2. 最小二乘：$\hat{y}=\hat{w}x+b$ 证：$\sum_{i=1}^{n}(y_i-\bar{y})^2=\sum_{i=1}^{n}(\hat{y}_i-\bar{y})^2+\sum_{i=1}^{n}(y_i-\hat{y}_i)^2$

左 $=\sum_{i=1}^{n}(y_i-\hat{y}_i+\hat{y}_i-\bar{y})^2$

$=\sum_{i=1}^{n}(y_i-\hat{y}_i)^2+\sum_{i=1}^{n}(y_i-\hat{y}_i)^2+\sum_{i=1}^{n}2(y_i-\hat{y}_i)(\hat{y}_i-\bar{y})$

只需证：$\sum_{i=1}^{n}2(y_i-\hat{y}_i)(\hat{y}_i-\bar{y})=0$

$f(w,b)=\sum_{i=1}^{n}(wx_i+b-y_i)^2$ 最小

$$\frac{\partial f}{\partial w}=2\sum_{i=1}^{n}(wx_i+b-y_i)\cdot x_i=0$$

$$\frac{\partial f}{\partial b}=2\sum_{i=1}^{n}(wx_i+b-y_i)=0$$

$\sum_{i=1}^{n}(\hat{y}_i-y_i)=0 \Rightarrow \sum_{i=1}^{n}(\hat{y}_i-y_i)\cdot(b-\bar{y})=0$

$\sum_{i=1}^{n}(\hat{y}_i-y_i)\cdot x_i=0 \Rightarrow \sum_{i=1}^{n}(\hat{y}_i-y_i)\cdot wx_i=0$

$\Rightarrow \sum_{i=1}^{n}(\hat{y}_i-y_i)(\hat{y}_i-\bar{y})=0$

故命题得证。

3.

$$P(Y=k)=e^{\beta_k X-\log Z}=\frac{e^{\beta_k X}}{Z}$$

$$\sum_{j=1}^{k}P(Y=j)=\sum_{j=1}^{k}\frac{e^{\beta_j X}}{Z}=\frac{\sum_{j=1}^{k}e^{\beta_j X}}{Z}=1$$

故：$Z=\sum_{j=1}^{k}e^{\beta_j X}$

故：$P(Y=j)=\frac{e^{\beta_j X}}{Z}=\frac{e^{\beta_j X}}{\sum_{j=1}^{k}e^{\beta_j X}}$