EEE213 Power Electronics and Electromechanism

3. Uncontrolled Rectifier – Three Phases

Review 3-phase systems.

a) 3 power supplies, each lag $\frac{2\pi}{3}$.

$$v_A = V_m \cos\omega t = \sqrt{2}\, V_s \cos\omega t$$

$$v_B = V_m \cos(\omega t - \frac{2\pi}{3})$$

$$v_C = V_m \cos(\omega t - \frac{4\pi}{3}) = V_m \cos(\omega t + \frac{2\pi}{3})$$

b) Phase voltages: $\dot{V}_A$, $\dot{V}_B$, $\dot{V}_C$

Line voltages: $\dot{V}_{AB}$, $\dot{V}_{BC}$, $\dot{V}_{CA}$.

eg: $\dot{V}_{AB} = \dot{V}_A - \dot{V}_B = \sqrt{3}\, V_m \angle \frac{\pi}{6}$

$$\Rightarrow v_{AB} = \sqrt{3}\, V_m \cos(\omega t + \frac{\pi}{6})$$

c) Total power.

$$P = 3V_{AN}\cdot I_A\cdot\cos\varphi$$

$$p_A = \dot{V}_{AN}\cdot\dot{I}_A = \sqrt{2}V_{AN}\cos\omega t \cdot \sqrt{2}I_A\cos(\omega t-\varphi)$$

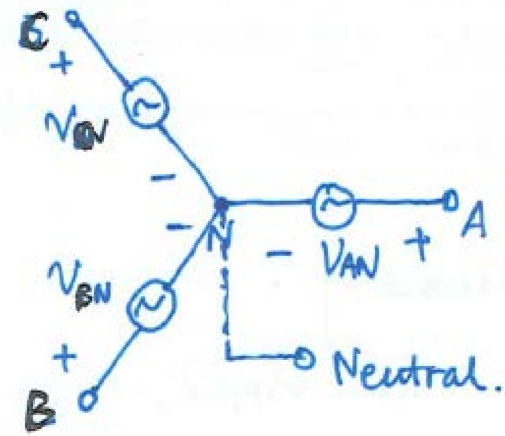
$$= V_{AN}\cdot I_A\left[\cos\varphi + \underline{\cos 2(\omega t-\varphi)}\right]$$

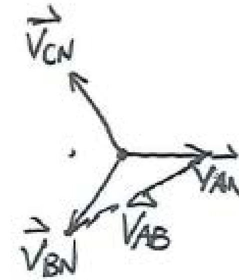
$p_B = \cdots$, $\quad p_C = \cdots$, Cancel the second part.

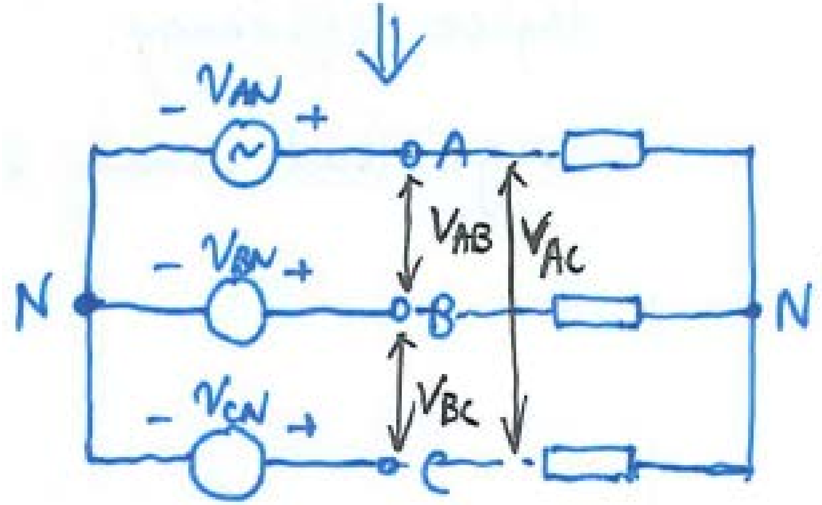
$$P = p_A + p_B + p_C = 3V_{AN}\cdot I_A\cdot\cos\varphi.$$

☆ V_{AN} is the effective phase voltage.

d) connections : star con.







Outline

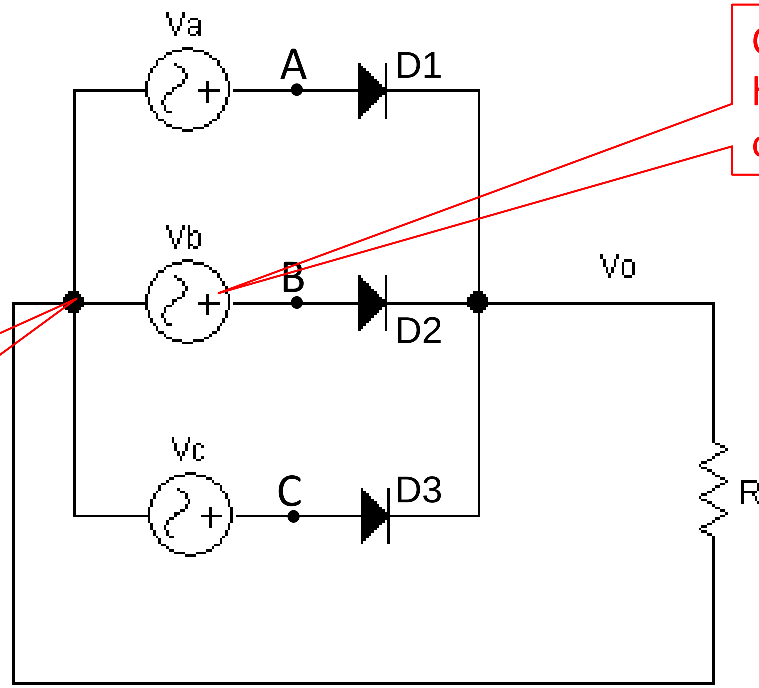
- Why 3-phase rectifiers?
- Half-bridge rectifiers
- Full-bridge rectifiers
  - With R load
  - With RL load
  - Harmonic analysis
- Design of rectifiers
  - Ratings of diodes
  - Filters: DC side and AC side
  - An example

1. Why three-phase rectifier?

- Single phase rectifiers are extensively used in low power applications particularly for power supplies to electronic circuits.
- Single phase rectifiers have several disadvantages:
  - Large output voltage and current form factor;
  - Large low frequency harmonic ripple current causing harmonic power loss and reduced efficiency;
  - Very large filter capacitor for obtaining smooth output dc voltage;
  - Low frequency harmonic current is injected in the input ac line which is difficult to filter. The situation becomes worse with capacitive loads.

2. Half bridge rectifier

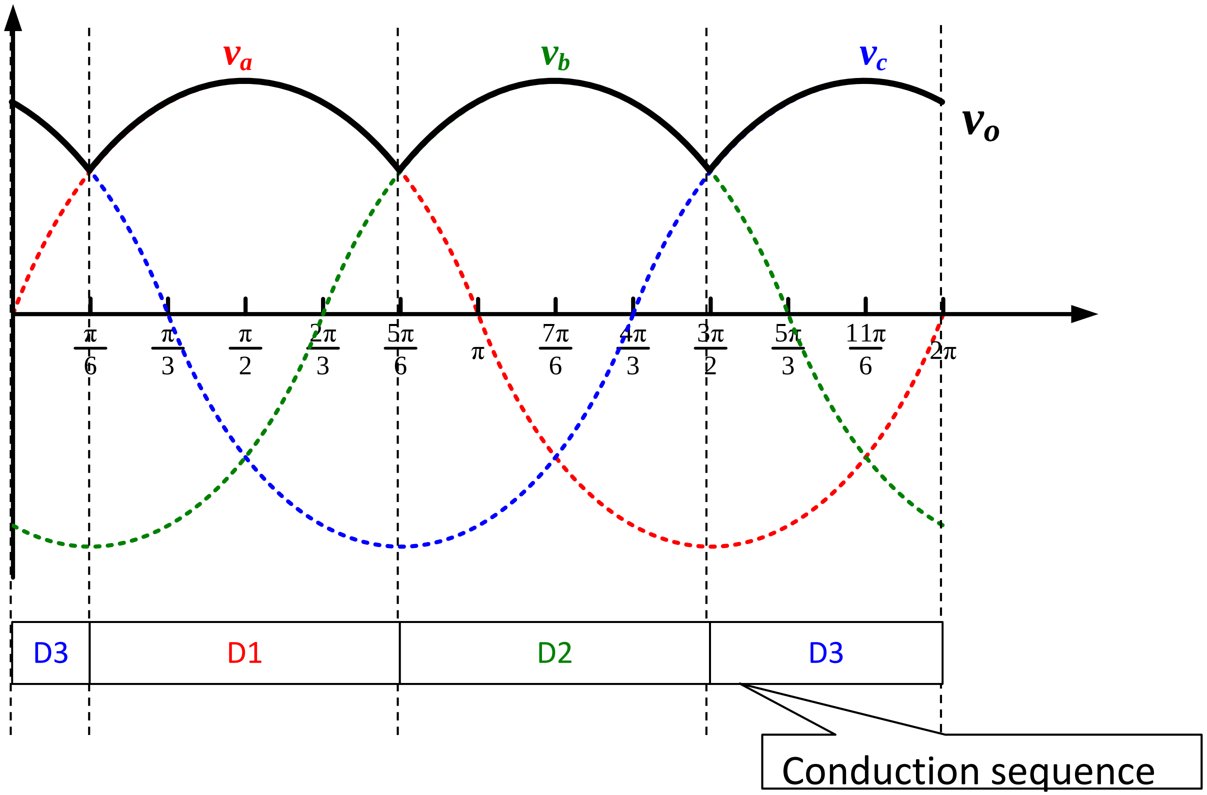
Compare v_a v_b and v_c, the highest one will support its corresponding diode to conduct.



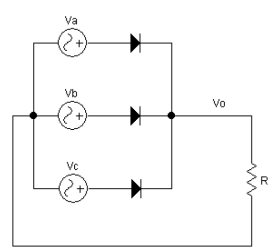
Negatives ends of the sources are connected together to ground (GND)

- Questions:
  - When does each diode conduct? How to determine that?
  - What is the waveform for output v_o?
  - What is the PIV (peak inverse voltage) on the diodes?

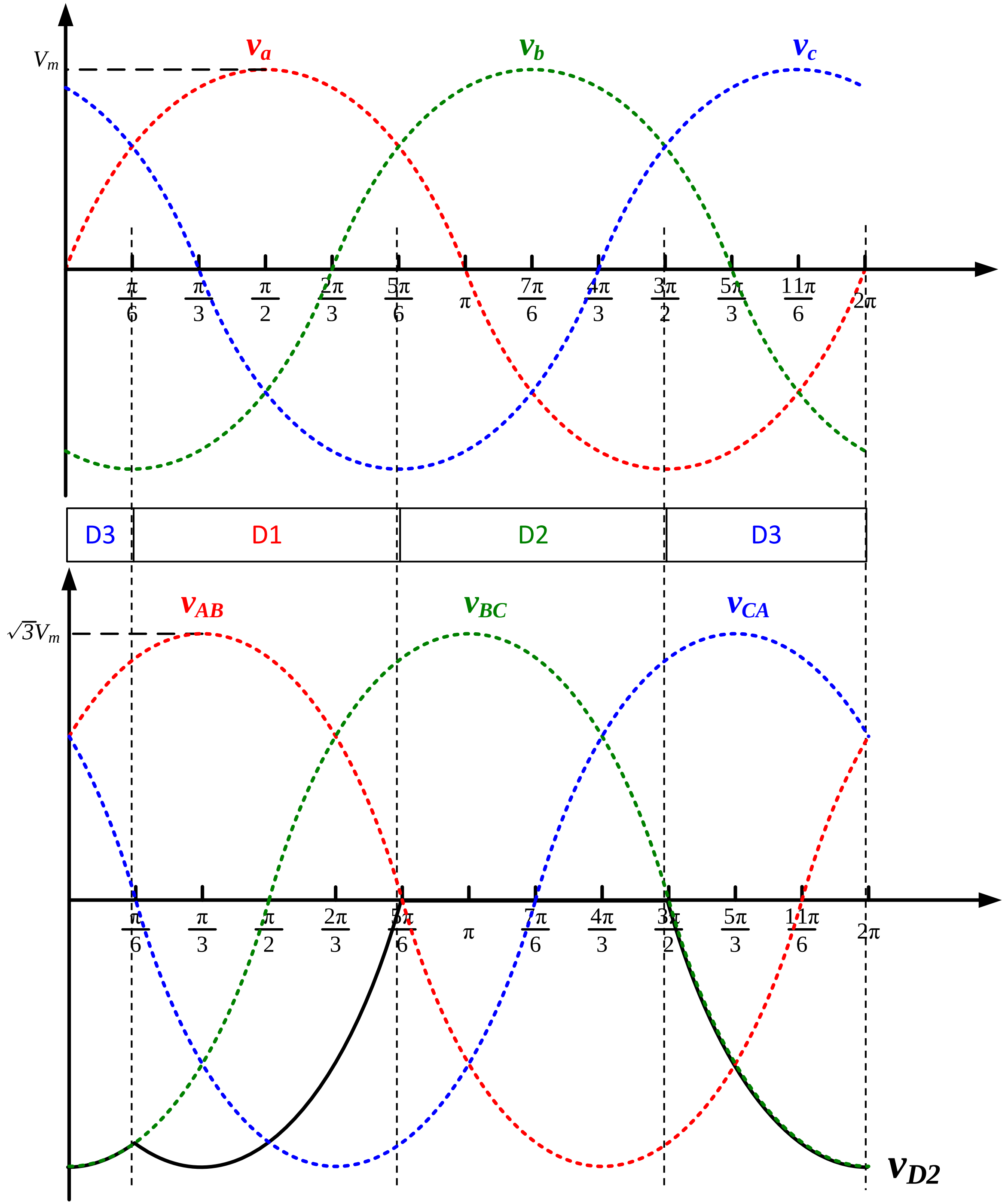
Compare v_a v_b and v_c during the whole period:

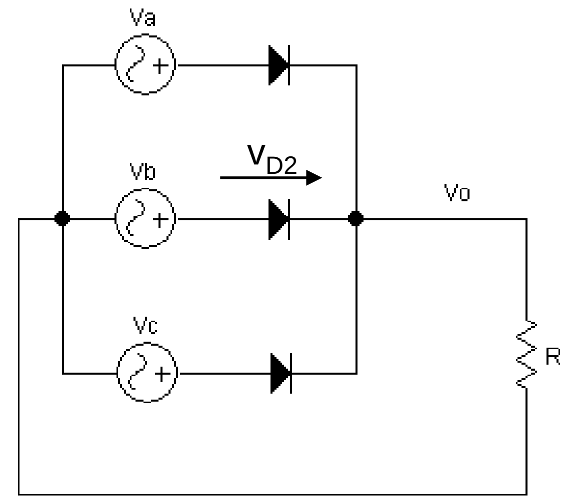


- Summary
  - Each diode conducts 120º in one period.



- The output voltage v_o:
- From 0 to π/6, blue line (v_c) is the highest one, so D3 conducts, D1 and D2 are stopped.
  - $v_o = v_c$ (phase voltage);
- From π/6 to 5π/6, red line (v_a) is the highest one, so D1 conducts, D2 and D3 are stopped.
  - $v_o = v_a$;
- From 5π/6 to 3π/2, green line (v_b) is the highest one, so D2 conducts, D1 and D3 are stopped.
  - $v_o = v_b$;
- From 3π/2 to 2π, blue line (v_c) is the highest one again, so D3 conducts again, D1 and D2 are stopped.
  - $v_o = v_c$





- The voltage on the diode v_D:
- From 0 to π/6, D3 conducts, D1 and D2 are stopped.
  - $v_{D2} = v_b - v_c = v_{bc}$ (line voltage);
- From π/6 to 5π/6, D1 conducts, D2 and D3 are stopped.
  - $v_{D2} = v_b - v_a = v_{ba} = -v_{ab}$;
- From 5π/6 to 3π/2, D2 conducts.
  - $v_{D2} = 0$;
- Summary: the PIV is the peak value of line voltage: $\sqrt{3}V_m$.

Calculation of the key parameters I

- 1. Average output voltage

$$V_o = \frac{1}{T}\int_0^T v_o(t)dt = 0.83V_m \qquad V_o = \frac{3}{2\pi}\int_{\pi/6}^{5\pi/6} V_m \cos\omega t d\omega t = \frac{3V_m}{\pi}\sin\frac{\pi}{3} = 0.83V_m$$

- 2. RMS of the output voltage

$$V_{RMS} = \sqrt{\frac{1}{T}\int_0^T v_o^2(t)dt} = 0.84V_m \qquad V_{RMS} = \sqrt{\frac{3}{2\pi}\int_{\pi/6}^{5\pi/6} V_m{}^2 \cos^2\omega t d\omega t} = 0.84V_m$$

- 3. Efficiency

$$\eta = \frac{P_o}{P_{RMS}} = \left(\frac{V_o}{V_{RMS}}\right)^2 = 96.7\%$$

- 4. Ripple factor

$$f_F = \frac{V_{RMS}}{V_o} = 1.01 \quad \Rightarrow \quad f_R = \frac{V_{ac}}{V_o} = \sqrt{f_F^2 - 1} = 0.18$$

Calculation of the key parameters II

- 5. Diode current

$$I_s = \sqrt{\frac{1}{\pi}\int_0^{\pi/3}\left(\frac{V_m}{R}\right)^2 \cos^2 \omega t \, d\omega t} = \frac{V_{RMS}}{\sqrt{3}R} = 0.48\frac{V_m}{R}$$

- 6. Transformer utilisation factor

$$f_T = TUF = \frac{P_{dc}}{P_s} = \frac{V_o^2 / R}{3V_s I_s} = \frac{V_o^2 / R}{3\left(V_m/\sqrt{2}\right)\left(0.48V_m / R\right)} = 0.66$$

- 7. Power factor

$$f_P = \frac{P_{RMS}}{P_s} = \frac{V_{RMS}^2 / R}{3V_s I_s} = \frac{V_{RMS}^2 / R}{3\left(V_m/\sqrt{2}\right)\left(0.48V_m / R\right)} = 0.68$$

- 8. Peak inverse voltage of each diode is equal to the peak value of the line voltage, which is

$$PIV = \sqrt{3}V_m$$

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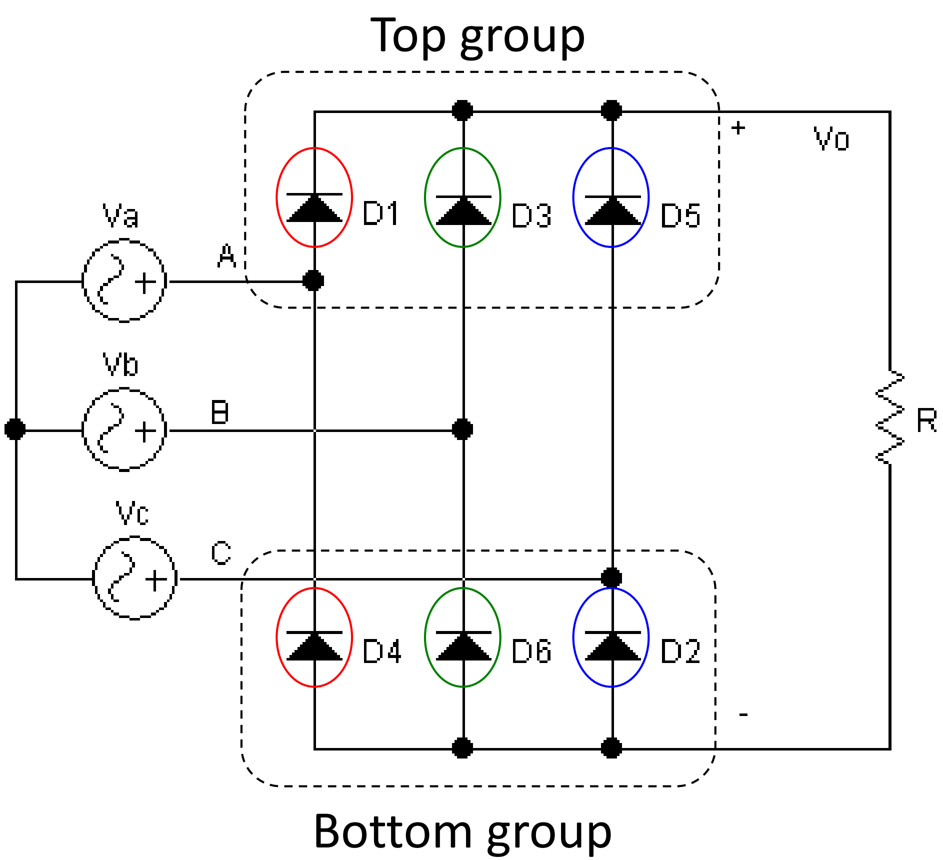
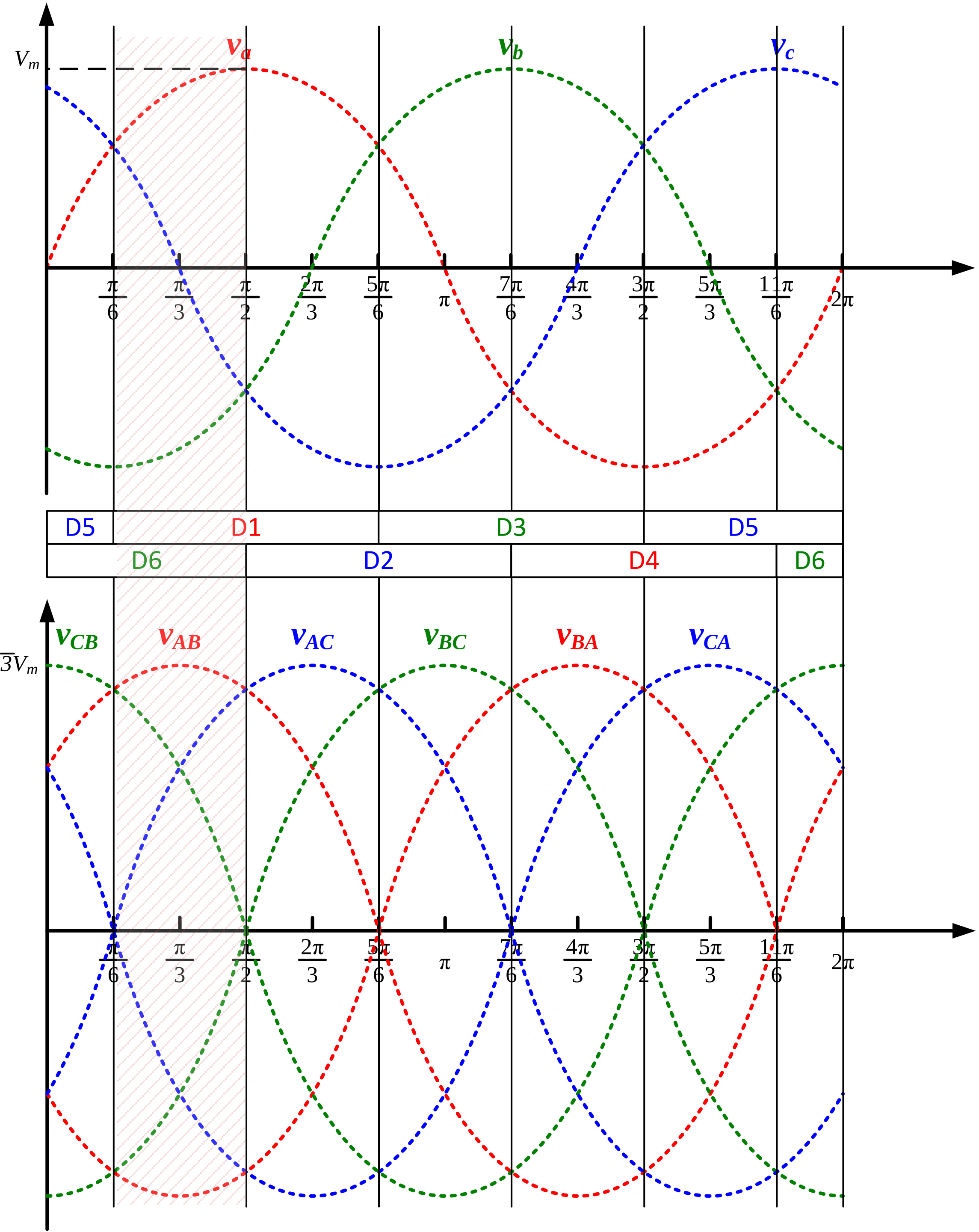
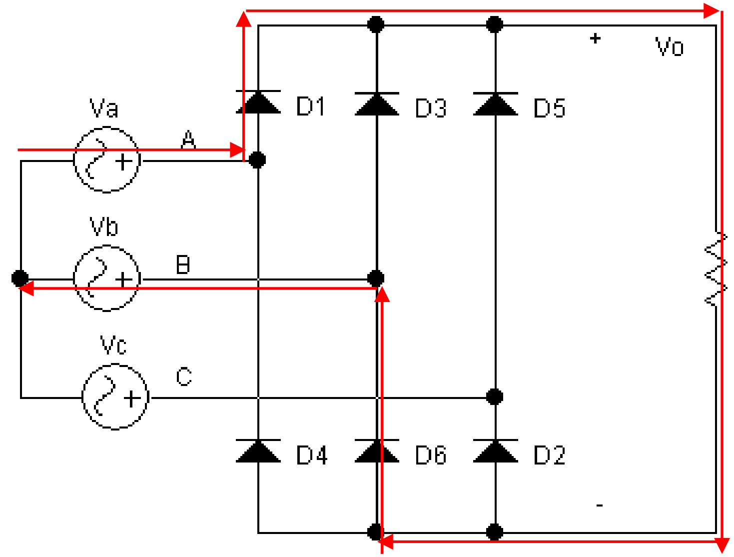
3.1 Full bridge rectifier

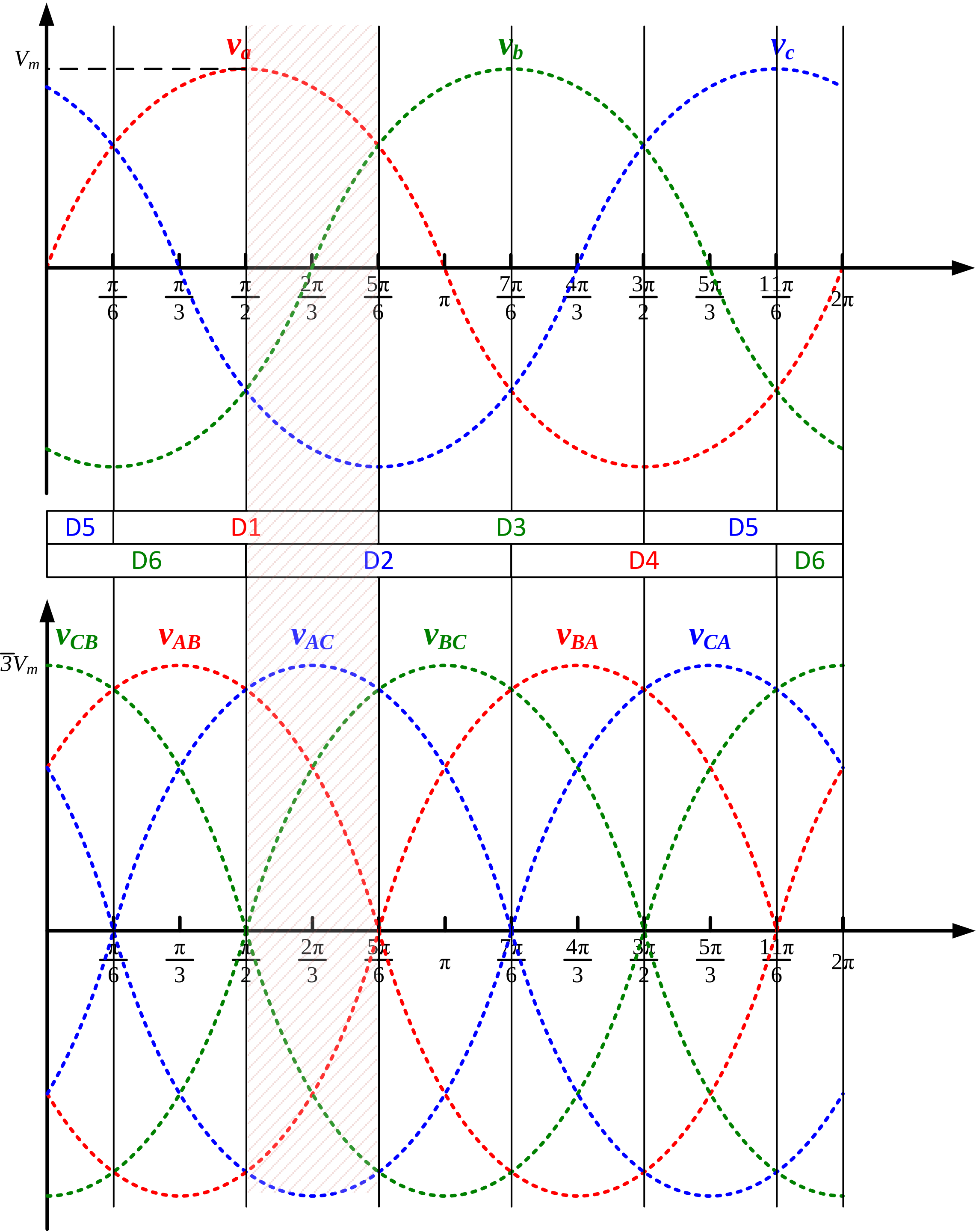


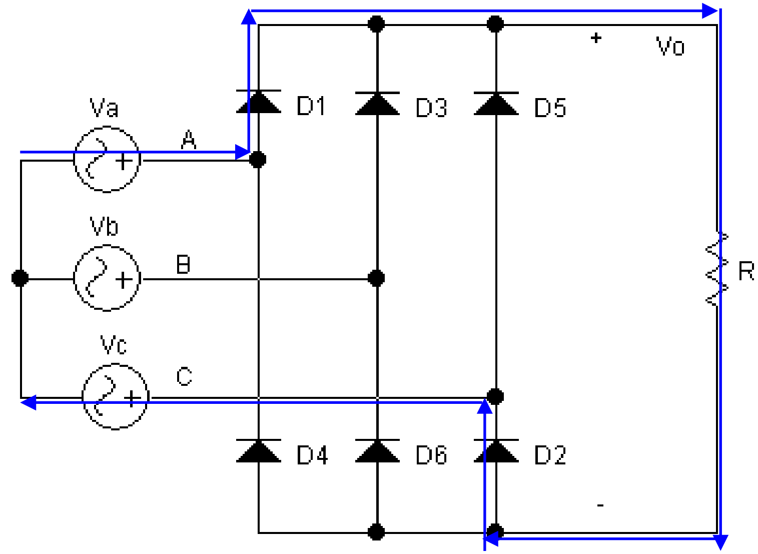
- Which diodes are conducting and how to determine that?
- To form a conduction loop, there always are two diodes conducting, one from top group and one from bottom group:
  - Top group: the one has the highest anode voltage;
  - Bottom group: the one has the lowest cathode voltage;



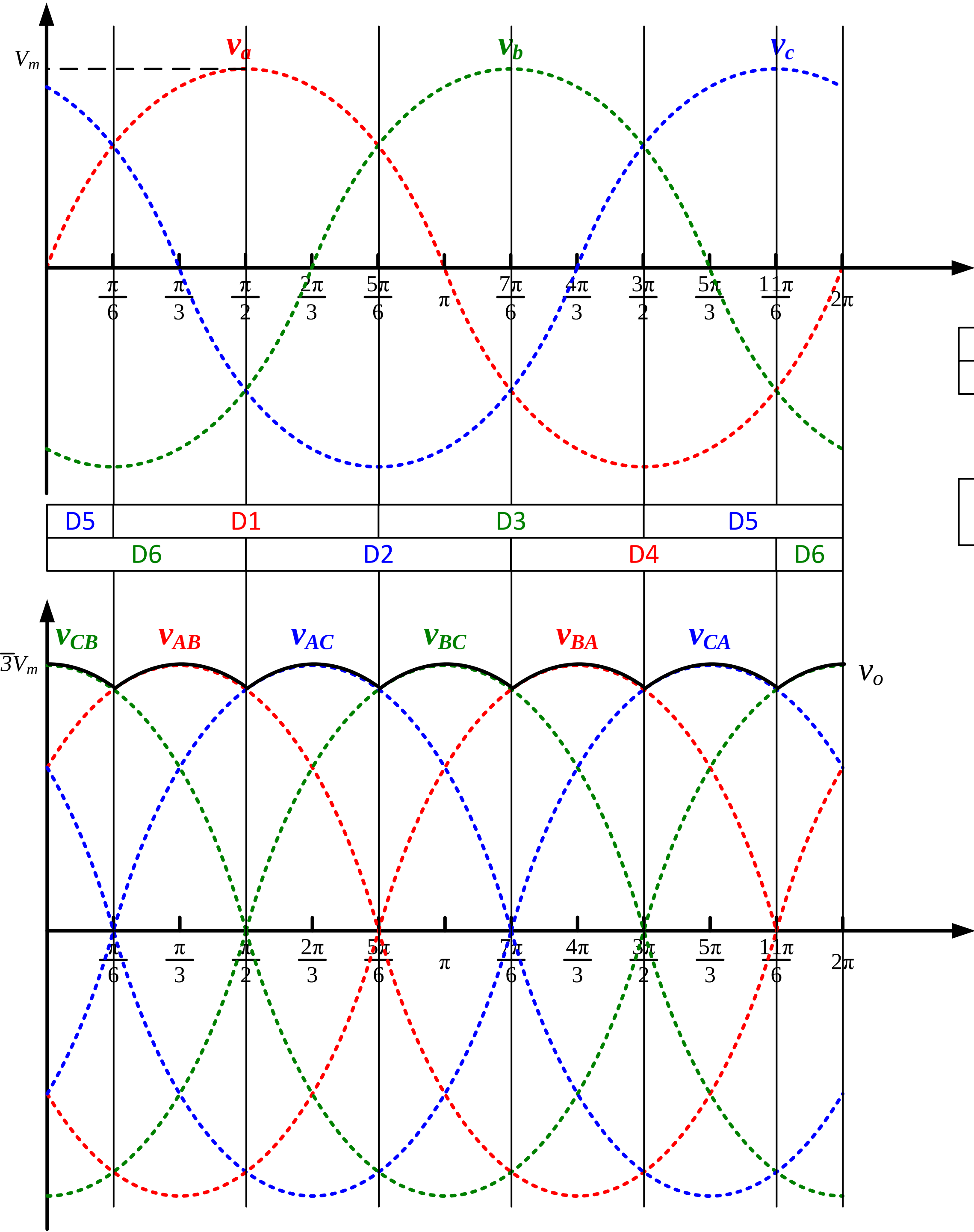


- From $\pi/6$ to $\pi/2$, v_a is the highest voltage and v_b is the lowest one.
  - So the top-group diode connected to v_a (D1) and the bottom-group diode connected to v_b (D6) are conducting;
  - The output voltage $v_o = v_a - v_b = v_{AB}$, the line voltage;





- From π/2 to 5π/6, v_a is still the highest voltage and v_c is the lowest one now.
  - So the top-group diode connected to v_a (D1) and the bottom-group diode connected to v_c (D2) are conducting;
  - The output voltage $v_o = v_a - v_c = v_{AC}$, the line voltage;

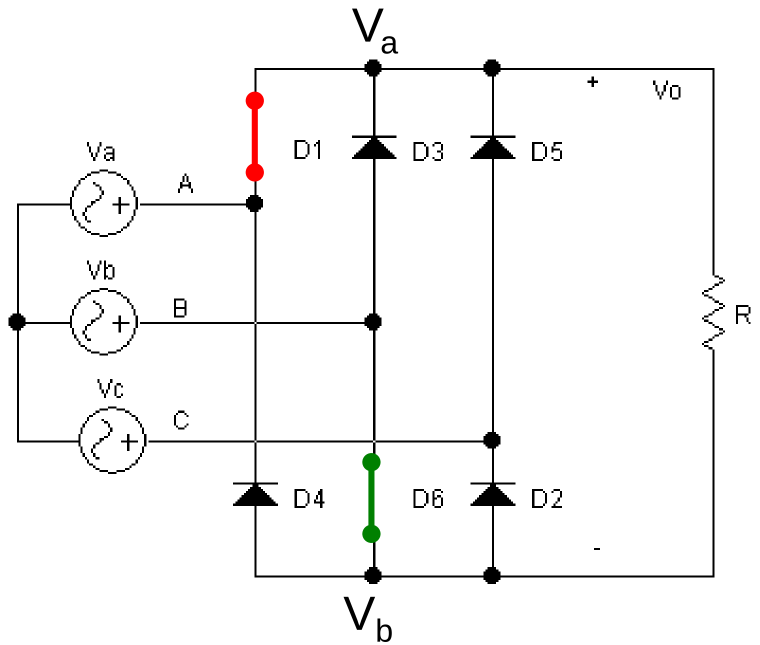


- Conduction sequence (Sequence of commutation)

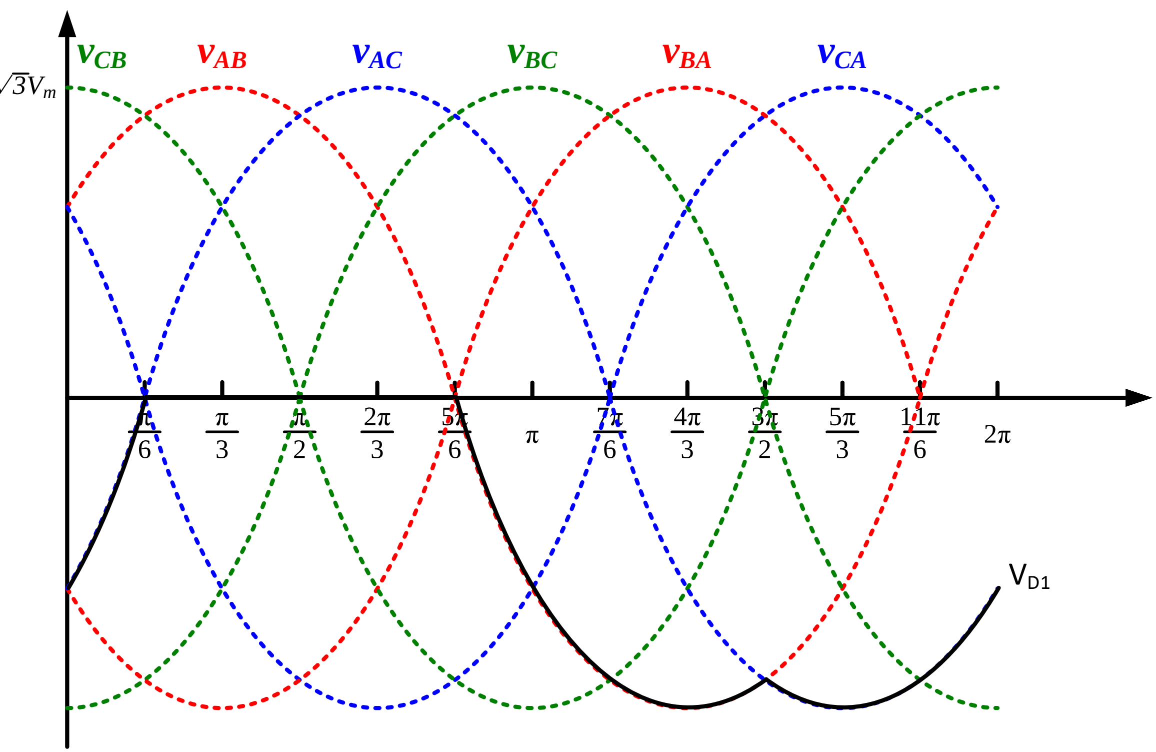
<table>
<tr><td>D5</td><td colspan="2">D1</td><td colspan="2">D3</td><td colspan="2">D5</td></tr>
<tr><td colspan="2">D6</td><td colspan="2">D2</td><td colspan="2">D4</td><td>D6</td></tr>
</table>

56	16	12	23	34	45	56

- Each diode conducts 120°, but in two slots
- Output voltage: 6-pulse ripple

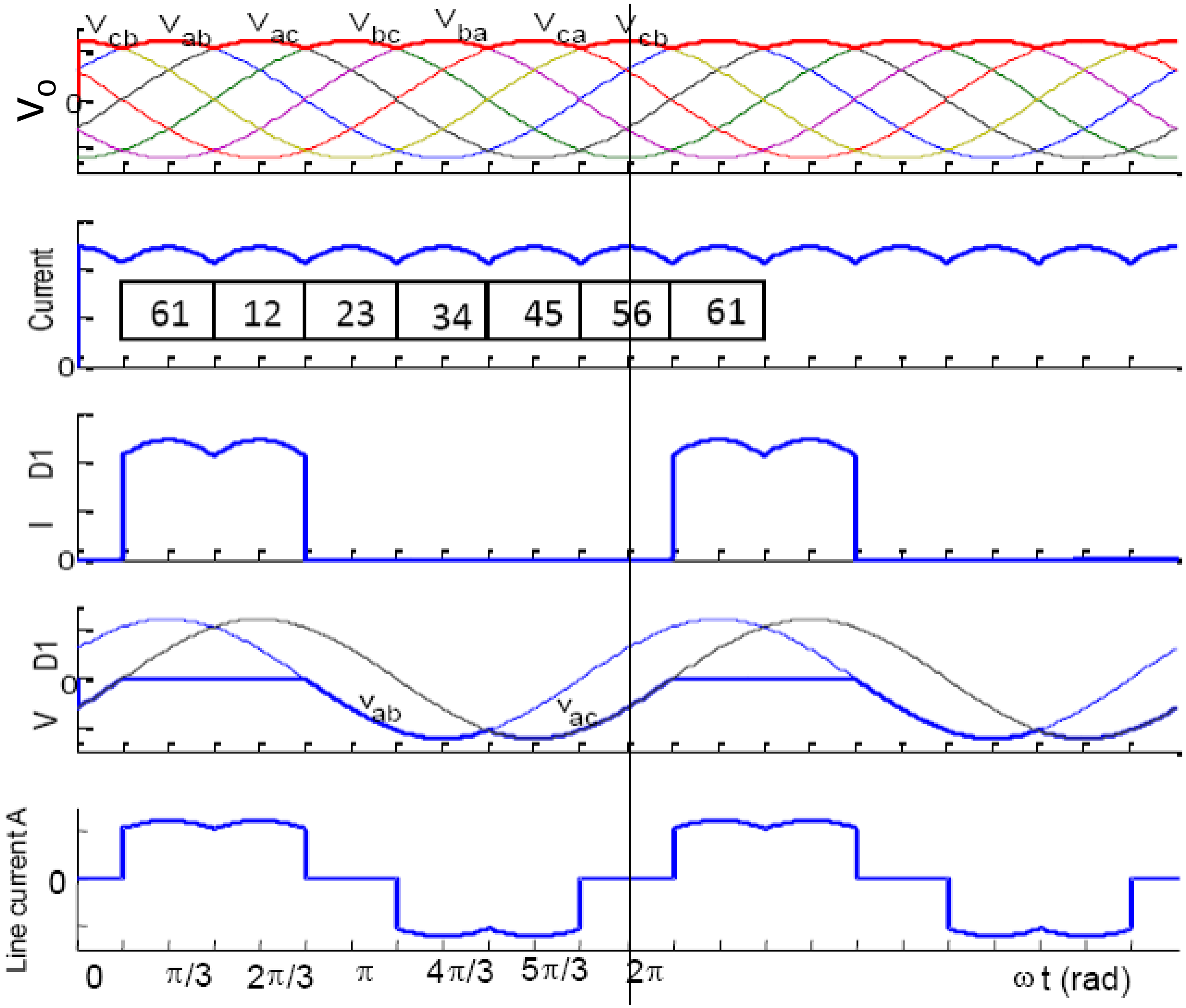


- (π/6, π/2), D1 and D6 are conducting;
  - $V_{D1} = 0$ (since D1 is conducting);
  - $V_{D3} = v_b - v_a = v_{BA}$;
  - $V_{D5} = v_c - v_a = v_{CA}$;
  - $V_{D6} = 0$ (since D6 is conducting);
  - $V_{D2} = v_b - v_c = v_{BC}$;
  - $V_{D4} = v_b - v_a = v_{BA}$;



- V_{D1} on the whole period (0, 2π):
  - (π/6, 5π/6), $V_{D1} = 0$ (since D1 is conducting);
  - (5π/6, 3π/2), D3 conducts, D1 stops. So $V_{D1} = v_a - v_b = v_{AB}$;
  - (3π/2, 2π) and (0, π/6), D5 conducts, D1 stops. So $V_{D1} = v_a - v_c = v_{AC}$;
  - The peak inverse voltage is the peak value of line voltage, so it is $\sqrt{3}V_m$

Waveforms



V_o
V_{cb} V_{ab} V_{ac} V_{bc} V_{ba} V_{ca} V_{cb}
0
Current
61 12 23 34 45 56 61
0
I_{D1}
0
V_{D1}
0
v_{ab}
v_{ac}
Line current A
0
0 π/3 2π/3 π 4π/3 5π/3 2π
ω t (rad)

Calculation of the key parameters I

- 1. Phase A voltage $v_a = V_m \sin \omega t$
- 2. Line voltage $v_{ab} = \sqrt{3} V_m \sin(\omega t + \frac{\pi}{6})$
- 3. The average voltage

$$V_0 = \frac{1}{2\pi} \times 6 \times \int_{\pi/6}^{\pi/6+\pi/3} v_{ab} d(\omega t) = \frac{3}{\pi} \int_{\pi/6}^{\pi/6+\pi/3} \sqrt{3} V_m \sin(\omega t + \frac{\pi}{6}) d(\omega t)$$

$$= \frac{3\sqrt{3} V_m}{\pi} (\cos \frac{\pi}{3} - \cos \frac{2\pi}{3})$$

$$= \frac{3\sqrt{6}}{\pi} V_s \approx 2.34 V_s = 1.35 V_{sl}$$

  – where V_s is the rms value of the phase voltage V_{sl} is the rms value of the line voltage

Calculation of the key parameters II

- 4. The RMS voltage

$$V = \sqrt{\frac{1}{2\pi} \times 6 \times \int_{\pi/6}^{\pi/6+\pi/3} v_{ab}^2 d(\omega t)} = 3V_m \sqrt{\frac{1}{\pi} \int_{\pi/6}^{\pi/6+\pi/3} \sin^2(\omega t + \frac{\pi}{6}) d(\omega t)}$$

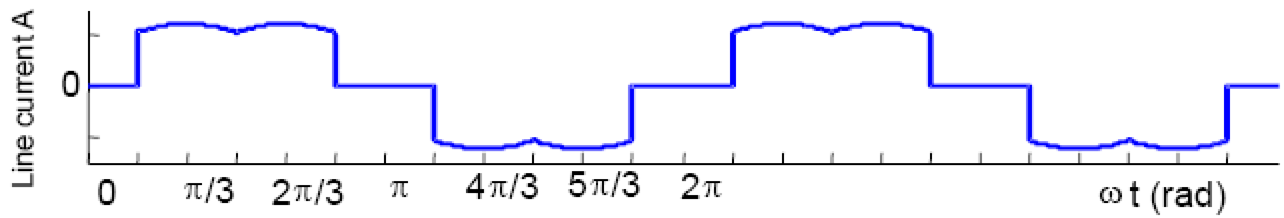
$$= 3V_m \sqrt{\frac{1}{\pi}(\frac{\pi}{6} + \frac{\sqrt{3}}{4})} = \sqrt{\frac{\pi^2}{18} + \frac{\pi}{4\sqrt{3}}} \cdot \frac{3\sqrt{6}}{\pi} V_s$$

$$\approx 1.0009 \cdot V_0$$

  - This is almost the same as V_0.
  - Hence, the efficiency is high.

Calculation of the key parameters III

- 5. Line current



- 6. The rms value of the line current (R load) is

$$I_{sl} = \sqrt{\frac{1}{2\pi} \times 4 \times \int_{\pi/6}^{\pi/6+\pi/3} \left(\frac{v_{ab}}{R}\right)^2 d(\omega t)} = \frac{V_m}{R}\sqrt{\frac{2}{\pi}\int_{\pi/6}^{\pi/6+\pi/3} 3\sin^2(\omega t + \frac{\pi}{6}) d(\omega t)}$$

$$= \frac{V_m}{R}\sqrt{\frac{6}{\pi}\int_{\pi/6}^{\pi/6+\pi/3} \sin^2(\omega t + \frac{\pi}{6}) d(\omega t)}$$

$$= \frac{\sqrt{3}V_m}{R}\sqrt{\frac{2}{\pi}\left(\frac{\pi}{6} + \frac{\sqrt{3}}{4}\right)} \approx 0.78 I_{ml}$$

  – where $I_{ml} = \sqrt{3}V_m / R$ is the peak line current (the same as the peak current through a diode).

Calculation of the key parameters IV

- 7. The rms value of the diode current is

$$I_D = I_{s1}/\sqrt{2} \approx 0.55 I_{m1}$$

  - Because

$$i_A = i_{D1} - i_{D4}$$

$$\Downarrow \text{square}$$

$$i_A^2 = i_{D1}^2 + i_{D4}^2 - 2 i_{D1} i_{D4} \quad \Leftarrow \quad i_{D1} i_{D4} = 0$$

$$= i_{D1}^2 + i_{D4}^2$$

$$\Downarrow \text{average}$$

$$I_A^2 = I_{D1}^2 + I_{D4}^2 = 2 I_D^2$$

$$\Downarrow \text{rms value}$$

$$I_{s1} = \sqrt{2} I_D$$

An example

A three-phase bridge rectifier is connected to a resistive load R=10 ohm. The RMS phase voltage is 100V.

- Determine
  - The average voltage
  - The rms voltage
  - The average current
  - The rectification ratio/efficiency
  - The output DC power
  - The peak line (diode) current
  - The rms line current
  - The rms diode current
  - The average diode current
  - The transformer utilization factor
  - The peak reverse voltage of diodes

An example

$V_s = 100\,V$

$V_m = \sqrt{2} V_s = 141\,V$

$V_{sL} = \sqrt{3} V_s = 172\,V$

$V_{mL} = \sqrt{6} V_s = 244.9\,V$

1) $V_o = 2.34 V_s = 234\,V$

2) $V_{rms} = 1.0009 V_o = 234.2\,V$

3) $I_o = \dfrac{V_o}{R} = 23.4\,A.$

4) $\eta = \dfrac{P_o}{P_{rms}} = \left(\dfrac{V_o}{V_{rms}}\right)^2 = 99.8\%$

5) $P_o = V_o I_o = 5.476\,kW.$

6) $I_{mL} = \dfrac{V_{mL}}{R} = \dfrac{\sqrt{6} V_s}{R} = 24.49\,A$

7) $I_{sL} = 0.78 I_{mL} = 19.1\,A$

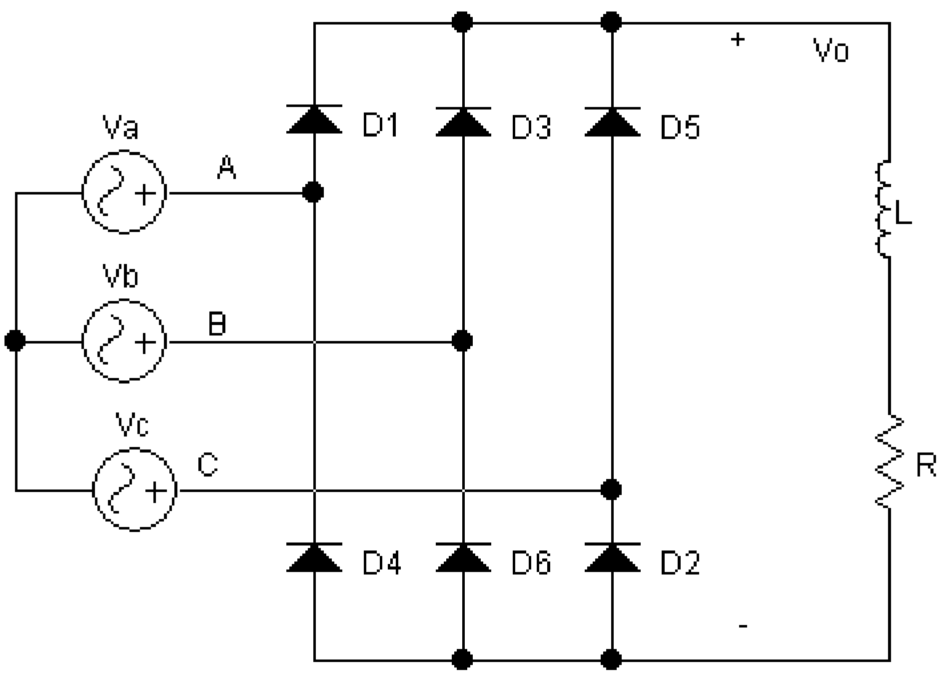
8) $I_D = \dfrac{I_{sL}}{\sqrt{2}} = 13.5\,A$

9) $I_{DO} = \dfrac{I_o}{3} = 7.8\,A$

11) $PIV = V_{mL} = 244.9\,V.$

- Determine
  1. 1. The average voltage
  2. 2. The rms voltage
  3. The average current
  4. The rectification ratio/efficiency
  5. The output DC power
  6. The peak line (diode) current
  7. The rms line current
  8. The rms diode current
  9. The average diode current
  10. The transformer utilization factor
  11. The peak reverse voltage of diodes

3.2 Full bridge rectifier with RL load



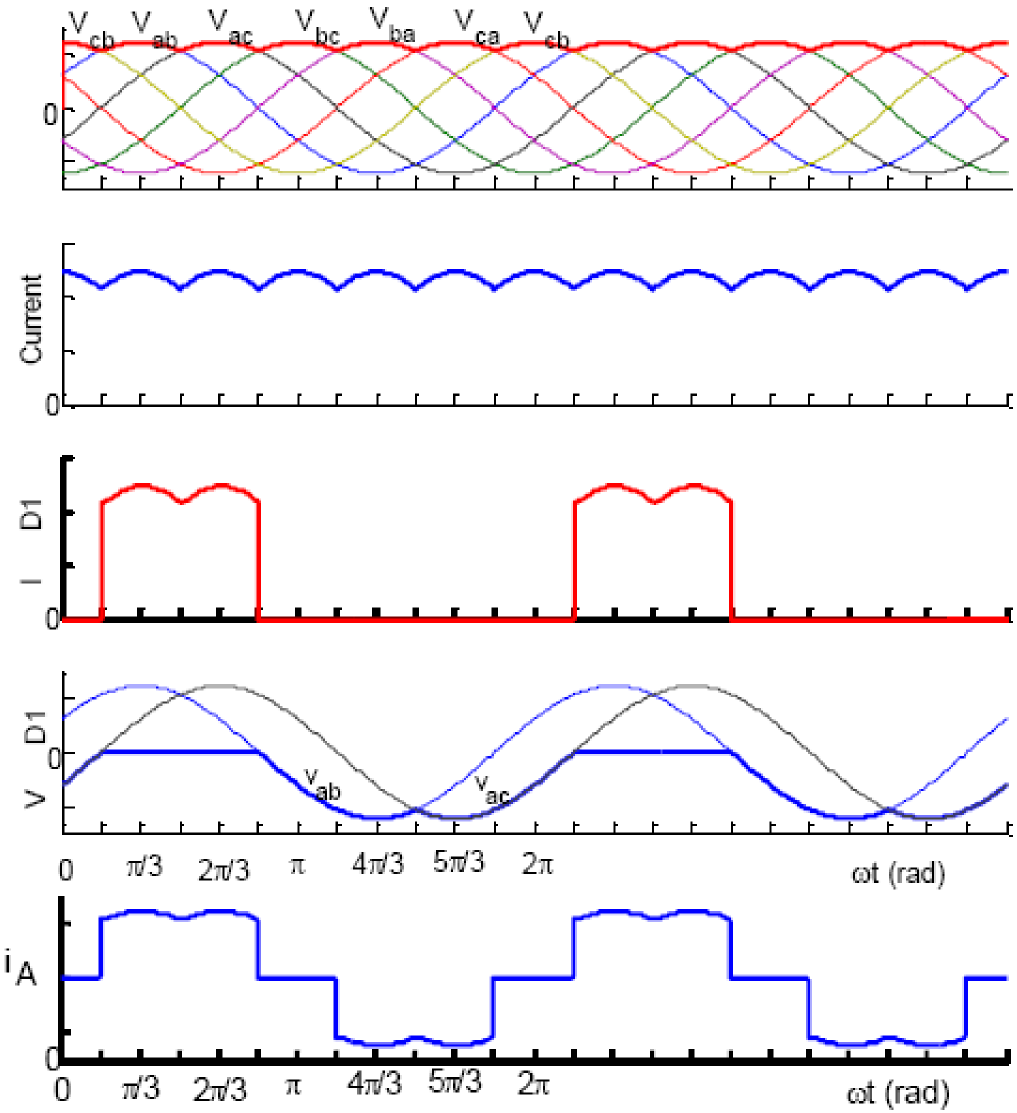
- Sequence of commutation

	D1	D3	D5	D1
D6	D2	D4	D6	

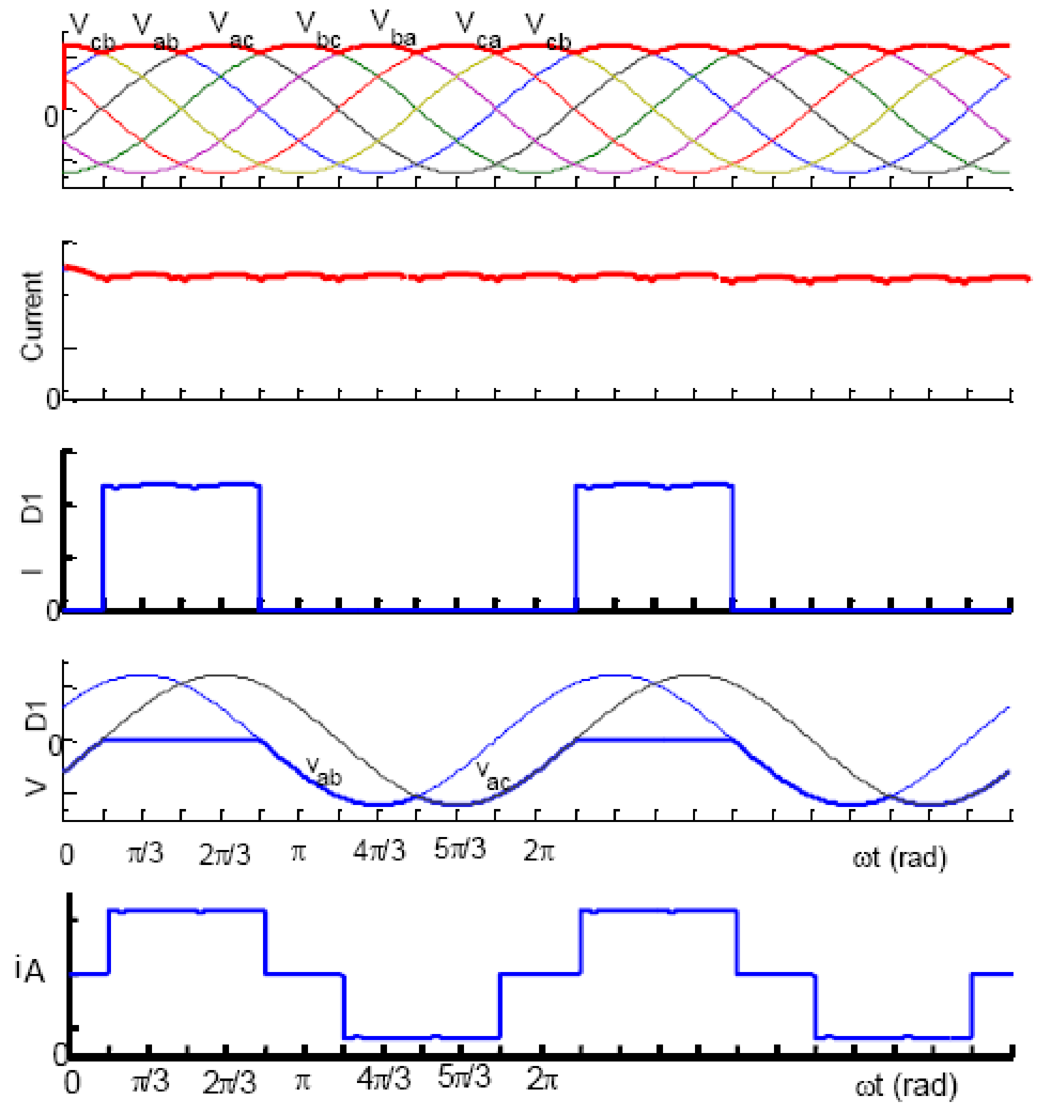
61	12	23	34	45	56	61

- An inductor is often used to smooth the load current.
- When the inductor is large enough, the current is a straight line.

Small L

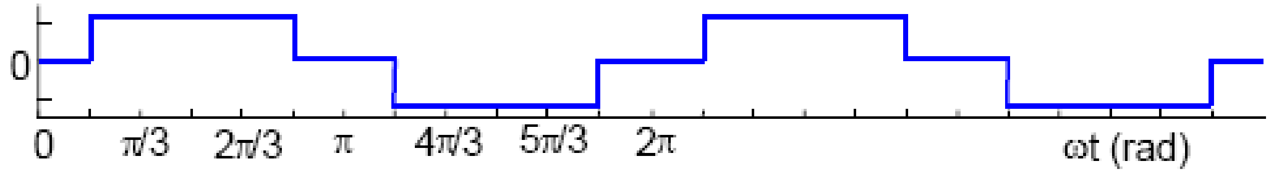


Large L



Harmonic analysis I

- Line current



- When the inductor is large enough, the ripple in the load current is negligible. The line current (phase A) can be described by

$$i_a(t) = \begin{cases} I_0 & (\frac{\pi}{6} \le \omega t \le \frac{5\pi}{6}) \\ -I_0 & (\frac{7\pi}{6} \le \omega t \le \frac{11\pi}{6}) \\ 0 & otherwise \end{cases}$$

- This can be expressed in a Fourier series as

$$i_a(t) = \frac{2\sqrt{3}}{\pi} I_0 \left(\sin \omega t - \frac{1}{5} \sin 5\omega t - \frac{1}{7} \sin 7\omega t + \frac{1}{11} \sin 11\omega t + \cdots \right)$$

Harmonic analysis II

- The rms value of the fundamental component is

$$I_{a1} = \frac{\sqrt{6}}{\pi} I_0$$

- The rms line current is

$$I_a = \sqrt{\frac{2}{2\pi}\int_{\pi/6}^{5\pi/6} I_0^2 d(\omega t)} = \sqrt{\frac{2}{3}} I_0$$

- The total harmonic distortion (THD) is

$$THD = \sqrt{(\frac{I_a}{I_{a1}})^2 - 1} \times 100\% = \sqrt{\frac{2/3}{6/\pi^2} - 1} \times 100\% = 31.08\%$$

- Since the fundamental component is in phase with the phase voltage, the displacement factor cosϕ is 1 and the power factor is

$$f_P = \frac{I_{a1}}{I_a}\cos\phi = \frac{\sqrt{6}}{\pi} / \sqrt{\frac{2}{3}} = \frac{3}{\pi} \approx 0.955$$

Comparison with the rectifier with R load

TABLE 3.2 Performance Parameters of Diode Rectifiers with a Resistive Load

Performance Parameters	Single-Phase Bridge Rectifier	Three-Phase Bridge Rectifier
Peak repetitive reverse voltage, V_{RRM}	$1.57V_{dc}$	$1.05V_{dc}$
Rms input voltage per transformer leg, V_s	$1.11V_{dc}$	$0.428V_{dc}$
Diode average current, $I_{F(AV)}$	$0.50I_{dc}$	$0.333I_{dc}$
Peak repetitive forward current, I_{FRM}	$1.57I_{dc}$	$3.14I_{dc}$
Diode rms current, $I_{F(RMS)}$	$0.785I_{dc}$	$0.579I_{dc}$
Form factor of diode current, $I_{F(RMS)}/I_{F(AV)}$	1.57	1.74
Rectification ratio, η	0.81	0.998
Form factor, FF	1.11	1.0009
Ripple factor, RF	0.482	0.042
Transformer rating primary, VA	$1.23P_{dc}$	$1.05P_{dc}$
Transformer rating secondary, VA	$1.23P_{dc}$	$1.05P_{dc}$
Output ripple frequency, f_r	$2f_s$	$6f_s$

4. Design of rectifiers

- Specifications of rectifiers
  - Output (DC) voltage V_0
  - Output (DC) power P_0
  - Other parameters, listed in the table on last slide
- Ratings of diodes
  - Rating of diodes is the criterion for choosing diodes
  - An example
- Filters: DC side and AC side
  - To smooth the waveform and minimise harmonics
  - An example

Rating of diodes

- The design of a rectifier always involves determining the ratings of semiconductor diodes;

- Rating of diodes is the criterion for choosing the suitable diode for the design, including:
  - Peak diode current: I_{ml}
  - Peak inverse voltage: PIV
  - Average diode current: I_{D0}
  - RMS current: I_D (less important)

A design example

- A three-phase bridge rectifier supplies a highly inductive load such that the average load current is I_0=60A and the ripple current is negligible.
  - Determine the ratings of the diodes if the phase voltage is 100V at 50Hz.
- Solution
  - The average current through a diode is I_{D0}=60/3=20A.
  - The rms diode current is

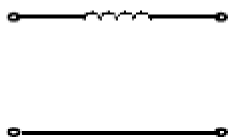
$$I_D = \sqrt{\frac{1}{2\pi}\int_0^{2\pi/3} I_0^2 d(\omega t)} = \frac{1}{\sqrt{3}} I_0 = 34.6 \text{ A}$$

  - The peak reverse voltage is

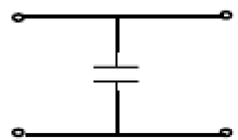
$$\sqrt{3}V_m = \sqrt{6}V_s = 244.9 \text{ V}$$

Filter

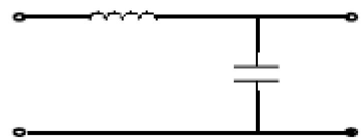
- DC side: In order to improve the quality of the output DC voltage, filters are often used on the DC side.



L filters

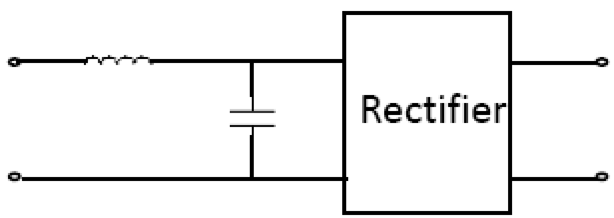


C filters



LC filters

- AC side: In order to reduce the harmonics on the AC input side, LC filters are often used on the AC side of rectifiers.



LC filters

A design example

- The single phase bridge rectifier is supplied from a 120V, 60Hz source. The load resistance is R=500ohm.
  - Calculate the value of a series inductor L that limits the RMS ripple current I_{ac} to less than 5% of I_{dc}.
- Solution
  - The load impedance is $Z = \sqrt{R^2 + (n\omega L)^2} \angle \theta_n$
  - and the load impedance angle is $\theta_n = \tan^{-1} \dfrac{n\omega L}{R}$
  - The Fourier series of the instantaneous current is

$$i_0(t) = I_{dc} - \frac{4V_m}{\pi\sqrt{R^2 + (n\omega L)^2}} \left[\frac{1}{3}\cos(2\omega t - \theta_2) + \frac{1}{15}\cos(4\omega t - \theta_4) \cdots \right]$$

  - where

$$I_{dc} = \frac{2V_m}{\pi R}$$

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A design example (cont.)

- The RMS value of the lowest order harmonic (n=2) of the ripple current is:

$$I_{ac} = \frac{4V_m}{\sqrt{2}\pi\sqrt{R^2 + (2\omega L)^2}}\left[\frac{1}{3}\right]$$

- Therefore, the ripple factor is

$$f_R = \frac{I_{ac}}{I_{dc}} = \frac{0.4714}{\sqrt{1^2 + (2\omega L / R)^2}}$$

- With the condition f_R=0.05 and R=500ohm, f=60Hz, the inductance value can be calculated:

L=6.22 H

- It can be observed that an inductance in the load offers a high impedance for the harmonics currents and acts like a filter in reducing the harmonics.