Practical Assignment № 4

Adaptive and Robust Control

STATE FEEDBACK ROBUST AND ROBUST ADAPTIVE CONTROL

variant number : 16

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A.Problem statement

Consider the plant presented in the "state-space" form

$$\dot{x} = Ax + bu,\ x(0) \tag{3.1}$$

$$y = c^T x \tag{3.2}$$

where $x \in R^n$ is the state vector, $u \in R$ is the control variable, $y \in R$ is the regulated variable,

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_0 & -a_1 & -a_2 & \cdots & -a_{n-1} \end{bmatrix}, b = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ b_0 \end{bmatrix}, c = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$a_i, i = \overline{0, n-1}$ are unknown parameters, b_0 is the coefficient which is known for the sake of simplicity.

B.Experimental part

● Experimental parameters（Group 16）

Var.	Matrix A	Transmission coef. b_0	Transient time, t_{tr}	Overshoot $\bar{\sigma}$,%	Input signal $g(t)$
16	$\begin{bmatrix} 0 & 1 \\ -7 & 6 \end{bmatrix}$	7	0.5	15	$3sign\left(\cos 0.2t\right) + 3$

➢ state of the system

From the previous theoretical background we can obtain the equation of state of the system as follows:

$$\begin{aligned} \dot{x} &= Ax + bu \\ y &= c^{\top} x \end{aligned} \tag{5.1}$$

where $x \in R^n$ is the state vector, u is the control, $y \in R$ is the adjustable variable,

➢ Robust controller

1. design the Robust controller

From the theoretical background we can obtain the Robust controller as follows:

$$
\begin{aligned}
&\dot{x}_M = A_M x_M + b_M g \\
&y_M = c_M^\top x_M \\
&A_M = A + b\theta^\top, b = \kappa b_M \\
&A_M^T P + P A_M = -Q \\
&\hat{\theta} = \gamma x b^T P e \\
&u = \hat{\theta}^T x + \frac{1}{\kappa} g
\end{aligned}
\tag{5.2}
$$

The simulink scheme is constructed from (5.1) and (5.2) as follows:

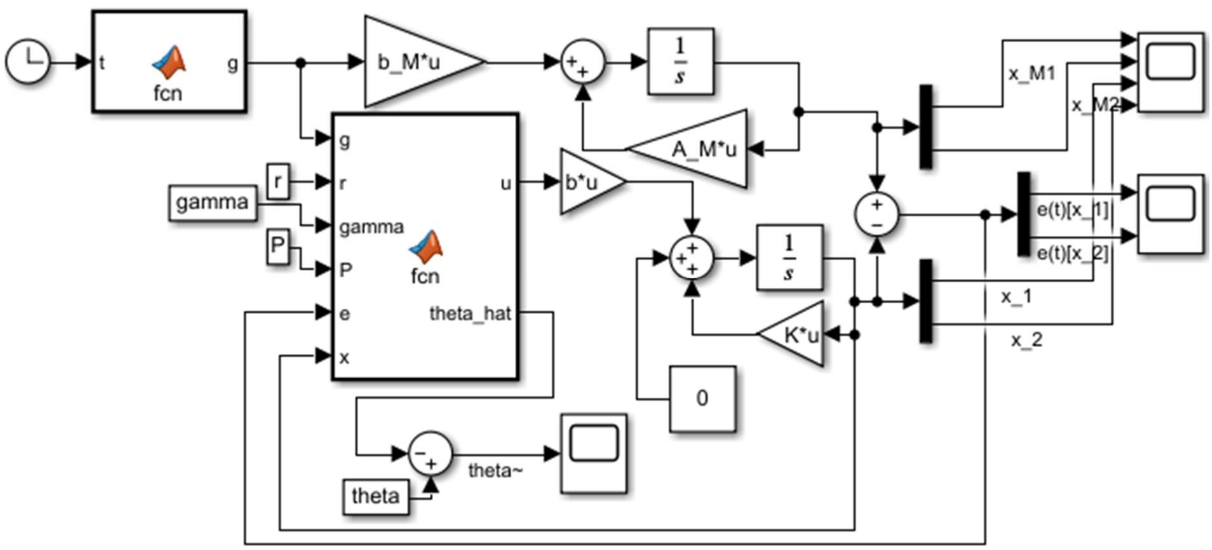

Figure 1: Robust controller simulation scheme

```
function g = fcn(t)
g = 3*sign(cos(0.2*t)) + 3;
```

Figure 2: Robust controller simulation scheme (g(t))

```
function [u, theta_hat] = fcn(g, r, gamma, P, e, x)
b= [0;7];
theta_hat = gamma*x*b'*P*e;
u = theta_hat'*x + 1/r*g;
```

Figure 3: Robust controller simulation scheme (controller(fcn))

2. Simulation

Make experiments for robust controllerfor three different gains γ assuming $\|\delta(t)\| \equiv 0$.The simulation results are shown in the following figure:

1) $\gamma = 0.01$

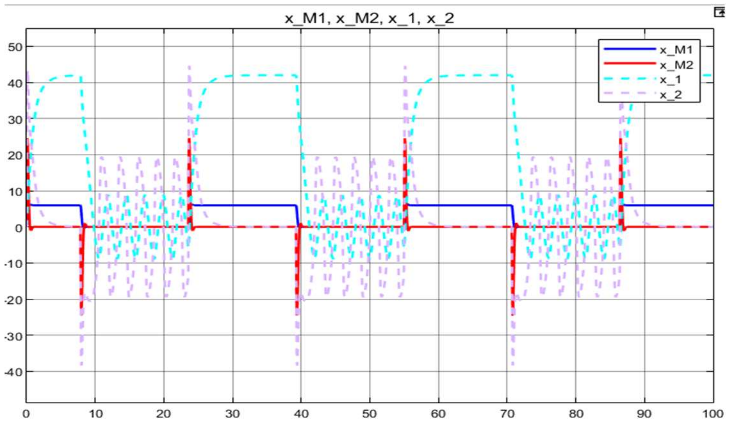

Figure 4： Robust controller simulation result （$x(t), x_m(t)$）

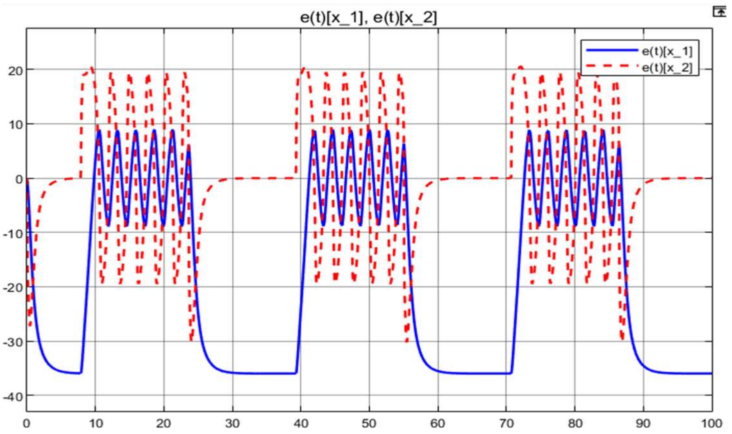

Figure 5： Robust controller simulation result （$\epsilon(t)$）

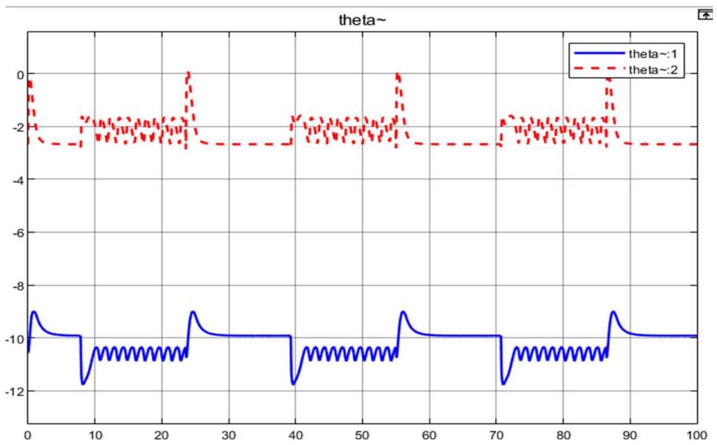

Figure 6：Robust controller simulation result （$\tilde{\theta}(t)$）

2) $\gamma = 1$

The simulation results are shown in the following figures:

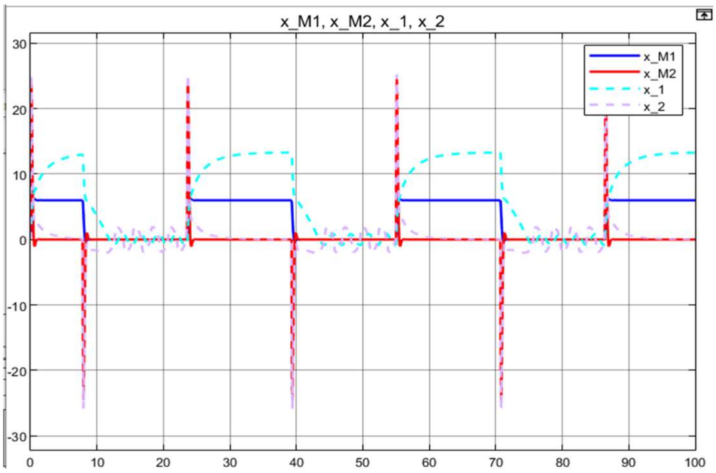

Figure 7： Robust controller simulation result （$x(t), x_m(t)$）

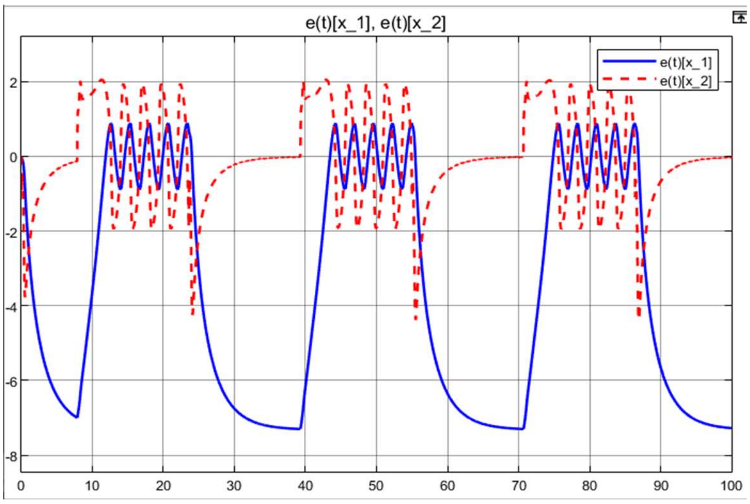

Figure 8： Robust controller simulation result （$\epsilon(t)$）

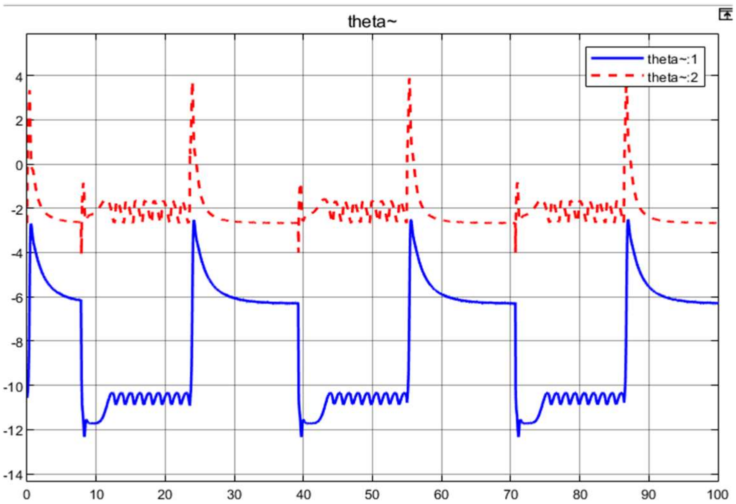

Figure 9：Robust controller simulation result （$\tilde{\theta}(t)$）

3) $\gamma = 100$

The simulation results are shown in the following figures:

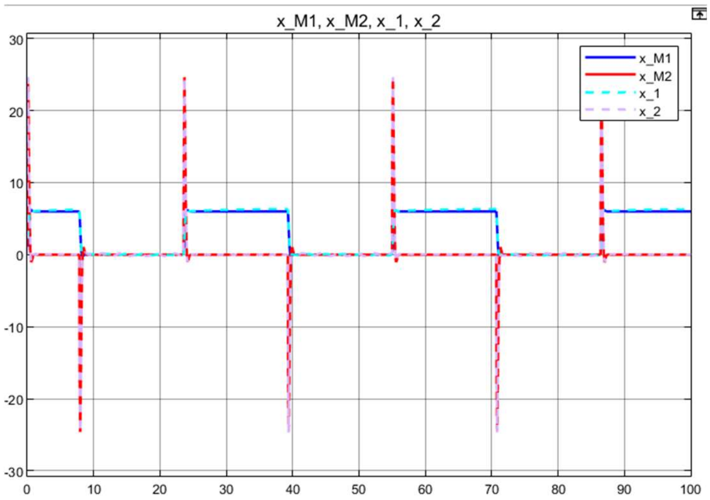

Figure 10： Robust controller simulation result （$x(t), x_m(t)$）

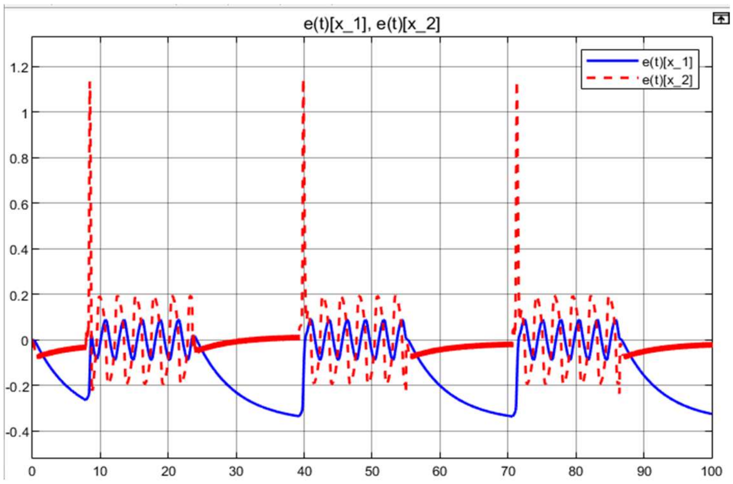

Figure 11 ： Robust controller simulation result （$\epsilon(t)$）

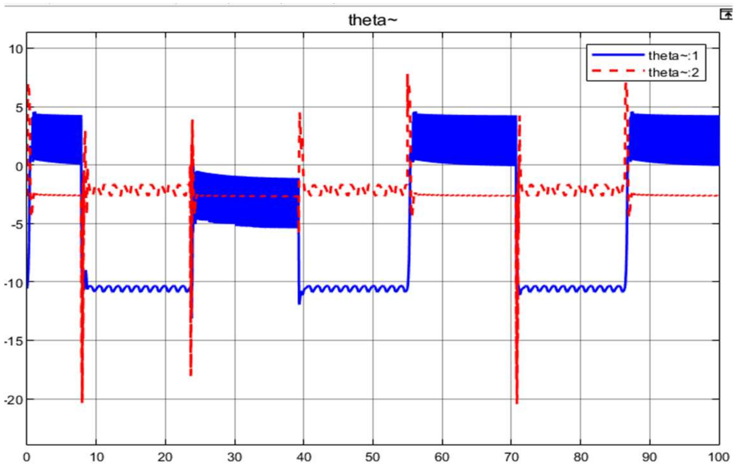

Figure 12：Robust controller simulation result （$\tilde{\theta}(t)$）

➢ Robust Adaptive Controller

1. design the Robust Adaptive controller

From the previous theoretical background we can obtain the Adaptive Controller as follows:

$$
\begin{aligned}
&\dot{x}_M = A_M x_M + b_M g \\
&y_M = c_M^\top x_M \\
&A_M = A + b\theta^\top, b = \kappa b_M \\
&A_M^T P + P A_M = -Q \\
&\dot{\hat{\theta}} = -\sigma\hat{\theta} + \gamma x b^T P e \\
&u = \hat{\theta}^T x + \frac{1}{\kappa} g
\end{aligned}
\tag{5.3}
$$

The simulink scheme is constructed from (5.1) and (5.3) as follows:

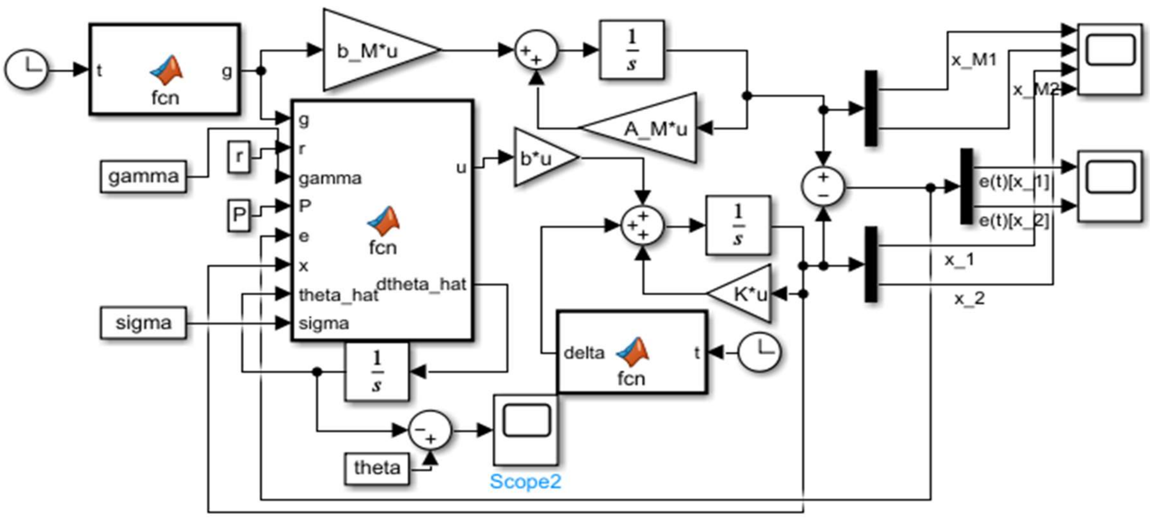

Figure 13: Robust Adaptive controller simulation scheme

```
function g = fcn(t)
g = 3*sign(cos(0.2*t)) + 3;
```

Figure 14: Robust Adaptive controller simulation scheme (reference signal g(t))

```
function [u, dtheta_hat] = fcn(g, r, gamma, P, e, x, theta_hat,sigma)
b= [0;7];
dtheta_hat = -1*sigma*theta_hat + gamma*x*b'*P*e;
u = theta_hat'*x + 1/r*g;
```

Figure 15: Robust Adaptive controller simulation scheme (controller)

```
function delta = fcn(t)
delta = [0.6*sin(10*t)+0.1*sin(50*t);0.5*cos(12*t)+0.2*sin(30*t)];
```

Figure 16: Robust Adaptive controller simulation scheme (disturbance delta)

2. Simulation

A. Make experiments for robust adaptive control for a certain parameter σ and two different γ used before without $\delta(t)$

1) $\sigma = 1 \gamma = 1$

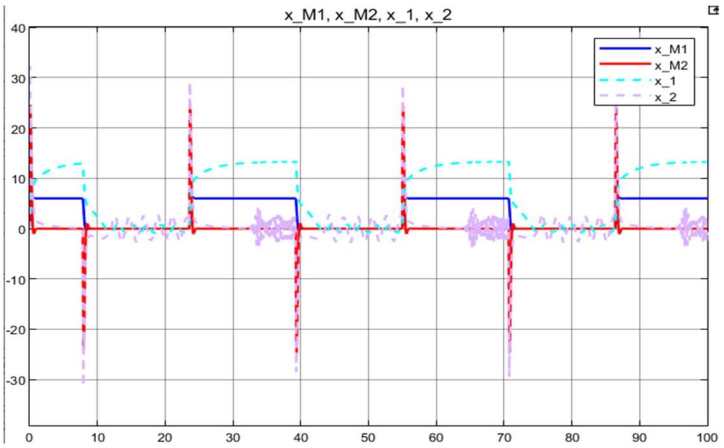

Figure 17： Robust Adaptive controller simulation result （$x(t), x_m(t)$）

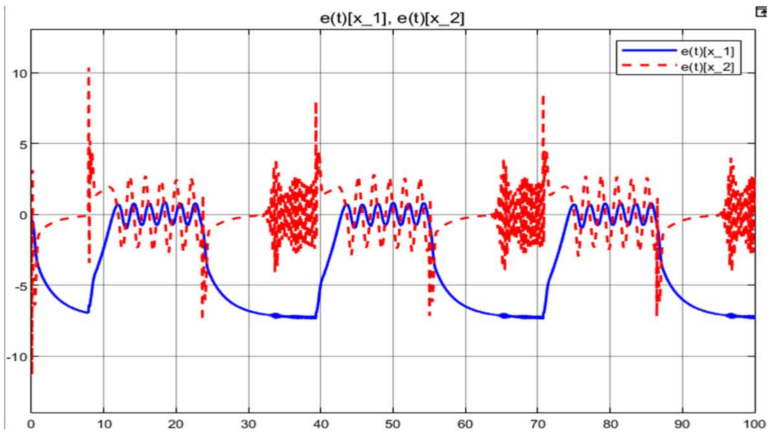

Figure 18： Robust Adaptive controller simulation result （$\epsilon(t)$）

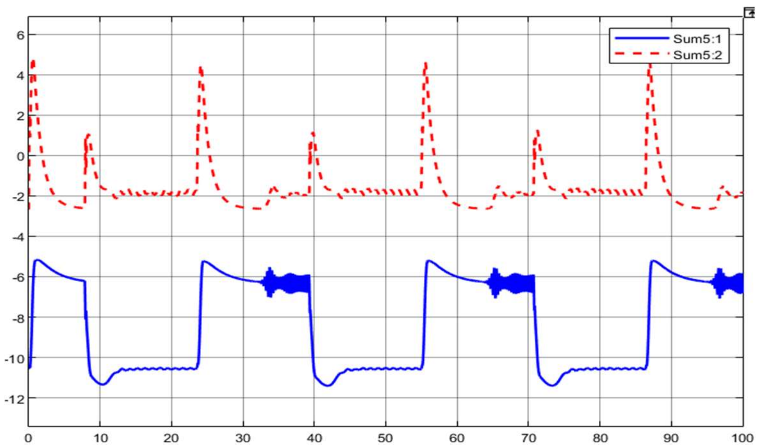

Figure 19：Robust Adaptive controller simulation result （$\tilde{\theta}(t)$）

2) $\sigma = 1\gamma = 100$

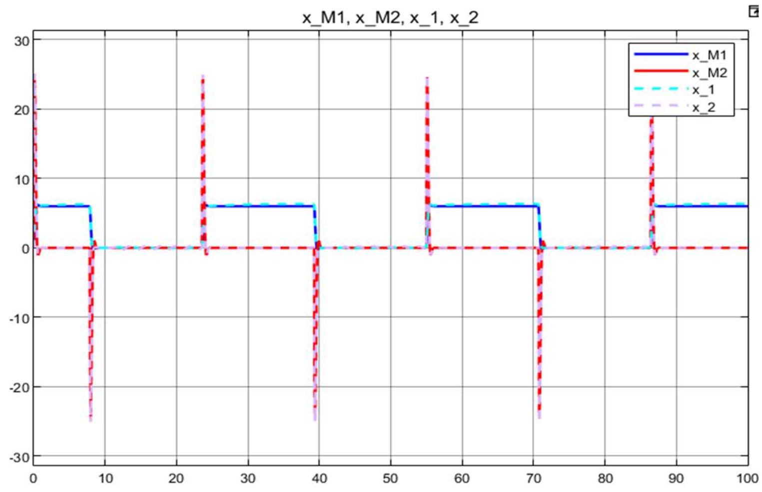

Figure 20： Robust Adaptive controller simulation result （$x(t), x_m(t)$）

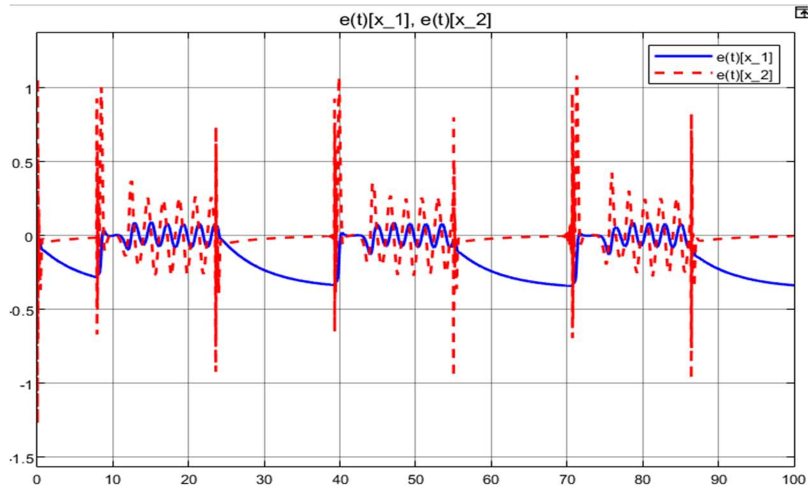

Figure 21： Robust Adaptive controller simulation result （$\epsilon(t)$）

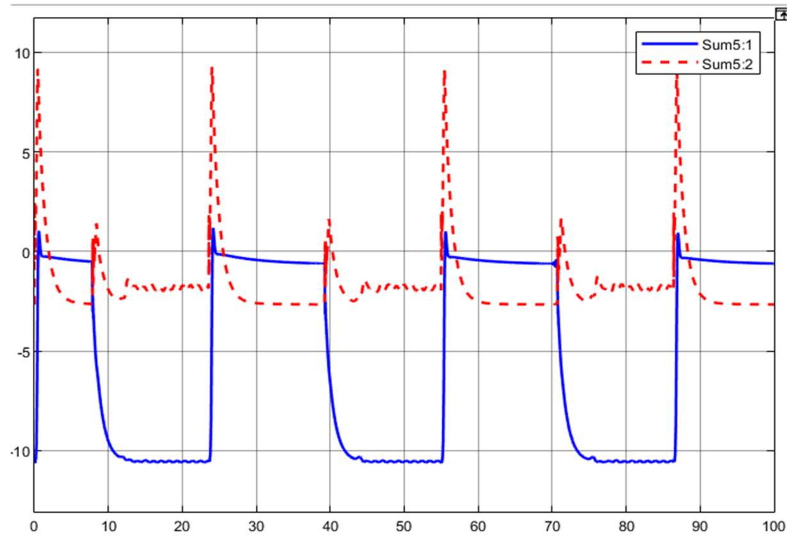

Figure 22：Robust Adaptive controller simulation result （$\tilde{\theta}(t)$）

B. Make experiments for robust adaptive control for a certain parameter σ and two different γ used before with $\delta(t)$

1) $\sigma = 1 \gamma = 1$

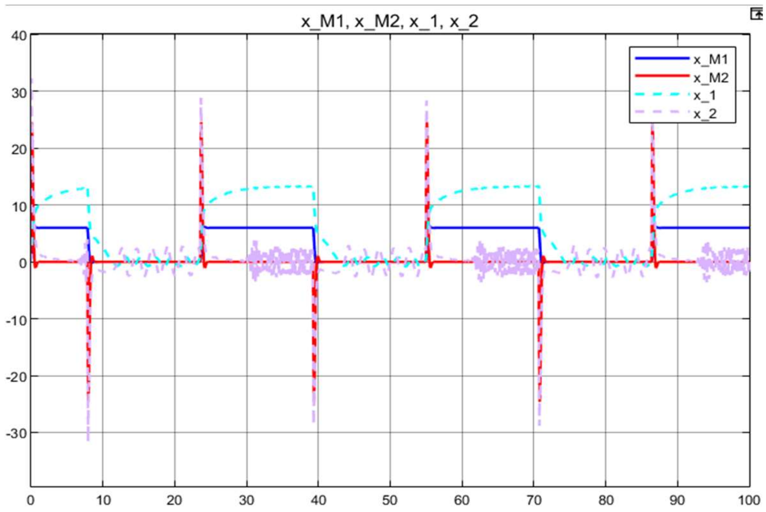

Figure 23 : Robust Adaptive controller simulation result ($x(t), x_m(t)$)

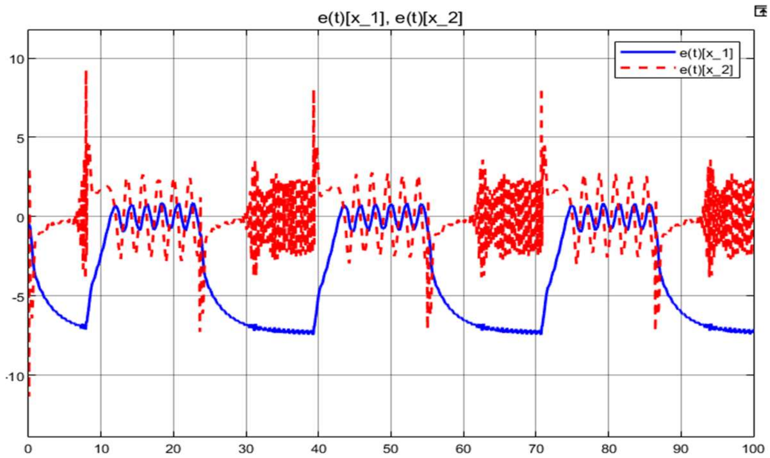

Figure 24 : Robust Adaptive controller simulation result ($\epsilon(t)$)

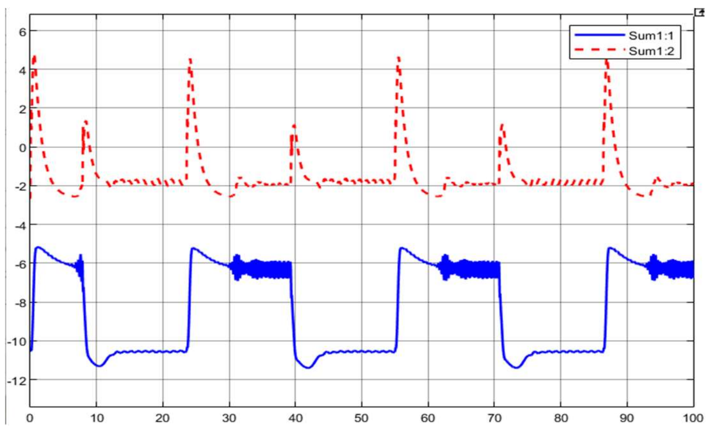

Figure 25: Robust Adaptive controller simulation result ($\tilde{\theta}(t)$)

2) $\sigma = 1 \gamma = 100$

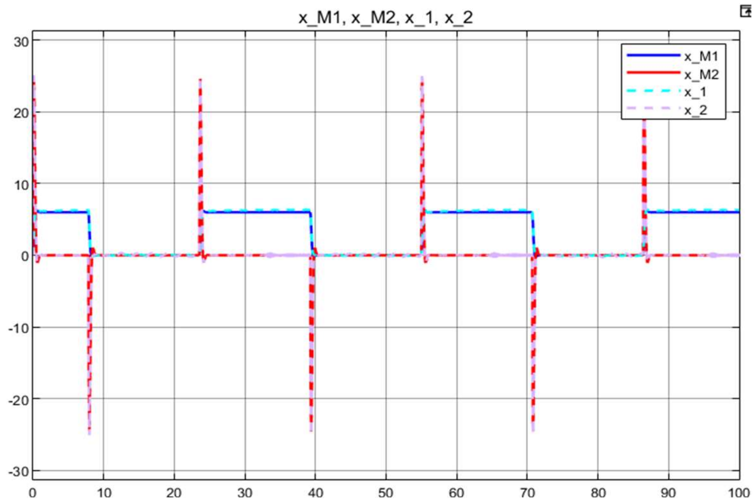

Figure 26： Robust Adaptive controller simulation result（$x(t), x_m(t)$）

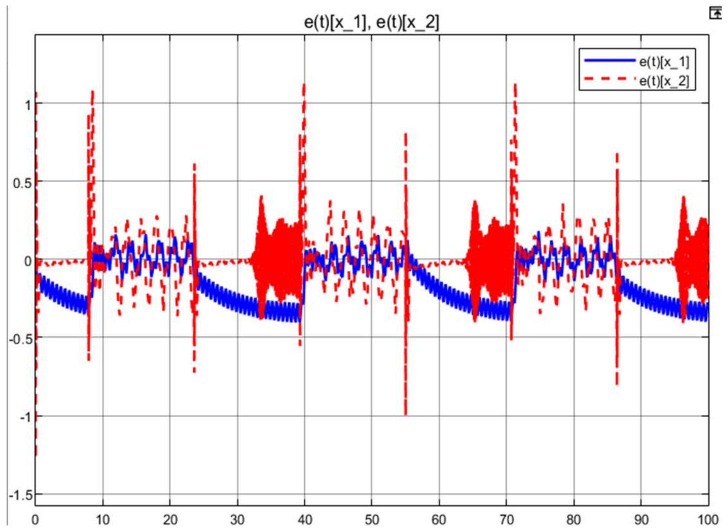

Figure 27： Robust Adaptive controller simulation result（$\epsilon(t)$）

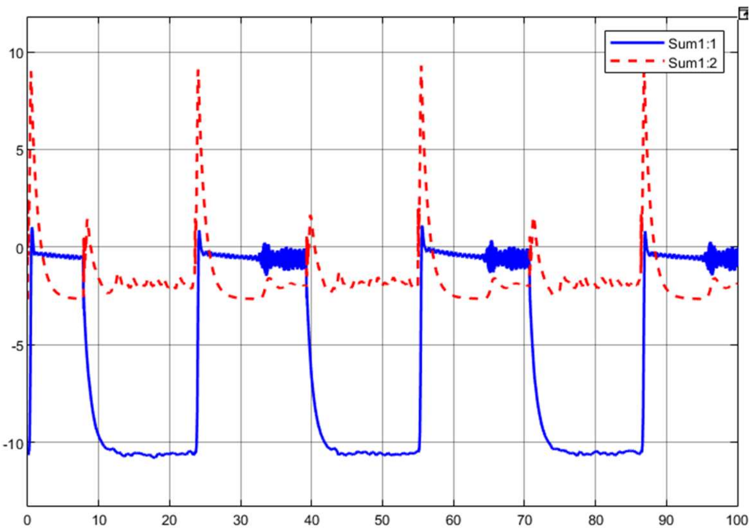

Figure 28：Robust Adaptive controller simulation result（$\tilde{\theta}(t)$）

C. Decrease σ and repeat this experiment for one of the selected gain γ.

1) without $\delta(t)$, $\sigma = 0.01 \gamma = 1$

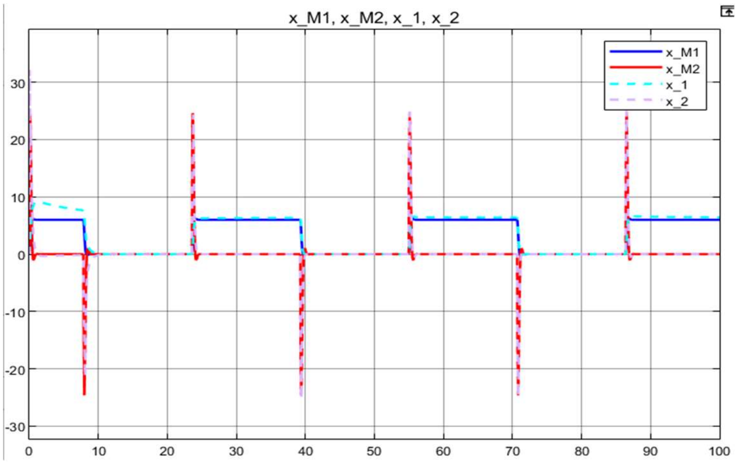

Figure 29： Robust Adaptive controller simulation result （$x(t), x_m(t)$）

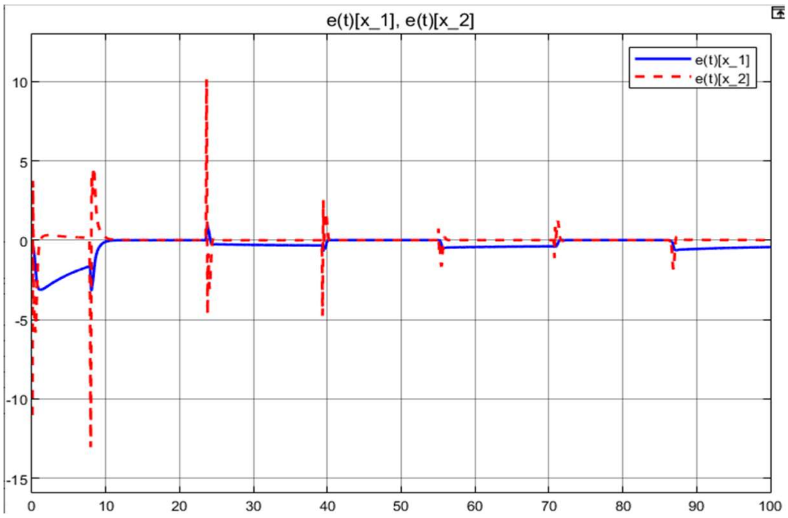

Figure 30： Robust Adaptive controller simulation result （$\epsilon(t)$）

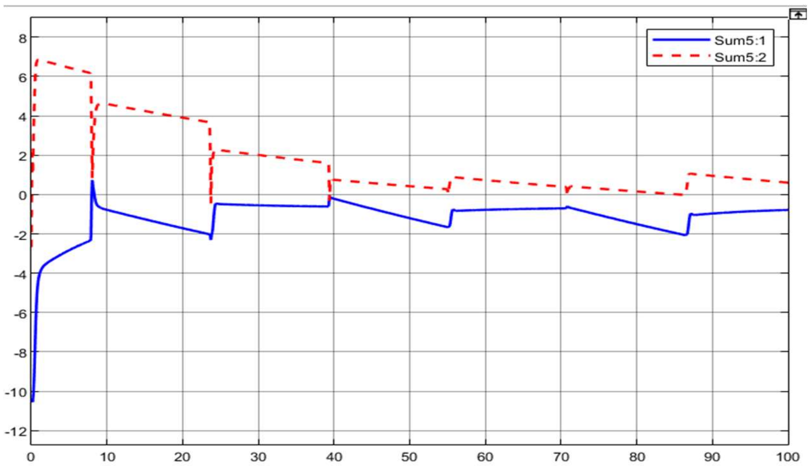

Figure 31：Robust Adaptive controller simulation result （$\tilde{\theta}(t)$）

2) with $\delta(t)$, $\sigma = 0.01\gamma = 1$

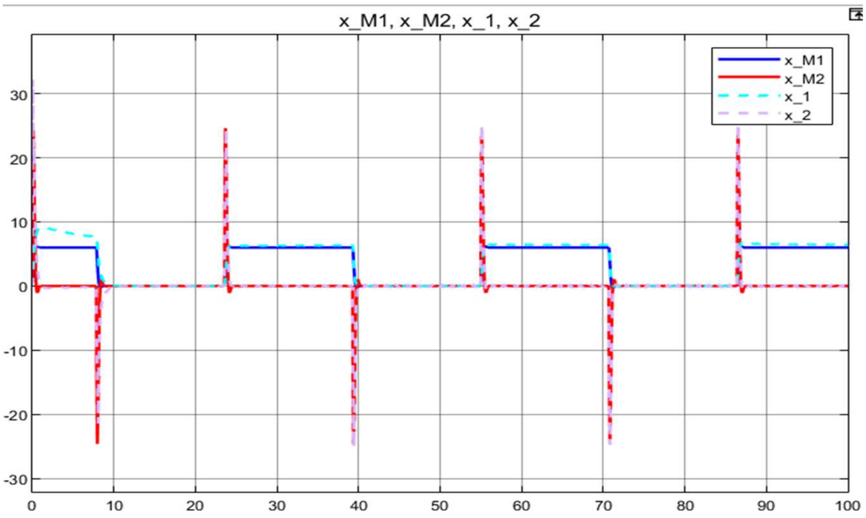

Figure 32： Robust Adaptive controller simulation result （$x(t), x_m(t)$）

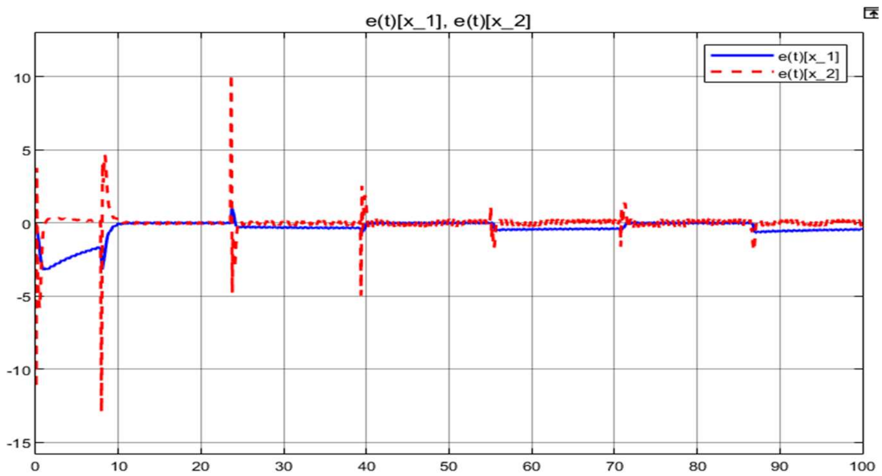

Figure 33： Robust Adaptive controller simulation result （$\epsilon(t)$）

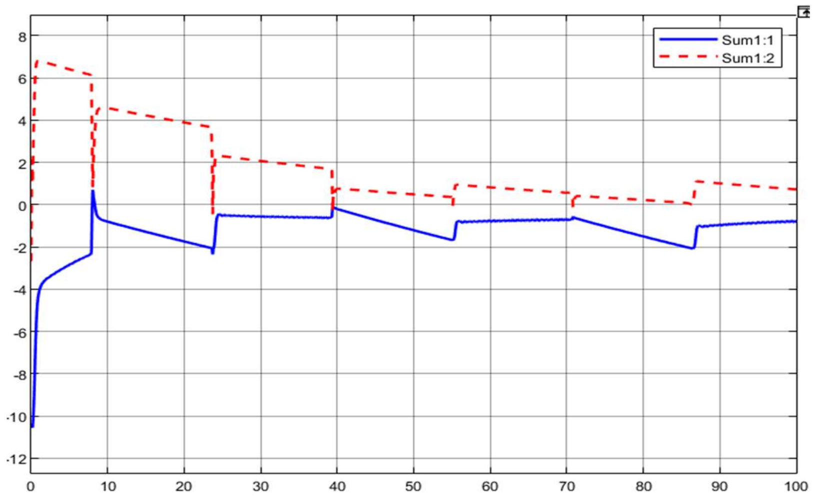

Figure 34: Robust Adaptive controller simulation result （$\tilde{\theta}(t)$）

C.Conclusions

- **Robust Controller**

From the simulation results we can show the following properties of the robust controller :

- Boundedness of all the signals in the closed-loop system;
  (Figure 4-12)
- Exponential convergence of the norm $\| e \|$ to residual set including zero origin $e^* = 0$. The radius of the set can be decreased by increasing gain γ;
  (Figure 5,8,11)
- Even in the disturbance-free case the steady state control error is not zero.
  (Figure 5,8,11)

- **Robust Adaptive Controller**

From the simulation results we can show the following properties of the adaptive robust controller :

- Boundedness of all the signals in the closed-loop system
  ;(Figure 17-34)
- Exponential convergence of the norms $\| e \|$ and $\| \tilde{\theta} \|$ to residual sets including origins $e^* = 0$ and $\tilde{\theta}^* = 0$, respectively. Radius of the sets can be reduced by increasing γ and decreasing σ;
  (Figure 18,19,21,22,24,25,27,28,30,31,33,34)
- In general case steady state values of e and $\tilde{\theta}$ are not zero, however they can be decreased by decreasing σ.
  (Figure 18,19,21,22,24,25,27,28,30,31,33,34)