

CFRM 523 Trading System Replication

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I. PRECIS

In the paper "The Cointegration Alpha: Enhanced Index Tracking and Long-Short Equity Market Neutral Strategies" by Carol Alexander and Anca Dimitriu, published in ISMA Discussion Papers in Finance, the authors assert that cointegration-based strategies offer a robust alternative to traditional correlation-based methods in index tracking and the construction of equity market neutral portfolios. The thesis of the work revolves around the premise that cointegration provides a more stable and reliable method to create portfolios that track indices closely and maintain market neutrality.

The authors demonstrate that cointegration-based strategies yield accurate index replication with lower volatility, and when applied to a set of DJIA stocks, these strategies produce encouraging returns with negligible correlation to the market, showing an innovative approach to managing systematic risk. They apply the cointegration technique not just to single index tracking but also to long-short market neutral strategies, revealing its flexibility in various investment scenarios.

One of the paper's main points is that cointegration-based strategies enhance index tracking by exploiting mean-reverting properties of stock price spreads, leading to a reduced need for frequent rebalancing, thus minimizing transaction costs. Another significant aspect is the paper's exploration of the practical challenges of implementing these strategies, including the sensitivity of results to stock selection methods and rebalancing rules. The authors conclude that while the strategies are promising, there's a need for further research, especially in stock selection rules optimization and the application of cointegration in varying market conditions. In this paper, I will discuss mainly different components' of index tracking portfolios and their performances with cointegration properties.

The target paper's relevance of this work to an analyst's research interests could be profound, particularly if the focus is on developing quantitative trading strategies that seek to achieve stable returns irrespective of market movements. By tying together the superior stability and risk management potential of cointegration techniques with practical implementation details, the paper provides a valuable framework that could be applied to current portfolio management challenges, especially in the context of growing market complexity and the need for sophisticated investment techniques.

II. LITERATURE REVIEW

In reviewing the literature on market efficiency and price relationships, particularly in equity and futures markets, the

work by Ackert and Racine (1998) provides a significant contribution to understanding the dynamics of cointegration within these financial instruments. Their study employs a no-arbitrage cost-of-carry model to investigate whether equity spot and futures markets are cointegrated when the cost of carry is considered. Their findings support the theoretical underpinning that markets exhibit a long-term equilibrium relationship, contingent on the inclusion of the cost of carry. This contrasts with earlier studies such as those by Brenner and Kroner (1995), which also explored the implications of cointegration in financial markets but did not specifically address equity markets with a focus on the role of the cost of carry. Ackert and Racine's results underline the importance of accounting for carrying costs to accurately model and predict the behavior of futures prices in relation to their underlying equity indices, contributing a pivotal piece to the puzzle of financial market behavior analysis and its impact on pricing models.

The book "Optimal Hedging Using Cointegration" by Carol Alexander investigates the application of cointegration—a statistical technique used to identify long-term equilibrium relationships between financial asset prices—for improving hedging strategies. Cointegration, unlike correlation that primarily measures short-term relationships, captures the long-term co-movements of asset prices, providing a more stable basis for constructing hedging portfolios. Alexander explores various financial markets and assets, demonstrating that cointegration can lead to more effective hedging compared to traditional correlation-based strategies. The analysis includes mathematical foundations, empirical tests, and practical applications, showcasing how cointegration can prevent over-hedging and reduce the need for frequent portfolio rebalancing, thus enhancing the long-term performance of investment portfolios.

The book "Forecasting in Cointegrated Systems" by Michael P. Clements and David F. Hendry examines the effects of imposing unit roots and cointegrating restrictions on forecast accuracy within linear systems of variables. It delves into the implications of different representations (levels, differences, cointegrated combinations) for forecasting error variances, illustrating that forecast performance varies significantly with the representation used. The text combines theoretical insights with empirical applications, utilizing asymptotic variance formulae and Monte Carlo simulations to analyze the behavior of forecast errors under various model specifications. This study highlights the importance of model specification in forecasting, showing that while certain restrictions may enhance short-term forecast accuracy, they might not hold advantages over longer horizons depending

on the system's dynamics and the nature of the variables involved.

III. DATA CONSTRUCTION

According to the target paper, to develop and evaluate various strategies based on cointegration, we utilized more recent daily prices of stocks within the Dow Jones Industrial Average (DJIA). I downloaded the data from Yahoo Finance API in Python. The time span of the data is from Jan 1, 2010 to Jan 1, 2016. The stocks are picked from DJIA at the date of Jan 1, 2010. I created an artificial 'reconstructed' DJIA historical series using the daily closing prices of the stocks currently in the DJIA, referencing the last recorded DJIA divisor value from December 31, 2015, which was 0.132246555. To calculate the index value for a given day within our dataset, we computed the average of all included stock prices, adjusting for the constant divisor (Fig. 2). Employing a reconstructed index, rather than the original, is warranted by the focus on the index's present composition. In essence, we are evaluating the performance of portfolios containing the present constituents of the DJIA against a market index derived from these same constituents. Moreover, this approach allows for a consistent methodology in accounting for dividends and adjustments for stock splits. The next step is to get the logarithm price and combine them together to construct a complete dataset (Fig. 2).

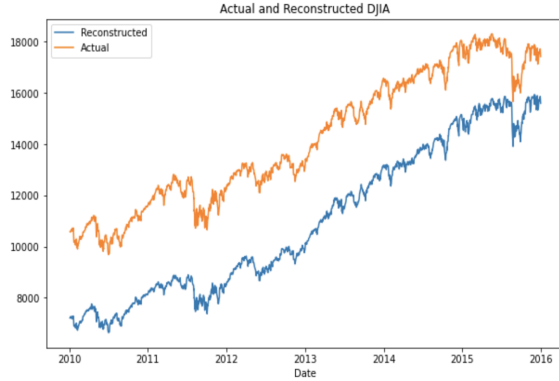


Fig. 1: Actual and Reconstructed DJIA

CSCO	CVX	DIS	GS	MSFT	NKE	PG	TRV	UNH	V	VZ	WBA	WMT	DJIA	
74513	44.988243	27.843178	135.576157	...	23.431589	13.761759	40.160057	35.336140	25.497286	19.899023	15.171669	24.284012	13.196314	7215.306848
689803	45.308904	27.773722	137.973129	...	23.439156	13.816513	40.173206	34.499023	25.456846	19.671289	15.199017	24.088703	13.064914	7225.085444
560406	45.312588	27.826127	136.500427	...	23.295315	13.732272	39.982647	34.009541	25.707541	19.407135	14.761981	23.906401	13.035712	7213.602055
635008	45.141895	27.834809	138.171616	...	23.052047	13.867049	39.765820	34.499023	26.684107	19.587751	14.674116	24.049637	13.043011	7267.437666
723164	45.221558	27.878219	136.538658	...	23.212040	13.839667	39.713238	34.449368	26.443426	19.641933	14.883362	24.082191	12.977311	7261.383800

Fig. 2: Stocks and DJIA Data(part)

IV. REPLICATION INDEX TRACKING PORTFOLIOS

A. Cointegration Exploration

For the cointegration test, the Augmented Dickey-Fuller (ADF) regression is run on the residual values. I ran the

test for each of the stocks in DJIA and to check if it is cointegrated with the index. The formula for this regression is:

$$\Delta\epsilon_t = \gamma\epsilon_{t-1} + \sum_{i=1}^p \alpha_i \Delta\epsilon_{t-i} + u_t$$

Where the null hypothesis of no cointegration ($\gamma = 0$) is tested against the alternative hypothesis ($\gamma < 0$). The graph below (Fig. 3) shows that there were only 6 companies' stocks co-integrated with the index with the critical value of 5% level. Their tickets are: DIS, HD, NKE, TRV, UNH, V.

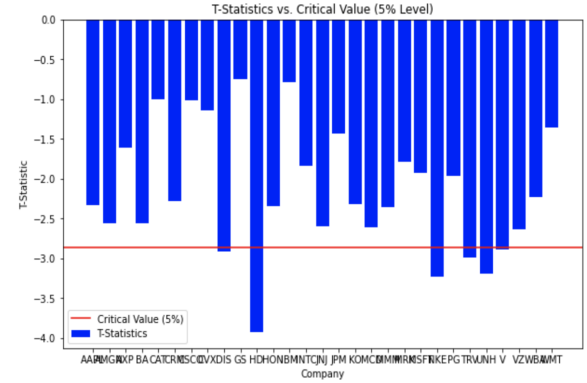


Fig. 3: Actual and Reconstructed DJIA

The Engle-Granger method is used to check if the residuals from cointegration regressions are stationary and it is confirmed to be true (Fig. 4). This approach is straightforward to apply, which is why we found it useful. However, it does have its downsides, As the target paper mentioned, like the fact that it may not work well with small samples or when there's a possibility of more than one cointegration relationship, but this isn't a problem here.

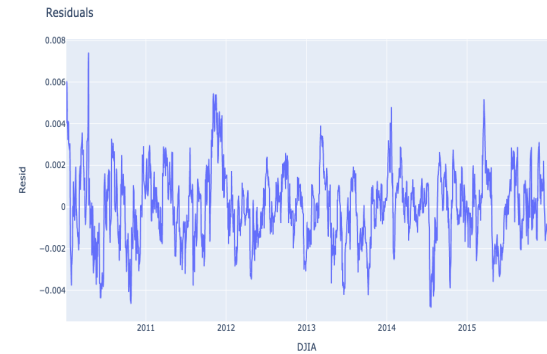


Fig. 4: Engle-Granger Test for DJIA

B. Weights Exploration

To start with, I regress the full time span (6 years) DJIA data on the stock components and gained a basic understanding on the weights distribution (Fig. 5). All of

the high weights' stocks did not pass the cointegration test, so we can conclude that there are no correlation between the weights and cointegration when we tried to construct index tracking portfolio.

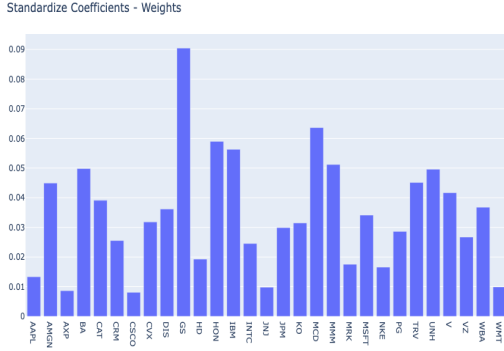


Fig. 5: Weights based on Full Time Span

C. Stocks Selection

I followed the target paper's stocks selection method. In conducting our analysis, we employed the most basic method of choosing stocks, which is to select them based on their price ranking within the index at the time the portfolio is created. We established portfolios that include the top 10, 15, 20, and 25 stocks, arranged in descending order by their relative importance in the index. I didn't make the composition dynamic, because I found that the index components changed during the period, meaning some companies were excluded by the index and new companies were in. This needs a more sophisticated stock system which I didn't find a free platform to implement. So I just stay with the 30 components of the index at the start year(2011).

D. Portfolio Dynamics

In the algorithm, I utilized 2 year calibration period before portfolio construction, and every 10 trading days I set up a portfolio weights rebalancing. For the transaction costs, I set up the same amount as the target paper's setting and calculated by the formula:

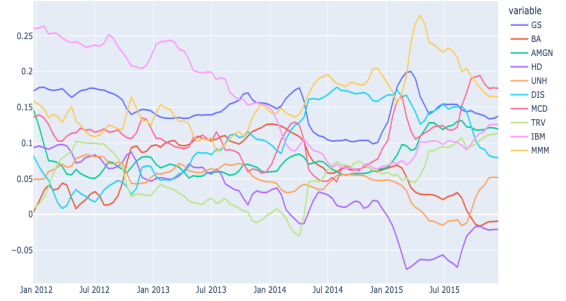
$$TC_{\tau} = 0.002 \sum_{k=1}^n |w_{k,\tau} - w_{k,\tau-10}| P_{k,\tau}$$

For the stock components' weights, the composition of each portfolio is determined by calculating the stock weights using the coefficients obtained from performing an ordinary least squares (OLS) regression. This regression uses the logarithm of the index price as the dependent variable and the logarithms of the stock prices within the portfolio as independent variables, over a predefined calibration period that precedes the point of portfolio assembly. As the target paper described, the formula is:

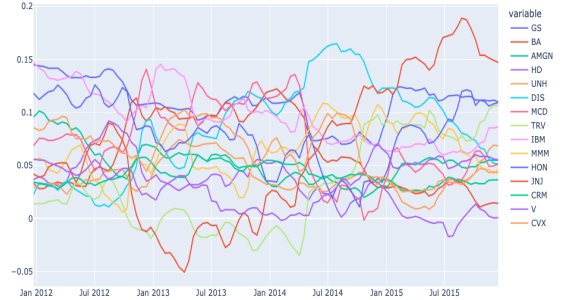
$$\log(\text{index}_t) = c_1 + \sum_{k=1}^n c_{k+1} * \log(P_{k,t}) + \varepsilon_t$$

The OLS Coefficients were then normalized to sum up to 1 to become the weights of the index tracking portfolio. Below are the weights graphs for 10, 15, 20, 25 stocks' tracking portfolios.

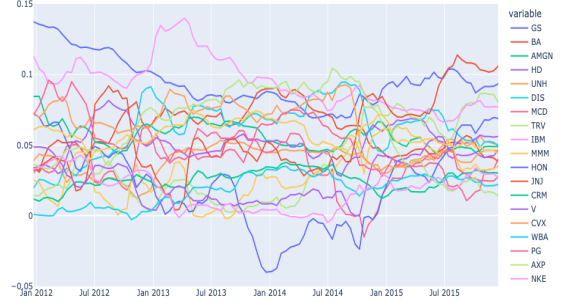
10 stocks Index Tracking Weights - Cointegration



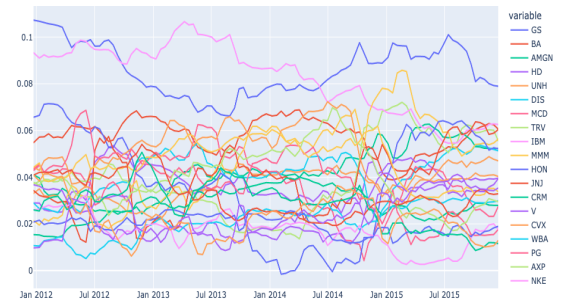
15 stocks Index Tracking Weights - Cointegration



20 stocks Index Tracking Weights - Cointegration



25 stocks Index Tracking Weights - Cointegration



The average cointegration test statistics for 10, 15, 20, 25 stocks index portfolios are -5.43, -6.04, -6.25, -7.30, which we can conclude that the more the components

involved, the more cointegrated the portfolio is with the index. The transaction cost percentages are 1.52%, 1.80%, 1.52%, 1.18%, which gives me a sense that maybe the more components involved, the more stable (less rebalancing efforts) the portfolio is. Since the 10 stocks portfolio has the same transaction costs as the 20 stocks portfolio, there may be some exclusions.

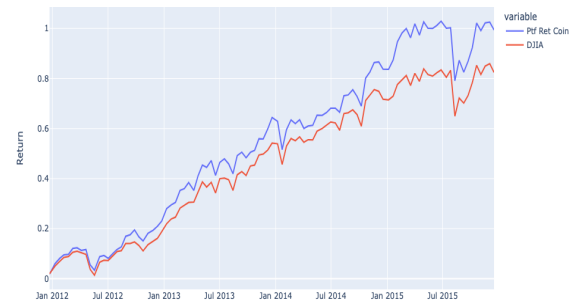
E. Return Analysis

The 15 stock tracking portfolios has the highest 100% return and the 10 stock portfolio has 99% return. The 20 and 25 stock portfolios has the return of approximately 86% return, while the index has the return of 82%, we can find a pattern that in my analysis, the less the stock components, the higher the return. It is accordance with the intuition that more stocks mean that the portfolio is more approximate the index components and thus more approximate the index return and cointegrated with the index with higher score. Moreover, I also calculated the exponentially weighted moving average (EWMA) with a smoothing parameter of 0.94 (Figure .6), it shows that 10 stock portfolio has the highest volatility. So we can conclude that the more stock components, the less volatility the portfolio has.

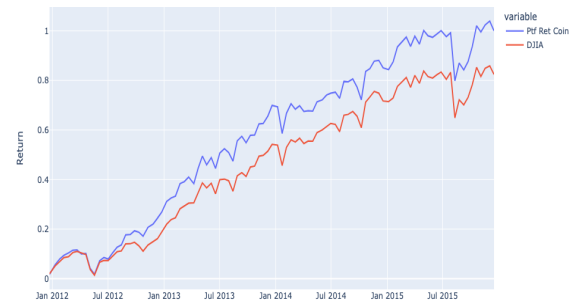
V. CONCLUSION AND NEXT STEPS

In conclusion, the research presented in "The Cointegration Alpha: Enhanced Index Tracking and Long-Short Equity Market Neutral Strategies" by Carol Alexander and Anca Dimitriu significantly advances the application of cointegration techniques in portfolio management. The study not only demonstrates the efficacy of these techniques in achieving more stable and efficient index tracking but also underscores their potential in reducing transaction costs through less frequent rebalancing requirements. Due to the short time, I have not complete the long-short strategy analysis, so this can be the next step to check the target paper's statistics. Also, a well formed backtesting can be conducted in the future, since I didn't find a free platform to conduct backtesting currently. In addition, if I have a completed trading system. I can make the stock selection process more dynamic. For example, during 2010 - 2016, there were some companies join the index components and some companies were excluded. Having a trading data pipeline can make this self-adjusted process happen. This paper's result are overall in accordance with the target paper's findings. However, there were several differences may be due to the data variance, since in this paper, I utilized more recent data and many factors should be considered in the change. For example, in the target paper, it said 10, 15 stocks portfolios were not cointegrated with the index, but I found that these portfolios were cointegrated very well with the index using the recent years data. By effectively merging theoretical insights with empirical analysis, this work provides a robust framework for investors and analysts looking to enhance portfolio management in increasingly complex market environments. The integration of cointegration into quantitative trading strategies promises to open new avenues for achieving superior risk-adjusted

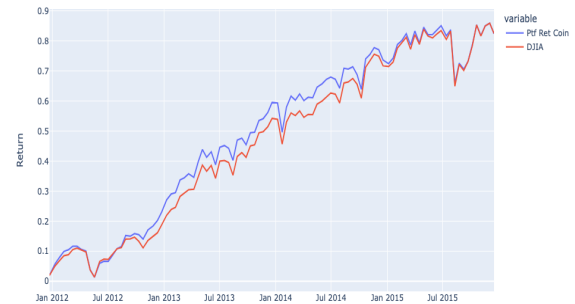
Out-of-Sample Tracking Performance



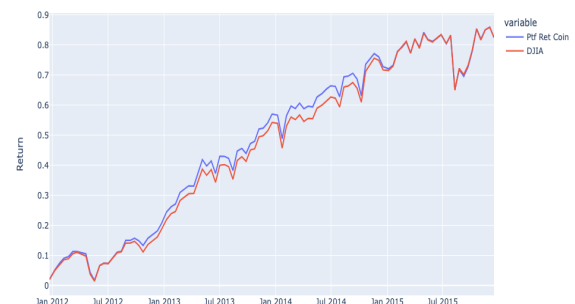
Out-of-Sample Tracking Performance



Out-of-Sample Tracking Performance



Out-of-Sample Tracking Performance



returns, making it a pivotal addition to the literature on financial market efficiency and portfolio theory.

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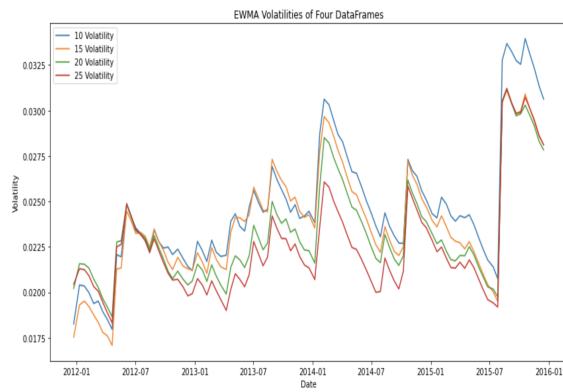


Fig. 6: EWMA Volatility

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