Xi'an Jiaotong-Liverpool University
西交利物浦大学

# EEE220 Instrumentation and Control System

*2018-19 Semester 2*


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# Lecture 22

# Outline

## Frequency Response Methods

- ❑ Introduction
- ❑ Frequency Response Plots – Polar Plot & Bode Plot
- ❑ Frequency Response Measurements
- ❑ Performance Specifications in the Frequency Domain
- ❑ Gain Margin and Phase Margin
- ❑ Compensators
- ❑ Frequency Response Methods Using Matlab

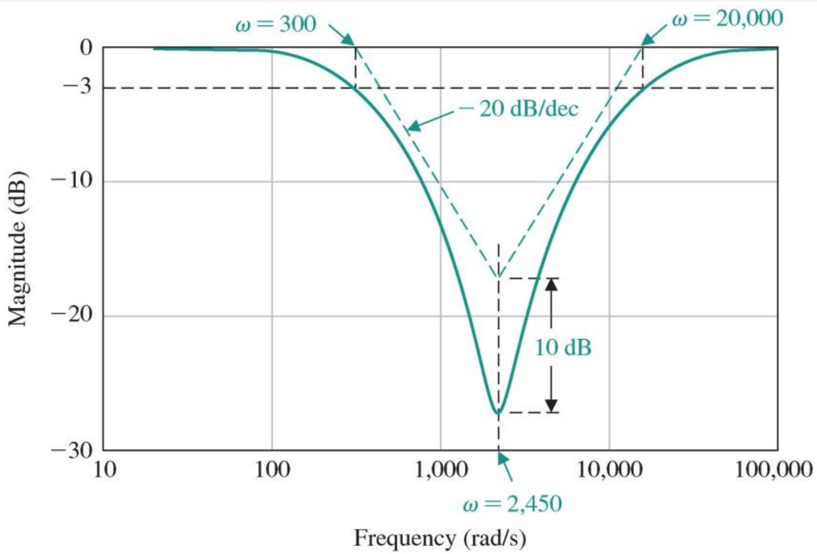
# Frequency Response Measurement

- A sine wave can be used to measure the open-loop frequency response of a system. In practice, a plot of amplitude versus frequency and phase versus frequency will be obtained. From these two plots, the transfer function (including both the loop TF and the closed-loop TF) of the system can be deduced.

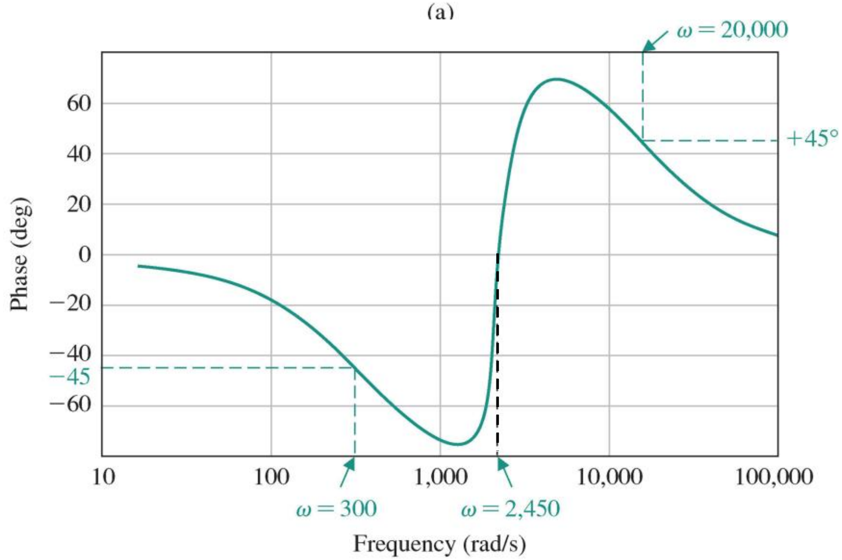
- A device called a **wave analyzer** can be used to measure the amplitude and phase variations as the frequency of the input sine wave is altered. Also, a device called a **transfer function analyzer** can be used to measure the loop transfer function and closed-loop transfer function.

- A typical signal analyzer instrument can perform frequency response measurement from DC to 100 kHz. Built-in analysis and modeling capabilities can derive poles and zeros from measured frequency responses or construct phase and magnitude responses from user-supplied models. This device can also synthesize the frequency response of a model of a system, allowing a comparison with an actual response.

# Example 22.1




(a)




(b)

- The magnitude declines at about -20 dB/decade as $\omega$ increase between 100 and 1000; and because the phase is -45° and the magnitude is -3dB at 300 rad/s → there must be a pole at $p_1 = 300$;

- Phase abruptly increases by nearly 180° and passing 0° at 2450 rad/s; also, the slope of the magnitude changes from -20 dB/decade to 20 dB/decade at 2450 rad/s → there must be a pair of quadratic zeros existing at $\omega_n = 2450$;

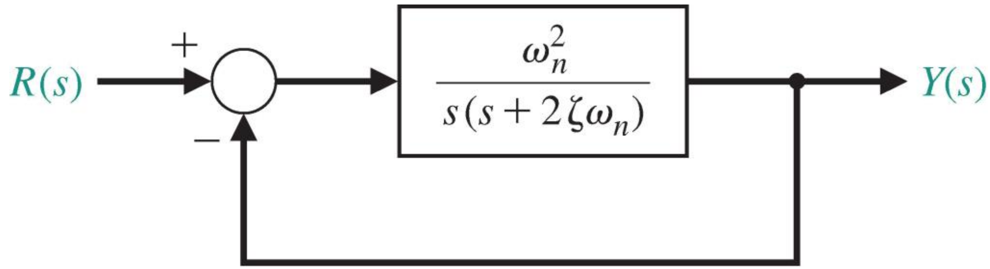
- Difference in the corner frequency of the asymptotes to the minimal response is 10 dB → $\zeta = 0.16$.

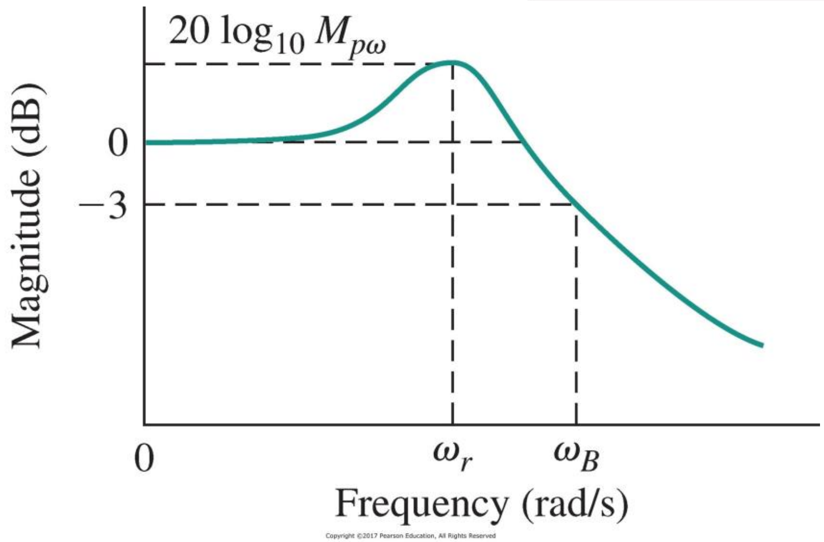
- Slope of magnitude returns to 0 dB/decade as $\omega$ exceeds 50000; specifically, magnitude is -3 dB and phase is 45° at 20000 rad/s → there must be a second pole at $p_2 = 20000$.

$$T(s) = \frac{(s/2450)^2 + (0.32/2450)s + 1}{(s/300 + 1)(s/20000 + 1)}$$

# Performance Specifications in the Frequency Domain

- For a second-order system with a pair of complex poles:



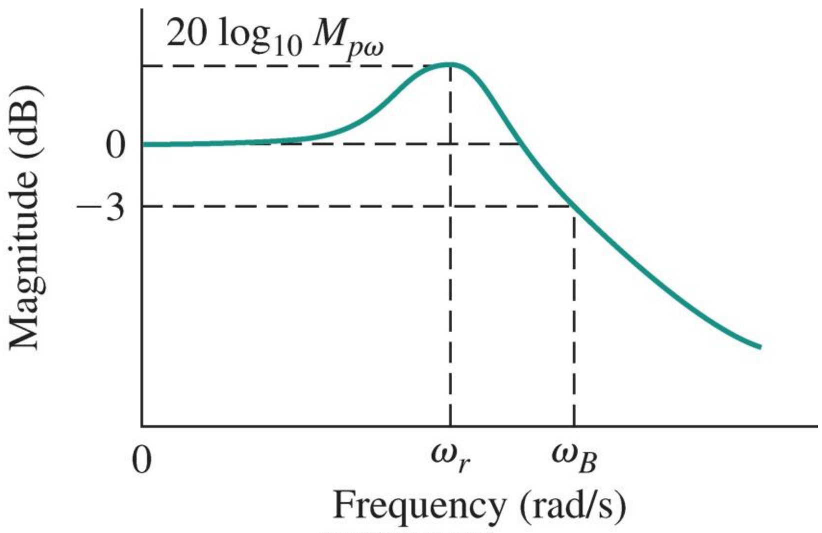

Two main frequency specifications:

1. **Maximum magnitude - $M_{p\omega}$**
2. **Bandwidth - $\omega_B$**

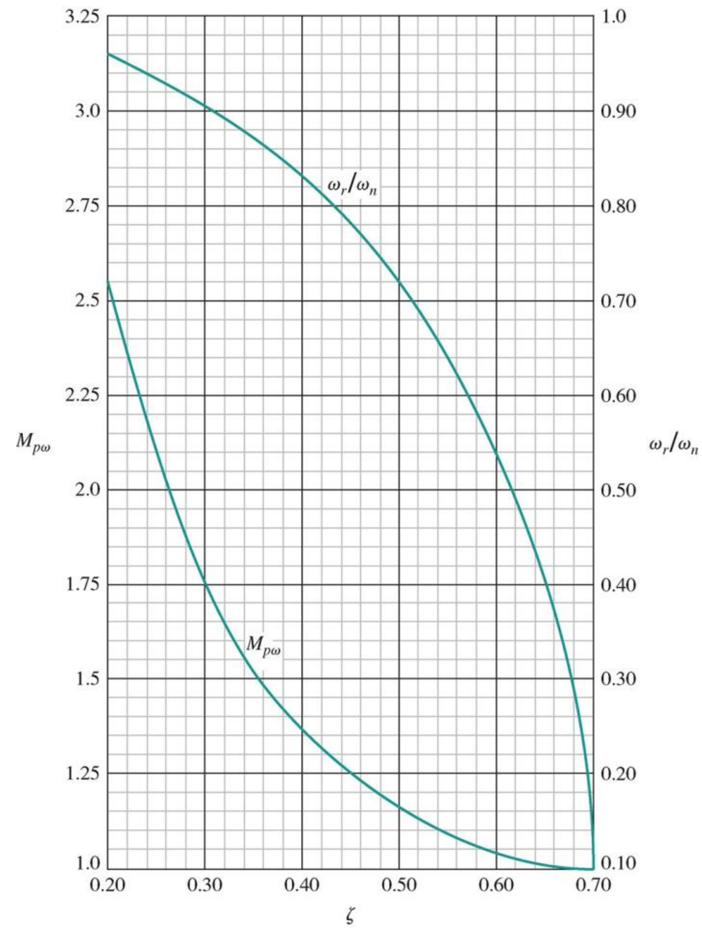
- For a higher-order system, if the frequency response is **dominated** by a pair of complex poles, the relationship between frequency response and time response will be valid.

# Maximum Magnitude

At the resonant frequency $\boldsymbol{\omega_r}$ a maximum value $\boldsymbol{M_{p\omega}}$ of the frequency response is attained.



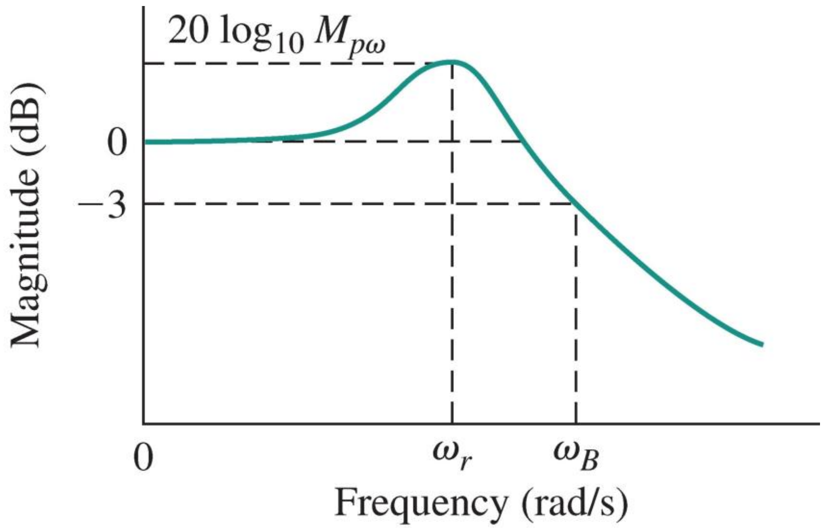

$$\boxed{M_{p\omega} = |G(j\omega_r)| = \left(2\zeta\sqrt{1-\zeta^2}\right)^{-1},} \quad \zeta < 0.707.$$

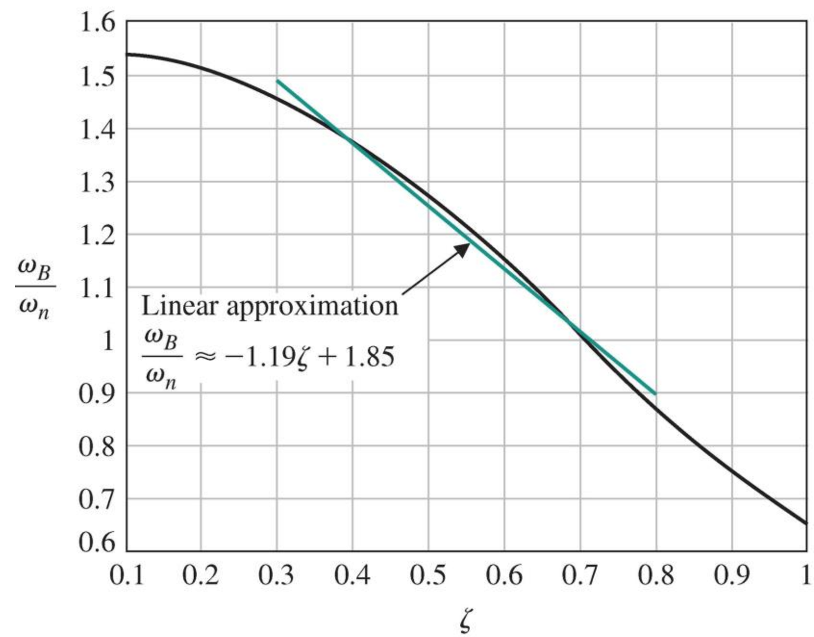



**Maximum magnitude $M_{p\omega}$ ↑ → Damping ratio $\zeta$ ↓ → Percent Overshoot P.O. ↑**

# Bandwidth

The bandwidth is the frequency at which the frequency response has declined 3 dB from its low-frequency value.



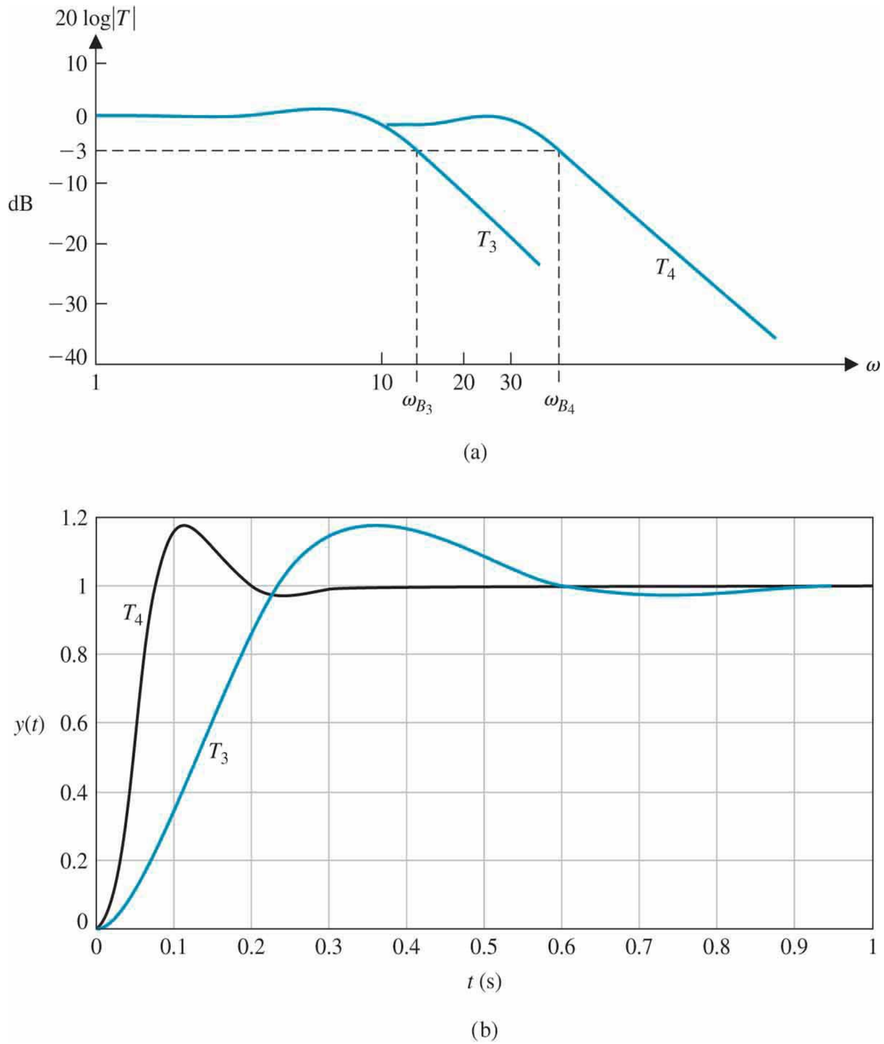




**Bandwidth $\omega_B$ ↑ (with a constant $\zeta$) → Natural frequency $\omega_n$ ↑ → Settling time $T_s$ ↓**

# Bandwidth vs. Settling Time

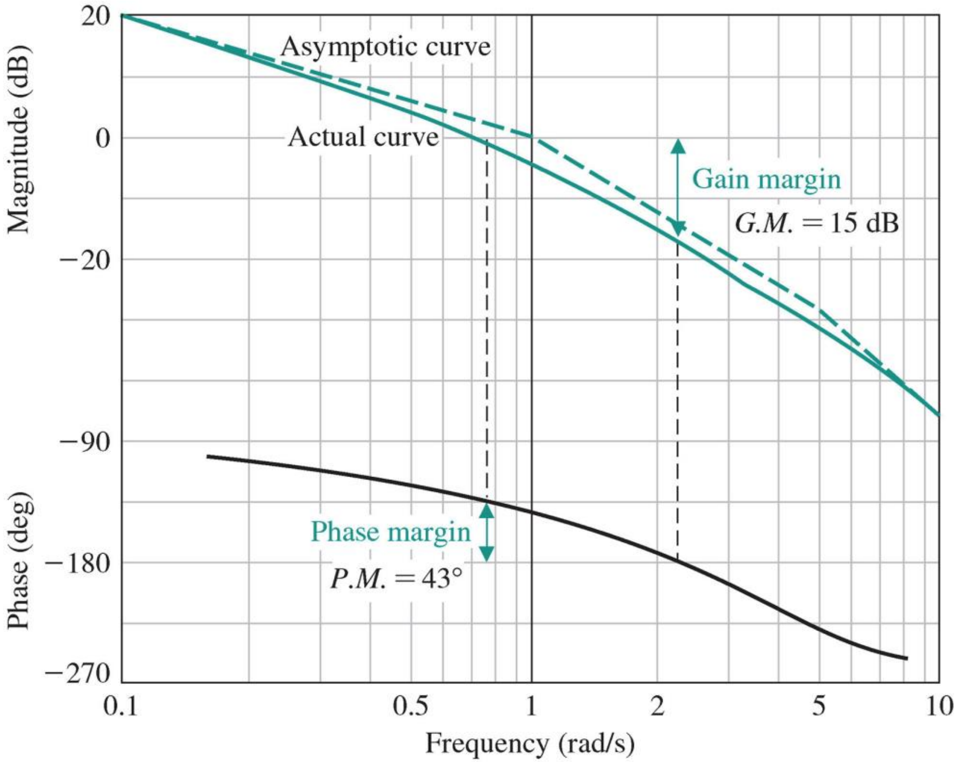
$$T_3(s) = \frac{100}{s^2 + 10s + 100}$$

$$T_4(s) = \frac{900}{s^2 + 30s + 900}$$



# Gain Margin and Phase Margin

- For a closed-loop control system, with loop transfer function $L(j\omega)$, the characteristic equation is $1 + L(j\omega) = 0 \rightarrow L(j\omega) = -1 + j0$.
- It indicates that when $|L(j\omega)| = 1$ (or $20\log(|L(j\omega)|) = 0\ \text{dB}$) and $\angle L(j\omega) = \pm 180^o$, the closed-loop system is marginally stable.

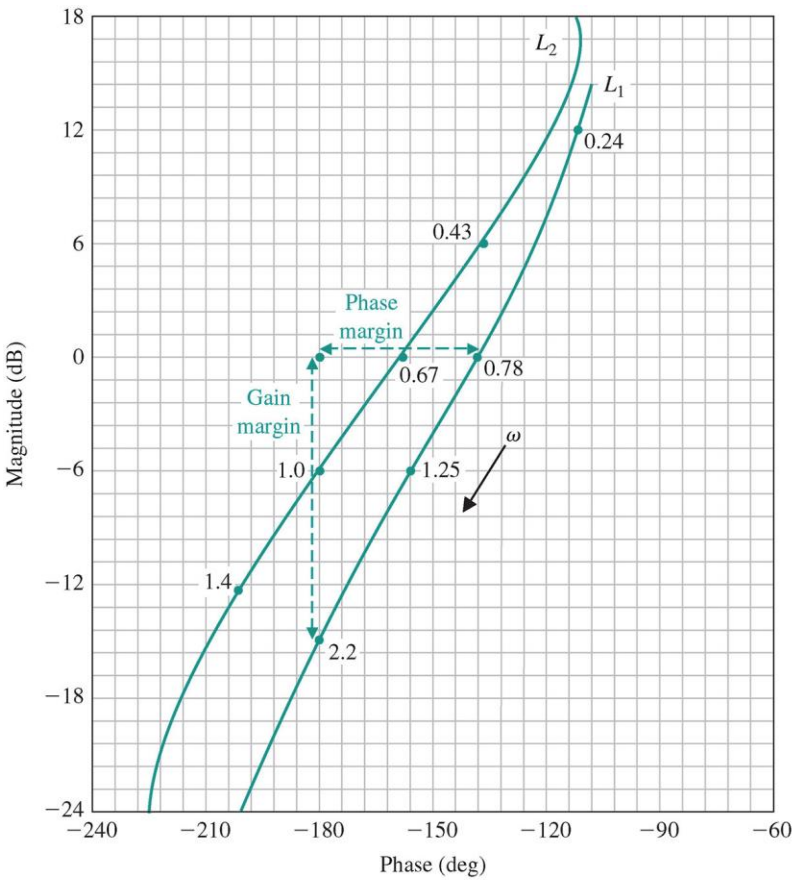




- The <u>gain margin</u> GM is the increase in the gain of the $L(j\omega)$ when phase = $\pm 180^o$ that will result in a marginally stable system;
- The <u>phase margin</u> $\emptyset_{pm}$ is the amount of phase shift of the $L(j\omega)$ at unity magnitude (0 dB) that will result in a marginally stable system.

# Relative Stability

- $L_1(j\omega) = \frac{1}{j\omega(j\omega+1)(0.2j\omega+1)}$, phase margin is 43°, gain margin is 15 dB
- $L_2(j\omega) = \frac{1}{j\omega(j\omega+1)^2}$, phase margin is 20°, gain margin is 5.7 dB



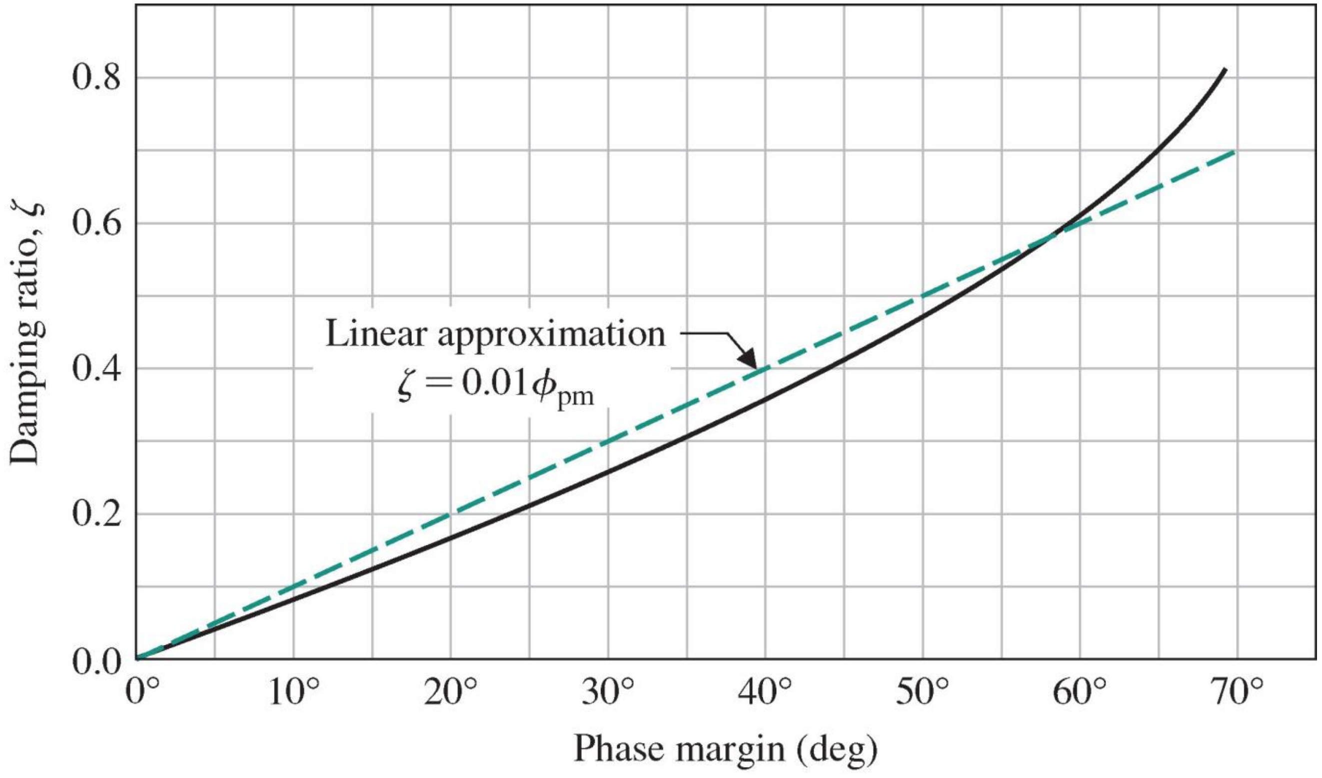


System 1 is MORE stable than system 2!

-- Gain margin or phase margin is actually a measure of **relative stability**.

# Phase Margin and Damping Ratio

$$\boxed{\zeta = 0.01\phi_{\text{pm}},}$$



# Compensator

- The performance of a feedback control system is of primary importance. A suitable control system is stable and result in an acceptable response to input commands, is less sensitive to system parameter changes, results in a minimum steady-state error for input commands, and is able to reduce the effect of undesirable disturbances.

- A feedback control system that provides an optimum performance without any necessary adjustments is rare.

- Thus, the design of a control system is concerned with the arrangement, or the plan, of the system structure and the selection of suitable components and parameters.

- The alteration or adjustment of a control system in order to provide a suitable performance is called compensation.

- A compensator is an additional component that is inserted into a control system to compensate for deficient performance.

# Compensator Structure

$$G_c(s) = \frac{K\prod_{i=1}^{M}(s+z_i)}{\prod_{j=1}^{n}(s+p_j)}$$

Compensator $G_c(s)$ can be chosen to alter either the shape of the root locus or the frequency response.

Compensator $G_c(s)$ is used with a process $G(s)$ so that the overall loop gain can be set to satisfy the steady state error requirement, and then $G_c(s)$ is used to adjust the system dynamics favorably without affecting the steady state error.

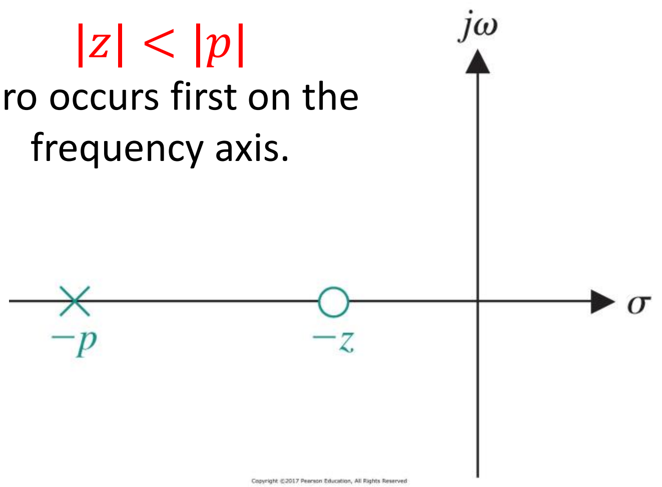
First order compensator with the transfer function

$$G_c(s) = \frac{K(s+z)}{(s+p)}$$

# Phase – Lead Compensator

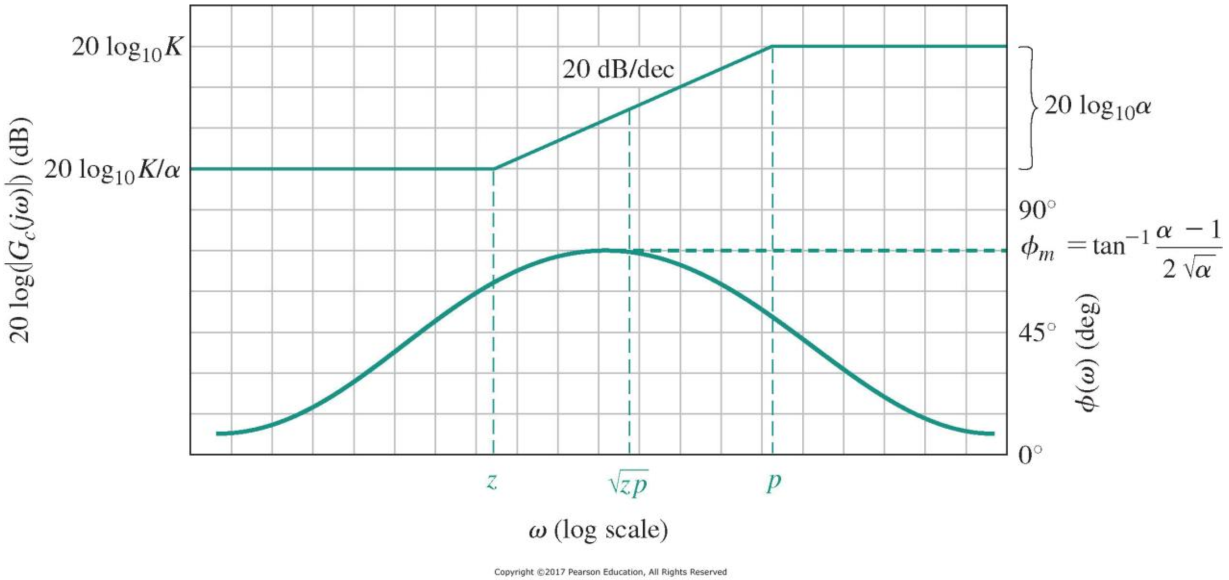
$|z| < |p|$

Zero occurs first on the frequency axis.




$$G_{c(j\omega)} = \frac{K(j\omega + z)}{(j\omega + p)} = \frac{K(1 + j\omega\alpha\tau)}{\alpha(1 + j\omega\tau)}$$

$$\tau = \frac{1}{p} \text{ and } \alpha = p/z > 1.$$



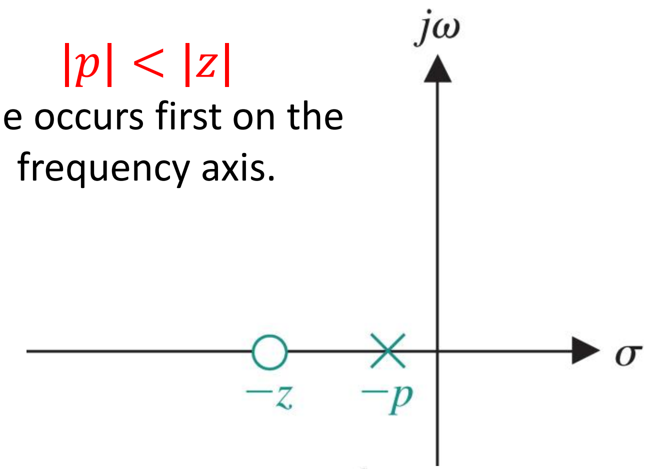

**The maximum phase lead $\phi_m$ occurs at $\omega_m$.**

$$\omega_m = \sqrt{zp} = \frac{1}{\tau\sqrt{\alpha}}.$$

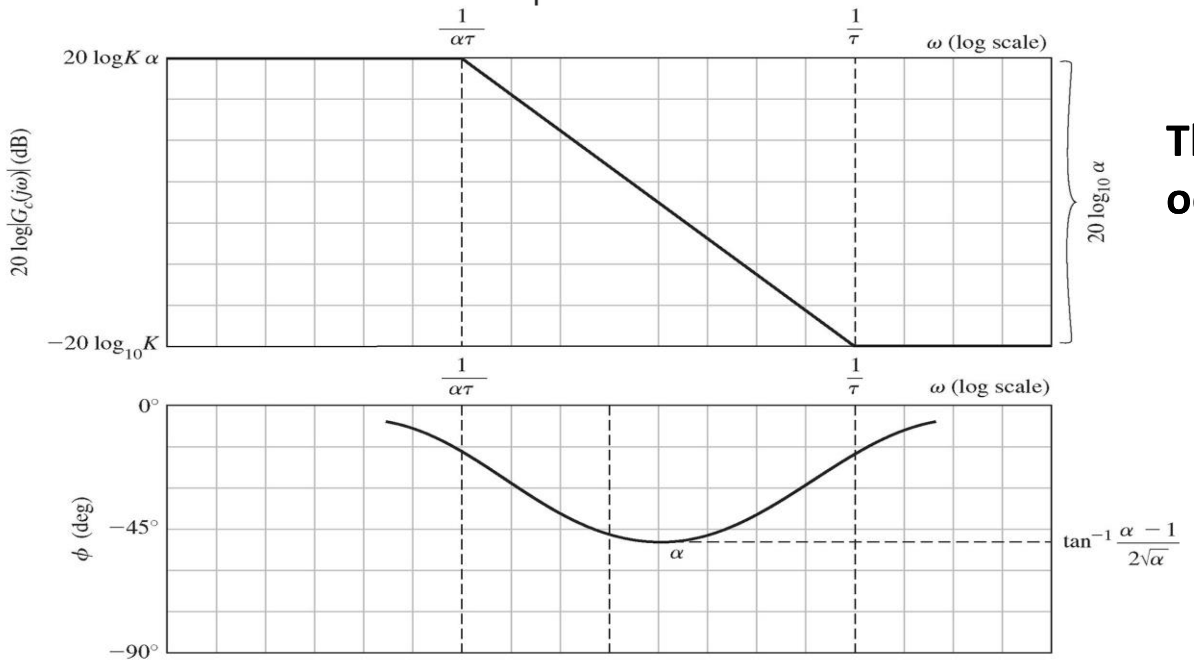
$$\sin\phi_m = \frac{\alpha - 1}{\alpha + 1}.$$

# Phase - Lag Compensator

$|p| < |z|$
Pole occurs first on the frequency axis.




$$G_c(j\omega) = \frac{K(j\omega + z)}{(j\omega + p)} = \frac{K\alpha(1 + j\omega\tau)}{1 + j\omega\alpha\tau}$$
$$\tau = \frac{1}{z} \text{ and } \alpha = z/p > 1.$$




**The maximum phase lag $\emptyset_m$ occurs at $\omega_m$.**

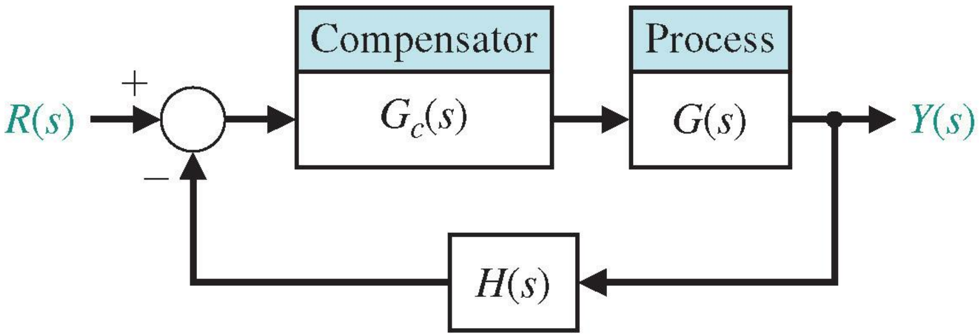
$$\omega_m = \sqrt{zp} = \frac{1}{\tau\sqrt{\alpha}}.$$

$$\sin\phi_m = \frac{\alpha - 1}{\alpha + 1}.$$

# Example 22.2

Let's consider the below system, we assume $G(s) = \frac{10}{s^2}$, and $H(s) = 1$. It is clear that without compensator, the closed-loop system is marginally stable. Design the compensator to satisfy the following specifications using frequency response methods:

1. Settling time (with 2% criterion) $T_s \le 4s$;
2. System damping ratio $\zeta \ge 0.45$.




**Solutions:**

Step 1.

$$T_s = \frac{4}{\zeta\omega_n} = 4;$$

$$\omega_n = \frac{1}{\zeta} = \frac{1}{0.45} = 2.22.$$

$$\omega_B = (-1.19\zeta + 1.85)\omega_n = 3.00$$

→ The bandwidth of the closed-loop system should be larger than 3 rad/s.

Step 2.

The phase margin of the system is required to be approximately

$$\phi_{pm} = \frac{\zeta}{0.01} = \frac{0.45}{0.01} = 45°.$$

The phase margin of the uncompensated system ($G(s)$) is $0^o$ → The compensator needs to provide a phase-lead angle of at least $45^o$ to the loop transfer function.

$$\frac{\alpha - 1}{\alpha + 1} = \sin \phi_m = \sin 45° \longrightarrow \alpha = 5.8$$

We use $\alpha = 6$. The gain added to the system at the frequency $\omega_m$ is

$$10 \log \alpha = 7.78 \text{ dB}$$

We want to have $\omega_m$ equal to the compensated slope crossing the 0 dB axis, thus, the compensated crossover frequency is located by evaluating the frequency where the uncompensated magnitude curve is equal to -7.78 dB. So,

$$\omega_m = 4.95$$

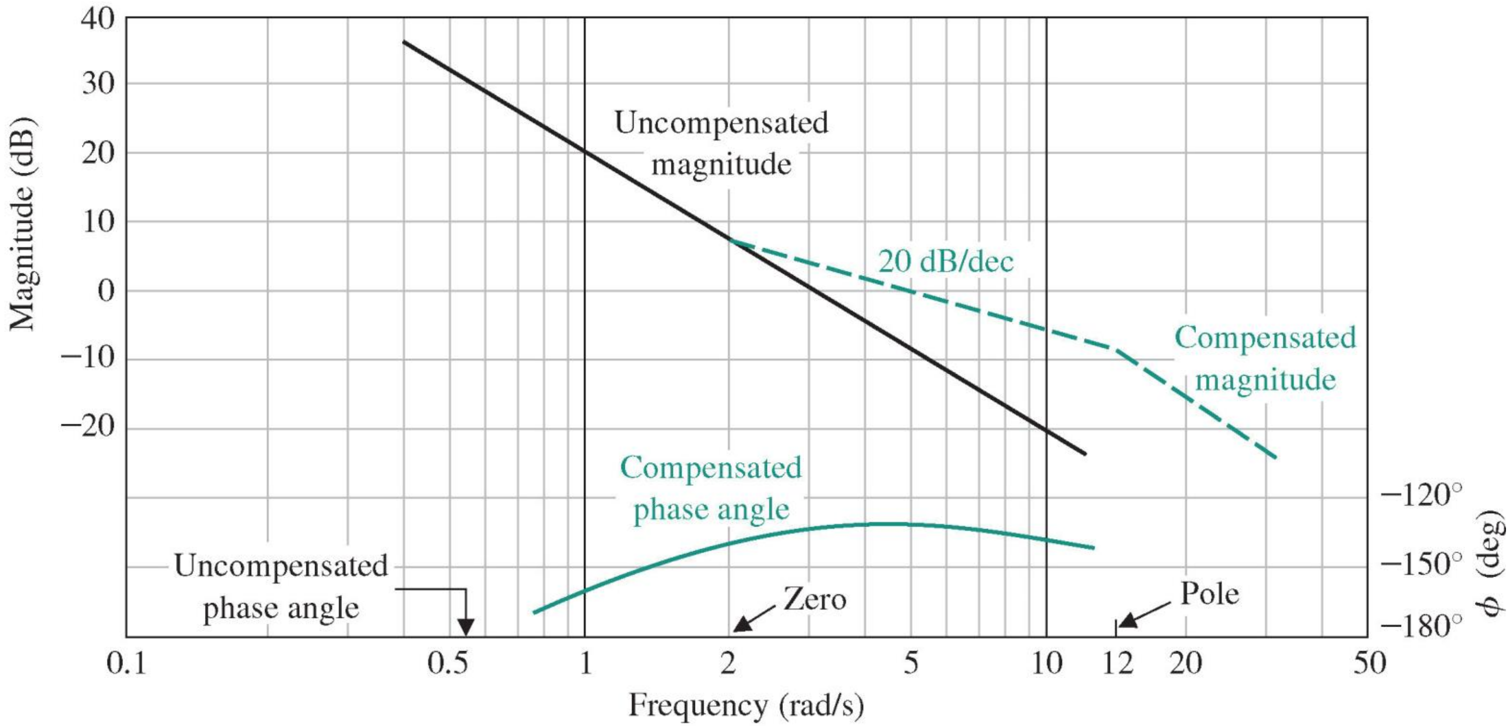
Step 3.

We can now determine the pole and zero of the compensator.

$$p = \omega_m \sqrt{\alpha} = 12.0 \qquad z = p/\alpha = 2.0$$

The compensator is $$G_c(s) = \frac{K}{6}\frac{(1+s/2.0)}{(1+s/12.0)}$$

Choose K=6 to keep the loop gain being still 10.



## Step 4.

Validate, and re-design if the specifications are not satisfied.

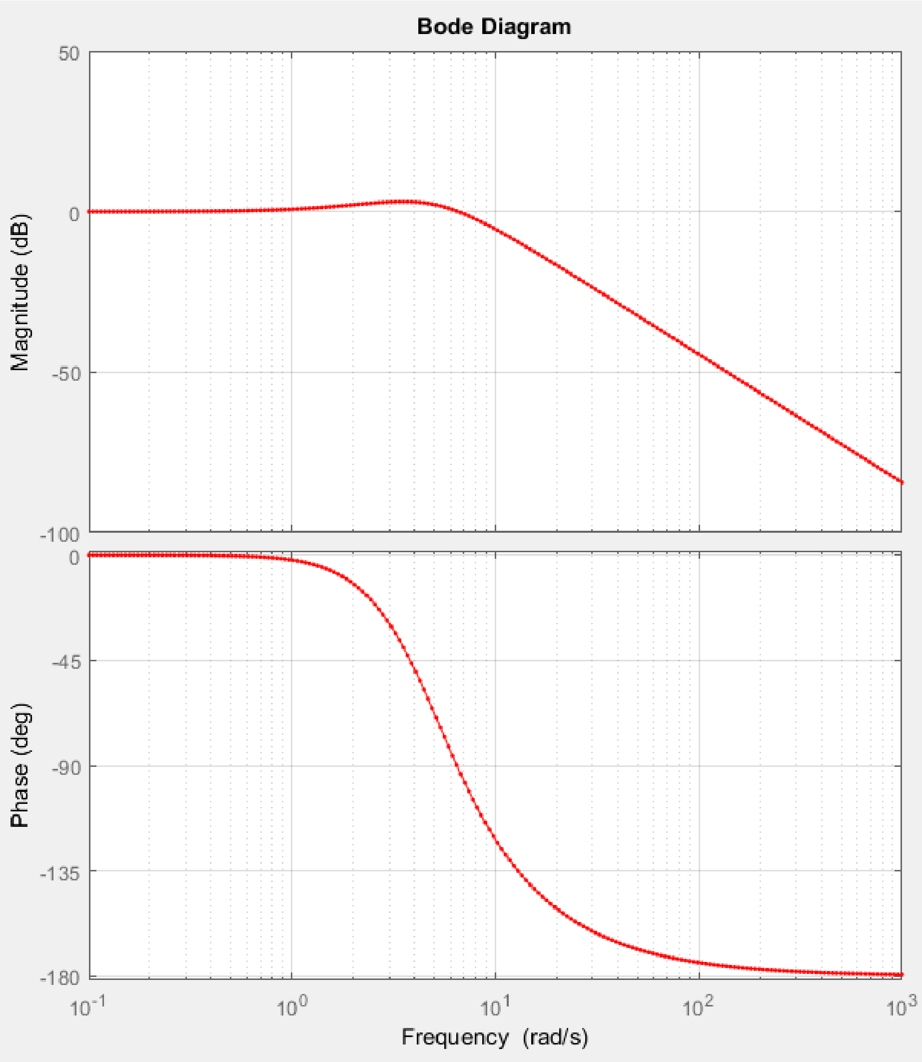
The total loop transfer function is

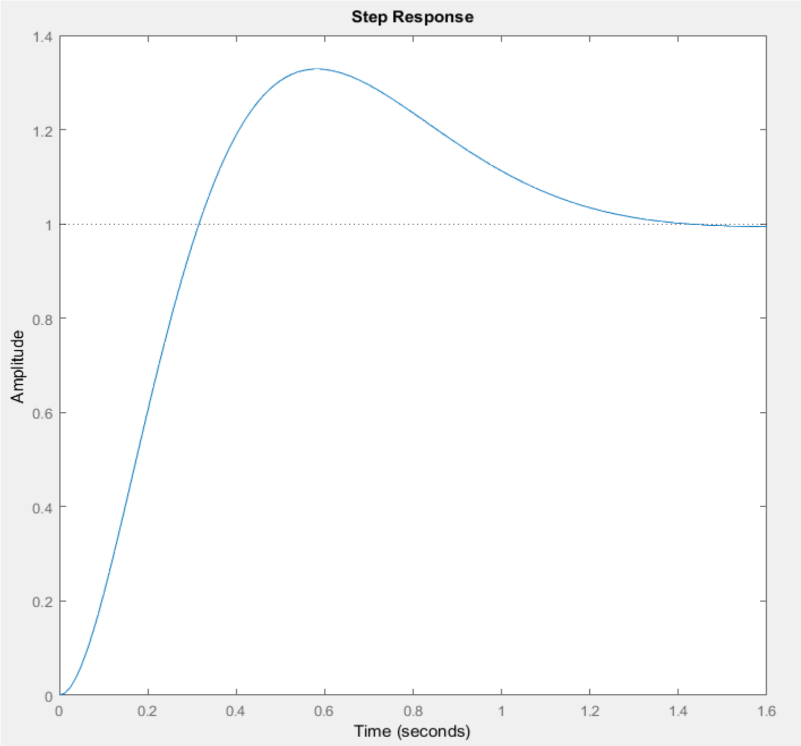
$$L(s) = \frac{10(1 + s/2)}{s^2(1 + s/12)} = \frac{60(s + 2)}{s^2(s + 12)}.$$

The closed-loop transfer function is

$$T(s) = \frac{60(s + 2)}{s^3 + 12s^2 + 60s + 120}$$

Obtain the bode plots for T(s) or step response of the system.

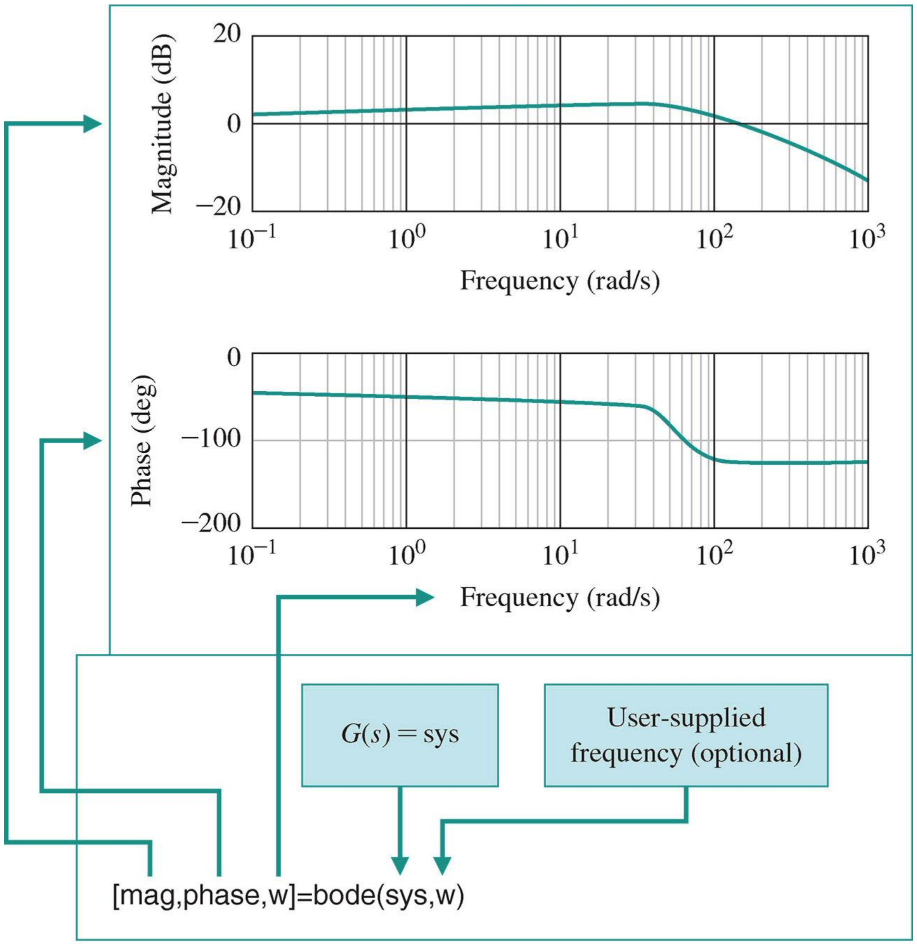


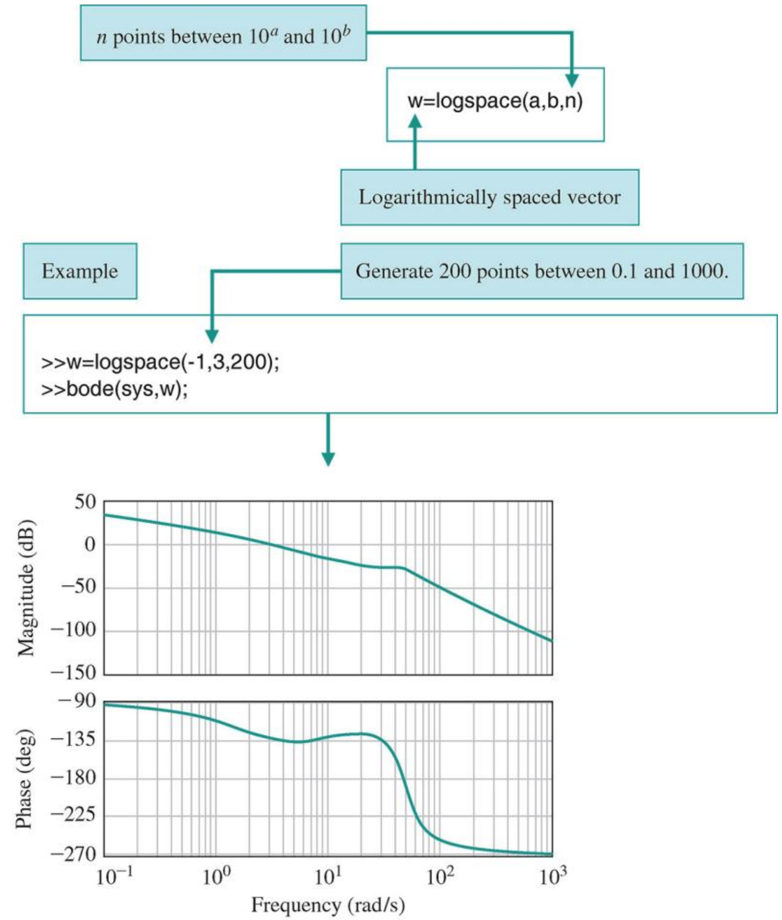

$$T_S = 1.3\ s; P.O. = 34\%;$$
$$\omega_B = 8.4\ rad/s$$

→ The design specifications have been satisfied.

# Frequency Response using Matlab





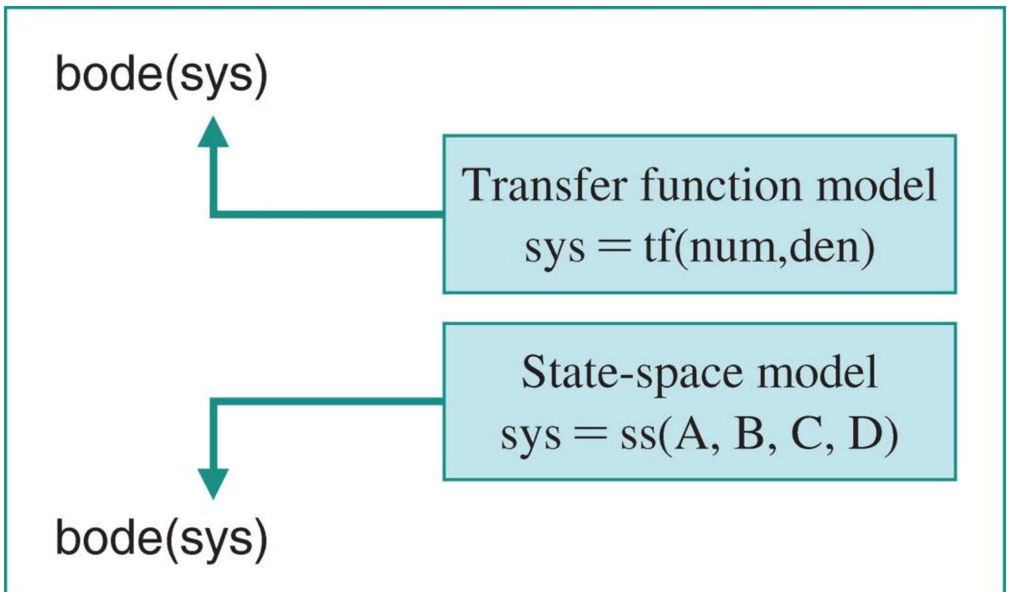



```
% Bode plot script for Figure 8.39
%
num=5*[0.1 1];
f1=[1 0]; f2=[0.5 1]; f3=[1/2500 .6/50 1];
den=conv(f1,conv(f2,f3));
%
sys=tf(num,den);
bode(sys)
```

Compute

$$s(1+0.5s)\left(1+\frac{0.6}{50}s+\frac{1}{50^2}s^2\right)$$





# Thank You !