EEE108 Electromagnetism and Electromechanics

Lecture 16

Linear DC Machines

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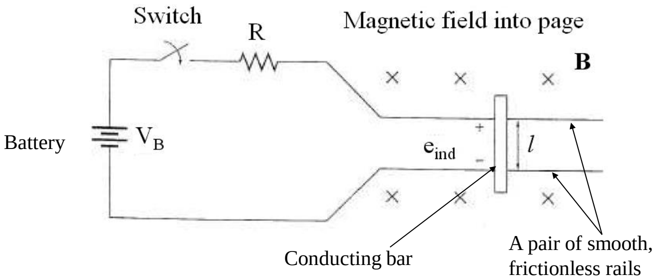
Today

Linear DC machines

- Four basic equations
- Starting behavior
- The Linear DC Machine as a motor
- The Linear DC Machine as a Generator
- The Linear DC Machine Starting Problems

Linear DC Machine



- A linear DC machine is about the simplest and easiest-to-understand version of a DC machine
- It operates according to the same principles and exhibits the same behavior as real generators/motors
- It serves as a good starting point in studying machines.

Linear DC Machine

Four Basic Equations

1. The force on a wire in the presence of a magnetic field:
$$\mathbf{F} = i(\mathbf{l} \times \mathbf{B})$$
F: force on wire
i: magnitude of current in wire
l: length of wire, in current's direction
B: magnetic flux density vector

2. The voltage induced on a wire moving in a magnetic field:
$$e_{ind} = (\mathbf{v} \times \mathbf{B}) \bullet \mathbf{l}$$
e_{ind}: voltage induced in wire
v: velocity of the wire
l: length of conductor in the magnetic field
B: magnetic flux density vector

3. Kirchhoff's voltage law:
EX: $V_B = e_{ind} + iR$

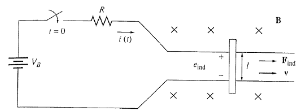
4. Newton's law:
For the bar: $F_{net} = ma$

Linear DC Machine

Starting the Linear DC Machine

The figure shows the linear DC machine under starting conditions:

To start → close the switch → a current flows in the bar: $i = \dfrac{V_B - e_{ind}}{R}$



$\boxed{V_B = e_{ind} + iR}$

In the very beginning of the starting, $e_{ind} = 0$, so $i = V_B/R$ $\boxed{e_{ind} = (\mathbf{v} \times \mathbf{B}) \bullet \mathbf{l}}$

When the current flows through the bar, the force is induced on the wire:

$F_{ind} = ilB$ the direction points to the right

As long as the force acts on the bar, the bar will accelerate to the right according Newton's law. $\boxed{F_{net} = ma}$

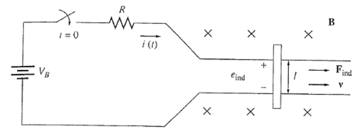
Then when the velocity of the bar begins to increase, a voltage is induced across the bar: $e_{ind} = vBl$ positive upward

The induced voltage reduces the current flowing in the bar (Lenz's law). Also can be seen by Kirchhoff's voltage law: $i \downarrow = \dfrac{V_B - e_{ind} \uparrow}{R}$

Linear DC Machine

Starting the Linear DC Machine

The result of this action is that eventually the bar will reach a constant steady-state speed where the net force on the bar is zero.



This will occur when e_{ind} has risen up to the voltage V_B. The steady state speed is:

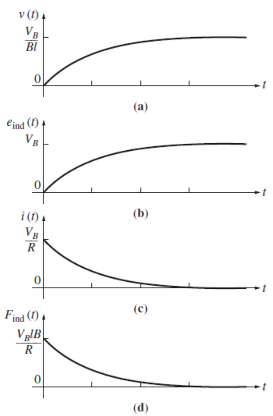
$V_B = e_{ind} = v_{ss}Bl \rightarrow v_{ss} = V_B/Bl$

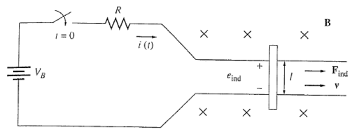
The bar will continue at this speed unless some external force disturbs it.

When the machine is started, velocity v, induced voltage e_{ind}, current i, and induced force F_{ind} is sketched in the figure.

Linear DC Machine

Starting the Linear DC Machine

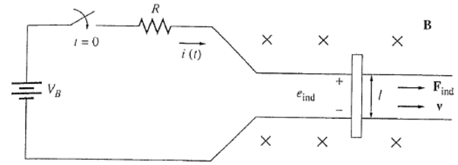




The linear DC machine on starting
(a) velocity v (t) as a function of time
(b) induced voltage $e_{ind}\,(t)$
(c) current $I\,(t)$
(d) induced force $F_{ind}\,(t)$

Linear DC Machine

Starting the Linear DC Machine



To summarize the starting behavior:

1. Closing the switch produces a current flow $i = V_B/R$.
2. The current flow produces a force on the bar given by $F = ilB$.
3. The bar accelerates to the right, producing an induced voltage e_{ind} as it speeds up.
4. This induced voltage reduces the current flow $i = [V_B - e_{ind}(\uparrow)\,]/R$.
5. The induced force is decreased $[F = i\,(\downarrow)\,lB]$ until eventually $F = 0$. At that point, $e_{ind} = V_B$, $i = 0$, and the bar moves at a constant no-load speed $v_{ss} = V_B/Bl$.

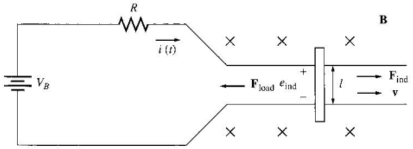
This is precisely the behavior observed in real motors on starting.

Linear DC Machine

The Linear DC Machine as a Motor

If the linear machine is initially running at the no-load steady-state conditions.

What will happen to this machine if an external load is applied to it?



Force F_{load} is added on the bar opposite the direction of the motion
→ a net force ($F_{net} = F_{load} - F_{ind}$) acts on the bar in the direction opposite the direction of motion.

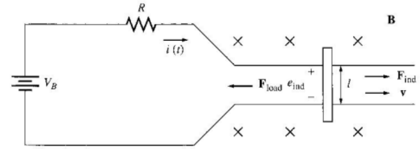
→ the bar slow down → as soon as the bar begins to slow down, the induced voltage on the bar drops (Lenz's law, also $e_{ind} = v{\downarrow}Bl$).

→ As the induced voltage decreases, the current flow in the bar rises:

$$i\uparrow = \frac{V_B - e_{\text{ind}}\downarrow}{R}$$

Linear DC Machine

The Linear DC Machine as a Motor

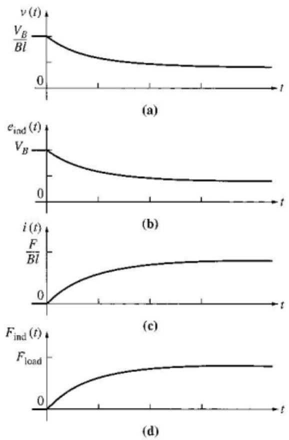


→ the induced force rises too ($F_{ind} = i{\uparrow}lB$).

→ The overall result of this chain of events is that the induced force rises until it is equal and opposite to the load force, and the bar again travels in a new lower steady speed.

Linear DC Machine

The Linear DC Machine as a Motor

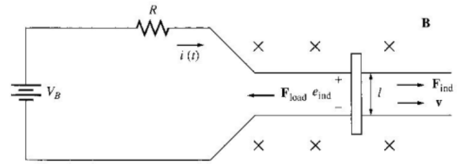


The linear DC machine operating at no-load conditions and then loaded as a motor

(a) velocity v (t) as a function of time
(b) induced voltage e_{ind} (t)
(c) current i (t)
(d) induced force F_{ind} (t)

Linear DC Machine

The Linear DC Machine as a Motor



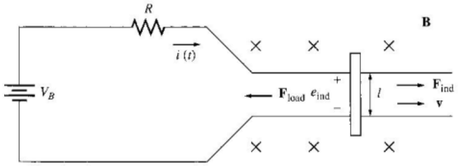
There is an induced force in the direction of motion of the bar, and the power is being converted from electrical form to mechanical form to keep the bar moving:
$P_{conv} = e_{ind}i = F_{ind}v$

The electric power of $e_{ind}i$ is consumed in the bar and is replaced by mechanical power ($F_{ind}v$).

Since power is converted from electrical to mechanical form, this bar is operating as a motor.

Linear DC Machine

The Linear DC Machine as a Motor



To summarize this behavior:

1. A force $\mathbf{F}_{load}$ is applied opposite to the direction of motion, which causes a net force $\mathbf{F}_{\text{net}}$ opposite to the direction of motion.
2. Acceleration $a = F_{net}/m$ is negative, so the bar slows down ($v\downarrow$).
3. The voltage $e_{ind} = v(\downarrow)Bl$ falls, and so $i = [V_B - e_{ind}(\downarrow)]/R$ increases
4. The induced force $F_{ind} = i(\uparrow)lB$ increases until $F_{ind} = F_{load}$ (with opposite directions) at a lower speed v.
5. The amount of electric power equal to $e_{ind}i$ is now being converted to mechanical power equal to $F_{ind}v$, and the machine is acting as a motor.

Linear DC Machine

Real DC Motor Behaves

A real DC motor in a precisely analogous fashion when it is loaded:

•As a load is added to its shaft, the motor begins to slow down, which reduces its internal voltage, increasing its current flow. The increased current flow increases its induced torque, and the induced torque will equal the load torque of the motor at a new, slower speed.

•The power converted from electrical form to mechanical form ($P_{conv} = F_{ind}v$) in a real motor is expressed by:

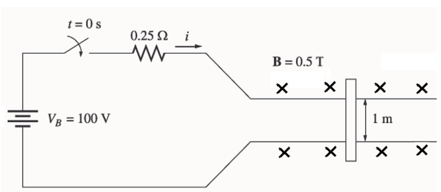
$$P_{conv} = \tau_{ind}\omega$$

Where the induced torque τ_{ind} is the rotational analog of the induced force F_{ind}, the angular velocity ω is the rotational analog of the linear velocity v.

Linear DC Machine

Example

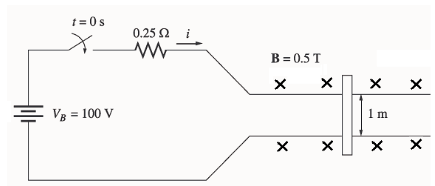
A linear machine has a magnetic flux density of 0.5 T directed into the page, a resistance of 0.25 Ω, a bar length l = 1.0 m, and a battery voltage of 100 V.
(a) What is the initial force on the bar at starting? What is the initial current flow?
(b) What is the no-load steady-state speed of the bar?
(c) If the bar is loaded with a force of 25 N opposite to the direction of motion, what is the new steady state speed? What is the efficiency of the machine under these circumstances?



Linear DC Machine

Example Solution

(a) What is the initial force on the bar at starting? What is the initial current flow?



The current in the bar at starting is

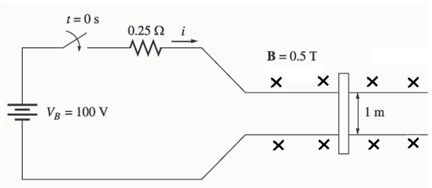
$$i = \frac{V_B}{R} = \frac{100}{0.25} = 400 \text{ A}$$

Therefore, the force on the bar at starting is
$F = i\,(\mathbf{l} \times \mathbf{B})$ = (400 A)(1 m)(0.5 T) = 200 N, to the right

Linear DC Machine

Example Solution

(b) What is the no-load steady-state speed of the bar?



The no-load steady-state speed of this bar can be found from the equation

$$V_B = e_{\text{ind}} = vBl$$

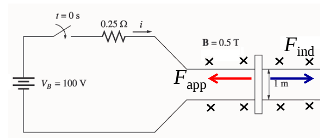
$$v = \frac{V_B}{Bl} = \frac{100 \text{ V}}{(0.5 \text{ T})(1 \text{ m})} = 200 \text{ m/s}$$

Linear DC Machine

Example Solution

(c) If the bar is loaded with a force of 25 N opposite to the direction of motion, what is the new steady state speed? What is the efficiency of the machine under these circumstances?

With a load of 25 N opposite to the direction of motion, the steady-state current flow in the bar will be given by



$$F_{\text{app}} = F_{\text{ind}} = ilB$$

$$i = \frac{F_{\text{app}}}{Bl} = \frac{25 \text{ N}}{(0.5 \text{ T})(1 \text{ m})} = 50 \text{ A}$$

The induced voltage in the bar will be

$$e_{\text{ind}} = V_B - iR = 100 \text{ V - } (50 \text{ A})(0.25 \ \Omega) = 87.5 \text{ V}$$

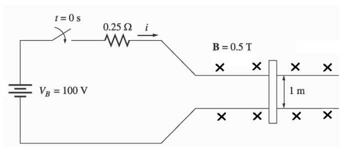
and the velocity of the bar will be

$$v = \frac{e_{\text{ind}}}{Bl} = \frac{87.5 \text{ V}}{(0.5 \text{ T})(1 \text{ m})} = 175 \text{ m/s}$$

Linear DC Machine

Example Solution

(c) If the bar is loaded with a force of 25 N opposite to the direction of motion, what is the new steady state speed? What is the efficiency of the machine under these circumstances?



The *input* power to the linear machine under these conditions is

$$P_{\text{in}} = V_B i = (100 \text{ V})(50 \text{ A}) = 5000 \text{ W}$$

The *output* power from the linear machine under these conditions is

$$P_{\text{out}} = e_{\text{ind}} i = (87.5 \text{ V})(50 \text{ A}) = 4375 \text{ W}$$

Therefore, the efficiency of the machine under these conditions is

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\% = \frac{4375 \text{ W}}{5000 \text{ W}} \times 100\% = 87.5\%$$

Where is the power of (5000-4375)=625 W ?

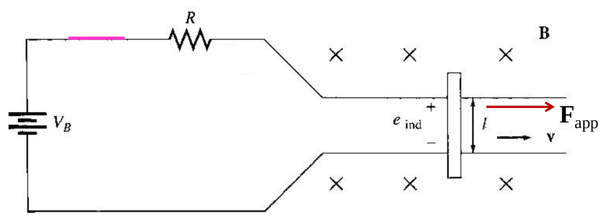
The machine is acting as a motor.

Linear DC Machine

The Linear DC Machine as a Generator

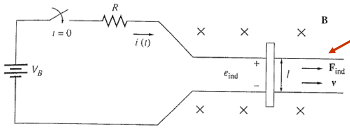
The linear machine is again operating under no-load steady-state condition.

A force is applied in the direction of motion, what happens?



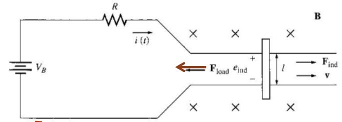
The linear DC machine as a generator

Linear DC Machine
The Linear DC Machine as a Generator



$F = 0$, $e_{ind} = V_B$, $i = 0$, and $v_{ss} = V_B/Bl$.

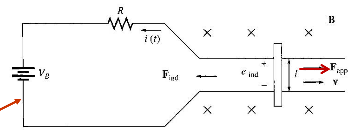
Starting a linear DC machine



$F_{ind} = F_{load}$ (with opposite directions) at a lower speed v.

Electric power $e_{ind}i$ is being converted to mechanical power $F_{ind}v$

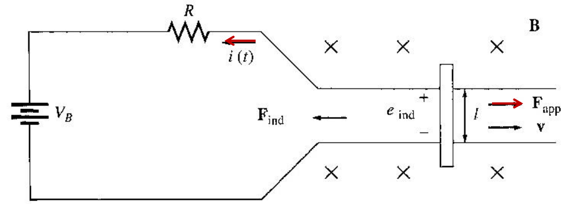
The linear DC machine as a motor



F = ?, v = ? Power = ?

The linear DC machine as a generator

Linear DC Machine
The Linear DC Machine as a Generator



⟶ A force in the direction of motion is applied.

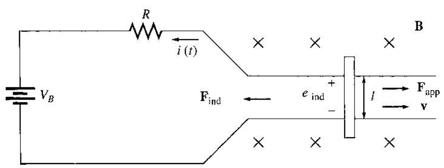
⟶ The applied force will cause the bar to accelerate in the direction of motion, and the velocity v of the bar will increase.

⟶ $e_{ind} = v\uparrow Bl$ will increase and will be larger than the battery voltage V_B.

⟶ The current will reverses direction when $e_{ind} > V_B$:

$$i = \frac{e_{ind} - V_B}{R}$$

Linear DC Machine
The Linear DC Machine as a Generator



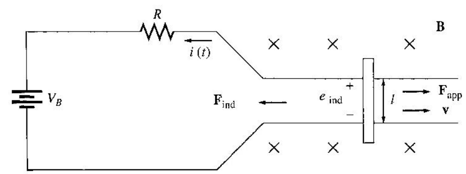
⟶ Since this current flows up through the bar, it induces a force in the bar: $F_{ind} = ilB$ to the left (right-hand rule) (opposes $\mathbf{F}_{app}$ on the bar)

⟶ Finally, the induced force will be equal and opposite to the applied force, the bar will be moving at higher speed than before.

⟶ Now the battery is charging. The machine is now serving as a generator.

⟶ The amount of the mechanical power $F_{ind}v$ is converted into electric power $e_{ind}i$.

Linear DC Machine
The Linear DC Machine as a Generator



To summarize this behavior:

1. A force $\mathbf{F}_{app}$ is applied in the direction of motion, $\mathbf{F}_{net}$ is in the direction of motion.
2. Acceleration $a = F_{net}/m$ is positive, so the bar speeds up ($v\uparrow$).
3. The voltage $e_{ind} = v(\uparrow)Bl$ increases, and so $i = [e_{ind}(\uparrow)-V_B]/R$ increases
4. The induced force $F_{ind} = i(\uparrow)lB$ increases until $F_{ind} = F_{app}$ (with opposite directions) at a higher speed v.
5. The amount of mechanical power equal to $F_{ind}v$ is now being converted to electric power $e_{ind}i$, and the machine is acting as a generator.

Linear DC Machine
Real DC Generator Behaves

A real DC generator behaves in precisely this manner:

A torque is applied to the shaft in the direction of motion, the speed of the shaft increase, the internal voltage increases, current flows out of the generator to the loads.

The amount of mechanical power converted to electric form in the real rotating generator:

$$P_{conv} = \tau_{ind}\omega$$

Linear DC Machine
The Linear DC Machine as a Generator and a Motor

It is interesting that the same machine acts as both motors and generators:

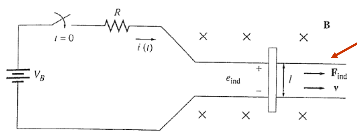
•The only difference between the two is whether the externally applied forces are in the direction of motion (generator) or opposite to the direction of motion (motor)

•Electrically, when $e_{ind} > V_B$, the machine acts as a generator, and when $e_{ind} < V_B$, the machine acts as a motor

•Whether the machine is a motor or a generator, both induced force (motor action), and induced voltage (generator action) are present at all times

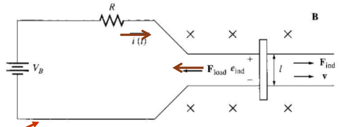
•The machine is a generator when it moved rapidly and a motor when it moved more slowly, but whether it was a motor or a generator, it always moved in the same direction.

Linear DC Machine
The Linear DC Machine as a Generator



Starting a linear DC machine

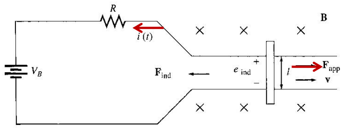
$F = 0$, $e_{ind} = V_B$, $i = 0$, and $v_{ss} = V_B/Bl$.



The linear DC machine as a motor

$F_{ind} = F_{load}$ (with opposite directions) at a lower speed v.

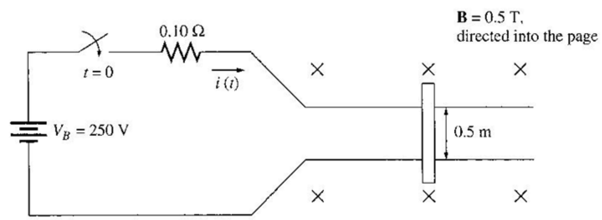
Electric power $e_{ind}i$ is being converted to mechanical power $F_{ind}v$



The linear DC machine as a generator

$F_{ind} = F_{app}$ (with opposite directions) at a higher speed v.
Mechanical power $F_{ind}v$ is being converted to electric power $e_{ind}i$

Linear DC Machine
Starting Problems

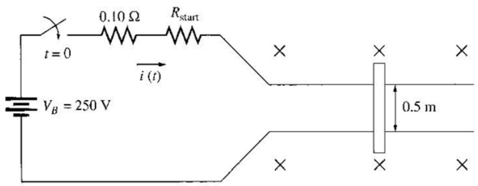


At starting conditions, the speed of the bar is zero, so $e_{ind} = 0$. the current flow at starting is: $i_{start} = V_B/R = 250/0.1 = 2500$ A

This starting is normally 10 times the rated current of the machine. Such current can cause severe damage to a motor.

Linear DC Machine

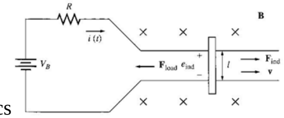
Starting Problems Solution



The easiest method for this simple linear machine is to insert an extra resistance into the circuit during starting to limit the current flow until e_{ind} build up enough to limit it.

Linear DC Machine

Example



A linear machine has the following characteristics

$B = 0.33$ T into page $R = 0.50\ \Omega$

$l = 0.5$ m $V_B = 120$ V

(a) If this bar has a load of 10 N attached to it opposite to the direction of motion, what is the steady-state speed of the bar?

(b) If the bar runs off into a region where the flux density falls to 0.30 T, what happens to the bar? What is its final steady-state speed?

(c) Suppose V_B is now decreased to 80 V with everything else remaining as in part (b). What is the new steady-state speed of the bar?

(d) From the results for parts (b) and (c), what are two methods of controlling the speed of a linear machine (or a real DC motor)?

Linear DC Machine

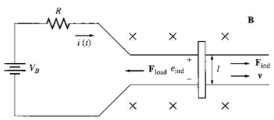
Example Solution

$B = 0.33$ T into page $R = 0.50\ \Omega$
$l = 0.5$ m $V_B = 120$ V

(a) With a load of 10 N opposite to the direction of motion, the steady-state current flow in the bar will be given by

$$F_{load} = F_{ind} = ilB$$

$$i = \frac{F_{load}}{Bl} = \frac{10\text{ N}}{(0.33\text{ T})(0.5\text{ m})} = 60.5\text{ A}$$



The induced voltage in the bar will be

$$e_{ind} = V_B - iR = 120\text{ V} - (60.5\text{ A})(0.50\ \Omega) = 89.75\text{ V}$$

and the velocity of the bar will be

$$v = \frac{e_{ind}}{Bl} = \frac{89.75\text{ V}}{(0.33\text{ T})(0.5\text{ m})} = 544\text{ m/s}$$

Linear DC Machine

Example Solution

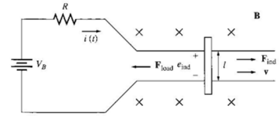
$B = 0.33$ T into page $R = 0.50\ \Omega$
$l = 0.5$ m $V_B = 120$ V

(b) If the flux density drops to 0.30 T while the load on the bar remains the same, there will be a speed transient until $F_{load} = F_{ind} = 10$ N again.

The new steady state current will be

$$F_{load} = F_{ind} = ilB$$

$$i = \frac{F_{load}}{Bl} = \frac{10\text{ N}}{(0.30\text{ T})(0.5\text{ m})} = 66.7\text{ A}$$



The induced voltage in the bar will be

$$e_{ind} = V_B - iR = 120\text{ V} - (66.7\text{ A})(0.50\ \Omega) = 86.65\text{ V}$$

and the velocity of the bar will be

$$v = \frac{e_{ind}}{Bl} = \frac{86.65\text{ V}}{(0.30\text{ T})(0.5\text{ m})} = 577\text{ m/s}$$

0.33T	60.5 A	89.75 V	544 m/s
0.30 T	66.7 A	86.65 V	577 m/s

$B\downarrow \Rightarrow v\uparrow$

Linear DC Machine

Example Solution

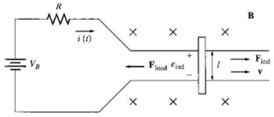
$B = 0.33$ T into page $R = 0.50\ \Omega$
$l = 0.5$ m $V_B = 120$ V

(c) If the battery voltage is decreased to 80 V while the load on the bar remains the same, there will be a speed transient until $F_{\text{load}} = F_{\text{ind}} = 10$ N again.

The new steady state current will be

$$F_{\text{load}} = F_{\text{ind}} = ilB$$

$$i = \frac{F_{\text{load}}}{Bl} = \frac{10\text{ N}}{(0.30\text{ T})(0.5\text{ m})} = 66.7\text{ A}$$



120 V	66.7 A	86.65 V	577 m/s
80 V	66.7 A	46.65 V	311 m/s

The induced voltage in the bar will be

$$e_{\text{ind}} = V_B - iR = 80\text{ V} - (66.7\text{ A})(0.50\ \Omega) = 46.65\text{ V}$$

and the velocity of the bar will be

$$v = \frac{e_{\text{ind}}}{Bl} = \frac{46.65\text{ V}}{(0.30\text{ T})(0.5\text{ m})} = 311\text{ m/s}$$

$$\boxed{V_B \downarrow \Rightarrow v \downarrow}$$

Linear DC Machine

Example Solution

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$l = 0.5$ m $V_B = 120$ V

(*d*) From the results of the two previous parts, we can see that there are two ways to control the speed of a linear DC machine:

- *reducing* the flux density B of the machine *increases* the steady-state speed, and
- *reducing* the battery voltage V_B *decreases* the stead-state speed of the machine.
- Both of these speed control methods work for real DC machines as well as for linear machines.

Summary

- Linear DC Machines:
  - ✓ Starting the Linear DC Machine
  - ✓ Linear DC Machine as a Motor
  - ✓ Linear DC Machine as a Generator
  - ✓ Starting Problem

Next

DC Machinery Fundamentals

Thanks for your attendance