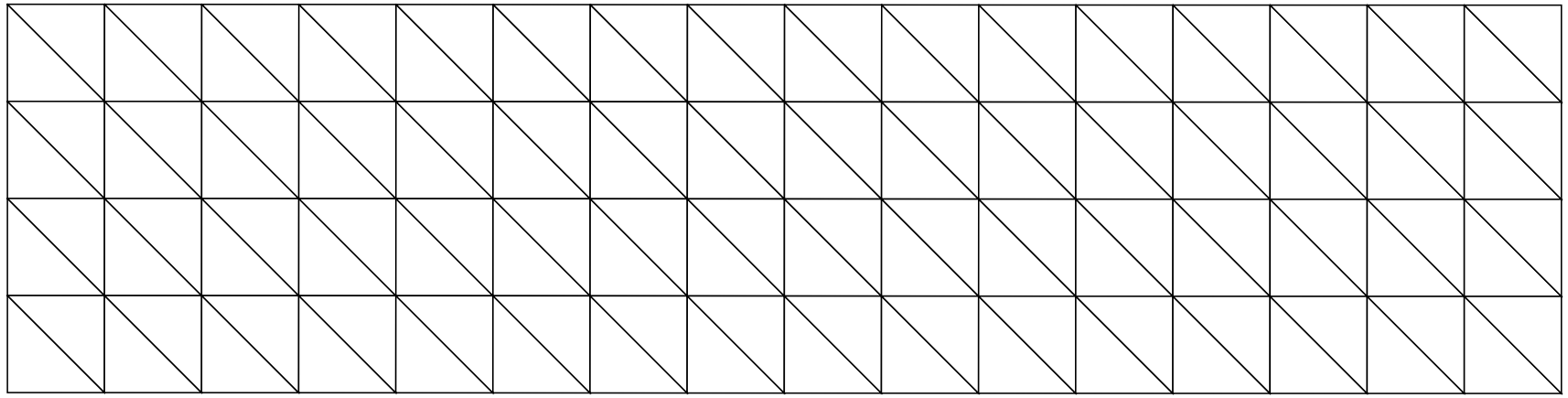
有限元方法5-2

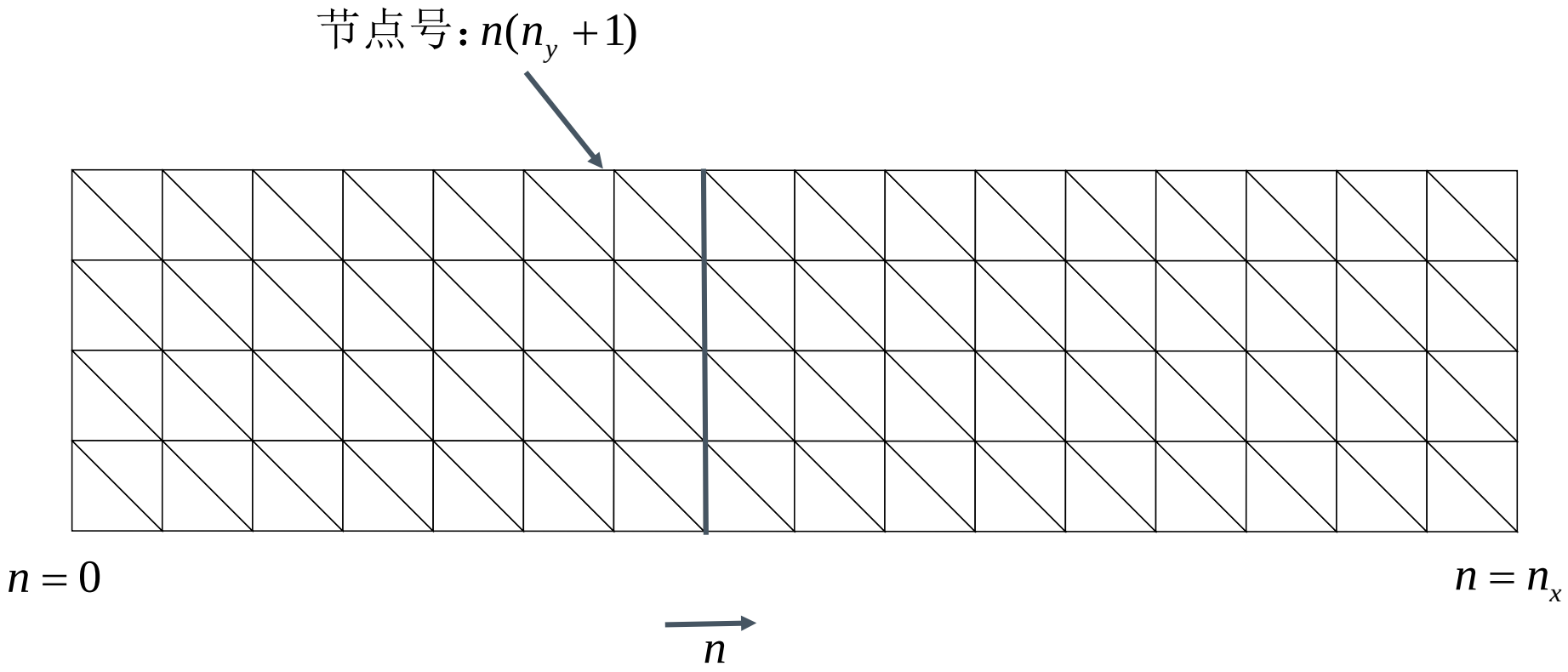
弹性力学平面应力问题

垂直单元数：n_y



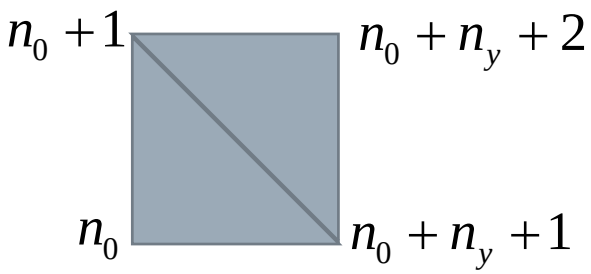
水平单元数：n_x

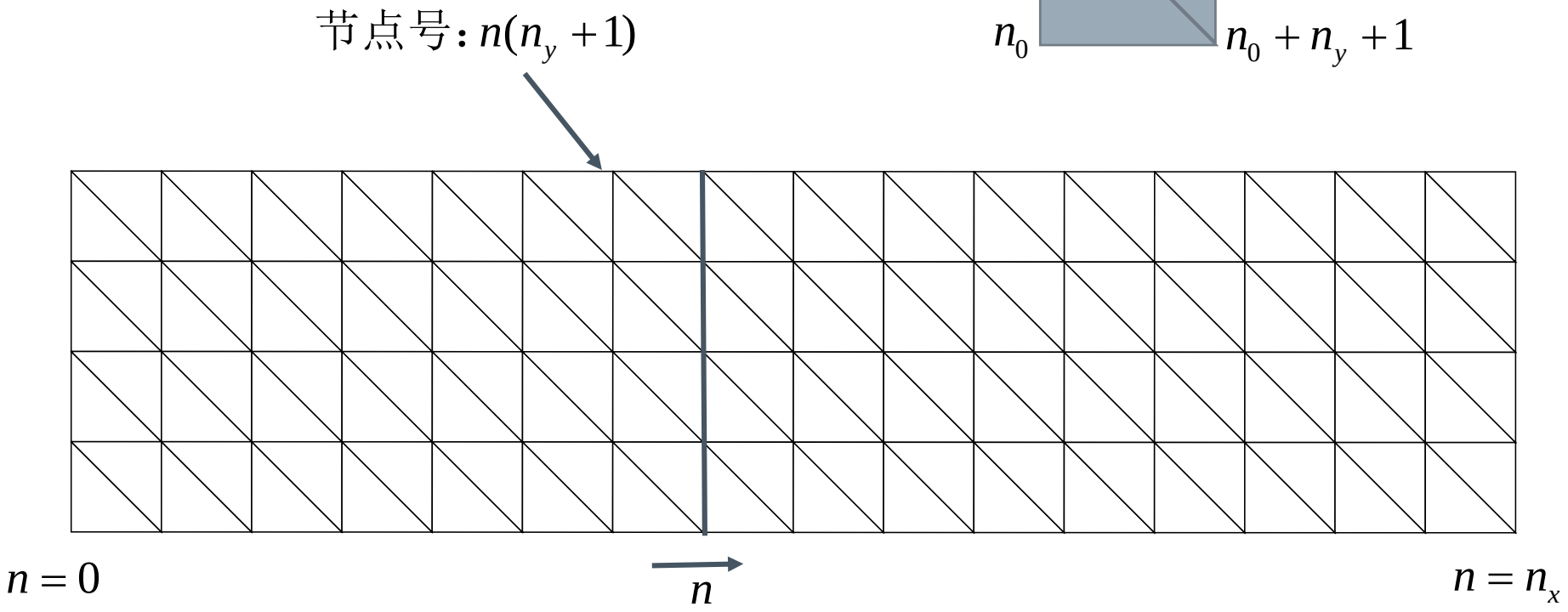
模型生成




```
coord(1,n*(ny+1)+(1:ny+1)) = dx*n;
coord(2,n*(ny+1)+(1:ny+1)) = dy*(0:ny);
```

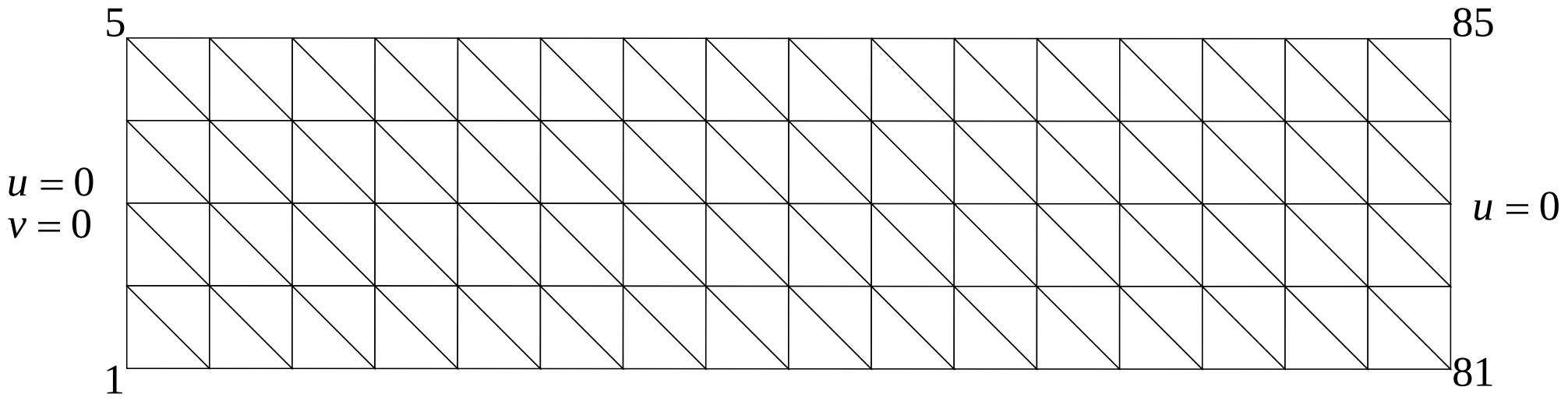
模型生成






```
n0 = n*(ny+1) + (1:ny);
element(:,2*n*ny+(1:ny)) = [n0+1; n0; n0+ny+1];
element(:,2*n*ny+ny+(1:ny)) = [n0+1; n0+ny+1; n0+ny+2];
```

模型生成

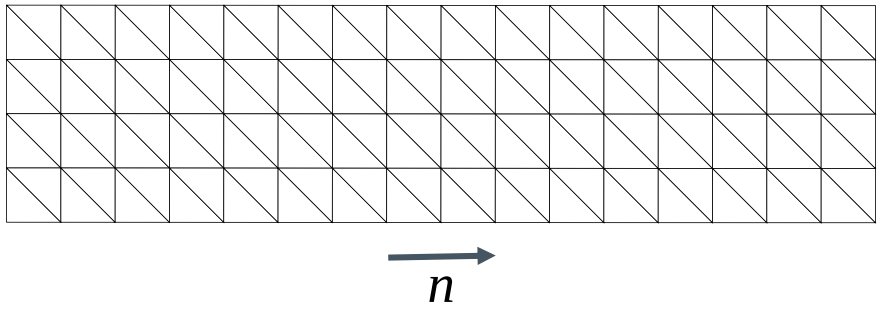


BC1= 1 2 3 4 5 1 2 3 4 5 81 82 83 84 85
1 1 1 1 1 2 2 2 2 2 1 1 1 1 1
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

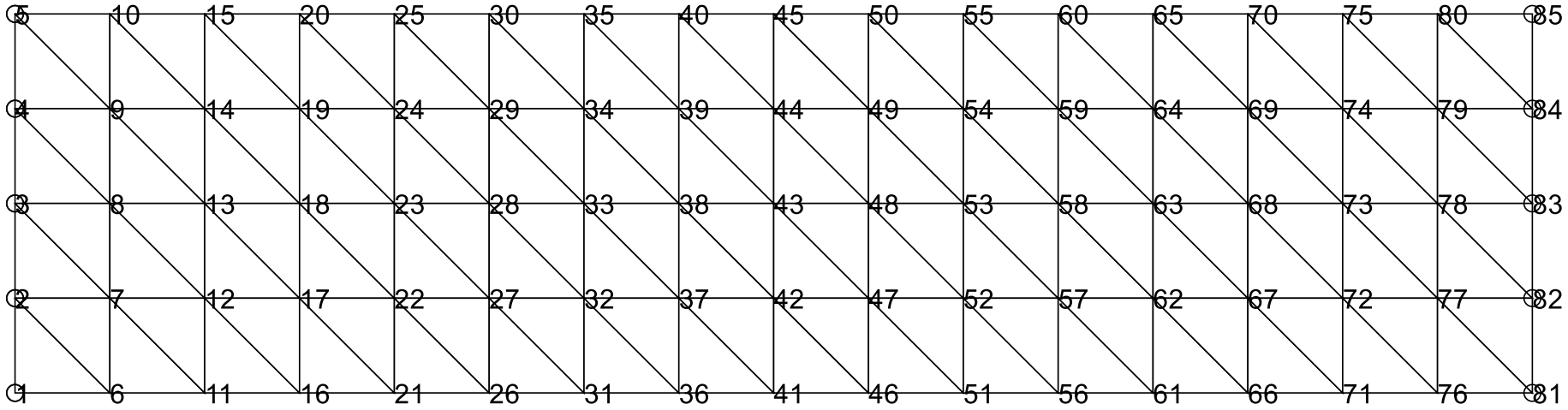
```
BC1=zeros(3,3*(ny+1));
BC1(:,1:ny+1) =           [1:ny+1;              ones(1,ny+1);   zeros(1,ny+1)];
BC1(:,ny+1+(1:ny+1)) =    [1:ny+1;              2*ones(1,ny+1); zeros(1,ny+1)];
BC1(:,2*(ny+1)+(1:ny+1))=[nx*(ny+1)+(1:ny+1); ones(1,ny+1);   zeros(1,ny+1)];
```

模型生成

```
function [coord,element,BC1]=CreateModel(a,b,delta,ShowOn)
nx = round(a/delta); ny = round(b/delta);
dx = a/nx; dy = b/ny;
coord=zeros(2,(nx+1)*(ny+1));
for n=0:nx
    coord(1,n*(ny+1)+(1:ny+1)) = dx*n;
    coord(2,n*(ny+1)+(1:ny+1)) = dy*(0:ny);
end
element=zeros(3,2*nx*ny);
for n=0:nx-1
    n0 = n*(ny+1) + (1:ny);
    element(:,2*n*ny+(1:ny)) = [n0+1; n0; n0+ny+1];
    element(:,2*n*ny+ny+(1:ny)) = [n0+1; n0+ny+1; n0+ny+2];
end
BC1=zeros(3,3*(ny+1));
BC1(:,1:ny+1) = [1:ny+1; ones(1,ny+1); zeros(1,ny+1)];
BC1(:,ny+1+(1:ny+1)) = [1:ny+1; 2*ones(1,ny+1); zeros(1,ny+1)];
BC1(:,2*(ny+1)+(1:ny+1)) = [nx*(ny+1)+(1:ny+1); ones(1,ny+1); zeros(1,ny+1)];
```



模型生成




```
if ~ShowOn
    return
end
hold on; axis equal off
for n=1:size(element,2)
    ix=element(:,n);
    x=coord(1,ix); y=coord(2,ix);
    fill(x,y,ones(size(x)));
end
plot(coord(1,BC1(1,:)),coord(2,BC1(1,:)),'ko');
for n=1:size(coord,2)
    text(coord(1,n),coord(2,n),num2str(n));
end
```

三角形单元公式

型函数

$$[N(x,y)]=\begin{bmatrix} N_1(x,y) & 0 & N_2(x,y) & 0 & N_3(x,y) & 0 \\ 0 & N_1(x,y) & 0 & N_2(x,y) & 0 & N_3(x,y) \end{bmatrix},\quad N_i(x,y)=\frac{1}{2\Delta}(a_i+b_i x+c_i y)$$

应变位移矩阵

$$[B]=\begin{bmatrix} B_1 & B_2 & B_3 \end{bmatrix},\quad [B_i]=\begin{bmatrix} N_{i,x} & 0 \\ 0 & N_{i,y} \\ N_{i,y} & N_{i,x} \end{bmatrix}=\frac{1}{2\Delta}\begin{bmatrix} b_i & 0 \\ 0 & c_i \\ c_i & b_i \end{bmatrix}$$

弹性矩阵

$$[D]=\frac{E}{1-\mu^2}\begin{bmatrix} 1 & \mu & 0 \\ \mu & 1 & 0 \\ 0 & & \frac{1-\mu}{2} \end{bmatrix}$$

单元矩阵和向量

$$[K_e]=\int_{\Omega_e}[B]^T[D][B]\mathrm{d}\Omega,\quad \{P_e\}=\int_{\Omega_e}[N]^T\{f\}\mathrm{d}\Omega$$

三角形单元公式

型函数 $[N(x,y)]=[N_1(x,y)\quad N_2(x,y)\quad N_3(x,y)]$

单元向量

$$\{P_e\}=\int_\Omega [N]^T\{f\}\mathrm{d}\Omega=\int_\Omega \begin{bmatrix} N_1(x,y) & 0 \\ 0 & N_1(x,y) \\ N_2(x,y) & 0 \\ 0 & N_2(x,y) \\ N_3(x,y) & 0 \\ 0 & N_3(x,y) \end{bmatrix} \begin{Bmatrix} 0 \\ -\rho g \end{Bmatrix} \mathrm{d}\Omega = \int_\Omega \begin{Bmatrix} 0 \\ -\rho g N_1(x,y) \\ 0 \\ -\rho g N_2(x,y) \\ 0 \\ -\rho g N_3(x,y) \end{Bmatrix} \mathrm{d}\Omega = \frac{\rho g \Delta t}{3} \begin{Bmatrix} 0 \\ -1 \\ 0 \\ -1 \\ 0 \\ -1 \end{Bmatrix}$$

$$\int_\Delta N_1(x,y)\mathrm{d}\Delta=\int_\Delta N_2(x,y)\mathrm{d}\Delta=\int_\Delta N_3(x,y)\mathrm{d}\Delta=\frac{\Delta}{3}$$

程序设计

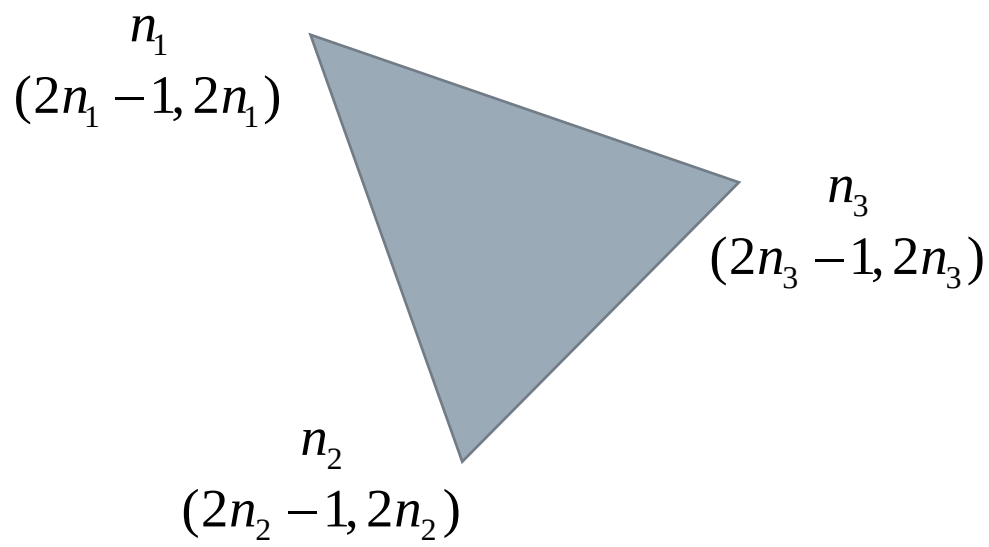
```
clc;clear all;close all;
YM = 210e9; PR = 0.25; thickness = 0.02;
EM = YM/(1-PR*PR)*[1 PR 0; PR 1 0; 0 0 (1-PR)/2];
a = 0.2; b = 0.001; delta = b/4;
[coord,element,BC1] = CreateModel(a,b,delta,0);
ngrid=size(coord,2);
nelem=size(element,2);
K=zeros(2*ngrid);
P=zeros(2*ngrid,1);
```

程序设计

```
for n=1:nelem
    ix=element(:,n);
    [ke,area] = EMPSTRI(EM,thickness,coord(1,ix),coord(2,ix));
    pe = (area*thickness/3)*9.8*7800*[0 -1 0 -1 0 -1]';
    LM = [2*ix(1)+[-1 0] 2*ix(2)+[-1 0] 2*ix(3)+[-1 0]];
    K(LM,LM)=K(LM,LM)+ke;
    P(LM)=P(LM)+pe;
end
```

$$\begin{Bmatrix} 2n_1+\begin{Bmatrix}-1\\0\end{Bmatrix} \\ 2n_2+\begin{Bmatrix}-1\\0\end{Bmatrix} \\ 2n_3+\begin{Bmatrix}-1\\0\end{Bmatrix} \end{Bmatrix}$$

$$\{P_e\}=\frac{\rho g \Delta t}{3}\begin{Bmatrix}1\\1\\1\end{Bmatrix}$$

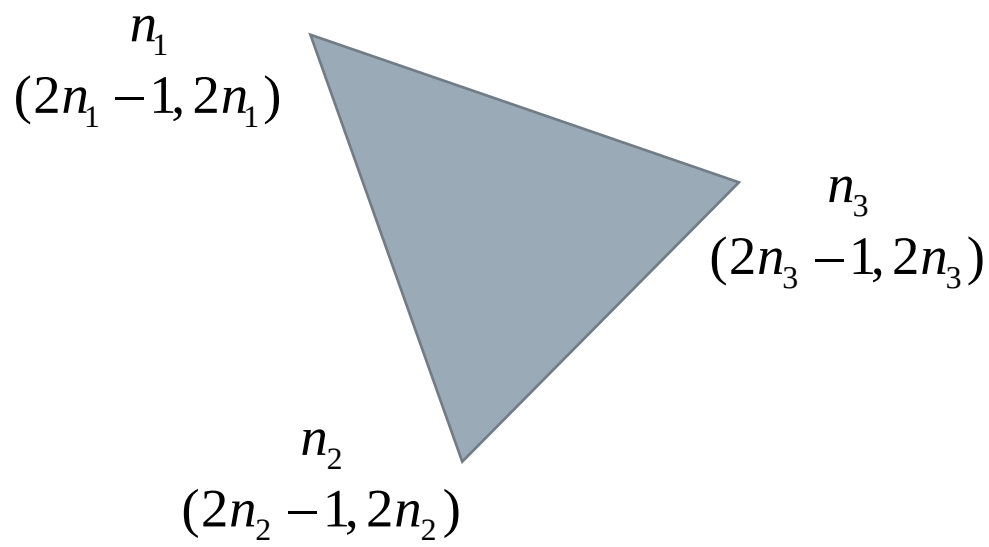


程序设计

```
for n=1:nelem
    ix=element(:,n);
    [ke,area] = EMPSTRI(EM,thickness,coord(1,ix),coord(2,ix));
    pe = (area*thickness/3)*9.8*7800*[0 -1 0 -1 0 -1]';
    LM = [2*ix(1)+[-1 0] 2*ix(2)+[-1 0] 2*ix(3)+[-1 0]];
    K(LM,LM)=K(LM,LM)+ke;
    P(LM)=P(LM)+pe;
end
```

$$\begin{Bmatrix} 2n_1+\begin{Bmatrix}-1\\0\end{Bmatrix} \\ 2n_2+\begin{Bmatrix}-1\\0\end{Bmatrix} \\ 2n_3+\begin{Bmatrix}-1\\0\end{Bmatrix} \end{Bmatrix}$$

$$\{P_e\}=\frac{\rho g \Delta t}{3}\begin{Bmatrix}1\\1\\1\end{Bmatrix}$$



程序设计

```
function [Ke,Area] = EMPSTRI(EM,t,x,y)
x = x(:); y = y(:);
b = [y(2)-y(3) y(3)-y(1) y(1)-y(2)];
c = [x(3)-x(2) x(1)-x(3) x(2)-x(1)];
Area = det([1 1 1; x' ;y'])/2;
B = zeros(3,6);
for j=1:3
    B(:,(1:2)+2*(j-1)) = (1/2/Area)*[b(j) 0; 0 c(j); c(j) b(j)];
end
Ke = Area*t*(B'*EM*B);
```

$$b_1 = y_2 - y_3 \quad c_1 = x_3 - x_2$$

$$[B] = \begin{bmatrix} B_1 & B_2 & B_3 \end{bmatrix}, \quad [B_i] = \frac{1}{2\Delta}\begin{bmatrix} b_i & 0 \\ 0 & c_i \\ c_i & b_i \end{bmatrix}$$

$$[K_e] = \int_{\Omega_e} [B]^T [D][B] \mathrm{d}\Omega$$

程序设计

1	2	3	4	5	1	2	3	4	5	81	82	83	84	85
1	1	1	1	1	2	2	2	2	2	1	1	1	1	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

```
nBC1 = size(BC1,2);
index1 = 2*(BC1(1,1:nBC1)-1) + BC1(2,1:nBC1);
index2=setdiff(1:(2*ngrid),index1);
q = zeros(2*ngrid,1);
q(index1) = BC1(3,:)';
K21 = K(index2,index1);
K22 = K(index2,index2);
P2 = P(index2) - K21*q(index1);
q(index2) = K22\P2;
```

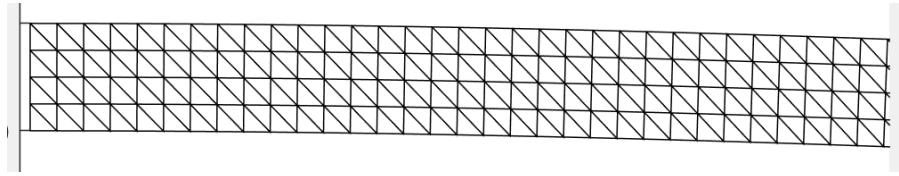
自由度编号=2×(节点号−1)+位移分量号

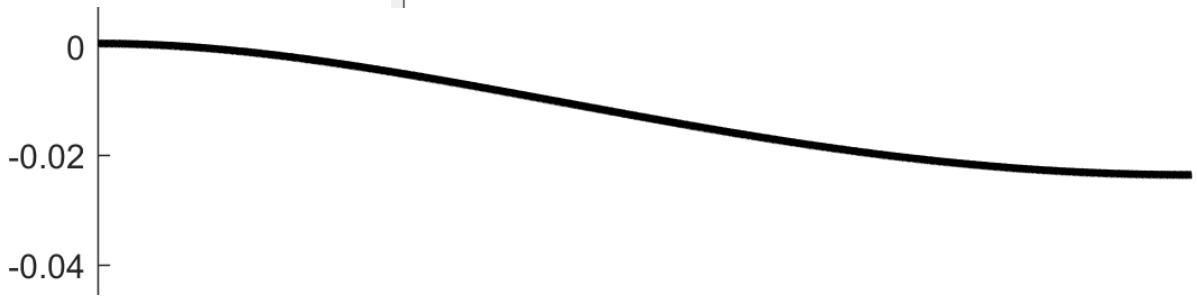
$$[K_2]\{q\}-\{p_2\}=0$$

$$\begin{bmatrix}K_{21} & K_{22}\end{bmatrix}\begin{Bmatrix}q_1\\ q_2\end{Bmatrix}-\{p_2\}=0$$

$$[K_{22}]\{q_2\}=\{p_2\}-[K_{21}]\{q_1\}$$

程序设计






```
hold on; axis equal
scale = 100;
for n=1:nelem
    ix=element(:,n);
    x=coord(1,ix); y=coord(2,ix);
    LM = [2*ix(1)+[-1 0] 2*ix(2)+[-1 0] 2*ix(3)+[-1 0]];
    uv = scale*reshape(q(LM),2,3);
    u = uv(1,:); v = uv(2,:);
    fill(x+u,y+v,ones(size(x)));
end
```