Project Proposal: Comparing Optimization and Estimation Techniques for Low-Thrust Spacecraft Rendezvous

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1 Introduction

Rendezvous of a chaser spacecraft with a target spacecraft in a predetermined orbit is a common problem in spaceflight operations. Spacecraft mass is frequently minimized during such operations to reduce overall mission expenditures. Advancements in low-thrust propulsion technology facilitate these minimizations by transforming propellant mass into a propulsive force more efficiently than other technologies. However, due to the small accelerations delivered by low-thrust systems, significant burn durations are required to alter the course of the spacecraft. Accordingly, low-thrust trajectory design must incorporate variables to describe the thrust vector in addition to the usual variables that describe a spacecraft state. The following proposed analysis attempts to blend these two traditional spaceflight problems and determine optimization and estimation techniques that allow rendezvous using low-thrust propulsion systems.

2 Problem Definition

Let the state vector, $\mathbf{x} = \{r,\ \theta,\ \dot{r},\ \dot{\theta},\ m\}$, store the spacecraft position as the radius, r, and angle θ, relative to an inertial reference frame centered on Earth. Similarly, the velocity is described by the time derivatives $\dot{r}$ and $\dot{\theta}$. Finally, the spacecraft mass, m, is included as mass varies during propulsive burns. The control vector, $\mathbf{u} = \{T,\ \alpha\}$ includes the thrust magnitude, T, and the orientation of the thrust vector relative to the inertial frame, α.

The dynamics governing the state are the well-known Keplerian equations of motion augmented with uncertainties which may include non-spherical gravity, solar radiation pressure, drag, or other forces. In equation form,

$$\mathbf{f} = \dot{\mathbf{x}} = \left\{ \begin{matrix} \dot{r} \\ \dot{\theta} \\ r\dot{\theta} - \mu/r^2 + \frac{T}{m}\left(C_\alpha C_\theta + S_\alpha S_\theta\right) \\ -2\dot{r}\dot{\theta}/r + \frac{T}{mr}\left(S_\alpha C_\theta - C_\alpha S_\theta\right) \\ -T/(I_{sp} g_0) \end{matrix} \right\} + \mathbf{w}\,, \tag{1}$$

where μ is the standard gravity parameter for Earth, $\mu = 3.986\text{e}05\ \text{km}^3/\text{s}^2$, I_{sp} is the propulsion system specific impulse, g_0 is the mean Earth gravitational acceleration, $g_0 = 9.80665\text{e-}03\ \text{km}/\text{s}^2$, and $\mathbf{w}$ is a vector of noise that encapsulates perturbations to the conic model.

The goal of the rendezvous problem is to identify an optimal control history, $\mathbf{u}^*$ that maximizes the spacecraft mass (i.e., minimizes the mass spent during the flight) and delivers the spacecraft to a desired state $\mathbf{x}_d$. The initial state, $\mathbf{x}(0) = \mathbf{x}_0$, is fixed, as is the final thrust magnitude, $T(t_f) = 0$.

3 Orbit Estimation

In the context of low-thrust optimization, recursive Bayesian updates are applied for each measurement received between successive trajectory subarcs. However, instead of using the traditional Extended Kalman Filter (EKF), which simply linearizes all the nonlinear transformations in the KF equations, the capabilities of an Unscented Kalman Filter (UKF) that uses a set of discretely sampled points to parameterize the system mean and covariance are leveraged. Essentially, the UKF represents a mixture between an EKF and Monte Carlo whereby the sampled or "sigma" points are used to propagate uncertainty through complete nonlinear transformations. Accordingly, no Jacobians are required to develop a state transition matrix that linearly predicts future uncertainty prior to a sequential Bayesian update. Additionally, the UKF implementation will incorporate process noise (at each node between trajectory subarcs) and *resample* sigma points to account for uncertainty due to suboptimal forces. The state and measurement update equations for the UKF are provided here,

State-Update

$$\begin{aligned} \bar{X}_t &= \sum_{i=0}^{2L} W_i^m \chi_{i,t/t-1} \\ \bar{P}_t &= Q + \sum_{i=0}^{2L} W_i^c (\chi_{i,t/t-1} - \bar{X}_t)(\chi_{i,t/t-1} - \bar{X}_t)^T \end{aligned} \tag{2}$$

Measurement-Update

$$\begin{aligned} K_t &= P_{xy} P_{yy}^{-1} \\ \hat{X}_t &= \bar{X}_t + K_t(y_t - \bar{y}_t) \\ P_t &= \bar{P}_t - K_t P_{yy} K_t^T \end{aligned} \tag{3}$$

where L is the number of sigma points, W^m are weights that capture distribution statistics, χ are sigma points, Q is the process-noise matrix, $\bar{y}_t$ are simulated measurements, and y_t are true measurements. The mapping of the measurement space (i.e. topocentric spherical coordinates) to inertial coordinates is given by,

$$\begin{aligned} \mathbf{r} &= \mathbf{o} + \rho \mathbf{u}_\rho \\ \dot{\mathbf{r}} &= \dot{\mathbf{o}} + \dot{\rho}\mathbf{u}_\rho + \rho\dot{\alpha}\cos\delta \mathbf{u}_\alpha + \rho\delta\mathbf{u}_\delta \end{aligned} \tag{4}$$

where $\mathbf{o}$ and $\dot{\mathbf{o}}$ are the ground station inertial position and velocity coordinates at measurement time.

4 Proposed Analysis

Given a fixed target vehicle trajectory and chaser vehicle initial conditions and covariance, the team will compare combinations of two optimization methods and two state estimation methods. Starting with an initial chaser state, a mass-optimal trajectory will be determined via either standard indirect methods or the Marsden-Weinstein-Meyer process. We expect the MWM process to be significantly faster and more efficient over standard indirect methods. The actual trajectory will be propagated forward and subjected to a stochastic process noise, simulating perturbing accelerations which are not reflected in the equations of motion. After a set time interval, a measurement will be recorded and subjected to stochastic measurement noise. The measurement will then be used to determine a state estimation and associated trajectory dispersion covariance via either an EKF or UKF. Using the state estimate, the same optimization method will be re-run to calculate a new control plan. This entire process will iterate until the chaser covariance ellipsoid overlaps with the target vehicle state and is under a given tolerance.

To compare and contrast the four total combinations of optimization and estimation, the team will run multiple trials of each set and compare mass spent and time of flight. Since the state estimate will be used to determine successful rendezvous, the team will also analyze the final error between the actual and estimated states. The analysis will be repeated for ranges of process and measurement noise to look for trends. Since the EKF propagatates mean and covariance linearly, we expect large errors due to the non-linear equations of motion. Conceptually, we expect prediction off of measurement updates using the UKF to produce miss distances between target and chaser vehicles that are inside the chaser covariance ellipsoid. If the predicted miss distance is outside the ellipsoid (i.e. due to a EKF measurement update), thrusting will be required to bring the miss inside the ellipsoid.