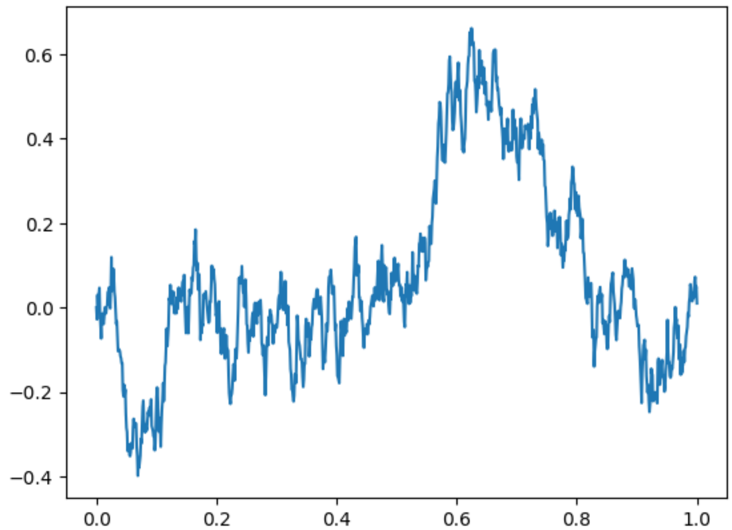
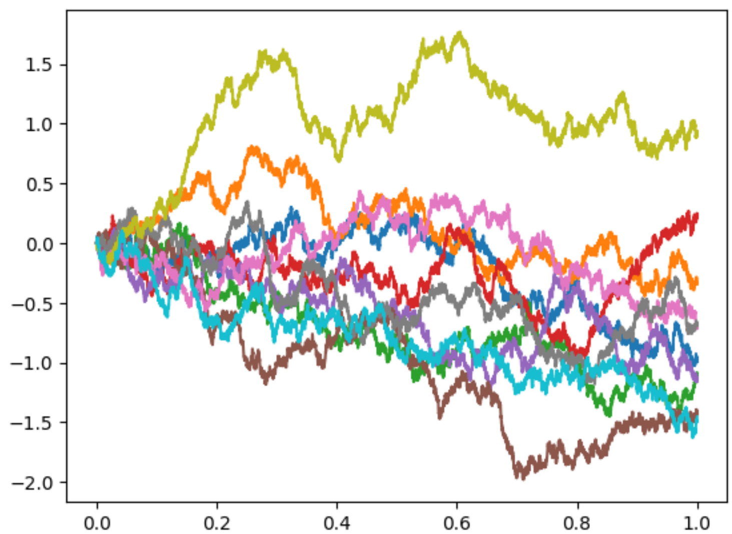
**Problem 1:**

There are three types of prices return, classical Brownian motion, arithmetic return system and geometric Brownian motion.
For the first one, classical Brownian Motion, if we do one for each time, with 1000 period, and with rt ~ N(0,sigma^2), we will have a plot like this



If we do 10 motions at one time, we will have the picture like



Now, let make it in financial sense, I assume the starting price of the stock of 100, with standard deviation of 0.1, we will have the outcome as

Classical brownian motion:

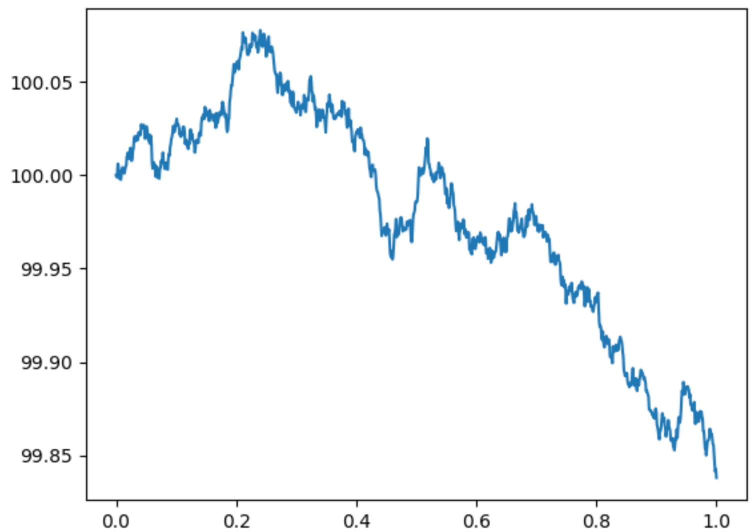
Expected mean of price: 100

Calculated mean of price: 99.97893381862005

Expected standard deviation of price: 0.1

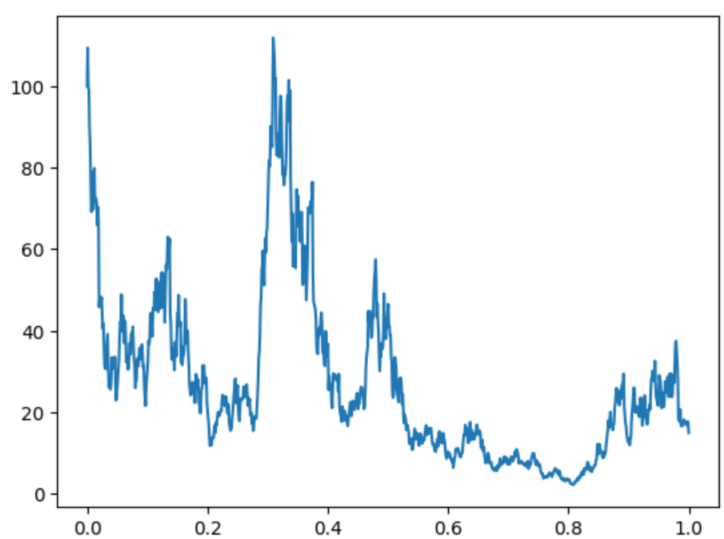
Calculated standard deviation of price: 0.05954861151237859

here is what the plot like




We can see that the price of the stock is around 101 and 99, exactly match our expectations. Next, let’s move on to Arithmetic, In the arithmetic return system, the price at time t is modeled as the product of the price at time t-1 and a random return, which is assumed to be normally distributed with mean 0 and standard deviation sigma. In other words, the price changes are proportional to the current price. The volatility of the arithmetic return system is larger than the classical Brownian motion because the price changes in the arithmetic return system are proportional to the size of the returns, whereas in the classical Brownian motion the price changes are proportional to the square root of the time step. This means that in the arithmetic return system, large returns will result in large price changes, whereas in the classical Brownian motion, large returns will result in smaller price changes. This difference in scaling results in a difference in volatility between the two models.

Here is the plot:



Here is the plot and outcomes:

Arithmetic return system:

Expected mean of price: 100.0

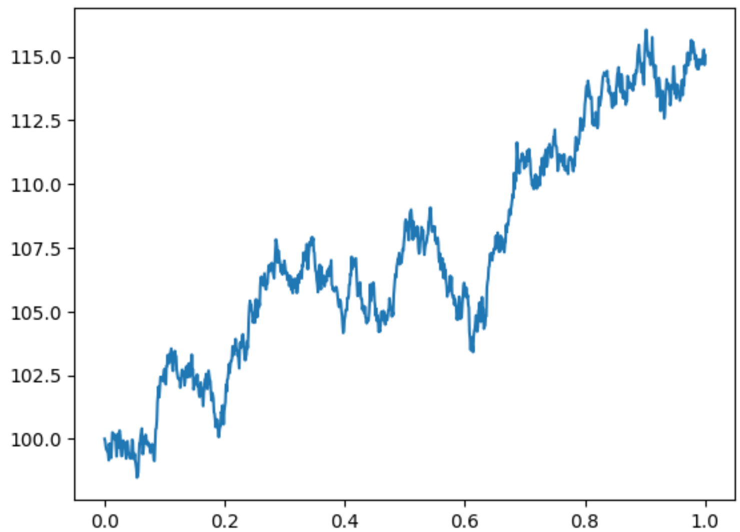
Calculated mean of price: 27.294276922006013

Expected standard deviation of price: 10.0

Calculated standard deviation of price: 21.091108536541974

Based on the reason above, we can see that the arithmetic return system has large deviation from the mean, which also indicates that there are large standard deviations.

Here is the Geometric Brownian Motion:




Geometric Brownian Motion:

Expected mean of price: 100.0

Calculated mean of price: 107.26067895592003

Expected standard deviation of price: 10.025052161544073

Calculated standard deviation of price: 4.570167270731065

From the data we can see, the volatility of a geometric Brownian motion model is larger than that of a classical Brownian motion model. This is because in a geometric Brownian motion model, the returns are compounded over time, leading to an exponential increase in the price. In contrast, in a classical Brownian motion model, the price changes are proportional to the square root of the time step, leading to a linear increase in the price. As a result, the volatility of a geometric Brownian motion model is typically larger, and the price changes are more pronounced.

**Problem 2**

Here is the arithmetic return for all price looks like:

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       AAPL      ABBV       ABT       ACN      ADBE       ADI       ADP  \
0  0.023338  0.011895  0.010082  0.010332  0.012125  0.051421  0.009460
1 -0.001203  0.007255 -0.002111  0.013359 -0.003211  0.037956  0.000438
2 -0.021083 -0.006583 -0.024756 -0.032118 -0.041303 -0.047911 -0.019147
3 -0.009170 -0.006897 -0.031123 -0.005083 -0.032556 -0.001360 -0.000690
4 -0.017626  0.010210  0.006473  0.000056 -0.008857  0.002320 -0.003384

       AMAT       AMD      AMGN  ...       TXN       UNH       UNP       UPS  \
0  0.060508  0.063561 -0.001916  ...  0.022170  0.007807  0.028399  0.015575
1  0.007998 -0.030566 -0.006161  ...  0.011853  0.003539  0.018496 -0.008682
2 -0.031864 -0.044651 -0.004847  ... -0.021942 -0.020494 -0.010623 -0.021071
3 -0.022873  0.013545 -0.004550  ...  0.012090 -0.005646  0.001284  0.001106
4 -0.021422  0.016541  0.002454  ... -0.000904 -0.011596 -0.019099 -0.001569

          V        VZ       WFC       WMT       XOM       ZTS
0  0.010739  0.002530  0.009634  0.002706 -0.014379  0.007344
1  0.004122  0.001590 -0.003639 -0.006681  0.002772 -0.012928
2 -0.018666  0.004211 -0.033756  0.039637 -0.003375 -0.009781
3 -0.008815  0.005506 -0.008789 -0.006838 -0.012965 -0.008491
4 -0.006419  0.002141 -0.005820 -0.011590 -0.013478 -0.005804
```

1. Here is the calculation for VaR(META) using Normal Distribution:
   The VaR for META at a confidence level of 5.0 % is: 0.06545287378451092
   Which mean a 5.0% confidence level means that there is only a 5.0% chance that META loss will exceed 0.06545287378451092

2. Using a normal distribution with an Exponentially Weighted Variance (Lambda = 0.94)
   Value at Risk (VaR): -0.06561787073963313
   Which mean a 5.0% confidence level means that there is only a 5.0% chance that META loss will exceed 0.06561787073963313

3. Using a MLE fitted T distribution.
   VaR for META: 0.00
   Which means only 5% chance that META loss will exceed 0
4. Using a fitted AR(1) model.

Value-at-Risk at 5.0% confidence level: -0.065466
Here is the VaR is negatively, which means that META may largely make profits.

5. Using a Historic Simulation:
   Value-at-Risk at 5.0% confidence level: 0.0537
   Which mean a 5.0% confidence level means that there is only a 5.0% chance that META loss will exceed 0.0537

Based on the result, we can say META could make profit in a large sense

**Problem 3**

For this problem, I used three different model for returns to calculate VaR, I process the data first to make it easier for latter analysis.

1. The exponentially weighted covariance with lambda = 0.97
   Portfolio A VaR: $6507.95
   Portfolio B VaR: $5181.87
   Portfolio C VaR: $4388.20
   Total VaR: $16078.02

This method assumes that recent data points are more relevant and assigns a higher weight to them. The higher VaR obtained with this method suggests that it may be more conservative than the other methods.

2. AR(1)
   Portfolio A VaR: $259.31
   Portfolio B VaR: $220.67
   Portfolio C VaR: $172.95
   Total VaR: $652.93

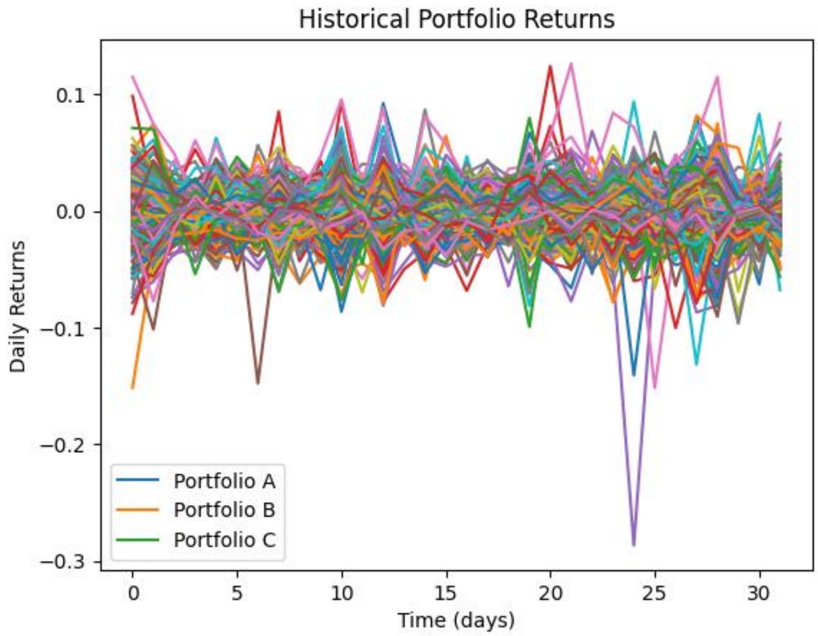
The AR(1) method resulted in the lowest VaR for each portfolio and the total VaR. This method assumes that the returns follow a first-order autoregressive process, where the current return depends on the previous return. The lower VaR obtained with this method suggests that it may be less conservative than the other methods.

3. MLE
   Portfolio A VaR: $1492.42
   Portfolio B VaR: $1268.86
   Portfolio C VaR: $1136.32
   Total VaR: $3897.60

The MLE method resulted in VaR values that are between the exponentially weighted covariance and AR(1) methods. This method estimates the parameters of a multivariate normal distribution from the data, assuming that the returns follow a normal distribution.

Overall, the results show that the exponentially weighted covariance method produces the highest VaR estimates, while the AR and MLE methods produce much lower VaR estimates. This is likely since the exponentially weighted covariance method takes into account the correlation between the different stocks in each portfolio, while the AR and MLE methods assume that the portfolio returns are independent and identically distributed.

Here is graph of Historical Portfolio Returns




The historical portfolio returns plot can give us some assumptions about the variance of future returns. We can also see that the three portfolios have different return patterns, with Portfolio A and B having similar returns, while Portfolio C appears to be less volatile. We can see that Portfolio A has generally had more volatile returns than the other two portfolios, which suggests that it may also have a higher level of risk.