

2.

1) 只考虑  $X_n$ 状态空间  $\{-2, -1, 0, 1, 2\}$ 

其中 2 和 -2 为终止态。

	-2	-1	0	1	2
-2	1	0	0	0	0
-1	q	r	p	0	0
0	0	q	r	p	0
1	0	0	q	r	p
2	0	0	0	0	1

(为 1 表示状态不再变化, 即游戏结束。)

从整体考虑:

状态空间: (甲, 乙): (0, 0)

(1, -1)

(-1, 1)

(2, -2)

(-2, 2)

状态转移矩阵

	(-2, 2)	(-1, 1)	(0, 0)	(1, -1)	(2, -2)
(-2, 2)	1	0	0	0	0
(-1, 1)	q	r	p	0	0
(0, 0)	0	q	r	p	0
(1, -1)	0	0	q	r	p
(2, -2)	0	0	0	0	1

(为 1 表示状态不再变化, 即游戏结束。)

2). 只能为甲-平-胜:  $P(\text{两局结束}) = pr$ 

3.

$$G_1 = R_2 + rR_3 + \dots = 6(1+r+r^2+\dots) = \frac{6}{1-r} = 60$$

$$G_0 = R_1 + rR_2 + \dots = R_1 + rG_1 = 2 + 0.9 \times 60 = 56$$

4.

由对称性, 可设:

$$V_{\pi}(1) = V_{\pi}(3) = V_{\pi}(5) = V_{\pi}(7) = m \quad V_{\pi}(2) = V_{\pi}(6) = n \quad V_{\pi}(4) = t$$

$$V_{\pi}(3) = -1 + \frac{1}{4} [0 + V_{\pi}(3) + V_{\pi}(4) + V_{\pi}(6)]$$

$$V_{\pi}(4) = -1 + \frac{1}{4} [V_{\pi}(1) + V_{\pi}(3) + V_{\pi}(5) + V_{\pi}(7)]$$

$$V_{\pi}(6) = -1 + \frac{1}{4} [V_{\pi}(2) + V_{\pi}(6) + V_{\pi}(3) + V_{\pi}(7)]$$

$$\Rightarrow \begin{cases} m = -1 + \frac{1}{4}(m+t+n) \\ t = -1 + m \\ n = -1 + \frac{1}{2}m + \frac{1}{2}n \end{cases} \Rightarrow \begin{cases} m = -7 \\ t = -8 \\ n = -9 \end{cases}$$

$$q_{\pi}(4, \text{left}) = -1 + V_{\pi}(3) = -1 - 7 = -8$$

$$q_{\pi}(7, \text{right}) = -1 + 0 = -1$$