# Chapter 1.3 Probability

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# 1.3.3 Basic Theorems of Probability: problem

- Consider the distribution of pass/fails in a course by student gender.
- Find $P(Male \cap Pass)$
- Find $P(Male \cup Pass)$

| | Pass | Fail | Tot |
|---|---|---|---|
| Male | 60 | 30 | 90 |
| Female | 9 | 1 | 10 |
| Tot | 69 | 31 | 100 |

# 1.3.3 Basic Theorems of Probability: problem sol

- Consider the distribution of pass/fails in a course by student gender.

| | Pass | Fail | Tot |
|---|---|---|---|
| Male | 60 | 30 | 90 |
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| Tot | 69 | 31 | 100 |

- Find $P(Male \cap Pass) = \frac{60}{100} = 0.6$
- Find $P(Male \cup Pass) = P(Male) + P(Pass) - P(Male \cap Pass) = \frac{69+90-60}{100} = 0.99$
- Note that $P(Male) + P(Pass) = 1.59 > 1$

# 1.3.4 Conditional Probability: problem

- Consider the distribution of pass/fails in a course by student gender.
- The probability of picking a student that is male and passed is $\frac{60}{100} = 0.6$. The probability of picking a student that is female and passed is $\frac{9}{100} = 0.09$.
- So boys are way better than girls?

| | Pass | Fail | Total |
|---|---|---|---|
| Male | 60 | 30 | 90 |
| Female | 9 | 1 | 10 |
| Total | 69 | 31 | 100 |

# 1.3.4 Conditional Probability: problem solution

- The probability of picking a student that is male and passed is $\mathrm{P}(\mathrm{M} \cap P) = \frac{60}{100} = 0.6$ and the probability of picking a student that is female and passed is $\mathrm{P}(\mathrm{F} \cap P) = \frac{9}{100} = 0.09$.

| | Pass | Fail | Total |
|---|---|---|---|
| Male | 60 | 30 | 90 |
| Female | 9 | 1 | 10 |
| Total | 69 | 31 | 100 |

- But this is only because there are more males than females P(M) = 0.9 and P(F) = 0.1!
- The conditional probabilities tell a different story: $\mathrm{P}(P|\mathrm{M}) = \frac{0.6}{0.9} = \frac{60}{90} = 0.33$ and $\mathrm{P}(P|\mathrm{F}) = \frac{0.09}{0.1} = \frac{9}{10} = 0.9$
- So girls are 2.7 times more likely to pass than boys

# 1.3.5 Independence: problem

- A company produces nails and screws. The probability of producing defective nails is 0.01 while for screws it is 0.05. They deliver a box with 20 nails and 40 screws.
  I. What is the probability that Mr Yung picks at random one piece and it is a defective screw?
  II. If Mr Yung picks a screw, what is the probability that it is defective?

# 1.3.5 Independence: problem solution

Let N and S denote nail and Screw, respectively and D= defective. The probabilities given are conditional ones:
$P(D|N) = 0.01$and $P(D|S) = 0.05$.

I. The probability of picking a defective screw is

$$P(D \cap S) = P(S)P(D|S) = \frac{40}{60}0.05 = \frac{4}{120} = 1/30$$

II. This is simply the probability that a screw is defective (no matter how many screws there are) because it is conditional on picking a screw. So it is simply

$$P(D|S) = 0.05$$

# 1.3.6 Summary

- Axioms and Rules of Probability
  - Axioms, complementation, addition
- Conditional Probability
  - Definition, multiplication rule
- Independence
  - Definition and conditions for