8803 - Mobile Manipulation: Kinematics

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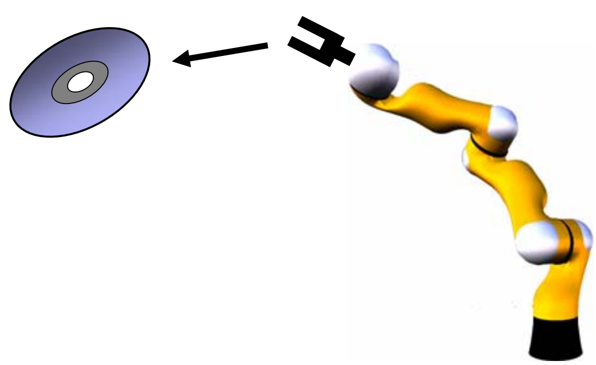
Our Task:

Today: Given joint angles $\rightarrow$ find end-effector & CD positions

Thursday: Given CD position $\rightarrow$ find robot joint angles



Coordinates

- **Coordinate System:** A set of numbers that specifies configuration
  - Points
  - Rigid Bodies
  - Articulated Manipulators
- **Degrees of Freedom:** Minimal number of independent coordinates

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  - Points
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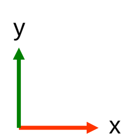
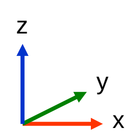
  - Articulated Manipulators
- **Degrees of Freedom:** Minimal number of independent coordinates
- How many DOF?
  - Point in the plane 2 (x,y)

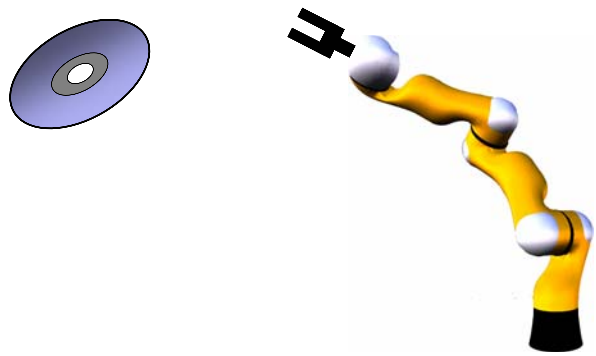


  - Point in 3D 3 (x,y,z)



Rigid Bodies

Set of points. Distance between any pair of points remains constant.



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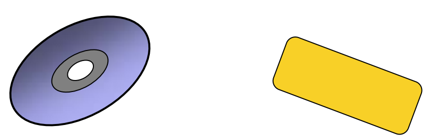

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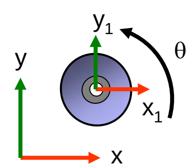
How many DOF?

Body in the plane 3



Why?

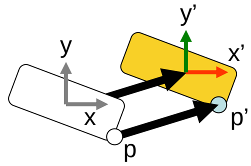
- Fixed to Local Coordinate Frame

- Rigid Body Displacements

Rigid Body Displacements

Must preserve **rigid** property, reflections are not allowed.

- **Translation:** Every point moves a fixed distance in a specified direction.



$$p'_x = p_x + t_x$$
$$p'_y = p_y + t_y$$

$$\begin{bmatrix} p'_x \\ p'_y \end{bmatrix} = \begin{bmatrix} p_x \\ p_y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

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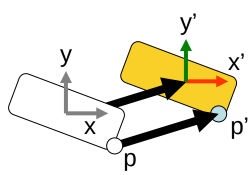

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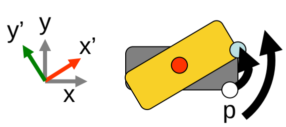
- **Rotation**: One point is fixed. Others move a specified **angle** relative to fixed point.



$$p'_x = p_x \cos\theta - p_y \sin\theta$$

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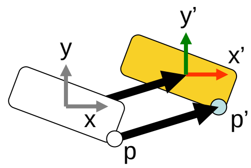

$$p'_x = p_x + t_x \qquad \begin{bmatrix} p'_x \\ p'_y \end{bmatrix} = \begin{bmatrix} p_x \\ p_y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$
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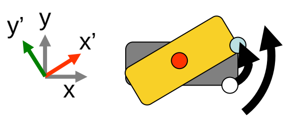

$$p'_x = p_x \cos\theta - p_y \sin\theta \qquad \begin{bmatrix} p'_x \\ p'_y \end{bmatrix} = \begin{bmatrix} \mathrm{c}\theta & -\mathrm{s}\theta \\ \mathrm{s}\theta & \mathrm{c}\theta \end{bmatrix} \begin{bmatrix} p_x \\ p_y \end{bmatrix}$$
$$p'_y = p_x \sin\theta + p_y \cos\theta$$

- Every displacement can be represented as **1 Translation** and/or **1 Rotation**

Displacements are a coordinate system (t_x,t_y,θ)

Homogeneous Transformations

Method for representing displacements and relative coordinates

$$\mathbf{T}_B^A = \begin{bmatrix} \mathbf{R}_B^A & & \mathbf{t}_B^A \\ & & \\ 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{p} = \begin{bmatrix} p_x \\ p_y \\ 1 \end{bmatrix}$$

Compact Representation. Allows for simple concatenation:

$$\mathbf{T}_C^A = \mathbf{T}_B^A \mathbf{T}_C^B$$

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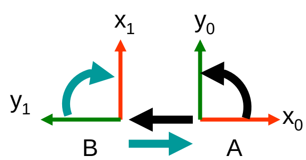
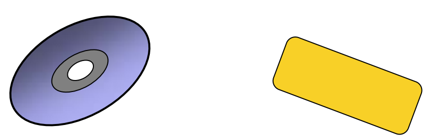


$$\begin{bmatrix} 0 & -1 & -1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

A to B B to A = Identity

Rigid Bodies



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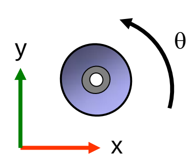
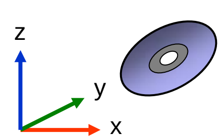


Body in the plane 3

Body in 3D 6



Representations of Rotation

- Fixed Axis (x,y,z) or (roll, pitch, yaw)
  - Convenient and intuitive, 12 variations
- Euler Angles (z,y,x), (z,y,z) - (moving axes)
  - Equivalent to reverse order fixed-axis
- Unit Quaternions (4 numbers)
  - Easy to compose
  - Meaningful Interpolation
  - Useful for numerical stability, sampling, optimization
- Angle-Axis (4 numbers = 3 axis + 1 angle)
- **Rotation Matrices (9 numbers – Orthonormal Matrix)**

Fixed Angles to Rotation Matrices

$$\mathbf{R}^A_{B\ XYZ}(\gamma,\beta,\alpha) = \mathbf{R}_Z(\alpha)\mathbf{R}_Y(\beta)\mathbf{R}_X(\gamma) =$$

$$\begin{bmatrix} c\alpha & -s\alpha & 0 \\ s\alpha & c\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\beta & 0 & s\beta \\ 0 & 1 & 0 \\ -s\beta & 0 & c\beta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\gamma & -s\gamma \\ 0 & s\gamma & c\gamma \end{bmatrix} =$$

$$\begin{bmatrix} c\alpha c\beta & c\alpha s\beta s\gamma - s\alpha c\gamma & c\alpha s\beta c\gamma + s\alpha s\gamma \\ s\alpha c\beta & s\alpha s\beta s\gamma + c\alpha c\gamma & s\alpha s\beta c\gamma - c\alpha s\gamma \\ -s\beta & c\beta s\gamma & c\beta c\gamma \end{bmatrix}$$

Homogeneous Transform 3D

$$\mathbf{T}_B^A = \begin{bmatrix} & \mathbf{R}_B^A & & \mathbf{t}_B^A \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

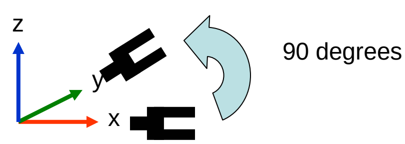
Interpretations:

- Maps p^B to p^A
- Transform operator: creates p_2^A from p_1^A
- Describes frame B relative to frame A

  t_B^A = position of the frame

$$\mathbf{R}_B^A = \begin{bmatrix} \mathbf{x}_B^A & \mathbf{y}_B^A & \mathbf{z}_B^A \end{bmatrix}$$

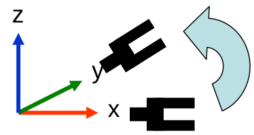
Exercise



You think you set the robot's joint angles such that the end effector should be at position (x_1, y_1, z_1) and turned 90º counter-clockwise from the initial position.

Your code gives you a homogeneous transform representing the current end-effector position relative to the initial one. What should it be?

Exercise



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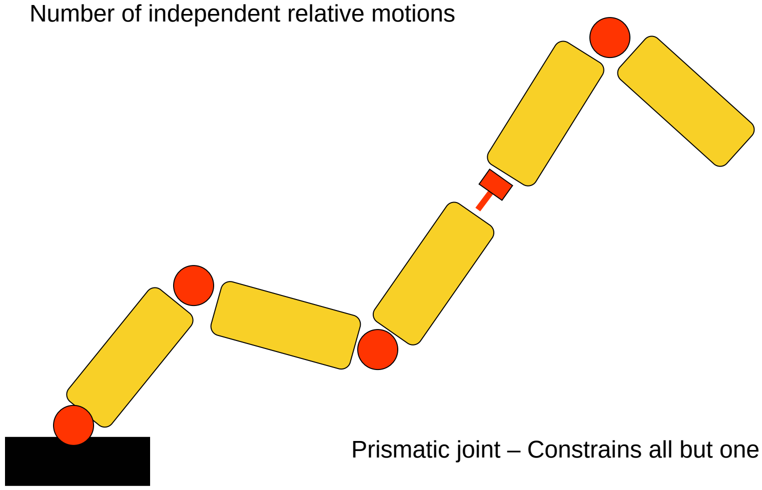
Your code gives you a homogeneous transform representing the current end-effector position relative to the initial one. What should it be?

$$\mathbf{T}^{initial}_{current} = \begin{bmatrix} 0 & -1 & 0 & x_1 \\ 1 & 0 & 0 & y_1 \\ 0 & 0 & 1 & z_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Robot Degrees of Freedom

Number of independent relative motions



Prismatic joint – Constrains all but one translation

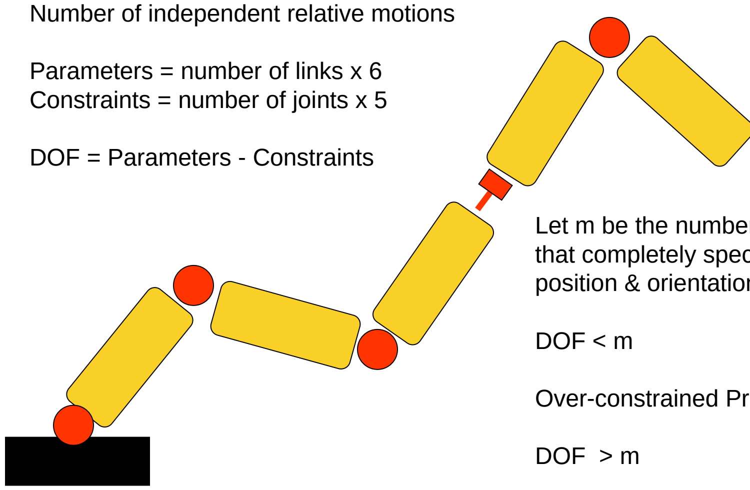
Revolute joint – Constrains all but one rotation

Robot Degrees of Freedom

Number of independent relative motions

Parameters = number of links x 6
Constraints = number of joints x 5

DOF = Parameters - Constraints



Let m be the number of parameters that completely specify end-effector position & orientation (typically 6)

DOF < m

Over-constrained Problem

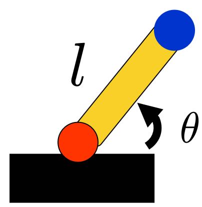
DOF > m

Redundant Degrees of Freedom

Forward Kinematics

- We have worked with:
  - Point coordinates
  - Workspace coordinates
- Now consider joint coordinates:
  - Kinematics relates Joint Coordinates to Workspace Coordinates

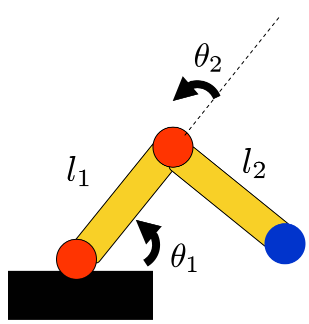
Forward Kinematics



1-DOF Robot Arm

$$p_x = l\cos\theta$$
$$p_y = l\sin\theta$$

Forward Kinematics

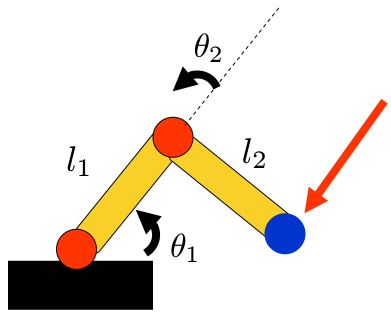


2-DOF Robot Arm

$$p_x = l_1 c_1 + l_2 c_{12}$$
$$p_y = l_1 s_1 + l_2 s_{12}$$

$$c_{12} = \cos(\theta_1 + \theta_2)$$
$$s_{12} = \sin(\theta_1 + \theta_2)$$

Forward Kinematics



2-DOF Robot Arm

$$\mathbf{T}_1^0 = \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{T}_2^1 = \begin{bmatrix} c_2 & -s_2 & l_1 \\ s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{T}_3^2 = \begin{bmatrix} 1 & 0 & l_2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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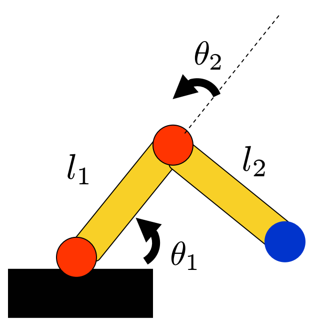
Forward Kinematics



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$$\mathbf{T}_2^0 = \mathbf{T}_1^0\mathbf{T}_2^1 = \begin{bmatrix} c_1c_2 - s_1s_2 & -c_1s_2 - s_1c_2 & l_1c_1 \\ s_1c_2 + c_1s_2 & -s_1s_2 + c_1c_2 & l_1s_1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_{12} & -s_{12} & l_1c_1 \\ s_{12} & c_{12} & l_1s_1 \\ 0 & 0 & 1 \end{bmatrix}$$

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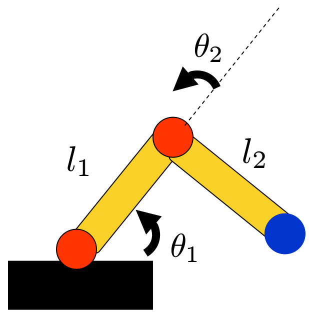
Forward Kinematics

2-DOF Robot Arm



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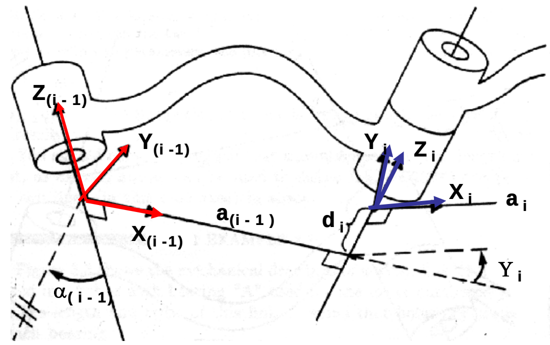
Denavit-Hartenberg (DH) Parameters

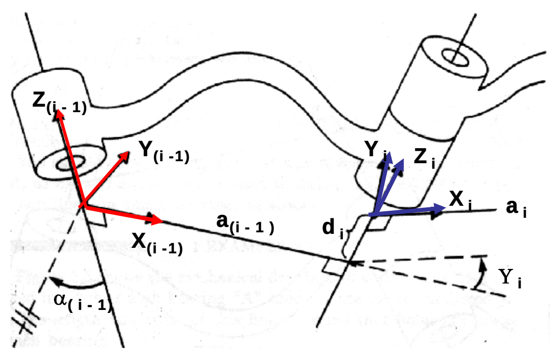
- 4 Parameters describe how frame (i) relates to frame (i-1)
- Compact description of manipulator kinematics
- Mechanical method for deriving transformations
- Widely used as a specification for robot manipulators

DH Parameters



First, label the coordinate frames

DH Parameters



Next, calculate the parameters:

$a_i =$ the distance from Z_i to Z_{i+1} measured along X_i
$\alpha_i =$ the angle between Z_i and Z_{i+1} measured about X_i
$d_i =$ the distance from X_{i-1} to X_i measured along Z_i
$\theta_i =$ the angle between X_{i-1} and X_i measured about Z_i

DH Parameters

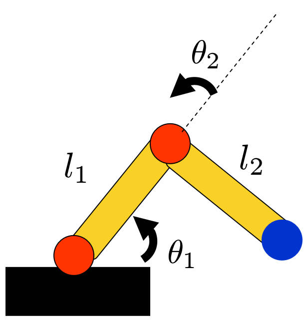
- Parameter Table:

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	0	l_1	0	θ_2
3	0	l_2	0	0

- DH Matrix:

$$\mathbf{T}_i^{i-1} = \begin{bmatrix} \mathrm{c}\theta_i & -\mathrm{s}\theta_i & 0 & \alpha_{i-1} \\ \mathrm{s}\theta_i \mathrm{c}\alpha_{i-1} & \mathrm{c}\theta_i \mathrm{c}\alpha_{i-1} & -\mathrm{s}\alpha_{i-1} & -\mathrm{s}\alpha_{i-1} d_i \\ \mathrm{s}\theta_i \mathrm{s}\alpha_{i-1} & \mathrm{c}\theta_i \mathrm{s}\alpha_{i-1} & \mathrm{c}\alpha_{i-1} & \mathrm{c}\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Forward Kinematics



2-DOF Robot Arm

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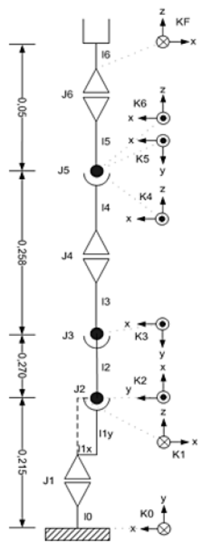
Special DH Parameters (6 DOF)



Joint i	Rotation α_{i-1}	Link length a_{i-1}	Joint angle θ_i	Link offset d_i
1	$-90°$	0	$\Theta_1+180°$	$l_0+l_{1,y}$
2	$90°$	$l_{1,x}$	$\Theta_2+90°$	0
3	$0°$	l_2	$\Theta_3+90°$	0
4	$90°$	0	Θ_4	l_3+l_4
5	$-90°$	0	Θ_5	0
6	$90°$	0	Θ_6	0
7	$0°$	0	$180°$	l_5