Practical Assignment № 2

Digital and Microcontroller Devices

Assignment Variant: 6

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Variant: 6

Variant	T	a	b	ζ	ω_d
6	0.35	3.5	10	0.55	8

Tasks:

1. Consider the open-loop transfer function of a plant is given by

$$G(s) = \frac{e^{-as}}{1 + bs}$$

with sampling time T . The parameters of the control object are given in the Table 1.

- According to variant 6, we get:

$$G(s) = \frac{e^{-3.5s}}{1 + 10s}, \qquad T = 0.35$$

2. Design a dead-beat digital controller for considered control object.

- The open-loop transfer function of a plant is given by

$$G(s) = \frac{e^{-3.5s}}{1 + 10s}.$$

Design a dead-beat digital controller for the system. While $T = 0.35$ s.

- Solution
  The transfer function of the system with a zero-order hold is given by

$$HG(z) = Z\left\{\frac{1 - e^{-sT}}{s} G(s)\right\} = (1 - z^{-1})Z\left\{\frac{e^{-3.5s}}{s(1 + 10s)}\right\}$$

or

$$HG(z) = (1 - z^{-1})z^{-3.5}Z\left\{\frac{1}{s(1 + 10s)}\right\} = (1 - z^{-1})z^{-10}Z\left\{\frac{1/10}{s(s + 1/10)}\right\}.$$

From z-transform tables we obtain

$$HG(z) = (1 - z^{-1})z^{-10} \frac{z(1 - e^{-0.1})}{(z - 1)(z - e^{-0.1})} = z^{-11} \frac{(1 - e^{-0.1})}{1 - e^{-0.1}z^{-1}}$$

or

$$HG(z) = \frac{0.095z^{-11}}{1 - 0.904z^{-1}}.$$

From Equations (9.3) and (9.5),

$$D(z) = \frac{1}{HG(z)} \frac{T(z)}{1 - T(z)} = \frac{1 - 0.904z^{-1}}{0.095z^{-1}} \frac{z^{-k}}{1 - z^{-k}}.$$

For realizability, we can choose $k \geq 11$. Choosing $k = 11$, we obtain

$$D(z) = \frac{1 - 0.904z^{-1}}{0.095z^{-11}} \frac{z^{-11}}{1 - z^{-11}}$$

or

$$D(z) = \frac{z^{11} - 0.904z^{10}}{0.095(z^{11} - 1)}.$$

- Simulink model

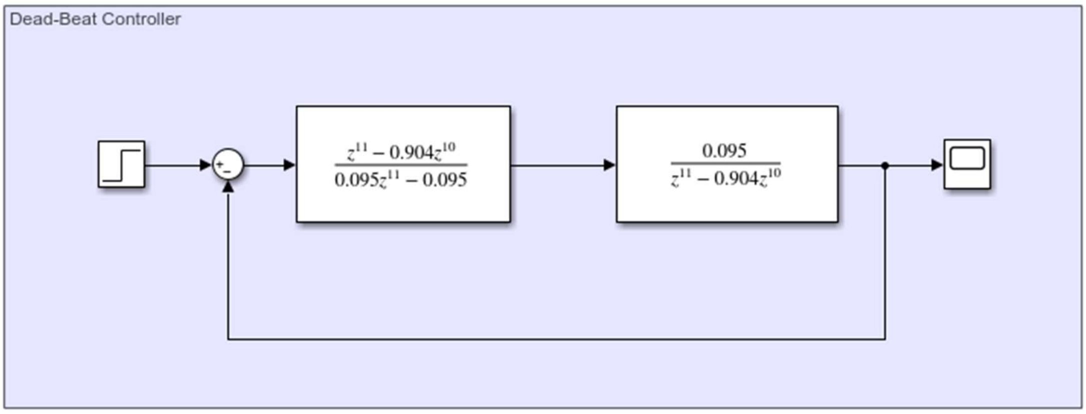

Figure 1: closed system with dead-beat digital controller

- Simulink result

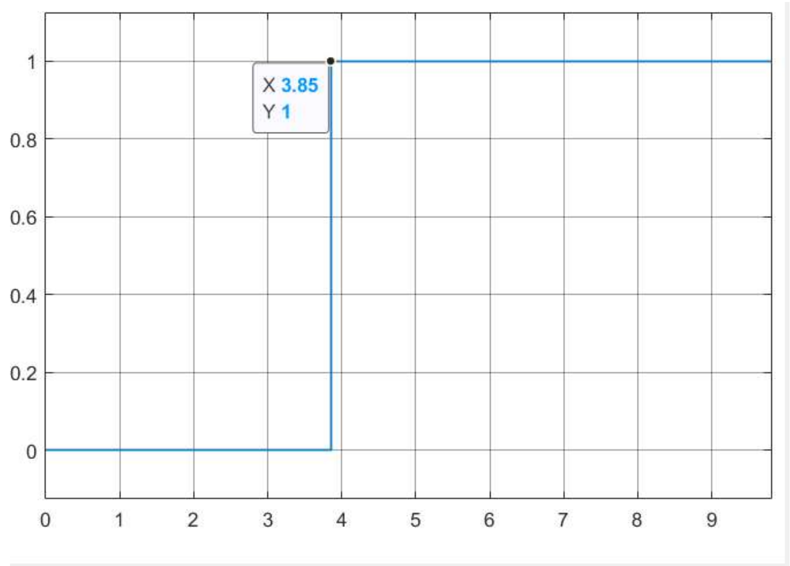

Figure 2: output of the closed system with dead-beat digital controller

- Conclusion:

  - The output response is unity after 3.85 s (tenth sample which we want) and stays at this value.

  - Although the dead-beat controller has provided an excellent response, the magnitude of the control signal (shown in Figure 3) may not be acceptable, and it may even saturate in practice.

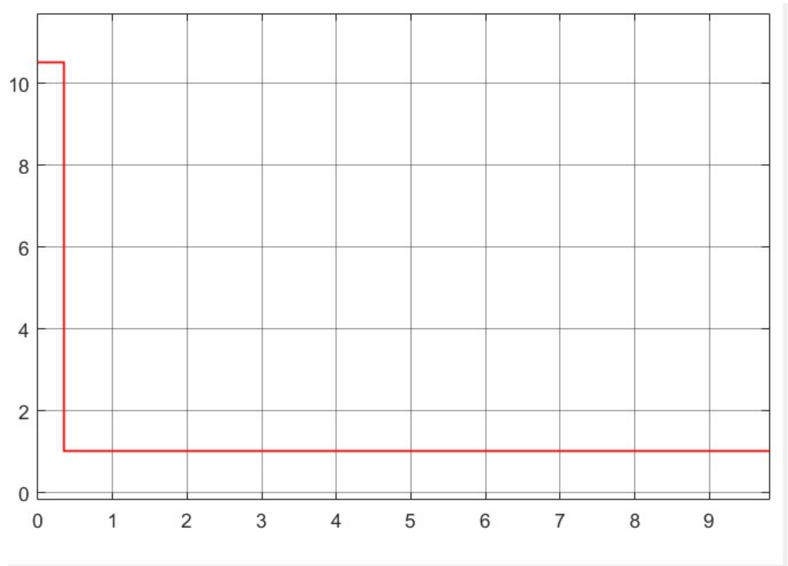

Figure 3: control signal of dead-beat digital controller

3. Design a Dahlin digital controller for considered control object.

- The open-loop transfer function of a plant is given by

$$G(s) = \frac{e^{-3.5s}}{1 + 10s}.$$

Design a Dahlin digital controller for the system. While $T = 0.35$ s.

- Solution

The transfer function of the system with a zero-order hold is given by

$$HG(z) = Z\left\{\frac{1 - e^{-sT}}{s} G(s)\right\} = (1 - z^{-1})Z\left\{\frac{e^{-3.5s}}{s(1 + 10s)}\right\}$$

or

$$HG(z) = (1 - z^{-1})z^{-3.5}Z\left\{\frac{1}{s(1 + 10s)}\right\} = (1 - z^{-1})z^{-10}Z\left\{\frac{1/10}{s(s + 1/10)}\right\}.$$

From z-transform tables we obtain

$$HG(z) = (1 - z^{-1})z^{-10}\frac{z(1 - e^{-0.1})}{(z - 1)(z - e^{-0.1})} = z^{-11}\frac{(1 - e^{-0.1})}{1 - e^{-0.1}z^{-1}}$$

or

$$HG(z) = \frac{0.095z^{-11}}{1 - 0.904z^{-1}}.$$

- For the controller, if we choose $q = 3.5$, then

$$D(z) = \frac{1}{HG(z)}\frac{T(z)}{1 - T(z)} = \frac{1 - 0.904z^{-1}}{0.095z^{-11}}\frac{z^{-k-1}(1 - e^{-0.1})}{1 - e^{-0.1}z^{-1} - (1 - e^{-0.1})z^{-k-1}}$$

or

$$D(z) = \frac{1 - 0.904z^{-1}}{0.095z^{-11}}\frac{0.095z^{-k-1}}{1 - 0.904z^{-1} - 0.095z^{-k-1}}$$

For realizability, if we choose $k = 10$, we obtain

$$D(z) = \frac{0.095z^{11} - 0.0858z^{10}}{0.095z^{11} - 0.0858z^{10} - 0.0090}.$$

- Simulink model

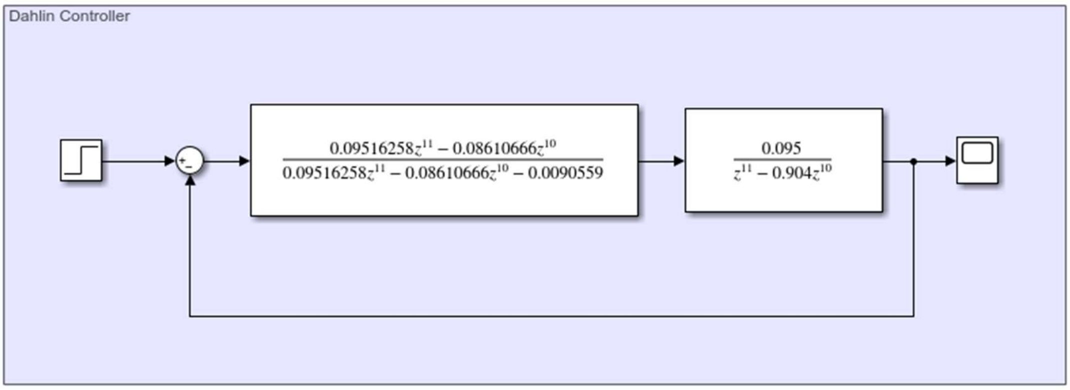

Figure 4: closed system with Dahlin digital controller

- Simulink result

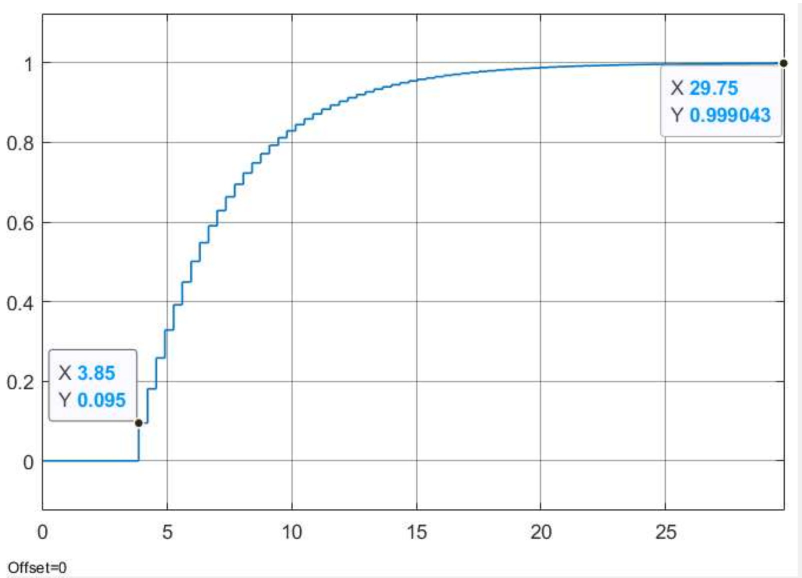

Figure 5: output of the closed system with Dahlin digital controller

- Conclusion

  - The Dahlin controller is a modification of the dead-beat controller and produces an exponential response which is smoother than that of the dead-beat controller. Figure5 shows the step response of the system. It is clear that the response is exponential as expected.

- ➢ Although the system response is slower than dead-beat controller, the controller signal is more acceptable.

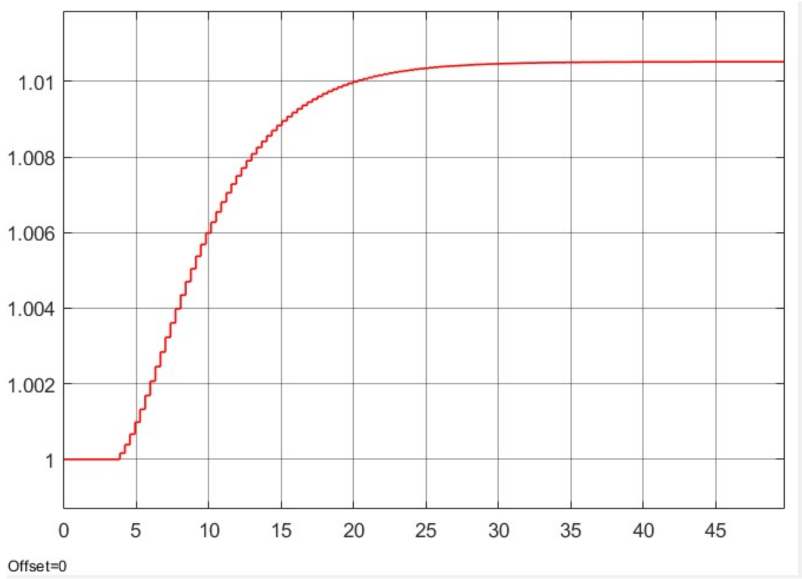

Figure 6: control signal of Dahlin digital controller

4. Consider the open-loop transfer function of a system together with a zero-order hold is given by

$$HG(z) = \frac{0.03(z + 0.75)}{z^2 - 1.5z + 0.5}$$

Design a digital controller with pole-placement approach so that the closed-loop system will have ζ and ω_d. The steady-state error to a step input should be zero. Also the steady-state error to a ramp input should be 0.2. Sampling time is T. All variables are given in table 1 .

- The open-loop transfer function of a system together with a zero-order hold is given by

$$HG(z) = \frac{0.03(z + 0.75)}{z^2 - 1.5z + 0.5}.$$

Design a digital controller so that the closed-loop system will have $\zeta = 0.55$ and $w_d = 8\text{rad/s}$. The steady-state error to a step input should be zero. Also, the steady-state error to a ramp input should be 0.2. While $T = 0.35$ s.

- Solution

The roots of a second-order system are given by

$$z_{1,2} = e^{-\zeta\omega_n T \pm j\omega_n T\sqrt{1-\zeta^2}} = e^{-\zeta\omega_n T}\left(\cos\ \omega_n T\sqrt{1-\zeta^2} \pm j\sin\ \omega_n T\sqrt{1-\zeta^2}\right)$$

Thus, the required pole positions are

$$z_{1,2} = e^{-0.55\times 9.58\times 0.35}(\cos\,(0.35\times 8) \pm j\sin\,(0.35\times 8)) = -0.149 \pm j0.0530.$$

The required controller then has the transfer function

$$T(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + \cdots}{(z + 0.149 + j0.0530)(z + 0.149 - j0.0530)}$$

which gives

$$T(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + \cdots}{1 + 0.298 z^{-1} + 0.025 z^{-2}}.$$

We now have to determine the parameters of the numerator polynomial. To ensure realizability, $b_0 = 0$ and the numerator must only have the b_1 and b_2 terms. Equation (9.6) then becomes

$$T(z) = \frac{b_1 z^{-1} + b_2 z^{-2}}{1 + 0.298 z^{-1} + 0.025 z^{-2}}.$$

- The other parameters can be determined from the steady-state requirements. The steady-state error is given by

$$E(z) = R(z)[1 - T(z)].$$

For a unit step input, the steady-state error can be determined from the final value theorem, i.e.

$$E_{ss} = \lim_{z\to 1} \frac{z-1}{z}\frac{z}{z-1}[v]$$

or

$$E_{ss} = 1 - T(1).$$

From (9.8), for a zero steady-state error to a step input,

$$T(1) = 1$$

From (9.7), we have

$$T(1) = \frac{b_1 + b_2}{1.323} = 1$$

or

$$b_1 + b_2 = 1.323,$$

and

$$T(z) = \frac{b_1 z + b_2}{z^2 + 0.298z + 0.025}.$$

If K_v is the system velocity constant, for a steady-state error to a ramp input we can write

$$E_{ss} = \lim_{z\to 1} \frac{(z-1)}{z} \frac{Tz}{(z-1)^2} [1 - T(z)] = \frac{1}{K_v}$$

or, using L 'Hospital's rule,

$$\left.\frac{dT}{dz}\right|_{z=1} = -\frac{1}{K_v T}.$$

Thus from (9.10),

$$\left.\frac{dT}{dz}\right|_{z=1} = \frac{b_1(z^2 + 0.298z + 0.025) - (b_1 z + b_2)(2z + 0.298)}{(z^2 + 0.298z + 0.025)^2} = -\frac{1}{K_v T} = -\frac{0.2}{0.35}$$
$$= -0.571,$$

giving

$$\frac{1.323b_1 - (b_1 + b_2)2.298}{1.323^2} = -0.571$$

or

$$0.975b_1 + 2.298b_2 = 0.999,$$

From (9.9) and (9.11) we obtain,

$$b_1 = 1.543 \text{ and } b_2 = -0.220.$$

Equation (9.10) then becomes

$$T(z) = \frac{1.543z - 0.220}{z^2 + 0.298z + 0.025}.$$

Equation (9.12) is the required transfer function. We can substitute in Equation (9.3) to find the transfer function of the controller:

$$D(z) = \frac{1}{HG(z)} \frac{T(z)}{1 - T(z)} = \frac{z^2 - 1.5z + 0.5}{0.03(z + 0.75)} \frac{T(z)}{1 - T(z)}$$

or,

$$D(z) = \frac{z^2 - 1.5z + 0.5}{0.03(z + 0.75)} \frac{1.543z - 0.220}{z^2 - 1.245z + 0.245}$$

which can be written as

$$D(z) = \frac{1.543z^3 - 2.5345z^2 + 1.1015z - 0.110}{0.03z^3 - 0.01485z^2 - 0.0206625z + 0.0055125}$$

The step response of the system with the controller is shown in Figure 6.

- Simulink model

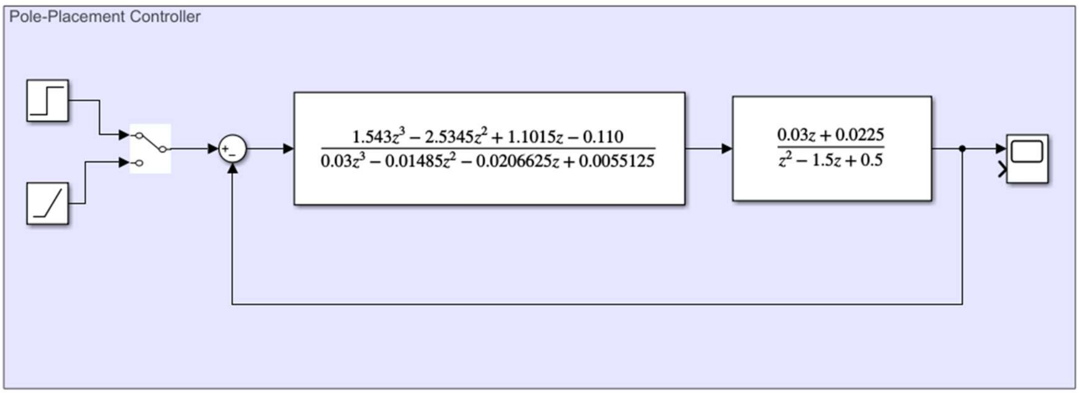

Figure 7: closed system with Pole-Placement control

- Simulink result

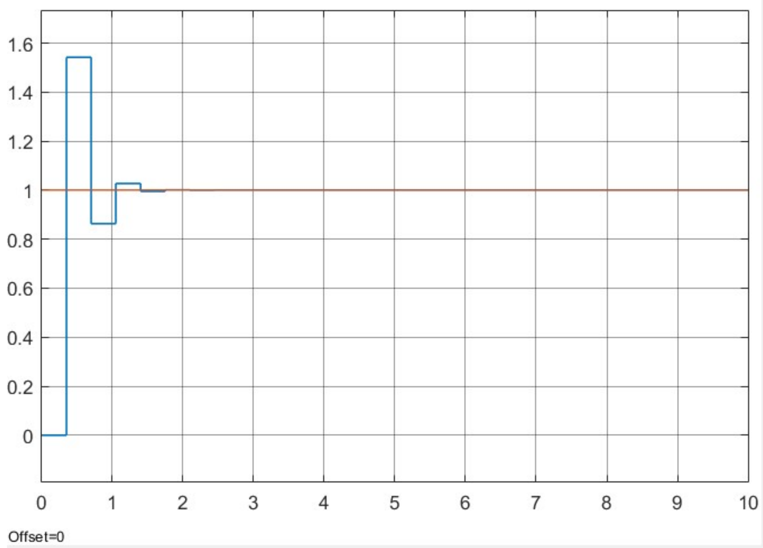

Figure 8: step response of the closed system and the step input.

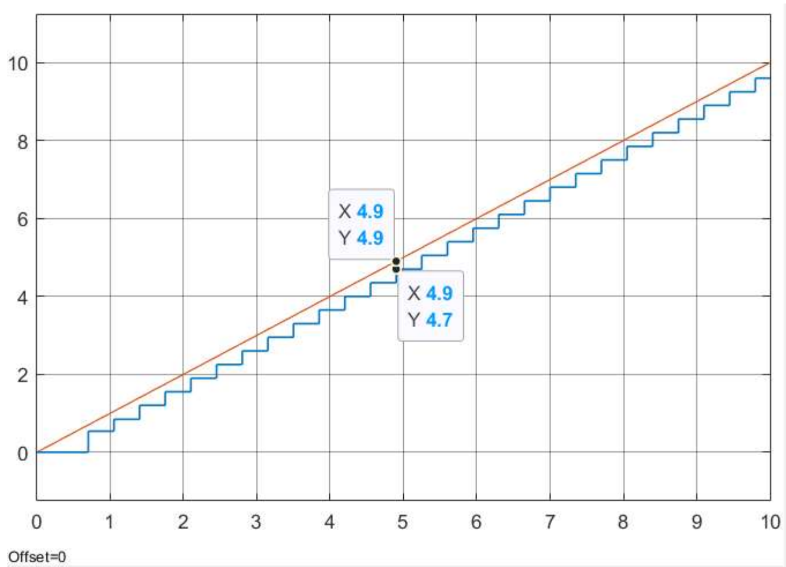

Figure 9: ramp response of the closed system and the ramp input.

- Conclusion

  - From the simulation results (Figure 8 & Figure 9), it is clear that the steady-state requirements are met which specify the steady-state error to a step input should be zero and to a ramp input should be 0.2.