Practice # 1

Kalman filter.

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Contents

1 Introduction

§1.1 Student information & & variant

- ⋆ Name: Xu Miao
- ⋆ ITMO Number: 293687
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- ⋆ Variant: $k = 7$

The variant data used in this practice is shown below

Table 1.1: Output value characteristic calculation

Parameter	values
k	7
Pendulum mass m	$3kg$
Pendulum length l	$1.5m$
Process noise covariance$Q$	0.005
Measurement noise covariance$R$	0.0025

§1.2 Introduction

In this report, I will solve the following tasks:

Part1. **Build a simulation model**

1) use Simulink to implement a linear time invariant or time variant Kalman Filter for estimating the angular position of the pendulum

Part2. **Simulation**

1) Use an initial condition $\frac{\pi}{18}$ radius for the angular position. Models for simulating simple pendulums and Kalman filters with covariance noise matrices constructed from variants $k = 7$. Determine the time when the estimation error becomes equal to zero.
2) Reduce the values of pair of noise covariances one order,and repeat step 1.
3) change the initial condition for the angular position and simulate the model

2 Notations

Table 2.1: Notation used in this report

Symbol	Definition
m	Pendulum mass
l	Pendulum length
Q	Process noise covariance
R	Measurement noise covariance
θ	Angle between pendulum and vertical
θ_0	The angle of the initial moment

⋆ Other notations instructions will be given in the text.

3 Solution of problem

§3.1 Part1.Build a simulation model

§3.1.i Mathematical model

Mathematical model of the pendulum

Consider a simple pendulum model as shown in Fig.3.1:

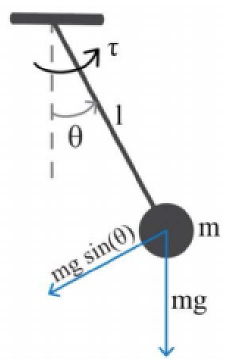

Fig. 3.1. Simple pendulum model

Analytical modeling of the system using the Euler-Lagrange equations results in a simple pendulum system without friction The system can be expressed as:

$$\frac{d^2\theta}{dt^2} + \frac{g}{l}\sin\theta = \frac{1}{ml^2}\tau \tag{3.1}$$

Linearization Model of Simple Pendulum System

As can be seen from Fig.3.2 below, when θ is small, the nonlinear term $\sin\theta$ in the system can be approximately expressed as θ:

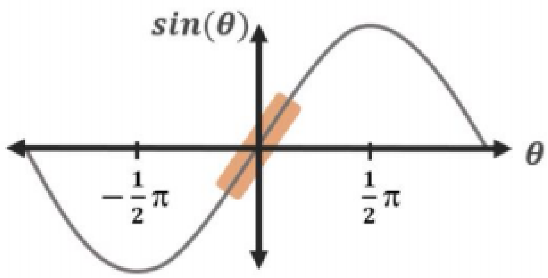

Fig. 3.2. Plot of the function $\sin\theta$

Since the system performance does not differ much when θ is small and the Kalman filter performs better for linear models, we linearize (3.1) as:

$$\frac{d^2\theta}{dt^2} + \frac{g}{l}\theta = \frac{1}{ml^2}\tau \tag{3.2}$$

Let $x_1 = \theta, x_2 = \dot{\theta}$, the state space model of the linearized system is constructed as follows:

$$\begin{aligned} \dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = Ax + B\tau, \\ y = Cx + D\tau, \end{aligned} \tag{3.3}$$

where:

$$A = \begin{bmatrix} 0 & 1 \\ -\frac{g}{l} & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \\ ml^2 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \end{bmatrix}, D = 0 \tag{3.4}$$

§3.1.ii Simulink Model

The measurement is noisy, we use **Gaussian noise** to simulate the **process noise** and **measurement noise** during the experiment.And use **Kalman filter** to eliminate the influence of noise.

Finally, the simulink model is constructed using equation (3.4) in combination with noise and Kalman filter as shown below:

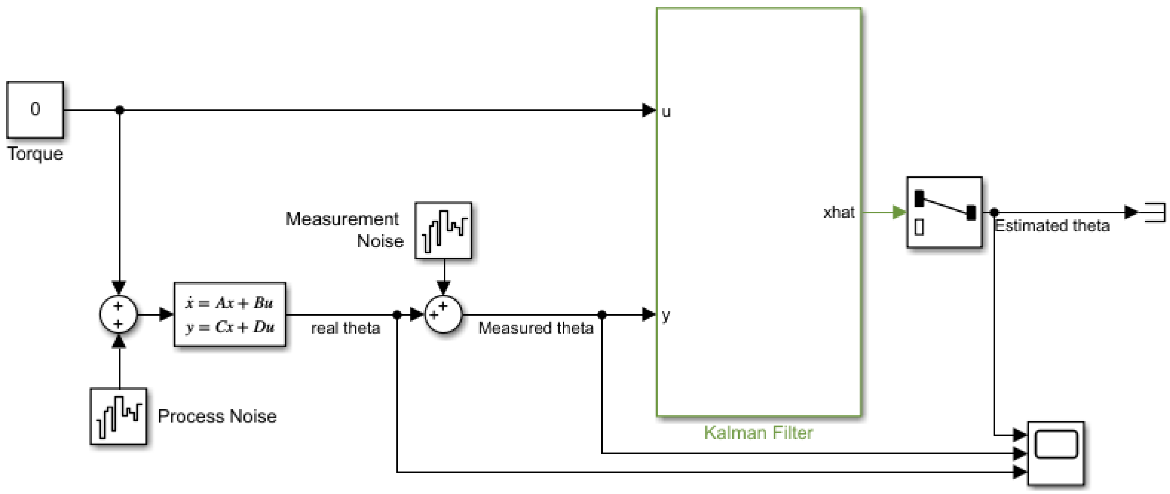

Fig. 3.3. Simulink Model

§3.2 Part2.Simulation

Linearization Model of Simple Pendulum System

§3.2.i Simulation Condition 1:

Simulation Data:

$$\begin{cases} \theta_0 = \frac{\pi}{18} \\ Q = 0.005 \\ R = 0.0025 \end{cases}$$

Simulation Result:

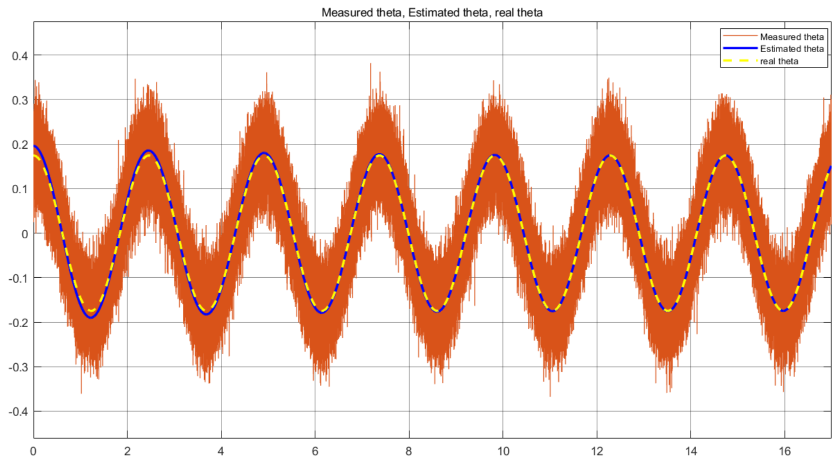

Fig. 3.4. Simulation Result1

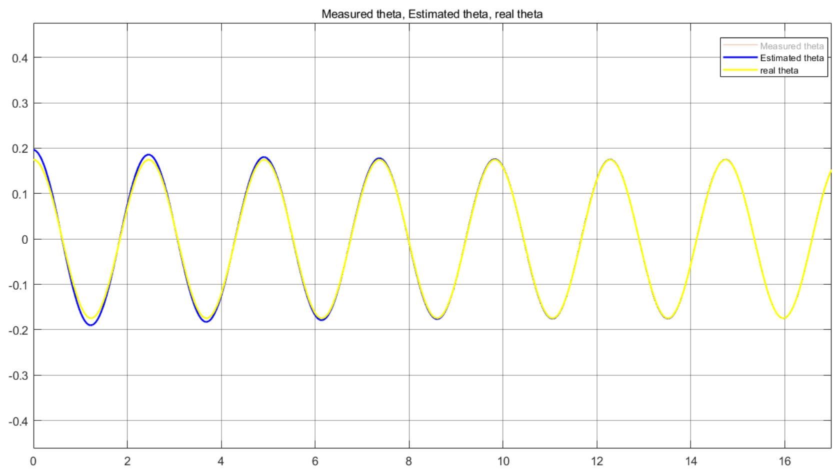

Fig. 3.5. Simulation Result2

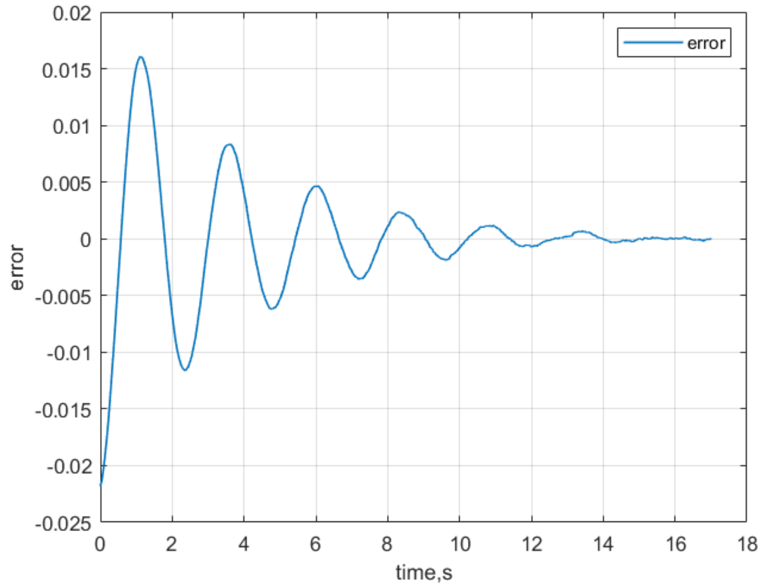

Fig. 3.6. Simulation Result3:Estimation error: Estimated θ - Real θ

§3.2.ii Simulation Condition 2:

Simulation Data:

reduce the variance of the noise by one order:

$$\begin{cases} \theta_0 = \frac{\pi}{18} \\ Q = 0.0005 \\ R = 0.00025 \end{cases}$$

Simulation Result:

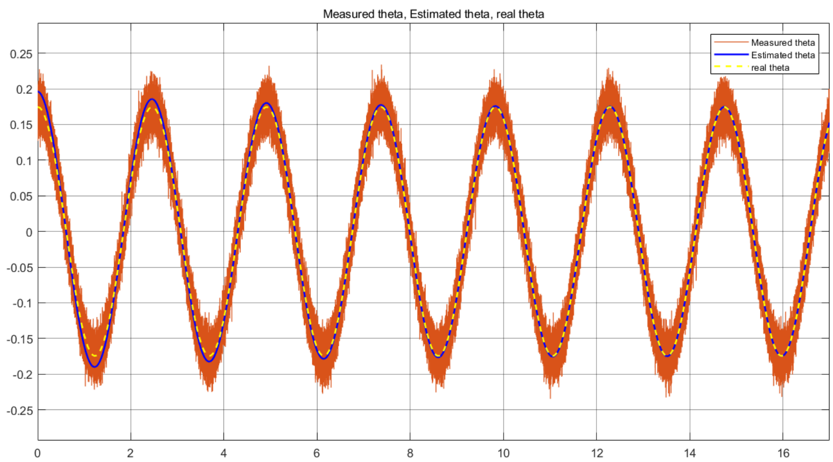

Fig. 3.7. Simulation Result1

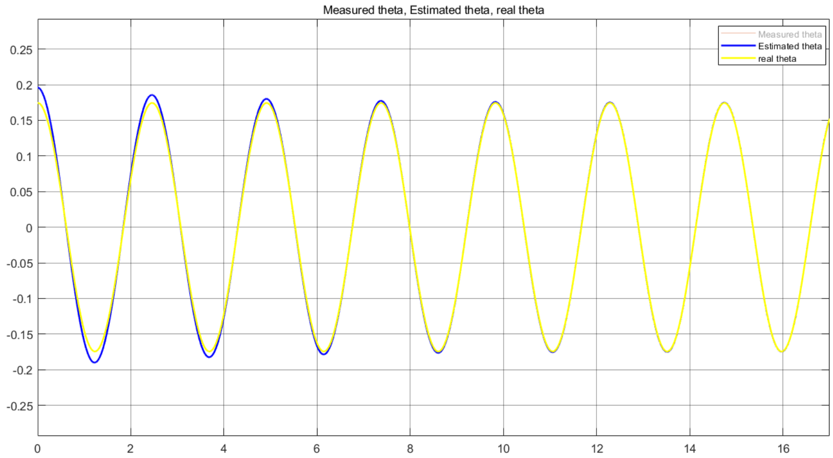

Fig. 3.8. Simulation Result2

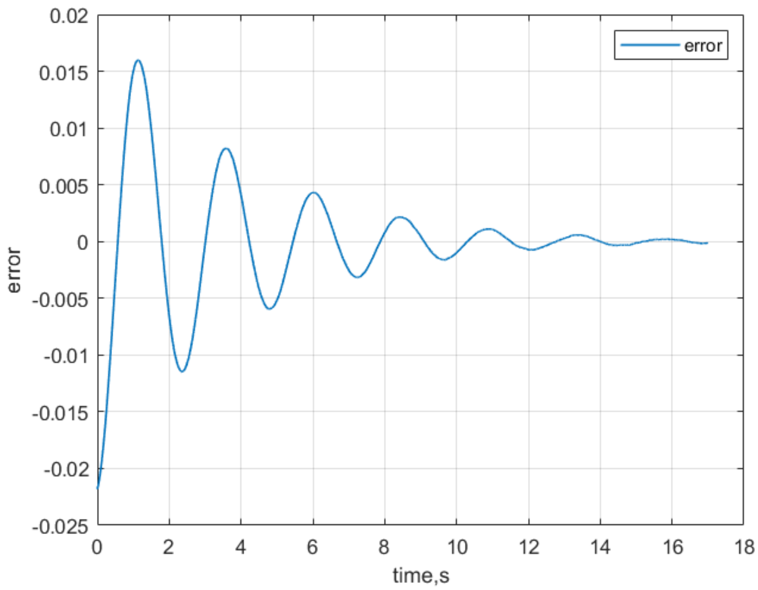

Fig. 3.9. Simulation Result3:Estimation error: Estimated θ - Real θ

§3.2.iii result comparison:

Comparing the effect of the noise variance reduction on the filter performance after the first-order reduction, the results of the error over time are shown in the following figure:

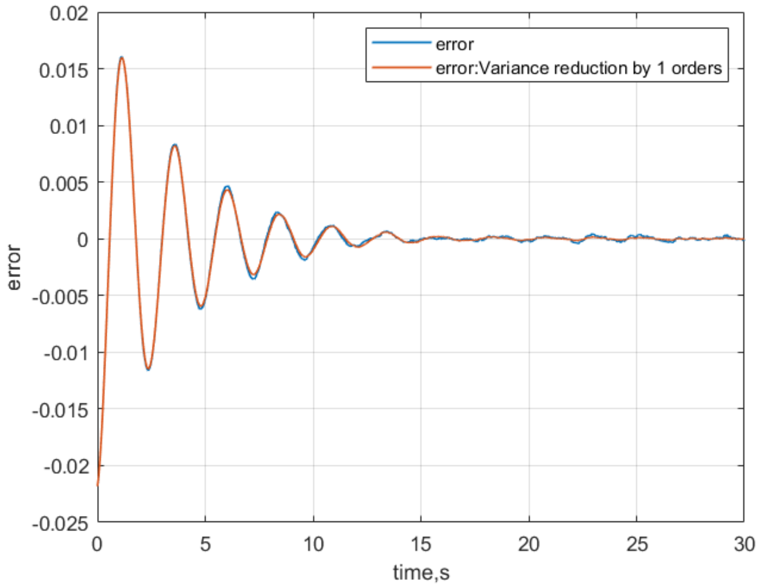

Fig. 3.10. Estimation error: Estimated θ - Real θ

Next, we compare the time it takes for different noise variance Kalman filters to bring the estimation error to zero. Because the system has continuous noise input, the error cannot become 0 and remain unchanged. We think that when the estimated error of the system $e < 5 \times 10^{-9}$, the estimated error is $0e = 0$,as shown in the table below:

Table 3.1: result comparison

noise variance	The time it takes for the observation error to be 0
$Q = 0.005, R = 0.0025$	26.8329s
$Q = 0.0005, R = 0.00025$	13.9886s

In order to ensure that the conclusions obtained from the experiments are more credible, we replace the initial angle and repeat the above experiments to observe the simulation results.

§3.2.iv Simulation Condition 3:

Simulation Data:

$$\begin{cases} \theta_0 = \frac{\pi}{2} \\ Q = 0.005 \\ R = 0.0025 \end{cases}$$

Simulation Result:

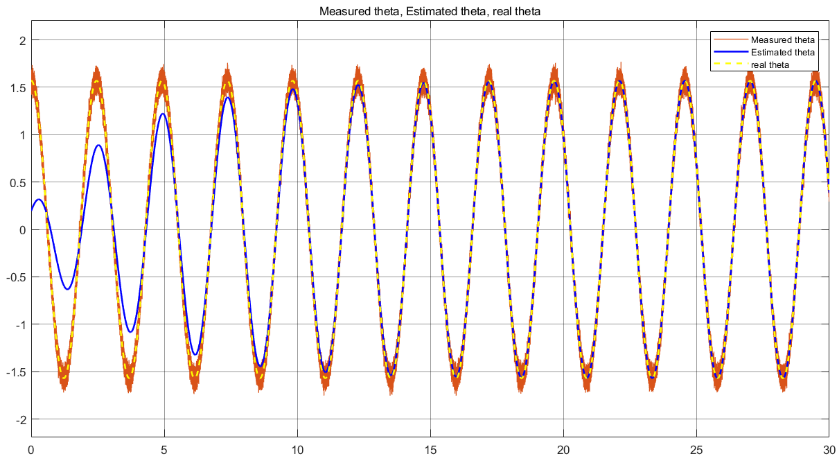

Fig. 3.11. Simulation Result1

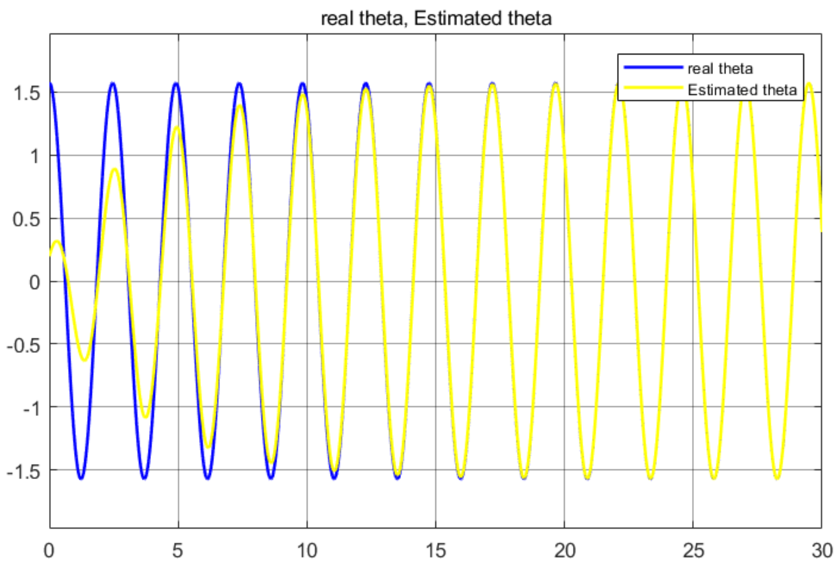

Fig. 3.12. Simulation Result2

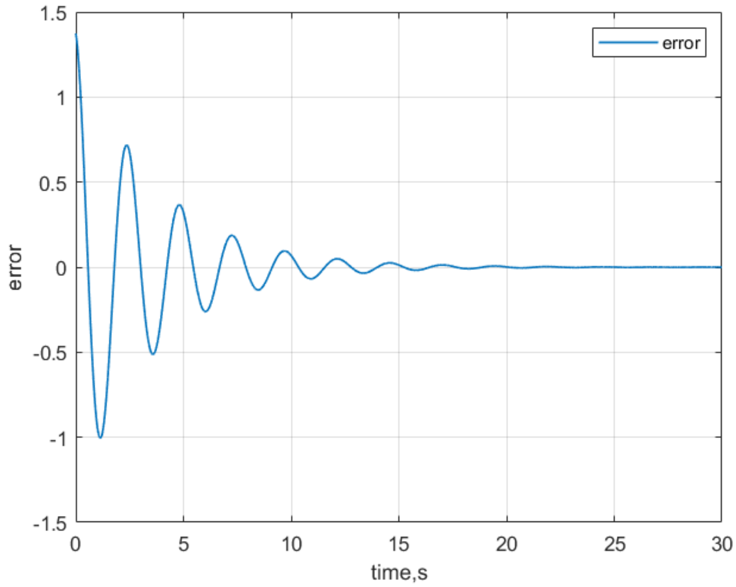

Fig. 3.13. Simulation Result3:Estimation error: Estimated θ - Real θ

§3.2.v Simulation Condition 4:

Simulation Data:

reduce the variance of the noise by one order:

$$\begin{cases} \theta_0 = \frac{\pi}{2} \\ Q = 0.0005 \\ R = 0.00025 \end{cases}$$

Simulation Result:

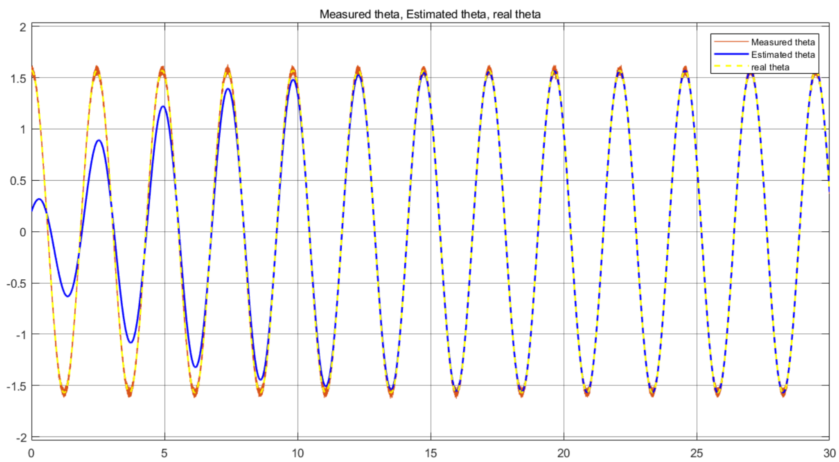

Fig. 3.14. Simulation Result1

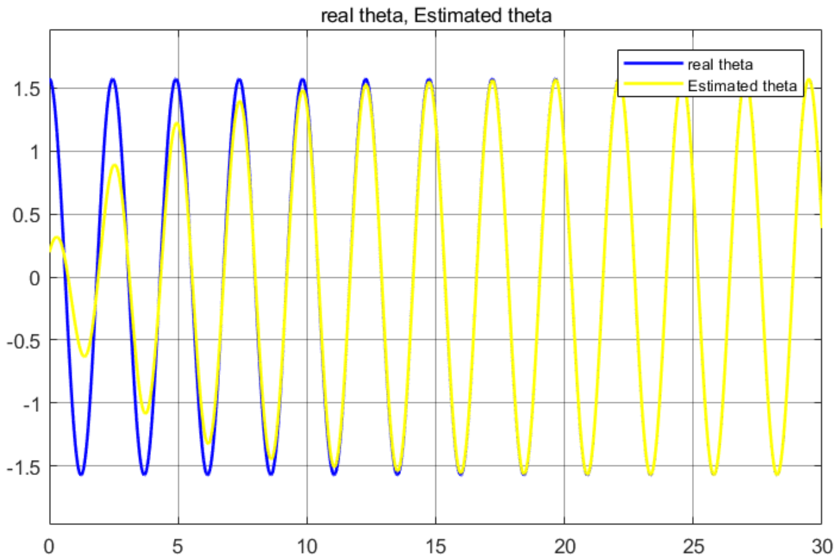

Fig. 3.15. Simulation Result2

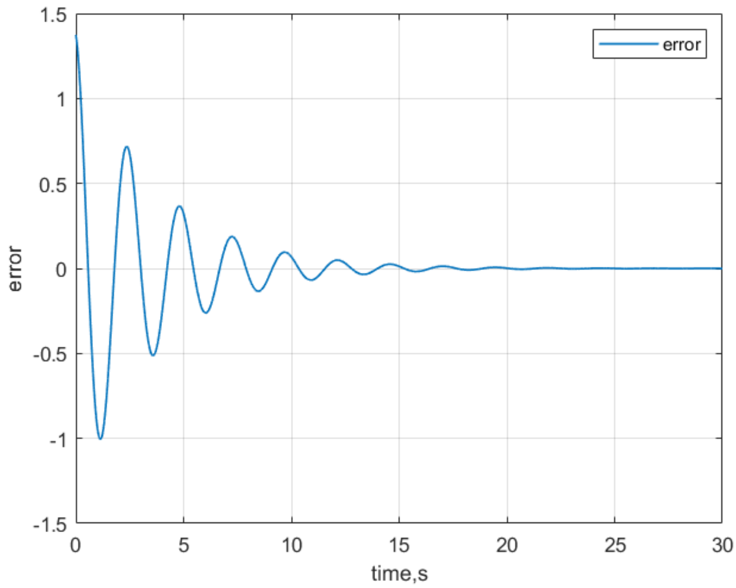

Fig. 3.16. Simulation Result3:Estimation error: Estimated θ - Real θ

§3.2.vi result comparison:

Comparing the effect of the noise variance reduction on the filter performance after the first-order reduction, the results of the error over time are shown in the following figure:

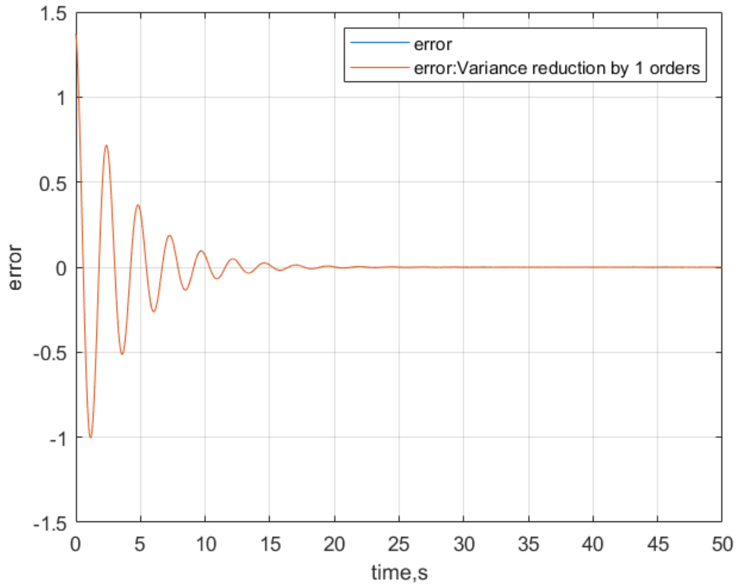

Fig. 3.17. Estimation error: Estimated θ - Real θ

Next, we compare the time it takes for different noise variance Kalman filters to bring the estimation error to zero. Because the system has continuous noise input, the error cannot become 0 and remain unchanged. We think that when the estimated error of the system $e < 5 \times 10^{-9}$, the estimated error is $0e = 0$,as shown in the table below:

Table 3.2: result comparison

noise variance	The time it takes for the observation error to be 0
$Q = 0.005, R = 0.0025$	27.5013s
$Q = 0.0005, R = 0.00025$	36.2442s

§3.3 Part3.Additional:nonlinear physical system model of the system

To illustrate why the Kalman filter requires system linearization, I model the nonlinear physical model of the pendulum system and observe the transformation of the estimation error:

§3.3.i Nonlinear simulink model:

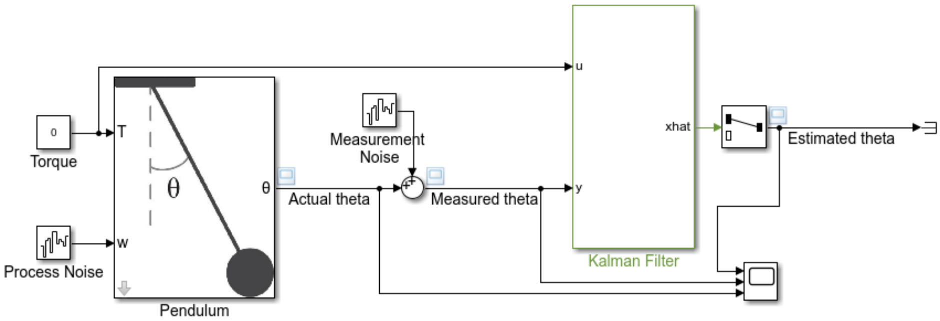

Fig. 3.18. Nonlinear simulink model

§3.3.ii estimation error:

Use the following data to simulate:

$$\begin{cases} \theta_0 = \frac{\pi}{18} \\ Q = 0.0005 \\ R = 0.00025 \end{cases}$$

The estimated error of the system is as follows:

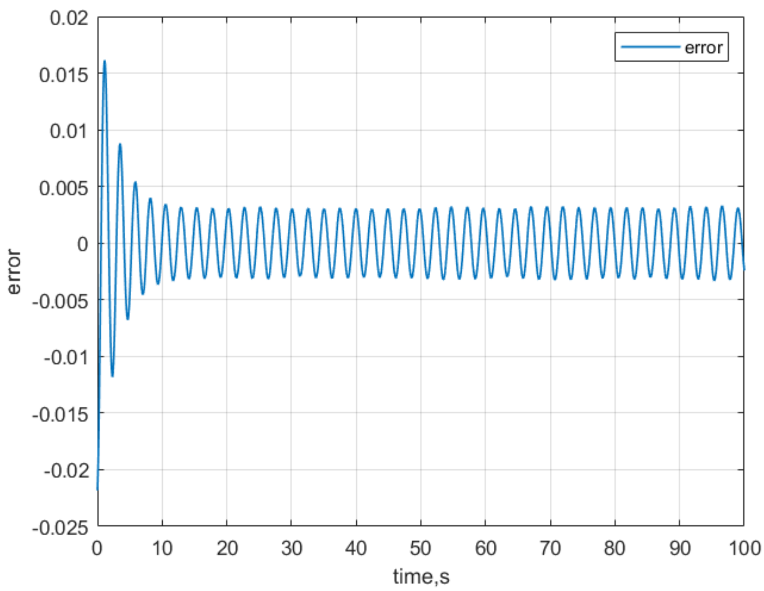

Fig. 3.19. Estimation error Estimated θ − Real θ

§3.4 Conclusion

From the whole experiment we can get the following properties of the Kalman filter:

- ⋆ The Kalman filter can effectively **reduce the oscillation** caused by the error caused by the measurement and movement process, making the estimation result smoother and closer to the real value(Fig.3.4,Fig.3.7,Fig.3.11,Fig.3.14)
- ⋆ The Kalman filter has **good** estimation results for **linear systems**, and the error can be small, but the estimation results for **nonlinear systems cannot converge**.(Fig.3.6,Fig.3.9,Fig.3.13,Fig.3.16,Fig.3.19)
- ⋆ As the variance of the system noise decreases, the Kalman filter estimate error converges faster(Table3.1,Table3.2)

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Appendix

A Complete source code

§A.1 parameter settings

Kalman_Filter_Simulink_Example_params.m

```
% Pendulum model
% Gravity
clear all
g = 9.81; % [m/s^2]
% Pendulum mass
m = 3; % [kg]
% Pendulum length
l = 1.5; % [m]

% State space representation
A = [0 1; -g/l 0];
B = [0; 1/(m*l^2)];
C = [1 0];
D = 0;

% Process noise covariance
Q = 0.0005;
% Measurement noise covariance
R = 0.00025;
% Sampling time
Ts = 0.0001; % [s]
```

§A.2 simulation

Simulation.m

```
% Process noise covariance
Q = 0.005;
% Measurement noise covariance
R = 0.0025;
sim('linear1.slx')
estimate = ans.theta.signals(2).values;
real = ans.theta.signals(3).values;
error = real-estimate;
time = ans.tout;
plot(time,error,"LineWidth",0.5)
grid on
for i = 1:1:length(time)
    if abs(error(i)) < 5e-9
        i
        tim = time(i)
```

```
                break
            end
        end
        % Process noise covariance
        Q = 0.0005;
        % Measurement noise covariance
        R = 0.00025;
        sim('linear1.slx')
        estimate = ans.theta.signals(2).values;
        real = ans.theta.signals(3).values;
        error = real-estimate;
        time = ans.tout;
        hold on
        plot(time,error,"LineWidth",0.5)
        grid on
        legend('error','error:Variance reduction by 1 orders')
        xlabel('time,s')
        ylabel('error')
        for i = 1:1:length(time)
            if abs(error(i)) < 5e-9
                i
                tim = time(i)
                break
            end
        end
```