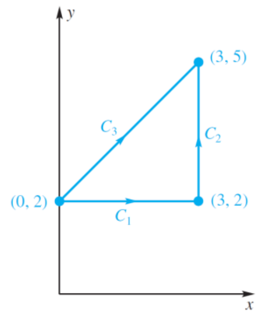
Review questions

1. Find the gradient for the following functions
   (1) $f(x, y, z) = \sin(xyz)$
   (2) $f(x, y, z) = x\mathrm{e}^{y}\cos z$
   (3) $f(x, y, z) = y^2\mathrm{e}^{-2z}$
2. Find div$\boldsymbol{F}$ and curl$\boldsymbol{F}$
   (1) $\boldsymbol{F} = < x^2, -2xy,\ yz^2 >$
   (2) $\boldsymbol{F} = < \mathrm{e}^{x}\cos y,\ \mathrm{e}^{x}\sin y,\ z >$
3. Assuming that the required partial derivatives exist and are continuous, show that
   (1) div(curl$\boldsymbol{F}$)=0
   (2) curl(grad$f$)=0
4. (1) Evaluate $\int_C xy^2dx + xy^2dy$ along the path $C = C_1 \cup C_2$.
   (2) Evaluate $\int_{C_3} xy^2dx + xy^2dy$ along C_3.



5. Evaluate $\int_C y^3dx + x^3dy$; C is the curve $x = 2t, y = t^2 - 3, -2 \le t \le 1$.
6. Evaluate $\int_C xzdx + (y + z)dy + xdz$; C is the curve $x = \mathrm{e}^{t}, y = \mathrm{e}^{-t}, z = \mathrm{e}^{2t}, 0 \le t \le 1$.
7. Find the work done by the force $\boldsymbol{F}$ in moving a particle along the curve C.
   (1) $\boldsymbol{F}(x, y) = (x^3 - y^3)\mathbf{i} + xy^2\mathbf{j}$; C is the curve $x = t^2, y = t^3, -1 \le t \le 0$.
   (2) $\boldsymbol{F}(x, y, z) = < 2x - y, 2z, y - z >$; C is the curve $x = \sin\frac{\pi t}{2}, y = \sin\frac{\pi t}{2}, z = t, 0 \le t \le 1$.
8. Determine whether the given field $\boldsymbol{F} \cdot d\boldsymbol{r}$ is exact. If so, find f such that $\boldsymbol{F} = \nabla f$.
   (1) $\boldsymbol{F}(x, y) = < 12x^2 + 3y^2 + 5y,\ 6xy - 3y^2 + 5x >$.
   (2) $\boldsymbol{F}(x, y) = < 4y^2\cos(xy^2), 8x\cos(xy^2) >$.
   (3) $\boldsymbol{F}(x, y, z) = < 3x^2, 6y^2, 9z^2 >$.
9. (1) $\int_{(-1,2)}^{(3,1)} (y^2 + 2xy)dx + (x^2 + 2xy)dy$
   (2) $\int_{(0,0,0)}^{(\pi,\pi,0)} (\cos x + 2yz)dx + (\sin y + 2xz)dy + (z + 2xy)dz$.
10. Sketch the region S and evaluate the line integral by Green's theorem.
    (1) $\oint_C 2xydx + y^2dy$, where C is the closed curve formed by $y = \frac{x}{2}$ and $y = \sqrt{x}$ between (0,0) and (4,2).
    (2) $\oint_C xydx + (x + y)dy$, where C is the triangle with vertices (0,0), (2,0) and (0,1).

11. Find the area of the region S by using $A(s) = \frac{1}{2}\oint_C xdy - ydx$.

   (1) S is bounded by the curves *y=4x* and $y = 2x^2$.

   (2) S is bounded by $(x+y)^2 = ax$ and *x*-axis.

12. Evaluate the surface integral $\iint_S g(x,y,z)dA$, where *g(x,y,z)=x* and *S: x+y+2z=4*, $0 \le x \le 1, 0 \le y \le 1$.

13. Find the area of the surface $z = x^2 + y^2$ below the plane *z=9*.

14. Evaluate the surface integral $\iint_S \boldsymbol{F} \cdot \boldsymbol{n} dA$.

   (1) $\boldsymbol{F} =< -x^2, y^2, 0 >, S: \boldsymbol{r}(u,v) =< u, v, 3u - 2v >, 0 \le u \le \frac{3}{2}, -2 \le v \le 2$.

   (2) $\boldsymbol{F} =< \tan(xy), x, y >, S: y^2 + z^2 = 1, 2 \le x \le 5, y \ge 0, z \ge 0$.

15. Evaluate the surface integral $\iint_S \boldsymbol{F} \cdot \boldsymbol{n} dA$ by the divergence theorem.

   (1) $\boldsymbol{F} =< x^2, 0, z^2 >$, *S* is the surface of the box $|x| \le 1, |y| \le 3, 0 \le z \le 2$.

   (2) $\boldsymbol{F} =< x, 2y + z, z + x^2 >$, *S* is the surface of $1 \le x^2 + y^2 + z^2 \le 4$.

   (3) $\boldsymbol{F} =< x^2, y^2, z^2 >$, *S* is the surface of the solid bounded by $x + y + z = 4, x = 0, y = 0, z = 0$.

16. Verify Stokes's theorem for $\boldsymbol{F} =< y, -x, yz >$ if *S* is the paraboloid $z = x^2 + y^2$ with the circle $x^2 + y^2 = 1, z = 1$ as its boundary. Here we take the normal vector of the surface as upward.

17. Use Stokes's theorem to calculate $\iint_S (curl\boldsymbol{F}) \cdot \boldsymbol{n} dA$, where $\boldsymbol{F} =< xz^2, x^3, \cos(xz) >$; *S* is the part of the ellipsoid $x^2 + y^2 + 3z^2 = 1$ above the *xy*-plane and ***n*** is the upward normal vector.