3. $E|X-\lambda|$

$= \sum_{k:k<\lambda}(\lambda-k)P(X=\lambda) + \sum_{k:k>\lambda}(k-\lambda)P(X=\lambda)$

$= 2\sum_{k:k<\lambda}(\lambda-k)P(X=\lambda) + E(X-\lambda)$

$= 2\sum_{k:k<\lambda}(\lambda-k)P(X=\lambda)$

$= 2\lambda\sum_{k:k<\lambda}\frac{e^{-\lambda}\lambda^k}{k!} - 2\sum_{k:k<\lambda}k\frac{e^{-\lambda}\lambda^k}{k!}$

$= 2\lambda e^{-\lambda}\left(\sum_{k:k<\lambda}\frac{\lambda^k}{k!} - \sum_{k:k<\lambda}\frac{\lambda^{k-1}}{(k-1)!}\right)$

$= 2\lambda e^{-\lambda}\frac{\lambda^{[\lambda]}}{[\lambda]!} = \frac{2e^{-\lambda}\lambda^{[\lambda]+1}}{[\lambda]!}$

5. 记试验成功的次数为Y

由Poisson过程的随机分流性.

$Y \sim P(p\lambda)$ $\therefore P(Y=k) = \frac{(\lambda p)^k e^{-\lambda p}}{k!}$ $k=0,1,2,\dots$

$EY = p\lambda$

6. 设雌昆虫数目为N，

每只昆虫下卵数目为X_i

总虫卵数目为Y，则 $Y=\sum_{i=1}^{N}X_i$

$P(Y=k) = \sum_{i=0}^{\infty}P(X=k|N=i)P(N=i)$

$= \sum_{i=0}^{\infty}\frac{e^{-\lambda}\lambda^i}{i!}\frac{e^{-\mu i}(\mu i)^k}{k!}$ $k=0,1,2,\dots$

7. $P(X=k) = \frac{e^{-\lambda_1}\lambda_1^k}{k!}p + \frac{e^{-\lambda_2}\lambda_2^k}{k!}(1-p)$ $k=0,1,2,3,\dots$

10. $\cancel{P(X=k) = \sum_{i=k}^{+\infty}\frac{e^{-\lambda}\lambda^i}{i!}\cdot C_i^k p^k(1-p)^{i-k} \quad k=0,1,2,\dots}$

$\cancel{P(Y=k) = \sum_{i=k}^{+\infty}\frac{e^{-\lambda}\lambda^i}{i!}C_i^k(1-p)^k p^{i-k} \quad k=0,1,2,\dots}$

10. (a) 由Poisson分布的随机分流性

$X\sim P(\lambda p)$ $P(X=k) = \frac{e^{-\lambda p}(\lambda p)^k}{k!}$ $k=0,1,2,\dots$

$Y\sim P(\lambda(1-p))$ $P(Y=k) = \frac{e^{-\lambda(1-p)}(\lambda(1-p))^k}{k!}$ $k=0,1,2,\dots$

(b) $P(X=x, Y=y)$

$= \sum_{k=0}^{\infty}P(X=x, Y=y|N=k)P(N=k)$

$= P(X=x, Y=y|N=x+y)P(N=x+y)$

$= C_{x+y}^{x}p^x(1-p)^y\cdot\frac{e^{-\lambda}\lambda^{x+y}}{(x+y)!}$

$= \frac{(x+y)!}{x!\,y!}p^x(1-p)^y\cdot\frac{e^{-\lambda}\lambda^{x+y}}{(x+y)!}$

$= \frac{e^{-p\lambda}(p\lambda)^x}{x!}\frac{e^{-(1-p)\lambda}((1-p)\lambda)^y}{y!}$

$= P(X=x)P(Y=y)$

$\therefore X$, Y相互独立

12. $P(M=k) = P(X_1=k, X_2<k) + P(X_1<k, X_2=k) + P(X_1=k, X_2=k)$

$= 2\frac{e^{-\lambda}\lambda^k}{k!}\sum_{j=0}^{k-1}\frac{e^{-\lambda}\lambda^j}{j!} + \frac{e^{-2\lambda}\lambda^{2k}}{(k!)^2}$ $k=0,1,2,\dots$

$P(N=k) = P(X_1>k, X_2=k) + P(X_1=k, X_2>k) + P(X_1=k, X_2=k)$

$= 2\frac{e^{-\lambda}\lambda^k}{k!}\sum_{j=k+1}^{+\infty}\frac{e^{-\lambda}\lambda^j}{j!} + \frac{e^{-2\lambda}\lambda^{2k}}{(k!)^2}$ $k=0,1,2,\dots$

13. $Cov(N_t, N_s)$

不妨设 $t \le s$

则 $Cov(N_t, N_s)$

$= Cov(N_t, N_t + N_s - N_t)$

$= Cov(N_t, N_t) + Cov(N_t, N_s - N_t)$

$= Cov(N_t, N_t) = DN_t$

$= \lambda t$

$\therefore Cov(N_t, N_s) = \lambda(t \wedge s)$

14. $X \sim P(10800)$

设消费 i 元的人有 X_i 个.

由 Poisson 过程的随机分流性:

$X_i \sim P(10800 \times C_{10}^{i}(0.15)^i(0.85)^{10-i})$ 且相互独立

设营业额为 Y.

则 $Y = \sum_{i=0}^{10} i X_i$

$EY = \sum_{i=0}^{10} i EX_i$

$= \sum_{i=0}^{10} i \times 10800 \times C_{10}^{i}(0.15)^i(0.85)^{10-i}$

$= 10800 \times E\eta_i = 10800 \times 1.5 = 16200$

$DY = \sum_{i=0}^{10} i^2 DX_i$

$= \sum_{i=0}^{10} i^2 \times 10800 \times C_{10}^{i}(0.15)^i(0.85)^{10-i}$

$= 10800 \times E\eta_i^2$

$E\eta_i^2 = D\eta_i + (E\eta_i)^2$

$= 10 \times 0.15 \times 0.85 + (10 \times 0.15)^2$

$= 3.525$

$\therefore DY = 38070$

$\therefore \sqrt{DY} \approx 195.12$

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2. ① $f_\theta(x) \ge 0 \quad \forall x \in R.$

② $\int_{-\infty}^{+\infty} f_\theta(x)\,dx$

$= \int_0^{+\infty} f_\theta(x)\,dx$

$= \int_0^{+\infty} \theta^2 x e^{-\theta x}\,dx$

$= \theta^2 \int_0^{+\infty} x e^{-\theta x}\,dx$

$\cancel{= \frac{1}{2}\theta^2 \int_0^{+\infty} e^{-\theta x}\,dx^2}$

$= -\theta \int_0^{+\infty} x e^{-\theta x}\,d(-\theta x)$

$= -\theta \int_0^{+\infty} x\,d(e^{-\theta x})$

$= -\theta\left(x e^{-\theta x}\Big|_0^{+\infty} - \int_0^{+\infty} e^{-\theta x}\,dx\right)$

$= \theta \int_0^{+\infty} e^{-\theta x}\,dx = -e^{-\theta x}\Big|_0^{+\infty} = 1$

由①②定义了一个密度函数

当 $x<0$ 时，$F(x)=\int_{-\infty}^{x} f_\theta(x)dx=0$

当 $x\geq 0$ 时，$F(x)=\int_{-\infty}^{x} f_\theta(x)dx$

$=\int_0^x f_\theta(x)dx$

$=-\theta\left(xe^{-\theta x}\Big|_0^x-\int_0^x e^{-\theta x}dx\right)$

$=-\theta\left(xe^{-\theta x}+\frac{1}{\theta}e^{-\theta x}\Big|_0^x\right)$

$=-\theta xe^{-\theta x}-e^{-\theta x}+1$

$\therefore F(x)=\begin{cases}-\theta xe^{-\theta x}-e^{-\theta x}+1 & x\geq 0\\ 0 & x<0\end{cases}$

$P(X\geq 1)=1-F(1)$

$=\theta e^{-\theta}+e^{-\theta}=(\theta+1)e^{-\theta}$

$F(-a)=$

5. (a) $\int_{-\infty}^{-a} f(x)dx=\int_{\infty}^{a} f(-x)d(-x)$

$=-\int_{\infty}^{a} f(-x)dx=\int_a^{\infty} f(x)dx$

$=1-\int_{-\infty}^{a} f(x)dx=1-F(a)$

$\therefore F(a)+F(-a)=1$

$\because F(a)-F(-a)=\int_{-a}^{a} f(x)dx=2\int_0^a f(x)dx$

$\therefore 2F(-a)=1-2\int_0^a f(x)dx$

$\therefore F(-a)=\frac{1}{2}-\int_0^a f(x)dx$

$\therefore F(-a)=1-F(a)=\frac{1}{2}-\int_0^a f(x)dx$

(b) $P(|X|<a)=P(-a<X<a)$

$=\int_{-a}^{a} f(x)dx$

$=\int_{-\infty}^{a} f(x)dx-\int_{-\infty}^{-a} f(x)dx$

$=F(a)-F(-a)$

由(a) $F(-a)=1-F(a)$

$\therefore P(|X|<a)=F(a)-F(-a)$

$=F(a)-1+F(a)=2F(a)-1$

(c) 由(b)

$P(|X|>a)=1-P(|X|\leq a)$

$=1-P(|X|<a)$

$=1-(2F(a)-1)$

$=2(1-F(a))$