Term Paper

Adaptive and Robust Control

Synthesis of an adaptive observer of the state of
a linear plant with improved parametric convergence

variant number : 2

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Contents

1. Plant and input signal parameters. （variant）

№	a_1	a_0	b_1	b_0	$u(t)$	Scheme with improved parametric convergence
2	2	2	1	8	$4\cos(6t+11)\sin(3t+7)$	Lion

2. List of the systems components.

● Plant

Consider the asymptotically stable plant:

$$\begin{cases} \dot{x} = Ax + bu,\ x(0), \\ y = Cx, \end{cases}$$

where x is the unmeasurable state vector, u and y are the measurable input and output signals respectively,

$$A = \begin{bmatrix} -a_{n-1} & 1 & \cdots & 0 \\ -a_{n-2} & 0 & & 0 \\ \vdots & & \ddots & 1 \\ -a_0 & 0 & & 0 \end{bmatrix}, b = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ b_m \\ \vdots \\ b_0 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}$$

$a_i, i = \overline{0, n-1}, b_j, j = \overline{0, m}$ are the unknown coefficients.

● observation system with gradient-based adaptation algorithm

➢ matching condition

$$A_0 = A + \bar{\theta}C.$$

where:

$$\bar{\theta} = \begin{bmatrix} k_{n-1} - a_{n-1} \\ k_{n-2} - a_{n-2} \\ \vdots \\ k_0 - a_0 \end{bmatrix}$$

➢ gradient-based adaptation algorithm

$$\hat{y} = \hat{\theta}^T \omega$$
$$\varepsilon = y - \hat{y}$$
$$\dot{\hat{\theta}} = \gamma \omega \varepsilon$$
$$\hat{x} = \sum_{i=0}^{n-1} \hat{\theta}_{i+1}(sI - A_0)^{-1} e_{n-i}[y] + \sum_{j=0}^{m} \hat{\theta}_{j+1+n}(sI - A_0)^{-1} e_{m-j}[u].$$

where:

$$\theta^T = [k_0 - a_0, k_1 - a_1, \dots, k_{n-1} - a_{n-1}, b_0, b_1, \dots, b_m]$$
$$\omega^T = \left[\frac{1}{K(s)}[y], \frac{s}{K(s)}[y], \dots, \frac{s^{n-1}}{K(s)}[y], \frac{1}{K(s)}[u], \frac{s}{K(s)}[u], \dots, \frac{s^m}{K(s)}[u]\right]$$
$$H(s) = \frac{1}{K(s)} = \frac{1}{s^n + k_{n-1}s^{n-1} + k_{n-2}s^{n-2} + \cdots + k_0}$$

| I is the $n \times n$ identity matrix

A_0 is a Hurwitz matrix presented in canonical form:

$$A_0 = \begin{bmatrix} -k_{n-1} & 1 & \cdots & 0 \\ -k_{n-2} & 0 & & 0 \\ \vdots & & \ddots & 1 \\ -k_0 & 0 & & 0 \end{bmatrix}.$$

$$e_i^T = [0, \dots, 0, 1_i, 0, \dots, 0]$$

➢ observation system

$$\hat{x} = \sum_{i=0}^{n-1} \hat{\theta}_{i+1}(sI - A_0)^{-1} e_{n-i}[y] + \sum_{j=0}^{m} \hat{\theta}_{j+1+n}(sI - A_0)^{-1} e_{m-j}[u].$$

● adaptive observation system with modified adaptation algorithm

➢ matching condition

$$A_0 = A + \bar{\theta} C.$$

where:

$$\bar{\theta} = \begin{bmatrix} k_{n-1} - a_{n-1} \\ k_{n-2} - a_{n-2} \\ \vdots \\ k_0 - a_0 \end{bmatrix}$$

➢ modified adaptation algorithm

$$\begin{aligned} \hat{y} &= \hat{\theta}^T \omega \\ \varepsilon &= y - \hat{y} \\ \dot{\hat{\theta}} &= \gamma \Xi^T (\bar{\bar{\Xi}} - \Xi \hat{\theta}) \end{aligned}$$

where:

$$\theta^T = [k_0 - a_0, k_1 - a_1, \dots, k_{n-1} - a_{n-1}, b_0, b_1, \dots, b_m]$$
$$\bar{\Xi} = \mathrm{col}\left(H_1(s)[\varepsilon + \omega^T \hat{\theta}], H_2(s)[\varepsilon + \omega^T \hat{\theta}], \dots, H_q(s)[\varepsilon + \omega^T \hat{\theta}]\right).$$
$$H_i(s) = \frac{1}{K_i(s)} = \frac{1}{s^n + k_{n-1}^i s^{n-1} + k_{n-2}^i s^{n-2} + \dots + k_0^i}$$

I is the $n \times n$ identity matrix

A_0 is a Hurwitz matrix presented in canonical form:

$$A_0 = \begin{bmatrix} -k_{n-1} & 1 & \cdots & 0 \\ -k_{n-2} & 0 & & 0 \\ \vdots & & \ddots & 1 \\ -k_0 & 0 & & 0 \end{bmatrix}.$$

$$e_i^T = [0, \dots, 0, 1_i, 0, \dots, 0]$$

➢ observation system

$$\hat{x} = \sum_{i=0}^{n-1} \hat{\theta}_{i+1} (sI - A_0)^{-1} e_{n-i}[y] + \sum_{j=0}^{m} \hat{\theta}_{j+1+n} (sI - A_0)^{-1} e_{m-j}[u].$$

3. Simulation schemes with listings.

- **observation system with gradient-based adaptation algorithm**

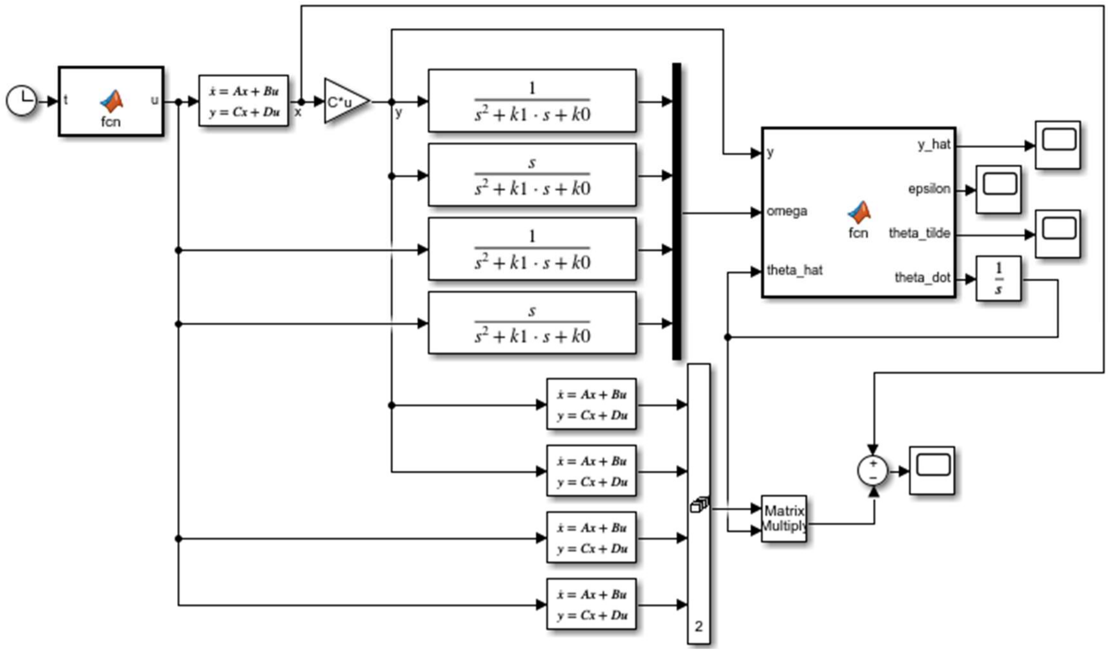


Figure 1： observation system with gradient-based adaptation algorithm simulation scheme1

```
function u = fcn(t)
u = 4*cos(6*t+11)*sin(3*t+7);
```

Figure 2： observation system with gradient-based adaptation algorithm simulation scheme2

```
function [y_hat,epsilon,theta_tilde,theta_dot] = fcn(y,omega,theta_hat, theta)
gamma = 10;
y_hat = theta_hat'*omega;
epsilon = y-y_hat;
theta_tilde = theta - theta_hat;
theta_dot = gamma*omega*epsilon;
```

Figure 3： observation system with gradient-based adaptation algorithm simulation scheme3

● adaptive observation system with modified adaptation algorithm

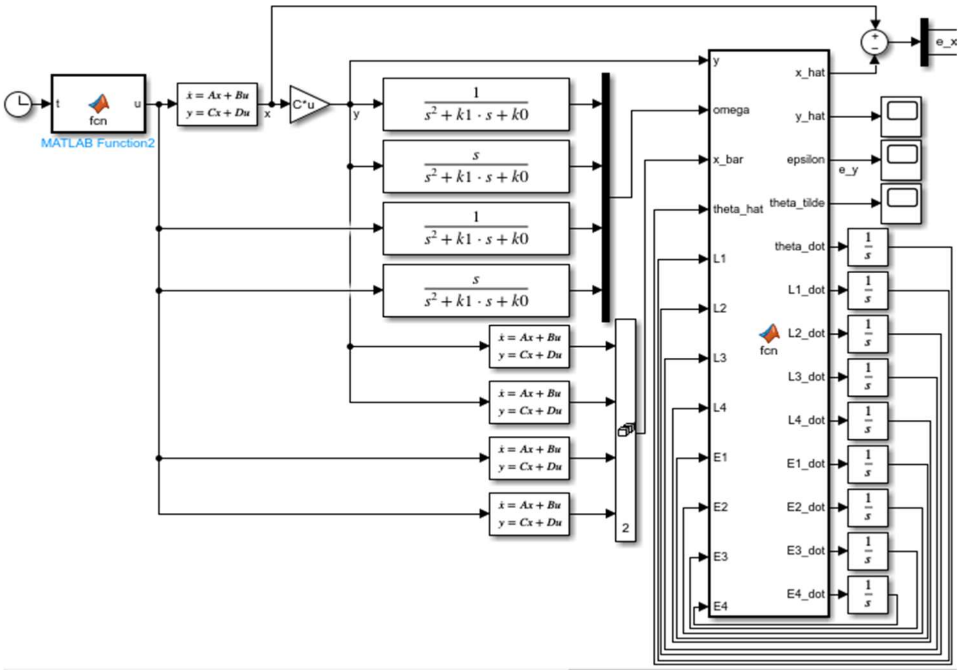


Figure 4:adaptive observation system with modified adaptation algorithm simulation scheme1

```
function u = fcn(t)
u = 4*cos(6*t+11)*sin(3*t+7);
```

Figure 5:adaptive observation system with modified adaptation algorithm simulation scheme2

```
function [x_hat,y_hat,epsilon,theta_tilde,theta_dot,L1_dot,L2_dot,L3_dot,L4_dot,E1_dot,E2_dot,E3_dot,
gamma = 500000;mu1=1;mu2=2;mu3=3;mu4=4;
y_hat = theta_hat'*omega;
epsilon = y-y_hat;
theta_tilde = theta - theta_hat;
%theta_dot = gamma*omega*epsilon;
L1_dot = -mu1*L1 + omega;
L2_dot = -mu2*L2 + omega;
L3_dot = -mu3*L3 + omega;
L4_dot = -mu4*L4 + omega;
L = [L1';L2';L3';L4'];

E1_dot = -mu1*E1 + epsilon+omega'*theta_hat;
E2_dot = -mu2*E2 + epsilon+omega'*theta_hat;
E3_dot = -mu3*E3 + epsilon+omega'*theta_hat;
E4_dot = -mu4*E4 + epsilon+omega'*theta_hat;
E = [E1';E2';E3';E4'];

x_hat=x_bar*theta_hat;

theta_dot = gamma*L'*(E-L*theta_hat);
```

Figure 6:adaptive observation system with modified adaptation algorithm simulation scheme3

4. Transients in the adaptive observation system with gradient-based adaptation algorithm. $\gamma = 100$

- output signal error$\epsilon_y = y - \hat{y} \gamma = 100$

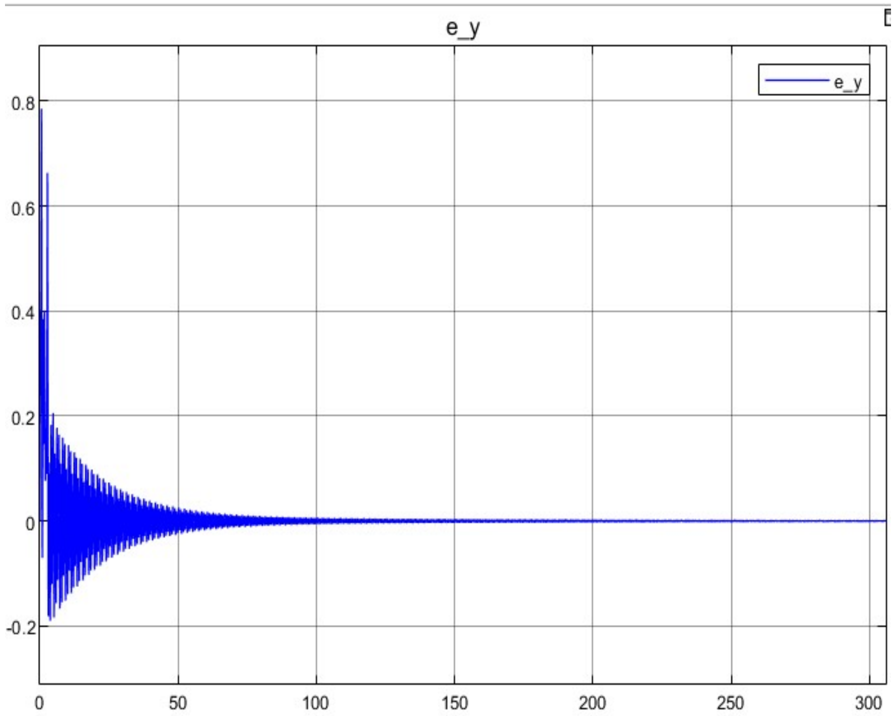
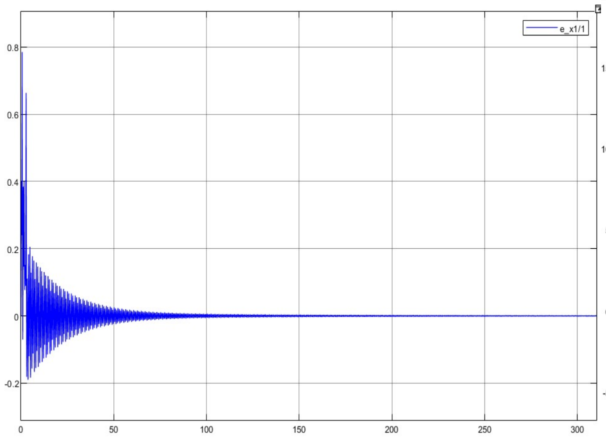


Figure 7:output signal error

- observation error$\epsilon_x = x - \hat{x} \gamma = 100$



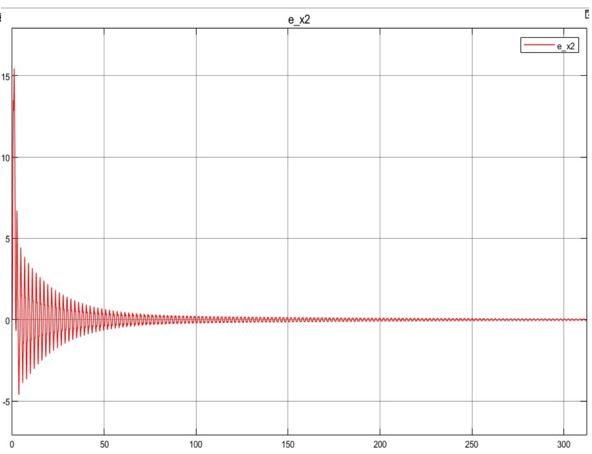


Figure 8(a):observation errorx_1 Figure 8(b):observation errorx_2

- **identification (parametric) error** $\tilde{\theta} = \theta - \hat{\theta}\gamma = 100$

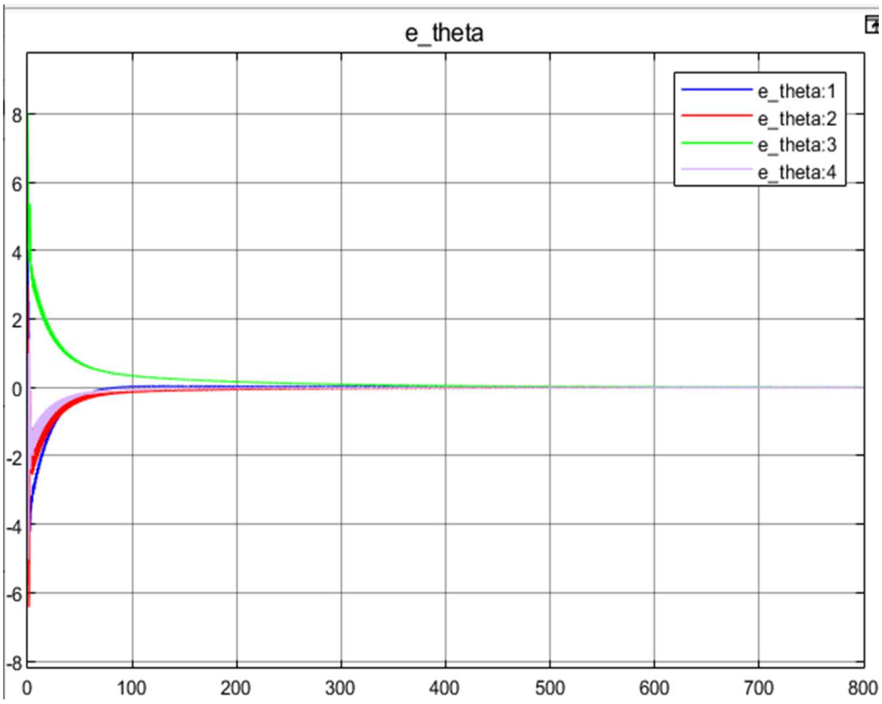
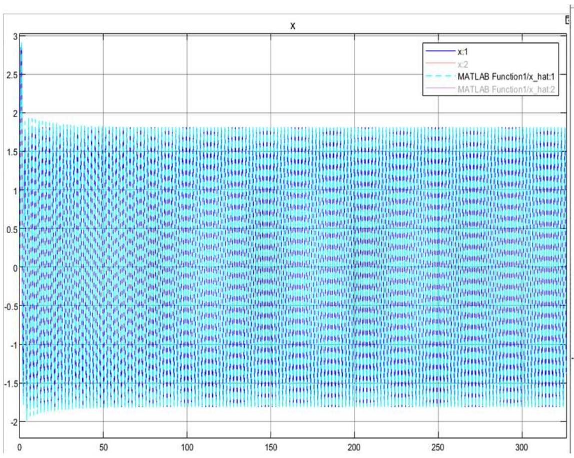


Figure 9:identification (parametric) error

- **state vector,state vector estimates** $x, \hat{x}\gamma = 100$



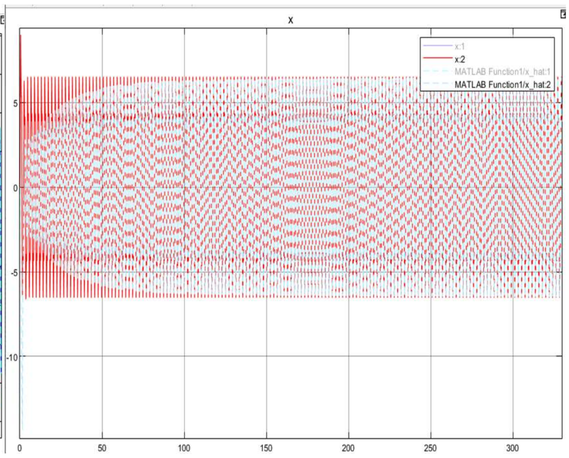


Figure 10(a):$x, \hat{x}$

Figure 10(b):$x, \hat{x}$

- regressor$\omega \gamma = 100$

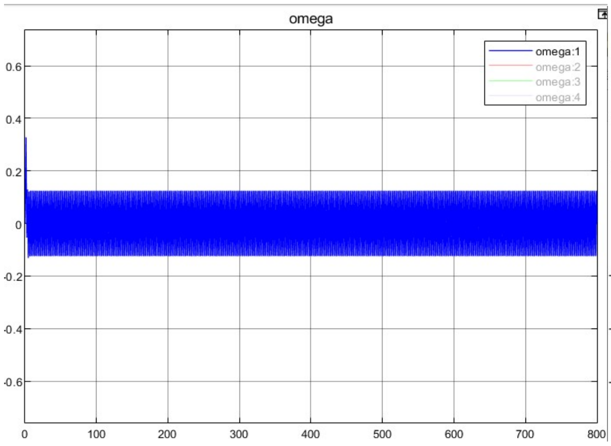


Figure 11(a):regressorω_1

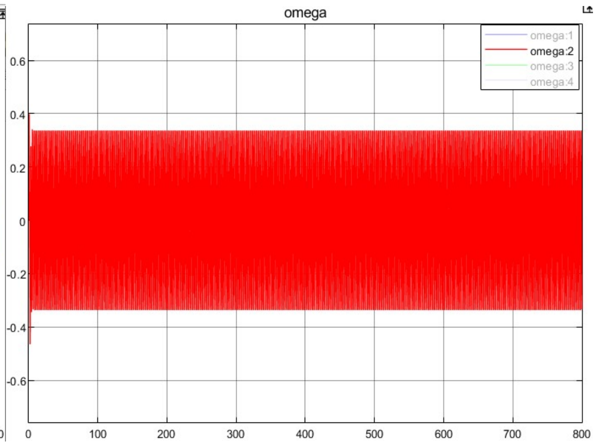


Figure 11(b):regressorω_2

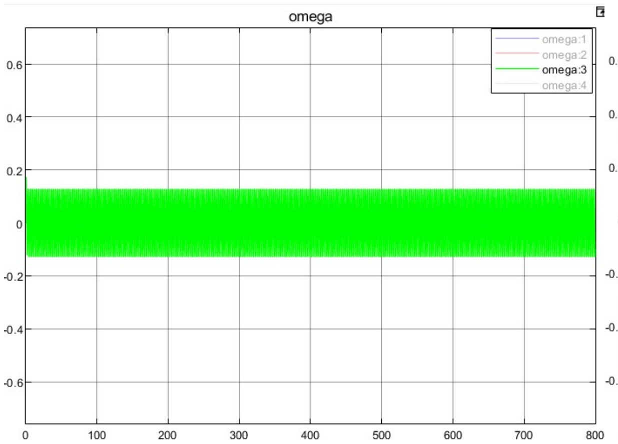


Figure 11(c):regressorω_3

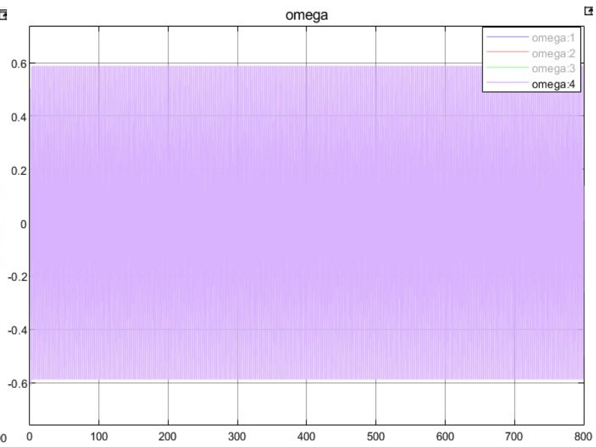


Figure11(d):regressorω_4

5. Transients in the adaptive observation system with modified adaptation algorithm. $\gamma = 1000 \gamma = 50000$

$\gamma = 500000$

- $\gamma = 1000$

1. output signal error $\epsilon_y = y - \hat{y}$

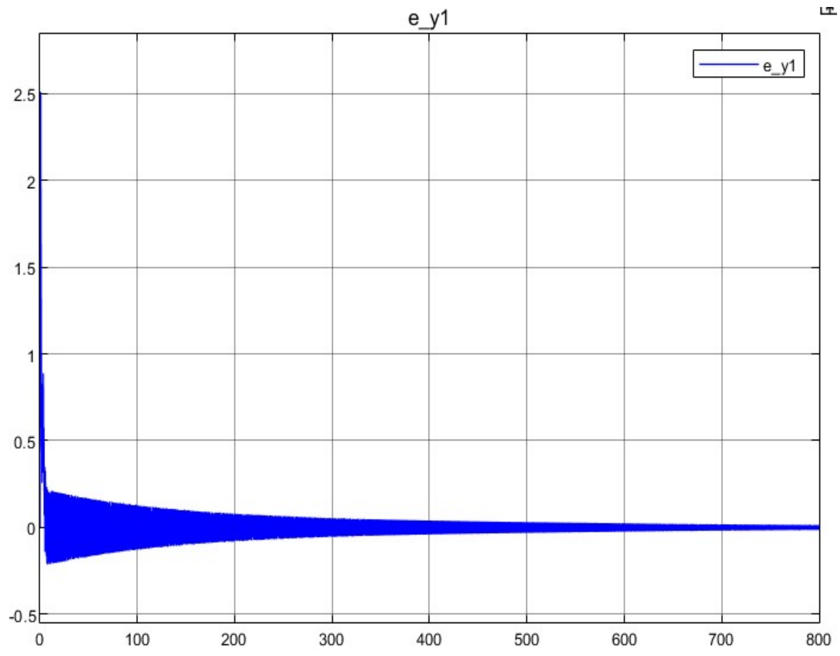
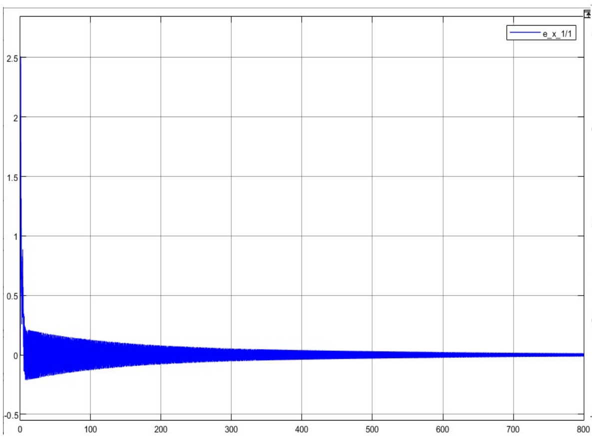


Figure 12:output signal error

2. observation error $\epsilon_x = x - \hat{x}$



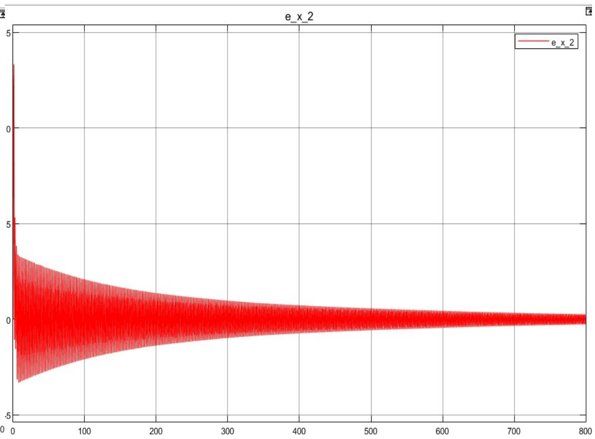


Figure 13(a):observation error x_1

Figure 13(b):observation error x_2

3. identification (parametric) error $\tilde{\theta} = \theta - \hat{\theta}$

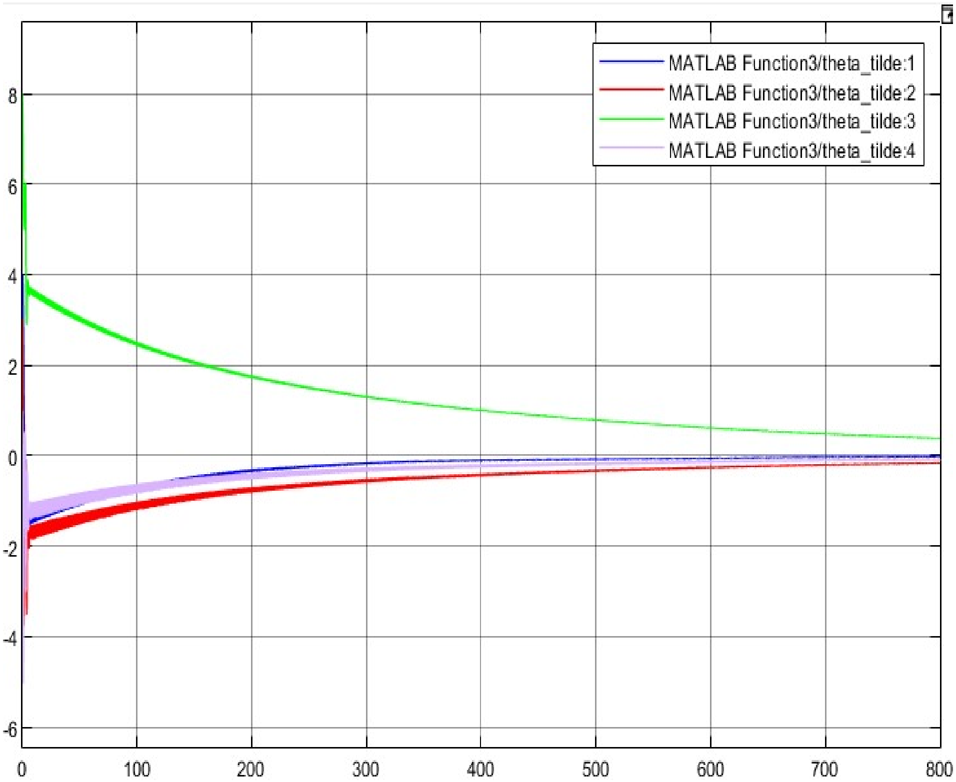
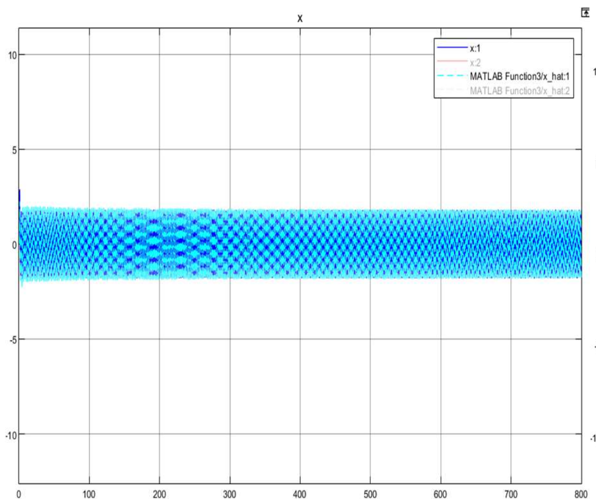


Figure 14:identification (parametric) error

4. state vector,state vector estimates $x, \hat{x}$



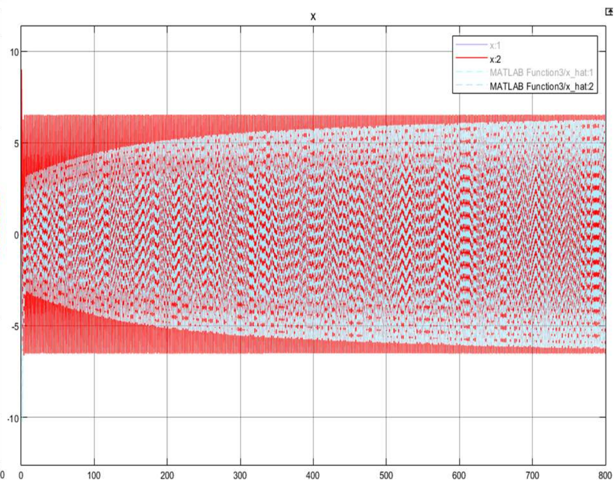


Figure 15(a): $x, \hat{x}$

Figure 15(b): $x, \hat{x}$

5. regressorω

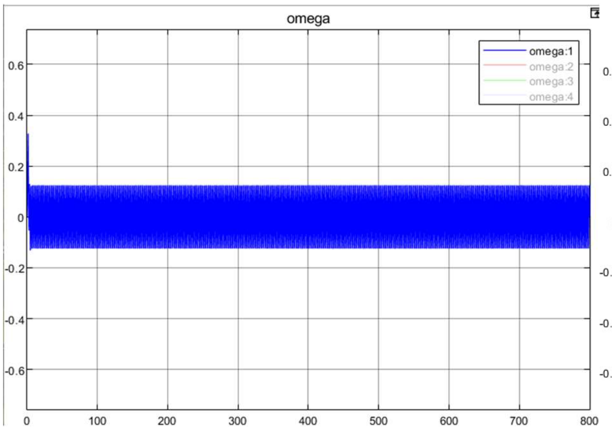


Figure 16(a):regressorω_1

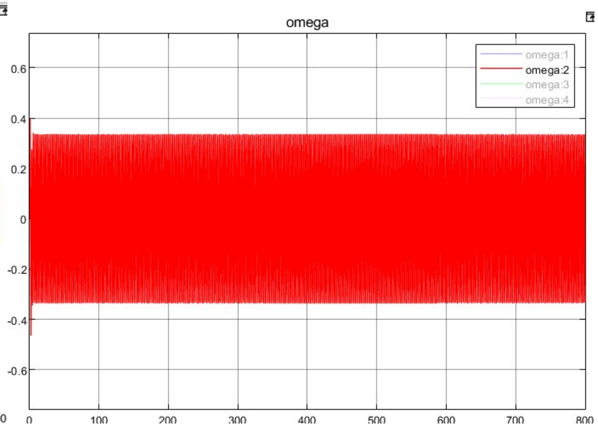


Figure 16(b):regressorω_2

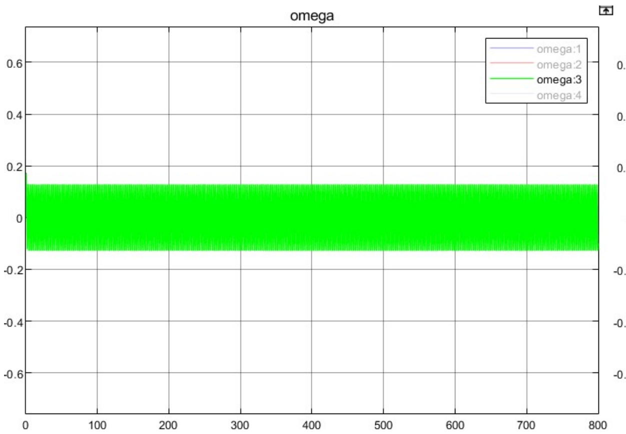


Figure 16(c):regressorω_3

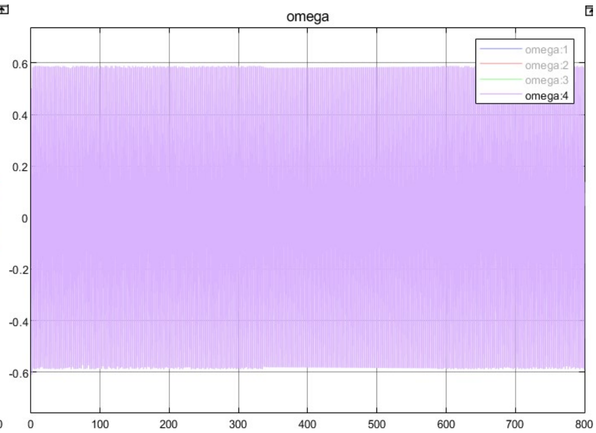


Figure16(d):regressorω_4

- $\gamma = 50000$

1. output signal error$\epsilon_y = y - \hat{y}$

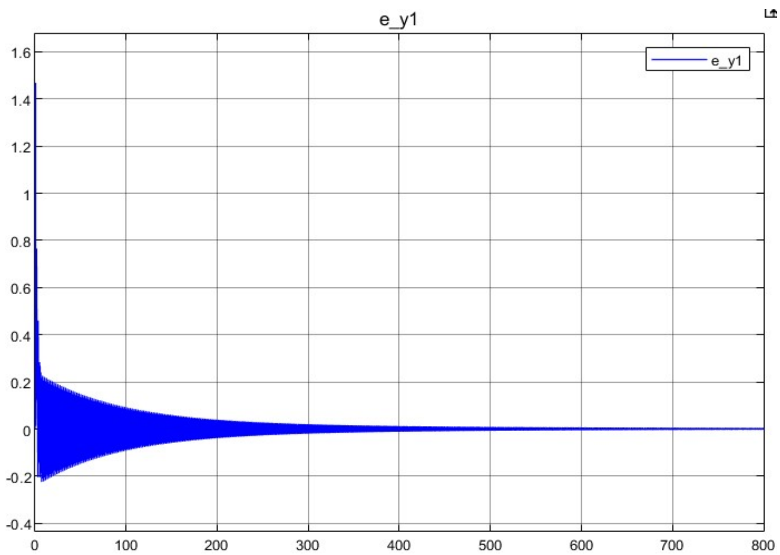
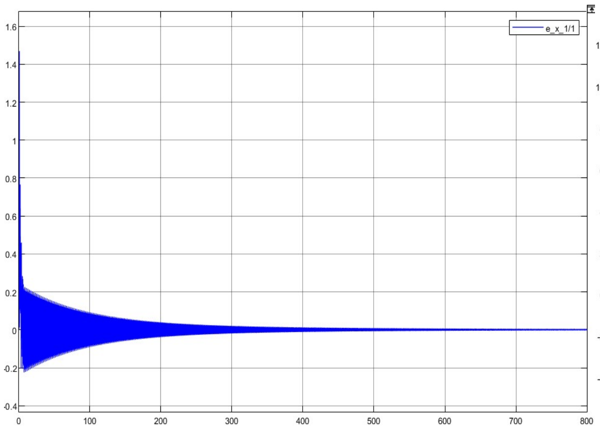


Figure 17:output signal error

2. observation error $\epsilon_x = x - \hat{x}$



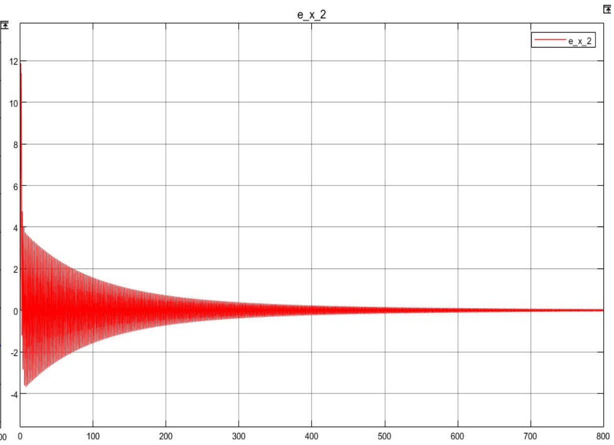


Figure 18(a):observation error x_1 Figure 18(b):observation error x_2

3. identification (parametric) error $\tilde{\theta} = \theta - \hat{\theta}$

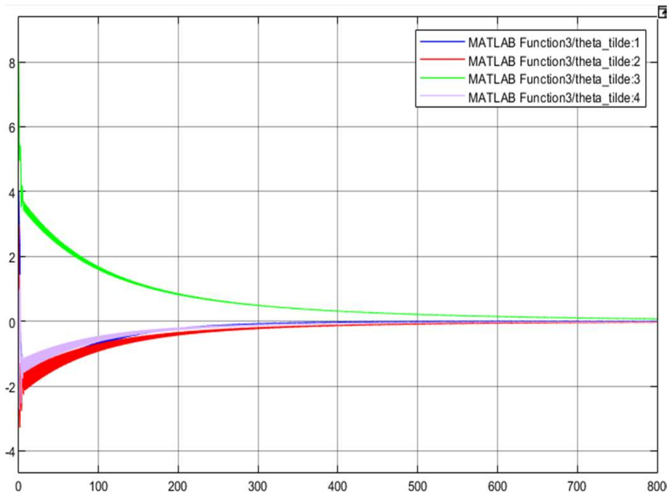
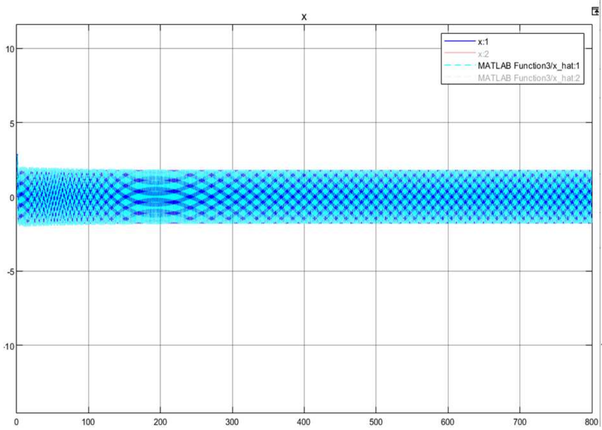


Figure 19:identification (parametric) error

4. state vector,state vector estimates $x, \hat{x}$



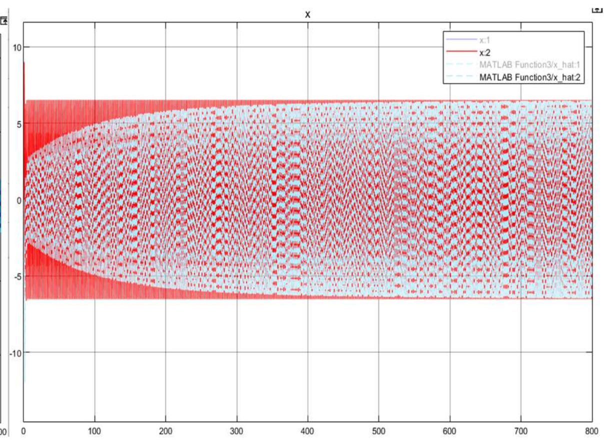


Figure 20(a): $x, \hat{x}$ Figure 20(b): $x, \hat{x}$

5. regressorω

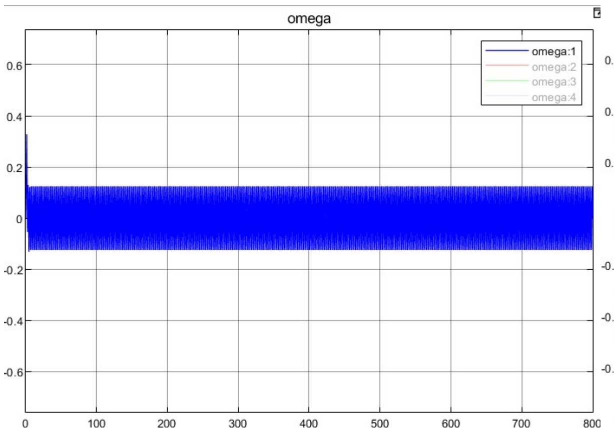


Figure 21(a):regressorω_1

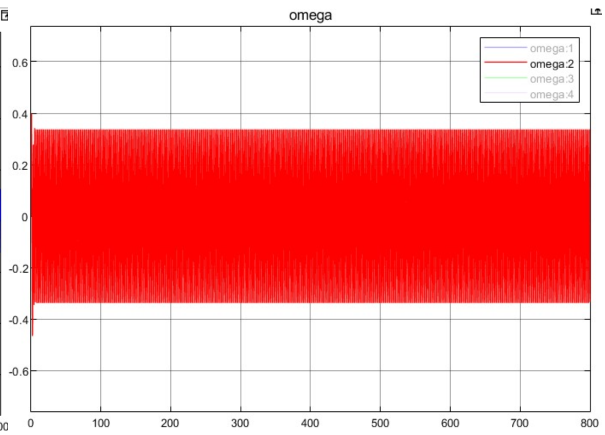


Figure 21(b):regressorω_2

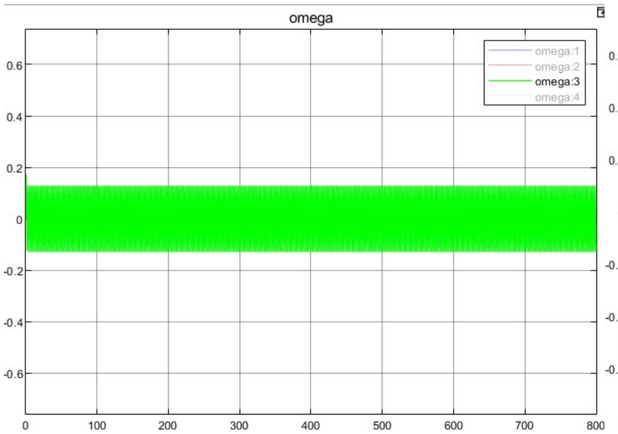


Figure 21(c):regressorω_3

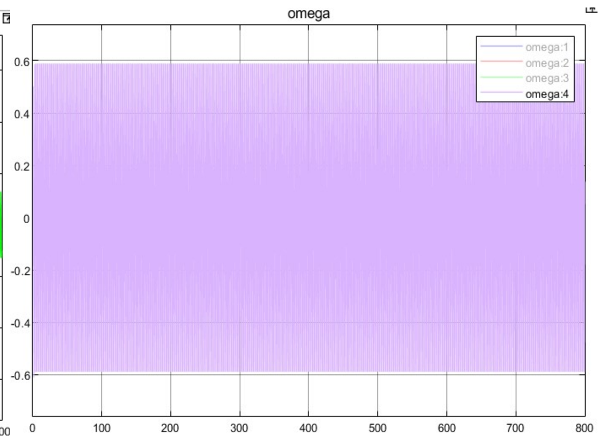


Figure21(d):regressorω_4

- $\gamma = 500000$

1. output signal error$\epsilon_y = y - \hat{y}$

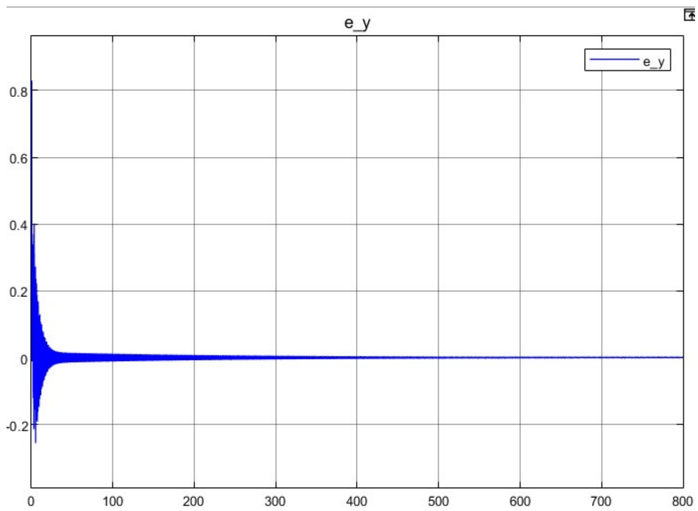
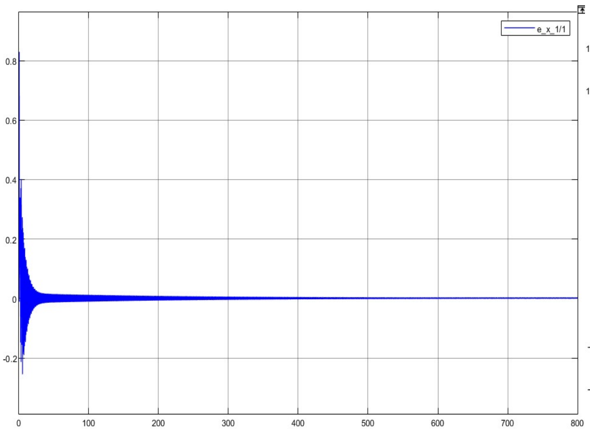


Figure 22:output signal error

2. **observation error $\epsilon_x = x - \hat{x}$**



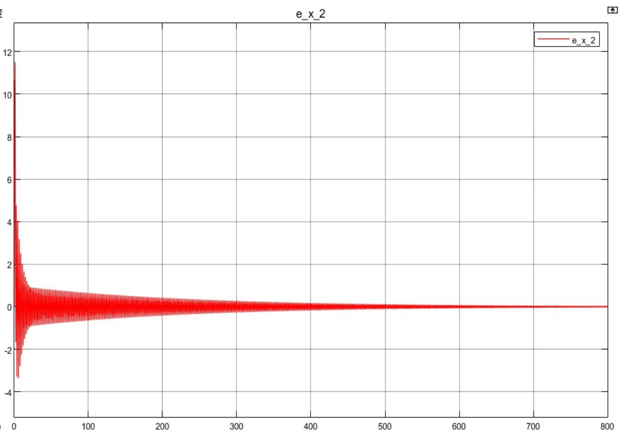


Figure 23(a):observation error x_1

Figure 23(b):observation error x_2

3. **identification (parametric) error $\tilde{\theta} = \theta - \hat{\theta}$**

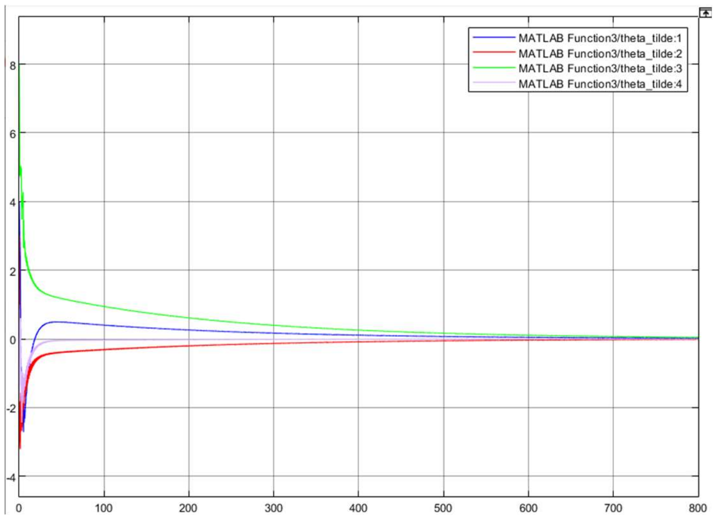
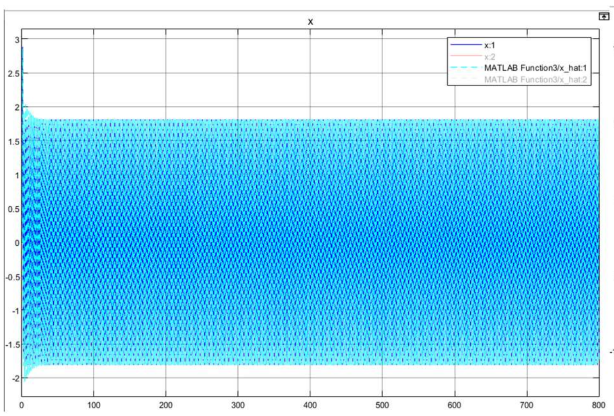


Figure 24:identification (parametric) error

4. **state vector,state vector estimates $x, \hat{x}$**



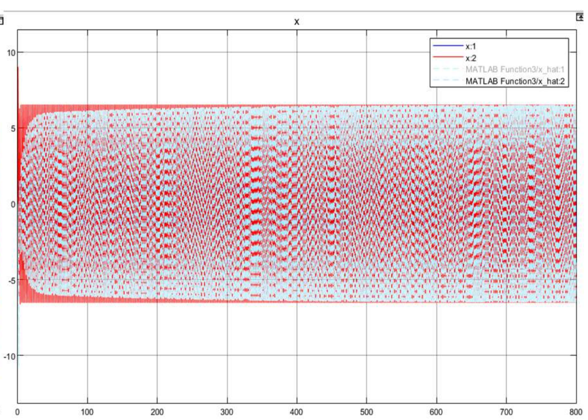


Figure 25(a): $x, \hat{x}$

Figure 25(b): $x, \hat{x}$

5. regressorω

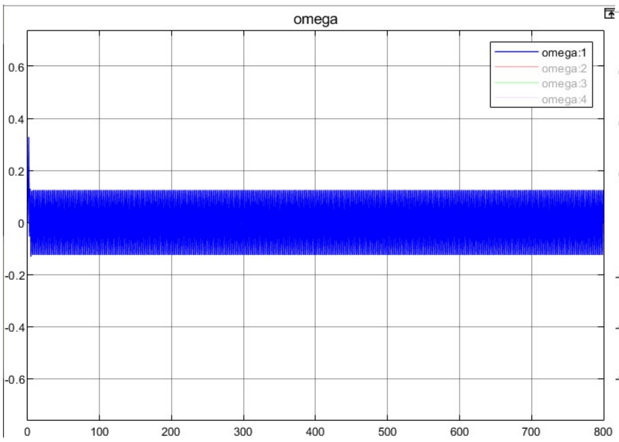


Figure 26(a):regressorω_1

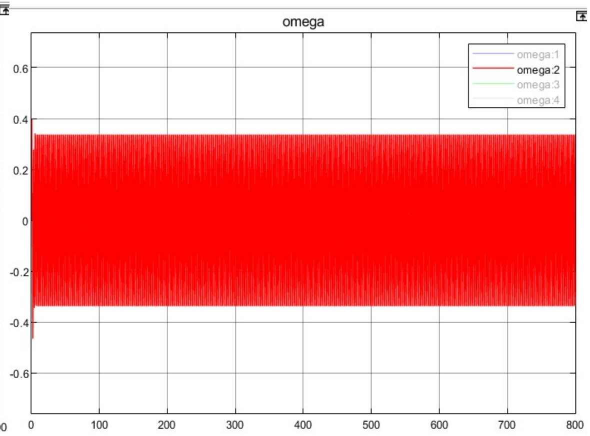


Figure 26(b):regressorω_2

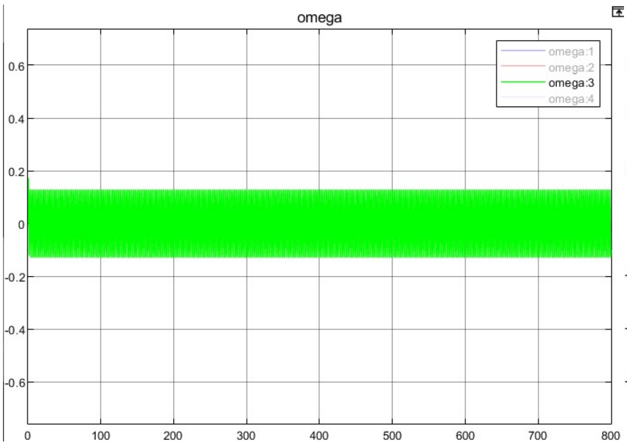


Figure 26(c):regressorω_3

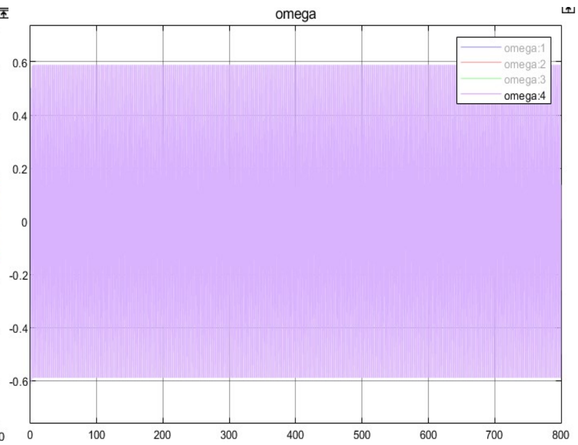


Figure26(d):regressorω_4

6. Appendix（Code）

```
clear all
a1 = 2;a0 = 2;b1 = 1;b0 = 8;k1 = 5;k0 = 6;
A = [-a1 1;-a0 0];b = [b1;b0];C = [1 0];
theta = [k0-a0;k1-a1;b0;b1];
A0 = [-k1 1;-k0 0];
```

7. Conclusions.

From the simulation results we can show the following properties of the observation system with Gradient-based adaptation algorithm:

- all signals in the observer are bounded;
  (Figure 7-11)
- the identification ε approaches zero asymptotically;
  (Figure 7-9)
- the parametric error $\tilde{\theta}$ approaches zero exponentially fast, if the vector ω satisfies the persistent excitation condition. This condition depends on the number of harmonics (spectral lines) in the signal u;
  (Figure 9)
- if the error $\tilde{\theta}$ converges towards zero, then the state vector $\hat{x}$ estimation also converges towards x.
  (Figure 8-10)

From the simulation results we can show the following properties of the adaptive observation system with modified adaptation algorithm:

- if ω and $\dot{\omega}$ are bounded, then the signals $\varepsilon, \tilde{\theta}$ are also bounded;
  (Figure12，14，16，17，19，21， 22，24，26)
- the error ε converges towards zero asymptotically;|
  (Figure12，17，22)
- parametric errors $\tilde{\theta}$ converge towards zero exponentially if the vector ω (matrix Ξ) satisfies the persistent excitation condition.
  (Figure 14，19，24)
- if the vector ω (matrix Ξ) satisfies the condition, then the convergence rate of parametric errors $\tilde{\theta}$ towards zero can be arbitrarily increased by increasing the value of coefficient γ.
  (Figure 14，19，24)