Tutorial 8 Fourier's theorem

1. $f(x+2L)=f(x)$ for all *x* and $f(x)=\begin{cases}-1, & -L<x<0,\\ 1, & 0\le x\le L,\end{cases}$,
   (a) Sketch the function $f(x)$ in the range *-2L<x<2L*.
   (b) To what does the series converge when $x=-L, 0, \frac{L}{2}, \frac{3L}{2}$.

2. The function $f(x)$ is defined $f(x)=\begin{cases}-x, & -L\le x<0\\ x, & 0\le x<L\end{cases}$ and $f(x+2L)=f(x)$.
   (a) Sketch $f(x)$ in *-3L<x<3L*.
   (b) State the values the Fourier series will converge to at $x=-\frac{L}{2}, 0, \frac{L}{3}, L$.
   (c) Find the Fourier series of $f(x)$ and give the first three non-zero terms.
   (d) By choosing an appropriate value for x in the Fourier series for $f(x)$, show that $\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}=\frac{\pi^2}{8}$.

3. The function $f(x)$ is defined $f(x) = \begin{cases} x, & -L \le x < 0 \\ L, & 0 \le x < L \end{cases}$, $f(x+2L) = f(x)$.
   (a) Sketch $f(x)$ in *-3L<x<3L.*
   (b) To what does the series converge when $x = -\frac{L}{2}, 0, \frac{L}{2}, L$.
   (c) Find its Fourier series.
   (d) Give the first three non-zero terms.