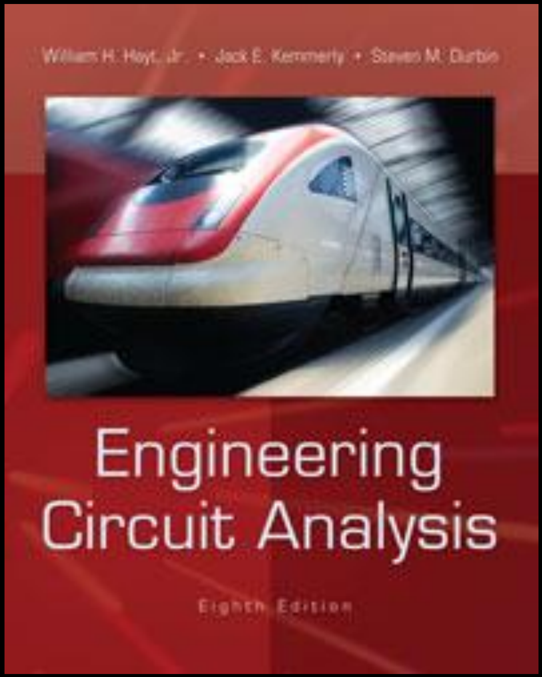
Chapter 5
Handy Circuit Analysis Techniques



Linear Elements and Circuits

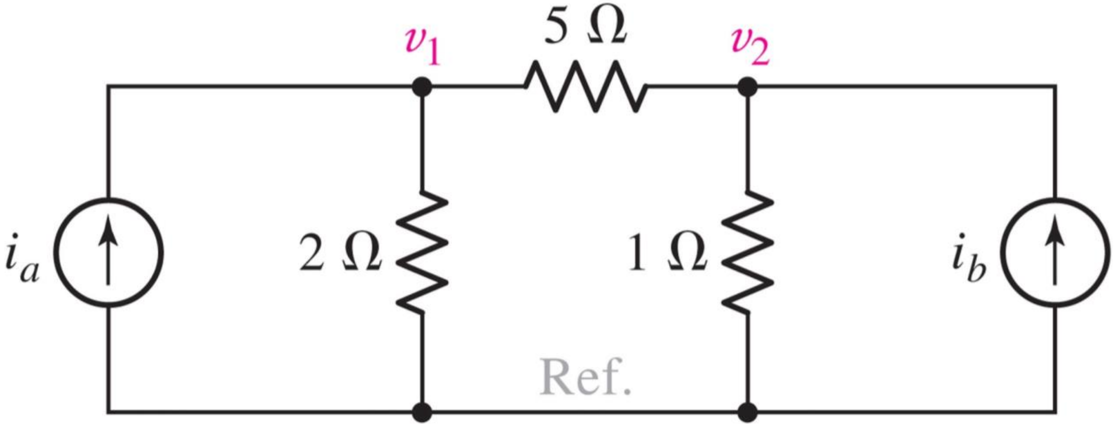
- a linear circuit element has a linear voltage-current relationship:
  - if $i(t)$ produces $v(t)$, then $Ki(t)$ produces $Kv(t)$
  - if $i_1(t)$ produces $v_1(t)$ and $i_2(t)$ produces $v_2(t)$, then $i_1(t) + i_2(t)$ produces $v_1(t) + v_2(t)$,
- resistors, sources are linear elements[1]
- a linear circuit is one with only linear elements

[1]Dependent sources need linear control equations to be linear elements.

The Superposition Concept

For the circuit shown, the solution can be expressed as:

$$\begin{bmatrix} 0.7 & -0.2 \\ -0.2 & 1.2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} i_a \\ i_b \end{bmatrix}$$



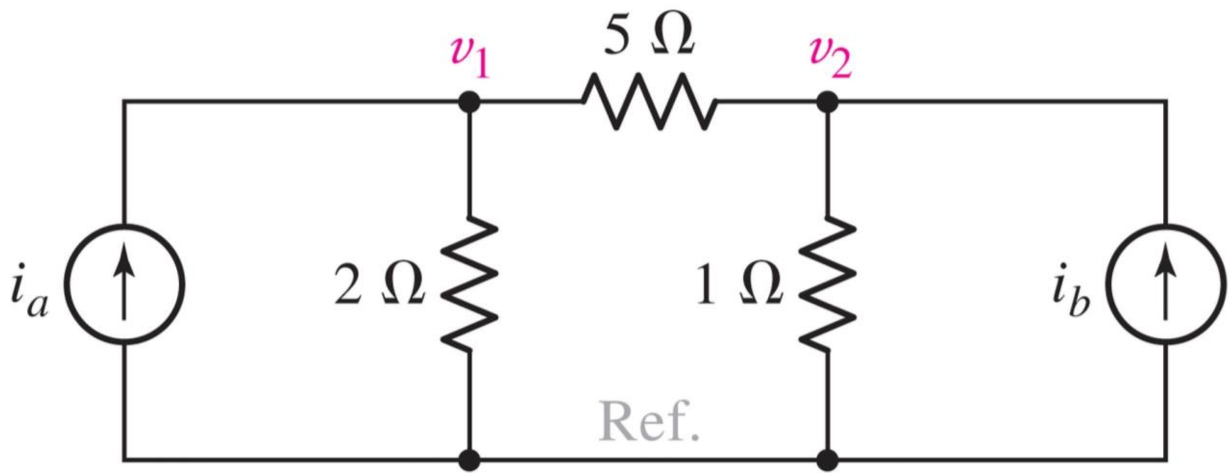
Question: How much of v_1 is due to source A, and how much is because of source B?

We use the superposition principle to answer.

The Superposition Concept

If we define A as

$$A = \begin{bmatrix} 0.7 & -0.2 \\ -0.2 & 1.2 \end{bmatrix}$$



then

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = A^{-1}\begin{bmatrix} i_a \\ i_b \end{bmatrix} = A^{-1}\begin{bmatrix} 0 \\ i_b \end{bmatrix} + A^{-1}\begin{bmatrix} i_a \\ 0 \end{bmatrix}$$

Experiment B

Experiment A

Superposition:
the response is the sum
of experiments A and B.

The Superposition Theorem

In a linear network, the **voltage across** or the **current through** any element may be calculated by *adding algebraically* all the individual voltages or currents caused by the separate independent sources acting “alone”, i.e. with

- all other independent voltage sources replaced by short circuits and
- all other independent current sources replaced by open circuits.

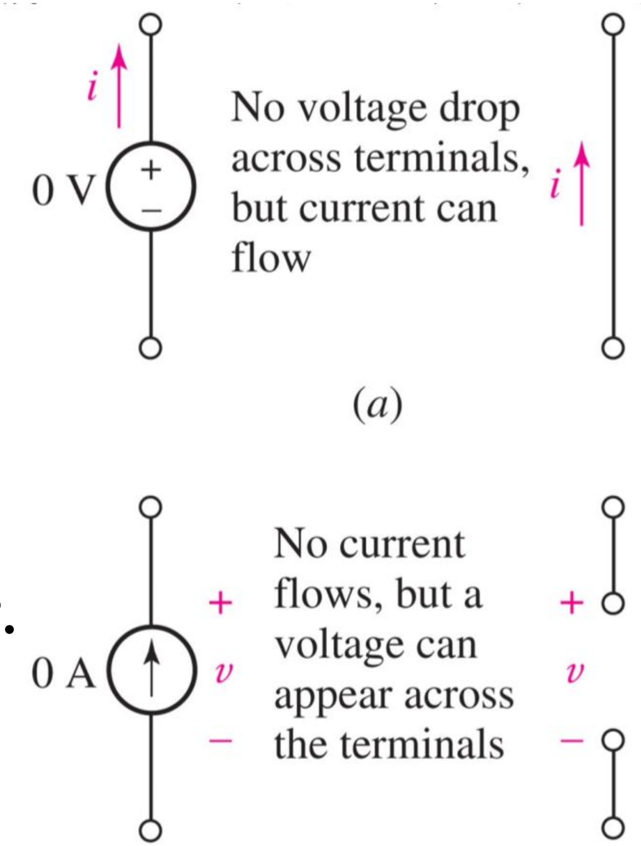
Applying Superposition

- Leave one source ON and turn all other sources OFF:
  - voltage sources: *set v=0.*

    These become *short circuits*.
  - current sources: *set i=0.*

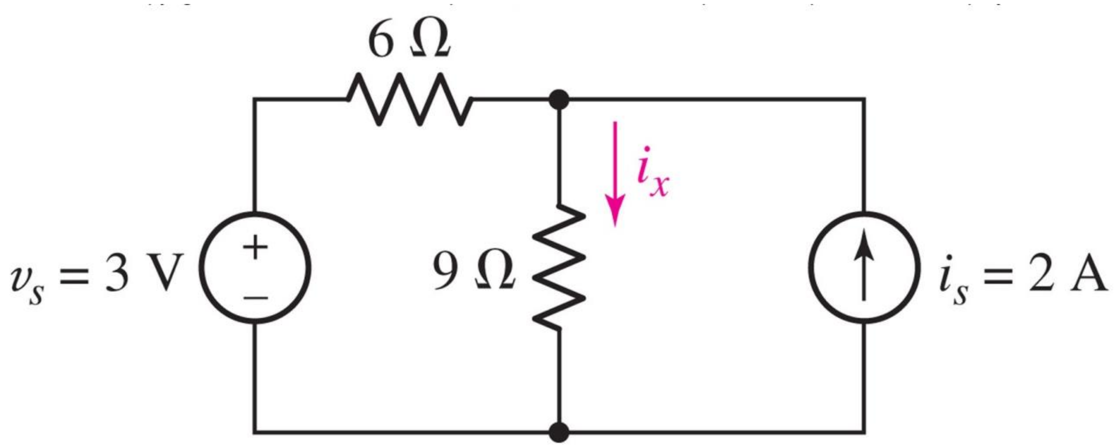
    These become *open circuits*.

    Find the response from this source.
- Add the resulting responses to find the total response.

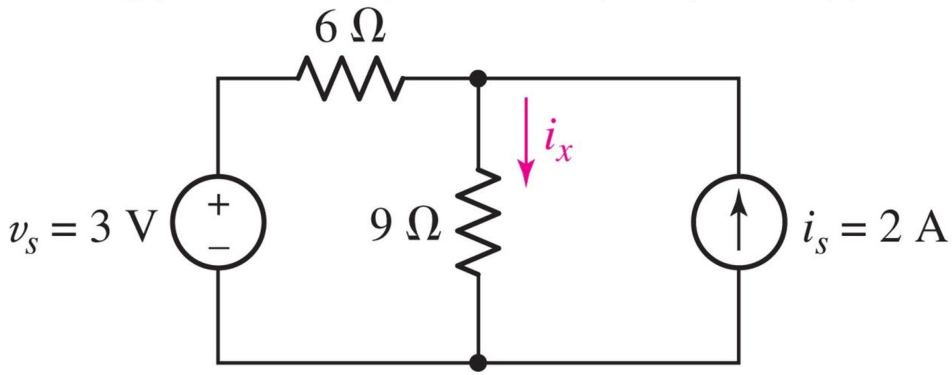


Superposition Example (part 1 of 4)

Use superposition to solve for the current i_x

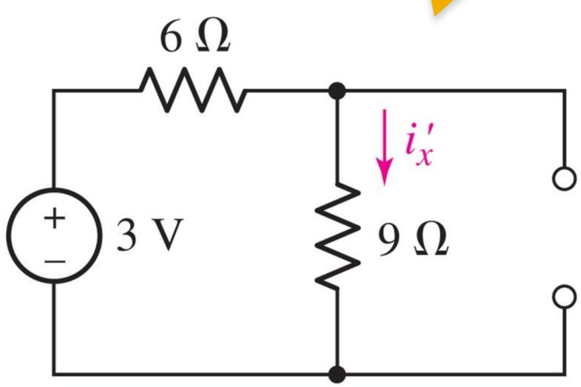


Superposition Example (part 2 of 4)

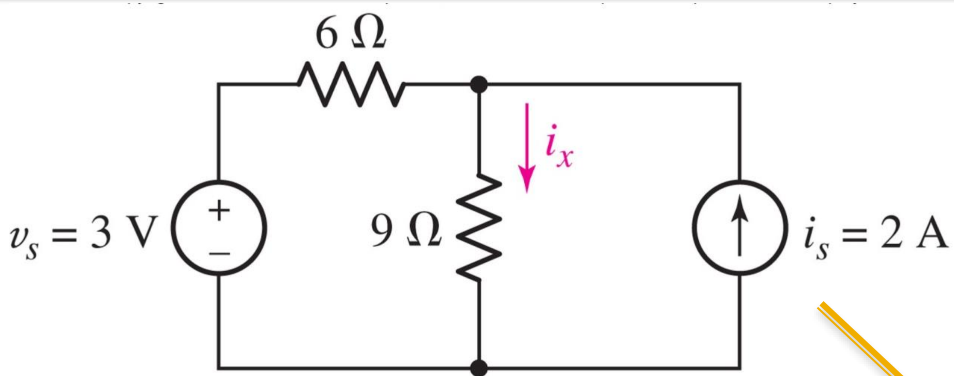


First, turn the current source off:

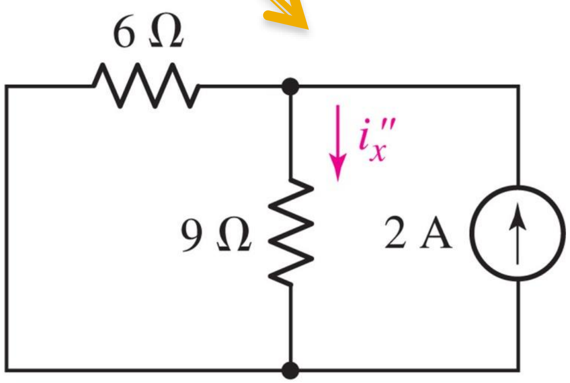
$$i_x' = \frac{3}{6+9} = 0.2$$



Superposition Example (part 3 of 4)

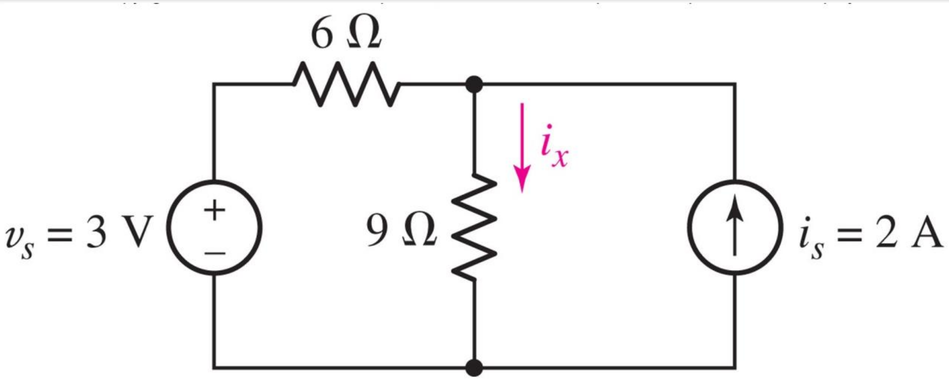


Then, turn the voltage source off:



$$i_x'' = \frac{6}{6+9}(2) = 0.8$$

Superposition Example (part 4 of 4)

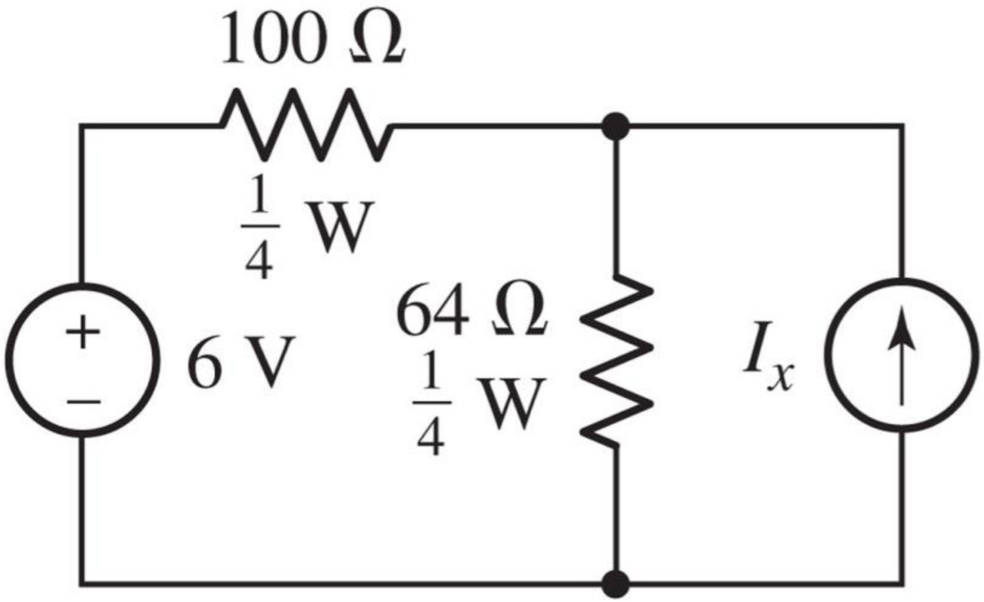


Finally, combine the results:

$$i_x = i_x' + i_x'' = 0.2 + 0.8 = 1.0$$

Example: Power Ratings

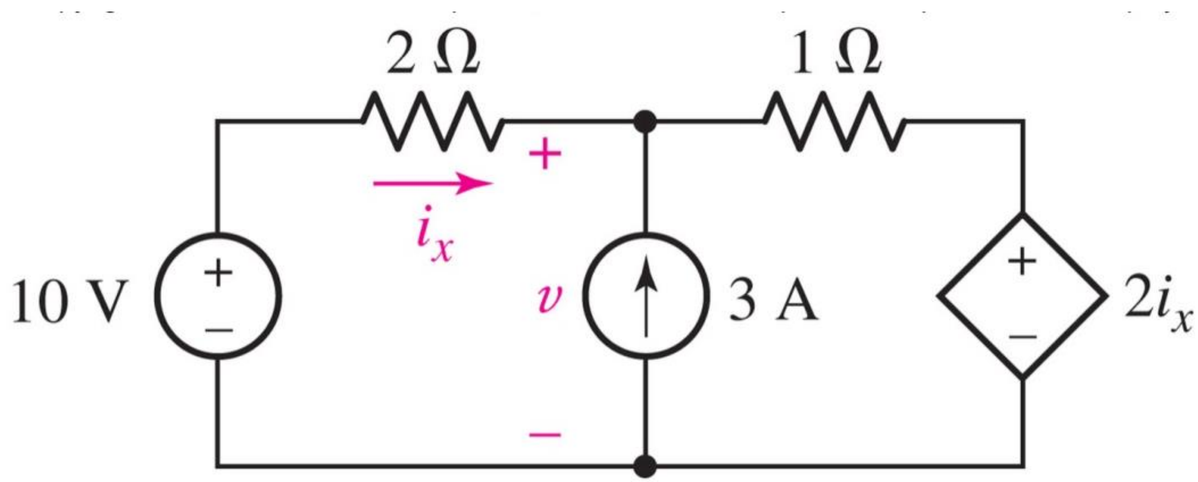
Determine the maximum *positive* current to which the source I_x can be set before any resistor exceeds its power rating.



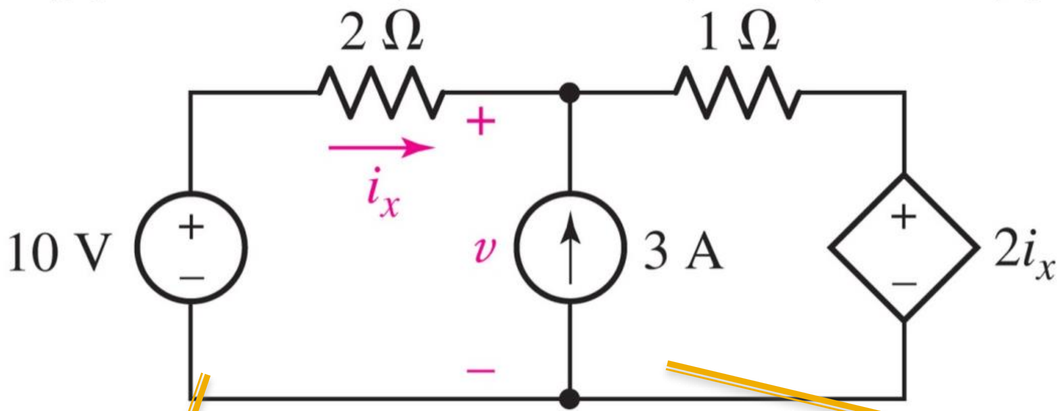
Answer: $I_x < 42.49$ mA

Superposition with a Dependent Source

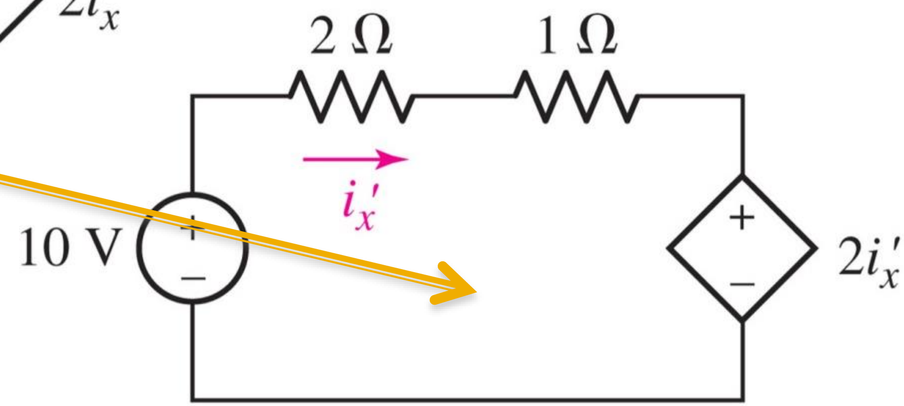
- When applying superposition to circuits with *dependent* sources, these *dependent* sources are never "turned off."



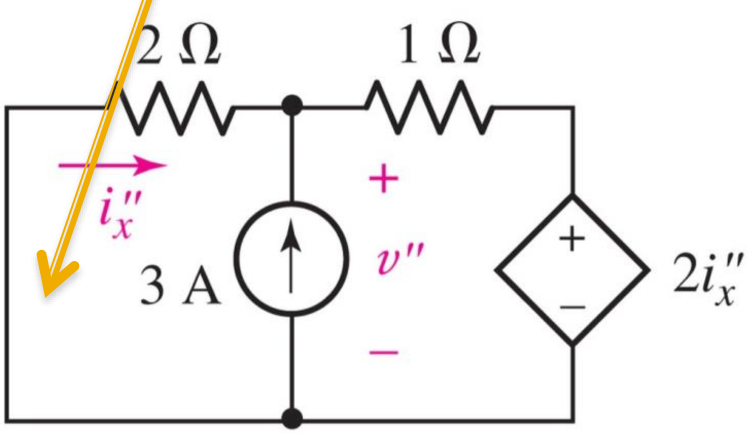
Superposition with a Dependent Source



current source off



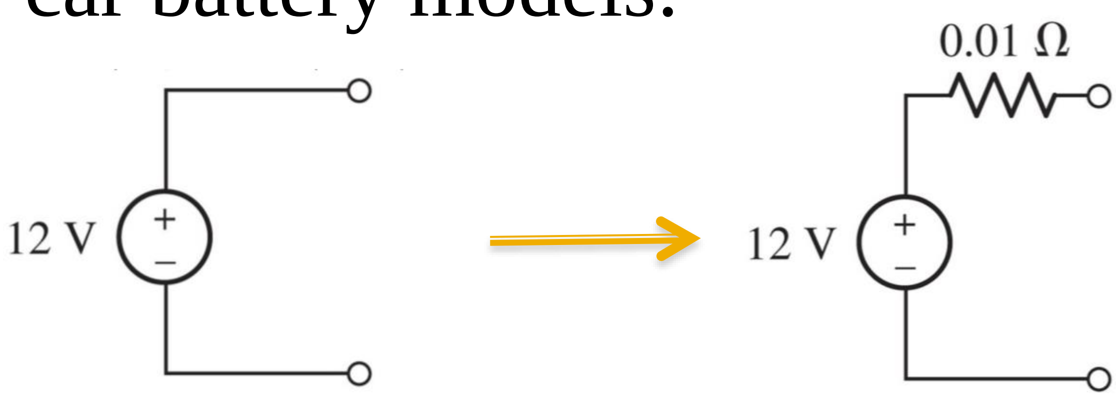
voltage source off



$$i_x = i_x' + i_x'' = 2 + (-0.6) = 1.4\ \text{A}$$

Practical Voltage Sources

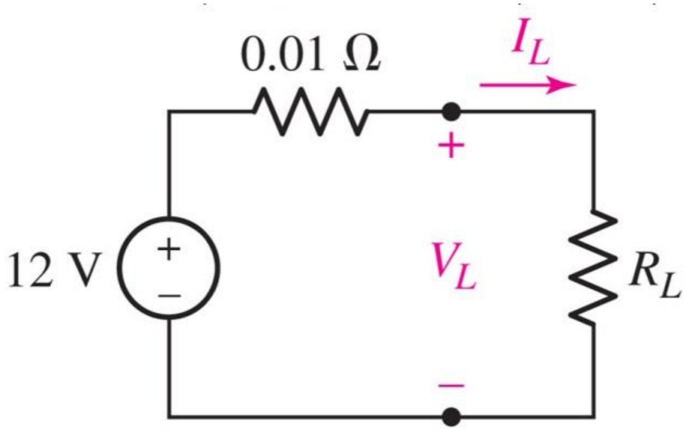
- Ideal voltage sources: a first approximation model for a battery.
- Why do real batteries have a current limit and experience voltage drop as current increases?
- Two car battery models:

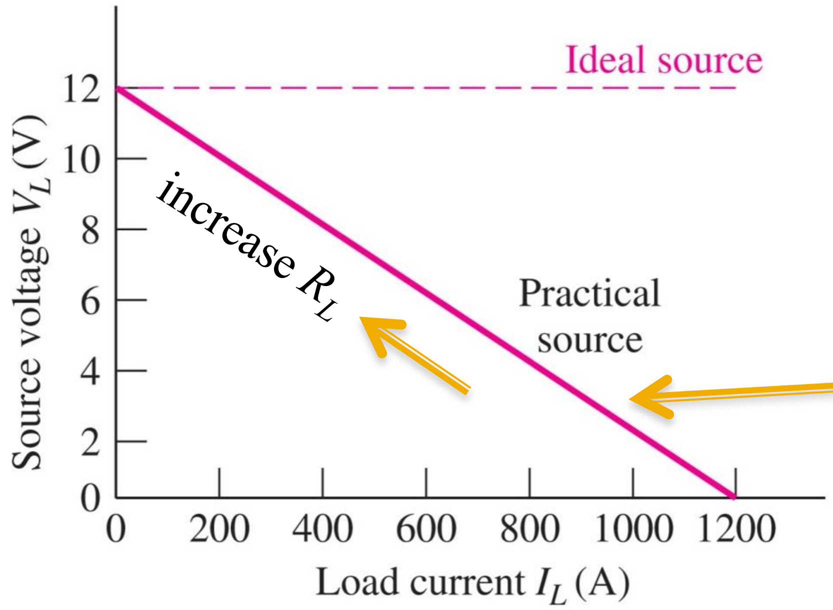


Practical Source: Effect of Connecting a Load

For the car battery example:

$$V_L = 12 - 0.01\ I_L$$

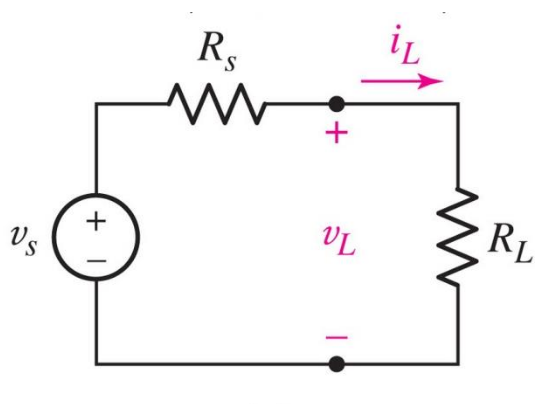


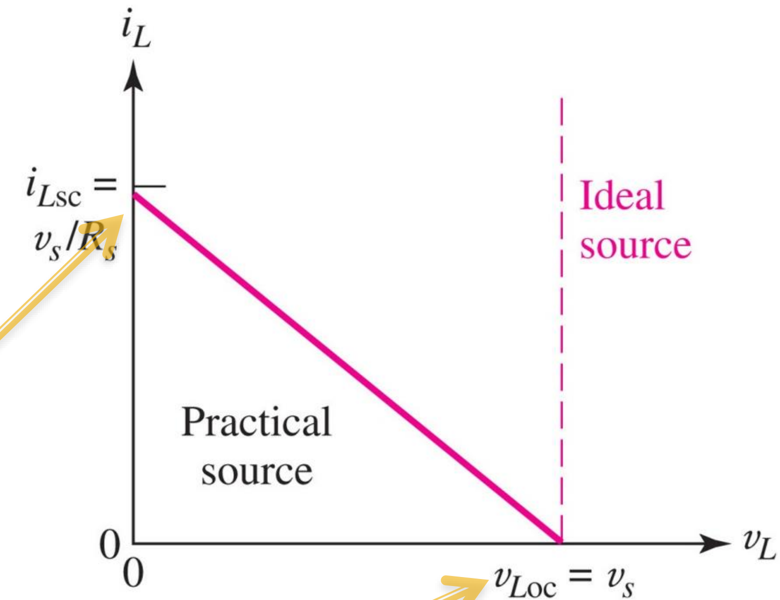


This line represents all possible R_L

Practical Voltage Source

The source has an internal resistance or output resistance, which is modeled as R_s



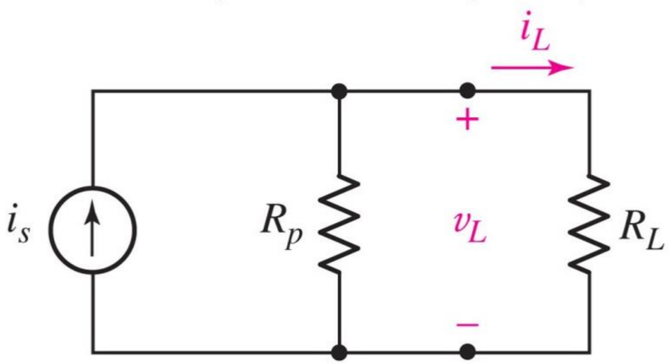


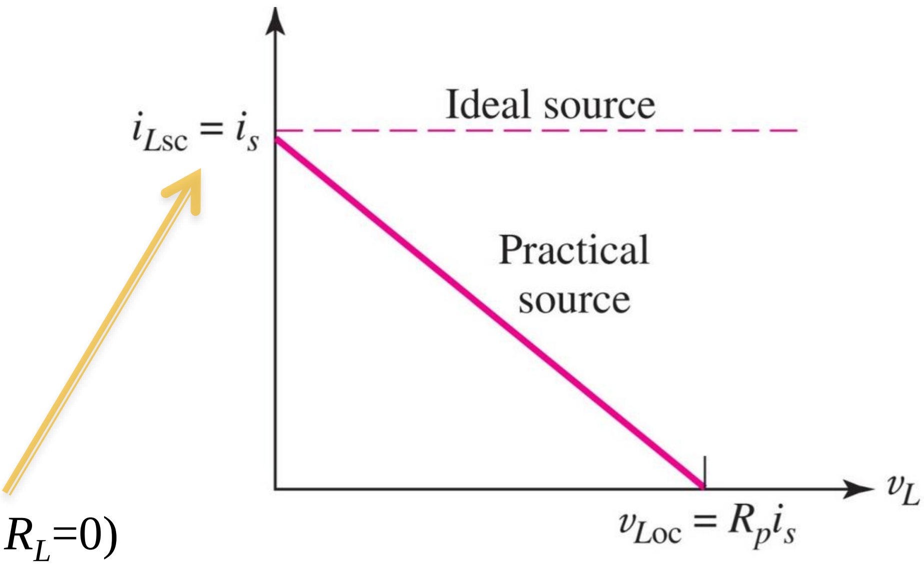
short circuit current (when R_L=0)

open circuit voltage (when R_L=∞)

Practical Current Source

The source has an internal *parallel* resistance which is modeled as R_p





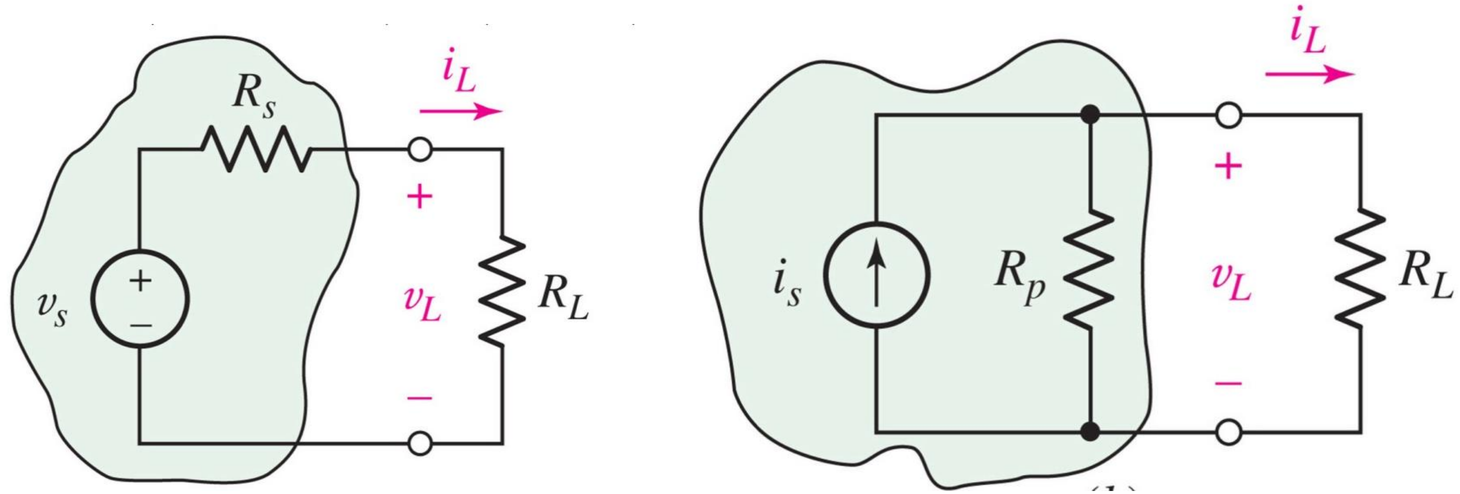
short circuit current (when R_L=0)

open circuit voltage (when R_L=∞)

Source Transformation and Equivalent Sources

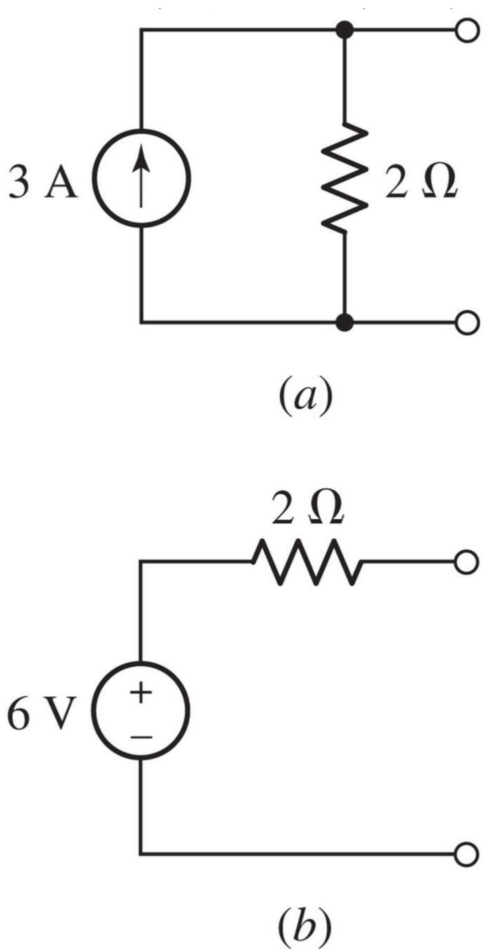
The sources are equivalent if

$$R_s = R_p \text{ and } v_s = i_s R_s$$



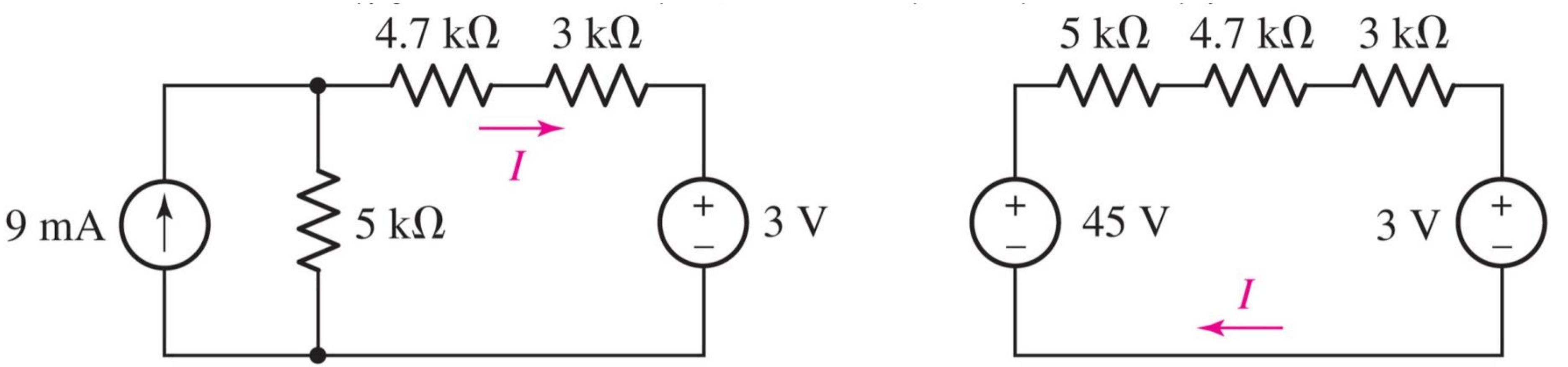
Source Transformation

- The circuits (a) and (b) are equivalent at the terminals.
- If given circuit (a), but circuit (b) is more convenient, switch them!
- This process is called
  source transformation.



Example: Source Transformation

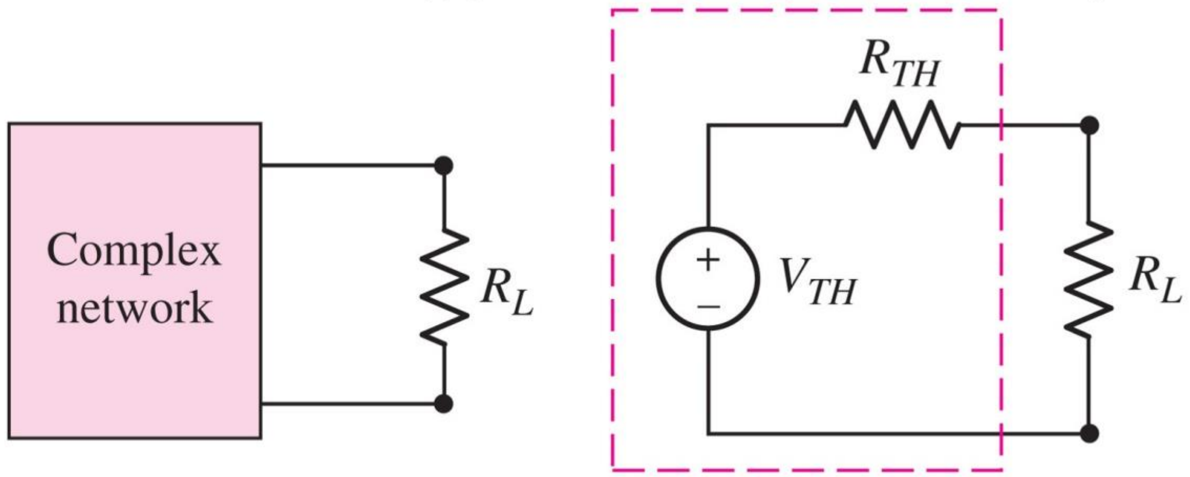
We can find the current I in the circuit below using source transformation, as shown.



$$I = (45-3)/(5+4.7+3) = 3.307 \text{ mA}$$

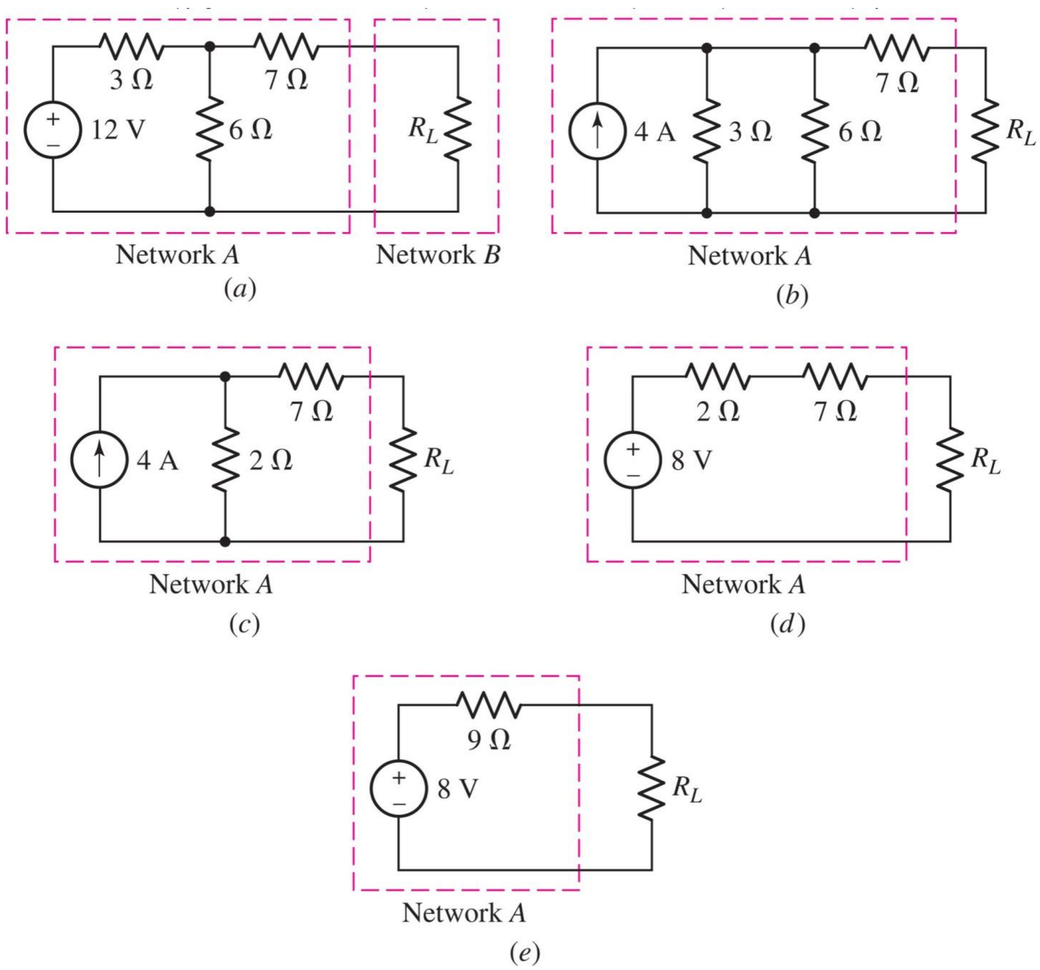
Thévenin Equivalent Circuits

Thévenin's theorem: a linear network can be replaced by its Thévenin equivalent circuit, as shown below:



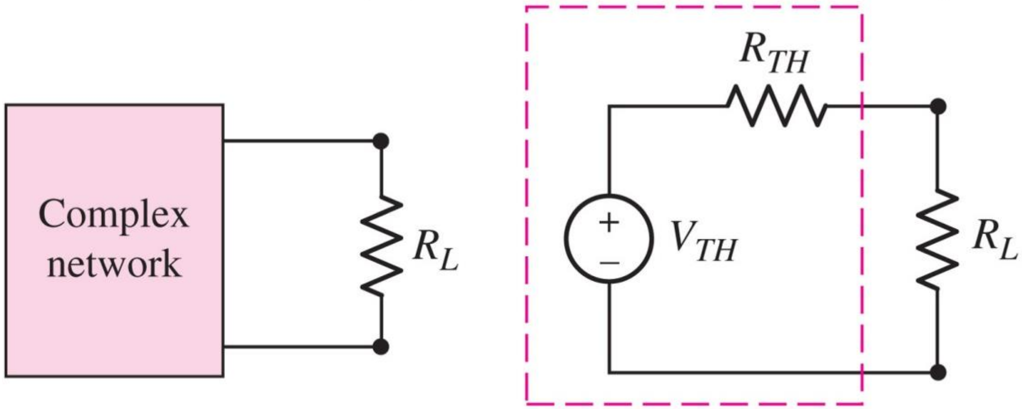
Thévenin Equivalent using Source Transformation

- We can repeatedly apply source transformation on network A to find its Thévenin equivalent circuit.
- This method has limitations- not all circuits can be source transformed.



Finding the Thévenin Equivalent

- Disconnect the load.
- Find the open circuit voltage v_{oc}
- Find the equivalent resistance R_{eq} of the network with all independent sources turned off.

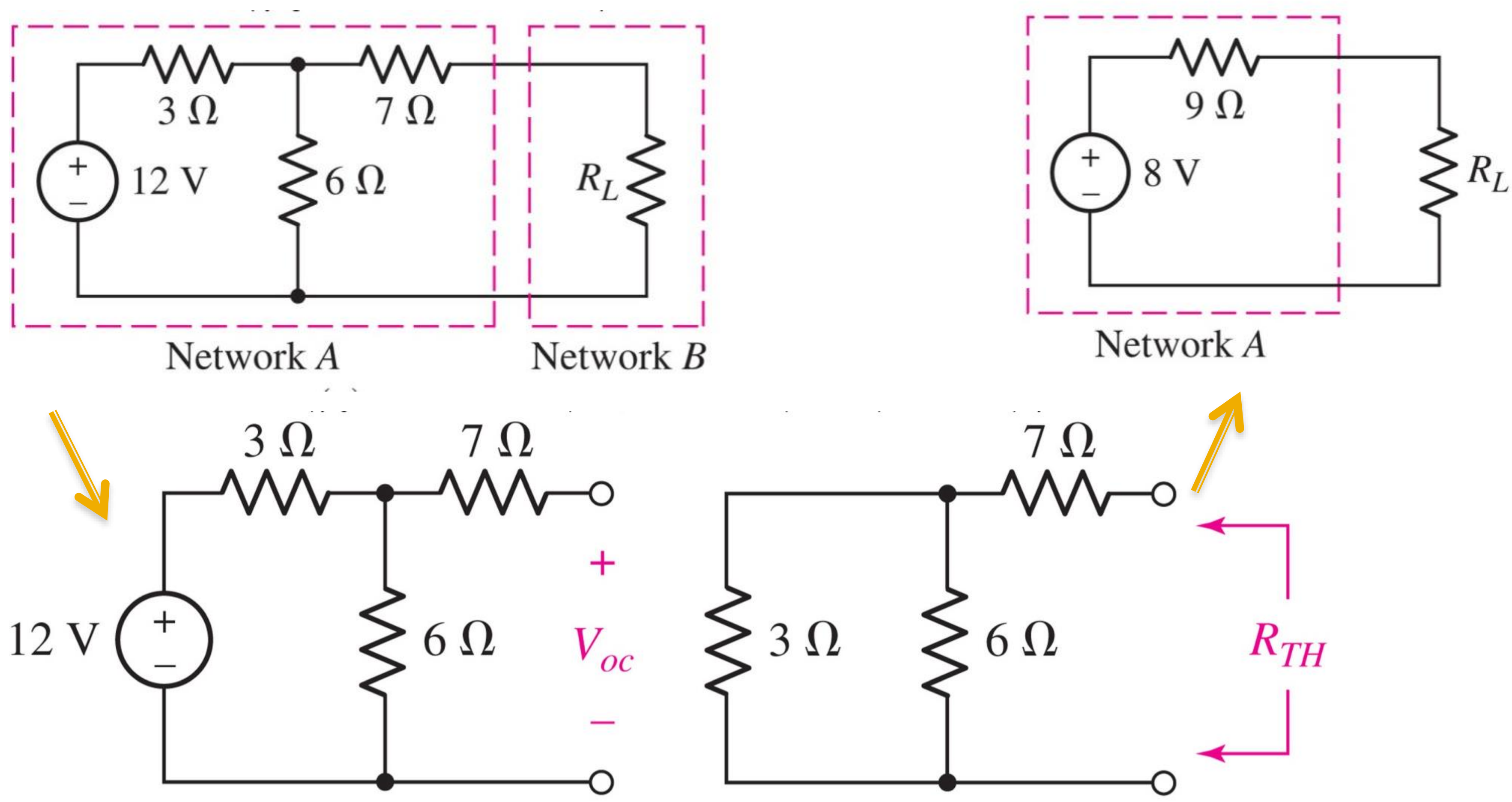


Then:

$V_{TH}=v_{oc}$ and

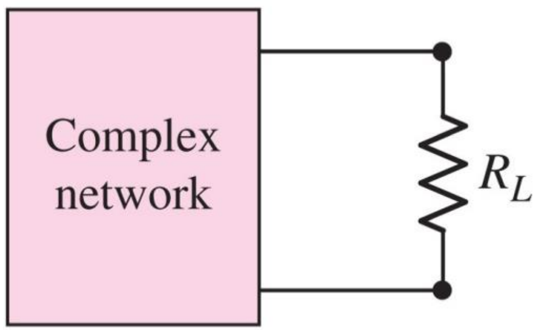
$R_{TH}=R_{eq}$

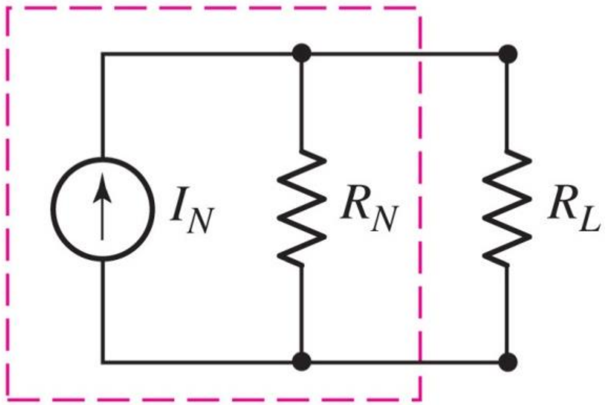
Thévenin Example



Norton Equivalent Circuits

Norton's theorem: a linear network can be replaced by its Norton equivalent circuit, as shown below:



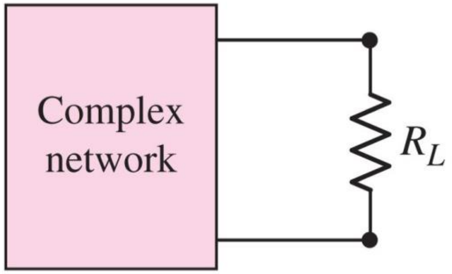


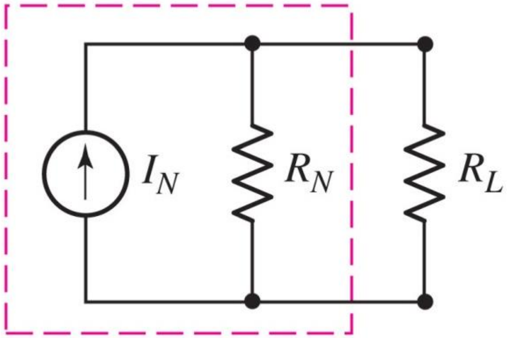
Finding the Norton Equivalent

- Replace the load with a short circuit.
- Find the short circuit current i_{sc}
- Find the equivalent resistance R_{eq} of the network with all independent sources turned off.

Then:

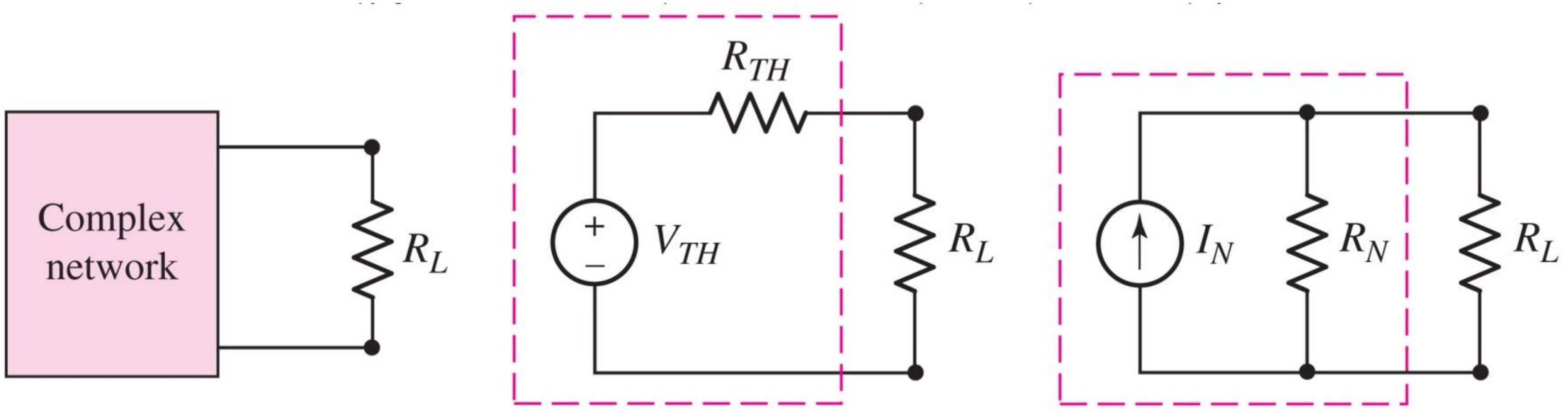
$I_N = i_{sc}$ and $R_N = R_{eq}$





Source Transformation: Norton and Thévenin

- The Thévenin and Norton equivalents are source transformations of each other!

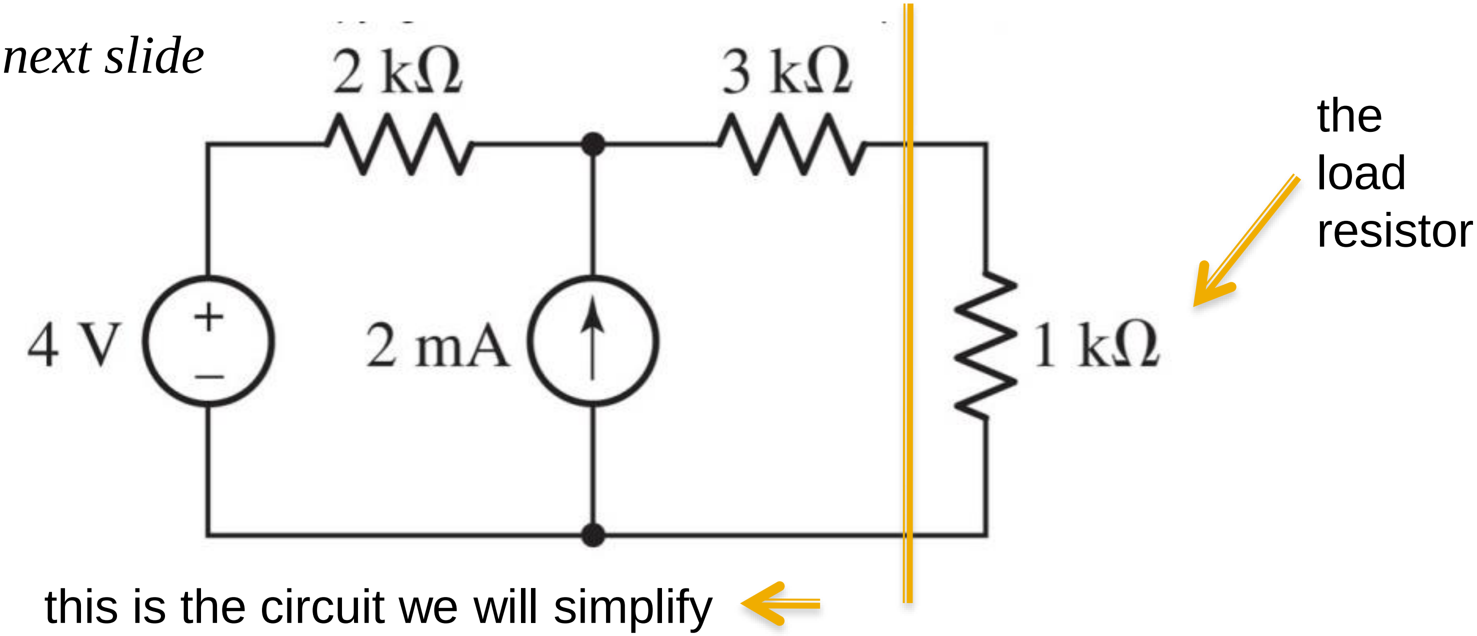


$$R_{TH} = R_N = R_{eq} \text{ and } v_{TH} = i_N R_{eq}$$

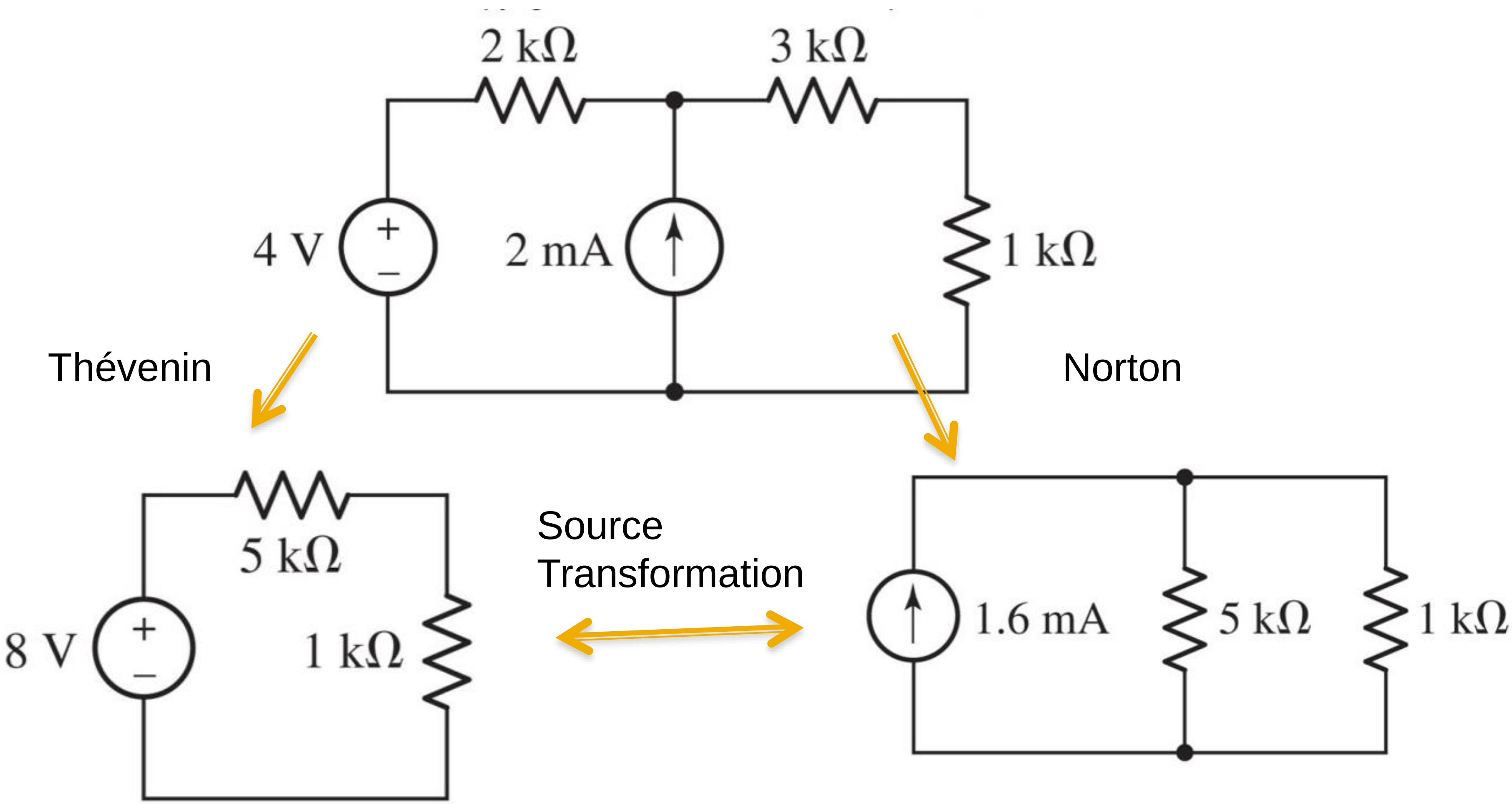
Example: Norton and Thévenin

Find the Thévenin and Norton equivalents for the network faced by the 1-kΩ resistor.

Answer: next slide



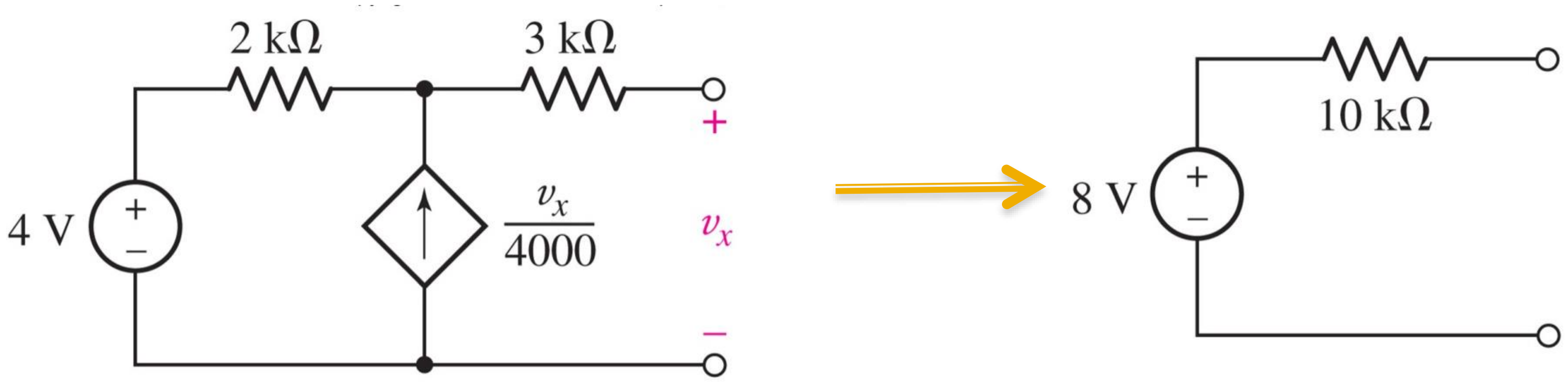
Example: Norton and Thévenin



Thévenin Example: Handling Dependent Sources

One method to find the Thévenin equivalent of a circuit with a dependent source: find V_{TH} and I_N and solve for $R_{TH}=V_{TH}/I_N$

Example:

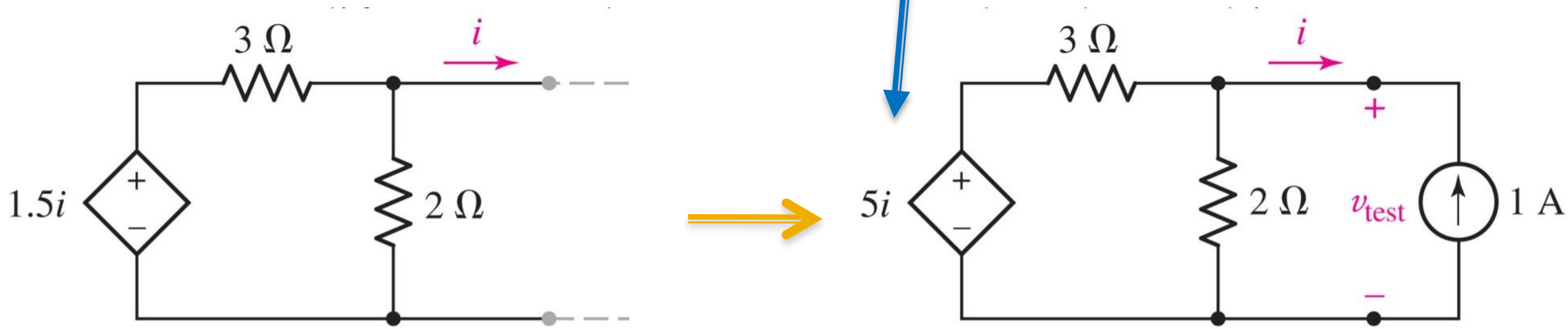


Thévenin Example: Handling Dependent Sources

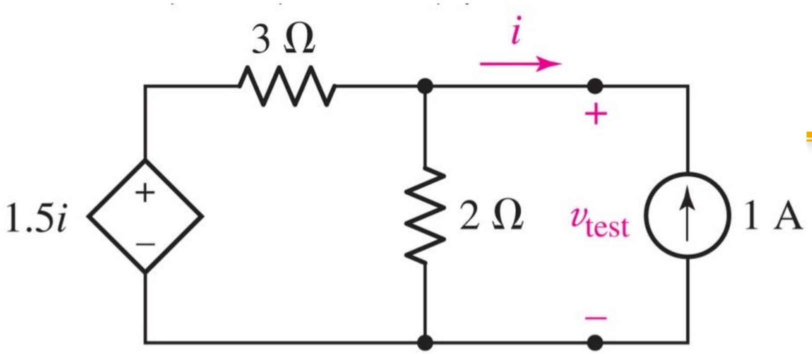
Finding the ratio V_{TH} / I_N fails when both quantities are zero!

Solution: apply a test source.

Should be 1.5i



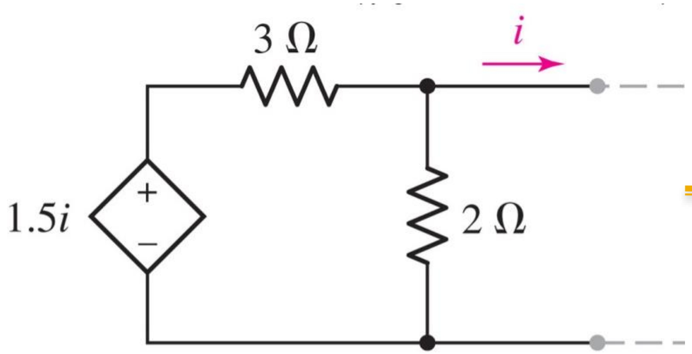
Thévenin Example: Handling Dependent Sources

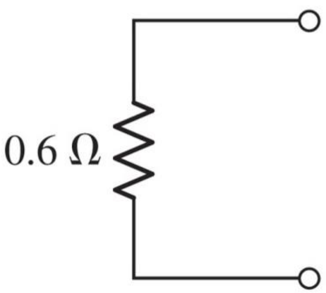


$$\frac{v_{test}}{2} + \frac{v_{test} - (1.5i)}{3} = 1$$

$$i = -1$$

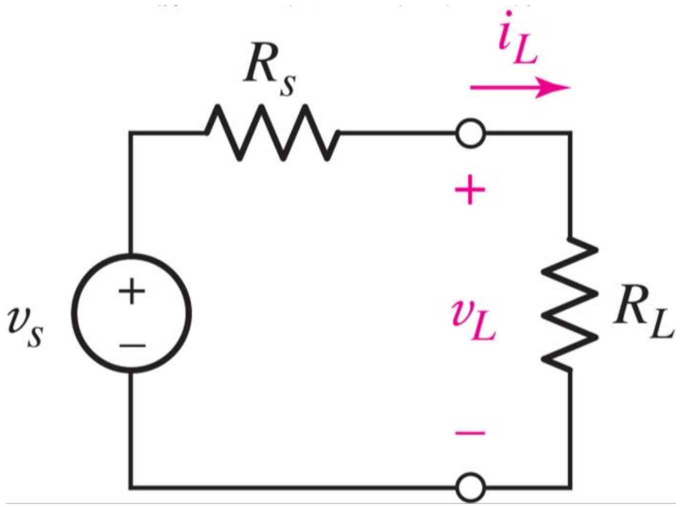
Solve: v_{test} =0.6 V, and so $R_{TH} = 0.6\ \Omega$





Maximum Power Transfer

What load resistor will allow the practical source to deliver the maximum power to the load?



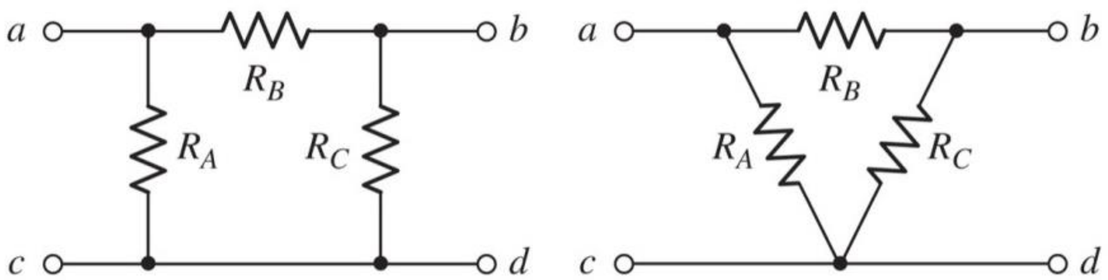
Answer: $R_L = R_s$

[Solve $dp_L/dR_L = 0$.]

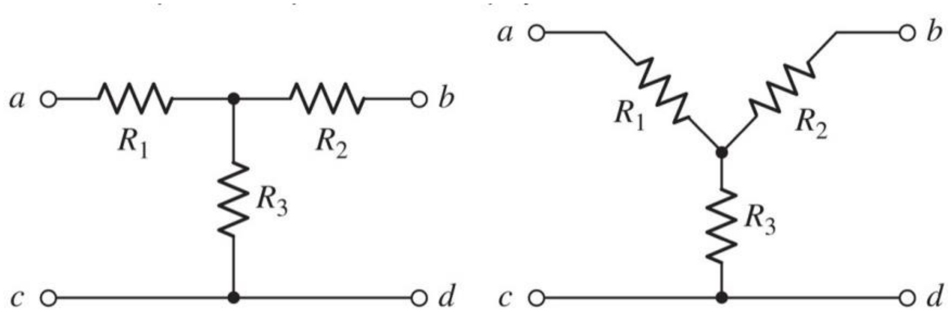
[Or: $p_L = i(v_s - iR_s)$, set $dp_L/di = 0$ to find $i_{max} = v_s/2R_s$. Hence $R_L = R_s$]

Δ-Y (delta-wye) Conversion

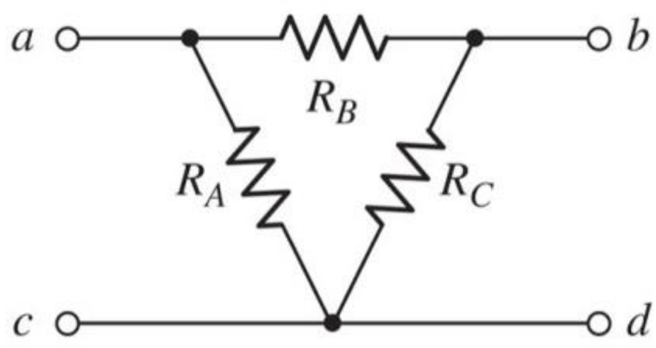
- The following resistors form a Δ:



- The following resistors form a Y:



Δ-Y (delta-wye) Conversion

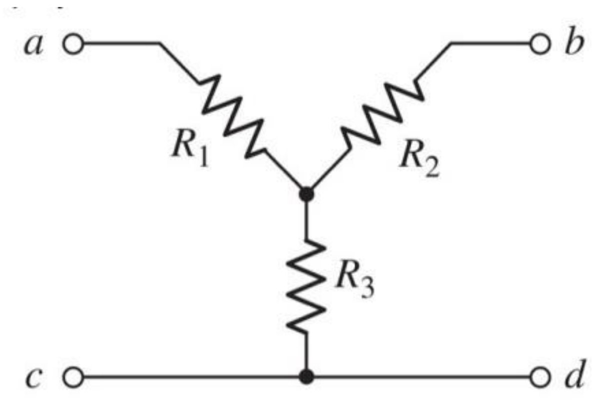


this Δ is equivalent to the Y if

$$R_A = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_B = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

$$R_C = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$



this Y is equivalent to the Δ if

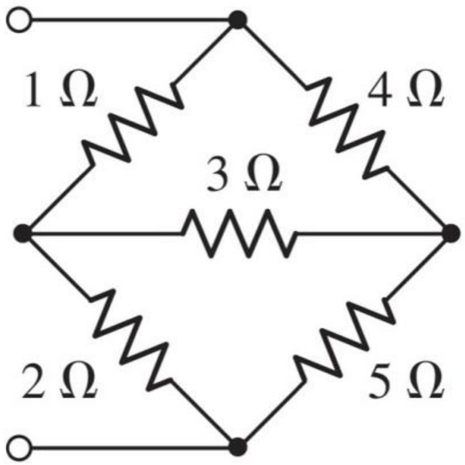
$$R_1 = \frac{R_AR_B}{R_A + R_B + R_C}$$

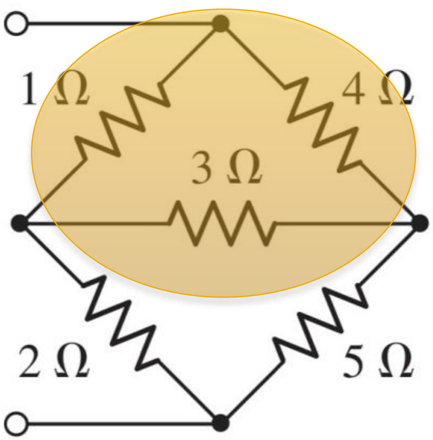
$$R_2 = \frac{R_BR_C}{R_A + R_B + R_C}$$

$$R_3 = \frac{R_CR_A}{R_A + R_B + R_C}$$

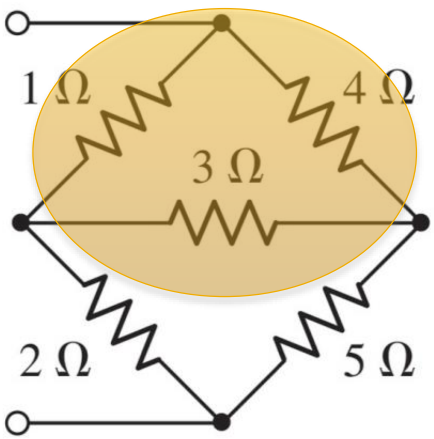
Example: Δ-Y Conversion

- How do we find the equivalent resistance of the following network? Convert a Δ to a Y

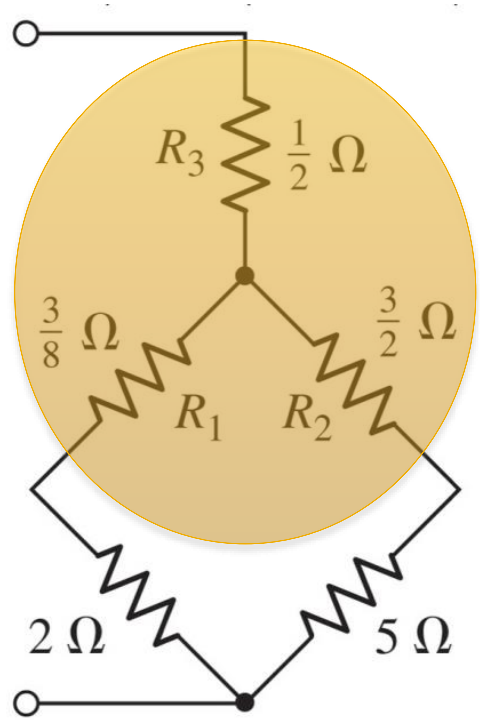


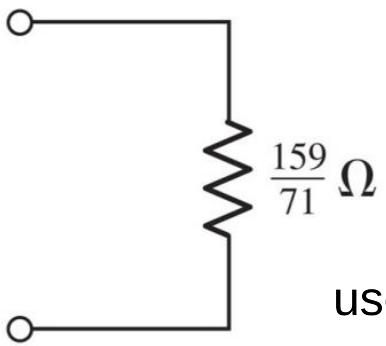


Example: Δ-Y Conversion



use the Δ to Y equations





use standard serial and parallel combinations