EEE225 Advanced Electrical Circuits and Electromagnetics

Lecture 13 More about EM Waves

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Plane wave in boundless dielectric medium

- Assume that:
  - i) the dielectric medium is of infinite extend;
  - ii) there is only one wave propagating along the z direction;

  => only the forward wave is propagating.
- Then, the *x* and *y* components are:

$$\tilde{E}_x(z) = E_{xf} e^{-j(\beta z - \theta_{xf})}$$
$$\tilde{E}_y(z) = E_{yf} e^{-j(\beta z - \theta_{yf})}$$

  - Using the Maxwell's equation (1), get the x and y of **H** field as:

$$\tilde{H}_x(z) = -\sqrt{\frac{\epsilon}{\mu}}\,\tilde{E}_y(z)$$
$$\tilde{H}_y(z) = \sqrt{\frac{\epsilon}{\mu}}\,\tilde{E}_x(z)$$

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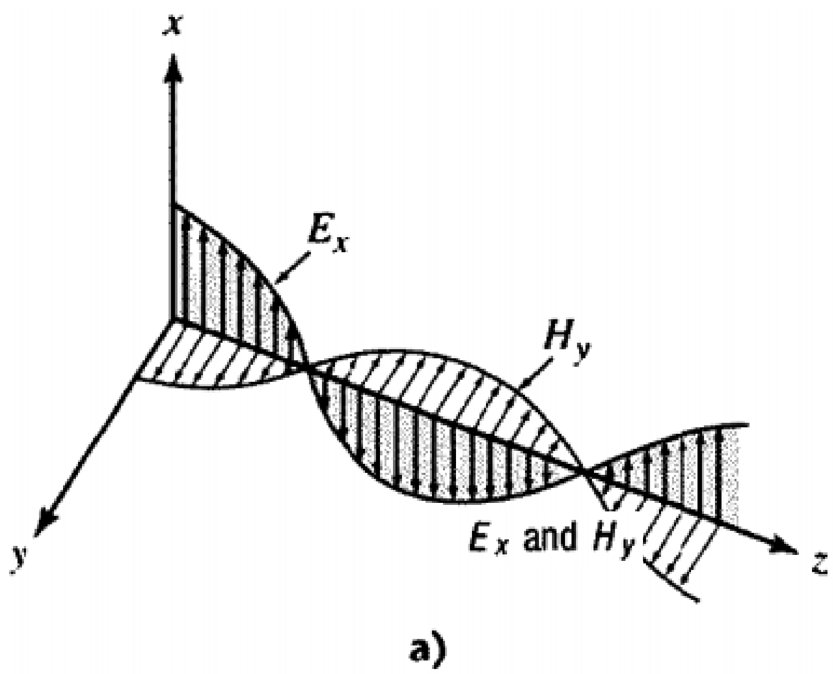
Plane wave in boundless dielectric medium

- The **E** and **H** relationship can also be written as

$$\vec{\mathbf{a}}_z \times \tilde{\mathbf{E}} = \sqrt{\frac{\mu}{\epsilon}}\,\tilde{\mathbf{H}} = \eta\tilde{\mathbf{H}}$$

  – where $\eta = \sqrt{\frac{\mu}{\epsilon}}$ has the unit of Ω. It is called the intrinsic (or wave) impedance.

  – Intrinsic impedance for the wave in free space is $\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377\Omega$



Plane wave in free space

- Free space (or vacuum) is a special case of a dielectric medium in which $\mu = \mu_0$ and $\varepsilon = \varepsilon_0$
- We can simply replace μ with μ_0 and ε with ε_0 to get:
  - Phase constant in free space:

    $$\beta_0 = \omega\sqrt{\mu_0\epsilon_0} = \frac{\omega}{c}$$

    - where $c = 1/\sqrt{\mu_0\epsilon_0} = 3 \times 10^8$ m/s is the speed of light.
  - Wave speed in free space:

    $$u_p = \frac{\omega}{\beta_0} = c \longrightarrow$$

    meaning that *an electromagnetic wave propagates in free space travelling with the speed of light*.
  - Wavelength in free space:

    $$\lambda_0 = \frac{2\pi}{\beta_0} = \frac{c}{f}$$

  - Intrinsic impedance of free space:

    $$\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 120\pi \approx 377\ \Omega$$

Plane wave in imperfect dielectric medium

- Imperfect dielectric medium is the general medium whose conductivity is not zero. It is also called conducting medium.
- In this case, the phasor form Helmholtz's equations for the time-harmonic waves should be:

$$\nabla^2 \tilde{\vec{\mathbf{E}}} = \boxed{(j\omega\mu\sigma - \omega^2\mu\epsilon)}\tilde{\vec{\mathbf{E}}}$$
$$\nabla^2 \tilde{\vec{\mathbf{H}}} = \boxed{(j\omega\mu\sigma - \omega^2\mu\epsilon)}\tilde{\vec{\mathbf{H}}}$$

- The complex coefficient may be expressed in somewhat compact form as: $j\omega\mu\sigma - \omega^2\mu\epsilon = j\omega\mu(\sigma + j\omega\epsilon)$

$$= -\omega^2\mu\epsilon\left[1 - j\frac{\sigma}{\omega\epsilon}\right]$$
$$= -\omega^2\mu\hat{\epsilon}$$

  – where $\hat{\epsilon} = \epsilon\left[1 - j\frac{\sigma}{\omega\epsilon}\right] = \epsilon' - j\epsilon''$ is called the ***complex permittivity*** of the medium. It is a function of frequency, and its real and imaginary part describe the dielectric and conducting properties of the medium.

Plane wave in imperfect dielectric medium

- In terms of the complex permittivity, we can express the wave equations as

$$\nabla^2\tilde{\vec{\mathbf{E}}} = -\omega^2\mu\hat{\epsilon}\tilde{\vec{\mathbf{E}}} \qquad \nabla^2\tilde{\vec{\mathbf{E}}} = \hat{\gamma}^2\tilde{\vec{\mathbf{E}}}$$

$$\nabla^2\tilde{\vec{\mathbf{H}}} = -\omega^2\mu\hat{\epsilon}\tilde{\vec{\mathbf{H}}} \qquad \nabla^2\tilde{\vec{\mathbf{H}}} = \hat{\gamma}^2\tilde{\vec{\mathbf{H}}}$$

  - where $\hat{\gamma}^2 = -\omega^2\mu\hat{\epsilon}$ is called the *propagation constant*.
- Assume
  - The wave propagates in z-direction;
  - TEM wave, i.e. independent of variations with respect to x and y
- Then we can have:

$$\frac{d^2\tilde{E}_x(z)}{dz^2} = \hat{\gamma}^2\tilde{E}_x$$

Recall

$$\boxed{\frac{d^2\tilde{E}_x}{dz^2} + \omega^2\mu\epsilon\tilde{E}_x = 0}$$

- Solve it get:

$$\tilde{E}_x(z) = \hat{E}_f e^{-\hat{\gamma}z} + \hat{E}_b e^{\hat{\gamma}z}$$

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Plane wave in imperfect dielectric medium

$$\tilde{E}_x(z) = \hat{E}_f e^{-\hat{\gamma} z} + \hat{E}_b e^{\hat{\gamma} z}$$

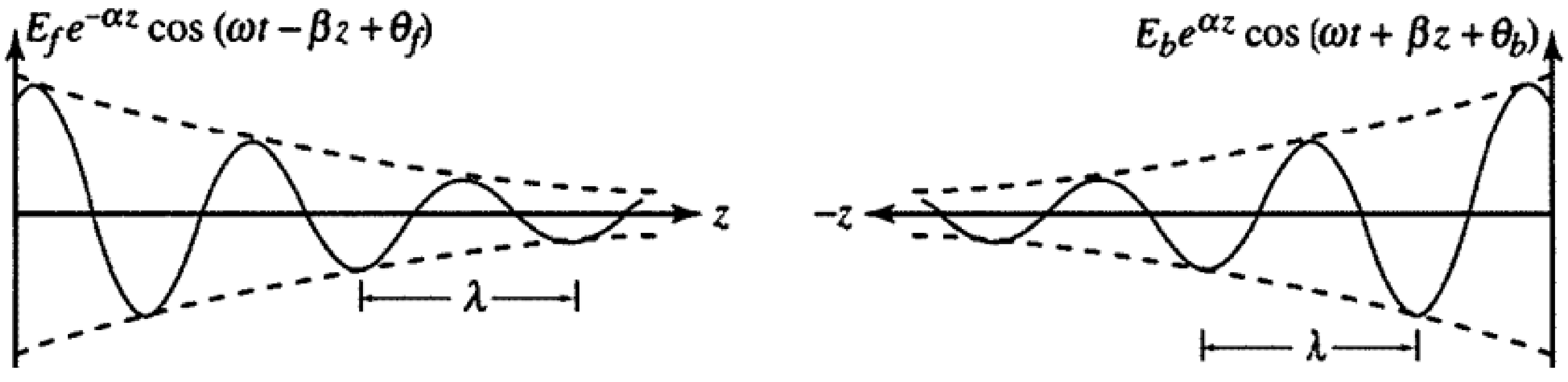
- $\boldsymbol{E_f}$ and $\boldsymbol{E_b}$ are complex coefficients independent of t and z.
- Propagation constant γ can be written as:

$$\hat{\gamma} = j\omega\sqrt{\mu\hat{\epsilon}} = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} = \alpha - j\beta$$

  - Real part α is the attenuation constant
  - Imaginary part β is the phase constant

- Separate the expression of γ into α and β,

$$E_x(z,t) = E_f e^{-\alpha z}\cos(\omega t - \beta z + \theta_f) + E_b e^{\alpha z}\cos(\omega t + \beta z + \theta_b)$$



Skin depth

All the waves in conducting medium are attenuated wave.

- So: how far can the wave propagate in a conducting medium before its amplitude becomes insignificant? – ***Skin depth***
- The skin depth is the distance travelled by the wave in a conducting medium at which its amplitude falls to 1/e of its value on the surface of that conducting medium.
- If we denote the skin depth by δ_c, the amplitude of the wave falls to 1/e when $\alpha\delta_c = 1$. Thus $\delta_c = \dfrac{1}{\alpha}$
- In good conductors, the wave attenuates very fast and the fields are confined to the region near the surface of the conductor – ***Skin effect***

Plane wave in good conductors

- The conducting medium behaves as a good conductor when:

$$\frac{\sigma}{\omega\epsilon} \geq 10$$

- In this case, some simplification can be done to the general expression for conducting medium:

$$\hat{\epsilon} \approx \frac{\sigma}{j\omega} \quad \Rightarrow \quad \hat{\gamma} = j\omega\sqrt{\mu\hat{\epsilon}} \approx j\omega\sqrt{\frac{\mu\sigma}{j\omega}} = \sqrt{j\omega\mu\sigma} = \sqrt{\omega\mu\sigma}\,\underline{/45^\circ}$$

- Thus, some parameters can be calculated:

$$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} \qquad \delta_c = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\sigma}} = \sqrt{\frac{1}{f\pi\mu\sigma}}$$

$$\hat{\eta} = \sqrt{\frac{\mu}{\hat{\epsilon}}} = \sqrt{\frac{\omega\mu}{\sigma}}\,\underline{/45^\circ}$$

Plane wave in good dielectrics

- The conducting medium behaves as a good dielectrics when:

$$\frac{\sigma}{\omega\epsilon} \le 0.1$$

- In this case, some simplification can be done to the general expression for conducting medium:

$$\sqrt{\hat{\epsilon}} = \sqrt{\epsilon\left[1 - j\frac{\sigma}{\omega\epsilon}\right]} \approx \sqrt{\epsilon}\left[1 - j\frac{\sigma}{2\omega\epsilon}\right] \Rightarrow \hat{\gamma} = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}} + j\omega\sqrt{\mu\epsilon}$$

- Thus, some parameters can be calculated:

$$\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}$$

$$\beta = \omega\sqrt{\mu\epsilon}$$

$$\hat{\eta} = \sqrt{\frac{\mu}{\epsilon}\left[1 + j\frac{\sigma}{2\omega\epsilon}\right]} \approx \sqrt{\frac{\mu}{\epsilon}}$$

Example 1

- A 1.8 kHz wave propagates in a medium characterized by μr = 1.6, εr = 25 and σ = 2.5 S/m. The electric field intensity in the region is given by

$$\tilde{\boldsymbol{E}} = 0.1e^{-\alpha z}\cos(2\pi f t - \beta z)\,\overrightarrow{\boldsymbol{a}_x}\,,\mathrm{V/m}$$

  - (a) Determine the propagation constant, attenuation constant, the wavelength of the wave, the intrinsic impedance, the phase velocity and the skin depth.
  - (b) Obtain an expression for H field.

Bounded plane wave

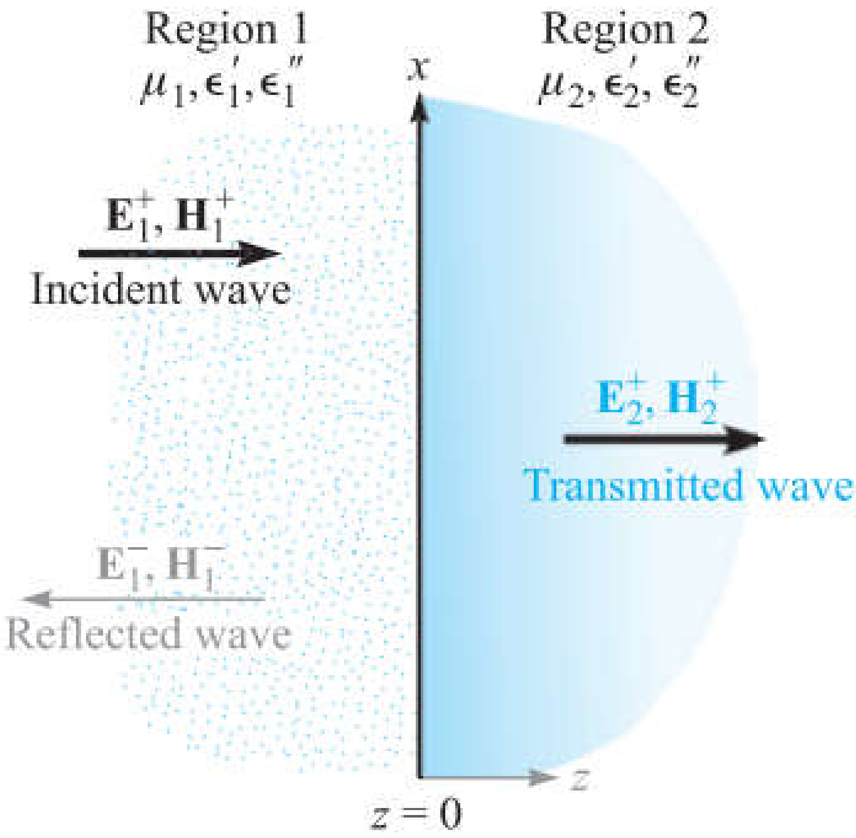
- A plane wave incident on a boundary establishes reflected and transmitted waves having the indicated propagation directions.
  - All fields are parallel to the boundary;
  - with electric fields along *x* and magnetic fields along *y*.

$$E^+_{xs1}(z) = E^+_{x10}e^{-jkz}$$

$$H^+_{ys1}(z) = \frac{1}{\eta_1}E^+_{x10}\,e^{-jk_1 z}$$

$$E^-_{xs1}(z) = E^-_{x10}\,e^{jk_1 z}$$

$$H^-_{xs1}(z) = -\frac{E^-_{x10}}{\eta_1}\,e^{jk_1 z}$$



$$E^+_{xs2}(z) = E^+_{x20}\,e^{-jk_2 z}$$

$$H^+_{ys2}(z) = \frac{1}{\eta_2}E^+_{x20}\,e^{-jk_2 z}$$

Boundary conditions

- The boundary conditions are now easily satisfied, and in the process the amplitudes of the transmitted and reflected waves may be found in terms of $E_{x10}{}^{+}$
- The total electric field intensity is continuous at $z = 0$:

$$E_{xs1}^{+} + E_{xs1}^{-} = E_{xs2}^{+} \qquad (z = 0)$$

Therefore $E_{x10}^{+} + E_{x10}^{-} = E_{x20}^{+}$

- The total magnetic field intensity is also continuous at $z = 0$:

$$H_{ys1}^{+} + H_{ys1}^{-} = H_{ys2}^{+} \qquad (z = 0)$$

Therefore $\dfrac{E_{x10}^{+}}{\eta_1} - \dfrac{E_{x10}^{-}}{\eta_1} = \dfrac{E_{x20}^{+}}{\eta_2}$

Reflection and transmission coefficients

- Solve the two boundary conditions, get:

$$\left.\begin{aligned} E_{x10}^{+} + E_{x10}^{-} &= E_{x20}^{+} \\ \frac{E_{x10}^{+}}{\eta_1} - \frac{E_{x10}^{-}}{\eta_1} &= \frac{E_{x20}^{+}}{\eta_2} \end{aligned}\right\} \Rightarrow E_{x10}^{-} = E_{x10}^{+}\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

- The ratio of the amplitudes of the reflected and incident electric fields defines the *reflection coefficient Γ*.

$$\Gamma = \frac{E_{x10}^{-}}{E_{x10}^{+}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = |\Gamma| e^{j\phi}$$

- Similarly, relative amplitude of the transmitted electric field intensity yielding the *transmission coefficient, τ*.

$$\tau = \frac{E_{x20}^{+}}{E_{x10}^{+}} = \frac{2\eta_2}{\eta_1 + \eta_2} = 1 + \Gamma = |\tau| e^{j\phi_t}$$

Perfect dielectric – conductor interface

- Let region 1 be a perfect dielectric and region 2 be a perfect conductor.
  - For a perfect conductor, $\eta_2 = 0$;
  - And no fields can exist in the perfect conductor, $E_{x20}{}^{+} = 0$;
- Therefore
  - Transmission coefficient $\tau = 0$;
  - Reflection coefficient $\Gamma = -1$;
  - And the field has: $E^{+}_{x10} = -E^{-}_{x10}$
- The total **E** field in region 1 is

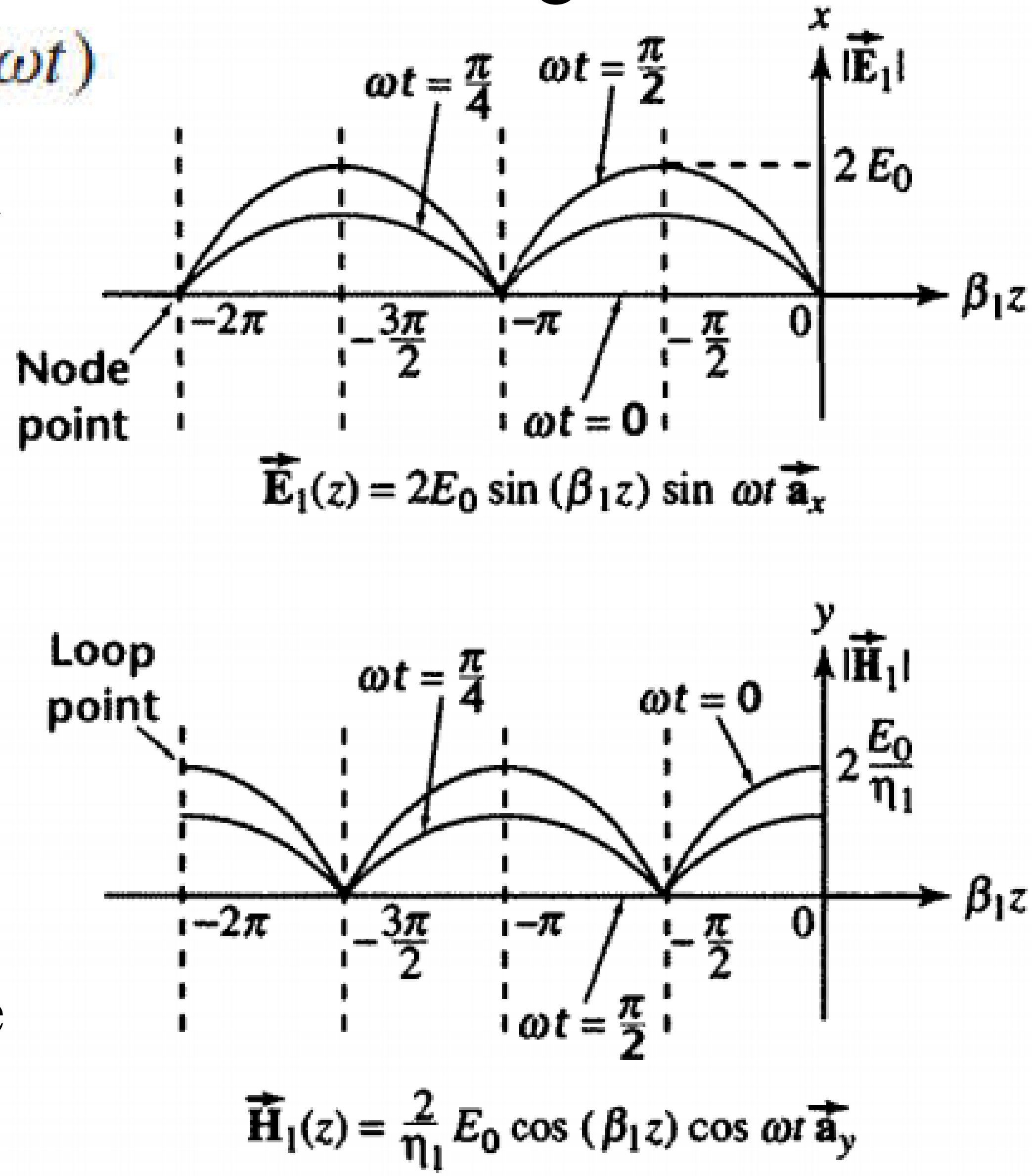
$$
\begin{aligned}
E_{xs1} &= E^{+}_{xs1} + E^{-}_{xs1} \\
&= E^{+}_{x10}\, e^{-j\beta_1 z} - E^{+}_{x10}\, e^{j\beta_1 z} = (e^{-j\beta_1 z} - e^{j\beta_1 z})\, E^{+}_{x10} \\
&= -j2\,\sin(\beta_1 z)\, E^{+}_{x10}
\end{aligned}
$$

Standing wave

- The instantaneous form of the total **E** field in region 1 is

$$\mathcal{E}_{x1}(z,t) = 2E_{x10}^{+}\sin(\beta_1 z)\sin(\omega t)$$

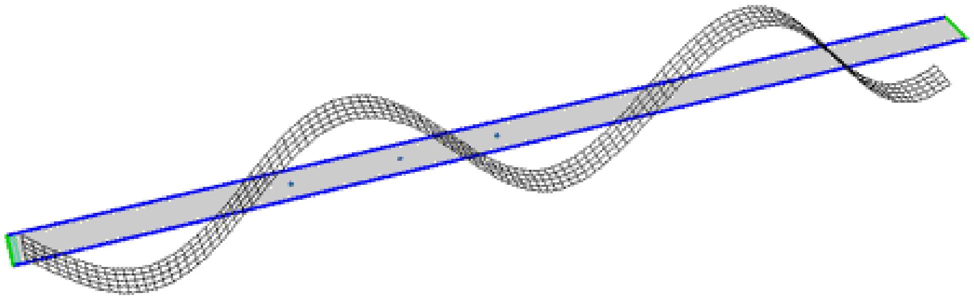
- The factors involving time and distance are separate trigonometric terms.
  - Whenever $\omega t = m\pi$, $Ex1(t)$ is zero at all positions.
  - On the other hand, spatial nulls in the standing wave pattern occur for all times wherever $\beta_1 z = m\pi$.
  - Same for **H** field, with 90° phase difference.

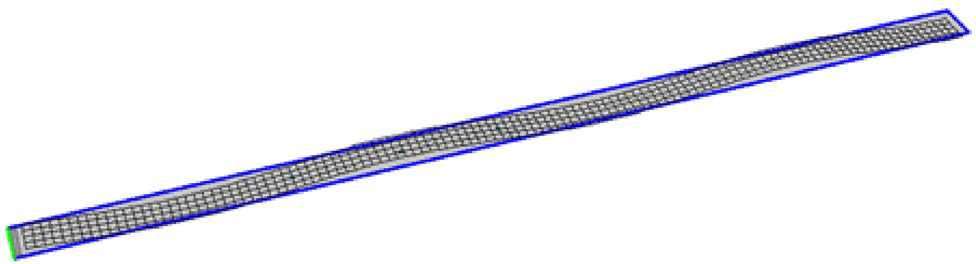


Travelling wave and Standing wave

Travelling Wave

Standing Wave





$$\mathcal{E}_{x1}(z,t) = E_{x10}^{+}\cos(\omega t - \beta_1 z)$$

$$\mathcal{E}_{x1}(z,t) = 2E_{x10}^{+}\sin(\beta_1 z)\sin(\omega t)$$

Perfect dielectric - dielectric interface

- With perfect dielectrics in both regions 1 and 2; η_1 and η_2 are both real positive quantities and $\alpha_1 = \alpha_2 = 0$.
- We can calculate the transmission and reflection coefficients and obtain the **E** and **H** fields in both region.

- Example: for two regions are shown:

$$\eta_1 = 100\ \Omega$$

$$\eta_2 = 300\ \Omega$$

$$E^+_{x10} = 100\ \text{V/m}$$

- Calculate the incident, reflected, and transmitted waves and the power they carry.

Mixed wave

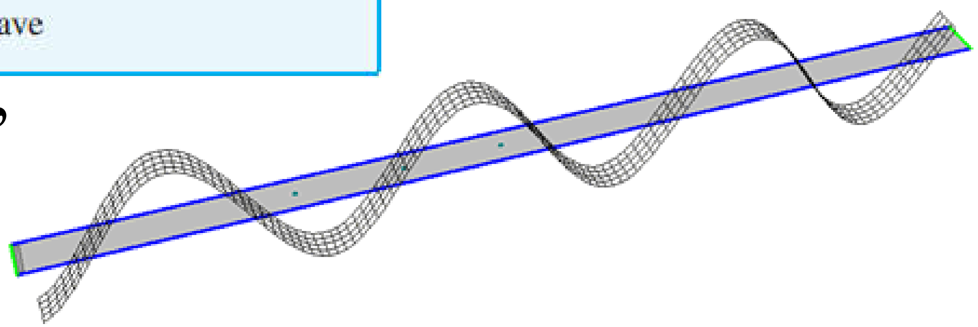
- In cases where $|\Gamma| < 1$, some energy is transmitted into the second region and some is reflected. Region 1 therefore supports a field that is composed of both a traveling wave and a standing wave.
- Total **E** field phasor in region 1 is

$$E_{x1T} = E_{x1}^{+} + E_{x1}^{-} = E_{x10}^{+} e^{-j\beta_1 z} + \Gamma E_{x10}^{+} e^{j\beta_1 z} = \left(e^{-j\beta_1 z} + |\Gamma| e^{j(\beta_1 z + \phi)}\right) E_{x10}^{+}$$

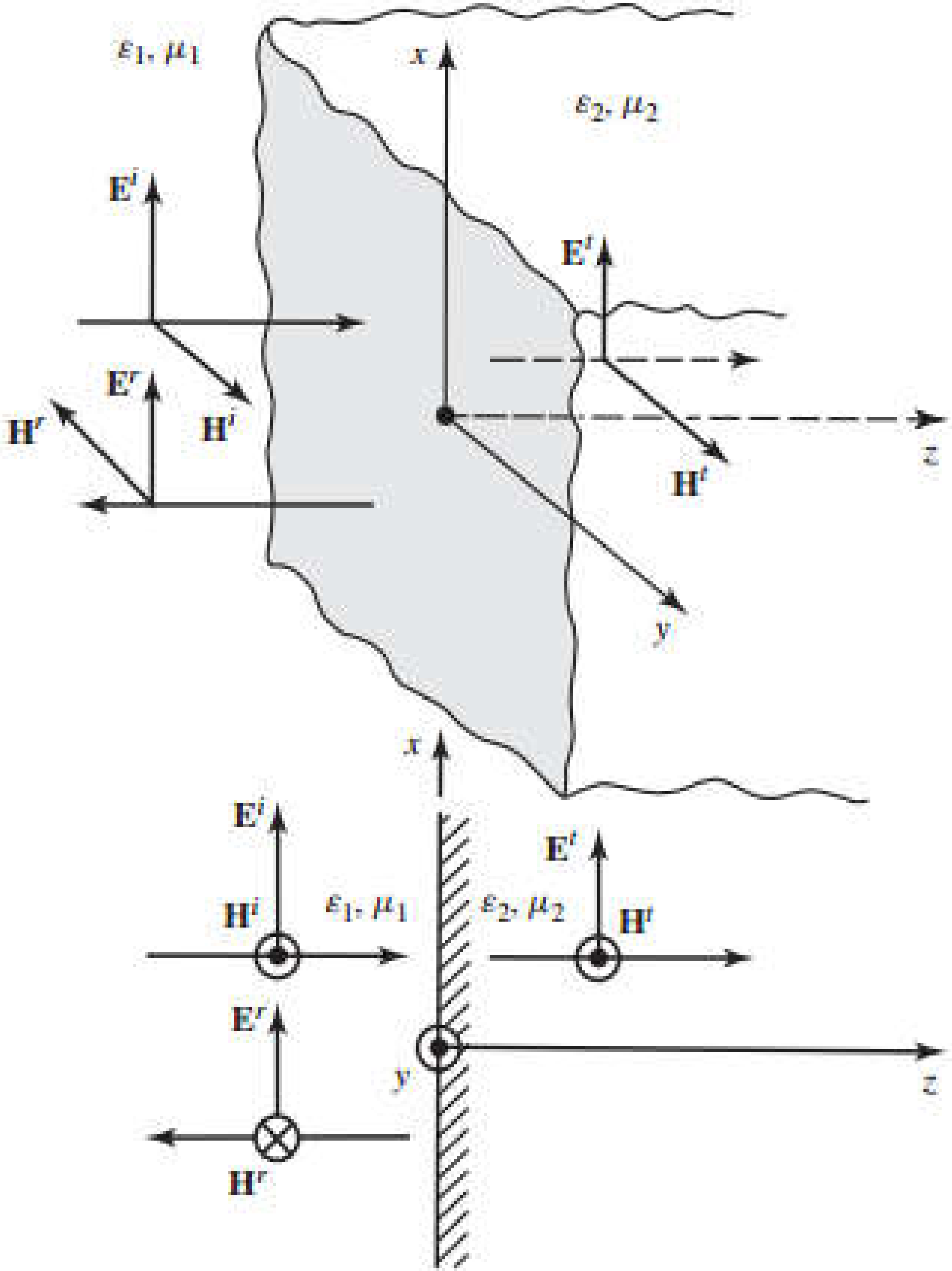
$$\Rightarrow \boxed{\mathcal{E}_{x1T}(z,t) = \underbrace{(1-|\Gamma|)E_{x10}^{+}\cos(\omega t - \beta_1 z)}_{\text{traveling wave}} + \underbrace{2|\Gamma|E_{x10}^{+}\cos(\beta_1 z + \phi/2)\cos(\omega t + \phi/2)}_{\text{standing wave}}}$$

- Define "Standing Wave Ratio"

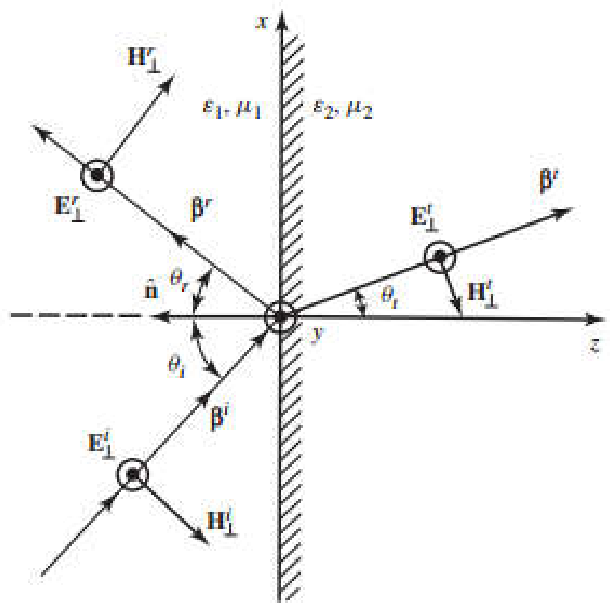
$$SWR = \frac{|E_{x1t}|_{max}}{|E_{x1t}|_{min}} = \frac{1+|\Gamma|}{1-|\Gamma|}$$



Normal incident VS Oblique incident



- Normal incident: the propagation direction is perpendicular to the interface, making **E** and **H** are all pure tangential components.
- Oblique incident: the incident wave has some angle to the normal direction of the interface, making analyses more complex.



Quiz

- 1. For a wave normally incident from free space onto a planar interface with a perfect conductor, the reflection coefficient is
  - (a) 0; (b) 1;
  - (c) -1; (d) Depends on the frequency of the wave

- 2. The standing wave ratio for perfect transmission (no reflection) is
  - (a) 0; (b) 1;
  - (c) -1; (d) 2.

Poynting Theorem

- With some derivation, we can have:

$$\nabla \cdot \boxed{(\vec{\mathbf{E}} \times \vec{\mathbf{H}})} + \vec{\mathbf{J}} \cdot \vec{\mathbf{E}} + \vec{\mathbf{H}} \cdot \frac{\partial \vec{\mathbf{B}}}{\partial t} + \vec{\mathbf{E}} \cdot \frac{\partial \vec{\mathbf{D}}}{\partial t} = 0$$

Differential form of Poynting's theorem

Poynting vector: with the unit of power density, W/m^2, is the instantaneous flow of power per unit area.
Defined as: **S** = **E x H**, where **S** is the Poynting vector, normal to the plane containing **E** and **H**.

- With some more modifications, get:

$$\oint_s \vec{\mathbf{S}} \cdot \vec{ds} + \int_v \vec{\mathbf{J}} \cdot \vec{\mathbf{E}}\, dv + \frac{d}{dt}\int_v w_m\, dv - \frac{d}{dt}\int_v w_e\, dv = 0$$

Integral form of Poynting's theorem

– where $w_m = \frac{1}{2}\vec{\mathbf{B}} \cdot \vec{\mathbf{H}} = \frac{1}{2}\mu H^2$

$w_e = \frac{1}{2}\vec{\mathbf{D}} \cdot \vec{\mathbf{E}} = \frac{1}{2}\epsilon E^2$

Poynting Theorem

$$\oint_s \vec{\mathbf{S}} \cdot \overrightarrow{ds} + \int_v \vec{\mathbf{J}} \cdot \vec{\mathbf{E}}\, dv + \frac{d}{dt}\int_v w_m\, dv - \frac{d}{dt}\int_v w_e\, dv = 0$$

- The first term represents the power crossing the closed surface *S* bounding the volume *v*.
- The second integral represents the power supplied to the charged particles by the field.
- The third term represents the rate of change of stored magnetic energy.
- The final term represents the rate of change of stored energy in the electric field.

$$-\oint_s \vec{\mathbf{S}} \cdot \overrightarrow{ds} = \int_v \vec{\mathbf{J}} \cdot \vec{\mathbf{E}}\, dv + \frac{d}{dt}\int_v (w_m + w_e)\, dv$$

- The negative sign on the left indicates that the net power must flow into volume v in order to account for (a) the power dissipation in the region as heat and (b) the increase in the energy stored in electric and magnetic fields.

Poynting Theorem – Example

- Example 7: The electric field intensity in a dielectric medium is given as $\mathbf{E} = E_0 \cos(\omega t - kz)\, \mathbf{a_x}$ V/m, where E_0 is its peak value, and k is a constant quantity.
- Determine:
  - (a) the magnetic field intensity in the region;
  - (b) the direction of power flow;
  - (c) the average power density.

EM Power and Poynting Theorem

- Electromagnetic waves carry with them Electromagnetic power. Energy is transported through space to distant receiving points by EM waves.
- Poynting Theorem:

$$-\oint_{\text{area}} (\mathbf{E} \times \mathbf{H}) \cdot d\mathbf{S} = \int_{\text{vol}} \mathbf{J} \cdot \mathbf{E}\, dv + \frac{d}{dt} \int_{\text{vol}} \frac{1}{2} \mathbf{D} \cdot \mathbf{E}\, dv + \frac{d}{dt} \int_{\text{vol}} \frac{1}{2} \mathbf{B} \cdot \mathbf{H}\, dv$$

  – On the right-hand side:
    - the first integral is the total instantaneous power dissipated within the volume;
    - the second integral is the time rate of increase of the energy stored in the electric field instantaneously;
    - the third integral is the time rate of increase of the energy stored in the magnetic field instantaneously.

  – On the left-hand side: the surface integral of **E**x**H** is the total power flowing *into* this volume, the direction is given by the minus sign.

Poynting vector – instantaneous power density

- $-\oiint_{area}(\boldsymbol{E}\times\boldsymbol{H})\cdot d\boldsymbol{S}$ is the total power flowing into the volume, so the total out-flowing power should be:

$$P_{out} = \oiint_{area}(\boldsymbol{E}\times\boldsymbol{H})\cdot d\boldsymbol{S} \quad (\mathrm{W})$$

- The integrand, known as the Poynting vector, is:

$$\boldsymbol{S} = \boldsymbol{E}\times\boldsymbol{H} \quad (\mathrm{W/m^2})$$

  – ***S*** is interpreted as an ***instantaneous power density***;
  – The direction of ***S*** indicates the direction of the power flow at a point
    - "Poynting" sounds like "pointing", and it is quite true.

Average Power Density

- **S** is given by the cross product of **E** and **H**: $E_x\mathbf{a}_x \times H_y\mathbf{a}_y = S_z\mathbf{a}_z$
- In an unbounded dielectric, **E** & **H** fields are given as:

$$\left.\begin{aligned} E_x &= E_{x0}e^{-\alpha z}\cos(\omega t - \beta z) \\ H_y &= \frac{E_{x0}}{|\eta|}e^{-\alpha z}\cos(\omega t - \beta z - \theta_\eta) \end{aligned}\right\} \Longrightarrow S_z = E_x H_y = \frac{E_{x0}^2}{|\eta|}e^{-2\alpha z}\cos(\omega t - \beta z)\cos(\omega t - \beta z - \theta_\eta)$$

- The time-average power density is obtained by integrate S_z over one cycle and divide by period T.

$$\langle S_z \rangle = \frac{1}{T}\int_0^T \frac{1}{2}\frac{E_{x0}^2}{|\eta|}e^{-2\alpha z}[\cos(2\omega t - 2\beta z - 2\theta_\eta) + \cos\theta_\eta]\,dt$$

  - Finally we get:

$$\boxed{\langle S_z \rangle = \frac{1}{2}\frac{E_{x0}^2}{|\eta|}e^{-2\alpha z}\cos\theta_\eta} \Longleftrightarrow \boxed{\langle \mathbf{S} \rangle = \frac{1}{2}\mathrm{Re}(\mathbf{E}_s \times \mathbf{H}_s^*) \quad \mathrm{W/m^2}}$$

Time domain | Phasor form

Example 2

- The far fields of a short vertical current element Idl located at the origin of a spherical coordinate system in free space is

$$\mathbf{E}(R,\theta) = \mathbf{a}_\theta E_\theta(R,\theta) = \mathbf{a}_\theta\left(j\frac{60\pi I\,d\ell}{\lambda R}\sin\theta\right)e^{-j\beta R} \qquad \text{(V/m)}$$

$$\mathbf{H}(R,\theta) = \mathbf{a}_\phi \frac{E_\theta(R,\theta)}{\eta_0} = \mathbf{a}_\phi\left(j\frac{I\,d\ell}{2\lambda R}\sin\theta\right)e^{-j\beta R} \qquad \text{(A/m),}$$

  - Where λ is the wavelength.
  - a) Write the expression for average power density;
  - b) Find the total average power radiated by the source.

The Polarization of a Wave

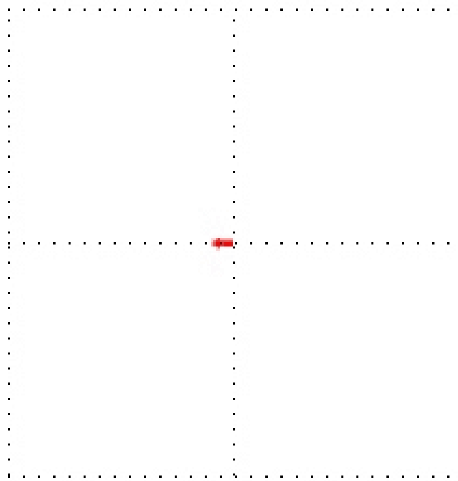
- Recall: for the wave propagating along the z axis, **E** was taken to lie along x, which then required **H** to lie along y.
  - This orthogonal relationship between **E**, **H**, and **S** is always true for a uniform plane wave. But the directions of **E** and **H** within the plane perpendicular to $\mathbf{a}_z$ may change.
  - Specifying only the electric field direction is sufficient, since magnetic field is readily found from **E** using Maxwell's equations.
- *Wave polarization* is defined as the time-dependent electric field vector orientation at a fixed point in space.
  - i.e. "polarization is the curve traced out, at a given observation point as a function of time, by the end point of the arrow representing the instantaneous electric field."

Linear Polarization

- Previously, with **S** in $\mathbf{a_z}$ direction,

$$\boldsymbol{E} = \boldsymbol{a_x} E_m e^{-\beta z}$$

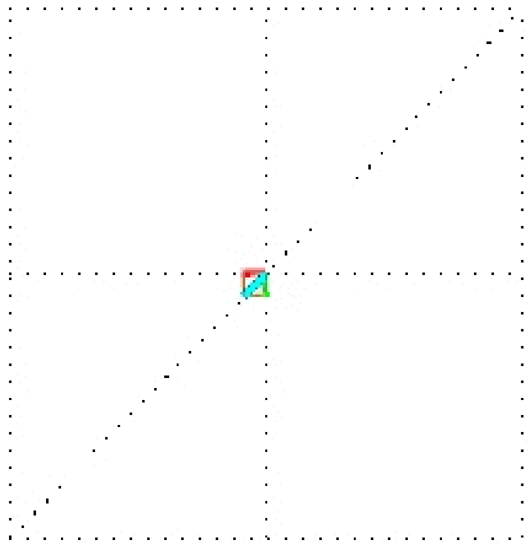
$$\boldsymbol{H} = \boldsymbol{a_y} \frac{E_m}{\eta} e^{-\beta z}$$



- More generally, the **E** field could be oriented in any fixed direction in the *xy* plane, so the total field in phasor form is:

$$\mathbf{E}_s = (E_{x0}\mathbf{a}_x + E_{y0}\mathbf{a}_y)e^{-\alpha z}e^{-j\beta z}$$

$$\mathbf{H}_s = \left[-\frac{E_{y0}}{\eta}\mathbf{a}_x + \frac{E_{x0}}{\eta}\mathbf{a}_y\right]e^{-\alpha z}e^{-j\beta z}$$



Linear Polarization

- The power density in the wave is

$$\langle \mathbf{S}_z \rangle = \frac{1}{2}\mathrm{Re}\{\mathbf{E}_s \times \mathbf{H}_s^*\} = \frac{1}{2}\mathrm{Re}\left\{\frac{1}{\eta^*}\right\}(|E_{x0}|^2 + |E_{y0}|^2)e^{-2\alpha z}\mathbf{a}_z \ \mathrm{W/m^2}$$

  – It indicates that our linearly polarized plane wave can be considered as two distinct plane waves having *x* and *y* polarizations, whose electric fields are combining *in phase* to produce the total **E**.

  – The same is true for the magnetic field components.

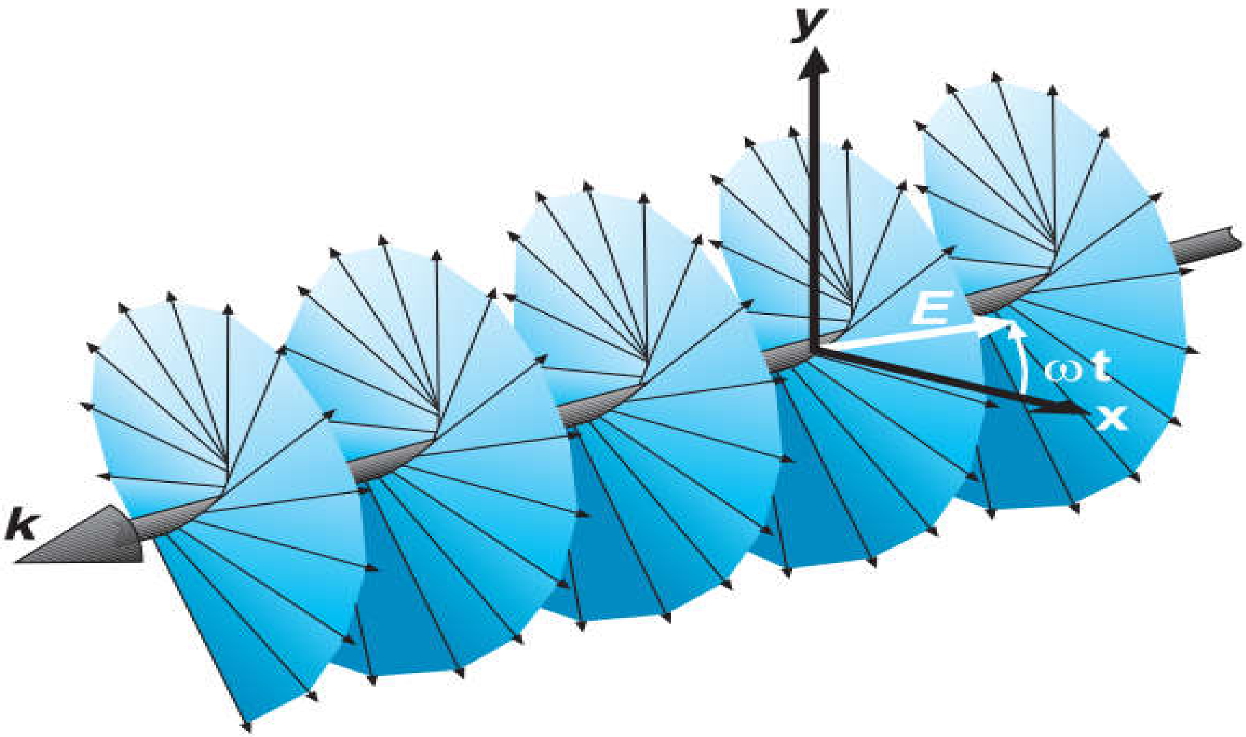
- Therefore:: *any polarization state can be described in terms of mutually perpendicular components of the electric field and their relative phasing.*

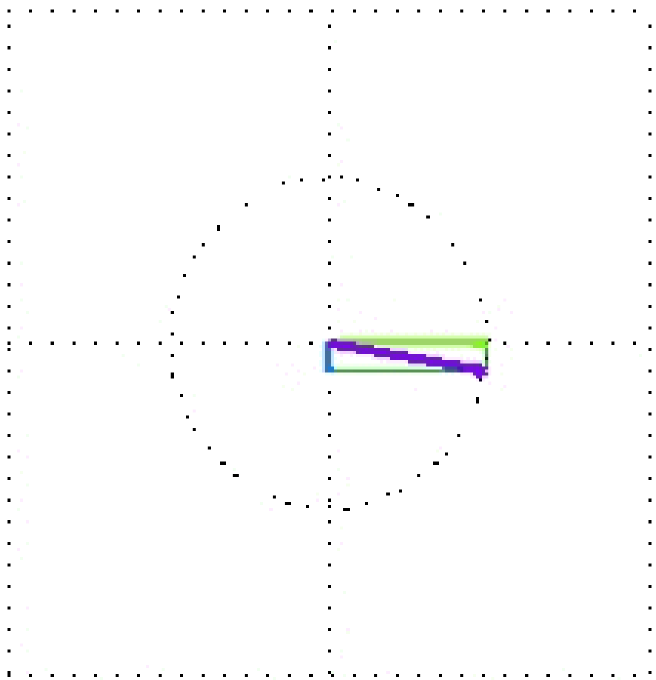
$$\mathbf{E}_s = (E_{x0}\mathbf{a}_x + E_{y0}\boxed{e^{j\phi}}\mathbf{a}_y)e^{-j\beta z}$$

Circular Polarization

- If the two orthogonal linearly polarized components have the same magnitude and are in phase quadrature, then the resultant time-dependent **E** vector rotates in the *x*-*y* plane, and its tip follows a perfect circle. Mathematically,

$$\mathbf{E}(z,t) = \mathbf{a}_x E_0 \cos(\omega t - \beta z) + \mathbf{a}_y E_0 \cos\left[\omega t - \beta z \pm (2n+1)\frac{\pi}{2}\right]$$

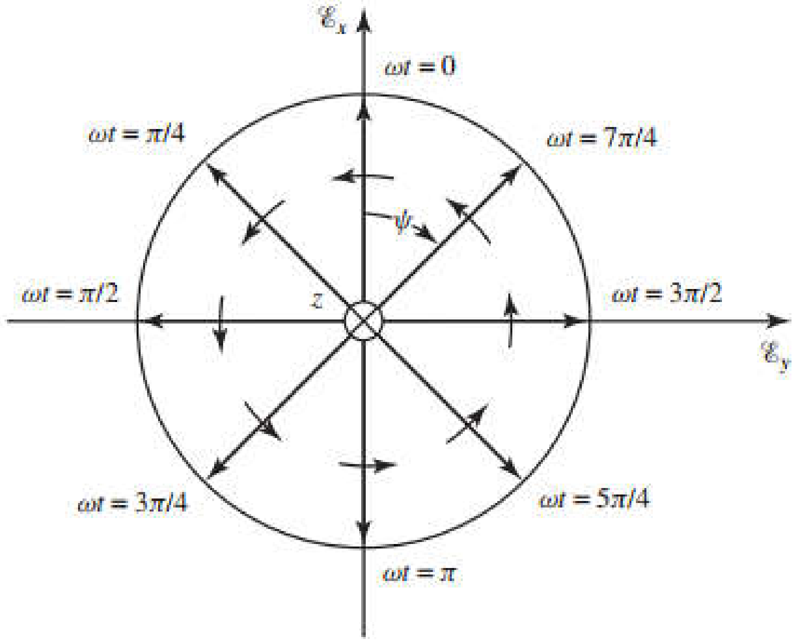


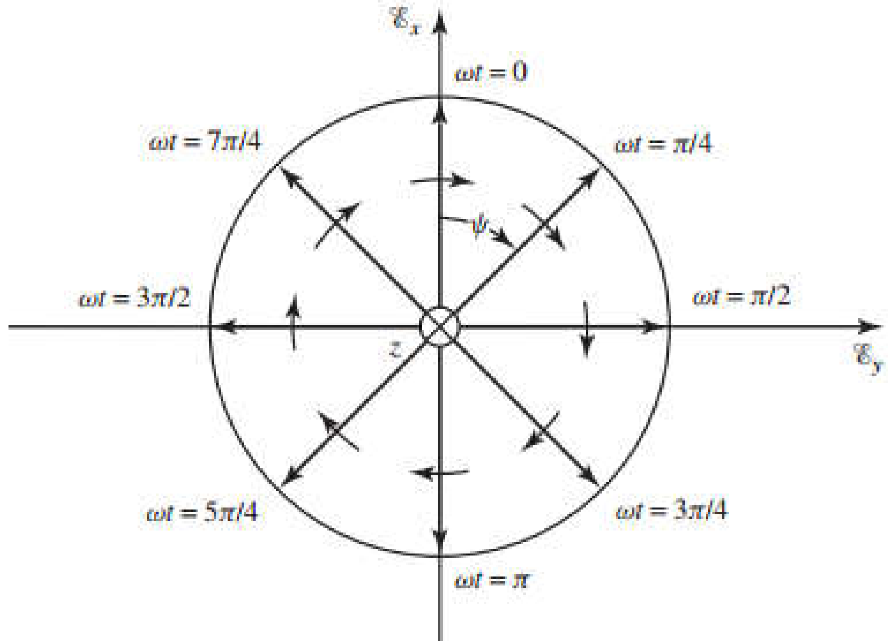


Circular Polarization

- The wave exhibits *left circular polarization* (l.c.p.) if, when orienting the left hand with the thumb in the direction of propagation, the fingers curl in the rotation direction of the field with time.

- The wave exhibits *right circular polarization* (r.c.p.) if, with the right-hand thumb in the propagation direction, the fingers curl in the field rotation direction.





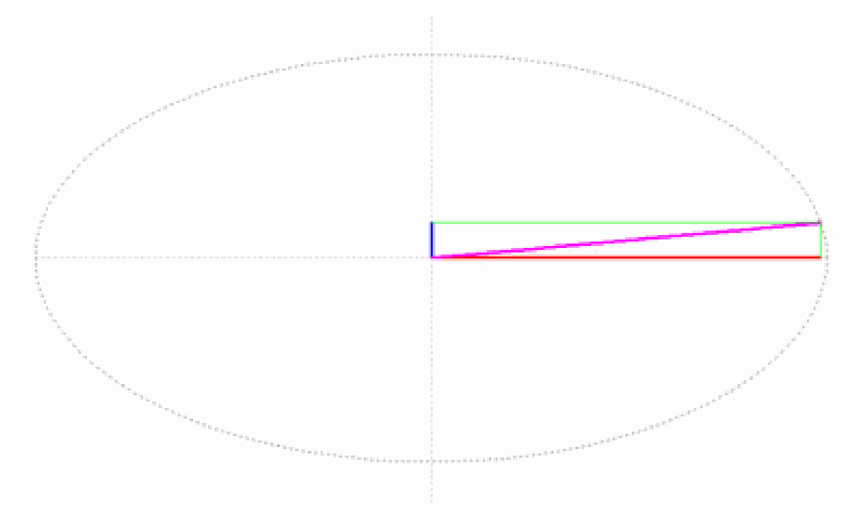
Elliptical Polarization

- Most generally,

$$\mathbf{E}_s = (E_{x0}\mathbf{a}_x + E_{y0}e^{j\phi}\mathbf{a}_y)e^{-j\beta z}$$

$$\mathbf{E}(z,t) = E_{x0}\cos(\omega t - \beta z)\mathbf{a}_x + E_{y0}\cos(\omega t - \beta z + \phi)\mathbf{a}_y$$

- The tip of the vector traces out the shape of an ellipse over time $t = 2\pi/\omega$. The wave is said to be *elliptically polarized*.
  - Special case 1: $\varphi = 0$
    - Linear polarization
  - Special case 2: $E_{x0} = E_{y0}$
    - Circular polarization



Quiz

- 3. The electric field of a plane wave propagating in a nonmagnetic medium is given by $\mathbf{E} = 3\sin(2\pi 10^7 t - 0.4\pi x)\mathbf{a}_y$ V/m. Calculate the average power density of the travelling wave:
  - (a) P_{av} = 144, mW/m2; (b) P_{av} = 72, mW/m2
  - (c) P_{av} = 36, mW/m2; (d) P_{av} = 10, mW/m2

- 4. The electric field of a travelling wave is given as

$$\mathbf{E}(z,t) = 1\cos(\omega t - \beta z)\mathbf{a}_x + 1\sin(\omega t - \beta z)\mathbf{a}_y$$

  Describe the polarization of the wave.
  - (a) linear, at 45^o to the x -axis
  - (b) some elliptical polarization but not circular
  - (c) circular, right-handed
  - (d) circular, left-handed