EEE319 Optimisation
Lecture 4 Linear Programming (3)

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Outline

- Last week
  - Tableau

- This week
  - Linear programming with mixed constraints
  - Big M method

Mixed Constraints

- Mixed constraints from an example

Max $Z = 400x_1 + 200x_2$

s.t. $x_1 + x_2 = 30$

$2x_1 + 8x_2 \geq 80$

$x_1 \leq 20$

$x_1, x_2 \geq 0$

Mixed Constraints

- Inserting slack variable, surplus variable and artificial variable into constraints, where inequality is transformed into equations:

$x_1 + x_2 + a_1 = 30$	artificial variable
$2x_1 + 8x_2 - s_1 + a_2 = 80$	surplus + artificial variables
$x_1 + s_2 = 20$	slack variables
$Z = 400x_1 + 200x_2 - Ma_1 - Ma_2$	$-Ma_i$ because of maximisation

Mixed Constraints

- Arranging all the terms of objective function on one side

$$x_1 + x_2 + a_1 = 30$$
$$2x_1 + 8x_2 - s_1 + a_2 = 80$$
$$x_1 + s_2 = 20$$
$$Z - 400x_1 - 200x_2 + Ma_1 + Ma_2 = 0$$

Mixed Constraints

- Establishing initial tableau

$$x_1 + x_2 + a_1 = 30$$

$$2x_1 + 8x_2 - s_1 + a_2 = 80$$

$$x_1 + s_2 = 20$$

$$Z - 400x_1 - 200x_2 + Ma_1 + Ma_2 = 0$$

Basic Variables	x_1	x_2	s_1	s_2	a_1	a_2	Z	
	1	1	0	0	1	0	0	30
	2	8	−1	0	0	1	0	80
	1	0	0	1	0	0	0	20
	−400	−200	0	0	M	M	1	0

Sometimes a dashed line is used to separate the constraints and objective function

Mixed Constraints

- Eliminating the big M from the bottom row by row operations (very critical step)

$Basic\ Variables$	x_1	x_2	s_1	s_2	a_1	a_2	Z	
	1	1	0	0	1	0	0	30
	2	8	-1	0	0	1	0	80
	1	0	0	1	0	0	0	20
	-400	-200	0	0	M	M	1	0

$Basic\ Variables$	x_1	x_2	s_1	s_2	a_1	a_2	Z	
	1	1	0	0	1	0	0	30
	2	8	-1	0	0	1	0	80
	1	0	0	1	0	0	0	20
	$-M-400$	$-M-200$	0	0	0	M	1	$-30M$

$-M*R_1+R_4$

Mixed Constraints

- Eliminating the big M from the bottom row by row operations (very critical step)-continued

$Basic\ Variables$	x_1	x_2	s_1	s_2	a_1	a_2	Z	
	1	1	0	0	1	0	0	30
	2	8	-1	0	0	1	0	80
	1	0	0	1	0	0	0	20
	-400	-200	0	0	M	M	1	0

$Basic\ Variables$	x_1	x_2	s_1	s_2	a_1	a_2	Z	
	1	1	0	0	1	0	0	30
	2	8	-1	0	0	1	0	80
	1	0	0	1	0	0	0	20
	$-3M-400$	$-9M-200$	M	0	0	0	1	$-110M$

$-M*R_2+R_4$

Mixed Constraints

- Adding basic variables (coefficient on each column has got only one ‘1’)

Basic Variables	x_1	x_2	s_1	s_2	a_1	a_2	Z	
a_1	1	1	0	0	1	0	0	30
a_2	2	8	-1	0	0	1	0	80
s_2	1	0	0	1	0	0	0	20
Z	$-3M-400$	$-9M-200$	M	0	0	0	1	$-110M$

Setting non-basic variables to zero, $x_1 = x_2 = s_1 = 0$, $a_1 = 40$, $a_2 = 80$, $s_2 = 20$. They are feasible. Ignore Z so far.

Mixed Constraints

- Selecting the pivot column with the smallest negative value (M is a very big value, e.g. M=1,000,000)

$Basic\ Variables$	x_1	x_2	s_1	s_2	a_1	a_2	Z	
a_1	1	1	0	0	1	0	0	30
a_2	2	8	-1	0	0	1	0	80
s_2	1	0	0	1	0	0	0	20
Z	$-3M-400$	$-9M-200$	M	0	0	0	1	$-110M$

Mixed Constraints

- Selecting the pivot row with the smallest ratio (M is a very big value, e.g. M=1,000,000)

$Basic\ Variables$	x_1	x_2	s_1	s_2	a_1	a_2	Z	
a_1	1	1	0	0	1	0	0	30
a_2	2	8	-1	0	0	1	0	80
s_2	1	0	0	1	0	0	0	20
Z	$-3M-400$	$-9M-200$	M	0	0	0	1	$-110M$

Mixed Constraints

- Highlighting the intersected value

$Basic\ Variables$	x_1	x_2	s_1	s_2	a_1	a_2	Z	
a_1	1	1	0	0	1	0	0	30
a_2	2	8	-1	0	0	1	0	80
s_2	1	0	0	1	0	0	0	20
Z	$-3M-400$	$-9M-200$	M	0	0	0	1	$-110M$

Mixed Constraints

- Changing the intersected value to 1

$Basic\ Variables$	x_1	x_2	s_1	s_2	a_1	a_2	Z	
a_1	1	1	0	0	1	0	0	30
a_2	$1/4$	1	$-1/8$	0	0	$1/8$	0	10
s_2	1	0	0	1	0	0	0	20
Z	$-3M-400$	$-9M-200$	M	0	0	0	1	$-110M$

Mixed Constraints

- Changing the other value in that column to zero

$Basic\ Variables$	x_1	x_2	s_1	s_2	a_1	a_2	Z	
a_1	1	1	0	0	1	0	0	30
a_2	$1/4$	1	$-1/8$	0	0	$1/8$	0	10
s_2	1	0	0	1	0	0	0	20
Z	$-3M-400$	$-9M-200$	M	0	0	0	1	$-110M$

$Basic\ Variables$	x_1	x_2	s_1	s_2	a_1	a_2	Z	
a_1	$3/4$	0	$1/8$	0	1	$-1/8$	0	20
a_2	$1/4$	1	$-1/8$	0	0	$1/8$	0	10
s_2	1	0	0	1	0	0	0	20
Z	$-\frac{3}{4}M-350$	0	$-\frac{1}{8}M-25$	0	0	$\frac{9}{8}M$	1	$-20M+2000$

R_1-R_2;

R_4+(9M+200)R_2

Mixed Constraints

- x_2 enters basic variable

$Basic\ Variables$	x_1	x_2	s_1	s_2	a_1	a_2	Z	
a_1	$3/4$	0	$1/8$	0	1	$-1/8$	0	20
x_2	$1/4$	1	$-1/8$	0	0	$1/8$	0	10
s_2	1	0	0	1	0	0	0	20
Z	$-\frac{3}{4}M-350$	0	$-\frac{1}{8}M-25$	0	0	$\frac{9}{8}M$	1	$-20M+2000$

Mixed Constraints

- Selecting pivot column and row again

$Basic\ Variables$	x_1	x_2	s_1	s_2	a_1	a_2	Z	
a_1	$3/4$	0	$1/8$	0	1	$-1/8$	0	20
x_2	$1/4$	1	$-1/8$	0	0	$1/8$	0	10
s_2	1	0	0	1	0	0	0	20
Z	$-\frac{3}{4}M - 350$	0	$-\frac{1}{8}M - 25$	0	0	$\frac{9}{8}M$	1	$-20M + 2000$

Mixed Constraints

- Pushing other values to zero

$Basic\ Variables$	x_1	x_2	s_1	s_2	a_1	a_2	Z	
a_1	$3/4$	0	$1/8$	0	1	$-1/8$	0	20
x_2	$1/4$	1	$-1/8$	0	0	$1/8$	0	10
s_2	1	0	0	1	0	0	0	20
Z	$-\frac{3}{4}M-350$	0	$-\frac{1}{8}M-25$	0	0	$\frac{9}{8}M$	1	$-20M+2000$

$Basic\ Variables$	x_1	x_2	s_1	s_2	a_1	a_2	Z	
a_1	0	0	$1/8$	$-3/4$	1	$-1/8$	0	5
x_2	$1/4$	1	$-1/8$	0	0	$1/8$	0	10
s_2	1	0	0	1	0	0	0	20
Z	$-\frac{3}{4}M-350$	0	$-\frac{1}{8}M-25$	0	0	$\frac{9}{8}M$	1	$-20M+2000$

Mixed Constraints

- Pushing other values to zero

Basic Variables	x_1	x_2	s_1	s_2	a_1	a_2	Z	
a_1	$3/4$	0	$1/8$	0	1	$-1/8$	0	20
x_2	$1/4$	1	$-1/8$	0	0	$1/8$	0	10
s_2	1	0	0	1	0	0	0	20
Z	$-\frac{3}{4}M-350$	0	$-\frac{1}{8}M-25$	0	0	$\frac{9}{8}M$	1	$-20M+2000$

Basic Variables	x_1	x_2	s_1	s_2	a_1	a_2	Z	
a_1	0	0	$1/8$	$-3/4$	1	$-1/8$	0	5
x_2	0	1	$-1/8$	$-1/4$	0	$1/8$	0	5
s_2	1	0	0	1	0	0	0	20
Z	$-\frac{3}{4}M-350$	0	$-\frac{1}{8}M-25$	0	0	$\frac{9}{8}M$	1	$-20M+2000$

Mixed Constraints

- Pushing other values to zero

$Basic\ Variables$	x_1	x_2	s_1	s_2	a_1	a_2	Z	
a_1	$3/4$	0	$1/8$	0	1	$-1/8$	0	20
x_2	$1/4$	1	$-1/8$	0	0	$1/8$	0	10
s_2	1	0	0	1	0	0	0	20
Z	$-\frac{3}{4}M - 350$	0	$-\frac{1}{8}M - 25$	0	0	$\frac{9}{8}M$	1	$-20M + 2000$

$Basic\ Variables$	x_1	x_2	s_1	s_2	a_1	a_2	Z	
a_1	0	0	$1/8$	$-3/4$	1	$-1/8$	0	5
x_2	0	1	$-1/8$	$-1/4$	0	$1/8$	0	5
s_2	1	0	0	1	0	0	0	20
Z	0	0	$-\frac{1}{8}M - 25$	$\frac{3}{4}M + 350$	0	$\frac{9}{8}M$	1	$-5M + 9000$

Mixed Constraints

- x_1 enters basic variables

$Basic\ Variables$	x_1	x_2	s_1	s_2	a_1	a_2	Z	
a_1	0	0	$1/8$	$-3/4$	1	$-1/8$	0	5
x_2	0	1	$-1/8$	$-1/4$	0	$1/8$	0	5
x_1	1	0	0	1	0	0	0	20
Z	0	0	$-\frac{1}{8}M-25$	$\frac{3}{4}M+350$	0	$\frac{9}{8}M$	1	$-5M+9000$

Mixed Constraints

- Selecting pivoting column and row again

$Basic\ Variables$	x_1	x_2	s_1	s_2	a_1	a_2	Z	
a_1	0	0	$1/8$	$-3/4$	1	$-1/8$	0	5
x_2	0	1	$-1/8$	$-1/4$	0	$1/8$	0	5
x_1	1	0	0	1	0	0	0	20
Z	0	0	$-\frac{1}{8}M-25$	$\frac{3}{4}M+350$	0	$\frac{9}{8}M$	1	$-5M+9000$

Mixed Constraints

- Changing the value to 1

$Basic\ Variables$	x_1	x_2	s_1	s_2	a_1	a_2	Z	
a_1	0	0	$1/8$	$-3/4$	1	$-1/8$	0	5
x_2	0	1	$-1/8$	$-1/4$	0	$1/8$	0	5
x_1	1	0	0	1	0	0	0	20
Z	0	0	$-\frac{1}{8}M-25$	$\frac{3}{4}M+350$	0	$\frac{9}{8}M$	1	$-5M+9000$

$Basic\ Variables$	x_1	x_2	s_1	s_2	a_1	a_2	Z	
a_1	0	0	1	-6	8	-1	0	40
x_2	0	1	$-1/8$	$-1/4$	0	$1/8$	0	5
x_1	1	0	0	1	0	0	0	20
Z	0	0	$-\frac{1}{8}M-25$	$\frac{3}{4}M+350$	0	$\frac{9}{8}M$	1	$-5M+9000$

Mixed Constraints

- Changing the other values in that column to 0

Basic Variables	x_1	x_2	s_1	s_2	a_1	a_2	Z	
a_1	0	0	1	-6	8	-1	0	40
x_2	0	1	$-1/8$	$-1/4$	0	$1/8$	0	5
x_1	1	0	0	1	0	0	0	20
Z	0	0	$-\frac{1}{8}M-25$	$\frac{3}{4}M+350$	0	$\frac{9}{8}M$	1	$-5M+9000$

Basic Variables	x_1	x_2	s_1	s_2	a_1	a_2	Z	
a_1	0	0	1	-6	8	-1	0	40
x_2	0	1	0	-1	1	0	0	10
x_1	1	0	0	1	0	0	0	20
Z	0	0	0	200	$M+200$	$M-25$	1	10000

Mixed Constraints

- s_1 enters basic variables

$Basic\ Variables$	x_1	x_2	s_1	s_2	a_1	a_2	Z	
s_1	0	0	1	-6	8	-1	0	40
x_2	0	1	0	-1	1	0	0	10
x_1	1	0	0	1	0	0	0	20
Z	0	0	0	200	$M+200$	$M-25$	1	10000

Z=10000, when x_1=20 and x_2=10

Simplex Method

- This is also called big M method.

Summary

- Optimisation from an example
- No constraints
- No confirmation of size of errors yet

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