# Question 1

How many scatters with $^1\mathrm{H}$ and with $^{235}\mathrm{U}$, on the average, would it take for $2\ \mathrm{MeV}$ neutrons to reach an average thermal energy of $0.025\ \mathrm{eV}$?

## Answer

The average logarithmic energy loss is

$$\xi = \overline{\ln \frac{E}{E'}} = 1 + \frac{\alpha}{1-\alpha} \ln \alpha, \alpha = \left(\frac{A-1}{A+1}\right)^2$$

So the average scatter time is

$$n = \frac{1}{\xi} \ln \frac{E_i}{E_f}$$

For this exercise, the total logarithmic energy loss is

$$\ln \frac{E_i}{E_f} = \ln\left(\frac{2\ \mathrm{MeV}}{0.025\ \mathrm{eV}}\right) = 18.198$$

For $^1\mathrm{H}$, $A_{^1\mathrm{H}} = 1.007\,83\ \mathrm{u}, \xi = 0.99983$, thus

$$n(^1\mathrm{H}) = \frac{18.198}{0.99983} = 18$$

For $^{235}\mathrm{U}$, $A_{^{235}\mathrm{U}} = 235.044\ \mathrm{u}, \xi = 0.008485$, thus

$$n(^{235}\mathrm{U}) = \frac{18.198}{0.008485} = 2145$$

# Question 2

Discuss the relative merits of water and graphite for use in a thermal reactor.

## Answer

Water:

- much less expensive
- can also be used as the coolant for a power reactor
- boils at lower temperatures and thus require a pressure vessel
- $\sigma_a$ is relatively large ($0.664\,\mathrm{b}$)
- larger $\xi$ and slows neutrons with fewer scatters
- much more inert chemically
- cannot be used as a structural element of the core
- can produce radioactive $^3\mathrm{H}$ and $^{16}\ \mathrm{N}$ by activation
- can dissociate and produce explosive hydrogen-oxygen gas mixtures

Graphite:

- reactor-grade (high purity) graphite is expensive
- cannot be used as the coolant

- solid to very high temperatures; thus no need for a pressure vessel
- $\sigma_a$ is very small ($34\,\mathrm{mb}$)
- smaller $\xi$ and requires more scatters to slow neutrons
- burns and thus hot graphite must be isolated from oxygen
- can be used as structural elements of the core
- does not activate and produce radionuclides
- stores energy and must be periodically annealed to avoid a sudden energy release (Wigner effect).

# Question 3

Plot the thermal fission factor for uranium as a function of its atom-% enrichment in $^{235}\mathrm{U}$.

## Answer

The thermal fission factor is

$$\eta = \nu \frac{\bar{\Sigma}_f}{\bar{\Sigma}_a} = \frac{\nu_{235}\bar{\sigma}_f^{235}}{\bar{\sigma}_a^{235} + \bar{\sigma}_a^{238}(N^{238}/N^{235})}$$

By definition of enrichment, $e$,

$$\frac{N^{235}}{N^{238}+N^{235}} = e \Rightarrow \frac{N^{238}}{N^{235}} = \frac{e}{1-e}$$

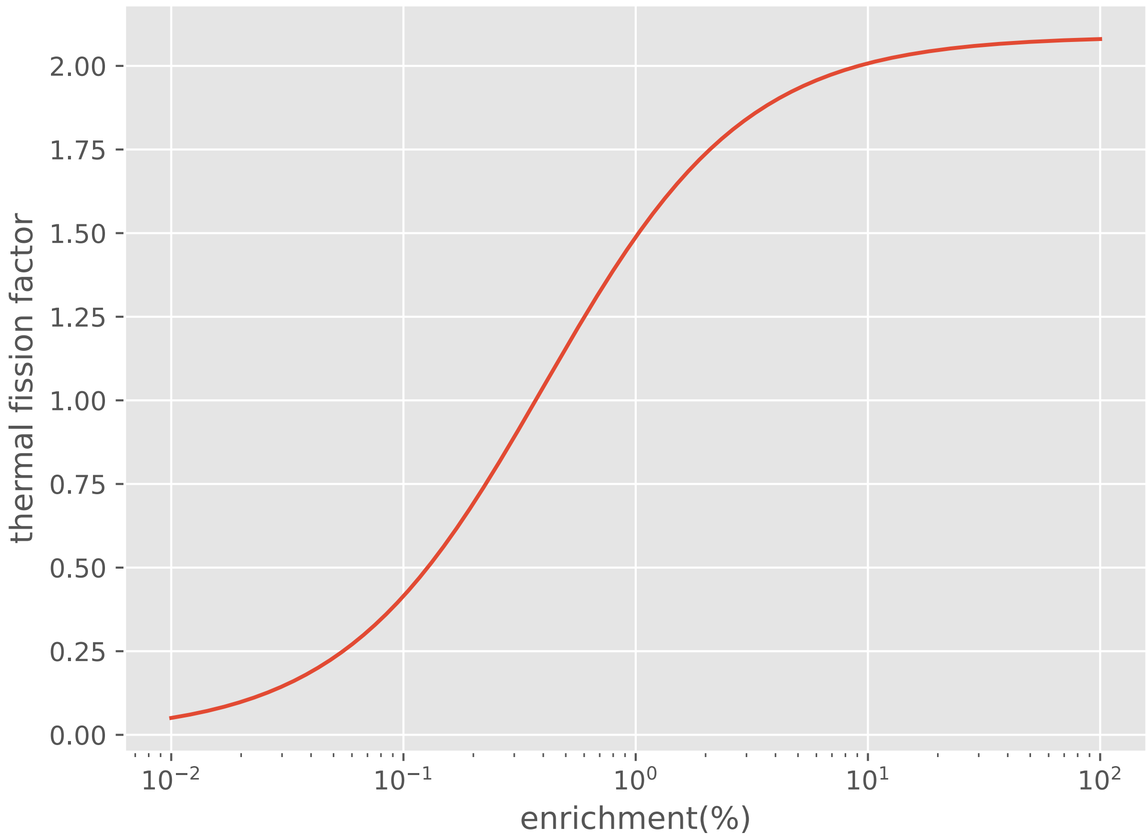
From Tabel 10.1 and Table 10.2 of our text book,

$$\bar{\sigma}_a^{235} = 592.6\ \mathrm{b}, \sigma_a^{238} = 2.382\ \mathrm{b}, \sigma_f^{235} = 505.9\ \mathrm{b}, \nu^{235} = 2.4367$$

Bring these in we get the relationship of $\eta$ and $e$:

$$\eta = \frac{\nu_{235}\sigma_f^{235}}{\sigma_a^{235} + \sigma_a^{238}\frac{e}{1-e}}$$

By those python code, we can plot it out:




```
# import matplotlib.pyplot as plt
# import numpy as np
# import astropy.units as u
sigma_a_235 = 592.6#*u.barn
sigma_a_238 = 2.382#*u.barn
sigma_f_235 = 505.9#*u.barn
nu235=2.4367

e = np.logspace(-4,-0,1000)
eta = nu235*sigma_f_235/(sigma_a_235+sigma_a_238*(1-e)/e)
plt.plot(e*100,eta)
plt.xlabel('enrichment(%)')
plt.xscale('log')
plt.ylabel('thermal fission factor')
plt.yscale('linear')
# plt.yticks((0,1,2))
plt.grid(True)
plt.savefig('hw3c_3.svg')
plt.show()
```

# Question 4

Consider a homogeneous mixture of fully enriched $^{235}\mathrm{U}$ and graphite. Plot $k_\infty$ versus $N^{235}/N^c$. What is the fuel-to-moderator ratio that yields the maximum value of $k_\infty$?

## Answer

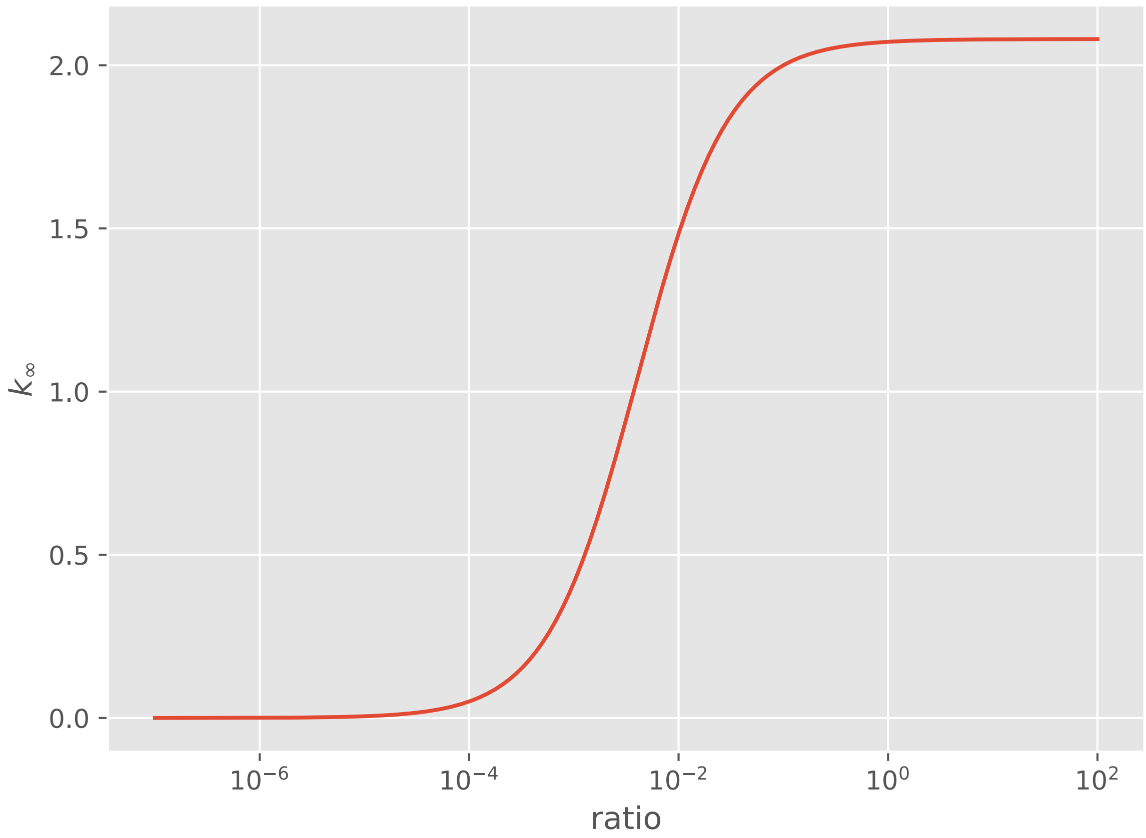
$$k_\infty = \eta\epsilon p f$$

For a dilute mixture of fully enriched uranium and carbon, $\epsilon p \approx 1$, thus

$$k_\infty = \eta f = \frac{\nu^{235}\bar{\sigma}_f^{235}}{\bar{\sigma}_a^{235} + \bar{\sigma}_a^c \frac{N^c}{N^{235}}} = \frac{\nu^{235}\bar{\sigma}_f^{235}}{\bar{\sigma}_a^{235} + \bar{\sigma}_a^c r^{-1}}$$

$$\bar{\sigma}_a^c = (\sqrt{\pi}/2)\sigma_a^c = 3.422 \text{ mb}$$

Bring these in and we can plot it out by the code below

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sigma_a_235 = 592.6#*u.barn
sigma_a_238 = 2.382#*u.barn
sigma_f_235 = 505.9#*u.barn
nu235=2.4367

e = np.logspace(-4,-0,1000)
eta = nu235*sigma_f_235/(sigma_a_235+sigma_a_238*(1-e)/e)
plt.plot(e*100,eta)
plt.xlabel('ratio')
plt.xscale('log')
plt.ylabel(r'$k_\infty$')
plt.yscale('linear')
# plt.yticks((0,1,2))
plt.grid(True)
plt.savefig('hw3c_3.svg')
plt.show()
```

# Question 5

Two detectors, placed symmetrically on either side of a fission source, record two fission fragments $f_1$ and $f_2$ from a fission event. If the flight time for $f_1$ is $20\%$ greater than that for $f_2$, calculate the ratio of the masses of $f_1$ and $f_2$ . Which has the most energy?

## Answer

Because the detectors are placed symmetrically, the flight time is.

$$t_{\text{flight}} = \frac{L}{v}$$

One can derive

$$(t_1 - t_2)/t_2 = 0.2 \Rightarrow t_1/t_2 = 1.2.$$

So $v_1/v_2 = t_2/t_2$. For the conservation of momentum

$$m_1 v_1 = m_2 v_2 \Rightarrow m_1/m_2 = v_2/v_1 = t_1/t_2.$$

So for the energy ratio

$$E_1/E_2 = m_1/m_2 (v_1/v_2)^2 = t_2/t_1 = 5/6$$

So the second fragment $f_2$ has the most energy.