In [ ]:
```python
import astropy.units as u
import astropy.constants as const
import numpy as np

# Use to obtain data
import pandas as pd
import urllib
# the service URL
livechart = "https://nds.iaea.org/relnsd/v1/data?"

# There have been cases in which the service returns an HTTP Error 403: Forbidden
# use this workaround
import urllib.request
def lc_pd_dataframe(url):
    req = urllib.request.Request(url)
    req.add_header('User-Agent', 'Mozilla/5.0 (X11; Ubuntu; Linux x86_64; rv:77.0) Gecko/20100101 Firefox/77.0')
    return pd.read_csv(urllib.request.urlopen(req))
```

Quesiton 1

Assume $^{235}\mathrm{U}$ splits into two stable nuclides $^{100}\mathrm{Ru}$, $^{132}\mathrm{Xe}$, and three neutrons. About 86% of the energy is in the kinetic energy of the fission fragments. A fully enriched $^{235}\mathrm{U}$ bomb is expected to give a prompt explosive yield of about $20\,\mathrm{kg}$ of TNT. How many kilograms of $^{235}\mathrm{U}$ are required (assuming the actual explosive yield efficiency is around 30%)?

Answer

The energy of this reaction have been calculated in the previous question. I copied the result here.

$$\Delta E = 192\,\mathrm{MeV}$$

In [ ]:
```python
m_235U = lc_pd_dataframe(livechart+"fields=ground_states&nuclides=235U")['atomic_mass'][0]*1e-6*u.u
m_100Ru = lc_pd_dataframe(livechart+"fields=ground_states&nuclides=100Ru")['atomic_mass'][0]*1e-6*u.u
m_132Xe = lc_pd_dataframe(livechart+"fields=ground_states&nuclides=132Xe")['atomic_mass'][0]*1e-6*u.u
m_n = lc_pd_dataframe(livechart+"fields=ground_states&nuclides=1n")['atomic_mass'][0]*1e-6*u.u
m_e = (const.m_e).to('u')
Delta_m = m_235U - (6*m_e + 3*m_n + m_100Ru + m_132Xe)
Delta_E = (Delta_m*const.c**2).to("MeV")
Delta_Ep = Delta_E*0.86
Delta_Ep
```

Out[ ]: $165.24487\ \mathrm{MeV}$

And the energy convert to explosion $\Delta E' = 0.86\Delta E = 165\,\mathrm{MeV}$.

Accoriding to Wikipedia, a ton of TNT can release about 4.184 gigajoules. So, the energy of explosion of $20\,\mathrm{kg}$ is E_0 where

$$E_0 = 4.184\,\mathrm{GJ} \cdot \frac{20\,\mathrm{kg}}{1\,\mathrm{t}}$$

$$

In [ ]:
```
E0 = (4.184*u.GJ*20*u.kg/u.t).to('MJ')
E0
```

Out[ ]: $83.68\ \mathrm{MJ}$

Thus $E_0 = 83.68\,\mathrm{MJ}$.

The number of $^{235}\mathrm{U}$ required is $N = E_0/\Delta E'$.

In [ ]:
```
N = (E0/Delta_Ep).si
N
```

Out[ ]: 3.1607001×10^{18}

So $N = 3.16 \times 10^{18}$. The mass of $^{235}\mathrm{U}$ is

$$m = N \cdot m_{^{235}\mathrm{U}}$$

In [ ]:
```
m = (N*m_235U).to('mg')
m
```

Out[ ]: $1.2336201\ \mathrm{mg}$

So the mass of $^{235}\mathrm{U}$ is $1.23\,\mathrm{mg}$.

Question 2

Describe two methods used for separating 235U from natural uranium.

Answer

Two methodologies are employed for isolating $^{235}\mathrm{U}$ from its natural form, each initiating with gaseous uranium hexafluoride as the primary substrate.

1. **Gaseous Diffusion:** This process subjects the gas to a sequence of semipermeable membranes. Through thousands of membrane passages, the concentration of ^{235}U incrementally escalates.

2. **Centrifugation Technique:** Involves introducing the gas into ultracentrifuges, which rotate at an incredibly high pace—thousands of revolutions per minute. Similar to gaseous diffusion, repeated cycles lead to a stepwise augmentation in $^{235}\mathrm{U}$ concentration.

Both techniques, albeit distinct in their operational mechanics, share the objective of progressively enriching $^{235}\mathrm{U}$ via iterative processes.

Question 3

What is the meaning of critical mass or critical size? How to roughly estimate the critical size of $^{233}\mathrm{U}$ bomb?

Answer

Critical mass is the smallest amount of fissile material needed for a sustained nuclear chain reaction. Critical size is the least include enough fissionable material to reach critical mass.

The fission cross section of $^{233}\mathrm{U}$ is $\sigma_f = 1.9\,\mathrm{b}$. We know that the density of uranium is $19\,\mathrm{g/cm^3}$. Thus the mean free path is

$$\lambda = \frac{1}{\sigma_f N} = \frac{m_{^{233}\mathrm{U}}}{\sigma_f \rho}$$

In [ ]:
```
sigma_f= 1.9*u.barn
rho = 19*u.g/(u.cm**3)
```

```
lam = (m_235U/(sigma_f*rho)).si
lam
```

Out[ ]: $0.10811624\ \mathrm{m}$

Thus the critical size is $R_c = \lambda/2$ and the critical mass is $m_c = \frac{4\pi R_c^3}{3}\rho$.

In [ ]:
```
R_c = lam/2
display(R_c)
m_c = (4*np.pi*R_c**3/3*rho).to(u.kg)
m_c
```

$0.05405812\ \mathrm{m}$

Out[ ]: $12.572598\ \mathrm{kg}$

So the critical size is $0.05\,\mathrm{m}$ and the critical mass is $12.6\,\mathrm{kg}$.