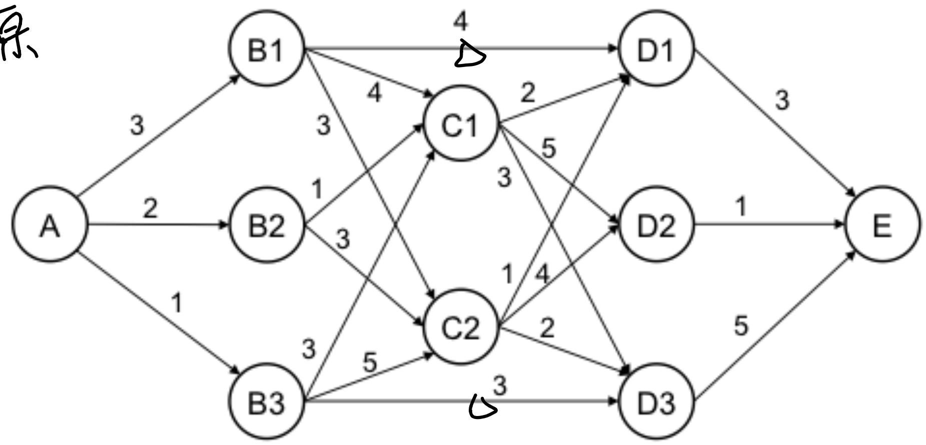
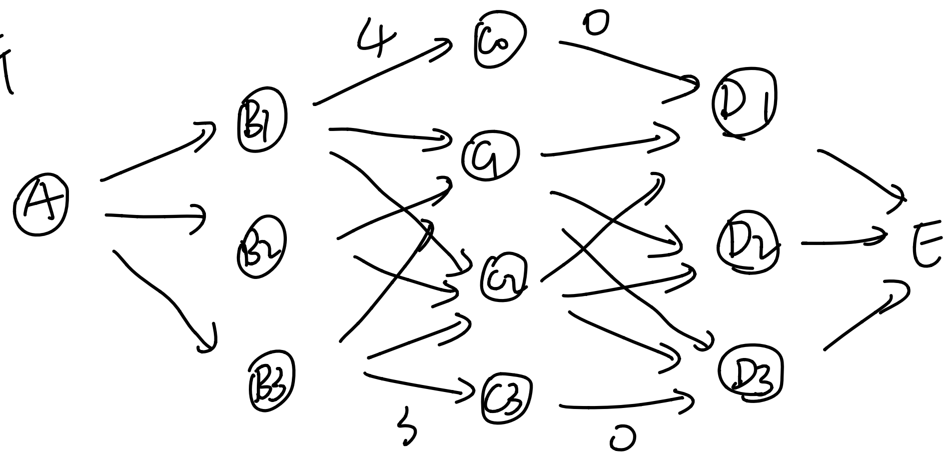
1、原



新



(a) 阶段：

$$\overset{1}{A \rightarrow} \overset{2}{B \rightarrow} \overset{3}{C \rightarrow} \overset{4}{D \rightarrow} \overset{5}{E}$$

状态：

$S_1=\{A\}$

$S_2=\{B_1, B_2, B_3\}$

$S_3=\{C_0, C_1, C_2, C_3\}$

$S_4=\{D_1, D_2, D_3\}$

$S_5=\{E\}$

决策：

$U_1(A)=\{B_1, B_2, B_3\}$

$U_2(B_1)=\{C_0, C_1, C_2\}$

$U_2(B_2)=\{C_1, C_2\}$

$U_2(B_3)=\{C_1, C_2, C_3\}$

$U_3(C_0)=D_1$ $\quad U_3(C_3)=D_3$

$U_3(C_1)=U_3(C_2)=\{D_1, D_2, D_3\}$

$U_4(D_1)=U_4(D_2)=U_4(D_3)=E$

指标函数：$f_k(S_k)$ = 第k阶段，从S_k到E的最优距离.

$$\min \quad f_1(A) = V_{1,5}(S_1, P_{1,5}) = \sum_{k=1}^{5} d_k(s_k, u_k)$$

$$s.t. \quad s_{k+1} = T_k(s_k, u_k) = u_k(s_k)$$

$$s_k \in S_k \ ; \ u_k(s_k) \in U_k(s_k) \ , \quad k = 1, 2, \cdots 5$$

(b) 逆序解法:

$k=5 \quad f_5(E) = 0$

$k=4 \quad f_4(D_1) = d_4(D_1, E) + f_5(E) = 3$

$f_4(D_2) = d_4(D_2, E) + f_5(E) = 1$

$f_4(D_3) = d_4(D_3, E) + f_5(E) = 5$

$k=3 \quad f_3(C_1) = \min\limits_{s_k \in \{D_1, D_2, D_3\}} (d_3(C_1, s_k) + f_4(s_k)) = 5$

$f_3(C_2) = \min\limits_{s_k \in \{D_1, D_2, D_3\}} (d_3(C_2, s_k) + f_4(s_k)) = 4$

$f_3(C_0) = d_3(C_0, D_1) + f_4(D_1) = 3$

$f_3(C_3) = d_3(C_3, D_3) + f_4(D_3) = 5$

$k=2 \quad f_2(B_1) = \min\limits_{s_k \in \{C_0, C_1, C_2\}} (d_2(B_1, s_k) + f_3(s_k)) = 7$

$f_2(B_2) = \min\limits_{s_k \in \{C_1, C_2\}} (d_2(B_2, s_k) + f_3(s_k)) = 6$

$f_2(B_3) = \min\limits_{s_k \in \{C_1, C_2, C_3\}} (d_2(B_3, s_k) + f_3(s_k)) = 8$

$k=1 \quad f_1(A) = \min\limits_{s_k \in \{B_1, B_2, B_3\}} (d_1(A, s_k) + f_2(s_k)) = 8$

$\therefore$ $A \to E$ 最短路径长为 8，是 $A \to B_2 \to C_1 \to D_1 \to E$

2、 3个阶段：分别决定 x_1, x_2, x_3 取值

状态：当前可以赋给 x_1, x_2, x_3 的值

决策：当前状态赋为 S_k 多少值，$x_k = U_k(S_k)$

允许决策集：$0 \le x_1 \le \frac{S_1}{2}$，$0 \le x_2 \le S_2$，$0 \le x_3 \le \frac{S_3}{10}$

则 $S_1 = 6$，$S_2 = S_1 - 2x_1$，$S_3 = S_2 - x_2$

并且 $x_1, x_2, x_3 \geqslant 0$

阶段指标：$8x_1^2$, $4x_2^2$, x_3^3

$\therefore$ $k=4$，$S_4 = 0$

$k=3$，$f_3(S_3) = \max\limits_{0 \le x_3 \le \frac{S_3}{10}} x_3^3 + 0 = \frac{S_3^3}{1000}$

$k=2$，$f_2(S_2) = \max\limits_{0 \le x_2 \le S_2} (4x_2^2 + f_3(S_3))$

$= \max\limits_{0 \le x_2 \le S_2} \left(4x_2^2 + \frac{(S_2 - x_2)^3}{1000}\right)$

$= \begin{cases} \frac{S_2^3}{1000} & S_2 \geqslant 4000 \\ 4S_2^2 & S_2 < 4000 \end{cases}$ （边界取到）

$\to x_2^* = 0$

$\to x_2^* = S_2$

$k=2$ $f_1(S_1) = \max\limits_{0 \le x_1 \le \frac{6}{2}} (8x_1^2 + f_2(S_2)) = \max\limits_{0 \le x_1 \le \frac{6}{2}} \left(8x_1^2 + \frac{(6 - 2x_1)^3}{1000}\right)$

① $s_2 < 4000$ 时，

$$f_1(s_1) = \max_{0 \le x_1 \le \frac{b}{2}} (8x_1^2 + 4(b-2x_1)^2) = \max\{4b^2, 2b^2\} = 4b^2$$

此时：$x_1^* = 0$，$\underline{x_2^* = s_2 = b}$，$x_3^* = s_3 = 0$，最优值 $z^* = 4b^2$.

$\therefore b < 4000$

② $s_2 \geq 4000$ 时，

令 $h(x) = 8x^2 + \frac{(b-2x)^3}{1000}$，$h'(x) = 16x - \frac{6}{1000}(b-2x)$，"V"

$\therefore f_1(s_1) = \max\left\{\frac{b^3}{1000}, 2b^2\right\} = \frac{b^3}{1000}$ $(b<4000)$

此时：$x_1^* = 0$，$x_2^* = 0$，$x_3^* = \frac{b}{10}$，$z^* = \frac{b^3}{1000}$.

3. 值迭代：

① $f_1(v_1) = \infty$ · $f_1(v_2) = 6$ $f_1(v_3) = \infty$ $f_1(v_4) = 2$

② $f_2(v_1) = \min\{0+\infty, 2+6, 3+\infty, \infty+2\} = 8$ $(v_1 \to v_2 \to v_5)$

$f_2(v_2) = \min\{2+\infty, 0+6, 2+\infty, 5+2\} = 6$ $(v_2 \to v_5)$

$f_2(v_3) = \min\{3+\infty, 2+6, 0+\infty, 1+2\} = 3$ $(v_3 \to v_4 \to v_5)$

$f_2(v_4) = \min\{\infty+\infty, 5+6, 1+\infty, 2+0\} = 2$ $(v_4 \to v_5)$

③ $f_3(V_1) = \min\{0+8,\ 2+6,\ 3+3,\ 10+2\} = 6$ $(V_1 \to V_3 \to V_4 \to V_5)$

$f_3(V_2) = \min\{2+8,\ 0+6,\ 2+3,\ 5+2\} = 5$ $(V_2 \to V_3 \to V_4 \to V_5)$

$f_3(V_3) = \min\{3+8,\ 2+6,\ 0+3,\ 1+2\} = 3$ $(V_3 \to V_4 \to V_5)$

$f_3(V_4) = \min\{3+8,\ 2+6,\ 1+3,\ 0+2\} = 2$ $(V_4 \to V_5)$

④ 重复了③，停止。

$\therefore \begin{cases} 1-3-4-5 \\ 2-3-4-5 \\ 3-4-5 \\ 4-5 \end{cases}$

策略迭代法：

初始策略： 1-2-3-4-5

则 $f_1(V_1) = 2 + f_1(V_2)$ $\qquad \hat{f}_1(V_1) = 7$

$f_1(V_2) = 2 + f_1(V_3)$ $\qquad \hat{f}_1(V_2) = 5$

$f_1(V_3) = 1 + f_1(V_4)$ $\qquad \hat{f}_1(V_3) = 3$

$f_1(V_4) = 2 + f_1(V_5)$ $\qquad \hat{f}_1(V_4) = 2$

$\because \quad c_{i p_2(v_i)} + \hat{f}_1(v_{p_2(v_i)}) = \min\limits_{1 \le j \le 5} \{ c_{ij} + \hat{f}_1(v_j) \}$

$\therefore \quad p_2(v_1) = 3 \quad p_2(v_2) = 3 \quad p_3(v_3) = 4 \quad p_4(v_4) = 5$

则 $f_2(v_1) = 3 + f_1(v_3)$ $\quad \hat{f}_2(v_1) = 6$

$f_2(v_2) = 2 + f_1(v_3)$ $\quad \hat{f}_2(v_2) = 5$

$f_2(v_3) = 1 + f_1(v_4)$ $\quad \hat{f}_2(v_3) = 3$

$f_2(v_4) = 2 + f_1(v_5)$ $\quad \hat{f}_2(v_4) = 2$

$\therefore \quad p_3(v_1) = 3, \quad p_3(v_2) = 3, \quad p_3(v_3) = 4, \quad p_3(v_4) = 5$

停止迭代，得最优解.

$$\begin{cases} 1-3-4-5 \\ 2-3-4-5 \\ 3-4-5 \\ 4-5 \end{cases}$$