# Composition-based Multi-Relational Graph Convolutional Networks

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Presented by - Sripath Mishra, Te-Lin Wu 01/19, CS249, Winter 2021

## Outline

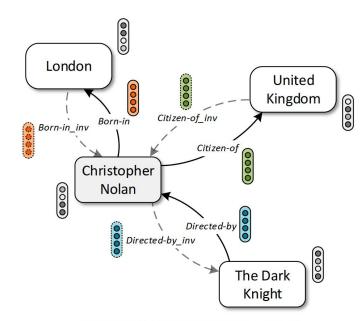
- Introduction
- Related Works
- Background
- Main Method: CompGCN
- Experiments
- Conclusions & Summaries

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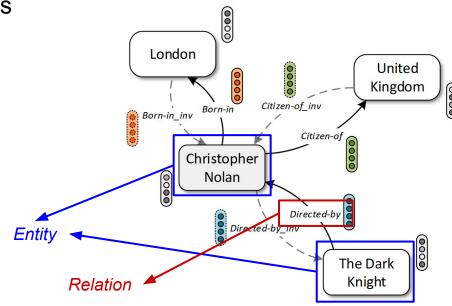
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- Graph Convolutional Neural Network (GCN) is proven powerful
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  - Graphs with *relations*
  - o Eg. Knowledge Graphs



Relational Graph with Embeddings

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- Graph Convolutional Neural Network (GCN) is proven powerful
- Most GCN handles undirected graphs
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  - Graphs with *relations*
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**Relational Graph with Embeddings** 

- Cons of prior works on multi-relational graphs
  - Over-parameterization
  - Only handles node representations
    - Can not perform link predictions
- Existing techniques on knowledge graph embeddings (KE)
  - Only does link predictions

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  - Over-parameterization
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  - Only does link predictions
- Solution:
  - Combine both: GCN + KE

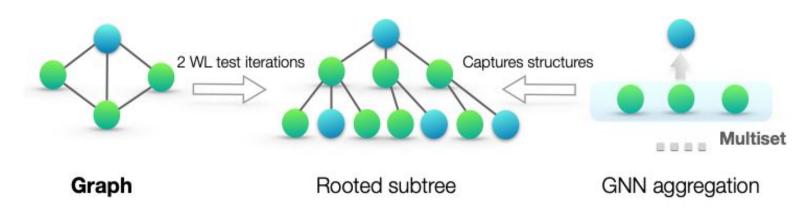
# CompGCN

Compositional-based Graph Convolutional Network

## Outline

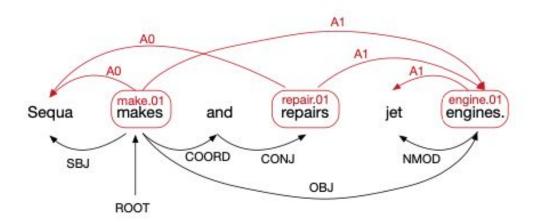
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Graph Convolutional Networks



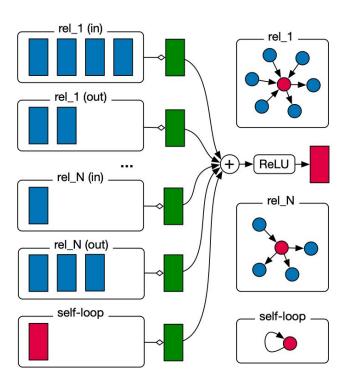
Middle panel: **rooted subtree structures** (at the blue node) that the *WL test* uses to distinguish different graphs. Right panel: if a GNN's aggregation function captures the *full multiset* of node neighbors, the GNN can capture the rooted subtrees in a recursive manner and be as powerful as the WL test.

GCNs for Multi-Relational Graph



An example sentence annotated with semantic (top) and syntactic dependencies (bottom).

GCNs for Multi-Relational Graphs

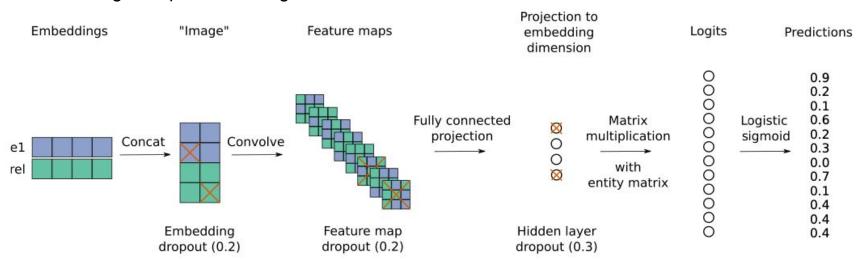


Update in the **R-GCN** model.

Activations (d-dimensional vectors) from neighboring nodes (dark blue) are gathered and then transformed *for each relation type individually* (for both in- and outgoing edges). The resulting representation (green) is accumulated in a (normalized) sum and passed through an activation function (such as the ReLU).

[Schlichtkrull et al., 2017]

Knowledge Graph Embedding



In the **ConvE** model, the entity and relation embeddings are first reshaped and concatenated (steps 1, 2); the resulting matrix is then used as input to a convolutional layer (step 3); the resulting feature map tensor is vectorised and projected into a k-dimensional space (step 4) and matched with all candidate object embeddings (step 5).

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## GCN on Undirected Graphs

- Undirected Graph: G = (V, E, X)
  - V: set of vertices
  - E: set of edges
  - $X \in \mathbb{R}^{|v|^*d0}$ :  $d_0$ -dimensional input features of each node
- The node representation for a single GCN layer: H = f(AXW)
  - $\circ$   $\hat{A} = D^{-1/2}(A+I)D^{+1/2}$
  - $\begin{array}{ll}
    \circ & D_{ii} = \sum_{j} (A+I)_{ij}, \\
    \circ & W \in \mathbb{R}^{d0xd1}
    \end{array}$

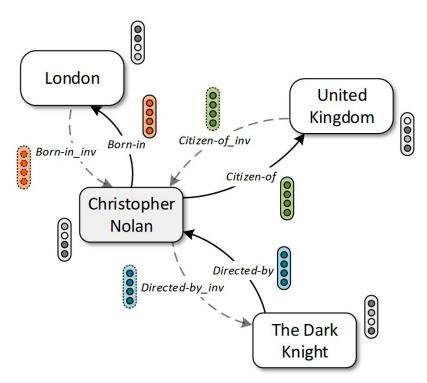
  - f: some activation function

## GCN on Multi-Relational Graphs

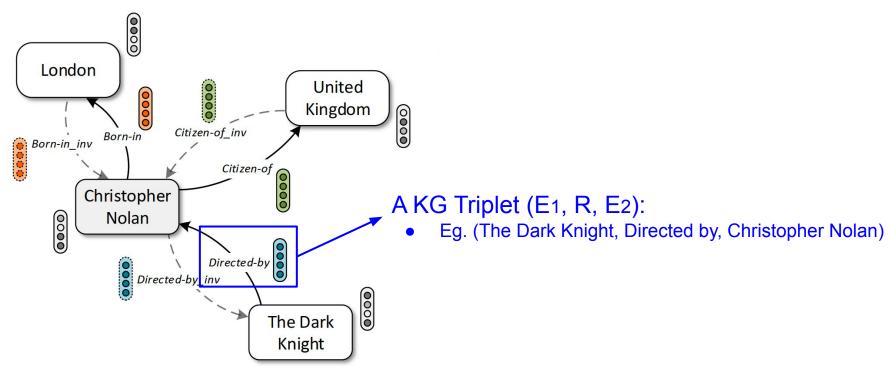
- Multi-Relational Graph: G = (V, R, E, X)
  - R: set of relations
  - (u, v, r): relation  $r \in R$  from node u to v.
  - o Inverse edge: (v, u, r<sup>-1</sup>)
- k-th layer representations of a relational graph: H<sup>k+1</sup>=f(ÂH<sup>k</sup>W<sup>k</sup><sub>r</sub>)
  - W<sup>k</sup><sub>r</sub>: relation-specific parameters

## Outline

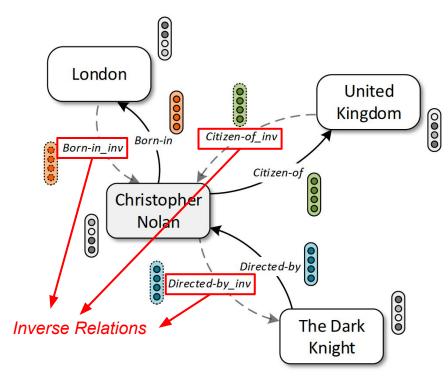
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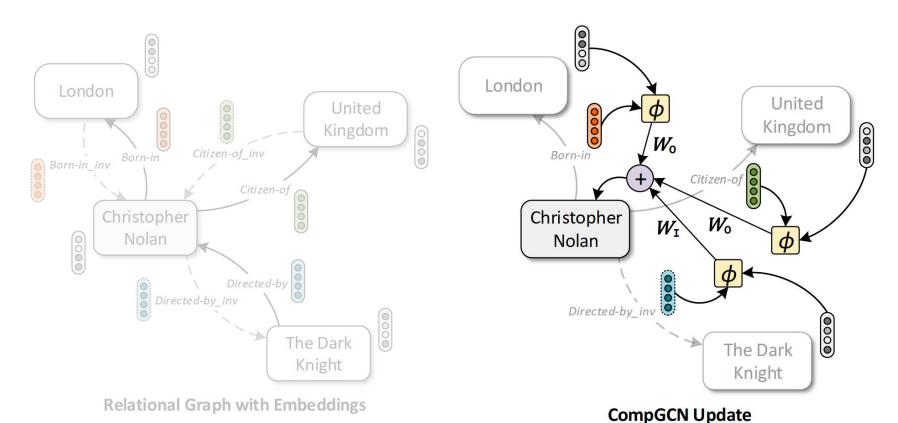
**Relational Graph with Embeddings** 

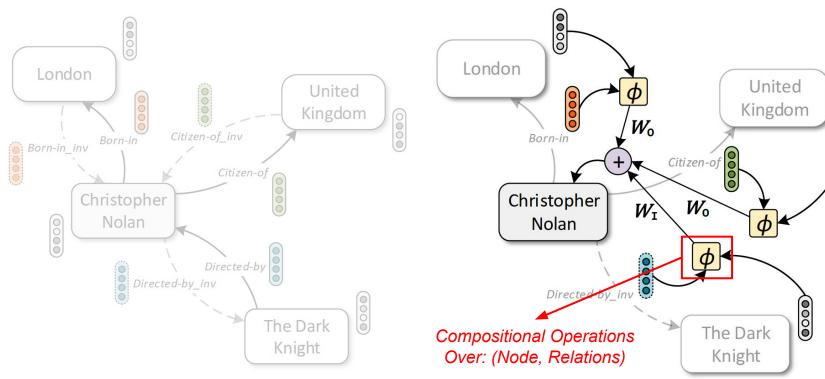


**Relational Graph with Embeddings** 



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**Relational Graph with Embeddings** 

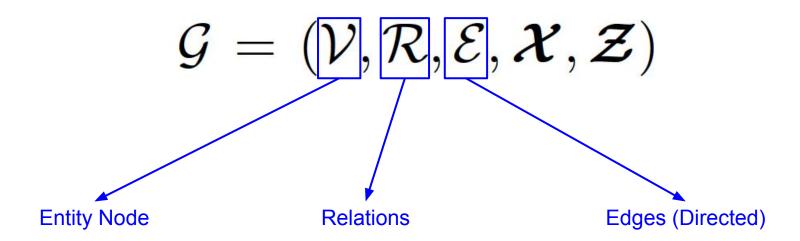
**CompGCN Update** 

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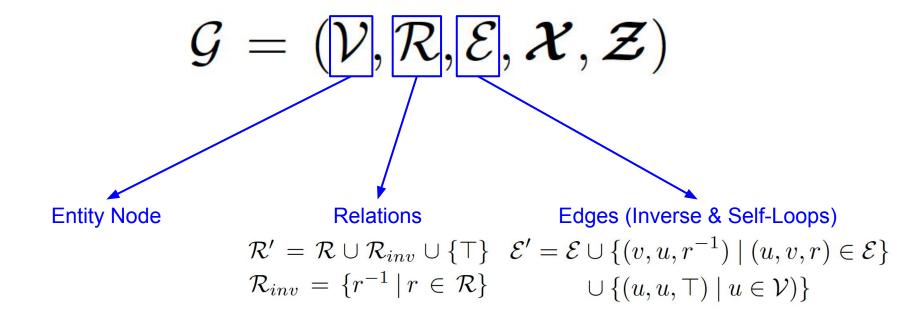
• Multi-Relational Graph:

$$\mathcal{G} = (\mathcal{V}, \mathcal{R}, \mathcal{E}, \mathcal{X}, \mathcal{Z})$$

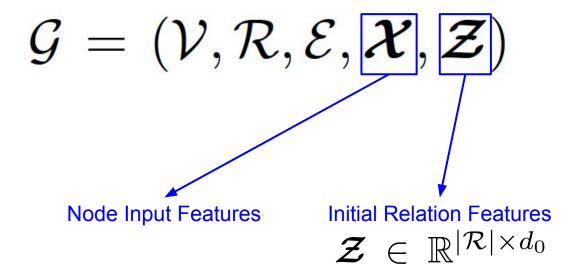
Multi-Relational Graph:



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Multi-Relational Graph:



# Representing Relations as Vectors

- Alleviates over-parameterization on relational graphs
- Allows utilizing any available relational features as initializations

# Representing Relations as Vectors

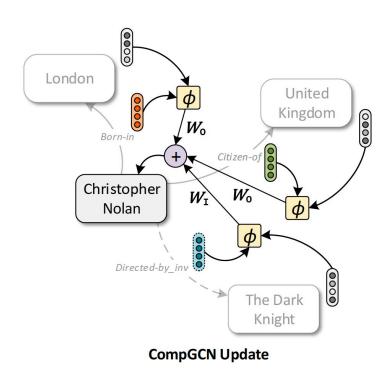
- Alleviates over-parameterization on relational graphs
- Allows utilizing any available relational features as initializations
- **Compositional Operations:**

$$e_o = \phi(e_s, e_r)$$

$$\phi: \mathbb{R}^d \times \mathbb{R}^d \to \mathbb{R}^d$$

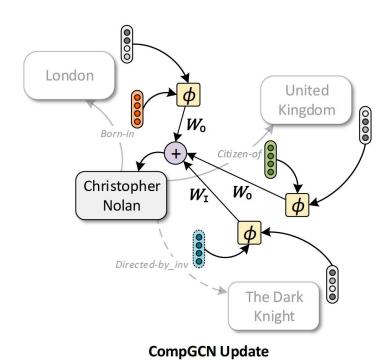
#### **Non-Parameterized Operations**

- Subtraction
- Multiplication Circular-Correlation  $[{\pmb a}\star{\pmb b}]_k=\sum_{i=0}^{d-1}a_ib_{(k+i)\,\mathrm{mod}\,d}$



#### **Standard Update Rule:**

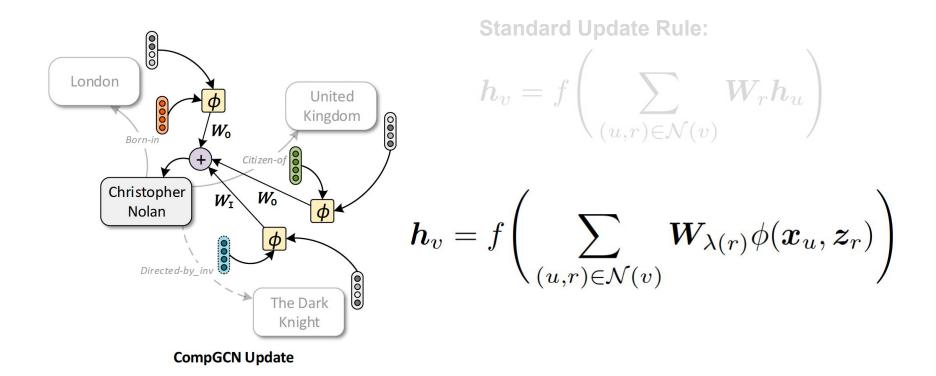
$$\boldsymbol{h}_v = f\left(\sum_{(u,r)\in\mathcal{N}(v)} \boldsymbol{W}_r \boldsymbol{h}_u\right)$$

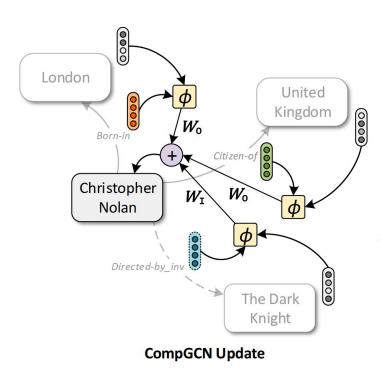


#### **Standard Update Rule:**

$$m{h}_v = f \left( \sum_{(u,r) \in \mathcal{N}(v)} m{W}_r m{h}_u 
ight)$$

Suffers from over-parameterization as each r needs its W!



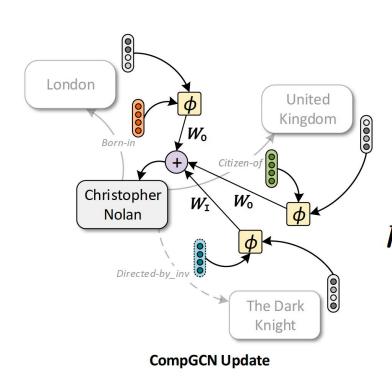


**Standard Update Rule:** 

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$$\boldsymbol{h}_v = f\left(\sum_{(u,r)\in\mathcal{N}(v)} \boldsymbol{W}_{\lambda(r)}\phi(\boldsymbol{x}_u, \boldsymbol{z}_r)\right)$$

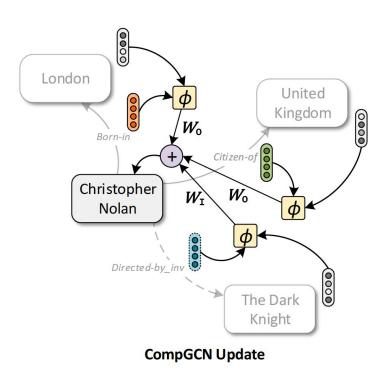
Number of feature dimensions:  $(\mathcal{O}(|\mathcal{R}|d))$ 



#### Relation-Specific Weights:

$$egin{aligned} oldsymbol{W}_{dir(r)} &= egin{cases} oldsymbol{W}_O, & r \in \mathcal{R} \ oldsymbol{W}_I, & r \in \mathcal{R}_{inv} \ oldsymbol{W}_S, & r = op (\textit{self-loop}) \end{cases} \ oldsymbol{h}_v &= f \left( \sum_{(u,r) \in \mathcal{N}(v)} oldsymbol{W}_{\lambda(r)} \phi(oldsymbol{x}_u, oldsymbol{z}_r) 
ight) \ \lambda(r) = \operatorname{dir}(r) \end{cases}$$

Number of feature dimensions:  $(\mathcal{O}(|\mathcal{R}|d))$ 



#### Relation-Specific Weights:

$$m{W}_{ ext{dir}(r)} = egin{cases} m{W}_O, & r \in \mathcal{R} \ m{W}_I, & r \in \mathcal{R}_{inv} \ m{W}_S, & r = \top \textit{(self-loop)} \end{cases}$$

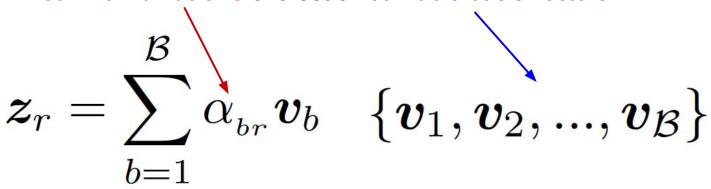
$$h_v = f\left(\sum_{(u,r)\in\mathcal{N}(v)} W_{\lambda(r)}\phi(x_u, z_r)\right)$$

#### Relational Embedding Update:

$$oldsymbol{h}_r = oldsymbol{W}_{ ext{rel}} oldsymbol{z}_r \,\, oldsymbol{W}_{ ext{rel}} \in \mathbb{R}^{d_1 imes d_0}$$

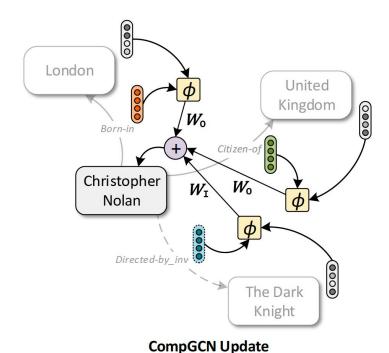
## Scaling the Number of Relations

- Relational Vectors are:
  - Linear Combinations of a set of learnable basis vectors



Learnable Scalar Weights:  $lpha_{hr} \in \mathbb{R}$ 

## Stacking CompGCN (k-Layers)



#### Node Embedding Update:

$$\boldsymbol{h}_{v}^{k+1} = f\left(\sum_{(u,r)\in\mathcal{N}(v)} \boldsymbol{W}_{\lambda(r)}^{k} \phi(\boldsymbol{h}_{u}^{k}, \boldsymbol{h}_{r}^{k})\right)$$

#### Relational Embedding Update:

$$oldsymbol{h}_r^{k+1} = oldsymbol{W}_{ ext{rel}}^k oldsymbol{h}_r^k$$

#### **GCN** Comparisons

Methods	Node Embeddings	Directions	Relations	Relation Embeddings	Number of Parameters
GCN Kipf & Welling (2016)	✓				$\mathcal{O}(Kd^2)$
Directed-GCN Marcheggiani & Titov (2017)	$\checkmark$	$\checkmark$			$\mathcal{O}(Kd^2)$
Weighted-GCN Shang et al. (2019)	$\checkmark$		$\checkmark$		$\mathcal{O}(Kd^2+K \mathcal{R} )$
Relational-GCN Schlichtkrull et al. (2017)	✓	✓	✓		$\mathcal{O}(\mathcal{B}Kd^2+\mathcal{B}K \mathcal{R} )$
COMPGCN (Proposed Method)	✓	✓	✓	<b>√</b>	$\mathcal{O}(Kd^2+\mathcal{B}d+\mathcal{B} \mathcal{R} )$

• **K**: Number of GCN Layers

• **d:** Embedding dimension

• **B**: Number of bases

• |R|: Total number of relations

## Reduction of CompGCN

**Proposition 4.1.** CompGCN generalizes the following Graph Convolutional based methods: **Kipf-GCN** (Kipf & Welling, 2016), **Relational GCN** (Schlichtkrull et al., 2017), **Directed GCN** (Marcheggiani & Titov, 2017), and **Weighted GCN** (Shang et al., 2019).

*Proof.* For Kipf-GCN, this can be trivially obtained by making weights  $(W_{\lambda(r)})$  and composition function  $(\phi)$  relation agnostic in Equation 5, i.e.,  $W_{\lambda(r)} = W$  and  $\phi(h_u, h_r) = h_u$ . Similar reductions can be obtained for other methods as shown in Table 2.

# Reduction of CompGCN

Methods	$oldsymbol{W}^k_{\lambda(r)}$	$\phi(m{h}_u^k,m{h}_r^k)$
Kipf-GCN (Kipf & Welling, 2016)	$oldsymbol{W}^k$	$\boldsymbol{h}_u^k$
Relational-GCN (Schlichtkrull et al., 2017)	$\boldsymbol{W}_r^k$	$\boldsymbol{h}_u^k$
Directed-GCN (Marcheggiani & Titov, 2017)	$oldsymbol{W}^k_{ ext{dir}(r)} \ oldsymbol{W}^k$	$\boldsymbol{h}_u^k$
Weighted-GCN (Shang et al., 2019)	$oldsymbol{W}^k$	$lpha_r^k oldsymbol{h}_u^k$

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## Benchmarking Tasks

- Link Predictions
- Node Classifications
- Graph Classifications

#### Benchmarking Tasks

#### Link Predictions

- Inferring missing facts based on known facts in KGs
- Datasets: FB15k-237 (FreeBase), WN18-RR (WordNet)
- Metrics: MRR & MR & Hits@N

#### Node Classifications

- Predicting labels of nodes
- Based on node features and connections
- Datasets: MUTAG & AM

#### Graph Classifications

- Learns representations of graphs, and predicts the types of graphs
- Datasets: MUTAG & PTC

#### **Dataset Statistics**

	<b>Link Prediction</b>		Node Classif	ication	<b>Graph Classification</b>			
	FB15k-237	WN18RR	MUTAG (Node)	AM	MUTAG (Graph)	PTC		
Graphs	1	1	1	1	188	344		
Entities	14,541	40,943	23,644	1,666,764	17.9 (Avg)	25.5 (Avg)		
Edges	310,116	93,003	74,227	5,988,321	39.6 (Avg)	29.5 (Avg)		
Relations	237	11	23	133	4	4		
Classes	-	-	2	11	2	2		

#### **GCN Baselines**

- Relational-GCN (R-GCN) (Schlichtkrull et al., 2017)
- Directed-GCN (D-GCN) (Marcheggiani & Titov, 2017)
- Weighted-GCN (W-GCN) (Shang et al., 2019)

#### **GCN Baselines**

- Relational-GCN (R-GCN) (Schlichtkrull et al., 2017)
  - Relation-specific weight matrices
  - Linear combinations of a set of basis matrices
- Directed-GCN (D-GCN) (Marcheggiani & Titov, 2017)
  - Separate weight matrices for incoming, outgoing edges and self-loops
  - Relation-specific biases
- Weighted-GCN (W-GCN) (Shang et al., 2019)
  - Learnable scalar weights assigned to each relation
  - Multiplies the weights with an "incoming message"

#### Task-Specific Baselines

- Link Predictions
  - TransE, DistMult, KBGAN, ComplEx, ... etc.
- Node Classifications
  - o Feat, WL, RDF2Vec, ... etc.
- Graph Classifications
  - Deep Graph CNN, Graph Isomorphism Network, ... etc.

TransE Objective: 
$$\mathcal{L} = \sum_{(h,\ell,t) \in S} \sum_{(h',\ell,t') \in S'_{(h-\ell,t)}} \left[ \gamma + d(\boldsymbol{h} + \boldsymbol{\ell}, \boldsymbol{t}) - d(\boldsymbol{h'} + \boldsymbol{\ell}, \boldsymbol{t'}) \right]_{+}$$

#### Goals of Experiments

- Q1. CompGCN compared to existing methods on Link Predictions
- Q2. Compare different GCN encoders and compositional operators
- Q3. CompGCN scalability
- Q4. CompGCN performance on node & graph classification

#### **Results - Link Predictions**

	FB15k-237				WN18RR					
	MRR	MR	H@10	H@3	H@1	MRR	MR	H@10	H@3	H@1
TransE (Bordes et al., 2013)	.294	357	.465	-	-	.226	3384	.501	-	-
DistMult (Yang et al., 2014)	.241	254	.419	.263	.155	.43	5110	.49	.44	.39
ComplEx (Trouillon et al., 2016)	.247	339	.428	.275	.158	.44	5261	.51	.46	.41
R-GCN (Schlichtkrull et al., 2017)	.248	-	.417		.151	-	-	-		-
KBGAN (Cai & Wang, 2018)	.278	-	.458		-	.214	-	.472	-	-
ConvE (Dettmers et al., 2018)	.325	244	.501	.356	.237	.43	4187	.52	.44	.40
ConvKB (Nguyen et al., 2018)	.243	311	.421	.371	.155	.249	3324	.524	.417	.057
SACN (Shang et al., 2019)	.35	-	.54	.39	.26	.47	-	.54	.48	.43
HypER (Balažević et al., 2019)	.341	250	.520	.376	.252	.465	5798	.522	.477	.436
RotatE (Sun et al., 2019)	.338	177	.533	.375	.241	.476	3340	.571	.492	.428
ConvR (Jiang et al., 2019)	.350	-	.528	.385	.261	.475	-	.537	.489	.443
VR-GCN (Ye et al., 2019)	.248	-	.432	.272	.159	120	<u>\</u>	-	: <u>-</u>	: <u>=</u>
COMPGCN (Proposed Method)	.355	197	.535	.390	.264	.479	3533	.546	.494	.443

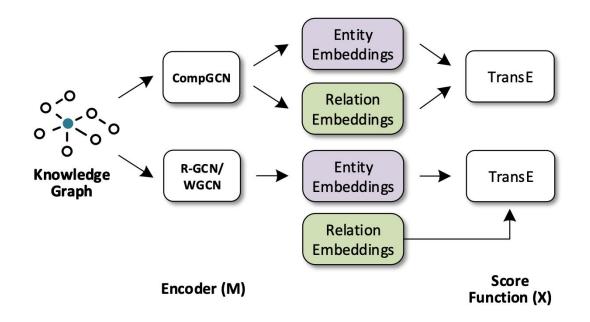
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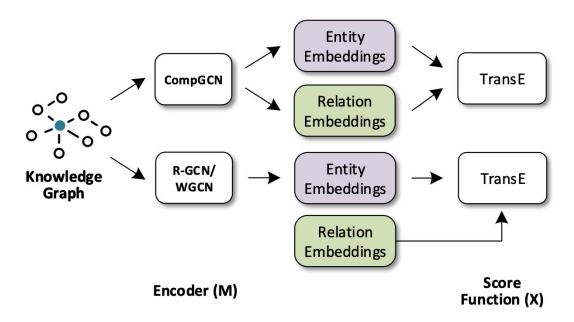
#### **Results - Link Predictions**

	FB15k-237					WN18RR					
	MRR	MR	H@10	H@3	H@1	-	MRR	MR	H@10	H@3	H@1
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COMPGCN (Proposed Method)	.355	197	.535	.390	.264		.479	3533	.546	.494	.443

#### **GCN** Comparisons



## **GCN** Comparisons



- Subtraction (Sub):  $\phi(e_s, e_r) = e_s e_r$ .
- Multiplication (Mult):  $\phi(e_s, e_r) = e_s * e_r$ .
- Circular-correlation (Corr):  $\phi(e_s, e_r) = e_s \star e_r$

#### GCN Comparison Results - Link Predictions

 X + M (Y): method M is used for obtaining entity/relation embeddings with X as scoring function. Y is the composition operator.

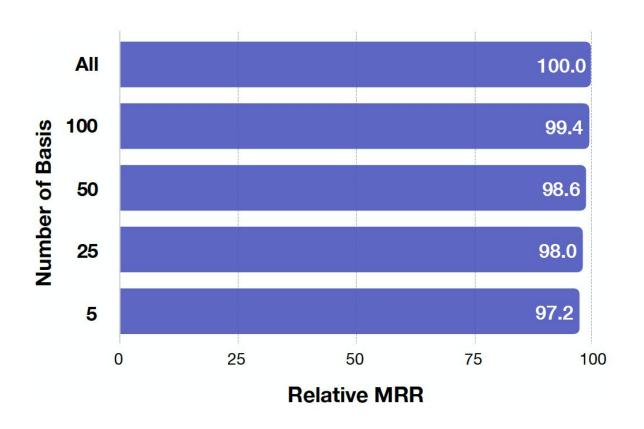
Scoring Function $(=X) \rightarrow$	8	TransF	E	DistMult			ConvE			
<b>Methods</b> ↓	MRR	MR	H@10	MRR	MR	H@10	MRR	MR	H@10	
X	0.294	357	0.465	0.241	354	0.419	0.325	244	0.501	
X + D-GCN	0.299	351	0.469	0.321	225	0.497	0.344	200	0.524	
X + R-GCN	0.281	325	0.443	0.324	230	0.499	0.342	197	0.524	
X + W-GCN	0.267	1520	0.444	0.324	229	0.504	0.344	201	0.525	
X + COMPGCN (Sub)	0.335	194	0.514	0.336	231	0.513	0.352	199	0.530	
X + COMPGCN (Mult)	0.337	233	0.515	0.338	200	0.518	0.353	216	0.532	
X + COMPGCN (Corr)	0.336	214	0.518	0.335	227	0.514	0.355	197	0.535	
X + COMPGCN (B = 50)	0.330	203	0.502	0.333	210	0.512	0.350	193	0.530	

# CompGCN Code Demo

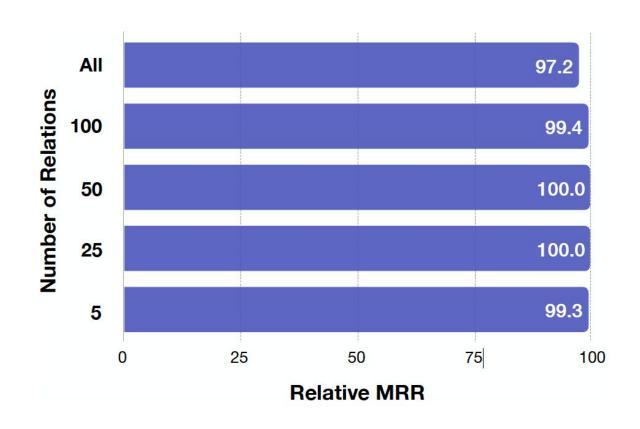
#### **Evaluating the Scalability**

- Effect of different number of basis vectors
- Scaling up different number of relations
- Compare to prior work R-GCN
  - Basis matrices instead of basis vectors

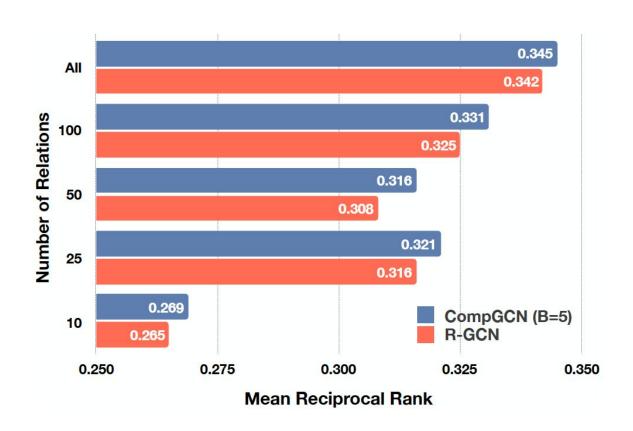
#### **Number of Base Vectors**



#### **Number of Relations**



#### Compared to R-GCN



# Node & Graph Classifications

	Node Classification		Graph Classification					
	MUTAG (Node)	AM		MUTAG (Graph)	PTC			
Feat*	77.9	66.7	PACHYSAN <sup>†</sup>	$\textbf{92.6} \pm \textbf{4.2}$	$60.0 \pm 4.8$			
$WL^*$	80.9	87.4	$DGCNN^\dagger$	85.8	58.6			
RDF2Vec*	67.2	88.3	$\mathrm{GIN}^\dagger$	$89.4 \pm 4.7$	$64.6 \pm 7.0$			
R-GCN*	73.2	89.3	R-GCN	$82.3 \pm 9.2$	$67.8 \pm 13.2$			
SynGCN	$74.8 \pm 5.5$	$86.2 \pm 1.9$	SynGCN	$79.3 \pm 10.3$	$69.4 \pm 11.5$			
WGCN	$77.9 \pm 3.2$	$90.2 \pm 0.9$	WGCN	$78.9 \pm 12.0$	$67.3 \pm 12.0$			
COMPGCN	$\textbf{85.3} \pm \textbf{1.2}$	$90.6 \pm 0.2$	COMPGCN	$89.0 \pm 11.1$	$71.6 \pm 12.0$			

Read Out:  $m{h}_{\mathcal{G}} = rac{1}{|\mathcal{V}|} \sum_{v \in \mathcal{V}} m{h}_v$ 

#### Outline

- Introduction
- Related Works
- Background
- Main Method: CompGCN
- Experiments
- Conclusions & Summaries

#### Summaries of CompGCN

- Multi-Relational Knowledge Graphs
- Representing Relations as Vectors
- Compositional Operators
- CompGCN Model
  - General
  - Alleviates over-parameterization
  - Scalable with basis vectors
- Extensive Experiments

#### CompGCN Future Works

- More complex compositional operators
  - Learnable eg. ConvE
  - Superior compositional operations
- Interpretable relational vectors

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#### Paper: CompGCN at ICLR 2020

Published as a conference paper at ICLR 2020

# COMPOSITION-BASED MULTI-RELATIONAL GRAPH CONVOLUTIONAL NETWORKS

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#### ABSTRACT

Graph Convolutional Networks (GCNs) have recently been shown to be quite successful in modeling graph-structured data. However, the primary focus has been on handling simple undirected graphs. Multi-relational graphs are a more general and prevalent form of graphs where each edge has a label and direction associated with it. Most of the existing approaches to handle such graphs suffer from over-parameterization and are restricted to learning representations of nodes only. In this paper, we propose COMPGCN, a novel Graph Convolutional framework which jointly embeds both nodes and relations in a relational graph. COMPGCN leverages a variety of entity-relation composition operations from Knowledge Graph Embedding techniques and scales with the number of relations. It also generalizes several of the existing multi-relational GCN methods. We evaluate our proposed method on multiple tasks such as node classification, link prediction, and graph classification, and achieve demonstrably superior results. We make the source code of COMPGCN available to foster reproducible research.

# Thank You & QA