8803 - Mobile Manipulation: Control

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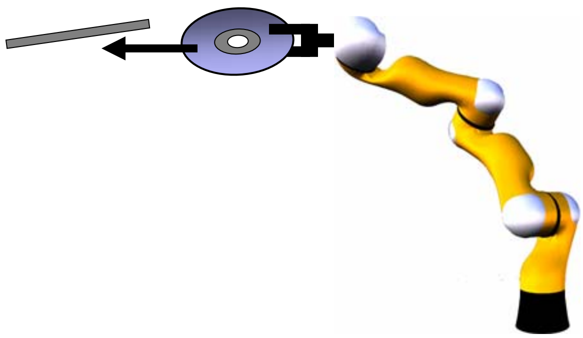
How do we make the robot move?

Today: Control of Dynamical Systems & Robots

Thursday: Strategies for Combining Position & Force



The Robot is a Dynamic System

What is a Dynamic System?

- A system whose variables are time-dependent
- Inputs and Outputs vary with time

How can we describe a dynamic system?

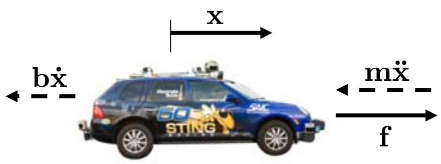
- Input-output differential equations
- Free-body diagrams
- State-variable matrix
- Transfer functions
- Block diagrams

Control

What is Control?

Given the system state, choose input to get a desired output.

Car Cruise Control



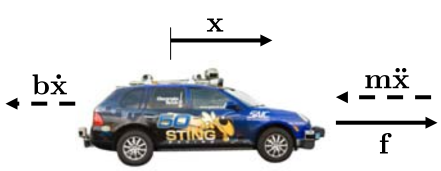
$$m\ddot{x} + b\dot{x} = f$$

$$\ddot{x} + \frac{b}{m}\dot{x} = \frac{f}{m}$$

State Variable Form:

$$\ddot{x} = \left[-\frac{b}{m}\right]\dot{x} + \left[\frac{1}{m}\right]f$$

Car Cruise Control (Open Loop)



$$m\ddot{x} + b\dot{x} = f$$

Steady State → Let $\ddot{x} = 0$

$$0 + b\dot{x}_{ss} = f$$

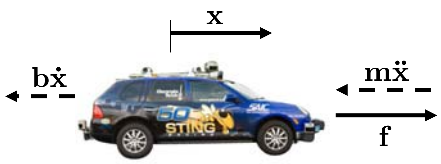
$$\dot{x}_r = \dot{x}_{ss} = \frac{f}{b}$$

$$\boxed{u = f = b\dot{x}_r}$$

State Variable Form:

$$\ddot{x} = \left[-\frac{b}{m}\right]\dot{x} + \left[\frac{1}{m}\right]f$$

Car Cruise Control (Open Loop)



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$$\boxed{f = b\dot{x}_r}$$

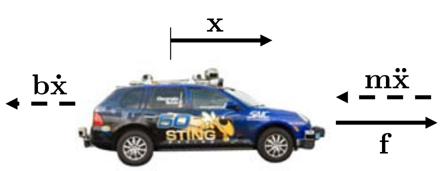
$$\dot{x}_r \rightarrow \boxed{\text{Ctrl}} \xrightarrow{f} \boxed{\text{Plant}} \xrightarrow{\dot{x}}$$

State Variable Form:

$$\ddot{x} = \left[-\tfrac{b}{m}\right]\dot{x} + \left[\tfrac{1}{m}\right] f$$

$$\boxed{\ddot{x} = \left[-\tfrac{b}{m}\right]\dot{x} + \left[\tfrac{b}{m}\right] \dot{x}_r}$$

Car Cruise Control (Open Loop)



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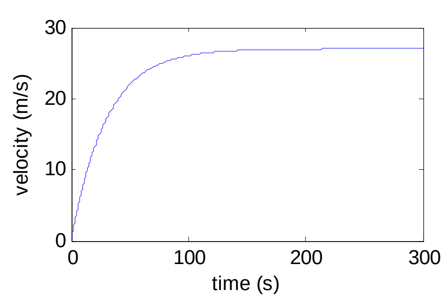


m = 1500kg b = 50Ns/m v_r = 27m/s

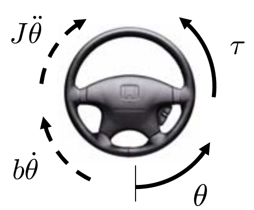
Car Steering

$$
\begin{aligned}
J\ddot{\theta} &+ b\dot{\theta} = \tau \\
\ddot{\theta} &= -\frac{b}{J}\dot{\theta} + \frac{1}{J}\tau
\end{aligned}
$$

State Variable Form:

$$
\begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{b}{J} \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{J} \end{bmatrix} \tau
$$

Car Steering (Proportional Control)



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\begin{aligned}
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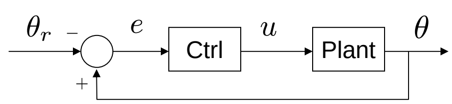
\end{aligned}
$$

$$\boxed{\tau = -K_p(\theta - \theta_r)}$$

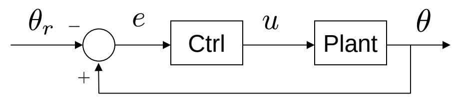
$$
\ddot{\theta} = -\frac{b}{J}\dot{\theta} - \frac{K_p}{J}\theta + \frac{K_p}{J}\theta_r
$$



State Variable Form:

$$
\begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{K_p}{J} & -\frac{b}{J} \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{K_p}{J} \end{bmatrix} \theta_r
$$

Car Steering (Proportional Control)



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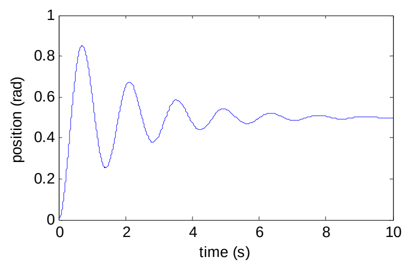
$$J\ddot{\theta} + b\dot{\theta} = \tau$$

$$\ddot{\theta} = -\frac{b}{J}\dot{\theta} + \frac{1}{J}\tau$$

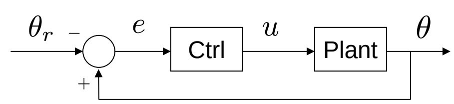
$$\boxed{\tau = -K_p(\theta - \theta_r)}$$

$$\ddot{\theta} = -\frac{b}{J}\dot{\theta} - \frac{K_p}{J}\theta + \frac{K_p}{J}\theta_r$$

J = .1kgm^2 b = .1Nms θ_r = .5rad



Car Steering (Proportional Control)



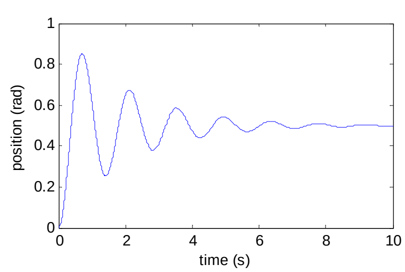
$$\boxed{\tau = -K_p(\theta - \theta_r)}$$

$$\ddot{\theta} = -\frac{b}{J}\dot{\theta} - \frac{K_p}{J}\theta + \frac{K_p}{J}\theta_r$$

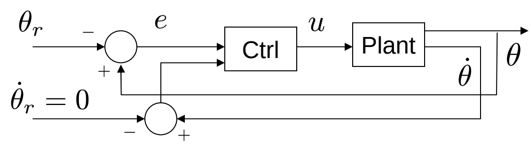
Stability?

Exponential Convergence?

Limitations?



Car Steering (PD Control)



State Variable Form:

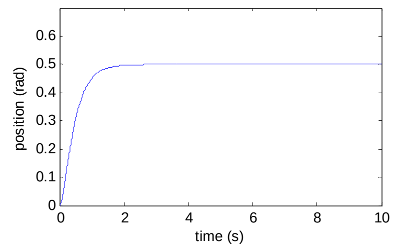
$$\begin{bmatrix}\dot{\theta}\\ \ddot{\theta}\end{bmatrix} = \begin{bmatrix}0 & 1\\ -\frac{K_p}{J} & -\frac{b+K_d}{J}\end{bmatrix}\begin{bmatrix}\theta\\ \dot{\theta}\end{bmatrix} + \begin{bmatrix}0\\ \frac{K_p}{J}\end{bmatrix}\theta_r$$

$$J\ddot{\theta} + b\dot{\theta} = \tau$$

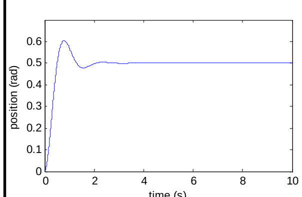
$$\ddot{\theta} = \frac{b}{J}\dot{\theta} + \frac{1}{J}\tau$$

$$\boxed{\tau = -K_d(\dot{\theta} - \dot{\theta}_r) - K_p(\theta - \theta_r)}$$

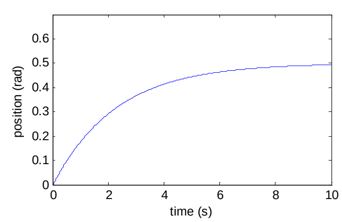
$$\ddot{\theta} = -\frac{b}{J}\dot{\theta} - \frac{K_d}{J}\dot{\theta} - \frac{K_p}{J}\theta + \frac{K_p}{J}\theta_r$$



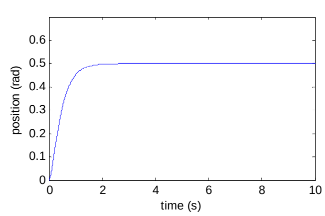
PD Control



Under-damped



Over-damped



Critically damped (Just Right)

PD Control (Selecting Gains)

$$\begin{aligned}\ddot{\theta} &= -\frac{b+K_d}{J}\dot{\theta} - \frac{K_p}{J}(\theta-\theta_r) \\ \ddot{\theta} &= -K_d'\dot{\theta} - K_p'(\theta-\theta_r)\end{aligned} \quad\Rightarrow\quad \ddot{\theta} + K_d'\dot{\theta} + K_p'\theta = K_p\theta_r$$

Characteristic Polynomial: $s^2 + K_d's + K_p'$

Standard Form: $s^2 + 2\xi\omega_n s + \omega_n^2$

Damping Ratio: ξ
Natural Frequency: ω_n

$$\begin{aligned}K_p' &= \omega_n^2 \\ K_d' &= 2\xi\omega_n\end{aligned} \quad\xrightarrow{\xi=1}\quad \begin{aligned}K_p' &= \omega_n^2 \\ K_d' &= 2\omega_n = 2\sqrt{K_p'}\end{aligned}$$

PD Control (Selecting Gains)

Starting Point:
$$K_p' = \omega_n^2$$
$$K_d' = 2\omega_n = 2\sqrt{K_p'}$$

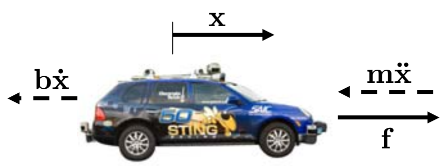
What is ω_n ?

Gains are limited by:
- structural flexibility
- time delay
- sampling rate
- actuator saturation

Optimal Control – Linear Quadratic Regulator

Tuning!

Car Cruise Control (Closed Loop)



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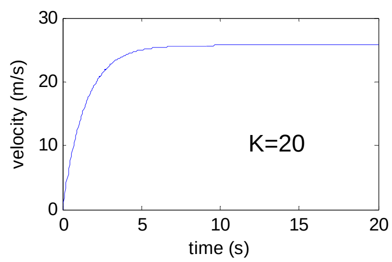
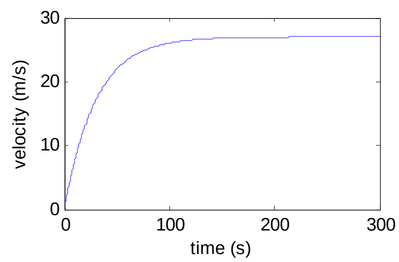
$$\boxed{f = -bK_p(\dot{x} - \dot{x}_r)}$$



Open Loop:



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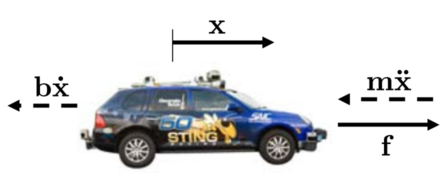

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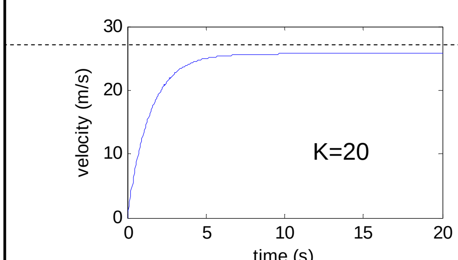

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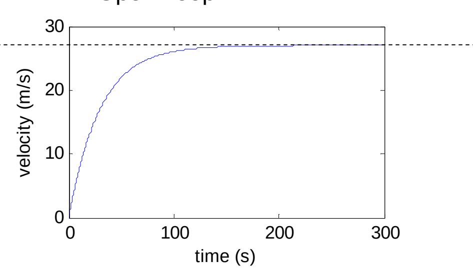
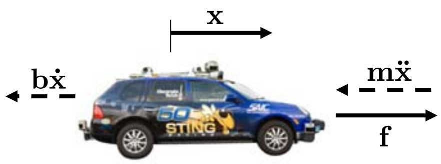
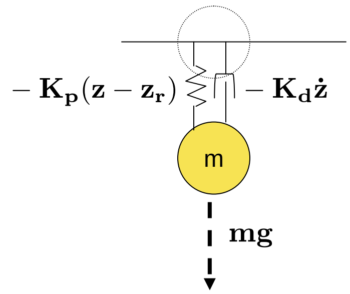


Another Example

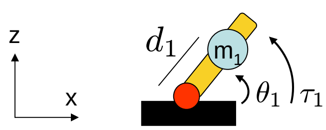


$$m\ddot{x} + b\dot{x} \quad = \quad f$$

$$\boxed{f = -bK_p(\dot{x} - \dot{x}_r)}$$



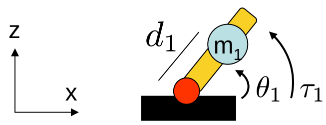
Manipulator Control (Gravity Compensation)



$$x_1 = d_1 \cos(\theta_1)$$

$$\ddot{\theta}_1 = -d_1 \cos(\theta_1) m_1 g + \tau_1$$

Manipulator Control (Gravity Compensation)



$$x_1 = d_1 \cos(\theta_1)$$

$$\ddot{\theta}_1 = -d_1 \cos(\theta_1) m_1 g + \tau_1$$

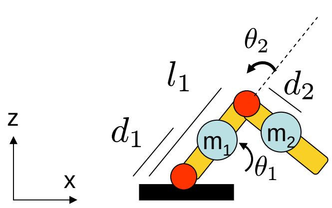
Acceleration due to gravity:

$$\tau_g = -d_1 \cos(\theta_1) m_1 g$$

How do we do position control?

$$\tau_1 = d_1 c_1 m_1 g - K_p(\theta_1 - \theta_{1d}) - K_d \dot{\theta}_1$$

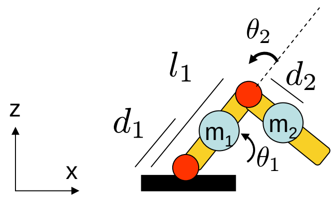
Manipulator Control (Gravity Compensation)



$$\begin{aligned} x_1 &= d_1 c_1 \\ x_2 &= l_1 c_1 + d_2 c_2 \end{aligned}$$

Torque due to gravity:

Manipulator Control (Gravity Compensation)

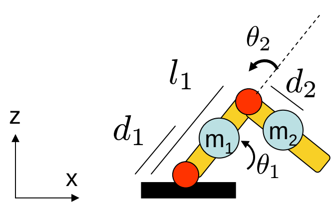


$$\begin{aligned} x_1 &= d_1 c_1 \\ x_2 &= l_1 c_1 + d_2 c_2 \end{aligned}$$

Torque due to gravity:

$$\begin{aligned} \tau_2 &= -d_2 c_{12} m_2 g \\ \tau_1 &= -d_1 c_1 m_1 g - (l_1 c_1 + d_2 c_{12}) m_2 g \end{aligned}$$

Manipulator Control (Gravity Compensation)



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How do we do position control?

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} d_2 c_{12} m_2 \\ d_1 c_1 m_1 + (l_1 c_1 + d_2 c_{12}) m_2 \end{bmatrix} g - \mathbf{K}_p \begin{bmatrix} \theta_1 - \theta_{1d} \\ \theta_2 - \theta_{2d} \end{bmatrix} - \mathbf{K}_d \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

Gains are typically diagonal matrices

Manipulator Control (Full Model)

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) = \tau$$

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Augmented PD:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}}_\mathbf{d} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}_\mathbf{d} + \mathbf{G}(\mathbf{q}) \boxed{- \mathbf{K}_\mathbf{d}(\dot{\mathbf{q}} - \dot{\mathbf{q}}_\mathbf{d}) - \mathbf{K}_\mathbf{p}(\mathbf{q} - \mathbf{q}_\mathbf{d})} = \tau$$

Computed Torque:

$$\mathbf{M}(\mathbf{q})(\ddot{\mathbf{q}}_\mathbf{d} \boxed{- \mathbf{K}_\mathbf{d}(\dot{\mathbf{q}} - \dot{\mathbf{q}}_\mathbf{d}) - \mathbf{K}_\mathbf{p}(\mathbf{q} - \mathbf{q}_\mathbf{d})}) + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) = \tau$$

More Jacobians (Virtual Work)

$\delta \mathbf{x} = \mathbf{J}\delta \mathbf{q}$ $\quad \mathbf{W} = \mathbf{F^T x}$ $\quad \delta \mathbf{W} = \mathbf{F^T}\delta \mathbf{x}$ $\quad \delta \mathbf{W} = \tau^{\mathbf{T}}\delta \mathbf{q}$

$$\mathbf{F^T}\delta \mathbf{x} = \tau^{\mathbf{T}}\delta \mathbf{q}$$

$$\mathbf{F^T J}\delta \mathbf{q} = \tau^{\mathbf{T}}\delta \mathbf{q}$$

Open Loop Force Control $\boxed{\tau = \mathbf{J^T F}}$

More Jacobians (Virtual Work)

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$$\mathbf{F^T J}\delta \mathbf{q} = \tau^{\mathbf{T}}\delta \mathbf{q}$$

Open Loop Force Control $\boxed{\tau = \mathbf{J^T F}}$

Velocity Jacobian for Link Mass **i**: $\mathbf{J_{vi}}$ $\quad \mathbf{J_{vi}^T g} = ?$

$$\mathbf{G} = -(\mathbf{J}_{v1}^T(m_1\mathbf{g}) + \mathbf{J}_{v2}^T(m_2\mathbf{g})...\mathbf{J}_{vn}^T(m_n\mathbf{g}))$$

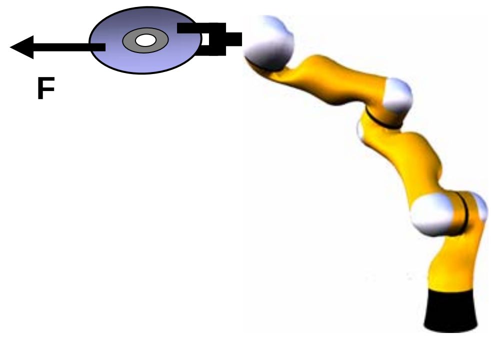
$$\boxed{\tau = \mathbf{G(q)} + \mathbf{J^T F}}$$

$\tau = \mathbf{J^T F}$ Manipulator Workspace Control

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) = \tau$$

$$\bar{\mathbf{M}}(\mathbf{q})\ddot{\mathbf{x}} + \bar{\mathbf{C}}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{x}} + \bar{\mathbf{G}}(\mathbf{q}) = \mathbf{F}$$

$$\begin{aligned} \bar{\mathbf{M}} &= J^{-T} M J^{T} \\ \bar{\mathbf{C}} &= J^{-T}(CJ^{-1} + M\dot{J}^{-1}) \\ \bar{\mathbf{G}} &= J^{-T} G \\ \mathbf{F} &= J^{-T} \tau \end{aligned}$$



Khatib '80, Murray '94

$\tau = \mathbf{J^T F}$ Manipulator Workspace Control

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) = \tau$$

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Computed Torque:

$$\mathbf{F} = \bar{\mathbf{M}}(\mathbf{q})(\ddot{\mathbf{x}}_\mathbf{d} + \boxed{\mathbf{K_d}(\dot{\mathbf{x}} - \dot{\mathbf{x}}_\mathbf{d}) - \mathbf{K_p}(\mathbf{x} - \mathbf{x_d})}) + \bar{\mathbf{C}}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{x}} + \bar{\mathbf{G}}(\mathbf{q})$$

$$\tau = \mathbf{J^T F}$$

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$\tau = \mathbf{J^T F}$ Manipulator Workspace Control

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\bar{\mathbf{G}} &= J^{-T} G \\
\mathbf{F} &= J^{-T} \tau
\end{aligned}$$

Computed Torque:

$$\mathbf{F} = \bar{\mathbf{M}}(\mathbf{q})(\ddot{\mathbf{x}}_\mathbf{d} \boxed{- \mathbf{K_d}(\dot{\mathbf{x}} - \dot{\mathbf{x}}_\mathbf{d}) - \mathbf{K_p}(\mathbf{x} - \mathbf{x_d})}) + \bar{\mathbf{C}}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{x}} + \bar{\mathbf{G}}(\mathbf{q})$$

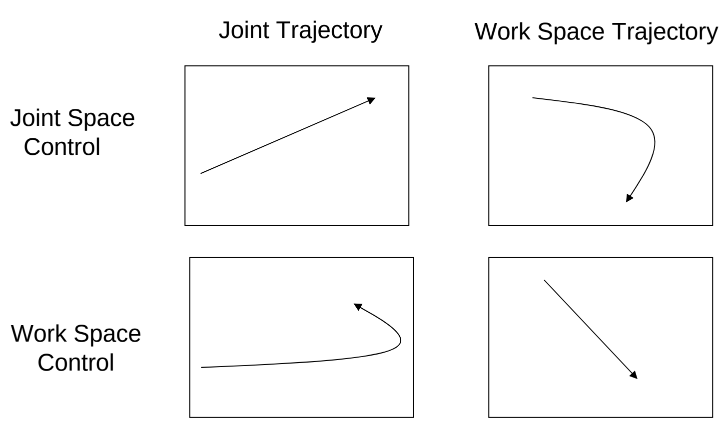
$$\tau = \mathbf{J^T F}$$

Gravity Augmented PD:

$$\mathbf{F} = \bar{\mathbf{G}}(\mathbf{q}) \boxed{- \mathbf{K_d}(\dot{\mathbf{x}} - \dot{\mathbf{x}}_\mathbf{d}) - \mathbf{K_p}(\mathbf{x} - \mathbf{x_d})}$$

$$\tau = \mathbf{J^T F}$$

Joint Space vs. Work Space (Step Response)



Summary

- Dynamic Systems change over time
- Open Loop Control
- Closed Loop PID Control
- Gravity Compensation & Linearization
- Joint Space Position Control
- Work Space Position Control
- Open Loop Force Control

Thursday

- How do we close the loop on force control?
- What is the relationship between force/position control?
- Can we do both?