EEE104 – Digital Electronics (I)

Lecture 8

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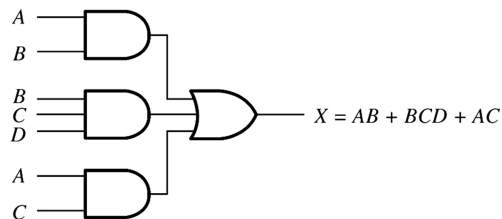
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In This Session

- Boolean Algebra and Logic Simplification
  - Standard Form of Boolean Algebra

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Standard Form of Boolean Expression

The Sum-of-Products (SOP) Form

- A **sum-of-products (SOP)** is the sum of product terms by Boolean addition, e.g. $AB + \bar{B}C\bar{D}$

- The **domain** of a Boolean expression is the set of variables in the expression, e.g. A, B, C, D.

- An SOP can be implemented by AND-OR logic.

$$X = AB + BCD + AC$$

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Standard Form of Boolean Expression

The Sum-of-Products (SOP) Form

- A **standard SOP** expression is one in which **all** the variables in the domain appear in each product term, e.g. $A\bar{B}CD + \bar{A}\bar{B}C\bar{D} + AB\bar{C}D$

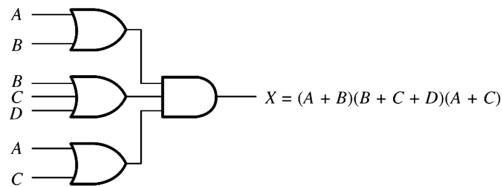- A general SOP can be converted to standard SOP by repeatedly using Boolean algebra rule 6 $(A + \bar{A} = 1)$

$$\bar{A}\bar{B} + AB\bar{C}D$$
$$= \bar{A}\bar{B}(C + \bar{C}) + AB\bar{C}D$$
$$= \bar{A}\bar{B}C(D + \bar{D}) + \bar{A}\bar{B}\bar{C}(D + \bar{D}) + AB\bar{C}D$$
$$= \bar{A}\bar{B}CD + \bar{A}\bar{B}C\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D} + AB\bar{C}D$$

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Standard Form of Boolean Expression

The Product-of-Sums (POS) Form

- A **product-of-sums (POS)** is the product of sum terms by Boolean multiplication, e.g. $(A+B)(\overline{B}+C+\overline{D})$

- A POS expression can be implemented by OR-AND logic.

$$X = (A + B)(B + C + D)(A + C)$$

Standard Form of Boolean Expression

The Product-of-Sums (POS) Form

- A **standard POS** expression is one in which **all** the variables in the domain appear in each sum term, e.g. $(A+\overline{B}+C+D)(\overline{A}+\overline{B}+C+\overline{D})(A+B+\overline{C}+\overline{D})$

- A general POS can be converted to standard POS by repeatedly using Boolean algebra rule 8 $(A\cdot\overline{A}=0)$ and rule 12 $(A+B)(A+C)=A+BC$

$$(\overline{A}+\overline{B})(A+B+\overline{C})$$
$$=(\overline{A}+\overline{B}+C\overline{C})(A+B+\overline{C})$$
$$=(\overline{A}+\overline{B}+C)(\overline{A}+\overline{B}+\overline{C})(A+B+\overline{C})$$

Boolean Expressions: A Review

- A **literal** is a variable or its complement.

$$A\overline{B}CD + \overline{A}\overline{B}C\overline{D} + \overline{A}B\overline{C}D$$

- A **product term** is the product of literals.

- A **sum-of-products (SOP)** is the sum of product terms.

- A **standard SOP** is one in which all variables appear in each product term.

Boolean Expressions: A Review

- A **sum term** is the sum of literals.

$$(A + B + \overline{C})(\overline{A} + B + C)(\overline{A} + \overline{B} + C)$$

- A **product-of-sums (POS)** is the product of sum terms.

- A **standard POS** is one in which all variables appear in each sum term.

Standard Form of Boolean Expression

Binary Representation

- The **binary value of a standard product term** is one which makes the product term equal to 1, e.g. it is 1010 for $A\overline{B}C\overline{D}$

- The **binary value of a sum term** is one which makes the sum term equal to 0, e.g. it is 0101 for $A+\overline{B}+C+\overline{D}$

Boolean Expressions and Truth Tables

Converting SOP to Truth Table

1. Convert the SOP to standard form.

2. Find the binary values for each product term.

3. Set its output to 1.

$$\overline{A}\,\overline{B}C + A\overline{B}\,\overline{C} + ABC$$

Inputs			Output
A	B	C	X
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Boolean Expressions and Truth Tables

Converting POS to Truth Table

$$(A + B + C)(A + \overline{B} + C)(A + \overline{B} + \overline{C})(\overline{A} + B + \overline{C})(\overline{A} + \overline{B} + C)$$

1. Convert the POS to standard form.

2. Find the binary value for each sum term.

3. Set its output to 0.

Inputs			Output
A	B	C	X
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Standard Form of Boolean Expression

Converting Standard SOP to Standard POS

$$\overline{A}\,\overline{B}\,\overline{C} + \overline{A}B\overline{C} + \overline{A}BC + A\overline{B}C + ABC$$

- Determine the binary values which represent the product terms, e.g. $000 + 010 + 011 + 101 + 111$

- Determine all of the binary values not included, e.g. $001, 100,$ and $110.$

- Write the equivalent sum terms in POS form, e.g.

$$(A + B + \overline{C})(\overline{A} + B + C)(\overline{A} + \overline{B} + C)$$

Boolean Expressions and Truth Tables

Determine SOP expressions from Truth Tables

1. List the binary values of the input variables for which the output is 1.

2. Replace each 1 with the variable and each 0 with the variable complement.

Inputs			Output
A	B	C	X
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

$$011 \longrightarrow \overline{A}BC$$
$$100 \longrightarrow A\overline{B}\,\overline{C}$$
$$110 \longrightarrow AB\overline{C}$$
$$111 \longrightarrow ABC$$

$$X = \overline{A}BC + A\overline{B}\,\overline{C} + AB\overline{C} + ABC$$

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Boolean Expressions and Truth Tables

Determine POS expressions from Truth Tables

1. List the binary values of the input variables for which the output is 0.

2. Replace each 1 with the variable complement and each 0 with the variable.

Inputs			Output
A	B	C	X
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

$$000 \longrightarrow A + B + C$$
$$001 \longrightarrow A + B + \overline{C}$$
$$010 \longrightarrow A + \overline{B} + C$$
$$101 \longrightarrow \overline{A} + B + \overline{C}$$

$$X = (A + B + C)(A + B + \overline{C})$$
$$(A + \overline{B} + C)(\overline{A} + B + \overline{C})$$

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