清华大学 TSINGHUA UNIVERSITY 数学作业纸

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7. X可能的取值为0,1,2,3

Y可能的取值为1,2,3

(X, Y)的联合分布列和及

边缘分布列为：

| X \ Y | 1 | 2 | 3 | $p_i^X$ |
|---|---|---|---|---|
| 0 | $\frac{2}{27}$ | $\frac{2}{9}$ | 0 | $\frac{8}{27}$ |
| 1 | 0 | $\frac{2}{9}$ | $\frac{2}{9}$ | $\frac{4}{9}$ |
| 2 | 0 | $\frac{2}{9}$ | 0 | $\frac{2}{9}$ |
| 3 | $\frac{1}{27}$ | 0 | 0 | $\frac{1}{27}$ |
| $p_j^Y$ | $\frac{1}{9}$ | $\frac{2}{3}$ | $\frac{2}{9}$ | 1 |

其中 $P(X=0, Y=1) = \frac{2}{3^3} = \frac{2}{27}$

$P(X=0, Y=2) = \frac{A_2^2 C_3^1}{3^3} = \frac{6}{27} = \frac{2}{9}$

$P(X=1, Y=2) = \frac{C_3^1 C_2^1}{3^3} = \frac{6}{27} = \frac{2}{9}$

$P(X=1, Y=3) = \frac{A_3^3}{3^3} = \frac{6}{27} = \frac{2}{9}$

$P(X=2, Y=2) = \frac{C_3^2 C_2^1}{3^3} = \frac{6}{27} = \frac{2}{9}$

$P(X=3, Y=1) = \frac{1}{3^3} = \frac{1}{27}$

9. ①设对于每一次试验，卡片上的号码为Y，则Y可取 $1,2,\dots,n$. $P(Y=i)=\frac{1}{n}$, $i=1,2,\dots,n$

$EY = \sum_{i=1}^{n} \frac{i}{n} = \frac{n+1}{2}$

$EX = kEY = \frac{k(n+1)}{2}$ （定理2.2）

②如果不放回，每一次试验的期望仍为 $\frac{n+1}{2}$

这是因为抽签问题每一次概率相等（习题课已证）

故由定理2.2，$EX = kEY = \frac{k(n+1)}{2}$ 不变

12. 每次试验3颗骰子点数和为奇数的概率 $p=\frac{1}{2}$

试验需进行次数设为X，则

$X \sim Ge(p)$

$\therefore EX = \frac{1}{p} = 2$

16. $\because X \sim B(n,p)$

$\therefore P(X=k) = C_n^k p^k (1-p)^{n-k}$

$k = 0,1,2,\dots,n$

$\therefore P(Y=k) = (-1)^k C_n^k p^k (1-p)^{n-k}$

$EY = \sum_{k=0}^{n} (-1)^k C_n^k p^k (1-p)^{n-k}$

$= \sum_{k=0}^{n} C_n^k (-p)^k (1-p)^{n-k}$

$= (-p+1-p)^n$

$= (1-2p)^n$

$\therefore EY = (1-2p)^n$

19. (a) $EX = \sum_{n=1}^{\infty} nP(X=n)$

$$\begin{aligned} = & \boxed{P(X=1)} \\ + & P(X=2) + \boxed{P(X=2)} \\ + & P(X=3) + P(X=3) + \boxed{P(X=3)} \\ + & \cdots \end{aligned}$$

$= P(X=1) + P(X=2) + P(X=3) + \cdots$

$+P(X=2)+P(X=3)+\cdots$

$+P(X=3)+\cdots$

$+\cdots$

$=\sum_{n=1}^{\infty}P(X\geq n)$

20. X的取值为 2, 3, 4, ⋯

$P(X\geq 2k+1)=2p^k q^k \quad k=1,2,3,\cdots$

$P(X\geq 2k)=p^{k-1}q^{k-1} \quad k=1,2,3,\cdots$

$\therefore EX=\sum_{n=1}^{\infty}P(X\geq n)$

$=\sum_{k=1}^{\infty}P(X\geq 2k+1)+\sum_{k=1}^{\infty}P(X\geq 2k)+P(X\geq 1)$

$=2\times\frac{pq}{1-pq}+\frac{1}{1-pq}+1$

$=\frac{1+2pq}{1-pq}+1$

$=\frac{2+pq}{1-pq}$

22. $f_{X_1}(x_1)=\begin{cases}\frac{1}{2} & x_1=1\\ \frac{1}{2} & x_1=0\\ 0 & \text{其他}\end{cases}$

$f_{X_2}(x_2)=\begin{cases}\frac{1}{2} & x_2=1\\ \frac{1}{2} & x_2=0\\ 0 & \text{其他}\end{cases}$

$f_{X_3}(x_3)=\begin{cases}\frac{1}{2} & x_3=1\\ \frac{1}{2} & x_3=0\\ 0 & \text{其他}\end{cases}$

设 $B=\{(0,0),(0,1),(1,0),(1,1)\}$

$f_{X_1X_2}(x_1,x_2)=\begin{cases}\frac{1}{4} & (x_1,x_2)\in B\\ 0 & \text{否则}\end{cases}$

$f_{X_1X_3}(x_1,x_3)=\begin{cases}\frac{1}{4} & (x_1,x_3)\in B\\ 0 & \text{否则}\end{cases}$

$f_{X_2X_3}(x_2,x_3)=\begin{cases}\frac{1}{4} & (x_2,x_3)\in B\\ 0 & \text{否则}\end{cases}$

可以验证

$\forall x_1,x_2,x_3$ 有

$f_{X_1X_2}(x_1,x_2)=f_{X_1}(x_1)f_{X_2}(x_2)$

$f_{X_1X_3}(x_1,x_3)=f_{X_1}(x_1)f_{X_3}(x_3)$

$f_{X_2X_3}(x_2,x_3)=f_{X_2}(x_2)f_{X_3}(x_3)$

$\therefore X_1,X_2,X_3$ 两两独立

但对于 $(x_1,x_2,x_3)=(1,1,1)$

$f(x_1,x_2,x_3)=\frac{1}{4}\neq\frac{1}{8}=\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}$

$=f_{X_1}(x_1)f_{X_2}(x_2)f_{X_3}(x_3)$

$\therefore X_1,X_2,X_3$ 不相互独立

补充题：$P(X=k)=P(X\leq k)-P(X\leq k-1)$

$=\frac{C_k^n-C_{k-1}^n}{C_N^n}=\frac{C_{k-1}^{n-1}}{C_N^n}$

$EX=\sum_{k=n}^{N}k\frac{C_{k-1}^{n-1}}{C_N^n}=\frac{n}{C_N^n}\sum_{k=n}^{N}C_k^n=\frac{n}{C_N^n}\left[\sum_{k=n+1}^{N}\left(C_{k+1}^{n+1}-C_k^{n+1}\right)+1\right]=\frac{nC_{N+1}^{n+1}}{C_N^n}=\frac{n(N+1)}{n+1}$