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EEE109: Electronic Circuits

Basic BJT Amplifiers – Part 1

Contents

- Understand the concept of an analog signal and the principle of a linear amplifier.
  - Investigate how a transistor circuit can amplify a small, time-varying input signal.

- Discuss and compare the three basic transistor amplifier configurations.
  - Analyze the common-emitter amplifier.
  - Analyze the emitter-follower amplifier.
  - Analyze the common-base amplifier.

- Analyze multitransistor or multistage amplifiers.
  circuit.

- Understand the concept of signal power gain in an amplifier

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General Definition of Amplifiers

Amplifiers - Definition

- An ideal amplifier is a unit with two input terminals and two output terminals. A signal (a voltage or current that varies with time) is applied to the input terminals and an exact copy of the signal but of larger magnitude is produced at the output terminals. That is if the input is S(t) then the output is $\mathrm{A} \times \mathrm{S}(\mathrm{t})$ where A is a constant numeric value that is usually greater than one.
- There are **four possibilities** because we can consider the input signal source to be a **current** or a **voltage** source. Similarly the output may act as a current or a voltage source. This gives four cases if we define

$\mathrm{i}_{\mathrm{in}}(t)$ is the input current. $\mathrm{v}_{\mathrm{in}}(t)$ is the input voltage

$\mathrm{i}_{\mathrm{out}}(t)$ is the output current. $\mathrm{v}_{\mathrm{out}}(t)$ is the output voltage

Amplifier - Classification

- Possible amplifiers are

$$v_{out}(t) = A \times v_{in}(t)$$ voltage amplifier

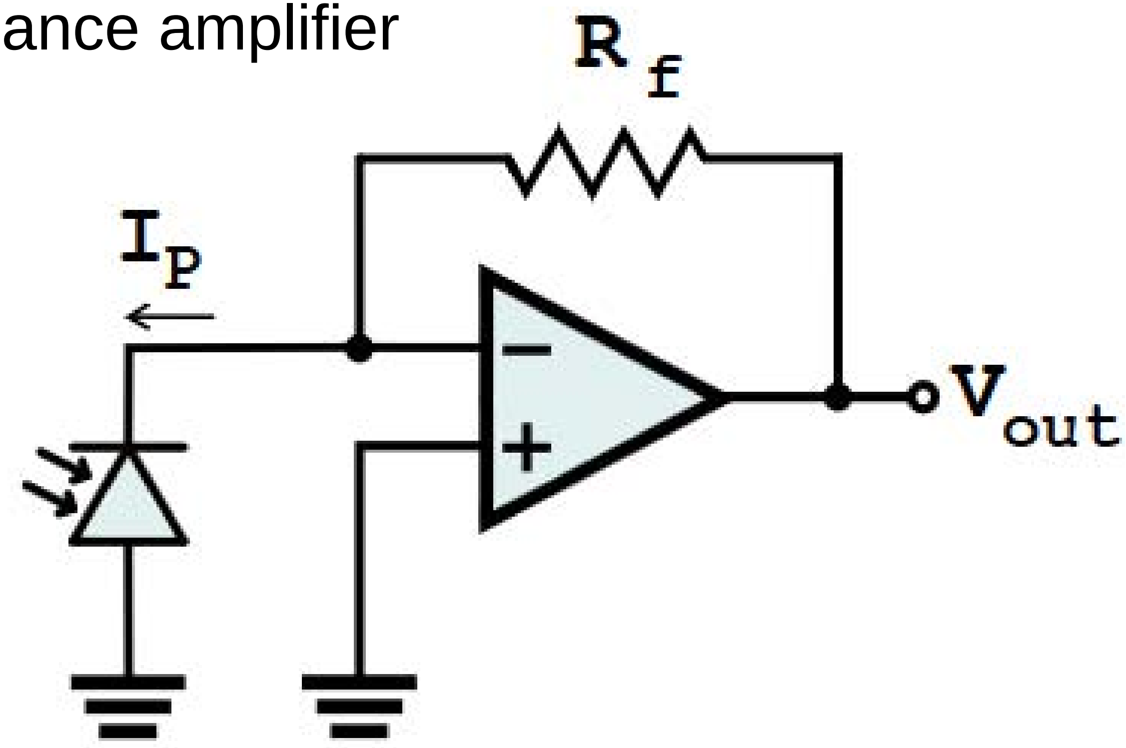
$$i_{out}(t) = A \times i_{in}(t)$$ current amplifier

$$v_{out}(t) = A \times i_{in}(t)$$ transimpedance amplifier

$$i_{out}(t) = A \times v_{in}(t)$$ transconductance amplifier

Although all have applications by far the largest number of cases examined are **voltage amplifiers** (and Thévénin and Nortons Theorems allow many of the others to be manipulated to this form).

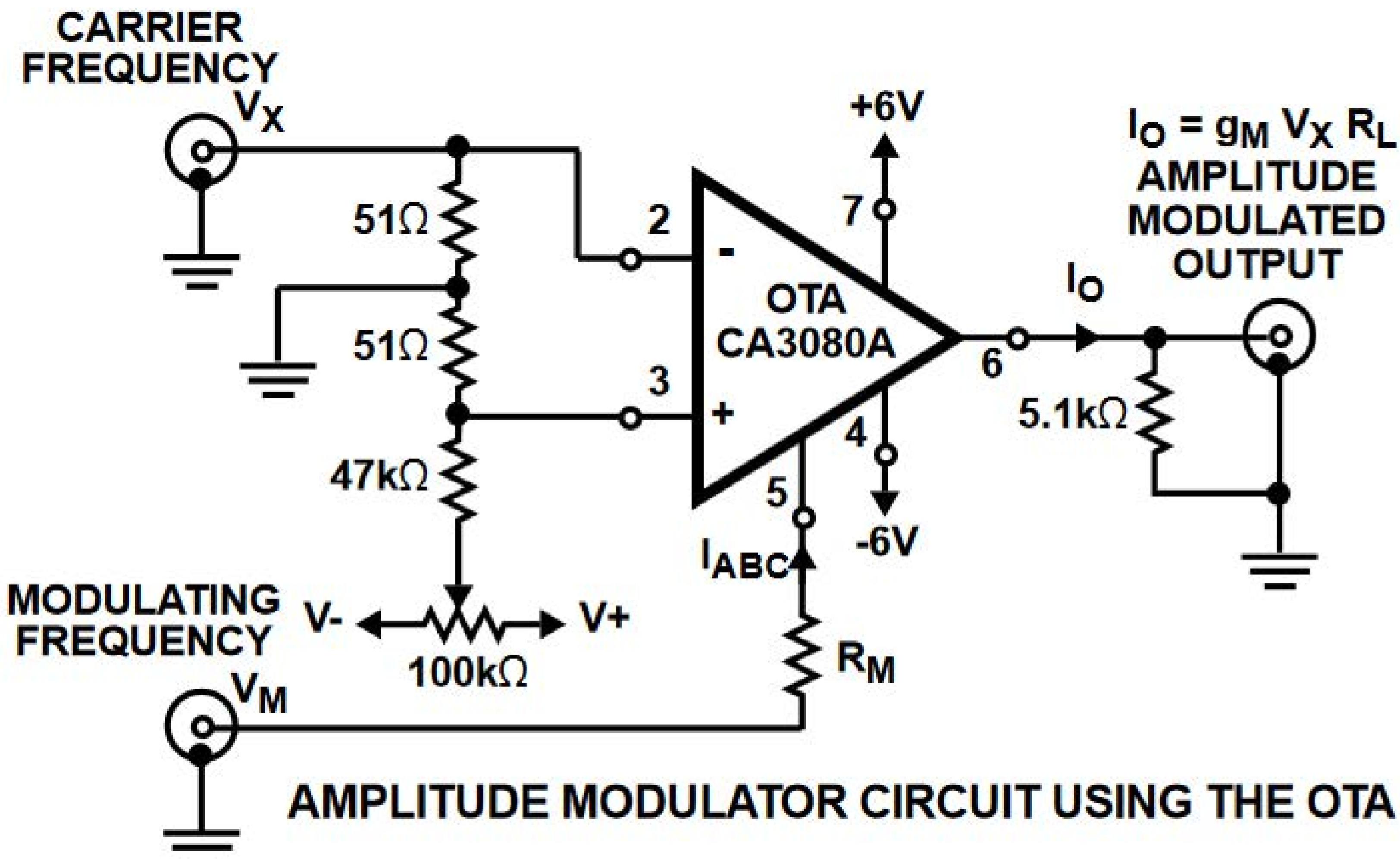
transimpedance amplifier



In electronics, a transimpedance amplifier (TIA) is a current-to-voltage converter, most often implemented using an operational amplifier. The TIA can be used to amplify the current output of Geiger – M ü ller tubes, photomultiplier tubes, accelerometers, photo detectors and other types of sensors to a usable voltage.

- Wiki

transconductance amplifier



AMPLITUDE MODULATOR CIRCUIT USING THE OTA

- from CA3080A Manual

Amplifier – Circuit Representation (1)

- Any amplifier can be considered to behave as the generic amplifier although it may not do so in an exact manner. The generic four terminal voltage amplifier is shown in Figure 2.1.

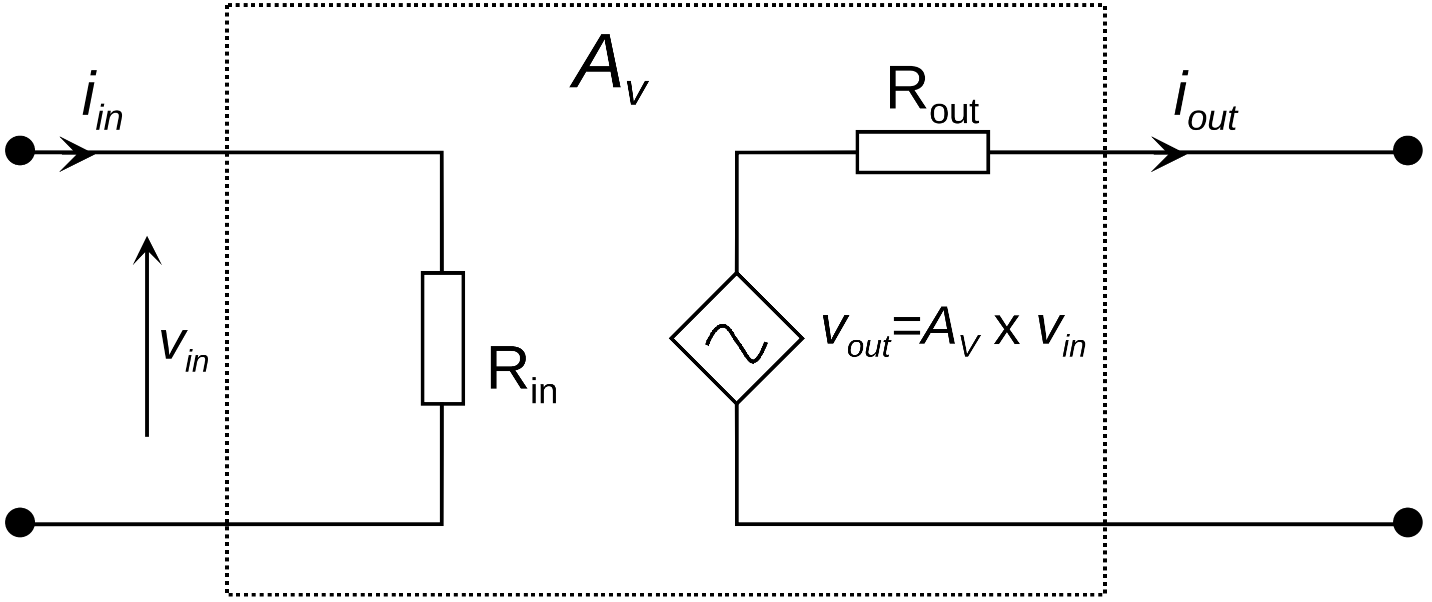


Figure 2.1

Amplifier - Circuit Representation (2)

- **Notes:**
  Usually the input signal is supplied from a source which can be regarded as a Thévénin circuit as in Figure 2.2. Therefore **is not the same as the source voltage**

- Usually the output signal is applied to a resistive load as in Figure 2.2. Therefore **is not the same as the voltage across the load**.

- In real circuits it is unusual for the input and output circuits to be totally isolated from each other. One of the most common arrangements – BUT NOT THE ONLY ONE – is for one input terminal and one output terminal to be joined.

Amplifier - Circuit Representation (3)

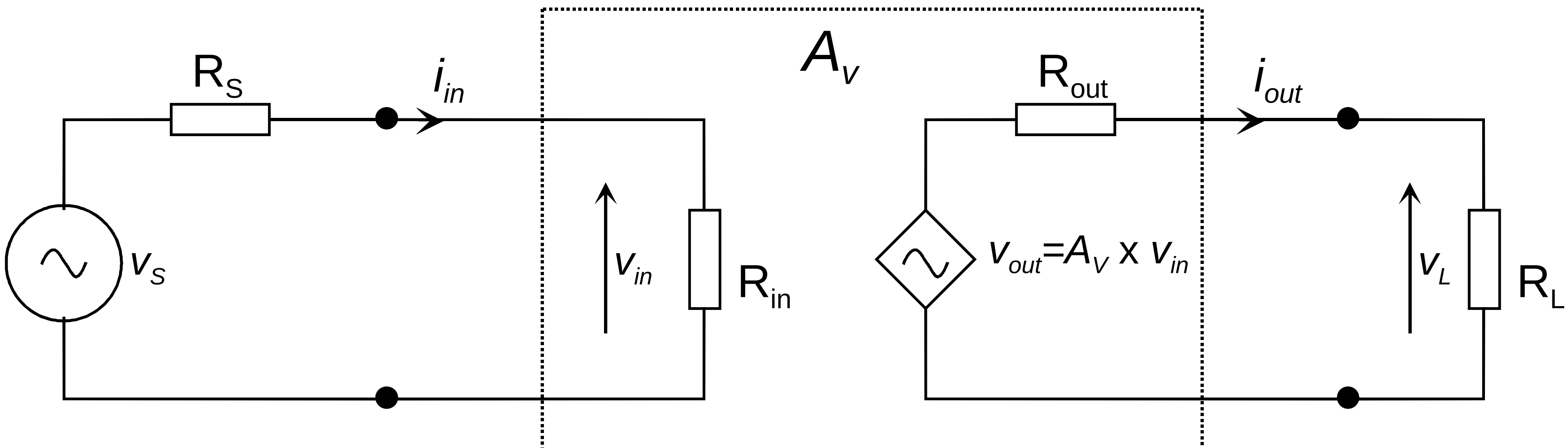


Figure 2.2

Amplifier – Input (1)

- R_S and R_{in} form a potential divider, therefore $v_{in} = v_S \frac{R_{in}}{R_S + R_{in}}$
- For the largest possible output (largest amplification of the signal, v_S) v_{in} must be as large as possible. Therefore **R_{in} should be much larger than R_S** - for general purpose voltage amplification a large input resistance is required. **However** you will learn in later years that in some situations other problems can arise and for these **matching is necessary -** the output resistance of the signal source must equal, match, the input resistance of the circuit to which it is connected requiring that $R_S = R_{in}$.

Amplifier – Input (2)

- In voltage amplifier design usually one of two cases arises

either maximum voltage output is required so

$R_{in} >> R_s$ – very high input resistance

or it is required that

$R_{in} = R_s$ – source resistance and input resistance are matched

Amplifier – Output

- R_{out} and R_L form a potential divider, therefore

$$v_L = A_v v_{in} \frac{R_L}{R_{out} + R_L}$$

- If R_L cannot be varied because the amplifier is required to drive a specified load then to get the largest possible output requires that R_L should be much larger than R_{out}. For general purpose voltage amplification **a small output resistance is required**. Again in some situations **matching** is necessary and the output resistance must match the load.

Amplifier – Matching (1)

- For simple voltage amplification often the output must be as large as possible. However often R_{out} is fixed and it is necessary to get maximum **power** possible in the load by choosing R_L. How does R_L affect the power in the load?

$$P_L = v_L \times i_{out} = \frac{v_L^2}{R_L} = A_v^2 \times v_{in}^2 \times \left(\frac{R_L}{R_{out} + R_L} \right)^2 \times \frac{1}{R_L}$$

- If the gain, input signal and output resistance are all fixed then only R_L affects the power output. Therefore differentiate P_L with respect to R_L to find how power varies with value of R_L

Amplifier – Matching (2)

$$P_L = \frac{K \times R_L}{(R_{out} + R_L)^2} \quad \text{so} \quad \frac{dP_L}{dR_L} = K\left(\frac{1}{(R_{out} + R_L)^2} - 2\frac{R_L}{(R_{out} + R_L)^3}\right)$$

A function is a maximum (or minimum) when the first differential is z

$$\frac{dP_L}{dR_L} = 0$$

which is
when

$$\frac{1}{(R_{out} + R_L)^2} = 2\frac{R_L}{(R_{out} + R_L)^3}$$

Re-arranging reduces this to $R_{out} = R_L$ and checking the second differential shows that this is the maximum case.

Amplifier – Matching (3)

- Hence for **maximum power output** from a circuit **the load should equal the output resistance**. **Note** that this is maximum power, not maximum voltage or maximum current.

maximum power output

$$R_{out} = R_L$$

Amplifier – Output Resistance (1)

- Usually voltage amplifier design requirements will lead one of two cases

either R_L is a fixed requirement and the maximum output voltage is required so

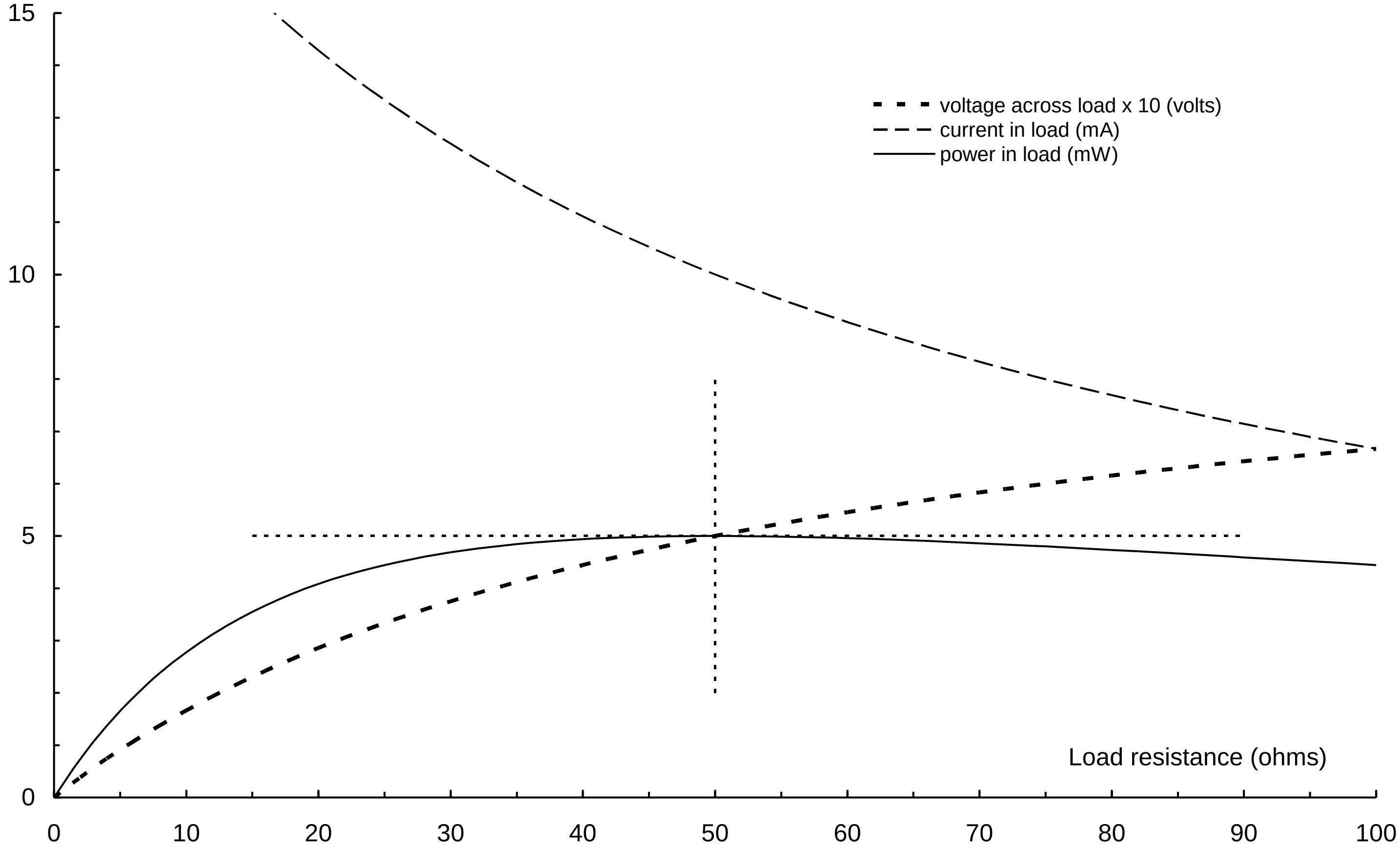
$R_{out} << R_L$ – very low output resistance

or

R_{out} is fixed and R_L can be selected; maximum power is required so

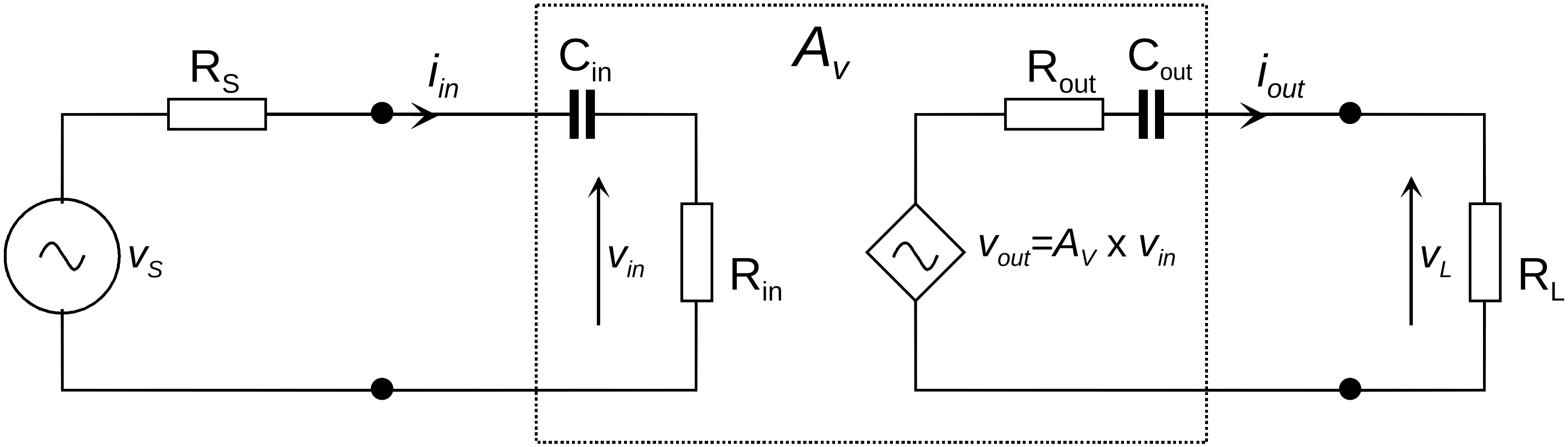
$R_{out} = R_L$ – output and load resistances are matched

Amplifier – Output Resistance (2)



Amplifier – AC Version

Later in the course the a.c coupled version of the amplifier will be considered.



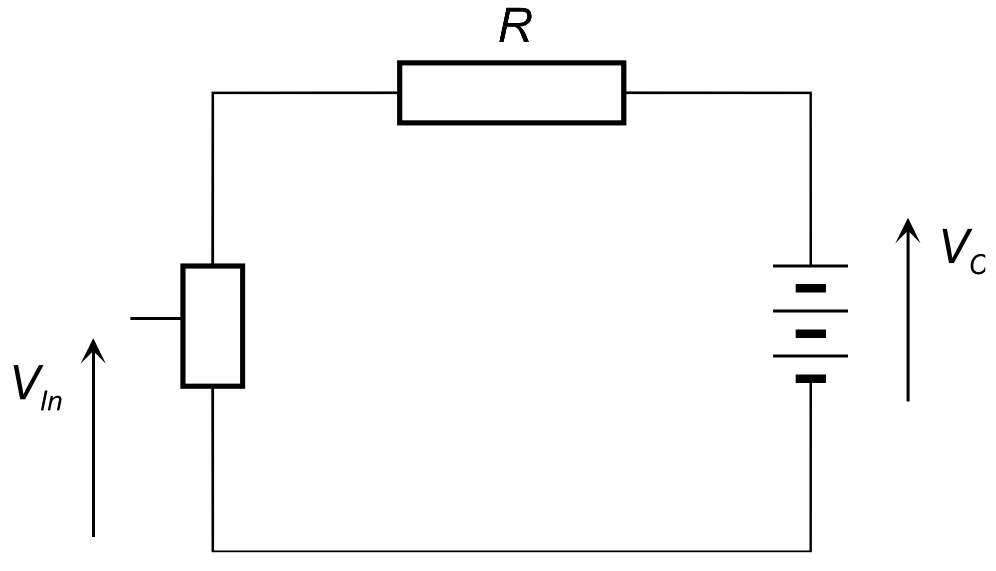
Bipolar Transistor Revisited

- Transistor in a circuit
- Bipolar transistor
- Transistor equivalent circuit

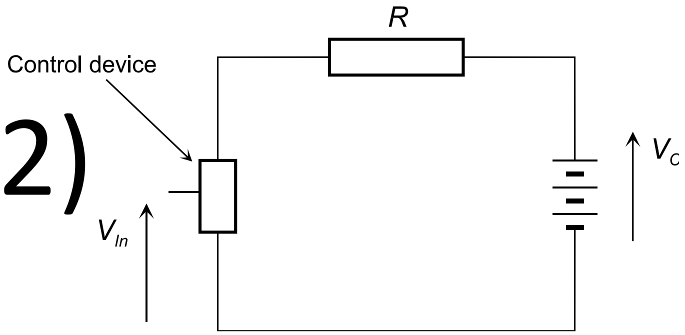
Transistor in a Circuit (1)

- This is a very brief outline from the viewpoint of the transistor as an electronic circuit element – ***not construction and physics of operation***

- Basic circuit with a transistor



Transistor in a Circuit (2)



- The **battery** is a store of energy (potential energy), the resistor absorbs energy.

- The box *'Control device'* with input signal V_{in} is some form of ***electronic control valve,*** usually a transistor.

- A change in the input, ΔV_{In} and ΔI_{In}, causes a change in the energy dissipated in resistor R by controlling the energy flow from the battery.

- If the control device requires **very little energy change** at its **input** to produce a **large change** in the **current through the resistor** there will be a gain – **amplification**.

Transistor in a Circuit (3)

If ΔI_R is the **change in current** through the resistor and ΔV_R is the **change in voltage** across the resistor then

$$A_i = \frac{\Delta I_R}{\Delta I_{In}}$$ and is the current gain

$$A_V = \frac{\Delta V_R}{\Delta V_{In}}$$ and is the voltage gain

$$A_P = A_V \times A_I = \frac{\Delta I_R}{\Delta I_{In}} \times \frac{\Delta V_R}{\Delta V_{In}}$$ and is the power gain

The transistor is the solid state electronic analogue of a control valve. *It is used to control (or switch) energy flow from an electrical energy supply into a load in which the energy is dissipated or stored.*

Bipolar Transistor (1)

They have **three terminals**; base, collector and emitter for **bipolar transistors.**

***Non-linear Transistor Characteristics*:**

- All transistors in all forms and materials have non-linear characteristics. Initially we examine bipolar transistors as these are the most suitable for laboratory work. In a **simplistic** form non-linear means the transistor's behaviour **cannot** be represented by **straight line equations** such as

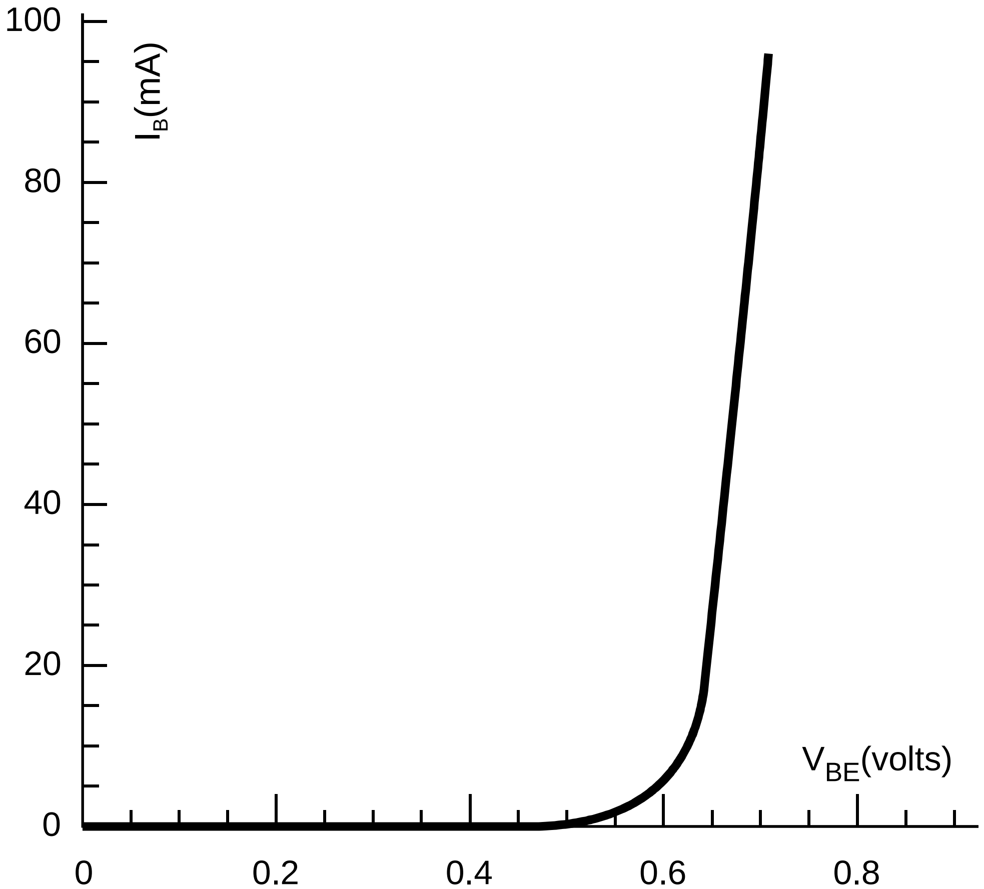
$$y = m \times x \quad \textbf{or} \quad y = m \times x + c$$

Bipolar Transistor (2)

Circuit analysis in other courses used **linear components** that **obey** **Ohm's Law**. The current through a resistor is related to the voltage across it by $V = I \times R$. Ohm's Law is a **linear equation.**

Capacitors and inductors also obey **Ohm's Law** ***in a time varying form***. For transistors the various current and voltages cannot be related by simple straight line equations.

Bipolar Transistor (3)

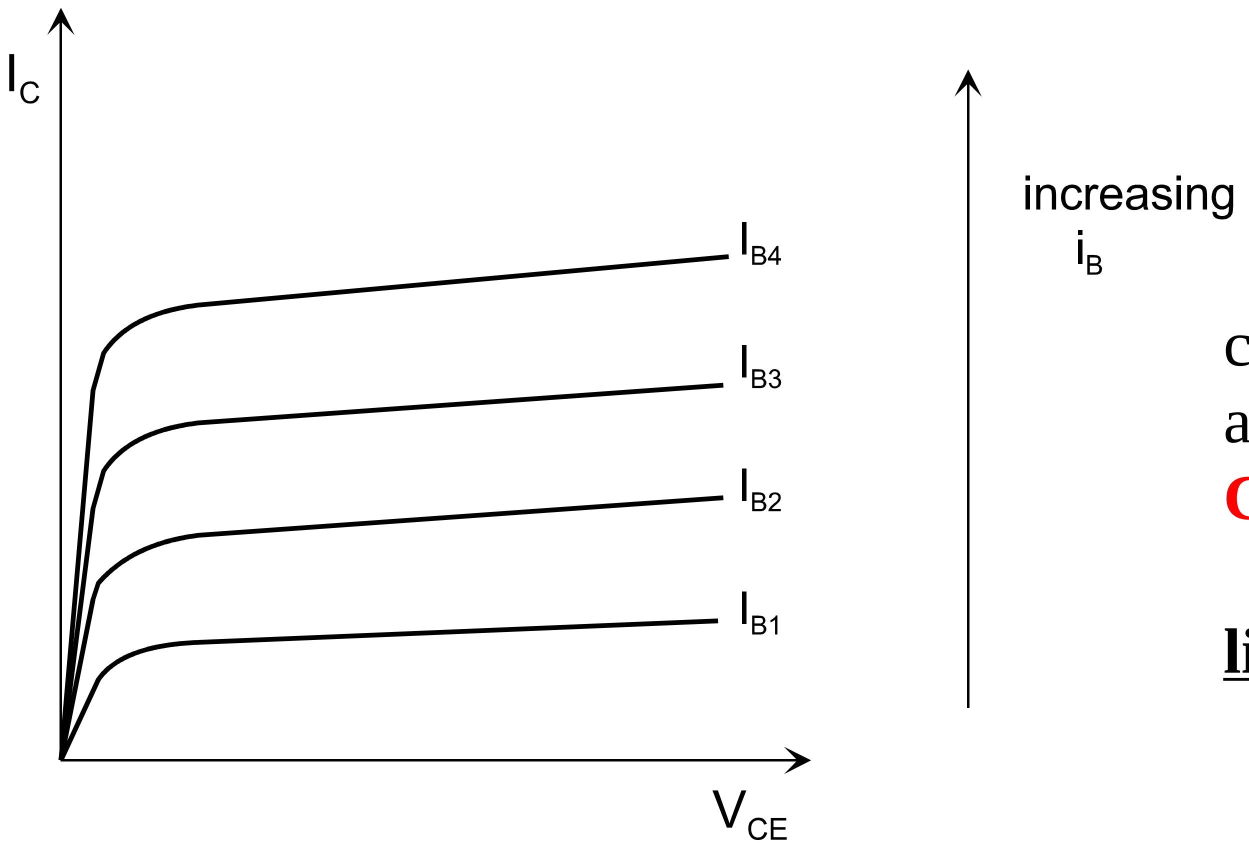


$$I_B = I_S.\exp\left(\frac{qV_{BE}}{kT} + 1\right)$$

the variation of I_B with V_{BE} is the same as the variation of a diode current with voltage – **not a straight line, not linear**

Typical input characteristic, base current as a function of base-emitter voltage

Bipolar Transistor (4)



Output characteristics are also called **transfer Characteristics**

– they are **non-linear**

Typical output characteristics, collector current as a function of collector-emitter voltage

Bipolar Transistor (5)

Why is non-linearity a problem?

• Many applications require the circuit output to be proportional to the input.

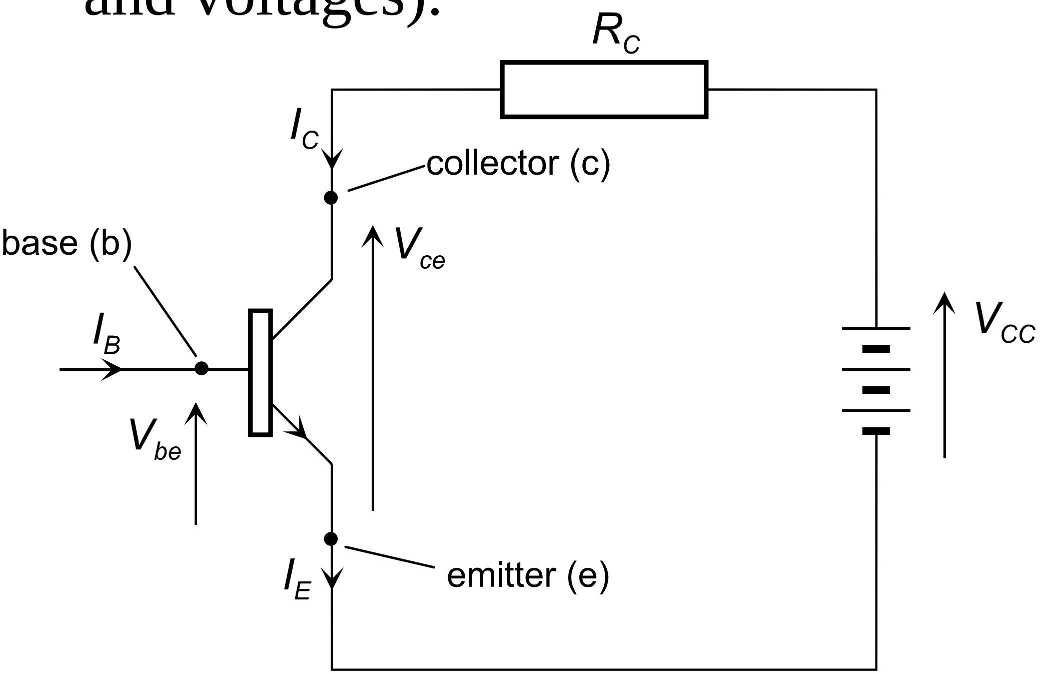
e.g. $V_{out} = k \times V_{in}$ to give **faithful reproduction** of the signal

• In design synthesis and in analysis, we require equations that have analytical solutions.

• For analysis super-position is used, it only works for linear systems

Bipolar Transistor (6)

An npn transistor in a simple circuit (for pnp reverse all currents and voltages).



Rc is in series with the collector.
The current Ic flows through Rc and the collector lead.

The transistor, Rc and the supply form a loop; using Kirchoff's voltage law

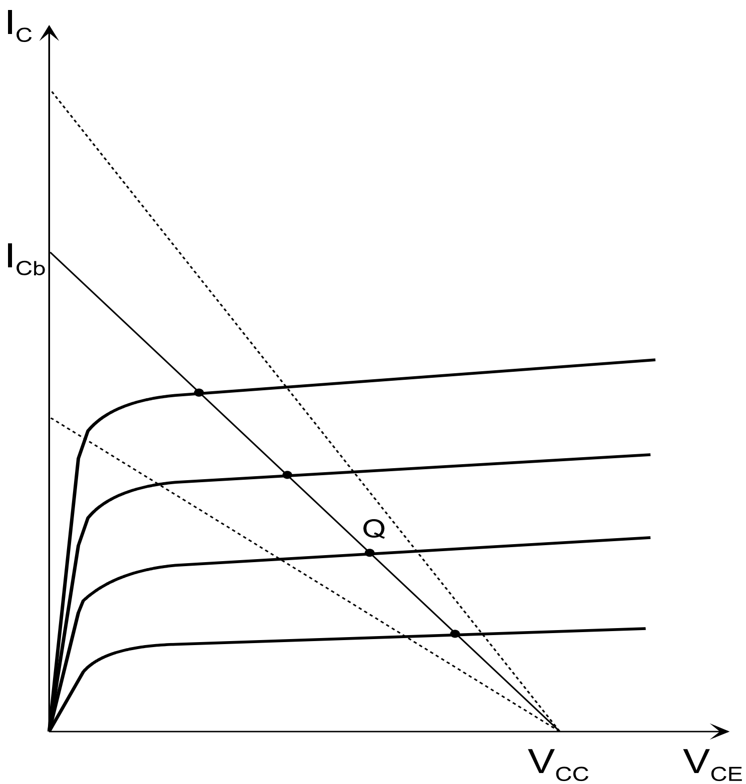
$$V_{CE} = V_{CC} - I_C \times R_C$$

Bipolar Transistor (7)

- I_C and V_{CE} must satisfy the linear relationship of this equation. V_{CC} and R_C are constants – values of circuit components.
- **BUT** I_C and V_{CE} **must also satisfy** the equation (curve, characteristic) which describes the behaviour of the transistor.
- That is the equation of the output characteristic for the particular value of current flowing into the base. There are now **two equations**, the straight line and the transistor output characteristic (which is complicated), these are **simultaneous equations**.

Bipolar Transistor (8) – Solving simultaneous equation (a)

One method of solution to find **the operating point** is **graphical**.
Draw line $V_{CE} = V_{CC} - (I_C \times R_C)$, called a **load line**, and the characteristic on the same axes
(line $I_C = (V_{CC} - V_{CE}) / R_C$ for the axes are shown).



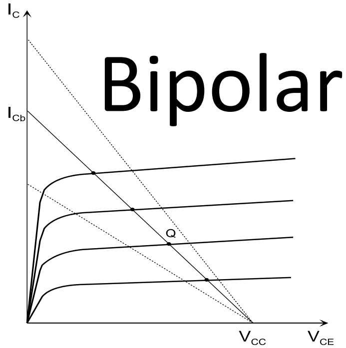
The straight line is easily determined.
When

$$I_C = 0 \quad V_{CC} = V_{CE}$$

and if $V_{CE} = 0 \quad I_C = I_{Cb} = \dfrac{V_{CC}}{R_C}$

These two points are on the required straight line – **it must go through both**!

*The broken lines are for different values of R_C



Bipolar Transistor (9) – Solving simultaneous equation (b)

For a specified value of base current, I_B, the values of I_C and V_{CE} are related by the characteristic curve for that value of base current. I_C and V_{CE} are also related by the straight line. The **only** values of I_C and V_{CE} which satisfy both of these are **where the characteristic and the line cross.**

If the transistor **transfer characteristics**, $I_C = f\{I_B, V_{CE}\}$, are known it is possible to determine the values of I_C and V_{CE}. **Note that the transfer characteristic** used is set by I_B which itself is set by the way additional circuits cause the transistor to behave – **to be examined later** – and I_B is related to V_{BE} by the **input (diode) characteristic** (next page)

Bipolar Transistor (10) – Solving simultaneous equation (c)

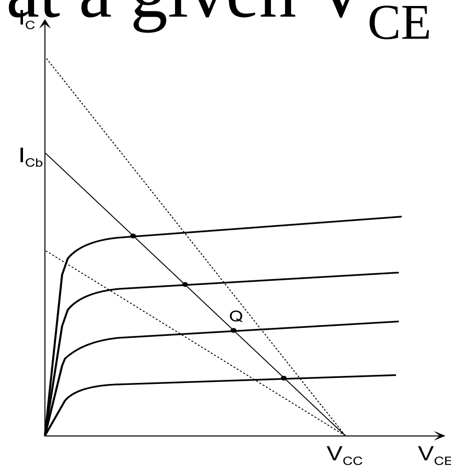
As I_B is changed the output characteristic to be used changes but the load line remains the same, the operating point moves along the line and the value of I_C changes.

Conversely if we force I_C to a value as I_C changes I_B will change. **A change in I_B results in a change in I_C (and vice versa)**

The ratio of I_B to I_C for the transistor at a given V_{CE} is defined as β,

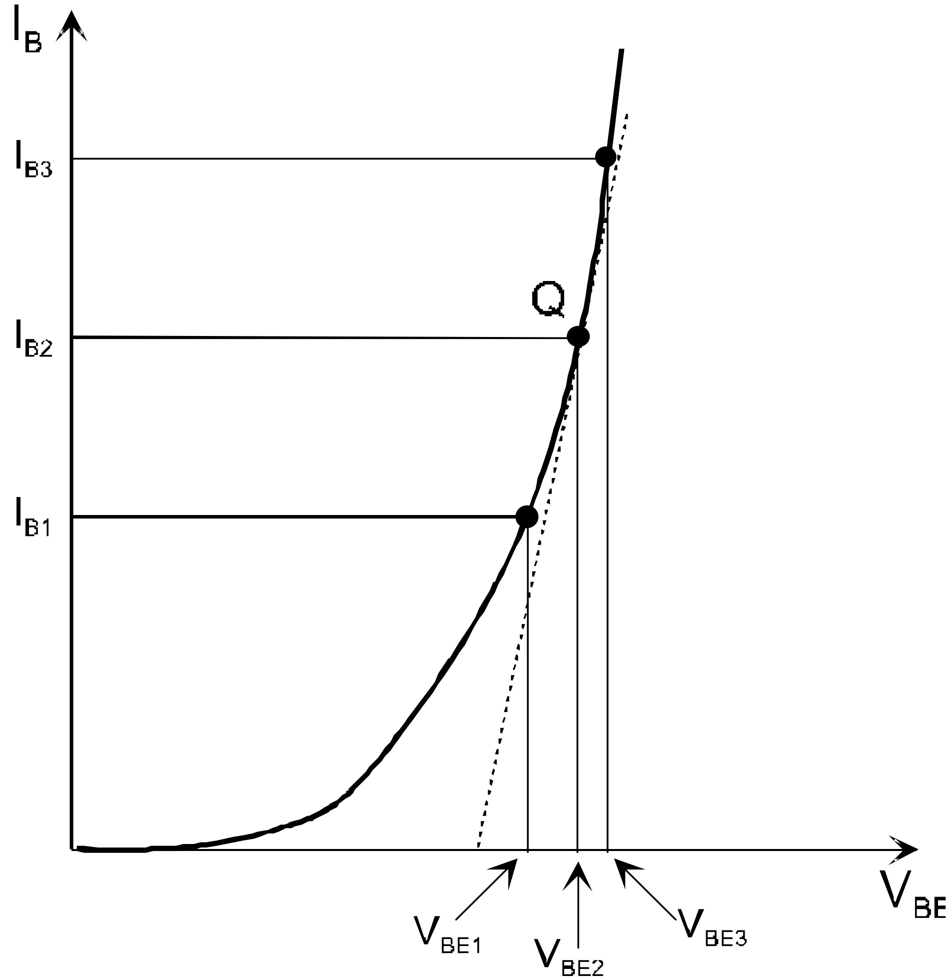
where

$$\beta = \frac{I_C}{I_B}$$



Bipolar Transistor (11) – Solving simultaneous equation (d)

β is known as the <u>static current gain</u> of the transistor, it is also called the DC gain denoted by h_{FE} or β_{DC}.



Bipolar Transistor (12)

- **Two common electronic engineering tasks**

Circuit Design - so that the transistor is at a required operating point and amplifies an input signal.

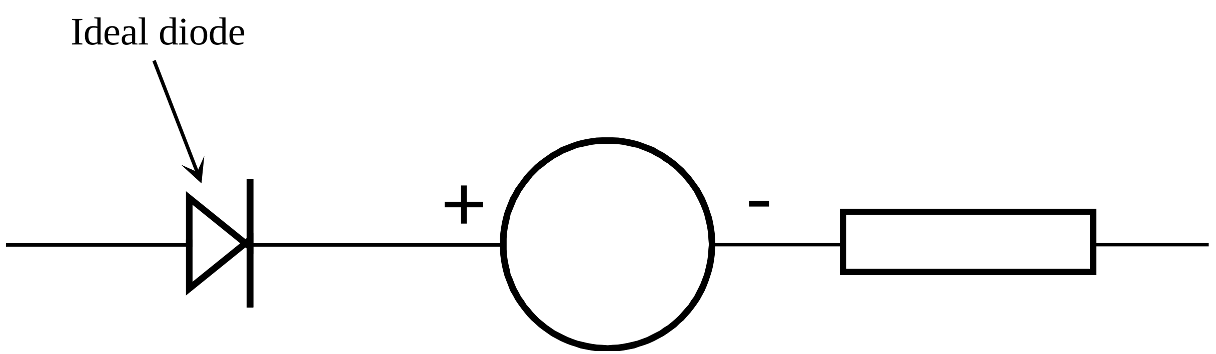
Circuit Analysis – determine the operating point and amplification factor from the circuit.
It often assists in design and analysis if the transistor is represented by an equivalent circuit.

Bipolar Transistor Equivalent Circuit

- h-parameter model
- Hybrid Pi model

Transistor Equivalent Circuit (1)

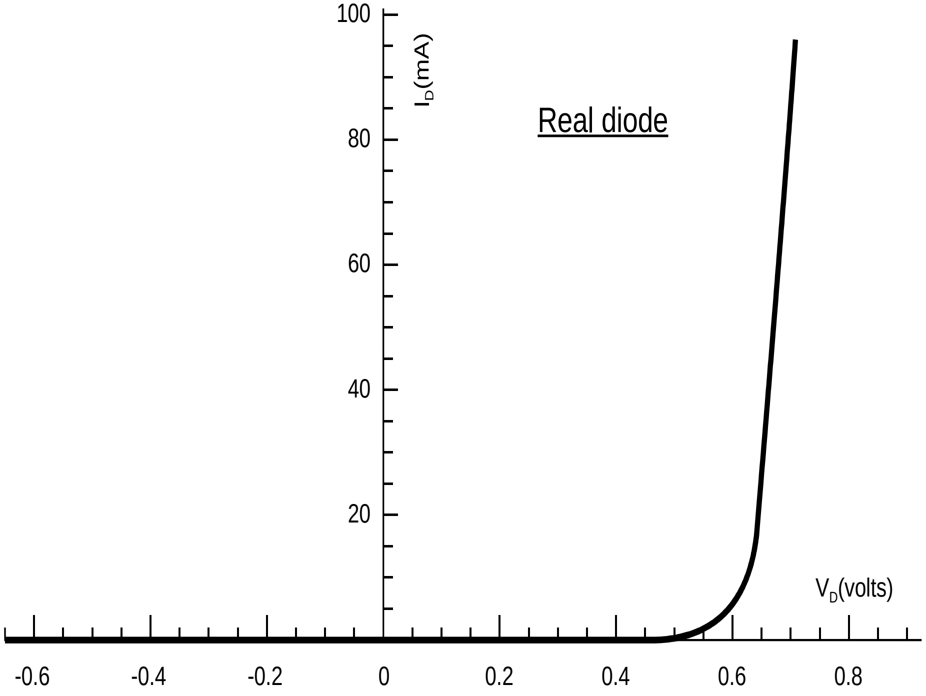
It was possible to replace the diode by an **equivalent circuit** using linear circuit components, this corresponds to **straight line approximations** to the characteristic curves.

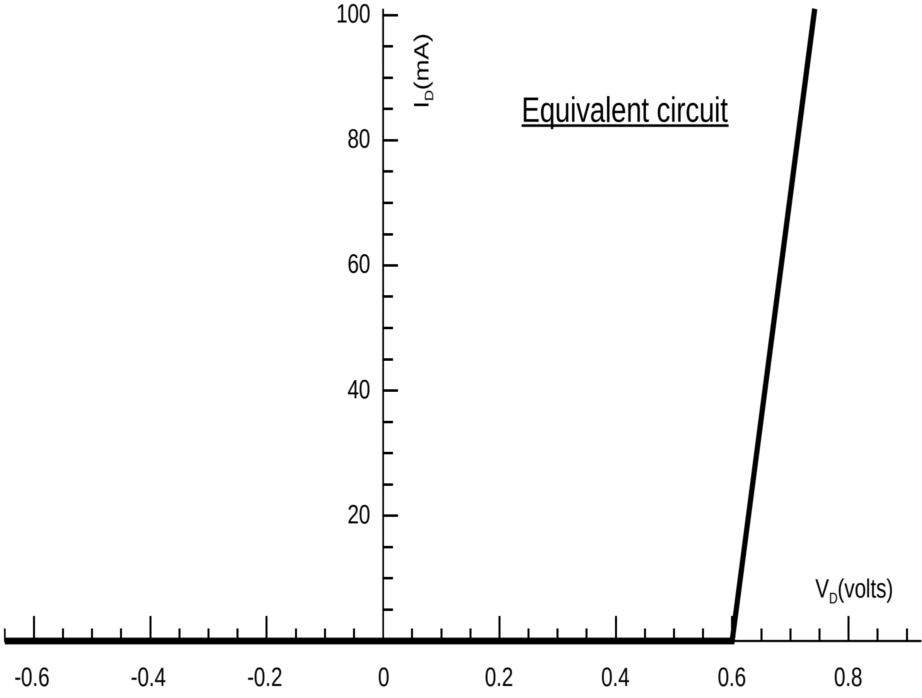


An equivalent circuit for a diode

Ideal diode means no resistance, no voltage drop across it when forward biased; no current flow when reverse biased (a perfect switch operated by the voltage across it).

Transistor Equivalent Circuit (2)





Characteristics for real diode and for the equivalent circuit

Transistor Equivalent Circuit (3)

<u>Perform a similar task for the transistor</u>
– the more complicated curves mean that we divide the transistor operation into two parts**.**

Large signal or steady state or d.c.
Sets operation in a small selected area of the characteristics. This year (and often in practice) this will be done without an equivalent circuit – work with the characteristics.

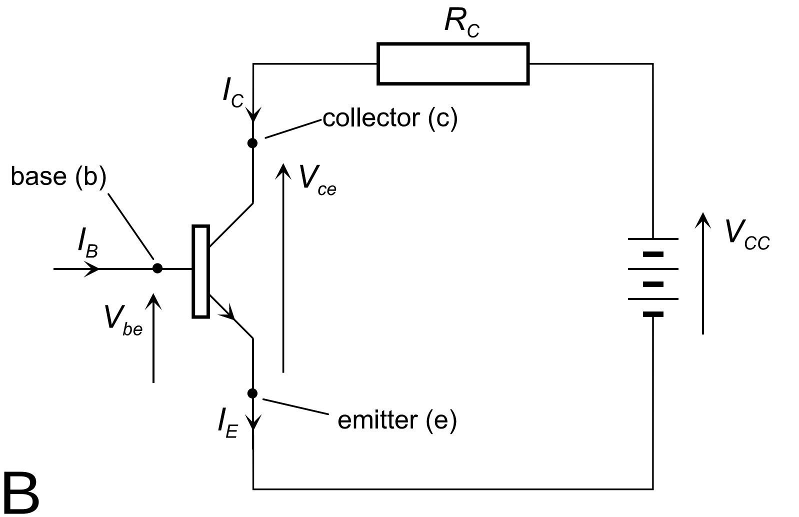
Small signal or dynamic system or a.c.
Consideration of the circuit behaviour when small changes in conditions are made at relatively high speed. The transistor's dynamic characteristics – effects of rapid changes.

Transistor Equivalent Circuit (4)

We will return to this division into two parts later, first obtain the small signal equivalent circuit. It should help your understanding if you can follow the development of the equivalent circuit but if you find it difficult jump straight to the actual equivalent circuit.

Kirchoff's current law (if I_B, I_C and I_E are in the directions in the figure on slide 27)

$$I_B + I_C = I_E$$

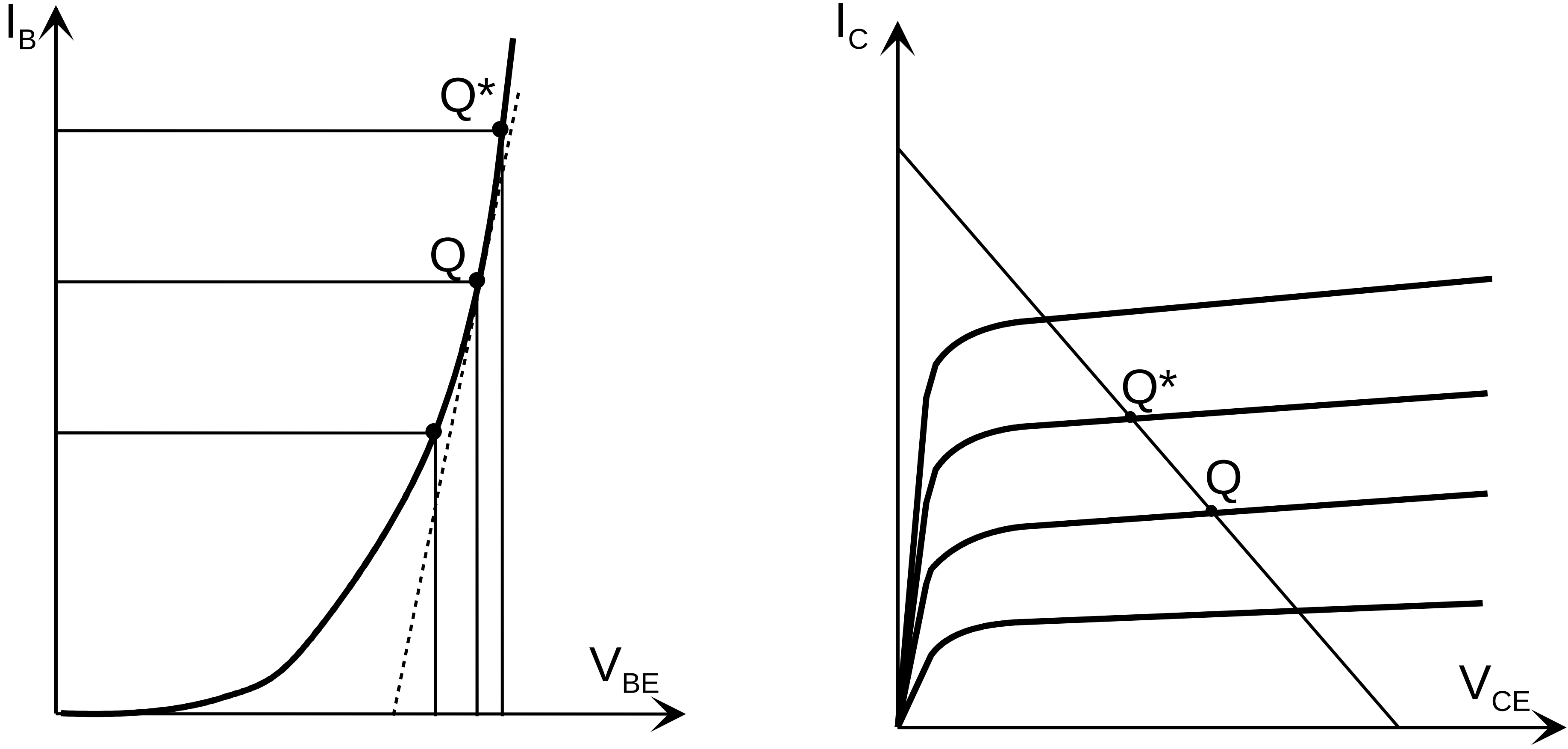


Using the expression for β

$$I_E = (1+\beta)\times I_B$$

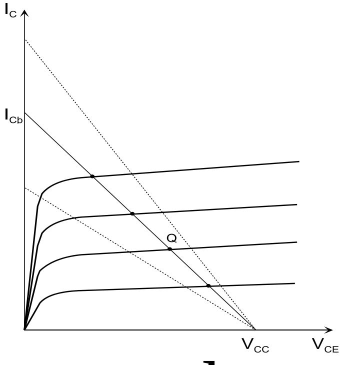
Transistor Equivalent Circuit (5)

Consider the transistor set at the operating point Q on the input and output characteristics. A small change ΔV_{BE} causes a change in the base current of ΔI_B determined from the input characteristic. *The change in I_B* moves the operating point to new positions, Q*, on both characteristics.



Transistor Equivalent Circuit (6)

On the output characteristics the new point, Q*, is on the load line where it intersects the characteristic for the **<u>new base current</u>**. i.e. the change ΔI_B moves the operating point changing ΔI_C and ΔV_{CE}.



If ΔI_B is small compared to I_B the consequent changes ΔI_C and ΔV_{CE} will be small compared to I_C and V_{BE} respectively and we can approximate the transistor behaviour using linear equations. This **linear behaviour** (model) is **<u>only valid for small changes</u>** in the transistor currents and voltages.

Transistor Equivalent Circuit (7)

Writing $I_C = f_1\{I_B, V_{CE}\}$ and $V_{BE} = f_2\{I_B, V_{CE}\}$ then

$$\Delta I_C = \left.\frac{\partial f_1}{\partial I_B}\right|_{V_{CE}} \times \Delta I_B + \left.\frac{\partial f_1}{\partial V_{CE}}\right|_{I_B} \times \Delta V_{CE} = \frac{\partial I_C}{\partial I_B} \times \Delta I_B + \frac{\partial I_C}{\partial V_{CE}} \times \Delta V_{CE}$$

$$\Delta V_{BE} = \left.\frac{\partial f_2}{\partial I_B}\right|_{V_{CE}} \times \Delta I_C + \left.\frac{\partial f_2}{\partial V_{CE}}\right|_{I_B} \times \Delta V_{CE} = \frac{\partial V_{BE}}{\partial I_B} \times \Delta I_B + \frac{\partial V_{BE}}{\partial V_{CE}} \times \Delta V_{CE}$$

Transistor Equivalent Circuit (8)

This *mathematical linearisation* of transistor behaviour leads to an equivalent circuit that describes the transistor behaviour for small changes. *'Change'* implies variation in time so changes correspond to a.c. signals and this leads to the **small signal a.c. equivalent circuit**.

One convention

• use capital (upper case) letters and subscripts for d.c. conditions

• use small (lower case) letters and subscripts for a.c. conditions

Transistor Equivalent Circuit (9)

Hence I_C is the steady d.c. collector current and i_c is the varying a.c. collector current. Drop the Δ notation as ΔI_C is the change (variation) in I_C so ΔI_C is i_c . In this notation the equations become

$$i_c = \frac{\partial i_c}{\partial i_b} \times i_b + \frac{\partial i_c}{\partial \nu_{ce}} \times \nu_{ce} \quad \text{and} \quad \nu_{be} = \frac{\partial \nu_{be}}{\partial i_b} \times i_b + \frac{\partial \nu_{be}}{\partial \nu_{ce}} \times \nu_{ce}$$

$\frac{\partial i_c}{\partial i_b}$ defines change in collector current due to change in

base current. The symbol h_{fe} is used for this and is the **small signal current gain**.

Transistor Equivalent Circuit (10)

Define

$$\frac{\partial i_c}{\partial i_b} = h_{fe}$$ = small signal current gain (a number, no units)

$$\frac{\partial i_c}{\partial v_{ce}} = h_{oe}$$ = output admittance (units of Ω^{-1})

$$\frac{\partial v_{be}}{\partial i_b} = h_{ie}$$ = input resistance (units of Ω)

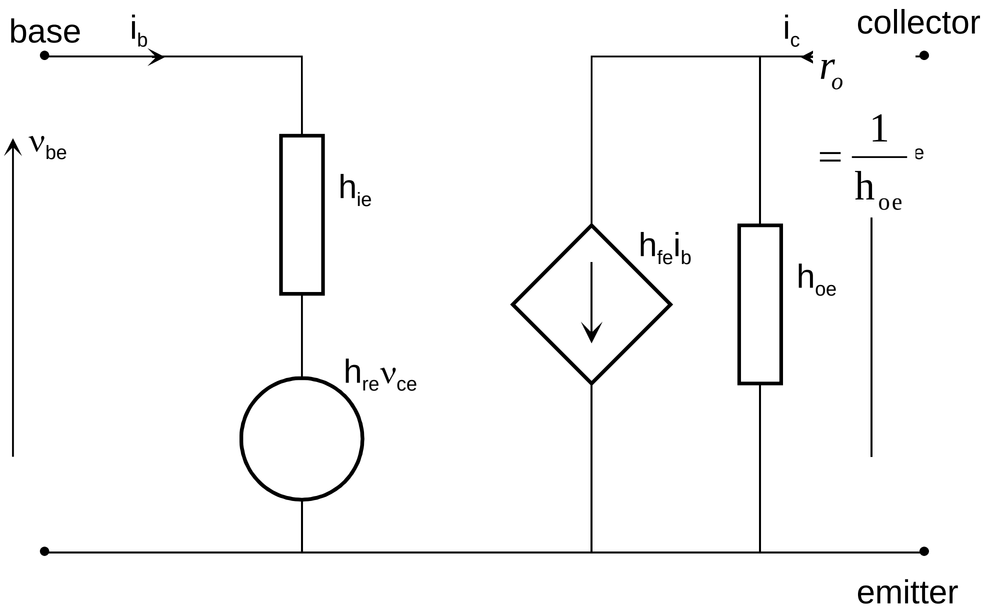
h stands for hybrid

$$\frac{\partial v_{be}}{\partial v_{ce}} = h_{re}$$ = voltage source amplitude factor (a number, no units)

Transistor Equivalent Circuit (11)

These are the "h" parameters for a linearised transistor model, **"h" = hybrid** because the parameters have mixed (different) units.

After setting the operating point the transistor can be represented by the **<u>small signal equivalent circuit</u>** for examination of the circuit behaviour for small changes in currents and voltages. The h parameters are the circuit elements of this model, the model is a circuit which behaves as the linearised characteristics.



Transistor Equivalent Circuit (12)

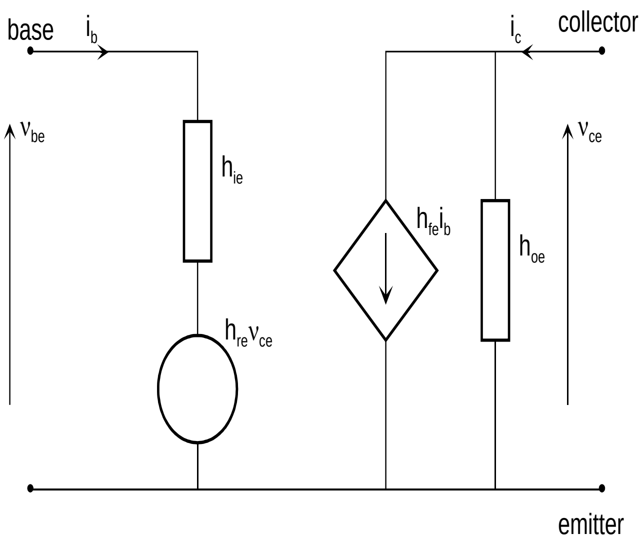
The emitter is the reference level for base ***and*** collector voltages. The emitter is common to the input and output parts of the circuit, this is a <u>**common emitter**</u> equivalent circuit.

For most purposes it is reasonable to assume that $\dfrac{\partial v_{be}}{\partial v_{ce}} = h_{re} \cong 0$

Reason:

As the base current changes by a small amount (and *vce* changes) the exponential input characteristic is so steep that *vbe* is very small, VBE changes very little.

Remember if any change in VBE is so small it can be ignored then for most purposes VBE can be regarded as a constant value of about 0.7 volts



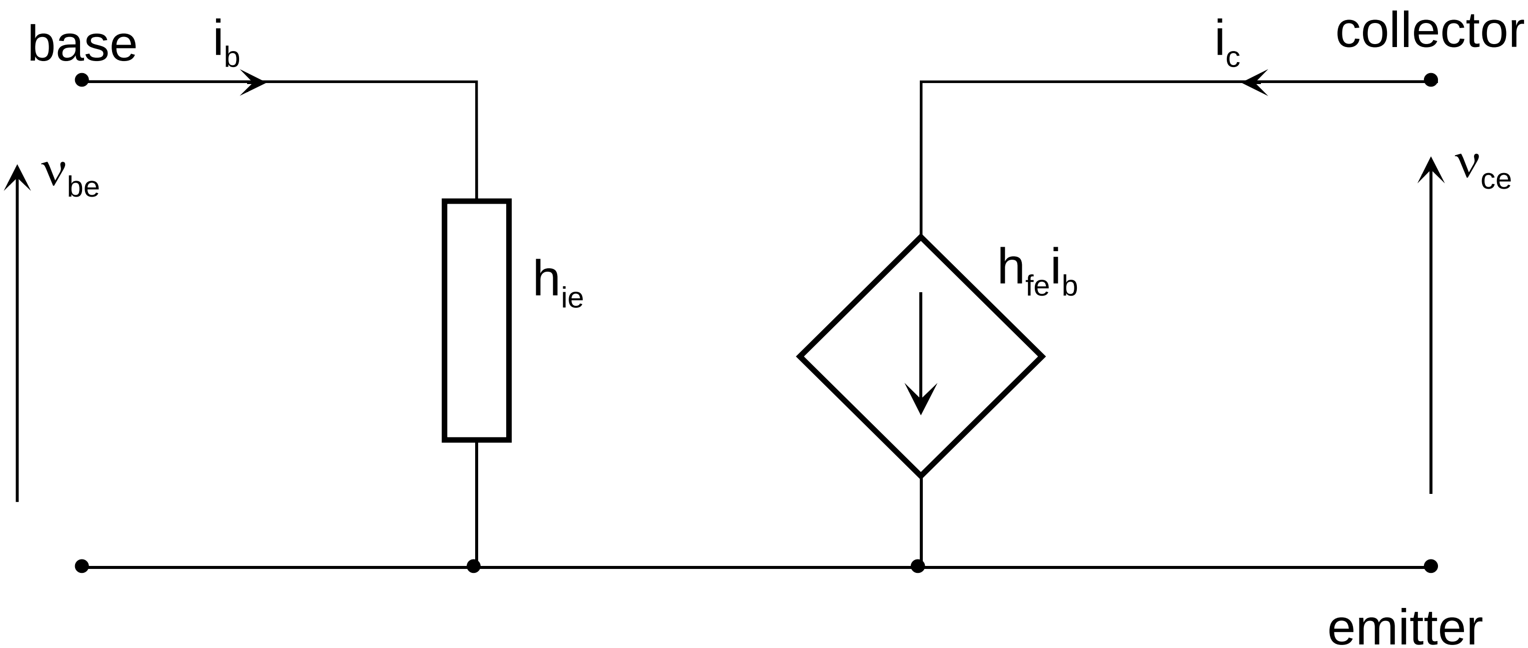
For a transistor with ideal characteristics output admittance is approx 0

$$\frac{\partial i_c}{\partial v_{ce}} = h_{oe} \cong 0$$

Transistor Equivalent Circuit (13)

In this semester, it will always be assumed that h_{re} is zero.
In almost all cases h_{oe} will be assumed to be zero.

In the most simple case the small signal a.c. equivalent circuit is



Note the value of the current generator in the collector side is set by the base current

Hybrid Pi Model

The Hybrid Pi model – An Introduction (1)

- We now appreciate the application of the ***h* parameter** transistor model to the study of AC transistor operation.

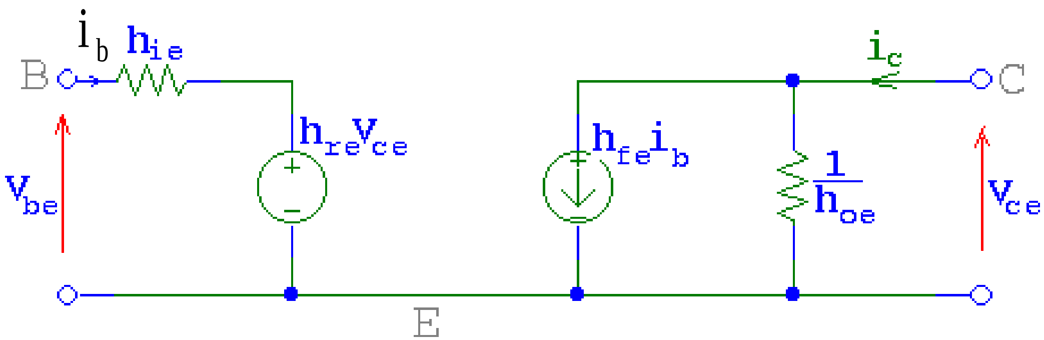
- The **hybrid pi model** of the transistor, which is more versatile than the *h parameter model* (as will be shown later), can be related to the *h* parameters.

- This is particularly important, as manufacturers typically supply values for the *h parameters* in their transistor **specification sheets**, and NOT the hybrid pi model parameters.

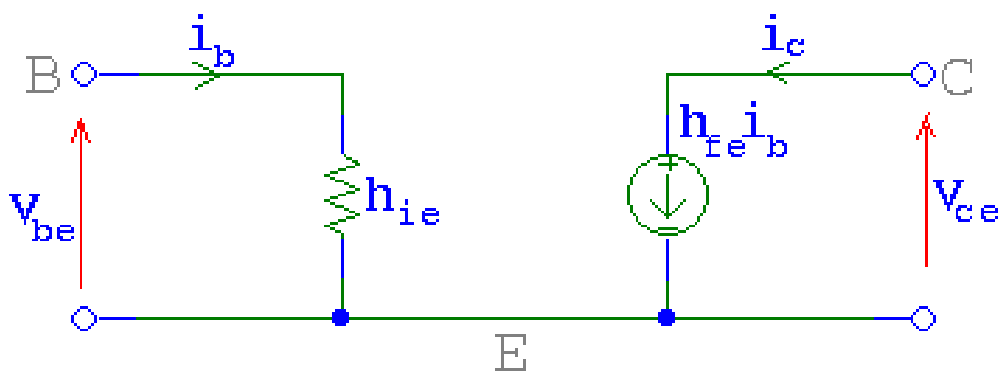
- Therefore one needs to be able to **derive** the **hybrid pi model** parameters **from** the ***h* parameters**.

The Hybrid Pi model – An Introduction (2)

For the CE BJT amplifier, the *h* parameter model is shown below

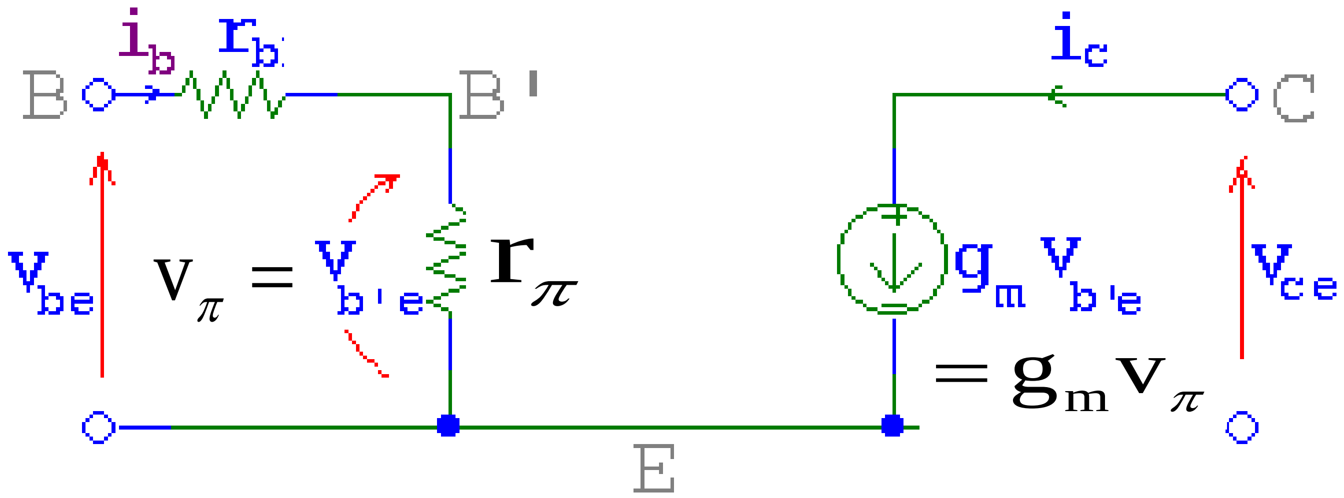


As h_{re} is typically a relatively small quantity and $1/h_{oe}$ is very large, the above model can be simplified, with only a minor effect on any calculations made.



The Hybrid Pi model – An Introduction (3)

The corresponding simplified hybrid pi model is



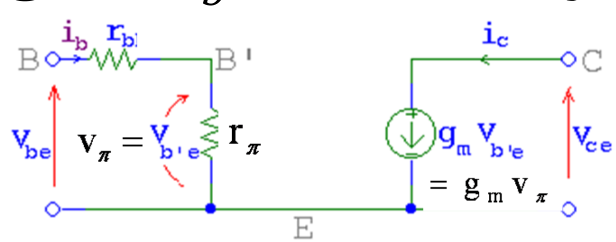
B' is NOT physically accessible BUT represents the internal base node at the junction, separated from the external node B by r_b .

g_m = transconductance of the BJT transistor.

The Hybrid Pi model – An Introduction (4)

The parameter r_b is the series resistance of the semiconductor material between the external base terminal B and an idealized internal base region B'. Typically, r_b is **a few tens of ohms** and is usually **much smaller** than r_π ; therefore ,r_b is normally negligible (a short circuit) at low frequencies. However, at high frequencies, r_b may not be negligible, since the input impedance becomes capacitive, as we will see in later on.

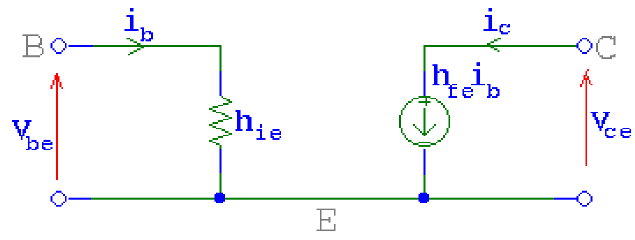
In the lecture notes, when we use the hybrid-π equivalent circuit model, we will **neglect** r_b unless they are specifically included.



The Hybrid Pi model – An Introduction (5)

Comparing the 2 models, it can be seen that

- $h_{ie} = r_b + r_\pi$ .....(1.1)

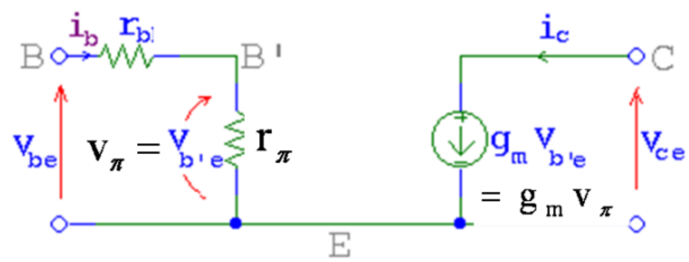


- $h_{fe}\, i_b = g_m v_{be} = g_m\, r_\pi\, i_b$

where $g_m = |I_c| / V_T$

OR

- $h_{fe} = g_m\, r_\pi$ ....(1.2)



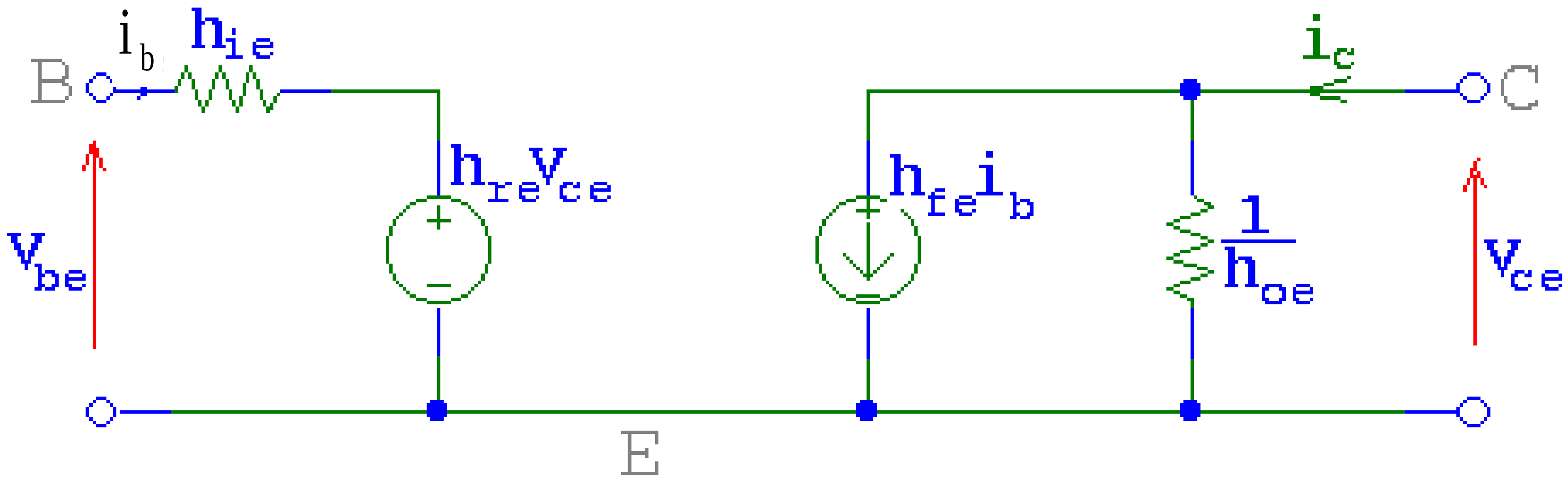
For a BJT

- g_m = |Ic|(mA) / 0.026V at room temperature ..(1.3)

NB: The above simplified models are ONLY applicable at LOW frequencies (up to midband range of frequencies) !!

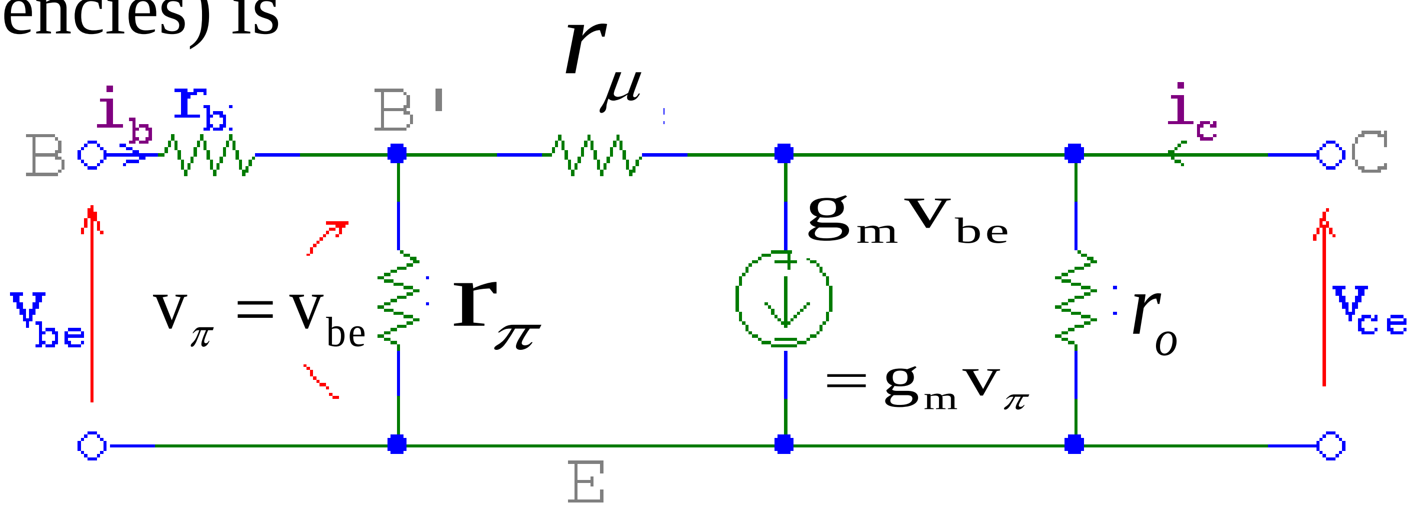
The Hybrid Pi model – An Introduction (6)

What about h_{oe} and h_{re} ?? How do they relate to the hybrid pi model ? To answer this question, we must return to the complete h-parameter model.

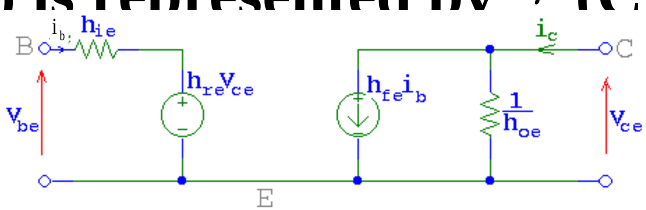


The Hybrid Pi model – An Introduction (7)

The corresponding complete hybrid pi model (at LOW frequencies) is



Note the effect of h_{oe} in the h parameter model is represented partly by r_o (i.e. the finite output resistance of the transistor). The effect of h_{re} in the h parameter model (showing some feedback from the output back to the input) is represented by r_μ (C = output, B' = input)

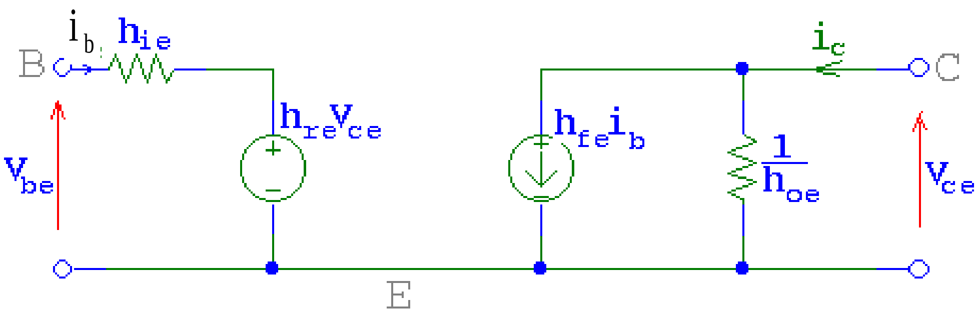


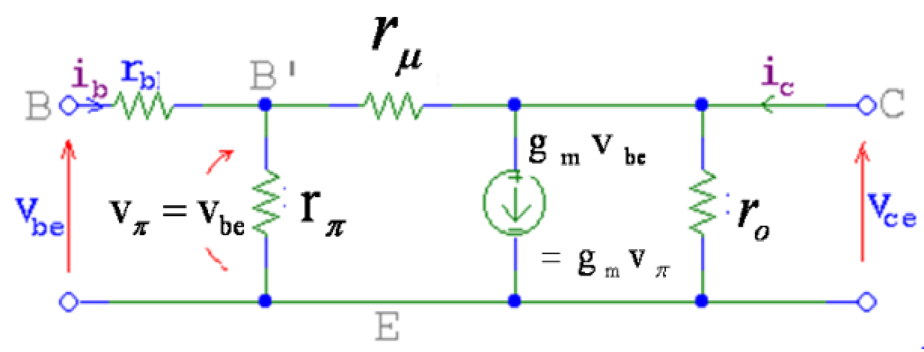
The Hybrid Pi model – An Introduction (8)

The parameter r_μ is the **reverse-biased diffusion resistance** of the base-collector junction. This resistance is typically on the **order of megohms** and can normally be neglected (an open circuit). However, the resistance does provide some feedback between the output and input, meaning that the base current is a slight function of the collector-emitter voltage.

In the lecture notes, when we use the hybrid-π equivalent circuit model, we will **neglect** r_μ unless they are specifically included.

The Hybrid Pi model – An Introduction (9)





$$h_{re} = V_{be}/V_{ce}\Big|_{i_b=0} = V_\pi/V_{ce} = r_\pi/(r_\pi + r_\mu) \approx r_\pi/r_\mu \qquad (1.4)$$

$$h_{oe} = i_c/V_{ce}\Big|_{i_b=0} \qquad (\text{assuming } r_\pi << r_\mu)$$

$$\text{Under these conditions } i_c = (V_{ce}/r_o) + V_{ce}/(r_\pi + r_\mu) + g_m V_\pi$$

$$\text{But from (1.4) for } i_b = 0,\ V_\pi = h_{re}V_{ce}$$

$$\text{Therefore, } h_{oe} = i_c/V_{ce} = 1/r_o + 1/r_\mu + g_m h_{re} \quad (\text{assuming } r_\pi << r_\mu)$$

$$\text{As } g_m = h_{fe}/r_\pi,\ \ h_{re} \approx r_\pi/r_\mu \qquad (\text{assuming } r_\pi << r_\mu)$$

$$\text{Therefore, } h_{oe} = 1/r_o + (1/r_\mu)(1 + h_{fe}) \qquad (1.5)$$

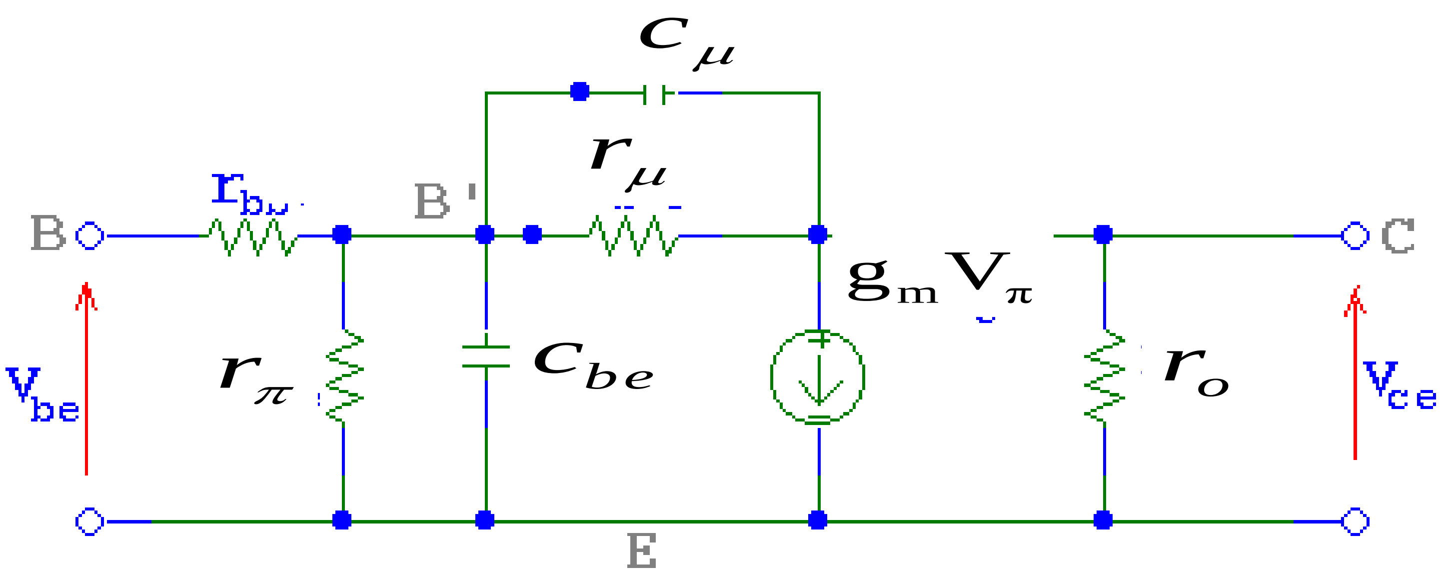
The Hybrid Pi model –

High Frequency Analysis of Transistor Operation

The h parameter model is only suitable at **low frequencies**. The high frequency behaviors of a transistor can easily be taken into consideration in the hybrid pi model if the following addition are made:

The **emitter diffusion capacitance** is added between terminals E and B' and the **collector transition capacitance** is placed between C & B'

The final hybrid pi model becomes



Summary

If the CE h paramaters at **LOW frequencies** are known at the collector current I_c, the hybrid pi model circuit parameters can then be calculated from the following equations, in the order given (derived from equations (1.1) - (1.5) above)

$$g_m = |I_c|(\mathrm{mA})/0.026 \quad \text{at room temperature} \tag{1.6}$$

$$r_\pi = h_{fe}/g_m \tag{1.7}$$

$$r_b = h_{ie} - r_\pi \tag{1.8}$$

$$r_\mu \approx r_\pi / h_{re} \tag{1.9}$$

$$1/r_o = h_{oe} - (1/r_\mu)(1 + h_{fe}) \tag{1.10}$$

Example

Typical values of h parameters for a BJT transistor at room temperature and I_C = 1.3mA are

- $h_{ie} = 2K1$
- $h_{re} = 10^{-4}$
- $h_{oe} = 10^{-5}$ A/V
- $h_{fe} = 100$

$$g_m = |I_c|(mA)/0.026$$

$$r_\pi = h_{fe}/g_m \qquad r_\mu \approx r_\pi / h_{re}$$

$$r_b = h_{ie} - r_\pi$$

$$1/r_o = h_{oe} - (1/r_\mu)(1 + h_{fe})$$

The corresponding hybrid pi model circuit parameters are $g_m =$

- $\mathbf{r_\pi} =$
- $\mathbf{r_b} =$
- $\mathbf{r_\mu} =$
- $\mathbf{r_o} =$

Coming Up

- Understand the concept of an analog signal and the principle of a linear amplifier.
  - Investigate how a transistor circuit can amplify a small, time-varying input signal.

- Discuss and compare the three basic transistor amplifier configurations.
  - Analyze the common-emitter amplifier.
  - Analyze the emitter-follower amplifier.
  - Analyze the common-base amplifier.

- Analyze multitransistor or multistage amplifiers. circuit.

- Understand the concept of signal power gain in an amplifier

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EEE109: Electronic Circuits

Basic BJT Amplifiers – Part 2

Contents

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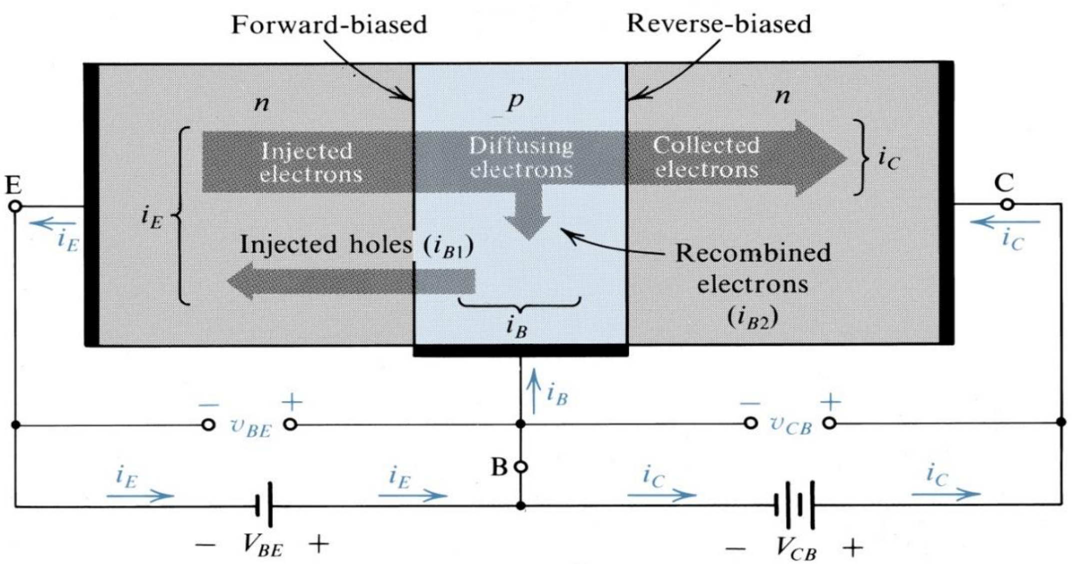
circuit.

- Understand the concept of signal power gain in an amplifier

Analyse the Common-Emitter Amplifier

Physical Mechanism: BJT in Active Mode



- Operation
  - Forward bias of EBJ injects electrons from emitter into base (small number of holes injected from base into emitter)
  - Most electrons shoot through the base into the collector across the reverse bias junction (think about band diagram)
  - Some electrons recombine with majority carrier in (P-type) base region

Physical Mechanism: Collector Current

- Electrons that diffuse across the base to the CBJ junction are swept across the CBJ depletion region to the collector b/c of the higher potential applied to the collector.

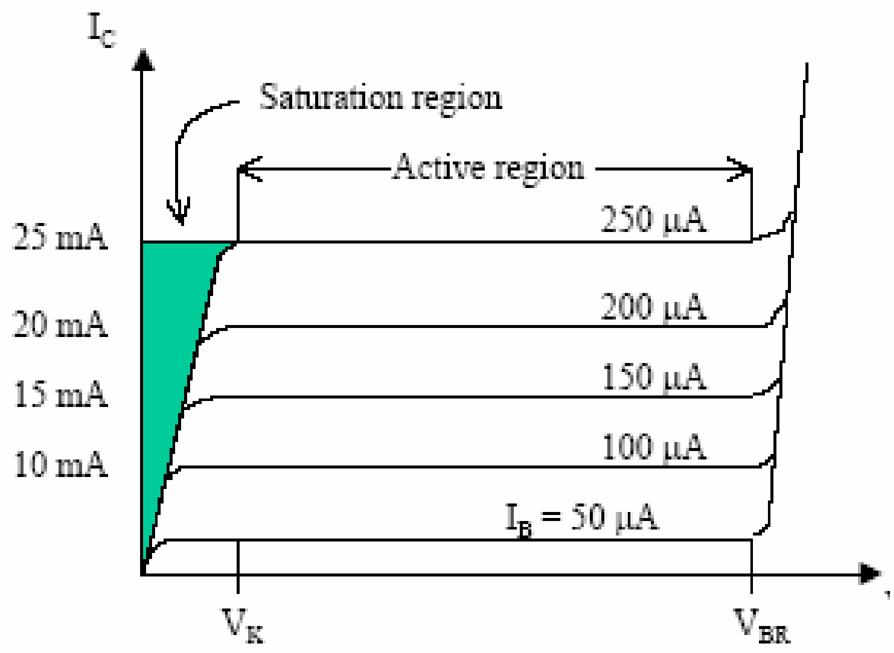
$\boxed{i_C = I_s e^{v_{BE}/V_T}}$ where the saturation current is $I_S = qA_E D_n n_{p0}/W$

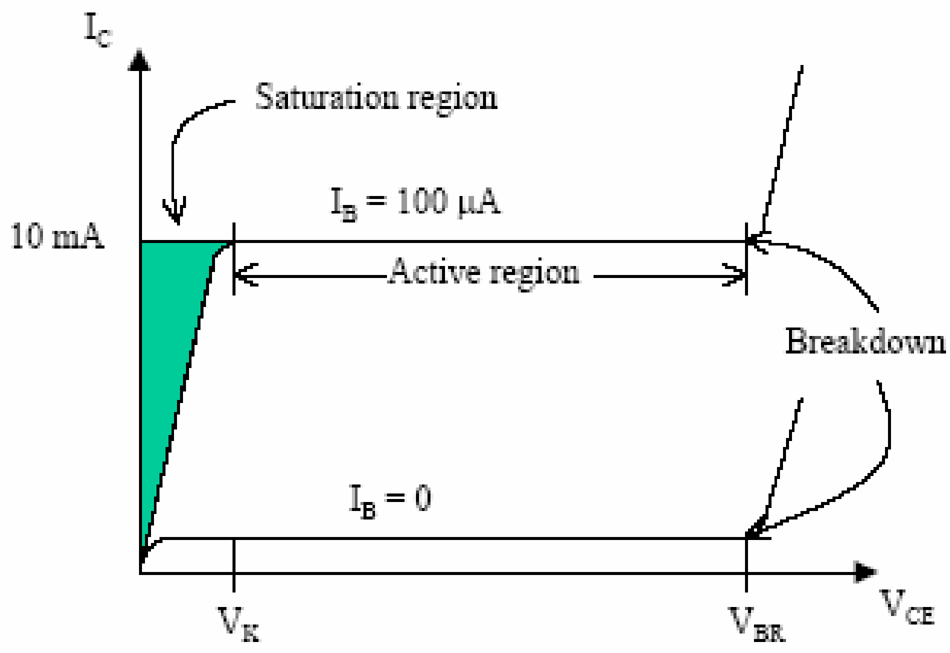
and we can rewrite the saturation current as:

$$\boxed{I_S = \frac{qA_E D_n n_i^2}{N_A W}}$$

- Note that i_C is independent of v_{CB} (potential bias across CBJ) ideally
- Saturation current is
  - inversely proportional to W and directly proportional to A_E
    - Want short base and large emitter area for high currents
  - dependent on temperature due to n_i^2 term

Physical Mechanism: Collector Current





Physical Mechanism: Base Current

- Base current i_B composed of two components:
  - holes injected from the base region into the emitter region

$$i_{B1} = \frac{qA_E D_p n_i^2}{N_D L_P} e^{v_{BE}/V_T}$$

  - holes supplied due to recombination in the base with diffusing electrons and depends on minority carrier lifetime t_b in the base

$$i_{B2} = \frac{Q_n}{\tau_b}$$

And the Q in the base is $$Q_n = \frac{qA_E W n_i^2}{N_A} e^{v_{BE}/V_T}$$

So, current is $$i_{B2} = \frac{qA_E W n_i^2}{N_A \tau_b} e^{v_{BE}/V_T}$$

- Total base current is $$\boxed{i_B = \left(\frac{qA_E D_p n_i^2}{N_D L_P} + \frac{qA_E W n_i^2}{N_A \tau_b} \right) e^{v_{BE}/V_T}}$$

Beta

- Can relate i_B and i_C by the following equation

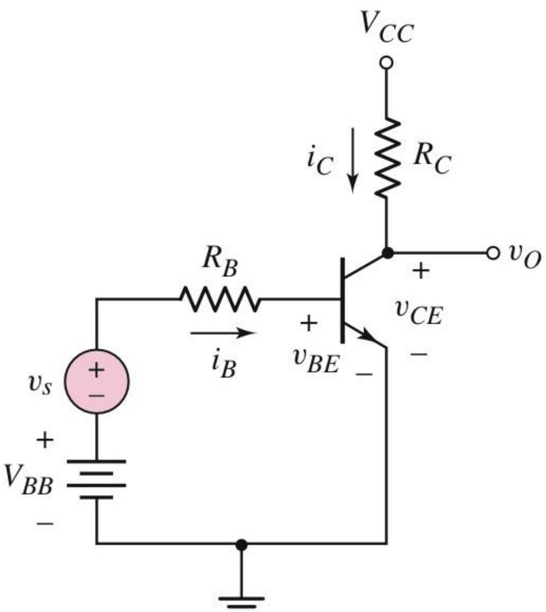
$$\boxed{i_B = \frac{i_C}{\beta} = \frac{I_S}{\beta} e^{v_{BE}/V_T}}$$

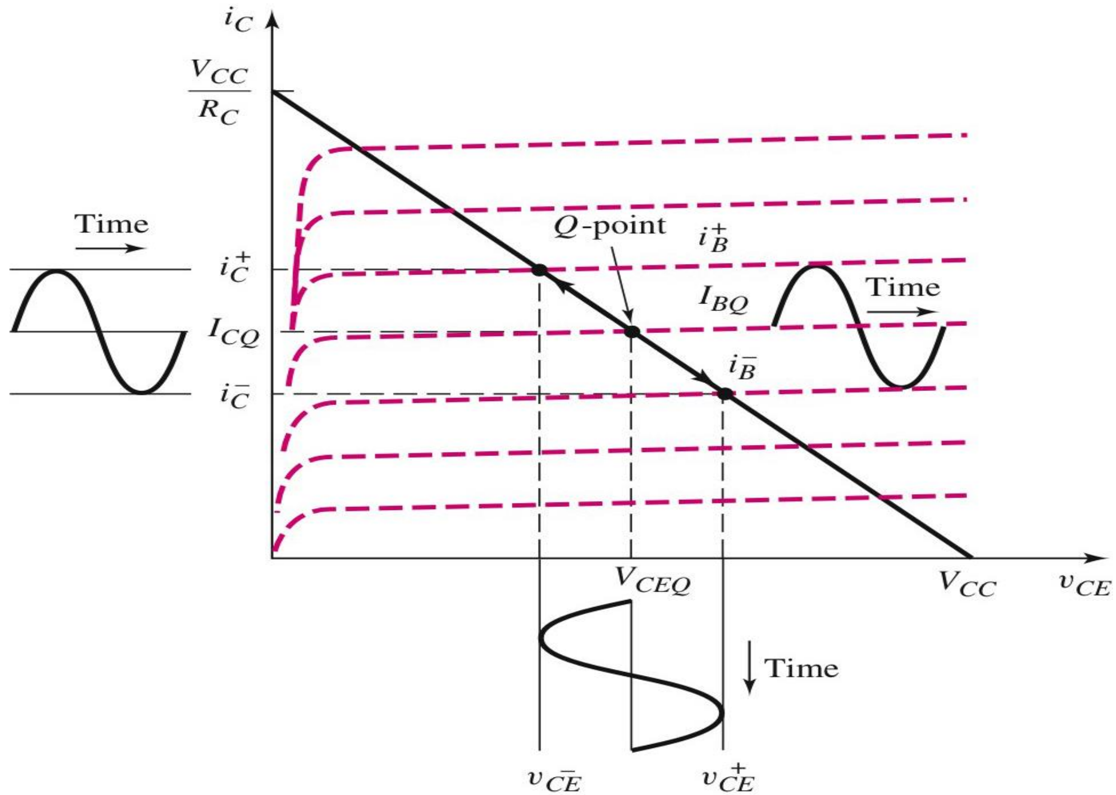
and *beta* is

$$\beta = \frac{1}{\frac{D_p}{D_n}\frac{N_A}{N_D}\frac{W}{L_p} + \frac{1}{2}\frac{W^2}{D_n \tau_b}}$$

  - Beta is constant for a particular transistor
  - On the order of 100-200 in modern devices (but can be higher)
  - Called the common-emitter current gain
- For high current gain, want small W, low N_A, high N_D

Common Emitter with Time-Varying Input





I_B Versus V_{BE} Characteristic

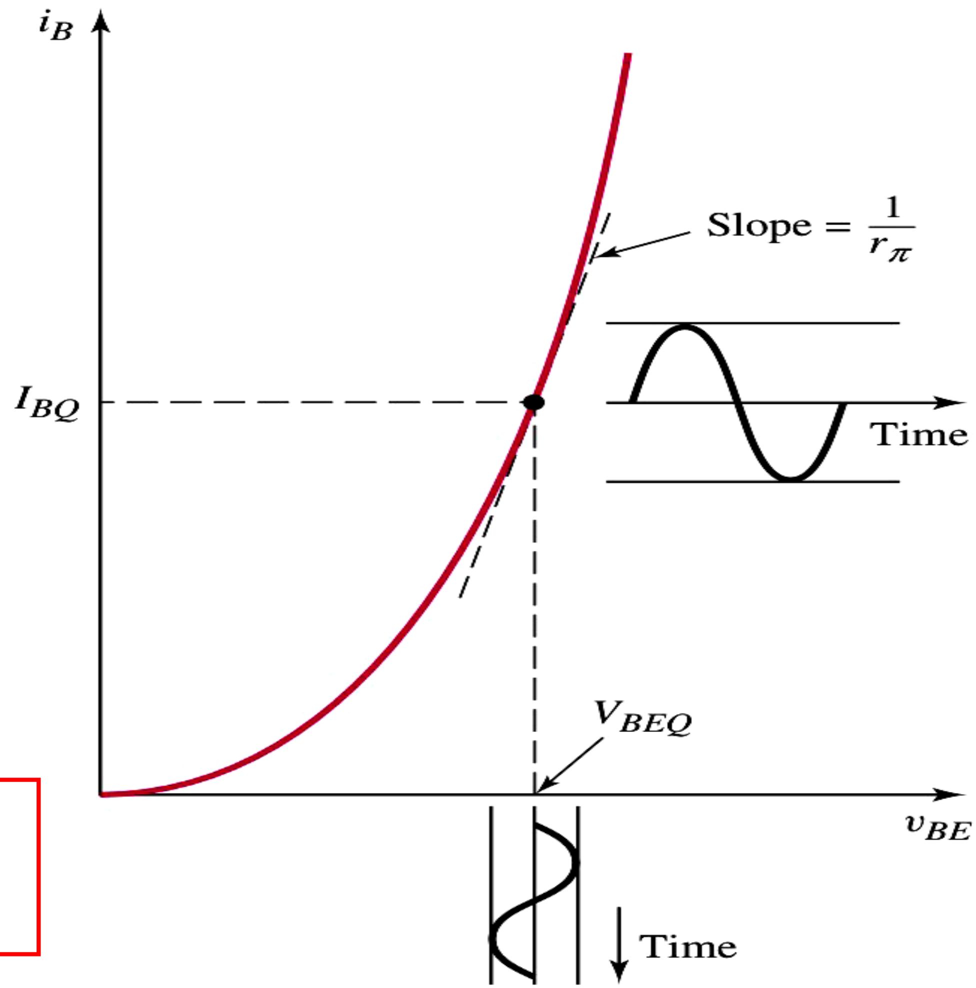
$$\boxed{i_B = \frac{i_C}{\beta} = \frac{I_S}{\beta} e^{v_{BE}/V_T}}$$

If $v_{be} \ll V_T$,

We can expand the exponential term in a Talor series, keeping only the **linear term**.

$$\boxed{i_B \cong I_{BQ}(1 + \frac{v_{be}}{V_T}) = I_B + i_b}$$

The approximation is what is meat by small signal!



Small Signal Implications

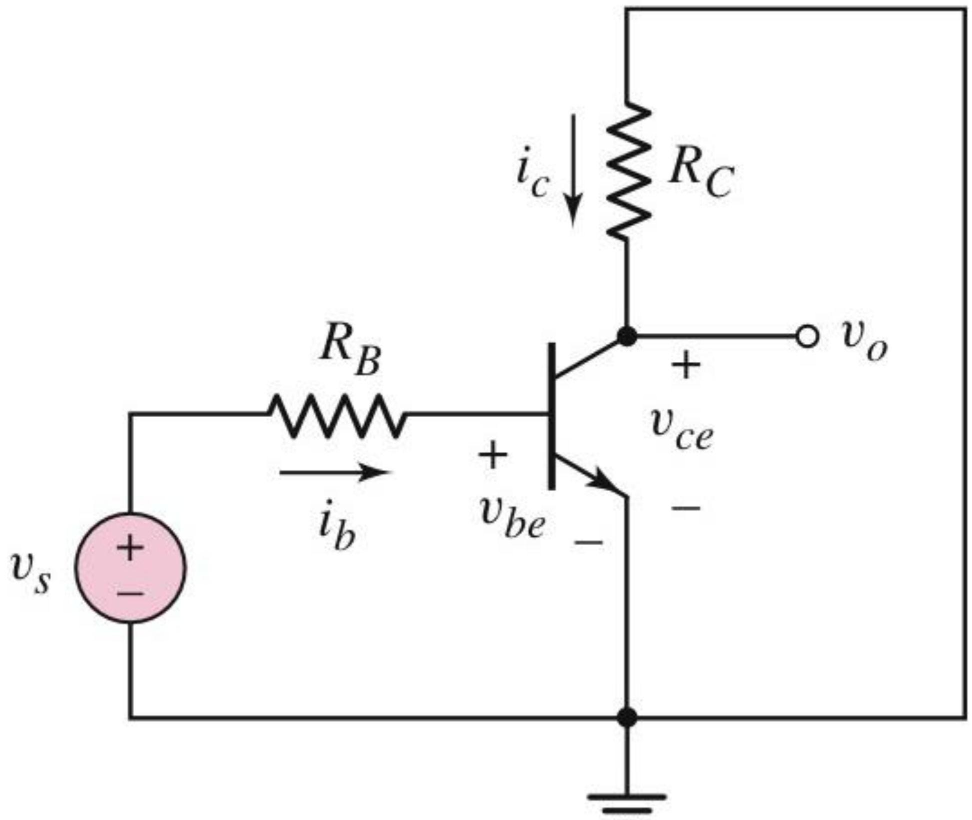
$$i_B \cong I_{BQ}(1+\frac{v_{be}}{V_T}) = I_B + i_b$$

$$i_b = \left(\frac{I_{BQ}}{V_T}\right)v_{be}$$

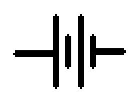
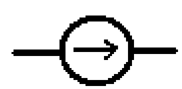
The **two** linear equations have two interpretations:

1. The *total instantaneous values* of current i_B can be written as an ac current <u>**superimposed**</u> on a dc quiescent value.
2. If v_{be} is sufficiently small, i_b and v_{be} have linear relationship.

ac Equivalent Circuit for Common Emitter



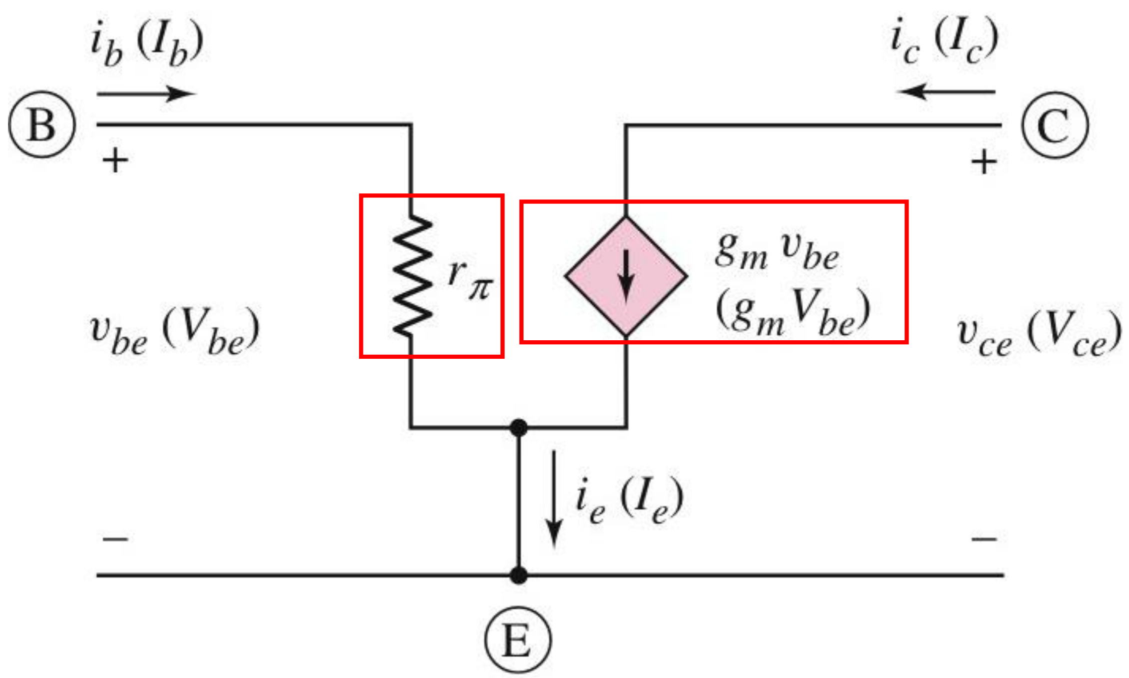
Transformation of Elements

Element	DC Model	AC Model
Resistor	R	R
Capacitor	Open	C
Inductor	Short	L
Diode	$+V_\gamma, r_f -$ 	$r_d = V_T/I_D$
Independent Constant Voltage Source	$+ V_S -$ 	Short
Independent Constant Current Source	I_S 	Open

Small-Signal Hybrid π Model for npn BJT

$$\boxed{i_B = \frac{i_C}{\beta} = \frac{I_S}{\beta} e^{v_{BE}/V_T}}$$

$$\boxed{i_C = I_s e^{v_{BE}/V_T}}$$



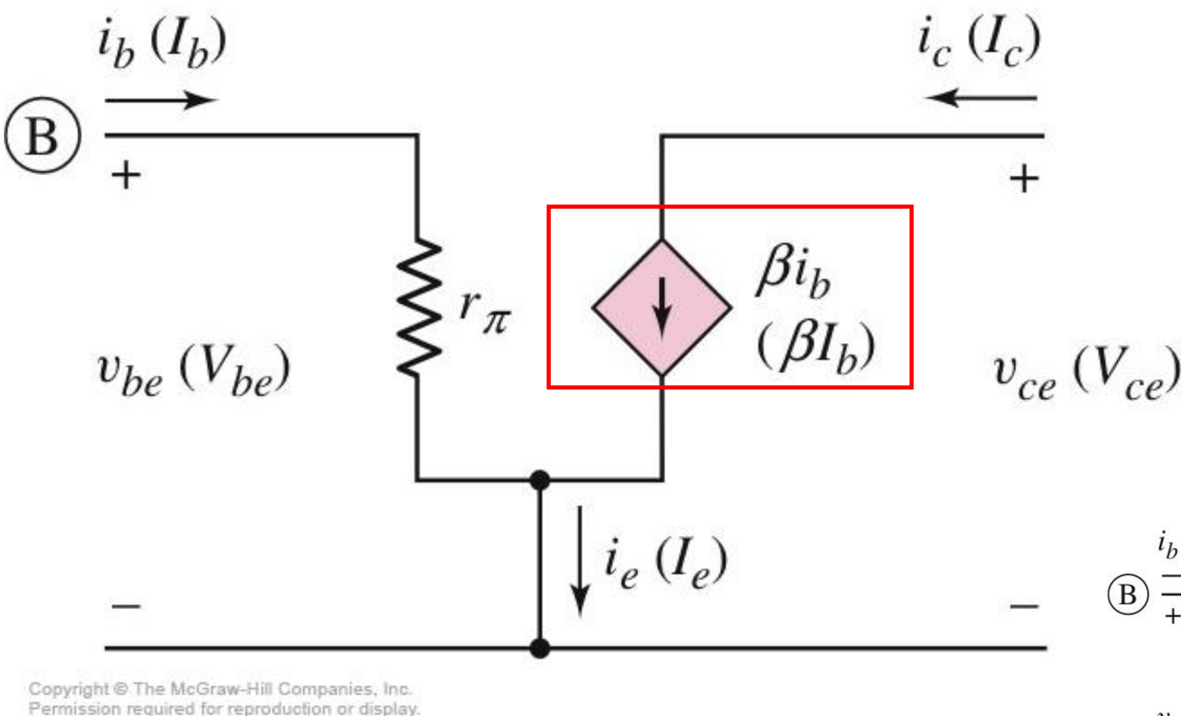
$$g_m = \frac{I_{CQ}}{V_T}$$

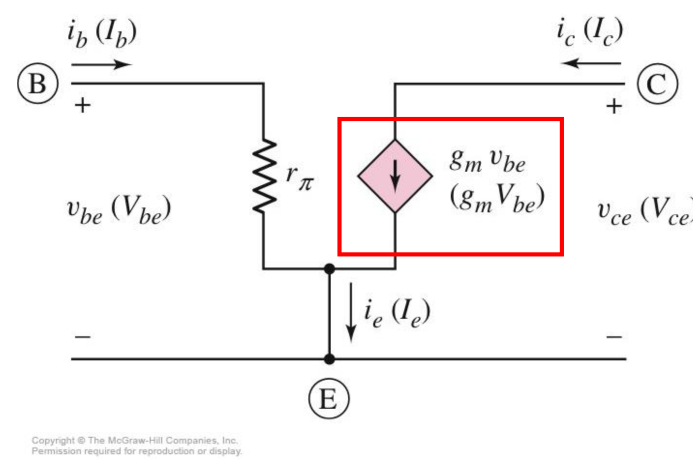
$$r_\pi = \frac{\beta V_T}{I_{CQ}}$$

$$\boxed{g_m r_\pi = \beta}$$

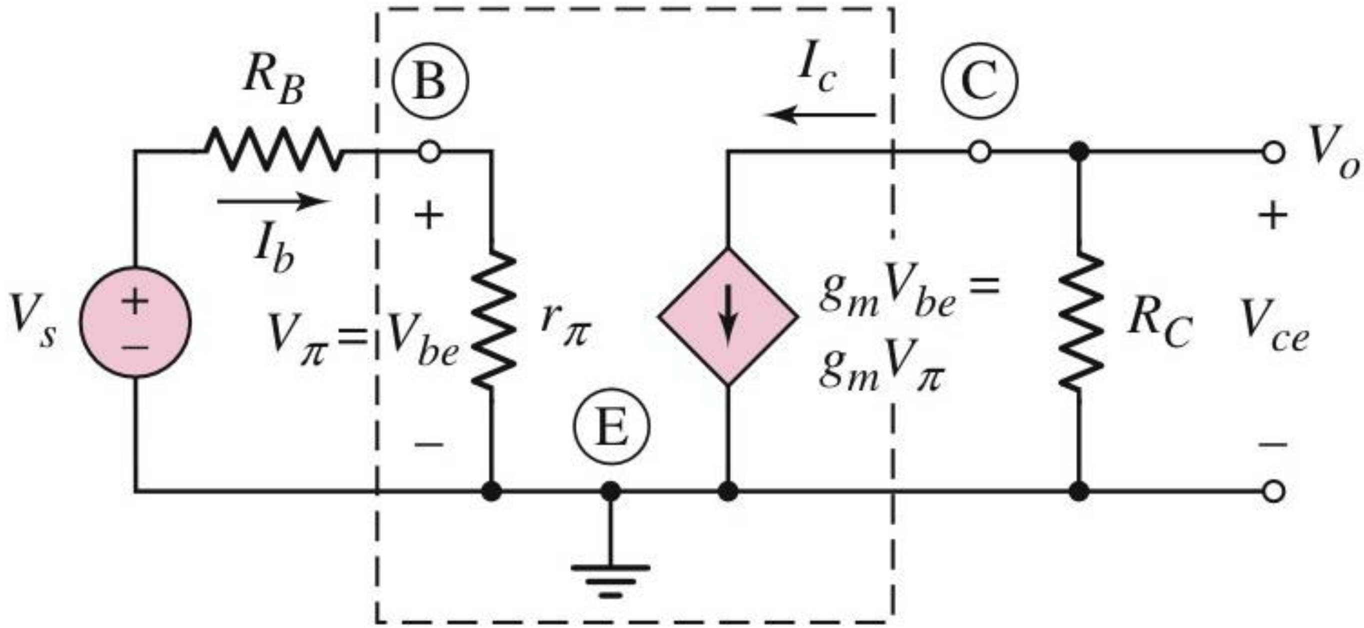
Phasor signals are shown in parentheses.

Small-Signal Equivalent Circuit Using Common-Emitter Current Gain





Small-Signal Equivalent Circuit for npn Common Emitter circuit

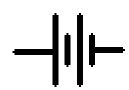
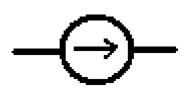


$$A_v = -(g_m R_C)\left(\frac{r_\pi}{r_\pi + R_B}\right)$$

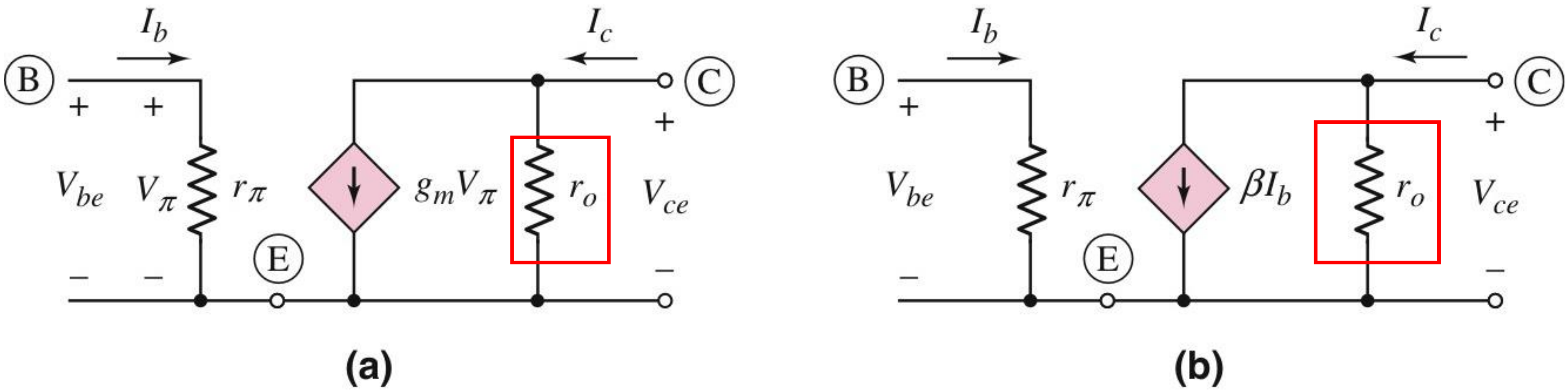
Problem-Solving Technique: BJT AC Analysis

1. Analyze circuit with only dc sources to find Q point.
2. Replace each element in circuit with small-signal model, including the hybrid π model for the transistor.
3. Analyze the small-signal equivalent circuit after setting dc source components to zero.

Transformation of Elements

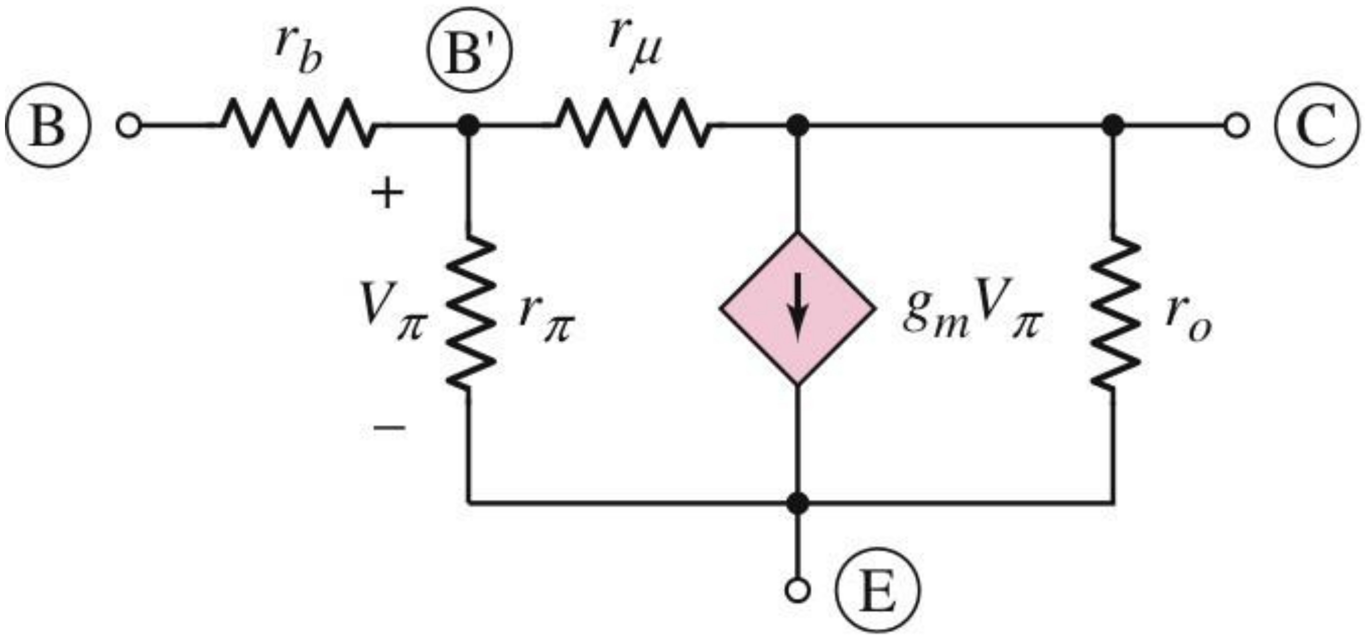
Element	DC Model	AC Model
Resistor	R	R
Capacitor	Open	C
Inductor	Short	L
Diode	$+V_\gamma$, r_f – 	$r_d = V_T/I_D$
Independent Constant Voltage Source	+ V_S - 	Short
Independent Constant Current Source	I_S 	Open

Hybrid π Model for npn with Early Effect



$$\boxed{r_o = \frac{V_A}{I_{CQ}}}$$

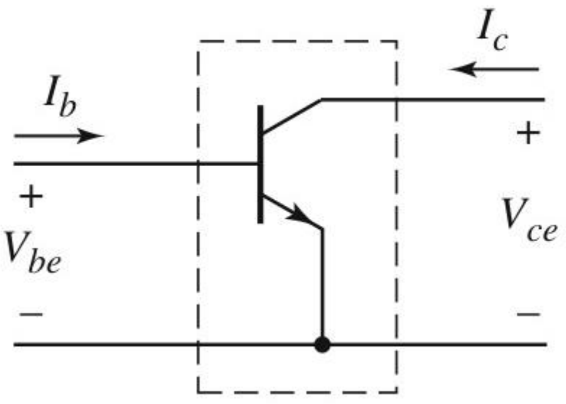
Expanded Hybrid π Model for npn



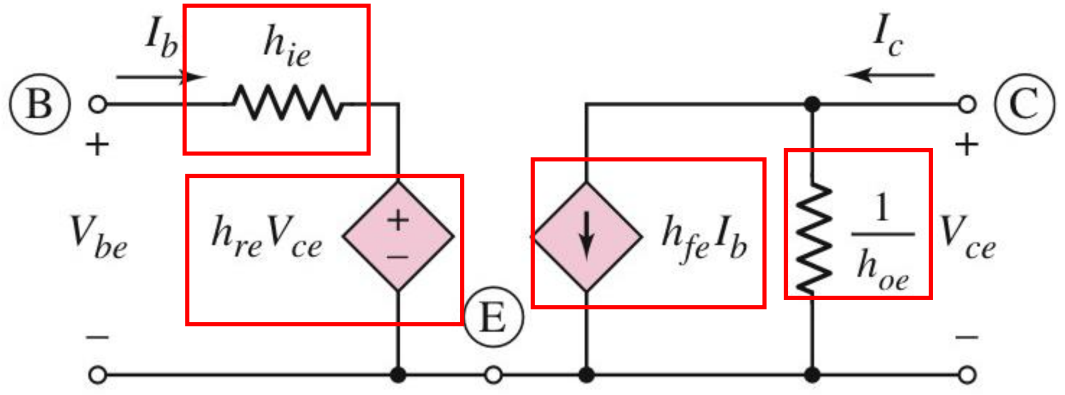
r_b is the series resistance of the semiconductor material between the external base terminal B and an idealised internal base region B'.

r_μ is the reverse-biased diffusion resistance of the base-collector junction.

h-Parameter Model for npn



(a)



(b)

$$h_{ie} = r_b + r_\pi \| r_\mu$$

$$h_{re} \cong \frac{r_\pi}{r_\mu}$$

$$h_{fe} = \beta$$

$$h_{oe} = \frac{1+\beta}{r_\mu} + \frac{1}{r_o}$$

C-E Amplifier Properties and Examples

- Common-Emitter (C-E) Amplifier Properties and Example

  - H-parameter Model

- Common-Emitter (C-E) Amplifier Properties and Example

  - Hybrid pi Model

Common-Emitter (C-E) Amplifier Properties and Example <u>(H-parameter Model)</u>

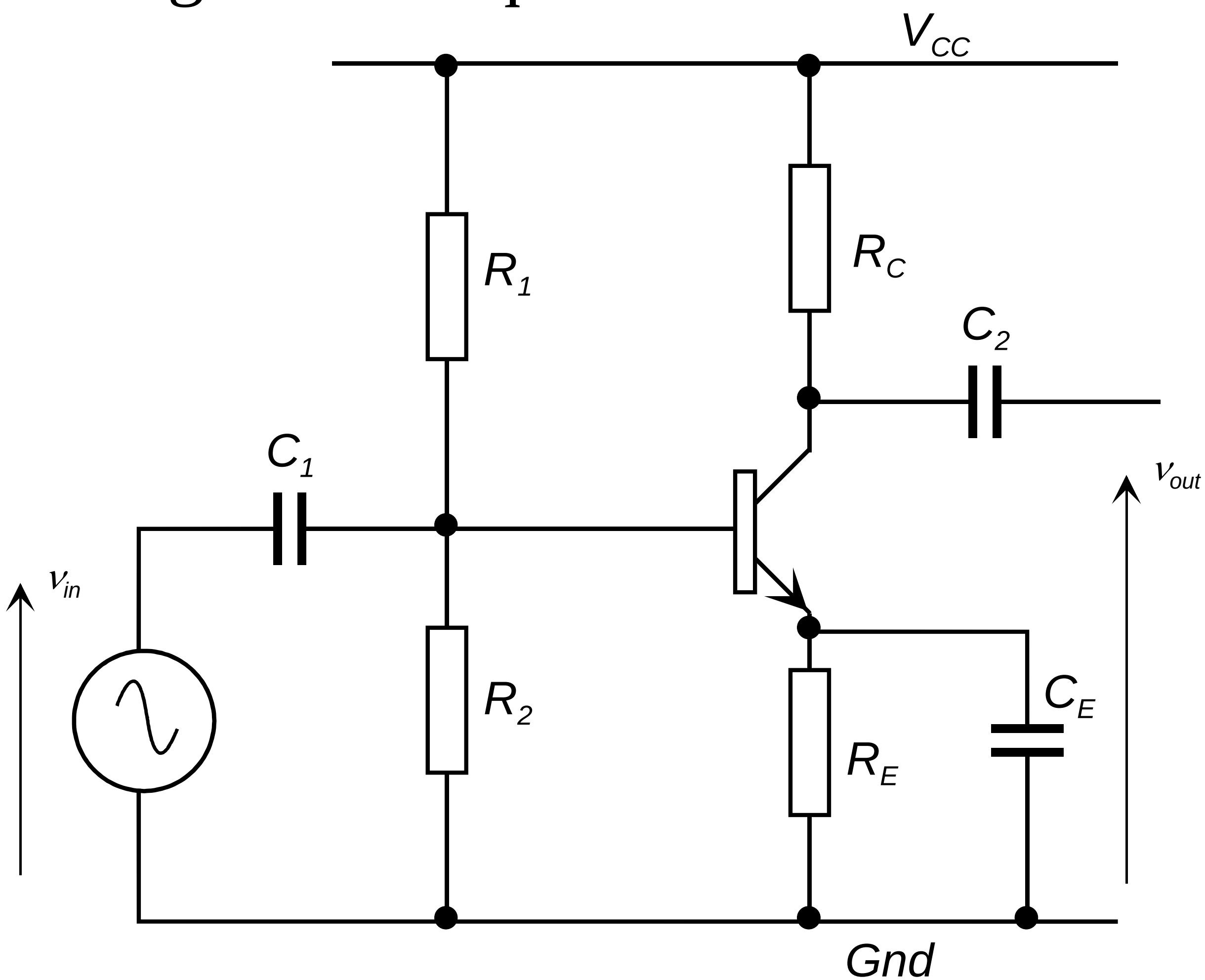
- Common Emitter Transistor Amplifier Properties

✓ Equivalent circuit
✓ Input circuit
✓ Output circuit
✓ Approximation and simplification
✓ Maximum power gain
✓ Gain in decibel

■ Appendix: Decibels and gain

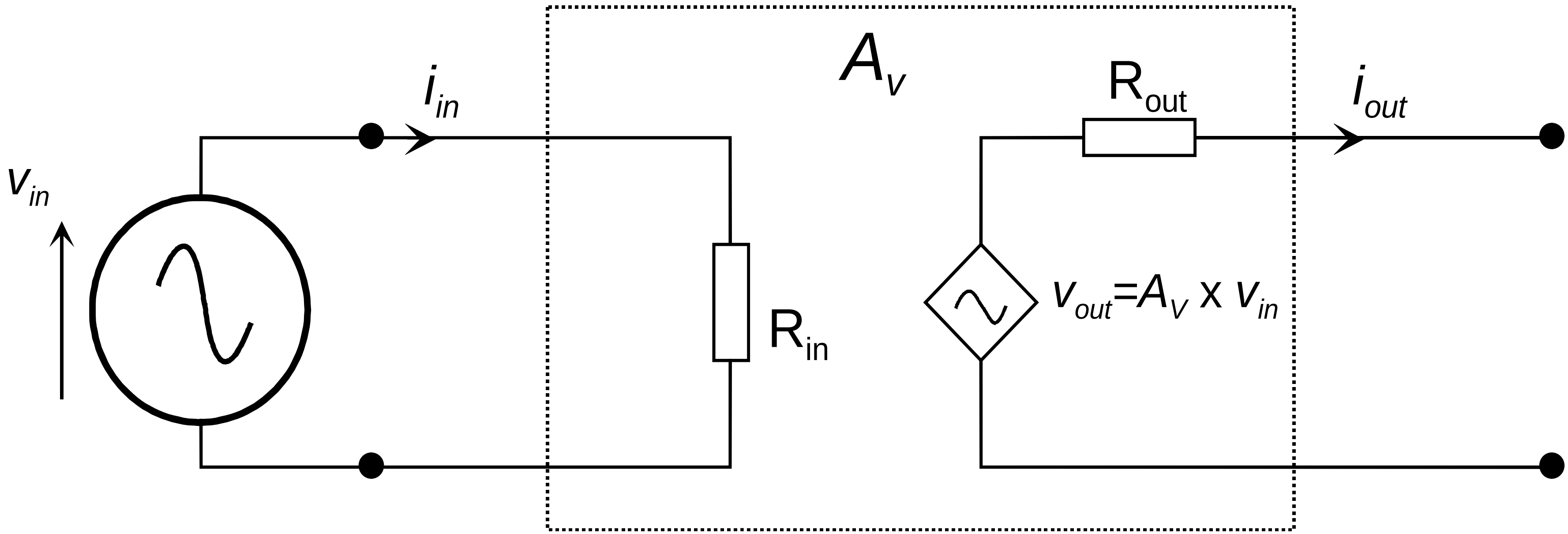
Common Emitter Transistor – Equivalent Circuit (1)

Determination of the a.c. behaviour of the common emitter amplifier using the a.c. equivalent circuit.



Common Emitter Transistor – Equivalent Circuit (2)

Any amplifier can be considered to behave as the generic amplifier although it may not do so in an exact manner.

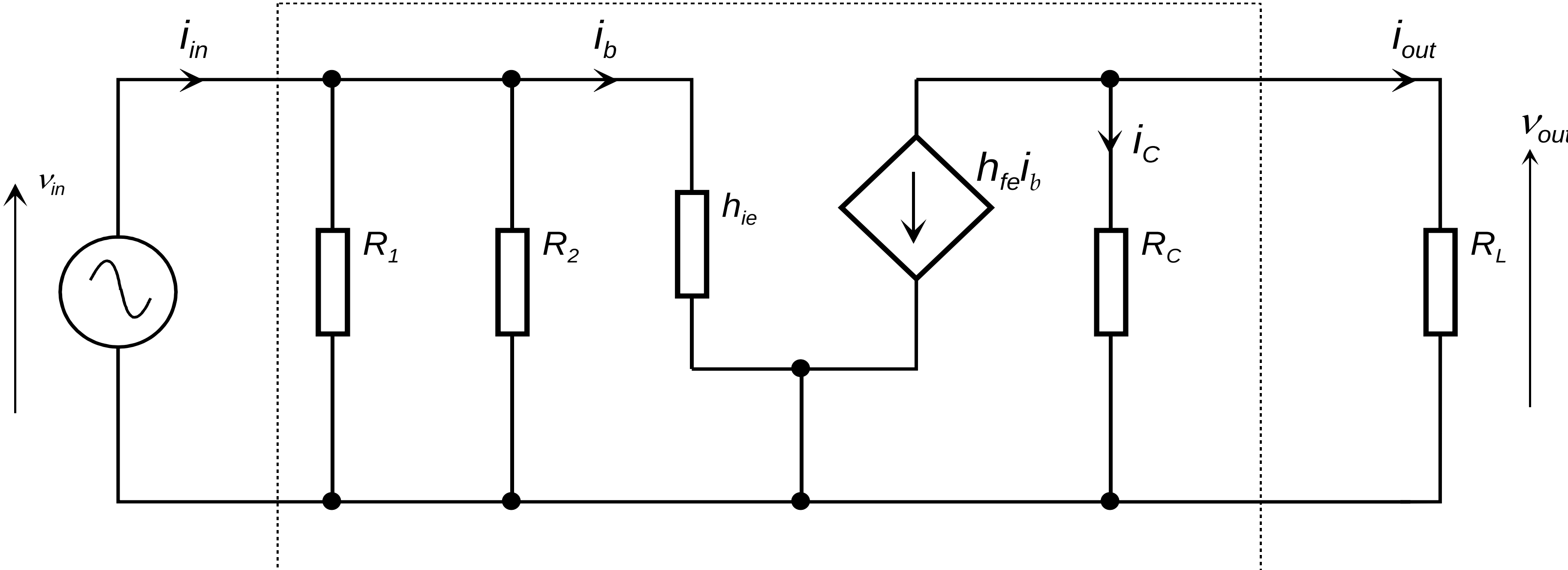


For a common emitter amplifier the primary concerns are

- Voltage gain (open circuit)
- Power gain
- Input Resistance
- Output Resistance

Common Emitter Transistor – Equivalent Circuit (3)

The common-emitter amplifier will usually have a load at the output, a resistor between the output and the common supply. At mid-frequency the capacitors are short circuits so the equivalent circuit becomes:



The voltage gain A_V (open circuit) was determined as

$$A_v = \frac{v_{out}}{v_{in}} = -h_{fe}\frac{R_C}{h_{ie}} \qquad 1.1$$

Common Emitter Transistor – Input Circuit

From equivalent circuit for an input signal ν_{in} , $i_b = \dfrac{\nu_{in}}{h_{ie}}$

R_{in} is the **input resistance** of the generic amplifier and for this circuit is R_1, R_2 and h_{ie} in parallel

$$R_{in} = \frac{R_1 R_2 h_{ie}}{\left[R_1 R_2 + h_{ie} R_1 + h_{ie} R_2\right]} \qquad 1.2$$

The **input power** is

$$P_{in} = \frac{\nu_{in}^2}{R_{in}} \qquad 1.3$$

giving

$$P_{in} = \frac{\nu_{in}^2 \left[R_1 R_2 + h_{ie} R_1 + h_{ie} R_2\right]}{R_1 R_2 h_{ie}} \qquad 1.4$$

As $\nu_{in} = i_b h_{ie}$

$$P_{in} = \frac{i_b^2 h_{ie} \left[R_1 R_2 + h_{ie} R_1 + h_{ie} R_2\right]}{R_1 R_2} \qquad 1.5$$

Common Emitter Transistor – Output Circuit

The **output voltage** is $v_{out} = i_{out} R_L$ 1.6

Use Kirchoff's current law at the collector node (with directions shown) and Ohm's Law to obtain

$$\nu_{out} = \frac{-h_{fe} i_b R_c R_L}{R_C + R_L} \qquad 1.7$$

Output power is $P_{out} = \frac{\nu_{out}{}^2}{R_L} = \frac{h_{fe}^2 i_b{}^2 R_C{}^2 R_L}{(R_C + R_L)^2}$ 1.8

Power gain is $A_p = \frac{P_{out}}{P_{in}} = \frac{h_{fe}^2 R_C^2 R_L}{(R_C + R_L)^2} \frac{R_1 R_2}{h_{ie}[R_1 R_2 + R_1 h_{ie} + R_2 h_{ie}]}$ 1.9

Common Emitter Transistor – Approximation and Simplification (1)

This is complicated but allows consideration of choices necessary to design an amplifier for a specified purpose. Except for choice of transistor the values of h_{ie} and h_{fe} are fixed. R_C and R_L are usually set by the application requirements. So for high power gain it is necessary to have the term

$$\frac{R_1 R_2}{\left[R_1 R_2 + R_1 h_{ie} + R_2 h_{ie}\right]}$$

as large as possible. It is always less than 1 but can be made close to 1 if

$R_1 R_2 >> R_1 h_{ie}$ and $R_1 R_2 >> R_2 h_{ie}$, that is $R_2 >> h_{ie}$ and $R_1 >> h_{ie}$

(Earlier R_1 and R_2 were required to be small enough for the current through them to be much greater than I_B – an upper limit. This new requirement gives a lower limit for *'small'*).

Common Emitter Transistor – Approximation and Simplification (2)

If the additional inequalities are met then $\dfrac{R_1R_2}{h_{ie}\left[R_1R_2 + R_1h_{ie} + R_2h_{ie}\right]} \approx \dfrac{1}{h_{ie}}$

h_{ie} is usually of order 1k for a bipolar transistor and it is usual to choose $R_1 + R_2$ in the 10k to 500k range.

With the approximations

$$A_p = \frac{P_{out}}{P_{in}} = \frac{h_{fe}^2 R_C^2 R_L}{\left(R_C + R_L\right)^2 h_{ie}} \qquad 1.10$$

and the input resistance reduces to $R_{in} \approx h_{ie}$

Common Emitter Transistor – Output Resistance (1)

A general approach to determining the output resistance R_o of the equivalent generic amplifier commonly uses one of three methods:

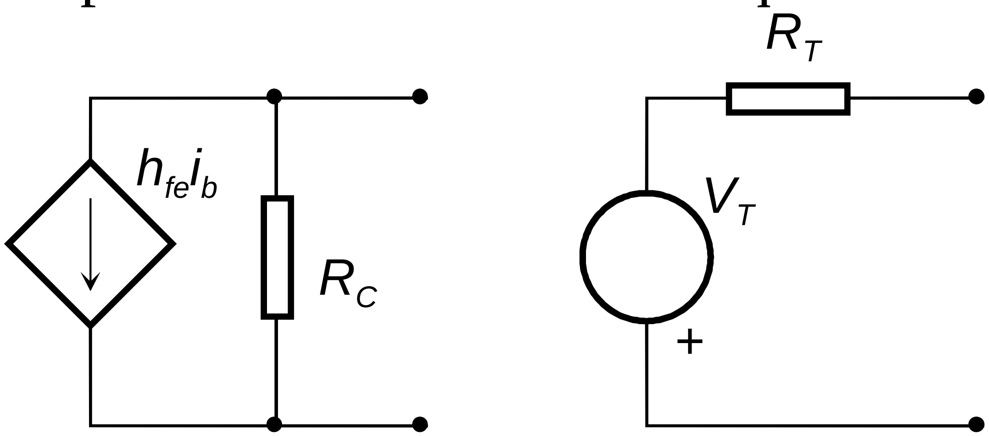
1. $$\frac{\textit{open circuit output voltage}}{\textit{short circuit output current}} = \frac{v_{oc}}{i_{sc}}$$

2. short circuit the input, connect a supply at the output and measure the current flowing into the amplifier at the output.

3. derive the ratio of a change in output voltage to the change in output current for a small change in applied load

Common Emitter Transistor – Output Resistance (2)

However for the common emitter amplifier with capacitor C_E present the equivalent circuit leads to a very simple evaluation. The output circuit of the amplifier and the Thévenin equivalent are



(a) (b)

R_T is the output resistance of the circuit. Remembering Thévenin's and Norton's Theorems then Thévenin and Norton resistances have the same value. Therefore

$$R_{out} = R_T = R_C \quad 1.11$$

Also

$$V_T = h_{fe} i_b R_C \quad 1.12$$

Common Emitter Transistor – Summary of Results

Initially it was stated that for a common emitter amplifier the primary concerns are

Voltage gain (open circuit)
Power gain
Input Resistance
Output Resistance

The results are

$$A_v = \frac{\nu_{out}}{\nu_{in}} = -h_{fe}\frac{R_C}{h_{ie}} \qquad 1.13$$

$$A_p = \frac{P_{out}}{P_{in}} \approx \frac{h_{fe}^2 R_C^2 R_L}{(R_C + R_L)^2 h_{ie}} \qquad 1.14$$

$$R_{in} = \frac{R_1 R_2 h_{ie}}{[R_1 R_2 + h_{ie} R_1 + h_{ie} R_2]} \approx h_{ie} \qquad 1.15$$

$$R_{out} = R_C \qquad 1.16$$

Common Emitter Transistor – Maximum Power Gain

Maximum power transfer to a load (examined in circuit theory) is when the Thévenin (or Norton) resistance equals the load resistance. Hence maximum power in the load requires $R_L = R_C$ It also requires $R_1 >> h_{ie}$ and $R_2 >> h_{ie}$ as these give high power gain (the approximations made are better and less signal power is lost in these resistors).

If these inequalities hold $R_{in} \cong h_{ie}$ and if also $R_L = R_C$

$$A_{pmax} = \frac{h_{fe}^2 R_C}{4h_{ie}} = \frac{h_{fe}^2 R_L}{4h_{ie}} \qquad 1.17$$

If R_L is similar to h_{ie} (around 1k), power gain is about $\frac{h_{fe}^2}{4}$ typically around 2000.

C-E Amplifier Properties
- H-parameter Model

Voltage gain:

$$A_v = -h_{fe}\frac{R_C}{h_{ie}}$$

Output Resistance:

$$R_{out} = R_C$$

Input Resistance:

$$R_{in} = \frac{R_1 R_2 h_{ie}}{\left[R_1 R_2 + h_{ie} R_1 + h_{ie} R_2\right]}$$

Approximation:

$$R_{in} \approx h_{ie}$$

Power Gain:

$$A_p = \frac{h_{fe}^2 R_C^2 R_L}{(R_C + R_L)^2}\frac{R_1 R_2}{h_{ie}\left[R_1 R_2 + R_1 h_{ie} + R_2 h_{ie}\right]}$$

Approximation:

$$A_p \approx \frac{h_{fe}^2 R_C^2 R_L}{(R_C + R_L)^2 h_{ie}}$$

* Assume $R_2 >> h_{ie}$ and $R_1 >> h_{ie}$, $R_1 + R_2$ in the 10k to 500k range. h_{ie} is typically 1k.

Common Emitter Transistor – Gain in Decibel (1)

Decibel Gain - Commonly gain is expressed in decibels (**a note on dBs is in Appendix**)

$$G_{dB} = 10\,log_{10}\left(\frac{P_{out}}{P_{in}}\right) = 10log_{10}\left(A_p\right) \qquad 1.18$$

If the gain is 2000 then $G_{dB} = 10\,log_{10}\left(2000\right) \cong 33dB$

Often voltage gain is expressed in dBs – strictly this is wrong as dBs are a measure of a ratio of two powers. The power gain may be written

$$A_P = \frac{P_{out}}{P_{in}} = \frac{\nu_{out}^2}{R_{out}}\frac{R_{in}}{\nu_{in}^2} = \left(\frac{\nu_{out}}{\nu_{in}}\right)^2 \frac{R_{in}}{R_{out}}$$

Common Emitter Transistor – Gain in Decibel (2)

If it is assumed that

$$R_{in} = R_{out}$$ (*<u>not true in most cases</u>* – see 1.15 and 1.16)

then

$$G_{dB} = 10\,log_{10}\left(\left(\frac{\nu_{out}}{\nu_{in}}\right)^2\right) = 20log_{10}\left(\frac{\nu_{out}}{\nu_{in}}\right)$$

$$G_{dB} = 20\,log_{10}\left(\left|A_V\right|\right) \qquad 1.19$$

where

$$A_V = \frac{\nu_{out}}{\nu_{in}} = \text{ the voltage gain}$$

Common Emitter Transistor – Gain in Decibel (3)

It is common to express amplifier voltage gain in decibels as in 1.19 even although **this is not strictly correct**. For example for the amplifier being examined in 1.1 gave

$$A_v = \frac{\nu_{out}}{\nu_{in}} = -h_{fe}\frac{R_C}{h_{ie}} \qquad 1.1$$

If $R_C = h_{ie}$ and $h_{fe} = 100$ then $A_V = h_{fe}$ and

$$G_{dB} = 20\,log_{10}(100) = 20 \times 2 = 40dB$$

Reminder: previous value for power gain was 33dB

Calculating voltage gain this way gives a value in dB, which is useful but is not the power gain.

Common-Emitter (C-E) Amplifier Properties and Example - Hybrid pi Model

- Common Emitter Amplifier Circuit Hybrid Pi Model

- ✓ Basic common emitter amplifier circuit
- ✓ Small signal equivalent circuit
- ✓ Example – 1.1
- ✓ Example – 1.2
- ✓ Circuit with emitter resistor
- ✓ Example – 1.3
- ✓ Example – 1.4

- Basic Common-Emitter Circuit

Figure below shows the basic common-emitter circuit with voltage-divider biasing. The signal from the signal source is coupled into the base of the transistor through the **coupling capacitor Cc**, which provides **dc isolation** between the amplifier and the signal source. The **dc transistor biasing** is establishing by **R_1** and **R_2**, and is not disturbed when the signal source is capacitively coupled to the amplifier.

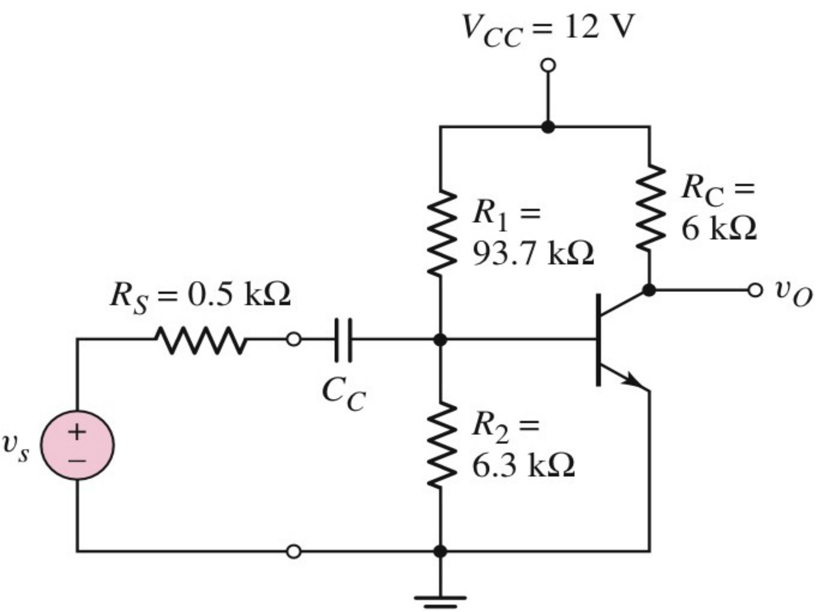


Figure 1.1: A common-emitter circuit with a voltage-divider biasing circuit and a coupling capacitor

Basic Common-Emitter Amplifier Circuit Pi Model

- Small-signal Equivalent Circuit

If the signal source is a sinusoidal voltage at frequency f , then the magnitude of the capacitor impedance is

$$|Z_c| = \frac{1}{2\pi f\, Cc} \qquad (1.1)$$

The small-signal equivalent circuit in which the coupling capacitor is assumed to be a **short circuit** is shown in Figure 1.2.

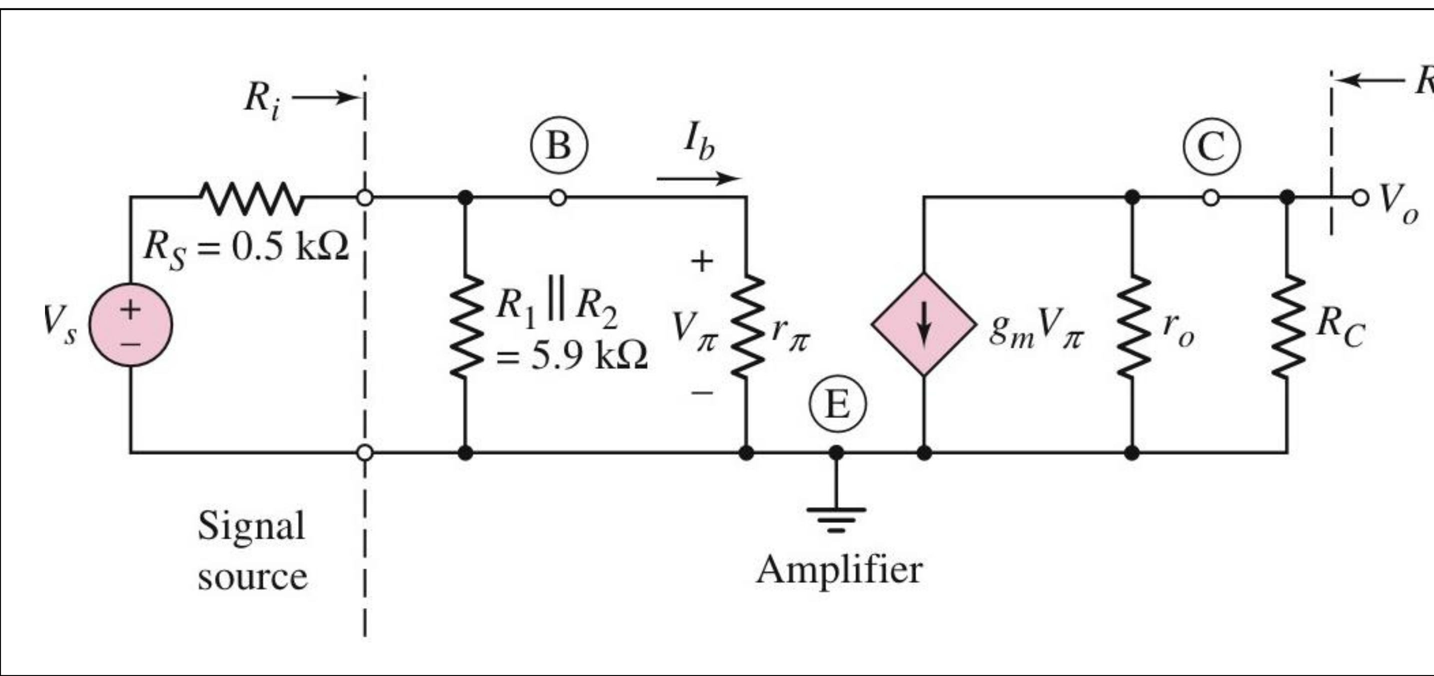
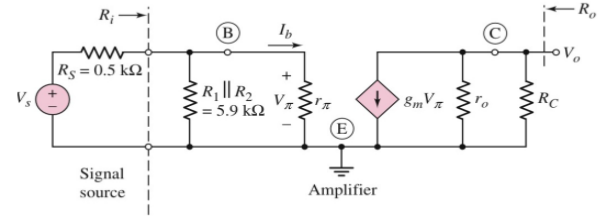


Figure 1.2

Example - 1.1



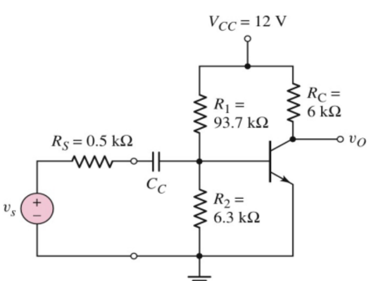
Determine the **small-signal voltage gain**, **input resistance**, and **output resistance** of the circuit shown in figure 1.1.

Assume the transistor parameters are : $\beta = 100, V_{BE}(on) = 0.7V$, and $V_A = 100V$

DC solution is given for this example: $I_{CQ} = 0.95mA$ and $V_{CEQ} = 6.31V$

AC solution:

The small-signal hybrid-π parameters for the equivalent circuit are



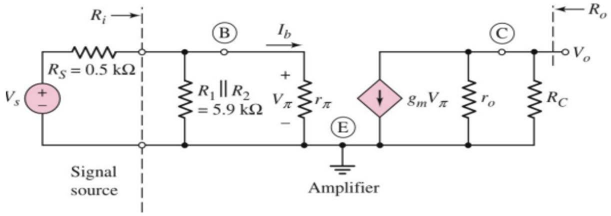
$$r_\pi = \frac{V_T \beta}{I_{CQ}} = \frac{(0.026)(100)}{(0.95)} = 2.74K\Omega$$

$$g_m = |I_c|(mA)/0.026$$

$$r_\pi = h_{fe}/g_m \qquad r_\mu \approx r_\pi / h_{re}$$

$$r_b = h_{ie} - r_\pi$$

$$1/r_o = h_{oe} - (1/r_\mu)(1 + h_{fe})$$



$$g_m = \frac{I_{CQ}}{V_T} = \frac{(0.95)}{(0.026)} = 36.5\text{mA/V}$$

and

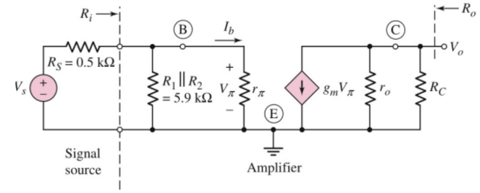
$$r_o = \frac{V_A}{I_{CQ}} = \frac{(100)}{(0.95)} = 105K\Omega$$

Assuming that Cc acts as short circuit, figure 1.2 shows the small-signal equivalent circuit. The small-signal output voltage is

$$V_o = -(g_m V_\pi)(r_o \,//\, R_c)$$

The dependent current $g_m V_\pi$ flows through the parallel combination of r_o and R_C, but in a direction that produces a negative output voltage. We can relate the control voltage V_π to the input voltage V_S by a voltage divider, we have

$$V_\pi = \left(\frac{R_1 \,//\, R_2 \,//\, r_\pi}{R_1 \,//\, R_2 \,//\, r_\pi + R_S}\right).V_S$$



We can then write the **small-signal voltage gain** as

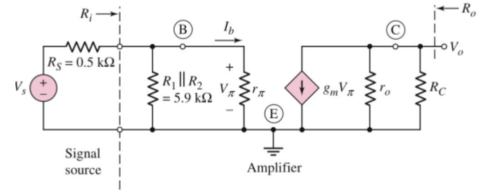
$$A_v = \frac{V_o}{V_S} = -g_m\left(\frac{R_1 // R_2 // r_\pi}{R_1 // R_2 // r_\pi + R_S}\right)\left(r_o // R_C\right)$$

or

$$A_v = -(36.5)\left(\frac{5.9 // 2.74}{5.9 // 2.74 + 0.5}\right)(105 // 6) = -163$$

We can also calculate R_i , which is the **resistance to the amplifier**. From figure 2.2, we see that

$$R_i = R_1 // R_2 // r_\pi = 5.9 // 2.74 = 1.87K\Omega$$



The output resistance R_o is found by setting the independent source V_S equal to zero. In this case, there is no excitation to the input portion of the circuit so $V_\pi = 0$, which implies that $g_m V_\pi = 0$ (am open circuit). The **output resistance** looking back into the output terminals is then

$$R_o = r_o // R_C = 105 // 6 = 5.68 K\Omega$$

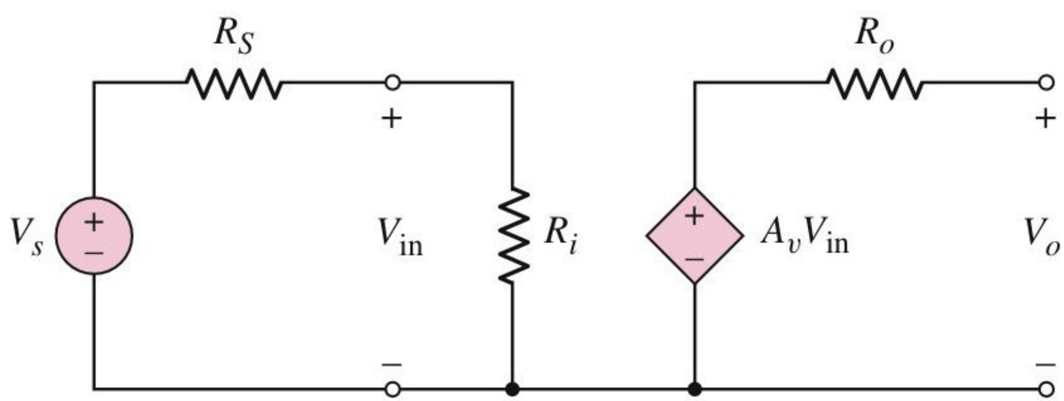
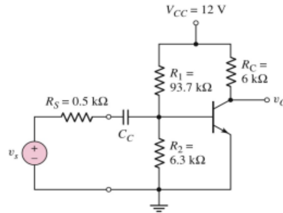


Figure 1.3: two-port equivalent circuit for the amplifier

Example - 1.2



The circuit parameters in figure 2.1 are changed to V_{cc}=5V, R_1= 35.2 Kohms, R_2= 5.83 Kohms, R_c= 10 Kohms, R_s=0. assume the transistor parameter are same as listed in example 1.1. Determine the **quiescent collector current** and **collector-emitter voltage**, and find the **small-signal voltage gain**.

$$R_{TH} = R_1 \| R_2 = 35.2 \| 5.83 = 5\ k\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right) \cdot V_{CC} = \left(\frac{5.83}{5.83 + 35.2}\right)(5)$$

or

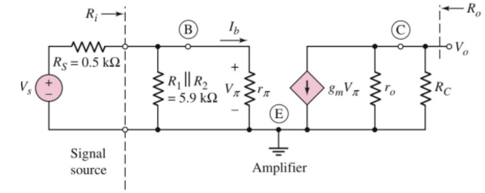
$$V_{TH} = 0.7105\ V$$

Then

$$I_{BQ} = \frac{V_{TH} - V_{BE}(on)}{R_{TH}} = \frac{0.7105 - 0.7}{5}$$

or

$$I_{BQ} = 2.1\ \mu A$$



and

$$I_{CQ} = \beta I_{BQ} = (100)(2.1\ \mu A) = 0.21\ mA$$

$$V_{CEQ} = V_{CC} - I_{CQ} R_C = 5 - (0.21)(10)$$

and

$$V_{CEQ} = 2.9\ V$$

Now

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.21}{0.026} = 8.08\ mA$$

$$r_o = \frac{V_A}{I_{CQ}} = \frac{100}{0.21} = 476\ k\Omega$$

And

$$A_v = -g_m (r_o \| R_C) = -(8.08)(476 \| 10)$$

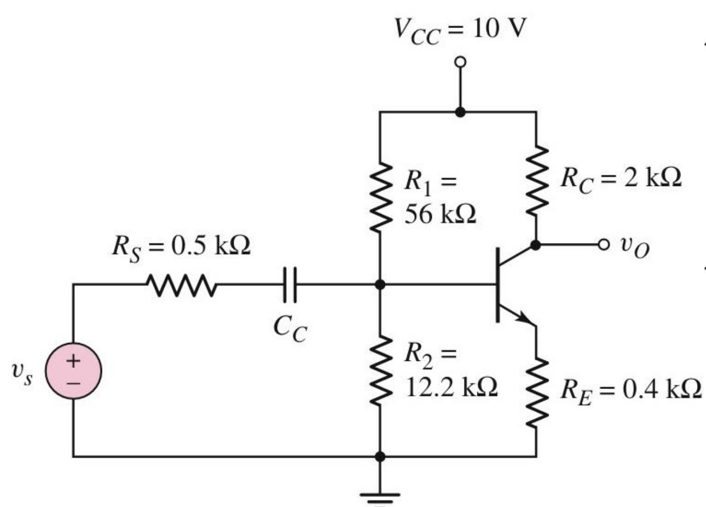
so

$$A_v = -79.1$$

- Circuit with Emitter Resistor (1)

For the circuit in figure 1.1, the bias resistors R1 and R2 in conjunction with if $V_{BE} = 0.7V$, then $i_B = 9.5\mu A$ and $i_C = 0.95mA$.

But if changed to $V_{BE} = 0.6V$, then $i_B = 26\mu A$, which is sufficient to drive the transistor into **saturation**. Therefore, the circuit shown in figure 1.1 is not practical. An improved **dc biasing design** includes an emitter resistor.



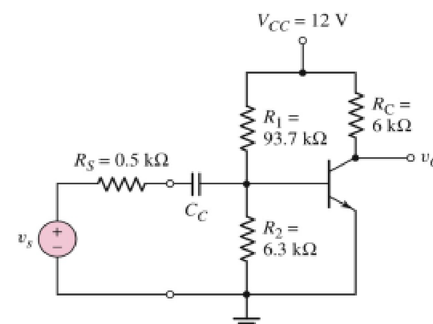


Figure 1.4: An npn common-emitter circuit with an emitter transistor

Basic Common-Emitter Amplifier Circuit Pi Model

- Circuit with Emitter Resistor (2)

Figure below shows the **small-signal hybrid-pi equivalent circuit** (three terminals of the transistor).

Sketch the hybrid-pi equivalent circuit between the three terminals and then sketch in the remaining circuit elements around these terminals.

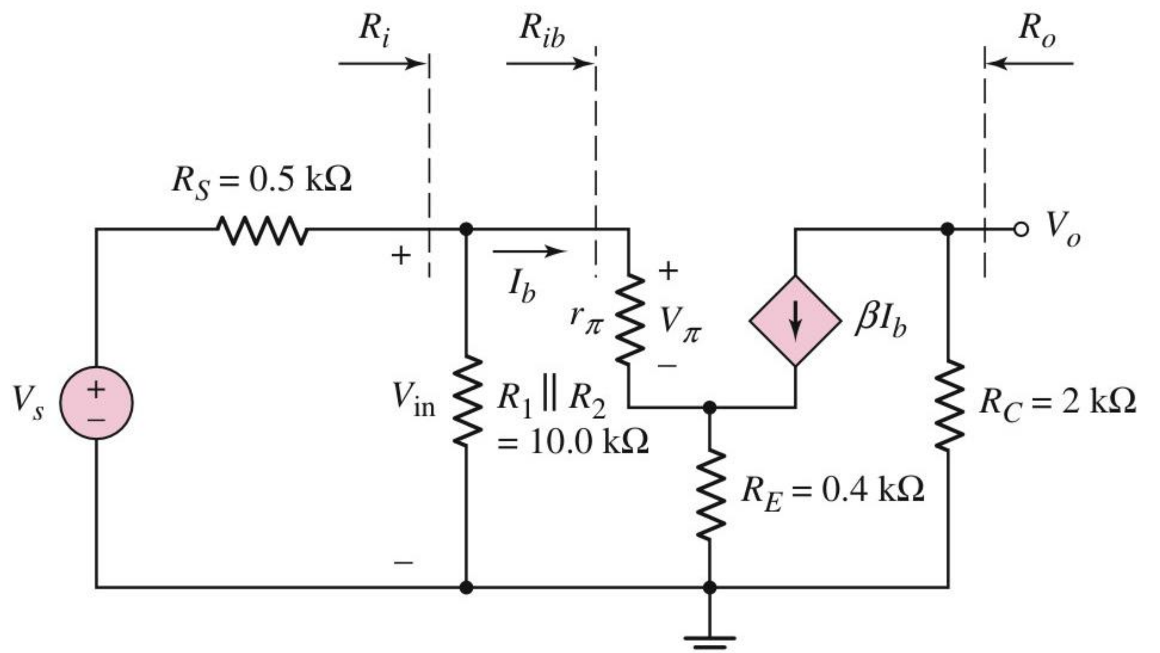


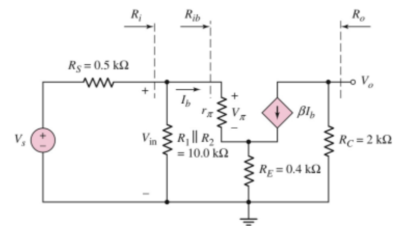
Figure 1.5: The small-signal equivalent circuit of the circuit shown in figure 1.4

Basic Common-Emitter Amplifier Circuit Pi Model

- Circuit with Emitter Resistor (3)

In this case, we are using the equivalent circuit with the **current gain parameter** β , and we are assuming that the **Early voltage is infinite** so the transistor **output resistance** r_o can be neglected (an open circuit).

The a.c. outpt voltage is



$$V_o = -(\beta I_b)R_C \qquad (1.2)$$

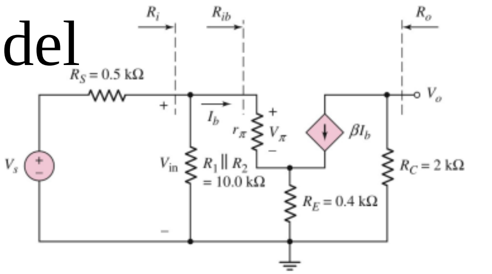
To find the **small-signal voltage gain**, it is worthwhile finding the input resistance first (R_{ib}) . We can write the following loop equation

$$V_{in} = I_b r_\pi + (I_b + \beta I_b)R_E \qquad (1.3)$$

The **input resistance** R_{ib} is then defined as, and found to be,

$$R_{ib} = \frac{V_{in}}{I_b} = r_\pi + (1+\beta)R_E \qquad (1.4)$$

Basic Common-Emitter Amplifier Circuit Pi Model



- Circuit with Emitter Resistor (4)

The **input resistance to the amplifier** is now

$$R_i = R_1 // R_2 // R_{ib} \qquad (1.5)$$

We can again relate V_{in} to V_s through a **voltage-divider** equation as

$$V_{in} = \left(\frac{R_i}{R_i + R_S} \right) . V_S \qquad (1.6)$$

Combining Equations (1.2), (1.4),and (1.6), we find the **small-signal voltage gain** is

$$A_v = \frac{V_o}{V_S} = -\frac{(\beta I_b) R_C}{V_S} = -\beta R_C \left(\frac{V_{in}}{R_{ib}} \right) . \left(\frac{1}{V_S} \right) \qquad (1.7a)$$

or

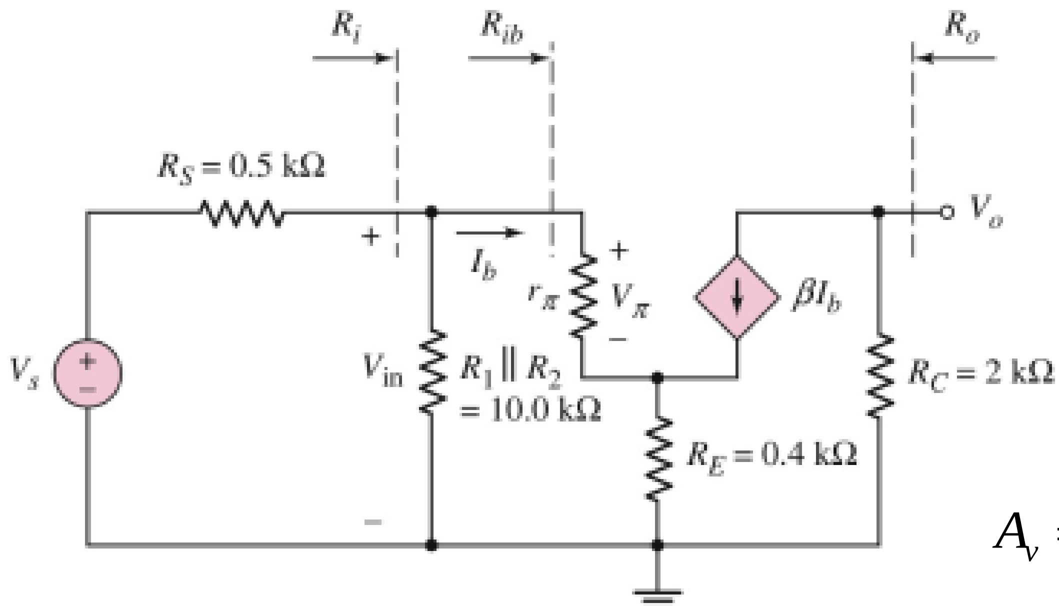
$$A_v = \frac{-\beta R_C}{r_\pi + (1+\beta) R_E} \left(\frac{R_i}{R_i + R_s} \right) \qquad (1.7b)$$

Basic Common-Emitter Amplifier Circuit Pi Model

- Circuit with Emitter Resistor (5)

From this equation, we see that if $R_i >> R_s$ and if $(1+\beta)R_E >> r_\pi$, then the small-signal voltage gain is approximately

$$A_v \cong \frac{-\beta R_C}{(1+\beta)R_E} \cong \frac{-R_C}{R_E} \qquad (1.8)$$



$$A_v = \frac{-\beta R_C}{r_\pi + (1+\beta)R_E}\left(\frac{R_i}{R_i + R_s}\right)$$

Example - 1.3

Determine the **small-signal voltage gain** and **input resistance** of a common-emitter circuit with an emitter resistor.

Assume the transistor parameters are : $\beta = 100, \mathrm{V_{BE}}(\mathrm{on}) = 0.7\mathrm{V}$, and $\mathrm{V_A} = \infty$

DC solution:

From a dc analysis of the circuit, we determine that $I_{CQ} = 2.16\mathrm{mA}$ and $V_{CEQ} = 4.81\mathrm{V}$, which shows that the transistor is biased in the forward-active mode

AC solution:

The small-signal hybrid-π parameters for the equivalent circuit are

$$r_\pi = \frac{V_T \beta}{I_{CQ}} = \frac{(0.026)(100)}{(2.16)} = 1.2K\Omega$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{2.16}{(0.026)} = 83.1\text{mA/V}$$

and

$$r_o = \frac{V_A}{I_{CQ}} = \infty$$

The input resistance to be base can be determined as

$$R_{ib} = r_\pi + (1+\beta)R_E = 1.20 + (101)(0.4) = 41.6K\Omega$$

And the input resistance to be amplifier is now found to be

$$R_i = R_1 // R_2 // R_{ib} = 10 // 41.6 = 8.06K\Omega$$

Using the exact expression for the voltage gain, we find

$$A_v = \frac{-(100)(2)}{1.20+(101)(0.4)}\left(\frac{8.06}{8.06+0.5}\right) = -4.53$$

If we use the approximation given by equation (1.8), we obtain

$$A_v = \frac{-R_C}{R_E} = \frac{-2}{0.4} = -5$$

Example - 1.4

For the circuit in figure 1.6 , let R_E=0.6Kohms, R_C=5.6Kohms, R_1= 250 Kohms, R_2= 75 Kohms, $V_{BE}(on)$= 0.7 V, and $\beta = 120$.

(a)For $V_A = \infty$, determine the input resistance looking into the base of the transistor and determine the small-signal voltage gain.

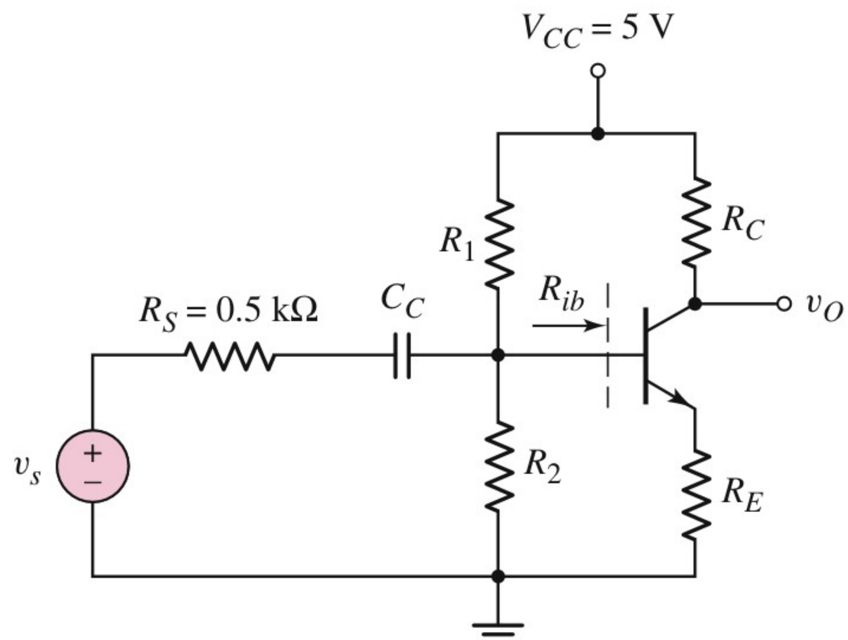


Figure 1.6

$$R_{TH} = R_1 \parallel R_2 = 250 \parallel 75 = 57.7\ k\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right)(V_{CC}) = \left(\frac{75}{75 + 250}\right)(5)$$

or

$$V_{TH} = 1.154\ V$$

$$I_{BQ} = \frac{V_{TH} - V_{BE}(on)}{R_{TH} + (1 + \beta) R_E}$$

or

$$I_{BQ} = 3.48\ \mu A$$

$$I_{CQ} = \beta I_{BQ} = (120)(3.38\ \mu A) = 0.418\ mA$$

(a)

Now

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.418}{0.026} = 16.08\, mA/V$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(120)(0.026)}{0.418} = 7.46\, k\Omega$$

We have

$$V_o = -g_m V_\pi R_C$$

We find

$$R_{ib} = r_\pi + (1+\beta) R_E = 7.46 + (121)(0.6)$$

or

$$R_{ib} = 80.1\ k\Omega$$

Also

$$R_1 \| R_2 = 250 \| 75 = 57.7\ k\Omega$$

$$R_1 \| R_2 \| R_{ib} = 57.7 \| 80.1 = 33.54\ k\Omega$$

We find

$$V_s' = \left(\frac{R_1 \parallel R_2 \parallel R_{ib}}{R_1 \parallel R_2 \parallel R_{ib} + R_S} \right) \cdot V_s = \left(\frac{33.54}{33.54 + 0.5} \right) \cdot V_s$$

or

$$V_s' = (0.985) V_s$$

Now

$$V_s' = V_\pi \left[1 + \left(\frac{1+\beta}{r_\pi} \right) R_E \right] = V_\pi \left[1 + \left(\frac{121}{7.46} \right) (0.6) \right]$$

or

$$V_\pi = (0.0932) V_s' = (0.0932)(0.985) V_s$$

So

$$A_v = \frac{V_o}{V_s} = -(16.08)(0.0932)(0.985)(5.6)$$

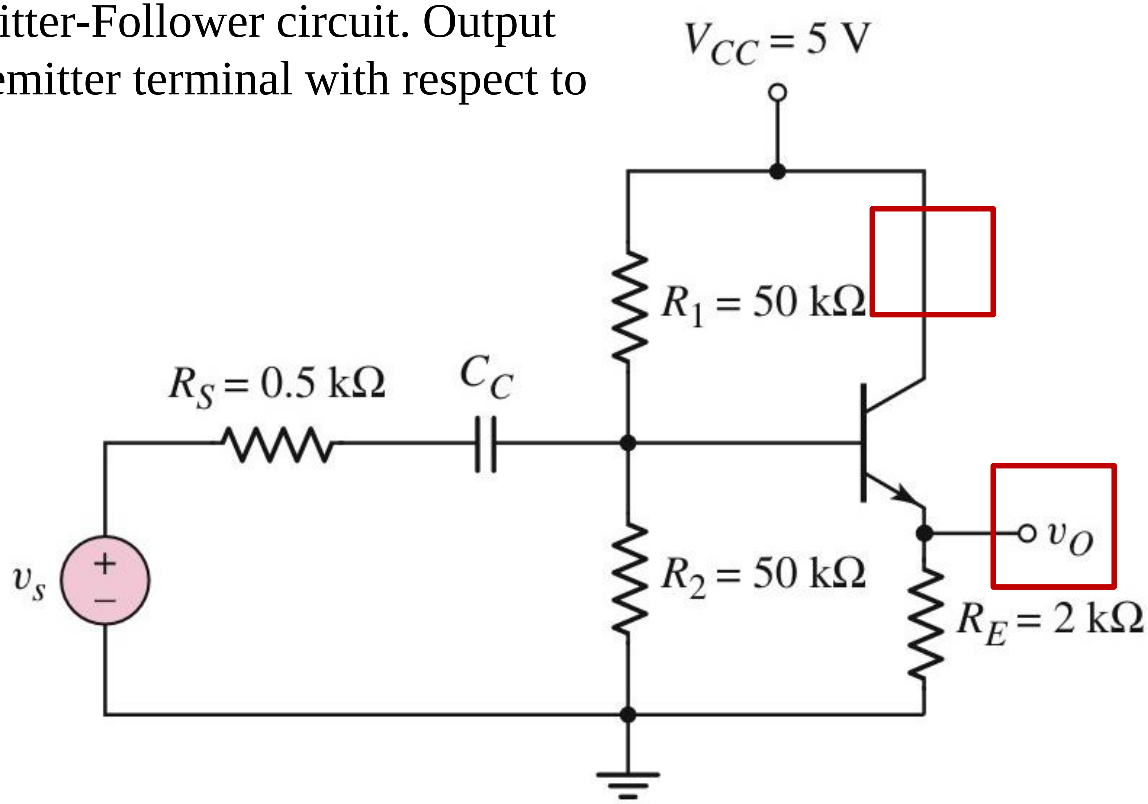
or

$$A_v = -8.27$$

Analyse the Common-Collector (C-C) Amplifier

Common-Collector or Emitter-Follower Amplifier

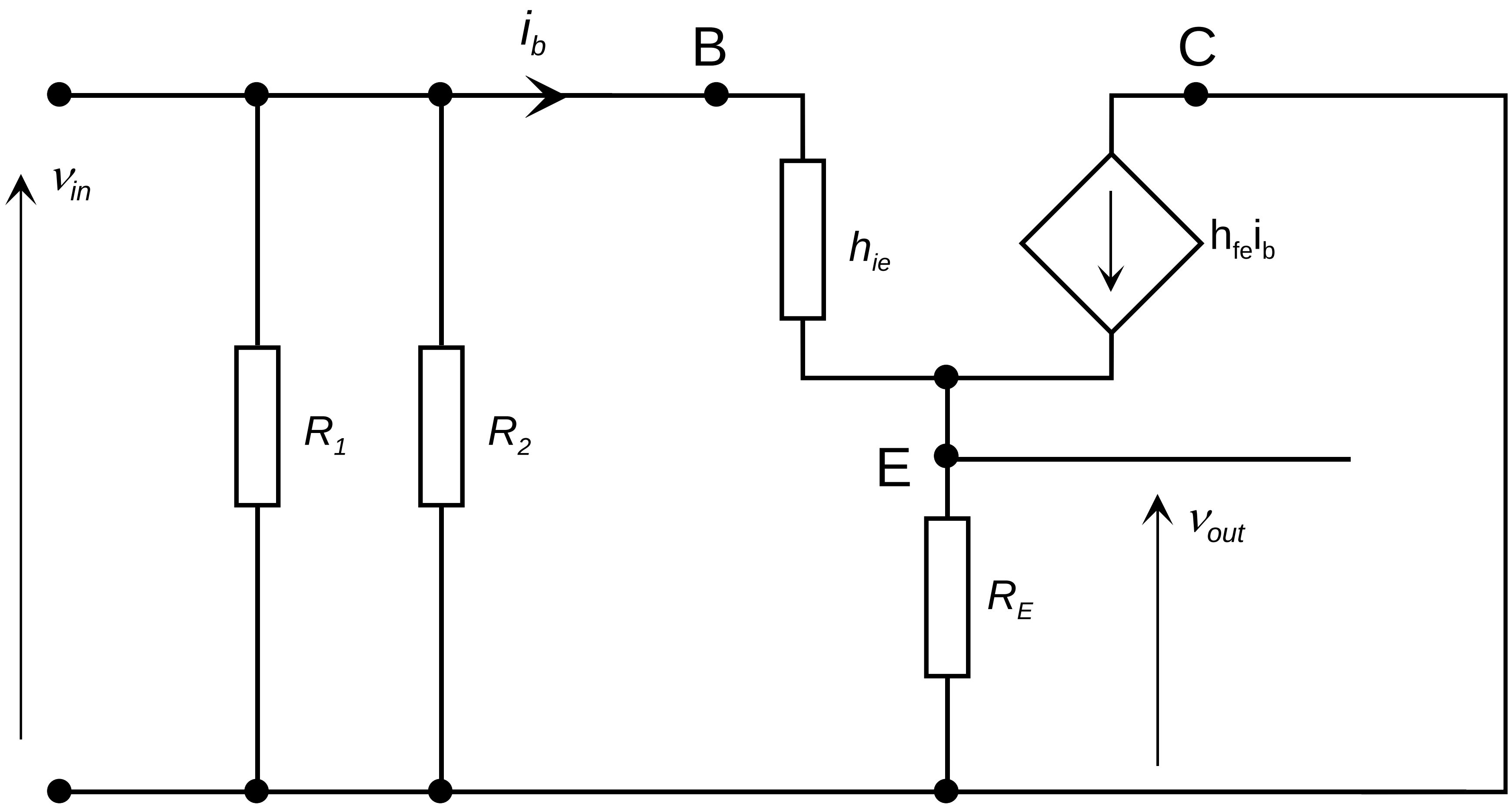
Figure 2.1 : Emitter-Follower circuit. Output signal is at the emitter terminal with respect to ground



The **output signal** is taken off of the **emitter** with respect to ground and the **collector** is connected directly to **Vcc**. Since Vcc is at signal ground in the ac equivalent circuit, we have the name ***common-collector*** *(emitter follower).*

C-C Amplifier Equivalent Circuit

- H-parameter Model



C-C Amplifier Properties
- H-parameter Model

Voltage Gain:

$$A_V = \frac{v_{out}}{v_{in}} = \frac{1}{\dfrac{h_{ie}}{(1+h_{fe})R_E} + 1}$$

Approximation:

$$\boxed{A_V \approx 1}$$

The output voltage at the ***emitter*** is ***in phase***, and essentially ***equal*** to the input signal voltage

Input Resistance:

$$R_{in} = \frac{R_b\left(h_{ie} + (1+h_{fe})R_E\right)}{R_b + h_{ie} + (1+h_{fe})R_E}$$

Approximation:

$$R_{in} \approx \frac{R_b(1+h_{fe})R_E}{R_b + (1+h_{fe})R_E}$$

*typically 10k or higher

C-C Amplifier Properties
- H-parameter Model (Cont')

Output Resistance:

$$R_{out} = \frac{R_E h_{ie}}{h_{ie} + (1 + h_{fe})R_E}$$

Approximation:

$$R_{out} \approx \frac{h_{ie}}{(1 + h_{fe})}$$

* h_{ie} is typically 1k and h_{fe} typically 100 or more. R_E is usually at least 10k.

* With h_{ie} ~ 1k and h_{fe} ~ 100 then R_{out} ~ 10 Ω.

Current Gain:

$$A_i = \frac{R_{in}}{R_L} = \frac{R_b(h_{ie} + (1 + h_{fe})R_E)}{R_L(h_{ie} + R_b + (1 + h_{fe})R_E)}$$

Approximation:

$$A_i \approx \frac{R_b(1 + h_{fe})R_E}{R_L(R_b + (1 + h_{fe})R_E)}$$

$$\approx (1 + h_{fe})\frac{R_E}{R_L}$$

* Assuming $h_{ie} << (1 + h_{fe})R_E$ and $R_b > (1 + h_{fe})R_E$

C-C Amplifier Hybrid-pi Parameter Properties and Examples

- Common-collector Amplifier

✓ Definition
✓ Small signal voltage gain
✓ Example 2.1
✓ Example 2.2
✓ Input resistance
✓ Output resistance
✓ Small signal current gain
✓ Example 2.3

Common-collector Amplifier – Small Signal Voltage Gain (1)

The hybrid-pi model of the bipolar transistor can also be used in the small-signal analysis of this circuit. Figure below shows the small-signal equivalent circuit of the circuit shown in figure 2.1. The collector terminal is at signal ground and the transistor output resistance r_o is in parallel with the dependent current source.

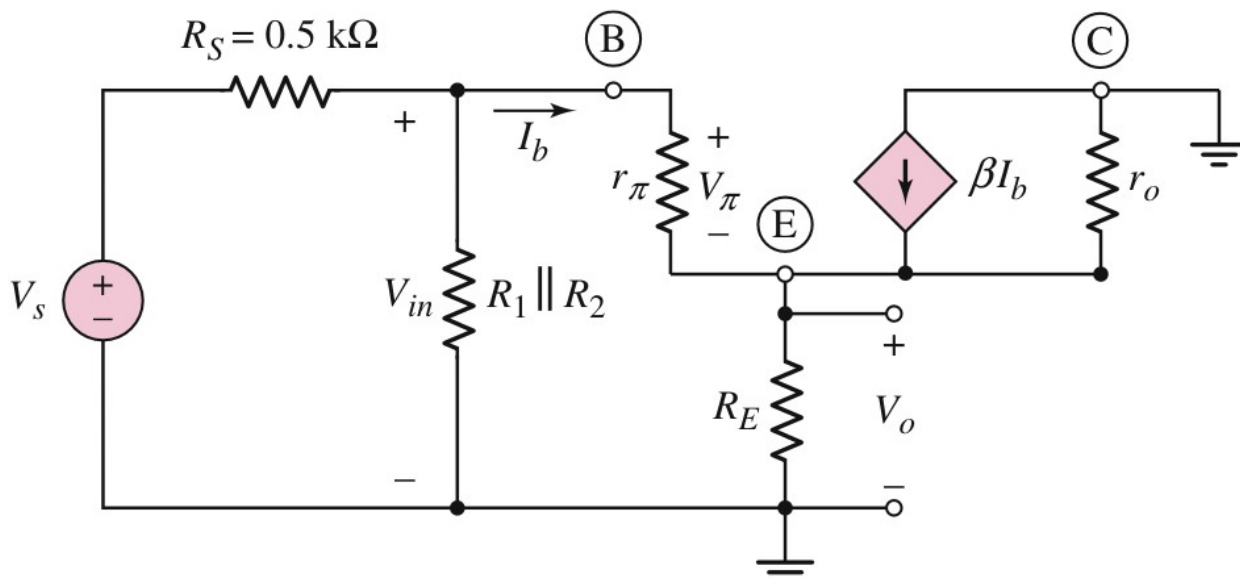


Figure 2.2: Small-signal equivalent circuit of the emitter-follower

Common-collector Amplifier – Small Signal Voltage Gain (2)

Figure below shows the equivalent circuit rearranged so that all signal grounds are at the same point

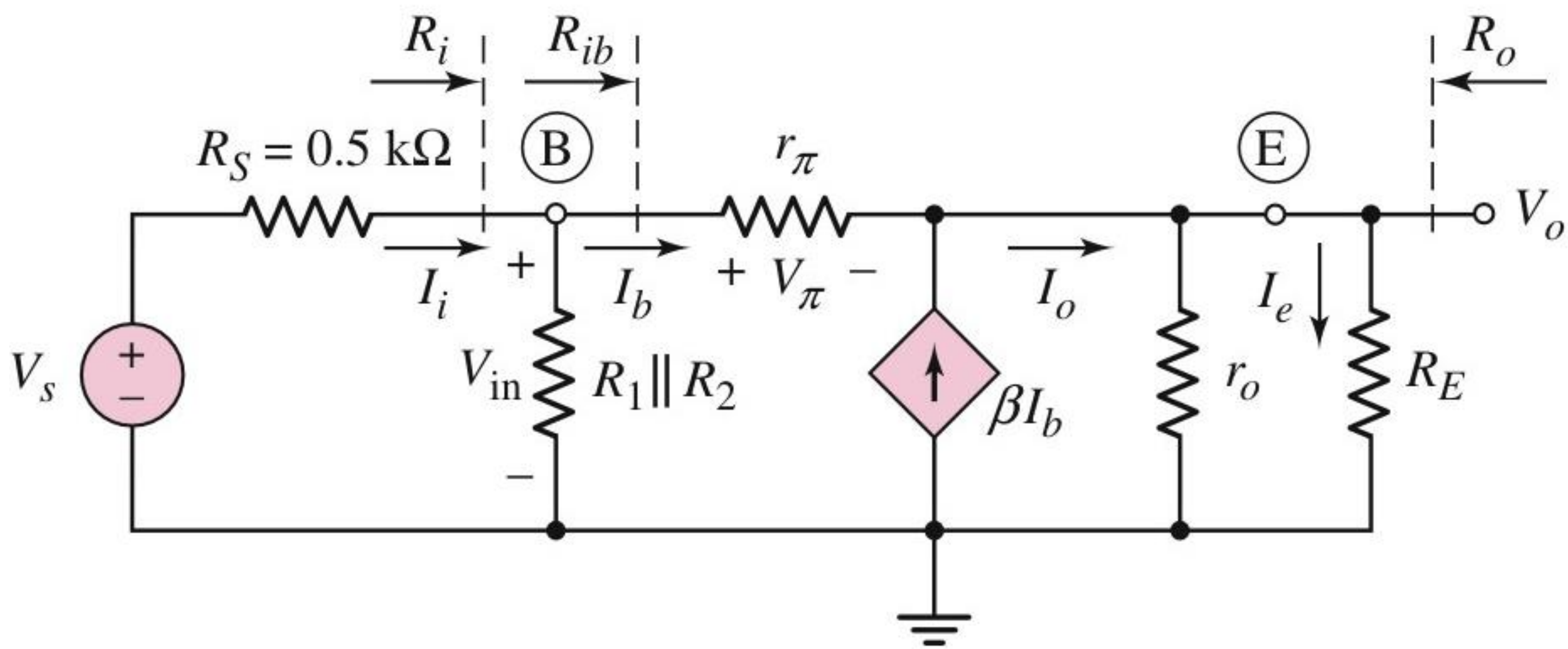
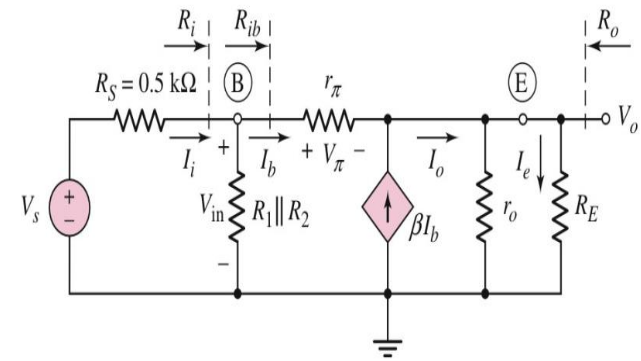


Figure 2.3: Small-signal equivalent circuit of the emitter-follower with all signal grounds are at the same point

Common-collector Amplifier – Small Signal Voltage Gain (3)

We see that

$$I_o = (1+\beta) I_b \qquad (2.1)$$



So the output voltage can be written as

$$V_o = I_b (1+\beta)(r_o \,//\, R_E) \qquad (2.2)$$

Writing a KVL equation around the base-emitter loop, we obtain

$$V_{in} = I_b \left[r_\pi + (1+\beta)(r_o \,//\, R_E) \right] \qquad (2.3a)$$

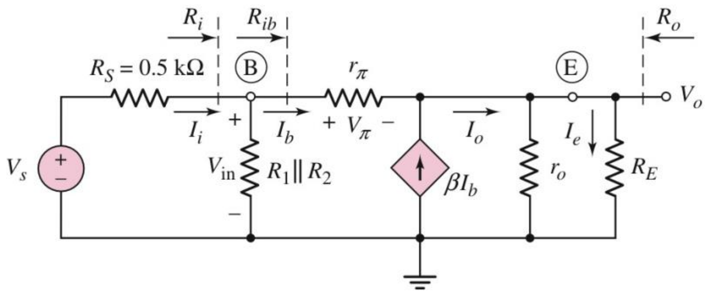
or

$$R_{ib} = \frac{V_{in}}{I_b} = r_\pi + (1+\beta)(r_o \,//\, R_E) \qquad (2.3b)$$

Common-collector Amplifier – Small Signal Voltage Gain (4)

We can also write

$$V_{in} = \left(\frac{R_i}{R_i + R_s} \right) . V_S \qquad (2.4)$$



Where $R_i = R_1 \,//\, R_2 \,//\, R_{ib}$

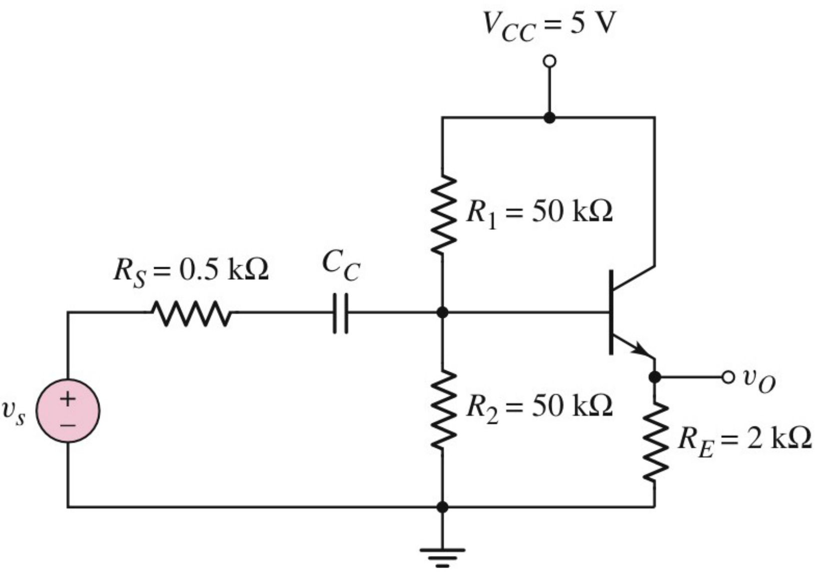
Combining Equations (2.2), (2.3b), and (2.4), the small-signal voltage gain is

$$A_v = \frac{V_o}{V_s} = \frac{(1+\beta)(r_o \,//\, R_E)}{r_\pi + (1+\beta)(r_o \,//\, R_E)} \left(\frac{R_i}{R_i + R_s} \right) \qquad (2.5)$$

Common-collector Amplifier – Example 2.1 (1)

Calculate the small-signal voltage gain of an emitter-follower circuit. For the circuit shown in figure 2.1, assume the transistor parameters are:

$\beta = 100$, $V_{BE}(on) = 0.7V$, and $V_A = 80V$



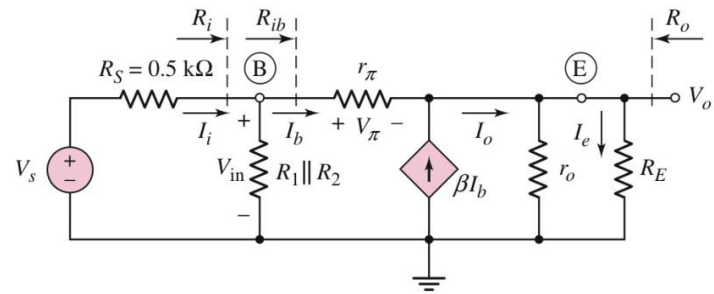
solution:

The dc analysis shows that $I_{CQ} = 0.793mA$ and $V_{CEQ} = 3.4V$. The small-signal hybrid-pi parameters are determined to be

$$r_\pi = \frac{V_T \beta}{I_{CQ}} = \frac{(0.026)(100)}{0.793} = 3.28K\Omega$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.793}{0.026} = 30.5mA/V$$

Common-collector Amplifier – Example 2.1 (2)



The small-signal voltage gain is then

$$A_v =$$

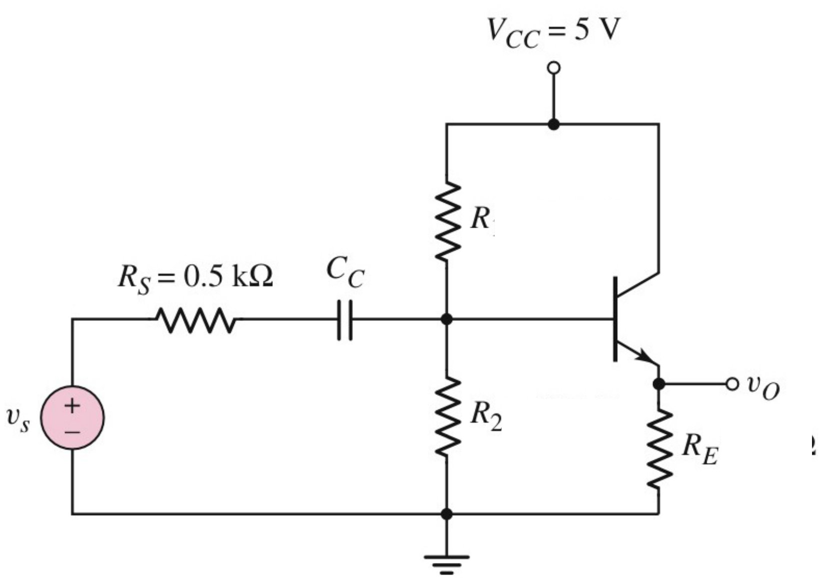
or

Common-collector Amplifier – Example 2.2

For the circuit shown in figure 2.1, let $V_{cc} = 5V$, $\beta = 120$, $V_A = 100V$, $R_E = 1K\Omega$

$V_{BE}(on) = 0.7V$, $R_1 = 25K\Omega$, and $R_2 = 50K\Omega$,

a) Determine the small-signal voltage gain . b) Find the input resistance looking into the base of the transistor.



Common-collector Amplifier – Example 2.2 (2)

$$g_m = \frac{I_{CQ}}{V_T} = \frac{2.29}{0.026} = 88.1\ mA/V$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(120)(0.026)}{2.29} = 1.36\ k\Omega$$

$$r_o = \frac{V_A}{I_{CQ}} = \frac{100}{2.29} = 43.7\ k\Omega$$

$$V_s' = \left(\frac{R_1 \| R_2 \| R_{ib}}{R_1 \| R_2 \| R_{ib} + R_S}\right) \cdot V_s$$

$$R_{ib} = r_\pi + (1+\beta)(R_E \| r_o) = 1.36 + (121)(1 \| 43.7)$$

or

$$R_{ib} = 120\ k\Omega \text{ and } R_1 \| R_2 = 16.7\ k\Omega$$

Then

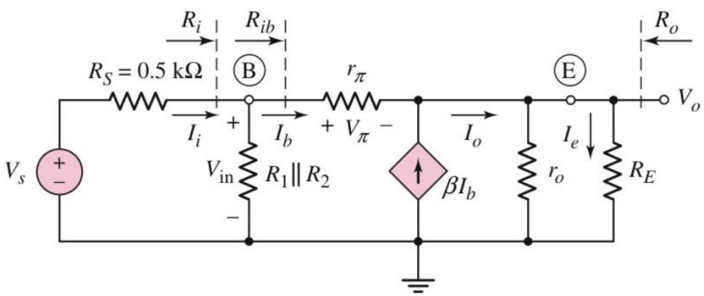
$$R_1 \| R_2 \| R_{ib} = 16.7 \| 120 = 14.7\ k\Omega$$

Now

$$V_s' = \left(\frac{14.7}{14.7 + 0.5}\right) \cdot V_s = (0.967) V_s$$

and

$$V_o = \left(\frac{V_\pi}{r_\pi} + g_m V_\pi\right)(R_E \| r_o) = V_\pi \left(\frac{1+\beta}{r_\pi}\right) R_E \| r_o$$



Example 2.2 (3)

We have

$$V_s' = V_\pi + V_o$$

then

$$V_\pi = \frac{V_s'}{1+\left(\frac{1+\beta}{r_\pi}\right)R_E \parallel r_o} = \frac{(0.967)V_s}{1+\left(\frac{1+\beta}{r_\pi}\right)R_E \parallel r_o}$$

We then obtain

$$A_v = \frac{V_o}{V_s} = \frac{(0.967)\left(\frac{1+\beta}{r_\pi}\right)R_E \parallel r_o}{1+\left(\frac{1+\beta}{r_\pi}\right)R_E \parallel r_o}$$

$$= \frac{(0.967)(1+\beta)R_E \parallel r_o}{r_\pi + (1+\beta)R_E \parallel r_o}$$

Now

$$R_E \parallel r_o = 1 \parallel 43.7 = 0.978\ k\Omega$$

Then

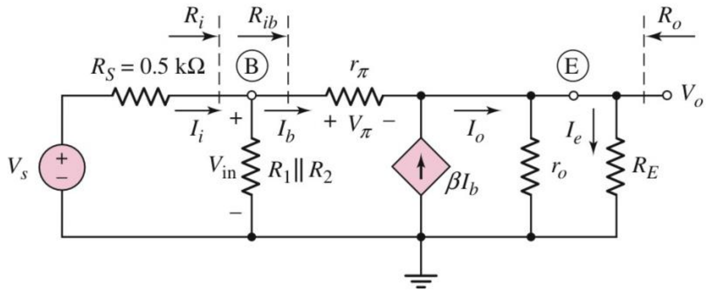
$$A_v = \frac{(0.967)(121)(0.978)}{1.36+(121)(0.978)} = 0.956$$

(b)

$$R_{ib} = r_\pi + (1+\beta)(R_E \parallel r_o)$$

or

$$R_{ib} = 1.36 + (121)(0.978) = 120\ k\Omega$$

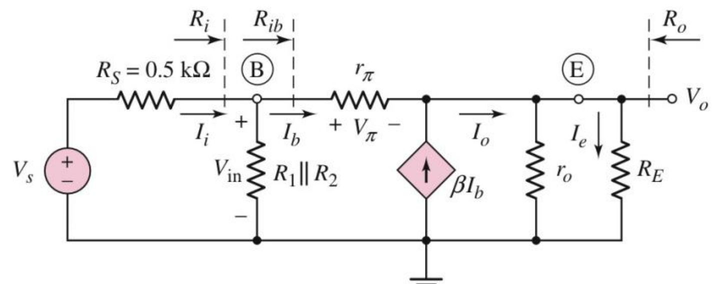


Common-collector Amplifier – Input Resistance

The input impedance, or small-signal input resistance for low-frequency signals, of the emitter-follower is determined in the same manner as for the common-emitter circuit. The input resistance R_{ib} was given by equation (2.3b)

$$R_{ib} = r_\pi + (1+\beta)(r_o \,//\, R_E)$$

Since the emitter current is $(1+\beta)$ times the base current, the effective impedance in the emitter is multiplied by $(1+\beta)$. We saw this same effect when an emitter resistor was included in a common-emitter circuit. This multiplication by $(1+\beta)$ is again called the resistance reflection rule.



Common-collector Amplifier – Output Resistance (1)

Initially, to find the output resistance of the emitter-follower circuit shown in figure 2.1, we will assume that the input signal source is ideal and that $R_s = 0$. The Figure below is derived from the small-signal equivalent circuit shown in figure 2.3 by setting the independent voltage source V_s equal to zero, which means that V_s acts as a short circuit.

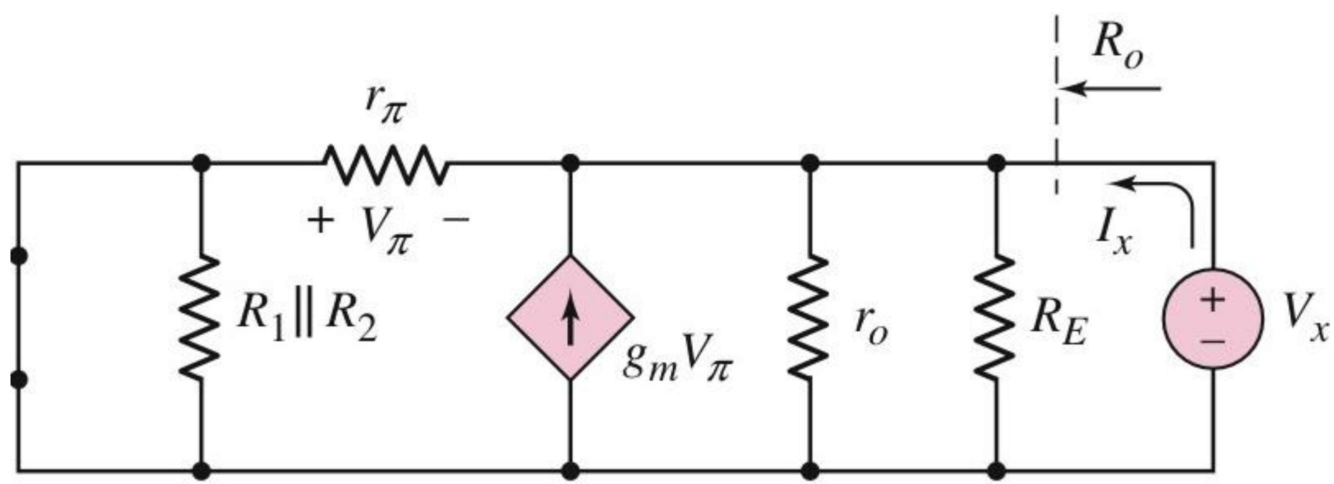


Figure 2.4: Small-signal equivalent circuit of the emitter-follower used to determine the output resistance.

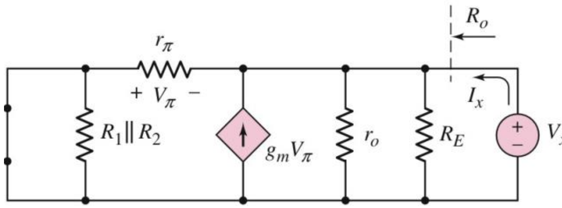
Common-collector Amplifier – Output Resistance (2)

A test voltage V_x is applied to the output terminal and the resulting test current is I_x . The output resistance, R_o , is given by

$$R_o = \frac{V_x}{I_x} \qquad (2.6)$$

In this case, the control voltage V_π is not zero, but is a function of the applied test voltage. From figure 2.4, we see that $V_\pi = -V_x$ .summing currents at the output node, we have

$$I_x + g_m V_\pi = \frac{V_x}{R_E} + \frac{V_x}{r_o} + \frac{V_x}{r_\pi} \qquad (2.7)$$



Since $V_\pi = -V_x$, equation (2.7) can be written as

$$\frac{I_x}{V_x} = \frac{1}{R_o} = g_m + \frac{1}{R_E} + \frac{1}{r_o} + \frac{1}{r_\pi} \qquad (2.8)$$

Common-collector Amplifier – Output Resistance (3)

Or the output resistance is given by

$$R_o = \frac{1}{g_m} // R_E // r_o // r_\pi \qquad (2.9)$$

The output resistance may also be written in a slightly different form, Equation (2.8) can be written in the form

$$\frac{1}{R_o} = \left(g_m + \frac{1}{r_\pi} \right) + \frac{1}{R_E} + \frac{1}{r_o} = \left(\frac{1+\beta}{r_\pi} \right) + \frac{1}{R_E} + \frac{1}{r_o} \qquad (2.10)$$

Or the output resistance can be written in the form

$$R_o = \frac{r_\pi}{1+\beta} // R_E // r_o \qquad (2.11)$$

This is an important result and is called the inverse resistance reflection rule and is the inverse of the reflection rule looking to the base.

Common-collector Amplifier – Output Resistance (4)

We can determine the output resistance of the emitter-follower circuit taking into account a nonzero source resistance. The circuit in figure below is derived from the small-signal equivalent circuit shown in figure 2.3 and can be used to find R_o

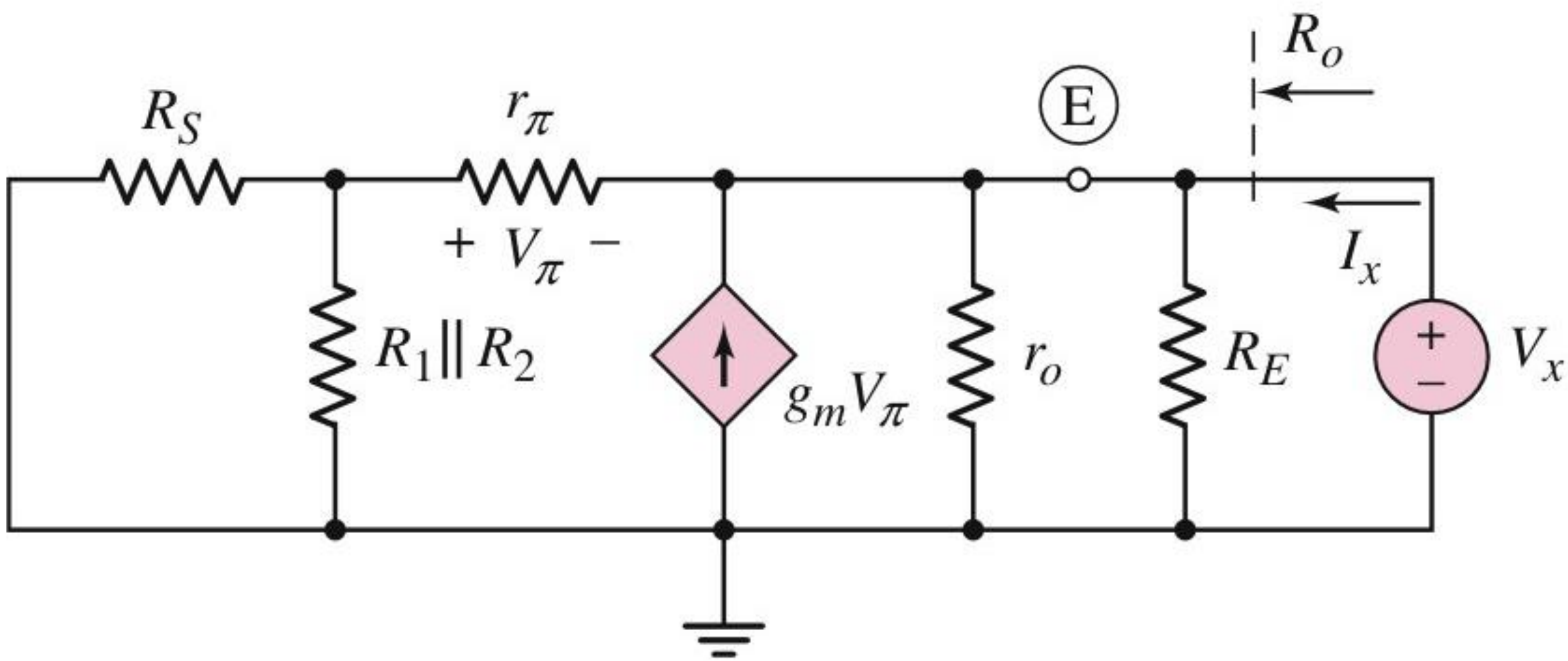
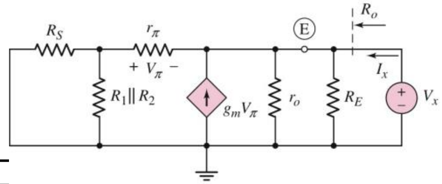


Figure 2.5: Small-signal equivalent circuit of the emitter-follower used to determine the output resistance including the effect of the source resistance R_s

Common-collector Amplifier – Output Resistance (5)

The independent source V_s is set equal to zero and test voltage V_x is applied to the output terminals. Again, the control voltage V_π is not zero, but is a function of the test voltage. Summing currents at the output node, we have



$$I_x + g_m V_\pi = \frac{V_x}{R_E} + \frac{V_x}{r_o} + \frac{V_x}{r_\pi + R_1 // R_2 // R_s} \qquad (2.12)$$

The control voltage can be written in terms of the test voltage by a voltage divider equation as

$$V_\pi = -\left(\frac{r_\pi}{r_\pi + R_1 // R_2 // R_s} \right) . V_x \qquad (2.13)$$

Equation (2.12) can then be written as

$$I_x = \left(\frac{g_m r_\pi}{r_\pi + R_1 // R_2 // R_s} \right) . V_x + \frac{V_x}{R_E} + \frac{V_x}{r_o} + \frac{V_x}{r_\pi + R_1 // R_2 // R_s} \qquad (2.14)$$

Common-collector Amplifier – Output Resistance (6)

Noting that $g_m r_\pi = \beta$, we find

$$\frac{I_x}{V_x} = \frac{1}{R_o} = \left(\frac{1+\beta}{r_\pi + R_1 // R_2 // R_s} \right) + \frac{1}{R_E} + \frac{1}{r_o} \qquad (2.15)$$

or

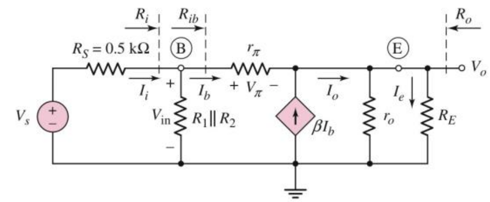
$$R_o = \left(\frac{r_\pi + R_1 // R_2 // R_s}{1+\beta} \right) // R_E // r_o \qquad (2.16)$$

In this case, the source resistance and bias resistances contribute to the output resistance

Common-collector Amplifier – Small Signal Current Gain (1)

We can determine the small-signal current gain of an emitter-follower by using the input resistance and the concept of current dividers. The small-signal current gain is defined as

$$A_i = \frac{I_e}{I_i} \qquad (2.16)$$



Where I_e and I_i are the output and input current phasors.

Using current divider equation, we can write the base current in terms of the input current, as follows:

$$I_b = \left(\frac{R_1 // R_2}{R_1 // R_2 + R_{ib}} \right) I_i \qquad (2.17)$$

Since $g_m V_\pi = \beta I_b$, then

$$I_0 = (1+\beta) I_b = (1+\beta) \left(\frac{R_1 // R_2}{R_1 // R_2 + R_{ib}} \right) I_i \qquad (2.18)$$

Common-collector Amplifier – Small Signal Current Gain (2)

Writing the load current in terms of I_o produces

$$I_e = \left(\frac{r_o}{r_o + R_E}\right) I_0 \qquad (2.19)$$

Combining equations (2.18) and (2.19) , we obtain the small-signal current gain, as follows:

$$A_i = \frac{I_e}{I_i} = (1+\beta)\left(\frac{R_1 // R_2}{R_1 // R_2 + R_{ib}}\right)\left(\frac{r_o}{r_o + R_E}\right) \qquad (2.20)$$

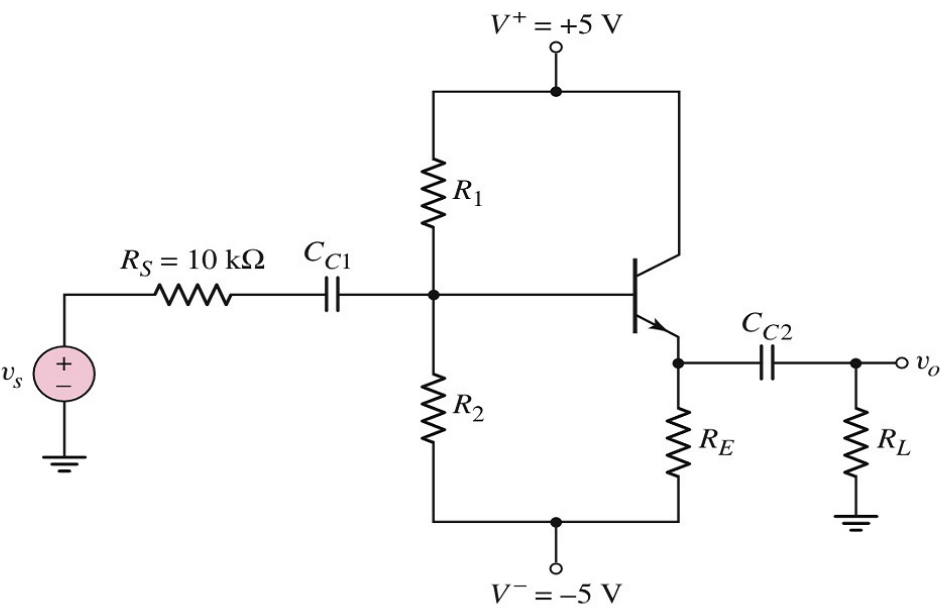
If we assume that $R_1 // R_2 >> R_{ib}$ and $r_o >> R_E$,then

$$A_i \cong (1+\beta) \qquad (2.21)$$

Which is current gain of the transistor.

Common-collector Amplifier – Example 2.3

For the circuit shown in figure below, let $\beta = 100, V_A = 125V$, and $V_{BE}(on) = 0.7V$ Assume $R_s = 0$, and $R_L = 1K\Omega$, a) Design a bias-stable circuit such that $I_{CQ} = 125mA$, and $V_{CEQ} = 4V$, b) What is small-signal Current gain $A_i = i_o / i_i$ c)What is output resistance looking back into the output terminal



We have

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right)(10) - 5$$

or

$$V_{TH} = \frac{1}{R_1}(481) - 5$$

We can write $I_{BQ} = \dfrac{V_{TH} - 0.7 - (-5)}{R_{TH} + (1+\beta)R_E}$

Or

$$0.0125 = \frac{\frac{1}{R_1}(481) - 5 - 0.7 + 5}{48.1 + (101)(4.76)}$$

which yields

$R_1 = 65.8\ k\Omega$

Since $R_1 \| R_2 = 48.1\ k\Omega$, we obtain

$R_2 = 178.8\ k\Omega$

(b)

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{1.25} = 2.08\ k\Omega$$

$$r_o = \frac{V_A}{I_{CQ}} = \frac{125}{1.25} = 100\ k\Omega$$

We may note that

$$g_m V_\pi = g_m (I_b r_\pi) = \beta I_b$$

Also

$$R_{ib} = r_\pi + (1+\beta)(R_E \parallel R_L \parallel r_o)$$
$$= 2.08 + (101)(4.76 \| 1 \| 100)$$

or

$$R_{ib} = 84.9\ k\Omega$$

Now

$$I_o = \left(\frac{R_E \| r_o}{R_E \| r_o + R_L} \right)(1+\beta) I_b$$

where

$$I_b = \left(\frac{R_1 \| R_2}{R_1 \| R_2 + R_{ib}} \right) \cdot I_s$$

We can then write

$$A_I = \frac{I_o}{I_s} = \left(\frac{R_E \| r_o}{R_E \| r_o + R_L} \right)(1+\beta)\left(\frac{R_1 \| R_2}{R_1 \| R_2 + R_{ib}} \right)$$

We have

$$R_E \| r_o = 4.76 \| 100 = 4.54\ k\Omega$$

so

$$A_I = \left(\frac{4.54}{4.54+1} \right)(101)\left(\frac{48.1}{48.1+84.9} \right)$$

or

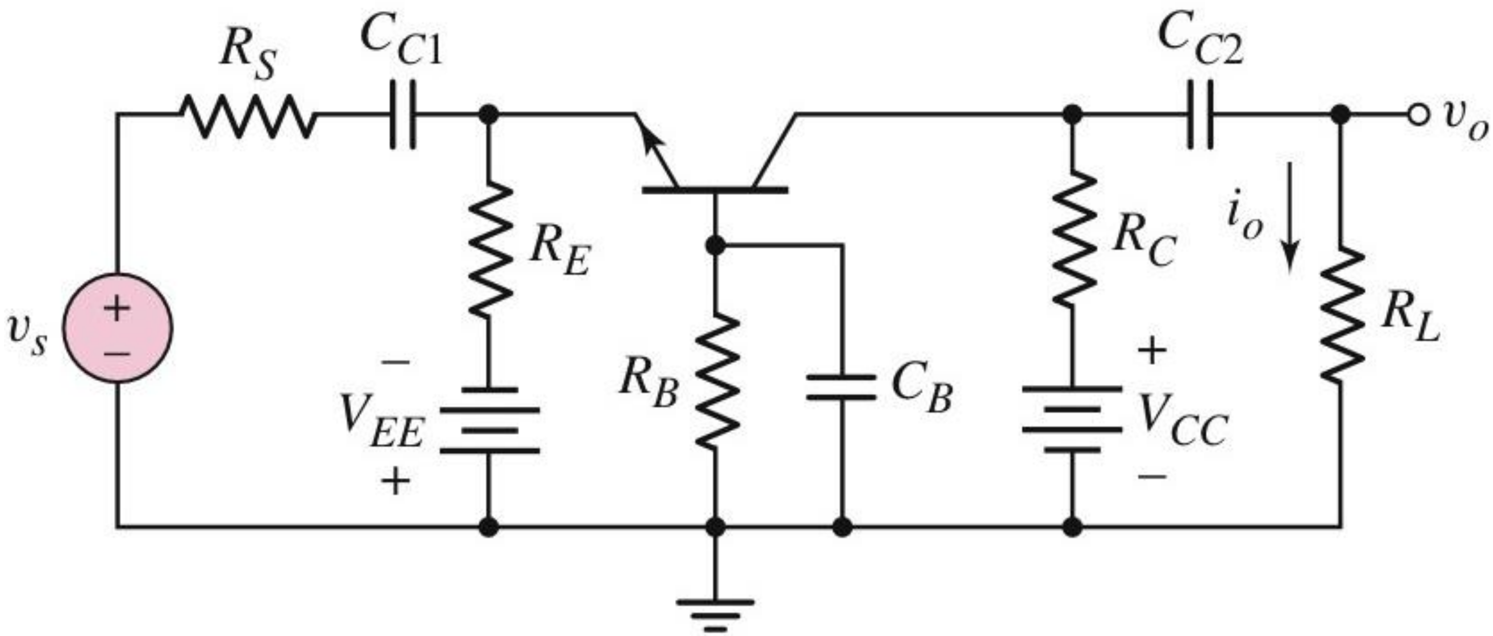
$$A_I = 29.9$$

(c) $$R_o = R_E \,\|\, r_o \left\| \frac{r_\pi}{1+\beta} = 4.76 \right\| 100 \left\| \frac{2.08}{101} \right.$$

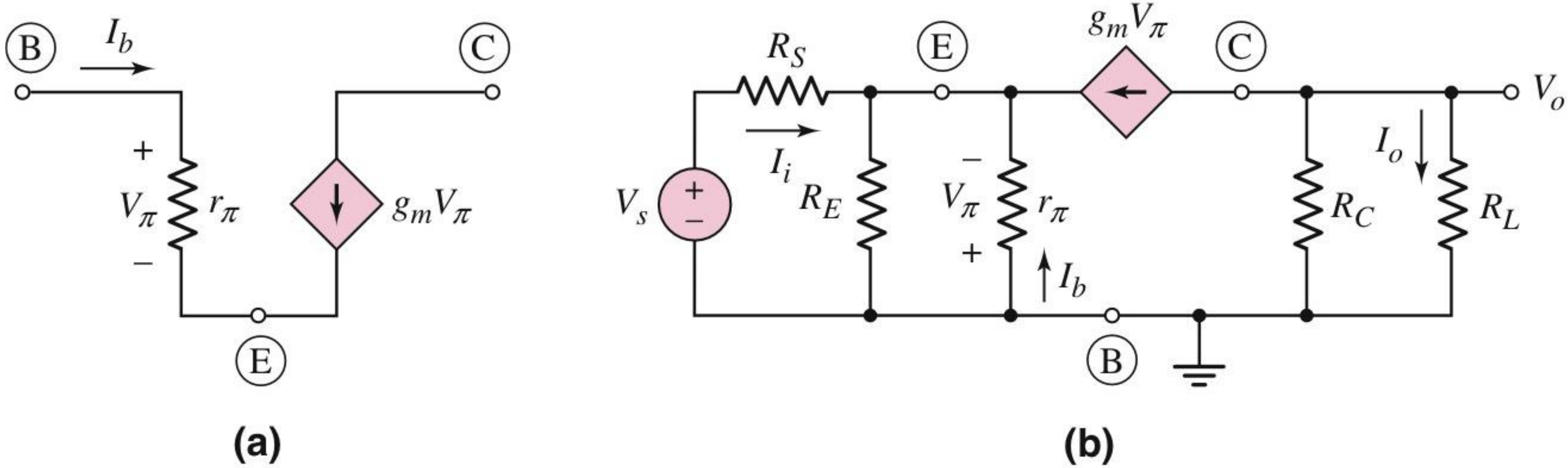
or $R_o = 20.5\ \Omega$

Analyse the Common-Base Amplifier

Common-Base Amplifier



Small-Signal Equivalent Circuit: Common Base

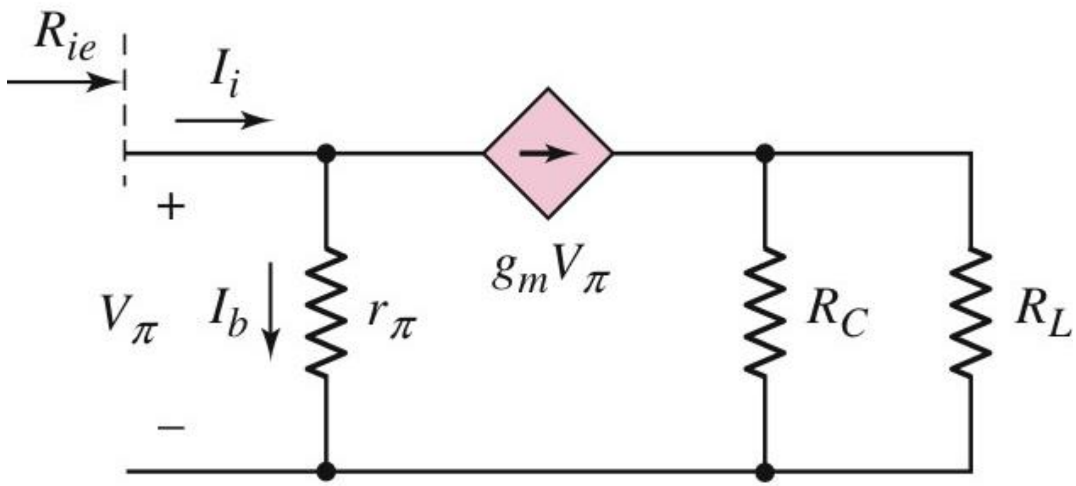


$$A_v = g_m (R_C \| R_L)$$

$$A_i = g_m \left(\frac{R_C}{R_C + R_L}\right)\left[\frac{r_\pi}{1+\beta} \| R_E\right]$$

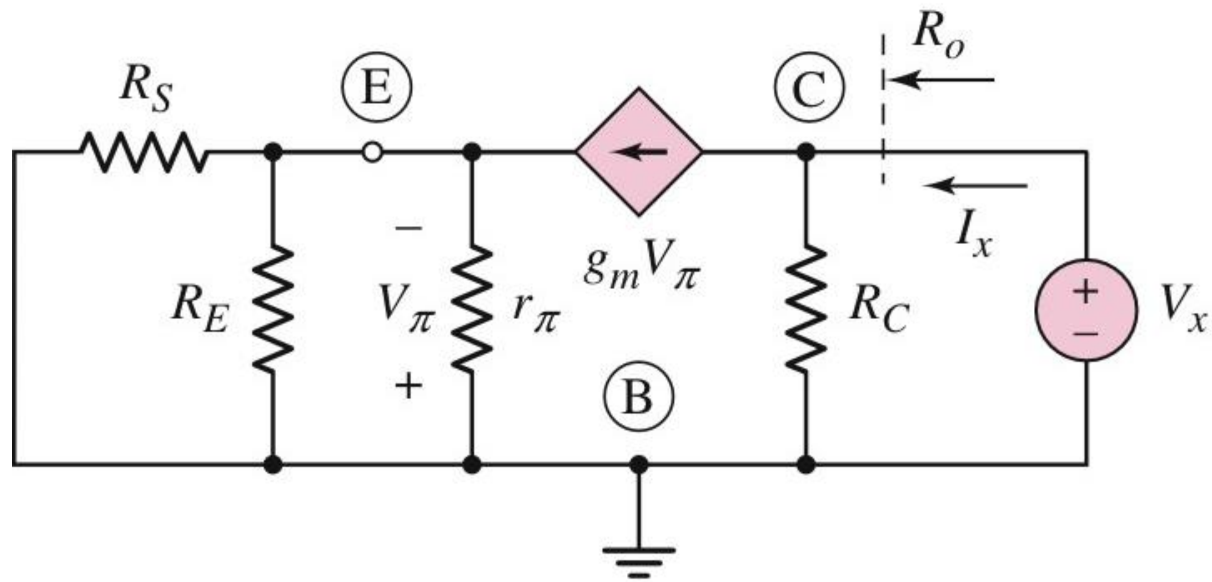
If R_E approaches infinity and R_L approaches zero, the current gain becomes the short-circuit current gain given by $A_{io} = \alpha$.

Input Resistance: Common Base



$R_{ie} = r_\pi/(1+\beta)$

Output Resistance: Common Base



$R_O = R_C$

Summary and Comparison

Configuration	**Voltage gain**	**Current gain**	**Input resistance**	**Output Resistance**
Common emitter	$A_V > 1$	$A_I > 1$	Moderate	Moderate to high
Emitter Follower	$A_V \cong 1$	$A_i > 1$	High	Low
Common base	$A_V > 1$	$A_i \cong 1$	Low	Moderate to high

Contents of Chapter

- Understand the concept of an analog signal and the principle of a linear amplifier.
  - Investigate how a transistor circuit can amplify a small, time-varying input signal.

- Discuss and compare the three basic transistor amplifier configurations.
  - Analyze the common-emitter amplifier.
  - Analyze the emitter-follower amplifier.
  - Analyze the common-base amplifier.