EEE225 Advanced Electrical Circuits and Electromagnetics

Lecture 12 Transmission Lines

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Room EE322

Content

- What is transmission line (TX)?
  - Parallel-plates and coaxial cable
- Wave propagation in the TX lines
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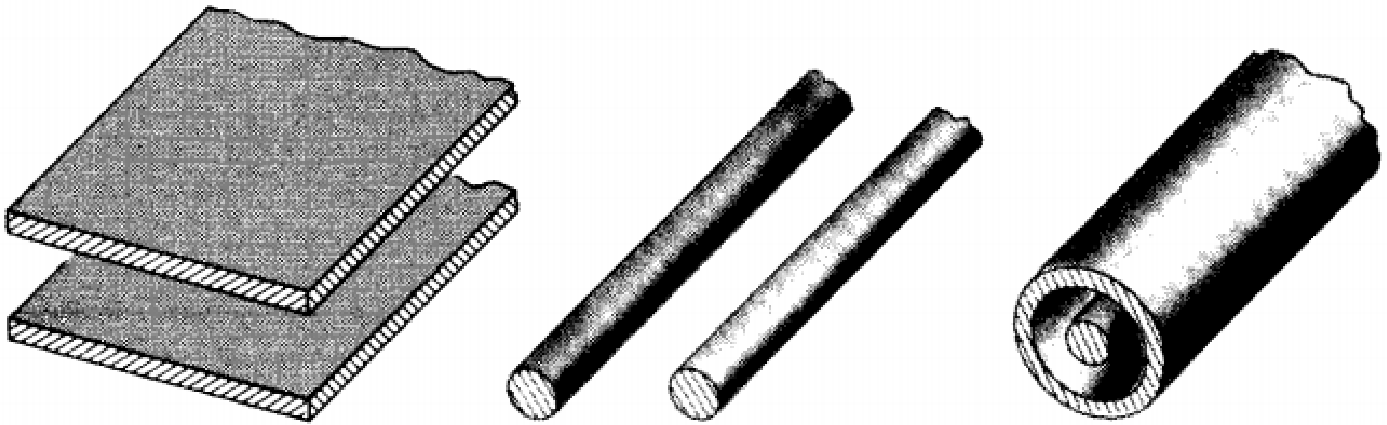
Transmission Line (TX)

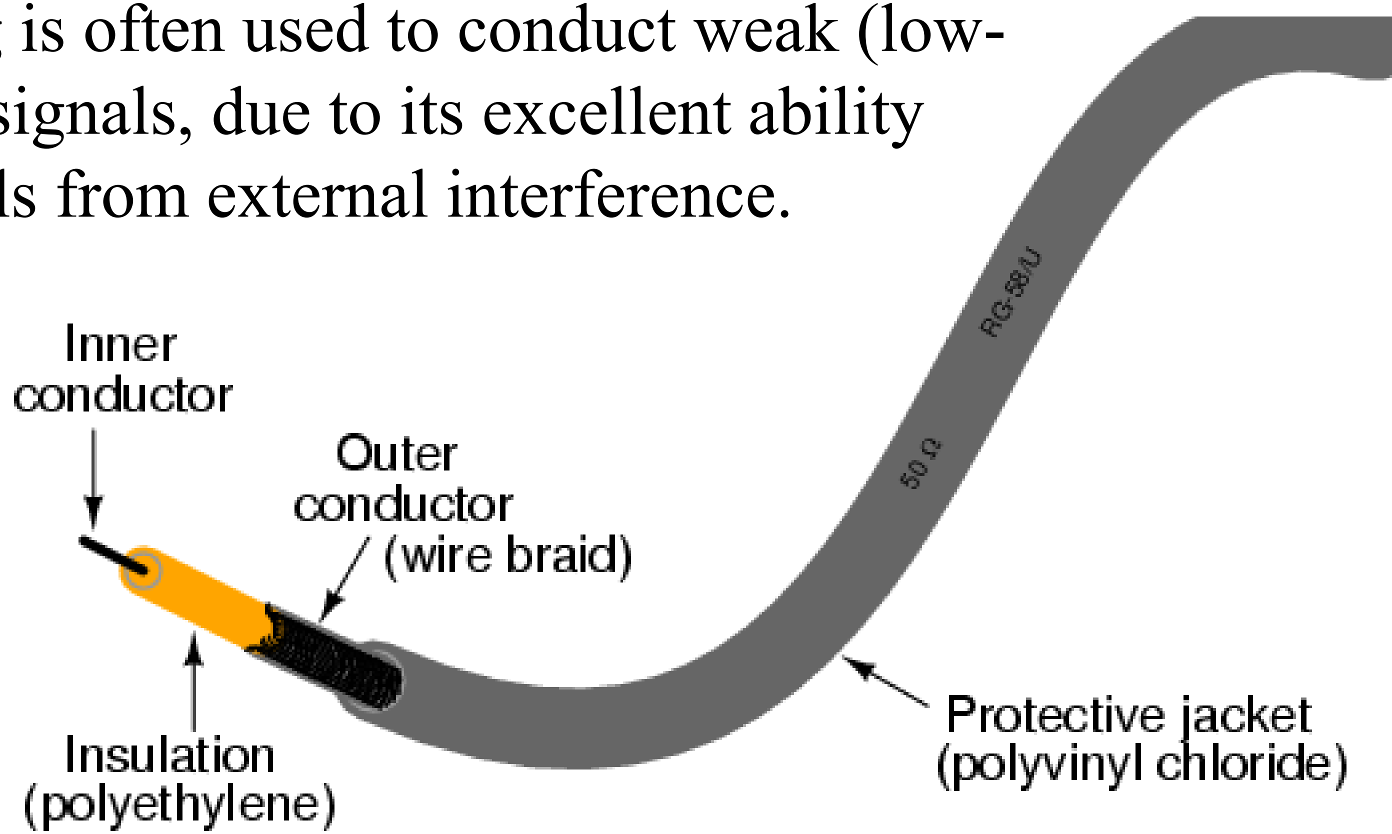
- A transmission line – In the most general sense Transmission Lines (TX) are interconnections that convey energy or information between two points. In this course we confine ourselves to TX used for guiding electromagnetic energy in the form of waves, or information, from one point to another.
  - Transmission lines are used for efficient transmission of power and information from the source to the load.



(a) Parallel-plate transmission line. (b) Two-wire transmission line. (c) Coaxial transmission line.

Example: coaxial Cable

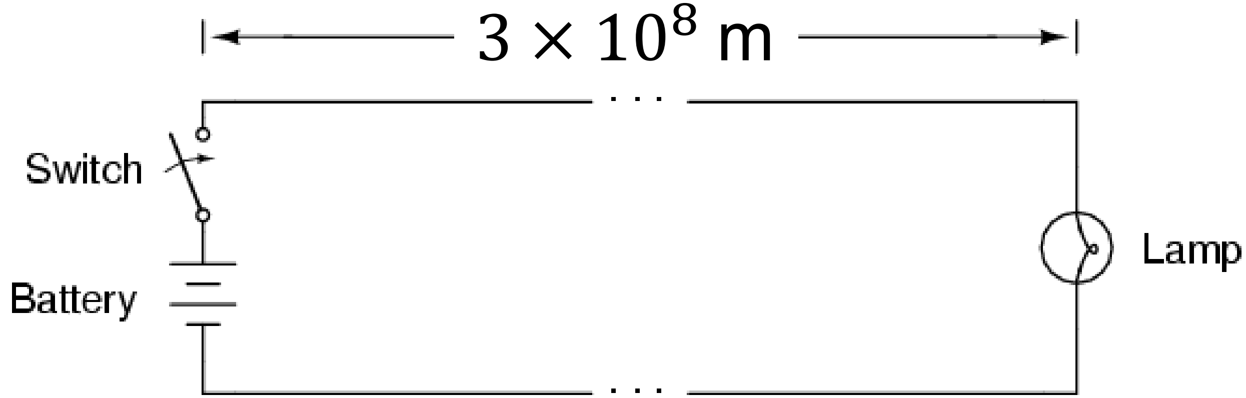
- Coaxial cable – a two-conductor cable made of a single conductor surrounded by a braided wire jacket, with a plastic insulating material separating the two.
  - This type of cabling is often used to conduct weak (low-amplitude) voltage signals, due to its excellent ability to shield such signals from external interference.



Wave propagation in the TX

- Suppose a simple circuit:
  - The overall effect of electrons pushing against each other happens at the speed of light (approximately 3×10^8 m/s).
  - When the switch is closed, the lamp immediately lights.

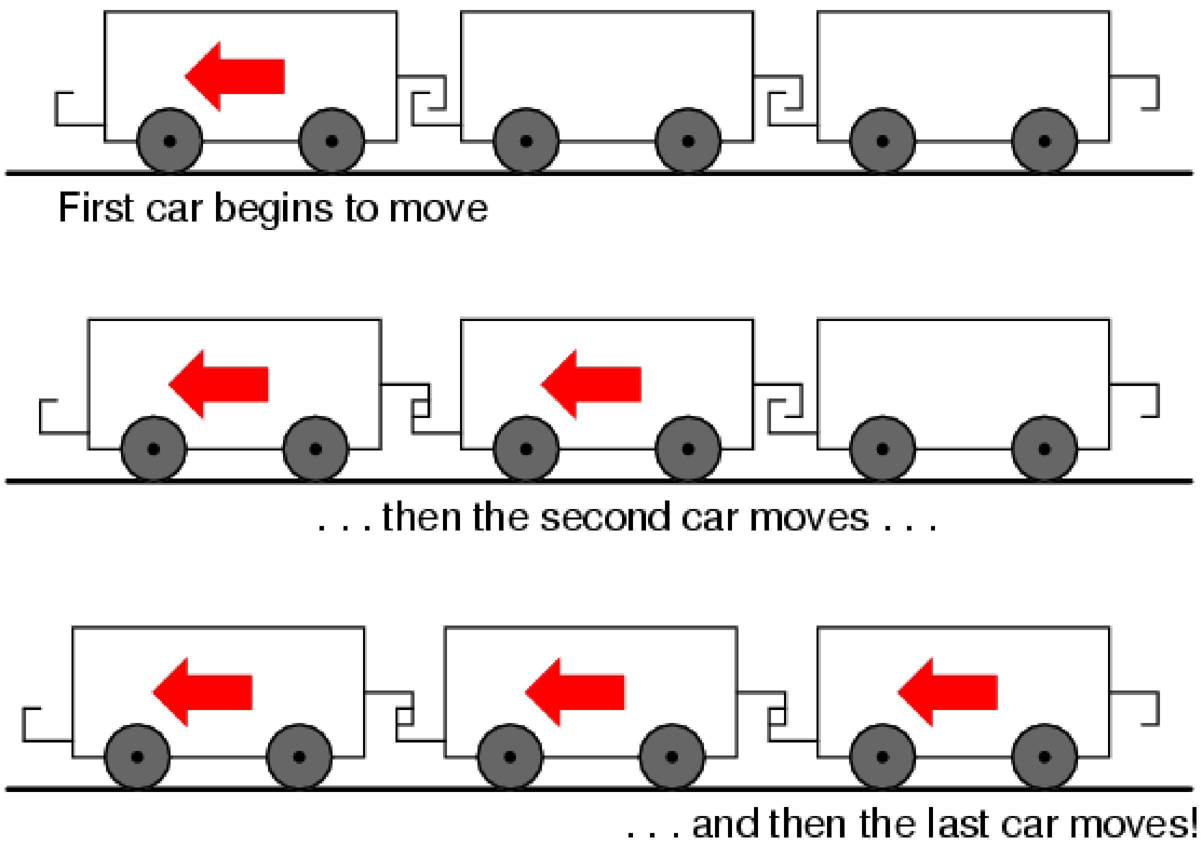
- Suppose a circuit with very long connection wire:
  - It should introduce a time delay into the circuit, delaying the switch's action on the lamp.



At the speed of light, lamp responds after 1 second

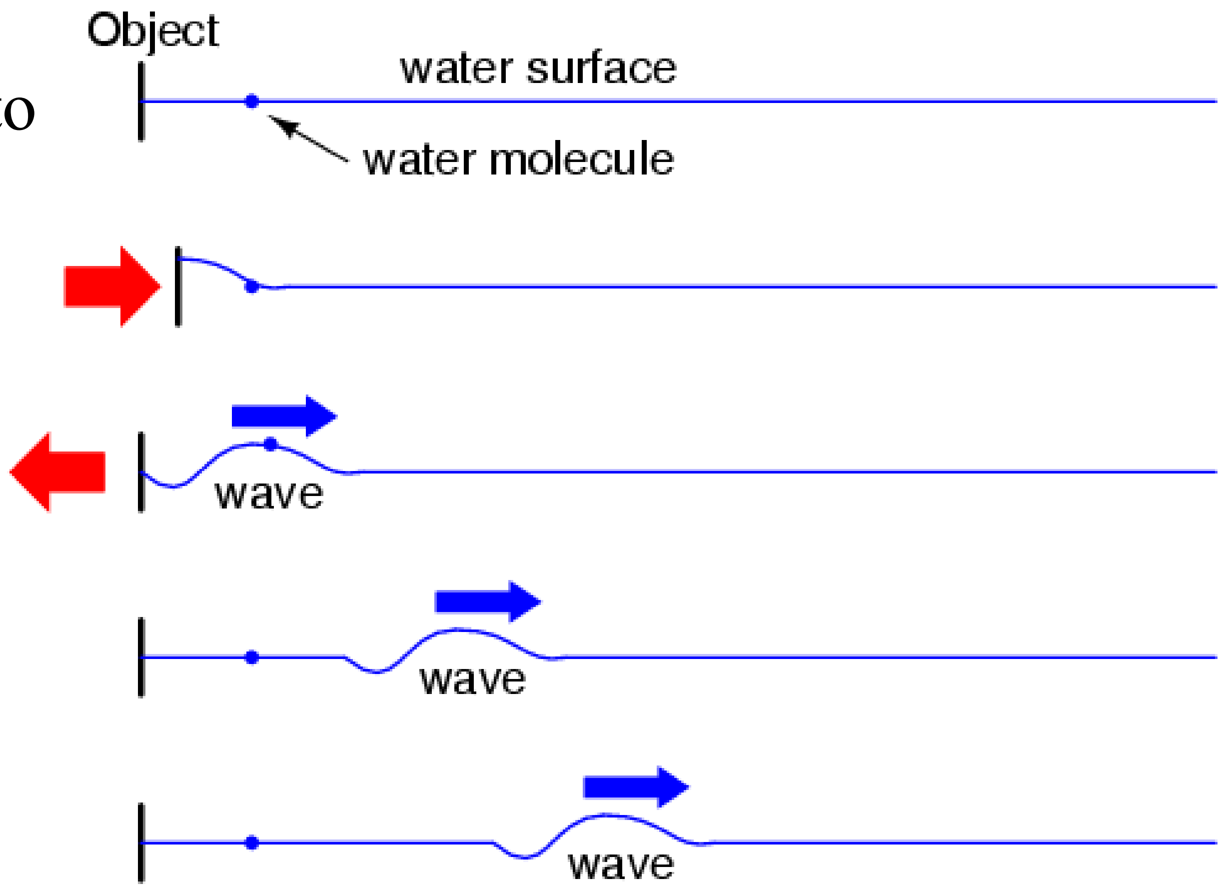
Wave propagation in the TX (cont.)

- Explanation 1: imagine the electrons within a conductor as rail cars in a train: linked together with a small amount of "slack adjuster" in the couplings.
  - Actually, motion is transferred from car to car (from electron to electron) at a maximum velocity limited by the coupling slack.



Wave propagation in the TX (cont.)

- Explanation 2: imagine the waves in water. Suppose a flat, wall-shaped object is suddenly moved horizontally along the surface of water, so as to produce a wave ahead of it.
  - The wave will travel as water molecules bump into each other, transferring wave motion along the water's surface.



TEM wave along a Parallel-plate TX line

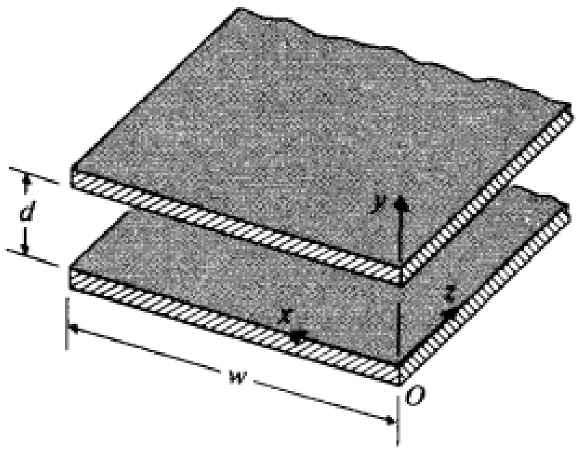


Figure 1: parallel-plate in xyz coordinates

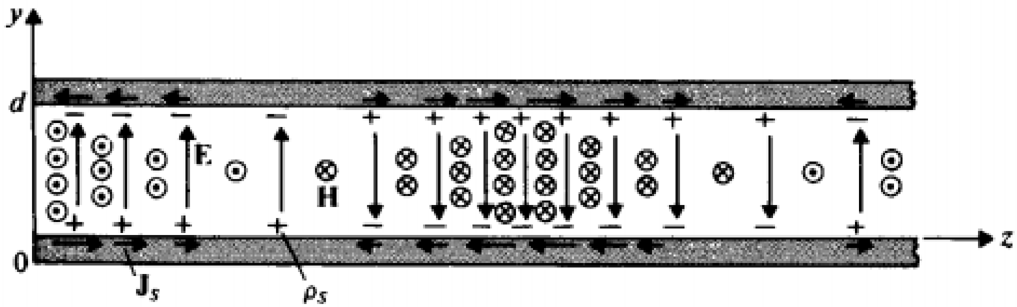


Figure 2: **E**, **H**, ρ_s and $\mathbf{J}_s$ distributions

- TEM wave propagating in the z direction along a ***lossless*** line (phasor forms):

$$\boldsymbol{E} = \boldsymbol{a_y} E_y = \boldsymbol{a_y} E_0 e^{-j\beta z} = \boldsymbol{a_y} E_0 e^{-\gamma z}$$

$$\boldsymbol{H} = \boldsymbol{a_x} H_x = -\boldsymbol{a_x} \frac{E_0}{\eta} e^{-j\beta z} = -\boldsymbol{a_x} \frac{E_0}{\eta} e^{-\gamma z}$$

- The boundary conditions should be satisfied at the interfaces of the dielectric and the plates:
  - At y=0 (lower plate), $\boldsymbol{a_n} = \boldsymbol{a_y}$

$$\boldsymbol{a_y} \cdot \boldsymbol{D} = \rho_{sl} = \varepsilon E_y = \varepsilon E_0 e^{-j\beta z}$$

$$\boldsymbol{a_y} \times \boldsymbol{H} = \boldsymbol{J_{sl}} = -\boldsymbol{a_z} H_x = \boldsymbol{a_z} \frac{E_0}{\eta} e^{-j\beta z}$$

  - At y=d (upper plate), $\boldsymbol{a_n} = -\boldsymbol{a_y}$

$$-\boldsymbol{a_y} \cdot \boldsymbol{D} = \rho_{su} = -\varepsilon E_y = -\varepsilon E_0 e^{-j\beta z}$$

$$-\boldsymbol{a_y} \times \boldsymbol{H} = \boldsymbol{J_{su}} = \boldsymbol{a_z} H_x = -\boldsymbol{a_z} \frac{E_0}{\eta} e^{-j\beta z}$$

Parameters of a Parallel-plate TX line: L & C

- Inductor for unit length
  - Should be L_l, but sometimes write as L for simplicity
  - Derivation:
  - Result:

$$L_l = \frac{\mu d}{w} \quad (H/m)$$

- Capacitor for unit length
  - Should be C_l, but sometimes write as C for simplicity
  - Derivation:
  - Result:

$$C_l = \frac{\varepsilon w}{d} \quad (F/m)$$

TEM wave along a Parallel-plate TX line

- Field phasors **E** and **H** satisfy the two Maxwell's curl equations:

1. $\nabla \times \boldsymbol{E} = -j\omega\mu\boldsymbol{H}$, with $\boldsymbol{E} = \boldsymbol{a_y}E_y$ and $\boldsymbol{H} = \boldsymbol{a_x}H_x$, get:

$\Rightarrow$ $\frac{dE_y}{dz} = j\omega\mu H_x$, taking the integral of y from 0 to d, we have:

$$\frac{d}{dz}\int_0^d E_y dy = j\omega\mu \int_0^d H_x dy$$

$$-\frac{dV(z)}{dz} = j\omega\mu d H_x = j\omega\mu d J_{su}(z) = j\omega\mu d \frac{I(z)}{w} = j\omega L_l I(z)$$

2. $\nabla \times \boldsymbol{H} = j\omega\varepsilon\boldsymbol{E}$, with $\boldsymbol{E} = \boldsymbol{a_y}E_y$ and $\boldsymbol{H} = \boldsymbol{a_x}H_x$, get:

$\Rightarrow$ $\frac{dH_x}{dz} = j\omega\varepsilon E_y$, taking the integral of x from 0 to w, we have:

$$\frac{d}{dz}\int_0^w H_x dx = j\omega\varepsilon \int_0^w E_y dx$$

$$-\frac{dI(z)}{dz} = -j\omega\varepsilon w E_y(z) = j\omega \frac{\varepsilon w}{d}\left[-E_y(z) d\right] = j\omega C_l V(z)$$

Telegraphist's equations and the solutions

- To get the equations of one variable only, take derivative again:

$$\boxed{\begin{aligned}\frac{dV(z)}{dz} &= -j\omega L_l I(z)\\ \frac{dI(z)}{dz} &= -j\omega C_l V(z)\end{aligned}} \longrightarrow \boxed{\begin{aligned}\frac{d^2V(z)}{dz^2} &= -\omega^2 L_l C_l V(z)\\ \frac{d^2I(z)}{dz^2} &= -\omega^2 L_l C_l I(z)\end{aligned}}$$

Telegraphist's equations

General wave equations

- Define the propagation constant $\gamma = \sqrt{-\omega^2 L_l C_l} = j\beta$, where β is the phase constant $\beta = \omega\sqrt{\mu\varepsilon}$, solve the telegraphist's equations get:

$$V(z) = \boxed{V_0^+ e^{-j\beta z}} + \boxed{V_0^- e^{j\beta z}}$$
$$I(z) = \boxed{I_0^+ e^{-j\beta z}} + \boxed{I_0^- e^{j\beta z}}$$

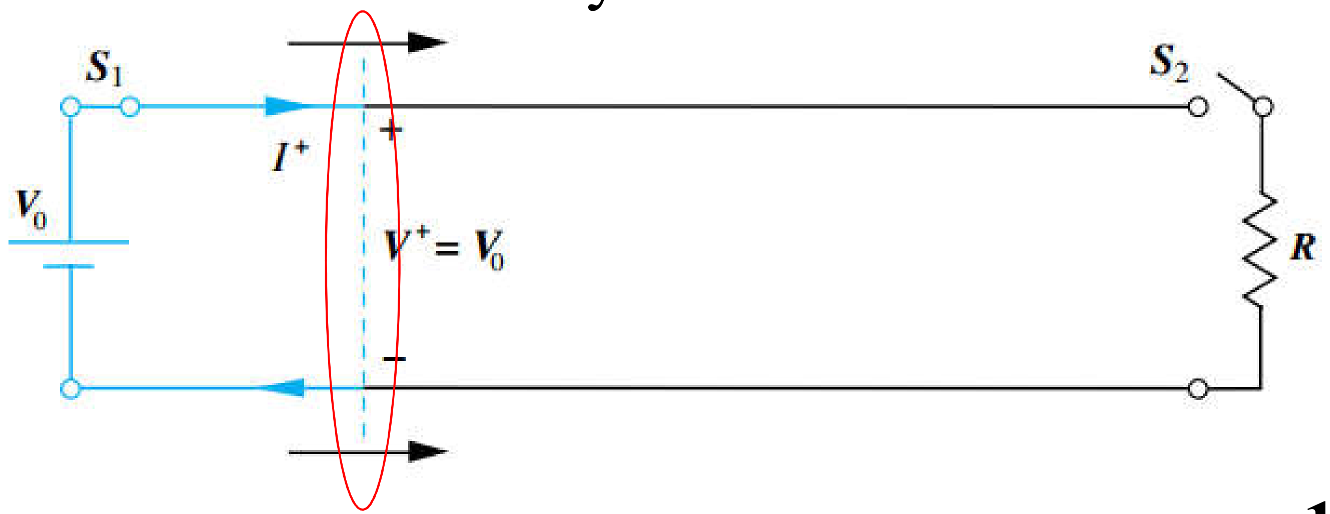
+z direction Forward travelling

-z direction Backward travelling

Circuit model of the TX

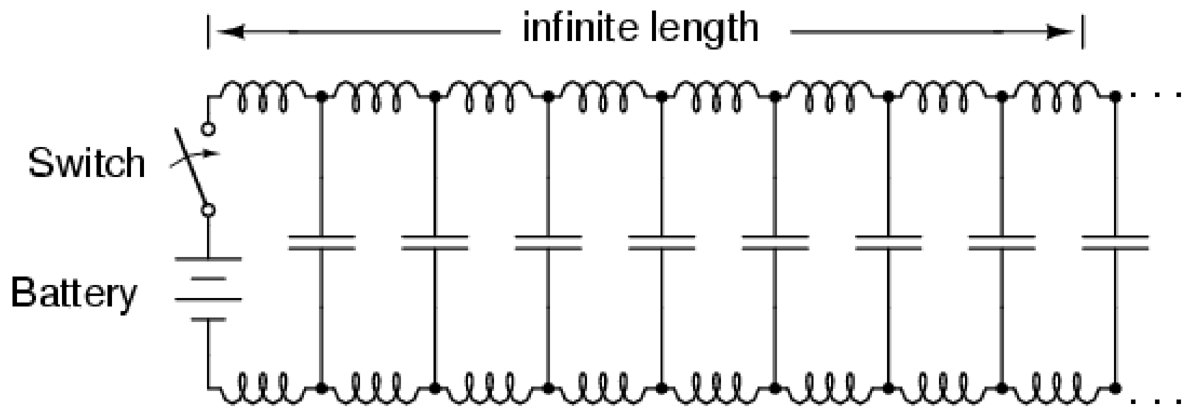
- Consider a lossless line.
  - By lossless, we mean that all power that is launched into the line at the input end eventually arrives at the output end.
  - A battery having voltage V_0 is connected to the input by closing switch S_1 at time $t = 0$. When the switch is closed, the effect is to launch voltage, $V^+ = V_0$.
  - This voltage does not instantaneously appear everywhere on the line, but rather begins to travel from the battery toward the load at a certain velocity.

"Wavefront" - the instantaneous boundary between the charged section and uncharged section.

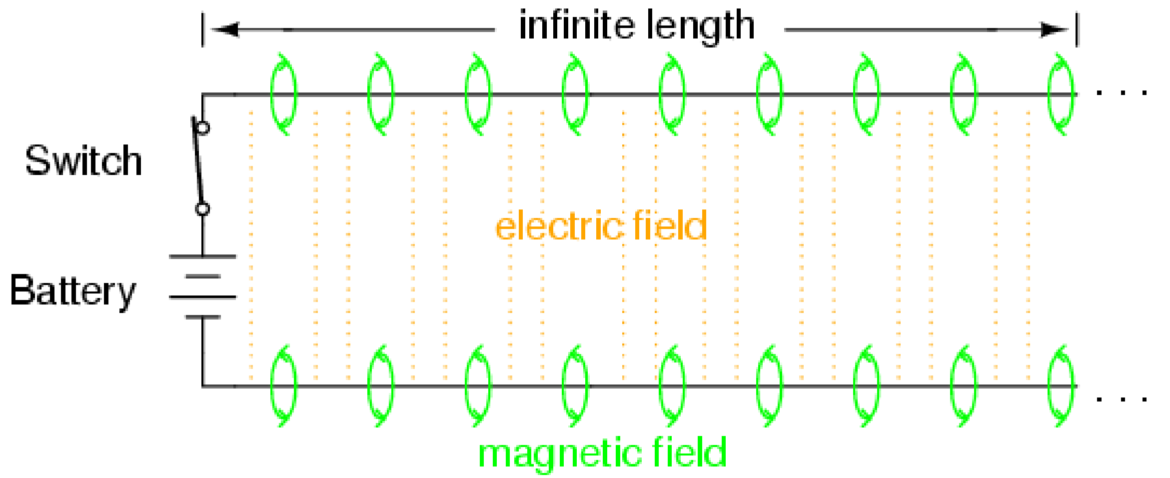


Circuit model of the TX

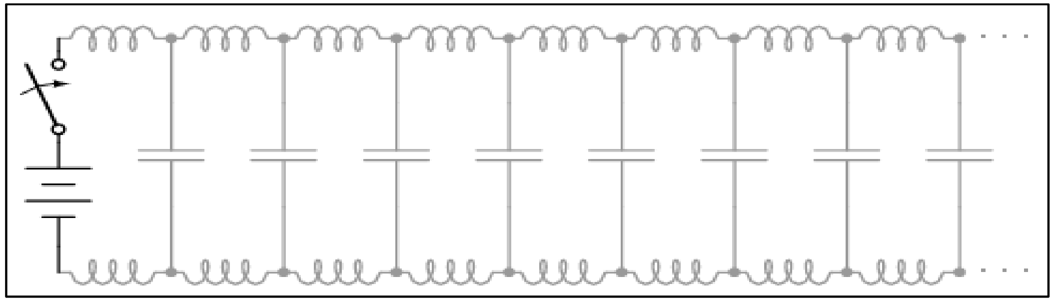
- Equivalent circuit showing lumped capacitance and inductance



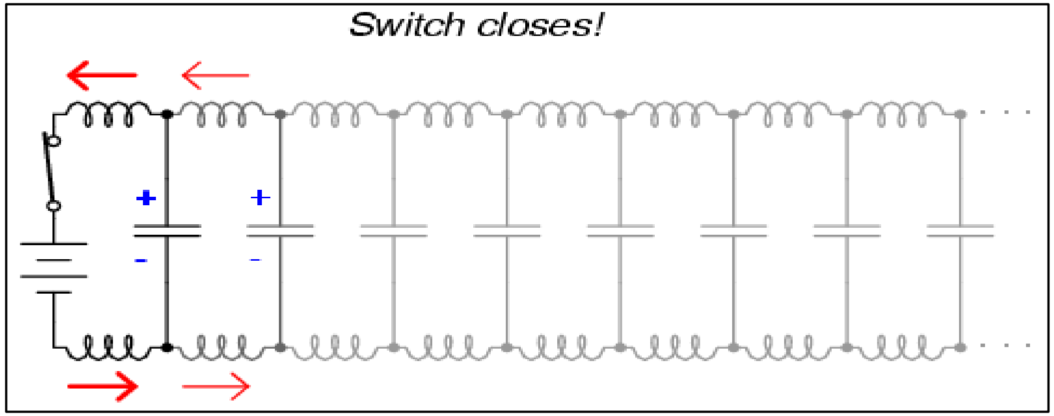
- Voltage charges capacitance; current charges inductance.



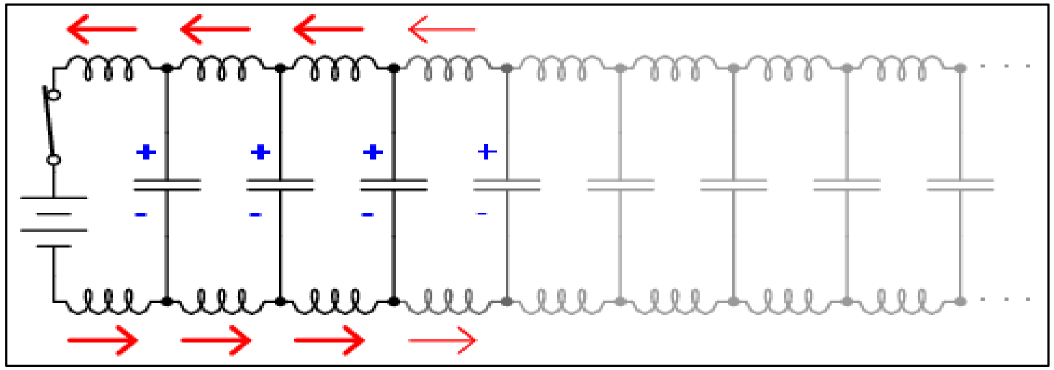
1. Uncharged transmission line.



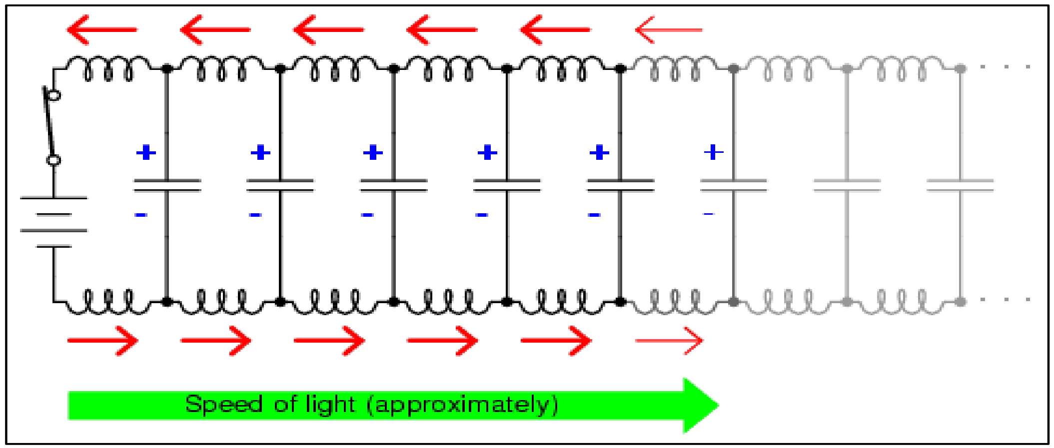
2. Begin wave propagation.



3. Continue wave propagation.

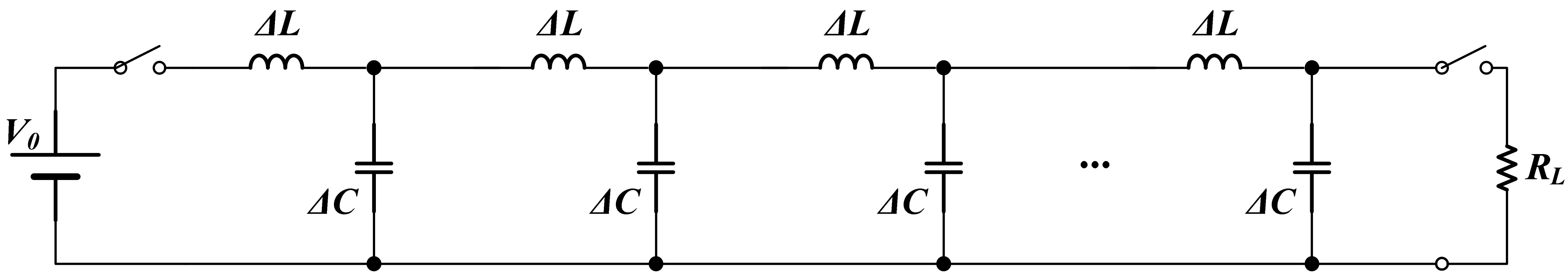


4. Propagate at speed of light.



Circuit model of the TX – Lossless TX

- TX structures (such as parallel plate, two-wire, coaxial cable) possess capacitance and inductance that are expressed on a per-unit-length basis.
- A model for TX using lumped capacitors and inductors can be constructed.
  - For a lossless TX, just L and C are needed to represent the it.



Lumped-element model of a transmission line.

Lossy Parallel-plate TX line

- Lossless parallel-plate:
  - Assuming the conductor is perfect ($\sigma_c = \infty$) and the dielectric is ideal ($\sigma_d = 0$), so the resistivity of the metals ($R = 0$) and the leakage between plates ($G = 0$)
- Lossy parallel-plate ($\sigma_c \neq \infty$ and $\sigma_c \neq \infty$):
  - The R and G can be solved $\begin{cases} G_l = \dfrac{\sigma_d w}{d} \\ R_l = \dfrac{2}{w}\sqrt{\dfrac{\pi f \mu}{\sigma_c}} \end{cases}$

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Parameters for TX – Example: Coaxial line

- Coaxial transmission line.
  - Geometry: center conductor of radius *a* and outer conductor of radius *b*; the dielectric material between them has the relative permittivity of ε_r and conductivity σ_r.
  - Inductance: $L = \frac{\mu}{2\pi} \ln \frac{b}{a}$ (H/m).
  - Capacitance: $C = \frac{2\pi\epsilon}{\ln(b/a)}$ (F/m),
  - Conductance: $G = \frac{2\pi\sigma}{\ln(b/a)}$ (S/m),
  - To determine the resistance, assume $\boldsymbol{J}_{si}$ is the surface current on the center conductor and $\boldsymbol{J}_{so}$ is the surface current on the outer conductor.
    - There must have: $I = 2\pi a J_{si} = 2\pi b J_{so}$.
    - The power dissipated in a unit length of the center and outer conductors are:

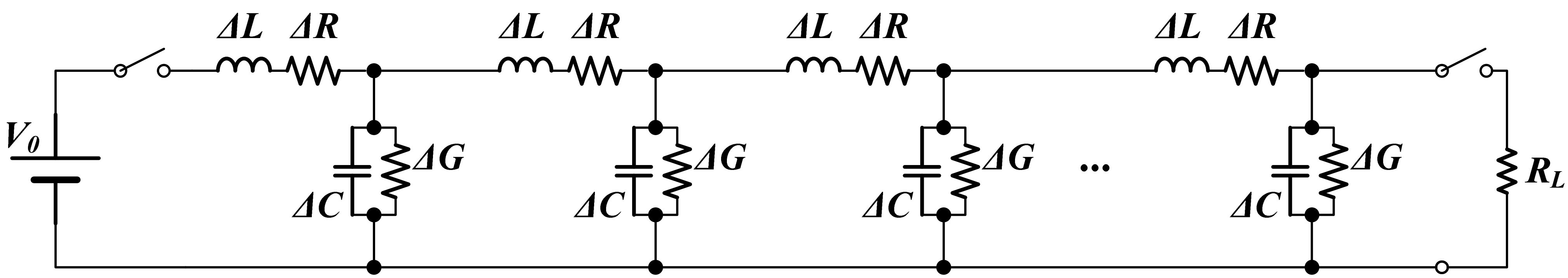
$$\left.\begin{aligned} P_{\sigma i} &= 2\pi a p_{\sigma i} = \frac{1}{2} I^2 \left(\frac{R_s}{2\pi a}\right), \\ P_{\sigma o} &= 2\pi b p_{\sigma o} = \frac{1}{2} I^2 \left(\frac{R_s}{2\pi b}\right). \end{aligned}\right\} R = \frac{R_s}{2\pi}\left(\frac{1}{a} + \frac{1}{b}\right) = \frac{1}{2\pi}\sqrt{\frac{\pi f \mu_c}{\sigma_c}}\left(\frac{1}{a} + \frac{1}{b}\right) \quad (\Omega/\text{m}).$$

Parallel-plate VS Coaxial cable

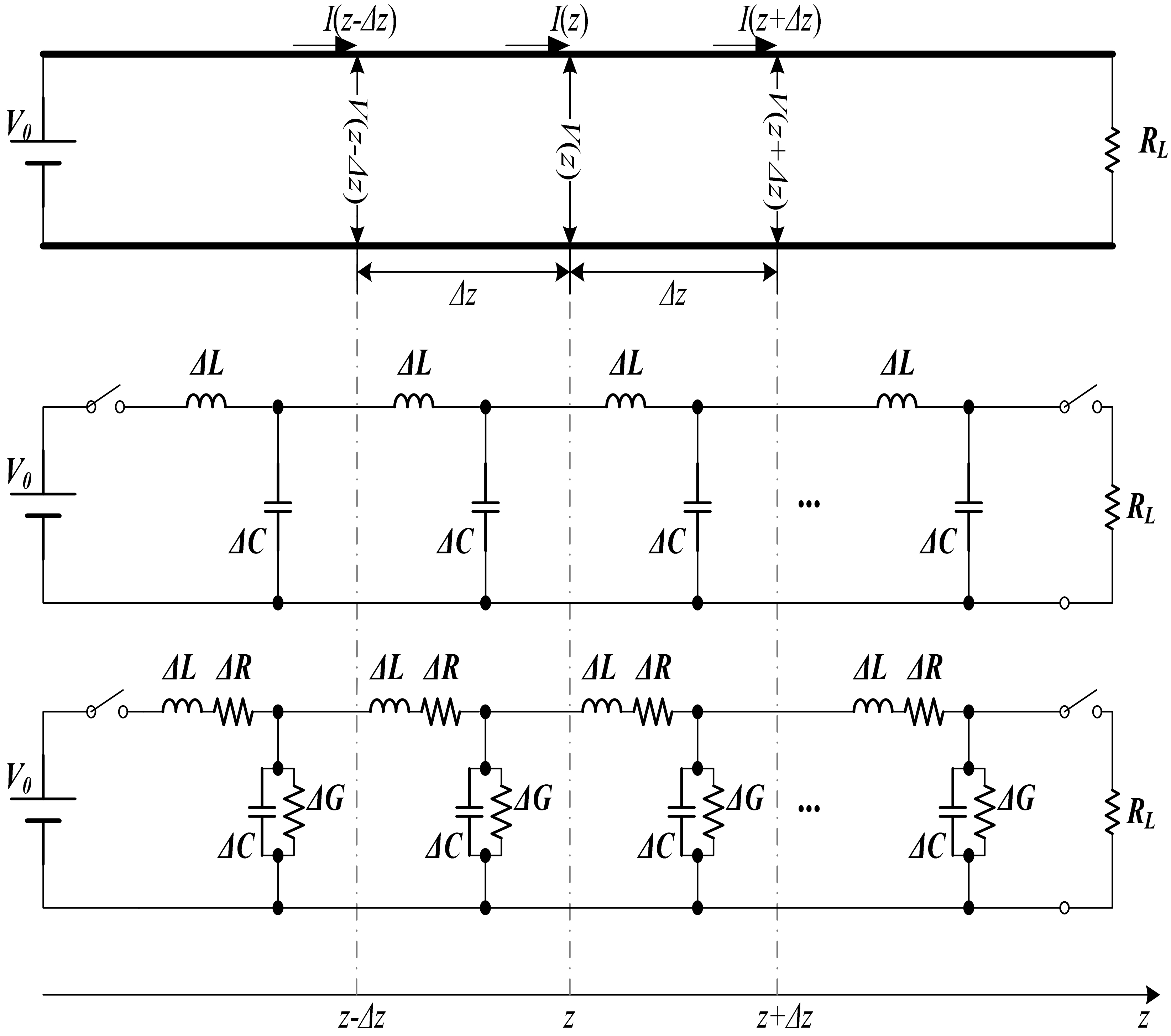
	Parallel-plate	Coaxial cable
L_l	$\frac{\mu d}{w}$	$\frac{\mu}{2\pi} ln\left(\frac{b}{a}\right)$
C_l	$\frac{\varepsilon w}{d}$	$\frac{2\pi\varepsilon}{ln\left(\frac{b}{a}\right)}$
G_l	$\frac{\sigma_d w}{d}$	$\frac{2\pi\sigma_d}{ln\left(\frac{b}{a}\right)}$
R_l	$\frac{2}{w}\sqrt{\frac{\pi f\mu}{\sigma_c}}$	$\frac{1}{2}\sqrt{\frac{f\mu}{\pi\sigma_c}}\left(\frac{1}{a}+\frac{1}{b}\right)$

Circuit model of the TX – General TX

- TX structures possess not just capacitance and inductance, but also resistance and admittance, that are all expressed on a per-unit-length basis.
- A model for TX using lumped elements can be constructed.
  - For a general (lossy) TX, all four elements are needed to represent it.



Lumped-element model of a transmission line.



Transmission line (two conductors)

Lossless LC model

Lossy RGLC model

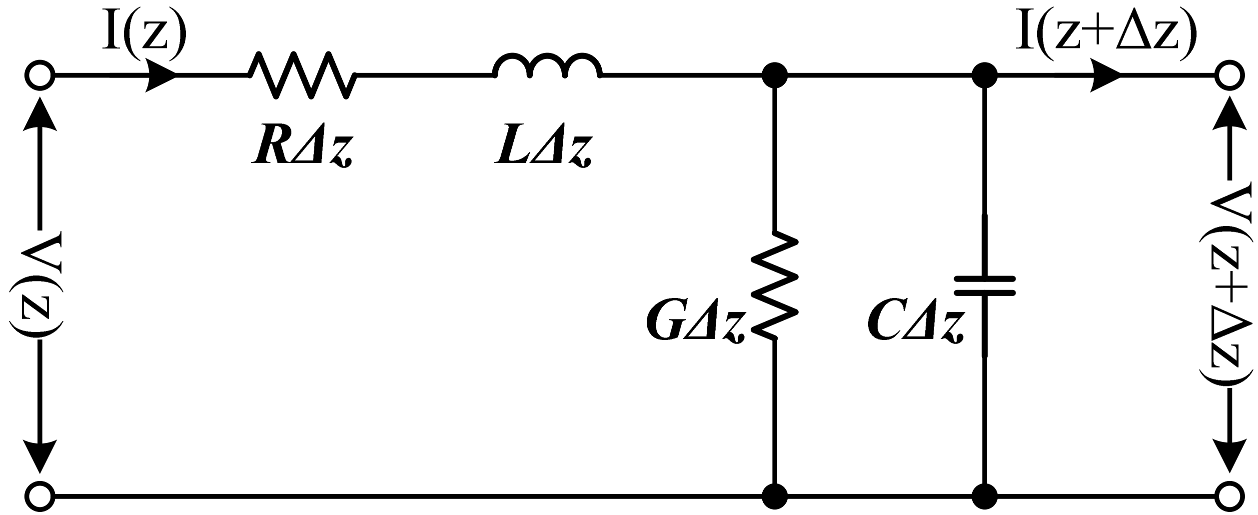
Circuit model of the TX – Telegraphist's equations

- The circuit model contains the primary constants of the transmission line: L, R, C and G.
  - all of which have values that are specified *per unit length*.
  - The dielectric (imperfect) may possess conductivity, σ_d, in addition to a dielectric constant, ε_r. => G and C.
  - The conductor (imperfect) may possess conductivity, σ_c => R and L.

- R and G are responsible for power loss in transmission.
  - i.e., Lossless TX has R = 0 and G = 0.
  - R and G are functions of frequency => in high frequency or low frequency, they have simplified expressions.

Circuit model of the TX – Telegraphist's equations

- Consider a line section of length Δz containing resistance $R\Delta z$, inductance $L\Delta z$, conductance $G\Delta z$, and capacitance $C\Delta z$.
  - Because the section of the line looks the same from either end, divide the series elements in half to produce a symmetrical network.



  - KVL: $V(z) - R\Delta z I(z) - L\Delta z \dfrac{\partial I(z)}{\partial t} - V(z+\Delta z) = 0$
  - KCL: $I(z) - G\Delta z V(z+\Delta z) - C\Delta z \dfrac{\partial V(z+\Delta z)}{\partial t} - I(z+\Delta z) = 0$

Circuit model of the TX – Telegraphist's equations

$$KVL:\quad V(z) - R\Delta z I(z) - L\Delta z\frac{\partial I(z)}{\partial t} - V(z+\Delta z) = 0$$

$$\Longrightarrow\quad RI(z) + L\frac{\partial I(z)}{\partial t} = -\frac{V(z+\Delta z) - V(z)}{\Delta z} = -\frac{\partial V(z)}{\partial z} \qquad (1)$$

$$KCL:\quad I(z) - G\Delta z V(z+\Delta z) - C\Delta z\frac{\partial V(z+\Delta z)}{\partial t} - I(z+\Delta z) = 0$$

$$\Longrightarrow\quad GV(z) + C\frac{\partial V(z)}{\partial t} = -\frac{I(z+\Delta z) - I(z)}{\Delta z} = -\frac{\partial I(z)}{\partial z} \qquad (2)$$

- Equations (1) and (2) describe the evolution of current and voltage in any transmission line. They are called the ***telegraphist's equations***.
- General wave equations for TX:

Substitute (2) to (1) get: $$\frac{\partial^2 V}{\partial z^2} = LC\frac{\partial^2 V}{\partial t^2} + (LG+RC)\frac{\partial V}{\partial t} + RGV \qquad (3)$$

Substitute (1) to (2) get: $$\frac{\partial^2 I}{\partial z^2} = LC\frac{\partial^2 I}{\partial t^2} + (LG+RC)\frac{\partial I}{\partial t} + RGI \qquad (4)$$

Circuit model of the TX – Propagation constant

- Equations (1) and (2) can be expressed for time harmonic fields, so we get ***Time harmonic TX equations***.

$$-\frac{dV}{dz} = (R + j\omega L)I(z) \qquad (1')$$

$$-\frac{dI}{dz} = (G + j\omega C)V(z) \qquad (2')$$

- Combine them to solve for V(z) and I(z), get

$$\frac{d^2V(z)}{dz^2} = \gamma^2 V(z) \quad \text{or} \quad \frac{d^2I(z)}{dz^2} = \gamma^2 I(z) \qquad (5)$$

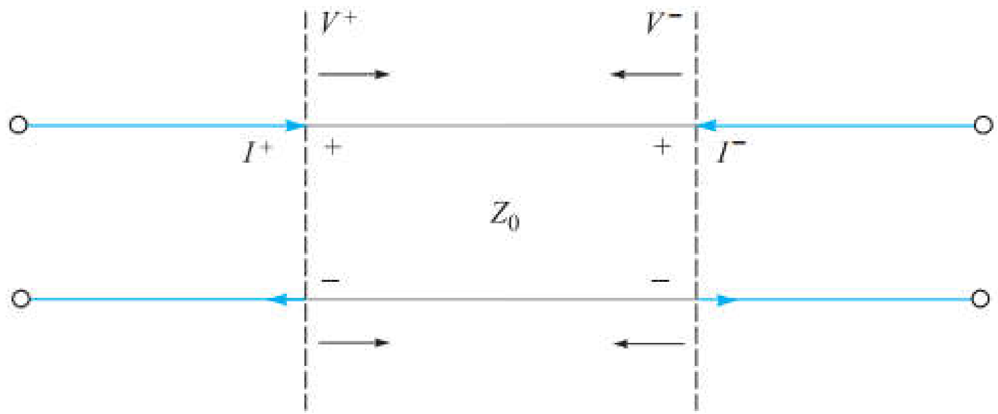
  – where $\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$ is the ***propagation constant***.

    - α – attenuation constant (Np/m)
    - β – phase constant (rad/m)

Circuit model of the TX – Characteristic impedance

- The solutions to the wave equation (5) are

$$V(z) = V^+(z) + V^-(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$
$$I(z) = I^+(z) + I^-(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$$



  - where the "+" and "-" superscripts denotes waves travelling in the +z and –z directions.
  - Wave amplitudes have: $\dfrac{V_0^+}{I_0^+} = -\dfrac{V_0^-}{I_0^-} = \dfrac{R + j\omega L}{\gamma}.$
  - For an infinite line, the term containing $e^{\gamma z}$ vanishes, there are no reflected wave. So: $V(z) = V^+(z) = V_0^+ e^{-\gamma z},$ $I(z) = I^+(z) = I_0^+ e^{-\gamma z}.$
  - The ratio of the voltage and the current for an infinitely long line is independent of z, is called the characteristic impedance of the line.

$$Z_0 = \frac{R + j\omega L}{\gamma} = \frac{\gamma}{G + j\omega C} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

Circuit model of the TX – Lossless line

- Lossless line (R = G = 0)
  - A) Propagation constant

$$\gamma = \alpha + j\beta = j\omega\sqrt{LC};$$
$$\alpha = 0,$$
$$\beta = \omega\sqrt{LC} \qquad \text{(a linear function of } \omega\text{).}$$

  - B) Phase velocity

$$u_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} \qquad \text{(constant).}$$

  - C) Characteristic impedance

$$Z_0 = R_0 + jX_0 = \sqrt{\frac{L}{C}};$$
$$R_0 = \sqrt{\frac{L}{C}} \qquad \text{(constant),}$$
$$X_0 = 0.$$

Circuit model of the TX – Low loss line

- Low loss line ($R << \omega L$, $G << \omega C$)
  - The low loss conditions are more easily satisfied at high frequencies.
  - A) Propagation constant

$$\gamma = \alpha + j\beta = j\omega\sqrt{LC}\left(1+\frac{R}{j\omega L}\right)^{1/2}\left(1+\frac{G}{j\omega C}\right)^{1/2}$$

$$\cong j\omega\sqrt{LC}\left(1+\frac{R}{2j\omega L}\right)\left(1+\frac{G}{2j\omega C}\right)$$

$$\cong j\omega\sqrt{LC}\left[1+\frac{1}{2j\omega}\left(\frac{R}{L}+\frac{G}{C}\right)\right];$$

$$\alpha \cong \frac{1}{2}\left(R\sqrt{\frac{C}{L}}+G\sqrt{\frac{L}{C}}\right)\Big]$$

$$\beta \cong \omega\sqrt{LC}$$

  - B) Phase velocity

$$u_p = \frac{\omega}{\beta} \cong \frac{1}{\sqrt{LC}} \qquad \text{(approximately constant).}$$

  - C) Characteristic impedance

$$Z_0 = R_0 + jX_0 = \sqrt{\frac{L}{C}}\left(1+\frac{R}{j\omega L}\right)^{1/2}\left(1+\frac{G}{j\omega C}\right)^{-1/2}$$

$$\cong \sqrt{\frac{L}{C}}\left[1+\frac{1}{2j\omega}\left(\frac{R}{L}-\frac{G}{C}\right)\right];$$

$$R_0 \cong \sqrt{\frac{L}{C}},$$

$$X_0 \cong -\sqrt{\frac{L}{C}}\,\frac{1}{2\omega}\left(\frac{R}{L}-\frac{G}{C}\right) \cong 0.$$

Circuit model of the TX – Velocity

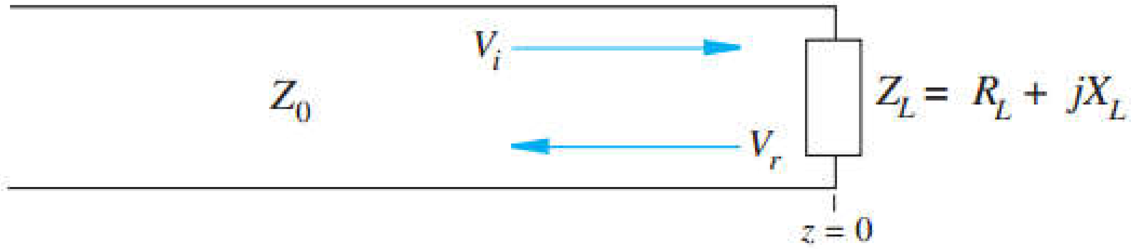
- A lossless line
  - The velocity of wave propagation on a lossless line (guided wave) is equal to the velocity of unguided plane wave in the dielectric of the line.

$$u_p = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu\varepsilon}}$$

- A lossy line
  - On a lossy line, the relationship $LC = \mu\varepsilon$ holds, but the velocity of wave propagation is no longer $1/\sqrt{LC}$.
  - On a low loss line, it can approximately equal to $1/\sqrt{LC}$.

Wave reflection – Reflection coefficient

- A transmission line of characteristic impedance Z_0 is terminated by a load having impedance of $Z_L = R_L + jX_L$.



  – The voltage incident on the load is V_i, and the voltage reflected is V_r, at $z = 0$: $V_L = V_{0i} + V_{0r}$
  – Additionally, the current through the load is the sum of the incident and reflected currents:

$$I_L = I_{0i} + I_{0r} = \frac{1}{Z_0}[V_{0i} - V_{0r}] = \frac{V_L}{Z_L} = \frac{1}{Z_L}[V_{0i} + V_{0r}]$$

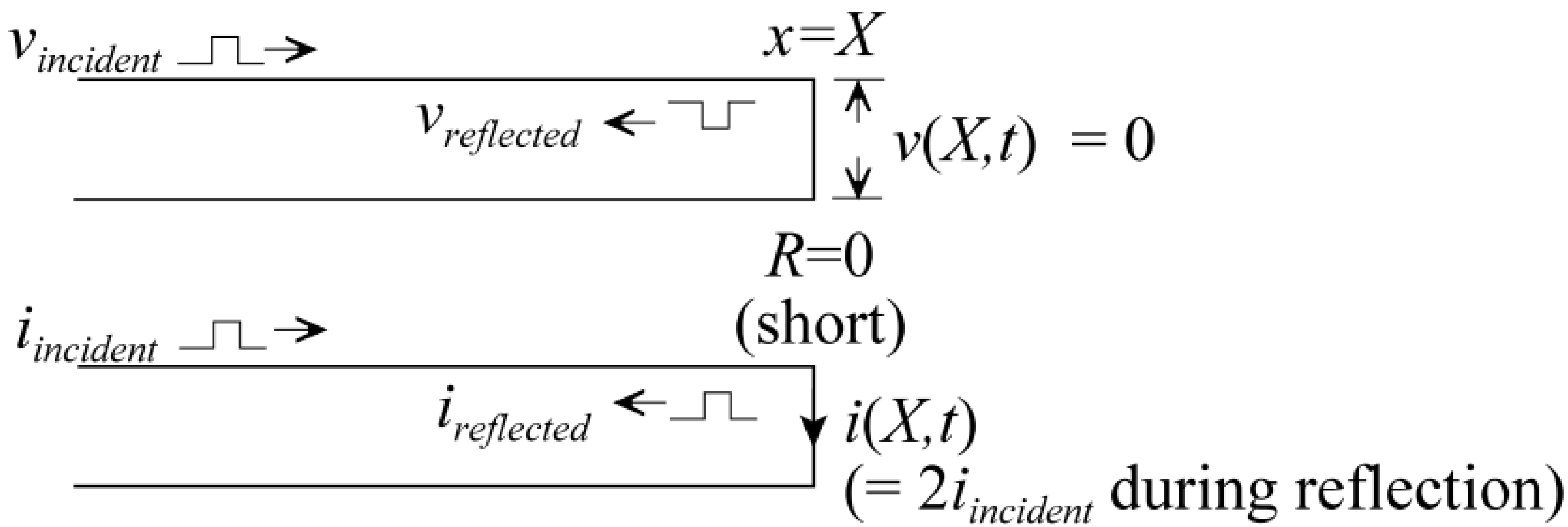
- Defined the ***reflection coefficient***, ***Γ***, as

$$\Gamma \equiv \frac{V_{0r}}{V_{0i}} = \frac{Z_L - Z_0}{Z_L + Z_0} = |\Gamma|e^{j\phi_r}$$

Notice the complex nature of Γ—meaning that, a reflected wave will experience a reduction in amplitude and a phase shift, relative to the incident wave.

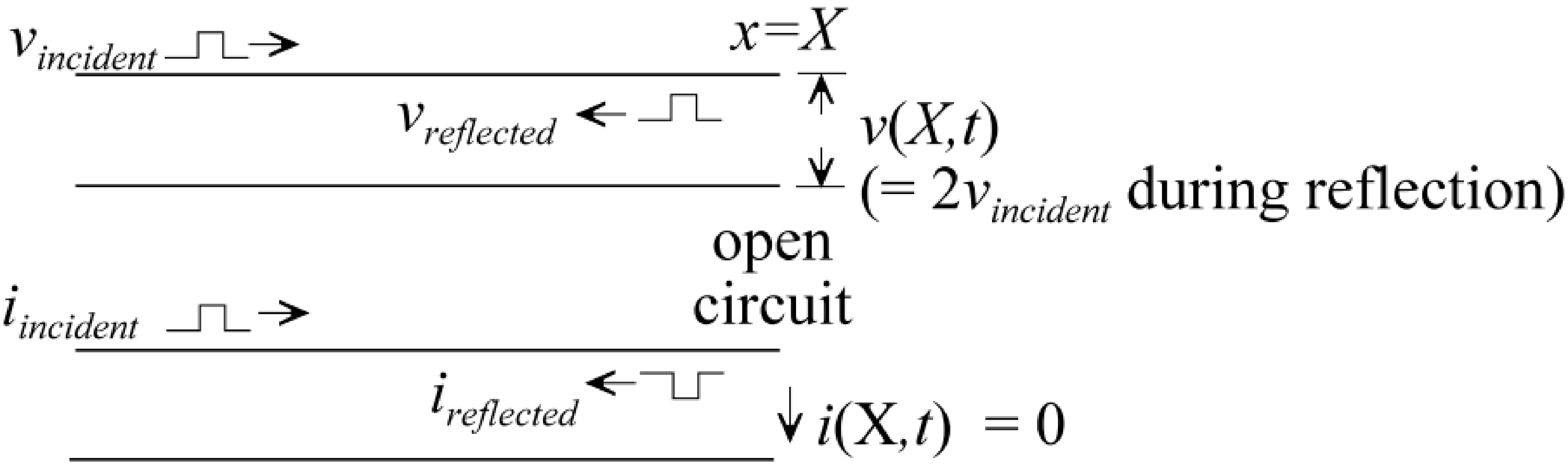
Reflections From the End of a TX (1)

- TX terminated with a short circuit
  - Reflected voltage wave: equal in amplitude and opposite in sign to the incident voltage wave;
  - For the reflected wave: the voltage and the current are 180° out of phase



Reflections From the End of a TX (2)

- TX terminated with an open circuit
  - Reflected current wave: equal in amplitude and opposite in sign to the incident current wave;
  - For the reflected wave: the voltage and the current are 180° out of phase



Wave reflection – Impedance matching

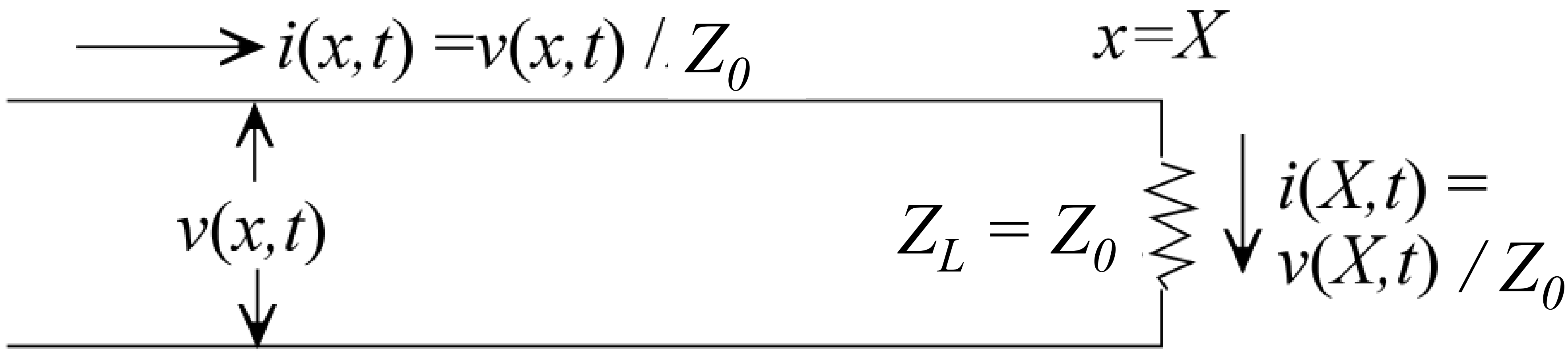
- Write $V_L = V_{0i} + \Gamma V_{0i}$
- Define the ***transmission coefficient***, $\boldsymbol{\tau}$, as:

$$\tau \equiv \frac{V_L}{V_{0i}} = 1 + \Gamma = \frac{2Z_L}{Z_0 + Z_L} = |\tau| e^{j\phi_t}$$

- Matching
  - The main objective in transmitting power to a load is to configure the line/load combination such that there is no reflection. The load therefore receives all the transmitted power.
  - The condition for this is $\Gamma = 0$, which means that the load impedance must be equal to the line impedance: $Z_0 = Z_L$.
  - In such cases the load is said to be matched to the line

Reflections From the End of a TX (3)

- TX terminated with a matched load
  - At any point along the transmission line, the current *i* and the voltage *v* are related to each other exactly as if the current were flowing through a load Z_0 connecting the two conductors.
    At the point where the loading is connected, the ratio of voltage to current is equal to $Z_L = Z_0$, just as it would be if the line went on forever, so there is no reflected signal from such a termination.



Wave reflection – Varied Z_L

- When $0 \le Z_L \le \infty$, Γ ranges from -1 to +1 in value.
  - $Z_L = 0$, terminate the TX by a short circuit:

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = -1$$

  - $Z_L = \infty$, terminate the TX by an open circuit:

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = 1$$

  - $Z_L = Z_0$, matching load:

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = 0$$

Wave reflection – *Varied Z_L*

- The table lists all 5 possible cases for the reflection of a pulse of amplitude V_0

Termination Resistance	Reflected Voltage Pulse Amplitude	Reflected Voltage Pulse Sign
$R = 0$ (short circuit)	V_0	inverted
$R < Z_{char}$	$< V_0$	inverted
$R = Z_{char}$	0	—
$R > Z_{char}$	$< V_0$	uninverted
$R = \infty$ (open circuit)	V_0	uninverted

Wave reflection – Reflected power fraction

- The reflected power fraction at the load is

$$\frac{\langle \mathcal{P}_r \rangle}{\langle \mathcal{P}_i \rangle} = \Gamma\Gamma^* = |\Gamma|^2$$

- The fraction of the incident power that is transmitted into the load is

$$\frac{\langle \mathcal{P}_t \rangle}{\langle \mathcal{P}_i \rangle} = 1 - |\Gamma|^2$$

  – Important: the reader should be aware that the transmitted power fraction is not $|\tau|^2$.

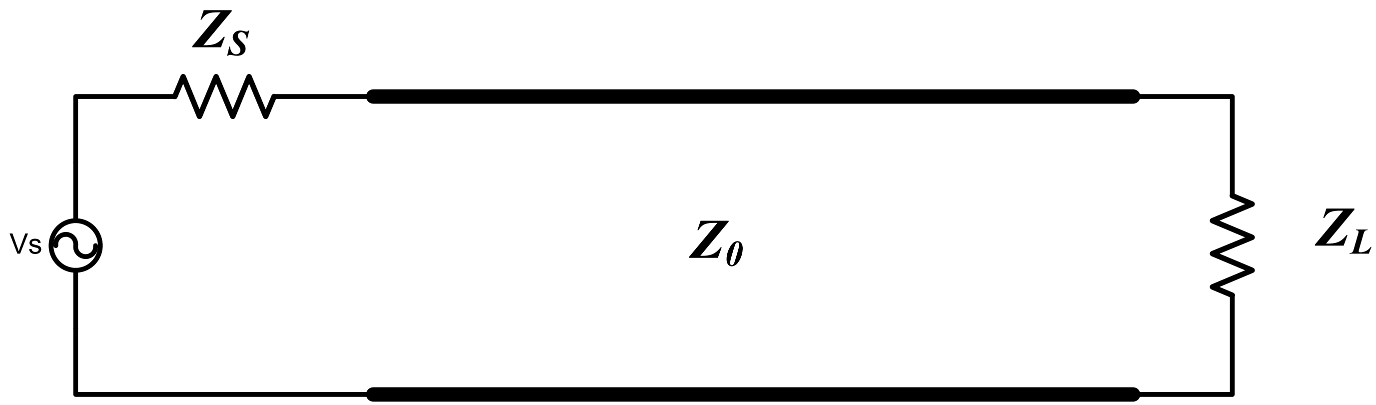
- Example: A 50 Ω lossless transmission line is terminated by a load impedance, $Z_L = 50 - j75\ \Omega$.If the incident power is 100 mW, find the power dissipated by the load.

Quiz

- 1. For a lossless line (R,G=0) the characteristic impedance depends on
  - (a) The ratio L/C
  - (b) The angular frequency
  - (c) The ratio L/C and the angular frequency
  - (d) None of these
- 2. For a coaxial line, the conductivity of the dielectric between the two conductors leads to the following parameter being non-zero
  - (a) R; (b) G;
  - (c) L; (d) C.

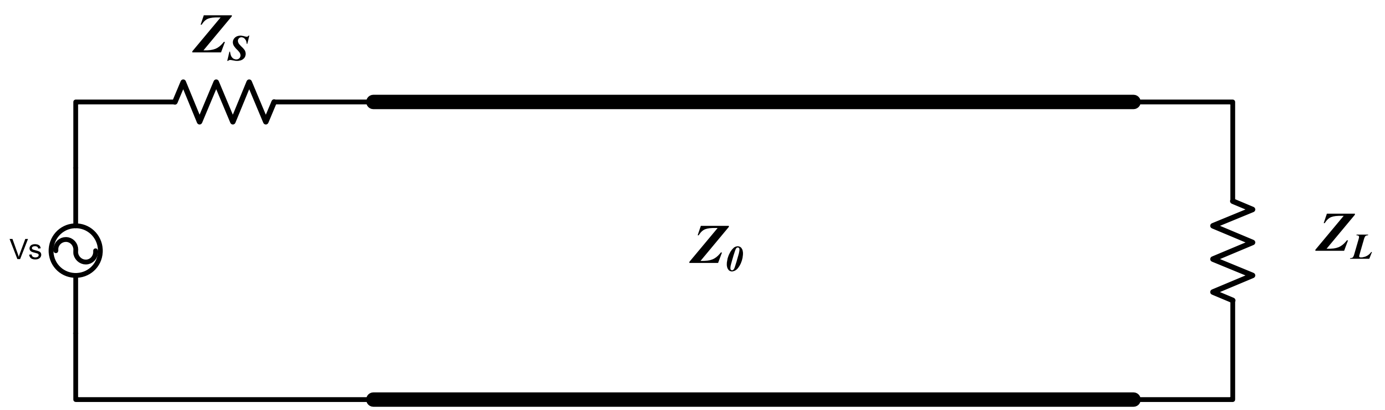
Transients on TX lines

- Lossless line
  - Length l, wave speed u_p, characteristic impedance Z_0, terminated by Z_L at the receiving end and Z_S at the sending end;

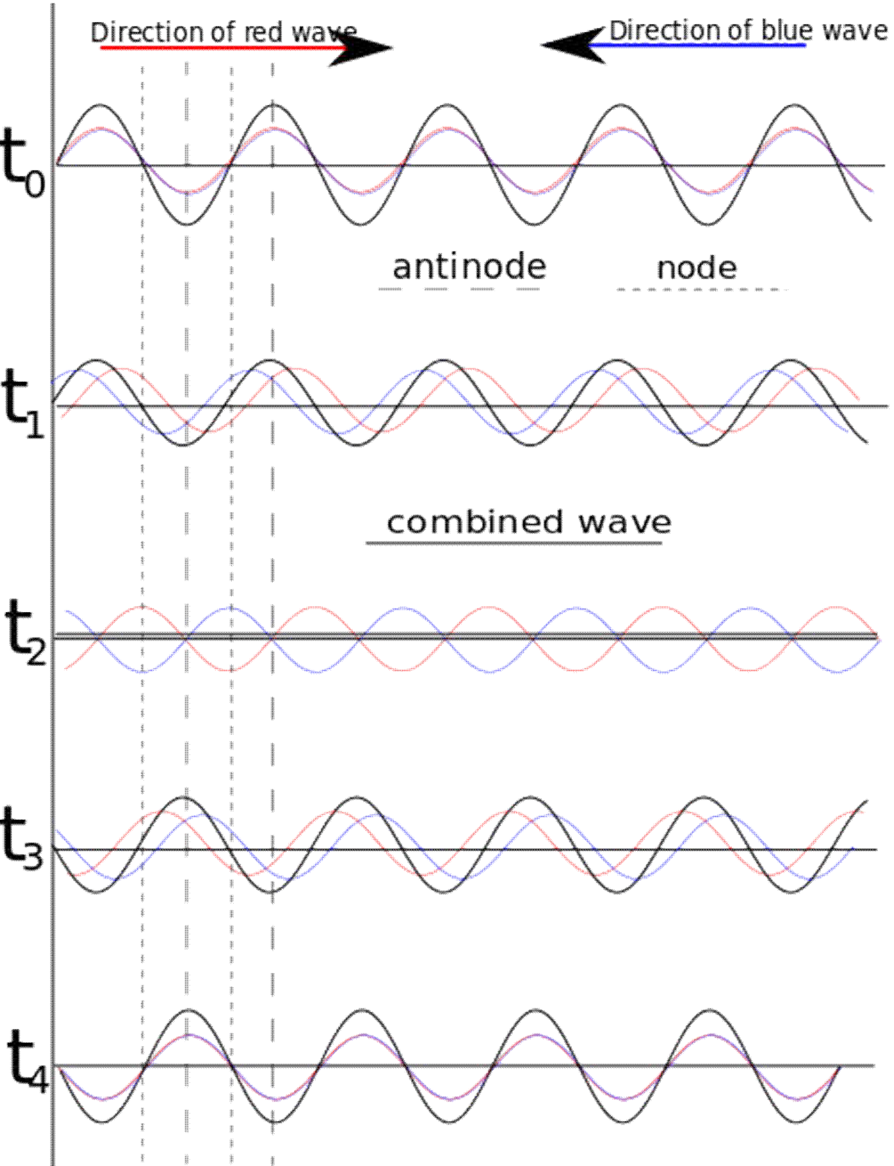


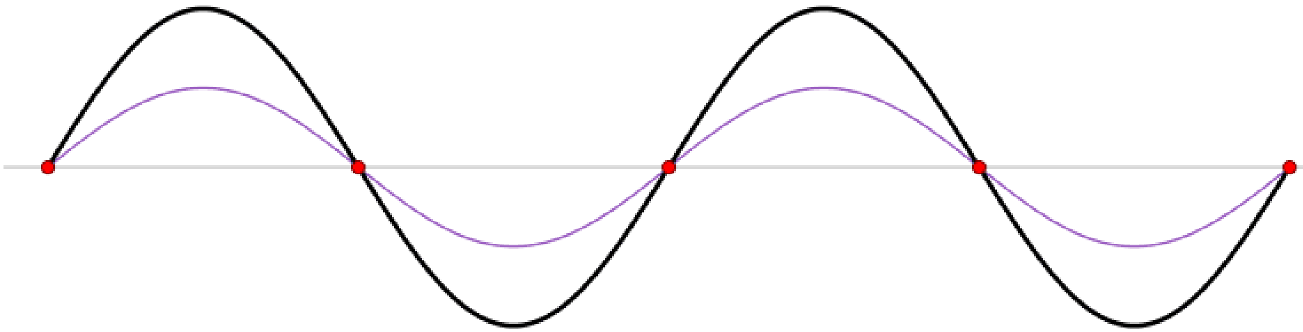
Transients on TX lines

- Lossy line
  - Length l, wave speed u_p, attenuation rate A, characteristic impedance Z_0, terminated by Z_L at the receiving end and Z_S at the sending end;



Standing Waves





- A standing wave in a transmission line is a wave in which the distribution of current / voltage is formed by the superposition of two waves of the same *freq*. propagating in opposite directions.
- It is formed when a wave is transmitted into a transmission line with an impedance mismatch, i.e., discontinuity, such as an open circuit or a short circuit.

Voltage Standing Wave Ratio (VSWR)

- On a lossless line, the ***voltage standing wave ratio*** (VSWR) is defined as the ratio between the maximum and minimum values of the voltage or current on the TX:

$$VSWR = \frac{V_{max}}{V_{min}} = \frac{I_{max}}{I_{min}}$$

- Consider the incident wave V_i and reflected wave V_r

$$VSWR = \frac{|V_i| + |V_r|}{|V_i| - |V_r|} = \frac{1 + |V_r|/|V_i|}{1 - |V_r|/|V_i|}$$

  - where $\frac{|V_r|}{|V_i|} = |\Gamma|$, so $VSWR = \frac{1+|\Gamma|}{1-|\Gamma|}$
  - At the same time:

$$|\Gamma| = \frac{VSWR - 1}{VSWR + 1}$$

VSWR – Example 1

- A signal of 10V is applied to a 50Ω co-axial transmission line terminated in a 200Ω load. Find:
  - (a) the voltage reflection co-efficient
  - (b) the magnitude of the reflected voltage
  - (c) the magnitude of the reflected current
  - (d) the VSWR.

VSWR – Example 2

- A lossless 100Ω transmission line is terminated in $50 + j75\Omega$. Find:
  - (a) the voltage reflection co-efficient
  - (b) VSWR