Combinations of Multisets and Counting Integer Solutions

$$R = \{\infty \cdot U_1, \infty \cdot U_2, \cdots, \infty \cdot U_t\},$$

Definition 4.39. Let t and s be non-negative integers. Define the symbol $\left(\!\binom{t}{s}\!\right)$ (read "t multichoose s") to denote the number of s-submultisets of a multiset M with t types of elements and with infinite repetition numbers. In other words, $\left(\!\binom{t}{s}\!\right)$ is the number of ways to select s objects from a group of t objects, when order is irrelevant and repetition is allowed. Note that, either by an appropriate interpretation of the definition or by fiat, we have $\left(\!\binom{0}{0}\!\right) = 1$.

可以表为以下几种情形

① placing s identical balls into t distinct boxes

s balls. identical

② The number of non-negative integer solutions to

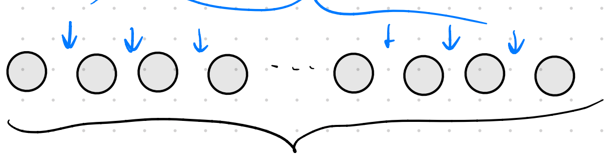
$$x_1 + x_2 + \cdots + x_t = s$$

t boxes. distinct

(挡板法) $\left(\!\binom{t}{s}\!\right) = \binom{t+s-1}{t-1} = \binom{t+s-1}{s-1}$

if you have an inexhaustible supply of t objects, then the number of ways of choosing s of them, with repetition allowed but with no regard to the order, is s+t−1 choose t

t-1个挡板插入s+t-1个空



s+t-1个球

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每种至少选一

可以表为以下几种情形

placing s identical balls into t distinct boxes each box gets at least one ball

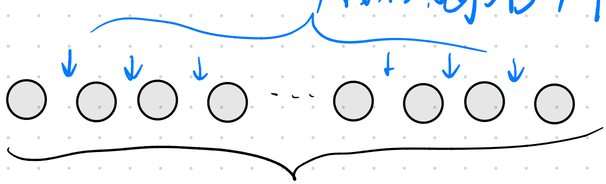
The number of non-negative integer solutions to $x_i \geq 1$

$$x_1 + x_2 + \cdots + x_t = s$$

(挡板法)

$$\left(\!\binom{t}{s-t}\!\right) = \binom{s-1}{s-t} = \binom{s-1}{t-1}.$$

t-1个挡板,插入s-1个空



s

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Example 4.47. I have 13 plants that I want to arrange on a shelf. (They will be arranged one next to another in a row). I have one each of Alstroemeria, Aquilegia, Arabis, Arctotis, and Aster as well as eight identical Violas. In how many ways can I do this if no two of the plants whose name starts with an A are next to each other?

$$\overset{x_1}{\underline{\quad\quad}} \quad A \quad \overset{x_2}{\underline{\quad\quad}} \quad A \quad \overset{x_3}{\underline{\quad\quad}} \quad A \quad \overset{x_4}{\underline{\quad\quad}} \quad A \quad \overset{x_5}{\underline{\quad\quad}} \quad A \quad \overset{x_6}{\underline{\quad\quad}}$$

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 8,$$

$y_i = x_i - 1$

$$x_1 + y_2 + \cdots + y_5 + x_6 = 4,$$

$$\left(\!\binom{6}{4}\!\right) = \binom{6+4-1}{6-1} = \binom{9}{5} = \binom{9}{4}$$

$$5!\binom{9}{4} = \frac{9!}{4!} = 15,120.$$