EEE108 Electromagnetism and Electromechanics

Lecture 15

Transducers; Linear Actuators; Basic Concept of Electrical Machines Fundamentals

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Today

- Transducers
- Linear Actuators

- Basic concept of electrical machines fundamentals
  - •Three basic elements: AC circuits
    DC circuits
  - •Rotational motion: Angular velocity
    Torque, Work, Power
  - •Three powers: Real, Reactive, and Apparent powers

Electric Machines

An electric machinery is a device that can convert either mechanical energy to electric energy or electric energy to mechanical energy.

The devices operate on similar principles but with various structures depending on their function:

Transducers: generally operate under linear input-output conditions and with relatively small signals. Used for measurement, control, … such as microphones, pickups, sensors, …

Force-producing devices: includes solenoids, relays and electromagnets

Linear actuators

Continuous energy-conversion devices: includes generators and motors

Transformers are usually studied together with generators and motors because they operate on the same principle.

Electrical Transducers

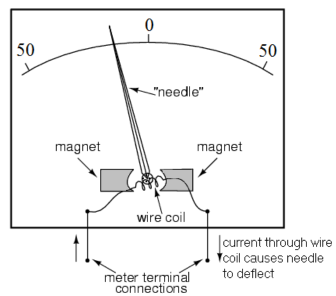
An electrical transducer changes electrical input to a mechanical or electrical output

Mechanical: analog scale
Electrical: numbers on a digital scale

A moving-coil transducer

A device that changes an input electrical signal into a mechanical output signal

Components
- •Permanent magnets
- •Moving coil around an iron core
- •Spring
- •Pointer
- •Scale

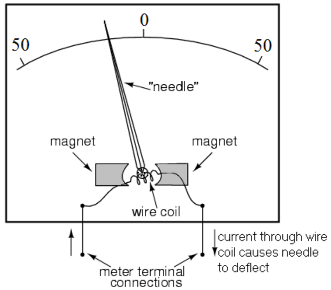


Moving Coil Transducer

How does it work?

Current flows through the coil around the iron core, creating a magnetic field that works with/against the permanent magnets. Like poles repel and unlike poles attract.

The needle position represents a balance between magnetic field strength and torsional force in the spring.



It can be calibrated to measure amps, volts, ohms, pounds, pressure, etc.

The meter reading is always proportional to the amount of current in the moving coil.

Wisely design, good maker, good material, …

A Linear Actuator

•The electric solenoid actuator

•Two basic laws govern solenoids:
Faraday's Law and Ampere's Law

•There are two main categories of solenoids:
Rotary and Linear

Linear solenoids have applications in appliances: vending machines, door locks, coin changers, circuit breakers, pumps, medical apparatus, automotive transmissions, postal machines…

Rotary solenoids have applications in machine tools, lasers, photo processing, media storage, medical apparatus, fire door closures, postal machines…

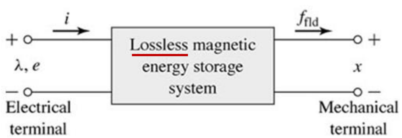
Electromechanical Energy Conversion

Various techniques have evolved to calculate the net forces or torques of concern in the electric machines.

In very simple cases, we can use the Lorentz force law: **F** = q(**E** + **v** x **B**)

In real electromechanical energy conversion devices, …

Energy method: based on the principle of conservation of energy.



$$\begin{pmatrix}\text{Energy input}\\ \text{from electric}\\ \text{sources}\end{pmatrix} = \begin{pmatrix}\text{Mechanical}\\ \text{energy output}\end{pmatrix} + \begin{pmatrix}\text{Increase in energy}\\ \text{stored in the system}\end{pmatrix}$$

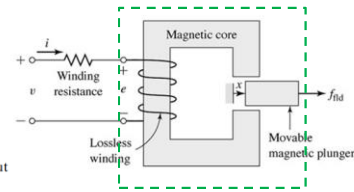
Electromechanical Energy Conversion

Energy method: based on the principle of conservation of energy
not require detailed knowledge of the field distribution

The most devices: * constructed of rigid, nondeforming structures
* performance determined by the net force/torque, acting on the moving component. It is rarely necessary to calculate the details of the internal force distribution.

The time rate of change of the stored energy in the magnetic field, W_{fld} :

$$\frac{dW_{fld}}{dt} = \underbrace{ei}_{\text{electric power input}} - \underbrace{f_{fld}\frac{dx}{dt}}_{\text{mechanical power output}}$$



With $e = \dfrac{d\lambda}{dt} \Rightarrow dW_{fld} = i\,d\lambda - f_{fld}\,dx$

Electromechanical Energy Conversion

For the lossless magnetic-energy-storage system:

$$\frac{dW_{fld}}{dt} = ei - f_{fld}\frac{dx}{dt} \quad \text{with} \quad e = \frac{d\lambda}{dt} \quad \text{we can have:} \quad \boxed{dW_{fld} = \underbrace{id\lambda}_{dW_{elec}} - \underbrace{f_{fld}dx}_{dW_{mech}}}$$

$$dW_{elec} = dW_{mech} + dW_{fld}$$

This equation together with Faraday's law forms the basis of the energy method.

where $dW_{elec} = id\lambda = eidt$: differential electric energy input

$dW_{mech} = f_{fld}dx$: differential mechanical energy output

dW_{fld} : differential change in megnetic stored energy

Electromechanical Energy Conversion

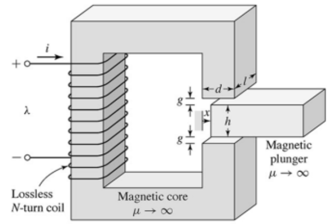
Example

The relay is made from infinitely-permeable magnetic material with a movable plunger, also of infinitely-permeable material. The height of the plunger is much greater than the air-gap length ($h >> g$).

Calculate the magnetic stored energy W_{fld} as a function of plunger position ($0 < x < d$) for N = 1000 turns, g = 2.0 mm, d = 0.15 m, l = 0.1 m and i = 10 A.

The energy stored in the system:

$$W_{fld} = \frac{1}{2}Li^2$$



Electromechanical Energy Conversion

Example

Solution

The self-inductance: $L = \frac{N\Phi_B}{I}$

If $\Re_c << \Re_g$: $\Phi \approx \frac{F}{\Re_g} = \frac{F\mu_0 A_g}{l_g} = NI\frac{\mu_0 A_g}{l_g}$

$$\boxed{L = \frac{\mu_0 N^2 A_g}{2g}}$$

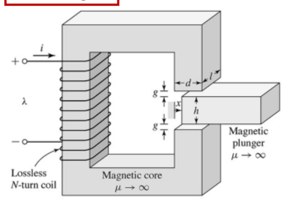
In the case shown: $W_{fld} = \frac{1}{2}L(x)i^2$

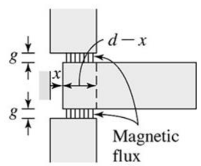
The inductance is: $L(x) = \frac{\mu_0 N^2 A_g}{2g}$

where A_g is the gap cross-sectional area:

$A_g = l(d-x) = ld(1-x/d)$

Then we have: $L(x) = \frac{\mu_0 N^2 ld(1-x/d)}{2g}$





Electromechanical Energy Conversion

Example

Solution

$$L(x) = \frac{\mu_0 N^2 ld(1-x/d)}{2g}$$

Then we have:

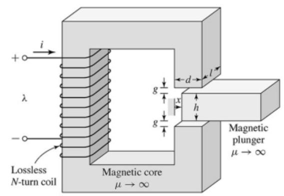
$$W_{fld} = \frac{1}{2}L(x)i^2 = \frac{1}{2}\frac{\mu_0 N^2 ld(1-x/d)}{2g}i^2$$

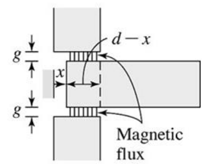
$$= \frac{1}{2}\frac{(4\pi\times10^{-7})\times1000^2\times0.1\times0.15\times(1-x/d)}{2\times2\times10^{-3}}10^2$$

$$= 236(1-\frac{x}{d}) \quad \text{J}$$

Position: x

λ

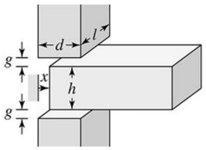




Determination of Magnetic Force from Energy

Rewrite $dW_{fld} = id\lambda - f_{fld}dx \quad \Rightarrow \quad dW_{fld}(\lambda, x) = id\lambda - f_{fld}dx \quad (1)$

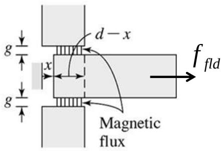
Then: $dW_{fld}(\lambda, x) = \left.\frac{\partial W_{fld}}{\partial \lambda}\right|_{x=C} d\lambda + \left.\frac{\partial W_{fld}}{\partial x}\right|_{\lambda=C} dx \quad (2)$



since λ and x are independent variables, then the equations 1 and 2 must be equal for all values of $d\lambda$ and dx:

$$i = \left.\frac{\partial W_{fld}}{\partial \lambda}\right|_{x=C} \quad (3) \quad \text{and}$$

$$f_{fld} = -\left.\frac{\partial W_{fld}}{\partial x}\right|_{\lambda=C} \quad (4)$$



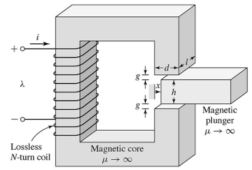
Once we know W_{fld} as a function of λ and x,

Equation (3) can be used to solve for $i(\lambda, x)$,

Equation (4) can be used to get the mechanical force $f_{fld}(\lambda, x)$.

Determination of Magnetic Force from Energy

For linear magnetic systems with $\lambda = L(x)i$, the energy stored in the magnetic system: $W_{fld} = \frac{1}{2}\frac{\lambda^2}{L(x)}$ then we have:



$$f_{fld} = -\frac{\partial}{\partial x}\left(\frac{1}{2}\frac{\lambda}{L(x)}\right)_{\lambda=C} = \frac{\lambda^2}{2[L(x)]^2}\frac{dL(x)}{dx}$$

With $\lambda = L(x)i \quad \Rightarrow \quad f_{fld} = \frac{i^2}{2}\frac{dL(x)}{dx}$ This is the basis of the electric solenoid actuator.

It is usually assumed that the current will be kept constant as the plunger moves, so i is independent of x.

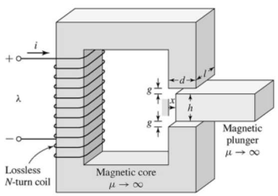
Therefore, the calculation of the force is reduced to finding the inductance as a function of x, the displacement of the plunger.

Determination of Magnetic Force from Energy

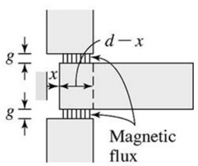
From: $L(x) = \frac{\mu_0 N^2 ld(1 - x/d)}{2g}$

$$\frac{dL(x)}{dx} = -\mu_0 \frac{N^2}{2g} l$$

Then we have: $f_{fld} = \frac{i^2}{2}\frac{dL(x)}{dx} = -\frac{i^2}{2}\frac{\mu_0 N^2 l}{2g}$



Note that the negative sign indicates that the force is in the $-x$ direction, drawing the plunger into the gap.



Equilibrium Position of a Plunger

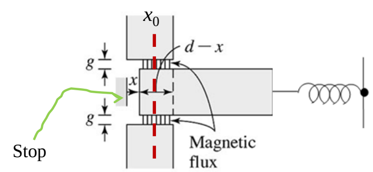
$$f_{fld} = \frac{i^2}{2}\frac{dL(x)}{dx} = -\frac{i^2}{2}\frac{\mu_0 N^2 l}{2g}$$

The force is proportional to i^2, so the polarity of the current flow does not affect the direction of the force. To extract the plunger, a spring may be needed.

With a spring constant K, the equilibrium position at $x = x_0$ is given by

$$\mu_0 \frac{N^2}{4g} li^2 = K(d - x_0)$$

$$x_0 = d - \mu_0 \frac{N^2}{4gK} li^2$$



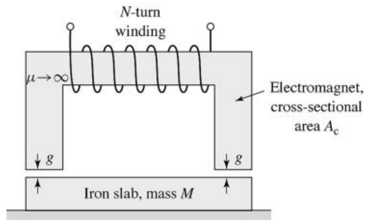
Determination of Magnetic Force from Energy

Example

An N-turn electromagnet is to be used to lift a slab of iron of mass M. The surface roughness of the iron is such that when the iron and the electromagnet are in contact, there is a minimum air gap of $g_{min} = 0.18$ mm in each leg. The electromagnet cross-section area $A_c = 32$ cm^2 and coil resistance is 2.8 ohms.

Calculate the minimum coil voltage which must be used to lift a slab of mass 95 kg against the force of gravity.

Neglect the reluctance of the iron.



Determination of Magnetic Force from Energy

Example

The coil inductance : $L = \dfrac{\mu_0 N^2 A_c}{2g}$

The lifting force : $f_{fld} = \dfrac{i^2}{2}\dfrac{dL(g)}{dg} = -\left(\dfrac{\mu_0 N^2 A_c}{4g^2}\right) i^2$

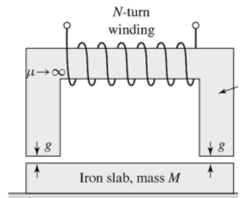
$\Rightarrow \quad i = \dfrac{2g}{N}\sqrt{\dfrac{f_{fld}}{\mu_0 A_c}}$

the minus sign indicates that the force acts in the direction to reduce the gap.

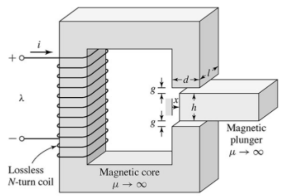
The reqiured force : $95\ \text{kg} \times 9.8\ \text{m/sec}^2 = 931\ \text{N}$

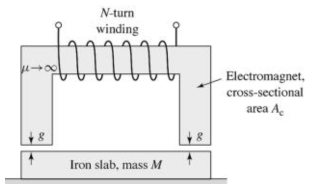
Setting $g = g_{min} \Rightarrow i_{min} = \dfrac{2g_{min}}{N}\sqrt{\dfrac{f_{fld}}{\mu_0 A_c}} = 385\ \text{mA}$

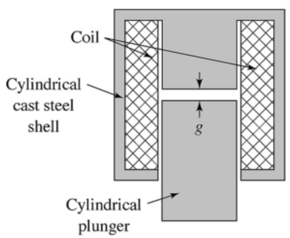
$v_{min} = i_{min} R = 385 \times 10^{-3} \times 2.8 = 1.08 \quad \text{V}$

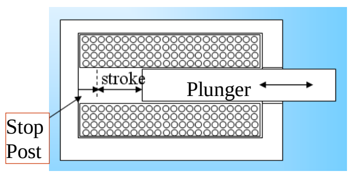


Geometry of Linear Actuators

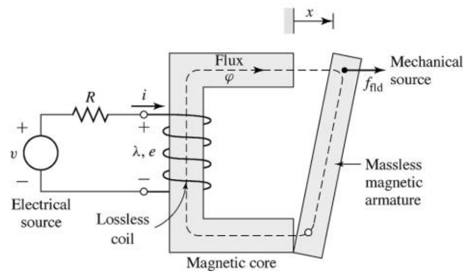








Geometry of Linear Actuators



Summary

- An electrical transducer changes electrical input to a mechanical or electrical output.
- Solenoids convert electrical energy into force and motion.
- When the coil is energized with electric current an electro-magnetic force is created around the coil. Enclosed solenoids are designed to direct that magnetic force through the steel housing and into the stop and plunger.
- The stop and the plunger become the opposite pole faces. These opposite poles are attracted to one another and this creates the force and motion in the plunger.
- The amount of force created is related to the amount of electrical current applied. Other factors such as the number of turns of wire in the coil, the size of the solenoid, air gap and the magnetic character of the steel used will affect the amount of force developed.
- Solenoid geometry can vary significantly.
- Return springs are often used to return the plunger and / or push rod back to its original position.

Electric Machines

Continuous Energy-conversion Devices

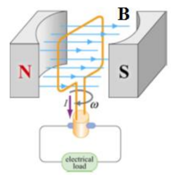
An electric machinery is a device that can convert either mechanical energy to electric energy or electric energy to mechanical energy.

Generators: mechanical energy to electric energy

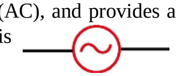
Rotating

Motors: electric energy to mechanical energy

Rotating



Alternating Current Sources (AC Sources)

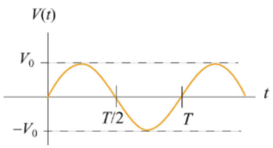
If a coil rotates in the presence of a magnetic field, the induced emf varies sinusoidally with time and leads to an alternating current (AC), and provides a source of AC power. The symbol for an AC voltage source is 

An example of an AC source is: $V(t) = V_0 sin\omega t$
where the maximum value of V_0 is called the ***amplitude***. The voltage varies between V_0 and $-V_0$.

T: the period.

f: the frequency, defined as $f = 1/T$, has the unit of inverse seconds (s^{-1}), or hertz (Hz).

ω: the angular frequency, defined to be $\omega = 2\pi f$.



When a voltage source is connected to a circuit, the current is produced, and written as: $I(t) = I_0 sin(\omega t - \varphi)$
with the same frequency as the voltage source. The amplitude I_0 and phase φ depends on the circuit.

Three elements: Resistors Inductors Capacitors

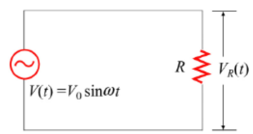
Simple AC Circuits

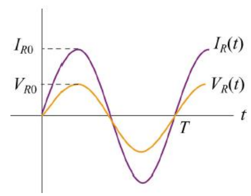
Purely Resistive Load

$V_R(t) = V(t)$ and $I_R(t)R = V_R(t)$

Then $I_R(t) = \dfrac{V_0 \sin\omega t}{R} = I_{R0} \sin\omega t$

$\Rightarrow \varphi = 0$





The root-mean-square (rms) current:

$I_{rms} = I_{R0}/\sqrt{2}$

The rms voltage:

$V_{rms} = V_{R0}/\sqrt{2}$

The power dissipated in the resistor:

$P_R(t) = I_R(t)V(t) = I_R^2(t)R$

The average power dissipated over one period:

$$P = \frac{1}{2} I_{R0}^2 R = I_{rms}^2 R = I_{rms} V_{rms} = \frac{V_{rms}^2}{R}$$

Simple AC Circuits

Purely Resistive Load

The power dissipated in the resistor:

$$P_R(t) = I_R(t)V(t) = I_R^2(t)R$$

The average power dissipated over one period:

$$P = \frac{1}{T}\int_0^T I_R^2(t)R dt = \frac{1}{T}\int_0^T \left(I_{R0}^2 \sin^2 \omega t\right)R dt = \frac{I_{R0}^2 R}{\omega T}\int_0^T \left(\sin^2 \omega t\right) d(\omega t)$$

$$= \frac{I_{R0}^2 R}{\omega T}\left[\frac{\omega t}{2} - \frac{1}{4}\sin(2\omega t)\right]_0^T = \frac{I_{R0}^2 R}{\omega T}\left[\frac{\omega T}{2} - \underline{\frac{1}{4}\sin(2\omega T)}\right]$$

$$= \frac{1}{2} I_{R0}^2 R = I_{rms}^2 R = I_{rms}V_{rms} = \frac{V_{rms}^2}{R}$$

$$2\omega T = 2(2\pi f)T = 4\pi \frac{1}{T}T = 4\pi$$

$$\text{Then } \frac{1}{4}\sin(2\omega T) = \frac{1}{4}\sin(4\pi) = 0$$

Simple AC Circuits

Purely Inductive Load

$V_L(t) = V(t)$

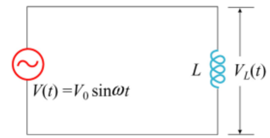
From Faraday's law :

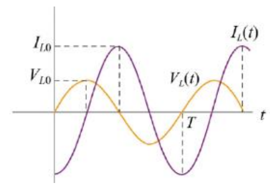
$$V_L(t) = L\frac{dI_L}{dt} \Rightarrow \frac{dI_L}{dt} = \frac{V(t)}{L} = \frac{V_{L0}}{L}\sin \omega t$$

$$I_L = \int dI_L = \frac{V_{L0}}{L}\int \sin \omega t \, dt = -\left(\frac{V_{L0}}{\omega L}\right)\cos \omega t$$

$$I_L = \left(\frac{V_{L0}}{\omega L}\right)\sin(\omega t - \pi/2)$$

$\Rightarrow \varphi = +\pi/2$ $\boxed{I(t) = I_0 \sin(\omega t - \varphi)}$





The amplitude of the current through the inductor :

$$I_{L0} = \frac{V_{L0}}{\omega L} = \frac{V_{L0}}{X_L}$$

where $X_L = \omega L$ called the ***inductive reactance.***

SI units of X_L is ohms (Ω)

Simple AC Circuits

Purely Capacitive Load

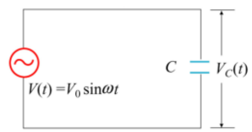
$V_C(t) = V(t)$

$Q(t) = CV(t) = CV_C(t) = CV_{C0}\sin \omega t$

$$I_C(t) = \frac{dQ}{dt} = \omega CV_{C0}\cos \omega t$$

$$I_C(t) = \omega CV_{C0}\sin(\omega t + \frac{\pi}{2})$$

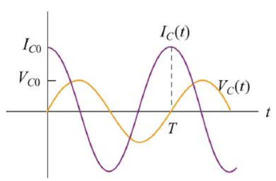
$\Rightarrow \varphi = -\pi/2$ $\boxed{I(t) = I_0 \sin(\omega t - \varphi)}$



The amplitude of the current through the capacitor :

$$I_{C0} = \omega CV_{C0} = \frac{V_{C0}}{X_C}$$

where $X_C = \frac{1}{\omega C}$ called the ***capacitive reactance.***



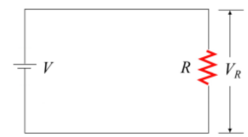
SI units of X_C is ohms (Ω)

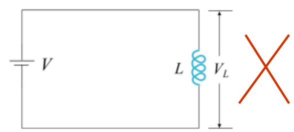
Three Elements in DC Circuits

$V_R = V$ and $I_R R = V_R$

The power dissipated in the resistor:

$P_R = I_R V = I_R^2 R$





The circuit is shorted.

$X_L = \omega L = 0$

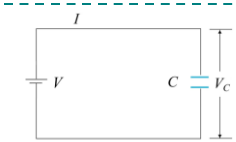
$V_C = V$

$Q = CV$

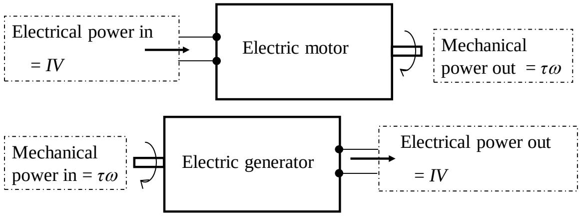
$I_C = 0 \Rightarrow I = 0$

The circuit is opened.

$$X_C = \frac{1}{\omega C} = \infty$$



Rotational Motion



•In general, a three-dimensional vector is required to completely describe the rotation of an object in space.

•In real electrical machine, they normally rotate around an axis, called shaft of the machine. So their rotation is restricted to one angular dimension, either clockwise (CW) or counterclockwise (CCW).

•In the module of EEE108, a counterclockwise angle of rotation is taken as positive, and a clockwise one is negative.

•Now, all the concepts of the rotation motion reduces to scalars.

Rotational Motion

Angular velocity ω

Is defined as the rate of change of the **angular displacement** φ with respect to time: $\omega = d\varphi/dt$

SI units: radians per second.

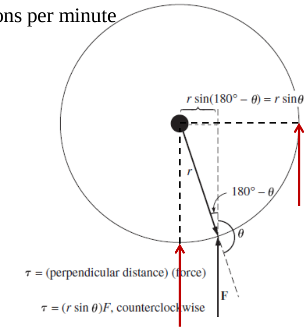
Engineers: revolutions per second or revolutions per minute

Torque τ

Is defined as the product of the force applied to the object and the smallest distance between the line of action of the force and the object's axis of rotation:

τ = (Force applied) (perpendicular distance)

SI units: Newton-meters



Rotational Motion

Work W

For rotational motion, work is the application of a torque through an angle:

$$W = \int \tau d\varphi$$

If the torque is constant:

$$W = \tau\varphi$$

SI units: Joules

Power P

Power is the rate of doing work:

$$P = \frac{dW}{dt} = \frac{d}{dt}(\tau\varphi)$$

Assuming constant torque:

$$P = \tau\frac{d\varphi}{dt} = \tau\omega$$

SI units: Joules per second

(watts, W)

Three Powers in AC Circuits

$V = V_{rms}$

$I = I_{rms}$

Real power P

Real power is the power assumed by the load:

$$P = VI\cos\varphi \qquad \text{Watts (W)}$$

Reactive power Q

Reactive power is the product of the voltage, current and the sine of the phase angle between them:

$$Q = VI\sin\varphi \qquad \text{Volt amperes reactive (Var)}$$

Reactive power is the power component that is exchange back and forth between a source and a load. Conventionally, inductive reactive power is defined as positive power and capacitive reactive power is defined as negative reactive power.

Apparent power S

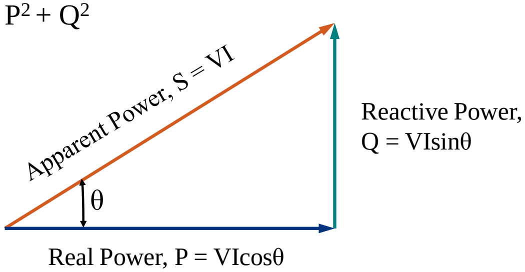
Apparent power is the product of the voltage and current without accounting of the phase angle:

$$S = VI \qquad \text{Volt amperes (V A)}$$

Three Powers in AC Circuits

Power triangle

$S^2 = P^2 + Q^2$



Today

- Transducers
- Linear Actuators
- **Basic concept of electrical machines fundamentals**
  - Three basic elements: AC circuits
    DC circuits
  - Rotational motion: Angular velocity
    Torque, Work, Power
  - Three powers: Real, Reactive, and Apparent powers

Next

Linear DC machines

Thanks for your attendance