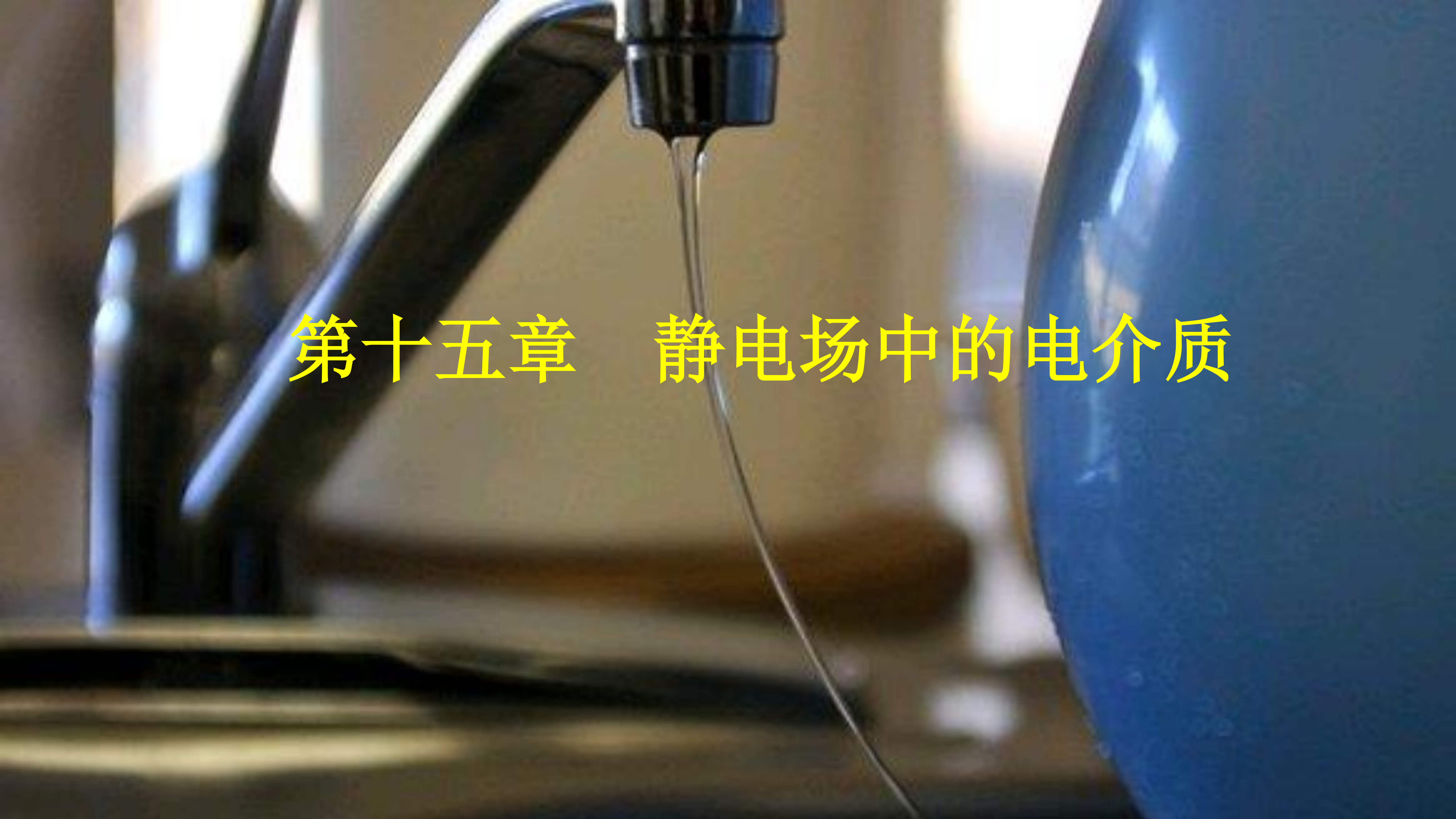
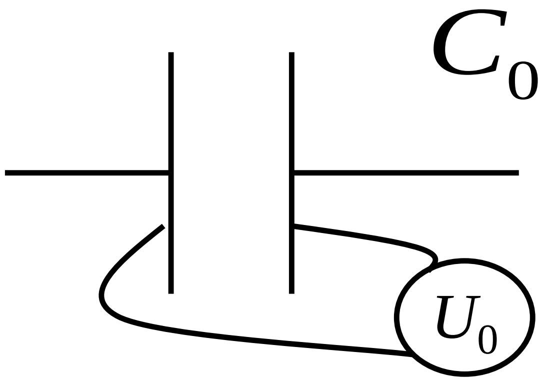
第十五章　静电场中的电介质



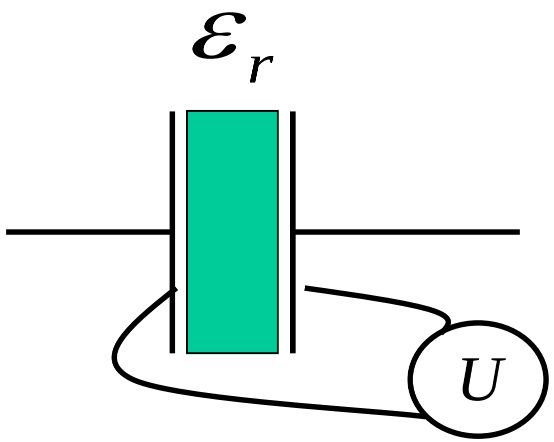
第十五章　静电场中的电介质

15.1 电介质对电场的影响

实验现象



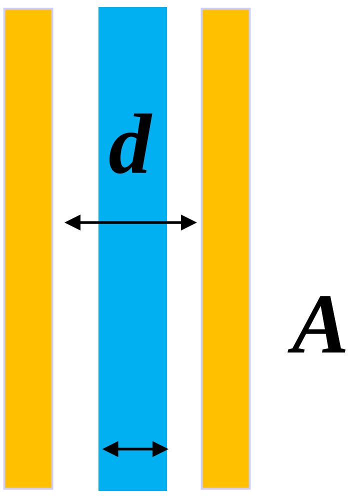
$$U = \frac{U_0}{\varepsilon_r}$$



$$\boxed{C = C_0 \varepsilon_r}$$

一般 $\varepsilon_r > 1$

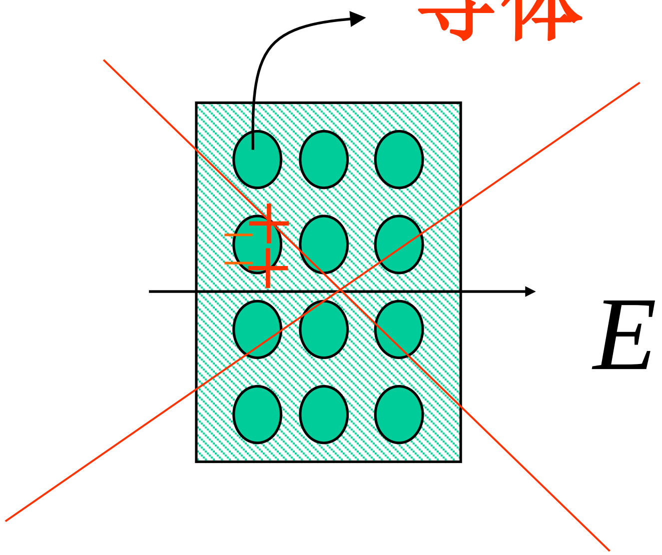
（演示实验）



插入导体板

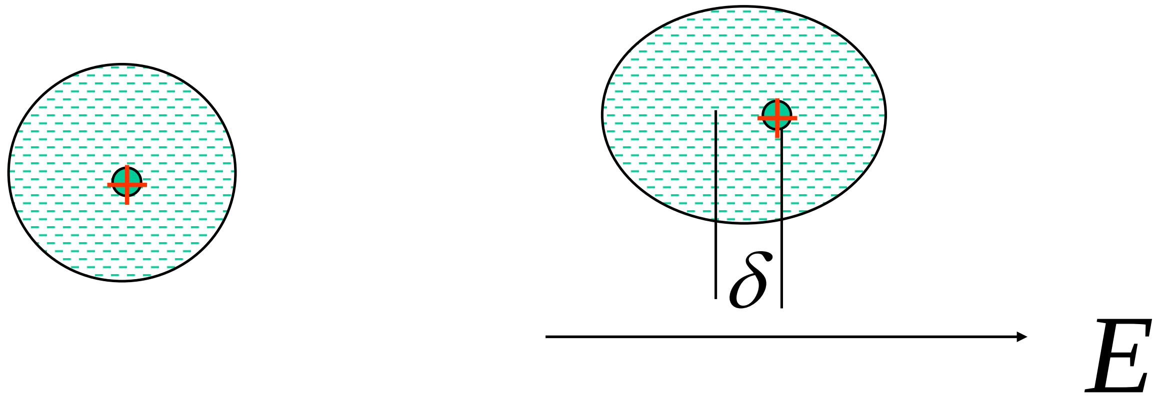
$$U' = E(d - d_1)$$

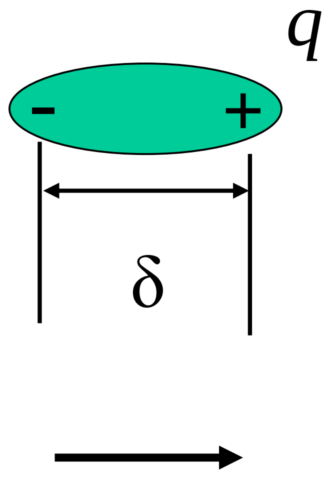
以为绝缘体（电介质）



对电场反应的是分子或原子

无极分子non-polar molecules



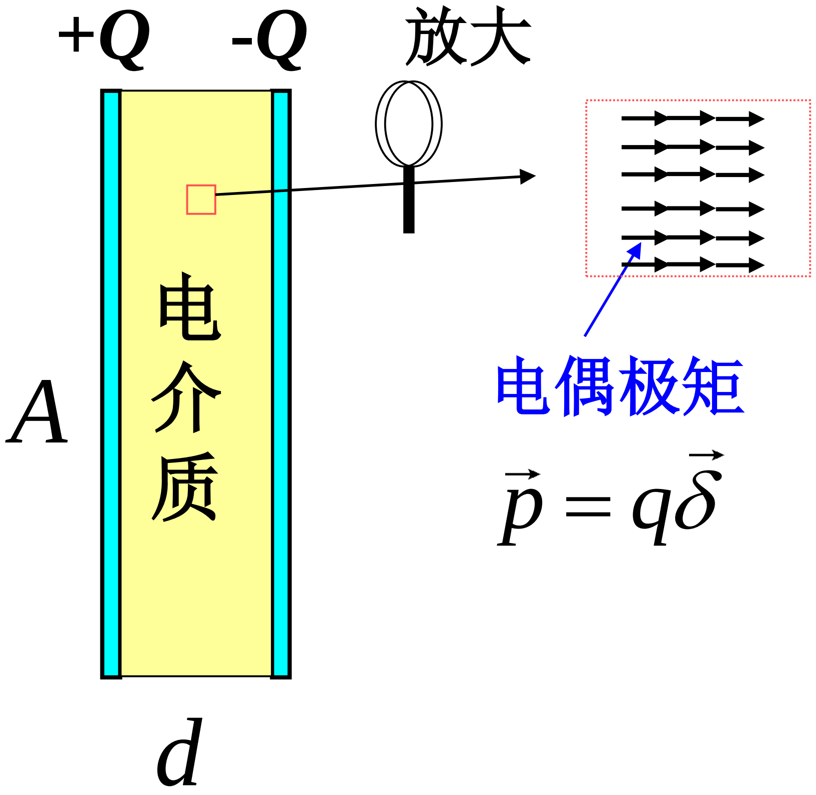


分子正负电荷中心在电场作用下错位

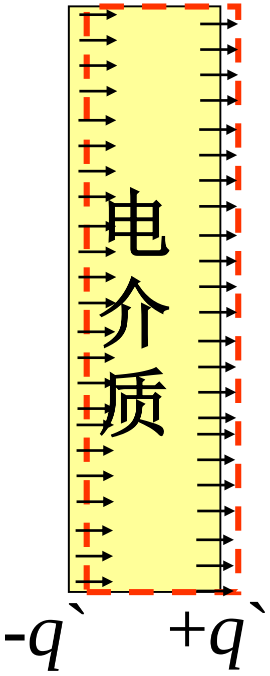
$$\vec{p} = q\vec{\delta}$$ 分子电偶极矩

位移极化 **(displacement) electronic polarization**

二. 电介质分子对电场的影响



$$\vec{p} = q\vec{\delta}$$



$$E < E_0 \quad U < U_0$$

$$U = \frac{U_0}{\varepsilon_r}$$

相对介电常数

气体~1　　云母、瓷、纸~ 5

水~80　　钛酸钡~ 10^3-10^4

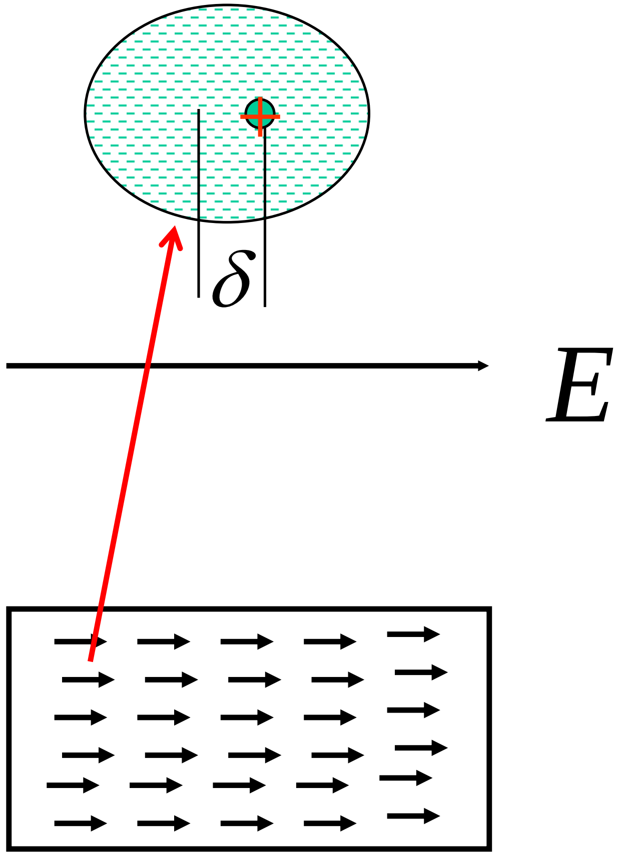
表面出现极化电荷

(Polarization charges)

电介质对电场的影响通过极化电荷体现出来

（电介质演示实验）

15.2 极化强度$\vec{P}$ (Polarization vector)



极化强度:

单位体积的总电偶极矩

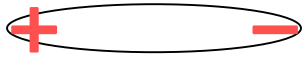
$$\vec{P} = N\vec{p}$$

分子数密度

单个分子位移极化的电偶极矩

平均电偶极矩

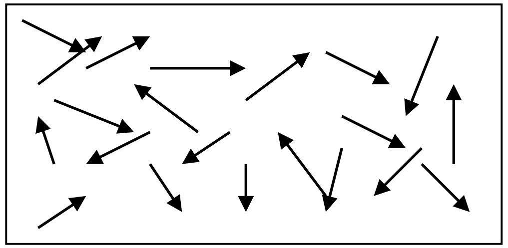
有极分子 polar molecules



$\vec{p}_{固有}$

有极分子

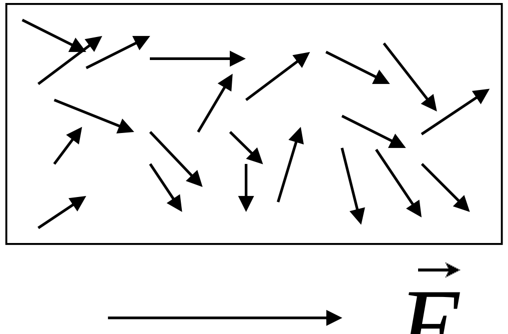
无外场时：



热运动→无规取向

单位体积的总电偶极矩处处为零

有外场时：



$\vec{E}$

力矩：$\vec{p}\times\vec{E}$

符合经典玻尔兹曼分布

能量：$-\vec{p}\cdot\vec{E}$

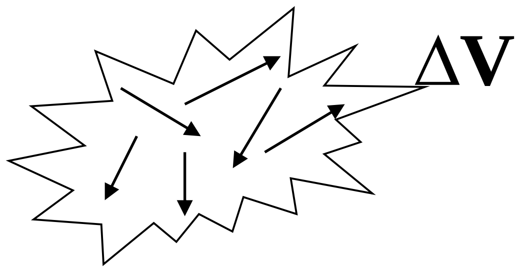
$\boxed{\propto e^{\vec{p}\cdot\vec{E}/kT}}$

取向极化

(orientation polarization)

极化强度 $\vec{P}$　单位体积的总电偶极矩

$$\boxed{\vec{P} = \lim \frac{\sum_i \vec{p}_i}{\Delta V}}$$



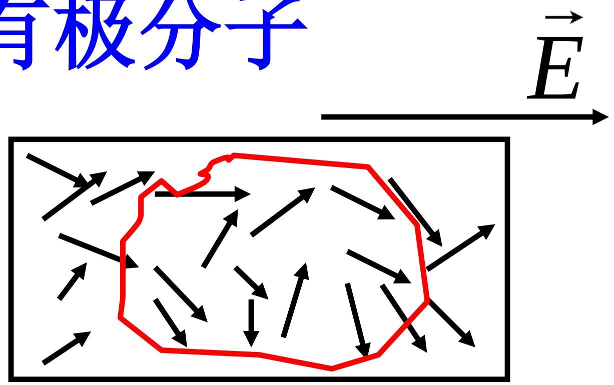
宏观上无限小微观上无限大的体积元 ΔV

$\vec{p}_i$ 每个分子的电偶极矩

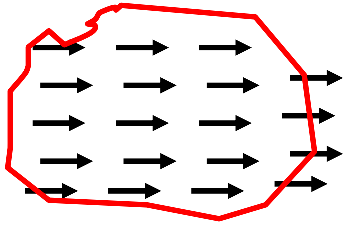
无外场时,无序排列, 求和为零　$\vec{P} = 0$

无极分子　上式 $= N\vec{p}$

有极分子



$\vec{p}_{固有}$



$$\vec{P} = \lim \frac{\sum_i \vec{p}_i}{\Delta V} = N\vec{p}$$

平均电偶极矩

$$\boxed{\vec{p} = \frac{\vec{P}}{N}}$$

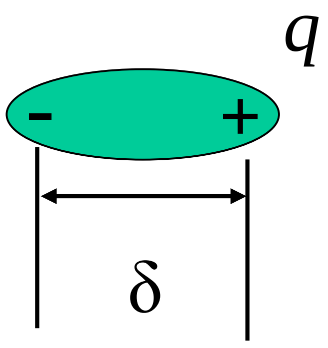
考察小区域, 电场均匀

均匀极化

$\vec{P}$ 均匀

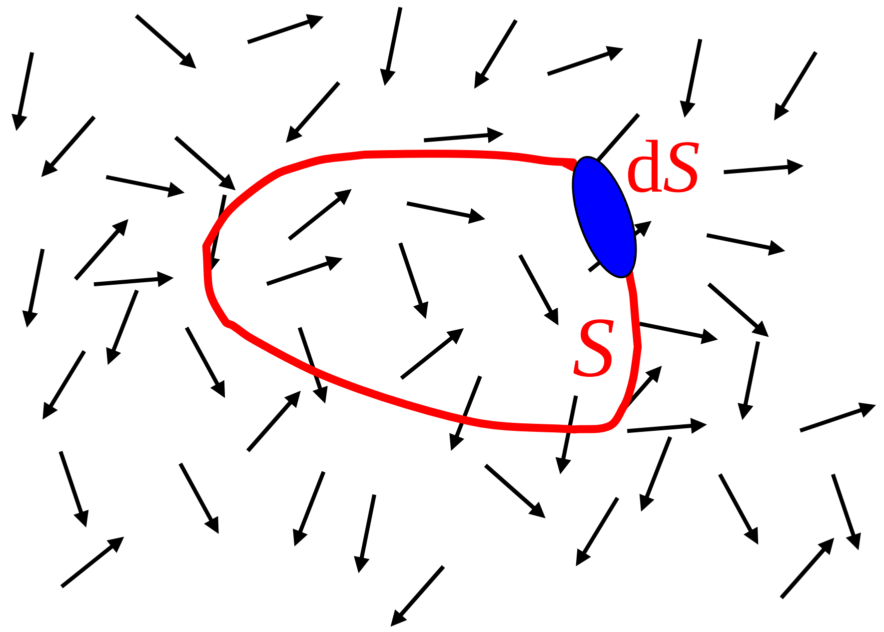
$$\boxed{\vec{P}_{MAX} = N\vec{p}_{固有}}$$

15.3 极化强度与极化电荷



平均电偶极矩

$$\vec{p} = q\vec{\delta}$$

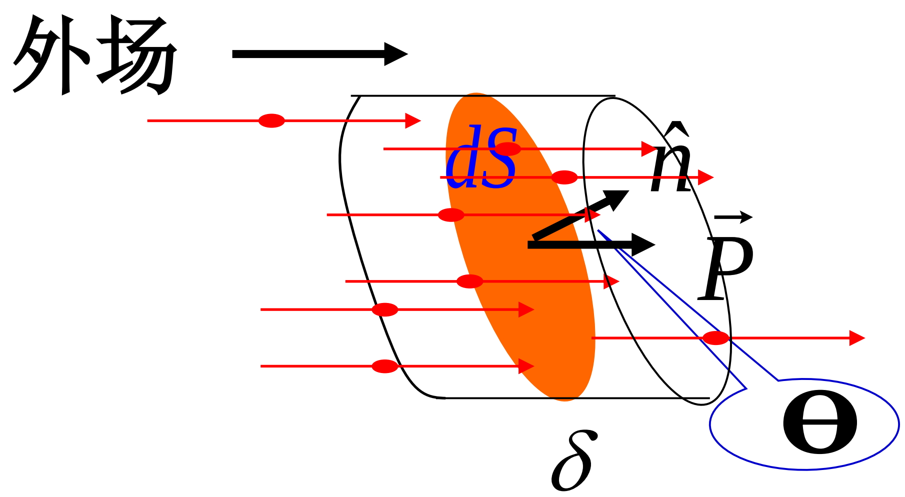


在已极化的介质内任意作一闭合面*S*

S 内有多少极化电荷?

平均电偶极矩穿过***S*** 的分子对***S***内的极化电荷有贡献

小面元dS贡献求和



穿出dS的偶极子数目等于圆柱内分子数目

$$= N\delta dS\cos\theta$$

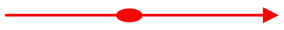
穿出dS的极化电荷

$$dq' = qN\delta dS\cos\theta$$

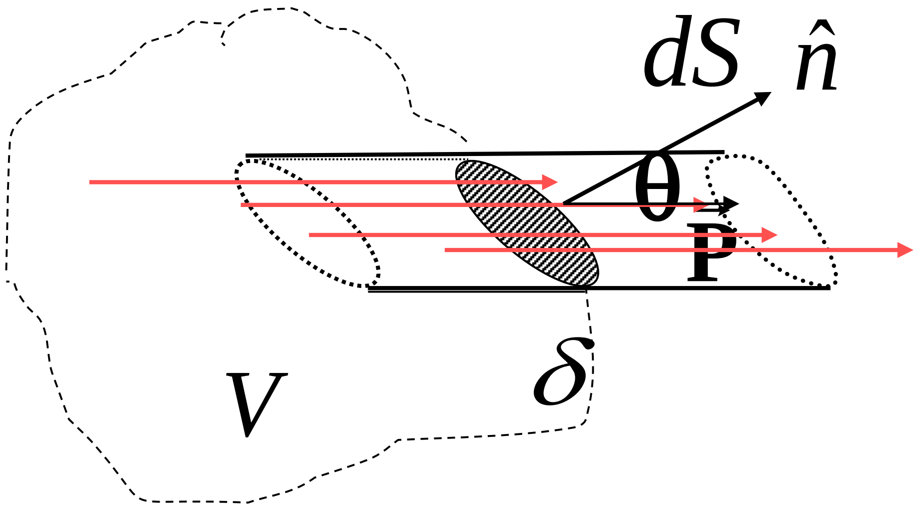
$$= PdS\cos\theta$$

$$= \vec{P}\cdot d\vec{S}$$

在dS 附近薄层内认为介质均匀极化

平均的电偶极矩 

$$\vec{p} = q\vec{\delta} = \vec{P}/N$$



小面元***dS***对面内极化电荷的贡献

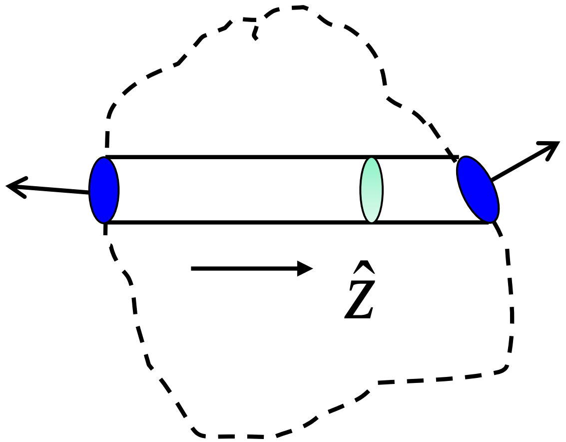
$$\boxed{dq' = -\vec{P}\cdot\hat{n}dS}$$

在***S***所围的体积内的极化电荷 q'与 $\vec{P}$ 的关系

$$\boxed{q' = -\oint\!\!\!\oint_S \vec{P}\cdot\hat{n}dS}$$

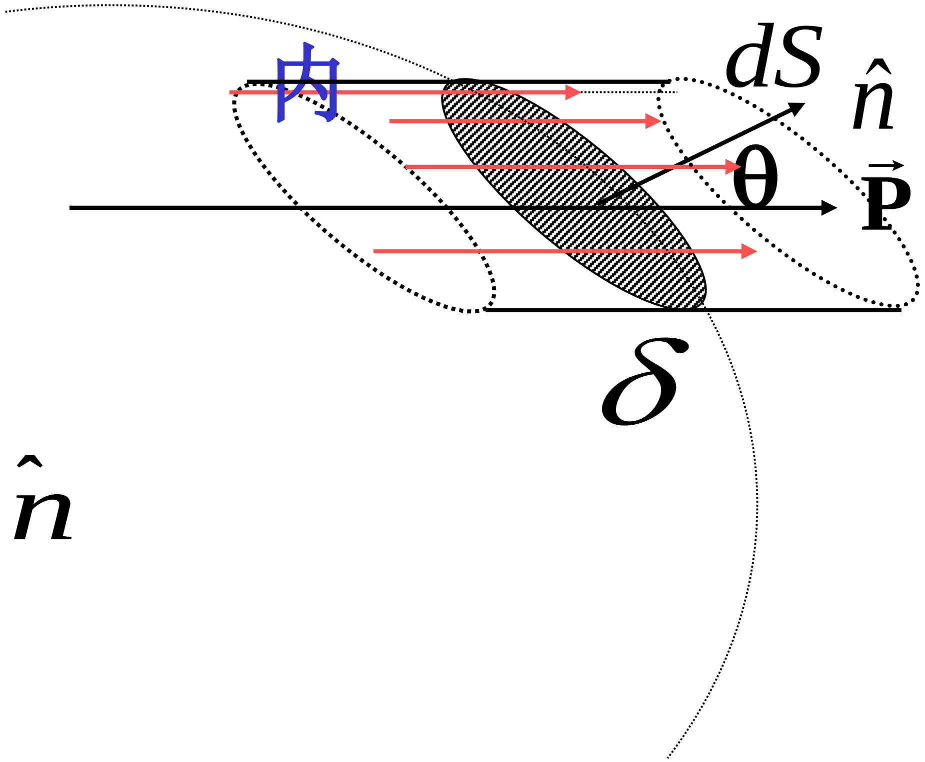
- 均匀极化处, 没有束缚电荷

$$q' = -\oiint_S \vec{P}\cdot\hat{n}dS = -P\oiint_S \hat{z}\cdot\hat{n}dS \quad = 0$$



- 均匀极化时束缚电荷只在表面
- 非均匀极化时束缚电荷可同时分布在内部和表面

电介质表面极化电荷面密度



$$dq' = \vec{P} \cdot \hat{n} dS$$

$$\sigma' = \frac{dq'}{dS} = \vec{P} \cdot \hat{n}$$

$$\boxed{\sigma' = \vec{P} \cdot \hat{n}}$$

$\hat{n}$ 介质外法线方向

15.4 电介质的极化规律

1.各向同性线性电介质 isotropy linearity

$$\boxed{\vec{P} = \chi_e \varepsilon_0 \vec{E}}$$

χ_e 介质的电极化率

无量纲的纯数 与 $\vec{E}$ 无关

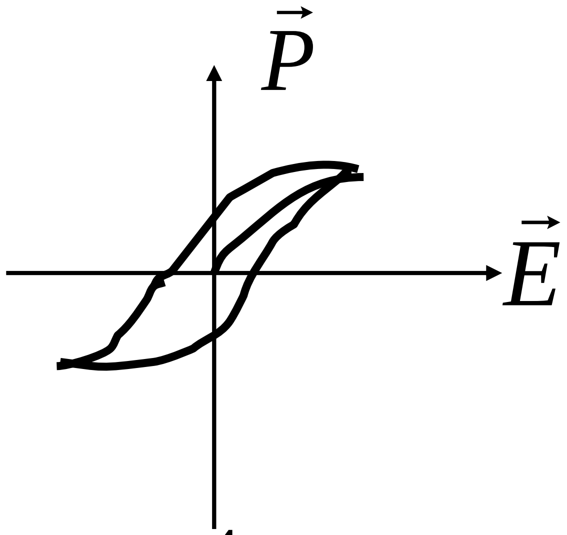
$$\boxed{\chi_e = \varepsilon_r - 1}$$

2.各向异性线性电介质 anisotropy

χ_e 与 $\vec{E}$ 、与晶轴的方位有关

弱场性质 张量描述

3. 铁电体　ferroelectrics　　类似于铁磁体



$\vec{P}$ 与 $\vec{E}$间非线性，

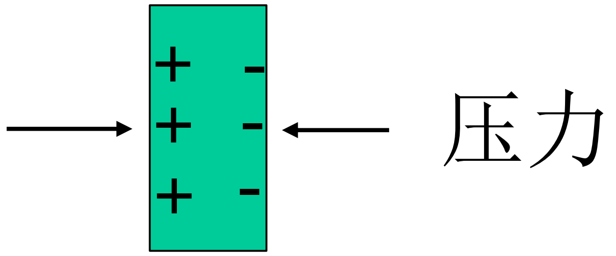
没有单值关系。

1) 电滞现象

2) 介电常数很大　ε_r　$10^2 \cdots 10^4$

3) 居里点

4) 压电现象



超声换能器

（演示实验）

4 有电介质时静电场的计算

自由电荷和极化电荷产生电场的叠加

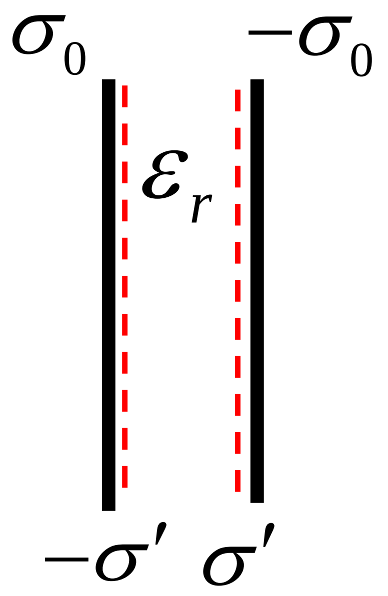
$$\boxed{\vec{E} = \vec{E}_0 + \vec{E}'}$$

$$q' = -\oiint_S \vec{P} \cdot \hat{n} dS \qquad \sigma' = \vec{P} \cdot \hat{n}$$

$$\vec{P} = \varepsilon_0 (\varepsilon_r - 1) \vec{E}$$

例1 平行板电容器

充满均匀各向同性线性电介质.
求:板内的场



解:均匀极化

表面出现束缚电荷

所有电荷场叠加

$$E_0 = \frac{\sigma_0}{\varepsilon_0} \qquad E' = \frac{\sigma'}{\varepsilon_0}$$

$$E = E_0 - E'$$

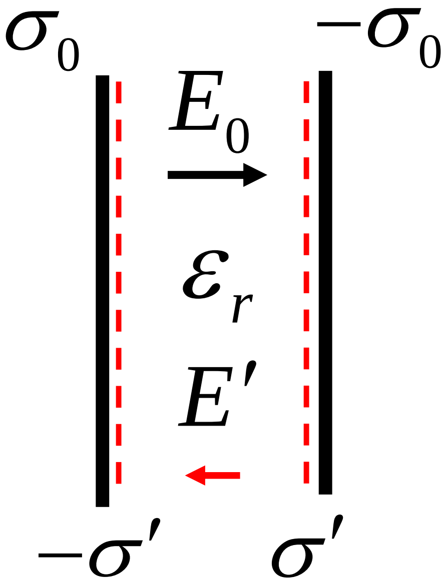
$$\sigma' = P_n = P = \varepsilon_0\left(\varepsilon_r - 1\right)E$$

$$E = \frac{\sigma_0}{\varepsilon_0} - \frac{\varepsilon_0(\varepsilon_r - 1)E}{\varepsilon_0}$$

$$E = \frac{\sigma_0}{\varepsilon_0\varepsilon_r}$$

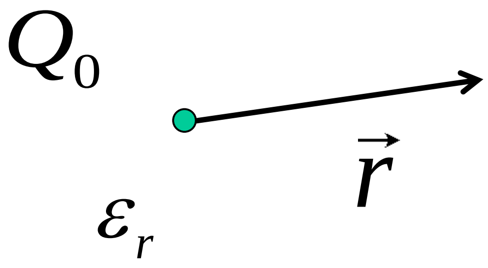
$$E = \frac{E_0}{\varepsilon_r}$$

普遍？



各向同性均匀电介质充满电场空间

例2 点电荷周围充满电介质时的电场



$$\boxed{\vec{E}=\frac{Q_0}{4\pi\varepsilon_0\varepsilon_r}\frac{\vec{r}}{r^3}}$$

学习***D***的高斯定理以后请自行验证

15.5 电位移矢量 $\vec{D}$ **electric displacement vector**

一. 电位移矢量

$$\boxed{\vec{D} = \varepsilon_0 \vec{E} + \vec{P}}$$

各向同性线性介质 $\boxed{\vec{P} = \varepsilon_0\left(\varepsilon_r - 1\right)\vec{E}}$

介质方程 $\boxed{\vec{D} = \varepsilon_0 \varepsilon_r \vec{E}}$ $\varepsilon = \varepsilon_0 \varepsilon_r$

量纲 $\left[\vec{D}\right] = \left[\vec{P}\right] = \left[\sigma\right]$ 单位 C/m^2

二. $\vec{D}$ 的高斯定理

$$\oiint_S \vec{E}\cdot d\vec{S} = \frac{\sum_i q_i}{\varepsilon_0} = \frac{\sum_i q_i' + \sum_i q_{oi}}{\varepsilon_0}$$

$$-\oiint_S \vec{P}\cdot d\vec{S}$$

$$\boxed{\vec{D} = \varepsilon_0 \vec{E} + \vec{P}}$$

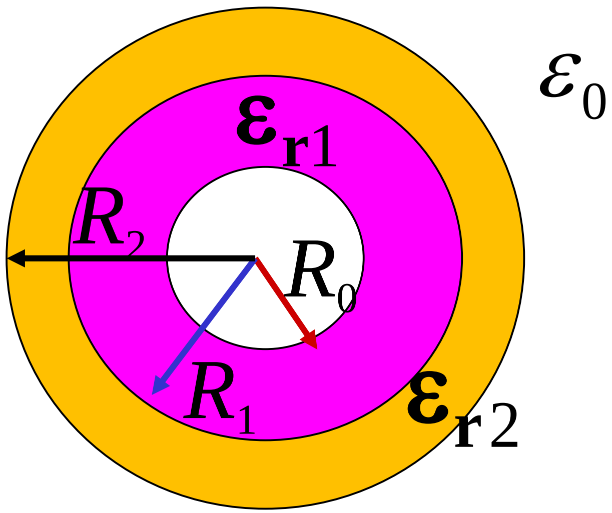
$$\boxed{\oiint_S \vec{D}\cdot d\vec{S} = \sum_i q_{0i}}$$

自由电荷

某种对称性的情况下，可以首先解出$\vec{D}$

即 $\vec{D} \Rightarrow \vec{E} \Rightarrow \vec{P} \Rightarrow \sigma' \Rightarrow q'$

例1 导体球置于均匀各向同性介质中（同心）如图示，导体带电 $\boldsymbol{Q}$



求：

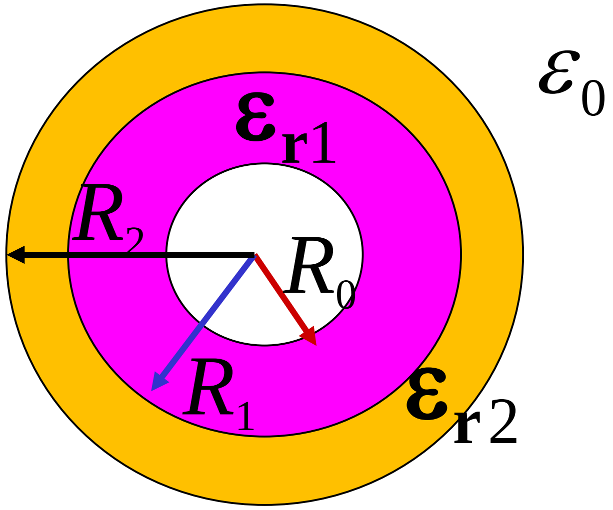
1. 场的分布
2. 紧贴导体球表面处的极化电荷
3. 两介质交界处的极化电荷

解：根据球对称性

$$\oiint_S \vec{D}\cdot d\vec{S} = \sum_i q_{0i}$$

$$\vec{D} = \begin{cases} 0 & r < R_0 \\ \dfrac{Q}{4\pi r^2}\hat{r} & r > R_0 \end{cases}$$

1) 场的分布



$$\boxed{\vec{E} = \frac{\vec{D}}{\varepsilon_0 \varepsilon_r}}$$

$$\boxed{\vec{P} = \left(1 - \frac{1}{\varepsilon_r}\right)\vec{D}}$$

按区域分别有

$$\vec{E}_0 = 0 \quad \vec{P}_0 = 0$$

$$\vec{E}_1 = \frac{Q}{4\pi\varepsilon_0\varepsilon_{r1} r^2}\hat{r} \quad \vec{P}_1 = \left(1 - \frac{1}{\varepsilon_{r1}}\right)\frac{Q}{4\pi r^2}\hat{r}$$

1换成2

球对称极化, 不是均匀极化

极化电荷体密度?

$$-q' = \oiint_S \vec{P} \cdot d\vec{S}$$

$$\boxed{q_0 = \oiint_S \vec{D} \cdot d\vec{S}}$$

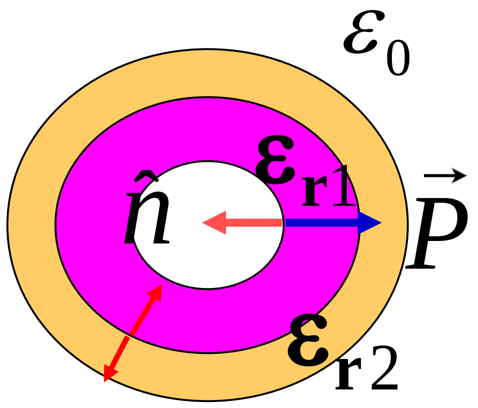
$$\vec{P} = \left(1 - \frac{1}{\varepsilon_r}\right)\vec{D}$$

自由电荷密度为零处
极化电荷密度亦为零

思考：点电荷Q周围极化电荷？

2) 求紧贴导体球表面处的极化电荷

$$\sigma' = \vec{P}\cdot\hat{n}\Big|_{r=R_0} = -P_1\big|_{r=R_0}$$



$$= -\frac{Q}{4\pi R_0^2}\left(1-\frac{1}{\varepsilon_{r1}}\right)$$

$$q' = \sigma'\cdot 4\pi R_0^2 = \left(\frac{1}{\varepsilon_{r1}}-1\right)Q$$

3) 两介质交界处极化电荷

$$\sigma'\big|_{r=R_1} = (P_1 - P_2)\big|_{r=R_1} = \frac{Q}{4\pi R_1^2}\left(\frac{1}{\varepsilon_{r2}}-\frac{1}{\varepsilon_{r1}}\right)$$

例2 一无限大各向同性均匀介质平板厚度为 d

相对介电常数为 ε_r 内部均匀分布体电荷密度为

ρ_0 的自由电荷

求：介质板内、外的 D E P

解：取坐标系如图

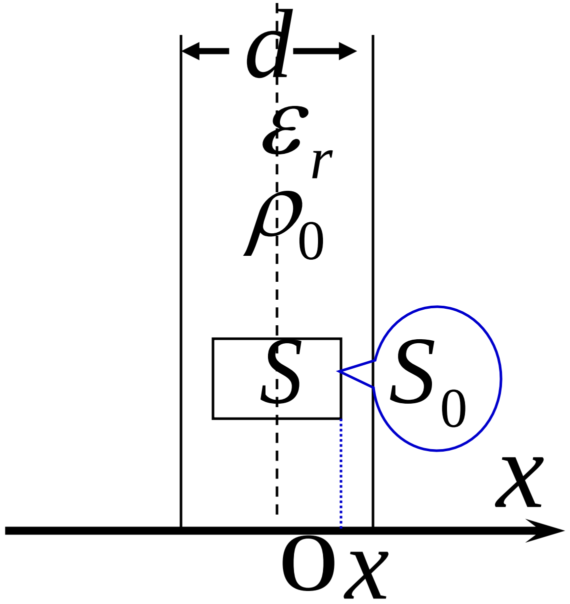
中心面对称 $\vec{D}$ $\vec{E}$ $\vec{P}\perp$ 平板

$x=0$ 处 $E=0$

以 $x=0$ 处的面为对称

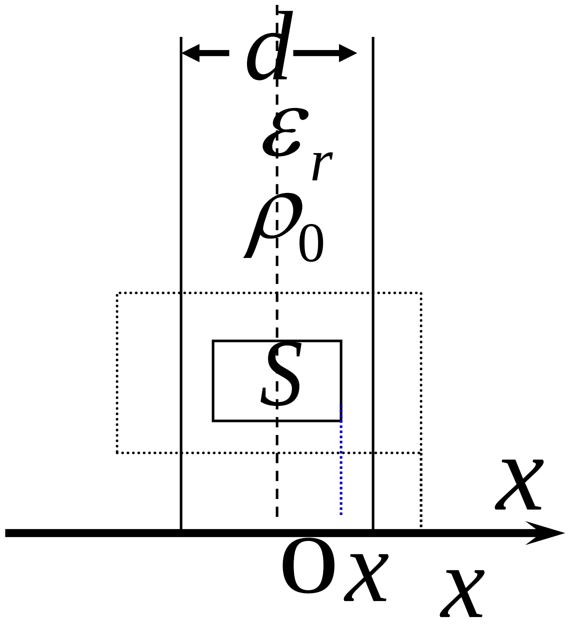
过场点作正柱形高斯面 S

底面积设 S_0



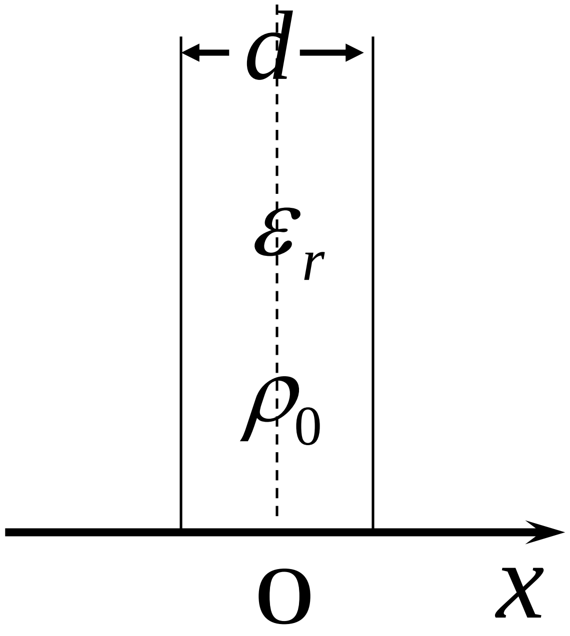
$$|x| \le \frac{d}{2} \qquad 2DS_0 = \rho_0 2|x|S_0$$

$$\boxed{D = \rho_0 |x|}$$



$$|x| \ge \frac{d}{2} \qquad 2DS_0 = \rho_0 S_0 d$$

$$\boxed{D = \frac{\rho_0}{2} d}$$



$$|x| \le \frac{d}{2} \qquad \boxed{D = \rho_0 |x|}$$

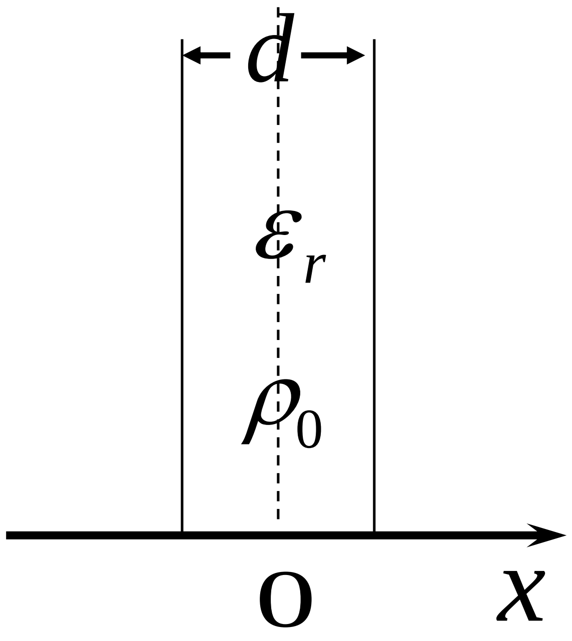
$$E = \frac{D}{\varepsilon_0 \varepsilon_r} = \frac{\rho_0 |x|}{\varepsilon_0 \varepsilon_r} \qquad P = (\varepsilon_r - 1)\frac{\rho_0 |x|}{\varepsilon_r}$$

$$|x| \ge \frac{d}{2} \qquad \boxed{D = \frac{\rho_0}{2} d}$$

$$E = \frac{D}{\varepsilon_0} = \frac{\rho_0 d}{2\varepsilon_0}$$

均匀场

$$P = \varepsilon_0(\varepsilon_r - 1)E = 0$$



极化体电荷密度?

$$\vec{P} = \left(1 - \frac{1}{\varepsilon_r}\right)\vec{D}$$

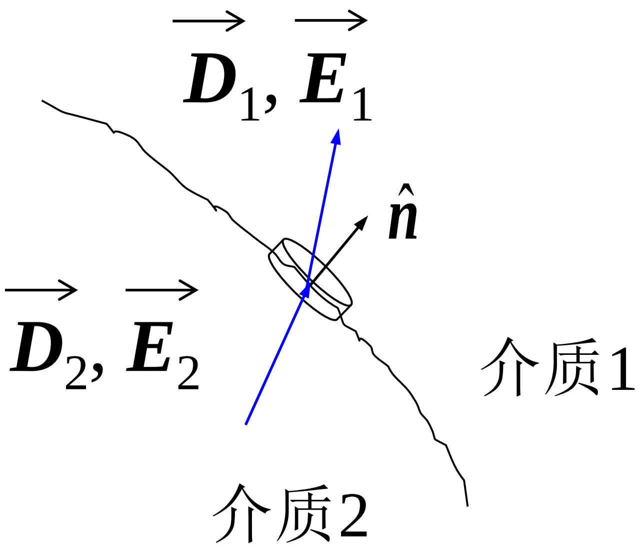
$$\rho' = -\left(1 - \frac{1}{\varepsilon_r}\right)\rho_0$$

极化电荷面密度?

$$\sigma' = P_{x=\pm d/2}$$

$$\sigma' = \left(1 - \frac{1}{\varepsilon_r}\right)\frac{\rho_0 d}{2}$$

三. 边值关系**



$$\oiint_S \vec{D}\cdot d\vec{S} = \iint_{底面1} \vec{D}\cdot d\vec{S} + \iint_{底面2} \vec{D}\cdot d\vec{S}$$

$$= \vec{D}_1 \cdot \hat{n}_1 \Delta S + \vec{D}_2 \cdot \hat{n}_2 \Delta S$$

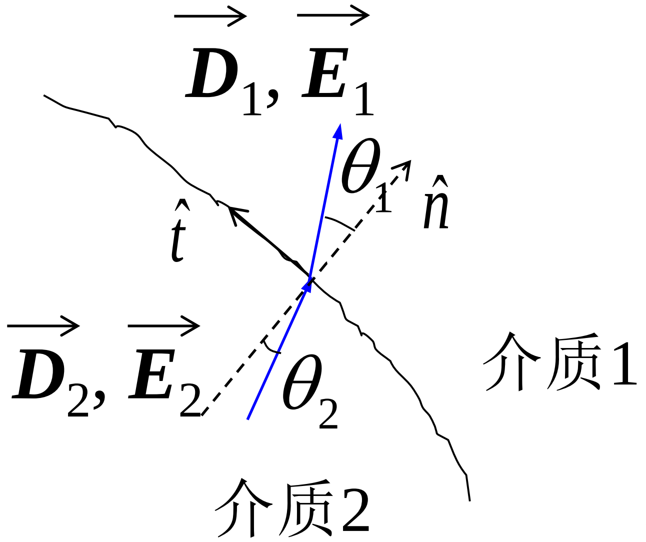
$$= (D_{1n} - D_{2n})\Delta S$$

$$(D_{1n} - D_{2n})\Delta S = q_0$$

$$\boxed{D_{1n} - D_{2n} = \sigma_0}$$

如果分界面上没有自由电荷

$$\boxed{D_{1n} = D_{2n}}$$



$$\oint_L \vec{E} \cdot \mathrm{d}\vec{l} = (E_{1t} - E_{2t})\Delta l = 0$$

$$\boxed{E_{1t} = E_{2t}}$$

如果分界面上没有自由电荷

$$D_{1n} = D_{2n} \quad \rightarrow \quad \varepsilon_1 E_{1n} = \varepsilon_2 E_{2n}$$

$$\tan\theta = \frac{E_t}{E_n}$$

又 $E_{1t} = E_{2t}$

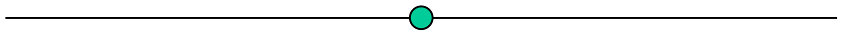
$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{E_{1t}E_{2n}}{E_{1n}E_{2t}} = \frac{E_{2n}}{E_{1n}} = \frac{\varepsilon_1}{\varepsilon_2}$$

电位移线或电场的折射定理

如下图, 电场为何?

~~D 球对称?~~

ε_2



q

ε_1

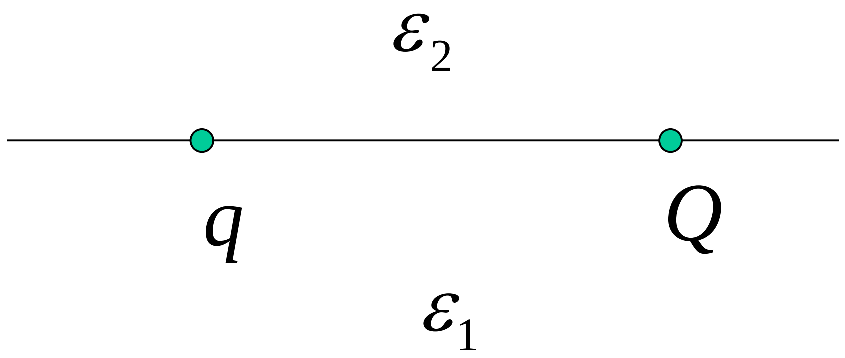
E 球对称!

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q+q'}{r^2}\hat{r}$$

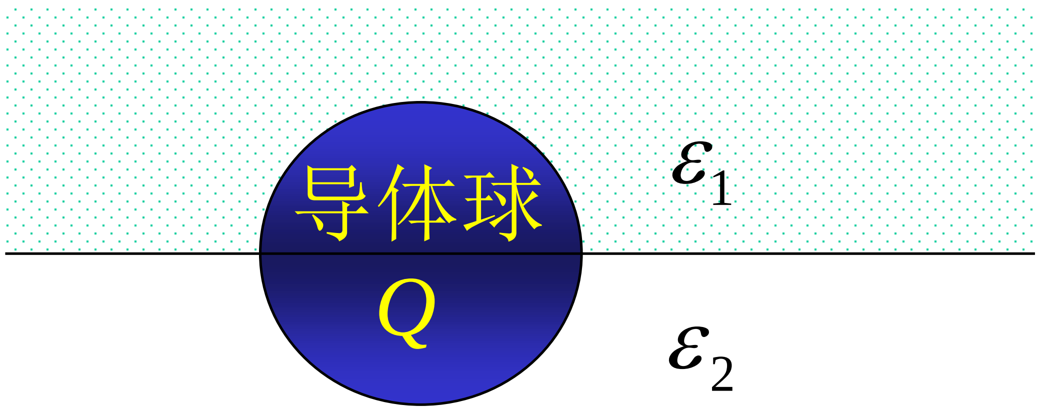
$$\oiint_S \vec{D}\cdot d\vec{S} = q \qquad D_i = \varepsilon_i E$$

$$2\pi r^2(D_1 + D_2) = q \qquad E = \frac{1}{4\pi(\varepsilon_1+\varepsilon_2)}\frac{q}{r^2}$$

$$2\pi r^2(\varepsilon_1+\varepsilon_2)E = q$$



电场的叠加原理



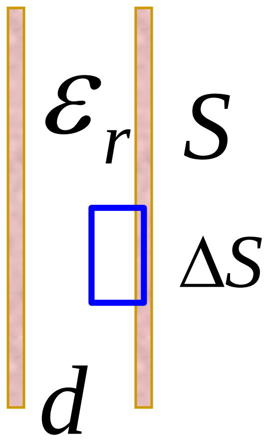
自由电荷和极化电荷
球对称分布

E球对称！ 自洽解

胡友秋、程福臻、叶邦角
电磁学与电动力学（上册）

15.6 有电介质时静电场的能量

一. 有介质时的电容器的电容



设带电 Q

$$E = \frac{E_0}{\varepsilon_r} \qquad U = \frac{U_0}{\varepsilon_r}$$

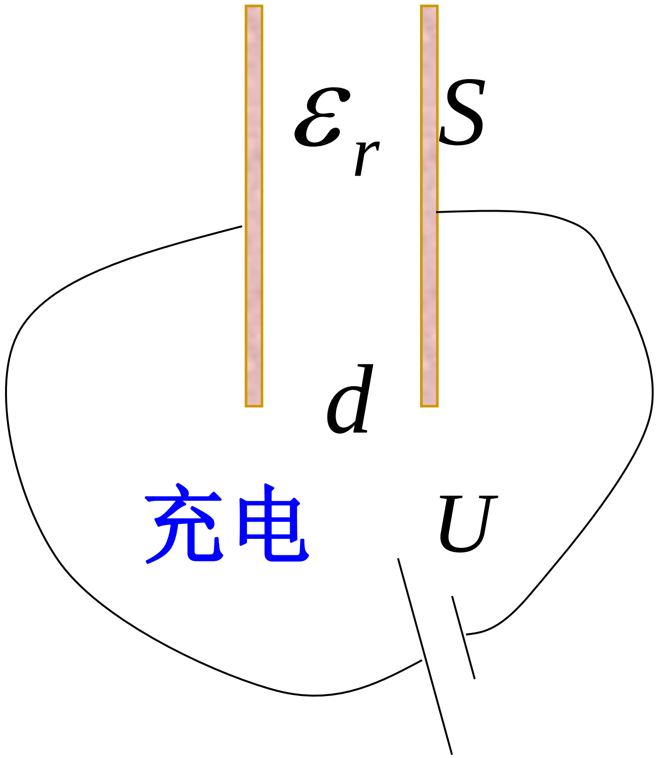
$$C = \frac{Q}{U} \qquad \boxed{\varepsilon_r = \frac{C}{C_0}}$$

电容率

$$E_0 = \frac{\sigma_0}{\varepsilon_0} \qquad D = \varepsilon_0 \varepsilon_r E = \varepsilon_0 E_0 = \sigma_0 = \frac{Q}{S}$$

导体内 $D = 0$

二. 场能密度



有介质时电势能？

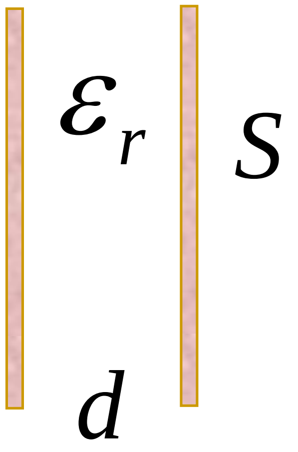
电池做功 = 电容器储能

$$W_e = A = \int \overset{\frac{dq}{dt}}{I} U dt = \int U dq$$

$$W_e = \int_0^Q \frac{q}{C} dq = \frac{Q^2}{2C} = \frac{1}{2} CU^2 = \frac{1}{2} QU$$

有无介质 电容器电能公式相同

能量储存于场中



$$W_e = \frac{1}{2}QU = \frac{1}{2}DSEd = \frac{1}{2}DEV$$

电场能量密度为 $w_e = \dfrac{W_e}{V}$

$$\boxed{w_e = \frac{1}{2}\vec{D}\cdot\vec{E}}$$

普遍