CS5340 Project 1 Report

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1 Description of EM algorithm to vanishing point estimation

E-step: Given a gradient image G, the E-step aims to obtain the posterior distribution $P(M|G, \Psi^{old})$ over all four edge model configurations M, where $\Psi = (\alpha, \beta, \gamma)$ is the camera rotation parameters. Using the factorized form of the posterior in [1], we have

$$P(M|G, \Psi^{old}) \propto \prod_p P(E_p|m_p)P(\phi_p|m_p, V^{old})P(m_p) \tag{1}$$

where V^{old} is the VP locations. Since we are dealing with edge pixels, $P(E_p|m_p) = 1$. Furthermore, for an edge pixel p,

$$P(\phi_p|m_p, V^{old}) = \begin{cases} \mathcal{N}(\theta_1 - \theta_2|\mu, \sigma), & \text{if } m_p \text{ is a VP model} \\ \frac{1}{2\pi}, & \text{otherwise} \end{cases} \tag{2}$$

with θ_1 being the normal direction of the edge estimated from an edge model m_p, and θ_2 being the gradient direction of the pixel. It is also noted that the rotation matrix R is a function of α, β, γ, therefore R is equivalent to Ψ in the actual computation of the posterior.
In addition, it is sufficient for the E-step to obtain a set of weights w_{pm} for each pixel p with respect to a VP direction model m, and w_{pm} is calculated by (normalized over all edge models)

$$w_{pm} = \frac{1}{Z_p}P(E_p|m_p)P(\phi_p|m_p, V^{old})P(m_p) = \frac{1}{Z_p}P(\phi_p|m_p, V^{old})P(m_p) \tag{3}$$

We also obtain pixel assignments in the E-step. For each pixel, its assignment to one of the models is done by

$$assignment_p = \underset{m_p}{\operatorname{argmax}}\, P(\phi_p|m_p, V^{old})P(m_p) \tag{4}$$

M-step: the M-step amounts to evaluate the camera rotation matrix R that maximizes the expected log-posterior $Q(\Psi; \Psi^{old})$, which simplifies to minimize

a weighted least-squares error obtained by only summing over the three VP models over all pixels [1].
The objective error function used in the least-squares optimization is defined as

$$error = \sum_{p} \sum_{m_p \in \{vp_1, vp_2, vp_3\}} w_{pm}(\theta_1 - \theta_2)^2 \tag{5}$$

where w_{pm} is obtained in the E-step, $\theta_1 - \theta_2$ is again the difference between the normal direction of the edge estimated from an edge model m_p the gradient direction of the pixel p.

The EM steps are iterated until the change in the sum of weighted errors in the M-step falls below 1e-2.

2 Vanishing points locations

In this section, the vanishing point (VP) locations obtained after EM algorithm reaches convergence are shown below.

For image P1030001.jpg, the VP locations are shown in Figure 1. The 3 VP locations obtained after running EM algorithm are:
VP1 (-346.00, 319.31)
VP2 (1011.04, 314.11)
VP3 (280.83, -6718.91)

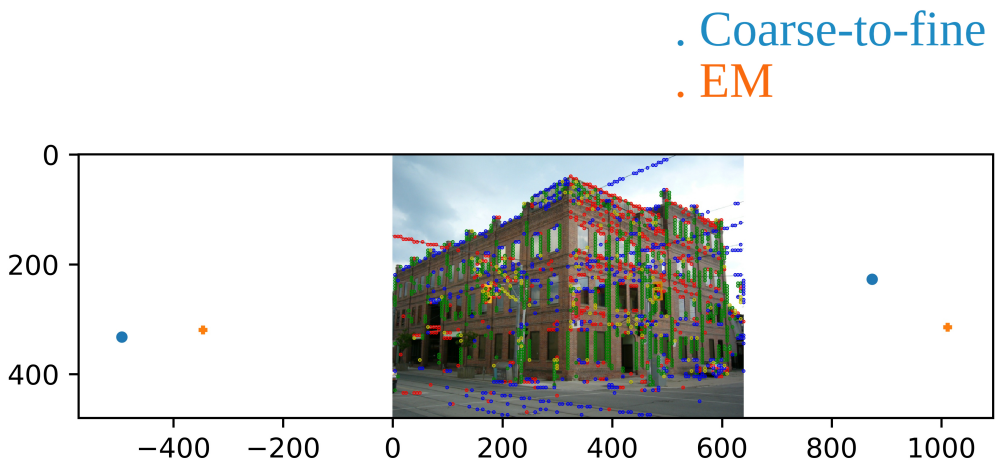

Figure 1: VP locations of image P1030001.jpg. Note that only 2 VPs are shown in the plot, the third VP is located beyond the plot view.

For image P1080055.jpg, the VP locations are shown in Figure 2. The 3 VP locations obtained after running EM algorithm are:
VP1 (-354.68, 304.72)
VP2 (996.45, 264.60)
VP3 (-94.93, -13305.14)

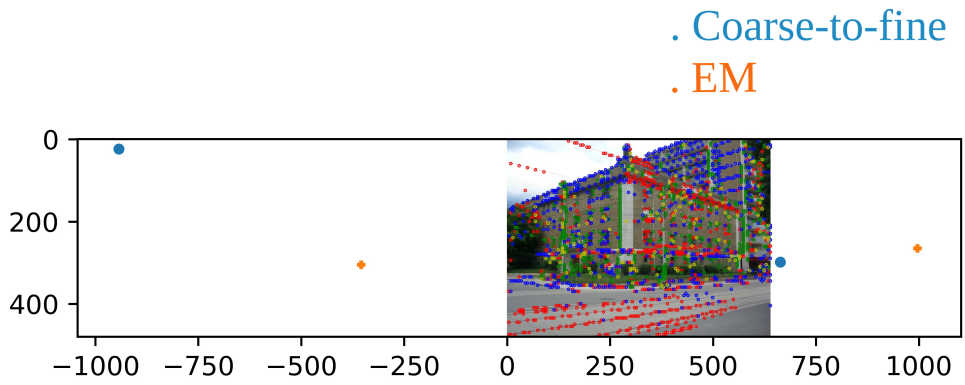

Figure 2: VP locations of image P1080055.jpg. Note that only 2 VPs are shown in the plot, the third VP is located beyond the plot view.

3 Assignments of each pixel

In this section, the assignments of each pixel to its corresponding VP are shown below. In each of the plot, the red, blue, and green dots denote the assignments of pixels to one of the three VPs, and the yellow dots represent pixels that are outliers which do not belong to any of the three VPs. For image P1030001.jpg, the pixel assignments are shown in Figure 3, and for image P1080055.jpg, the pixel assignments are shown in Figure 4.

4 Results analysis

In the VP location plots in Figure 5 and 6, 2 of 3 VP locations are shown, with the third VP located at the axis perpendicular to the other 2 VPs, but is beyond the plot view to show. As can been seen from these plots, the interpolated dashed lines of the same color edge pixels meet at the same vanishing point, which verifies that the same color pixels belong to the same VP. In addition, it

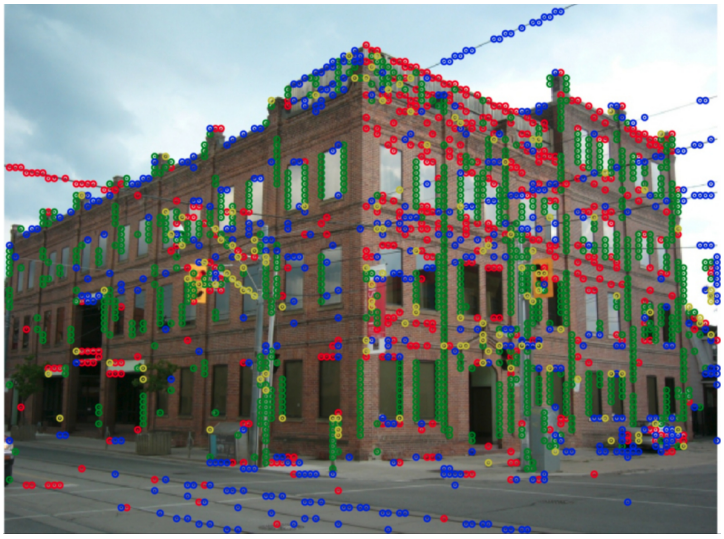

Figure 3: Pixel assignments of image P1030001.jpg

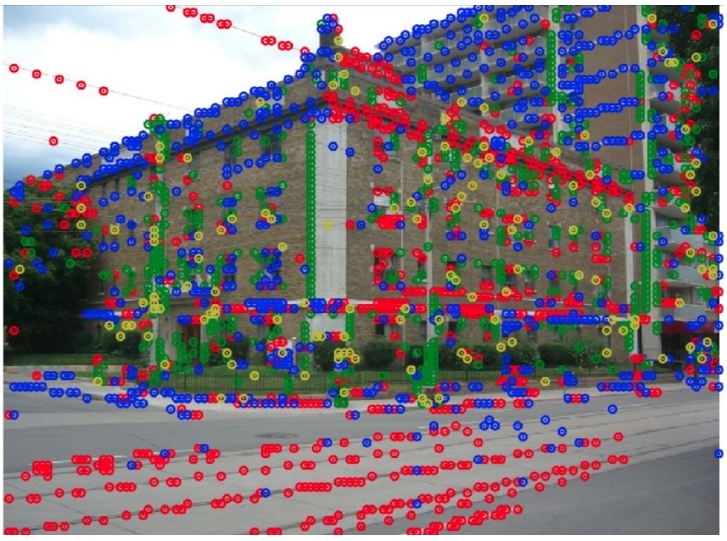

Figure 4: Pixel assignments of image P1080055.jpg

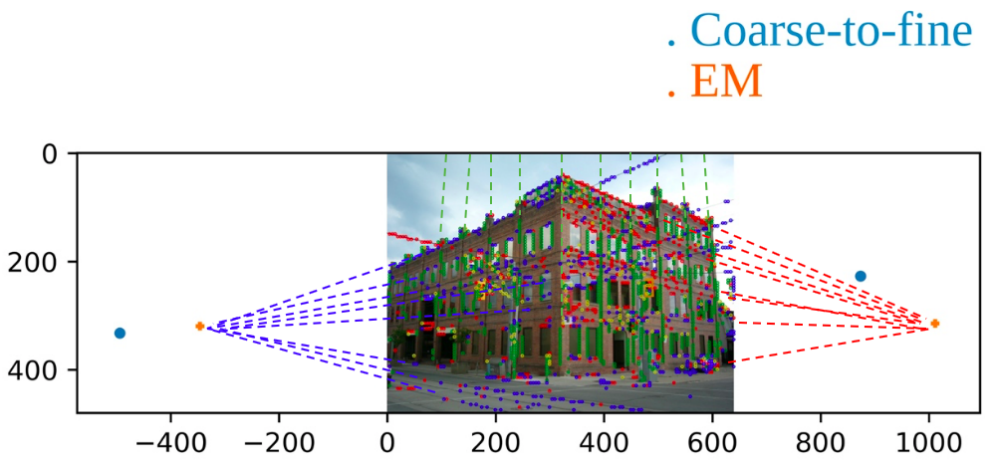

Figure 5: VP locations of image P1030001.jpg with dashed lines indicating edge pixel VP direction. Note that only 2 VPs are shown in the plot, the third VP is located beyond the plot view.

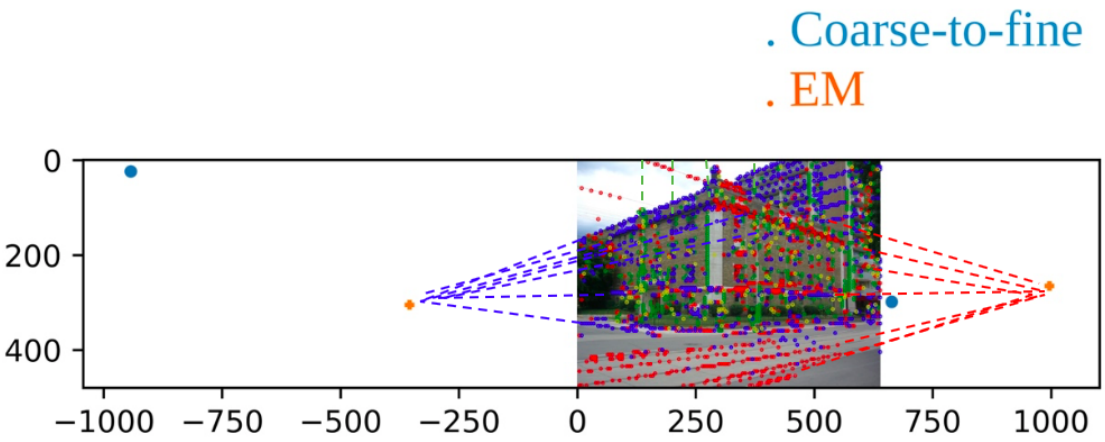

Figure 6: VP locations of image P108055.jpg with dashed lines indicating edge pixel VP direction. Note that only 2 VPs are shown in the plot, the third VP is located beyond the plot view.

can also be seen from the plots that the VP locations after EM (in orange dots) do not deviate too much from those obtained from the coarse-to-fine initialization (in blue dots) [2], which helps demonstrate that the VP locations after EM make sense.

References

[1] Grant Schindler and Frank Dellaert. Atlanta world: an expectation maximization framework for simultaneous low-level edge grouping and camera calibration in complex man-made environments. *CVPR*, 2004.

[2] Faraz M. Mirzaei and Stergios I. Roumeliotis. Optimal estimation of vanishing points in a manhattan world. *Proceedings of the 2011 International Conference on Computer Vision, ICCV '11*, pages 2454 – 2461, 2011.