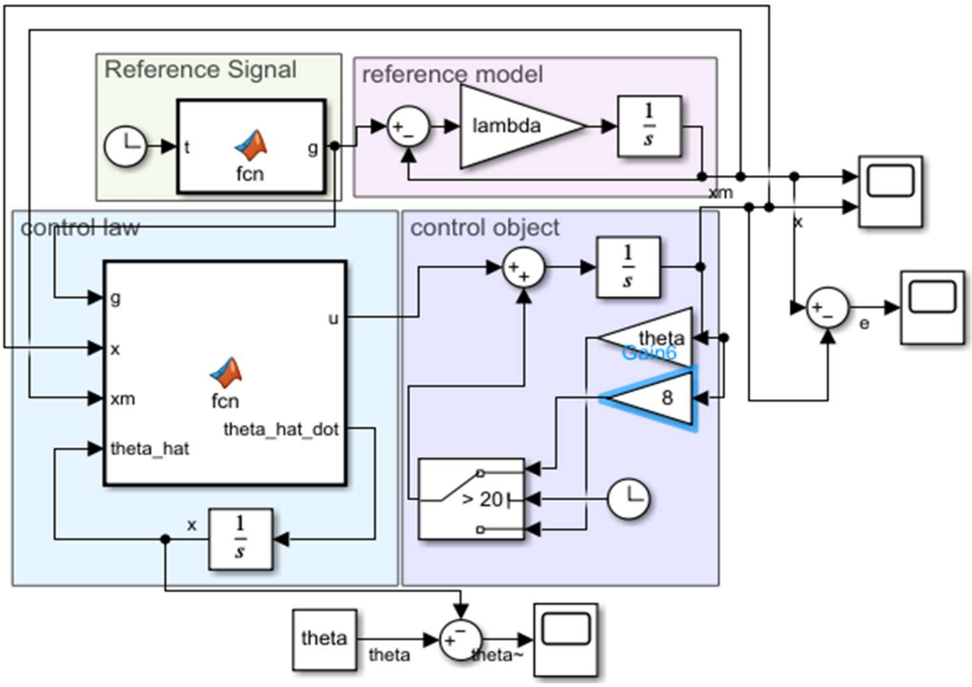

Figure 5: Adaptive Controller simulation scheme

2. Simulation

Make a simulation experiment, in which the plant parameter θ is increased rapidly (controller parameters are not changed)

we change the parameter θ as follows:

$$\theta : 7 \rightarrow 8 (t = 20)$$

The optimal γ is selected through the experiment, so that θ has the maximum convergence speed and does not fall into the shock. The optimal γ is:

$$\gamma = 2$$

The simulation results are shown in the following figure:

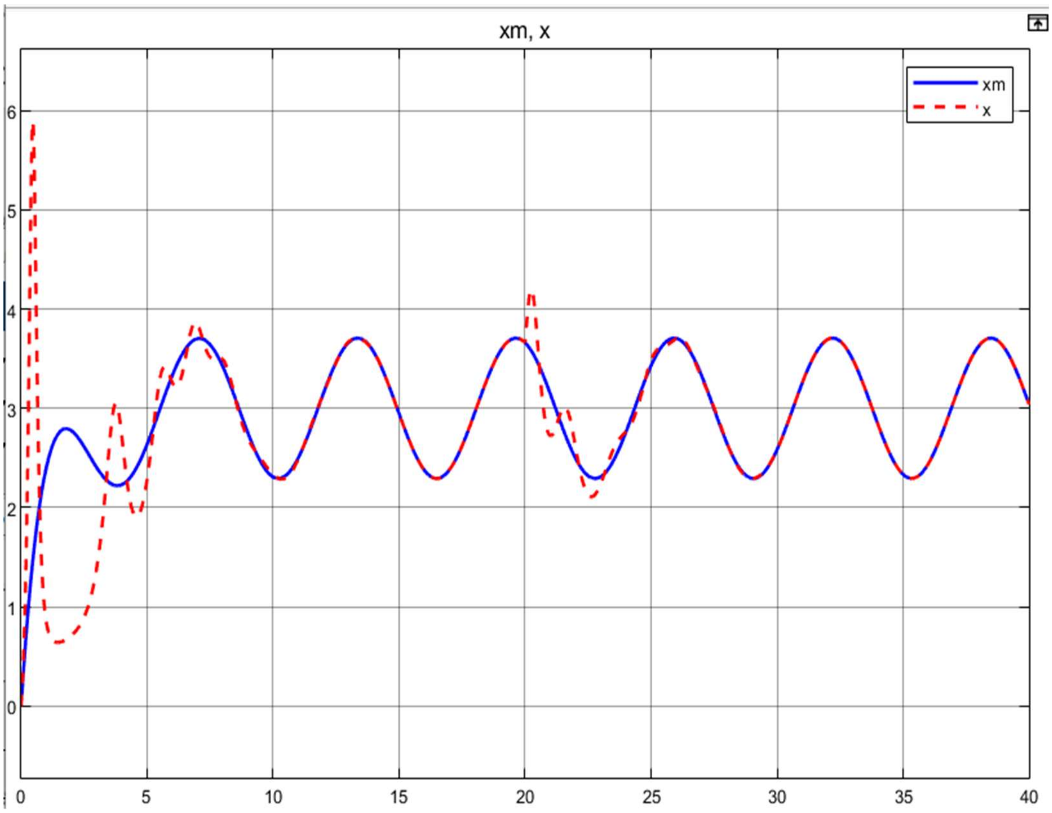

Figure 6： Adaptive Controller simulation result （$x(t), x_m(t)$）

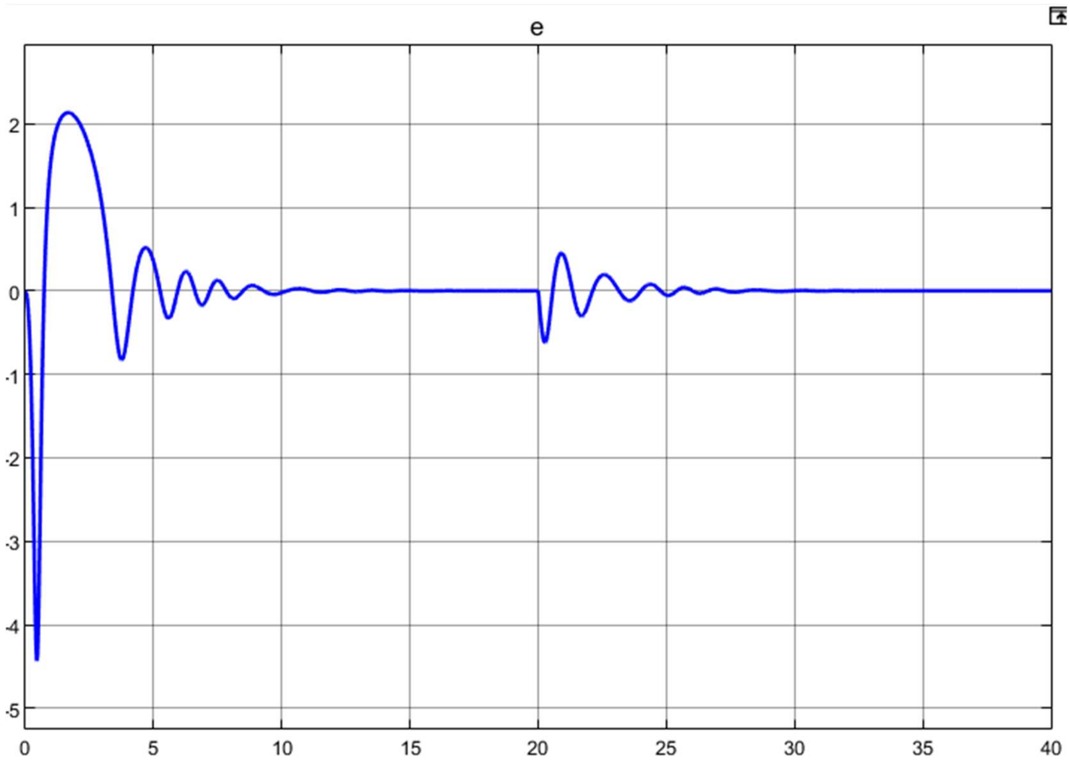

Figure 7： Adaptive Controller simulation result （$\epsilon(t)$）

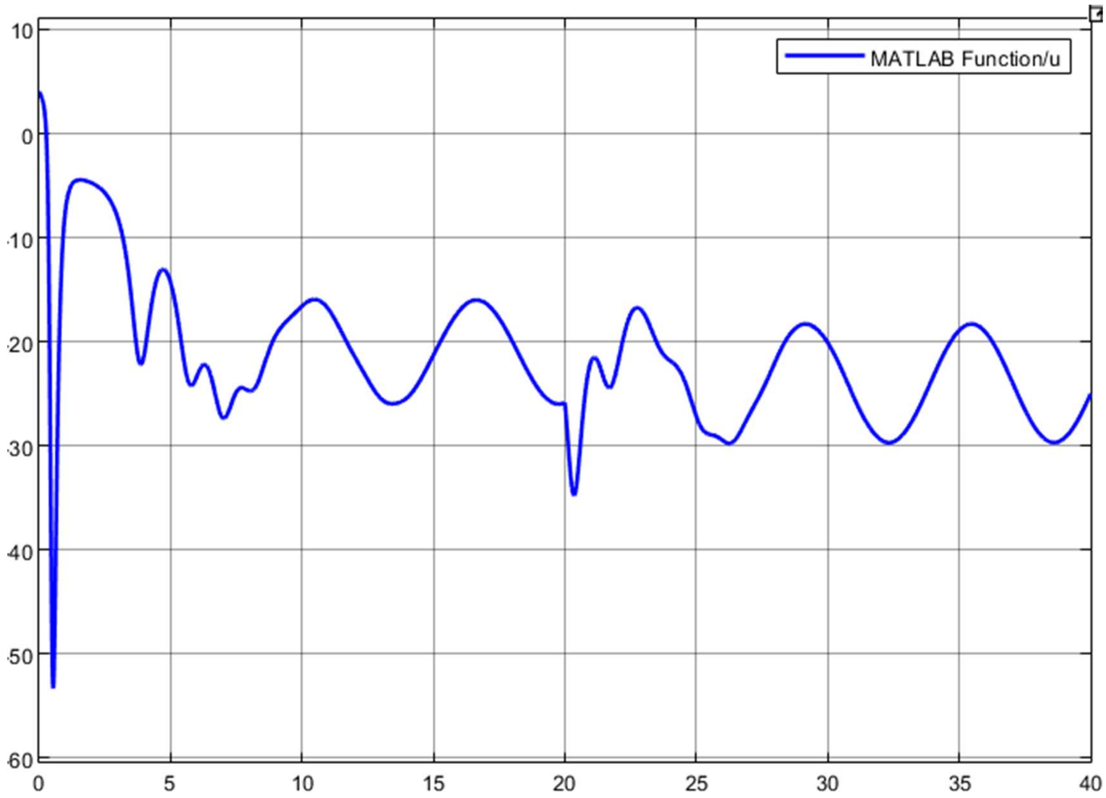

Figure 8： Adaptive Controller simulation result （$u(t)$）

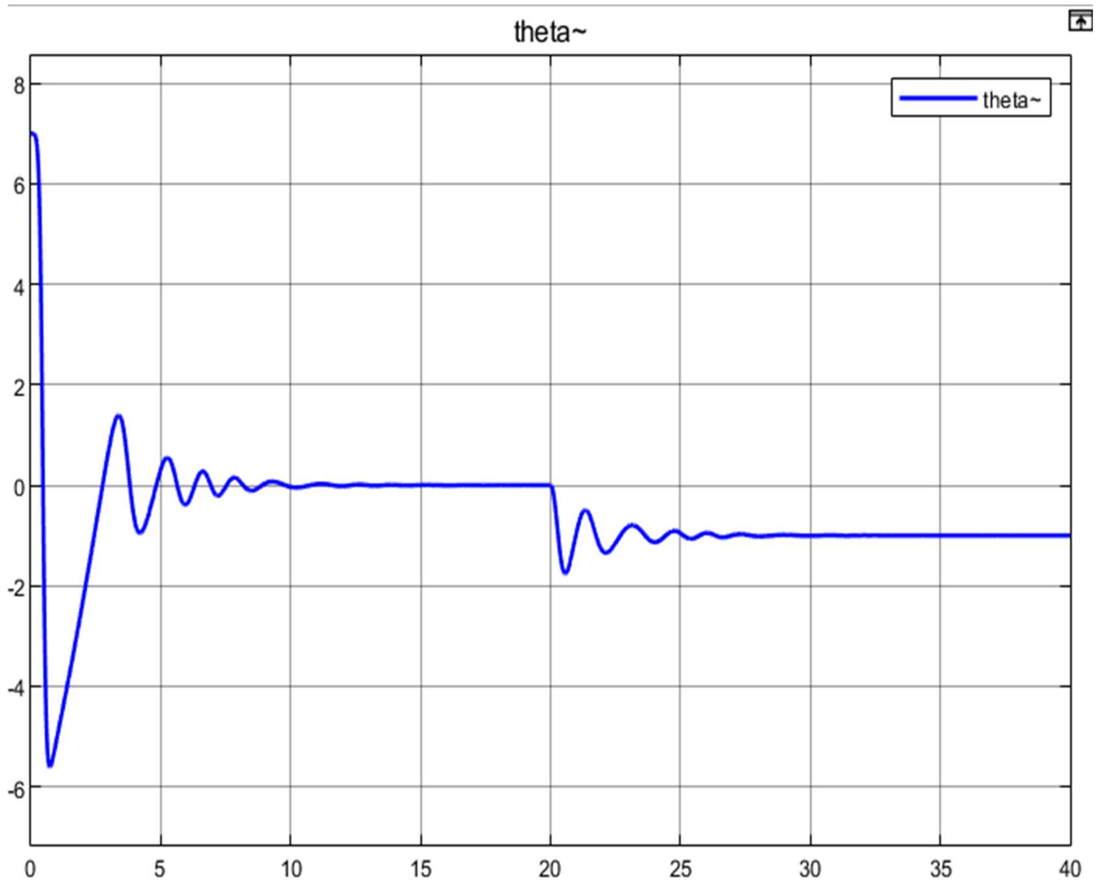

Figure 9：Adaptive Controller simulation result （$\tilde{\theta}(t)$）

3. Conclusion

Comparing Fig. 2 and Fig. 6, Fig. 3 and Fig. 7, and Fig. 4 and Fig. 8, we can find:

Compared with the non-adaptive controller our designed adaptive controller in the case of sudden change of plant parameters:

1. the system stability is still maintained by being able to
2. all signals are bounded
3. the tracking error ϵ is converged asymptotically to 0

Therefore, the designed adaptive controller meets the requirements of the technical specification.

4. Make experiments for different gains

- Small gain $\gamma = 0.01$

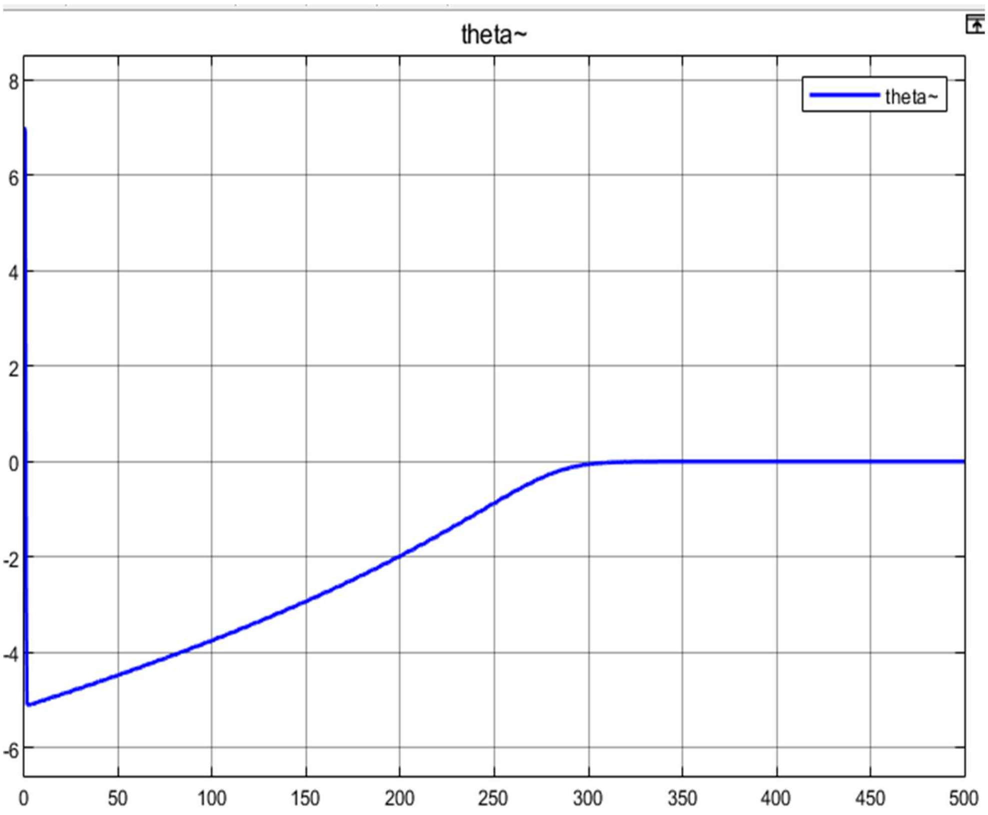

Figure 10：Adaptive Controller simulation result with small gain（$\tilde{\theta}(t)$， $\gamma = 0.01$）

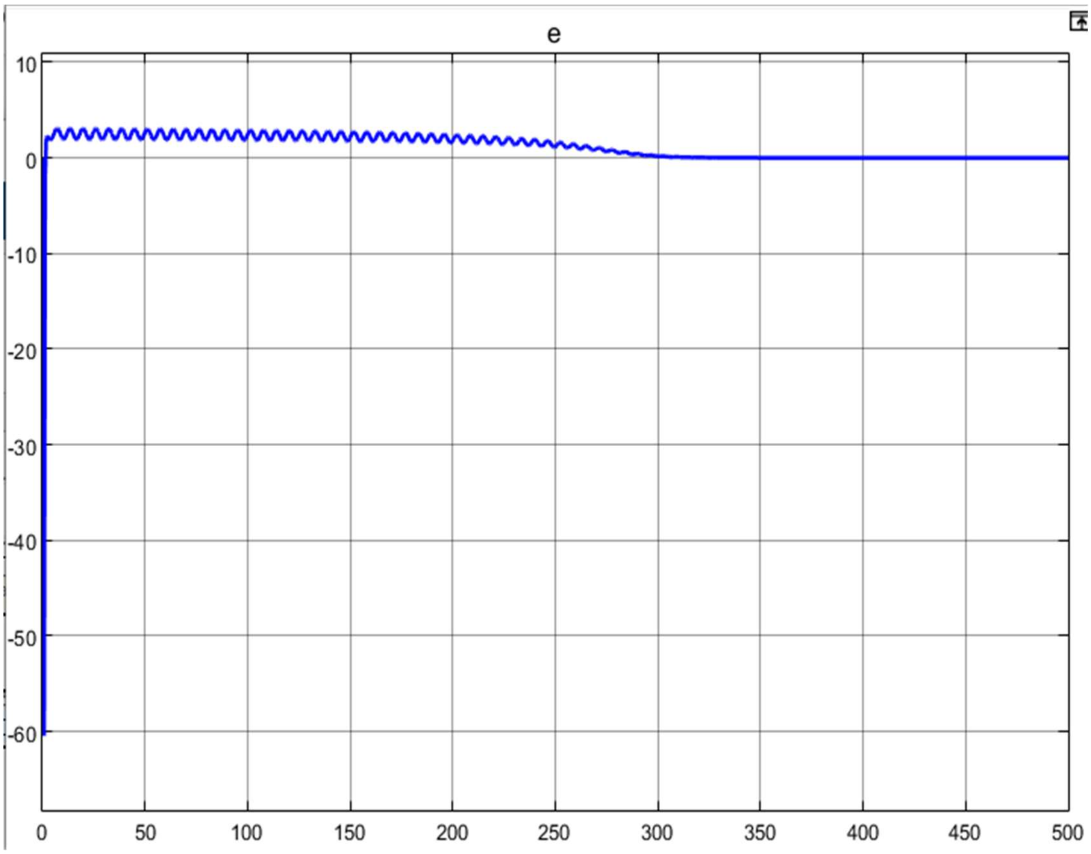

Figure 11： Adaptive Controller simulation result with small gain （$\epsilon(t)$， $\gamma = 0.01$）

- **Optimal gain$\gamma = 2$**

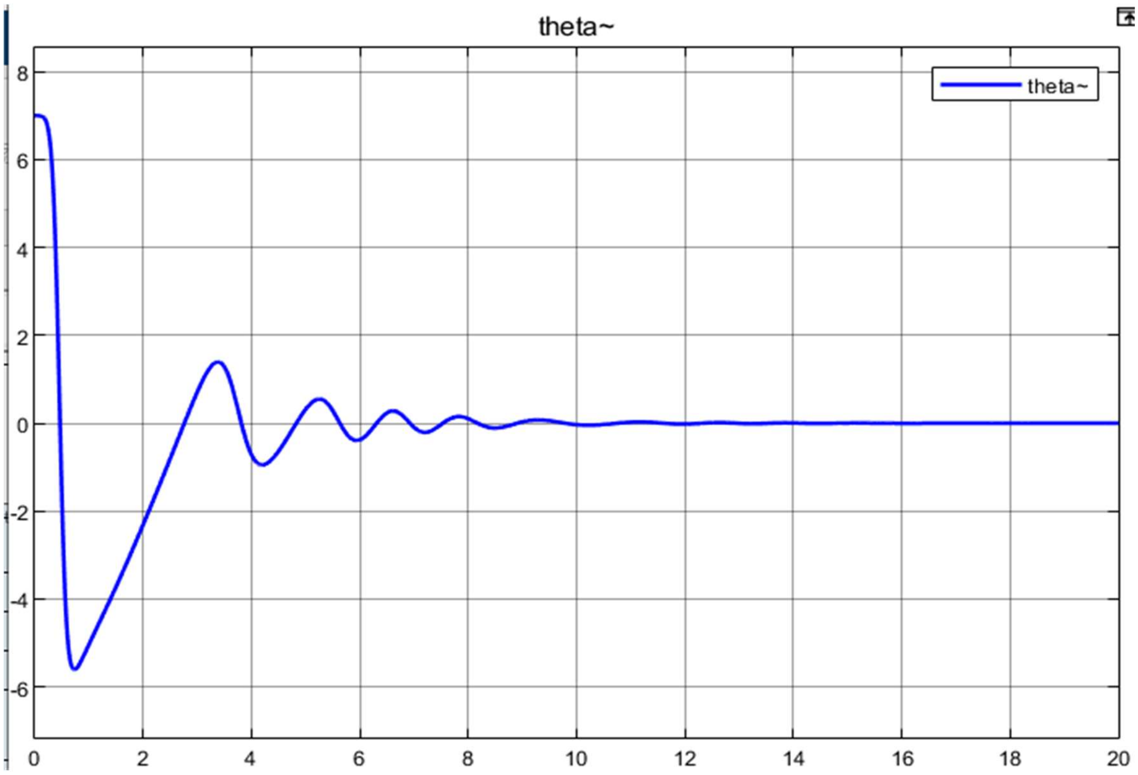

Figure 12： Adaptive Controller simulation result with optimal gain （$\tilde{\theta}(t)$， $\gamma = 2$）

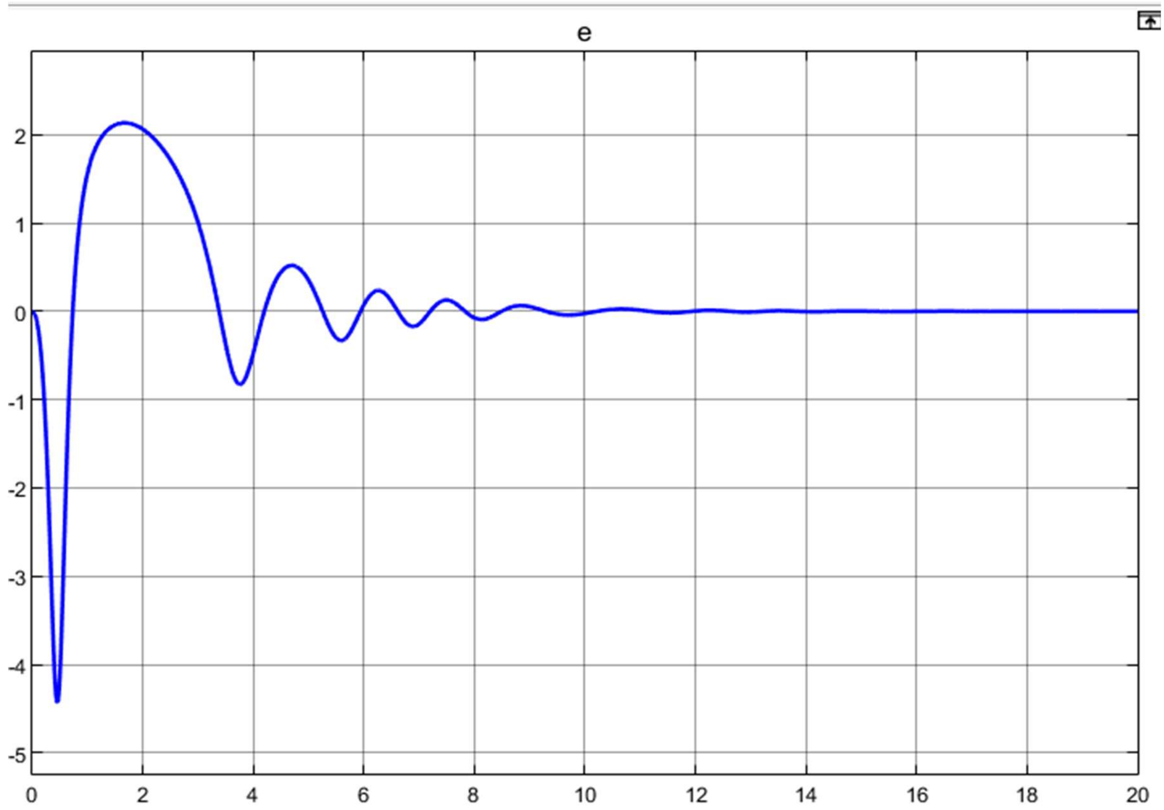

Figure 13： Adaptive Controller simulation result with optimal gain （$\epsilon(t)$， $\gamma = 2$）

- **Large gain** $\gamma = 50$

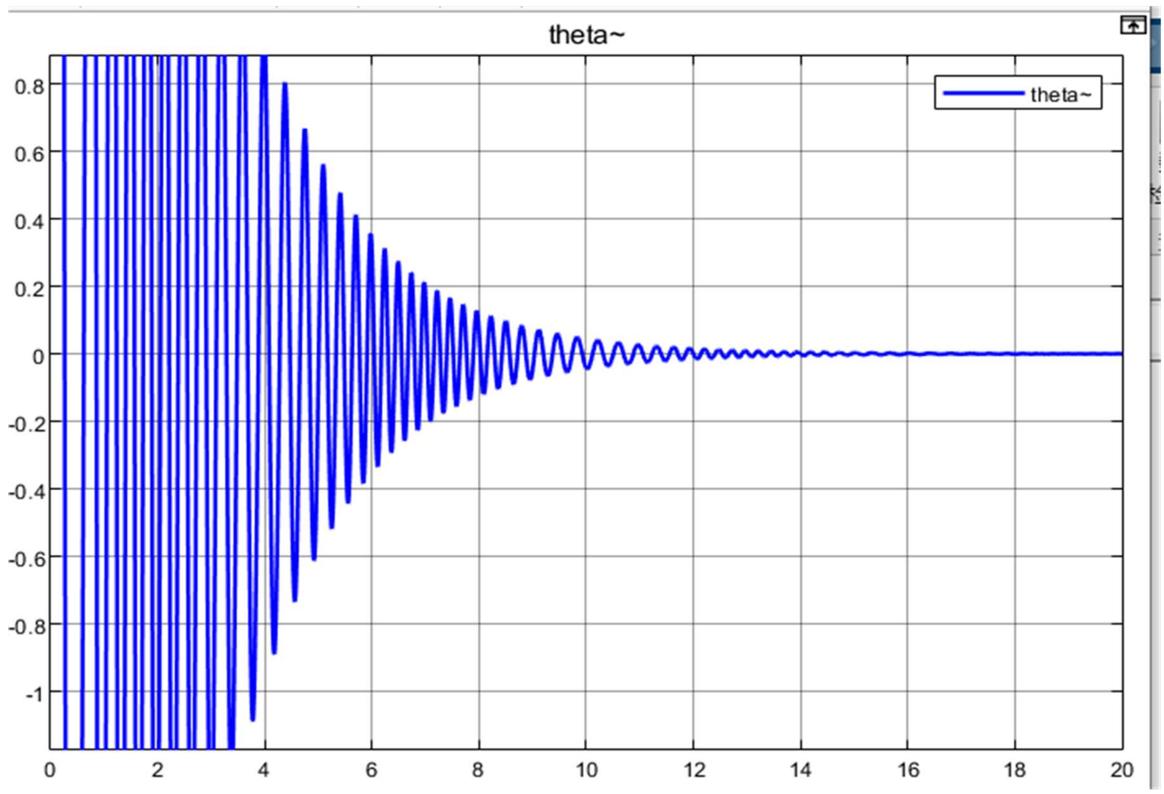

Figure 14：Adaptive Controller simulation result with large gain （$\tilde{\theta}(t)$， $\gamma = 50$）

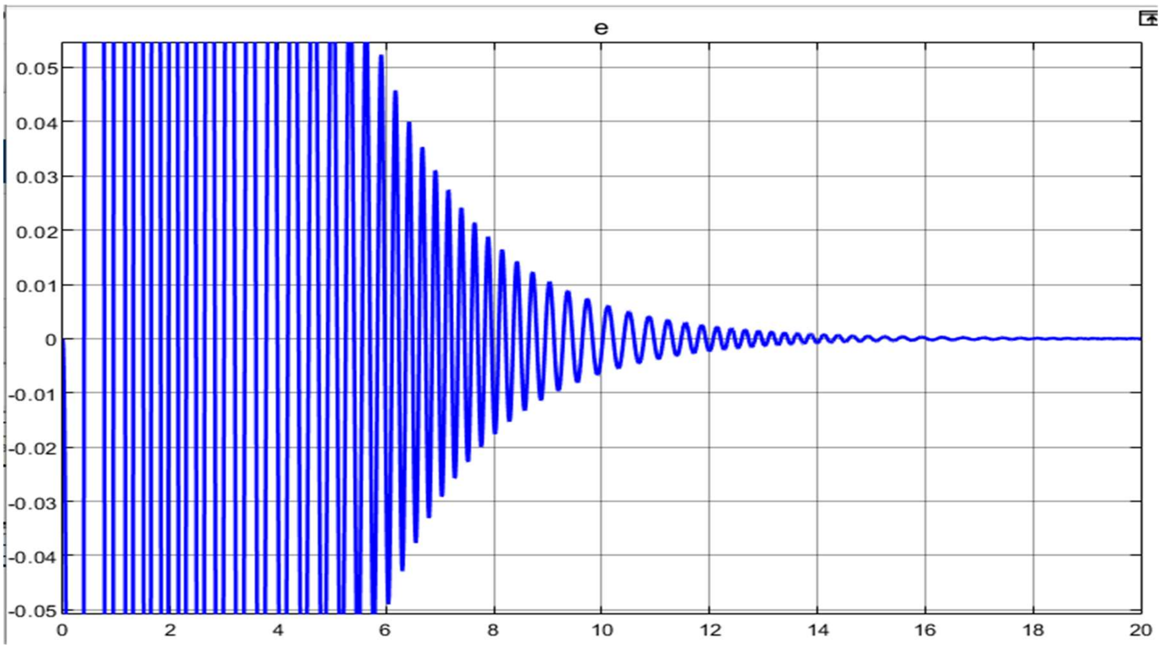

Figure 15 : Adaptive Controller simulation result with large gain ($\epsilon(t)$, $\gamma = 50$)

- **Conclusions**

As can be seen in Figures 10-15: Small γ lead to slow aperiodic parameters convergence, while for large γ one can observe the oscillations of parameters tuning .So that's why we need the adaptive gain γ needs experiments to determine the optimal γ.

D. Conclusions

- non-adaptive controllers cannot meet the control needs when the system parameters are not known.
- Therefore, when facing a system with unknown parameters, the adaptive controller can be designed using the method of this experiment.
- from the experimental results, the designed adaptive controller satisfies the technical specification requirements.
- it is worth noting that: the adaptive gain γ needs experiments to determine the optimal γ, for which the rate of convergence of $\tilde{\theta}$ to zero is maximum. Small γ lead to slow aperiodic parameters convergence, while for large γ one can observe the oscillations of parameters tuning .