Xi'an Jiaotong-Liverpool University
西交利物浦大学

EEE204 Continuous and Discrete Time Signals and Systems II

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Electrical and Electronic Engineering

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Week 6

Preparation for Mid-term Exam

Preparation for Mid-term Exam

- Representation of discrete-time (DT) signals
  - Analytical expression: $x[n], y[m], z[k]$;
  - The argument of any DT signal must be an integer;
  - Graphical illustration: elementary signals $(\delta[n], u[n], \cos(2\pi f n))$ and their combinations.
- Energy and power signals
  - Energy signals have finite energy and zero average power;
  - Power signals have infinite energy and finite average power (periodical signals);
  - Some signals have both infinite energy and power.

- Elementary signals and some important equations
  - $\delta[n]$, $u[n]$, a^n, sinusoid;
  - $x[n] = \sum_{k=-\infty}^{+\infty} x[k]\delta[n-k]$;
  - $\delta[n] = u[n] - u[n-1]$;
  - $\sum_{k=0}^{N-1} \delta[n-k] = u[n] - u[n-N]$.
- Periodic signals
  - $x[n] = x[n+N], \forall n,\ N \in Z^+,\ N \geqslant 1$;
  - Find the fundamental period for at least complex exponential and sinusoidal signals.

- Plots of advanced DT signals
  - Apply time domain transformation rules for signals in the form of $(x[\alpha n + \beta])$.
- Sampling theory
  - Derivation of DT signals obtained from sampling continuous signals both in time and frequency domains;
  - Nyquist rate, $f_s = 2f_{\text{max}}, \omega_s = 2\pi f_s = \dfrac{2\pi}{T_s}$;
  - Finding the maximum frequency of a continuous-time signal using CTFT is helpful.

Preparation for Mid-term Exam

- Discrete-time systems
  - Check the linearity of the system;
  - Check the time invariance of the system;
  - Check the causality of the system;
  - Check the stability of the system;
  - Check the memory of the system.
- Convolution sum: $y[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k]$
  - Evaluate convolution sum using three different approaches;
  - Plot signals in terms of the dummy variable k.

- z-transform
  - Find z-transform of DT signals;
  - Find the region of convergence of the resulting z-transform.
- Mathematical tools
  - Properties of trigonometric functions;
  - Euler's formula;
  - Properties of (infinite) power series;
  - CTFT and its properties.

z-Transform

We focus on the bilateral (double-side) z-transform of a discrete-time signal, which is defined as follows

$$X(z) \triangleq \sum_{n=-\infty}^{\infty} x[n]z^{-n},$$

or

$$X(\cdot) = Z\{x[\cdot]\},$$

or shorthand:

$$x[n] \overset{z}{\leftrightarrow} X(z).$$

z-Transform

$$X(z) = Z\{x[n]\} \triangleq \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

- Note capital letter for transform.
- In the maths literature, this is called a power series.
- It is a mapping from the space of discrete-time signals to the space of functions defined over (some subset of) the complex plane.
- We will also call the complex plane the z-plane.

- Any time we consider a summation or integral with infinite limits, we must think about convergence.
- Some infinite series do converge to a finite value, e.g., $1 + 1/2 + 1/4 + 1/8 + \cdots = \frac{1}{1-1/2} = 2$.
- Some infinite series simply do not converge.
- The infamous harmonic series is an infinite series that converges to infinity: $1 + 1/2 + 1/3 + 1/4 + \ldots = \infty$.

The region of convergence or ROC is defined as the set of values $z \in \mathbb{C}$ for which the sequence $x[n]z^{-n}$ is absolutely summable.

$$\left\{ z \in \mathbb{C} : \sum_{n=-\infty}^{\infty} |x[n]z^{-n}| \right\}$$

All absolutely summable sequences have convergent infinite series. But there are some sequences, such as $\sum_{n=1}^{\infty}(-1)^n/n$, that are not absolutely summable yet have convergent infinite series. These will not be included in our definition of ROC, but this will not limit the practical utility.

Example

Find the z-transform of $\delta[n]$ and ROC

Example

Find the z-transform of $\delta[n-k]$ and ROC

Example

Find the z-transform of $x[n] = \{9, \underline{3}, 0, \pi\}$ and ROC

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n},$$

$$= 9z^1 + 3z^0 + 0z^{-1} + \pi z^{-2},$$

$$= 9z + 3 + \pi z^{-2}.$$

Example

Find the z-transform of $x[n] = a^n u[n]$ and ROC

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} = \sum_{n=0}^{\infty} a^n z^{-n},$$

$$= \sum_{n=0}^{\infty} (az^{-1})^n = \frac{1}{1 - az^{-1}} = \frac{z}{z - a},$$

The series converges iff $|az^{-1}| < 1$, i.e., $|z| > |a|$.

Special case: $a = 1$ leaves just the unit step function.

$$u[n] \overset{z}{\leftrightarrow} U(z) = \frac{1}{1 - z^{-1}} = \frac{z}{z - 1}, |z| > 1.$$

Example

Find the z-transform of $x[n] = -a^n u[-n-1]$ and ROC

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n},$$

$$= \sum_{n=-\infty}^{-1} -a^n z^{-n} \overset{k=-n}{=\!=\!=} -\sum_{k=1}^{\infty} (a^{-1}z)^k,$$

$$= -\frac{a^{-1}z}{1-a^{-1}z} = -\frac{z}{a-z} = \frac{z}{z-a},$$

The series converges iff $|a^{-1}z| < 1$, i.e., $|z| < |a|$.

Note that the last two examples have the same formula for $X(z)$. The ROC is essential for resolving this ambiguity!

- Page 741–748 read section 10.0–10.1;
- Page 797, Q10.1: (a)–(d);
- Page 797, Q10.2;
- Page 797, Q10.3;
- Page 801, Q10.21: (a)–(h);
- Page 801, Q10.22: (a)–(d).

Thank you for your attention.