EEE108 Electromagnetism and Electromechanics

Lecture 7

Capacitors and Capacitance
Lab 1 Tutorial

Dr. Jinling Zhang

Dept. of Electrical and Electronic Engineering
University of Xi'an Jiaotong-Liverpool
Email: jinling.zhang@xjtlu.edu.cn

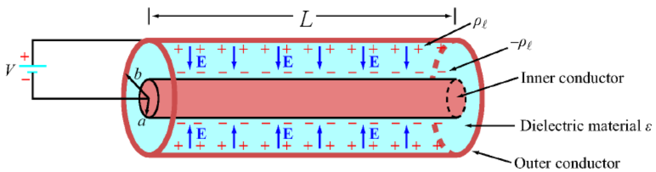
Today

- **Capacitors and Capacitance (2)**
- **Lab 1 Tutorial**

Cylindrical Capacitors

A solid cylindrical conductor of radius a surrounded by a coaxial cylindrical shell of inner radius b. The length of both cylinders is L and this length is much larger than $b - a$, the separation of the cylinders, so that edge effects can be neglected. The capacitor is charged so that the inner cylinder has charge $-Q$ while the outer shell has a charge $+Q$.

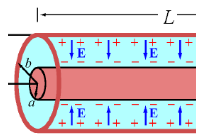
What is the capacitance?



Coaxial capacitor filled with insulating material of permittivity ε

Cylindrical Capacitors

$\boxed{\rho_l = Q/L}$



The potential difference :

$$\Delta\varphi = \varphi_b - \varphi_a = -\int_a^b (-E_r)dr = \frac{\rho_l}{2\pi\varepsilon}\int_a^b \frac{dr}{r} = \frac{\rho_l}{2\pi\varepsilon}\ln\left(\frac{b}{a}\right)$$

The capacitance : $C = \dfrac{Q}{|\Delta\varphi|} = \dfrac{\rho_l L}{\dfrac{\rho_l}{2\pi\varepsilon}\ln\left(\dfrac{b}{a}\right)} = \dfrac{2\pi\varepsilon\, L}{\ln(b/a)}$

Again, C depends only on the geometrical factors, L, a and b.

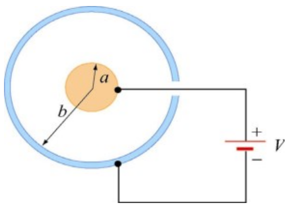
Capacitance per unit length $= \dfrac{C}{L} = \dfrac{2\pi\varepsilon}{\ln(b/a)}$ (F/m)

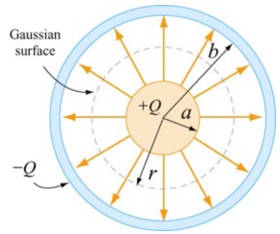
Energy stored in a cylindrical capacitor :

$$U = \frac{1}{2}C|\Delta\varphi|^2 = \frac{1}{2}CV^2 = \frac{1}{2}\frac{2\pi\varepsilon L}{\ln(b/a)}\left(\frac{\rho_l}{2\pi\varepsilon}\ln\left(\frac{b}{a}\right)\right)^2 = \frac{L\rho_l^{\,2}\ln(b/a)}{4\pi\varepsilon} = \frac{Q^2\ln(b/a)}{4\pi\varepsilon L}$$

Spherical Capacitors

A spherical capacitor which consists of two concentric spherical shells of radii a and b. The inner shell has a charge $+Q$ uniformly distributed over its surface, and the outer shell an equal but opposite charge $-Q$.
What is the capacitance of this configuration?





Spherical Capacitors

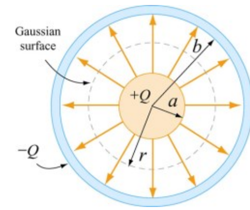
When $a < r < b$, by using Gauss's law :

$$\oint\!\!\oint_S \mathbf{E} \bullet d\mathbf{s} = E_r A = E_r(4\pi r^2) = \frac{Q}{\varepsilon} \Rightarrow E_r = \frac{1}{4\pi\varepsilon}\frac{Q}{r^2}$$

Then the potential difference between the two shells :

$$\Delta\varphi = \varphi_b - \varphi_a = -\int_a^b E_r dr = -\frac{Q}{4\pi\varepsilon}\int_a^b \frac{dr}{r^2}$$

$$= -\frac{Q}{4\pi\varepsilon}\left(\frac{1}{a} - \frac{1}{b}\right) = -\frac{Q}{4\pi\varepsilon}\left(\frac{b-a}{ab}\right)$$



The capacitance :

$$C = \frac{Q}{|\Delta\varphi|} = 4\pi\varepsilon\left(\frac{ab}{b-a}\right)$$

Again, the capacitance C depends only on the physical dimensions, a and b.

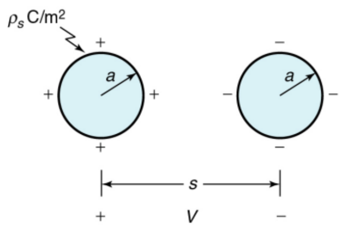
Two-wire Line

Determine the capacitance per unit of length for a two-wire line.

Solution

The conductors are infinite in length and have a uniform charge distribution ρ_s C/m² along their length and around their peripheries.

The assumption is acceptable when the ratio of s/a is large enough (usually a ratio of 5 ~ 10 is sufficient and is typical for transmission lines.).



With this assumption, we can utilize the result of an infinitely long rod of negligible radius.

Two-wire Line

An infinitely long rod of negligible radius has a uniform positive charge density in free space. The electric field is:

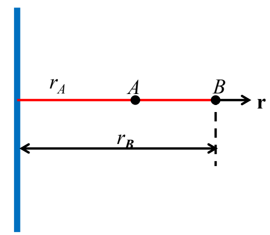
$$\mathbf{E} = \frac{\rho_L}{2\pi\varepsilon_0 r}\mathbf{a}_r$$

The potential difference at distance r_A and r_B is:

$$\varphi_{AB} = \varphi_A - \varphi_B = -\int_{r_B}^{r_A} \mathbf{E} \bullet d\mathbf{L} = \int_{r_A}^{r_B} \mathbf{E} \bullet d\mathbf{L}$$

$$= \int_{r_A}^{r_B} \frac{\rho_L}{2\pi\varepsilon_0 r}\mathbf{a}_r \bullet d\mathbf{L} = \int_{r_A}^{r_B} \frac{\rho_L}{2\pi\varepsilon_0 r} dr$$

$$= \frac{\rho_L}{2\pi\varepsilon_0} \ln r \Big|_{r_A}^{r_B} = \frac{\rho_L}{2\pi\varepsilon_0} \ln \frac{r_B}{r_A}$$

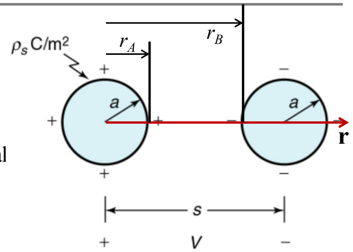


For an infinite charge source, the zero potential point is normally taken at a finite distance away from the source.

Two-wire Line

Solution Cont.

$$\boxed{\varphi_{AB} = \varphi_A - \varphi_B = \frac{\rho_L}{2\pi\varepsilon_0}\ln\frac{r_B}{r_A}}$$



By superposition, we can have the total voltage between the two cylinders:

$$V = 2\frac{\rho_L}{2\pi\varepsilon}\ln\left(\frac{r_B}{r_A}\right) = \frac{\rho_L}{\pi\varepsilon}\ln\left(\frac{s}{a}\right)$$

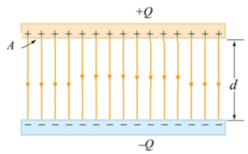
where $r_B = s - a \cong s$ and $r_A = a$, $\rho_L = 2\pi a\rho_s$ is the charge per unit of length.

Then the capacitance per unit length is:

$$C = \frac{\rho_L}{\Delta\varphi} = \frac{\rho_L}{V} = \frac{\rho_L}{\frac{\rho_L}{\pi\varepsilon}\ln\left(\frac{s}{a}\right)} = \frac{\pi\varepsilon}{\ln\left(\frac{s}{a}\right)} \quad \text{F/m}$$

The Procedure of Calculating Capacitance

(1) Identify the direction of the electric field using symmetry.
(2) Calculate the electric field everywhere between the two conductors.
(3) Compute the electric potential difference $\Delta\varphi$.
(4) Calculate the capacitance C using $C = Q/\Delta\varphi$.

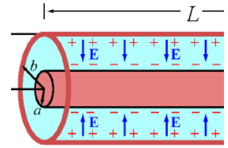


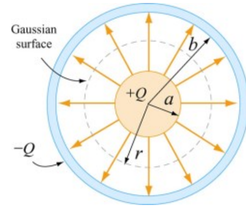
Parallel plates

$$C = \frac{\varepsilon A}{d}$$

$$C = \frac{2\pi\varepsilon L}{\ln(b/a)}$$

Cylindrical configuration





Spherical configuration

$$C = 4\pi\varepsilon\left(\frac{ab}{b-a}\right)$$

Dielectrics and Capacitance

Parallel Plate	Cylindrical	Spherical
$C = \frac{\varepsilon A}{d}$	$C = \frac{2\pi\varepsilon L}{\ln(b/a)}$	$C = 4\pi\varepsilon\left(\frac{ab}{b-a}\right)$

where $\varepsilon = \varepsilon_r\varepsilon_0$, ε_0 is the permittivity in vacuum $= 8.85\times10^{-12}$ F/m

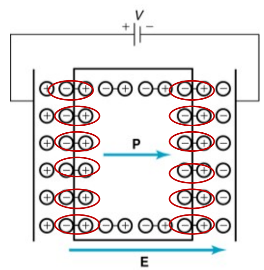
If permittivity $\uparrow \rightarrow C \uparrow$ with the same size

Large C results in more energy stored

Relative permittivity (dielectric constant) of common materials

Material	Relative Permittivity, ε_r
Air (at sea level)	1.0006
Petroleum oil	2.1
Polystyrene	2.6
Glass	4.5–10
Quartz	3.8–5
Bakelite	5
Mica	5.4–6

Dielectrics and Capacitance



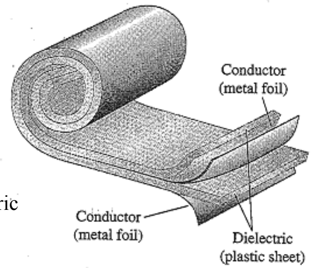
(b)

- A dielectric material is inserted between the plates.
- The bound charge dipoles attempt to align with the field, creating a polarization vector in the dielectric.
- A surface charge is created on the two surfaces of the dielectric. *More free charge is drawn from the battery.*

Dielectrics and Capacitance

Most capacitors have an insulating material, called dielectric, such as paper or plastic between their conducting plates.

Three reasons:
1. To maintain a physical separation of the plates
2. Increase the maximum possible potential difference between the conducting plates
3. Capacitance increases when the space between the conductors is filled with dielectrics



Example: using dielectric sheets to separate the conductors

Dielectrics and Breakdown

The electric energy density: $u_E = \dfrac{U_E}{\text{Volume}} = \dfrac{1}{2}\varepsilon E^2 \quad \text{J/m}^3$

$\boxed{E\uparrow \rightarrow u_E\uparrow}$

The results obtained so far are under the condition of the magnitude of E does not exceed a certain critical value – dielectric strength.

Beyond the critical value, the dielectric material can sustain permanent damage. This abrupt change in behavior is called dielectric breakdown.

The dielectric strength E_{ds} is the largest magnitude of E that the material can sustain without breakdown.

The dielectric strength depends on the material composition, as well as other factors such as temperature and humidity.

Dielectrics and Breakdown

Relative permittivity (dielectric constant) and dielectric strength of common materials

Material	Relative Permittivity, ε_r	Dielectric Strength, E_{ds} (MV/m)
Air (at sea level)	1.0006	3
Petroleum oil	2.1	12
Polystyrene	2.6	20
Glass	4.5–10	25–40
Quartz	3.8–5	30
Bakelite	5	20
Mica	5.4–6	200

$\varepsilon = \varepsilon_r\varepsilon_0$ and $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m

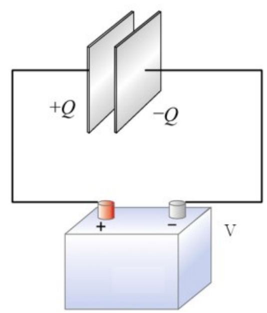
The breakdown field strength at which dry air loses its insulating ability and allows a discharge to pass through is 3×10^6 V/m . At this field strength, the electric energy density is:

$$u_E = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\left(3\times10^6\,\text{V/m}\right)^2 = 40\ \text{J/m}^3$$

Capacitors in Electric Circuits

Charging a Capacitor

A capacitor can be charged by connecting the plates to the terminals of a battery, which are maintained at a potential difference V called the terminal voltage.



The battery could be thought of as a charge pump that brings a charge Q from one plate to the other.

Capacitors in Electric Circuits

Parallel Connection

The potential difference V is the same across each capacitor:

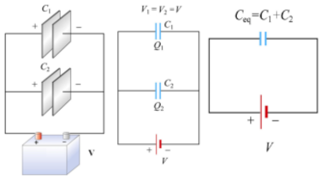
$$C_1 = \frac{Q_1}{V},\ C_2 = \frac{Q_2}{V}$$

Total charge Q supplied by the battery:

$$Q = Q_1 + Q_2 = C_1V + C_2V = (C_1 + C_2)V$$

The two capacitor can be replaced by a single equivalent capacitor:

$$C_{eq} = \frac{Q}{V} = C_1 + C_2$$



Generally, to any number of capacitors are connected in parallel:

$$\boxed{C_{eq} = C_1 + C_2 + C_3 + ... + C_N = \sum_{i=1}^{N} C_i} \qquad \text{(Parallel)}$$

Capacitors in Electric Circuits

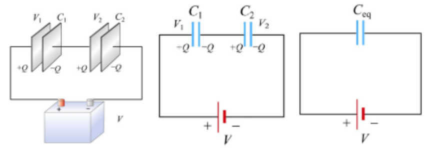
Series Connection

The potential difference across each capacitor:

$$V_1 = \frac{Q}{C_1},\ V_2 = \frac{Q}{C_2}$$

Total potential difference is the sum of the two potential difference:

$$V = V_1 + V_2 = \frac{Q}{C_1} + \frac{Q}{C_2}$$



The two capacitor can be replaced by a single equivalent capacitor: $C_{eq} = \frac{Q}{V}$, then $V = \frac{Q}{C_{eq}}$

so, we have: $\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} \rightarrow \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$

Generally, to any number of capacitors are connected in series:

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ... + \frac{1}{C_N} = \sum_{i=1}^{N} \frac{1}{C_i}}$$

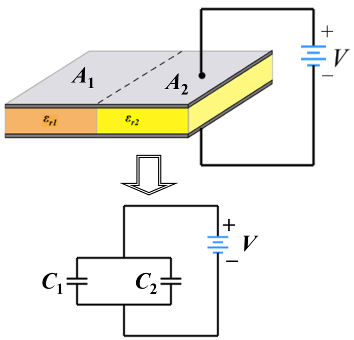
Capacitor Filled with Two Different Dielectrics

Two dielectrics with dielectric constants ε_{r1} and ε_{r2} each fill half the space between the plates of a parallel-plate capacitor. Each plate has an area A and the plates are separated by a distance d.
Find the capacitance of the system.

Solution

The potential difference on each half of the capacitor is the same, so the system can be treated as being composed of two capacitors connected in parallel. Thus, the capacitance of the system is

$$C = C_1 + C_2$$



Capacitor Filled with Two Different Dielectrics

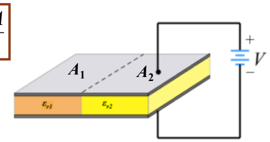
Solution Cont.

$$\boxed{C = \frac{\varepsilon A}{d}}$$

As $C_1 = \frac{\varepsilon_{r1}\varepsilon_0(A/2)}{d}$ and $C_2 = \frac{\varepsilon_{r2}\varepsilon_0(A/2)}{d}$

the capacitance of the system:

$$C = C_1 + C_2$$
$$= \frac{\varepsilon_{r1}\varepsilon_0(A/2)}{d} + \frac{\varepsilon_{r2}\varepsilon_0(A/2)}{d}$$
$$= \frac{\varepsilon_0 A}{2d}(\varepsilon_{r1} + \varepsilon_{r2})$$

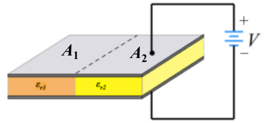


$$\boxed{\begin{array}{l} A = 500\ \text{cm}^2, V = 10\ \text{V}, \\ d = 2\ \text{mm},\ \varepsilon_{r1} = 2, \varepsilon_{r2} = 4 \end{array}}$$

$$C = \frac{\varepsilon_0 A}{2d}(\varepsilon_{r1} + \varepsilon_{r2}) = \frac{8.85\times10^{-12}\times500\times10^{-4}}{2\times2\times10^{-3}}(2+4)$$
$$= 663.7\ \text{pF}$$

Capacitor Filled with Two Different Dielectrics

Solution Cont.



If $A_1 \neq A_2$, $C_1 = \dfrac{\varepsilon_{r1}\varepsilon_0 A_1}{d}$ and $C_2 = \dfrac{\varepsilon_{r2}\varepsilon_0 A_2}{d}$

$$C = C_1 + C_2$$

$$= \frac{\varepsilon_{r1}\varepsilon_0 A_1}{d} + \frac{\varepsilon_{r2}\varepsilon_0 A_2}{d} = \varepsilon_0 \frac{\varepsilon_{r1}A_1 + \varepsilon_{r2}A_2}{d}$$

$$\boxed{A_1 = 100\ \text{cm}^2,\ A_2 = 400\ \text{cm}^2,\ V = 10\ \text{V},\ d = 2\ \text{mm},\ \varepsilon_{r1} = 2,\ \varepsilon_{r2} = 4}$$

$$C = \varepsilon_0 \frac{\varepsilon_{r1}A_1 + \varepsilon_{r2}A_2}{d}$$

$$= 8.85\times10^{-12}\,\frac{2\times100\times10^{-4} + 4\times400\times10^{-4}}{2\times10^{-3}}$$

$$= 796.5\ \ \text{pF}$$

$$\boxed{A = 500\ \text{cm}^2,\ V = 10\ \text{V},\ d = 2\ \text{mm},\ \varepsilon_{r1} = 2,\ \varepsilon_{r2} = 4}$$

$$C = \frac{\varepsilon_0 A}{2d}(\varepsilon_{r1} + \varepsilon_{r2})$$

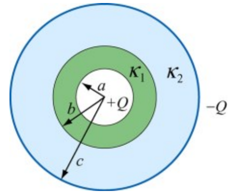
$$= \frac{8.85\times10^{-12}\times500\times10^{-4}}{2\times2\times10^{-3}}(2+4)$$

$$= 663.7\ \ \text{pF}$$

Spherical Capacitor Filled with Dielectrics

Consider a conducting spherical shell with an inner radius a and outer radius c. Let the space between two surfaces be filed with two different dielectric materials so that the dielectric constant is between a and b is $\varepsilon_{r1}=\kappa_1$, and between b and c is $\varepsilon_{r2}=\kappa_2$.

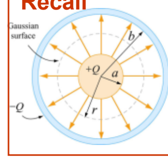
Find the capacitance of this system.



Solution

The system can be treated as two capacitors connected in series, since the total potential difference across the capacitors is the sum of potential differences across individual capacitors.

Recall



$$C = 4\pi\varepsilon\left(\frac{ab}{b-a}\right)$$

$$\Downarrow$$

$$C = 4\pi\varepsilon_r\varepsilon_0\left(\frac{r_1 r_2}{r_2 - r_1}\right)$$

Spherical Capacitor Filled with Dielectrics

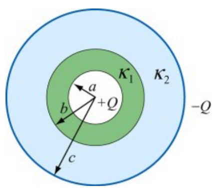
Thus, the equivalent capacitance is

$$\boxed{C = 4\pi\varepsilon_r\varepsilon_0\left(\frac{r_1 r_2}{r_2 - r_1}\right)}$$

$$\frac{1}{C} = \frac{1}{\dfrac{4\pi\varepsilon_0\kappa_1 ab}{(b-a)}} + \frac{1}{\dfrac{4\pi\varepsilon_0\kappa_2 bc}{(c-b)}} = \frac{\kappa_2 c(b-a) + \kappa_1 a(c-b)}{4\pi\varepsilon_0\kappa_1\kappa_2 abc}$$

$$\Downarrow$$

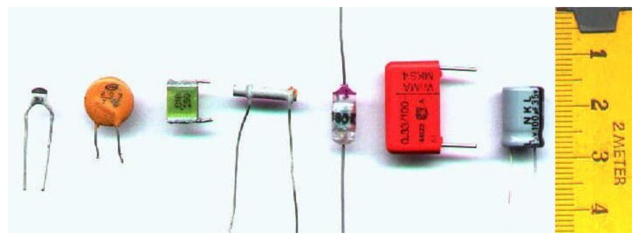
$$\boxed{C = \frac{4\pi\varepsilon_0\kappa_1\kappa_2 abc}{\kappa_2 c(b-a) + \kappa_1 a(c-b)}}$$



When κ_1 and κ_2 both are equal to 1 , the expression reduces to:

$$C = \frac{4\pi\varepsilon_0 abc}{c(b-a) + a(c-b)} = \frac{4\pi\varepsilon_0 abc}{b(c-a)} = \frac{4\pi\varepsilon_0 ac}{(c-a)}$$

Examples of Commercial Capacitors



Capacitor materials: from left: multilayer ceramic, ceramic disc, multilayer polyester film, tubular ceramic, polystyrene, metalized polyester film, aluminum electrolytic.

Examples of Commercial Capacitors

Various types of capacitor

Summary of Capacitors and Capacitance

- Capacitors and capacitance:

  A capacitor is a device to store electric charge and potential energy.

  The capacitance: $C = Q/|\Delta\varphi|$
- Capacitors in series and parallel:

  $$\frac{1}{C_{eq}} = \sum_{i=1}^{N} \frac{1}{C_i} \quad \text{(Series)} \qquad C_{eq} = \sum_{i=1}^{N} C_i \quad \text{(Parallel)}$$
- Energy in a capacitor:

  $$U = \frac{1}{2}CV^2 = \frac{1}{2}QV \ \text{(Energy stored)} \qquad u_E = \frac{1}{2}\varepsilon E^2 \ \text{(Energy density)}$$
- Dielectrics:

  When a dielectric material is filled into a capacitor, the capacitance increases by a factor ε_r: $C = \varepsilon_r C_0$, where C_0 is the capacitance in vacuum.

Lab 1 Tutorial

Lab. 1 Electrostatic Field Plotting

Objective:

➢To illustrate the pattern of electric field lines associated with systems of conductors.

➢To recognize:

i) electric charges as the sources and sinks of field lines;

ii) the electric field as the (negative) gradient of potential.

Lab. 1 Electrostatic Field Plotting

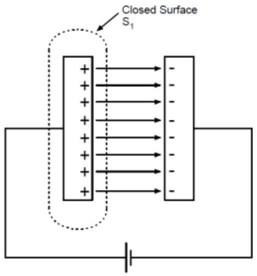


Figure 1 Charges and **E** Field with Parallel Plates

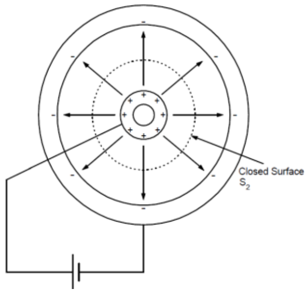


Figure 2 Charges and **E** Field with Concentric Cylinders

Lab. 1 Electrostatic Field Plotting

Schedule and Groups

March 20, 11:00-15:00 **Room 215/213, B4+**	**March 20, 15:00-18:00** **Room 215/213, B4+**

- Two students are in a group.
- Please check your time and group on ICE.
- Sign your name when you arrive!
- No signature, no mark!

Lab. 1 Electrostatic Field Plotting

Report Writing

The objective of writing a report is to convey information to the reader, not just me!! Your report should be clear, straightforward and you should not omit important data/information.

For the experiment the report should include the followings:

1. Cover page, which shows the experiment title, your name, ID number, the name who is in your group, the lab's date and the report submitting date.

2. Abstract (10%)
A short section of between 50 and 300 words which must be capable of being read and understood independently of the rest of the report. This section should briefly summarise
 _ The purpose and scope of the experiment,
 _ The experimental procedures that were carried out,
 _ The main conclusions.

Lab. 1 Electrostatic Field Plotting

Report Writing

3. Introduction (10%)
This section describes, in general terms, the scope of the experiment and its relevance to the field of study you are engaged in. A statement of objectives should be given along with general comments about how the experiment will be carried out.

You may like to put the equipments used in the experiment in this section.

4. Main body (45%) PROPER TITLE NOT "Main body"!!
In this section you explain how you performed the measurements and describe your findings.

- Experimental procedures;
- Theory and procedures;
- …

Report Writing

5. Error analysis and Discussion (5%)

6. Conclusion (5%)
It is a concise statement of what has been learnt from or confirmed by the experiment. This section must be consistent with earlier sections. Usually, a short paragraph is used here to summarise the most important findings.

7. References (5%)
You may refer to textbooks, research papers, magazine articles, data sheets and lecture notes as references. References should be listed in a separate section at the end of the report, in the proper form.

8. Appendix (20%) Don't miss any questions!
Please include your answers for the prelab questions in the appendix section of your report.

Report Submission

The report should be submitted by a softcopy (to ICE) before 5:00pm on 3rd April, 2018

Q & A

***Q*: Do we need to submit the hard copy of the hand drawn?**

***A*: No. Your report should include all information you want to submit.**

***Q*: Shall we use the same figure which was drawn by my another group member?**

***A*: No. You should do everything by yourself except sharing the experiment data.**

In Lab:

NO FOOD! NO DRINK!

Next

- **Mini-review of Electrostatics**
- **Electric Current**

Thanks for your attendance