EEE213 Power Electronics and Electromechanism

2. Uncontrolled Rectifier – Single Phase

Rectification
Controllability
Uncontrolled and Controlled
vs
Input power (source)
Single phase and Three phase

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Before we start …

- What is "Rectifier"?
  - A rectifier is an electrical device that converts alternating current (AC), current that periodically reverses direction, to direct current (DC), current that flows in only one direction.

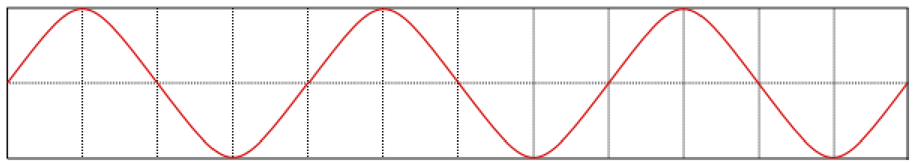
$$\boxed{\text{AC}} \xrightarrow[\text{Rectifier}]{\text{AC to DC}} \boxed{\text{DC}}$$

  - Rectifiers may be made of ***solid state diodes***, vacuum tube diodes, mercury arc valves, and other components.

Semiconductor devices

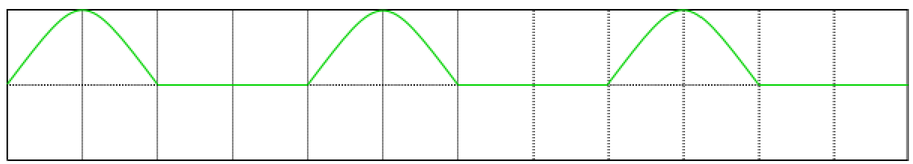
Classification of rectifiers

- Controllability
  - Uncontrolled
  - Controlled
- Input power (source)
  - Single phase
  - Three phase
- Waveform
  - Half-wave
  - Full-wave



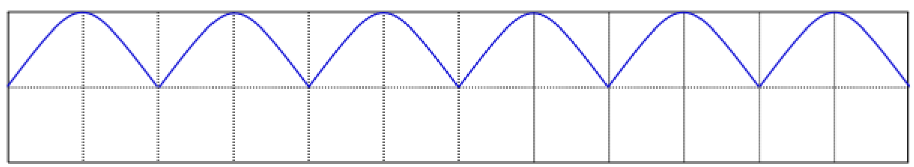
Sinusoidal Signal

Input



Half-wave Rectification

Half-wave



Full-wave Rectification

Full-wave

Importance of rectifiers

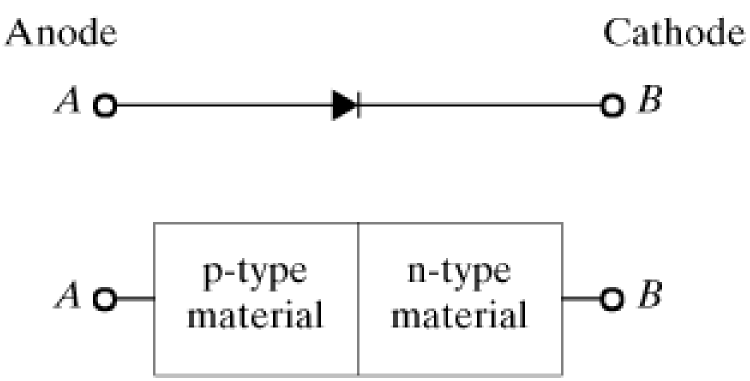
- The primary application of rectifiers is to derive DC power from an AC supply.
  - Virtually all electronic devices require DC, so rectifiers find uses inside the power supplies of virtually all electronic equipment;
  - Drive Electromechanical apparatus;
  - Electrolyte.
- Devices
  - Diodes (二极管) – not controllable;
  - Thyristors (晶闸管)
    - Silicon-controlled rectifiers (SCR): on-controllable
    - Gate turn-off Thyristors (GTO) : on/off-controllable

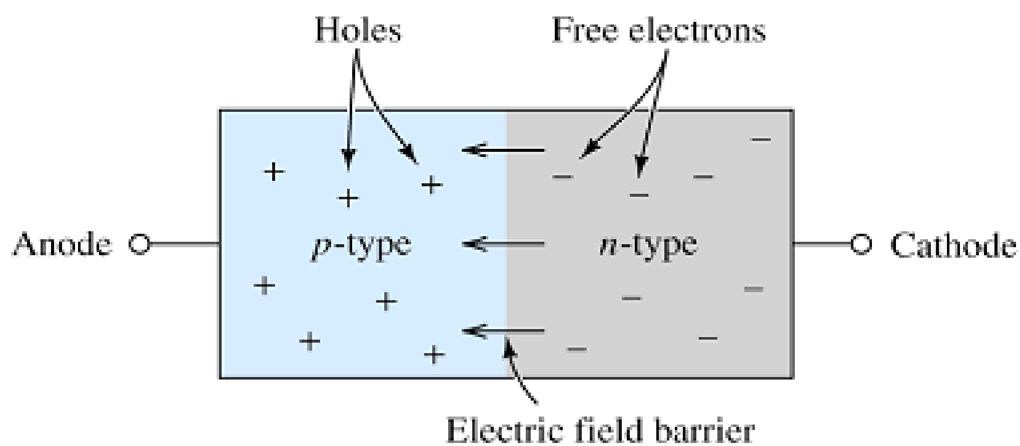
Outline

- 1. Diode
  - Diodes and power diodes
  - Static characteristics
  - Dynamic characteristics
  - Types of power diodes
- 2. Uncontrolled half-wave rectifier
  - Basic resistive loading
  - Parameters of the rectifier
  - An example of parameter calculation (R load)
  - RE load
  - RL load
  - RC load
- 3. Uncontrolled full-wave rectifier
  - Resistive loading
  - RL load
  - RLE load

1.1 Diodes

- A diode is a two-terminal, passive, non-linear device;
- It is constituted by a P-N junction;
- The P-type side is the anode, and the N-type side is the cathode.

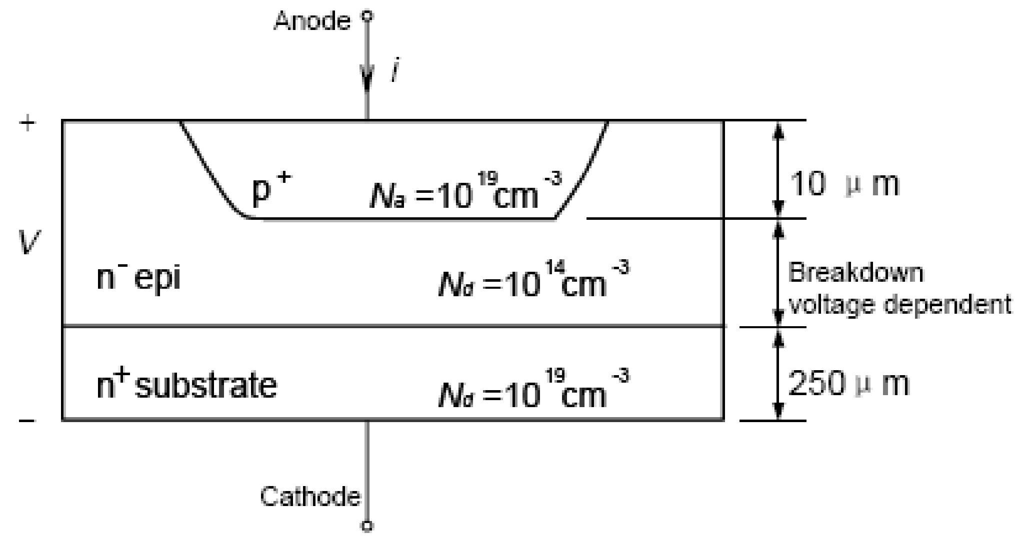


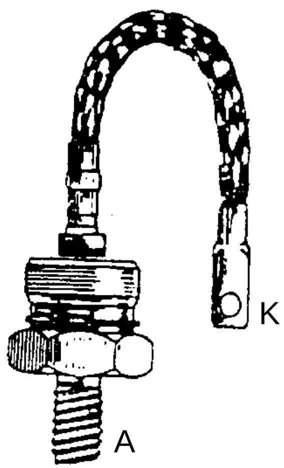


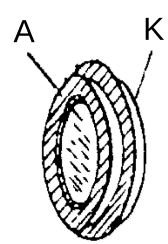
1.1 Power Diodes

- Features different from low-power (information electronic) diodes
  - Higher voltage / current withstand
  - Low leakage current
  - Low conduction loss

  - Larger size
  - Low speed

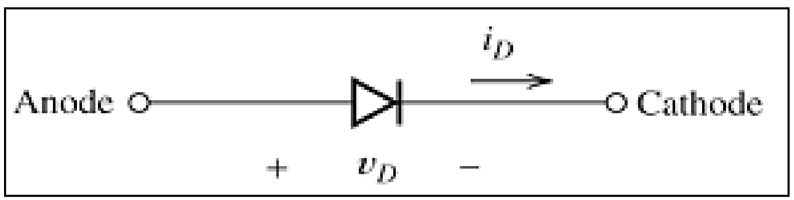


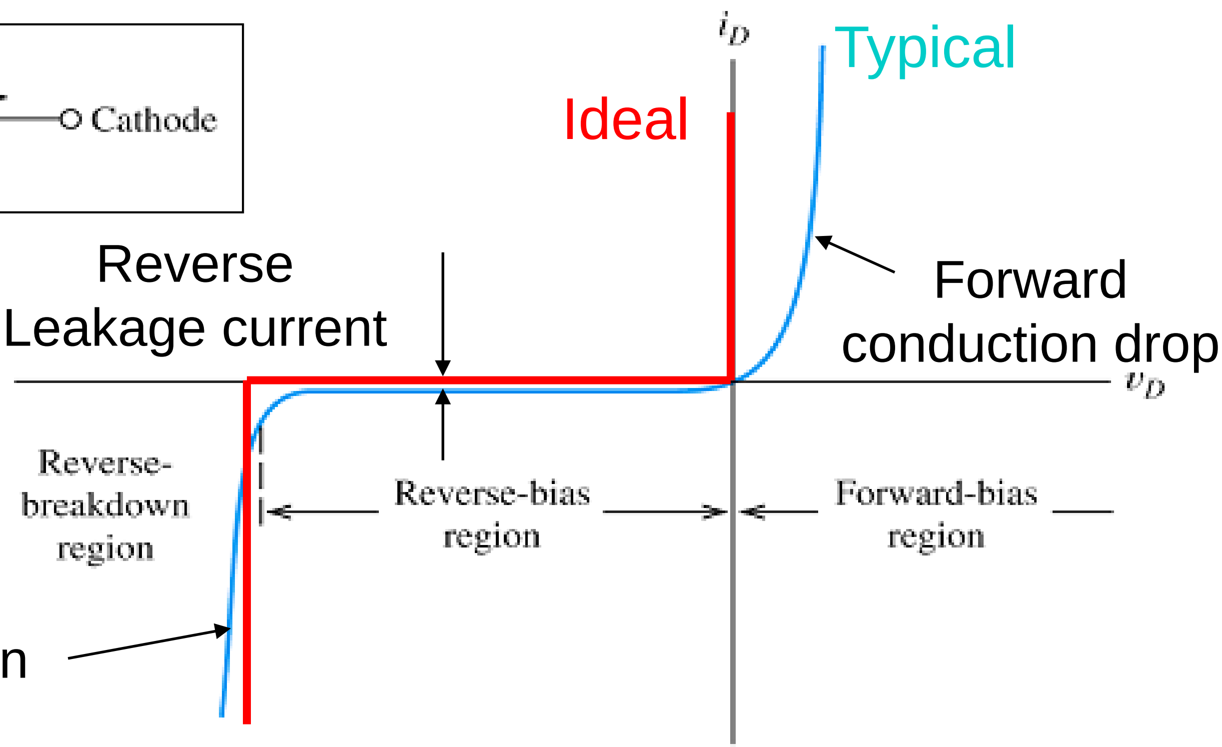




1.2 Static characteristics (I-V Relation)

- v_D = bias voltage, negative for reverse bias and positive for forward bias
- i_D = current through diode, from anode (P region) to cathode (N region)





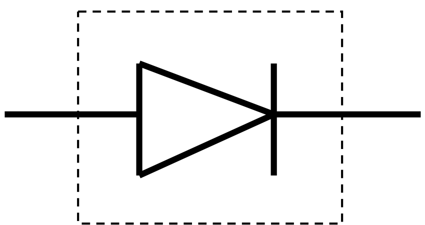
1.2 I-V Relation

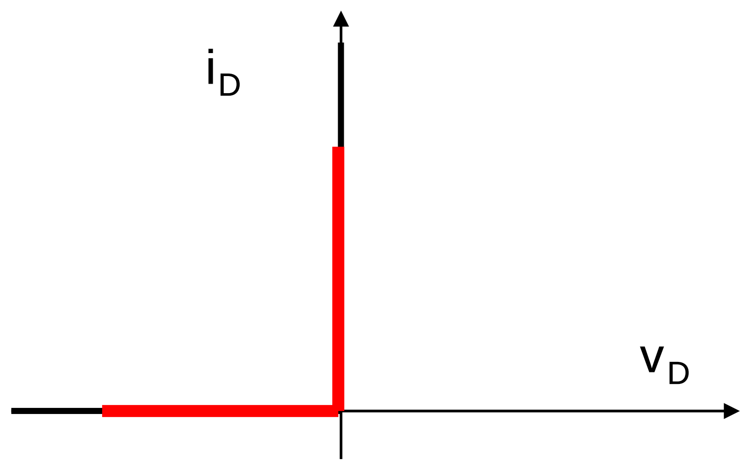
- Forward-biased region: a conductor (ON)
  - In forward bias, a diode has a threshold voltage V_D (门槛电压), at which current flow becomes significant and increases exponentially. V_D=0.3V for Ge, 0.7V for Si. \
- Reverse-biased region: an insulator (OFF)
  - In reverse bias, there is a negligible reverse saturation current I_s.
- Breakdown region:
  - If reverse bias is increased sufficiently to a breakdown voltage V_B, there is a sudden increase in reverse current, which is called breakdown.

Equivalent Circuit Representation I

- The ideal diode model
  - A perfect conductor or short circuited, if forward biased;
  - A perfect insulator or open circuited, if reverse biased.

The Ideal Diode Model

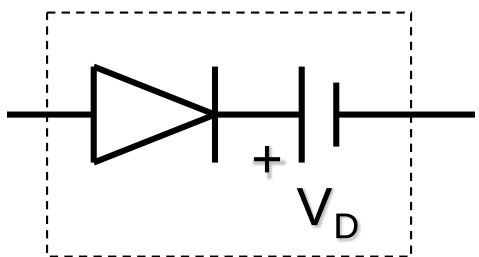


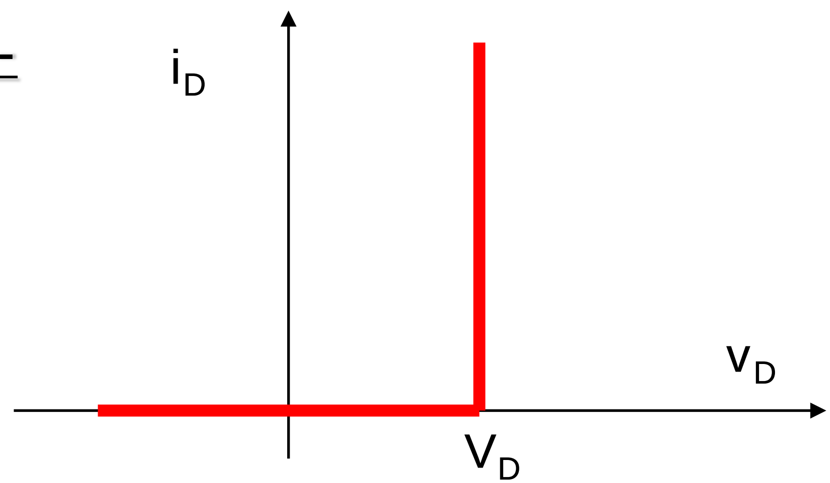


Equivalent Circuit Representation II

- The Constant-Voltage-Drop Model
  - A perfect conductor or short circuited, if forward bias voltage exceed the threshold voltage V_D
  - A perfect insulator or open circuited, if reverse biased.

The Constant-Voltage-Drop Model

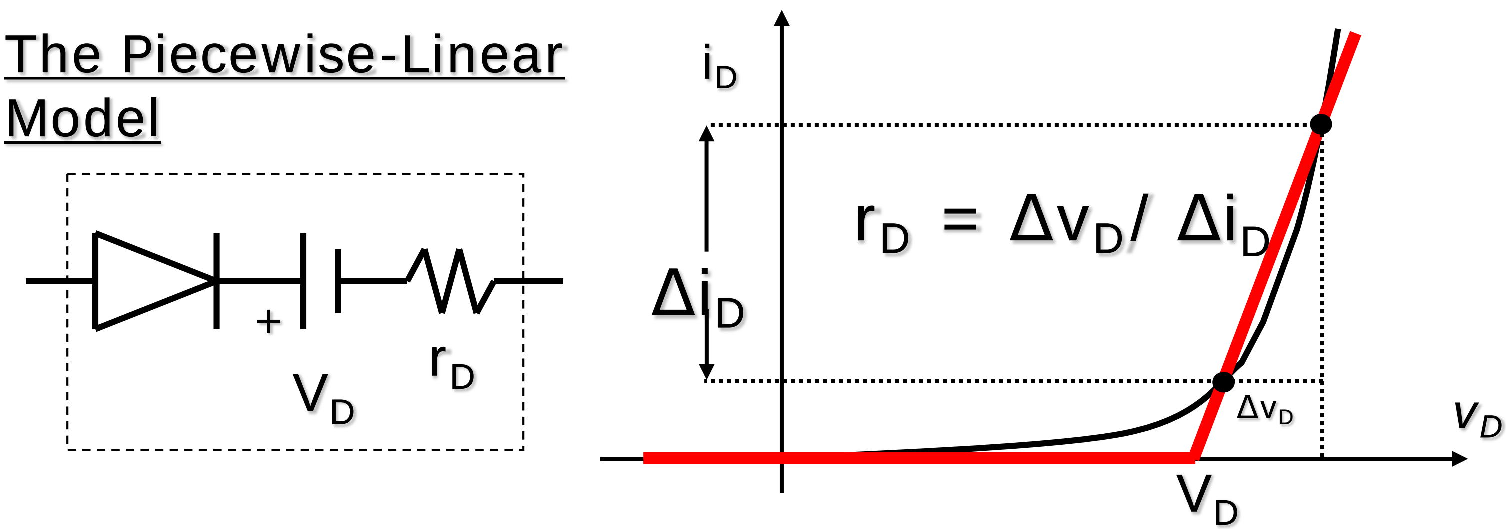




Equivalent Circuit Representation III

- The Piecewise-Linear Model
  - A resistive conductor, if forward bias voltage exceed the threshold voltage V_D
  - A perfect insulator or open circuited, if reverse biased.

The Piecewise-Linear Model



1.3 Junction capacitor (结电容)

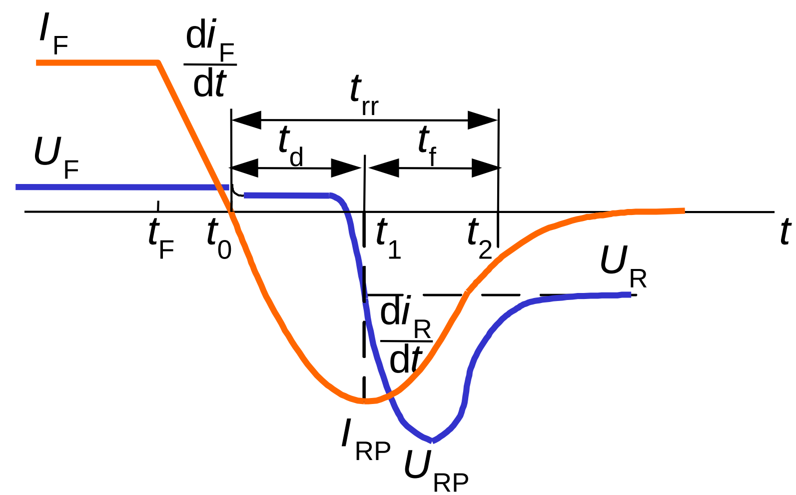
- The positive and negative charge in the depletion region is variable with the changing of external voltage.
  - Junction capacitor C_J
    - Potential barrier capacitor C_B (势垒电容)
    - Diffusion capacitor C_D (扩散电容)
- Junction capacitor influences the switching characteristics of diodes, especially the switching frequency.

1.3 Switching (dynamic) characteristics I

- Turn-off transient – reverse-recovery process:
  - Reverse-recovery time, reverse-recovery charge,
  - reverse-recovery peak current.

Reverse recovery time： $t_{rr}= t_d+ t_f$
Delay time： $t_d= t_1- t_0$,
Current falling time： $t_f= t_2- t_1$
Soft factor： $S_r = t_f /t_d$
Peak reverse recovery current:

$$I_{RP} = t_d \frac{di_F}{dt}$$



The current in a forward-biased junction diode is due to the net effect of majority and minority carriers. Once a diode is in a forward conduction mode and then its forward current is reduced to zero (due to the natural behavior of the diode circuit or application of a reverse voltage) , the diode continues to conduct due to minority carriers that remain stored in the PN-junction and the bulk semiconductor material. The minority carriers require a certain time to recombine with opposite charges and to be neutralized. This time is called **the reverse recovery time** of the diode.

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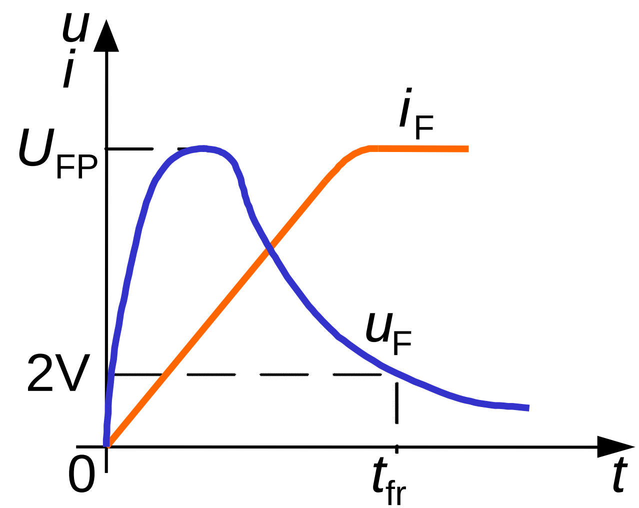
1.3 Switching (dynamic) characteristics II

- Turn-on transient –
  forward-recovery process:
  - forward-recovery time

Forward voltage drop will firstly peak at U_{FP}, then gradually reduce to some value close to steady state voltage drop.
Forward recovery time t_{fr}
If the rate of rise of the forward current is higher, the U_{FP} is higher.

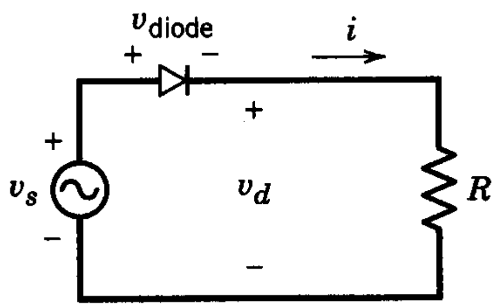


1.4 Types of power diodes

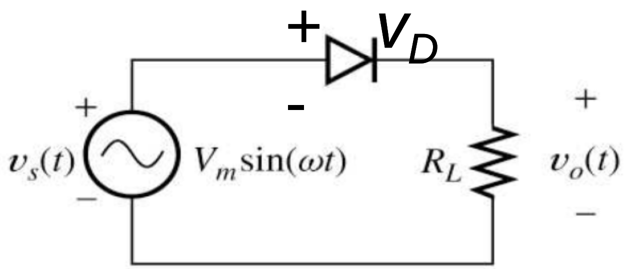
- General purpose diode (rectifier diode):
  - Standard recovery
- Fast recovery diode (FRD)
  - Reverse recovery time and charge specified. t_{rr} is usually less than 1μs, for many less than 100 ns —— ultra-fast recovery diode.
- Schottky diode (Schottky barrier diode-SBD)
  - A majority carrier device
  - Essentially no recovered charge, and lower forward voltage.
  - Restricted to low voltage (less than 200V)

2.1 Single phase uncontrolled half-wave rectifier

- A half-wave rectifier is the simplest rectifier.
  - It consists of only one diode.
  - It is not normally used in industrial applications, but it is very useful in understanding the principle of rectification

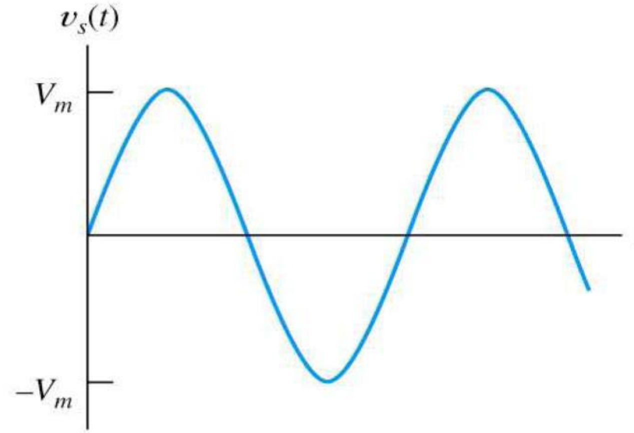


The output voltage and current of this rectifier are strongly influenced by the type of the load.



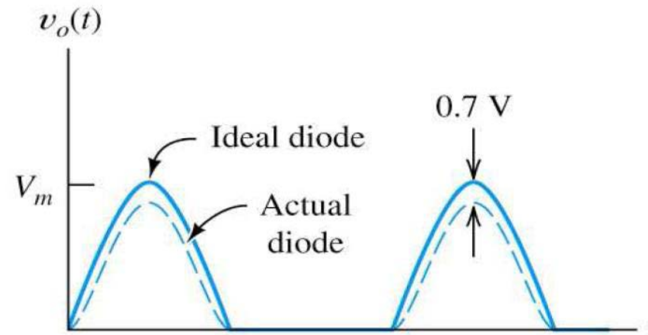
$$v_s = V_m \sin \omega t$$

Input voltage
v_s



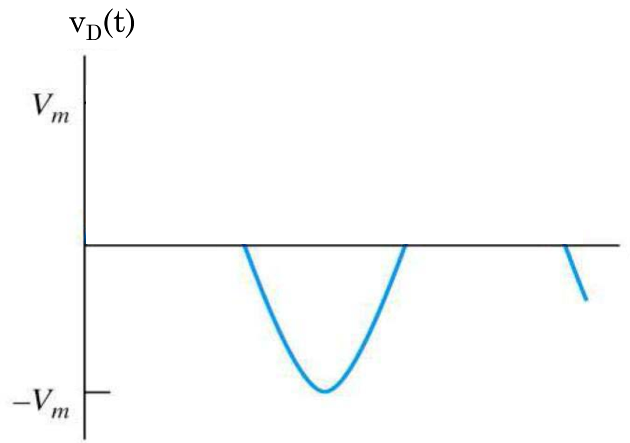
=

Output voltage
v_o



+

Voltage across
the diode v_D



About output voltage v_o

- A DC signal
- Discontinuous, which means discontinuous power / energy
- Full of harmonics: the Fourier series of v_0

$$\boxed{v_o = V_o + \sum_{n=1}^{\infty}\left(a_n \cos n\omega t + b_n \sin n\omega t\right)}$$

$$T = 2\pi / \omega$$

$$V_o = \frac{1}{T}\int_0^T v_o(t)dt = \frac{1}{T}\int_0^{T/2} V_m \sin\omega t dt = \frac{V_m}{\pi}$$

$$a_n = \frac{2}{T}\int_0^T v_o(t)\cos n\omega t dt = \frac{1}{T}\int_0^{T/2} V_m \sin\omega t \cos n\omega t dt = \begin{cases} \dfrac{2V_m}{\pi\left(1-n^2\right)} & n = 2,4,6\ldots \\ 0 & n = 1,3,5\ldots \end{cases}$$

$$b_n = \frac{2}{T}\int_0^T v_o(t)\sin n\omega t dt = \frac{1}{T}\int_0^{T/2} V_m \sin\omega t \sin n\omega t dt = \begin{cases} \dfrac{V_m}{2} & n = 1 \\ 0 & n = 2,3,4\ldots \end{cases}$$

$$\Rightarrow \boxed{v_o = \frac{V_m}{\pi} + \frac{V_m}{2}\sin\omega t - \frac{2V_m}{3\pi}\cos 2\omega t - \frac{2V_m}{15\pi}\cos 4\omega t - \ldots\ldots}$$

2.2 Parameters of rectifiers I

- The average (DC) output voltage

$$V_o = \frac{1}{T}\int_0^T v_o(t)dt$$

- The average (DC) output current

$$I_o = \frac{1}{T}\int_0^T i_o(t)dt$$

- The output DC power

$$P_o = V_o I_o$$

- The RMS output voltage

$$V_{RMS} = \sqrt{\frac{1}{T}\int_0^T v_o^2(t)dt}$$

- The RMS output current

$$I_{RMS} = \sqrt{\frac{1}{T}\int_0^T i_o^2(t)dt}$$

- The output AC power

$$P_{RMS} = V_{RMS} I_{RMS}$$

2.2 Parameters of rectifiers II

- Rectification ratio (or efficiency when ignoring the switching losses)

$$\eta = \frac{P_o}{P_{RMS}}$$

- The transformer (converter) utilization factor $f_T = \frac{P_o}{P_S}$, where $P_S = V_S I_S$ is the input power

- The effective (RMS) value of the AC component of the output voltage

$$V_{ac} = \sqrt{V_{RMS}^2 - V_o^2}$$

2.2 Parameters of rectifiers III

- The form factor to measure the shape of the output voltage

$$f_F = \frac{V_{RMS}}{V_o}$$

- The ripple factor to measure the ripple content

$$f_R = \frac{V_{ac}}{V_o} = \sqrt{f_F^2 - 1}$$

- The total harmonic distortion (THD), or the harmonic factor, of the input current to measure the distortion

$$THD = \sqrt{\left(\frac{I_S}{I_{S1}}\right)^2 - 1} \times 100\% = \frac{\sqrt{I_S^2 - I_{S1}^2}}{I_{S1}} \times 100\%$$

  – where I_s is the RMS value of i_s and I_{s1} is the fundamental component of i_s

2.2 Parameters of rectifiers IV

- The crest factor to measure the peak value i_m as compared to the RMS value I_{RMS}

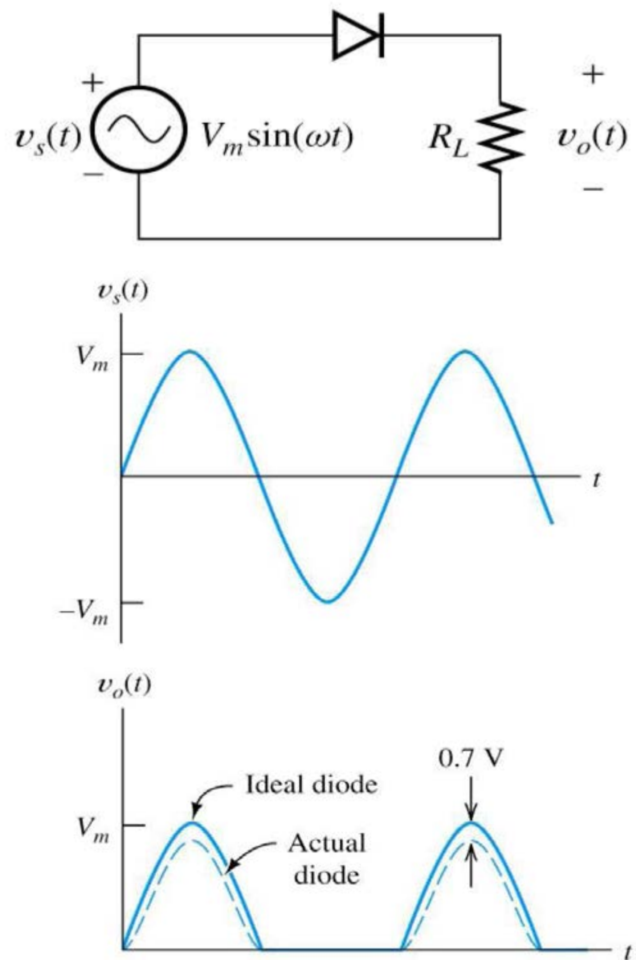
$$f_C = \frac{i_m}{I_{RMS}}$$

- The displacement angle ϕ is defined to be the phase difference between the fundamental components of the input current and voltage.
  - The displacement (power) factor is defined to be cos ϕ
  - The power factor

$$f_P = \frac{real\ \ power}{apparent\ \ power} = \frac{V_S I_{S1}}{V_S I_S}\cos\phi = \frac{I_{S1}}{I_S}\cos\phi$$

2.3 A worked example

- Calculate the parameters introduced before for the shown circuit.



$$v_o = \frac{V_m}{\pi} + \frac{V_m}{2}\sin\omega t - \frac{2V_m}{3\pi}\cos 2\omega t - \frac{2V_m}{15\pi}\cos 4\omega t - ......$$

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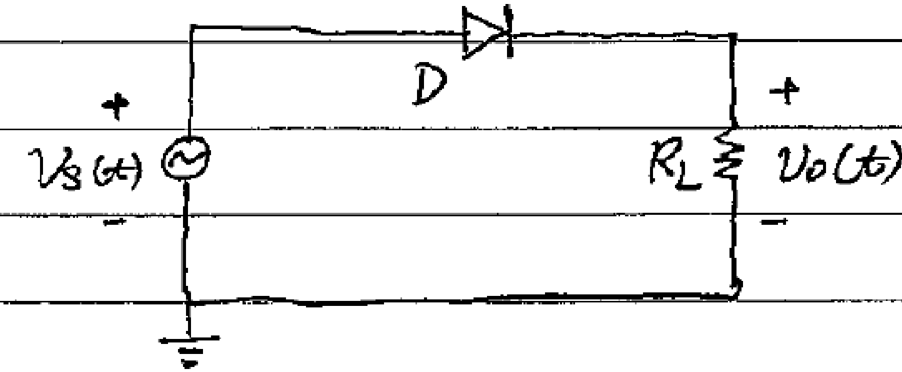
⇒ Calculate the parameters.

$v_S(t) = V_m \sin(\omega t)$.



* Solution.

1) Average DC output : V_o (or V_{dc})

$$V_o = \frac{1}{T}\int_0^{T/2} V_m \sin\omega t\, dt = \frac{V_m}{\pi} \approx 0.318\,V_m$$

$$\Rightarrow I_o = \frac{0.318\,V_m}{R_L}$$

2) RMS value (effective value) of output : V_{RMS}

$$V_{RMS} = \sqrt{\frac{1}{T}\int_0^{T/2} V_o^2(t)\, dt} = \sqrt{\frac{1}{T}\int_0^{T/2} V_m^2 \sin^2(\omega t)\, dt}$$

$$= \frac{V_m}{2} = 0.5\,V_m$$

$$\Rightarrow I_{RMS} = \frac{0.5\,V_m}{R_L}$$

3) Rectification Ratio (Efficiency): η

$$\eta = \frac{P_o}{P_{RMS}} = \frac{V_o^2/R_L}{V_{RMS}^2/R_L} = \left(\frac{V_o}{V_{RMS}}\right)^2 = \left(\frac{0.318}{0.5}\right)^2 \approx 40.5\%$$

4) Transformer utilisation factor: f_T.

$$f_T = \frac{P_o}{P_S} = \frac{P_o}{V_S \cdot I_S} \longrightarrow P_o = V_o \cdot I_o = \frac{(0.318 V_m)^2}{R_L}$$

Effective value of input signal $V_S = V_m/\sqrt{2}$.

Effective value of input current, equal to output current flow over R_L. $\Rightarrow I_S = I_{RMS} = \frac{0.5 V_m}{R_L}$

$$\Rightarrow f_T = \frac{(0.318 V_m)^2/R_L}{\left(V_m/\sqrt{2}\right)\left(0.5V_m/R_L\right)} = 0.287.$$

5) Form factor: f_F.

$$f_F = \frac{V_{RMS}}{V_0} = \frac{0.5}{0.318} = 1.57$$

6) Ripple factor: f_R

$$f_R = \frac{V_{ac}}{V_0} = \frac{\sqrt{V_{RMS}^2 - V_0^2}}{V_0} = \sqrt{\left(\frac{V_{RMS}}{V_0}\right)^2 - 1}$$

$$= \sqrt{f_F^2 - 1} = 1.21$$

effective value of

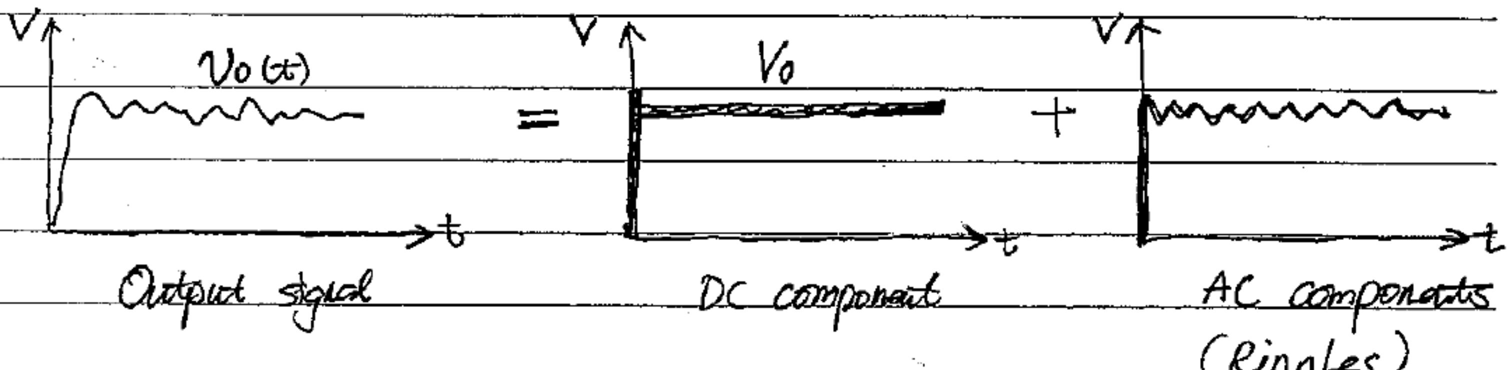
* V_{AC} is the AC components of the output signal.

i.e.: for signal $v_o = \boxed{\frac{V_m}{\pi}} + \boxed{\frac{V_m}{2}\sin\omega t} - \frac{2V_m}{3\pi}\cos 2\omega t - \cdots$

1st Harmonic

DC component $V_0 = \frac{V_m}{\pi}$

AC components. Effective value $V_{ac} = \sqrt{V_{RMS}^2 - V_0^2}$



7) Crest factor: f_c

$$f_c = \frac{I_{peak}}{I_s} = \frac{V_m/R_L}{0.5V_m/R_L} = 2.$$

8) Harmonic factor = Total Harmonic Distortion: THD.

$$THD = \sqrt{\left(\frac{I_s}{I_{s1}}\right)^2 - 1} \times 100\%$$

I_s: effective value (RMS) of total input. $I_s = I_{RMS} \cancel{R_L} = \frac{V_{RMS}}{R_L} = \frac{0.5V}{R_L}$

I_{s1}: effective value (RMS) of the fundamental component (line-frequency f_1) of the input current.

first harmonic. ω.

$$\Rightarrow I_{s1} = \frac{1}{\sqrt{2}} \cdot \frac{V_m}{2} \cdot \frac{1}{R_L}$$

So. $$THD = \sqrt{\left(\frac{0.5V_m/R_L}{0.5V_m/\sqrt{2}\cdot R_L}\right)^2 - 1} = \sqrt{2-1} = 1 = 100\%.$$

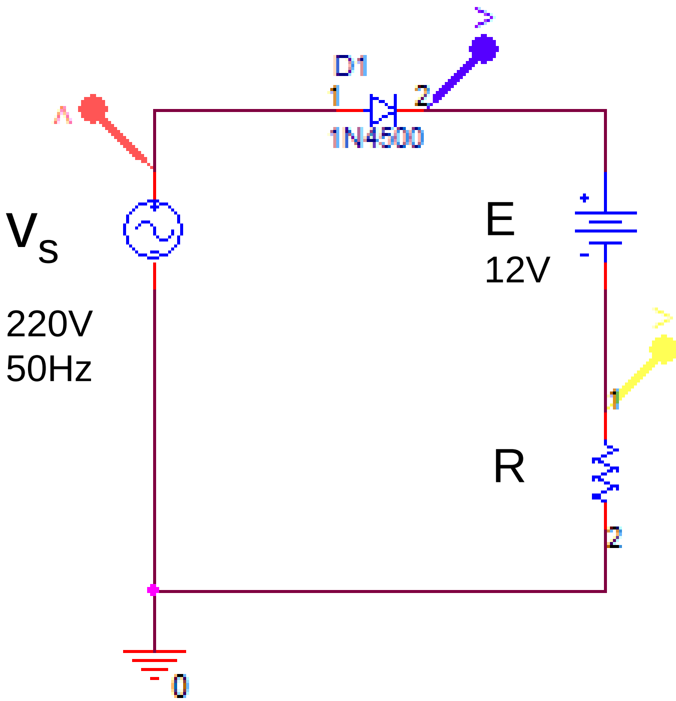
9) Power factor: f_p

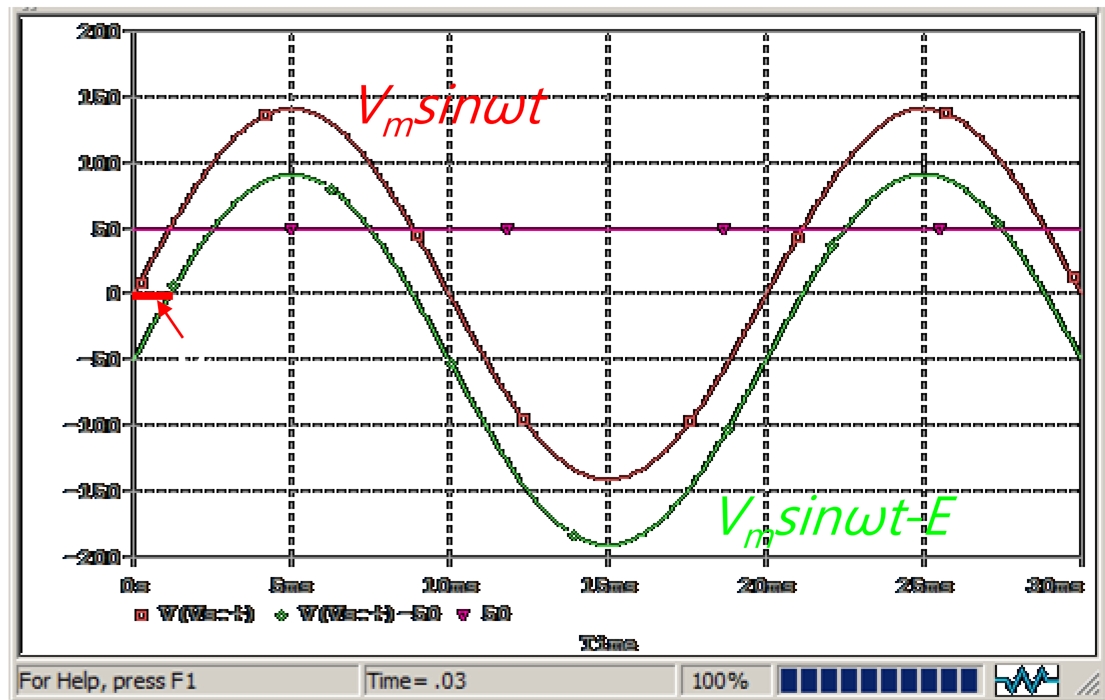
$$f_p = \frac{V_s I_{s1}}{V_s I_s} \cos\phi = \frac{I_{s1}}{I_s} \cos\phi = \frac{1}{\sqrt{2}} = 0.707.$$

$\cos\phi = 1$, because $\phi = 0$. no phase difference

2.4 A simple battery charger

- The circuit is to use the 220V/50Hz AC power to charge a 12V battery:





1. Find the loading resistor R to keep the average charging current at Id=5A
2. If the capacity of the battery is 1600mAh, What is the charging time?

* Solution:

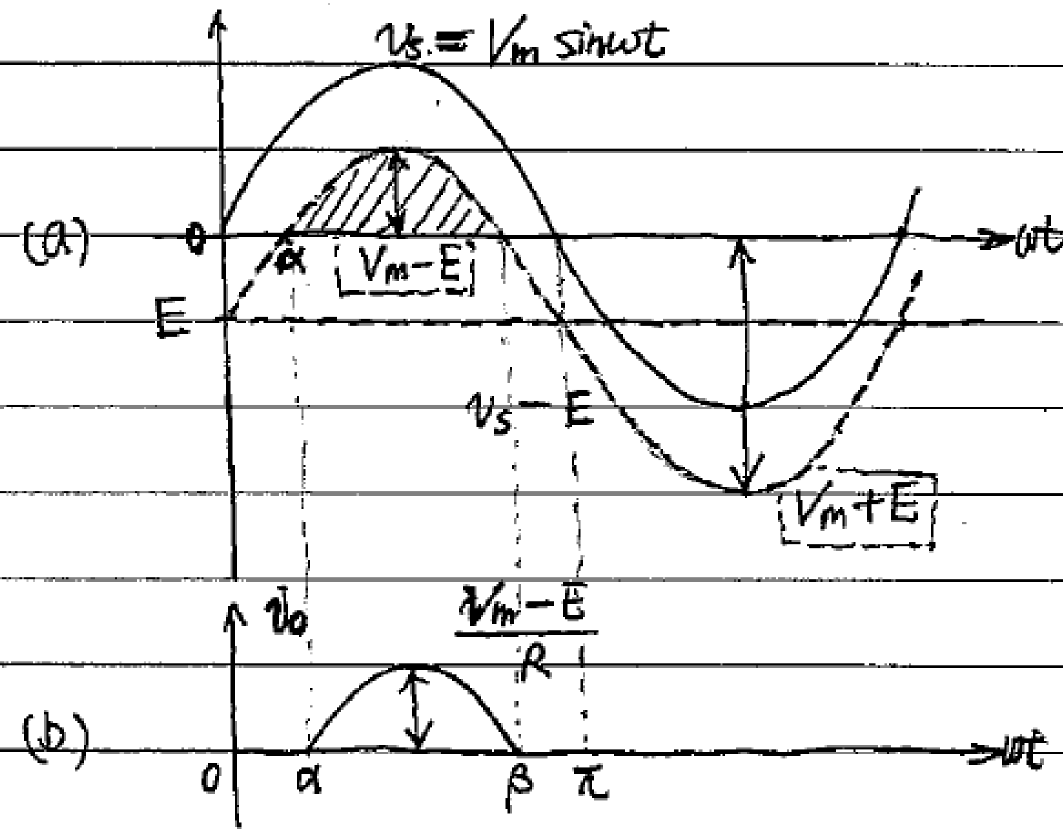
1. When $v_s > E$, diode D starts to conduct.

* In the shadow region in figure (a), $v_s - E > 0$, D conducts

* The angle D starts to conduct can be found from the condition:

$$\boxed{V_m \sin\alpha = E} \quad \cdots (1)$$

* Substitute the values $V_m = 311V$, $E = 12V$, in equation (1), got:



$$\alpha = \sin^{-1}\frac{E}{V_m} \approx 2.21° \ (0.038 \text{ radian}).$$

* The angle D stops to conduct $\beta = \pi - \alpha = 177.79°$.

* And conduction angle $\delta = \beta - \alpha = \pi - 2\alpha = 175.58°$

2. The averaging charging current $I_{dc} = I_o$ is:

$$I_o = \frac{V_o}{R} = \frac{1}{2\pi R}\int_{\alpha}^{\beta}(V_m \sin\omega t - E)\,d(\omega t)$$

$$= \frac{1}{2\pi R}(2V_m\cos\alpha + 2E\alpha - \pi E).$$

Therefore, the R can be calculated to provide $I_o = 5A$:

$$R = \frac{1}{2\pi I_o}(2V_m\cos\alpha + 2E\alpha - \pi E)$$

$$= 18.6\Omega.$$

3. The RMS charging current I_{RMS} is:

$$I_{RMS} = \frac{V_{RMS}}{R} = \frac{1}{R}\sqrt{\frac{1}{2\pi}\int_{\alpha}^{\beta}(V_m \sin\omega t - E)^2 \, d(\omega t)}$$

$$= \sqrt{\frac{1}{2\pi R^2}\left[\left(\frac{V_m^2}{2}+E^2\right)(\pi-2\alpha)+\frac{V_m^2}{2}\sin 2\alpha - 4V_m E\cos\alpha\right]}$$

$$= 7.95A.$$

The power dissipated on R is $P_R = I_{RMS}^2 \cdot R = 1175.3\,W$

4. The power delivered P_{dc} to the battery is

$$P_{dc} = E I_{dc} = 12 \times 5 = 60 W.$$

$$\text{charging time } h_o = \frac{\text{Capacity (in W·h)}}{P_{dc}}$$

* Battery's capacity are usually labelled as xxx mAh, which means with certain voltage, it can continuously discharge with xxx mA current for 1 hour.

Therefore, the capacity in W·h can be calculated by

$$\text{capacity (in W·h)} = \text{capacity (in A·h)} * E$$

$$= 1.6 \times 12 = 19.2 \text{ W·h}$$

$$* \text{ charging time } h_o = \frac{\text{Capacity (in W·h)}}{P_{dc}} = \frac{19.2}{60} = 0.32 h.$$

* It can also be calculated by

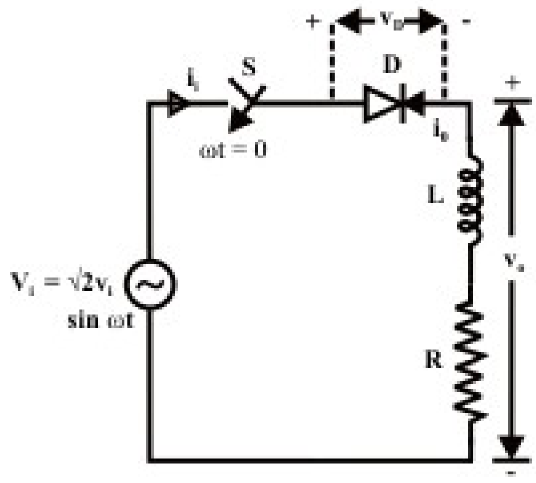
$$h_o = \frac{\text{Capacity (A·h)}}{I_{dc}} = \frac{1.6}{5} = 0.32 h.$$

5. The peak reverse voltage across the diode is

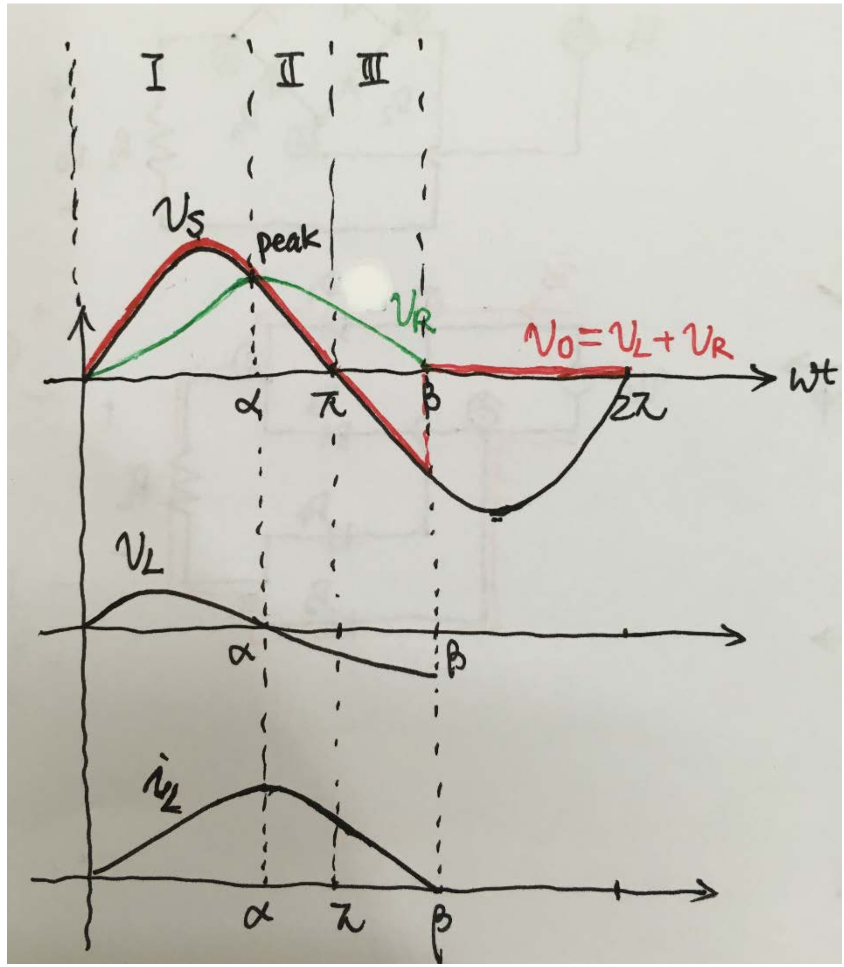
$$V_{rm} = V_m + E = 323 V.$$

2.5 With RL load

- The ripple factor of output current can be reduced by connecting an inductor in series with the load resistance.



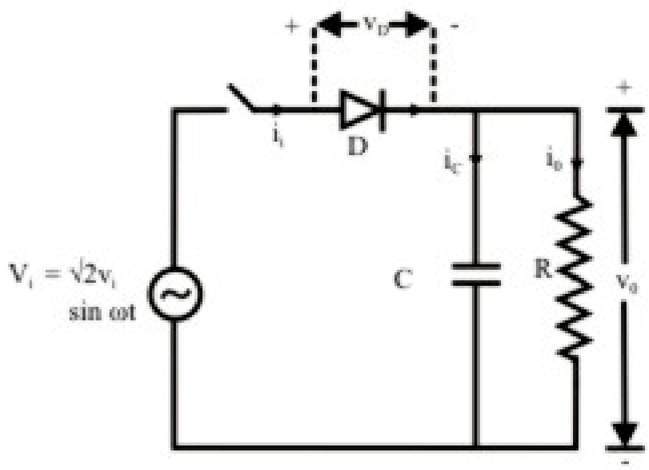
(a)



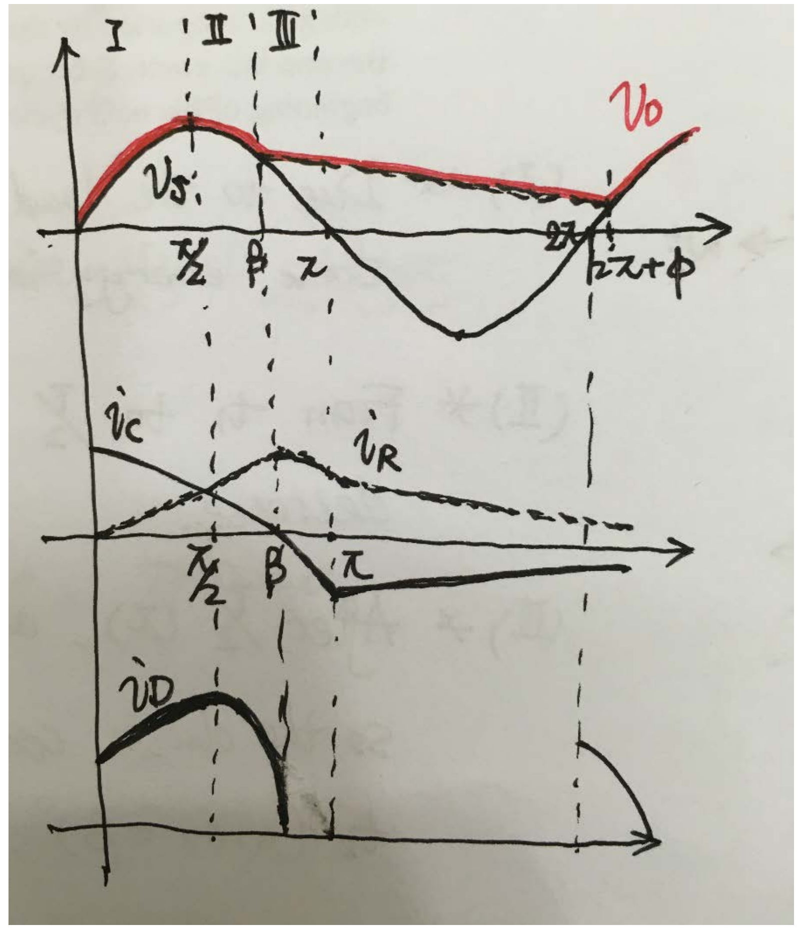
Current continues to flows for a while even after the input voltage has gone negative

2.6 With RC load

- The problem of poor ripple factor of the output voltage can also be solved by connecting a capacitor across the load resistance

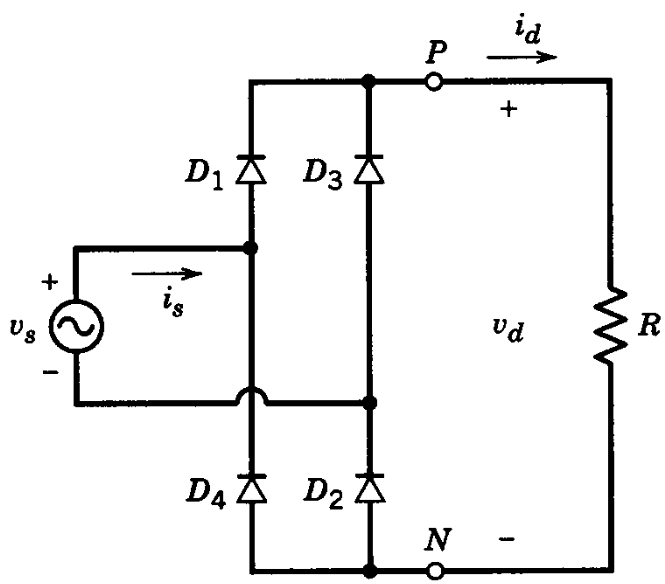


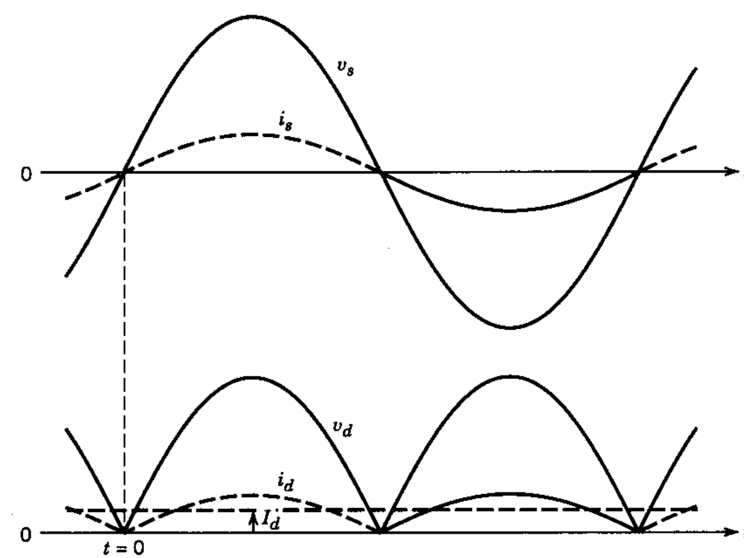
(a)

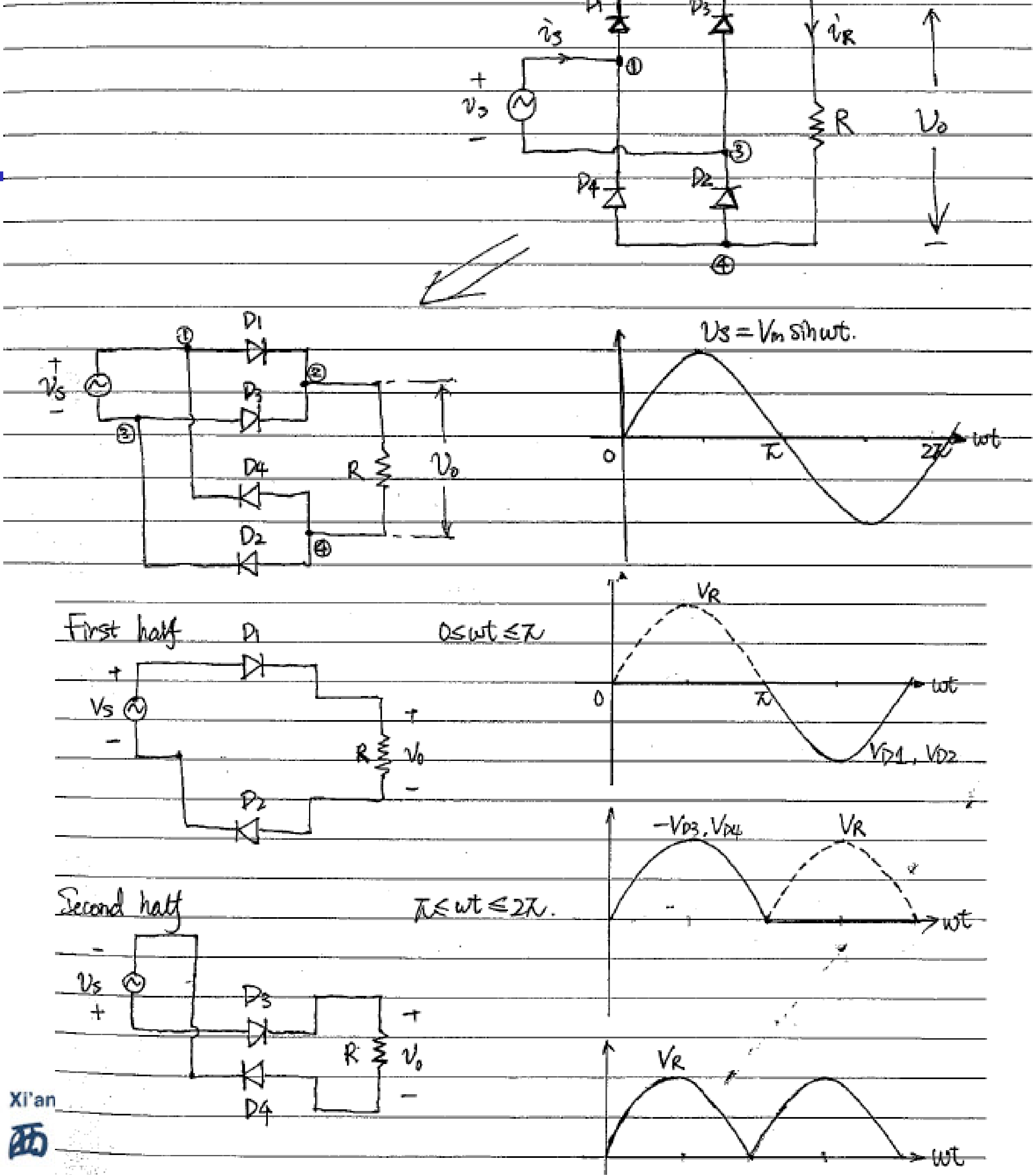


3.1 Single phase uncontrolled full-wave rectifier

- Bridge rectifier (using 4 diodes)
  - The positive half-cycle goes through D_1 and D_2, and the negative half-cycle goes through D_3 and D_4
  - The peak reverse voltage of the diodes is the same as the peak value of v_s







i_s
v_s
D1
D3
D4
D2
i_R
R
V_o
$v_s = V_m \sin\omega t.$
ωt
π
2π
First half
$0 \le \omega t \le \pi$
V_R
V_{D1}, V_{D2}
$-V_{D3}, V_{D4}$
Second half
$\pi \le \omega t \le 2\pi.$

Parameters calculation

1) $V_0 = V_{dc} = \frac{1}{2\pi}\int_0^{\pi} V_m \sin\omega t \, d\omega t + \frac{1}{2\pi}\int_{\pi}^{2\pi} V_m |\sin\omega t| \, d\omega t$

$= \frac{2}{\pi} V_m = 0.637 V_m.$

2) $V_{RMS} = \frac{V_m}{\sqrt{2}} = 0.707 V_m.$ V_{RMS} Equals to the effective value of input $v_s(t) = V_m/\sqrt{2} = V_{SRMS}$.

3) Efficiecy $\eta = \frac{P_{dc}}{P_{RMS}} = \left(\frac{V_0}{V_{RMS}}\right)^2 = 81\%$

4) Form factor $f_F = \frac{V_{RMS}}{V_0} = 1.11$

5) Ripple factor $f_R = \sqrt{f_F^2 - 1} = 0.482.$

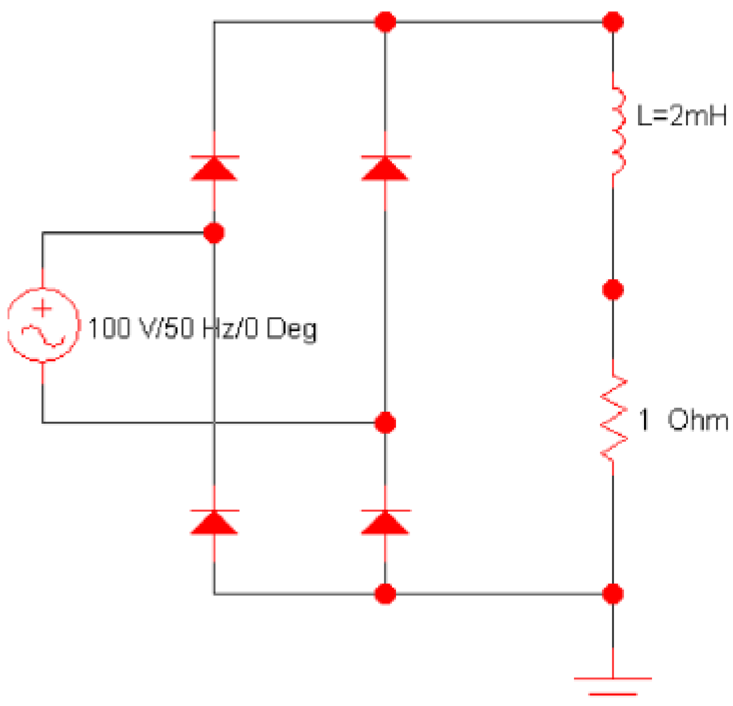
6) Crest factor $f_C = \frac{I_{peak}}{I_s} = \frac{V_m/R}{V_{RMS}/R} = \frac{1}{0.707} = 1.41.$

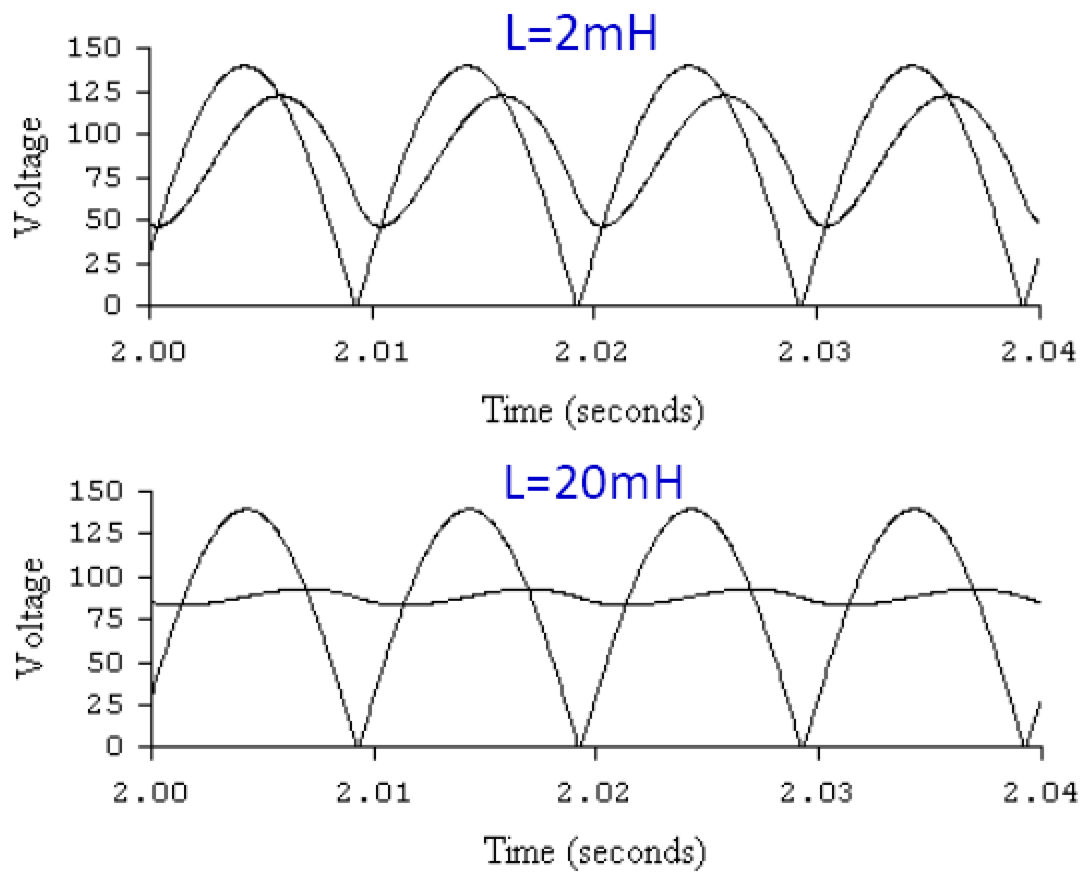
7) Fourier Series of v_0:

$v_0(t) = \frac{2V_m}{\pi} - \frac{4V_m}{3\pi}\cos 2\omega t - \frac{4V_m}{15\pi}\cos 4\omega t - \frac{4V_m}{35\pi}\cos 6\omega t - \cdots$

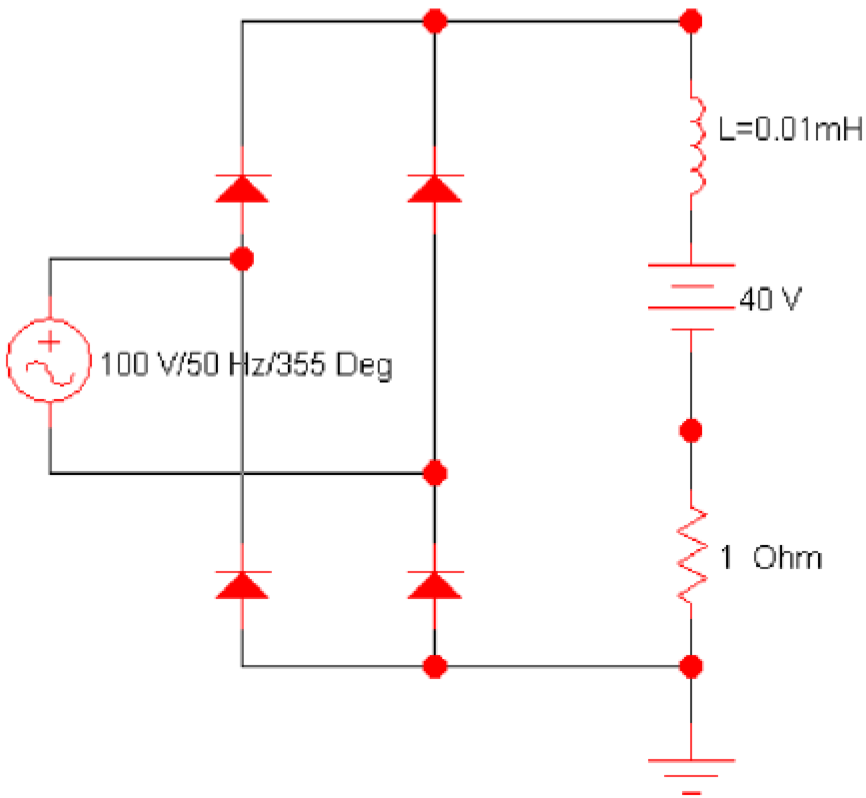
3.2 With RL load

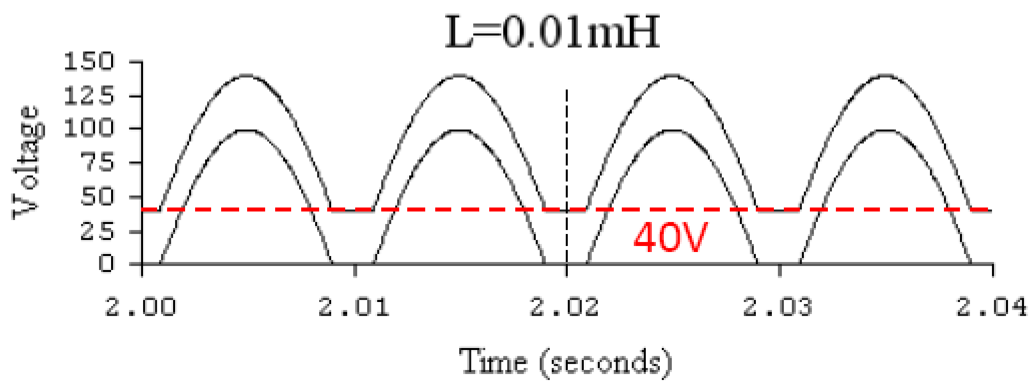
- An inductor is used to smooth the current
- The larger the inductor, the smoother the current



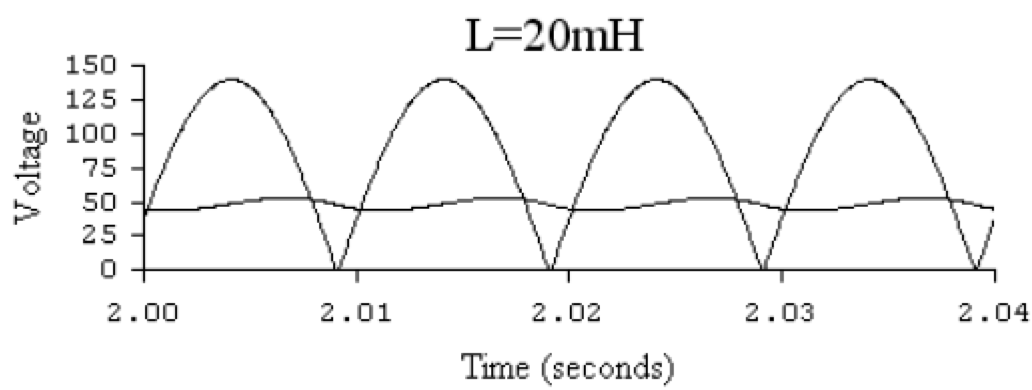


3.3 With RLE load





(b) discontinuous-current mode



(a) continuous-current mode

Different way of drawing bridge