Practice 4

Student Information

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k = 7

1. Find Equilibrium Points:

1)

$$\begin{aligned} \dot{x}_1 &= -x_1 + 14x_1^3 + x_2 \\ \dot{x}_2 &= -x_1 - 7x_2 \end{aligned} \tag{1}$$

$$\begin{aligned} \dot{x}_1 &= -x_1 + 14x_1^3 + x_2 = 0 \\ \dot{x}_2 &= -x_1 - 7x_2 = 0 \end{aligned} \tag{2}$$

Solution (2) we can get :

Equilibrium points :

$$(0;0)\,,(\frac{2}{7};-\frac{2}{49})\,,(-\frac{2}{7};\frac{2}{49}) \tag{3}$$

2)

$$\begin{aligned} \dot{x}_1 &= 7x_1 + x_1x_2 \\ \dot{x}_2 &= -x_2 + x_2^2 + x_1x_2 - x_1^3 \end{aligned} \tag{4}$$

$$\begin{aligned} \dot{x}_1 &= 7x_1 + x_1x_2 = 0 \\ \dot{x}_2 &= -x_2 + x_2^2 + x_1x_2 - x_1^3 = 0 \end{aligned} \tag{5}$$

Solution (5) we can get :

Equilibrium points :

$$(0;0)\,,(0;1)\,,(3.2219;-7) \tag{6}$$

3)

$$\begin{aligned} \dot{x}_1 &= 7x_2 \\ \dot{x}_2 &= -x_1 + x_2\left(1 - x_1^2 + 0.7x_1^4\right) \end{aligned} \tag{7}$$

$$\begin{aligned} \dot{x}_1 &= 7x_2 = 0 \\ \dot{x}_2 &= -x_1 + x_2\left(1 - x_1^2 + 0.7x_1^4\right) = 0 \end{aligned} \tag{8}$$

Solution (8) we can get :

Equilibrium points :

$$(0;0) \tag{9}$$

4)

$$\begin{aligned} \dot{x}_1 &= 7\left(x_1 - x_2\right)\left(1 - x_1^2 - x_2^2\right) \\ \dot{x}_2 &= \left(x_1 + x_2\right)\left(1 - x_1^2 - x_2^2\right) \end{aligned} \tag{10}$$

$$\begin{aligned} \dot{x}_1 &= 7\left(x_1 - x_2\right)\left(1 - x_1^2 - x_2^2\right) = 0 \\ \dot{x}_2 &= \left(x_1 + x_2\right)\left(1 - x_1^2 - x_2^2\right) = 0 \end{aligned} \tag{11}$$

Solution (11) we can get :

Equilibrium points :

1. $$\text{All points on the circle :} \qquad x_1^2 + x_2^2 = 1 \tag{12}$$

2. $$(0;0) \tag{13}$$

5)

$$\begin{aligned} \dot{x}_1 &= -x_1^3 + 7x_2 \\ \dot{x}_2 &= 7x_1 - x_2^3 \end{aligned} \tag{14}$$

$$\begin{aligned} \dot{x}_1 &= -x_1^3 + 7x_2 = 0 \\ \dot{x}_2 &= 7x_1 - x_2^3 = 0 \end{aligned} \tag{15}$$

Solution (11) we can get :

Equilibrium points :

$$(0;0)\,,(\sqrt{7};\sqrt{7})\,,(-\sqrt{7};-\sqrt{7}) \tag{16}$$

2. Discuss why these systems are nonlinear

1)

$$\begin{aligned}\dot{x}_1 &= -x_1 + 14x_1^3 + x_2 \\ \dot{x}_2 &= -x_1 - 7x_2\end{aligned} \tag{17}$$

There are nonlinear expressions in the system as follows:

$$14x_1^3 \tag{18}$$

2)

$$\begin{aligned}\dot{x}_1 &= 7x_1 + x_1x_2 \\ \dot{x}_2 &= -x_2 + x_2^2 + x_1x_2 - x_1^3\end{aligned} \tag{19}$$

There are nonlinear expressions in the system as follows:

$$x_1x_2, x_2^2, x_1x_2, -x_1^3 \tag{20}$$

3)

$$\begin{aligned}\dot{x}_1 &= 7x_2 \\ \dot{x}_2 &= -x_1 + x_2\left(1 - x_1^2 + 0.7x_1^4\right)\end{aligned} \tag{21}$$

There are nonlinear expressions in the system as follows :

$$-x_2x_1^2, 0.7x_1^4x_2 \tag{22}$$

4)

$$\begin{aligned}\dot{x}_1 &= 7\left(x_1 - x_2\right)\left(1 - x_1^2 - x_2^2\right) \\ \dot{x}_2 &= \left(x_1 + x_2\right)\left(1 - x_1^2 - x_2^2\right)\end{aligned} \tag{23}$$

There are nonlinear expressions in the system as follows :

$$-7x_1^3, -7x_1x_2^2, 7x_1^2x_2, 7x_2^3, -x_1^3, -x_1x_2^2, -x_1^2x_2, -x_2^3 \tag{24}$$

5)

$$\begin{aligned}\dot{x}_1 &= -x_1^3 + 7x_2 \\ \dot{x}_2 &= 7x_1 - x_2^3\end{aligned} \tag{25}$$

There are nonlinear expressions in the system as follows :

$$-x_1^3, -x_2^3 \tag{26}$$

3. Make simulation

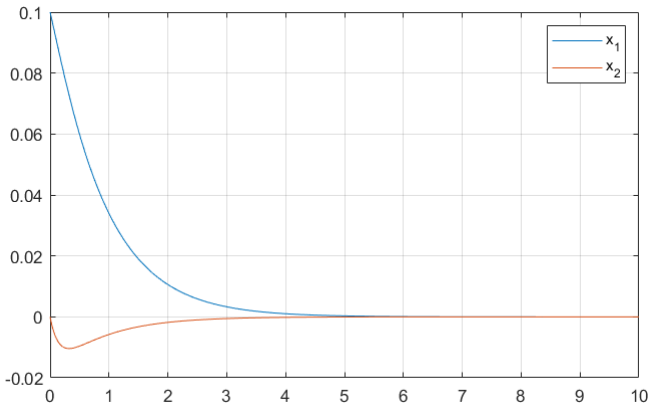
1)

$$\begin{aligned} \dot{x}_1 &= -x_1 + 14x_1^3 + x_2 \\ \dot{x}_2 &= -x_1 - 7x_2 \end{aligned} \tag{27}$$

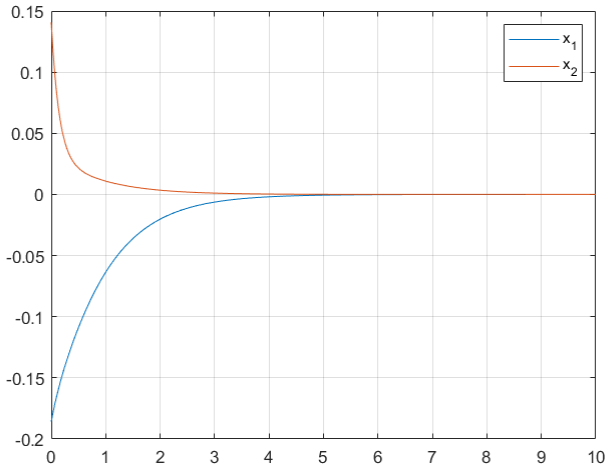
Equilibrium points :

$$(0;0)\,,(\frac{2}{7};-\frac{2}{49})\,,(-\frac{2}{7};\frac{2}{49}) \tag{28}$$

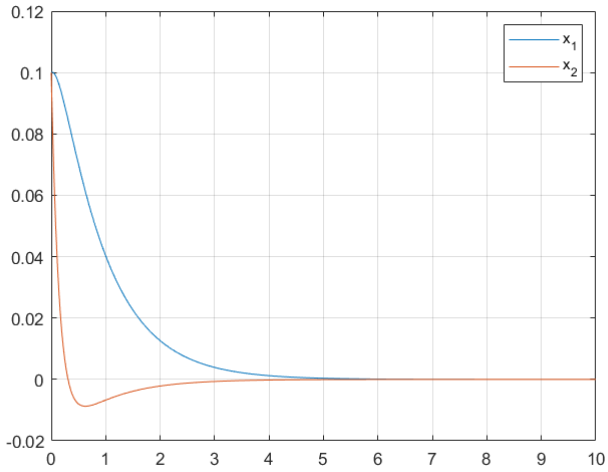
1. $x(0) = (0.1; 0)$



2. $x(0) = (-\frac{2}{7} + 0.1; \frac{2}{49} + 0.1)$



3. $x(0) = (0.1; 0.1)$



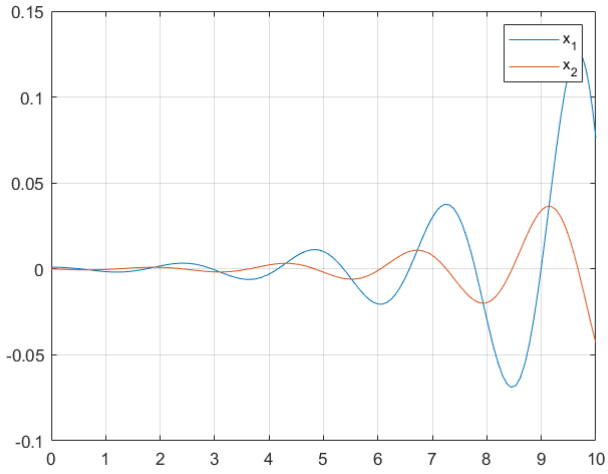
3)

$$\begin{aligned} \dot{x}_1 &= 7x_2 \\ \dot{x}_2 &= -x_1 + x_2\left(1 - x_1^2 + 0.7x_1^4\right) \end{aligned} \tag{29}$$

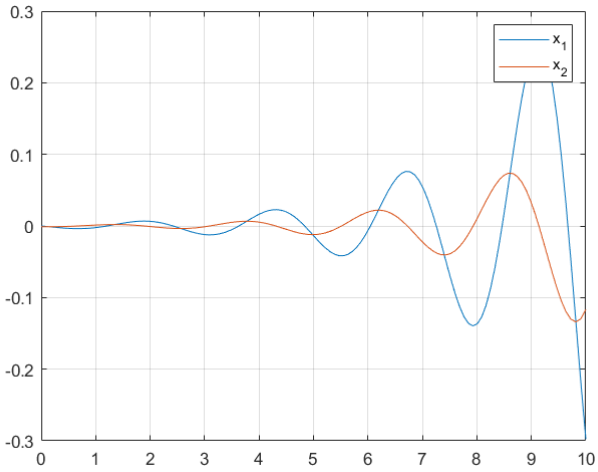
Equilibrium points :

$$(0;0) \tag{30}$$

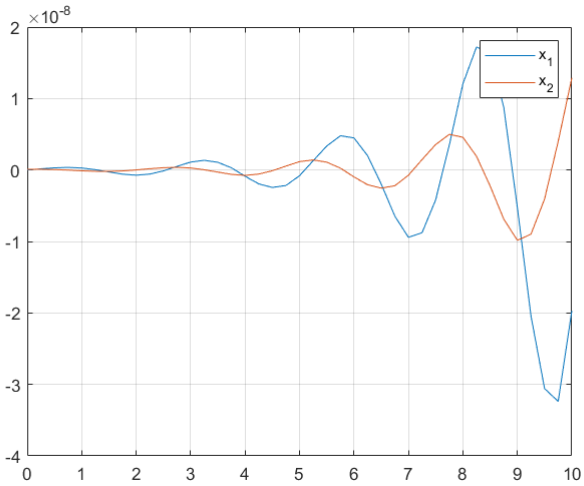
1. $x(0) = (0.001; 0)$



2. $x(0) = (0; -0.001)$



3. $x(0) = (0; 1e - 10)$



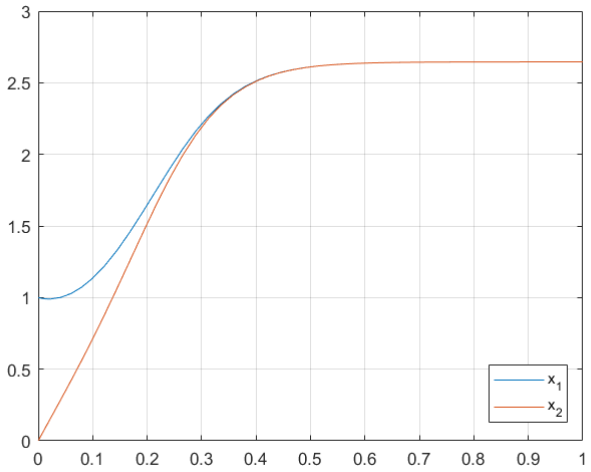
5)

$$\begin{aligned} \dot{x}_1 &= -x_1^3 + 7x_2 \\ \dot{x}_2 &= 7x_1 - x_2^3 \end{aligned} \tag{31}$$

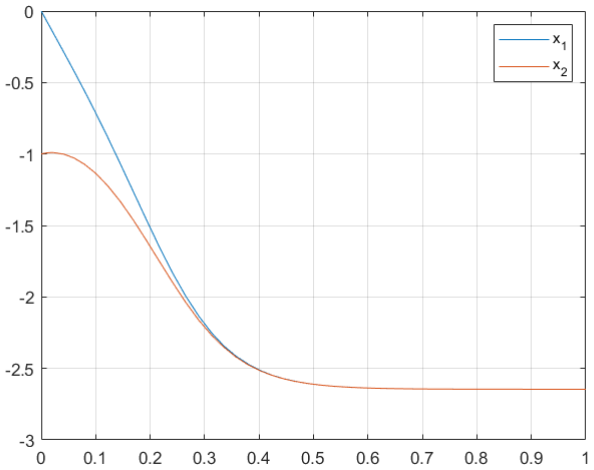
Equilibrium points :

$$(0; 0)\ ,\ (\sqrt{7}; \sqrt{7})\ ,\ (-\sqrt{7}; -\sqrt{7}) \tag{32}$$

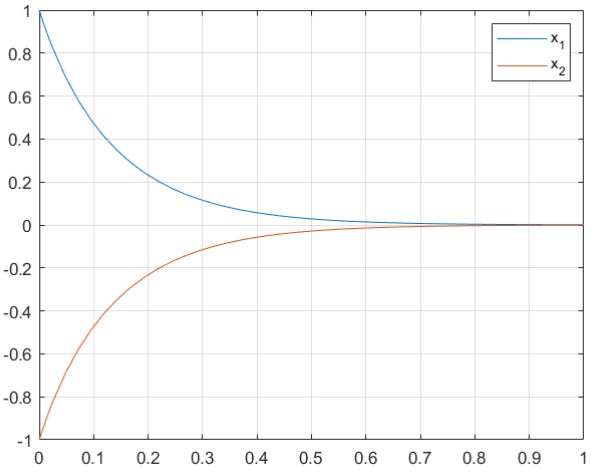
1. $x(0) = (1; 0)$



2. $x(0) = (0; -1)$



3. $x(0) = (1; -1)$



4. phase portrait (Optional)

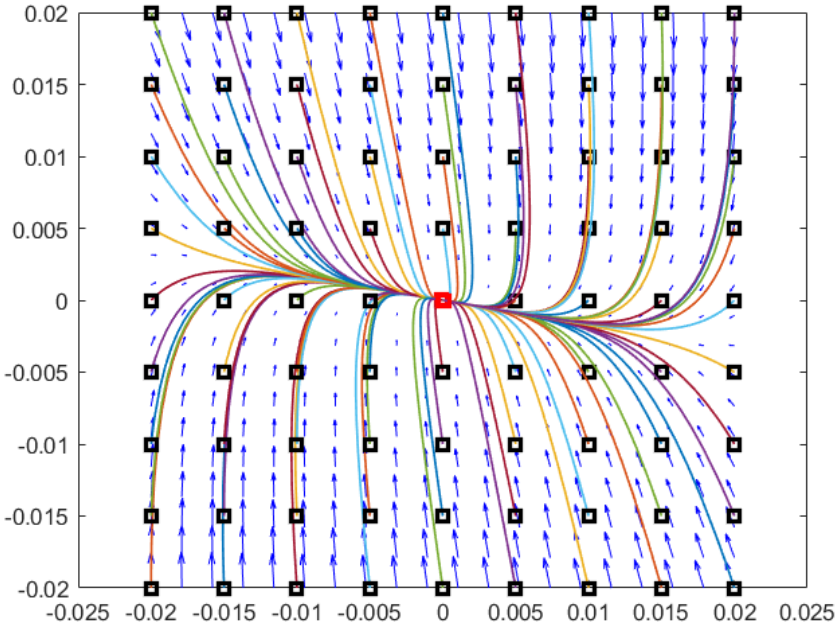
1)

$$\begin{aligned} \dot{x}_1 &= -x_1 + 14x_1^3 + x_2 \\ \dot{x}_2 &= -x_1 - 7x_2 \end{aligned} \tag{33}$$

Equilibrium points :

$$(0;0)\ ,(\frac{2}{7};-\frac{2}{49})\ ,(-\frac{2}{7};\frac{2}{49}) \tag{34}$$

Black is the initial point, red is the end point



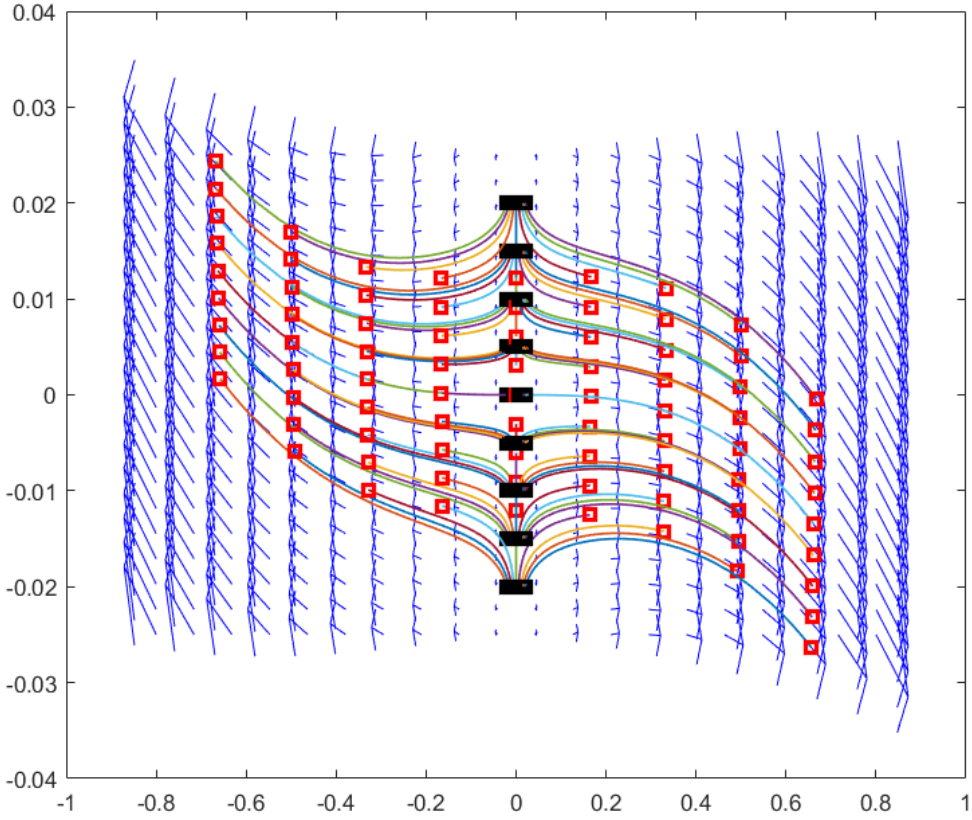
2)

$$\begin{aligned} \dot{x}_1 &= 7x_1 + x_1x_2 \\ \dot{x}_2 &= -x_2 + x_2^2 + x_1x_2 - x_1^3 \end{aligned} \tag{35}$$

Equilibrium points :

$$(0;0)\ ,(0;1)\ ,(3.2219;-7) \tag{36}$$

Black is the initial point, red is the end point



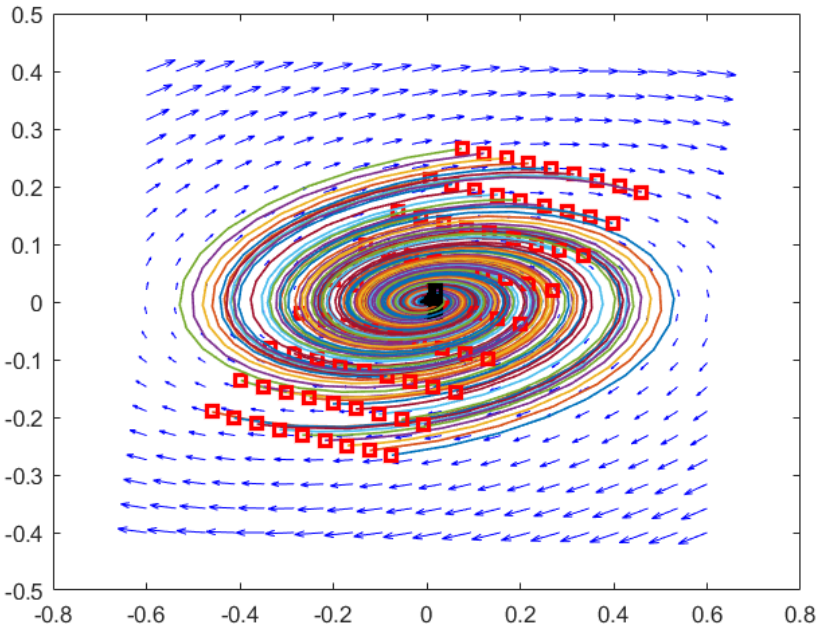
3)

$$\begin{aligned} \dot{x}_1 &= 7x_2 \\ \dot{x}_2 &= -x_1 + x_2\left(1 - x_1^2 + 0.7x_1^4\right) \end{aligned} \tag{37}$$

Equilibrium points :

$$(0;0) \tag{38}$$

Black is the initial point, red is the end point



4)

$$\begin{aligned} \dot{x}_1 &= 7\left(x_1 - x_2\right)\left(1 - x_1^2 - x_2^2\right) \\ \dot{x}_2 &= \left(x_1 + x_2\right)\left(1 - x_1^2 - x_2^2\right) \end{aligned} \tag{39}$$

Equilibrium points :

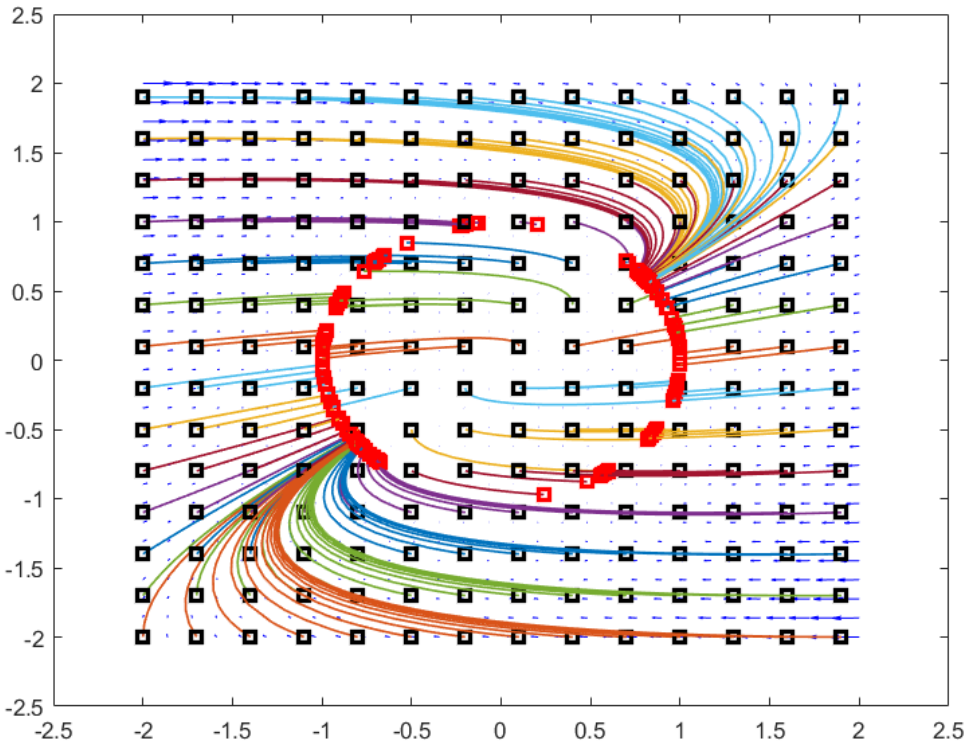
1.

$$\text{All points on the circle :} \qquad x_1^2 + x_2^2 = 1 \tag{40}$$

2.

$$(0;0) \tag{41}$$

Black is the initial point, red is the end point



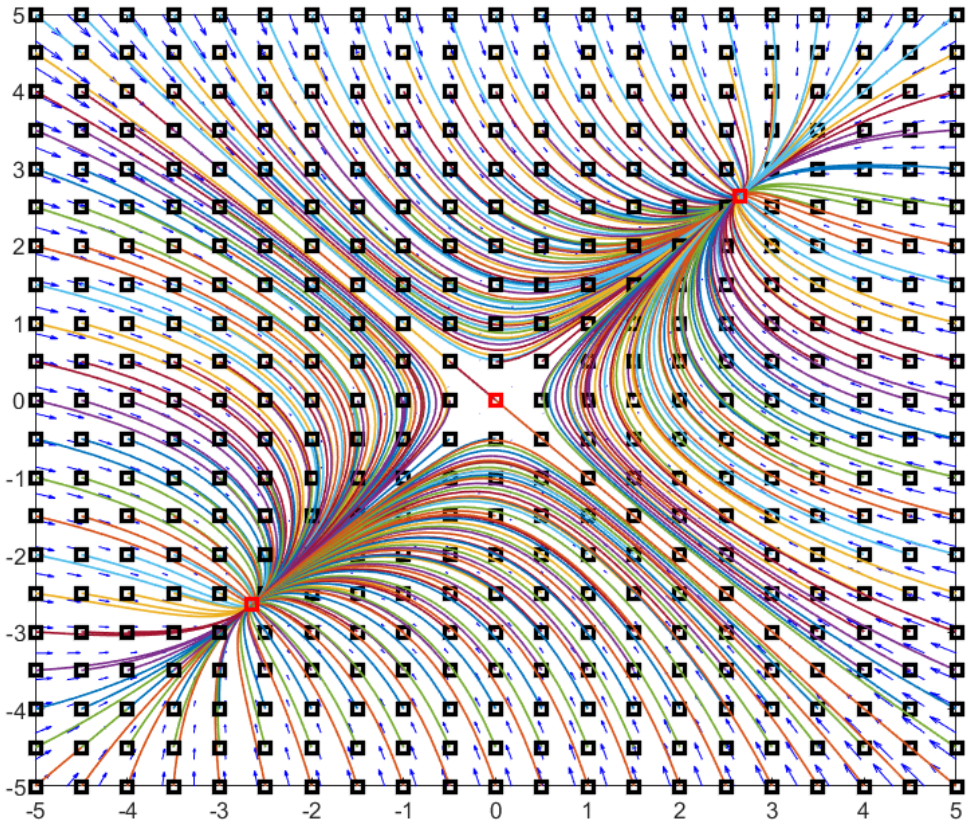
5)

$$\begin{aligned} \dot{x}_1 &= -x_1^3 + 7x_2 \\ \dot{x}_2 &= 7x_1 - x_2^3 \end{aligned} \tag{42}$$

Equilibrium points :

$$(0;0)\,,(\sqrt{7};\sqrt{7})\,,(-\sqrt{7};-\sqrt{7}) \tag{43}$$

Black is the initial point, red is the end point


5
4
3
2
1
0
-1
-2
-3
-4
-5
-5
-4
-3
-2
-1
0
1
2
3
4
5