

EEE104 – Digital Electronics (I)

Lecture 7

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In This Session

- Boolean Algebra and Logic Simplification
 - Boolean Operations
 - Laws and Rules
 - DeMorgan's Theorems
 - Logic Simplification

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Boolean Operations

Concepts

- **Boolean algebra** is the mathematics of digital systems.
- A **variable** is a symbol used to represent a logical quantity. It can have a 1 or 0 value.
- The **complement** is the inverse of a variable and is indicated by an overbar, e.g. \bar{A}
- A **literal** is a variable or its complement.

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Boolean Operations

Boolean Addition

- Equivalent to the OR operation with the basic rules:

$0 + 0 = 0$
$0 + 1 = 1$
$1 + 0 = 1$
$1 + 1 = 1$

- A **sum term** is a sum of literals, and is equal to 0 only if each of the literals is 0, e.g. $\bar{A} + B + C + \bar{D}$

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Boolean Operations

Boolean Multiplication

- Equivalent to the AND operation with the basic rules:

$$\begin{array}{l} 0 \cdot 0 = 0 \\ 0 \cdot 1 = 0 \\ 1 \cdot 0 = 0 \\ 1 \cdot 1 = 1 \end{array}$$

- A **product term** is the product of literals. It is equal to 1 only if each of the literals is 1, e.g. $\bar{A}BC\bar{D}$

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Laws of Boolean Algebra

Commutative Laws

$$A + B = B + A$$

$$AB = BA$$

Associate Laws

$$A + (B + C) = (A + B) + C$$

$$A(BC) = (AB)C$$

Distributive Laws (or Factoring)

$$A(B + C) = AB + AC$$

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Rules of Boolean Algebra

1. $A + 0 = A$	7. $A \cdot A = A$
2. $A + 1 = 1$	8. $A \cdot \bar{A} = 0$
3. $A \cdot 0 = 0$	9. $\bar{\bar{A}} = A$
4. $A \cdot 1 = A$	10. $A + AB = A$
5. $A + A = A$	11. $A + \bar{A}B = A + B$
6. $A + \bar{A} = 1$	12. $(A + B)(A + C) = A + BC$

- A, B, C can represent a single variable or a combination of variables.
- Rules 10-12 can be derived using simpler rules and the laws.

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Rules of Boolean Algebra

Rule 10: $A + AB = A$

$$A + AB = A(1 + B) \quad \text{Rule 2} \\ = A$$

Rule 11: $A + \bar{A}B = A + B$

$$A + \bar{A}B = A + AB + \bar{A}B \quad \text{Rule 10 or Rule 2} \\ = A + (A + \bar{A})B \\ = A + B \quad \text{Rule 6}$$

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Rules of Boolean Algebra

Rule 12:

$$(A + B)(A + C) = A + BC$$

$$(A + B)(A + C) = AA + AC + AB + BC$$

$$= A + AC + AB + BC \quad \text{Rule 7}$$

$$= A + AB + BC \quad \text{Rule 10 or Rule 2}$$

$$= A + BC \quad \text{Rule 10 or Rule 2}$$

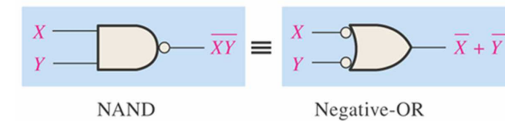
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DeMorgan's Theorems (1st)

- The complement of a product of variables is equal to the sum of the complements of variables.

$$\overline{XY} = \bar{X} + \bar{Y}$$

Inputs		Output	
X	Y	\overline{XY}	$\bar{X} + \bar{Y}$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0



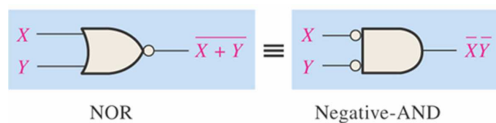
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DeMorgan's Theorems (2nd)

- The complement of a sum of variables is equal to the product of the complements of variables.

$$\overline{X + Y} = \bar{X}\bar{Y}$$

Inputs		Output	
X	Y	$\overline{X + Y}$	$\bar{X}\bar{Y}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

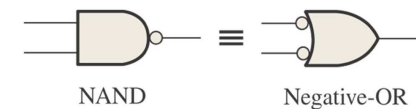


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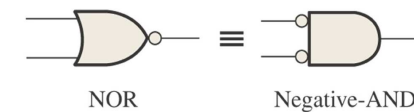
DeMorgan's Theorems

Break the bar, change the sign.

$$\overline{XY} = \bar{X} + \bar{Y}$$



$$\overline{X + Y} = \bar{X}\bar{Y}$$



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DeMorgan's Theorems

Application Example

- Each variable can also represent a combination of other variables.

$$\begin{aligned}\overline{(AB + C)(A + BC)} &= \overline{(AB + C)} + \overline{(A + BC)} \\ &= (\overline{AB})\overline{C} + \overline{A}(\overline{BC}) \\ &= (\overline{A} + \overline{B})\overline{C} + \overline{A}(\overline{B} + \overline{C})\end{aligned}$$

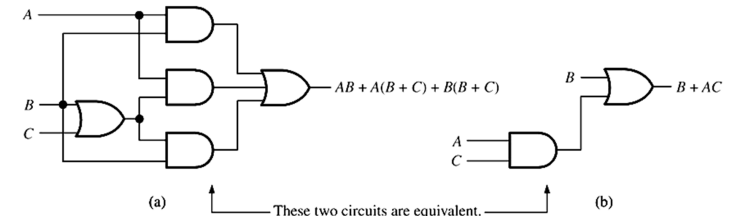
$$\begin{aligned}\overline{[A + B\overline{C}][D(E + \overline{F})]} &= \overline{[A + B\overline{C}]} \overline{[D(E + \overline{F})]} \\ &= (A + B\overline{C})(\overline{D} + \overline{(E + \overline{F})}) \\ &= (A + B\overline{C})(\overline{D} + E + \overline{F})\end{aligned}$$

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Simplification Using Boolean Algebra

- To use the fewest gates possible to implement a given expression.

$$\begin{aligned}AB + A(B + C) + B(B + C) \\ &= AB + AB + AC + BB + BC \\ &= AB + AC + B + BC \\ &= AB + AC + B = B + AC\end{aligned}$$



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