(5.4)

6. $\alpha\le 0$ 的情况无意义，

下面只考虑 $\alpha>0$ 的情况。

$x_1\cdots x_n$ 服 $N(\mu,1)$

则 $\bar{X}\sim N(\mu,\frac{1}{n})$

$\forall\mu\ge 0,\ P(|\bar{X}|<c)\le\alpha$

$\Leftrightarrow \forall\mu\ge 0,\ P(|\bar{X}|<c)_{max}\le\alpha.$

若 $c\le 0$，则恒成立.

若 $c>0$，则令 $\varphi(\mu)\triangleq$

$P(|\bar{X}|<c)=P(-c<\bar{X}<c)$

$=P\left(\frac{-c-\mu}{\sqrt{\frac{1}{n}}}<\frac{\bar{X}-\mu}{\sqrt{\frac{1}{n}}}<\frac{c-\mu}{\sqrt{\frac{1}{n}}}\right)$

$=\Phi\left(\frac{c-\mu}{\sqrt{\frac{1}{n}}}\right)-\Phi\left(\frac{-c-\mu}{\sqrt{\frac{1}{n}}}\right)$

$\varphi'(\mu)=\sqrt{n}f\left(\frac{-c-\mu}{\sqrt{\frac{1}{n}}}\right)-\sqrt{n}f\left(\frac{c-\mu}{\sqrt{\frac{1}{n}}}\right)$

$=\sqrt{n}f\left(\frac{c+\mu}{\sqrt{\frac{1}{n}}}\right)-nf\left(\frac{c-\mu}{\sqrt{\frac{1}{n}}}\right)<0$

$\therefore\ \mu>0$ 时，$\varphi(\mu)$ 单调递减.

$\therefore\ \varphi(\mu)_{max}=\varphi(0)$

$\therefore$ 只需找到 $\mu=0$ 时的最小 c 即可.

$\mu=0$ 时，$P(|\bar{X}|<c)=\Phi(\sqrt{n}c)-\Phi(-\sqrt{n}c)$

$=2\Phi(\sqrt{n}c)-1=\alpha$

$\therefore\ \Phi(\sqrt{n}c)=\frac{\alpha+1}{2}$

$\therefore\ C_{min}=\Phi^{-1}\left(\frac{\alpha+1}{2}\right)/\sqrt{n}$.

10. 令 $Y_1=\frac{X_1+X_2}{\sqrt{2}}\quad Y_2=\frac{X_1-X_2}{\sqrt{2}}$

$\therefore\ Y_1\sim N(0,1)$

$Y_2\sim N(0,1)$.

$\because\ Cov(Y_1,Y_2)=Cov\left(\frac{X_1}{\sqrt{2}},\frac{X_1}{\sqrt{2}}\right)-Cov\left(\frac{X_2}{\sqrt{2}},\frac{X_2}{\sqrt{2}}\right)$

$=0$

$\therefore\ Y_1,Y_2$ 独立. $\quad\therefore\ \frac{Y_2^2}{Y_1^2}\sim F(1,1)$

$P\left(\frac{(X_1+X_2)^2}{(X_1-X_2)^2+(X_1+X_2)^2}>k\right)$

$=P\left(1+\frac{(X_1-X_2)^2}{(X_1+X_2)^2}<\frac{1}{k}\right)$

$=P\left(\frac{Y_2^2}{Y_1^2}<\frac{1}{k}-1\right)=0.05$

查表可得 $F_{0.95}(1,1)=161.45$.

$\therefore\ F_{0.05}(1,1)=\frac{1}{161.45}$

$\therefore\ \frac{1}{k}-1=\frac{1}{161.45}\Rightarrow k\doteq 0.99384$

12. $x_{n+1}\sim N(\mu,\sigma^2)$

$\bar{x}_n\sim N(\mu,\frac{\sigma^2}{n})$

$x_{n+1}-\bar{x}_n\sim N(0,\frac{n+1}{n}\sigma^2)$

$A\triangleq\frac{x_{n+1}-\bar{x}_n}{\sqrt{\frac{n+1}{n}}\sigma}\sim N(0,1)$

$B\triangleq\frac{(n-1)S_n^2}{\sigma^2}\sim\chi^2(n-1)$

$\because S_n^2$ 与 $\bar{x}_n$ 相独立，S_n^2 与 x_{n+1} 相独立

$\therefore A$ 与 B 相独立

$$\therefore t=\frac{A}{B}=\frac{\frac{x_{n+1}-\bar{x}_n}{\sqrt{\frac{n+1}{n}}\sigma}}{\sqrt{\frac{\frac{(n-1)S_n^2}{\sigma^2}}{n-1}}}=\sqrt{\frac{n}{n+1}}\frac{x_{n+1}-\bar{x}_n}{S_n}\sim t(n-1)$$

$\therefore c=\sqrt{\frac{n}{n+1}}$，t分布自由度为 $n-1$

(55)

$$\because P(X_1=x_1,X_2=x_2,\cdots,X_n=x_n|T=t)$$

$$=\frac{P(X_1=x_1,X_2=x_2,\cdots,X_n=t-\sum_{i=1}^{n-1}x_i)}{P(T=t)}$$

$$=\frac{P(X_1=x_1)P(X_2=x_2)\cdots P(X_n=t-\sum_{i=1}^{n-1}x_i)}{P(T=t)}$$

令 $Y_i=X_i+1$，则 $Y_i\sim Ge(\theta)$ $i=1,2,\cdots,n$.

则 $\sum_{i=1}^n Y_i=\sum_{i=1}^n X_i+n=T+n\sim NB(n,\theta)$

$$\therefore P(T=t)=P(\sum_{i=1}^n Y_i=t+n)=C_{t+n-1}^{n-1}\theta^n q^t$$

$$\therefore 原式=\frac{\theta^{n-1}(1-\theta)^{\sum_{i=1}^{n-1}x_i}\ \theta(1-\theta)^{t-\sum_{i=1}^{n-1}x_i}}{C_{t+n-1}^{n-1}\theta^n q^t}$$

$$=\frac{(1-\theta)^t}{C_{t+n-1}^{n-1}q^t}$$ 与 θ 无关.

$\therefore T=\sum_{i=1}^n X_i$ 为充分统计量.

4. 利用

$X_1,X_2,\cdots,X_n$ 的联合分布为

$$f(x_1,\cdots,x_n|\mu)$$

$$=\prod_{i=1}^n\frac{1}{\sqrt{2\pi}}e^{-\frac{(x_i-\mu)^2}{2}}$$

$$=\left(\frac{1}{\sqrt{2\pi}}\right)^n e^{-\frac{\sum_{i=1}^n(x_i-\mu)^2}{2}}$$

$$T_0=\frac{1}{n}\sum_{i=1}^n X_i\sim N(\mu,\frac{1}{n})$$

$$g(t|\mu)=\frac{1}{\sqrt{\frac{2\pi}{n}}}e^{-\frac{(t-\mu)^2}{2\cdot\frac{1}{n}}}$$

$$=\frac{1}{\sqrt{\frac{2\pi}{n}}}e^{-\frac{n(\bar{x}-\mu)^2}{2}}$$

$$\frac{f(x_1,\cdots,x_n|\mu)}{g(t|\mu)}$$

$$=n^{\frac{1}{2}}(2\pi)^{-\frac{(n-1)}{2}}e^{\frac{n(\bar{x}-\mu)^2}{2}-\frac{\sum_{i=1}^n(x_i-\mu)^2}{2}}$$

$$=n^{-\frac{1}{2}}(2\pi)^{-\frac{n-1}{2}}e^{\frac{\sum_{i=1}^n(\bar{x}-\mu)^2-\sum_{i=1}^n(x_i-\mu)^2}{2}}$$

$$\because\sum_{i=1}^n(x_i-\mu)^2$$

$$=\sum_{i=1}^n(x_i-\bar{x}+\bar{x}-\mu)^2$$

$$=\sum_{i=1}^n(x_i-\bar{x})^2+\sum_{i=1}^n(\bar{x}-\mu)^2.$$

$\therefore$ 上式 $=n^{-\frac{1}{2}}(2\pi)^{-\frac{n-1}{2}}e^{-\frac{\sum_{i=1}^n(x_i-\bar{x})^2}{2}}$

与 μ 无关

$\therefore T_0=\bar{x}$ 为充分统计量

$\therefore T=nT_0=\sum_{i=1}^n X_i$ 为充分统计量

9.
(1) $L(\theta)=\prod_{i=1}^{n}p(x_i;\theta)$

$=\prod_{i=1}^{n}C_{x_i+r-1}^{r-1}\theta^{r}(1-\theta)^{x_i}$

$=\left(\prod_{i=1}^{n}C_{x_i+r-1}^{r-1}\right)\left(\theta^{nr}(1-\theta)^{\sum_{i=1}^{n}x_i}\right)$

∴充分统计量为 $\bar{x}$

(2) $L(m)=\prod_{i=1}^{n}p(x_i,m)$

$=\begin{cases}\left(\frac{1}{m}\right)^n & x_i\in\{1,\dots,m\}\ i=1,\dots,n.\\ 0 & \text{否则}\end{cases}$

$=\begin{cases}\left(\frac{1}{m}\right)^n & x_i\le m\quad i=1,\dots,n\\ 0 & \text{否则}\end{cases}$

$=\left(\frac{1}{m}\right)^n 1_{\{x_i\le m\mid i=1,\dots,n\}}$

$=\left(\frac{1}{m}\right)^n 1_{\{\max x_i\le m\}}$

∴充分统计量为 $\max x_i\quad i=1,2,\dots n$

(3) $L(\mu,\sigma)=\prod_{i=1}^{n}p(x_i;\mu,\sigma)$

$=\prod_{i=1}^{n}\frac{1}{\sqrt{2\pi}\sigma x_i}e^{-\frac{(\ln x_i-\mu)^2}{2\sigma^2}}1_{\{x_i>0\}\ i=1,2,3,\dots,n\}}$

$=\left(\prod_{i=1}^{n}\frac{1}{\sqrt{2\pi}x_i}1_{\{x_i>0;\ i=1,2,3,\dots,n\}}\right)\cdot\sigma^{-n}\cdot e^{-\frac{\sum_{i=1}^{n}(\ln x_i-\mu)^2}{2\sigma^2}}$

$=\left(\prod_{i=1}^{n}\frac{1}{\sqrt{2\pi}x_i}1_{\{x_i>0;\ i=1,2,\dots,n\}}\right)\cdot\sigma^{-n}e^{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}\ln^2 x_i-\frac{n\mu^2}{2\sigma^2}+\frac{2\mu\sum_{i=1}^{n}\ln x_i}{2\sigma^2}}$

∴充分统计量为 $\left(\sum_{i=1}^{n}\ln^2 x_i,\ \sum_{i=1}^{n}\ln x_i\right)$

(4) $L(\lambda)=\prod_{i=1}^{n}p(x_i,\lambda)$

$=\prod_{i=1}^{n}2\lambda x_i e^{-\lambda x_i^2}I\{x_i\ge 0\}$

$=\left(2\prod_{i=1}^{n}x_i I\{x_i\ge 0\}\right)\left(\lambda^n e^{-\sum_{i=1}^{n}x_i^2}\right)$

∴充分统计量为 $\sum_{i=1}^{n}x_i^2$

19. $L(\theta,\mu)$

$=\prod_{i=1}^{n}\frac{1}{\theta}e^{-\frac{x_i-\mu}{\theta}}1_{\{x_i>\mu\}}$

$=\frac{1}{\theta^n}e^{-\frac{\sum_{i=1}^{n}(x_i-\mu)}{\theta}}1_{\{x_{(1)}>\mu\}}$

$=\frac{1}{\theta^n}e^{-\frac{\sum_{i=1}^{n}x_i-n\mu}{\theta}}1_{\{x_{(1)}>\mu\}}$

$=\theta^{-n}e^{-\frac{\bar{x}-\mu}{\frac{\theta}{n}}}1_{\{x_{(1)}>\mu\}}$

取 $h(x_1,\dots,x_n)=1$

$g(T_1(x_1\dots x_n),T_2(x_1\dots x_n),\theta,\mu)$

$=\theta^{-n}e^{-\frac{\bar{x}-\mu}{\frac{\theta}{n}}}1_{\{x_{(1)}>\mu\}}$

∴ $T_1=\bar{x}$，$T_2=x_{(1)}$是充分统计量

∴ $(\bar{x},x_{(1)})$是充分统计量

(6.1)

2. X_i 服从 $E(\lambda)$ $i=1,2,3,\cdots,n$

X_i 服从 $G(1,\lambda)$

$\therefore \sum_{i=1}^{n} X_i \sim G(n,\lambda)$

令 $Y=\sum_{i=1}^{n}X_i$ $f(y,\lambda)=\frac{\lambda^n}{\Gamma(n)}y^{n-1}e^{-\lambda y}I_{\{y>0\}}\,dy$

$$E\bar{X}=\int_0^{+\infty}\frac{1}{n}y\cdot\frac{\lambda^n}{\Gamma(n)}y^{n-1}e^{-\lambda y}\,dy$$

$$=\frac{1}{n!\lambda}\int_0^{+\infty}(\lambda y)^n e^{-\lambda y}\,d(\lambda y)$$

$$=\frac{1}{n!}\cdot\frac{1}{\lambda}\cdot\Gamma(n+1)=\frac{1}{\lambda}$$

$$E\frac{1}{\bar{X}}=\int_0^{+\infty}\frac{1}{\frac{1}{n}y}\frac{\lambda^n}{\Gamma(n)}y^{n-1}e^{-\lambda y}\,dy$$

$$=n\int_0^{+\infty}\frac{\lambda^n}{\Gamma(n)}y^{n-2}e^{-\lambda y}\,dy$$

$$=\frac{n\lambda}{\Gamma(n)}\int_0^{+\infty}(\lambda y)^{n-2}e^{-\lambda y}\,d\lambda y$$

$$=\frac{n\lambda}{\Gamma(n)}\cdot\Gamma(n-1)=\frac{n}{n-1}\lambda\neq\lambda$$

$\therefore E\frac{1}{\bar{X}}\neq\lambda$

$\therefore \frac{1}{\bar{X}}$ 不是λ的无偏估计.

4. $E\left(c\sum_{i=1}^{n-1}(X_{i+1}-X_i)^2\right)$

$$=cE\left(\sum_{i=1}^{n-1}X_{i+1}^2+\sum_{i=1}^{n-1}X_i^2-2\sum_{i=1}^{n-1}X_{i+1}X_i\right)$$

$$=c\left(\sum_{i=1}^{n-1}EX_{i+1}^2+\sum_{i=1}^{n-1}EX_i^2-2\sum_{i=1}^{n-1}E(X_{i+1}X_i)\right)$$

$$=c\left((n-1)(\mu^2+\sigma^2)+(n-1)(\mu^2+\sigma^2)-2(n-1)\mu^2\right)$$

$$=2c(n-1)\sigma^2=\sigma^2$$

$\therefore c=\frac{1}{2(n-1)}$

(6.2)

2. (1)

$$EX=\frac{N}{2}=\bar{X}=\frac{1}{n}\sum_{i=1}^{n}X_i$$

$$N=\frac{2}{n}\sum_{i=1}^{n}X_i$$

$$=2\bar{X}$$

$\therefore \hat{N}_{矩}=2\bar{X}$

4. (1)

$$EX=\int_0^{\theta}x\frac{2}{\theta^2}(\theta-x)\,dx$$

$$=\frac{2}{\theta^2}\int_0^{\theta}\theta x-x^2\,dx$$

$$=\frac{2}{\theta^2}\int_0^{\theta}\frac{1}{2}\theta x^2-\frac{1}{3}x^3\,dx$$

$$=\frac{2}{\theta^2}\left(\frac{1}{2}\theta^3-\frac{1}{3}\theta^3\right)$$

$$=\frac{2}{\theta^2}\cdot\frac{1}{6}\theta^3=\frac{1}{3}\theta=\bar{X}$$

$\hat{\theta}_{矩}=3\bar{X}$

(10.3)

7.
(1) $E\bar{X} = EX_1 = \frac{3}{2}\theta$

$\therefore E(\frac{2}{3}\bar{X}) = \theta$

$\therefore E\hat{\theta} = \theta$

$\therefore \hat{\theta}$ 是 θ 的无偏估计

$D\hat{\theta} = D(\frac{2}{3}\bar{X}) = \frac{4}{9}D\bar{X} = \frac{4}{9}\cdot\frac{DX_1}{n} = \frac{4}{9}\cdot\frac{1}{n}\cdot\frac{\theta^2}{12}$

$= \frac{\theta^2}{27n}$

$\therefore \lim\limits_{n\to\infty} D\hat{\theta} = 0$

$E\hat{\theta} = \theta$

$\therefore \hat{\theta}$ 为 θ 的相合估计.

(2) $L(\theta) = \prod_{i=1}^{n} f(x_i;\theta) = \prod_{i=1}^{n}\frac{1}{\theta}\cdot 1_{\{\theta < x_i < 2\theta\}}$ $i=1,2,3,\dots$

$= \frac{1}{\theta^n} 1_{\{\theta < x_i < 2\theta\}}$ $i=1,2,3,\dots$

$= \frac{1}{\theta^n} 1_{\{\theta \le x_{(1)} \le x_{(n)} \le 2\theta\}}$

$\therefore L(\theta)$ 随 θ 单调递减，而 $\frac{1}{2}x_{(n)} \le \theta \le x_{(1)}$

$\therefore$ 当 $(2\theta) = x_{(n)}$ 时，$L(\theta)$ 最小.

$\therefore \hat{\theta}_{MLE} = \frac{1}{2}X_{(n)} = \frac{1}{2}\max\{X_1,\dots X_n\}$

$F_{X_{(n)}}(x) = P(X_n \le x) = P(\max\{X_1\cdots X_n\} \le x) = \prod_{i=1}^{n} P(X_i \le x)$

$$= \begin{cases} 0 & x < \theta \\ \left(\frac{x-\theta}{\theta}\right)^n & \theta \le x < 2\theta \\ 1 & x \ge 2\theta \end{cases}$$

$$\therefore f_{X_{(n)}}(x) = \begin{cases} n\left(\frac{x-\theta}{\theta}\right)^{n-1}\cdot\frac{1}{\theta} & \theta \le x < 2\theta \\ 0 & 其他 \end{cases}$$

$\therefore EX_{(n)} = \int_{\theta}^{2\theta} x\cdot n\left(\frac{x-\theta}{\theta}\right)^{n-1}\frac{1}{\theta}dx$

$= \frac{n}{\theta^n}\int_{\theta}^{2\theta} x(x-\theta)^{n-1}dx$

$= \frac{n}{\theta^n}\int_0^{\theta}(x+\theta)x^{n-1}dx$

$= \frac{n}{\theta^n}\int_0^{\theta} x^n + \theta x^{n-1}dx$

$= \frac{n}{\theta^n}\cdot\left(\frac{1}{n+1}\theta^{n+1} + \frac{1}{n}\theta^{n+1}\right)$

$= \theta\left(\frac{n}{n+1}+1\right)$

$= \frac{2n+1}{n+1}\theta$

$\therefore E\hat{\theta}_{MLE} = \frac{1}{2}EX_{(n)} = \frac{2n+1}{2n+2}\theta$

$\therefore \hat{\theta}_{MLE}$ 不是 θ 的无偏估计

$EX_{(n)}^2 = \int_{\theta}^{2\theta} x^2 n\left(\frac{x-\theta}{\theta}\right)^{n-1}\frac{1}{\theta}dx$

$= \frac{n}{\theta^n}\int_0^{\theta}(x+\theta)^2 x^{n-1}dx$

$= \frac{n}{\theta^n}\int_0^{\theta}(x^2+2\theta x+\theta^2)x^{n-1}dx$

$= \frac{n}{\theta^n}\int_0^{\theta} x^{n+1}+2\theta x^n+\theta^2 x^{n-1}dx$

$= \frac{n}{\theta^n}\left(\frac{\theta^{n+2}}{n+2}+\frac{2\theta^{n+2}}{n+1}+\frac{\theta^{n+2}}{n}\right)$

$= \theta^2\left(\frac{n}{n+2}+\frac{2n}{n+1}+1\right)$

$= \frac{4n^2+8n+2}{(n+2)(n+1)}\theta^2$

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$$DX_{(n)} = EX_{(n)}^2 - (EX_{(n)})^2$$

$$= \frac{4n^2+8n+2}{(n+2)(n+1)}\theta^2 - \frac{(2n+1)^2}{(n+1)^2}\theta^2$$

$$= \frac{(4n^2+8n+2)(n+1)-(n+2)(2n+1)^2}{(n+2)(n+1)^2}\theta^2$$

$$= \frac{n}{(n+2)(n+1)^2}\theta^2$$

$\therefore D\hat{\theta}_{MLE} = \frac{1}{4}DX_{(n)} = \frac{n}{4(n+2)(n+1)^2}\theta^2$

$\because \lim\limits_{n\to\infty} E\hat{\theta}_{MLE} = \theta$

$\lim\limits_{n\to\infty} D\hat{\theta}_{MLE} = 0$

$\therefore \hat{\theta}_{MLE}$ 是相合估计

8. (1) $L(\theta) = \prod\limits_{i=1}^{n} f(x_i;\theta)$

$$= \prod_{i=1}^{n} e^{-(x_i-\theta)} = e^{-\sum_{i=1}^{n} x_i + n\theta} I\{x_i>\theta \mid i=1,2,\dots,n\}$$

$$= e^{-\sum_{i=1}^{n} x_i + n\theta} I\{x_{(1)}>\theta\}$$

$L(\theta)$ 随 θ 单调递增　$\theta_{max} = x_{(1)}$

$\therefore \hat{\theta}_1 = X_{(1)}$

$$F_{X_{(1)}}(x) = P(X_{(1)}\le x) = P(\min\{X_1\cdots X_n\}\le x)$$

$$= 1-\prod_{i=1}^{n}(1-P(X_i\le x)) = 1-(1-P(X_1\le x))^n$$

$$P(X_i\le x) = \begin{cases} 1-e^{-(x-\theta)} & x>\theta \\ 0 & x\le\theta \end{cases}$$

$$F_{X_{(1)}}(x) = \begin{cases} 1-e^{-n(x-\theta)} & x>\theta \\ 0 & x\le\theta \end{cases}$$

$$f_{X_{(1)}}(x) = \begin{cases} ne^{-n(x-\theta)} & x>\theta \\ 0 & x\le\theta \end{cases}$$

$$EX_{(1)} = \int_{\theta}^{+\infty} x\cdot ne^{-n(x-\theta)}dx$$

$$= ne^{n\theta}\int_{\theta}^{+\infty} xe^{-nx}dx$$

$$= -e^{n\theta}\int_{\theta}^{+\infty} x\, d(e^{-nx})$$

$$= -e^{n\theta}\left(xe^{-nx}\Big|_{\theta}^{+\infty} - \int_{\theta}^{+\infty} e^{-nx}dx\right)$$

$$= e^{n\theta}\theta e^{-n\theta} + \frac{1}{n}$$

$$= \theta + \frac{1}{n}$$

$\therefore \hat{\theta}_1$ 不是无偏估计

$$EX_{(1)}^2 = \int_{\theta}^{+\infty} x^2\cdot ne^{-n(x-\theta)}dx$$

$$= n\int_{0}^{+\infty}(x+\theta)^2 e^{-nx}dx$$

$$= n\int_{0}^{+\infty} x^2e^{-nx}dx + 2n\theta\int_{0}^{+\infty} xe^{-nx}dx + n\theta^2\int_{0}^{+\infty} e^{-nx}dx$$

$$= \frac{1}{n^2}\int_{0}^{+\infty} t^2e^{-t}dt + \frac{2\theta}{n}\int_{0}^{+\infty} te^{-t}dt + \theta^2\int_{0}^{+\infty} e^{-t}dt$$

数 学 作 业 纸

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$= \frac{1}{n^2}\Gamma(3) + \frac{2\theta}{n}\cdot\Gamma(2) + \theta^2\cdot\Gamma(1)$

$= \frac{2}{n^2} + \frac{2\theta}{n} + \theta^2$

$DX_{(1)} = EX_{(1)}^2 - (EX_{(1)})^2$

$= \frac{2}{n^2} + \frac{2\theta}{n} + \theta^2 - \theta^2 - \frac{2}{n}\theta - \frac{1}{n^2}$

$= \frac{1}{n^2}$

$\because \lim_{n\to+\infty} E(X_{(1)}) = \theta$

$\lim_{n\to+\infty} DX_{(1)} = 0$

$\therefore$ $\hat{\theta}_1$是相合估计.

(2) $EX = \int_{\theta}^{+\infty} x e^{-(x-\theta)} dx$

$= \int_0^{+\infty} (x+\theta) e^{-x} dx$

$= \int_0^{+\infty} x e^{-x} dx + \theta\int_0^{+\infty} e^{-x} dx$

$= \Gamma(2) + \theta\Gamma(1)$

$= 1+\theta = \bar{X}$

$\therefore \hat{\theta}_2 = \bar{X} - 1$

$\hat{\theta}_2$是无偏估计.

$E\hat{\theta}_2 = \theta$

$D\hat{\theta}_2 = D(\bar{X}-1) = D\bar{X}$

$= \frac{DX_1}{n} = \frac{1}{n}$

$\because \lim_{n\to\infty} D\hat{\theta}_2 = 0$

$\therefore$ $\hat{\theta}_2$是相合估计