EEE340 Protective Relaying

Lecture 3 – Unsymmetrical Faults

Short Circuit

- Symmetrical Faults
  - Three-Phase Short Circuit
- Unsymmetrical Faults
  - Single Line-to-Ground Fault
  - Line-to-Line Fault
  - Double Line-to-Ground Fault

Short Circuit

短路种类 示意图 短路代表符号

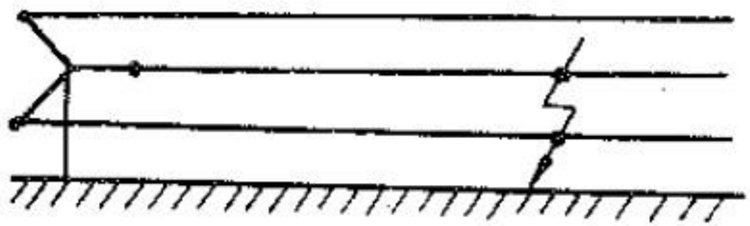
- ❑ Three-Phase Short Circuit

三相短路

$f^{(3)}$

- ❑ Double Line-to-Ground Fault

两相短路接地



$f^{(1,1)}$

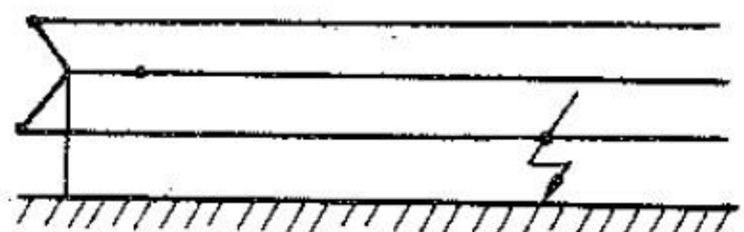
- ❑ Line-to-Line Fault

两相短路

$f^{(2)}$

- ❑ Single Line-to-Ground Fault

单相短路

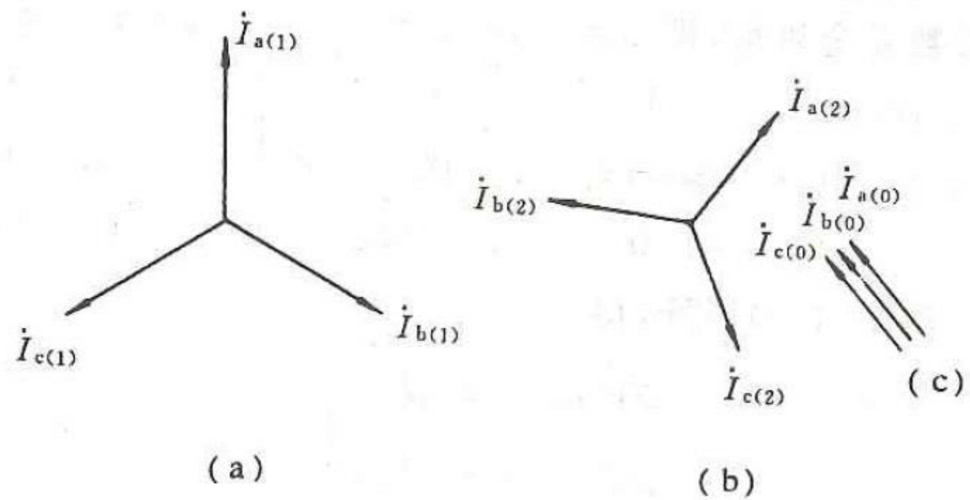


$f^{(1)}$

Symmetrical Components

❖ Assume that a set of unsymmetrical phasors is given, these phasors can be resolved into following three sets of sequence components:

- ***Zero-sequence*** components, consisting of three phasors with equal magnitudes and with zero phase displacement.
- ***Positive-sequence*** components, consisting of three phasors with equal magnitudes, 120 degree phase displacement, and positive sequence.
- ***Negative-sequence*** components, consisting of three phasors with equal magnitudes, 120 degree phase displacement, and negative sequence.



Symmetrical Components

❖ If we consider *phase a* as a reference, the relation between the three phasors and their symmetrical components is:

$$\begin{bmatrix} \dot{I}_{a(1)} \\ \dot{I}_{a(2)} \\ \dot{I}_{a(0)} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & a & a^2 \\ 1 & a^2 & a \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} \dot{I}_a \\ \dot{I}_b \\ \dot{I}_c \end{bmatrix}$$

$$a = e^{j120^o}, \quad a^2 = e^{j240^o}, \quad 1 + a + a^2 = 0, \quad a^3 = 1$$

$\dot{I}_{a(1)}, \dot{I}_{a(2)}, \dot{I}_{a(0)}$ are the positive, negative and zero sequence components for phase a respectively.

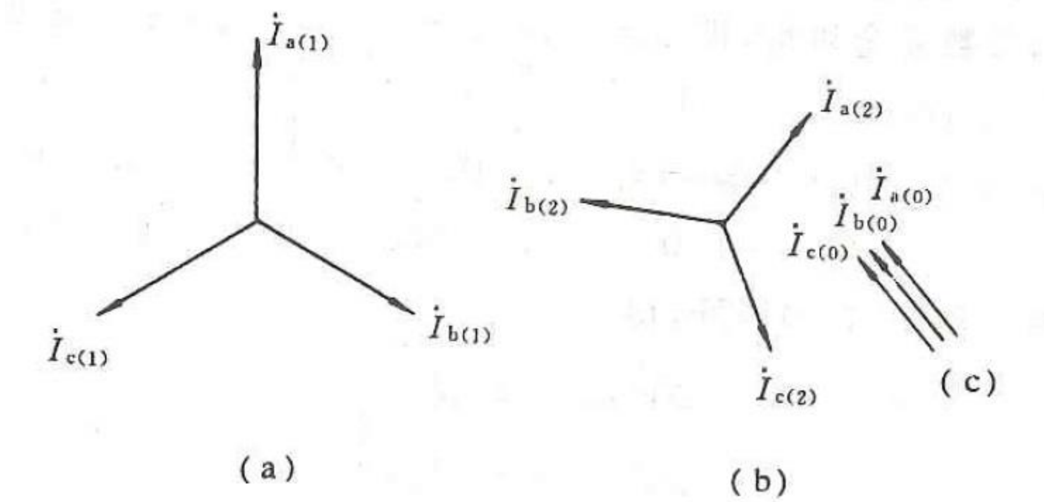
Symmetrical Components

- With the positive, negative and zero sequence components of phase *a* given, the symmetrical components for phase b and c can be calculated as:

$$\dot{I}_{b(1)} = a^2\dot{I}_{a(1)}, \dot{I}_{c(1)} = a\dot{I}_{a(1)}$$

$$\dot{I}_{b(2)} = a\dot{I}_{a(2)}, \dot{I}_{c(2)} = a^2\dot{I}_{a(2)}$$

$$\dot{I}_{b(0)} = \dot{I}_{c(0)} = \dot{I}_{a(0)}$$



Symmetrical Components

$$I_{abc} = S^{-1} I_{120} \qquad S^{-1} = \begin{bmatrix} 1 & 1 & 1 \\ a^2 & a & 1 \\ a & a^2 & 1 \end{bmatrix}$$

- This equation can be further extended as:

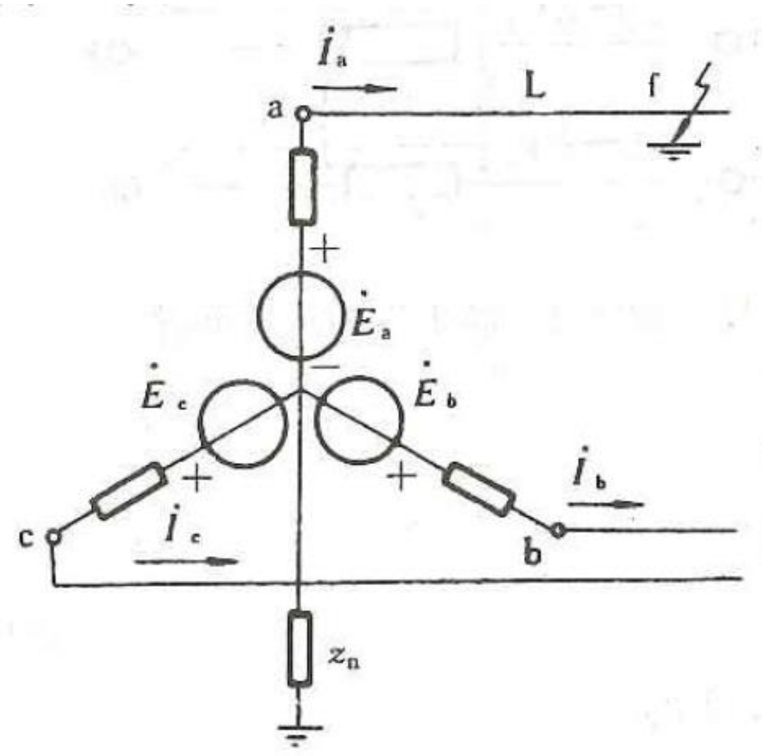
$$\dot{I}_a = \dot{I}_{a(1)} + \dot{I}_{a(2)} + \dot{I}_{a(0)}$$

$$\dot{I}_b = \dot{I}_{b(1)} + \dot{I}_{b(2)} + \dot{I}_{b(0)} = a^2 \dot{I}_{a(1)} + a\dot{I}_{a(2)} + \dot{I}_{a(0)}$$

$$\dot{I}_c = \dot{I}_{c(1)} + \dot{I}_{c(2)} + \dot{I}_{c(0)} = a\dot{I}_{a(1)} + a^2 \dot{I}_{a(2)} + \dot{I}_{a(0)}$$

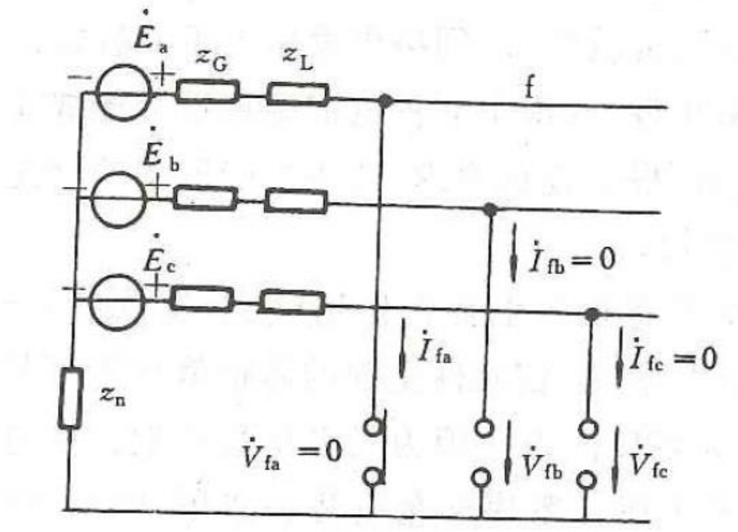
- This relation can be applied to both current and voltage phasors.

Example for unsymmetrical components

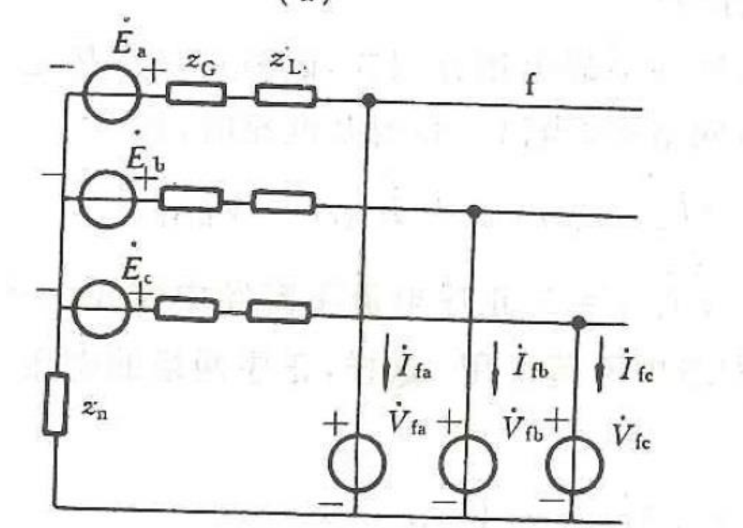


- A generator is connected with open transmission line, its neutral point is grounded through z_n;
- There is a single line-to-ground fault at location *f* of phase *a*;
- So the impedance of phase *a* to ground is zero (impedance of arc is neglected).
- The voltage between phase *a* and ground V_{fa}=0; But V_{fb}≠0, V_{fc}≠0.
- Except the fault location, parameters of other parts are still symmetrical.

Example for unsymmetrical components



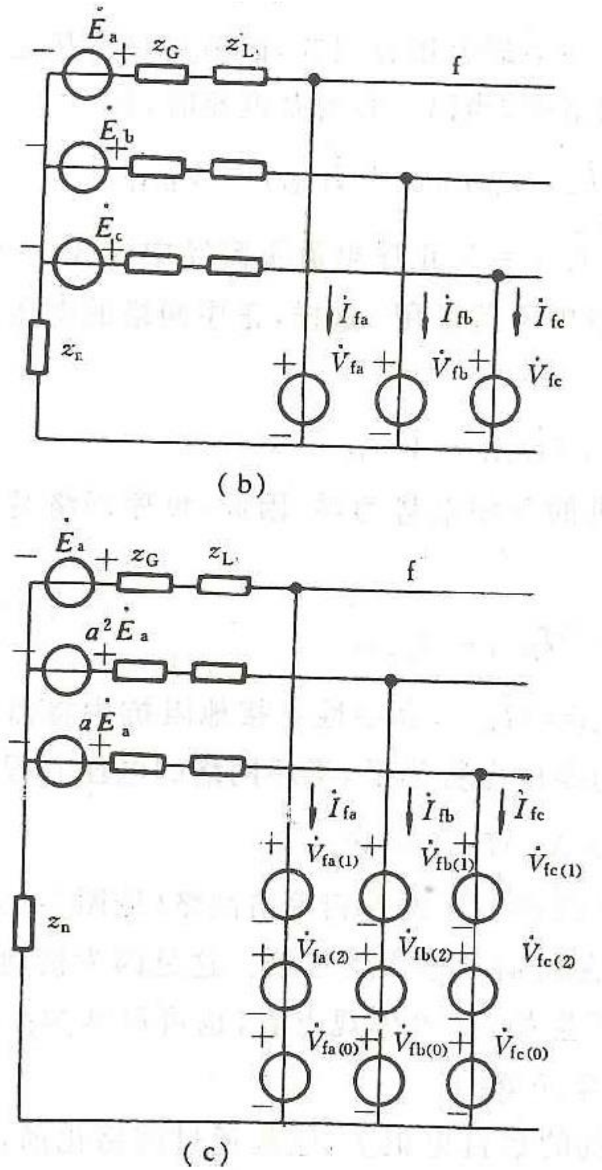
(a)



(b)

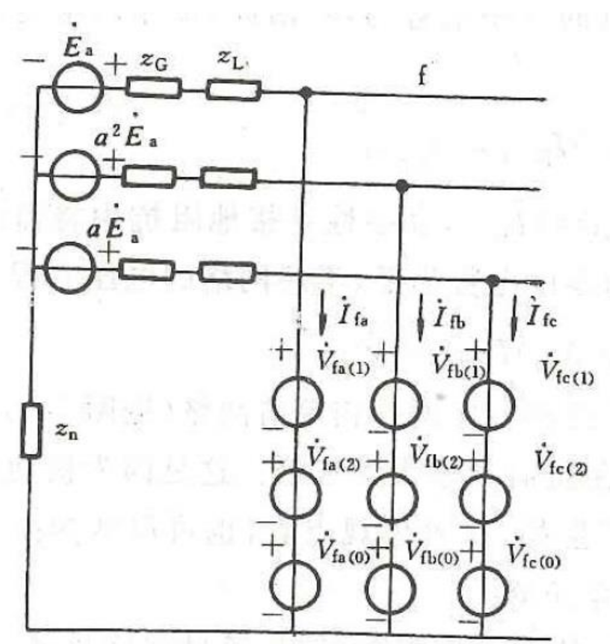
- At the fault location, a set of unsymmetrical voltage sources are connected as shown in (b);
- The voltage of the connected sources are just the same as the phase voltages of the fault location discussed before;
- This is equivalent to the single line-to-ground fault as shown in (a);
- That is to say the fault conditions can be equivalent to connection with a set of unsymmetrical voltage sources.

Example for unsymmetrical components

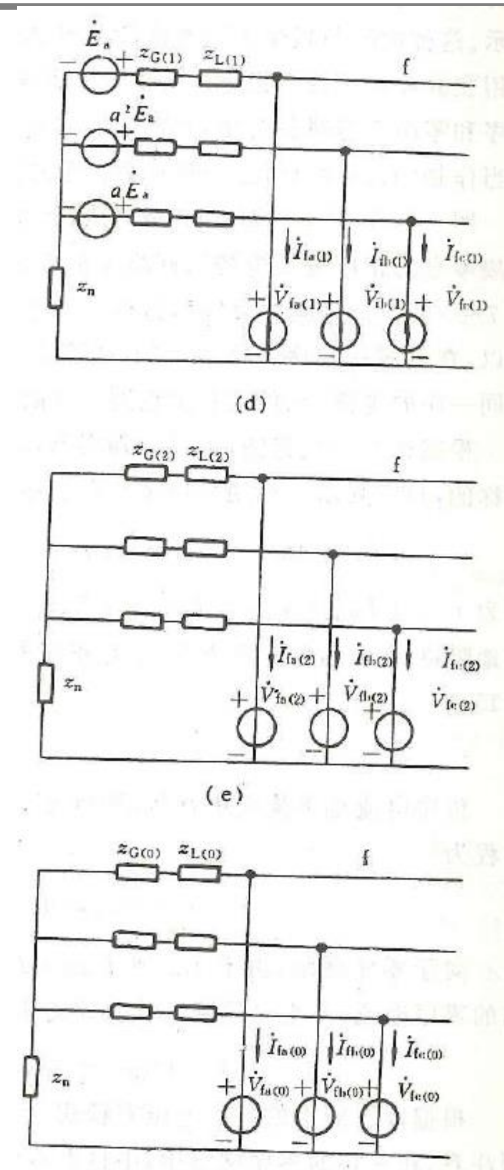


- The connected unsymmetrical voltage sources in (b)can be decomposed as three symmetrical components;
- Then the connection with the unsymmetrical voltage sources can be equivalent to connection with three sets of symmetrical components at the fault location as shown in (c);
- For one phase, such as phase *a*, the sum of the positive, negative and zero sequence voltage is equal to the voltage of this phase at the fault location.

Example for unsymmetrical components



(c)



(d) (e) (f)

- The connection with total three sets of symmetrical components is equivalent to sum of connection with each sequence component and corresponding sequence impedances.

Example for unsymmetrical components

- Diagram *(d)* is the positive sequence network where only positive sequence voltage sources are applied and only positive sequence currents flow through the positive sequence impedances of the components;

- Diagrams *(e)* and *(f)* are negative and zero sequence networks;

- Because the generator can only generate positive sequence voltage, negative sequence voltage sources exist only at the fault location in the negative sequence network with currents and impedances of the same sequence;

- Zero sequence voltage sources exist only at the fault location in the zero sequence network with currents and impedances of the same sequence.

Example for unsymmetrical components

- To write the voltage equations for the three sequence network;
- Because each sequence network is symmetrical for each phase, so the voltage equation of one phase is enough for analysis;
- In the positive sequence network, take phase *a* as the reference:

$$\dot{E}_a - (z_{G(1)} + z_{L(1)})\dot{I}_{fa(1)} - z_n(\dot{I}_{fa(1)} + \dot{I}_{fb(1)} + \dot{I}_{fc(1)}) = \dot{V}_{fa(1)}$$

$$\Downarrow$$

$$(\dot{I}_{fa(1)} + \dot{I}_{fb(1)} + \dot{I}_{fc(1)}) = 0$$

- The positive sequence currents do not flow through the neutral point.

$$\Downarrow$$

$$\dot{E}_a - (z_{G(1)} + z_{L(1)})\dot{I}_{fa(1)} = \dot{V}_{fa(1)}$$

Example for unsymmetrical components

- The negative sequence currents do not flow through the neutral point;
- The negative sequence voltage is zero for the generator;
- Then the negative sequence voltage equation is:

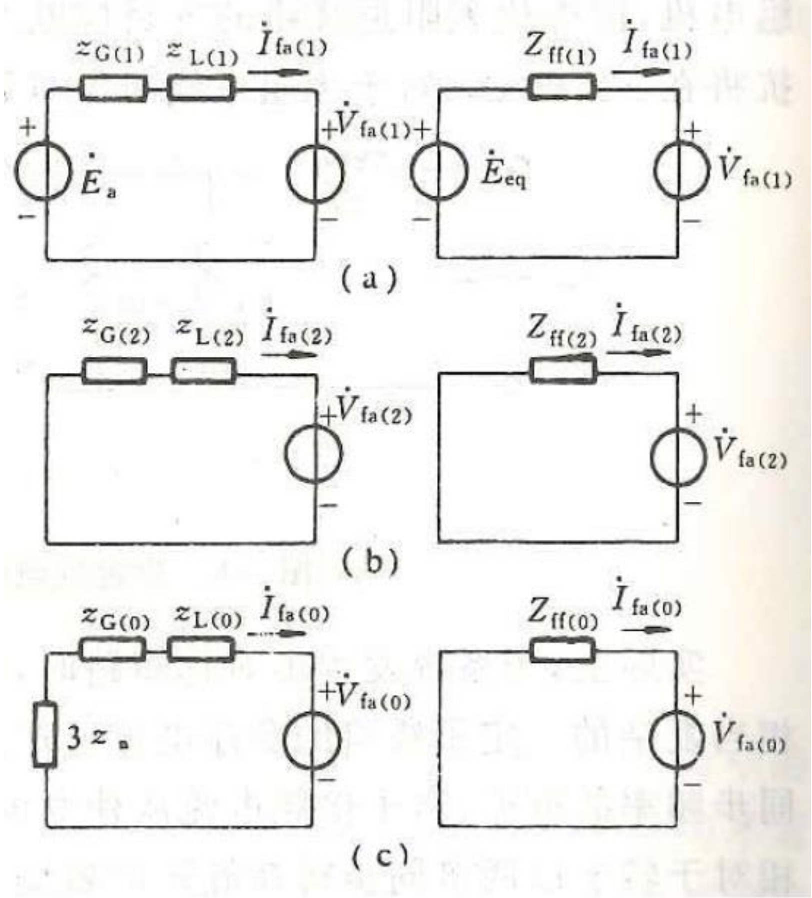
$$0-(z_{G(2)}+z_{L(2)})\dot{I}_{fa(2)}=\dot{V}_{fa(2)}$$

- For the zero sequence network : $(\dot{I}_{fa(0)}+\dot{I}_{fb(0)}+\dot{I}_{fc(0)})=3\dot{I}_{fa(0)}$
- The zero sequence voltage drop on the neutral impedance z_n is equal to three times of zero sequence currents flowing through z_n ;
- The zero sequence voltage is zero for the generator;
- Then the zero sequence voltage equation is:

$$0-(z_{G(0)}+z_{L(0)}+3z_n)\dot{I}_{fa(0)}=\dot{V}_{fa(0)}$$

Single Phase Equivalent Networks

- In single phase zero sequence network, the neutral grounded impedance z_n must be enlarged for three times because the zero sequence currents following through z_n is three times of single phase zero sequence current;



Example for unsymmetrical components

- Although the connections of real systems are complicated, it is always possible to simplify them and get following sequence voltage equations:

$$\dot{E}_{eq} - z_{ff(1)} \dot{I}_{fa(1)} = \dot{V}_{fa(1)}$$

$$0 - z_{ff(2)} \dot{I}_{fa(2)} = \dot{V}_{fa(2)}$$

$$0 - z_{ff(0)} \dot{I}_{fa(0)} = \dot{V}_{fa(0)}$$

- $\dot{E}_{eq}$ is the Thevinin equivalent voltage source in the positive sequence network.
- $z_{ff(1)}, z_{ff(2)}, z_{ff(0)}$ are input impedance of the fault location for positive, negative and zero sequence.
- $\dot{I}_{fa(1)}, \dot{I}_{fa(2)}, \dot{I}_{fa(0)}$ are positive, negative and zero sequence components for the current at the fault location.
- $\dot{V}_{fa(1)}, \dot{V}_{fa(2)}, \dot{V}_{fa(0)}$ are positive, negative and zero sequence components for the voltage at the fault location.

Calculation for Unsymmetrical Faults

- To deal with unsymmetrical faults, the unsymmetrical impedances at the fault location can be represented by unsymmetrical voltage and current phasors;
- The other parts of the system can be still considered with symmetrical impedances;
- For the systems with symmetrical three phase impedances, the positive, negative and zero sequence networks can be analyzed independently;
- For each different type of unsymmetrical fault, specific boundary conditions need to be identified and applied together with the sequence voltage equations to solve for solutions.

Calculation for Unsymmetrical Faults

$$\dot{E}_{eq} - z_{ff(1)}\dot{I}_{fa(1)} = \dot{V}_{fa(1)}$$

$$0 - z_{ff(2)}\dot{I}_{fa(2)} = \dot{V}_{fa(2)}$$

$$0 - z_{ff(0)}\dot{I}_{fa(0)} = \dot{V}_{fa(0)}$$

- When all components of the network are only represented by reactance:

$$\dot{E}_{eq} - jX_{ff(1)}\dot{I}_{fa(1)} = \dot{V}_{fa(1)}$$

$$-X_{ff(2)}\dot{I}_{fa(2)} = \dot{V}_{fa(2)}$$

$$-jX_{ff(0)}\dot{I}_{fa(0)} = \dot{V}_{fa(0)}$$

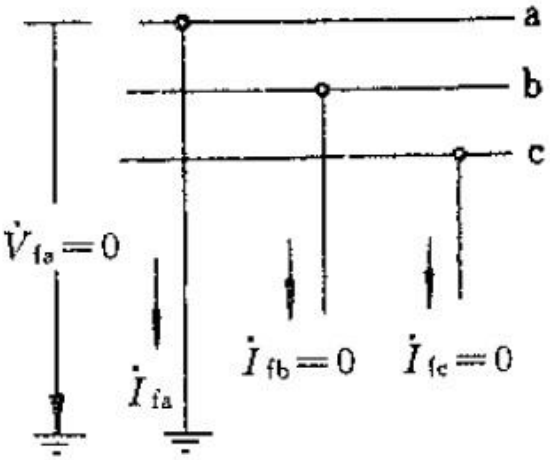
- $\dot{E}_{eq} = \dot{V}_f^{(0)}$ can be considered as the voltage at the fault location before the fault.
- There are totally six unknown quantities, so three additional equations have to be identified according to boundary conditions.

Single Line-to-Ground Fault

- The boundary conditions of single line-to-ground fault can be summarized as:

$$\dot{V}_{fa} = 0, \quad \dot{I}_{fb} = 0, \quad \dot{I}_{fc} = 0$$

- These can be further expressed as:



$$\dot{V}_{fa(1)} + \dot{V}_{fa(2)} + \dot{V}_{fa(0)} = 0$$

$$a^2\dot{I}_{fa(1)} + a\dot{I}_{fa(2)} + \dot{I}_{fa(0)} = 0$$

$$a\dot{I}_{fa(1)} + a^2\dot{I}_{fa(2)} + \dot{I}_{fa(0)} = 0$$

$$\dot{V}_{fa(1)} + \dot{V}_{fa(2)} + \dot{V}_{fa(0)} = 0$$

$$\dot{I}_{fa(1)} = \dot{I}_{fa(2)} = \dot{I}_{fa(0)}$$

Single Line-to-Ground Fault

- The positive sequence fault current then can be calculated as:

$$\dot{I}_{fa(1)} = \frac{\dot{V}_f^{(0)}}{j(X_{ff(1)} + X_{ff(2)} + X_{ff(0)})}$$

- With the positive sequence fault current given, all the other sequence components for fault current and voltage can be further calculated:

$$\dot{I}_{fa(2)} = \dot{I}_{fa(0)} = \dot{I}_{fa(1)}$$

$$\dot{V}_{fa(1)} = \dot{V}_f^{(0)} - jX_{ff(1)}\dot{I}_{fa(1)}$$

$$\dot{V}_{fa(2)} = -jX_{ff(2)}\dot{I}_{fa(1)}$$

$$\dot{V}_{fa(0)} = -jX_{ff(0)}\dot{I}_{fa(1)}$$

Interconnected Sequence Network

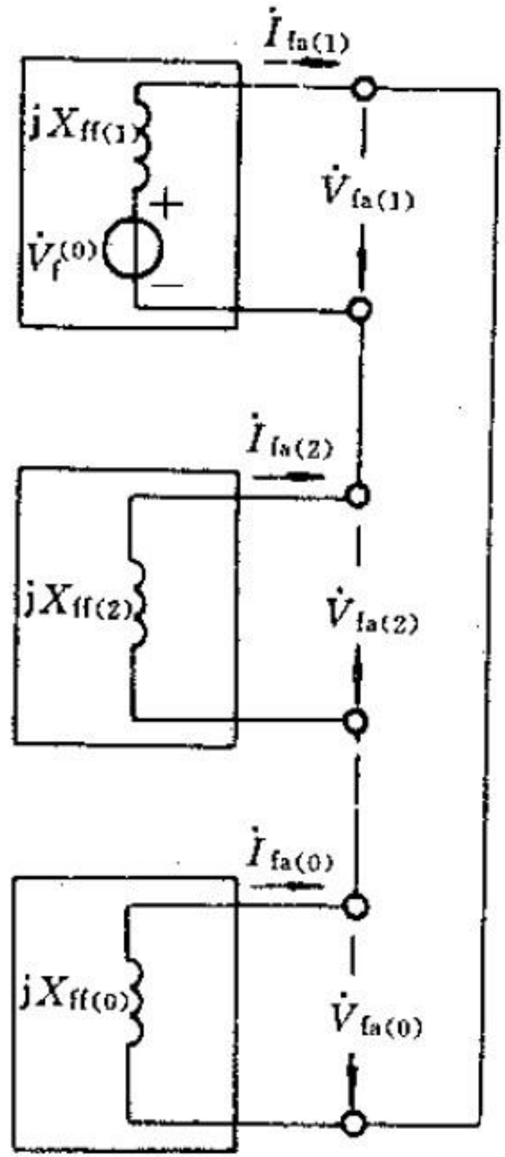
$$\dot{I}_{fa(2)} = \dot{I}_{fa(0)} = \dot{I}_{fa(1)}$$

$$\dot{V}_{fa(1)} = \dot{V}_f^{(0)} - jX_{ff(1)}\dot{I}_{fa(1)}$$

$$\dot{V}_{fa(2)} = -jX_{ff(2)}\dot{I}_{fa(1)}$$

$$\dot{V}_{fa(0)} = -jX_{ff(0)}\dot{I}_{fa(1)}$$

- According to the relations among different sequence components, all three sequence networks can be interconnected at the faulted ports with the same relations between sequence components.



Single Line-to-Ground Fault

- Then the fault current can be calculated as:

$$\dot{I}_{fa} = \dot{I}_{fa(1)} + \dot{I}_{fa(2)} + \dot{I}_{fa(0)} = 3\dot{I}_{fa(1)}$$

$$\dot{I}_{fa} = \frac{3\dot{V}_f^{(0)}}{j(X_{ff(1)} + X_{ff(2)} + X_{ff(0)})}$$

- The voltages of non-faulted phases can be calculated as:

$$\dot{V}_{fb} = a^2\dot{V}_{fa(1)} + a\dot{V}_{fa(2)} + \dot{V}_{fa(0)} = j[(a^2 - a)X_{ff(2)} + (a^2 - 1)X_{ff(0)}]\dot{I}_{fa(1)}$$

$$\dot{V}_{fc} = a\dot{V}_{fa(1)} + a^2\dot{V}_{fa(2)} + \dot{V}_{fa(0)} = j[(a - a^2)X_{ff(2)} + (a - 1)X_{ff(0)}]\dot{I}_{fa(1)}$$

Line-to-Line Fault

- The boundary conditions of line-to-ground fault can be summarized as:

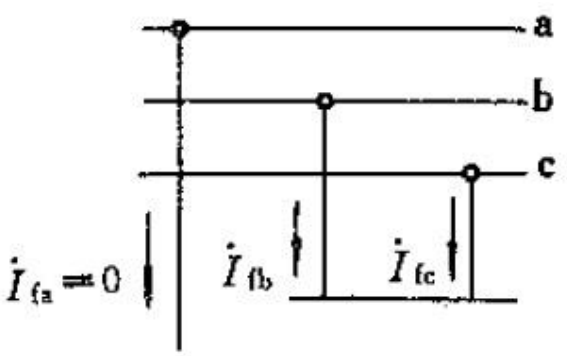
$$\dot{I}_{fa} = 0, \quad \dot{I}_{fb} + \dot{I}_{fc} = 0, \quad \dot{V}_{fb} = \dot{V}_{fc}$$

- These can be further expressed as:

$$\dot{I}_{fa(1)} + \dot{I}_{fa(2)} + \dot{I}_{fa(0)} = 0$$

$$a^2\dot{I}_{fa(1)} + a\dot{I}_{fa(2)} + \dot{I}_{fa(0)} + a\dot{I}_{fa(1)} + a^2\dot{I}_{fa(2)} + \dot{I}_{fa(0)} = 0$$

$$a^2\dot{V}_{fa(1)} + a\dot{V}_{fa(2)} + \dot{V}_{fa(0)} = a\dot{V}_{fa(1)} + a^2\dot{V}_{fa(2)} + \dot{V}_{fa(0)}$$



$$\dot{I}_{fa(0)} = 0 \qquad \dot{I}_{fa(1)} + \dot{I}_{fa(2)} = 0 \qquad \dot{V}_{fa(1)} = \dot{V}_{fa(2)}$$

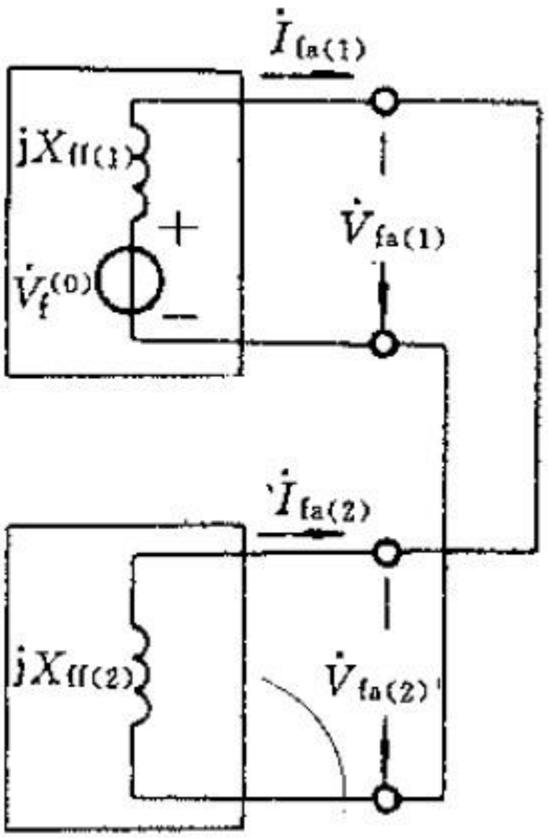
Interconnected Sequence Network

$$\dot{I}_{fa(0)} = 0$$

$$\dot{I}_{fa(1)} + \dot{I}_{fa(2)} = 0$$

$$\dot{V}_{fa(1)} = \dot{V}_{fa(2)}$$

- According to the relations among different sequence components, an interconnected sequence network can be constructed by the positive and negative sequence network. There is no zero sequence network.



Line-to-Line Fault

- Then the sequence fault current can be calculated as:

$$\dot{I}_{fa(1)} = \frac{\dot{V}_f^{(0)}}{j(X_{ff(1)} + X_{ff(2)})}$$

$$\dot{I}_{fa(2)} = -\dot{I}_{fa(1)}$$

$$\dot{V}_{fa(1)} = \dot{V}_{fa(2)} = -jX_{ff(2)}\dot{I}_{fa(2)} = jX_{ff(2)}\dot{I}_{fa(1)}$$

- The currents of faulted phases can be calculated as:

$$\dot{I}_{fb} = a^2\dot{I}_{fa(1)} + a\dot{I}_{fa(2)} + \dot{I}_{fa(0)} = (a^2 - a)\dot{I}_{fa(1)} = -j\sqrt{3}\dot{I}_{fa(1)}$$

$$\dot{I}_{fc} = -\dot{I}_{fb} = j\sqrt{3}\dot{I}_{fa(1)}$$

$$\left|\dot{I}_{fc}\right| = \left|\dot{I}_{fb}\right| = \sqrt{3}\left|\dot{I}_{fa(1)}\right|$$

Line-to-Line Fault

- Then the voltages of each phase at faulted location can be calculated as:

$$\dot{V}_{fa} = \dot{V}_{fa(1)} + \dot{V}_{fa(2)} + \dot{V}_{fa(0)} = 2\dot{V}_{fa(1)} = j2X_{ff(2)}\dot{I}_{fa(1)}$$

$$\dot{V}_{fb} = a^2\dot{V}_{fa(1)} + a\dot{V}_{fa(2)} + \dot{V}_{fa(0)} = -\dot{V}_{fa(1)} = -\frac{1}{2}\dot{V}_{fa}$$

$$\dot{V}_{fc} = \dot{V}_{fb} = -\dot{V}_{fa(1)} = -\frac{1}{2}\dot{V}_{fa}$$

- The faulted current is $\sqrt{3}$ times of the positive sequence current.

- The voltage of non-faulted phase at the faulted location is two times of the positive sequence voltage.

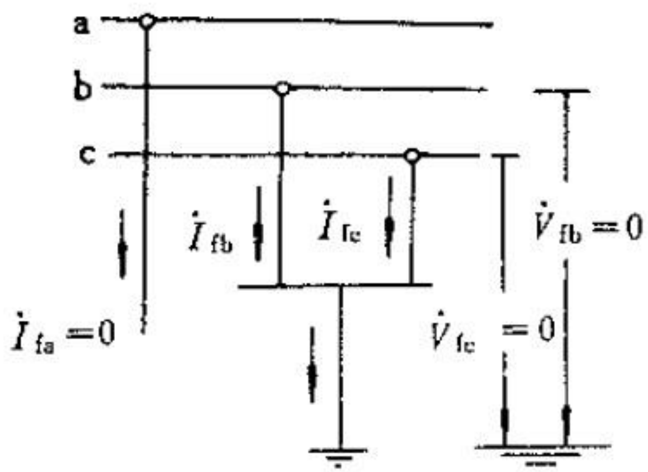
- The voltage of the faulted phase is only half of the non-faulted phase and the direction is opposite.

Double Line-to-Ground Fault

- The boundary conditions of double line-to-ground fault can be summarized as:

$$\dot{I}_{fa} = 0, \quad \dot{V}_{fb} = 0, \quad \dot{V}_{fc} = 0$$

- These can be further expressed as:



$$\dot{I}_{fa(1)} + \dot{I}_{fa(2)} + \dot{I}_{fa(0)} = 0$$

$$a^2\dot{V}_{fa(1)} + a\dot{V}_{fa(2)} + \dot{V}_{fa(0)} = 0$$

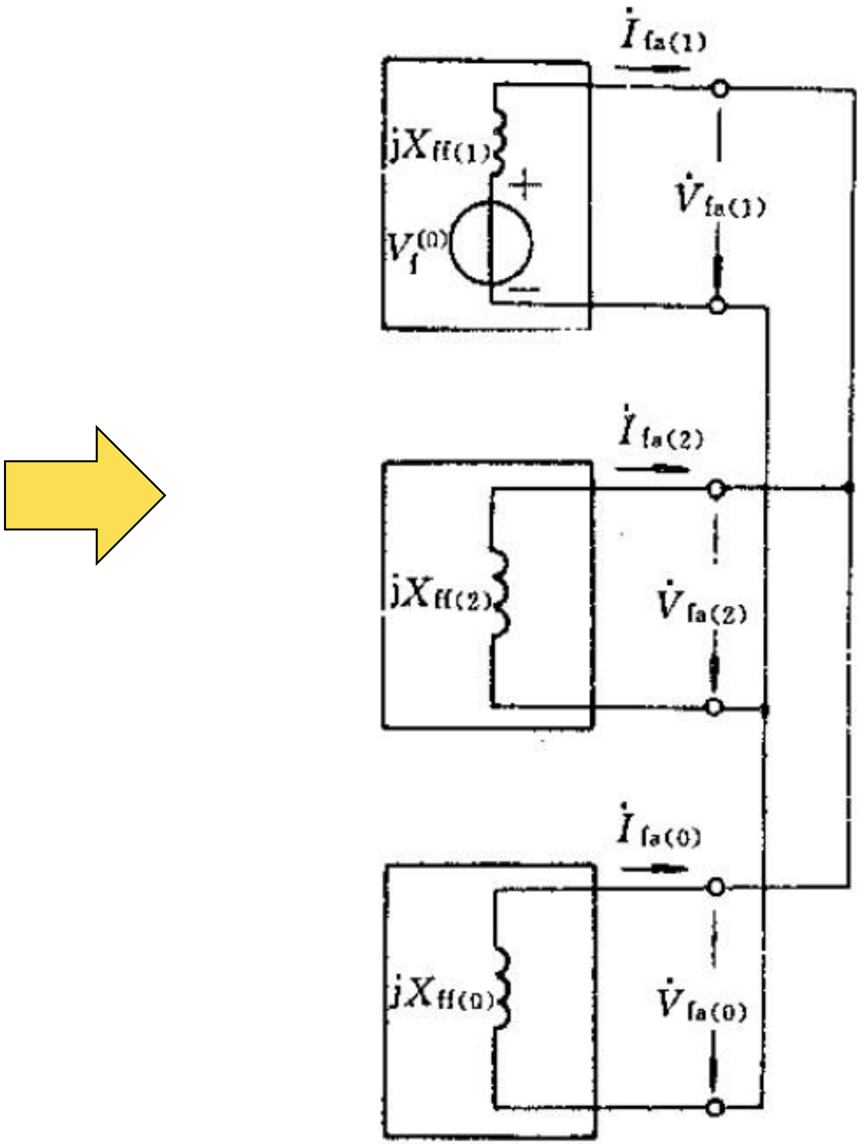
$$a\dot{V}_{fa(1)} + a^2\dot{V}_{fa(2)} + \dot{V}_{fa(0)} = 0$$

$$\dot{I}_{fa(1)} + \dot{I}_{fa(2)} + \dot{I}_{fa(0)} = 0 \qquad \dot{V}_{fa(1)} = \dot{V}_{fa(2)} = \dot{V}_{fa(0)}$$

Interconnected Sequence Network

$$\dot{I}_{fa(1)} + \dot{I}_{fa(2)} + \dot{I}_{fa(0)} = 0$$

$$\dot{V}_{fa(1)} = \dot{V}_{fa(2)} = \dot{V}_{fa(0)}$$



- According to the relations among different sequence components, all three sequence networks can interconnected at the faulted ports with the same relations between sequence components.

Double Line-to-Ground Fault

- Then the sequence fault current can be calculated as:

$$\dot{I}_{fa(1)} = \frac{\dot{V}_f^{(0)}}{j(X_{ff(1)} + X_{ff(2)} // X_{ff(0)})}$$

$$\dot{I}_{fa(2)} = -\frac{X_{ff(0)}}{X_{ff(2)} + X_{ff(0)}} \dot{I}_{fa(1)}$$

$$\dot{I}_{fa(0)} = -\frac{X_{ff(2)}}{X_{ff(2)} + X_{ff(0)}} \dot{I}_{fa(1)}$$

- The sequence voltages can be calculated as:

$$\dot{V}_{fa(1)} = \dot{V}_{fa(2)} = \dot{V}_{fa(0)} = j \frac{X_{ff(2)} X_{ff(0)}}{X_{ff(2)} + X_{ff(0)}} \dot{I}_{fa(1)}$$

Double Line-to-Ground Fault

- Then the fault current can be calculated as:

$$\dot{I}_{fb} = a^2\dot{I}_{fa(1)} + a\dot{I}_{fa(2)} + \dot{I}_{fa(0)} = (a^2 - \frac{X_{ff(2)} + aX_{ff(0)}}{X_{ff(2)} + X_{ff(0)})\dot{I}_{fa(1)}$$

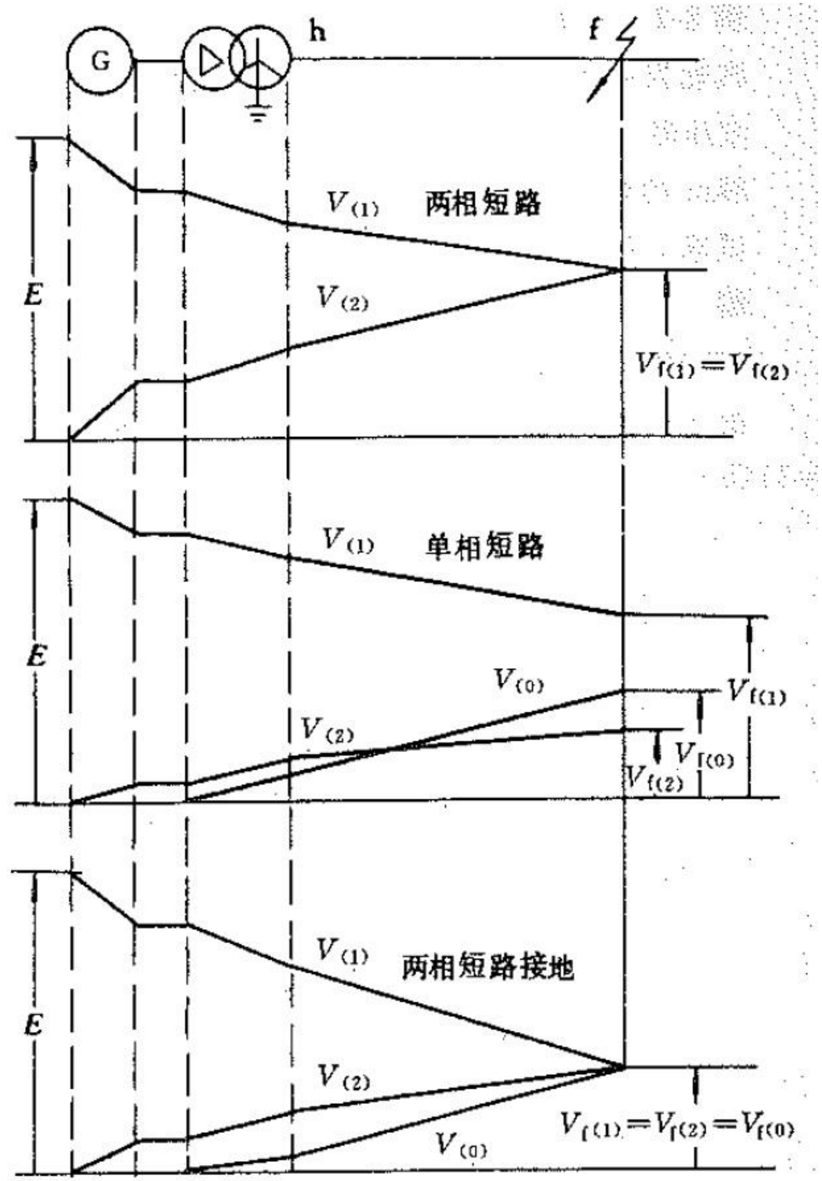
$$\dot{I}_{fc} = a\dot{I}_{fa(1)} + a^2\dot{I}_{fa(2)} + \dot{I}_{fa(0)} = (a - \frac{X_{ff(2)} + a^2X_{ff(0)}}{X_{ff(2)} + X_{ff(0)}})\dot{I}_{fa(1)}$$

- The voltage of non-faulted phase at the fault location can be calculated as:

$$\dot{V}_{fa} = 3\dot{V}_{fa(1)} = j\frac{3X_{ff(2)}X_{ff(0)}}{X_{ff(2)} + X_{ff(0)}}\dot{I}_{fa(1)}$$

Voltage and Current of Non-faulted Locations

- Firstly, to calculate the distribution of each sequence component in the network.
- And then compose the sequence components into corresponding voltage and current.
- The negative and zero sequence voltages are highest at the fault location, and decay with the distance from the fault location.



Next Lecture

Overcurrent Protection 1

Thanks for your attendance