1、

(a) 任意 $X_1=(x_1,y_1)\in R$, $X_2=(x_2,y_2)\in R$, $\lambda\in[0,1]$, 有:

$$f(\lambda X_1+(1-\lambda)X_2)=f(\lambda x_1+(1-\lambda)x_2,\ \lambda y_1+(1-\lambda)y_2)$$

$$=\max\{\lambda x_1+(1-\lambda)x_2,\ \lambda y_1+(1-\lambda)y_2\}$$

不妨设 $\max\{\lambda x_1+(1-\lambda)x_2,\ \lambda y_1+(1-\lambda)y_2\}$

$$=\lambda x_1+(1-\lambda)x_2$$

又: $x_1\le\max\{x_1,y_1\}$

$x_2\le\max\{x_2,y_2\}$

$\therefore f(\lambda X_1+(1-\lambda)X_2)=\lambda x_1+(1-\lambda)x_2$

$$\le\lambda\max\{x_1,y_1\}+(1-\lambda)\max\{x_2,y_2\}$$

$$=\lambda f(X_1)+(1-\lambda)f(X_2)$$

$\therefore f(x,y)$ 是凸函数

(b) $f(x,y)=\ln(e^x+e^y)$

$$\frac{\partial f}{\partial x}=\frac{e^x}{e^x+e^y},\quad \frac{\partial f}{\partial y}=\frac{e^y}{e^x+e^y}$$

$$\nabla f(x,y) = \left[\frac{e^x}{e^x+e^y} \quad \frac{e^y}{e^x+e^y} \right]^T$$

$$\nabla^2 f(x,y) = \begin{bmatrix} \frac{e^x e^y}{(e^x+e^y)^2} & -\frac{e^x e^y}{(e^x+e^y)^2} \\ -\frac{e^x e^y}{(e^x+e^y)^2} & \frac{e^x e^y}{(e^x+e^y)^2} \end{bmatrix}$$

$$= \frac{e^x e^y}{(e^x+e^y)^2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$ 是半正定矩阵

$\therefore$ $f(x,y)$ 是凸函数

2. $f(x) = -3x^2 + 21.6x + 1$, $x \in [0, 25]$

黄金分割法:

	8%*25=2						
k	|ak-bk|	ak	bk	tk'	tk	f(tk')	f(tk)
0	25	0	25				
1	15.45	0	15.45	9.55	15.45	-66.3275	-381.3875
2	9.5481	0	9.5481	5.9019	9.5481	23.98376917	-66.25968083
3	5.9007258	0	5.9007258	3.6473742	5.9007258	39.87326706	23.99998238
4	3.646648544	2.254077256	5.9007258	2.254077256	3.646648544	34.4454759	39.87347174
5	2.2536288	2.254077256	4.507706056	3.647097	4.507706056	39.87334562	37.40820915
6	1.392742599	3.114963457	4.507706056	3.114963457	3.646819854	39.17421866	39.8734237
		0.5(ak+bk)		f[0.5(ak+bk)]			
		3.811334757		39.74601286			

即 $x^* = 3.8113$, $f(x^*) = 39.7460$

Fibonacci法：

δ=8%*25=2,n=6,Fn=13									
k	Fn-k	Fn-k+1	|ak-bk|	ak	bk	tk'	tk	f(tk')	f(tk)
0	13	21	25	0	25				
1	8	13	15.38461538	0	15.38461538	9.615384615	15.38461538	-68.67455621	-376.7514793
2	5	8	9.615384615	0	9.615384615	5.769230769	9.615384615	25.76331361	-68.67455621
3	3	5	5.769230769	0	5.769230769	3.846153846	5.769230769	39.69822485	25.76331361
4	2	3	3.846153846	1.923076923	5.769230769	1.923076923	3.846153846	31.44378698	39.69822485
5	1	2				3.846153846	3.846153846	39.69822485	39.69822485
				x*		f(x*)			
				3.846153846		39.69822485			

$t_5 = 3.8462$， $f(t_5) = 39.6982$.

取 $t_5' = a_4 + (\frac{1}{2} + \varepsilon)(b_4 - a_4) = 3.8465$ $(\varepsilon = 0.0001)$

$f(t_5') = 39.6977$

$f(t_5) > f(t_5')$

∴ 取最终区间 $[1.9231, 3.8465]$

$x^* = \frac{1}{2}(1.9231 + 3.8465) = 2.8848$， $f(x^*) = 38.3455$

3、 令 $t = x - \pi$ ， $x = t + \pi$

$\varphi(t) = f(t+\pi) = \sin(t+\pi) = -\sin t$ ， $t \in [0, 2\pi]$

非精确搜索的Goldstein法：

① 选择 $\alpha > 1$， $0 < m_1 < m_2 < 1$， $a_0 = 0$， $b_0 = 2\pi$， $k = 0$

比如此处选则 $\alpha=2$，$m_1=0.3$，$m_2=0.7$

则 $g_1(t)=\varphi(0)+m_1\varphi'(0)t=-0.3t$

$g_2(t)=\varphi(0)+m_2\varphi'(0)t=-0.7t$ ，$t_0=\pi$

② 计算 $\varphi(t_k)$，若 $\varphi(t_k)\le g_1(t_k)$，到 ③.

否则，令 $a_{k+1}=a_k$，$b_{k+1}=t_k$，到 ④.

③ 若 $\varphi(t_k)\ge g_2(t_k)$，停止迭代，输出 t_k.

否则，令 $a_{k+1}=t_k$，$b_{k+1}=b_k$，到 ④、

④ 令 $t_{k+1}=0.5(a_{k+1}+b_{k+1})$，到 ⑤.

⑤ 用 $k+1$ 替代 k，回到 ②.

1. 判断以下函数是否为凸函数，并给出理由。

   (a) $f(x,y)=\max(x,y)$，$x,y\in\mathbb{R}$。

   (b) $f(x,y)=\ln(e^x+e^y)$，$x,y\in\mathbb{R}$。

2. 分别用黄金分割法和斐波那契法求函数

$$f(x)=-3x^2+21.6x+1$$

   在区间$[0,25]$上的极大值和极大点，要求最后的区间长度不大于初始区间的8%。

3. 给出用Goldstein法对$f(x)=\sin(x)$在区间$x\in[\pi,3\pi]$上做非精确搜索的步骤。