



Xi'an Jiaotong-Liverpool University

西交利物浦大學

# **EEE109: Electronic Circuits**

## **Basic FET Amplifiers**

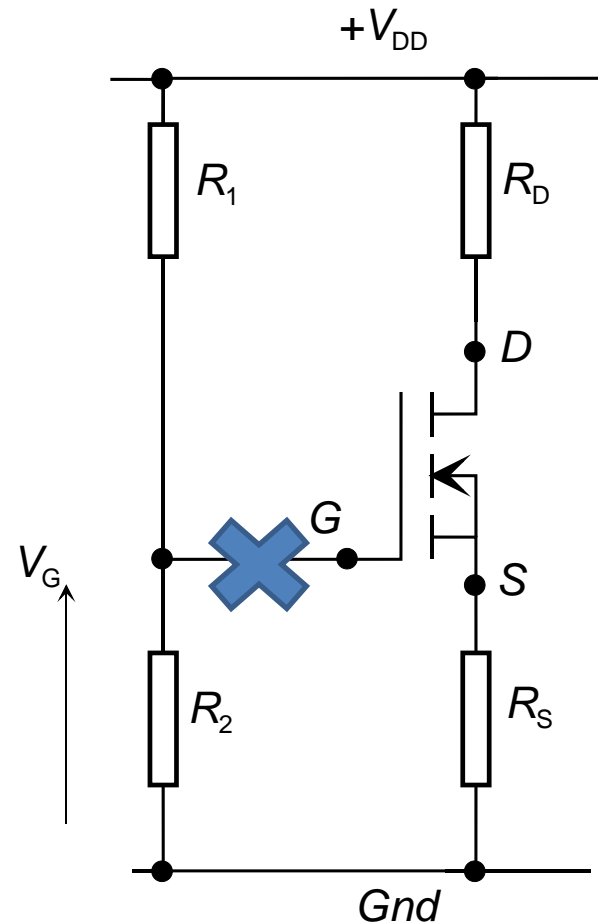
# Contents of Chapter 4

- Investigate a single-transistor circuit that can amplify a small, time-varying input signal
  - Develop small-signal models that are used in the analysis of linear amplifiers.
- Discuss and compare the three basic transistor amplifier configurations.
  - Analyze the common-source amplifier.
  - Analyze the source-follower amplifier.
  - Analyze the common-gate amplifier.
- Analyze multitransistor or multistage amplifiers.
- Develop the small-signal model of JFET devices and analyze basic JFET amplifiers.
- Design a two-stage MOSFET amplifier circuit.

# General Amplifier Characteristics

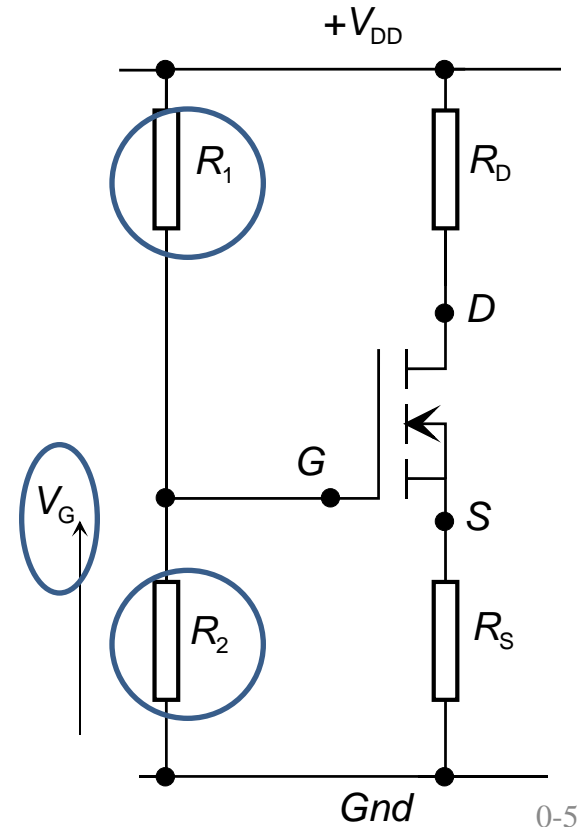
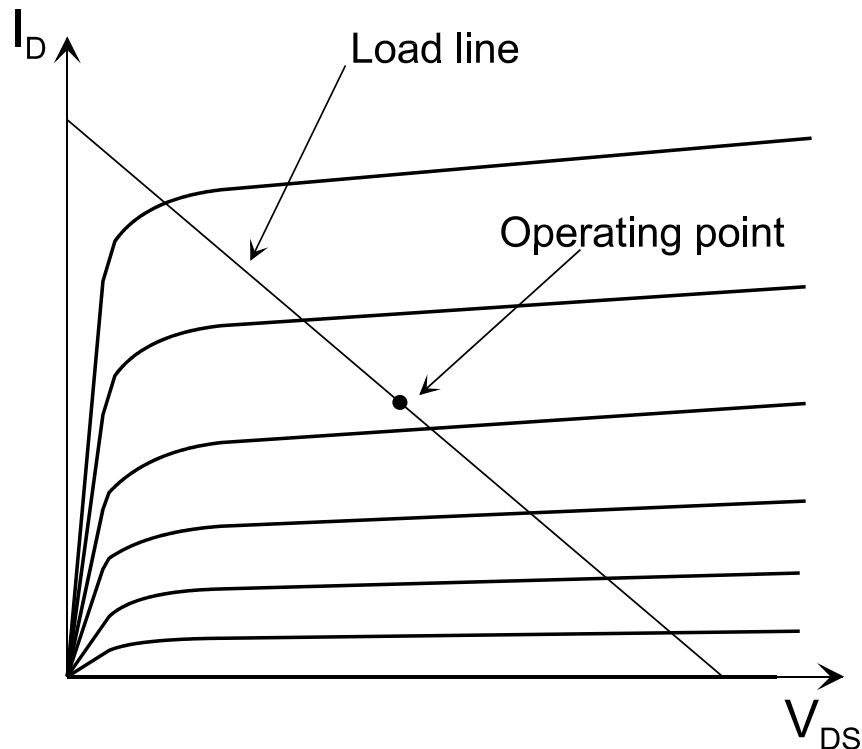
# Amplifier Characteristics (1)

The MOSFET may be used as a **switch** or biased to operate as an **amplifier**; circuits are similar to those for the bipolar transistor **EXCEPT** no current flows into the gate. Using four resistor biasing a common source amplifier is formed.



# Amplifier Characteristics (2)

The gate voltage  $V_G$  is set by the two resistors  $R_1$  and  $R_2$  and the voltage divider gives  $V_G$ . The MOSFET only conducts if  $V_G > V_T$ . The bias condition (operating point) gives the required values of  $V_{DS}$  and  $I_D$ .



# Circuit Notation Conventions

Table 4.1

Summary of notation

| Variable      | Meaning                    |
|---------------|----------------------------|
| $i_D, v_{GS}$ | Total instantaneous values |
| $I_D, V_{GS}$ | DC values                  |
| $i_d, v_{gs}$ | Instantaneous ac values    |
| $I_d, V_{gs}$ | Phasor values              |

# Small Signal Analysis Problem Solving Skills


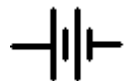

# Problem-Solving Technique:

## MOSFET AC Analysis

1. Analyze circuit with only the dc sources to find quiescent solution. Transistor must be biased in saturation region for linear amplifier.
2. Replace **elements** with small-signal model.
3. Analyze small-signal equivalent circuit, setting **dc sources to zero**, to produce the circuit to the time-varying input signals only.



# Transformation of Elements

| Element                             | DC Model  | AC Model        |
|-------------------------------------|---|-----------------|
| Resistor                            | R   | R               |
| Capacitor                           | Open  | C               |
| Inductor                            | Short   | L               |
| Diode                               | $+V_\gamma, r_f -$<br> | $r_d = V_T/I_D$ |
| Independent Constant Voltage Source | $+ V_S -$<br>          | Short           |
| Independent Constant Current Source | $I_S$<br>             | Open            |

# Practical Skill: General DC & AC Analysis (General Procedure)

- **DC analysis:**
  - Find DC equivalent circuit by replacing all capacitors by open circuits and inductors by short circuits.
  - Find Q-point from DC equivalent circuit by using appropriate large-signal transistor model.
- **AC analysis:**
  - Find AC equivalent circuit by replacing all capacitors by short circuits, inductors by open circuits, DC voltage sources by ground connections and DC current sources by open circuits.
  - Replace transistor by small-signal model
  - Use small-signal AC equivalent to analyze AC characteristics of amplifier.
  - Combine end results of DC and AC analysis to yield total voltages and currents in the network.

*Note: Since we are dealing with linear amplifiers (saturation mode), the principle of superposition holds*

# Practical Skill: General DC & AC Analysis (Equivalent Circuits)

- DC equivalent circuit:

- 1) replacing all capacitances with open circuits,
- 2) replacing inductances with short circuits,
- 3) reducing AC sources to zero:
  - ✓ Replacing AC voltage sources by short circuits and
  - ✓ Replacing AC current sources by open circuits.

- AC equivalent circuit:

- 1) reducing all DC sources to zero:
  - ✓ Replacing DC voltage sources with short circuits and
  - ✓ Replacing DC current sources with open circuits.
- 2) replacing all capacitances with short circuits,
- 3) replacing inductances with open circuits,
- 4) replacing transistor with small signal equivalent circuit.

# NMOS Transistor Small-Signal Parameters

- Values depends on Q-point

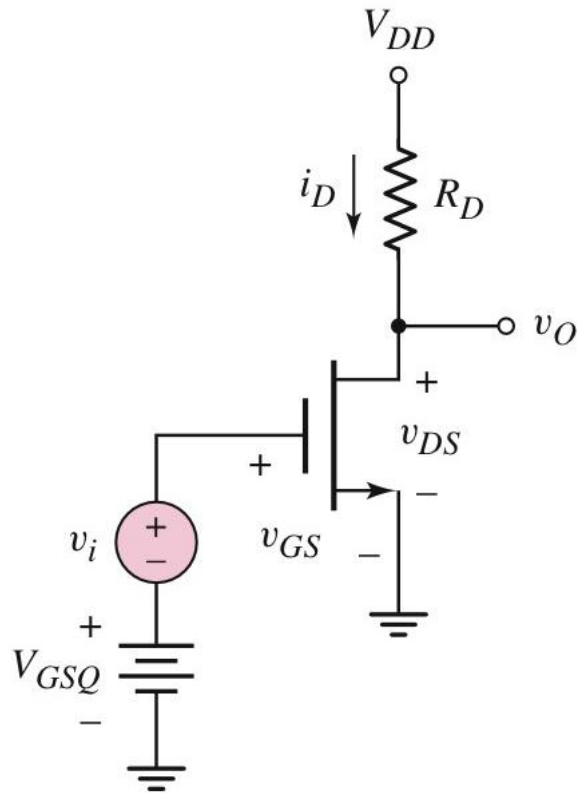
$$g_m = \frac{\partial i_D}{\partial v_{GS}} = \frac{i_d}{v_{gs}}$$

$$g_m = 2K_n(V_{GSQ} - V_{TN}) = 2\sqrt{K_n I_{DQ}}$$

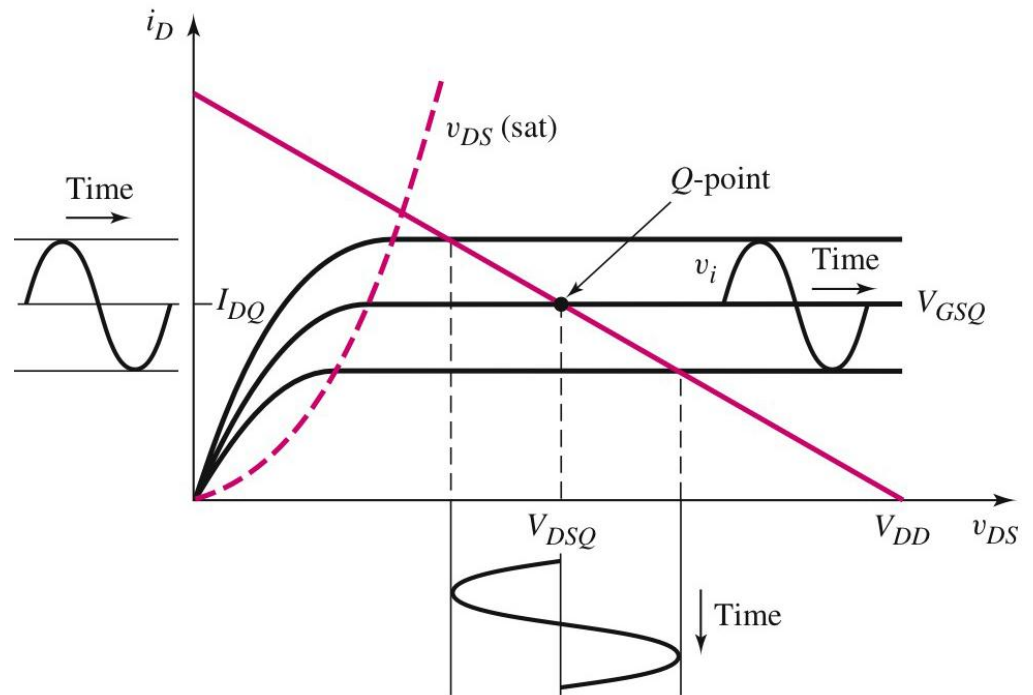
$$r_o = \left(\frac{\partial i_D}{\partial v_{DS}}\right)^{-1}$$

$$r_o = [\lambda K_n (V_{GSQ} - V_{TN})^2]^{-1} \cong [\lambda I_{DQ}]^{-1}$$

# NMOS Common-Source Circuit

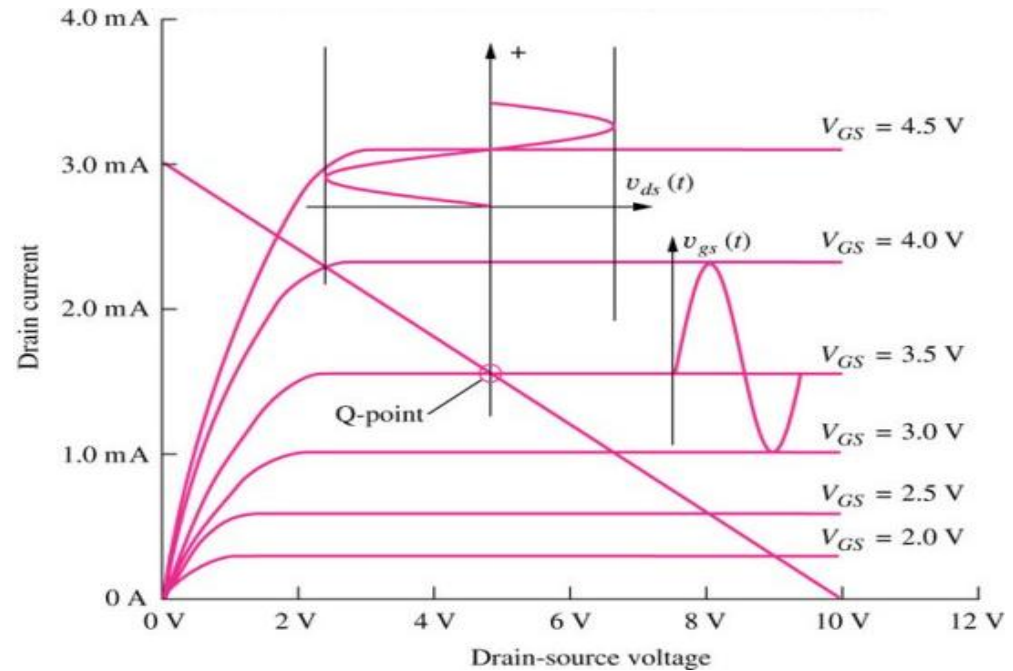
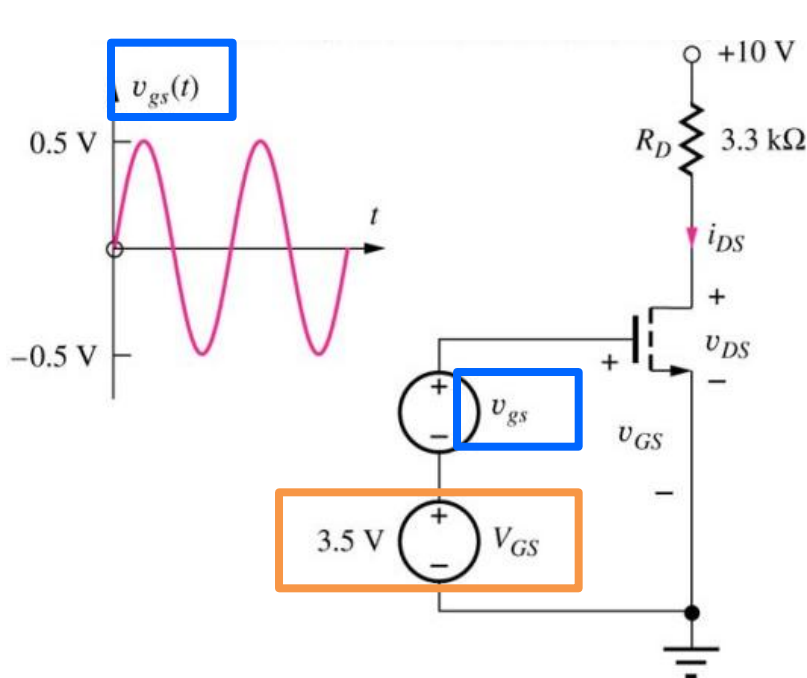


Copyright © The McGraw-Hill Companies, Inc.  
Permission required for reproduction or display.



Copyright © The McGraw-Hill Companies, Inc.  
Permission required for reproduction or display.

# MOSFET Amplifier Example



MOSFET is biased in active region by DC voltage source  $V_{GS}$ .

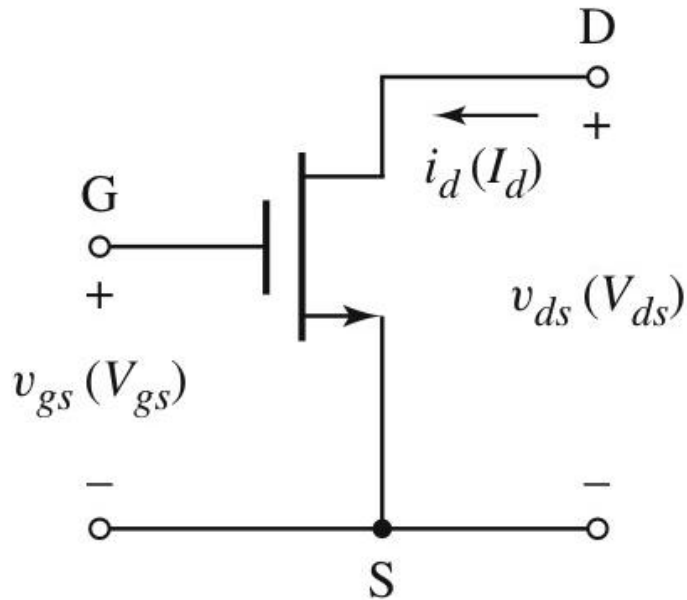
Q-point is set at  $(V_{DS}, I_D) = (4.8\text{ V}, 1.56\text{ mA})$  @  $V_{GS} = 3.5\text{ V}$ .

Total gate-source voltage is:  $v_{GS}(t) = V_{GS} + v_{gs}(t)$

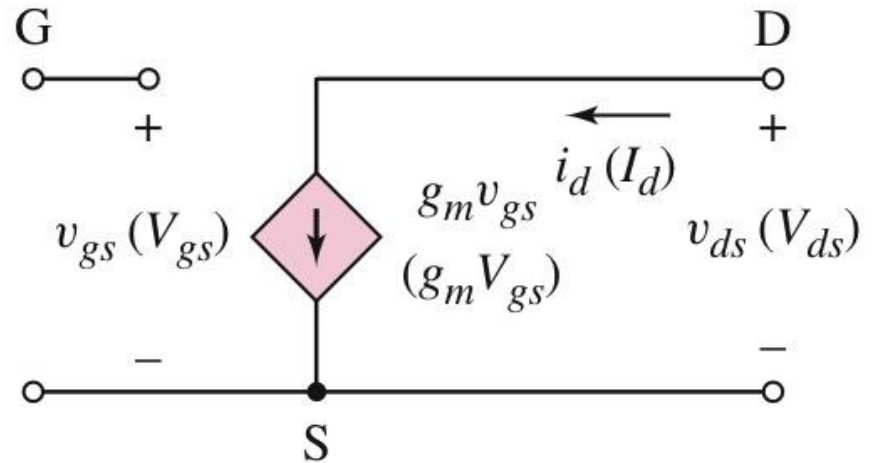
The input:  $1\text{ V}$   $p-p$  change in  $v_{GS}$

The output:  $1.25\text{ mA}$   $p-p$  change in  $i_D$  and  $4\text{ V}$   $p-p$  change in  $v_{DS}$ .

# Simple NMOS Small-Signal Transistor Equivalent Circuit (1)



(a)



(b)

Copyright © The McGraw-Hill Companies, Inc.  
Permission required for reproduction or display.

# Simple NMOS Small-Signal Transistor

## Equivalent Circuit (2)

$$v_{GS} = V_{GSQ} + v_i = V_{GSQ} + v_{gs} \quad (4.1)$$

where  $V_{GSQ}$  is the dc component and  $v_{gs}$  is the ac component. The instantaneous drain current is

$$i_D = K_n(v_{GS} - V_{TN})^2 \quad (4.2)$$

Substituting Equation (4.1) into (4.2) produces

$$i_D = K_n[V_{GSQ} + v_{gs} - V_{TN}]^2 = K_n[(V_{GSQ} - V_{TN}) + v_{gs}]^2 \quad (4.3(a))$$

or

$$i_D = \boxed{K_n(V_{GSQ} - V_{TN})^2} + \boxed{2K_n(V_{GSQ} - V_{TN})v_{gs}} + \boxed{K_nv_{gs}^2} \quad (4.3(b))$$

The first term in Equation (4.3(b)) is the dc or quiescent drain current  $I_{DQ}$ , the second term is the time-varying drain current component that is linearly related to the signal  $v_{gs}$ , and the third term is proportional to the square of the signal voltage. For a sinusoidal input signal, the squared term produces undesirable harmonics, or non-linear distortion, in the output voltage. To minimize these harmonics, we require

$$v_{gs} \ll 2(V_{GSQ} - V_{TN}) \quad (4.4)$$



# Simple NMOS Small-Signal Transistor Equivalent Circuit (3)

$$i_D = I_{DQ} + i_d \quad (4.5)$$

Again, small-signal implies linearity so that the total current can be separated into a dc component and an ac component. The ac component of the drain current is given by

$$i_d = 2K_n(V_{GSQ} - V_{TN})v_{gs} \quad (4.6)$$

The small-signal drain current is related to the small-signal gate-to-source voltage by the transconductance  $g_m$ . The relationship is

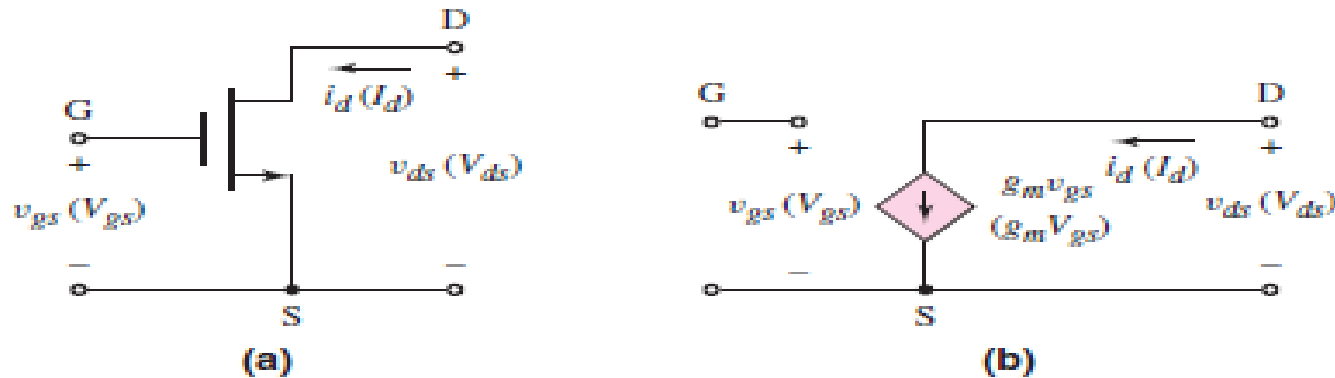
$$g_m = \frac{i_d}{v_{gs}} = 2K_n(V_{GSQ} - V_{TN}) \quad (4.7)$$

The transconductance is a transfer coefficient relating output current to input voltage and can be thought of as representing the gain of the transistor.

The transconductance can also be obtained from the derivative

$$g_m = \left. \frac{\partial i_D}{\partial v_{GS}} \right|_{v_{GS}=V_{GSQ}=\text{const.}} = 2K_n(V_{GSQ} - V_{TN}) \quad (4.8(a))$$

# Common Source with Channel Modulation



**Figure 4.5** (a) Common-source NMOS transistor with small-signal parameters and (b) simplified small-signal equivalent circuit for NMOS transistor

$$i_D = K_n[(v_{GS} - V_{TN})^2(1 + \lambda v_{DS})] \quad (4.16)$$

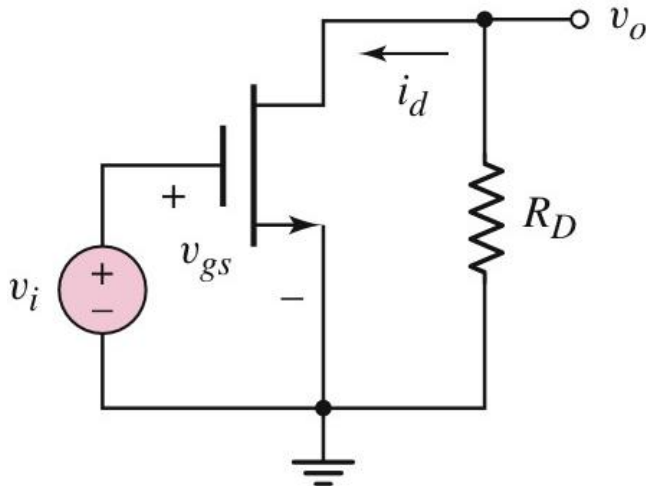
where  $\lambda$  is the channel-length modulation parameter and is a positive quantity. The small-signal output resistance, as previously defined, is

$$r_o = \left( \frac{\partial i_D}{\partial v_{DS}} \right)^{-1} \bigg|_{v_{GS}=V_{GSQ}=\text{const.}} \quad (4.17)$$

or

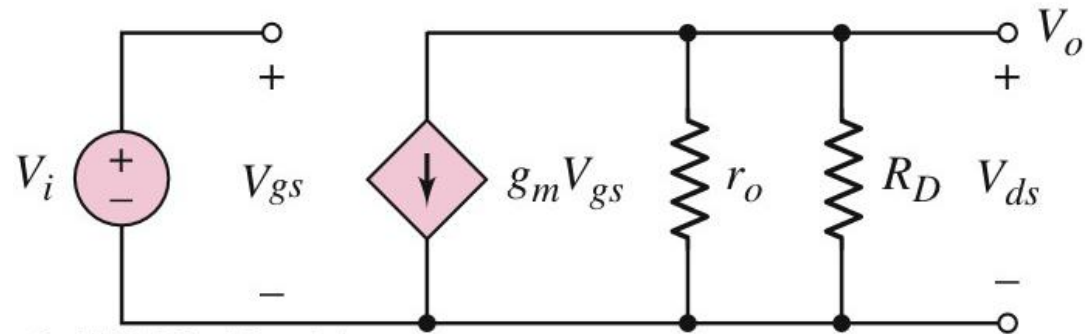
$$r_o = [\lambda K_n (V_{GSQ} - V_{TN})^2]^{-1} \cong [\lambda I_{DQ}]^{-1} \quad (4.18)$$

# NMOS Common-Source Circuit



Copyright © The McGraw-Hill Companies, Inc.  
Permission required for reproduction or display.

AC

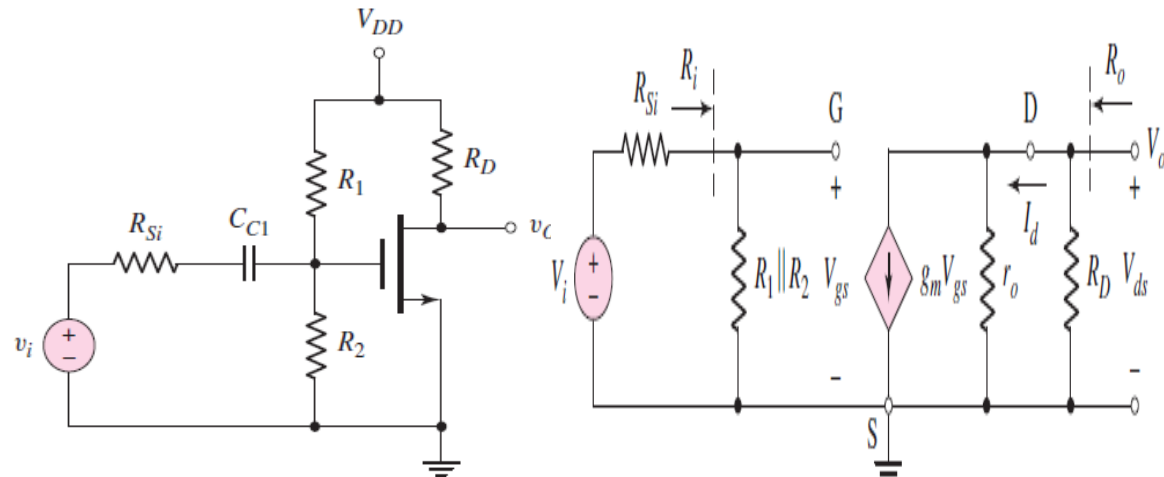
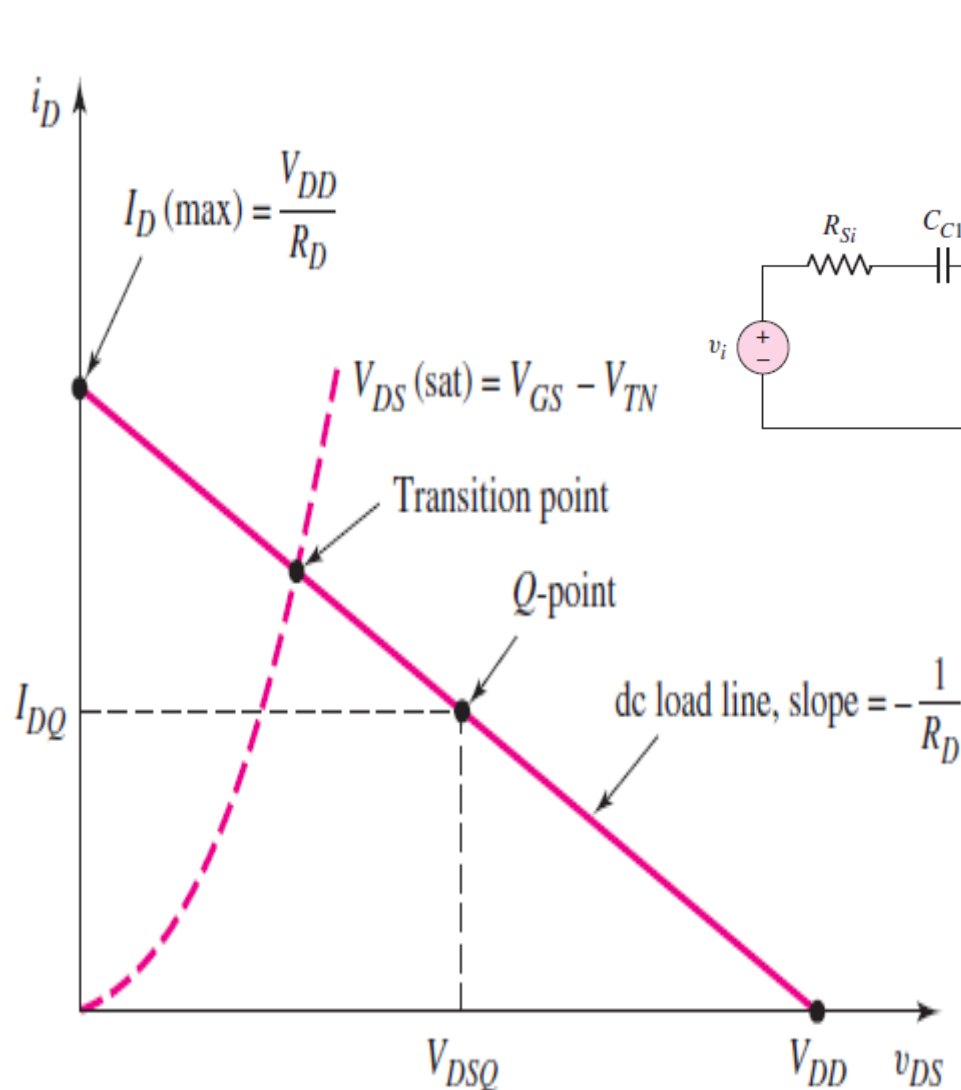


Copyright © The McGraw-Hill Companies, Inc.  
Permission required for reproduction or display.

Small-signal

$$A_v = V_o / V_i = -g_m (r_o \parallel R_D)$$

# NMOS Common-Source Circuit Analysis



$$V_o = -g_m V_{gs} (r_o \parallel R_D)$$

The input gate-to-source voltage is

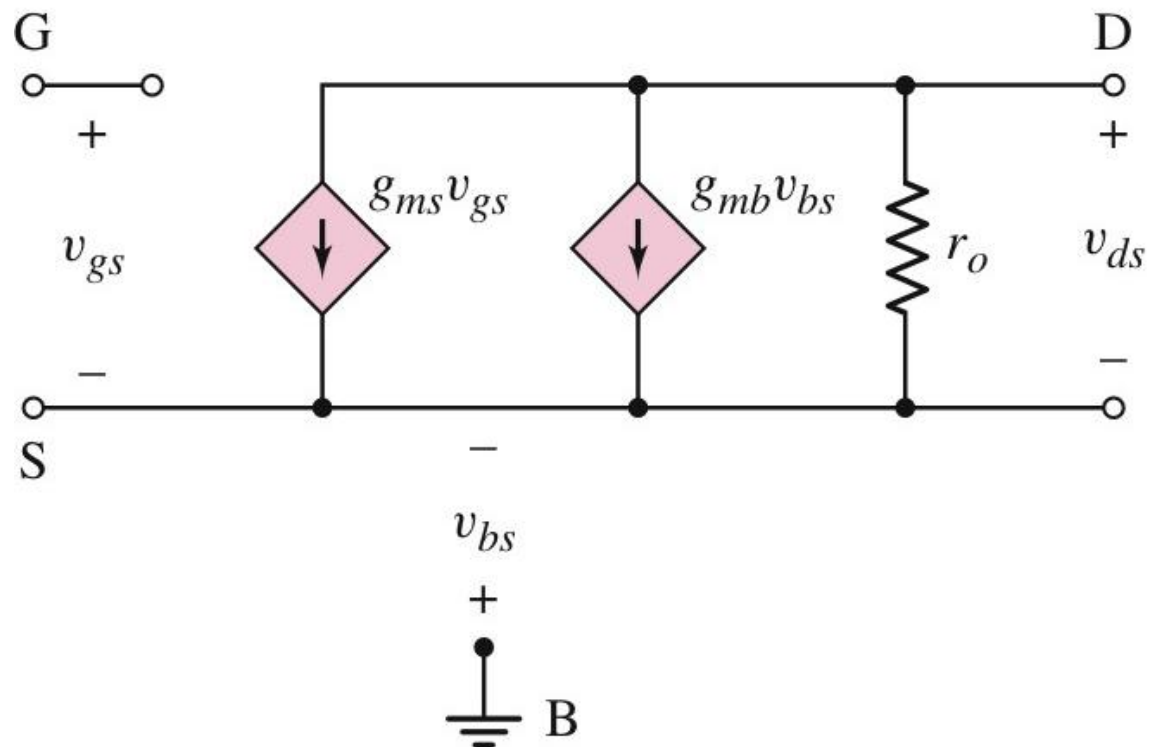
$$V_{gs} = \left( \frac{R_i}{R_i + R_{Si}} \right) \cdot V_i$$

so the small-signal voltage gain is

$$A_v = \frac{V_o}{V_i} = -g_m (r_o \parallel R_D) \cdot \left( \frac{R_i}{R_i + R_{Si}} \right)$$

# Modeling the Body Effects

# Modeling the Body Effects



Copyright © The McGraw-Hill Companies, Inc.  
Permission required for reproduction or display.

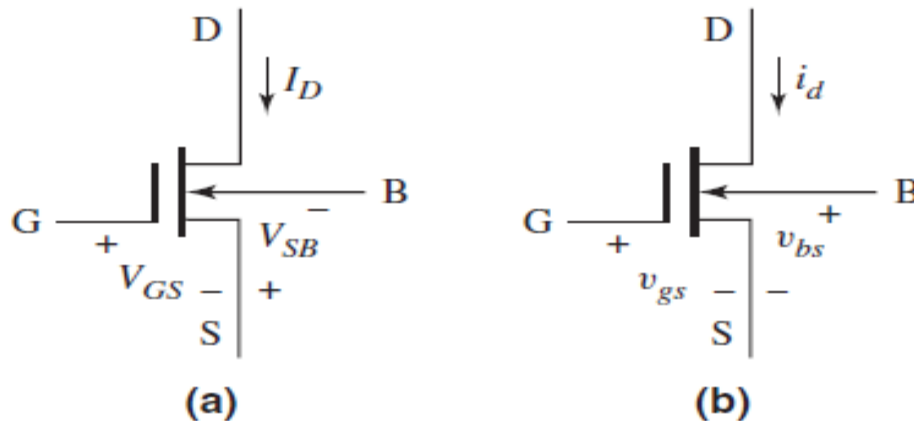
# Body Effect Consideration (1)

Modeling the Body Effect: As mentioned in Section 3.1.9, Chapter 3, the body effect occurs in a MOSFET in which the **substrate**, or **body**, is **not directly connected to the source**. For an NMOS device, the body is connected to the most negative potential in the circuit and will be at signal ground.

$$i_D = K_n(v_{GS} - V_{TN})^2$$

and the threshold voltage is given by

$$V_{TN} = V_{TNO} + \gamma[\sqrt{2\phi_f + v_{SB}} - \sqrt{2\phi_f}]$$



**Figure 4.12** The four-terminal NMOS device with (a) dc voltages and (b) ac voltages

# Body Effect Consideration (2)

$$g_{mb} = \left. \frac{\partial i_D}{\partial v_{BS}} \right|_{Q-pt} = \left. \frac{-\partial i_D}{\partial v_{SB}} \right|_{Q-pt} = - \left( \frac{\partial i_D}{\partial V_{TN}} \right) \cdot \left( \frac{\partial V_{TN}}{\partial v_{SB}} \right) \Big|_{Q-pt}$$

Using Equation (4.22), we find

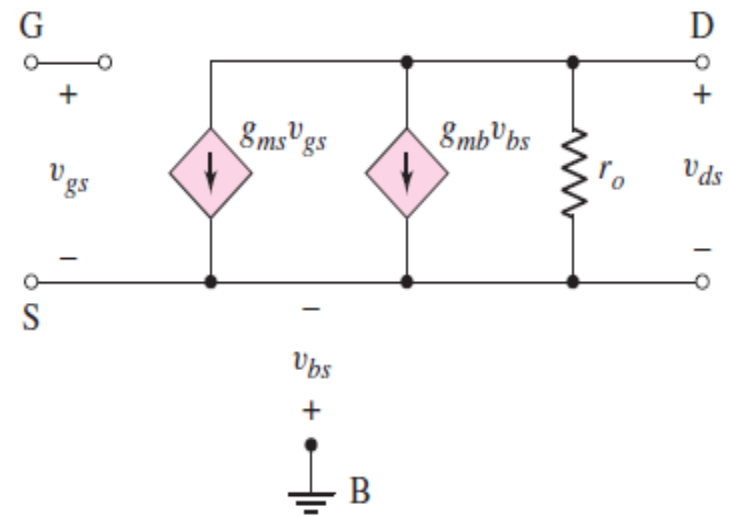
$$\frac{\partial i_D}{\partial V_{TN}} = -2K_n(v_{GS} - V_{TN}) = -g_m$$

and using Equation (4.23), we find

$$\frac{\partial V_{TN}}{\partial v_{SB}} = \frac{\gamma}{2\sqrt{2\phi_f + v_{SB}}} \equiv \eta$$

The back-gate transconductance is then

$$g_{mb} = -(-g_m) \cdot (\eta) = g_m \eta$$



Small-signal equivalent circuit of NMOS device including body effect Test

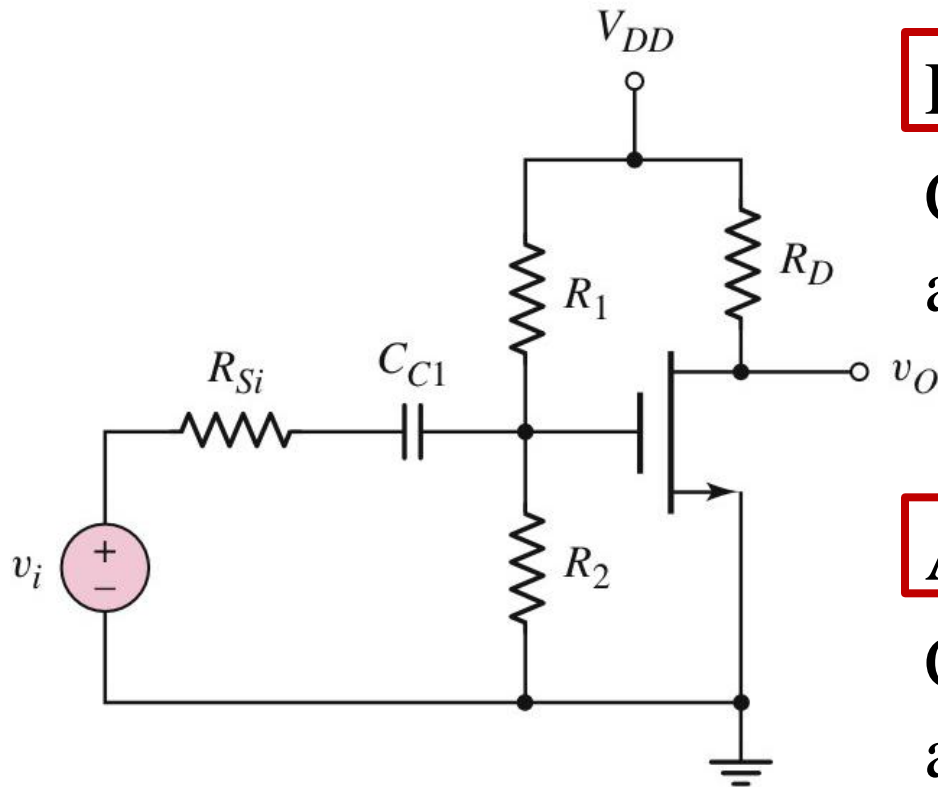


# Three Type of MOSFET Amplifier Circuit

- Common Source Amplifier
- Common Drain Amplifier
- Common Gate Amplifier

# Common Source Amplifier Circuit

# Common-Source Configuration



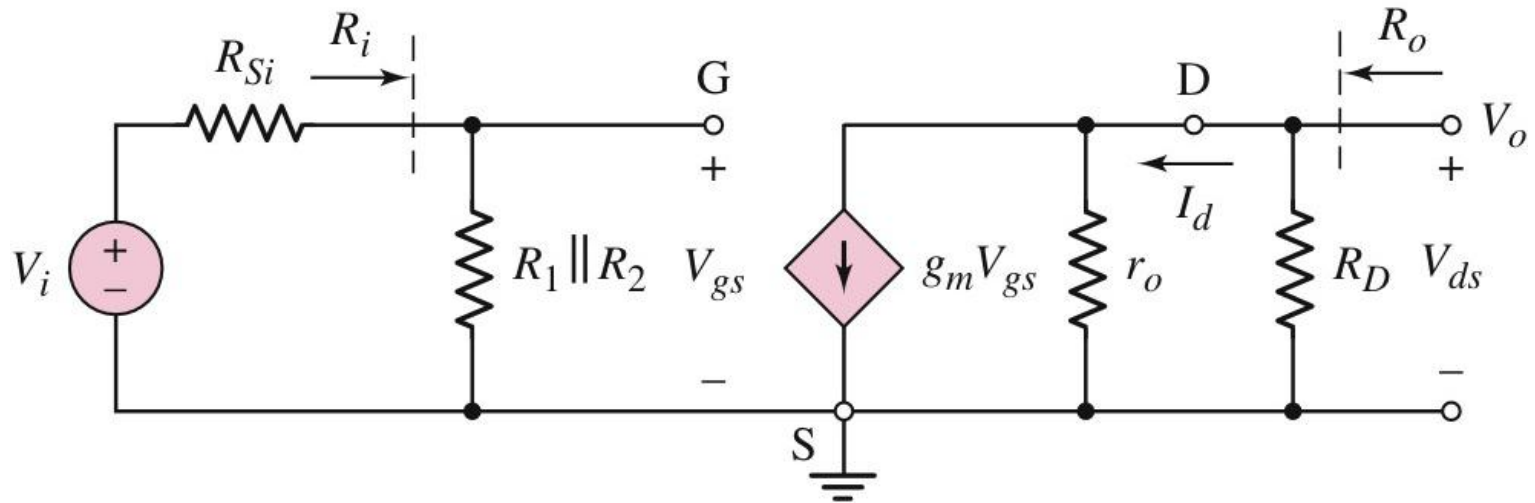
**DC** analysis:

Coupling capacitor is assumed to be open.

**AC** analysis:

Coupling capacitor is assumed to be a short. DC voltage supply is set to zero volts.

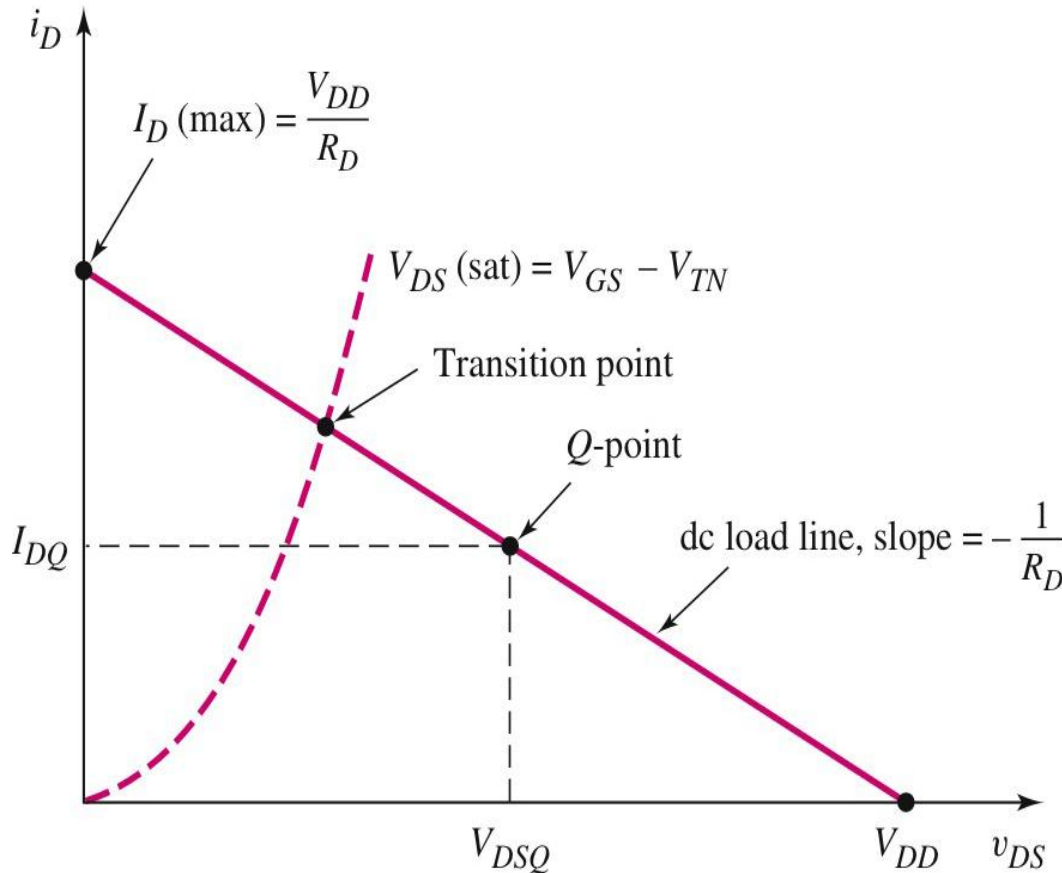
# Small-Signal Equivalent Circuit



Copyright © The McGraw-Hill Companies, Inc.  
Permission required for reproduction or display.

$$A_v = V_o / V_i = -g_m (r_o \parallel R_D) \left( \frac{R_i}{R_i + R_{Si}} \right)$$

# DC Load Line



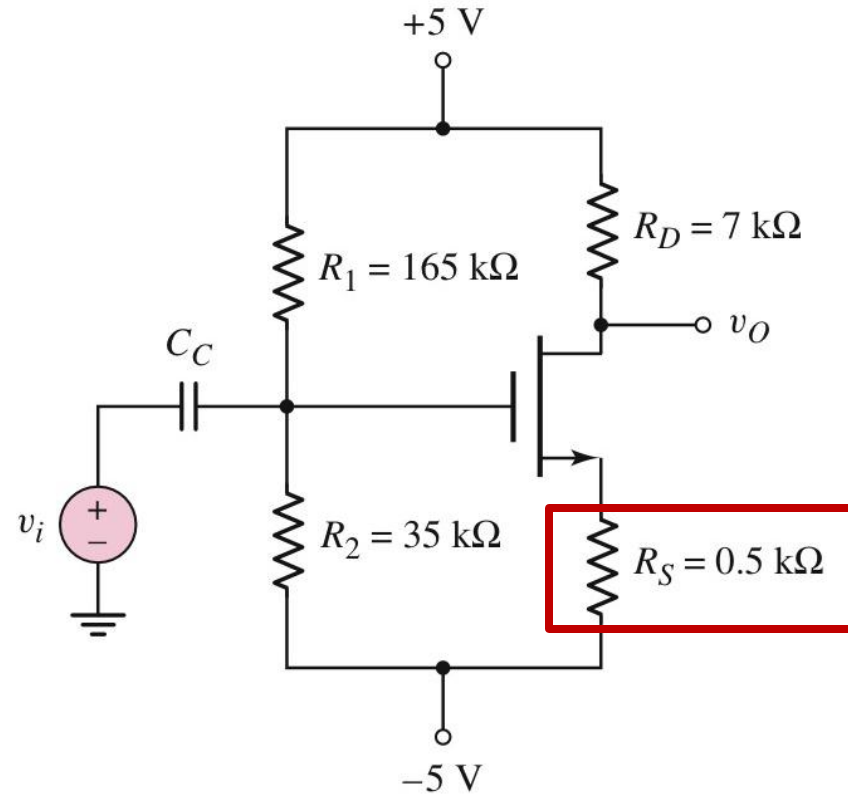
Copyright © The McGraw-Hill Companies, Inc.  
Permission required for reproduction or display.

Q-point near the middle of the saturation region for maximum symmetrical output voltage swing..

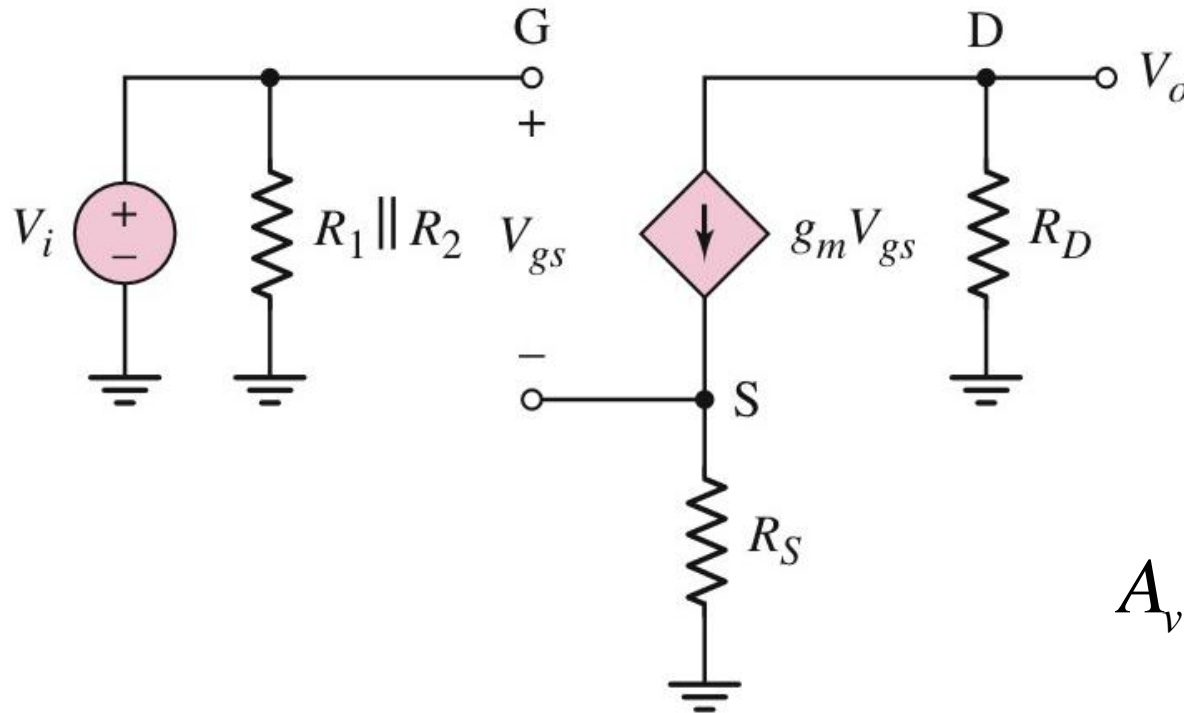
Small AC input signal for output response to be linear.

# The Effect of Source Resistance

# Common-Source Amplifier with Source Resistor



# Small-Signal Equivalent Circuit for Common-Source with Source Resistor



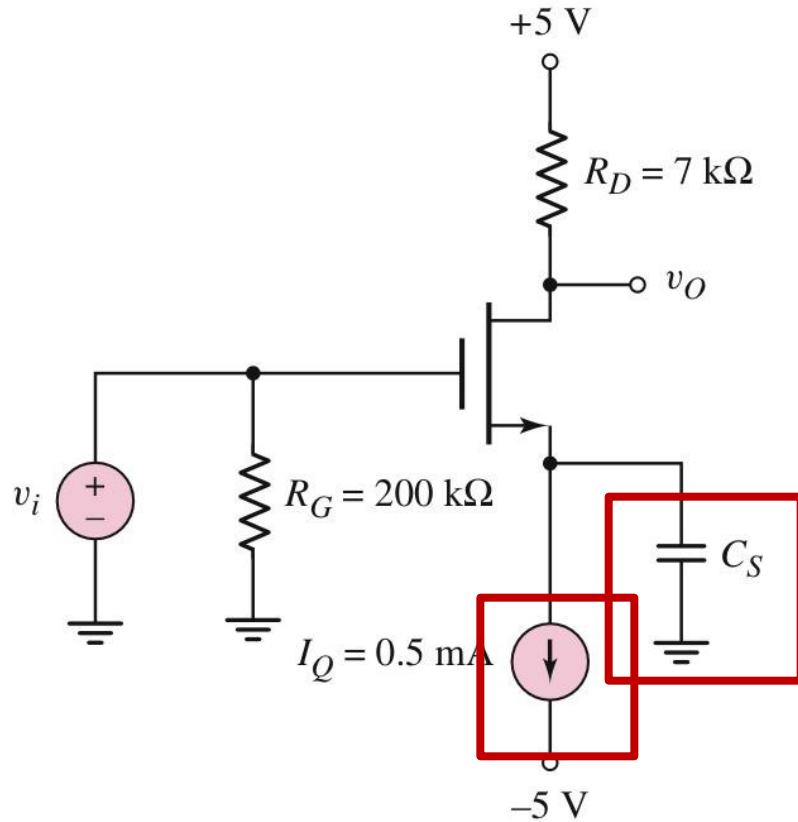
$$A_v = \frac{-g_m R_D}{1 + g_m R_S}$$

Copyright © The McGraw-Hill Companies, Inc.  
Permission required for reproduction or display.

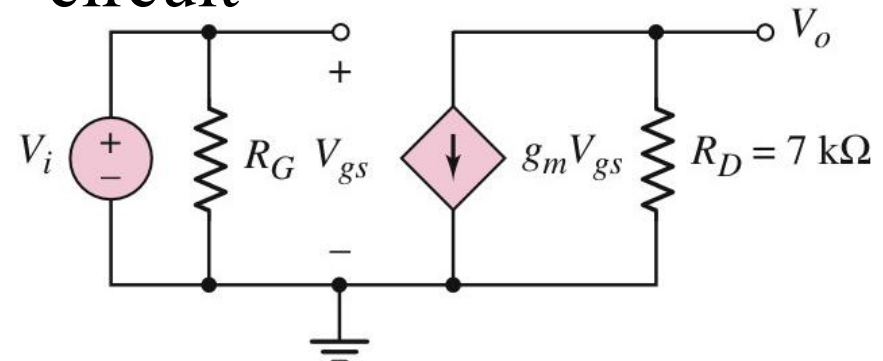
- The source resistor is added to stabilize the Q-point.
- Voltage gain is reduced by an increase in the denominator.



# Common-Source Amplifier with Bypass Capacitor



Small-signal equivalent circuit



Copyright © The McGraw-Hill Companies, Inc.  
Permission required for reproduction or display.

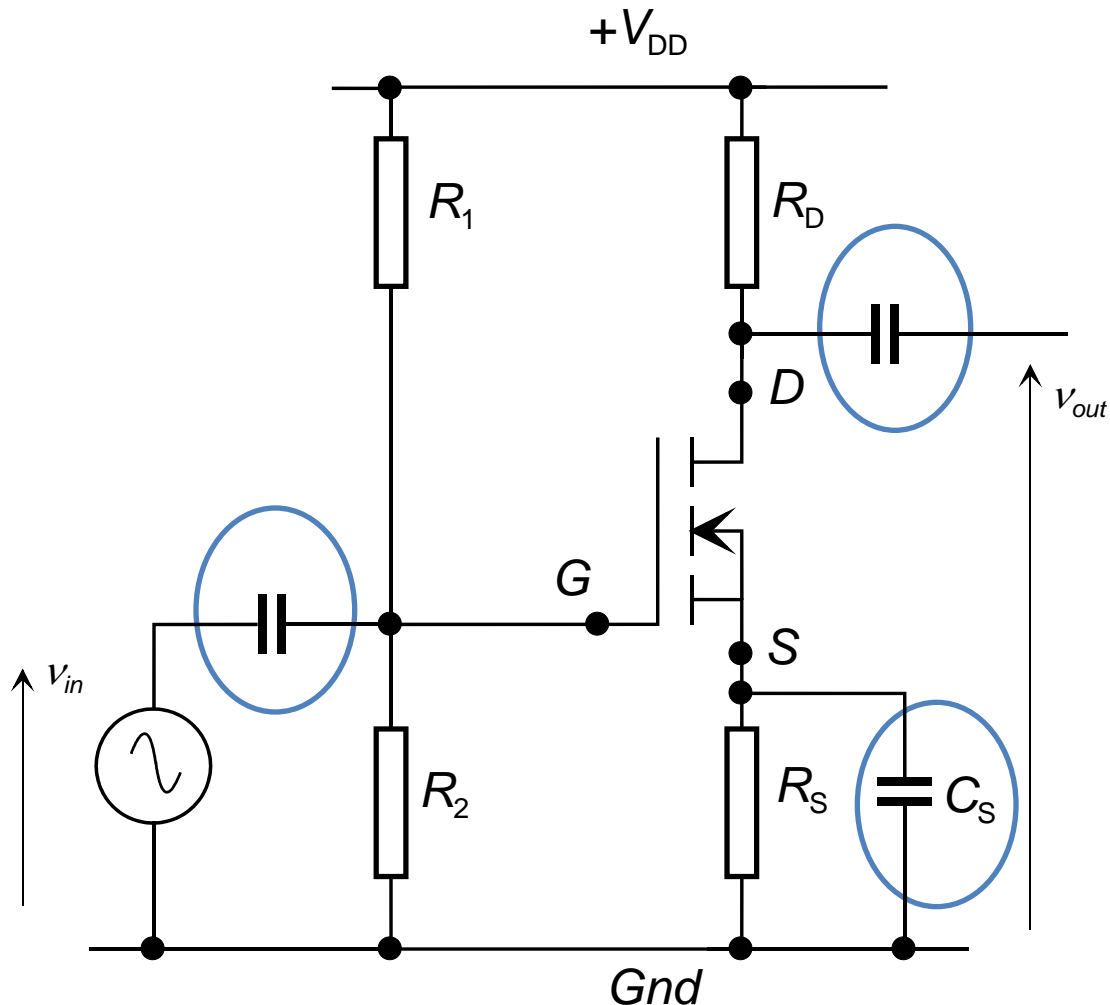
Copyright © The McGraw-Hill Companies, Inc.  
Permission required for reproduction or display.

- Bypass capacitor is added to minimize the loss in the voltage gain.
- The source resistor is replaced by the current source to further stabilize the Q-point.

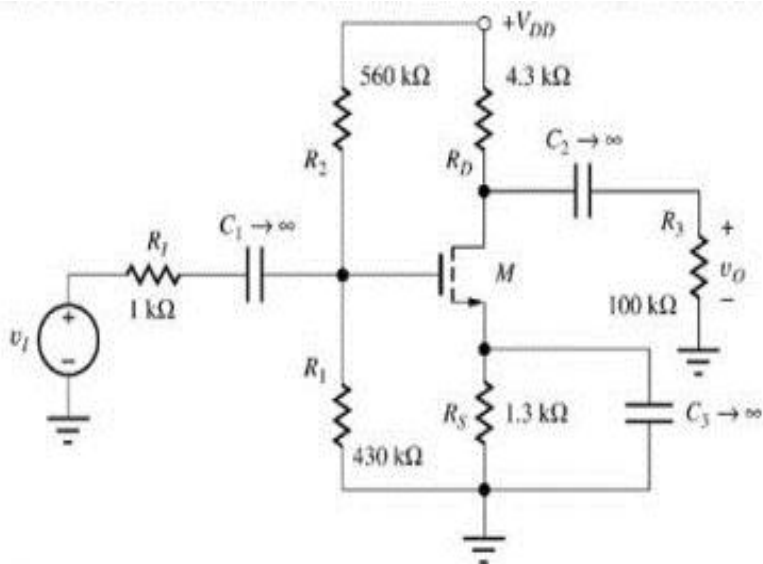
# Common Source Small Signal Analysis Parameters

# Small Signal Amplifier

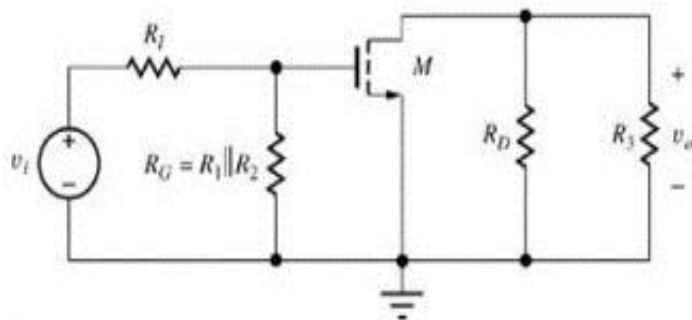
A **small-signal amplifier** is built by adding **input and output coupling capacitors** and a **source bypass capacitor**



# Small-Signal Analysis of Complete C-S Amplifier: AC Equivalent



(a)



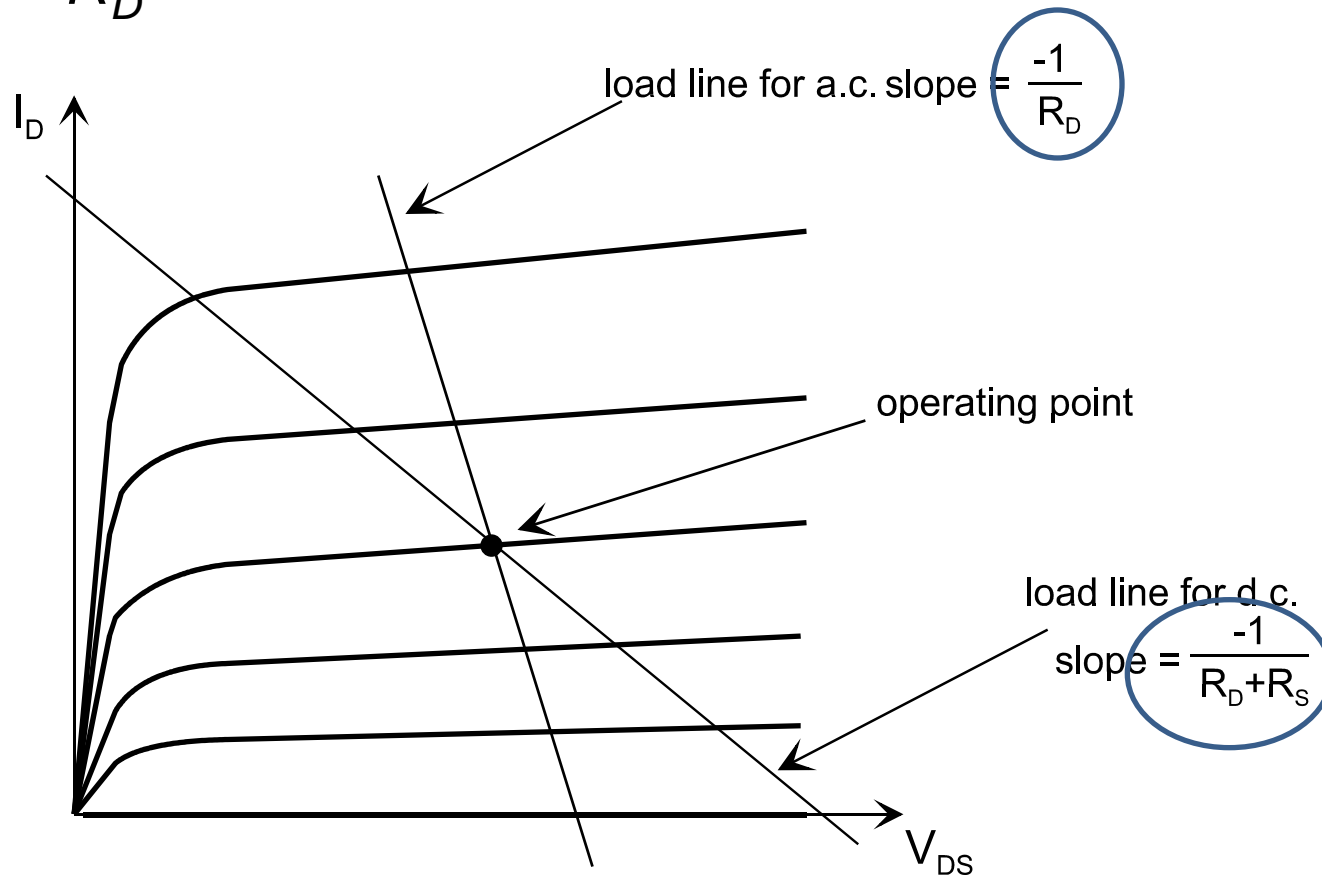
(b)

- AC equivalent circuit is constructed by assuming that all capacitances have zero impedance at signal frequency and dc voltage sources represent AC grounds.
- Assume that Q-point is already known.

$$R_G = R_1 \parallel R_2$$

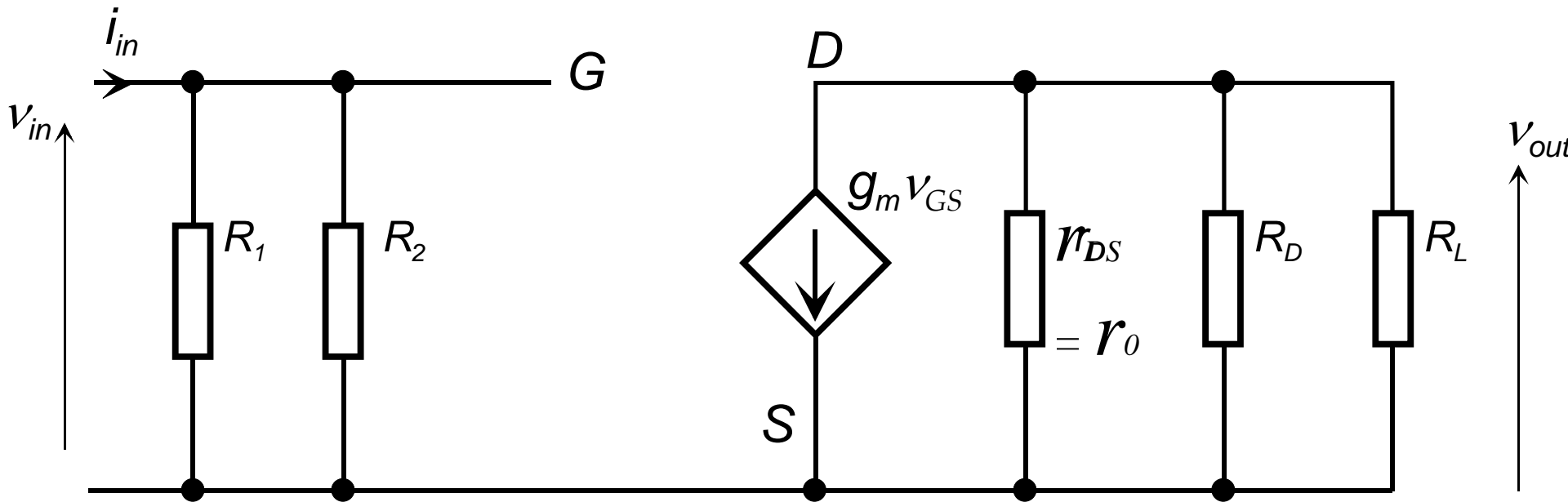
# A.c. Load Line

Note that when the **source bypass capacitor** is used, only the **drain resistor**  $R_D$  determines the a.c. response at the operating point, the a.c. load line slope is  $-\frac{1}{R_D}$  at the operating point.



# Small Signal Equivalent Circuit

The small signal equivalent circuit of the common-source amplifier is



# Input Resistance and Voltage Gain

The **input resistance** is  $R_{in} = \frac{v_{in}}{i_{in}}$  and  $i_{in} = \frac{v_{in}}{R_G}$  where  $R_G = \frac{R_1 R_2}{R_1 + R_2}$

Hence  $R_{in} = R_G = \frac{R_1 R_2}{R_1 + R_2}$

Large values may be selected for  $R_1$  and  $R_2$  – the upper limit is set by high frequency requirements

The **open circuit voltage gain** is

$$A_v = \frac{v_{out}}{v_{in}}$$

$$v_{out} = -g_m v_{gs} R_D$$

$$v_{in} = v_{gs}$$

Hence  $A_v = \frac{v_{out}}{v_{in}} = -g_m R_D$

# Output Resistance

Note  $g_m$  has **units of Siemens**,  $\Omega^{-1}$ , so  $A_v$  is dimensionless (has no units) as it should.

$$v_{oc} = -g_m v_{gs} R_D$$

$$i_{sc} = -g_m v_{gs}$$

Hence 
$$r_{out} = R_D$$

**Summary** - the generic (black box) amplifier properties of the common source amplifier are

$$R_{in} = R_G = \frac{R_1 R_2}{R_1 + R_2} \quad A_v = \frac{v_{out}}{v_{in}} = -g_m R_D \quad R_{out} = R_D$$



# Common Source Circuit Examples

# Example 1

Consider an n-channel enhancement-mode MOSFET with the following parameter:  $V_{TN} = 0.75\text{V}$ ,  $W = 40\mu\text{m}$ ,  $L = 4\mu\text{m}$ ,  $\mu_n = 650\text{cm}^2/\text{V-s}$ ,

$$t_{\text{ox}} = 450\text{\AA}, \text{ and } \epsilon_{\text{ox}} = (3.9)(8.85 \times 10^{-14})\text{F/cm}.$$

Determine the current when  $V_{GS} = 2V_{TN}$ , for the transistor biased on the saturation region.

## Solution:

The conduction parameter is determined by equation R4. First, consider the units involved in this equation, as follows:

$$K_n = \frac{W(\text{cm}) \cdot \mu_n \left( \frac{\text{cm}^2}{\text{V-s}} \right) \cdot \epsilon_{\text{ox}} \left( \frac{\text{F}}{\text{cm}} \right)}{2L(\text{cm}) \cdot t_{\text{ox}}(\text{cm})} = \frac{\text{A}}{\text{V}^2}$$

## Example 1 (Cont')

$$K_n = \frac{(40 \times 10^{-4})(650)(3.9)(8.85 \times 10^{-14})}{2(4 \times 10^{-4})(450 \times 10^{-8})} = 0.249 \text{ mA/V}^2$$

$$i_D = K_n (v_{GS} - V_{TN})^2 = (0.249)(1.5 - 0.75)^2 = 0.140 \text{ mA}$$

## Example 2

An n-channel enhancement-mode MOSFET with  $V_{TN} = 1\text{V}$  has a drain Current  $i_D = 0.8\text{mA}$  when  $V_{GS} = 3\text{V}$  and  $V_{DS} = 4.5\text{V}$ .

Calculate the drain current when :

a)  $V_{GS} = 2\text{V}$ ;  $V_{DS} = 4.5\text{V}$

b)  $V_{GS} = 3\text{V}$ ;  $V_{DS} = 1\text{V}$

Solution:

$$V_{TN} = 1\text{ V}, V_{GS} = 3\text{ V}, V_{DS} = 4.5\text{ V}$$

$$V_{DS} = 4.5 > V_{DS}(\text{sat}) = V_{GS} - V_{TN} = 3 - 1 = 2\text{ V}$$

Transistor biased in the saturation region

$$I_D = K_n (V_{GS} - V_{TN})^2 \Rightarrow 0.8 = K_n (3 - 1)^2 \Rightarrow K_n = 0.2\text{ mA/V}^2$$

## Example 2 (Cont')

(a)  $V_{GS} = 2\text{ V}, V_{DS} = 4.5\text{ V}$

Saturation region:

$$I_D = (0.2)(2-1)^2 \Rightarrow \underline{I_D = 0.2\text{ mA}}$$

(b)  $V_{GS} = 3\text{ V}, V_{DS} = 1\text{ V}$

Nonsaturation region:

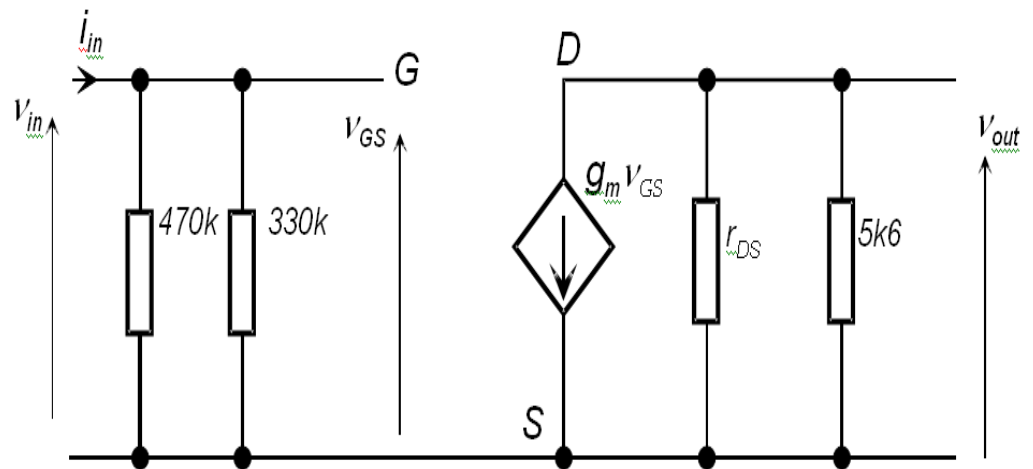
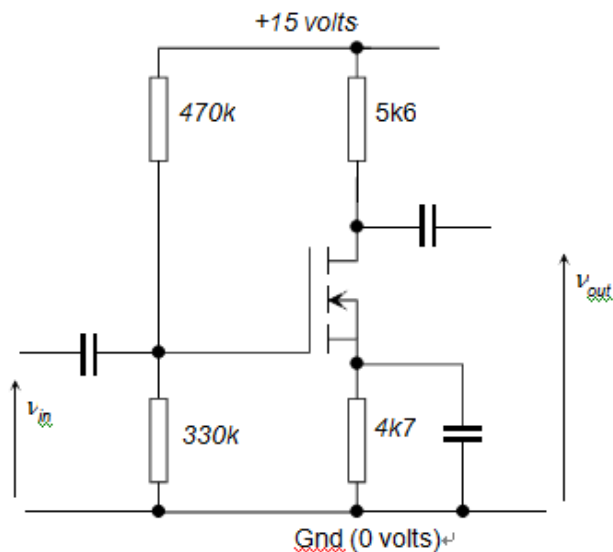
$$I_D = (0.2) \left[ 2(3-1)(1) - (1)^2 \right] \Rightarrow \underline{I_D = 0.6\text{ mA}}$$

# Example 3

A common source amplifier circuit based around a single n-channel MOSFET is shown in Figure Ex3.1. The equivalent hybrid pi model of the circuit is shown in the Figure Ex3.2.

- Calculate the input resistance.
- Calculate the output resistance.
- Calculate the open circuit voltage gain .
- Calculate the current gain when the amplifier has a load of 10k .

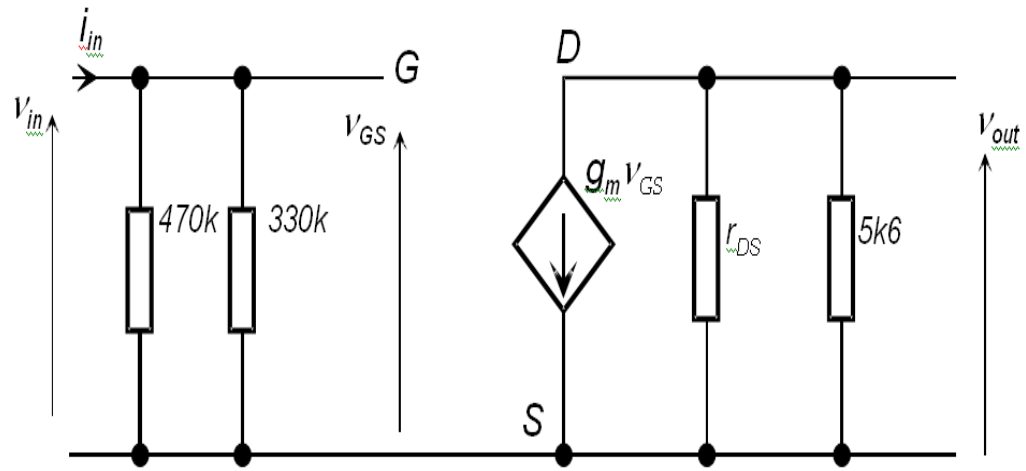
Assume the transconductance  $g_m = 30 \text{ mA/volts}$  and assume that  $r_{DS}$  is so large it may be neglected.



## Example 3 (Cont')

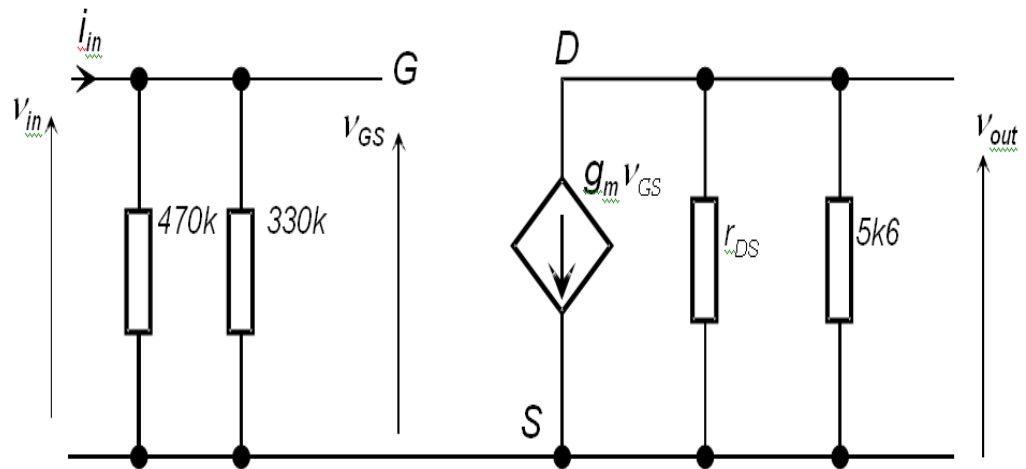
### Solution:

a) Input resistance is 470k and 330k in parallel  $R_{in} = \left( \frac{1}{470k} + \frac{1}{330k} \right)^{-1} = 194k$



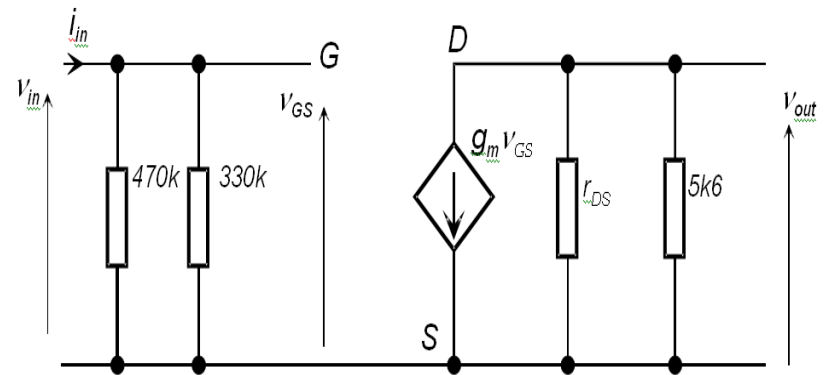
## Example 3 (Cont')

b) Ignoring  $r_{DS}$ , output resistance is  $R_{out} = 5k\Omega$





## Example 3 (Cont')



c) Open circuit voltage gain  $A_v = \frac{v_{out}}{v_{in}}$

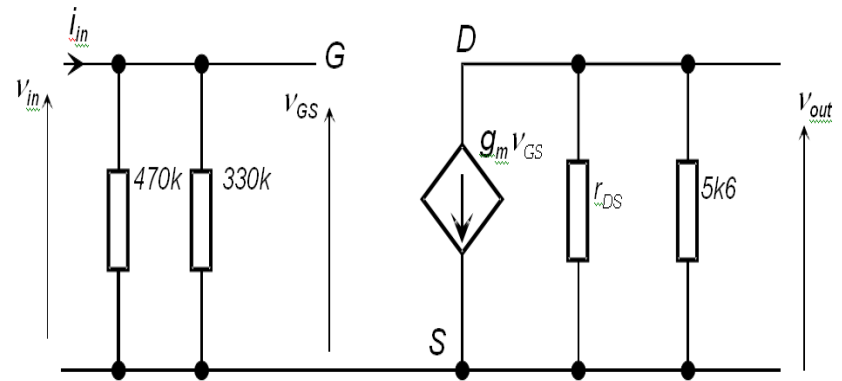
Since  $v_{out} = -g_m v_{gs} R_D$  and  $v_{in} = v_{gs}$

then

$$A_v = \frac{v_{out}}{v_{in}} = -g_m R_D$$

$$= -30 \times 10^{-3} \times 5.6 \times 10^3 = -168$$

## Example 3 (Cont')

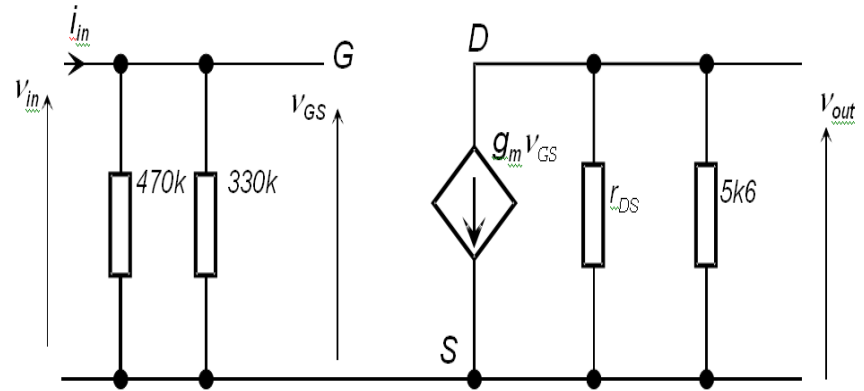


d) Input current is  $i_{in} = v_{in} / 194k \text{ mA}$

Combined output resistance with load is 5k6  
and 10k in parallel is  $R_C = \left( \frac{1}{5600} + \frac{1}{10000} \right)^{-1} = 3590\Omega$

$$\begin{aligned} \text{Output voltage, } v_{out} &= -g_m v_{gs} R_C \\ &= -30 \times 10^{-3} \times 3.59 \times 10^3 \times v_{in} \\ &= -107.7 v_{in} \end{aligned}$$

## Example 3 (Cont')



d) Output current

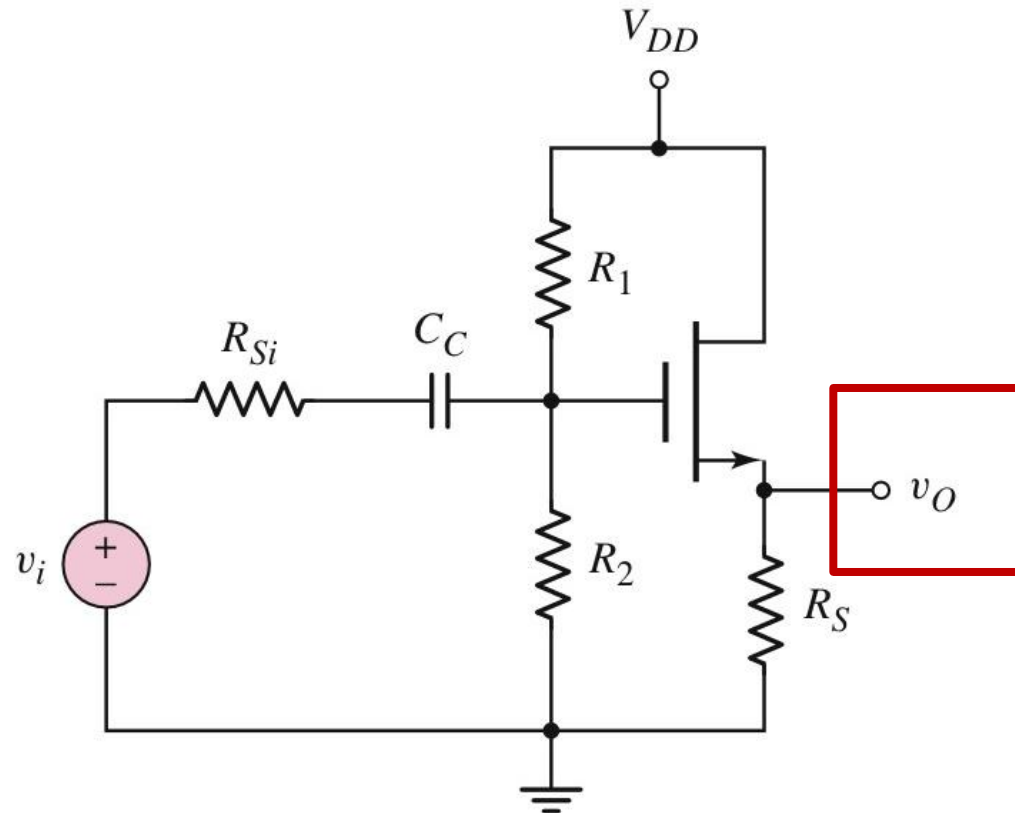
$$i_{out} = v_{out} / R_L = -107.7 v_{in} / 10^4 = \frac{-107.7 \times 194k \times i_{in}}{10k}$$

Hence current gain is

$$A_i = i_{out} / i_{in} = -2089$$

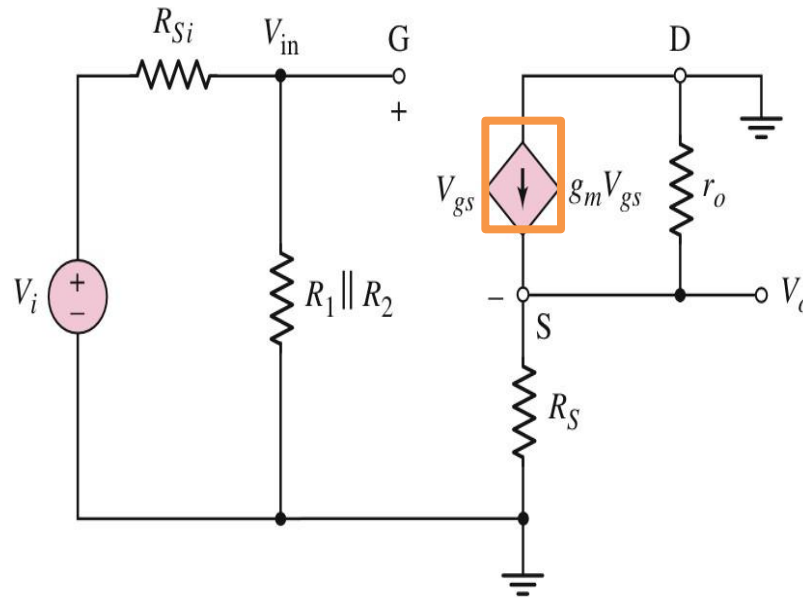
# Common Drain or Source Follower Amplifier Circuit

# NMOS Source-Follower or Common Drain Amplifier

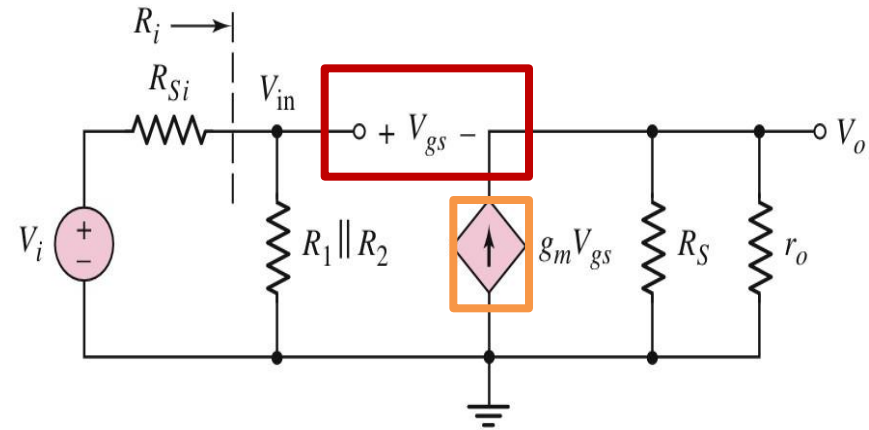


Copyright © The McGraw-Hill Companies, Inc.  
Permission required for reproduction or display.

# Small-Signal Equivalent Circuit for Source Follower



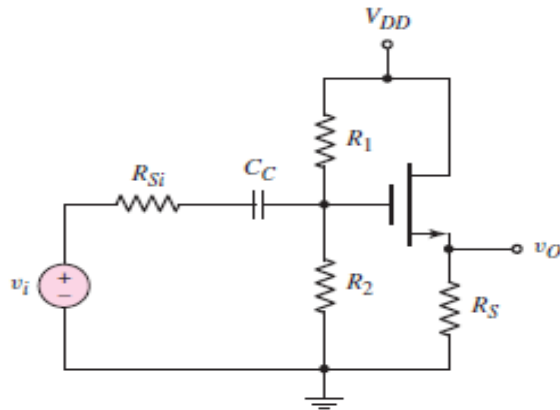
(a)



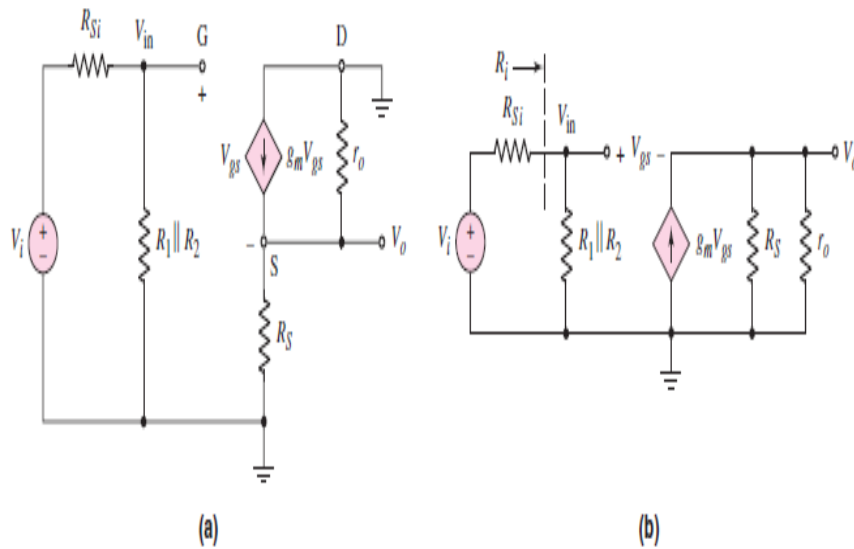
(b)

Copyright © The McGraw-Hill Companies, Inc.  
Permission required for reproduction or display.

# Common Drain Amplifier



**Figure 4.26** NMOS source-follower or common-drain amplifier



**Figure 4.27** (a) Small-signal equivalent circuit of NMOS source follower and (b) small-signal equivalent circuit of NMOS source follower with all signal grounds at a common point

$$V_o = (g_m V_{gs})(R_S \parallel r_o) \quad (4.30)$$

Writing a KVL equation from input to output results in the following:

$$V_{in} = V_{gs} + V_o = V_{gs} + g_m V_{gs}(R_S \parallel r_o) \quad (4.31(a))$$

Therefore, the gate-to-source voltage is

$$V_{gs} = \frac{V_{in}}{1 + g_m(R_S \parallel r_o)} = \left[ \frac{\frac{1}{g_m}}{\frac{1}{g_m} + (R_S \parallel r_o)} \right] \cdot V_{in} \quad (4.31(b))$$

Equation (4.31(b)) is written in the form of a voltage-divider equation, in which the gate-to-source of the NMOS device looks like a resistance with a value of  $1/g_m$ . More accurately, the effective resistance looking into the source terminal (ignoring  $r_o$ ) is  $1/g_m$ . The voltage  $V_{in}$  is related to the source input voltage  $V_i$  by

$$V_{in} = \left( \frac{R_i}{R_i + R_{Si}} \right) \cdot V_i \quad (4.32)$$

where  $R_i = R_1 \parallel R_2$  is the input resistance to the amplifier.

Substituting Equations (4.31(b)) and (4.32) into (4.30), we have the small-signal voltage gain:

$$A_v = \frac{V_o}{V_i} = \frac{g_m(R_S \parallel r_o)}{1 + g_m(R_S \parallel r_o)} \cdot \left( \frac{R_i}{R_i + R_{Si}} \right) \quad (4.33(a))$$

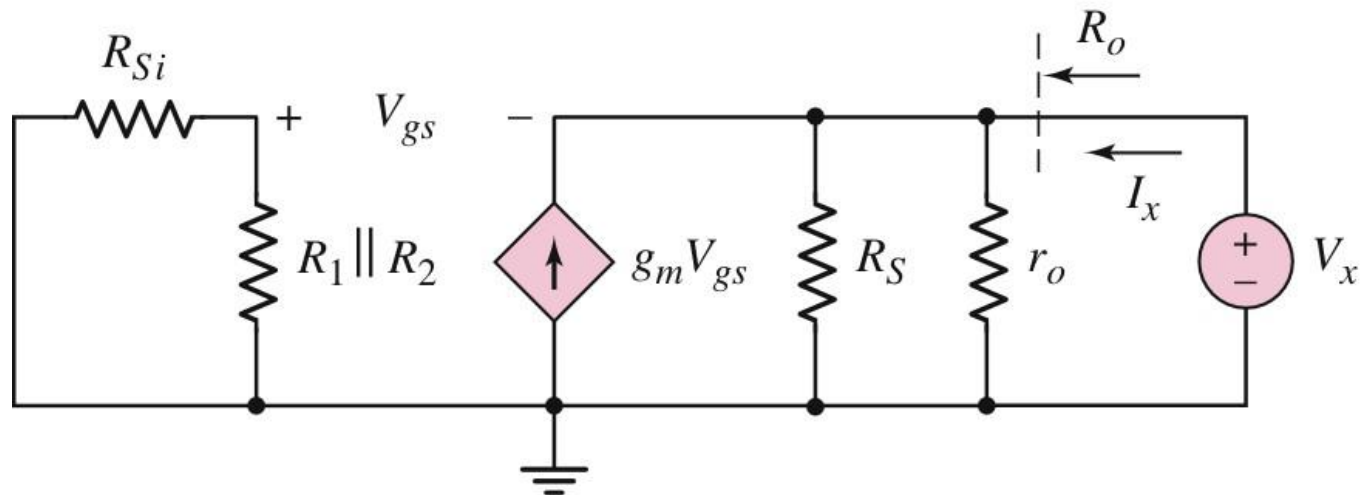
or

$$A_v = \frac{R_S \parallel r_o}{\frac{1}{g_m} + R_S \parallel r_o} \cdot \left( \frac{R_i}{R_i + R_{Si}} \right) \quad (4.33(b))$$

which again is written in the form of a voltage-divider equation. An inspection of Equation 4.33(b) shows that the magnitude of the voltage gain is always less than unity.

# Determining Output Impedance

## NMOS Source Follower



Copyright © The McGraw-Hill Companies, Inc.  
Permission required for reproduction or display.

$$R_o = \frac{1}{g_m} \parallel R_S \parallel r_o$$



# Input and Output Impedances

$$R_o = \frac{V_x}{I_x}$$

Writing a KCL equation at the output source terminal produces

$$I_x + g_m V_{gs} = \frac{V_x}{R_S} + \frac{V_x}{r_o}$$

Since there is no current in the input portion of the circuit, we see that  $V_{gs} = -V_x$ . Therefore, Equation (4.35) becomes

$$I_x = V_x \left( g_m + \frac{1}{R_S} + \frac{1}{r_o} \right)$$

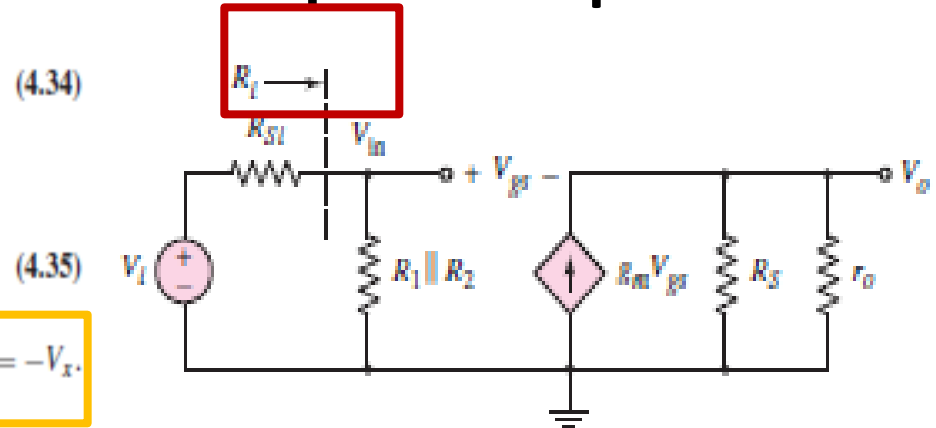
or

$$\frac{I_x}{V_x} = \frac{1}{R_o} = g_m + \frac{1}{R_S} + \frac{1}{r_o}$$

The output resistance is then

$$R_o = \frac{1}{g_m} \parallel R_S \parallel r_o$$

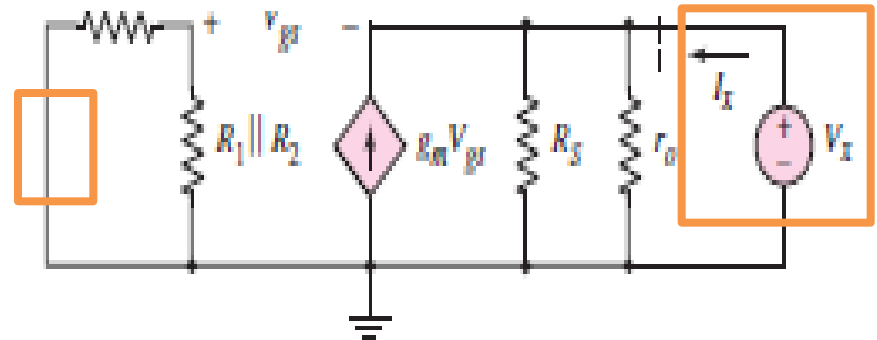
- The small-signal input resistance  $R_i$  as defined as the Thevenin equivalent resistance of the bias resistors (see figure above). So  $R_i = R_1 \parallel R_2$
- To calculate the small-signal output resistance, we set all independent small signal sources equal to zero, apply a test voltage to the output terminals, and measure a test current.



(4.36(a))

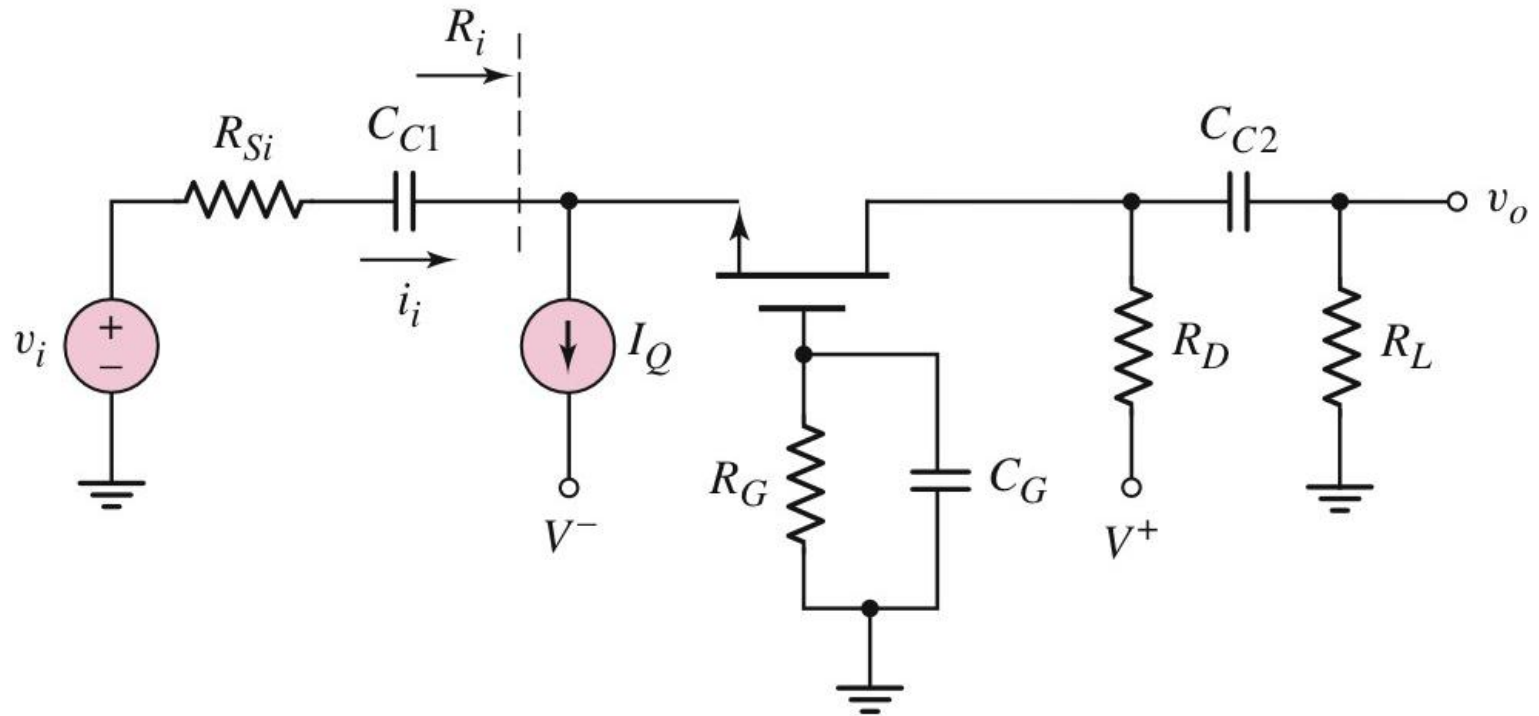
(4.36(b))

(4.37)



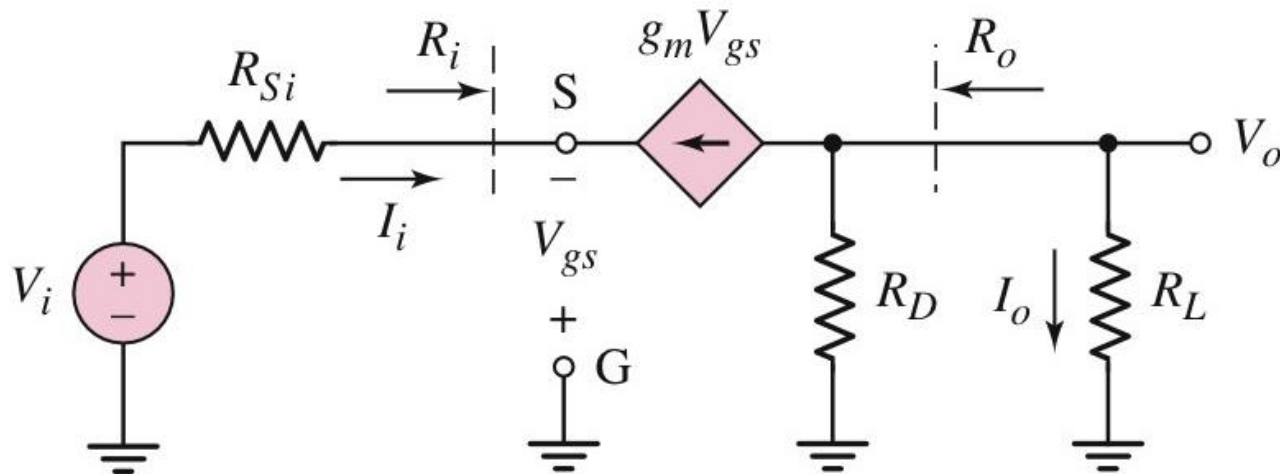
# Common Gate Amplifier Circuit

# Common-Gate Circuit



Copyright © The McGraw-Hill Companies, Inc.  
Permission required for reproduction or display.

# Small-Signal Equivalent Circuit for Common Gate



Copyright © The McGraw-Hill Companies, Inc.  
Permission required for reproduction or display.

$$A_v = \frac{g_m (R_D \parallel R_L)}{1 + g_m R_{Si}}$$

$$A_i = \frac{I_o}{I_i} = \left( \frac{R_D}{R_D + R_L} \right) \left( \frac{g_m R_{Si}}{1 + g_m R_{Si}} \right)$$

# Common Gate Amplifier

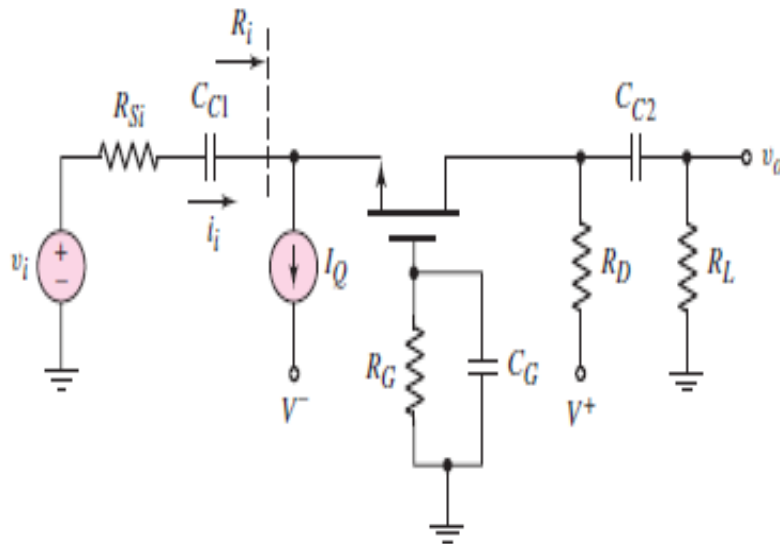


Figure 4.32 Common-gate circuit

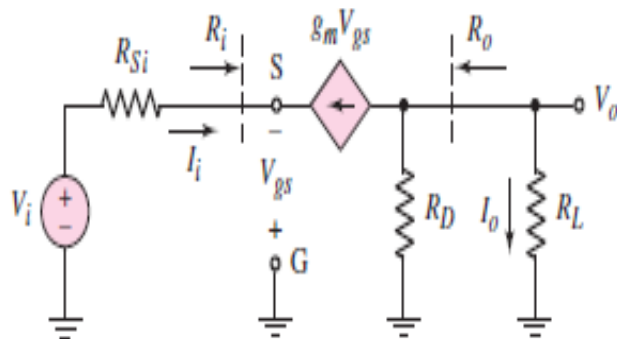


Figure 4.33 Small-signal equivalent circuit of common-gate amplifier

$$V_o = -(g_m V_{gs})(R_D \parallel R_L) \quad (4.38)$$

Writing the KVL equation around the input, we find

$$V_i = I_i R_{Si} - V_{gs} \quad (4.39)$$

where  $I_i = -g_m V_{gs}$ . The gate-to-source voltage can then be written as

$$V_{gs} = \frac{-V_i}{1 + g_m R_{Si}} \quad (4.40)$$

The small-signal voltage gain is found to be

$$A_v = \frac{V_o}{V_i} = \frac{g_m (R_D \parallel R_L)}{1 + g_m R_{Si}} \quad (4.41)$$

Also, since the voltage gain is positive, the output and input signals are in phase.

In many cases, the signal input to a common-gate circuit is a current. Figure 4.34 shows the small-signal equivalent common-gate circuit with a Norton equivalent circuit as the signal source. We can calculate a current gain. The output current  $I_o$  can be written

$$I_o = \left( \frac{R_D}{R_D + R_L} \right) (-g_m V_{gs}) \quad (4.42)$$

At the input we have

$$I_i + g_m V_{gs} + \frac{V_{gs}}{R_{Si}} = 0 \quad (4.43)$$

or

$$V_{gs} = -I_i \left( \frac{R_{Si}}{1 + g_m R_{Si}} \right) \quad (4.44)$$

The small-signal current gain is then

$$A_i = \frac{I_o}{I_i} = \left( \frac{R_D}{R_D + R_L} \right) \cdot \left( \frac{g_m R_{Si}}{1 + g_m R_{Si}} \right) \quad (4.45)$$

We may note that if  $R_D \gg R_L$  and  $g_m R_{Si} \gg 1$ , then the current gain is essentially unity.

# Input and Output Resistance

## 4.5.2 Input and Output Impedance

In contrast to the common-source and source-follower amplifiers, the common-gate circuit has a low input resistance because of the transistor. However, if the input signal is a current, a low input resistance is an advantage. The input resistance is defined, using Figure 4.33, as

$$R_i = \frac{-V_{gs}}{I_i} \quad (4.46)$$

Since  $I_i = -g_m V_{gs}$ , the input resistance is

$$R_i = \frac{1}{g_m} \quad (4.47)$$

This result has been obtained previously.

We can find the output resistance by setting the input signal voltage equal to zero. From Figure 4.33, we see that  $V_{gs} = -g_m V_{gs} R_{Si}$ , which means that  $V_{gs} = 0$ . Consequently,  $g_m V_{gs} = 0$ . The output resistance, looking back from the load resistance, is therefore

$$R_o = R_D \quad (4.48)$$

# Comparison of Three Amplifier Circuits

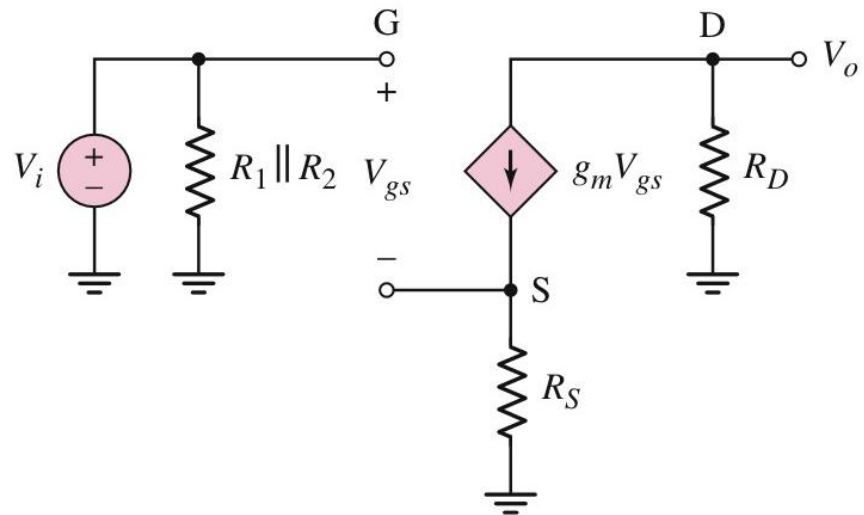
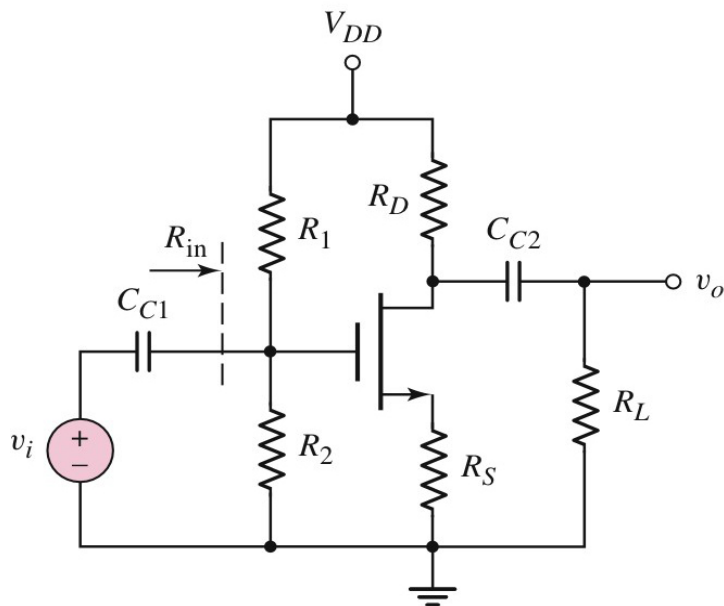
# Comparison of 3 Basic Amplifiers

| Configuration   | Voltage Gain    | Current Gain    | Input Resistance | Output Resistance |
|-----------------|-----------------|-----------------|------------------|-------------------|
| Common Source   | $A_v > 1$       | —               | $R_{TH}$         | Moderate to high  |
| Source Follower | $A_v \approx 1$ | —               | $R_{TH}$         | Low               |
| Common Gate     | $A_v > 1$       | $A_i \approx 1$ | Low              | Moderate to high  |



# Example 4

The parameters of the circuit shown in Figure P4.15 are  $V_{DD} = 12\text{ V}$ ,  $R_S = 0.5\text{ k}\Omega$ ,  $R_{in} = 250\text{ k}\Omega$ , and  $R_L = 10\text{ k}\Omega$ . The transistor parameters are  $V_{TN} = 1.2\text{ V}$ ,  $K_n = 1.5\text{ mA/V}^2$ , and  $\lambda = 0$ . (a) Design the circuit such that  $I_{DQ} = 2\text{ mA}$  and  $V_{DSQ} = 5\text{ V}$ . (b) Determine the small signal voltage gain. (The small signal output resistance  $r_o \cong \frac{1}{\lambda I_{DQ}}$ . The transconductance  $g_m = 2\sqrt{K_n I_{DQ}}$ .)



Copyright © The McGraw-Hill Companies, Inc.  
Permission required for reproduction or display.

# Solution

$$(a) \quad V_{DSQ} = V_{DD} - I_{DQ}(R_S + R_D)$$

$$5 = 12 - (2)(R_S + R_D) \Rightarrow R_S + R_D = 3.5 \text{ k}\Omega$$

$$R_S = 0.5 \text{ k}\Omega, \text{ then } R_D = 3 \text{ k}\Omega$$

$$I_{DQ} = K_n (V_{GSQ} - V_{TN})^2$$

$$2 = 1.5(V_{GSQ} - 1.2)^2 \Rightarrow V_{GSQ} = 2.355 \text{ V}$$

$$V_G = V_{GSQ} + I_{DQ}R_S = 2.355 + (2)(0.5) = 3.355 \text{ V}$$

$$V_G = \left( \frac{R_2}{R_1 + R_2} \right) \cdot V_{DD} = \frac{1}{R_1} \cdot R_{in} \cdot V_{DD}$$

$$3.355 = \frac{1}{R_1} (250)(12) \Rightarrow R_1 = 894 \text{ k}\Omega$$

$$R_1 \parallel R_2 = 894 \parallel R_2 = 250 \Rightarrow R_2 = 347 \text{ k}\Omega$$

$$(b) \quad g_m = 2\sqrt{(1.5)(2)} = 3.464 \text{ mA/V}$$

$$A_v = \frac{-g_m(R_D \parallel R_L)}{1 + g_m R_S} = \frac{-(3.464)(3 \parallel 10)}{1 + (3.464)(0.5)} = -2.93$$

# Chapter 4

- Investigate a single-transistor circuit that can amplify a small, time-varying input signal
  - Develop small-signal models that are used in the analysis of linear amplifiers.
- Discuss and compare the three basic transistor amplifier configurations.
  - Analyze the common-source amplifier.
  - Analyze the source-follower amplifier.
  - Analyze the common-gate amplifier.