Generator and Transformer Models, and The Per-unit System (Part I)
EEE210

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Overview

Table of Contents

3.1 Introduction

In this chapter we discuss the following topics

1. Models of generators and transformers for steady-state balanced operations.
2. One-line diagram of a power system showing generators, transformers, transmission lines, capacitors, reactors, and loads.
3. The per-unit system and the impedance diagram on a common MVA base.

Table of Contents

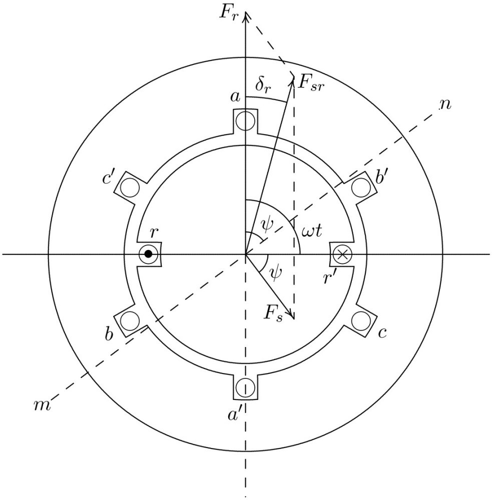


Figure 1: Elementary two-pole three-phase synchronous generator

3.2 Synchronous Generators

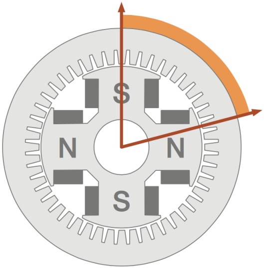
- Note that the coils aa', bb', and cc' are 120° apart. Axis of a-coil is the x-axis as shown in the Figure 1.
- These are concentrated full pitch windings[1]. In real machines, the windings are distributed among many slots, and often are not full pitch.
- We assume the windings produce a sinusoidal mmf around the rotor periphery (angle around the airgap with repect to winding axis).

[1] See next slide for the meaning of "winding pitch"

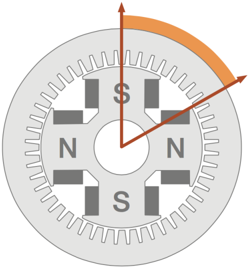
Meaning of "Winding pitch"

- The winding pitch of a generator is the number of slots spanned by each coil in the stator winding divided by the number of slots per pole.
- For example, in a 4-pole generator with a 48-slot stator (12 slots per pole), if each coil in the stator winding spans 12 slots then the ratio is $1/1$ and the winding is full pitch.

Stator coil configurations for 5/6 and 2/3 winding pitches



Each coil spans 10 slots
Winding pitch = 10/12 = 5/6



Each coil spans 8 slots
Winding pitch = 8/12 = 2/3

3.2.1 Generator Model

Assume the rotor is excited with DC current I_f producing a flux ϕ which rotates with the rotor at speed w. At time t the rotor would have moved an angle wt. Thus the flux linkage with coil a is

$$\lambda_a = N\phi \cos wt$$

The voltage induced in coil aa' is obtained from Faraday's law as

$$\begin{aligned} e_a = -\frac{d\lambda}{dt} &= \omega N\phi \sin \omega t \\ &= E_{max} \sin \omega t \\ &= E_{max} \cos(wt - \frac{\pi}{2}) \end{aligned}$$

where

$$E = \omega N\phi = 2\pi f N\phi$$

Therefore, the rms value of the generator voltage is

$$E = 4.44 f N \phi$$

The frequency is a function of speed and the number of poles, thus

$$f = \frac{P}{2} \frac{n}{60}$$

where

- n is the synchrounous speed in rpm,
- P is the number of poles (always an even number)

3.2.1 Generator Model

- If the phase a is connected to a load, then a current i_a will flow.
- Depending on the load, this current will have a phase angle, say ψ (see Figure 1) lagging the generator voltage e_a which is along the x-axis.
- Again, this is shown in the figure as the line mn.
- The same is true for phases b and c but they will lag the voltage in phase a by 120° and 240° respectively.

Since $e_a \propto \sin(wt)$, we have

$$
\begin{aligned}
i_a &= I_{max}\sin(wt-\psi)\\
i_b &= I_{max}\sin(wt-\psi-120^\circ)\\
i_c &= I_{max}\sin(wt-\psi-240^\circ)
\end{aligned}
$$

Since mmf is proportional to the current, we then have

$$
\begin{aligned}
F_a &= Ki_a = KI_{max}\sin(wt-\psi) = F_m\sin(wt-\psi)\\
F_b &= Ki_b = KI_{max}\sin(wt-\psi-120^\circ) = F_m\sin(wt-\psi-120^\circ)\\
F_c &= Ki_c = KI_{max}\sin(wt-\psi-240^\circ) = F_m\sin(wt-\psi-240^\circ)
\end{aligned}
$$

3.2.1 Generator Model

We now take components of these phasors along the line mn and in quadrature with it. Along mn we have

$$\begin{aligned} F_1 &= F_a \cos(wt - \psi) + \\ & F_b \cos(wt - \psi - 120^\circ) + \\ & F_c \cos(wt - \psi - 240^\circ) \end{aligned}$$

Using the identity $\sin\alpha\cos\alpha = \frac{1}{2}\sin(2\alpha)$, the above equation becomes

$$\begin{aligned} F_1 = \frac{F_m}{2} \quad [& \sin 2(wt - \psi) \\ + & \sin 2(wt - \psi - 120^\circ) \\ + & \sin 2(wt - \psi - 240^\circ)] = 0 \end{aligned}$$

3.2.1 Generator Model

Next, we consider the components of the mmf perpendicular to mn

$$\begin{aligned} F_1 &= F_a \sin(wt-\psi) + \\ & F_b \sin(wt-\psi-120^\circ) + \\ & F_c \sin(wt-\psi-240^\circ) \end{aligned}$$

Using the identity $sin^2\alpha = \frac{1}{2}(1-\cos 2\alpha)$ we have

$$\begin{aligned} F_1 = \frac{F_m}{2}[\,3 \quad &- \quad \cos 2(wt-\psi) \\ &+ \quad \cos 2(wt-\psi-120^\circ) \\ &+ \quad \cos 2(wt-\psi-240^\circ)\ ] = \frac{3}{2}F_m \end{aligned}$$

It is concluded that the resultant armature mmf has a constant amplitude perpendicular to line mn, and rotates at a constant speed and in synchronism with the field mmf F_r

- In the diagram below, note that F_s is perpendicular to line mn and rotates with it at the same speed.
- This indicates that the armature current I_a is producing a reactive voltage drop parallel to line mn due to inductance in the machine.

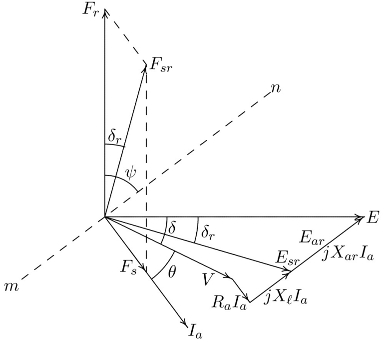


Figure 2: Combined phasor / vector for one phase of a cylindrical rotor generator

3.2.1 Generator Model

- The rotor field F_r produces the no-load generated voltage E (at zero armature current). Note that E lags F_r by 90°.
- E is called the excitation voltage, proportional to the field current.
- The voltage/current phasors for phase a are lagging the flux diagram by 90°.
- Note that Figure 2 is a hybrid combining spatial and temporal vectors.

3.2.1 Generator Model

- Assuming the armature carries a current to a load, now the armature reaction flux F_s is produced. This is perpendicular to mn
- The two fluxes (due to rotor and armature) combine together to form the resultant flux F_{sr}.
- The resultant flux induces the generated on-load emf E_{sr}
- The armature mmf F_s induces the voltage E_{ar} known as armature reaction voltage.
- In all cases, each mmf produces a voltage lagging the mmf by 90°.

3.2.1 Generator Model

- Note that the voltage E_{ar} leads F_s (hence I_a) by 90°.
- Thus we can theorize an inductor model for this relationship with reactance X_{ar}

$$E_{ar} = jX_{ar}I_a$$

- X_{ar} is known as the reactance of armature reaction. Thus we have the circuit equation

$$E = E_{sr} + jX_{ar}I_a$$

3.2.1 Generator Model

- The terminal voltage V is less than E_{sr} by the amount of voltage drop $R_a I_a$ and leakage reactance voltage drop $X_l I_l$. Thus

  $$E = V + [R_a + j(X_l + X_{ar})]I_a$$

  which can be simplified to

  $$E = V + [R_a + jX_s]I_a$$

  where $X_s = X_l + X_{ar}$ is known as synchronous reactance

Table of Contents

3.3.1 Power factor control

- Most generators are connected to a large power grid. This is called the infinite bus (its voltage, angle, and frequency are constant)
- Assuming the generator has a small leakage reactance and small armature resistance, then its model is shown below
- We have two ways of calculating the power (per phase)

$$P_{1\phi} = |V||I_a| \cos\theta \tag{1}$$

$$P_{1\phi} = \frac{|E||V|}{X_s} \sin\delta \tag{2}$$

- Equating both equations, we have

$$|E| \sin\delta = X_s |I_a| \cos\theta.$$

Assuming the power is constant, then from (1), $P_{1\phi} = |V||I_a|\cos\theta$, we have $|I_a|\cos\theta = \text{const}$. This locus is shown as the vertical dashed line in Figure 3.

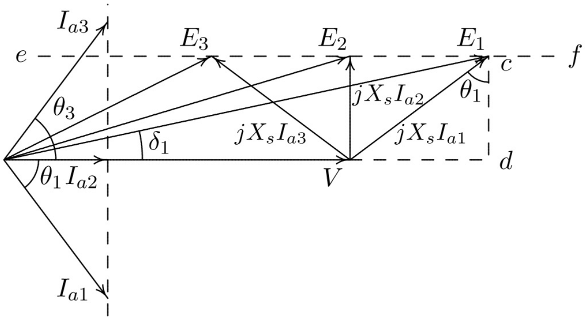


Figure 3: Variation of field current at constant power

3.3.1 Power factor control

- From (2), assuming the power is a constant, we have $|E| \sin \delta = \text{const}$. This locus is the horizontal dashed line shown.
- Note that for this load, the minimum armature current is I_{a2} when the power factor of the generator is unity.
- Note that E_2 is directly above the voltage V. Thus I_{a2} and V are in phase, producing unity power factor.
- If the excitation is increased, the emf of the generator is increased to some value say E_1 as shown.

3.3.1 Power factor control

- Clearly the voltage of the generator leads the current hence it is like an inductor consuming vars (lagging p.f.).
- On the other hand, if the excitation is reduced below that for unity power factor, the emf of the generator is smaller, say E_3 which lags I_{a3}.
- Now the power factor of the generator leading, i.e. it is like a capacitor (current leading voltage).

Example (3.1)

A 50-MVA, 30-kV, three-phase, 60-Hz synchronous generator has a synchronous reactance of $9\ \Omega$ per phase and a negligible resistance. The generator is delivering rated power at a 0.8 power factor lagging at the rated terminal voltage to an infinite bus.

(a) Determine the excitation voltage per phase E and the power angle δ.

(b) With the excitation held constant at the value found in (a), the driving torque is reduced until the generator is delivering $25\ MW$. Determine the armature current and the power factor.

(c) If the generator is operating at the excitation voltage of part (a), what is the steady-state maximum power the machine can deliver before losing synchronism?