Homework 4

2022-11-16

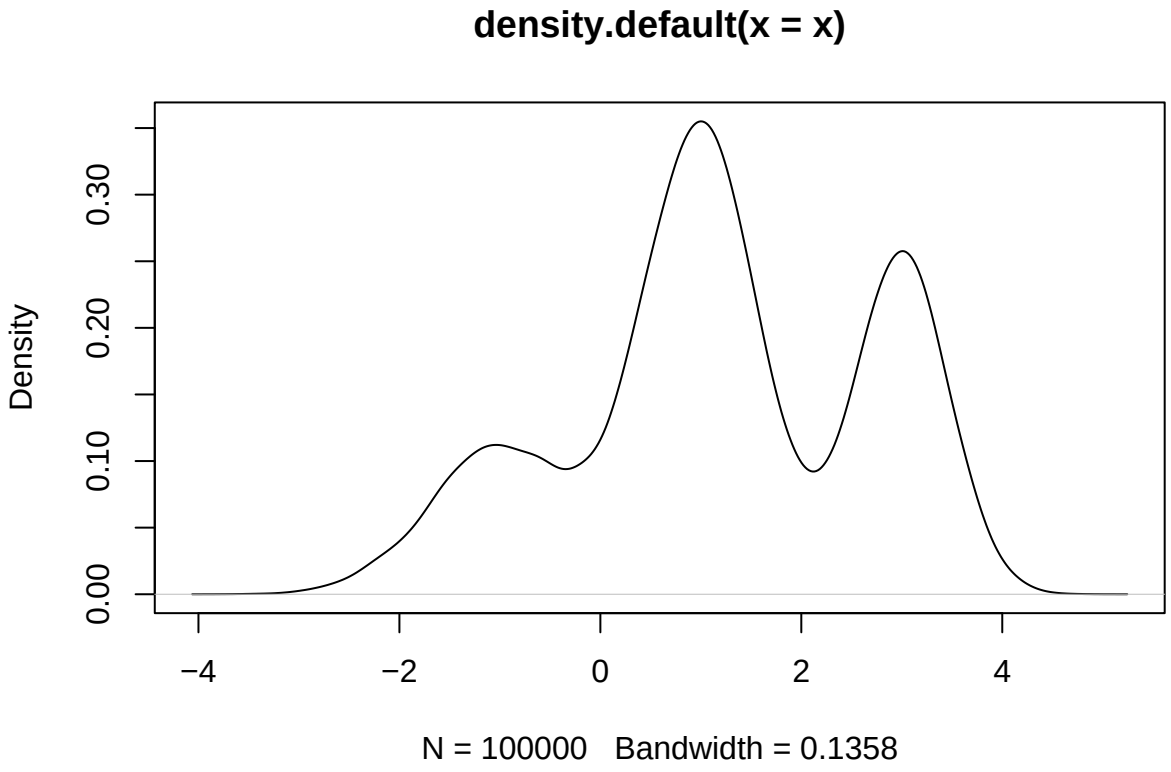
1) Finite Mixture Model

Composition sampling:

```
pi_1=0.2
pi_2=0.5
n=100000 # set this to the desired number of samples
u=runif(n)
z=ifelse(u<pi_1,1,ifelse(u<(pi_1+pi_2),2,3))
x=rep(NaN, n)
for (i in 1:n){
  if(z[i]==1){
    x[i]=rnorm(1, -1, sqrt(0.5))
  }
  if(z[i]==2){
    x[i]=rnorm(1, 1, sqrt(0.3))
  }
  if(z[i]==3){
    x[i]=rnorm(1, 3, sqrt(0.2))
  }
}
```

Plot the empirical density of x:

```
plot(density(x))
```



This is very close to the real density plot.

2) The Metropolis Algorithm

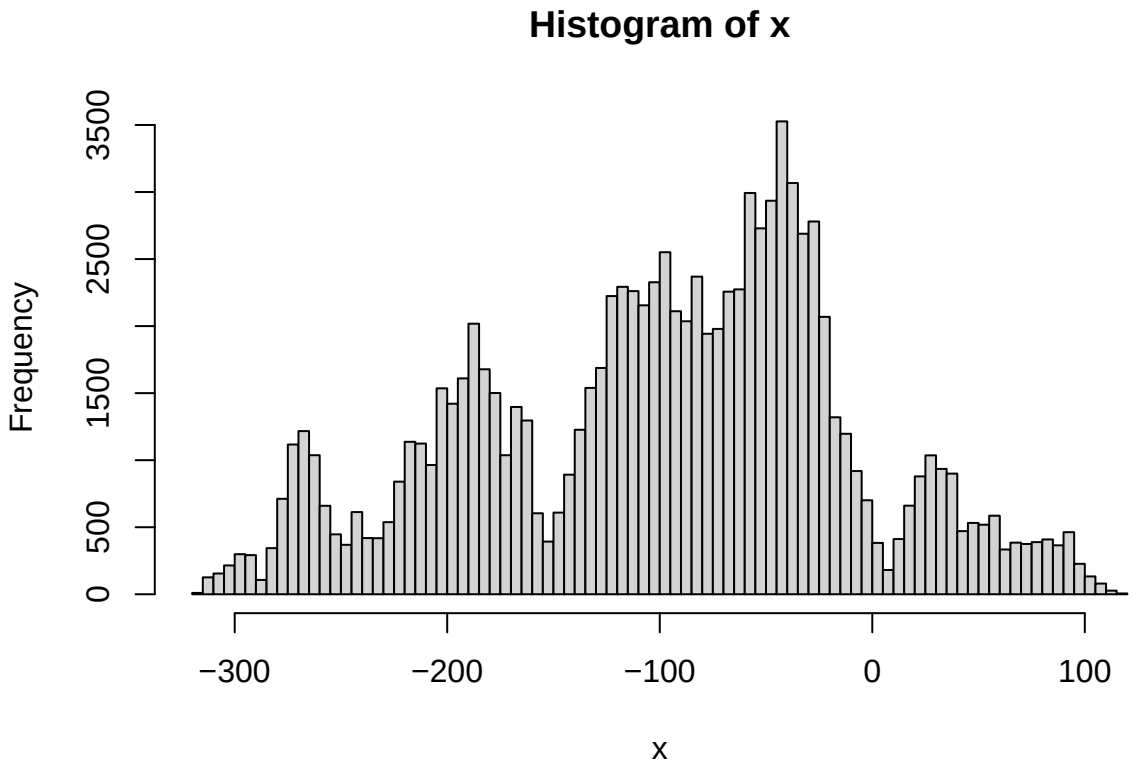
Create a function to evaluates the target density:

```
f=function(x){
  return(0.2*dnorm(1, -1, sqrt(0.5))+0.5*dnorm(1, 1, sqrt(0.3))+(1-0.2-0.5)*dnorm(1, 3, sqrt(0.2)))
}
```

Metropolis sampling:

```
n=100000
x=rep(NA,n)
x[1]=.5

for(i in 2:n){
  candidate=rnorm(1, x[i-1], sqrt(1))
  alpha=min(1, f(candidate)/f(x[i-1]))
  z=runif(1)<alpha
  x[i]=ifelse(z,candidate,x[i-1])
}
hist(x,100)
```


Histogram of x
Frequency
0
500
1500
2500
3500
−300
−200
−100
0
100
x