

Comparing Optimization and Estimation Techniques for Low-Thrust Spacecraft Rendezvous

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Low-thrust technologies offer efficient mass usage which can lower spacecraft mass and, thus, mission costs. However, due to the small forces generated by low-thrust engines, long-duration burns are required to significantly alter a spacecraft path. Accordingly, low-thrust trajectory design must incorporate additional variables to describe the thrust vector during the burn duration. The solution to this higher-dimensional trajectory design problem is often obtained via optimal control where the thrust vector constitutes the set of control variables. In this study, a rendezvous scenario between a low-thrust-equipped spacecraft and an uncontrolled “mothership” is considered. Uncertainties in the spacecraft position, velocity, and thrust vectors are mitigated via estimation algorithms and a mass-optimal trajectory is obtained that satisfies the natural dynamics and mission-imposed constraints. Several techniques for optimization and estimation are investigated and their effects on the optimal solution are discussed.

I. Nomenclature

A	=	amplitude of oscillation
a	=	cylinder diameter
C_p	=	pressure coefficient
C_x	=	force coefficient in the x direction
C_y	=	force coefficient in the y direction
c	=	chord
dt	=	time step
F_x	=	X component of the resultant pressure force acting on the vehicle
F_y	=	Y component of the resultant pressure force acting on the vehicle
f, g	=	generic functions
h	=	height
i	=	time index during navigation
j	=	waypoint index
K	=	trailing-edge (TE) nondimensional angular deflection rate

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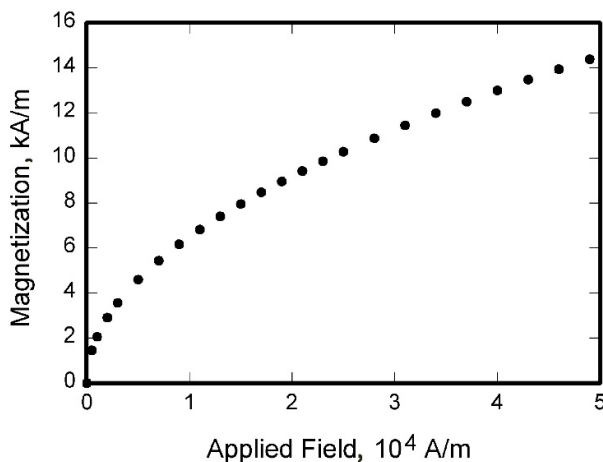


Fig. 1 Magnetization as a function of applied fields.

Place figure captions below all figures; place table titles above the tables. If your figure has multiple parts, include the labels “a),” “b),” etc. below and to the left of each part, above the figure caption. Please verify that the figures and tables you mention in the text actually exist. *Please do not include captions as part of the figures, and do not put captions in separate text boxes linked to the figures.* When citing a figure in the text, use the abbreviation “Fig.” except at the beginning of a sentence. Do not abbreviate “Table.” Number each different type of illustration (i.e., figures, tables, images) sequentially with relation to other illustrations of the same type.

Figure axis labels are often a source of confusion. Use words rather than symbols. As in the example to the right, write the quantity “Magnetization” rather than just “M.” Do not enclose units in parenthesis, but rather separate them from the preceding text by commas. Do not label axes only with units. As in Fig. 1, for example, write “Magnetization, A/m” or “Magnetization, A · m⁻¹,” not just “A/m.” Do not label axes with a ratio of quantities and units. For example, write “Temperature, K,” not “Temperature/K.”

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$$\int_0^{r_2} F(r, \varphi) dr d\varphi = [\sigma r_2 / (2\mu_0)] \int_0^\infty \exp(-\lambda|z_j - z_i|) \lambda^{-1} J_1(\lambda r_2) J_0(\lambda r_i) \lambda d\lambda \quad (1)$$

Be sure that the symbols in your equation are defined before the equation appears, or immediately following. Italicize symbols (*T* might refer to temperature, but *T* is the unit tesla). Refer to “Eq. (1),” not “(1)” or “equation (1)” except at the beginning of a sentence: “Equation (1) is. . .” Equations can be labeled other than “Eq.” should they represent

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Use only one space after periods or colons. Hyphenate complex modifiers: “zero-field-cooled magnetization.” Avoid dangling participles, such as, “Using Eq. (1), the potential was calculated.” [It is not clear who or what used Eq. (1).] Write instead “The potential was calculated using Eq. (1),” or “Using Eq. (1), we calculated the potential.”

Insert a zero before decimal points: “0.25,” not “.25.” Use “cm²” not “cc.” Indicate sample dimensions as “0.1 cm × 0.2 cm,” not “0.1 × 0.2 cm².” The preferred abbreviation for “seconds” is “s,” not “sec.” Do not mix complete spellings and abbreviations of units: use “Wb/m²” or “webers per square meter,” not “webers/m².” When expressing a range of values, write “7 to 9” or “7–9,” not “7~9.”

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Be aware of the different meanings of the homophones “affect” (usually a verb) and “effect” (usually a noun), “complement” and “compliment,” “discreet” and “discrete,” “principal” (e.g., “principal investigator”) and “principle” (e.g., “principle of measurement”). Do not confuse “imply” and “infer.”

The word “data” is plural, not singular (i.e., “data are,” not “data is”). The subscript for the permeability of vacuum μ_0 is zero, not a lowercase letter “o.” The term for residual magnetization is “remanence”; the adjective is “remanent”; do not write “remnance” or “remnant.” The word “micrometer” is preferred over “micron” when spelling out this unit of measure. A graph within a graph is an “inset,” not an “insert.” The word “alternatively” is preferred to the word “alternately” (unless you really mean something that alternates). Use the word “whereas” instead of “while” (unless you are referring to simultaneous events). Do not use the word “essentially” to mean “approximately” or “effectively.” Do not use the word “issue” as a euphemism for “problem.” When compositions are not specified, separate chemical symbols by en-dashes; for example, “NiMn” indicates the intermetallic compound Ni_{0.5}Mn_{0.5} whereas “Ni–Mn” indicates an alloy of some composition Ni_xMn_{1–x}.

Be aware of the different meanings of the homophones “affect” (usually a verb) and “effect” (usually a noun), “complement” and “compliment,” “discreet” and “discrete,” “principal” (e.g., “principal investigator”) and “principle” (e.g., “principle of measurement”). Do not confuse “imply” and “infer.”

Prefixes such as “non,” “sub,” “micro,” “multi,” and “ultra” are not independent words; they should be joined to the words they modify, usually without a hyphen. There is no period after the “et” in the abbreviation “et al.” The abbreviation “i.e.,” means “that is,” and the abbreviation “e.g.,” means “for example” (these abbreviations are not italicized). If desired, more detailed style and formatting instructions can be found in the AIAA style guide, AIAA Style (available from AIAA upon request).

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References