Comparing Optimization and Estimation Techniques for Low-Thrust Spacecraft Rendezvous

Andrew Cox, Mike Sparapany, Collin York, Waqar Zaidi

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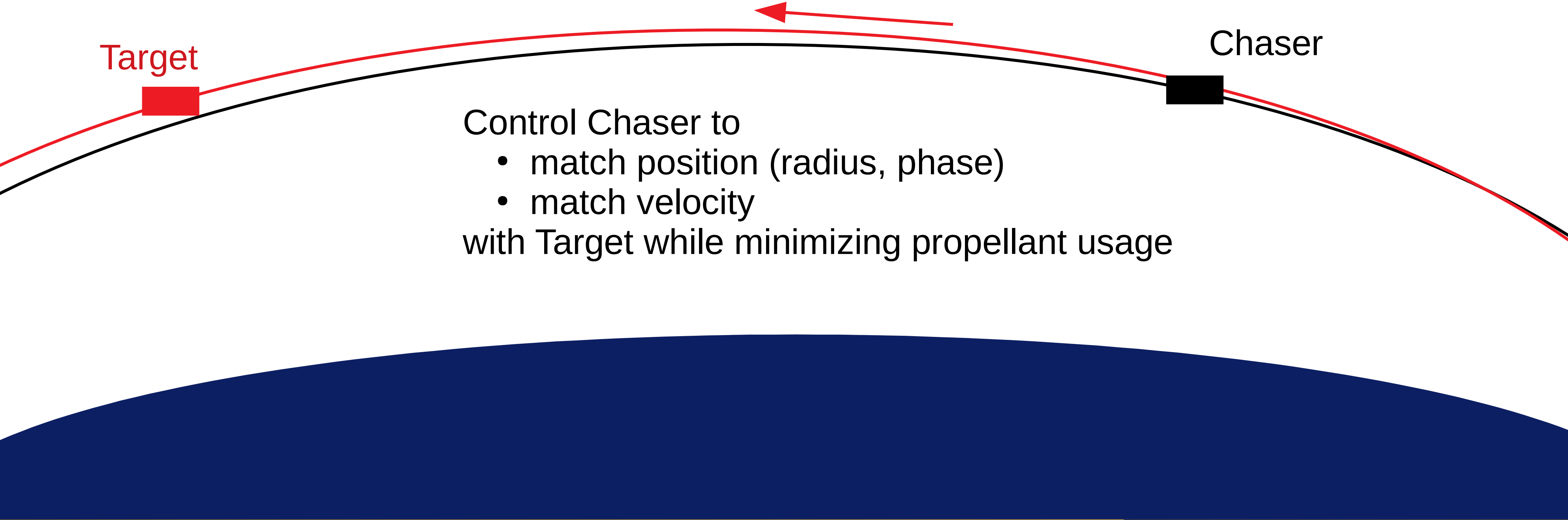
AAE 568 Course Project, Purdue University

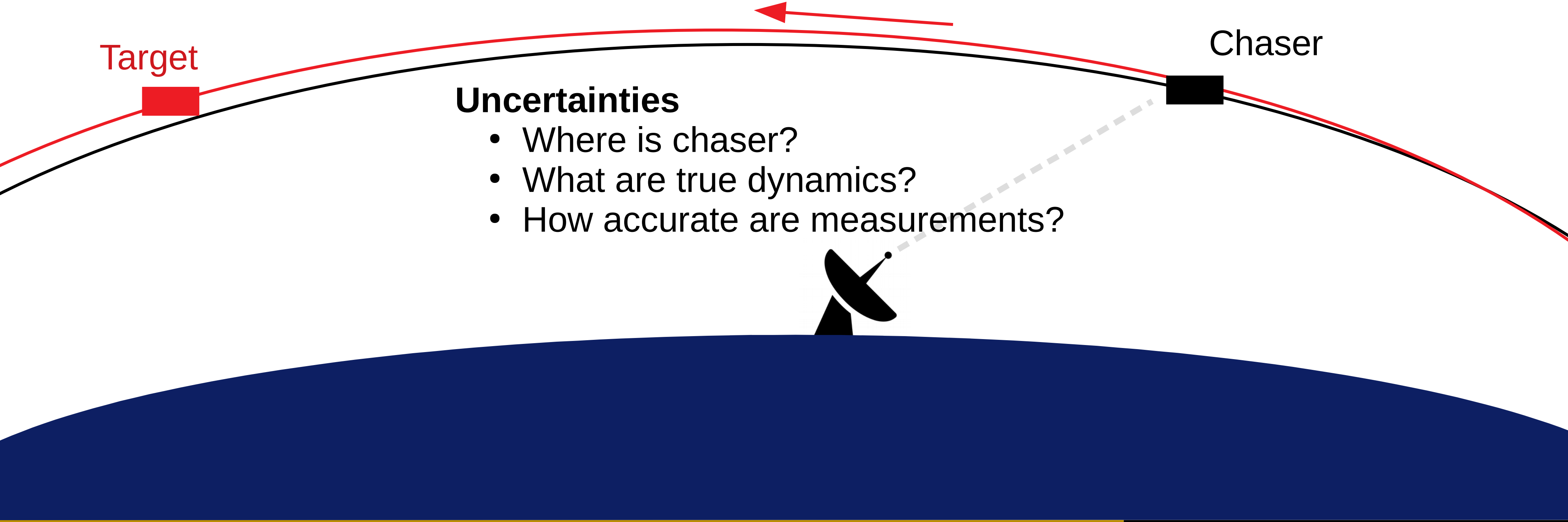
Problem Motivation



Control Chaser to
- match position (radius, phase)
- match velocity

with Target while minimizing propellant usage

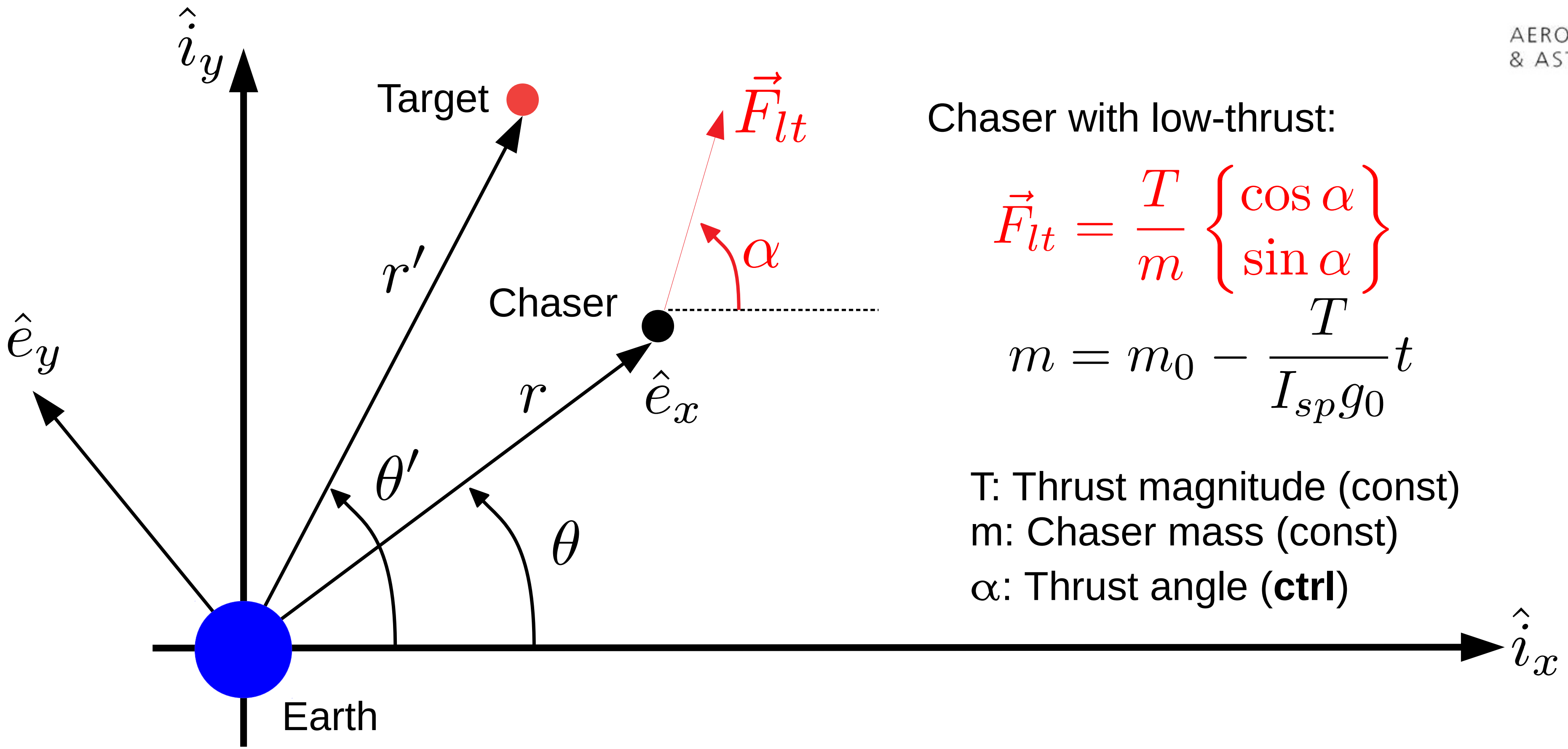
Problem Motivation, Cont'd



Uncertainties

- Where is chaser?
- What are true dynamics?
- How accurate are measurements?

Low-Thrust Control



Chaser with low-thrust:

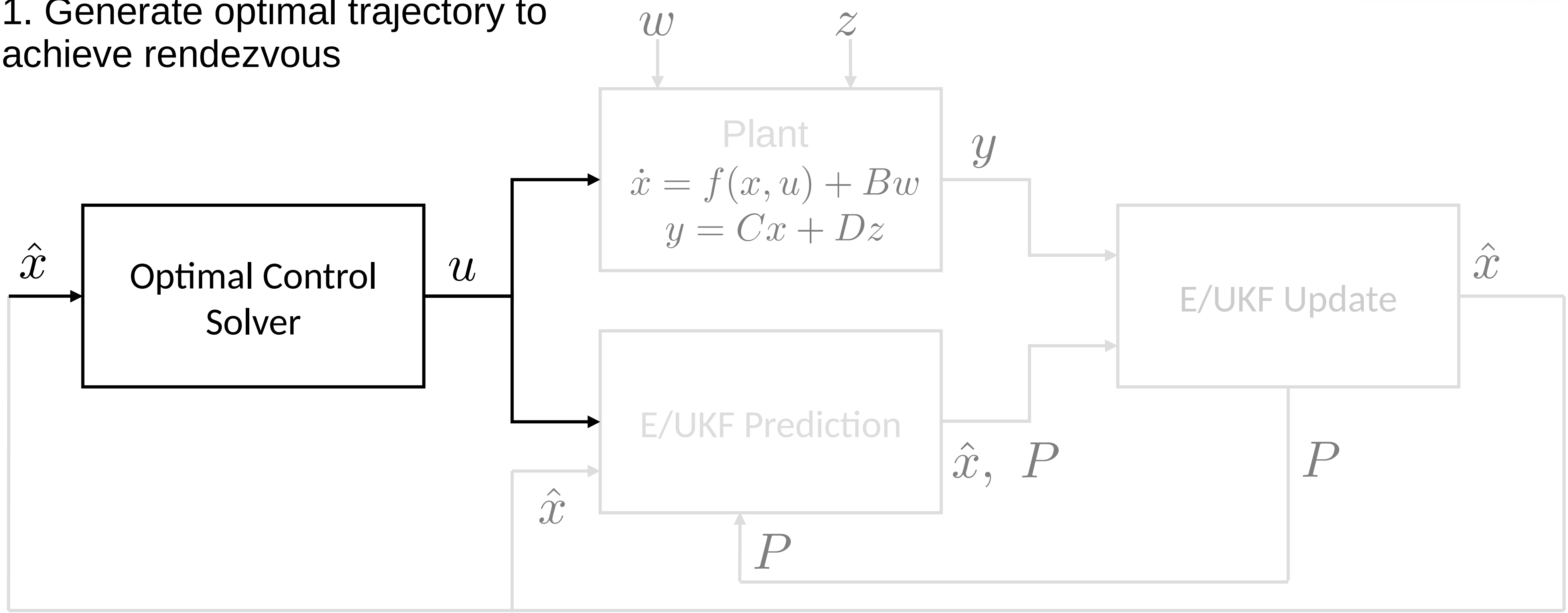
$$\vec{F}_{lt} = \frac{T}{m}\begin{Bmatrix}\cos\alpha \\ \sin\alpha\end{Bmatrix}$$

$$m = m_0 - \frac{T}{I_{sp}g_0}t$$

T: Thrust magnitude (const)
m: Chaser mass (const)
α: Thrust angle (**ctrl**)

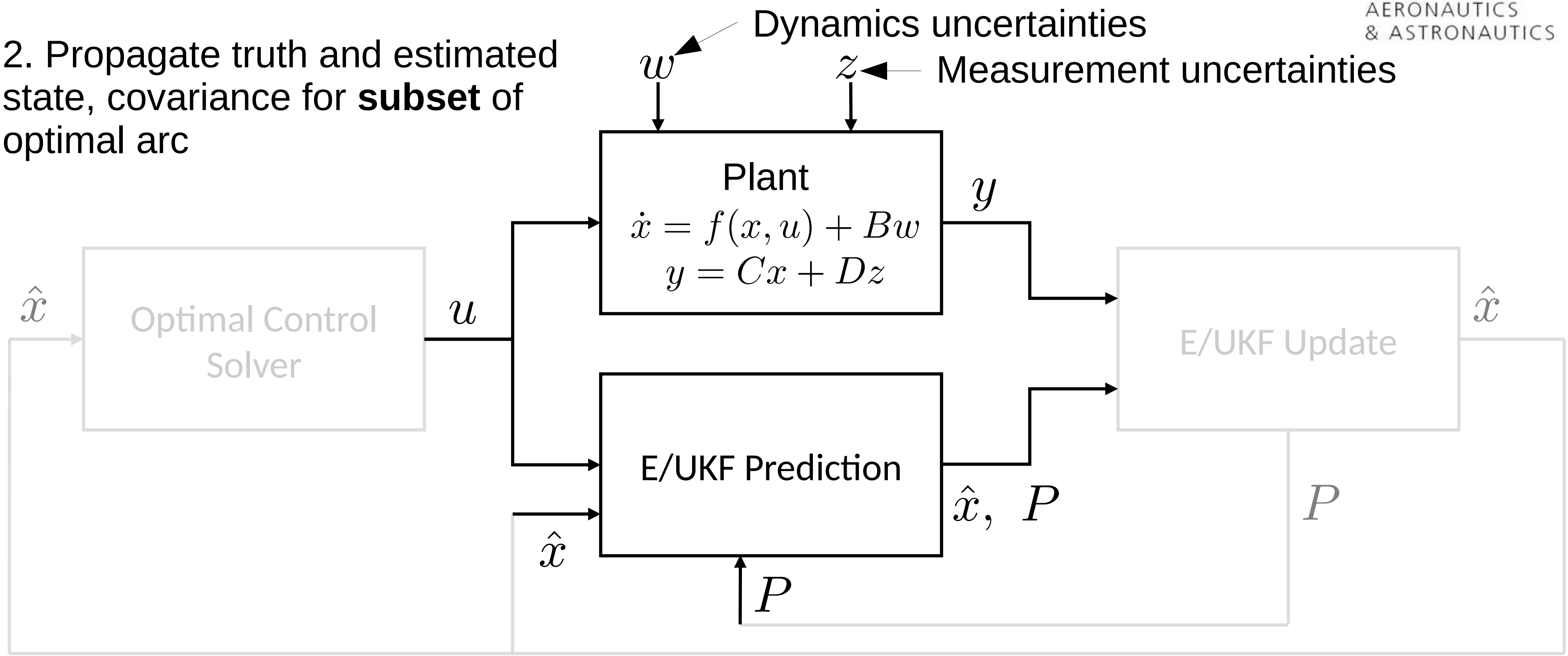
Problem Overview

1. Generate optimal trajectory to achieve rendezvous



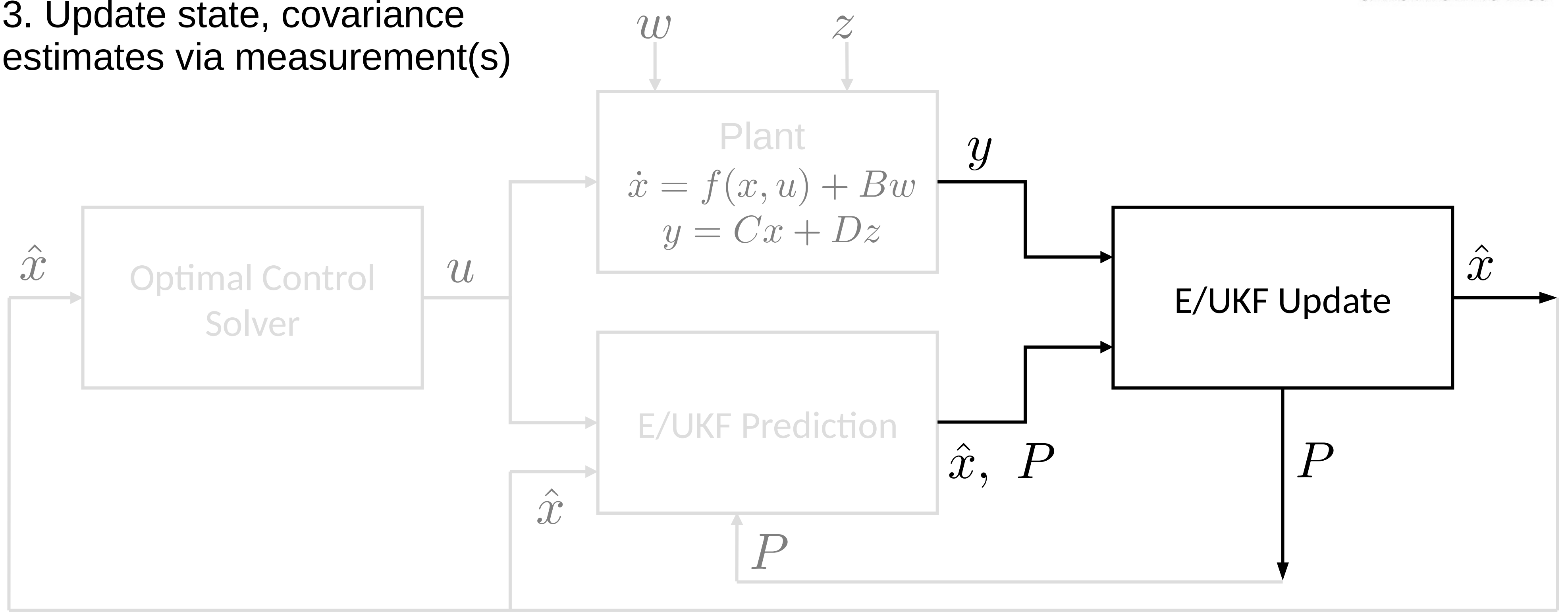
Problem Overview

2. Propagate truth and estimated state, covariance for **subset** of optimal arc



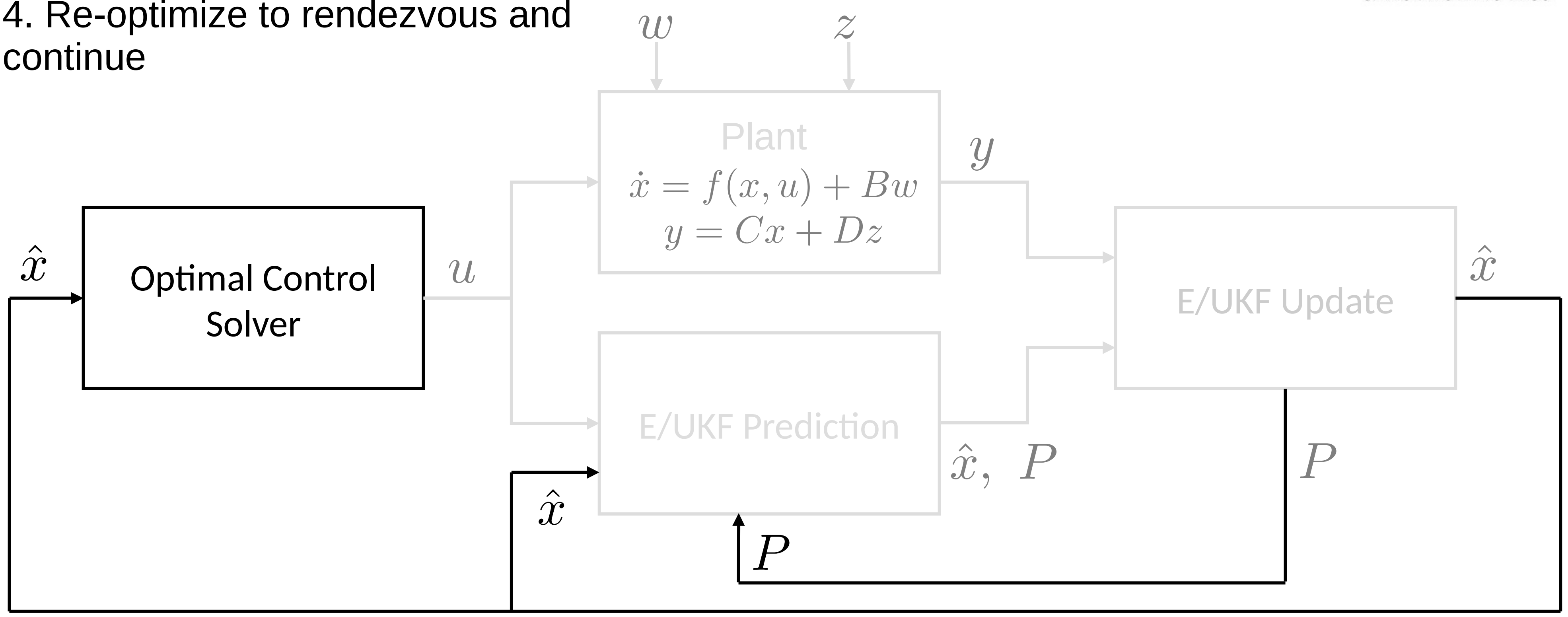
Problem Overview

3. Update state, covariance estimates via measurement(s)



Problem Overview

4. Re-optimize to rendezvous and continue



Optimization Problem Definition

Goal: minimize propellant usage = maximize m_f = minimize t_f

$$\min_{\alpha} \; J = t_f$$

Subject to:

$$\dot{\vec{x}} = \begin{Bmatrix} \dot{r} \\ \dot{\theta} \\ r\dot{\theta}^2 - \frac{\mu}{r^2} + \frac{T}{m}\left(C_\alpha C_\theta + S_\alpha S_\theta\right) \\ -2\frac{\dot{r}\dot{\theta}}{r} + \frac{T}{mr}\left(S_\alpha C_\theta - C_\alpha S_\theta\right) \end{Bmatrix} \qquad \vec{x}(t_f) = \begin{Bmatrix} r' \\ \theta_0' + \dot{\theta}' t \\ \dot{r}_0' \\ \dot{\theta}_0' \end{Bmatrix}$$

$$\vec{x}(t_0) = \begin{Bmatrix} r_0 & \theta_0 & \dot{r}_0 & \dot{\theta}_0 \end{Bmatrix}^T, \qquad t_0 = 0 \qquad\qquad t_f = \text{free}$$

Indirect Optimization

- Non-dimensionalize the problem by initial radius, mean motion, and spacecraft mass to help convergence
- Utilize Euler-Lagrange Equations

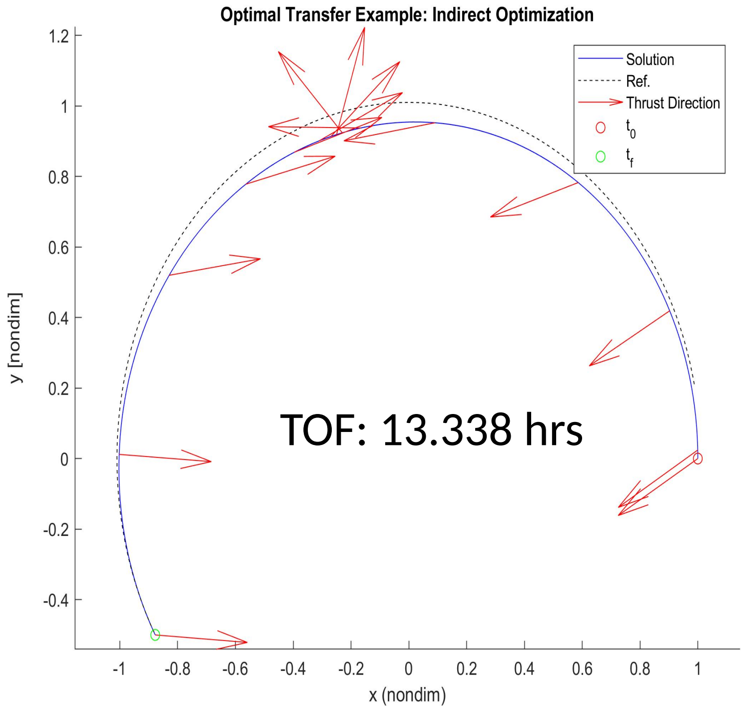
$$\dot{\lambda} = \left(\frac{\partial H}{\partial x}\right)^T \qquad \frac{\partial H}{\partial \alpha} = 0$$

- Two-Point Boundary Value Problem requires one more constraint from the Transversality Condition,

$$H_f \, \mathrm{d}t_f - \lambda_f^T \, \mathrm{d}x_f + \mathrm{d}g = 0$$

$$H_f \, \mathrm{d}t_f - \lambda_2 \left(\partial\theta_f + \frac{\partial\theta}{\partial t}\bigg|_{t_f} \mathrm{d}t_f\right) + \mathrm{d}t_f = 0$$

$$H_f - \lambda_2 \dot{\theta}' + 1 = 0$$



Direct Optimization

Model dynamics by piecewise 3rd-degree polynomials (control constant along segments)

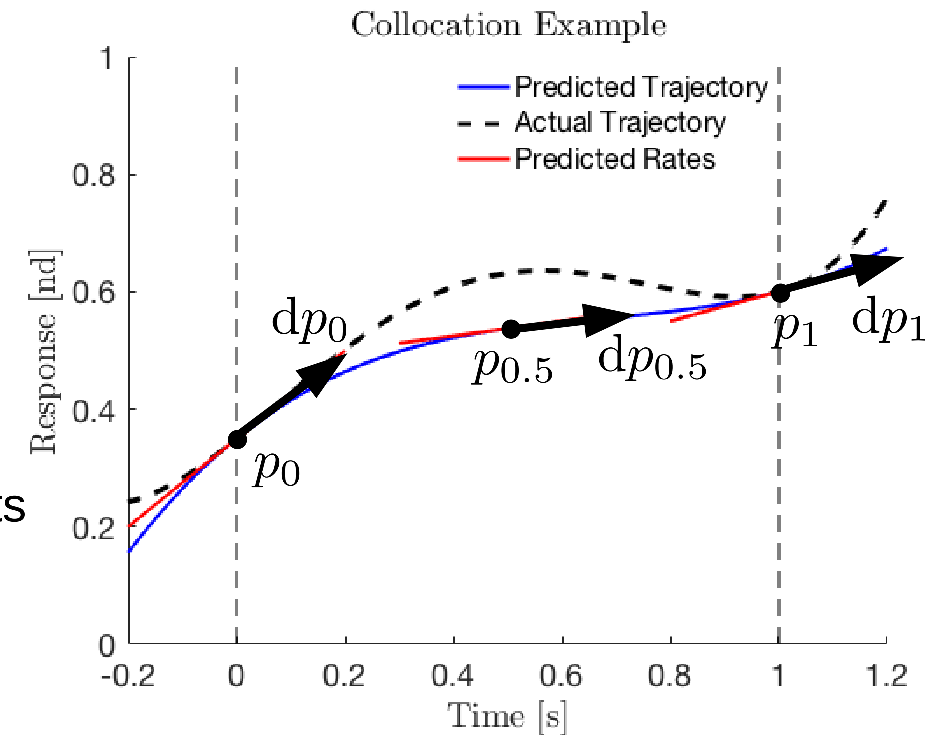
$$\tilde{p}_{0.5} = \frac{1}{2}(p_0 + p_1) + \frac{t_f(\mathrm{d}p_0 - \mathrm{d}p_1)}{8(N-1)}$$

$$\mathrm{d}\tilde{p}_{0.5} = -\frac{3(N-1)(p_0 - p_1)}{2t_f} - \frac{1(\mathrm{d}p_0 + \mathrm{d}p_1)}{4}$$

$$\mathrm{d}p_{0.5} = f(t, \tilde{p}_{0.5})$$

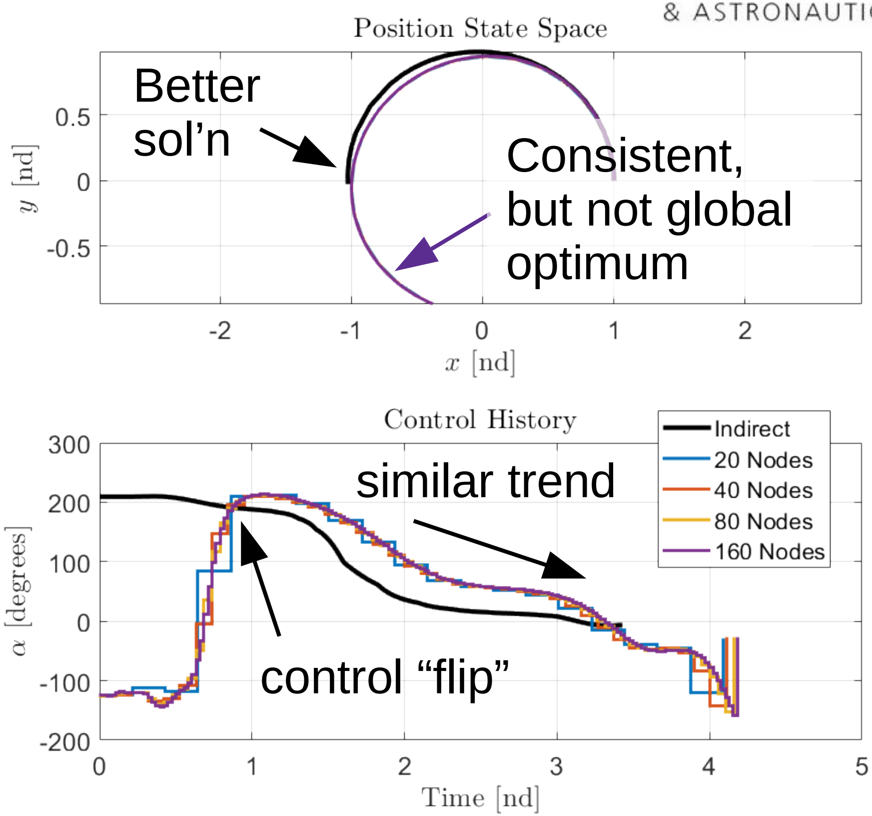
Constraint: $\mathrm{d}p_{0.5} - \mathrm{d}\tilde{p}_{0.5} = 0\ \forall$ polynomial midpoints

Driven by Sequential Quadratic Programming (SQP from MATLAB's *fmincon*)



Indirect vs. Direct Optimization

	Direct	Indirect
Pros	• No variational calculus • Easy to make initial guess (no costates) • Easy to find a potential (suboptimal) solution	• Root-solver is simpler • Solution guaranteed locally optimal • Convincing global optimum with engineering judgement
Cons	• Unsure if solution is optimal, or even real • Numerical optimizer is more complicated	• Initial guess hard to make • Sensitive to initial guess



Estimation Problem Definition

$$\dot{x} = f(x, \alpha) + Bw$$ Process equations with acceleration noise

$$y_k = Cx_k + Dz_k$$ Range & range-rate measurements

- w is mean-zero Gaussian white noise with $\sigma = 10^{-8}$ km/s^2 (J_2, lunar gravity at GEO)
- z is mean-zero Gaussian white noise with $\sigma_1 = 10$ m, $\sigma_2 = 1$ mm/s
- Continuous-discrete formulations to model continuous gravity perturbation and discrete measurement intervals

Goals:

- Determine appropriate sample time for Extended Kalman Filter (EKF)
- Compare EKF and Unscented Kalman Filter (UKF)

Extended Kalman Filter

Continuous Propagation Equations:

$$\dot{\hat{x}} = f(\hat{x}, \alpha)$$
$$\dot{P} = A(\hat{x}(t))P + PA(\hat{x}(t))^T + BWB^T$$
$$P(0) = P_0$$
$$\text{where } A(\hat{x}(t)) = \frac{\partial f}{\partial x}\bigg|_{\hat{x}(t)}$$

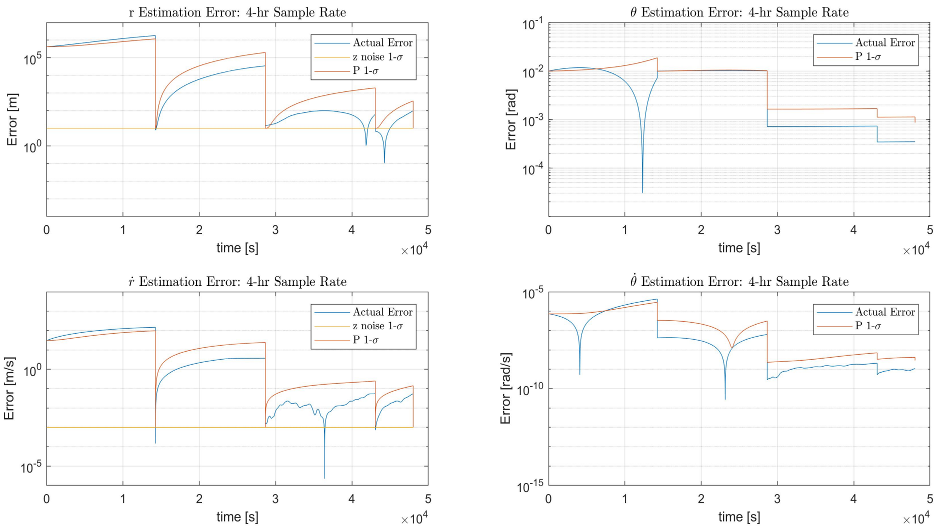
Discrete Update Equations:

$$\hat{x}(t_k^+) = \hat{x}(t_k^-) + L_k\left(y_k - C\hat{x}(t_k^-)\right)$$
$$P(t_k^+) = (I - L_k C)P(t_k^-)$$
$$\text{where } P(t_k^-)C^T\left(CP(t_k^-)C^T + DZD^T\right)^{-1}$$

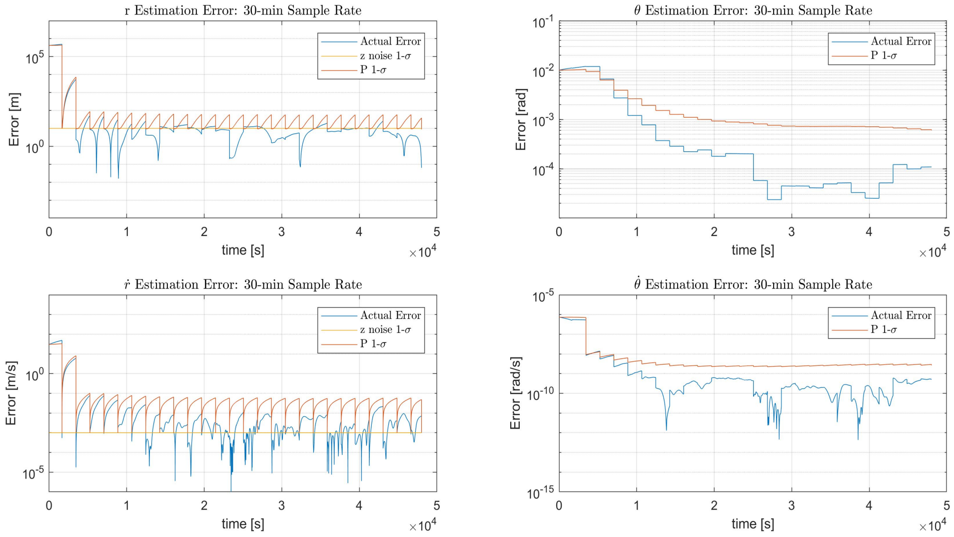
EKF Sampling Time Determination

PURDUE
AERONAUTICS & ASTRONAUTICS

4-Hr Sampling Rate



30-min Sampling Rate



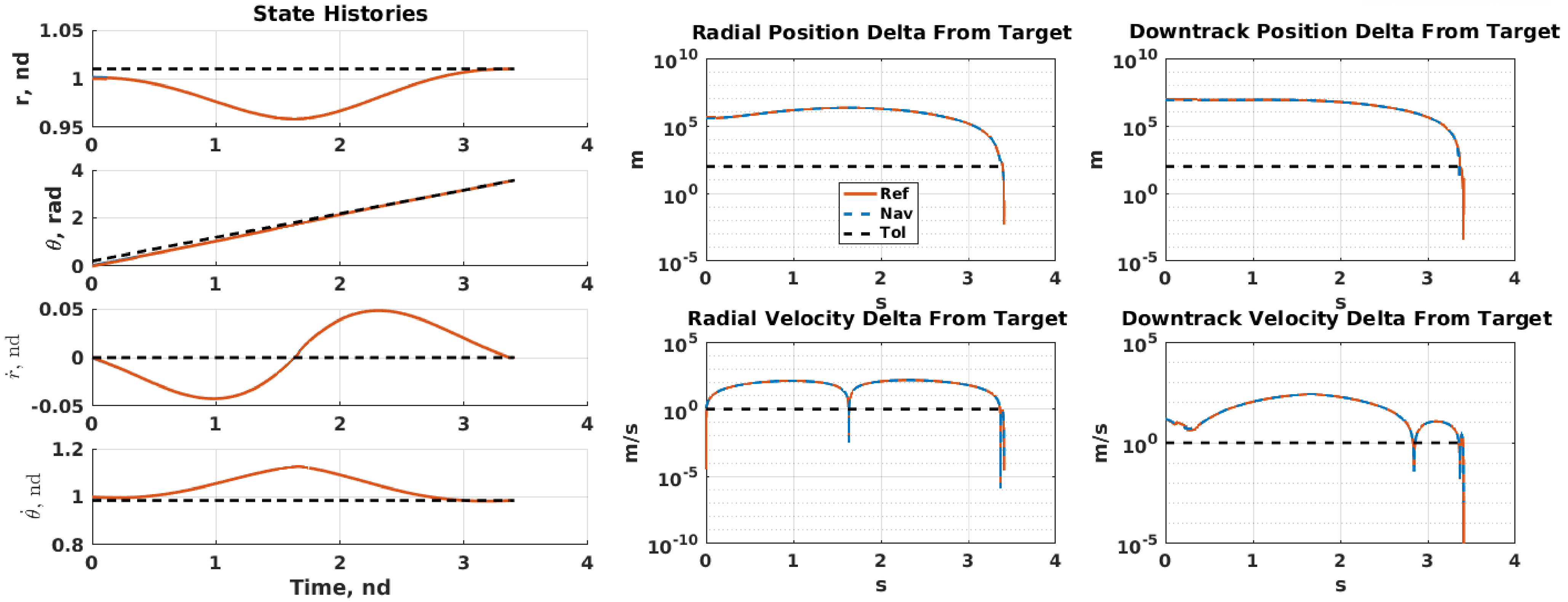
Unscented Kalman Filter

Waqar

Estimation Method Comparison

Waqar

Example: Indirect + EKF Results



Contributions

Andrew: Dynamics derivation, integration of components into mission loop

Michael: Direct optimization implementation, optimizer comparison

Collin: EKF implementation, indirect optimization implementation

Waqar: UKF implementation, estimator comparison

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