Long-Short Market Neutral Strategy and Index Tracking Portfolios Using Enhanced Metrics and Cointegration Tests

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I. PRECIS

In this paper, I build on the work of Carol Alexander and Anca Dimitriu, who demonstrated the advantages of cointegration-based strategies over traditional correlation-based methods for index tracking and equity market neutral portfolios. The fundamental idea behind this strategy is that cointegration offers a more stable and reliable method for creating portfolios that accurately track indices and maintain market neutrality, which can also be applied to long-short strategies. The hypotheses are that cointegration-based strategies will provide superior risk-adjusted returns, lower maximum drawdown, and higher overall returns compared to traditional methods. Additionally, it is hypothesized that dynamic stock selection based on cointegration will further enhance portfolio performance. To test these hypotheses, the study employs comprehensive performance metrics—Sharpe ratio, maximum drawdown, and overall return—to evaluate the effectiveness of both long-short and index tracking strategies. Cointegration tests are applied to determine long-term equilibrium relationships between asset prices, and dynamic stock selection is used to reflect changes in index components over time. The practical challenges of implementing these strategies, such as sensitivity to stock selection methods and rebalancing rules, are also examined, with an emphasis on reducing transaction costs through less frequent rebalancing. By integrating these elements, this paper aims to provide a valuable framework for investors and analysts seeking stable returns regardless of market movements, addressing current portfolio management challenges, and outperform the target index.

II. LITERATURE REVIEW

"Hedge Funds' Long-Short Strategy" by Ohsang Kwon explores one of the most common trading strategies employed by hedge funds, the long-short strategy. Using Markowitz's portfolio theory as a foundation, Kwon extends the theory to include assets with negative expected returns, thereby demonstrating scenarios where higher returns can be associated with lower risk and vice versa. The paper categorizes long-short strategies into two types: long-short equity and fixed-income arbitrage, each applicable under different conditions of asset return expectations and correlations. Long-short equity strategies are effective when one asset's expected return is positive and the other's is negative, with a high correlation between them. In contrast, fixed-income arbitrage is used when both assets have similar positive returns and a high correlation. The paper also addresses the leverage requirements and potential risks of these strategies, particularly when the actual correlations deviate from expectations.

The paper "Key Design Choices in Long/Short Equity" by AQR explores the strategic considerations involved in implementing a long/short equity strategy, aiming to convert stock selection views into efficient and diversifying returns. The paper examines key decisions such as choosing between systematic or discretionary approaches, single-name shorts or index hedging, and how to combine various signals into a cohesive portfolio. It highlights that long/short equity strategies can offer robust "cash-plus" returns, especially in high-interest rate environments, making them attractive to investors seeking resilient returns amidst market headwinds. The paper also emphasizes the historical performance advantage of long/short strategies over long-only approaches, underscoring their potential to generate excess returns.

The "Long-Short Equity Handbook" by Mallory Horejs provides an in-depth analysis of long-short equity strategies, which involve taking both long and short positions in equities to manage downside risk and enhance returns. The paper traces the origins of this strategy to Alfred Winslow Jones in 1949 and examines its evolution across various investment vehicles, including hedge funds, mutual funds, ETFs, and separate accounts. It categorizes long-short equity strategies into three types: stock-pickers who use fundamental analysis, those who hedge long positions through derivatives, and option-writing strategies. The paper highlights the differences in performance, risk profiles, fees, and tax implications across these vehicles. It also emphasizes that despite their higher fees, hedge funds typically deliver superior risk-adjusted returns compared to other investment vehicles. Finally, the handbook demonstrates how incorporating long-short equity funds can improve the risk-adjusted returns of a traditional stock-bond portfolio.

The "Long/Short Equity Strategy" paper authored by AIMA Canada outlines a hedge fund strategy that emphasizes shifting principal risk from market risk to manager risk through skilled stock selection. Rooted in the Jones model, this strategy combines long and short equity positions with modest leverage to optimize risk-adjusted returns. Managers identify mispriced stocks through rigorous fundamental and technical analysis, and actively manage portfolio exposures using a blend of stock selection and sector allocation. Historical performance data from 1990 to 2005 demonstrates that long/short equity strategies have provided superior risk-adjusted returns compared to traditional long-only equity investments. The paper underscores the importance of thorough due diligence in assessing manager skill and the effectiveness of risk management practices, highlighting the strategy's potential for delivering stable and attractive returns in diverse market conditions.

III. Data Construction

After construct index tracking portfolios within the Dow Jones Industrial Average (DJIA), I decide to extend the universe to S&P500. I utilized more recent daily prices of stocks within the universe and downloaded the data from Yahoo Finance API in Python. The data has a 4-year time spans, from 01-01-2019 to 01-01-2023. The first two years data is used to select candidate pairs from cointegration test, and the last two year data is used to backtest strategies. Logarithm price and and logarithm return are calculated using these datasets.

IV. S&P 500 Index Tracking Portfolios with Cointegration

A. Stocks Selection

Unlike directly implementing price ranking for stock selection in the previous replicate index tracking portfolio, I employ a cointegration test to select candidate stock components for my portfolios. However, this cointegration test is not the typical one where we want the log prices of the stocks to be cointegrated with the index. While it is true that a portfolio of cointegrated stocks can generate returns similar to the index, it does not guarantee that the portfolio will outperform the index. When the index price is high, the components might include high-price stocks to maintain cointegration rather than those with high returns. Therefore, I conduct a cointegration test between the index's log price and the returns of the stocks. My goal is to select stocks whose returns increase when the index price is high. Since the S&P 500 generally increases in the long term, I disregard market correlation drawbacks in this instance because, for an index tracking portfolio, my primary objective is to ensure the portfolio outperforms the index across various market trends. For cointegration, specifically, the Augmented Dickey-Fuller (ADF) regression is run on the residual values. I conducted this test for each stock in the S&P 500 index to determine if it is cointegrated with the index. To remind, the formula for this regression is:

$$\Delta\epsilon_t = \gamma\epsilon_{t-1} + \sum_{i=1}^{p} \alpha_i \Delta\epsilon_{t-i} + u_t$$

Where the null hypothesis of no cointegration ($\gamma = 0$) is tested against the alternative hypothesis ($\gamma < 0$). The graph below (Fig .1) shows that there were 234 companies' stocks co-integrated with the index with the critical value of 5% level.

The Engle-Granger method is also used to check if the residuals from cointegration regressions are stationary and it is confirmed to be true (Fig. 2). Fig. 3 shows the selected stocks that appear to be cointegrated with S&p 500.

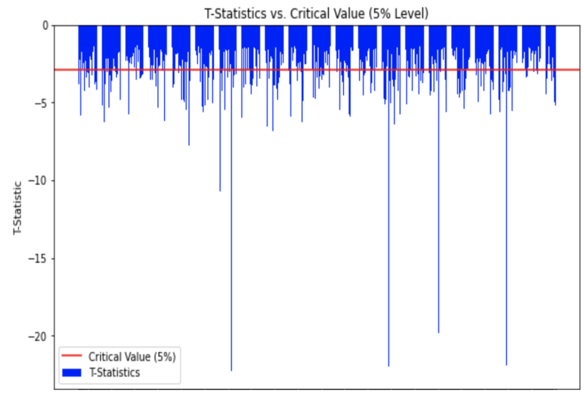


Fig. 1: T-statistics of the Stock Candidates

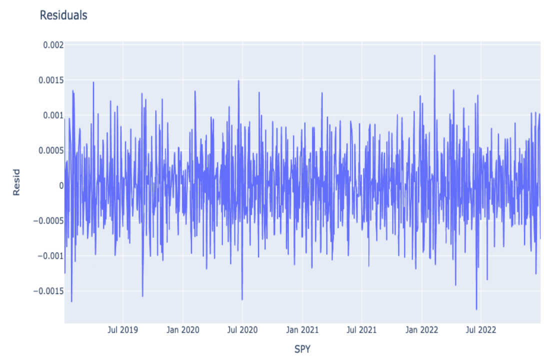


Fig. 2: Engle-Granger Test for SPY

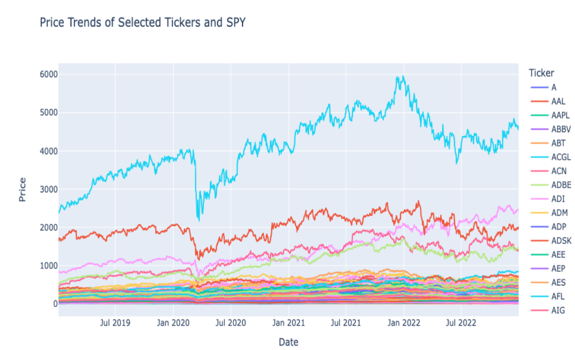


Fig. 3: Cointegrated Stocks

B. Trading Period

The decision to rebalance every 35 trading days is a practical compromise to minimize transaction costs while maintaining the effectiveness of the cointegration relationship in tracking the benchmark. This approach avoids the impracticality and high costs associated with daily or intraday rebalancing, instead opting for a more manageable and cost-effective trading period. Moreover, I check the trading period from 1 to 62 and find that trading period between 21 - 42 days have the best performances.

C. Portfolio Dynamics

In the algorithm, I utilized 2 year calibration period before portfolio construction. I also specify the trading days to set up a portfolio weights rebalancing. For the transaction costs, it is calculated by the formula:

$$TC_\tau = 0.002 \sum_{k=1}^{n} |w_{k,\tau} - w_{k,\tau-10}| \, P_{k,\tau}$$

For the stock components' weights, the composition of each portfolio is determined by calculating the stock weights using the coefficients obtained from performing an ordinary least squares (OLS) regression. This regression uses the logarithm of the index price as the dependent variable and the logarithms of the stock prices within the portfolio as independent variables, over a predefined calibration period(2 years) that precedes the point of portfolio assembly. The formula is:

$$\log(\text{index}_t) = c_1 + \sum_{k=1}^{n} c_{k+1} * \log(P_{k,t}) + \varepsilon_t$$

The OLS Coefficients were then normalized to sum up to 1 to become the weights of the index tracking portfolio(Fig. 4).

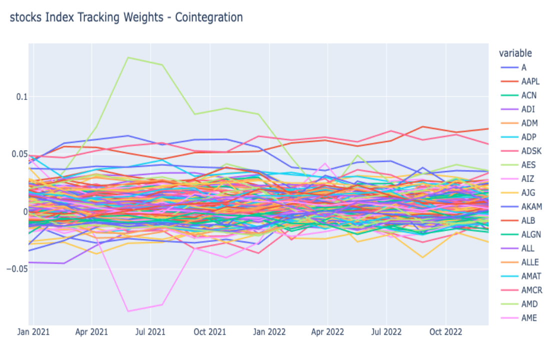
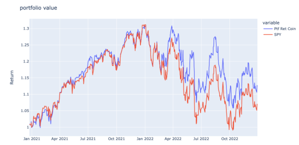


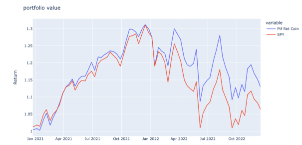
Fig. 4: Index Tracking Portfolio Weights Tracking

D. Performance Analysis

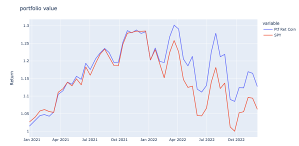
Index tracking portfolios with different trading periods have similar patterns but different return performances (Fig. 5). The total portfolio values range from 1.10 to 1.19, where 15 day period has the highest return of 19 % and 1 day period has the lowest return of 10 %. Maximum drawdown ranges from 15% to 21% compares to S&P 500's 18% to 24%. Sharpe ratio ranges from -3.34 to 0.21 compares to S&P 500's -3.46 to 0.14. For transaction cost, it ranges from 1% to 3 %, which implies ideal cointegration properties. In conclusion, all the index tracking portfolios outperform the S&P 500 index with the same rolling period.



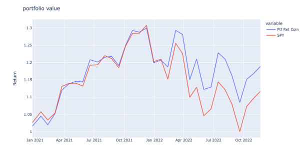
(a) 1 day



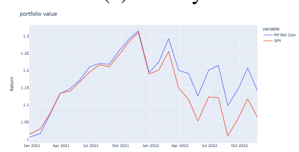
(b) 7 day



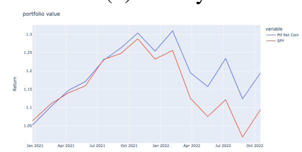
(c) 10 day



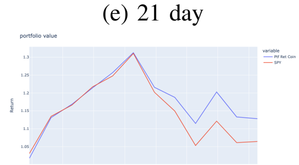
(d) 15 day



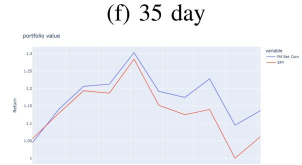
(e) 21 day



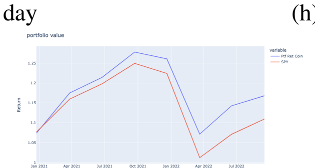
(f) 35 day



(g) 42 day



(h) 50 day



(i) 62 day

Fig. 5: Return for Different Rebalancing Days

V. Long-Short Market Neutral Strategy

A. Stocks Selection

While the market index and its component stocks typically exhibit a strong cointegration relationship, this may not hold true for portfolios designed to outperform the market index by a significant margin, such as 50%. The challenge in establishing a cointegration relationship for such artificial benchmarks can lead to unstable stock weights, higher transaction costs, and increased return volatility. Therefore, it is crucial to ensure that all portfolios tracking 'plus' or 'minus' benchmarks successfully pass the cointegration test to maintain stability and minimize costs. Therefore, I conduct cointegration test to check if the log price of the stock is cointegrated with the log price of the index usng ADF test. The new cointegration regression is:

$$\log(\text{index\_plus}_t) = a_1 + \sum_{k=1}^{n} a_{k+1} \cdot \log(P_{k,t}) + u_t$$

$$\log(\text{index\_minus}_t) = b_1 + \sum_{k=1}^{n} b_{k+1} \cdot \log(P_{k,t}) + u_t$$

I also set different threshold to select different candidate stocks for different index plus and minus portfolios.

B. Portfolio Dynamic: Fund of Funds

A Fund of Funds (FoF) is an investment vehicle that allocates its assets into a portfolio of other investment funds rather than investing directly in individual securities like stocks or bonds. This strategy aims to enhance diversification, reduce risk, and simplify the investment process for individual investors. Managed by professional portfolio managers, FoFs provide access to a wide range of asset classes, sectors, and regions, including specialized funds such as hedge funds or private equity funds that might otherwise be inaccessible to retail investors. This long short market neutral strategy is an example of FoFs.

To construct 'plus' and 'minus' bench marks, I add or subtract an annual excess return of x% from the benchmark returns, distributed uniformly across daily returns. I use the selected stocks and the benchmarks to run the linear regression model to get the weights of each component stocks. After getting the weights and then return for each index tracking plus or index tracking minus portfolios, for each index tracking plus portfolio, we use its return to minus the return of every index tracking minus portfolio to get the return of the new long short portfolio.

C. Cointegration Alpha

Cointegration alpha is an advanced alpha generation technique that leverages the long-term equilibrium relationships between asset prices, identified through cointegration analysis. Unlike traditional methods that rely on short-term correlation, cointegration captures the entire information set in level financial variables, allowing for more stable and reliable portfolio optimization. By constructing long short portfolios based on cointegrated assets, investors can achieve enhanced index tracking and market neutral strategies. These portfolios exhibit mean-reverting tracking errors, reduced volatility, and low correlation with the market, making them particularly resilient during market downturns. The enhanced stability of cointegrated relationships significantly reduces the need for frequent rebalancing, thus lowering associated transaction costs and enhancing the overall strategy's profitability.

In order to identify the cointegration alpha, I constructed a series of portfolios with trading period of 35 days, but with different cointegration score, index plus spread, and index minus spread(Fig. 6). These different variables score will generate different portfolios for long and short and thus can discover different arbitrage opportunities. Therefore, I ran cointegration score of -3, -2.86, -2.5, -2.4, -2.2, -2, -1.5, spread of 5%, 10%, 15%, 25%, 35%, 50%. I go over all the possibilities and some of them generate ideal alpha(Fig. 7).

D. Performance Analysis

After creating different long short portfolios with different index tracking portfolio pairs, I discover potential alpha in these portfolios. The portfolio return ranges from -11% to 25%. Sharpe ratio ranges from -0.22 to 0.68. Maximum drawdown ranges from $1*10^{-3}$ % to 6 %. The more excessive return, the more maximum drawdown comparatively.

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Fig. 6: Long Short Portfolios with Different Setups

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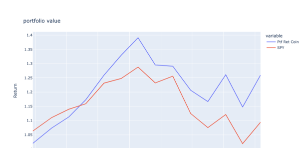
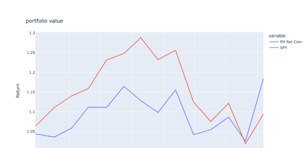
Fig. 7: Best Long Short Portfolios

I selected the index tracking plus portfolio with excessive return of 5%, cointegration score of -2.86, and the index tracking minus portfolio with excessive return of -50%, cointegration score of -2 and it generates the most return (25%) with sharpe ratio 0.68 and maximum drawdown of 1 %(Fig. 8, Fig. 9). Moreover, the market correlation of this long short portfolio is only -0.06, in contrast to the index tracking portfolio's 0.99. Apparently it outperforms S&P 500 index with 35 rolling days (0.09 sharpe ratio, 20% maximum drawdown) as well as index tracking portfolio with 35 rebalancing period (19% return, 0.21 sharpe ratio, 14% maximum drawdown).



(a) Index Tracking Plus Portfolio



(b) Index Tracking Minus Portfolio

Fig. 8: The Best Performance Long Short Portfolio

VI. Conclusion and Next Steps

In this paper, I have demonstrated the effectiveness of cointegration-based strategies for index tracking and long-short market neutral portfolios, building on the work of Carol Alexander and Anca Dimitriu. The results indicate that these strategies offer superior risk-adjusted returns, lower maximum drawdowns, and higher overall returns compared to traditional correlation-based methods. By employing dynamic stock selection based on cointegration and optimizing the rebalancing period, the constructed portfolios consistently

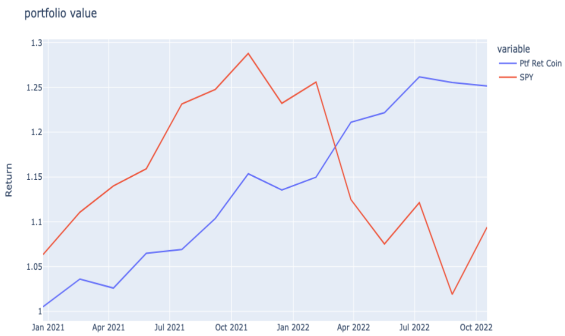


Fig. 9: Long Short Portfolio Value

outperformed the S&P 500 index. The next phase of this research will focus on identifying the feasible lifespan of cointegration relationships for individual stocks to ensure that the portfolio components remain effective over time. This will involve periodic testing and updating of the candidate stocks to maintain strong cointegration with the index, thereby enhancing the stability and performance of the portfolios. Additionally, the research will optimize the stock selection refresh period, incorporate advanced machine learning techniques to predict the persistence of cointegration relationships, and expand the universe of assets to validate the findings. Evaluating the real-world implementation of these strategies, considering factors like transaction costs and liquidity constraints, will further refine these strategies and provide robust tools for investors seeking stable and superior returns regardless of market conditions.

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