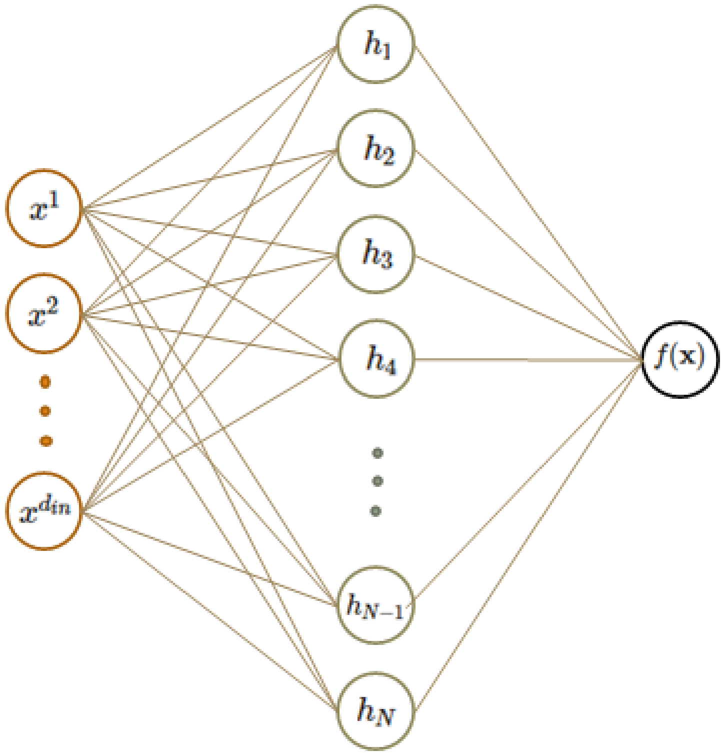
Neural networks and quantum field theory

RESULTS RECURRENCE

Correlation Functions



- Correlation function (n-pt functions)

$$G^{(n)}(x_1, \ldots, x_n) = \frac{\int df f_1 \ldots f_n e^{-S}}{Z}$$

- From experiment:

$$G^{(n)}(x_1, \ldots, x_n) = \mathbb{E}[f(x_1) \ldots f(x_n)]$$

- In GP theory (N is infinite):

$$G^{(n)}_{GP}(x_1, \ldots, x_n) = \sum_{p \in \text{Wick}(x_1, \ldots, x_n)} K(a_1, b_1) \ldots K(a_{n/2}, b_{n/2})$$

- In NGP theory (N is finite):

$$G^{(n)}(x_1, \ldots, x_n) = \frac{\int df f(x_1) \ldots f(x_n) \left[1 - \int d^{d_{in}}x\, g_k f(x)^k + O(g_k^2)\right] e^{-S_{GP}} / Z_{GP,0}}{\int df \left[1 - \int d^{d_{in}}x\, g_k f(x)^k + O(g_k^2)\right] e^{-S_{GP}} / Z_{GP,0}}$$

Experimental measurements

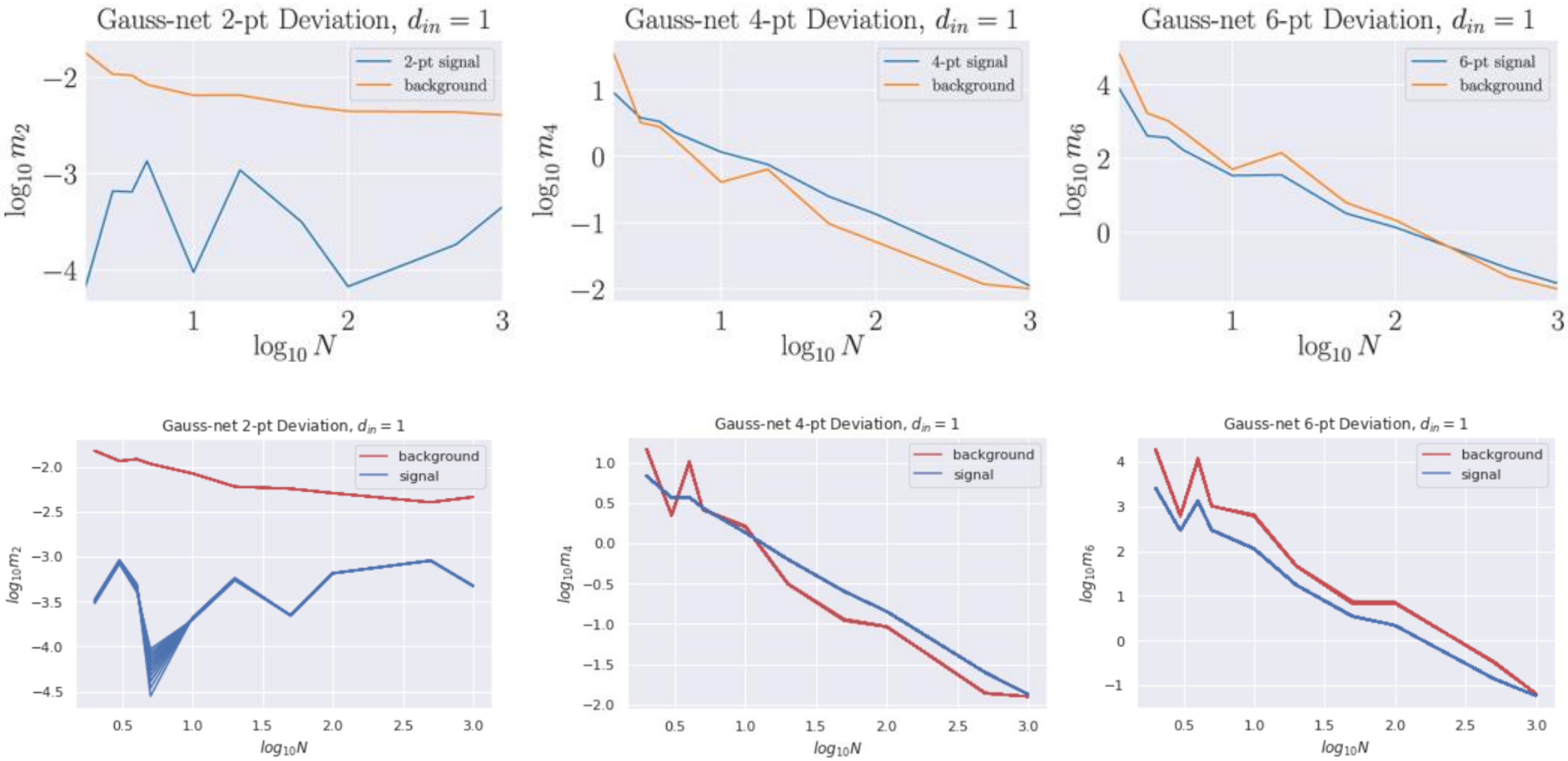
- normalized deviation m_n

$$m_n(x_1, \ldots, x_n) = \Delta G^{(n)}(x_1, \ldots, x_n) / G_{GP}^{(n)}(x_1, \ldots, x_n)$$

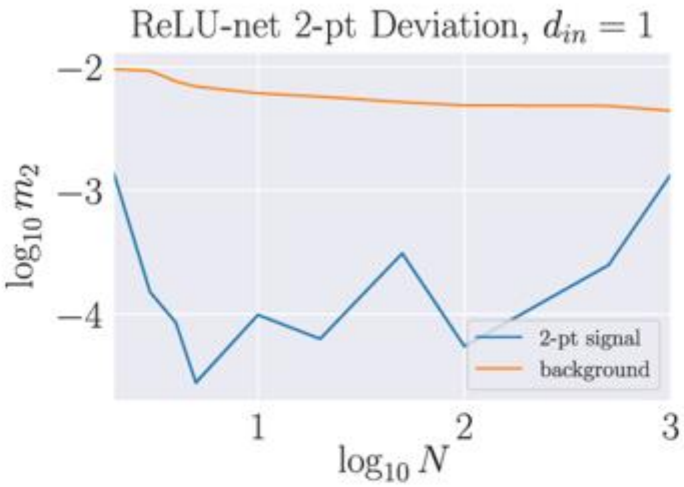
$$\Delta G^{(n)}(x_1, \ldots, x_n) = G^{(n)}(x_1, \ldots, x_n) - G_{GP}^{(n)}(x_1, \ldots, x_n)$$

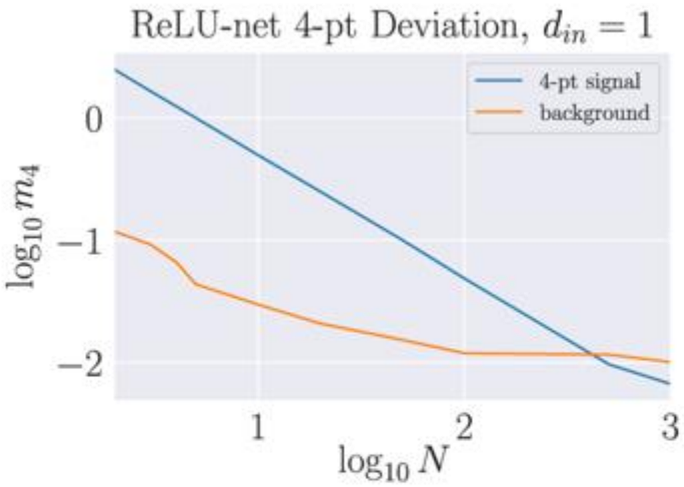
- Measure m_2, m_4, m_6 multiple times.
- Compute mean and std.
- Compare mean and std.

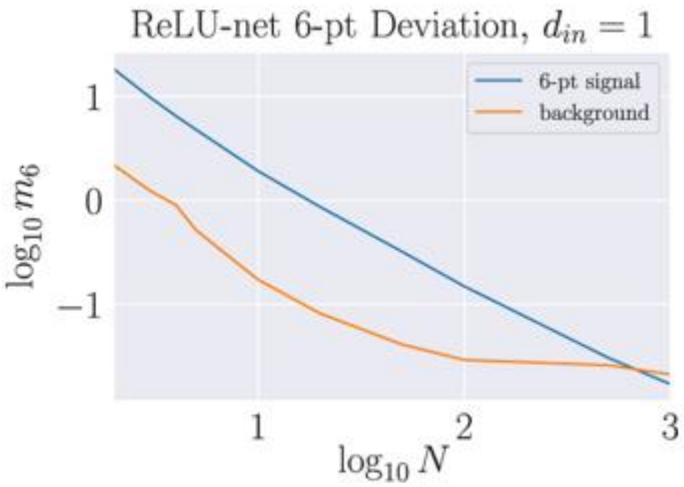
Falloff to GP Feynman diagrams at large width - Gauss-net

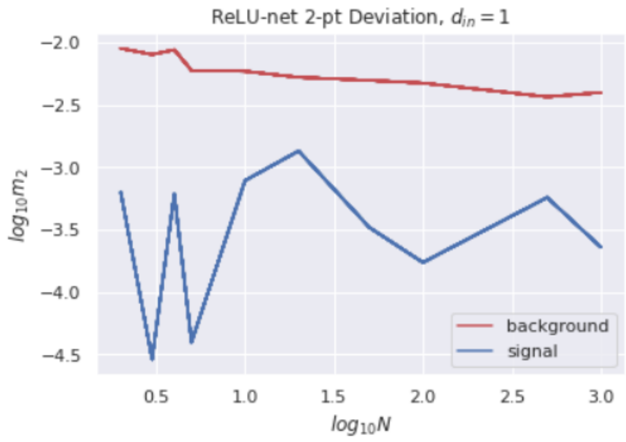


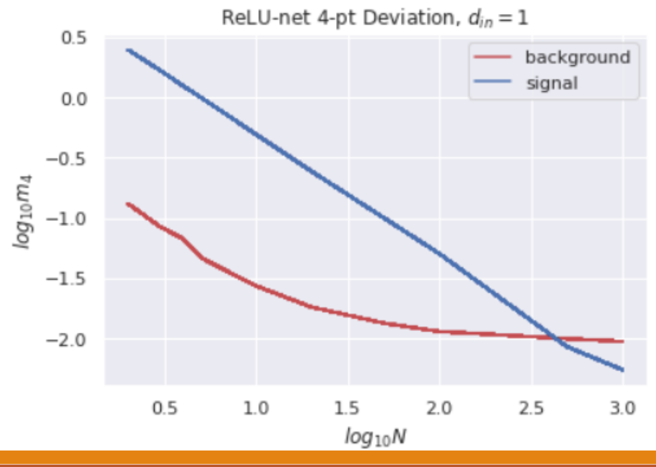
Falloff to GP Feynman diagrams at large width - ReLU-net

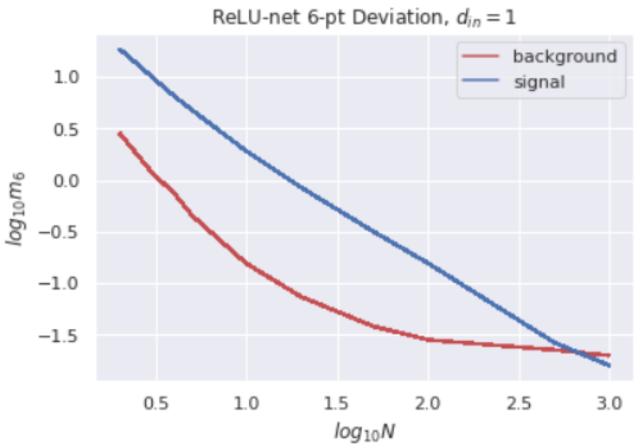




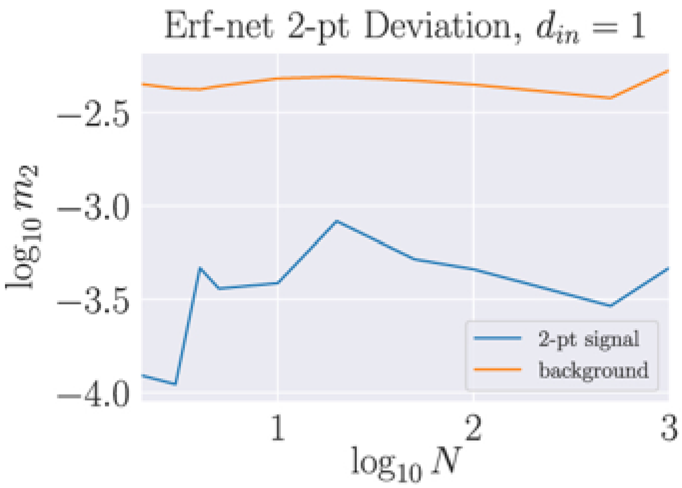


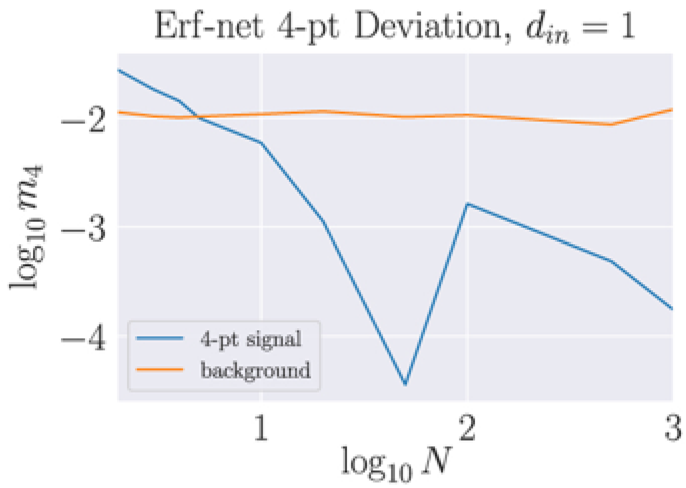


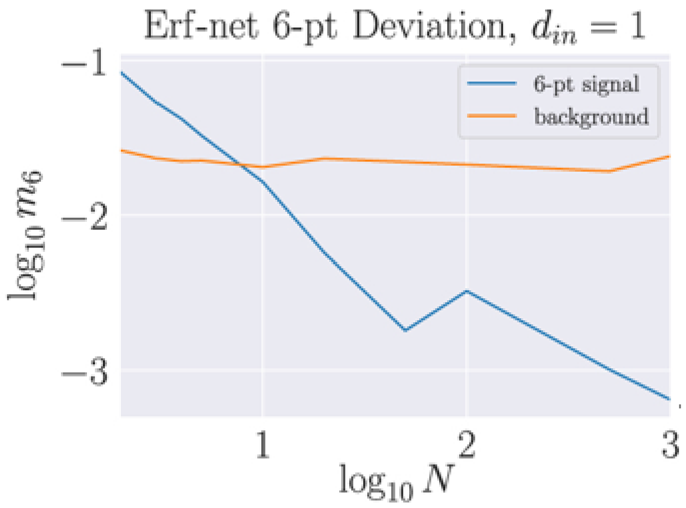


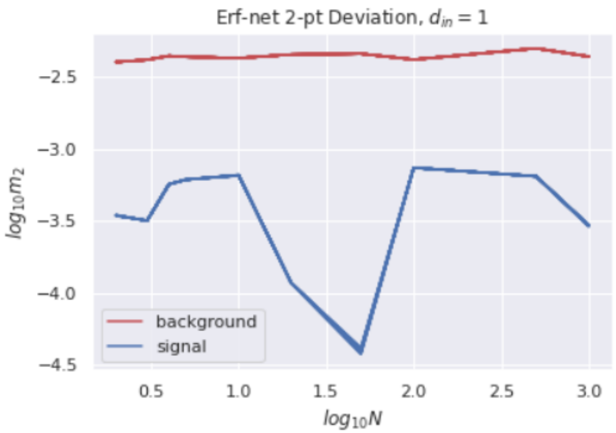


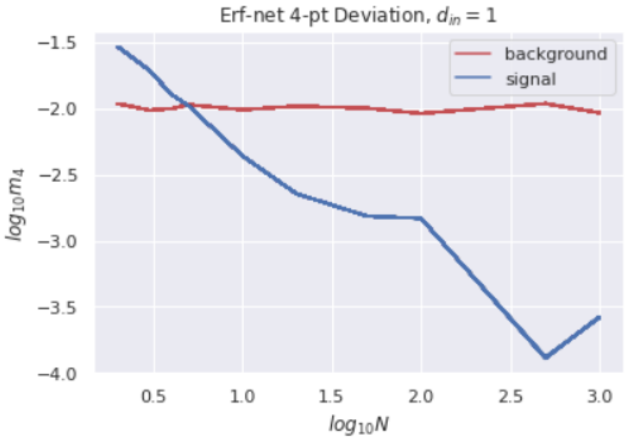
Falloff to GP Feynman diagrams at large width - Erf-net

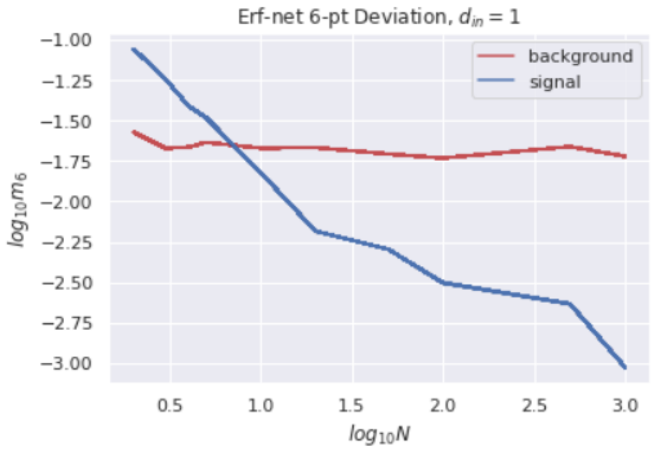












Scaling of the Non-GP part

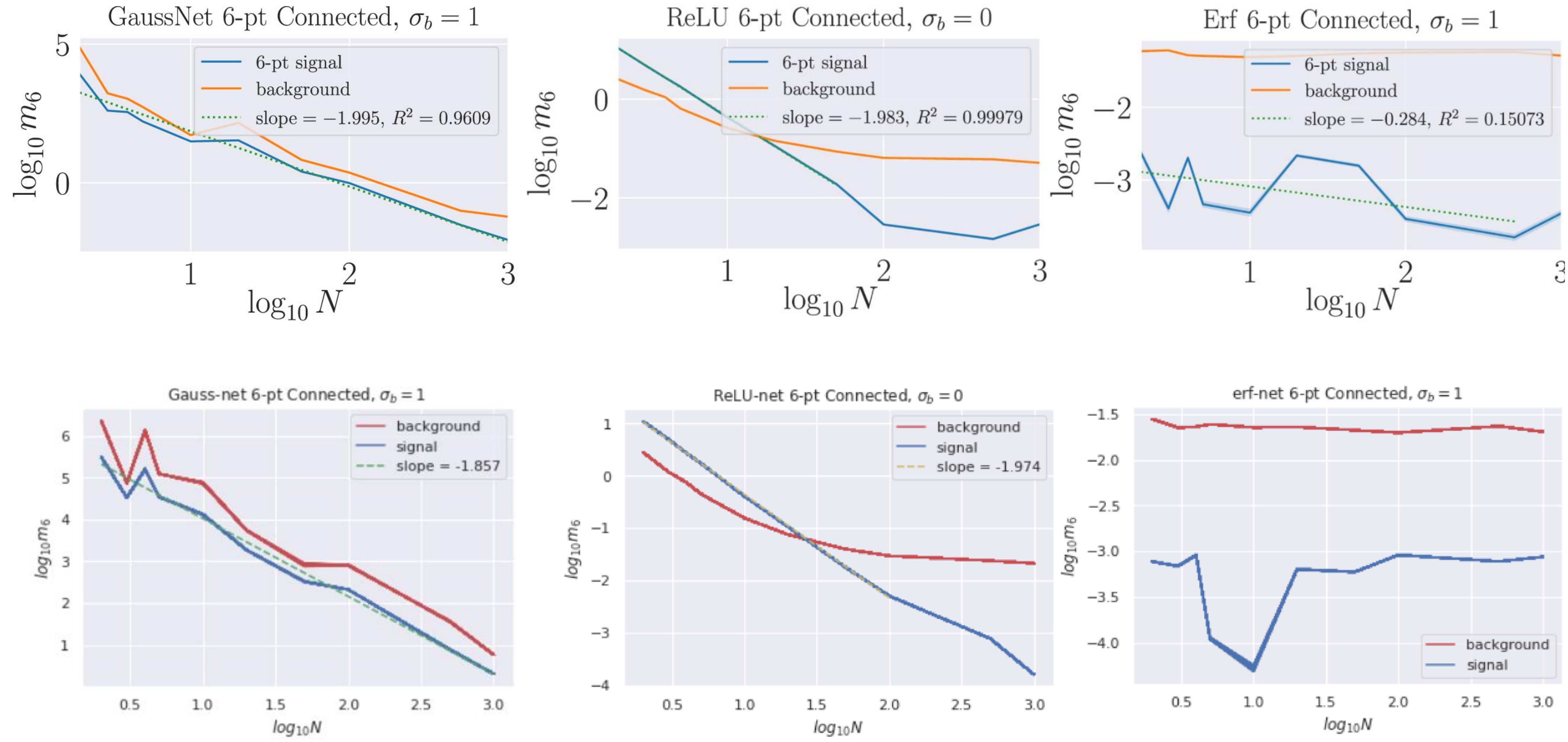
$$G^{(4)}(x_1,\ldots,x_4)|_{\text{connected}} = \Delta G^{(4)} \propto N^{-1}$$

$$G^{(6)}(x_1,\ldots,x_6)|_{\text{connected}} = \Delta G^{(6)} - \sum G^{(4)}(x_1,\ldots,x_4)|_{\text{connected}}\, G^{(2)}(x_5,x_6) \propto N^{-2}$$

Normalized:

$$\frac{G^{(4)}(x_1,\ldots,x_4)|_{\text{connected}}}{G^{(4)}(x_1,\ldots,x_4)} \propto N^{-1}$$

$$\frac{G^{(6)}(x_1,\ldots,x_6)|_{\text{connected}}}{G^{(6)}(x_1,\ldots,x_6)} \propto N^{-2}$$

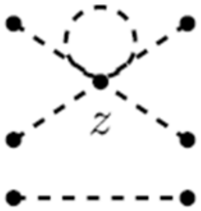


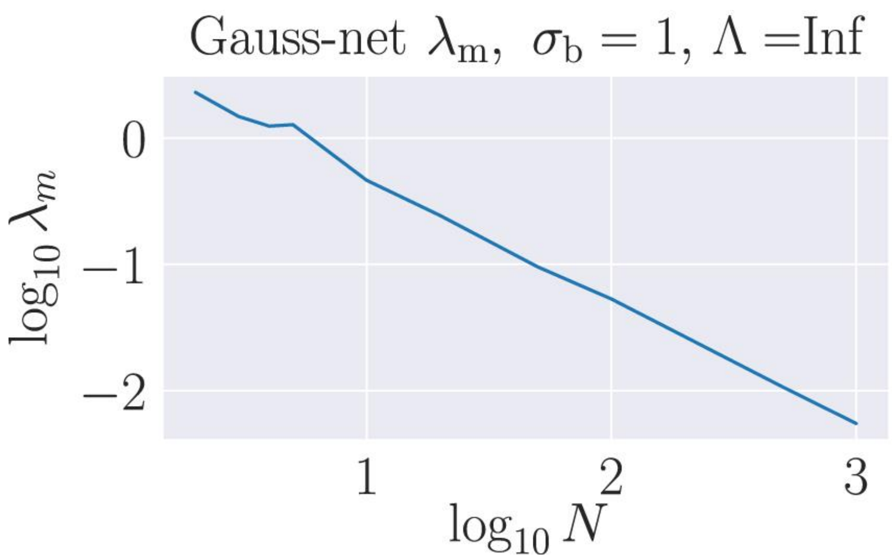
$$\delta G^{(6)}\Big|_{\text{connected}} = \sqrt{(\delta G^{(6)})^2 + (G^{(2)}\delta G^{(4)})^2}$$

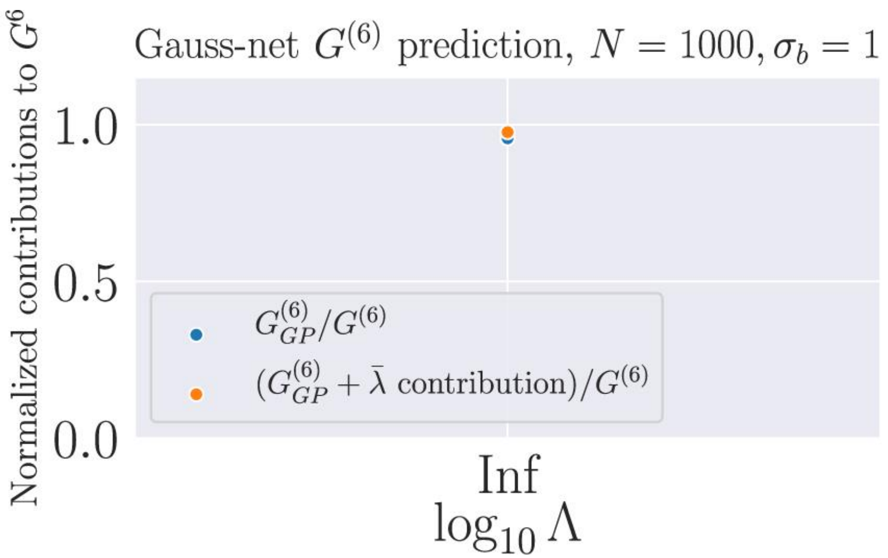
Coupling constants

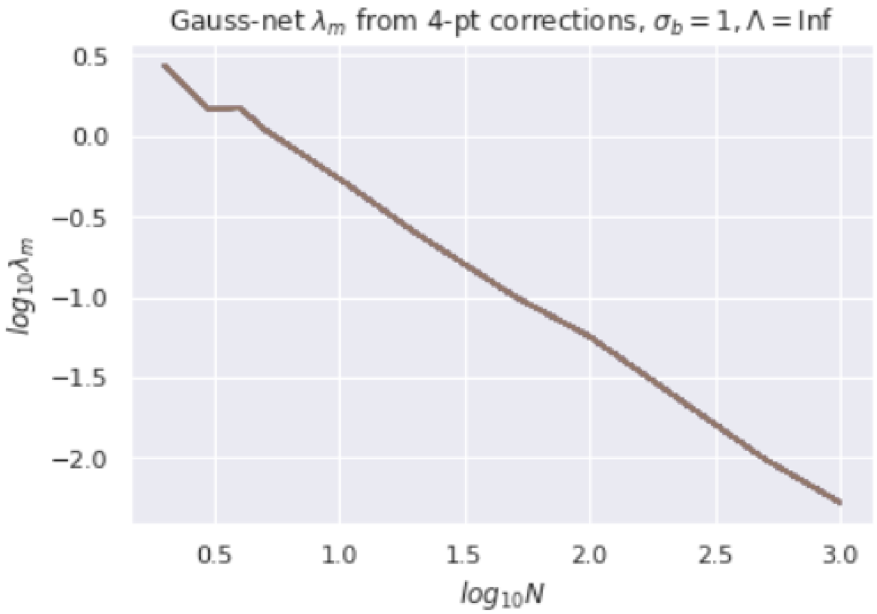
$$\Delta S = \int d^{d_{in}}x \left[\lambda f(x)^4 + \kappa f(x)^6\right]$$

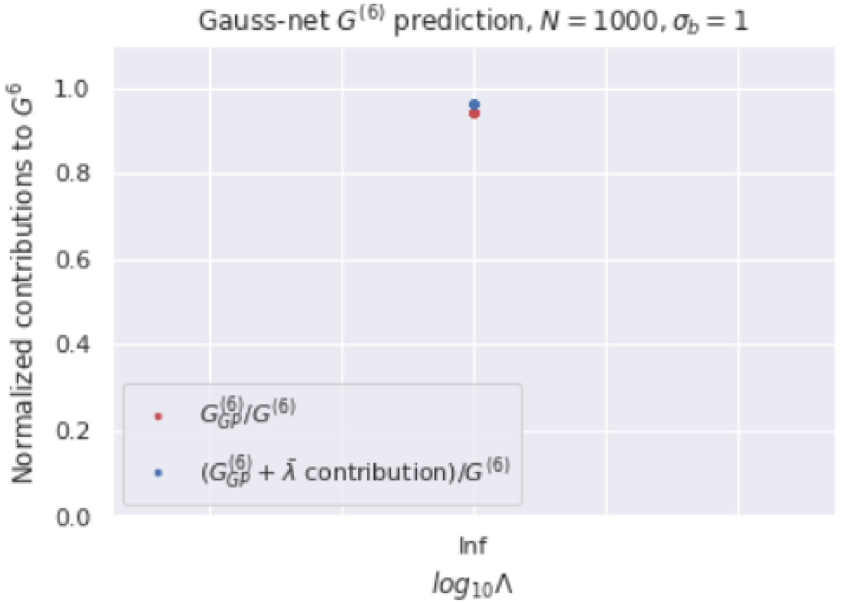
$G^{(4)}(x_1, x_2, x_3, x_4) \quad = \quad 3$  $- 24\,\lambda$  $- 360\,\kappa$ 

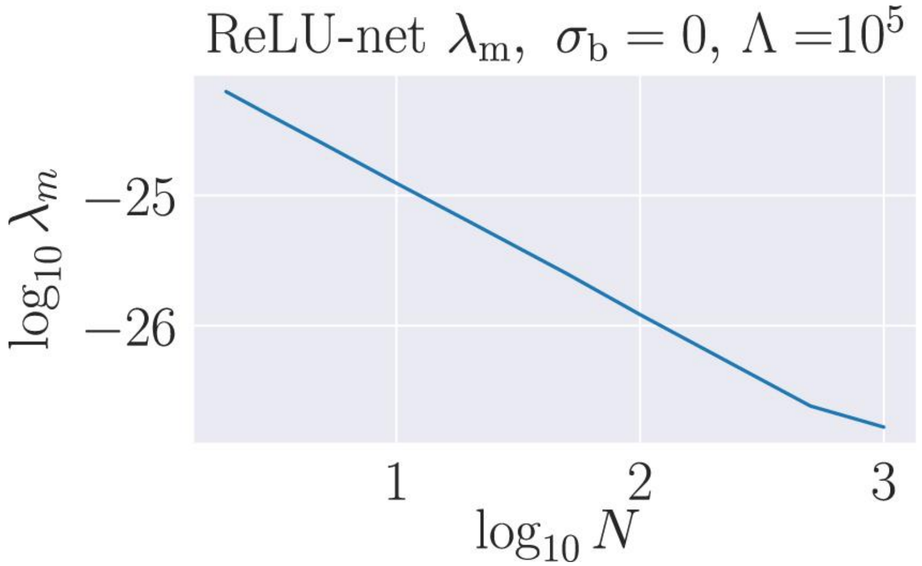
$G^{(6)}(x_1, x_2, x_3, x_4, x_5, x_6) \quad = 15$  $- \quad 360\,\lambda$  $- \kappa \Big[720$  $+ 5400$  $\Big]$

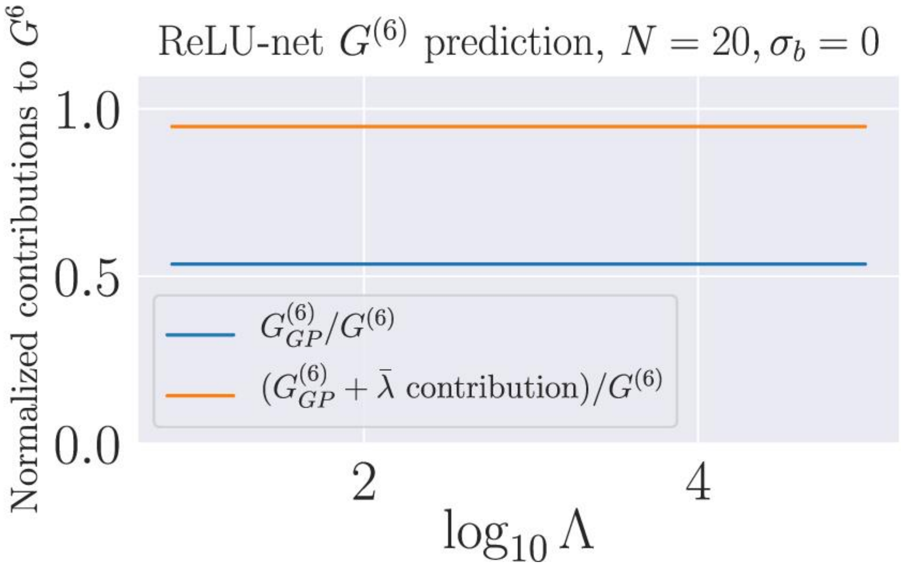

Gauss-net λ_{m}, $\sigma_{\mathrm{b}} = 1$, $\Lambda =$Inf
$\log_{10} \lambda_m$
0
-1
-2
1
2
3
$\log_{10} N$

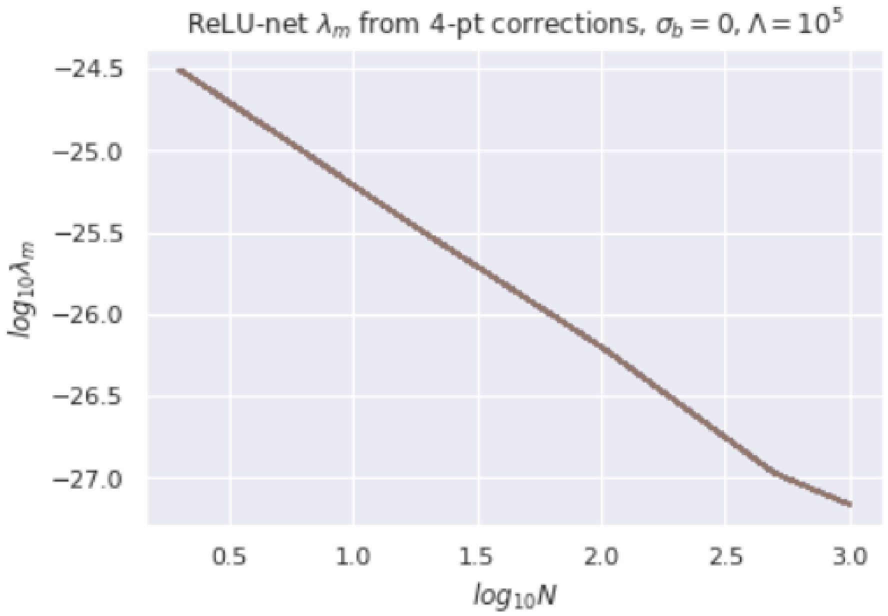

Gauss-net $G^{(6)}$ prediction, $N = 1000, \sigma_b = 1$
Normalized contributions to G^6
1.0
0.5
0.0
$G^{(6)}_{GP}/G^{(6)}$
$(G^{(6)}_{GP} + \bar{\lambda}$ contribution$)/G^{(6)}$
Inf
$\log_{10} \Lambda$

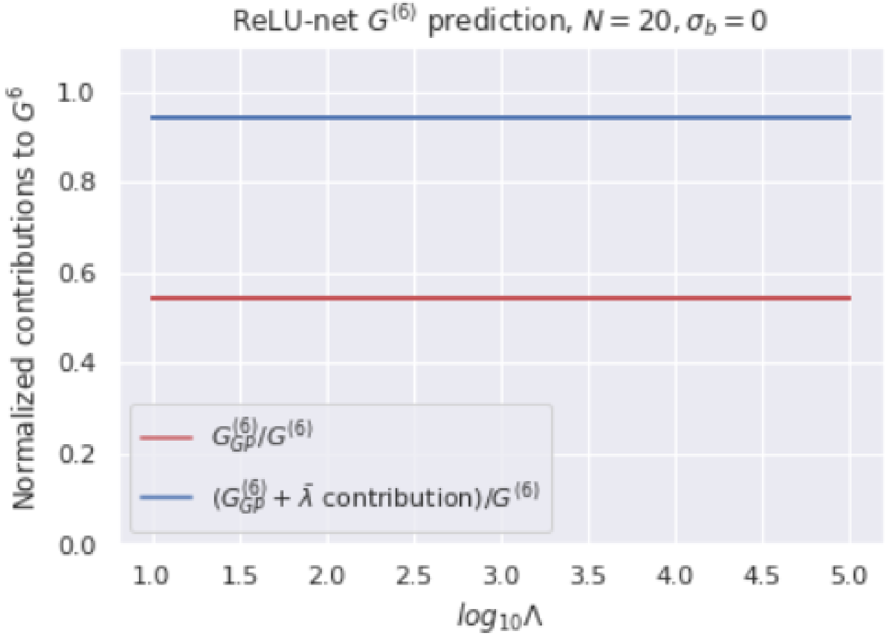

Gauss-net λ_m from 4-pt corrections, $\sigma_b = 1, \Lambda = \mathrm{Inf}$
$log_{10}\lambda_m$
0.5
0.0
-0.5
-1.0
-1.5
-2.0
0.5
1.0
1.5
2.0
2.5
3.0
$log_{10}N$

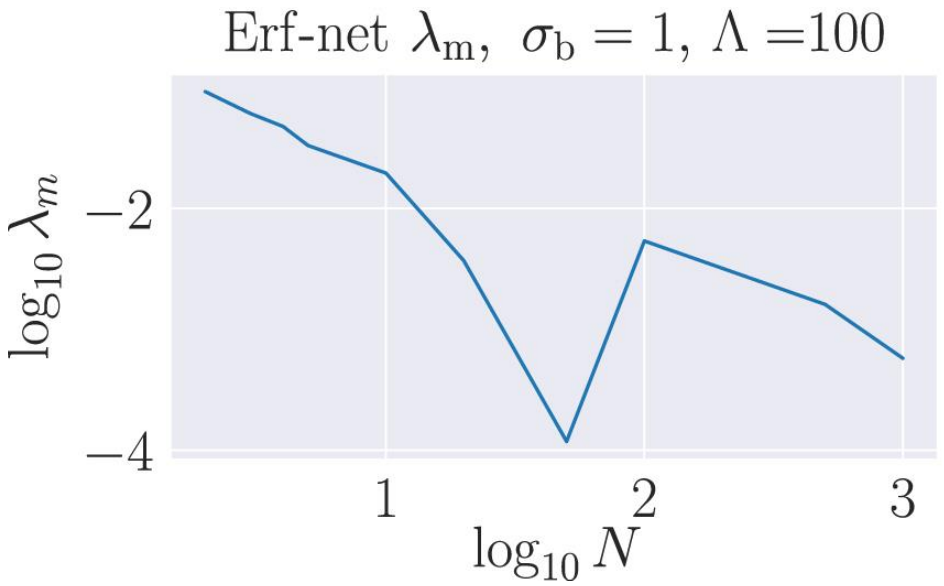

Gauss-net $G^{(6)}$ prediction, $N = 1000, \sigma_b = 1$
Normalized contributions to G^6
1.0
0.8
0.6
0.4
0.2
0.0
$G^{(6)}_{GP}/G^{(6)}$
$(G^{(6)}_{GP} + \bar{\lambda}$ contribution$)/G^{(6)}$
Inf
$log_{10}\Lambda$

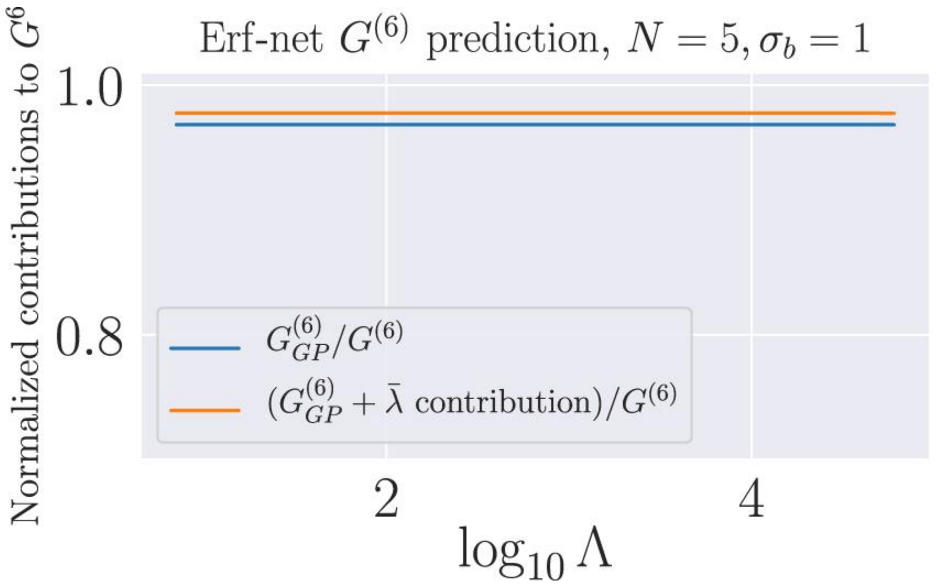

ReLU-net λ_{m}, $\sigma_{\mathrm{b}} = 0$, $\Lambda = 10^5$
$\log_{10}\lambda_m$
−25
−26
1
2
3
$\log_{10} N$

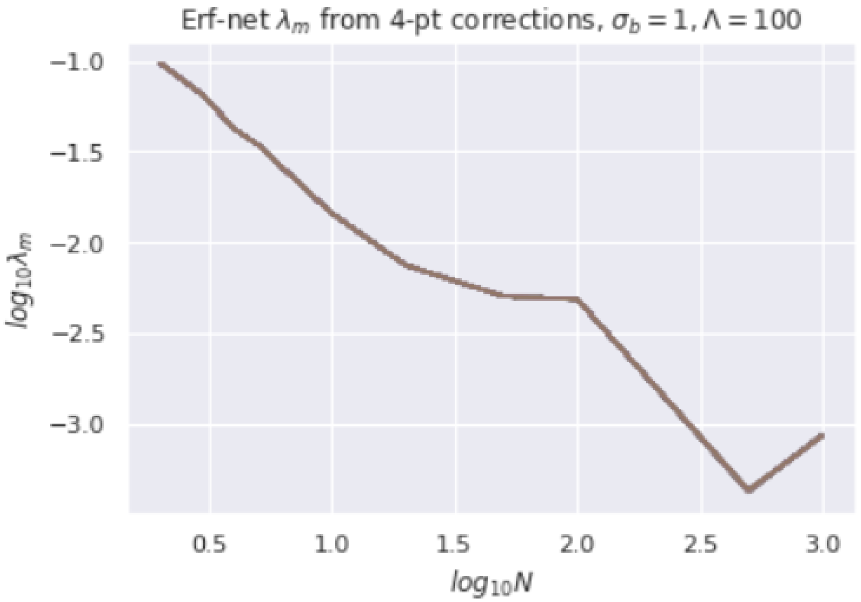

ReLU-net $G^{(6)}$ prediction, $N = 20, \sigma_b = 0$
Normalized contributions to G^6
1.0
0.5
0.0
2
4
$\log_{10}\Lambda$
$G^{(6)}_{GP}/G^{(6)}$
$(G^{(6)}_{GP} + \bar{\lambda}$ contribution$)/G^{(6)}$

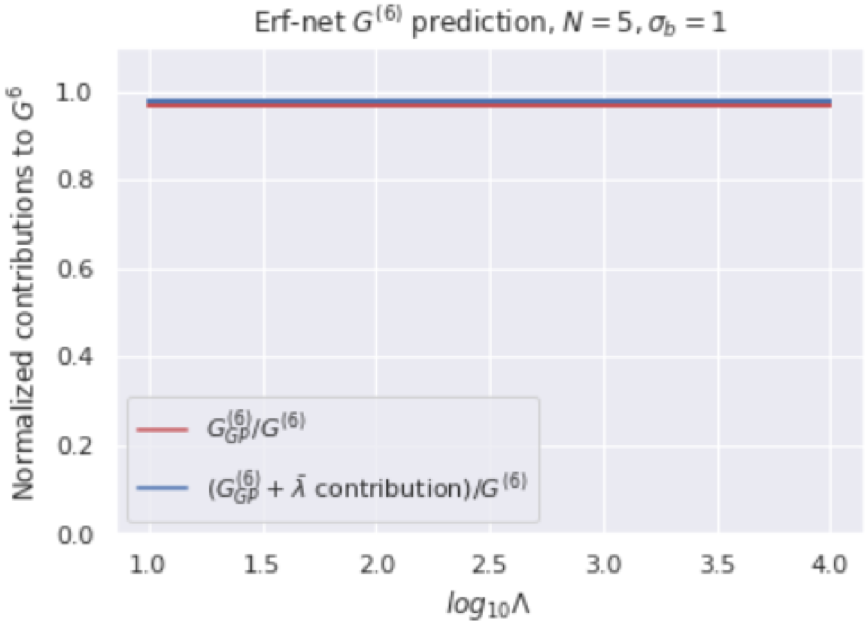

ReLU-net λ_m from 4-pt corrections, $\sigma_b = 0$, $\Lambda = 10^5$
$log_{10}\lambda_m$
−24.5
−25.0
−25.5
−26.0
−26.5
−27.0
0.5
1.0
1.5
2.0
2.5
3.0
$log_{10}N$


ReLU-net $G^{(6)}$ prediction, $N = 20, \sigma_b = 0$
Normalized contributions to G^6
1.0
0.8
0.6
0.4
0.2
0.0
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
$log_{10}\Lambda$
$G^{(6)}_{GP}/G^{(6)}$
$(G^{(6)}_{GP} + \bar{\lambda}$ contribution$)/G^{(6)}$


Erf-net $\lambda_{\rm m}$, $\sigma_{\rm b}=1$, $\Lambda=$100
$\log_{10}\lambda_m$
-2
-4
1
2
3
$\log_{10} N$


Erf-net $G^{(6)}$ prediction, $N=5, \sigma_b=1$
Normalized contributions to G^6
1.0
0.8
$G^{(6)}_{GP}/G^{(6)}$
$(G^{(6)}_{GP}+\bar{\lambda}$ contribution$)/G^{(6)}$
2
4
$\log_{10}\Lambda$


Erf-net λ_m from 4-pt corrections, $\sigma_b=1, \Lambda=100$
$log_{10}\lambda_m$
−1.0
−1.5
−2.0
−2.5
−3.0
0.5
1.0
1.5
2.0
2.5
3.0
$log_{10}N$

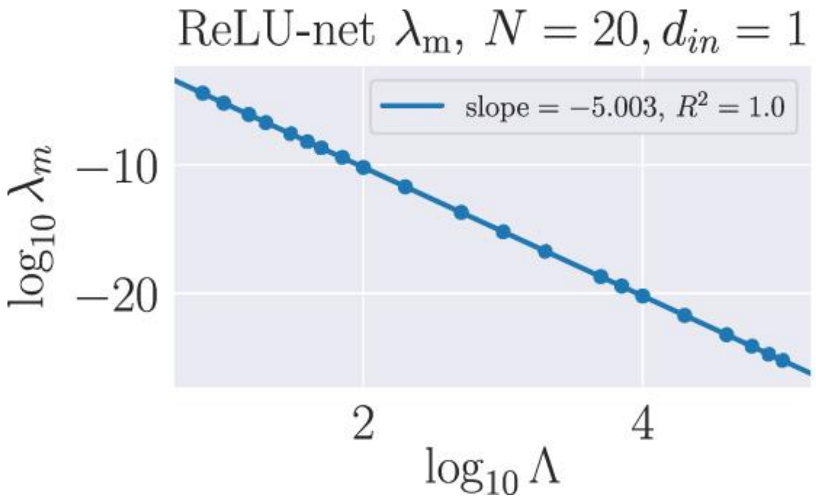

Erf-net $G^{(6)}$ prediction, $N=5, \sigma_b=1$
Normalized contributions to G^6
1.0
0.8
0.6
0.4
0.2
0.0
$G^{(6)}_{GP}/G^{(6)}$
$(G^{(6)}_{GP}+\bar{\lambda}$ contribution$)/G^{(6)}$
1.0
1.5
2.0
2.5
3.0
3.5
4.0
$log_{10}\Lambda$

Renormalization Group Equations

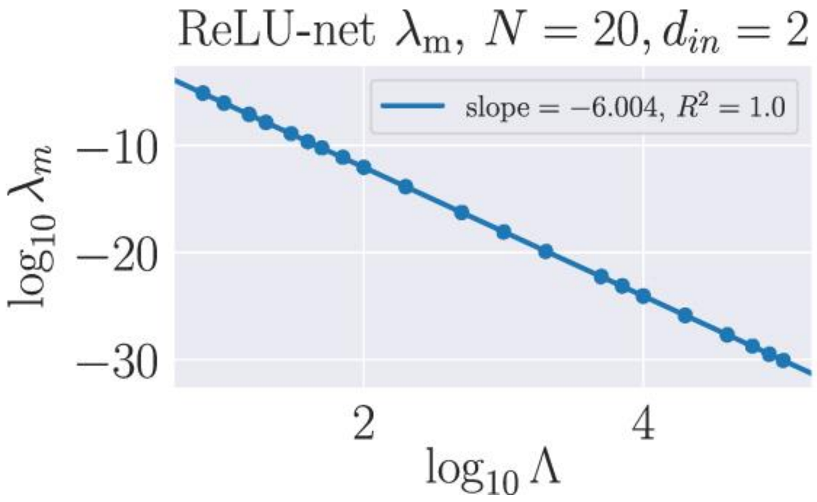
$$\Delta S = \int d^{d_{in}}x \left[\lambda f(x)^4 + \kappa f(x)^6\right]$$

$$\beta(\lambda) := \frac{\partial \lambda}{\partial \log(\Lambda)} = -(d_{\text{in}} + 4)\lambda \,.$$

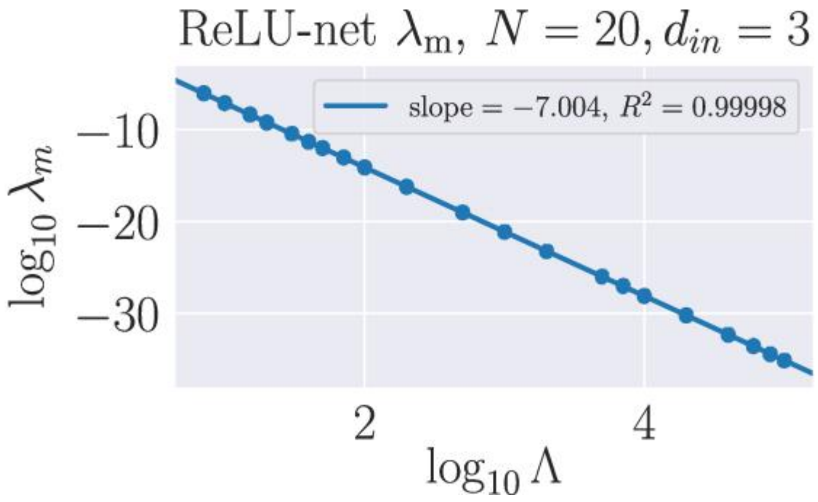
$$[\lambda] = -d_{in} - 4, \; [\kappa] = -d_{in} - 6$$



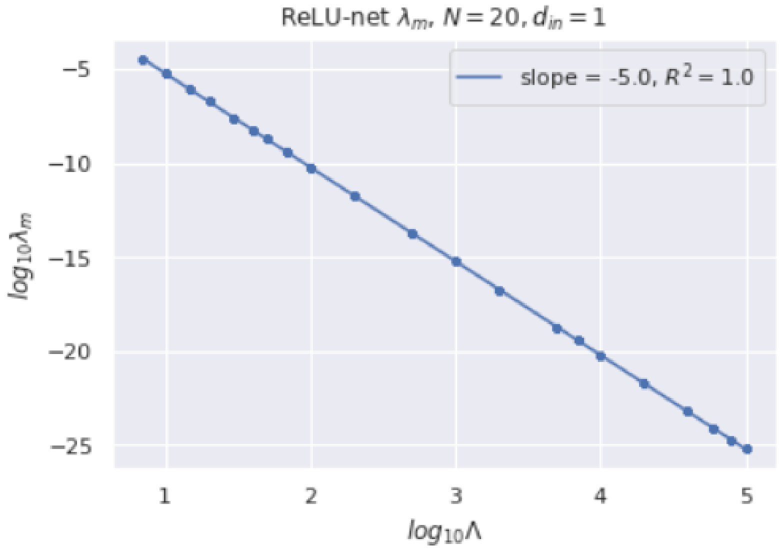
ReLU-net λ_{m}, $N = 20, d_{in} = 1$
slope = −5.003, $R^2 = 1.0$
$\log_{10} \lambda_m$
$\log_{10} \Lambda$



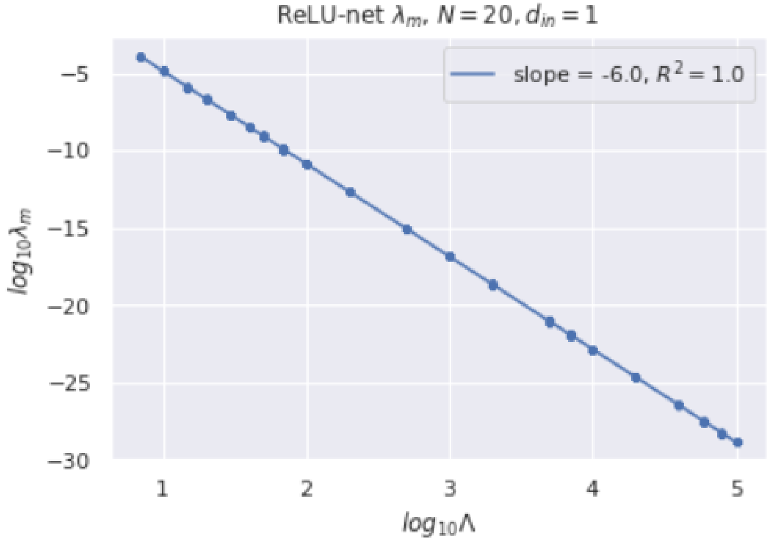
ReLU-net λ_{m}, $N = 20, d_{in} = 2$
slope = −6.004, $R^2 = 1.0$
$\log_{10} \lambda_m$
$\log_{10} \Lambda$



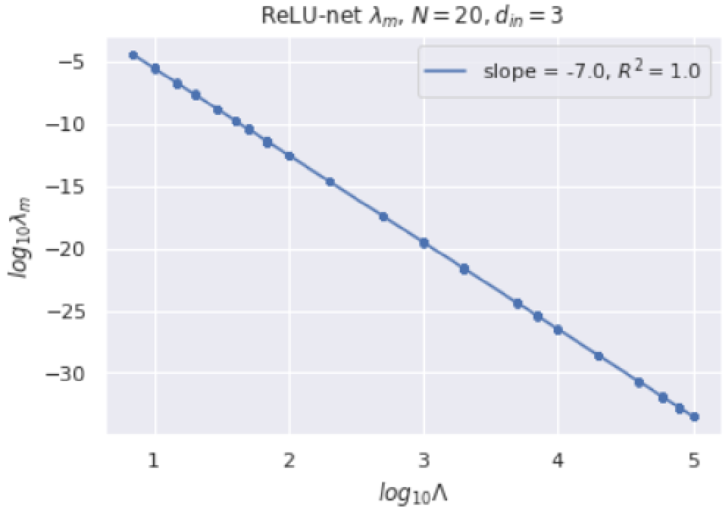
ReLU-net λ_{m}, $N = 20, d_{in} = 3$
slope = −7.004, $R^2 = 0.99998$
$\log_{10} \lambda_m$
$\log_{10} \Lambda$



ReLU-net λ_m, $N = 20, d_{in} = 1$
slope = -5.0, $R^2 = 1.0$
$log_{10}\lambda_m$
$log_{10}\Lambda$



ReLU-net λ_m, $N = 20, d_{in} = 1$
slope = -6.0, $R^2 = 1.0$
$log_{10}\lambda_m$
$log_{10}\Lambda$



ReLU-net λ_m, $N = 20, d_{in} = 3$
slope = -7.0, $R^2 = 1.0$
$log_{10}\lambda_m$
$log_{10}\Lambda$