MTH101: Tutorial 10

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Exercise 1.1

Using Laplace transform (without convolution) to solve

1. $y'' + 0.04y = 0.02t^2, \quad y(0) = -25, y'(0) = 0.$
2. $y'' + 3y' + 2y = r(t), \quad y(0) = y'(0) = 0,$

$$\text{where } r(t) = \begin{cases} 1 & \text{if } 0 < t < 1, \\ 0 & \text{if } t > 1. \end{cases}$$

Exercise 2.1

Find $f(t)$ *if* $\mathcal{L}\left[f\right]$ *equals*

$$\frac{e^{-as}}{s(s-2)}.$$

Exercise 2.2

Solve the following equation by the Laplace transform.

$$y(t) - \int_0^t y(\tau) \sin 2(t-\tau) d\tau = \sin 2t.$$

Exercise 2.3

Using Laplace transform and convolution to solve

$$y'' + 3y' + 2y = r(t), \quad y(0) = y'(0) = 0,$$

$$\text{where } r(t) = \begin{cases} 1 & \text{if } 0 < t < 1, \\ 0 & \text{if } t > 1. \end{cases}$$

Exercise 2.4

Solve the following initial value problem.

$$y'' + y = r(t), \quad y(0) = 0,\ y'(0) = 0,$$

$$r(t) = \begin{cases} \cos t, & \text{if } 0 \le t \le \pi, \\ 0, & \text{otherwise.} \end{cases}$$

Exercise 3.1

Find $\mathcal{L}\left[f\right]$ *for the following function.*

$$f(t) = te^{-kt} \sin t.$$

Exercise 3.2

Find $f(t)$ for the following $\mathcal{L}[f]$.

$$\mathcal{L}[f] = \frac{2s+6}{(s^2+6s+10)^2}.$$