Practice 6

Student Information

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> k = 7

1)

$$\begin{aligned} \dot{x}_1 &= x_1 + x_2 \\ \dot{x}_2 &= 2x_1^3 x_2 + x_1 - u \end{aligned} \tag{1}$$

1.1 Find feedback control laws

choose：

$$u = 2x_1^3 x_2 + v \tag{2}$$

Then ：

$$\begin{aligned} \dot{x}_1 &= x_1 + x_2 \\ \dot{x}_2 &= x_1 - v \end{aligned} \tag{3}$$

choose：

$$v = k_1 x_1 + k_2 x_2. \tag{4}$$

Then ：

$$\begin{aligned} \dot{x}_1 &= x_1 + x_2 \\ \dot{x}_2 &= (1 - k_1)x_1 - k_2 x_2 \quad \Rightarrow \dot{x} = \underbrace{\begin{bmatrix} 1 & 1 \\ 1 - k_1 & -k_2 \end{bmatrix}}_{\text{is Hurwtz}} x \end{aligned} \tag{5}$$

So, the feedback control law of this system is :

$$u = 2x_1^3 x_2 + k_1 x_1 + k_2 x_2. \tag{6}$$

1.2 Stabilize the systems with 0% overshooting

We need to choose k_1, k_2 to make the system stable,When the feedback controller is used, our system becomes linear, and we can use the analysis method of the linear system to solve.

1.2.1 Constructing linear system with feedback control laws

After feedback control, we have transformed the nonlinear system into a linear system

$$\dot{x} = \begin{bmatrix} 1 & 1 \\ 1-k_1 & -k_2 \end{bmatrix} \cdot x \tag{7}$$

To find k_1, k_2 we can use the modal control method

1.2.1 modal control

First, let $k_1 = 0, k_2 = 0$ to get the original uncontrolled system

Thus, we have the following system:

$$\begin{gathered} \dot{x} = A \cdot x \\ A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \end{gathered} \tag{8}$$

1. Determine whether the system is stable now:

Find the eigenvalues of matrix A:

$$\begin{gathered} \lambda_1 = -0.618 < 0 \\ \lambda_2 = 1.618 > 0 \end{gathered} \tag{9}$$

The system is now **unstable**

2. checking system for complete controllability

$$\text{rank}(N_c) = [B \;\; AB] = 2 \tag{10}$$

system is completely controllable

3. Form reference model

System order	Butterworth polynomial	Overshooting $\sigma, \%$	Settling time t_s^*, sec
$n = 1$	$s + \omega_0$	0	3
$n = 2$	$s^2 + 2\omega_0 s + \omega_0^2$	0	4.8

(11)

n=2 Newton polynomial

$$\begin{gathered} \omega = \frac{t_s^*}{t_S} = 2.4 \\ D_d(s) = s^2 + 4.8s + 5.76 \\ \Gamma = \begin{bmatrix} 0 & 1 \\ -5.76 & -4.8 \end{bmatrix}, H = [1 \quad 0] \end{gathered} \tag{12}$$

4. Finding matrix transfoemation M

The solution of the Sylvester matrix equation with respect to the matrix M:

```
M = sylv(-A,gamma,-B*H)
```

5. Calculation of matrix K

$$K = HM^{-1} = [12.56 \;\; 5.8] \tag{13}$$

6. Form control signal

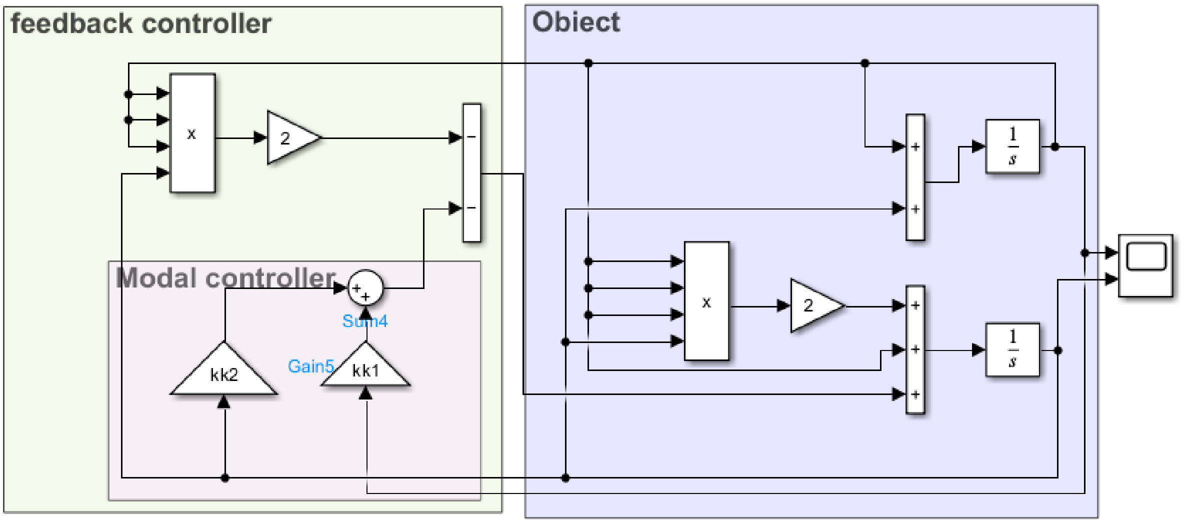
$$v(t) = -\mathrm{K}x(t) \tag{14}$$

1.2.3 control law

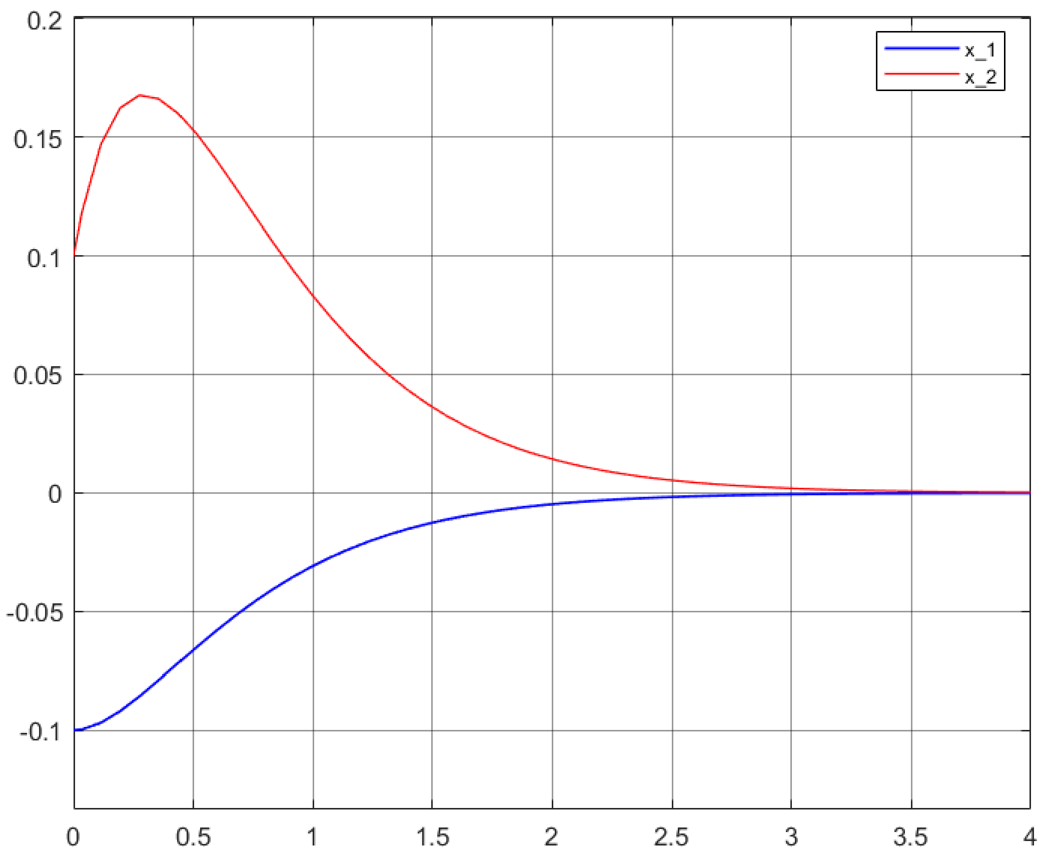
In summary, we can get the control signal of the feedback control

$$u = 2x_1^3 x_2 + 12.56x_1 + 5.8x_2. \tag{15}$$

1.3 Modelling



1.4 Simulation



2)

$$
\begin{aligned}
\dot{x}_1 &= -x_1 + x_2 - x_3 \\
\dot{x}_2 &= -x_1 x_3 - x_2 + u \\
\dot{x}_3 &= -x_1 + u
\end{aligned} \tag{16}
$$

2.1 change of coordinates

use the change of coordinates

$$
z = T(x) := \begin{pmatrix} x_1 \\ -x_1 + x_2 - x_3 \\ 2x_1 - 2x_2 + x_3 - x_1 x_3 \end{pmatrix} \tag{17}
$$

2.2 Find feedback control laws

$$
\begin{aligned}
\dot{z} &= \begin{bmatrix} \dot{x_1} \\ -\dot{x_1} + \dot{x_2} - \dot{x_3} \\ 2\dot{x_1} - 2\dot{x_2} + \dot{x_3} - \dot{x_1} x_3 - x_1 \dot{x_3} \end{bmatrix} \\
\begin{bmatrix} \dot{z_1} \\ \dot{z_2} \\ \dot{z_3} \end{bmatrix} &= \begin{bmatrix} z_2 \\ z_3 \\ z_1 + z_1^2 + \frac{1}{z_1+1}(-2z_2^2 - 2z_3 - 2z_1 z_3 + z_2 z_3) - (z_1+1)u \end{bmatrix}
\end{aligned} \tag{18}
$$

choose:

$$
u = \frac{1}{(z_1+1)^2}(z_1^3 + z_1^2 - 2z_2^2 - 2z_3 - 2z_1 z_3 + z_2 z_3) + \frac{1}{z_1+1} v \tag{19}
$$

Then :

$$
\begin{aligned}
\dot{z}_1 &= z_2 \\
\dot{z}_2 &= z_3 \\
\dot{z}_3 &= z_1 - v
\end{aligned} \tag{20}
$$

choose:

$$
v = k_1 z_1 + k_2 z_2 + k_3 z_3 \tag{21}
$$

Then :

$$
\begin{aligned}
\dot{z}_1 &= z_2 \\
\dot{z}_2 &= z_3 \\
\dot{z}_3 &= (1 - k_1) z_1 - k_2 z_2 - k_3 z_3
\end{aligned} \tag{22}
$$

$$
\begin{aligned}
\dot{z}_1 &= z_2 \\
\dot{z}_2 &= z_3 \\
\dot{z}_3 &= (1 - k_1) z_1 - k_2 z_2 - k_3 z_3
\end{aligned}
\quad \Rightarrow \dot{x} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 - k_1 & -k_2 & -k_3 \end{bmatrix}}_{\text{is Hurwtz}} x \tag{23}
$$

So, the feedback control law of this system is :

$$
u = \frac{1}{(z_1+1)^2}(z_1^3 + z_1^2 - 2z_2^2 - 2z_3 - 2z_1 z_3 + z_2 z_3) + \frac{1}{z_1+1}(k_1 z_1 + k_2 z_2 + k_3 z_3) \tag{24}
$$

2.2 Stabilize the systems with 0% overshooting

We need to choose k_1, k_2 to make the system stable,When the feedback controller is used, our system becomes linear, and we can use the analysis method of the linear system to solve.

2.2.1 Constructing linear system with feedback control laws

After feedback control, we have transformed the nonlinear system into a linear system

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1-k_1 & -k_2 & -k_3 \end{bmatrix} \cdot x \tag{25}$$

To find k_1, k_2, k_3 we can use the modal control method

1.2.1 modal control

First, let $k_1 = 0, k_2 = 0, k_3 = 0$ to get the original uncontrolled system

Thus, we have the following system:

$$\begin{gathered} \dot{x} = A \cdot x \\ A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \end{gathered} \tag{26}$$

1. Determine whether the system is stable now:

Find the eigenvalues of matrix A:

$$\begin{gathered} \lambda_1 = -0.5000 + 0.8660i \\ \lambda_2 = -0.5000 - 0.8660i \\ \lambda_3 = 1.0000 + 0.0000i \end{gathered} \tag{27}$$

The system is now **unstable**

2. checking system for complete controllability

$$\operatorname{rank}(N_c) = [B \;\; AB] = 3 \tag{28}$$

system is completely controllable

3. Form reference model

System order	Butterworth polynomial	Overshooting $\sigma, \%$	Settling time t_s^*, sec
$n=1$	$s+\omega_0$	0	3
$n=2$	$s^2+2\omega_0 s+\omega_0^2$	0	4.8
$n=3$	$s^3+3\omega_0 s^2+3\omega_0^2 s+\omega_0^3$	0	6.2

(29)

n=3 Newton polynomial

$$\begin{gathered} \omega = \frac{t_s^*}{t_S} = 2 \\ D_d(s) = s^3 + 6s^2 + 12s + 8 \\ \Gamma = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -8 & -12 & -6 \end{bmatrix}, H = [1 \quad 0 \quad 0] \end{gathered} \tag{30}$$

4. Finding matrix transfoemation M

The solution of the Sylvester matrix equation with respect to the matrix M:

```
M = sylv(-A,gamma,-B*H)
```

5. Calculation of matrix K

$$K = HM^{-1} = [9 \;\; 12 \;\; 6] \tag{31}$$

6. Form control signal

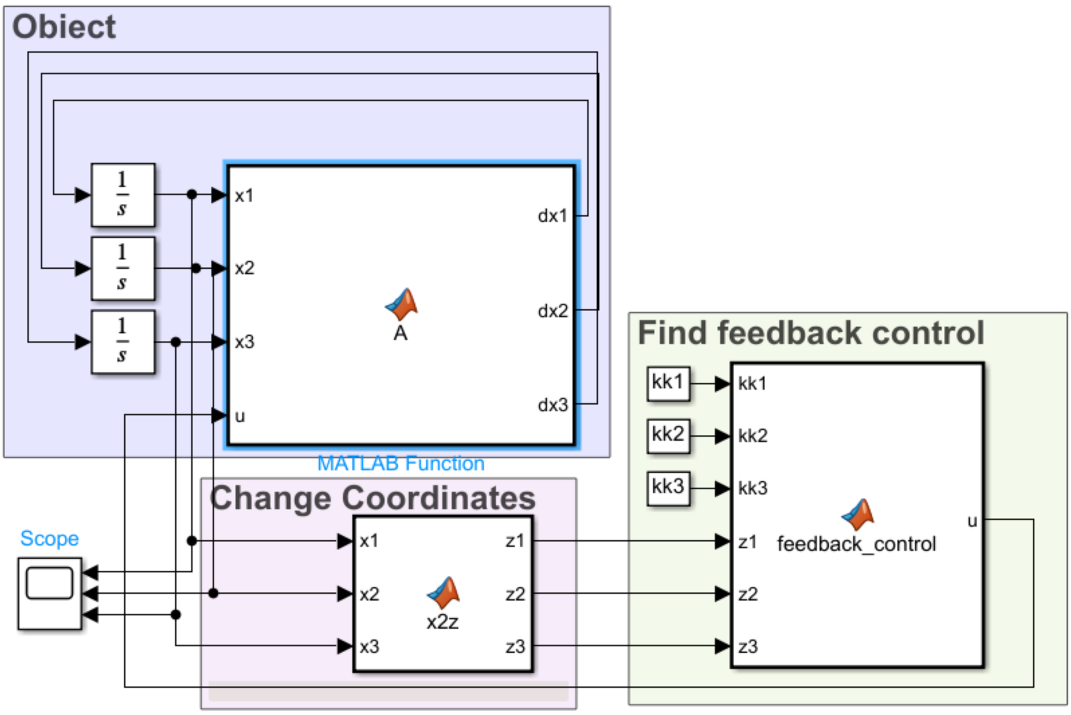
$$v(t) = -\mathrm{K}x(t) \tag{32}$$

1.2.3 control law

In summary, we can get the control signal of the feedback control

$$u = \frac{1}{(z_1+1)^2}(z_1^3 + z_1^2 - 2z_2^2 - 2z_3 - 2z_1z_3 + z_2z_3) + \frac{(9z_1 + 12z_2 + 6z_3)}{z_1+1}. \tag{33}$$

1.3 Modelling

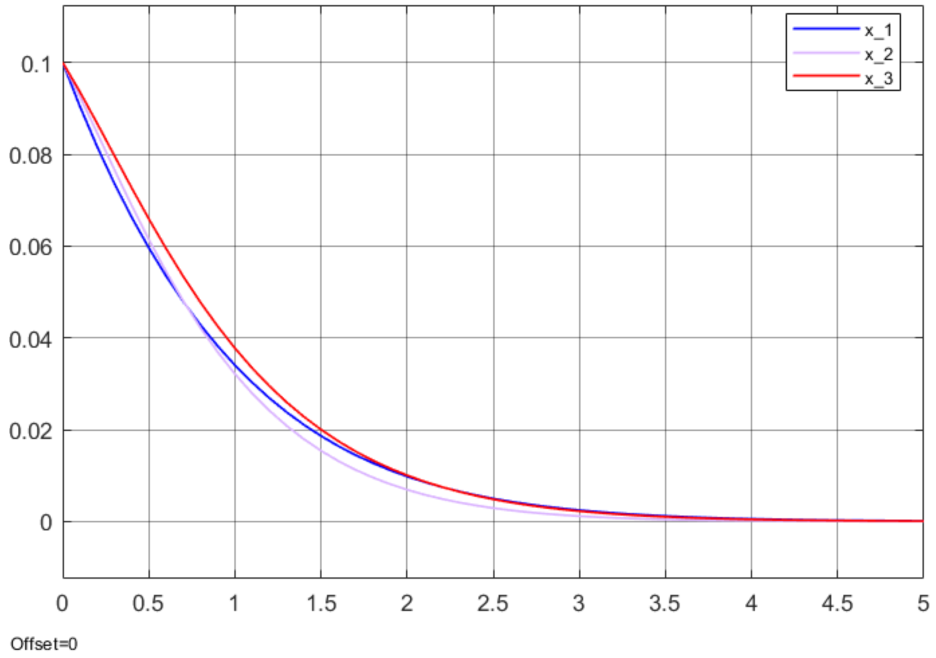



```
function [dx1,dx2,dx3] = A(x1,x2,x3,u)
dx1 = -x1+x2-x3;
dx2 = -x1*x3-x2+u;
dx3 = -x1+u;
```

```
function [z1,z2,z3] = x2z(x1,x2,x3)
z1 = x1;
z2 = -x1+x2-x3;
z3 = 2*x1-2*x2+x3-x1*x3;
```

```
function u  = feedback_control(kk1,kk2,kk3,z1,z2,z3)
u = 1/(z1+1)^2*(z1^3+z1^2-2*z2^2-2*z3-2*z1*z3+z2*z3)+ (kk1*z1+kk2*z2+kk3*z3)/(z1+1)
```

1.4 Simulation



linearization at a point approaches(using the Jacobian matrix)simple analysis

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1)

$$\dot{x}_1 = x_1 + x_2$$
$$\dot{x}_2 = 2x_1^3 x_2 + x_1 - u \tag{34}$$

Jacobian matrix :

$$J = \frac{\partial f}{\partial x} = \begin{bmatrix} 1 & 1 \\ 1 & 2x_1^3 \end{bmatrix} \tag{35}$$

2.1 at $x^\star = (0; 0)$

$$A_1 = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$
$$dx = A_1 \cdot x \tag{36}$$

Find the eigenvalues of matrix A:

$$\lambda_1 = -0.618 < 0$$
$$\lambda_2 = 1.618 > 0 \tag{37}$$

The system is now **unstable**

2)

$$
\begin{aligned}
\dot{x}_1 &= -x_1 + x_2 - x_3 \\
\dot{x}_2 &= -x_1 x_3 - x_2 + u \\
\dot{x}_3 &= -x_1 + u
\end{aligned} \tag{38}
$$

Jacobian matrix :

$$
J = \frac{\partial f}{\partial x} = \begin{bmatrix} -1 & 1 & -1 \\ -x_3 & -1 & -x_1 \\ -1 & 0 & 0 \end{bmatrix} \tag{39}
$$

2.1 at $x^{\star} = (0;0)$

$$
\begin{aligned}
A_1 &= \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \\
dx &= A_1 \cdot x
\end{aligned} \tag{40}
$$

Find the eigenvalues of matrix A:

$$
\begin{aligned}
\lambda_1 &= -1.6180 < 0 \\
\lambda_2 &= 0.6180 > 0 \\
\lambda_3 &= -1 < 0
\end{aligned} \tag{41}
$$

The system is now **unstable**

Conclusion

feedback linearization

advantages:

1. Basically **applicable to all nonlinear systems**, not limited by the system

Disadvantages:

1. The calculation is more **complicated**, and it is not easy to solve for very complex systems

linearization at a point

advantages:

1. This method is very **convenient and fast** when **roughly and simply** analyzing the **characteristics** of the equilibrium point (stable or unstable), and it can be seen from practice5 that in most cases, the characteristics of the equilibrium point obtained by this method are basically the same as the original system.

Disadvantages:

1. It can be seen from some experimental results that the performance at the equilibrium point of the linearized system and the original nonlinear system may be **similar** but **not all are exactly the same** (and within **a certain small range need to be found**). Meaning, this method is **not very suitable** for **high precision** and **large scale** analysis.

Both Approach

1. Linear systems have many well-established control laws and are easy to analyze and computerized (using matrices). After we use a certain method to linearize the nonlinear system, we can design its control scheme through many kinds of linear system control schemes, so the linearization of the nonlinear system is very important and very beneficial.