Fundamental of Power Systems Part I
EEE210

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Overview

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2.1 Introduction

- The concept of power is of central importance in electrical power systems and is the main topic of this chapter.
- This chapter presents the review of the power concepts encountered in the electric circuit theory.
- Also the transmission of complex power between two voltage sources is considered, and the dependency of real power on the voltage phase angle and the dependency of reactive power on voltage magnitude is established.

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2.2 Power in single-phase circuits

We define $v(t)$ and $i(t)$ as:

$$
\begin{aligned}
v(t) &= V_m \cos(wt + \theta_v) && (1)\\
i(t) &= I_m \cos(wt + \theta_i) && (2)\\
& && (3)
\end{aligned}
$$

Then the instantaneous power is given by:

$$
\begin{aligned}
p(t) &= v(t)i(t) \\
&= V_m I_m \cos(wt + \theta_v)\cos(wt + \theta_i) \\
&\quad \ldots \\
&= |V||I|\cos\theta\,[1 + \cos 2(wt + \theta_v)] + |V||I|\sin\theta\sin 2(wt + \theta_v)
\end{aligned}
$$

where

- $\theta = \theta_v - \theta_i$ is the *power factor angle*, which is the angle between the voltage and current.

Tutorial

Derive $p(t) = v(t)i(t)$ and show that the final equation is similar to the one shown in the previous slide.

- Note: the power factor is given by $\cos\theta$.
- For an inductive circuit, voltage leads the current, resulting in lagging power factor.
- For a capacitive circuit, current leads the voltage, resulting in leading power factor.

The instantaneous power can be expressed in two parts:

$$p(t) = p_R(t) + p_X(t)$$

where

$$\begin{aligned} p_R(t) &= |V||I|\cos\theta\,[1+\cos 2(wt+\theta_v)] \text{ and} \\ p_X(t) &= |V||I|\sin\theta \sin 2(wt+\theta_v). \end{aligned}$$

As such, in terms of $P = |V||I|\cos\theta$ and $Q = |V||I|\sin\theta$, the instantaneous power can be expressed as:

$$p(t) = P\,[1+\cos 2(wt+\theta_v)] + Q\sin 2(wt+\theta_v)$$

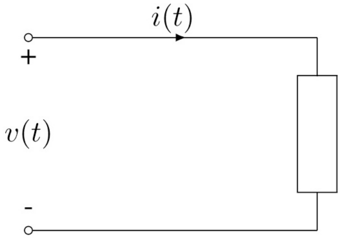

Figure 1: Sinusoidal source supplying a load

Example (2.1)

A supply voltage in Figure 1 is given by $v(t) = 100\cos wt$ and the load is inductive with impedance $Z = 1.25\angle 60^\circ\ \Omega$. Determine the expression for the instantaneous current $i(t)$ and the instantaneous power $p(t)$.

Tutorial: Write down your solution here

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2.3 Complex power

- The rms voltage phasor shown in (1) and (2) are

$$
\begin{aligned}
V &= |V|\angle\theta_v \\
I &= |I|\angle\theta_i
\end{aligned}
$$

- The term VI^* results in

$$
\begin{aligned}
VI^* &= |V||I|\angle\theta_v - \theta_i \\
&= |V||I|\angle\theta \\
&= |V||I|\cos\theta + j|V||I|\sin\theta
\end{aligned}
$$

which is identical to $S = VI^* = P + jQ$ where S is the complex power

For example, for an inductive load, the current lags the voltage and the phasor diagrams would be as shown in Figure 2

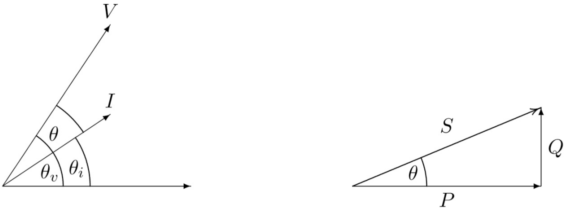

Figure 2: Phasor diagram and power triangle for an inductive load (lagging PF).

Likewise, for a capacitive load, the current would lead the voltage, with the phasor diagrams shown in Figure 3

Figure 3: Phasor diagram and power triangle for a capacitive load (leading PF).

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2.4 Complex power balance

From the conservation of energy, real power supplied by the source is equal to the sum of real powers absorbed by the load.

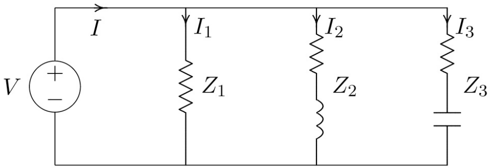

Figure 4: Three loads in parallel

Example (2.2)

Three impedances in parallel are supplied by a source of $V = 1200\angle 0^\circ$ V where the impedances are given by: $Z_1 = 60 + j0\,\Omega, Z_2 = 6 + j12\,\Omega$, and $Z_3 = 30 - j30\,\Omega$. Find the power absorbed by each load and the total complex power.

Tutorial: Write down your solution here

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2.5 Power factor correction

Adding a capacitor, usually in parallel, with an inductive load can improve the power factor.

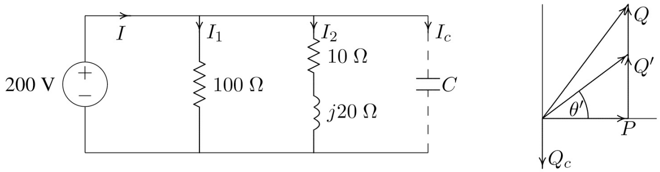

Figure 5: Circuit for Example 2.3 and the power triangle

Example (2.3)

Two loads $Z_1 = 100 + j0\,\Omega$ and $Z_2 = 10 + j20\,\Omega$ are connected across a 200-V rms, 60-Hz source as shown in Figure 5.

ⓐ Find the total real and reactive power, the power factor at the source, and the total current.

ⓑ Find the capacitance of the capacitor connected across the loads to improve the overall power factor to 0.8 lagging.

Tutorial: Write down your solution here

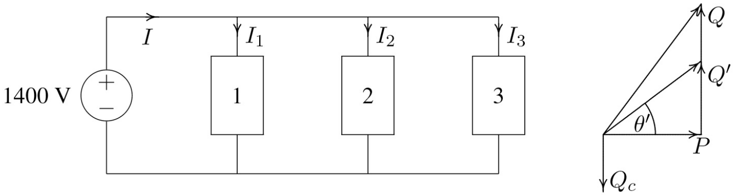

Figure 6: Circuit for Example 2.4

Example (2.4)

Three loads are connected in parallel across a 1400-V rms, 60-Hz single phase supply as shown in Figure 6.

- **Load 1** Inductive load, 125 kVA at 0.28 power factor.
- **Load 2** Capacitive load, 10 kW and 40 kvar.
- **Load 3** Resistive load of 15 kW.

(a) Find the total kW, kvar, kVA, and the supply power factor.

(b) A capacitor of negligible resistance is connected in parallel with the above loads to improve the power factor to 0.8 lagging. Determine the kvar rating of this capacitor and the capacitance in μF

Tutorial: Write down your solution here

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Considering two ideal voltage sources connected by a line impedance $Z = R + jX\ \Omega$ as shown in Figure 7.

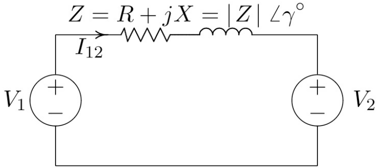

Figure 7: Two interconnected voltage sources

At the sending end, the real and reactive power are

$$P_{12} = \frac{|V_1||V_2|}{X}\sin(\delta_1 - \delta_2) \tag{4}$$

$$Q_{12} = \frac{|V_1|}{X}\left[|V_1| - |V_2|\cos(\delta_1 - \delta_2)\right] \tag{5}$$

Obervations from (4) and (5)

1. Usually $\delta_1 - \delta_2$ is very small (less than 10°, thus $P_{12} \propto \sin\delta$, i.e. small changes in power angle greatly change the real power, not the reactive power. If $\delta_1 > \delta_2$, then power flows from node 1 to node 2. If $\delta_1 < \delta_2$, then power flows in the opposite direction (from node 2 to 1).
2. Maximum power transfer occurs when $\delta = 90^\circ$ and is given by $P_{max} = \frac{|V_1||V_2|}{X}$.
3. Since $\delta \approx 0, Q \propto |V_1| - |V_2|$, thus small changes in $|V_1| - |V_2|$ greatly affect Q but not P.

From the observations,

1. In order to control real power, we need to change the power angle δ. This is done by increasing prime mover power (mechanical power driving the generator).
2. To control reactive power, we need to change the difference in voltage magnitude. This is done by changing the DC excitation of a generator.

Example (2.5)

Two voltage sources $V_1 = 120\angle -5^\circ$ V and $V_2 = 100\angle 0^\circ$ V are connected by a short line of impedance $Z = 1 + j7\,\Omega$ as shown in Figure 7. Determine the real and reactive power supplied or received by each source and the power loss in the line.

Tutorial: Write down your solution here

The End