

Question 2.2:

$$\text{Network 2: } \vec{a}^{(1)} = \tilde{W} \vec{a}^{(0)} + \tilde{b}$$

$$\text{Network 1: } \vec{a}^{(1)} = W^{(1)} \vec{a}^{(0)} + \vec{b}^{(1)}$$

$$\vec{a}^{(2)} = W^{(2)} \vec{a}^{(1)} + \vec{b}^{(2)}$$

$$\vec{a}^{(3)} = W^{(3)} \vec{a}^{(2)} + \vec{b}^{(3)}$$

$$\therefore \vec{a}^{(3)} = W^{(3)} (W^{(2)} \vec{a}^{(1)} + \vec{b}^{(2)}) + \vec{b}^{(3)}$$

$$= W^{(3)} [W^{(2)} (W^{(1)} \vec{a}^{(0)} + \vec{b}^{(1)}) + \vec{b}^{(2)}] + \vec{b}^{(3)}$$

$$= \boxed{W^{(3)} W^{(2)} W^{(1)} \vec{a}^{(0)} + W^{(3)} \vec{b}^{(1)} + W^{(3)} \vec{b}^{(2)}} \times$$

$$= W^{(3)} W^{(2)} W^{(1)} \vec{a}^{(0)} + W^{(3)} W^{(2)} \vec{b}^{(1)} + W^{(3)} \vec{b}^{(2)} + \vec{b}^{(3)}$$

$$\therefore \tilde{W} = W^{(3)} W^{(2)} W^{(1)}$$

$$\tilde{b} = W^{(3)} W^{(2)} \vec{b}^{(1)} + W^{(3)} \vec{b}^{(2)} + \vec{b}^{(3)}$$