Answer 1A

$$mean\ \bar{x} = 91.08$$
$$variance\ s^2 = 207.47$$

Thus the 95% confidence interval for the variance is given by:

$$\left(\frac{(n-1)s^2}{\chi^2_{n-1,1-\frac{\alpha}{2}}}, \frac{(n-1)s^2}{\chi^2_{n-1,\frac{\alpha}{2}}}\right)$$

Since $n - 1 = 123 > 100$, the upper bound and lower bound are reversed

$$\left(\frac{(n-1)s^2}{\chi^2_{n-1,\frac{\alpha}{2}}}, \frac{(n-1)s^2}{\chi^2_{n-1,1-\frac{\alpha}{2}}}\right) = \left(\frac{(124-1)\times 207.47}{\chi^2_{123,0.025}}, \frac{(124-1)\times 207.47}{\chi^2_{123,0.975}}\right)$$
$$= \left(\frac{123\times 207.47}{155.589}, \frac{123\times 207.47}{94.195}\right) = (164.01, 270.91)$$

The results generated in STATA is shown below:

Table 1. The 95% confidence interval of iqf generated in STATA.

Variable	Obs	Variance	[95% Conf.	Interval]
iqf	124	207.4731	164.0165	270.9189

This implies that we are 95% confident that the standard deviation of Full-scale IQ in the population is between 12.81 and 16.46.

Answer 1B

No lead exposure: lead_type = 1

$observatios\ n = 78$, $mean\ \bar{x} = 92.88$, $variance\ s^2 = 235.45$

The 95% confidence interval for the variance is given by:

$$\left(\frac{(n-1)s^2}{\chi^2_{n-1,1-\frac{\alpha}{2}}}, \frac{(n-1)s^2}{\chi^2_{n-1,\frac{\alpha}{2}}}\right) = \left(\frac{(78-1)\times 235.45}{\chi^2_{77,0.975}}, \frac{(78-1)\times 235.45}{\chi^2_{77,0.025}}\right)$$
$$= \left(\frac{77\times 235.45}{103.158}, \frac{77\times 235.45}{54.623}\right) = (175.75, 311.91)$$

This implies that we are 95% confident that the standard deviation of Full-scale IQ of people who have no lead exposure in the population is between 13.28 and 17.66.

Current lead exposure: lead_type = 2

$observatios\ n = 24$, $mean\ \bar{x} = 88.75$, $variance\ s^2 = 103.85$

The 95% confidence interval for the variance is given by:

$$\left(\frac{(n-1)s^2}{\chi^2_{n-1,1-\frac{\alpha}{2}}}, \frac{(n-1)s^2}{\chi^2_{n-1,\frac{\alpha}{2}}}\right) = \left(\frac{(24-1)\times 103.85}{\chi^2_{23,0.975}}, \frac{(24-1)\times 103.85}{\chi^2_{23,0.025}}\right)$$
$$= \left(\frac{23\times 103.85}{38.076}, \frac{23\times 103.85}{11.689}\right) = (62.73,204.34)$$

This implies that we are 95% confident that the standard deviation of Full-scale IQ of people who currently have lead exposure in the population is between 7.92 and 14.29.

Past lead exposure: lead_type = 3
$observatios\ n = 22,\ mean\ \bar{x} = 87.23,\ variance\ s^2 = 204.28$
The 95% confidence interval for the variance is given by:

$$\left(\frac{(n-1)s^2}{\chi^2_{n-1,1-\frac{\alpha}{2}}}, \frac{(n-1)s^2}{\chi^2_{n-1,\frac{\alpha}{2}}}\right) = \left(\frac{(22-1)\times 204.28}{\chi^2_{21,0.975}}, \frac{(22-1)\times 204.28}{\chi^2_{21,0.025}}\right)$$
$$= \left(\frac{21\times 204.28}{35.479}, \frac{21\times 204.28}{10.283}\right) = (120.91, 417.18)$$

This implies that we are 95% confident that the standard deviation of Full-scale IQ of people who previously have lead exposure in the population is between 11.00 and 20.42.

The results generated in STATA is shown below:
Table 2. The 95% confidence interval of iqf grouped by lead type.

-> lead_type = 1

Variable	Obs	Variance	[95% Conf.	Interval]
iqf	78	235.454	175.7493	331.9086

-> lead_type = 2

Variable	Obs	Variance	[95% Conf.	Interval]
iqf	24	103.8478	62.73042	204.3452

-> lead_type = 3

Variable	Obs	Variance	[95% Conf.	Interval]
iqf	22	204.2792	120.9132	417.1843

The results from Answer 1A and Answer 1B indicate that lead exposure might lead to lower full-scale IQ. In addition, the variance of the group who currently have lead exposure is smaller than that of the group who had lea exposure, suggesting that the case may occur more often for people who currently have lead exposure.

Answer 2A
Since there is additional information about whether the proportion of lung cancer death is higher or lower than in the general population, we will construct a two-sided

confidence interval. Since $np_0q_0 = 20 \times 0.25 \times 0.75 = 3.75 < 5$, we will use the exact test. The 95% confidence interval is given by

```
. ci proportions death, exact

                                                         — Binomial Exact —
    Variable |        Obs  Proportion    Std. Err.       [95% Conf. Interval]
-------------+---------------------------------------------------------------
       death |         20         .25     .0968246        .0865715    .4910459
```

Table 3. The 95% confidence interval of the proportion test.
The 95% confidence interval is (0.087, 0.491).

Answer 2B

Let p = the true proportion of deaths due to lung cancer. The hypothesis statement can be expressed as $H_o: p = 0.12$ versus $H_1: p \neq 0.12$. Since $np_0q_0 = 20 \times 0.12 \times 0.88 = 2.1 < 5$, we will use the exact test.

```
. power oneproportion .12 .25, test(binomial) n(20)

Estimated power for a one-sample proportion test
Binomial test
Ho: p = p0  versus  Ha: p != p0

Study parameters:

        alpha =    0.0500
            N =        20
        delta =    0.1300
           p0 =    0.1200
           pa =    0.2500

Estimated power and alpha:

        power =    0.2142
 actual alpha =    0.0067
```

The power of the hypothesis test is 0.2142.