Practical Assignment № 4

Optimal Control

variant number : 6

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Optimal control design with free and fixed initial state point

1. experimental values（Group 6）

- experimental values

№	Plant	Cost function	Initial condition and constraints
6	$\begin{cases}\dot{x}_1 = x_2, \\ \dot{x}_2 = u\end{cases}$	$J = \int_0^5 3u^2(\tau)d\tau$	$x_1(0) = x_2(0) = 0,$ $x_1(5) = 1, x_2(5) = 0$

2. Synthesize an optimal controller according the given cost function and simulate its work on a given time interval.

- Plant

$$\begin{cases}\dot{x}_1 = x_2 \\ \dot{x}_2 = u\end{cases}$$

- State-space function

$$\dot{X} = AX + Bu$$
$$A = \begin{bmatrix}0 & 1 \\ 0 & 0\end{bmatrix}$$
$$B = \begin{bmatrix}0 \\ 1\end{bmatrix}$$

- Hamiltonian:

$$H = \varphi_0 u^2 + \varphi \dot{x} = -3u^2 + \varphi_1 \dot{x}_1 + \varphi_2 \dot{x}_2 = -3u^2 + \varphi_1 x_2 + \varphi_2 u$$

- The Euler-Lagrange equations (Pontryagin's maximum principle):

$$\begin{cases}\dot{\varphi}_i = -\frac{\partial H}{\partial x_i} \\ \frac{\partial H}{\partial u} = 0\end{cases} \Longrightarrow \begin{cases}\dot{\varphi}_1 = 0 \\ \dot{\varphi}_2 = -\varphi_1 \\ -6u + \varphi_2 = 0\end{cases} \Longrightarrow \begin{cases}\dot{x} = Ax + Bu \\ \dot{\varphi}_1 = 0 \\ \dot{\varphi}_2 = -\varphi_1 \\ u = \frac{1}{6}\varphi_2\end{cases} \Longrightarrow \begin{cases}\dot{\varphi}_1 = 0 \\ \dot{\varphi}_2 = -\varphi_1 \\ \dot{x}_1 = x_2 \\ \dot{x}_2 = \frac{1}{6}\varphi_2\end{cases}$$

- **The Euler-Lagrange equations**

$$\begin{cases}\dot{\varphi}_1 = 0\\ \dot{\varphi}_2 = -\varphi_1\\ \dot{x}_1 = x_2\\ \dot{x}_2 = \frac{1}{6}\varphi_2\end{cases} \Longrightarrow \begin{bmatrix}\dot{\varphi}_1\\ \dot{\varphi}_2\\ \dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 0 & 0 & 0\\ -1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & \frac{1}{6} & 0 & 0\end{bmatrix}\begin{bmatrix}\varphi_1\\ \varphi_2\\ x_1\\ x_2\end{bmatrix} \Longrightarrow \begin{cases}\varphi_1(t) = C_1\\ \varphi_2(t) = -C_1 t + C_2\\ x_1(t) = \frac{1}{6}\left(-C_1\frac{t^3}{6} + C_2\frac{t^2}{2} + C_3 t + C_4\right)\\ x_2(t) = \frac{1}{6}\left(-C_1\frac{t^2}{2} + C_2 t + C_3\right)\end{cases}$$

- **Transversality conditions:**

$$\varphi_i(0) = -\frac{\partial G}{\partial x_i(0)}, \quad \varphi_i(t_f) = \frac{\partial G}{\partial x_i(t_f)}$$

$$\Downarrow$$

$$\varphi_1(0) = 0, \varphi_2(0) = 0$$
$$\varphi_1(t_f) = 0, \varphi_2(t_f) = 0$$

- **Transversality conditions:**

Bringing the boundary conditions into the calculation we can solve the unknowns in the equation，Finally we can obtain the control signal as follows：

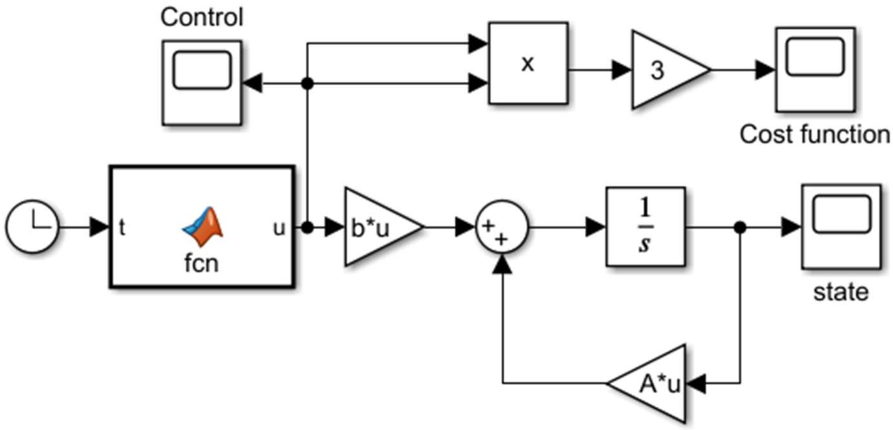
$$u(t) = \frac{6}{25} - \frac{12}{125}t$$

- **Code**

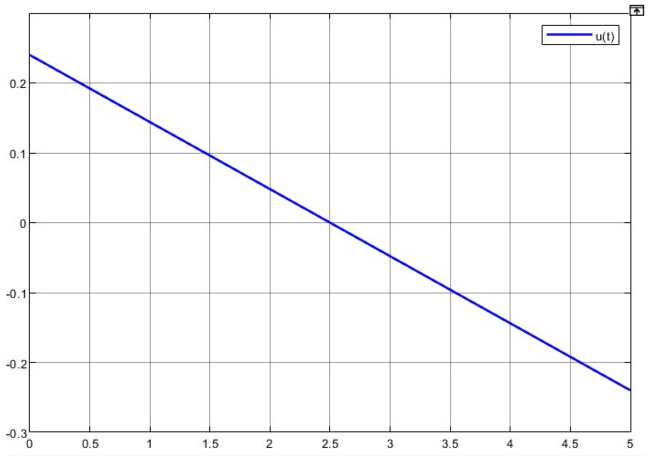
```
syms f1(t) f2(t) x1(t) x2(t)
ode1 = diff(f1) == 0;ode2 = diff(f2) == -f1;
ode3 = diff(x1) == x2;ode4 = diff(x2) == 1/6*f2;
odes = [ode1;ode2;ode3;ode4];
cond1 = x1(0) == 0;cond2 = x2(0) == 0;
cond3 = x1(5) == 1;cond4 = x2(5) == 0;
conds = [cond1;cond2;cond3;cond4];
[f1Sol(t),f2Sol(t),x1Sol(t),x2Sol(t)] = dsolve(odes,conds);
u = 1/6*f2Sol(t)
```

3. Plot the control, state, and cost function.

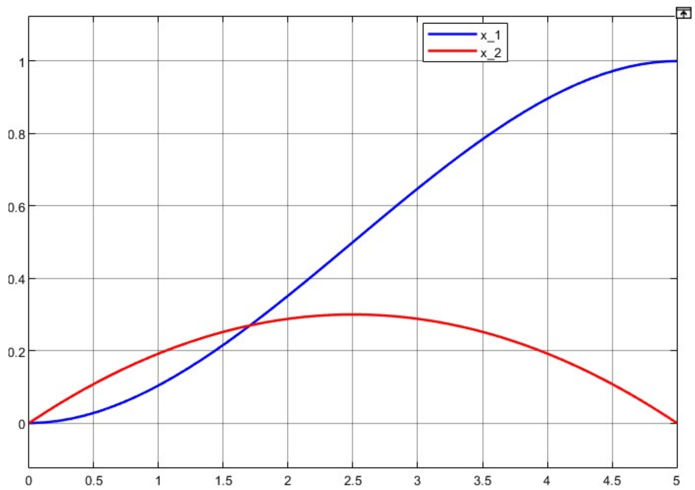
- **Model**



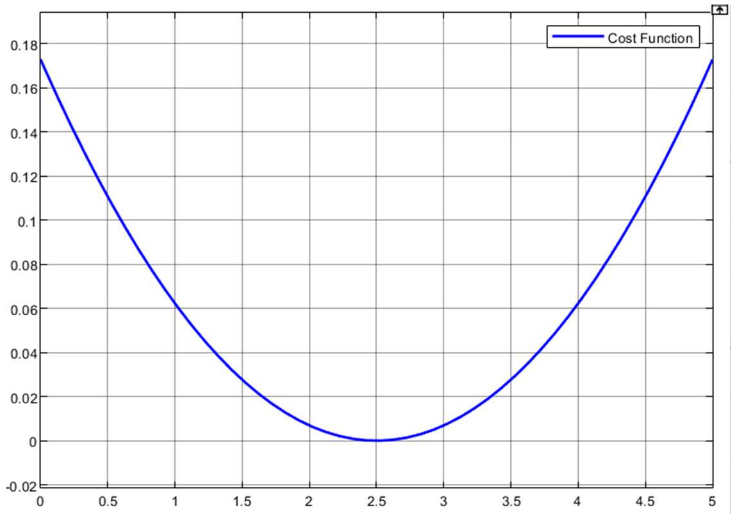
- **Control**



- **State**



- **Cost Function**



4. Plot the cost function for changed controller parameters from the optimal ones.

Change the form of the control signal, assuming the following:

$$u(t) = at^2 + bt \tag{1}$$

We already know:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = u \end{cases} \tag{2}$$

$$\begin{aligned} \varphi_1(0) = 0, \varphi_2(0) = 0 \\ \varphi_1(t_f) = 0, \varphi_2(t_f) = 0 \end{aligned} \tag{3}$$

By bringing (1) into (2) and using boundary condition (3), the solution is obtained:

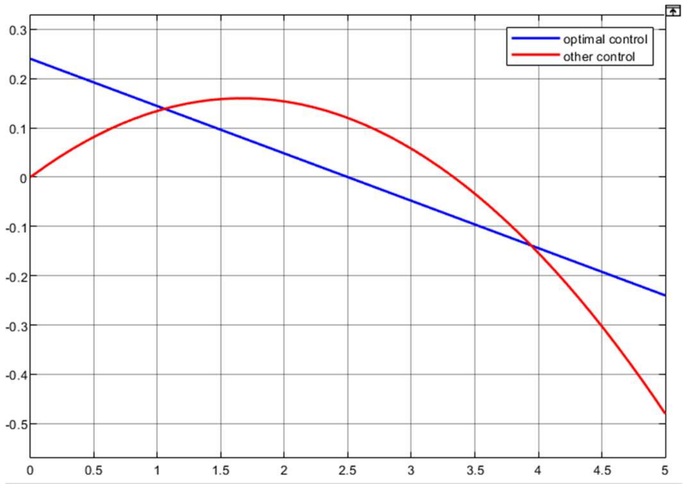
$$\begin{cases} a = -36/625 \\ b = 24/125 \end{cases}$$

So we have:

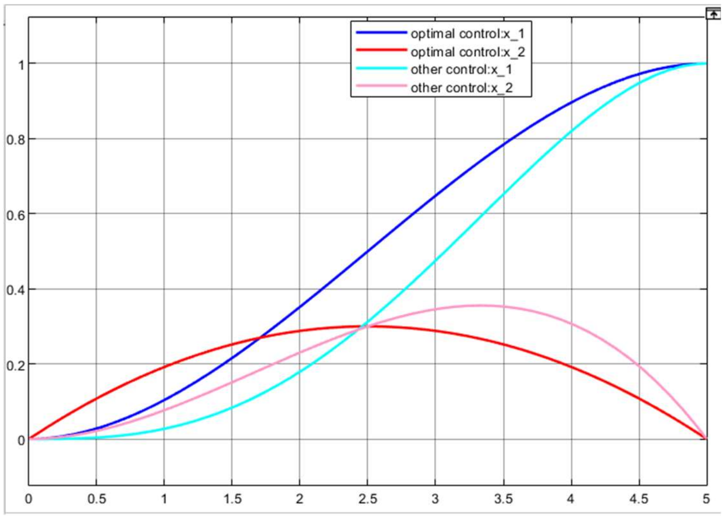
$$u(t) = -\frac{36}{625}t^2 + \frac{24}{125}t \tag{5}$$

The original control scheme is changed to Equation (5), and the simulation results are shown as follows:

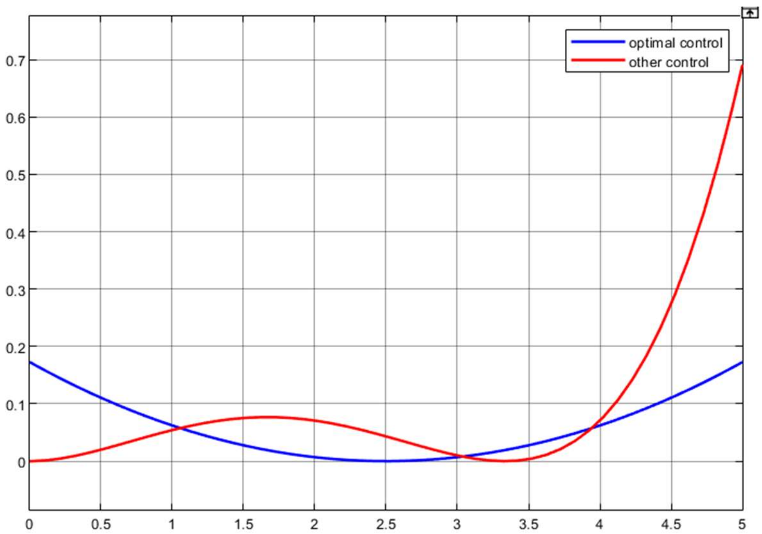
- **Control**



- **State**



- **Cost Function**



Conclusion :

By changing the parameters of the controller, we can also make the system reach the limit condition, but the value of the cost function is much larger than the value of the optimal control. It can be seen that the benefit of this optimal control method is to achieve the target while minimizing the consumption