Tutorial 10 D'Alembert's solution of wave equation and types of PDEs

1. Using the d'Alembert's solution, solve

$$\frac{\partial^2 y}{\partial t^2} = 9\frac{\partial^2 y}{\partial x^2}, \quad \text{for } 0 < x < 1, t > 0;$$

Where $y(0,t) = y(1,t) = 0$ for $t \geq 0$, and $y(x,0) = \sin(2\pi x)$, $y_t(x,0) = \sin(3\pi x)$.

2. Find the type of the following PDEs.
   (1) $u_{xx} + 4u_{yy} = 0$
   (2) $u_{xx} + 2u_{xy} + u_{yy} = 0$
   (3) $u_{xx} + 5u_{xy} + 4u_{yy} = 0$
   (4) $xu_{xx} - yu_{xy} = 0$
   (5) $u_{xx} - 4u_{xy} + 5u_{yy} = 0$