第二章 电势

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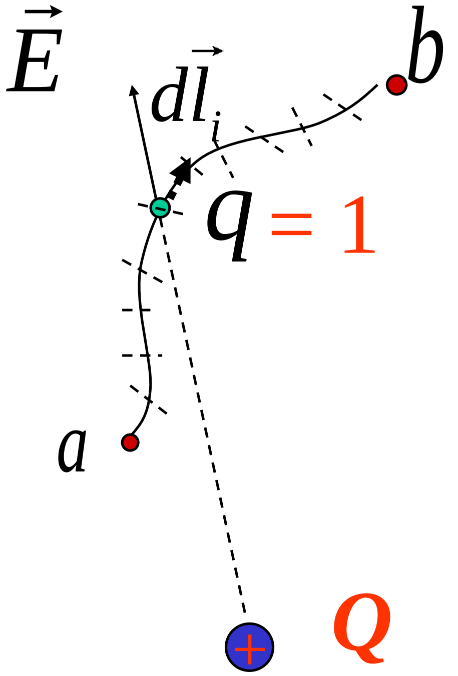
Electric potential

2.1 静电场的环路定理



单个电荷电场

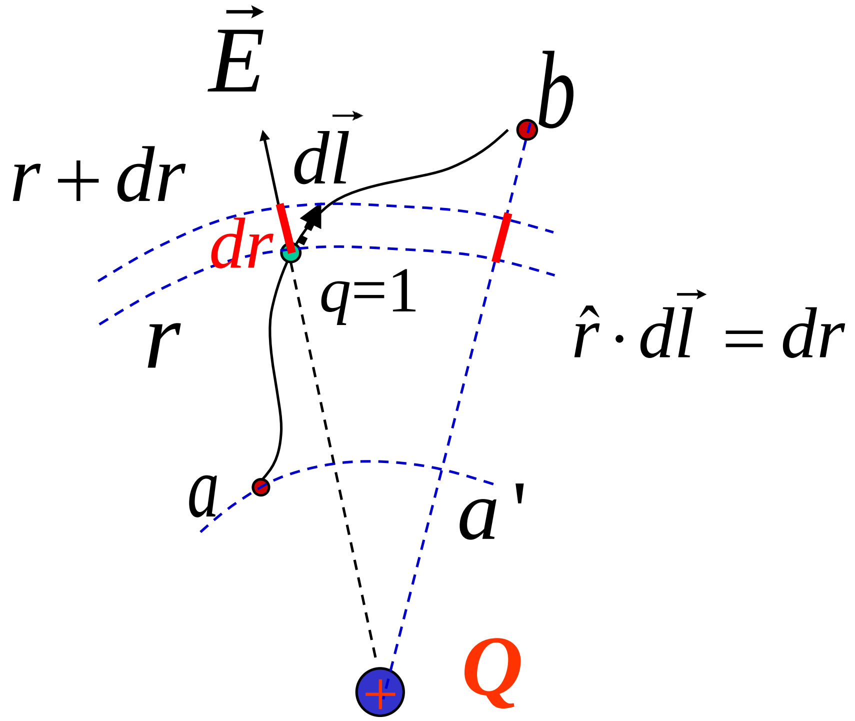
外力克服静电场力作功

$$W = -\int_{(a)}^{(b)} \vec{F} \cdot d\vec{l}$$

$$= -\sum_i \vec{F}_i \cdot d\vec{l}_i$$

$$W(\text{unit}) = -\int_{(a)}^{(b)} \vec{E} \cdot d\vec{l}$$

单个电荷电场



$$-\int_{(a)}^{(b)} \vec{E}\cdot d\vec{l} = -\frac{Q}{4\pi\varepsilon_0}\int_a^b \frac{\hat{r}\cdot d\vec{l}}{r^2}$$

$$= -\frac{Q}{4\pi\varepsilon_0}\int_a^b \frac{dr}{r^2}$$

$$= -\frac{Q}{4\pi\varepsilon_0}\int_{a'}^b \frac{dr}{r^2}$$

$$\boxed{-\int_{(a)}^{(b)} \vec{E}\cdot d\vec{l} = -\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_a}-\frac{1}{r_b}\right)}$$

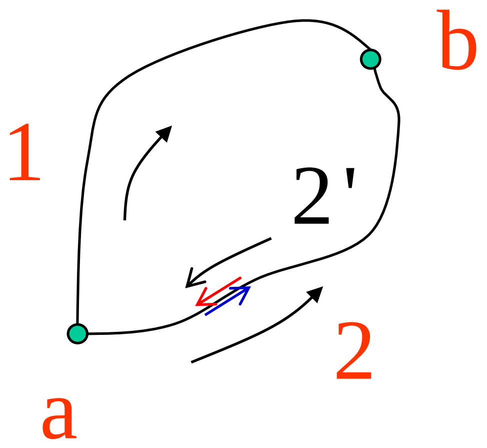
只和初末位置有关, 路径无关

多个点电荷场，
利用场强叠加原理

$$-\int_{(a)}^{(b)} \vec{E}\cdot d\vec{l} = -\int_{(a)}^{(b)} \sum_i \vec{E}_i \cdot d\vec{l}$$

$$= -\sum_i \int_{(a)}^{(b)} \vec{E}_i \cdot d\vec{l}$$

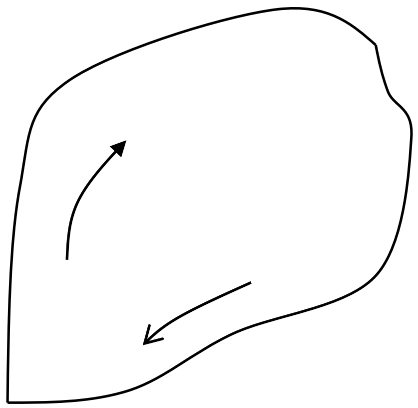
只和初末位置有关，路径无关



$$-\int_{(1)} \vec{E}\cdot d\vec{l} = -\int_{(2)} \vec{E}\cdot d\vec{l}$$

$$-\int_{(1)} \vec{E}\cdot d\vec{l} = \int_{(2')} \vec{E}\cdot d\vec{l}$$

$$\int_{(1)} \vec{E}\cdot d\vec{l} + \int_{(2')} \vec{E}\cdot d\vec{l} = 0$$



$$\boxed{\oint_L \vec{E}\cdot d\vec{l} = 0}$$

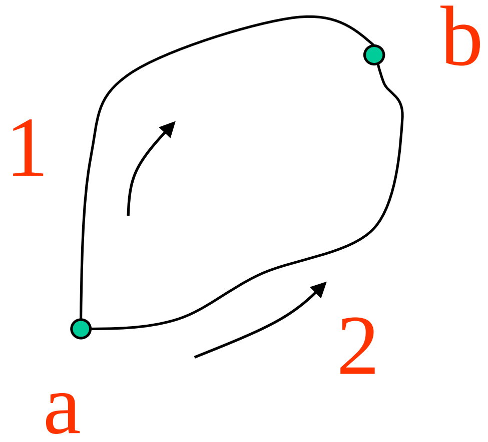
静电场环路定理

静电场的保守性

⇓

静电场线不能闭合

2.2　电势和叠加原理

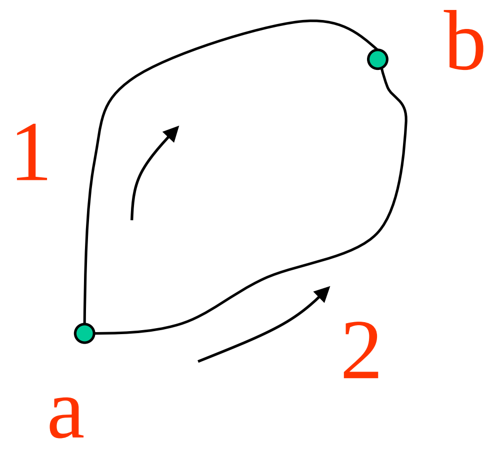


$$\phi(b)-\phi(a)=-\int_{(a)}^{(b)}\vec{E}\cdot d\vec{l}$$

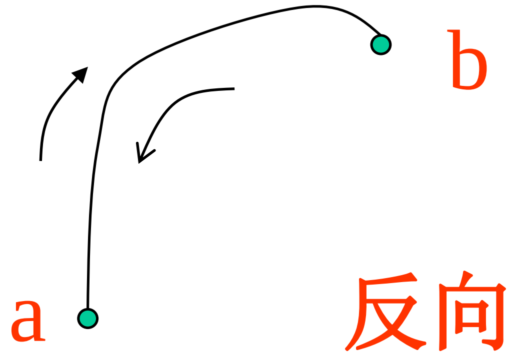
只和初末位置有关,路径无关

利用这个积分定义函数差 — 电势差

为什么是函数差？



$$\phi(b)-\phi(a)=-\int_{(a)}^{(b)}\vec{E}\cdot d\vec{l}$$



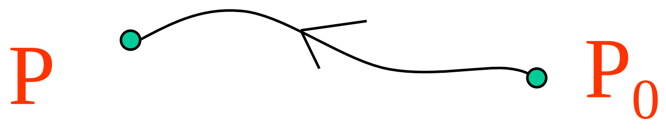
反向要变号

- 电势的相对值有意义
- 沿电场线电势是下降的

高压带电实验

设参考点 P_0

$$\phi(P_0)=0$$



电势
$$\boxed{\phi(P)=-\int_{P_0}^{P}\vec{E}\cdot d\vec{l}}$$

点电荷电势
$$\phi(\mathrm{b})-\phi(\mathrm{a})=-\int_{(a)}^{(b)}\vec{E}\cdot d\vec{l}=-\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_a}-\frac{1}{r_b}\right)$$

P_0 选在无限远处
$$\phi(\mathrm{P})=-\int_{\infty}^{P}\vec{E}\cdot d\vec{l}=\frac{Q}{4\pi\varepsilon_0}\frac{1}{r}$$

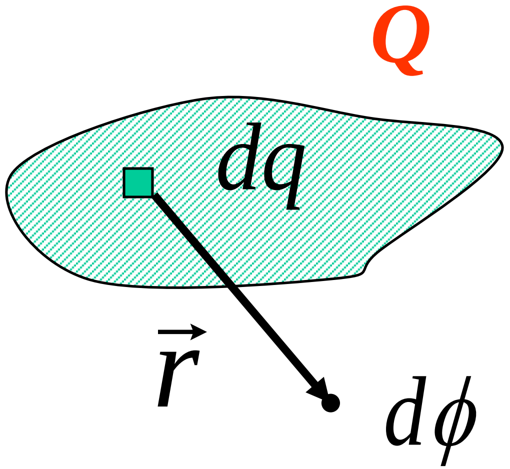
电势叠加原理

$$\phi = -\int_{p_0}^{p} \vec{E} \cdot d\vec{l} = -\int_{P_0}^{P} \sum_i \vec{E}_i \cdot d\vec{l}_i$$

$$= \sum_i \left(-\int_{P_0}^{P} \vec{E}_i \cdot d\vec{l}_i \right) = \sum_i \phi_i$$

点电荷系 $\phi = \sum_i \frac{1}{4\pi\varepsilon_0} \frac{q_i}{r_i}$

若带电体可看作是电荷连续分布的



$$\phi = \int_{(Q)} \frac{1}{4\pi\varepsilon_0} \frac{dq}{r}$$

电势叠加原理

体电荷分布 $dq = \rho dV$

面电荷分布 $dq = \sigma dS$

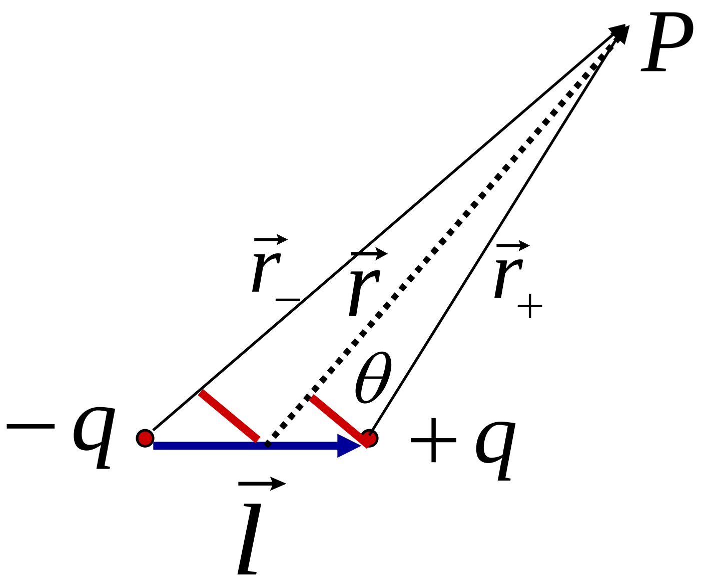
线电荷分布 $dq = \lambda dl$

例1 电偶极子势

解：

$$\phi = \phi_+ + \phi_-$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_+} - \frac{1}{r_-}\right)$$



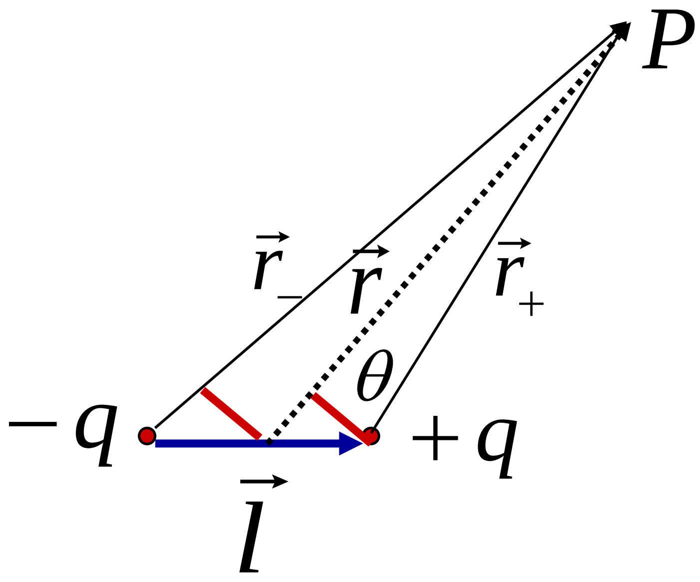
$$r \gg l$$

$$r_+ = r - \frac{l}{2}\cos\theta$$

$$r_- = r + \frac{l}{2}\cos\theta$$

$$r_- - r_+ \approx l\cos\theta$$

$$r_+ r_- = r^2$$



$$\phi = \frac{q}{4\pi\varepsilon_0}\frac{r_- - r_+}{r_- r_+} = \frac{ql\cos\theta}{4\pi\varepsilon_0 r^2}$$

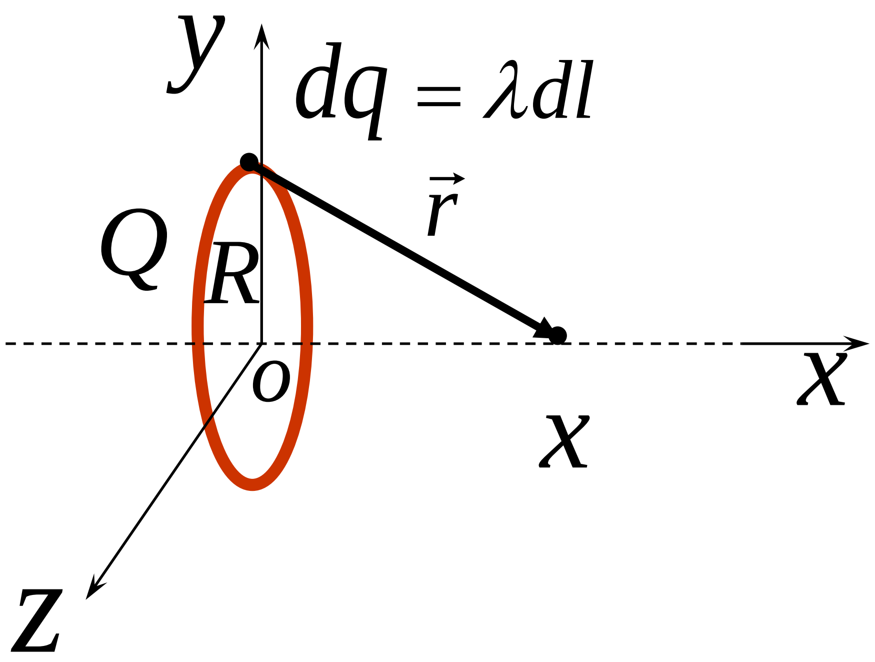
代入

$$\vec{p} = q\vec{l}$$

$$lr\cos\theta = \vec{l}\cdot\vec{r}$$

$$\phi = \frac{\vec{p}\cdot\vec{r}}{4\pi\varepsilon_0 r^3}$$

例2 均匀带电圆环轴线上的电势



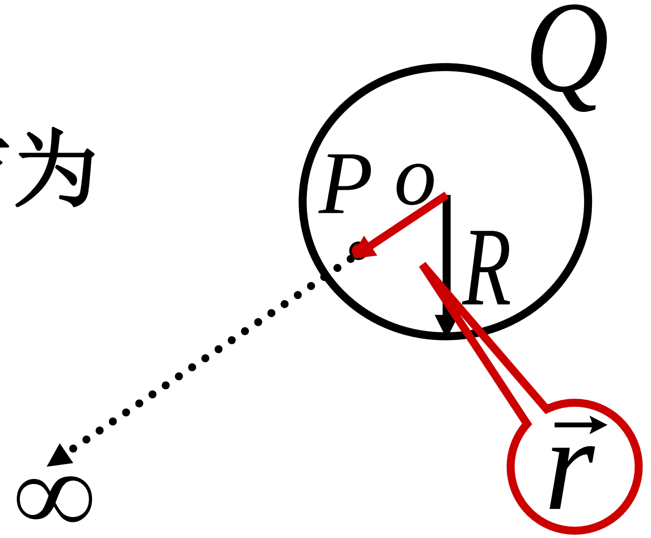
解：

在圆环上任取电荷元 dq

$$d\phi = \frac{dq}{4\pi\varepsilon_0 r}$$

$$\phi = \int \frac{dq}{4\pi\varepsilon_0 r} = \frac{Q}{4\pi\varepsilon_0 r} = \frac{Q}{4\pi\varepsilon_0}\frac{1}{\sqrt{R^2 + x^2}}$$

例3 计算均匀带电球面的电势



解：均匀带电球面电场的分布为

$$r < R \quad E = 0$$

$$r > R \quad \vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$$

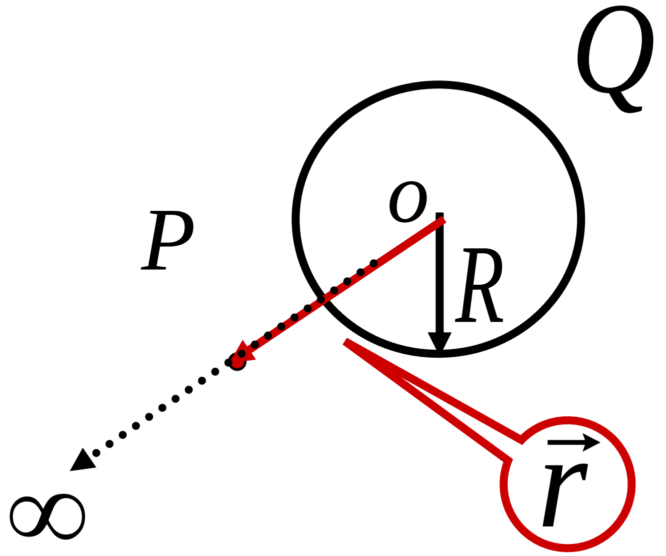
若场点在球内　即　$r < R$　如图

$$\phi = \int_P^\infty \vec{E}\cdot d\vec{l} = \int_r^R 0dl + \int_R^\infty \frac{Q}{4\pi\varepsilon_0 r^2}dr = \frac{Q}{4\pi\varepsilon_0 R}$$

等势体

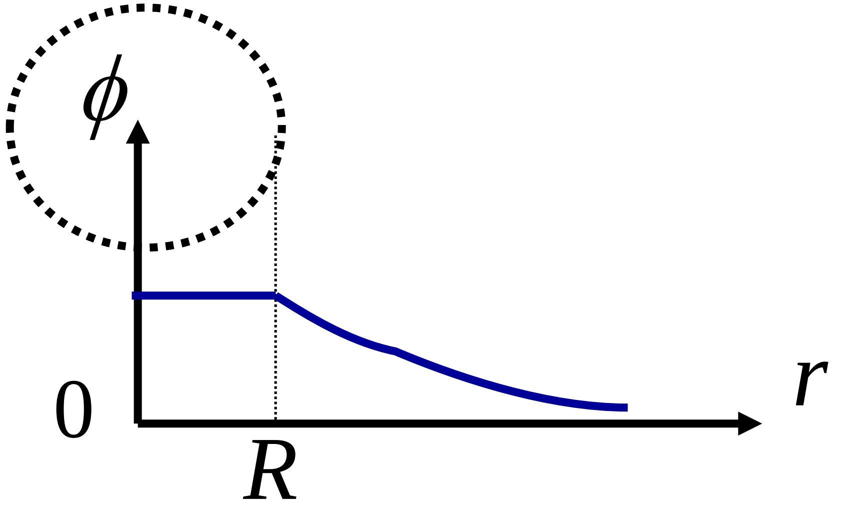
场点在球面外　即 $r > R$

$$\phi = \int_r^{\infty} \frac{Q}{4\pi\varepsilon_0 r^2} dr = \frac{Q}{4\pi\varepsilon_0 r}$$



与电量集中在球心的点电荷的电势分布相同

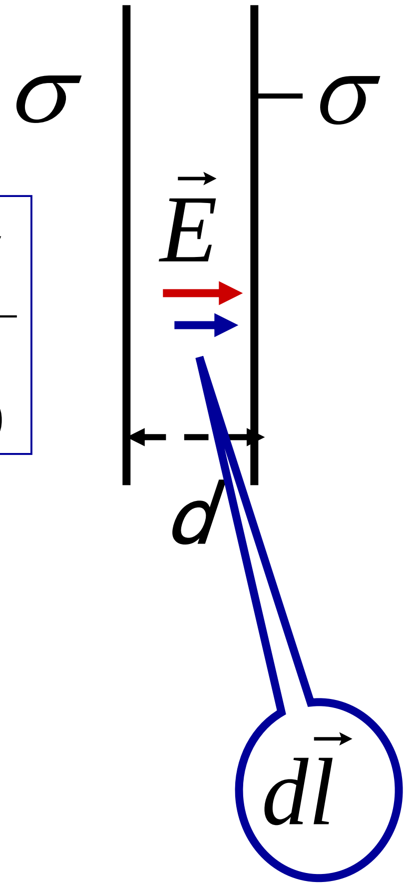
图示



例4. 平行板电容器两板间的电势差

解：

平行板电容器内部的场强为 $\boxed{E = \dfrac{\sigma}{\varepsilon_0}}$



两板间的电势差

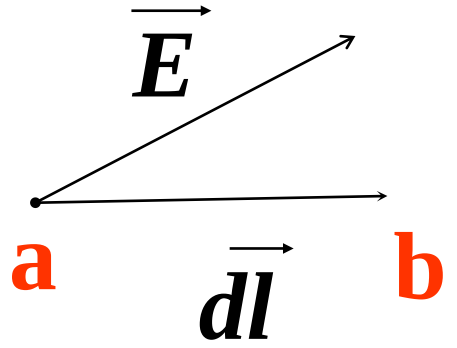
$$\Delta\phi = \int_{(+)}^{(-)} \vec{E} \cdot d\vec{l} = \int_{(+)}^{(-)} E dl$$

$$= E \int_{(+)}^{(-)} dl = Ed$$

2.3 电势梯度

一. 电场强度与电势梯度

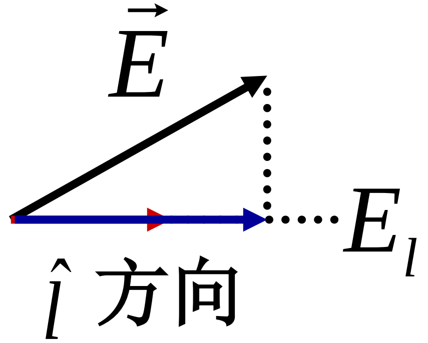
$$-\int_a^b \vec{E}\cdot d\vec{l} = \phi_b - \phi_a$$



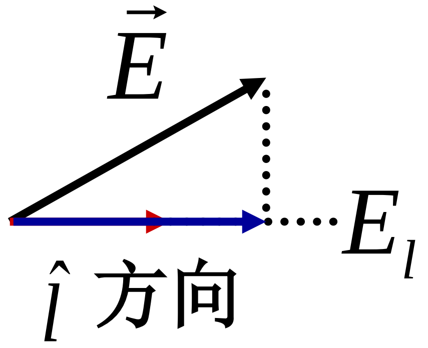
$$\downarrow$$

$$-E_l dl = d\phi$$

$$E_l = -\frac{d\phi}{dl}$$



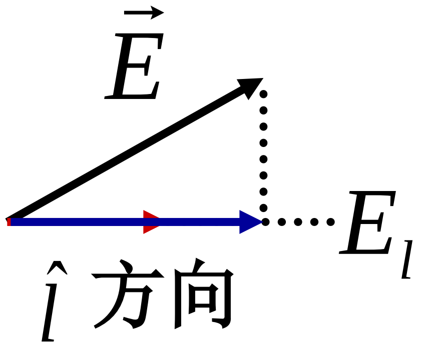
$$\boxed{E_l = -\frac{\partial \phi}{\partial l}}$$



即电场强度在 l 方向的分量值

等于电势在 l 方向的方向导数的负值

$$E_x = -\frac{\partial \phi}{\partial x} \qquad E_y = -\frac{\partial \phi}{\partial y} \qquad E_z = -\frac{\partial \phi}{\partial z}$$



$$E_l = -\frac{\partial \phi}{\partial l}$$

哪个方向导数最大？

沿电场方向

数学上最大的方向导数叫梯度 ***grad***

$$\boxed{\vec{E} = -grad\,\phi}$$

$$\boxed{\vec{E} = E_x\vec{i} + E_y\vec{j} + E_z\vec{k}}$$

$$\vec{E} = -\frac{\partial \phi}{\partial x}\vec{i} - \frac{\partial \phi}{\partial y}\vec{j} - \frac{\partial \phi}{\partial z}\vec{z}$$

在直角坐标系中

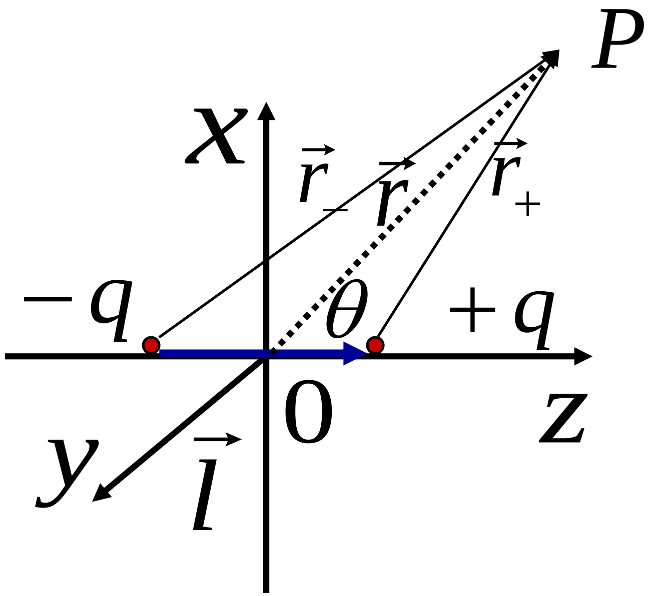
梯度算符

$$\nabla = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}$$

$$\boxed{\vec{E} = -\nabla\phi}$$

$$E_l = -\frac{\partial\phi}{\partial l} = -(\nabla\phi)\cdot\hat{l}$$

例5 利用电势梯度求电偶极子任一点电场强度



$$\phi = \frac{\vec{p}\cdot\vec{r}}{4\pi\varepsilon_0 r^3}$$

$$\phi = \frac{pz}{4\pi\varepsilon_0 (x^2+y^2+z^2)^{3/2}}$$

$$E_x = -\frac{\partial \phi}{\partial x} = \frac{pz}{4\pi\varepsilon_0} \frac{3x}{(x^2+y^2+z^2)^{5/2}} = \frac{p}{4\pi\varepsilon_0} \frac{3xz}{r^5}$$

$$E_y = -\frac{\partial \phi}{\partial y} = \frac{p}{4\pi\varepsilon_0} \frac{3yz}{r^5}$$

$$E_z = -\frac{\partial \phi}{\partial z} = -\frac{p}{4\pi\varepsilon_0} \left(\frac{1}{r^3} - \frac{3z^2}{r^5}\right)$$

$$\boxed{\vec{E} = E_x \vec{i} + E_y \vec{j} + E_z \vec{k}}$$

$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0 r^3}\left(-\vec{p} + \left(3\vec{p}\cdot\hat{r}\right)\hat{r}\right)}$$

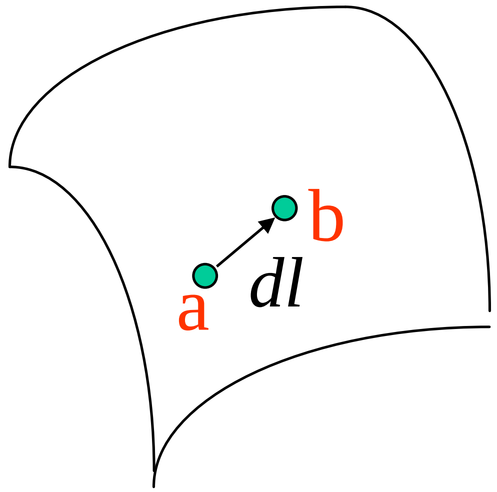
二. 等势面

由电势相等的点组成的面叫等势面

满足方程

$$\phi(x, y, z) = C$$

$$-E_l dl = d\phi = 0$$



电力线处处垂直等势面

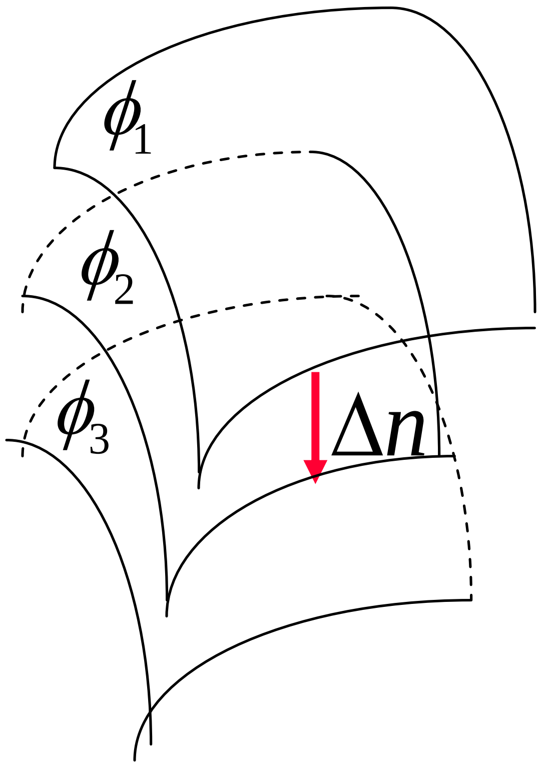
当常量 C 取等间隔数值时

一系列的等势面

$$\Delta\phi_{12} = \Delta\phi_{23}$$

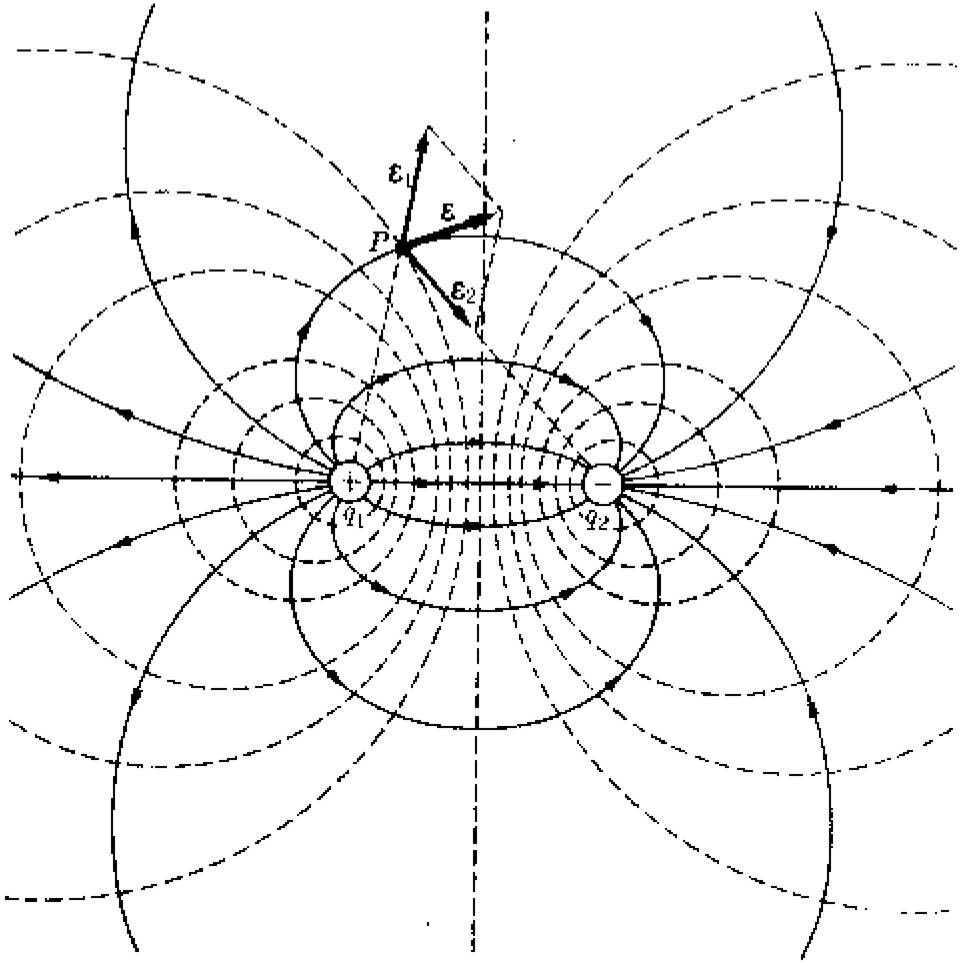
$$\phi_1 < \phi_2 < \phi_3$$

$$\Delta\phi \approx -E\Delta n$$

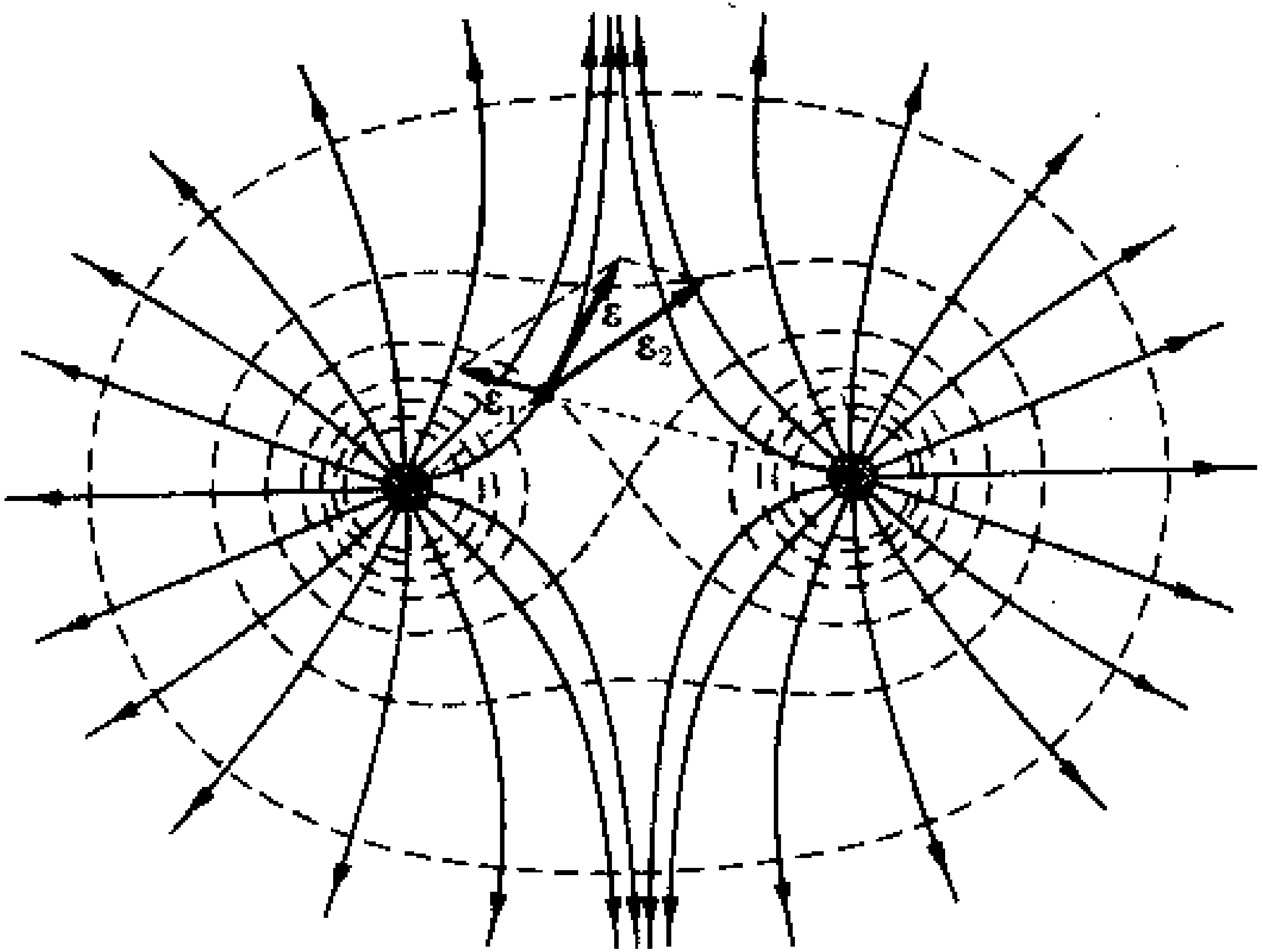


等势面的疏密反映了场的强弱

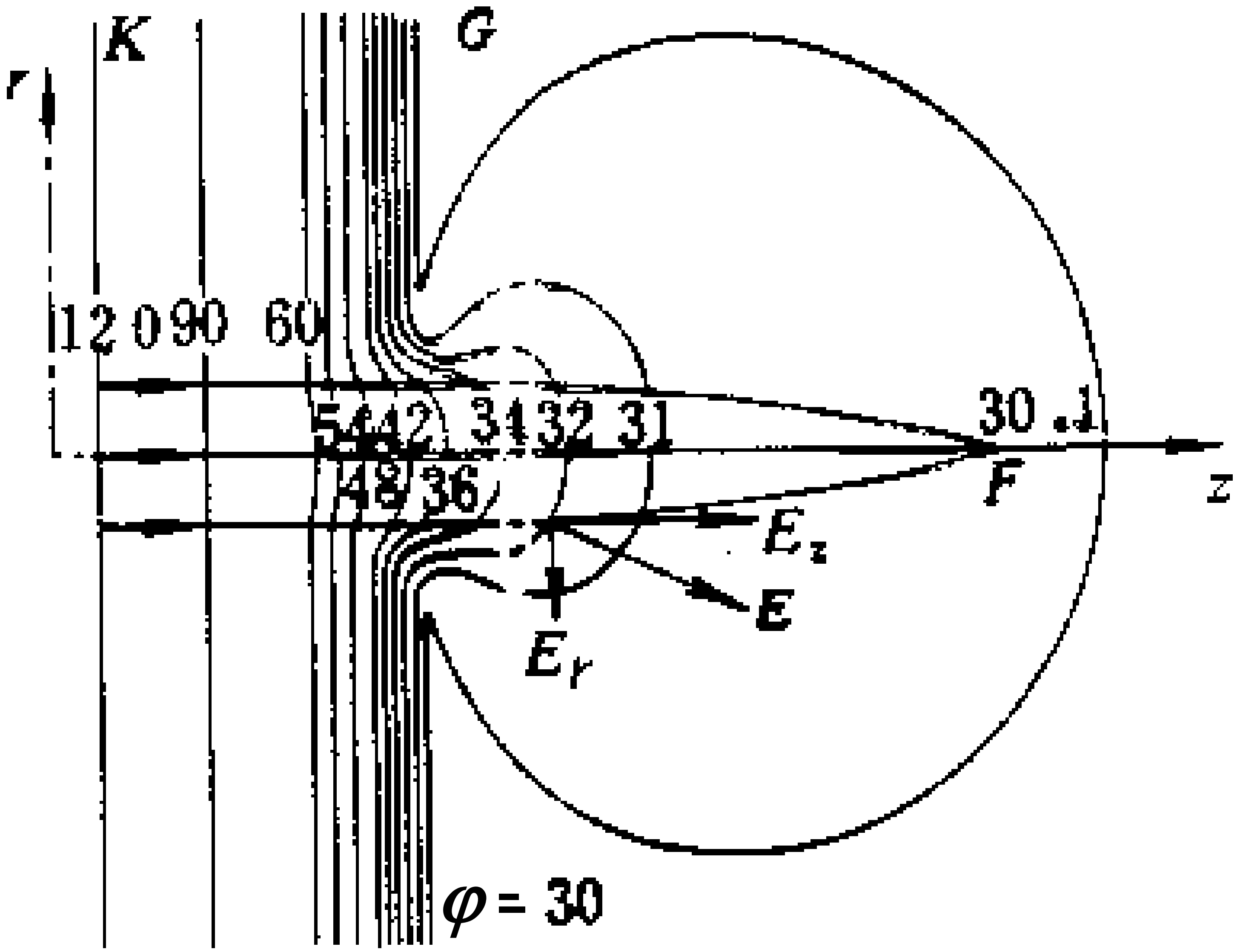
▲某些等势面：



电偶极子的电场线和等势面



两个等量的正电荷的电场线和等势面

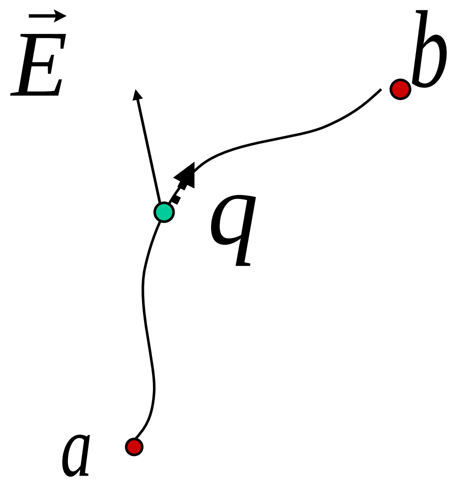


静电透镜的等势面

2.4　电荷系的静电能

1. 点电荷在静电场中的电势能

外力克服静电力作的功等于相应电势能的增量



$$W_{ab} = -\int_a^b \vec{f} \cdot d\vec{l} = W_b - W_a$$

电势和电势能取同一零点

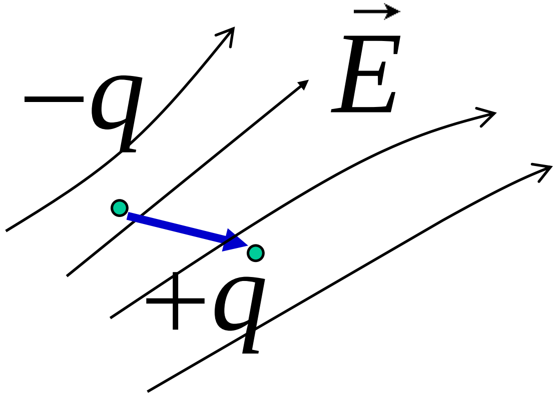
$$-q\int_a^b \vec{E} \cdot d\vec{l} = W_b - W_a$$

$$\boxed{W = q\phi}$$

$$-\int_a^b \vec{E} \cdot d\vec{l} = \phi_b - \phi_a$$

静电场电势能只与位置有关

2. 电偶极矩在静电场中电势能



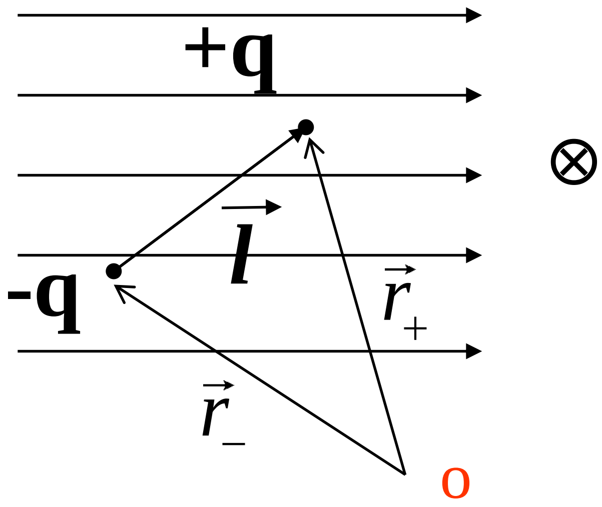
$$
\begin{aligned}
W &= +q\phi(\vec{r}_+) - q\phi(\vec{r}_-) \\
&= q[\phi(\vec{r}_+) - \phi(\vec{r}_-)] \\
&= -q\vec{E}\cdot\vec{l} = -\vec{p}\cdot\vec{E}
\end{aligned}
$$

$$\boxed{W = -\vec{p}\cdot\vec{E}}$$

电偶极子作为整体在外场中能量

没有考虑偶极子自能

另一个角度认识电偶极矩在静电场中电势能：



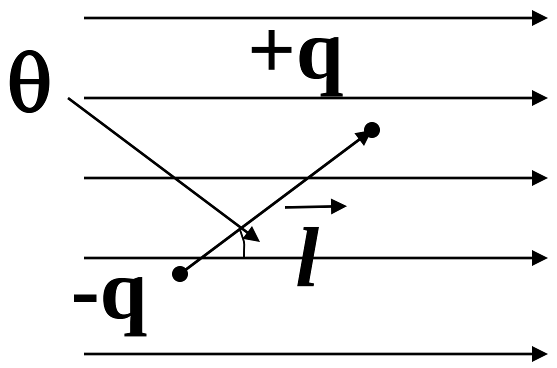
在均匀电场中

总电场力为零

电场力矩

$$\vec{M} = \vec{r}_+ \times q\vec{E} - \vec{r}_- \times q\vec{E} = q(\vec{r}_+ - \vec{r}_-) \times \vec{E} = \vec{p} \times \vec{E}$$

$$\boxed{\vec{M} = \vec{p} \times \vec{E}}$$



$\otimes$

$$\left|\vec{M}\right| = \left|\vec{p} \times \vec{E}\right| = pE\sin\theta$$

力矩垂直屏面，电偶极子只在屏面转动

θ=π/2 时电势能为零

外力矩克服电场力矩做功 $W_\theta = \int_{\pi/2}^{\theta} M d\theta$

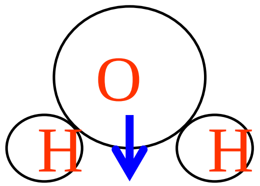
$$W_\theta = \int_{\pi/2}^{\theta} pE\sin\theta d\theta = -pE\cos\theta = -\vec{p}\cdot\vec{E}$$

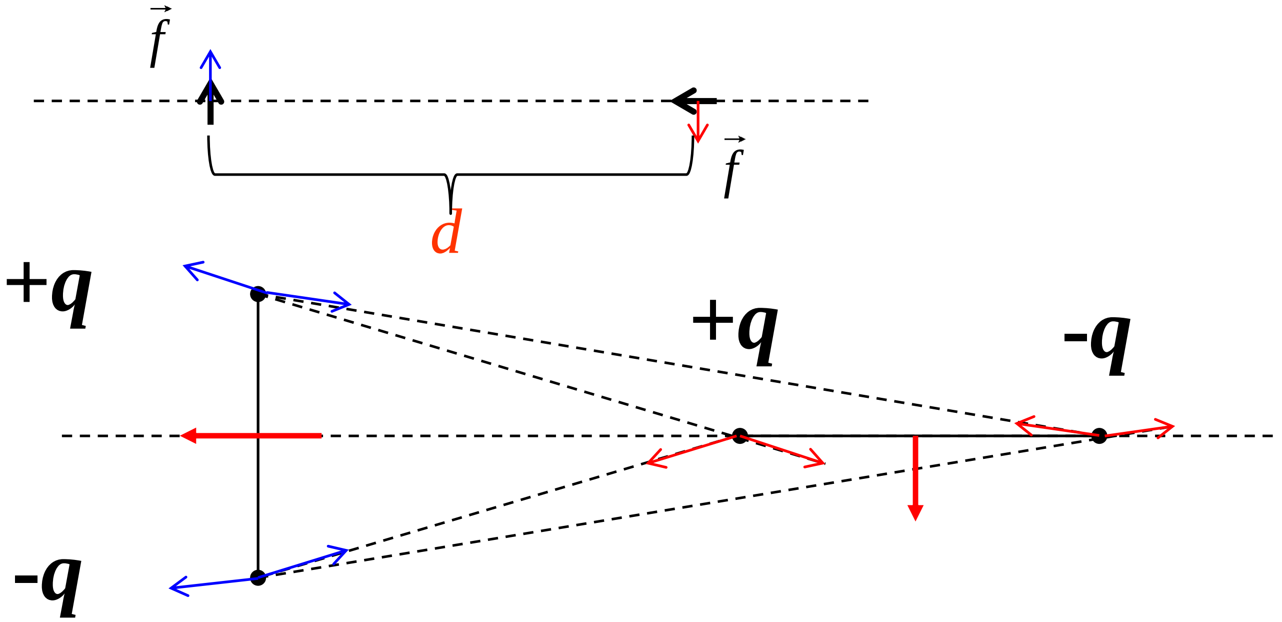
电偶极矩相互作用力*

极性分子之间的电偶极矩力

取向力 范德瓦尔斯（van der Waals）力

H_2O





相互作用力大小相等方向相反

不沿着一条直线 $\vec{d}\times\vec{f}$ $\otimes$

$\vec{p}\times\vec{E}$ $\odot$ 总力矩为零

3.　电荷系静电能**

两个电荷

$q_1 \quad r \quad q_2$

想象 $q_1\,q_2$ 初始时相距无限远

第一步　先把 q_1 摆在某处　　外力不作功

第二步　再把 q_2　从无限远移过来　使系统处于上面的状态，外力克服 q_1 的场作功

$$W = q_2\phi_{21}$$

q_1 在 q_2 所在处的电势

也可以先移动 q_2

$$W = q_1\phi_{12}$$

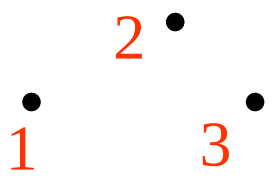
q_2 在 q_1 所在处的电势

$$= q_2\phi_{21}$$

$$W = \frac{1}{2}q_1\phi_{12} + \frac{1}{2}q_2\phi_{21} = \frac{1}{2}q_1\phi_1 + \frac{1}{2}q_2\phi_2$$

$$W = q_1\phi_{12} + q_2\phi_{23} + q_3\phi_{31}$$

三个电荷:



$$= \frac{1}{2}q_1\phi_{12} + \frac{1}{2}q_2\phi_{21} + \frac{1}{2}q_2\phi_{23} + \frac{1}{2}q_3\phi_{32} + \frac{1}{2}q_3\phi_{31} + \frac{1}{2}q_1\phi_{13}$$

$$= \frac{1}{2}q_1(\phi_{12} + \phi_{13}) + \frac{1}{2}q_2(\phi_{21} + \phi_{23}) + \frac{1}{2}q_3(\phi_{32} + \phi_{31})$$

$$= \frac{1}{2}q_1\phi_1 + \frac{1}{2}q_2\phi_2 + \frac{1}{2}q_3\phi_3$$

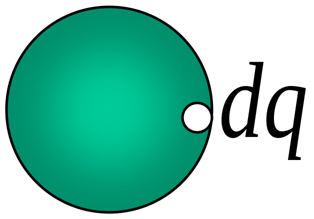
N个电荷:

$$\boxed{W = \frac{1}{2}\sum_i q_i\phi_i}$$

ϕ_i —— 除 q_i 以外的电荷在 q_i 处的电势

若带电体连续分布:

ϕ ：所有电荷在**dq**处的电势



$$\boxed{W = \frac{1}{2}\int_{(Q)} dq\phi}$$

例6 均匀带电球壳 带电量 Q 半径 r，求电势能

$$\phi = \frac{Q}{4\pi\varepsilon_0 r}$$

$$W = \frac{1}{2}\int_{(Q)} dq \frac{Q}{4\pi\varepsilon_0 r} = \frac{Q^2}{8\pi\varepsilon_0 r}$$

ϕ 应是**dq**以外的电荷在**dq**处产生的电势

dq 自身处电势？ $W=\frac{1}{2}\int_{(Q)} dq\phi$

体电荷分布

dq均匀带电球 半径为**R**

电势 $\propto \frac{dq}{4\pi\varepsilon_0 R} \sim \rho R^2 \xrightarrow{R\to 0} 0$

面电荷分布

dq 是薄圆片 电势 $\sim \sigma R \xrightarrow{R\to 0} 0$

dq 对ϕ的贡献总是零

线电荷分布时又如何? $W=\frac{1}{2}\int_{(Q)} dq\phi$

dq 自身处电势

dq 线度趋于零时是对数发散的

dq 以外电荷在 dq 处的电势

线电荷分布电势能仍然是发散的

点电荷自能发散

$$W=\frac{Q^2}{8\pi\varepsilon_0 r}\xrightarrow{r\longrightarrow 0}\infty$$

真空中静电场小结

1. 理论框架

库仑定律

\+ 叠加原理

$$\oiint_S \vec{E}\cdot d\vec{S} = \frac{\sum_i q_{i内}}{\varepsilon_0}$$

$$\oint_L \vec{E}\cdot d\vec{l} = 0$$

ϕ

- 静电场范围等价
- 场量描述更具普遍性

2. 计算静电场方法

$$\vec{E} \quad \phi$$

$$\boxed{\vec{f} = q\vec{E}} \qquad \boxed{W_e = q\phi}$$

- 点电荷公式+叠加原理
- 对称性 + 高斯定理

$$\phi = \int_{(P)}^{P(0)} \vec{E} \cdot d\vec{l}$$

$$\vec{E} = -\nabla \phi$$