1. 由多个基础坐标变换矩阵合成最终的坐标变换矩阵时，如果每次坐标系旋转都是相对于第一个固定坐标系的某个轴，则最终旋转矩阵的合成规则为依次左乘基础旋转变换矩阵，即：${}^{0}_{2}R={}^{1}_{2}R\cdot{}^{0}_{1}R$，请给出一种该公式的推导过程。

此处认为 ${}^{0}_{1}R$ 为绕固定 z 轴旋转 θ 角的变换矩阵、

${}^{1}_{2}R$ 为绕中间轴 y_1 旋转 α 角的变换矩阵.

定义：绕中间轴坐标变换，可依次得到 Σ_1 和 Σ_2

绕固定轴坐标变换，可依次得到 Σ_1 和 Σ_{2^*}

显然有：

$${}^{0}P = {}^{0}_{1}R \cdot {}^{1}P$$

$${}^{0}P = {}^{0}_{2}R \cdot {}^{2}P$$

$${}^{1}P = {}^{1}_{2}R \cdot {}^{2}P$$

$${}^{0}P = {}^{0}_{2^*}R \cdot {}^{2^*}P$$

$${}^{1}P = {}^{1}_{2^*}R \cdot {}^{2^*}P$$

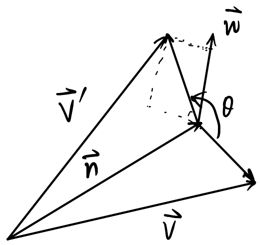
而： 绕 y 轴正转 $\alpha \Leftrightarrow$ 绕 z 轴正转 θ + 绕 y_1 轴正转 α + 绕 z 轴逆转 θ

故有：${}^{1}P = {}^{1}_{0}R \; {}^{1}_{2}R \; {}^{0}_{1}R \; {}^{2^*}P$

而： ${}^{0}P = {}^{0}_{1}R \cdot {}^{1}P$ ， ${}^{0}P = {}^{0}_{2^*}R \cdot {}^{2^*}P$

故：${}^{0}_{2^*}R = {}^{1}_{2}R \; {}^{0}_{1}R$

2. 坐标系{0}绕通过原点的直线逆时针旋转角(右手系)后得到坐标系{1}，请用的单位向量为[nx, ny, nz]^{T}和旋转角度表示坐标变换矩阵${}^{0}_{1}R$，并给出完整的推导过程。



$$\vec{V} = \vec{V}_{\parallel} + \vec{V}_{\perp}$$

$$\vec{V}_{\parallel} = (\vec{V}\cdot\vec{n})\,\vec{n}$$

$$\vec{V}_{\perp} = \vec{V} - (\vec{V}\cdot\vec{n})\vec{n}$$

$$\vec{w} = \vec{n}\times\vec{V}_{\perp} = \vec{n}\times\vec{V}$$

$$\vec{V}' = \vec{V}_{\parallel}' + \vec{V}_{\perp}' \qquad \vec{V}_{\parallel}' = \vec{V}_{\parallel}$$

$$\vec{V}_{\perp}' = \vec{w}\sin\theta + \vec{V}_{\perp}\cos\theta = \cos\theta\,(\vec{V} - (\vec{V}\cdot\vec{n})\cdot\vec{n}) + \sin\theta\,(\vec{n}\times\vec{V}).$$

$$\vec{V}' = \vec{V}_{\parallel}' + \vec{V}_{\perp}' = (\vec{V}\cdot\vec{n})\vec{n} + \cos\theta\,(\vec{V} - (\vec{V}\cdot\vec{n})\cdot\vec{n}) + \sin\theta\,(\vec{n}\times\vec{V}). =$$

$$= (1-\cos\theta)(\vec{V}\cdot\vec{n})\vec{n} + \cos\theta\cdot\vec{V} + \sin\theta\,(\vec{n}\times\vec{V})$$

$$P = [1, 0, 0] \Rightarrow p' = \begin{bmatrix} n_x^2(1-\cos\theta)+\cos\theta \\ n_xn_y(1-\cos\theta)+n_z\sin\theta \\ n_xn_z(1-\cos\theta)-n_y\sin\theta \end{bmatrix}^T$$

$$q = [0, 1, 0] \Rightarrow q' = \begin{bmatrix} n_xn_y(1-\cos\theta)-n_z\sin\theta \\ n_y^2(1-\cos\theta)+\cos\theta \\ n_yn_z(1-\cos\theta)-n_y\sin\theta \end{bmatrix}^T$$

$$r = [0, 0, 1] \Rightarrow r' = \begin{bmatrix} n_xn_z(1-\cos\theta)+n_y\sin\theta \\ n_yn_z(1-\cos\theta)-n_x\sin\theta \\ n_z^2(1-\cos\theta)+\cos\theta \end{bmatrix}^T$$

$$R = \begin{bmatrix} p' \\ q' \\ r' \end{bmatrix}$$