Xi'an Jiaotong-Liverpool University

EEE108 Electromagnetism and Electromechanics

Lecture 11

Faraday's Law
Magnetic Materials

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Last

* Magnetic flux

$$\Phi_B = \iint B\cos\phi ds = \iint \mathbf{B} \bullet d\mathbf{s}$$

* The magnetic force acting on a moving charge q

$$\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}$$

* The magnetic force acting on a wire

$$\mathbf{F}_B = I\mathbf{L} \times \mathbf{B}$$

* The magnitude of the magnetic force between two straight wires of carrying steady current of I_1 and I_2 and separated by r in free space

$$F_B = \frac{\mu_0 I_1 I_2 l}{2\pi r}$$

* The torque acting on a close loop of wire

$$\mathbf{T} = I\mathbf{A} \times \mathbf{B}$$

* The magnetic dipole moment of a closed loop

$$\mathbf{m} = I\mathbf{A} \quad \text{one turn}$$

$$\mathbf{m} = NI\mathbf{A} \quad \text{N turns}$$

Today

- Faraday's Law of Induction
- Magnetic Materials

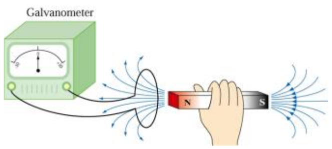
Faraday's Law of Induction

Fourth (Final) Maxwell's Equation

- The electric fields produced by stationary/moving charges.
- The magnetic fields produced by moving charges (currents).
- Imposing an **electric field** on a conductor gives rise to a current which in turn generates a **magnetic field**.
- Whether a magnetic field could produce an electric field?

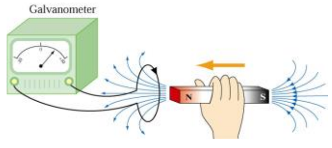
Faraday's Experiments

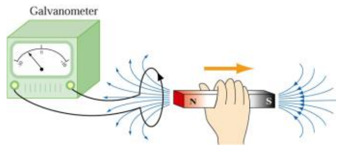
Electromagnetic Induction



No current when bar magnet is stationary with respect to the loop.

The phenomenon is known as ***electromagnetic induction***.





A current is induced when a relative motion exists between the bar magnet and the loop.

Faraday's law

Integral Form

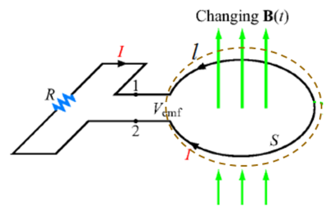
Faraday's law of induction :
The induced emf ε in a coil is proportional to the negative of the rate of change of magnetic flux : $\varepsilon = -\dfrac{d\Phi_B}{dt}$

For a coil with N turns, the total induced emf is N times as large : $\varepsilon = -N\dfrac{d\Phi_B}{dt}$

emf : a "driving force" for current : $\varepsilon = \int \mathbf{E} \bullet d\mathbf{l}$

Faraday's law in integral form

$$\boxed{\varepsilon = \oint_l \mathbf{E} \bullet d\mathbf{l} = -\frac{d\Phi_B}{dt}}$$



Loop in a changing **B** field

Faraday's law

Differential Form

We can write : $\dfrac{d\Phi_B}{dt} = \int_s \dfrac{\partial \mathbf{B}}{\partial t} \bullet d\mathbf{s}$

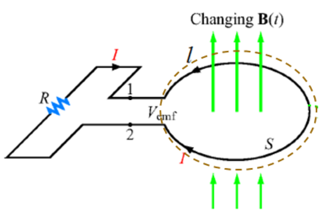
so $\oint_l \mathbf{E} \bullet d\mathbf{l} = -\dfrac{d\Phi_B}{dt} = -\int_s \dfrac{\partial \mathbf{B}}{\partial t} \bullet d\mathbf{s}$

By applying Kelvin - Stokes's theorem :

$$\oint_l \mathbf{E} \bullet d\mathbf{l} = \int_s (\nabla \times \mathbf{E}) \bullet d\mathbf{s} = -\int_s \frac{\partial \mathbf{B}}{\partial t} \bullet d\mathbf{s}$$

$$\boxed{\varepsilon = \oint_l \mathbf{E} \bullet d\mathbf{l} = -\frac{d\Phi_B}{dt}}$$

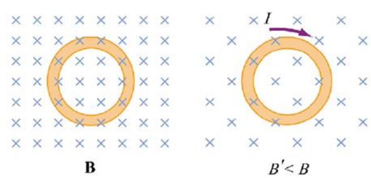
Most Popular: $\boxed{\varepsilon = -\dfrac{d\Phi_B}{dt}}$



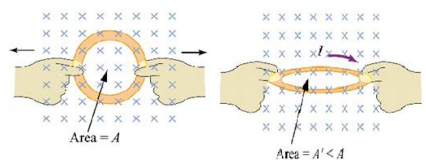
Loop in a changing **B** field

Faraday's Law

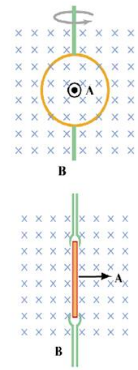
Three Ways of Change of Magnetic Flux



1. Vary the magnitude of **B** with time



2. Vary the magnitude of **A** with time



3. Vary the angle between **B** and **A** with time

Direction of Induced Current

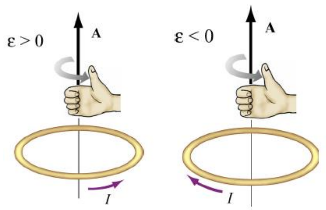
The direction of the **induced current** is determined by **Lenz's law**:

The induced current produces magnetic fields which tend to oppose the change in magnetic flux that induces such currents.

$$\boxed{\varepsilon = -\frac{d\Phi_B}{dt}}$$

$$\frac{d\Phi_B}{dt}: \begin{cases} > 0 \Rightarrow \text{induced emf } \varepsilon < 0 \\ < 0 \Rightarrow \text{induced emf } \varepsilon > 0 \\ = 0 \Rightarrow \text{induced emf } \varepsilon = 0 \end{cases}$$

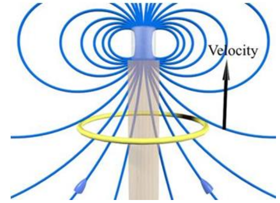
Lenz's law states that the induced emf must be in the direction that opposes the change.



Determination of the direction of induced current by the right-hand rule

Quiz 1

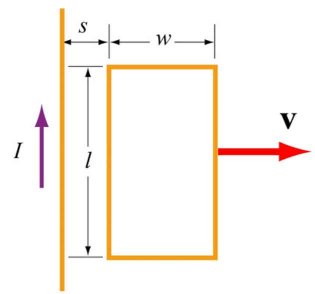
A coil moves up from underneath a magnet with its north pole pointing upward.
The current in the coil and the force on the coil:



1. Current clockwise; force up
2. Current counterclockwise; force up
3. Current clockwise; force down
4. Current counterclockwise; force down

Quiz 2

A rectangular loop of wire is pulled away from a long wire carrying current I in the direction shown in the sketch. The induced current in the rectangular circuit is:



1. Clockwise
2. Counterclockwise
3. Neither, the current is zero

Inductor in a Changing Magnetic Field

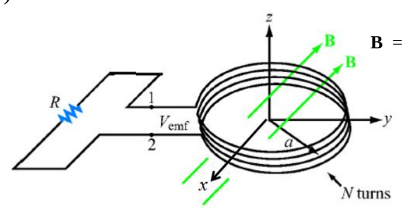
An **inductor** is formed by winding N turns of a thin conducting wire into a circular loop of radius a. The inductor loop is in the x-y plane with its center at the origin, and connected to a resistor R. In the presence of a magnetic field $\mathbf{B} = B_0(2\mathbf{a}_y + 3\mathbf{a}_z)\sin\omega t$, where ωt is the angular frequency.

Find

(a) the magnetic flux linking a single turn of the inductor

(b) V_{emf}, given that $N = 10$, $B_0 = 0.2$ T, $a = 10$ cm and $\omega = 103$ rad/s

(c) the induced current in the circuit for $R = 1$ kΩ (ignore the wire resistance)



Faraday's Law

Inductor in a Changing Magnetic Field

Solution

(a) the magnetic flux linking a single turn of the inductor

$$\Phi = \iint_S \mathbf{B} \bullet d\mathbf{s} = \iint_S [B_0(2\mathbf{a}_y + 3\mathbf{a}_z)\sin\omega t] \bullet \mathbf{a}_z ds = 3\pi a^2 B_0 \sin\omega t$$

(b) V_{emf}, given that N = 10, B_0 = 0.2 T, a = 10 cm and ω = 103 rad/s

$$V_{emf} = -N\frac{d\Phi}{dt} = -N\frac{d}{dt}(3\pi a^2 B_0 \sin\omega t) = -3\pi N\omega a^2 B_0 \cos\omega t$$

For $N = 10,\ a = 0.1\,\text{m},\ \omega = 10^3$ rad/s, and $B_0 = 0.2$ T

$$V_{emf} = -3\pi \times 10 \times 10^3 \times (0.1)^2 \times 0.2\cos 10^3 t \quad = -188.5\cos 10^3 t \quad \text{V}$$

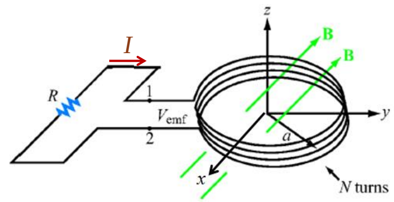
(c) the induced current in the circuit for R = 1 kΩ (ignore the wire resistance)

$$I = \frac{188.5}{1000}\cos 1000t = 0.19\cos 1000t \quad \text{A}$$

Faraday's Law

Inductor in a Changing Magnetic Field

At t = 0, what is the direction of the current?



$$V_{emf} = -N\frac{d\Phi}{dt} = -3\pi N\omega a^2 B_0 \cos\omega t \;\Rightarrow\; \boxed{\frac{d\Phi}{dt} = 3\pi\omega a^2 B_0 \cos\omega t}$$

At $t = 0,\ d\Phi/dt > 0$

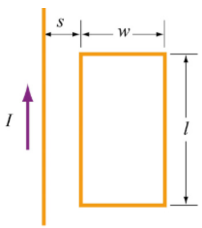
By Lenz's law, the current direction is shown in the Figure.

Induced Electric Field

Rectangular Loop Near a Wire with Changing Current

Without moving the rectangular loop of wire:

Suppose that the current is a function of time with $I(t) = a + bt$, where a and b are positive constants. What is the induced emf in the loop and the direction of the induced current?



Induced Electric Field

Rectangular Loop Near a Wire with Changing Current

Solution

At distance r the magnetic field created by the wire is:

$$B = \frac{\mu_0 I}{2\pi r}$$

B-field changed by changing source current

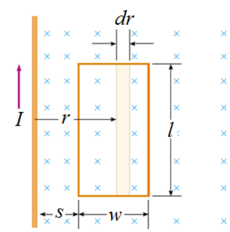
The magnetic flux through an area element $d\mathbf{A}$ is:

$$d\Phi_B = \frac{\mu_0 I}{2\pi r} dA \qquad dA = l\,dr$$

then the total magnetic flux is:

$$\Phi_B = \frac{\mu_0 I l}{2\pi}\int_s^{w+s}\frac{dr}{r} = \frac{\mu_0 I l}{2\pi}\ln r\Big|_s^{w+s}$$

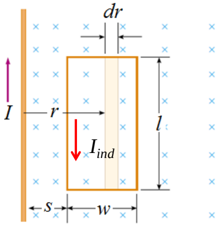
$$= \frac{\mu_0 I l}{2\pi}\ln\left(1 + \frac{w}{s}\right)$$



Induced Electric Field
Rectangular Loop Near a Wire with Changing Current

Solution Cont.

According to Faraday's law, the induced emf is:



$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left[\frac{\mu_0 Il}{2\pi}\ln\left(\frac{s+w}{s}\right)\right]$$

$$= -\frac{\mu_0 l}{2\pi}\ln\left(\frac{s+w}{s}\right)\cdot\frac{dI}{dt} = -\boxed{\frac{\mu_0 bl}{2\pi}\ln\left(\frac{s+w}{s}\right)}$$

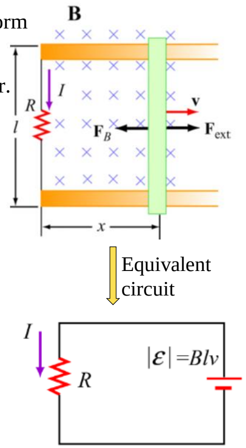
Note: $dI/dt = b$

$$\frac{d\Phi_B}{dt} > 0$$

The straight wire carrying a current I produces a magnetic flux into the page through the rectangular loop. By *Lenz's law*, the induced current in the loop must be flowing counterclockwise in order to produce a magnetic field out of the page to counteract the increase in inward flux.

Motional EMF

The conducting bar moves through a region of uniform magnetic field by sliding along two frictionless conducting rails and connected together by a resistor. If an external force be applied, the conductor moves to the right with a constant velocity.



The magnetic flux through the closed loop

$$\Phi_B = BA = Blx$$

According to Faraday's law, the induced emf is

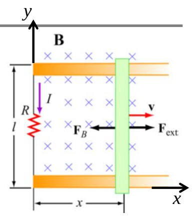
$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(Blx) = -Bl\frac{dx}{dt} = -Blv$$

The induced current with the direction of counterclockwise: $I = \frac{|\varepsilon|}{R} = \frac{Blv}{R}$

Motional EMF

Energy Conservation

$$\boxed{I = \frac{Blv}{R}}$$



The magnetic force acting on the bar:

$$\mathbf{F}_B = I(l\mathbf{a}_y)\times(-B\mathbf{a}_z) = -IlB\mathbf{a}_x = -\left(\frac{B^2l^2v}{R}\right)\mathbf{a}_x$$

which is in the opposite direction of $\mathbf{v}$.

For the bar to move at a constant velocity, the net force acting on it must be zero, so the external agent must supply a force

$$\mathbf{F}_{ext} = -\mathbf{F}_B = +\left(\frac{B^2l^2v}{R}\right)\mathbf{a}_x$$

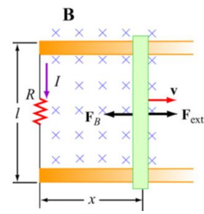
The power delivered by $\mathbf{F}_{ext}$ is equal to the power dissipated in the resistor

Motional EMF

With v_0, without F_{ext}

If at t=0, the speed of the rod is v_0, and the external agent stops pushing, the bar will slow down because of the magnetic force directed to the left.

From Newton's second law



$$F_B = -\frac{B^2l^2v}{R} = ma = m\frac{dv}{dt}$$

or

$$\frac{dv}{v} = -\frac{B^2l^2}{mR}dt = -\frac{dt}{\tau}$$

Where $\tau = \frac{mR}{B^2l^2}$. Upon integration, we obtain

$$v(t) = v_0 e^{-t/\tau}$$

The speed decreases exponentially without an external agent doing work.

Faraday's Law

Faraday's law describes how a time varying magnetic field creates an electric field. This aspect of electromagnetic induction is the operating principle behind many electric generators.

The line integral of the electric field around a closed loop is equal to the negative of the rate of change of the magnetic flux through the area enclosed by the loop.

Faraday's law in integral form

$$\oint \mathbf{E} \bullet d\mathbf{l} = -\frac{d\Phi_B}{dt}$$

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

Faraday's law in differential form

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Up to Now: Maxwell's Equations

	Integral Form	Differential Form
Gauss's law	$\oiint_S \mathbf{D} \bullet d\mathbf{s} = Q_{enc}$	$\nabla \bullet \mathbf{D} = \rho$
Gauss's law for Magnetism	$\oiint_S \mathbf{B} \bullet d\mathbf{s} = 0$	$\nabla \bullet \mathbf{B} = 0$
Ampere's law	$\oint_C \mathbf{H} \bullet d\mathbf{L} = I_{enc}$	$\nabla \times \mathbf{H} = \mathbf{J}$
Faraday's law	$\varepsilon = \oint_C \mathbf{E} \bullet d\mathbf{L} = -\frac{d\Phi_B}{dt}$	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$

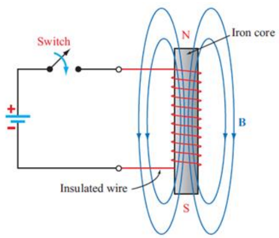
Magnetic Materials

How do magnetic materials affect magnetic field?

$$\mathbf{B}_0 = \mu_0 \mathbf{H}$$

$$\mathbf{B} = \mu \mathbf{H} = \mu_r \mu_0 \mathbf{H}$$

$$\mu_r = ?$$



With magnetic materials, their effect on **B** can be one of the following:

(i) reduce **B** below, $\mathbf{B} < \mathbf{B}_0$ (diamagnetic materials)

(ii) increase **B** a little above, $\mathbf{B} > \mathbf{B}_0$ (paramagnetic materials)

(iii) increase **B** a lot above, $\mathbf{B} >> \mathbf{B}_0$ (***ferromagnetic materials***)

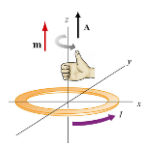
Electron Orbital and Spin Magnetic Moments

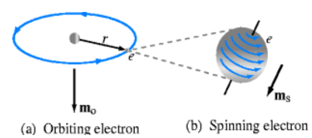
Magnetization in a material substance is associated with atomic current loops generated by two principal mechanisms:

1. Orbital motions of the electrons around the nucleus and similar motions of the protons around each other in the nucleus.
2. Electron spin.

The magnetic moment of the nucleus is much smaller than that of an electron, the total magnetic moment of an atom is dominated by the sum of the magnetic moments of its electrons.

The magnetic moment of a closed loop: $\mathbf{m} = I\mathbf{A}$





An electron generates (a) an orbital magnetic moment $\mathbf{m}_0$ as it rotates around the nucleus, and (b) a spin magnetic moment $\mathbf{m}_s$, as it spins about its own axis.

Magnetic Permeability

The ***magnetization vector*** $\mathbf{M}$ of a material is defined as the vector sum of the magnetic dipole moment of the atoms contained per unit volume.

The magnetic flux density produced by ***magnetization vector*** $\mathbf{M}$ is :

$$\mathbf{B}_{\mathrm{M}} = \mu_0 \mathbf{M}$$

In the presence of an externally applied magnetic field, the total magnetic flux density in the material is :

$$\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_{\mathrm{M}} = \mu_0 \mathbf{H} + \mu_0 \mathbf{M} = \mu_0 (\mathbf{H} + \mathbf{M})$$

In general, a material becomes magnetized in response to the exteral field $\mathbf{H}$, $\mathbf{M}$ can be expressed as :

$$\mathbf{M} = \chi_m \mathbf{H}$$

χ_m is a dimensionless quantity called the ***magnetic susceptibility.***

Then $\mathbf{B} = \mu_0 (\mathbf{H} + \chi_m \mathbf{H}) = \mu_0 (1 + \chi_m) \mathbf{H} = (1 + \chi_m) \mathbf{B}_0$

$$\boxed{\mathbf{B} = (1 + \chi_m) \mathbf{B}_0}$$

Magnetic Permeability

$$\mathbf{B} = \mu_0 (\mathbf{H} + \chi_m \mathbf{H}) = \mu_0 (1 + \chi_m) \mathbf{H} = (1 + \chi_m) \mathbf{B}_0$$
$$\mathbf{B} = \mu \mathbf{H}$$

Relative permeability of the material is defined as :

$$\mu_r = 1 + \chi_m$$

Magnetic Permeability of the material is defined as :

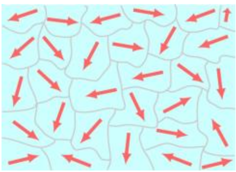
$$\mu = \mu_0 \mu_r = \mu_0 (1 + \chi_m) \quad \text{SI unit : H/m}$$

Paramagnetism	Diamagnetism
$\mu_r > 1$	$\mu_r < 1$
$\chi_{\mathrm{m}} > 0 \ (10^{-6} \sim 10^{-3})$	$\chi_{\mathrm{m}} < 0 \ (-10^{-9} \sim -10^{-5})$
Aluminum, Calcium, Magnesium, Tungsten	Copper, Diamond, Gold, Silver, Lead

Magnetic Permeability

Ferromagnetic Material

Ferromagnetic materials : iron, nickel, cobalt, …

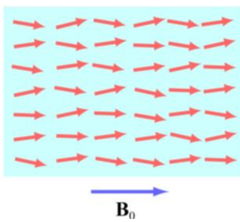


Unmagnetized domains

$\chi_m >> 1, \ \mu_r >> 1, \quad \Rightarrow \boxed{\mu >> \mu_0}$

Without an external magnetic field, the domains take on random orientations, resulting in no net magnetization.

Magnetized domains: A magnetized domain of a material is a microscopic region (on the order of 10^{-10} m^{3}) within which the magnetic moments of all its atoms (typically on the order of 10^{19} atoms) are aligned parallel to each other.



Magnetized domains

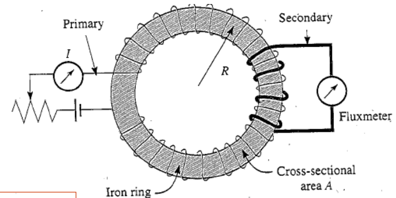
Magnetic Materials

B-H Curve

Recall $\mathbf{B} = \mu_0 \mathbf{H}$ free space
$\mathbf{B} = \mu \mathbf{H} = \mu_0 \mu_r \mathbf{H}$

The line or curve showing B as a function of H is called a magnetization curve or B-H curve.

To measure a magnetization curve for a ferromagnetic substances, say, for an iron sample, a ring may be cut from the sample to form an iron-cored toroid.



Recall Using Ampere's law :

$$\oint \mathbf{B} \bullet d\mathbf{L} = B \oint dl = B(2\pi r) = \mu_0 \mu_r NI$$

$$B = \frac{\mu_0 \mu_r NI}{2\pi r}$$

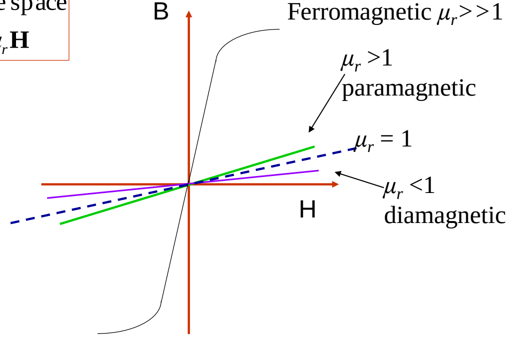
$$\Rightarrow H = \frac{NI}{2\pi R}$$

N is the turns in the primary side.

Magnetic Materials

B-H Curve

$$\mathbf{B}_0 = \mu_0 \mathbf{H} \quad \text{free space}$$
$$\mathbf{B} = \mu \mathbf{H} = \mu_0 \mu_r \mathbf{H}$$



Magnetic Materials

Magnetization Curves

The relative permeability at any point on the magnetization curve is:

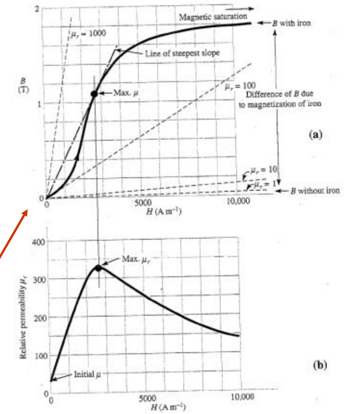
$$\mu_r = \frac{B}{\mu_0 H} = 7.96 \times 10^5 \frac{B}{H} \quad \text{dimensionless}$$

where B = ordinate of the point, T

H = abscissa of the point, A/m

Note: the relative permeability is not proportional to the slope of the curve (dB/dH) but to the ratio B/H.

Typical initial magnetization curve (ferromagnetic)



Magnetic Materials

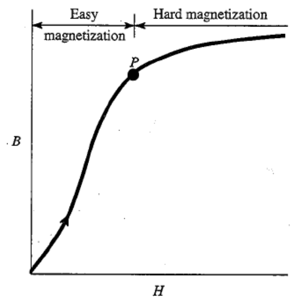
Saturation Magnetization

Is there a limit to the magnetization that a given material can reach?
YES!

The initial magnetization curve may be divided into two sections:

1. The steep section – easy (steep) magnetization
2. The flat section – hard (flat) magnetization

Very strong magnetic fields are required to reach the state of saturation, where all the moments of magnetic domains in the material are parallel to **H** and the magnetization curve flattens off completely.



A typical initial magnetization curve for a ferromagnetic sample, where the material is completely demagnetized and both B and H are zero before a field is applied.

Magnetic Materials

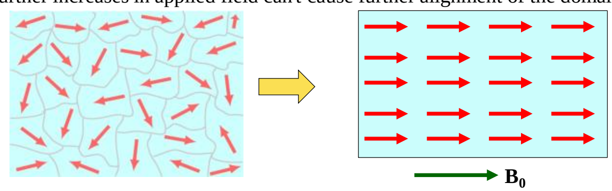
Saturation Magnetization

Phenomenon:

Saturation is the state reached when an increase in applied external magnetizing field **H** **cannot increase the magnetization of the material further**, so the total magnetic field **B** levels off. It is a characteristic particularly of ferromagnetic materials, such as iron, nickel, cobalt and their alloys.

Explanation:

Saturation occurs when practically all the magnetic domains are lined up, so further increases in applied field can't cause further alignment of the domains.

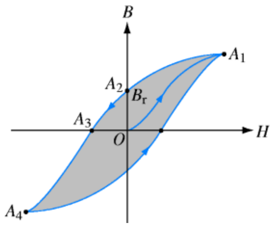


Unmagnetized domains Magnetized domains

Magnetic Materials

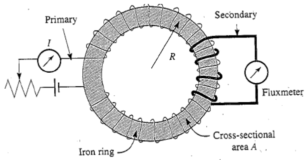
Magnetic Hysteresis

Having reached saturation, where we continue the experiment by reducing I and H. We cannot retrace the initial-magnetization curve – the effects of hysteresis begin to show. Hysteresis means that B lags behind H, so the magnetization curves for increasing and decreasing the applied field are not the same. The "tour" of what is called a ***hysteresis loop***.



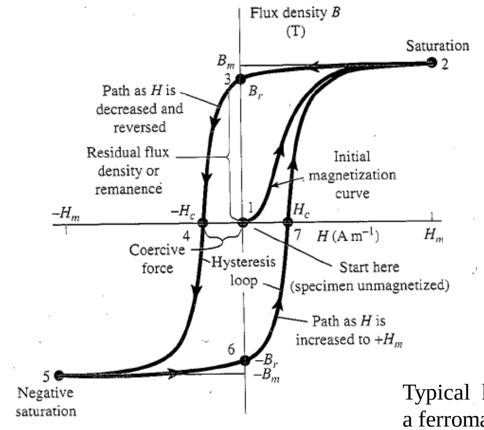
Ferromagnetic materials have some 'residual magnetism B_r' even when the current is zero.

The magnetization process in ferromagnetic materials depends not only on the external magnetic field, but on the magnetic history as well.



Magnetic Materials

Magnetic Hysteresis

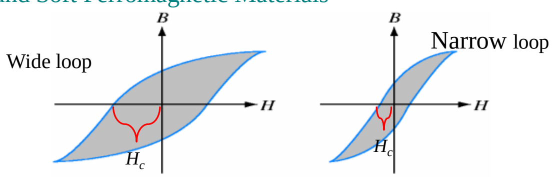


Typical hysteresis loop for a ferromagnetic material

Magnetic Materials

Magnetic Hysteresis

Hard and Soft Ferromagnetic Materials



(a) Hard material (b) Soft material

Hard materials have large coercive forces H_c and broad (fat) hysteresis loops.

They cannot be easily demagnetized by an external magnetic field.

Used in the fabrication of permanent magnets.

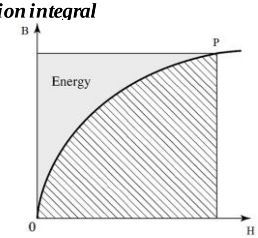
Soft materials have small coercive forces H_c and therefore narrow (thin) hysteresis loops.

Soft materials can be more easily magnetized and demagnetized by an external magnetic field.

Energy in a Magnet

The magnetic energy w_m per unit volume of a magnet brought to saturation from an originally unmagnetized condition is given by the integral of the initial-magnetization curve expressed by:

$w_m = \int_0^B H dB \qquad (\mathrm{J/m^3})$ ***Initial magnetization integral***

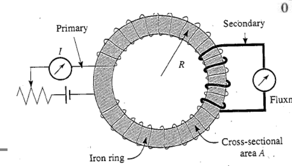


The dimensional relation :

$$H : \frac{\mathrm{A}}{\mathrm{m}}$$

$$B : \mathrm{T} = \frac{\mathrm{N}}{\mathrm{A \cdot m}} = \frac{\mathrm{N \cdot m}}{\mathrm{A \cdot m^2}} = \frac{\mathrm{J}}{\mathrm{A \cdot m^2}}$$

$$HB : \frac{\mathrm{A}}{\mathrm{m}} \times \frac{\mathrm{J}}{\mathrm{A \cdot m^2}} = \mathrm{J/m^3}$$



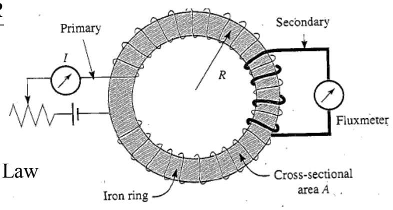
Energy in a Magnet

$$H = \frac{NI}{2\pi R} \Rightarrow I = \frac{H 2\pi R}{N}$$

$$e = -N\frac{d\Phi_B}{dt}$$

Ampere's Law

Faraday's Law



$$\int_0^t e \cdot I \cdot dt = \int_0^t N\frac{d\Phi_B}{dt} \cdot \frac{H \cdot L}{N} dt = \int_0^{\Phi_B} H \cdot L d\Phi_B = \int_0^B H \cdot L \cdot A dB = V\int_0^B H dB$$

where $V = L \cdot A$, the volume of material in the ring

Then the density of energy is :

$$w_m = \int_0^B H dB \quad \text{J/m}^3$$

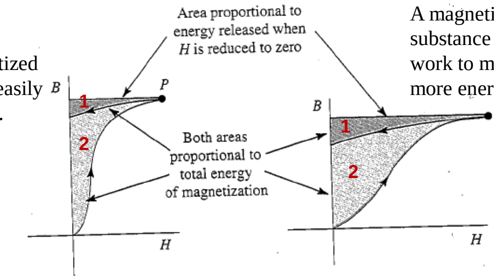
Energy in a Magnet

The area between the cure and the *B* axis is a measure of the energy density.

The upper darker areas show that energy is released when bringing *H* to zero.

Soft magnetized material is easily magnetized.

A magnetically hard substance takes more work to magnetize as more energy needed.



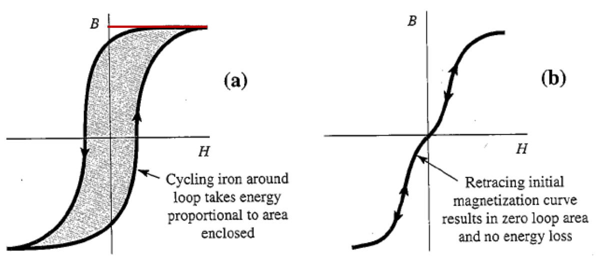
Soft magnetized material

Hard magnetized material

Energy in a Magnet

The energy proportional to the area of the loop is lost (appears as heat).

If no hysteresis is present and the initial magnetization curve is retraced, the area of the loop is zero. This magnetization-demagnetization process is then accomplished with no loss of energy as heat in the magnet.



Summary of Magnetic Materials

Magnetic materials affect B:

❖Diamagnetic-- reduce **B**

❖Paramagnetic -- increase **B** a little

❖Ferromagnetic -- increase **B** a lot -- magnetized domains

Ferromagnetic materials:

❖B-H curve

❖Saturation magnetization

❖Magnetic hysteresis -- hysteresis loop – hard and soft ferromagnetic materials

❖Energy in a magnet -- energy lost

Next Lecture

- Inductors and Inductance
  - Energy Stored in Magnetic Fields
  - *RL* Circuit

Thanks for your attendance