

1.6 Mean and Variance of a variable

1.6.1 Mean of a Discrete Random Variable

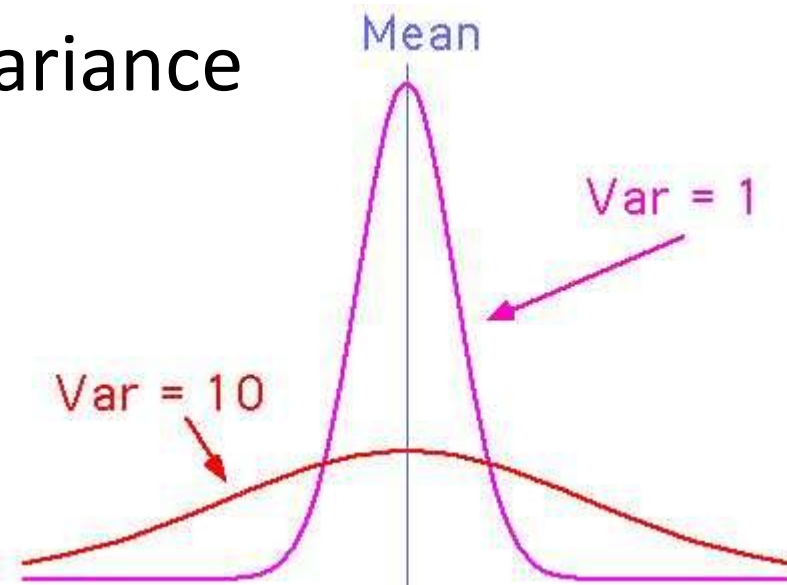
1.6.2 Variance of a Discrete Random Variable

1.6.3 Mean of a Continuous Random Variable

1.6.4 Variance of a Continuous Random Variable

1.6.5 Properties of Mean and Variance

1.6.6 Summary



1.6.1 Mean of a Discrete Random Variable: problem

The mean mass of chlorine is 35.5 unit. Chlorine atoms exist naturally in two types, one with 35 unit and another type with 37 unit. What is the fraction (proportion) of chlorine atoms that have mass of 35 unit?

[hint: the fraction is the probability]

1.6.1 Mean of a Discrete Random Variable: challenge

Solution

Let the fraction be x . $E(\text{mass}) = 35x + 37(1 - x) = 35.5$.

Therefore the mean mass is

$$35x + 37(1 - x) = 35.5. \text{ Solving, } x = 0.75 \quad \blacksquare$$

1.6.5 Properties of Mean and Variance: problem

Let X be a random variable with pdf

$$f(x) = \begin{cases} \frac{1}{2}, & 0 < x \leq 2 \\ 0 & \text{elsewhere} \end{cases}$$

Show that mean and variance of $Y = 3X - 2$ are such that

$$E(Y) = 3E(X) - 2 \text{ and } Var(Y) = 9Var(X)$$

1.6.5 Properties of Mean and Variance: solution 1

$$E(X) = \int_0^2 \frac{x}{2} dx = \left[\frac{x^2}{4} \right]_0^2 = 1,$$

$$E(Y) = \int_0^2 \frac{3X - 2}{2} dx = \frac{1}{2} \left[\frac{3x^2}{2} - 2x \right]_0^2 = 1 = 3E(X) - 2$$

1.6.5 Properties of Mean and Variance: solution 2

$$\text{var}(X) = \int_0^2 (x - 1)^2 \frac{1}{2} dx = \frac{1}{2} \left[\frac{(x-1)^3}{3} \right]_0^2 = \frac{1}{3}, E(X) = 1$$

$$\text{Var}(Y)$$

$$= \int_0^2 \frac{(3X - 2 - 1)^2}{2} dx = \frac{1}{2} \left[\frac{(3x - 3)^3}{9} \right]_0^2 = \frac{2 * 27}{18} = 3 = 9\text{Var}(X)$$