EEE104 – Digital Electronics (I)

Lecture 2

Dr. Ming Xu

Dept of Electrical & Electronic Engineering

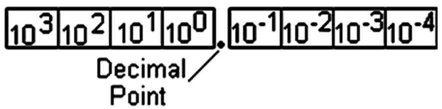
XJTLU

In This Session

- Binary numbers
- Conversion between decimal and binary numbers
- Binary arithmetic

Decimal Numbers

- The decimal numbering system has ten digits: 0-9
- Digits at different positions are assigned different **weights** which are powers of ten.
- The value of a decimal number is the sum of the weighted digits.



$$47 = (4 \times 10^1) + (7 \times 10^0)$$
$$= (4 \times 10) + (7 \times 1) = \mathbf{40 + 7}$$

Binary Numbers

Decimal Number	Binary Number			
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1
10	1	0	1	0
11	1	0	1	1
12	1	1	0	0
13	1	1	0	1
14	1	1	1	0
15	1	1	1	1

Counting in binary

1. Begin counting: 0, 1.
2. Include another bit position and continue: 10, 11.
3. Include a third bit position and continue: 100, 101, 110, 111.

Binary Numbers

- In decimal numbering system, with n digits you can count up to a number $10^n - 1$. e.g.

  1 digit for $10^1 - 1 = 9$

  2 digits for $10^2 - 1 = 99$
- In binary numbering system, **with n bits you can count up to a number 2^n – 1**. e.g.

  2 bits for $2^2 - 1 = 3$

  3 bits for $2^3 - 1 = 7$

  4 bits for $2^4 - 1 = 15$

  5 bits for $2^5 - 1 = 31$

Binary Numbers

The weighting structure

$$2^{n-1} \ldots 2^3\, 2^2\, 2^1\, 2^0 .\, 2^{-1}\, 2^{-2} \ldots 2^{-n}$$

↑ Binary point

1. **Least significant bit (LSB)**: the right-most bit in a binary number.
2. **Most significant bit (MSB)**: the left-most bit in a binary number.

Binary weights.

Positive Powers of Two (whole numbers)									Negative Powers of Two (fractional number)					
2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}	2^{-5}	2^{-6}
256	128	64	32	16	8	4	2	1	1/2	1/4	1/8	1/16	1/32	1/64
									0.5	0.25	0.125	0.0625	0.03125	0.015625

Binary-to Decimal Conversion

Add the weights of all bits that are 1.

$$\begin{array}{rccccccc} \text{Weight:} & 2^6 & 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \\ \text{Binary number:} & 1 & 1 & 0 & 1 & 1 & 0 & 1 \end{array}$$

$$\begin{aligned} 1101101 &= 2^6 + 2^5 + 2^3 + 2^2 + 2^0 \\ &= 64 + 32 + 8 + 4 + 1 = \mathbf{109} \end{aligned}$$

$$\begin{array}{rcccccc} \text{Weight:} & & & 2^{-1} & 2^{-2} & 2^{-3} & 2^{-4} \\ \text{Binary number:} & 0 & . & 1 & 0 & 1 & 1 \end{array}$$

$$\begin{aligned} 0.1011 &= 2^{-1} + 2^{-3} + 2^{-4} \\ &= 0.5 + 0.125 + 0.0625 = \mathbf{0.6875} \end{aligned}$$

Decimal-to-Binary Conversion

Sum-of-weight method

1. Find the greatest weight which is less than or equal to the number.
2. Subtract the weight from the number, and find the greatest weight which is less than or equal to the remainder.
3. Repeat this process until the remainder becomes zero.

$$\begin{aligned} 12 &= 8 + 4 = 2^3 + 2^2 \longrightarrow \mathbf{1100} \\ 25 &= 16 + 8 + 1 = 2^4 + 2^3 + 2^0 \longrightarrow \mathbf{11001} \\ 58 &= 32 + 16 + 8 + 2 = 2^5 + 2^4 + 2^3 + 2^1 \longrightarrow \mathbf{111010} \\ 82 &= 64 + 16 + 2 = 2^6 + 2^4 + 2^1 \longrightarrow \mathbf{1010010} \end{aligned}$$

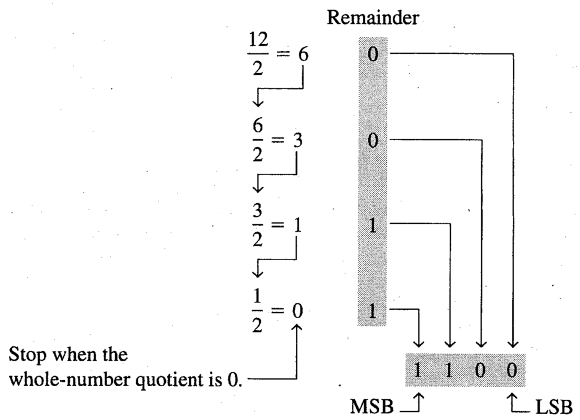
Decimal-to-Binary Conversion

Repeated division-by-2 method for whole numbers

1. Divide the number by 2.
2. Repeat dividing the resultant quotient by 2 until a zero quotient is produced.
3. The remainders generated by the divisions form the binary number.
4. The first remainder is the least significant bit (LSB).

Decimal-to-Binary Conversion

Repeated division-by-2 method for whole numbers



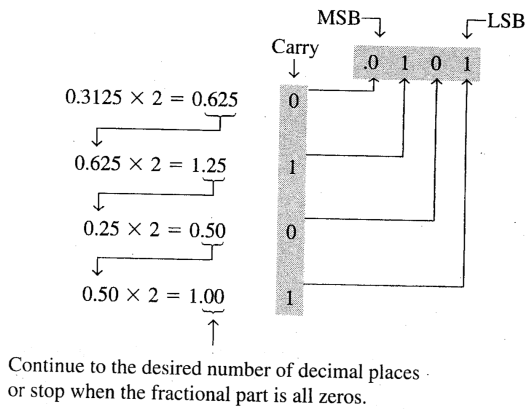
Decimal-to-Binary Conversion

Repeated multiplication by 2 for fractions

1. Multiply the number by 2.
2. Repeat multiplying the resultant fractional part of the product by 2 until the fractional product is zero or until the desired number of decimal places is reached..
3. The carries generated by the multiplications form the binary number.
4. The first carry is the most significant bit (MSB).

Decimal-to-Binary Conversion

Repeated multiplication by 2 for fractions



Binary Arithmetic

Binary Addition

$0 + 0 = 0$	Sum of 0 with a carry of 0
$0 + 1 = 1$	Sum of 1 with a carry of 0
$1 + 0 = 1$	Sum of 1 with a carry of 0
$1 + 1 = 10$	Sum of 0 with a carry of 1
$1 + 1 + 1 = 11$	Sum of 1 with a carry of 1

(a)
$$\begin{array}{r} 11 \\ +\,11 \\ \hline \mathbf{110} \end{array} \qquad \begin{array}{r} 3 \\ +\,3 \\ \hline 6 \end{array}$$

(b)
$$\begin{array}{r} 100 \\ +10 \\ \hline \mathbf{110} \end{array} \qquad \begin{array}{r} 4 \\ +\,2 \\ \hline 6 \end{array}$$

Binary Arithmetic

Binary Subtraction

$0 - 0 = 0$	
$1 - 1 = 0$	
$1 - 0 = 1$	
$10 - 1 = 1$	$0 - 1$ with a borrow of 1

(a)
$$\begin{array}{r} 11 \\ -\,01 \\ \hline \mathbf{10} \end{array} \qquad \begin{array}{r} 3 \\ -\,1 \\ \hline 2 \end{array}$$

(b)
$$\begin{array}{r} 101 \\ -\,011 \\ \hline \mathbf{010} \end{array} \qquad \begin{array}{r} 5 \\ -\,3 \\ \hline 2 \end{array}$$

Binary Arithmetic

Binary Multiplication

$0 \times 0 = 0$
$0 \times 1 = 0$
$1 \times 0 = 0$
$1 \times 1 = 1$

(a)
$$\begin{array}{r} 11 \\ \times\,11 \\ \hline 11 \\ +\,11 \\ \hline \mathbf{1001} \end{array} \qquad \begin{array}{r} 3 \\ \times\,3 \\ \hline 9 \end{array}$$

Partial products: 11, $+\,11$

(b)
$$\begin{array}{r} 111 \\ \times\,101 \\ \hline 111 \\ 000 \\ +\,111 \\ \hline \mathbf{100011} \end{array} \qquad \begin{array}{r} 7 \\ \times\,5 \\ \hline 35 \end{array}$$

Partial products: 111, 000, $+\,111$

Binary Arithmetic

Binary Division

Follow the same procedure as division in decimal.

(a)
$$\begin{array}{r} \mathbf{10} \\ 11\overline{)110} \\ 11 \\ \hline 000 \end{array} \qquad \begin{array}{r} 2 \\ 3\overline{)6} \\ 6 \\ \hline 0 \end{array}$$

(b)
$$\begin{array}{r} \mathbf{11} \\ 10\overline{)110} \\ 10 \\ \hline 10 \\ 10 \\ \hline 00 \end{array} \qquad \begin{array}{r} 3 \\ 2\overline{)6} \\ 6 \\ \hline 0 \end{array}$$