EEE 319 Lab Tutorial

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Content

1. Introduction

The most widely circulated journals that publish papers related to engineering optimization are *Engineering Optimization, ASME Journal of Mechanical Design, AIAA Journal, ASCE Journal of Structural Engineering, Computers and Structures* and so on.

The solution of most practical optimization problems requires the use of computers. Several commercial software systems are available to solve optimization problems that arise in different engineering areas. **MATLAB** is a popular software that is used for the solution of a variety of scientific and engineering problems.

1. Introduction

MATLAB has several toolboxes each developed for the solution of problems from a specific scientific area. The specific toolbox of interest for solving optimization and related problems is called the *optimization toolbox*. It contains a library of programs or m-files, which can be used for the solution of minimization, equations, least squares curve fitting, and related problems.

2. LAB-1-Optimization

Find the solution of the following nonlinear optimization problem using the MATLAB function *fmincon*:

$$\text{Minimize } f(x_1, x_2) = 9.82x_1x_2 + 2x_1$$

Minimization of function of several variables subject to constraints	Find $\mathbf{x}$ to minimize $f(\mathbf{x})$ subject to $\mathbf{c}(\mathbf{x}) \le \mathbf{0}$, $\mathbf{c}_{\text{eq}} = \mathbf{0}$ $[A]\mathbf{x} \le \mathbf{b}$, $[A_{\text{eq}}]\mathbf{x} = \mathbf{b}_{\text{eq}}$, $\mathbf{l} \le \mathbf{x} \le \mathbf{u}$	`fmincon`

2. LAB-1-Optimization

subject to

$$g_1(x_1, x_2) = \frac{2500}{\pi x_1 x_2} - 500 \leq 0$$

$$g_2(x_1, x_2) = \frac{2500}{\pi x_1 x_2} - \frac{\pi^2(x_1^2 + x_2^2)}{0.5882} \leq 0$$

$$g_3(x_1, x_2) = -x_1 + 2 \leq 0$$

$$g_4(x_1, x_2) = x_1 - 14 \leq 0$$

$$g_5(x_1, x_2) = -x_2 + 0.2 \leq 0$$

$$g_6(x_1, x_2) = x_2 - 0.8 \leq 0$$

2. LAB-1-Optimization

Result:

```
The values of function value and constraints at starting point
f=
 41.4960
c =
-215.7947
-540.6668
-5.0000
-7.0000
-0.2000
-0.4000
ceq =
   []
Optimization terminated: first-order optimality
measure less
```

2. LAB-1-Optimization

Result:

```
than options. TolFun and maximum constraint violation
is less
than options.TolCon.
Active inequalities (to within options.TolCon = 1e-006):
lower upper ineqlin ineqnonlin
              1
              2
x=
 5.4510 0.2920
fval =
 26.5310
The values of constraints at optimum solution
c=
 -0.0000
 -0.0000
 -3.4510
 -8.5490
 -0.0920
 -0.5080
ceq =
  []
```

3. LAB-1-Symbolic

Use the following MATLAB commands to construct a symbolic function:

$$syms\ x\ y$$

$$f = (y+1)\char`^3 + x*y\char`^2 + y\char`^2 - 4*x*y - 4*y + 1$$

Compute the first partials of *f* and Hessian of *f* by setting

$$fx = diff(f,x), fy = diff(f,y)$$

$$H = [diff(fx,x), diff(fx,y); diff(fy,x), diff(fy,y)]$$

3. LAB-1-Symbolic

We can use the '***subs***' command to evaluate the Hessian for any pair ***(x, y).*** For example, to evaluate the Hessian when **x=3** and y**=5**, set

$$H1 = subs(H, [x, y], [3, 5])$$

Use the MATLAB command to determine vectors x and y containing the x and y coordinates of the stationary points. Evaluate the Hessian at each stationary point and then determine whether the stationary point is a local maximum, local minimum, or saddle point.

3. LAB-1-Symbolic

Result:

```
H =
[        0, 2*x + 6*y + 8]
[ 2*y - 4, 2*x + 6*y + 8]
H1 =
[ 0, 44]
[ 6, 44]
```

4. LAB-2

Given an objective function:

$$f(X) = x_1^2 + 2x_2^2 - 0.3\ \cos(3\pi x_1) - 0.4\cos(4\pi x_2) + 0.7$$

(1) Find the contour of **f** on $\mathbf{x_1}$-$\mathbf{x_2}$ plane, with gradient.
(2) Generate the surface of **f**.

Contour: a line drawn on a map connecting points of equal height.

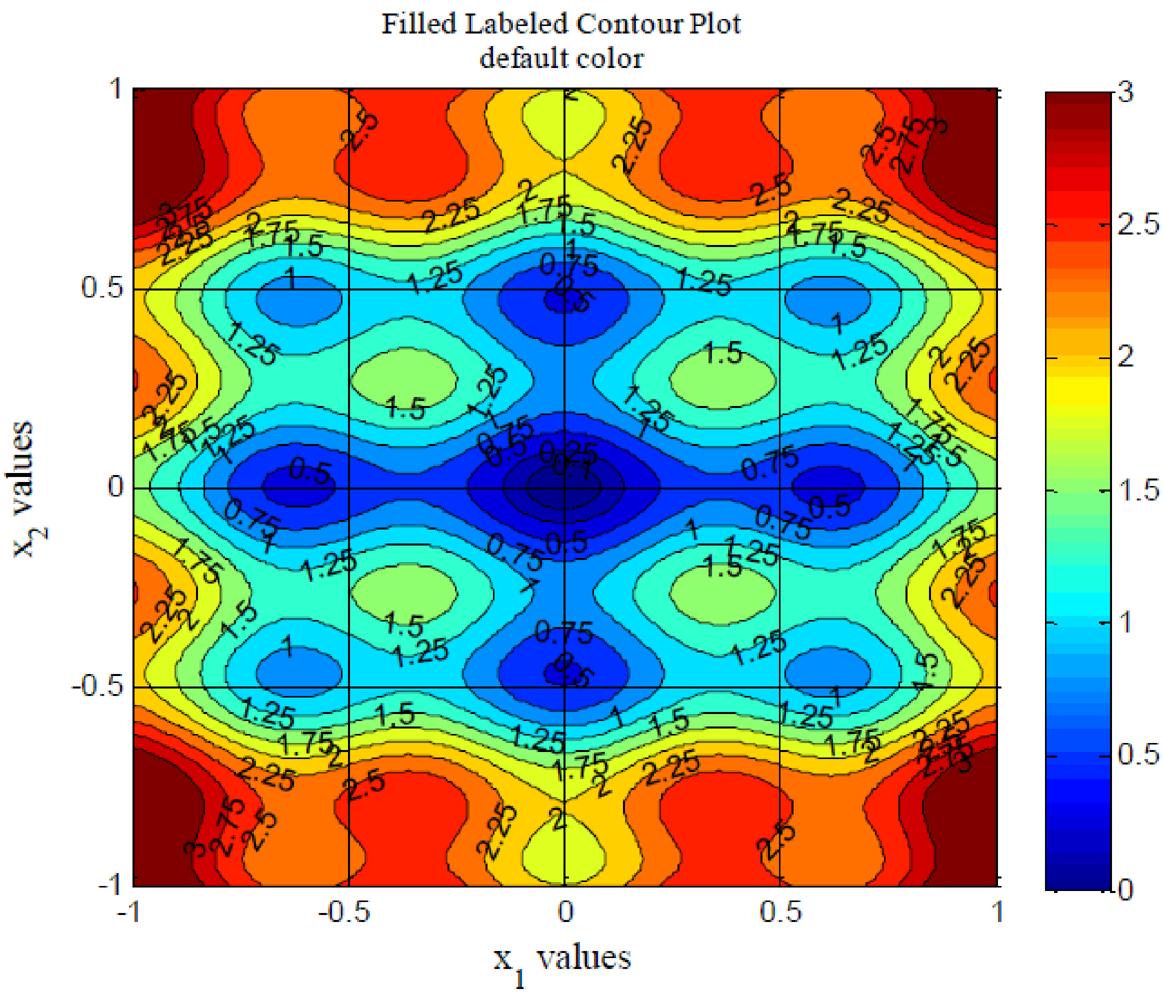
4. LAB-2

Please use ***help*** command to find the usage of the following functions:

MESHGRID	CLABEL
COLORBAR	COLORMAP
SET	GRADIENT
QUIVER	MESH
SURF	VIEW

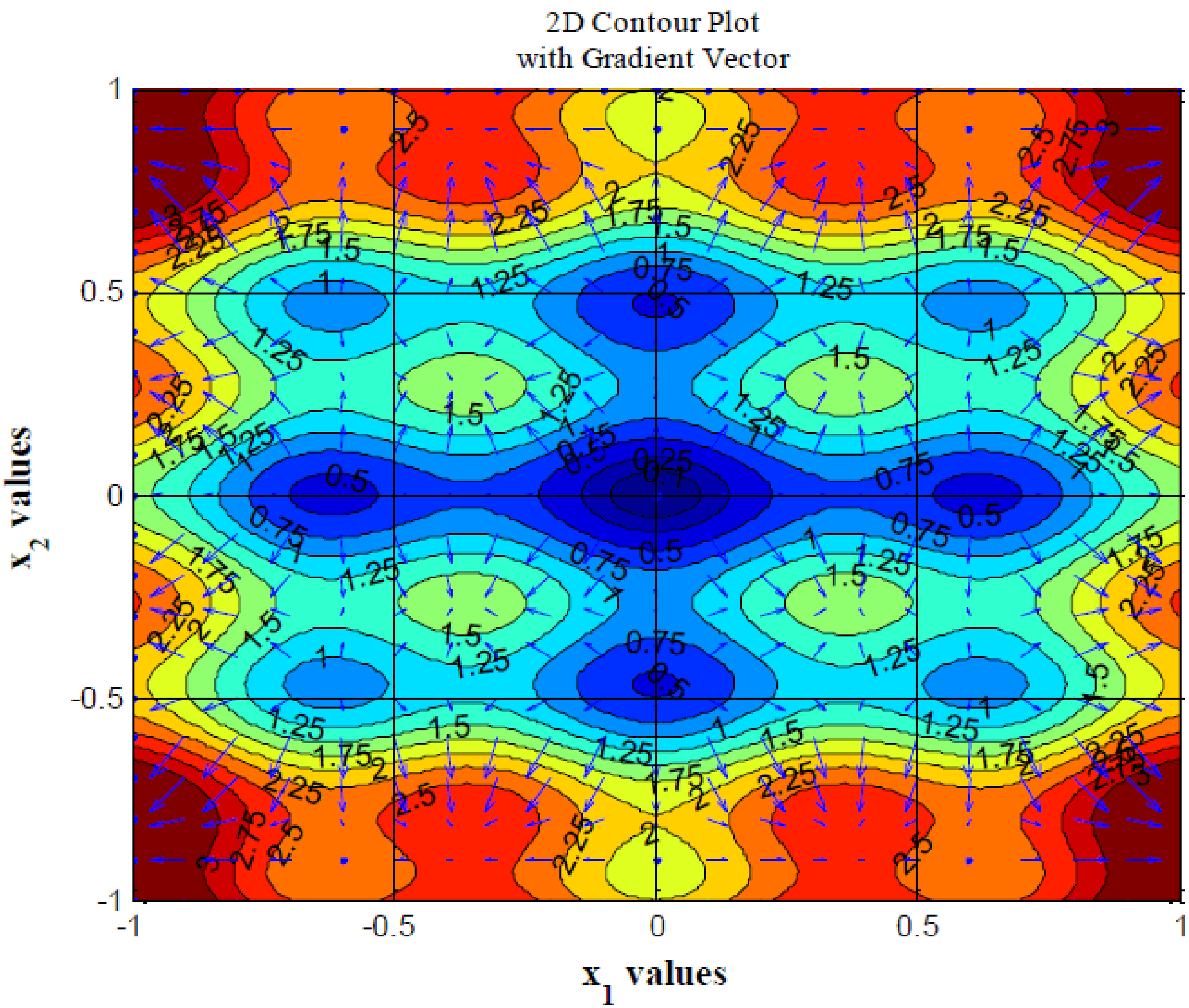
4. LAB-2

Result:



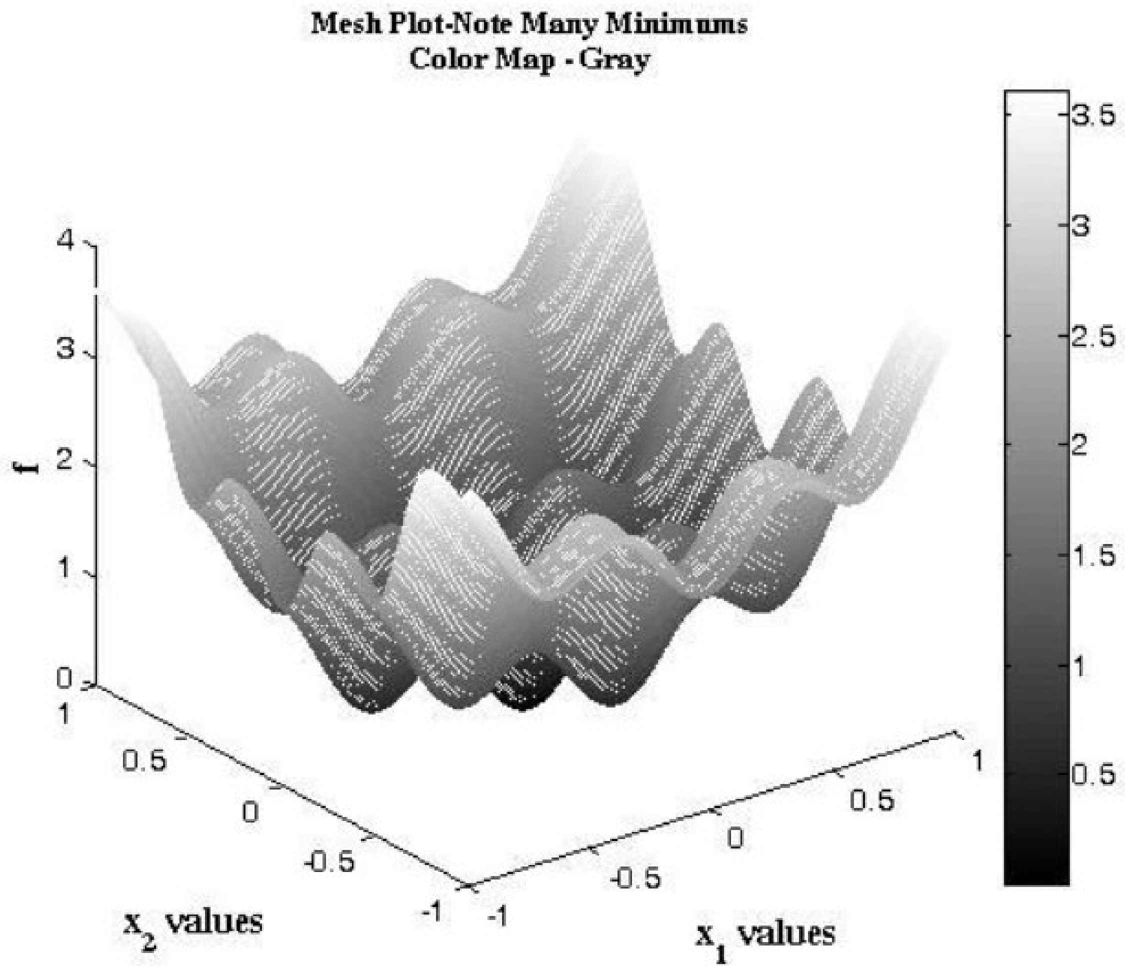
4. LAB-2

Result:



4. LAB-2

Result:



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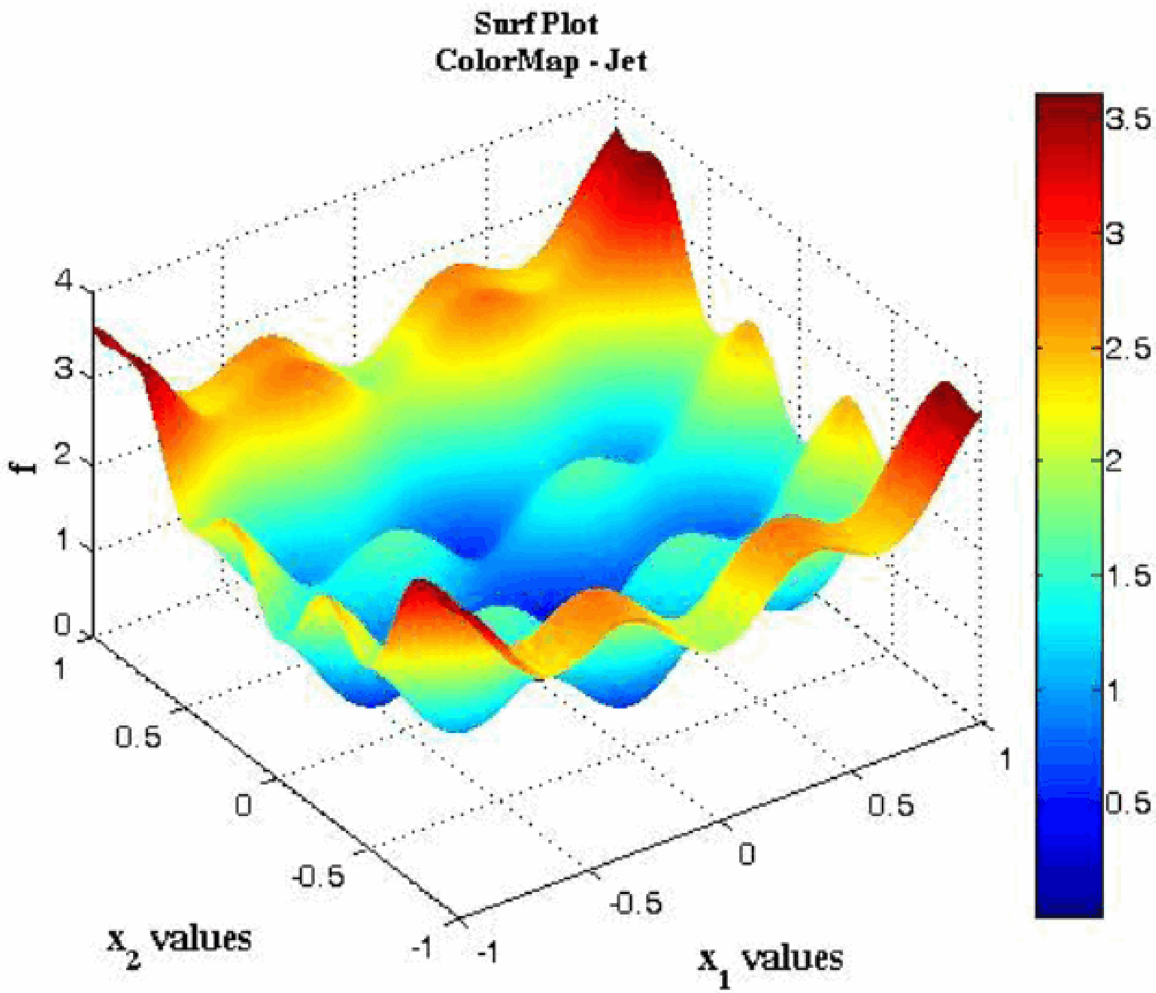
4. LAB-2

Result:



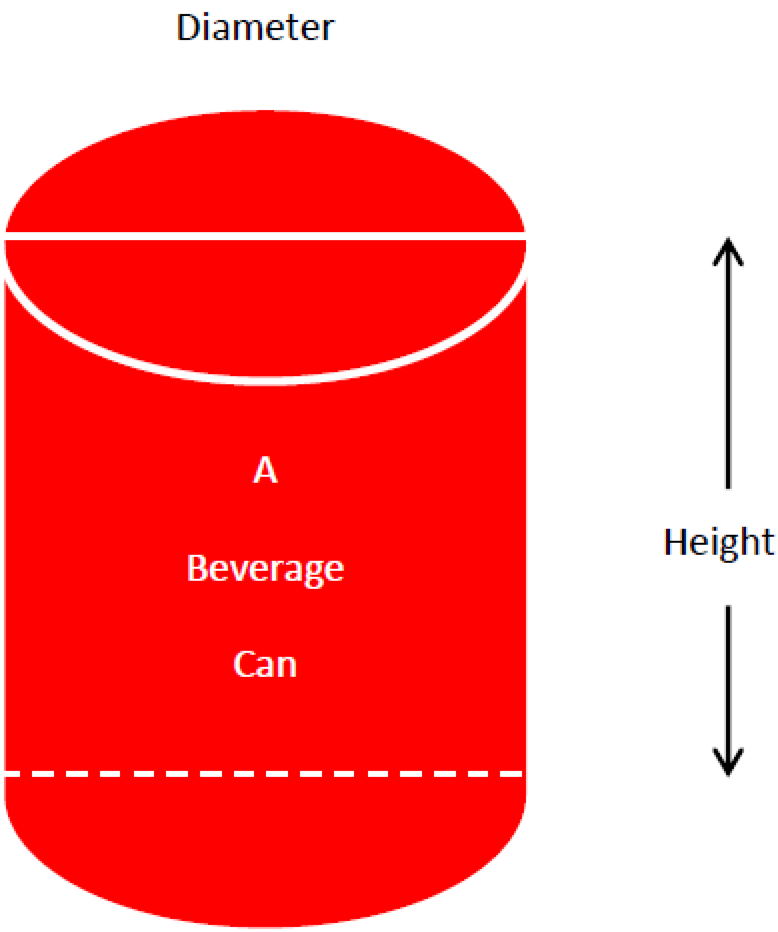
5. LAB-3

Design Problem:

New consumer research with deference to the obesity problem among the general population, suggests that people should drink no more than about 0.25 litre ($250cm^3$) of soda pop at a time. The fabrication cost of the re-designed soda can is proportional to the surface area, and can be estimated at $1.00 per square centimeter of the material used. A circular cross-section is the most plausible, given current tooling available for manufacture. For aesthetic reason, the height must be at least twice the diameter. Studies indicate that holding comfort requires a diameter between 5 and 8 cm. Please create a design that will cost least.

5. LAB-3

Design Problem:



5. LAB-3

Mathematical formulation:

$$\text{Minimize } \mathrm{f(d,h)} = \mathrm{C}(\pi dh + \pi d^2/4)$$

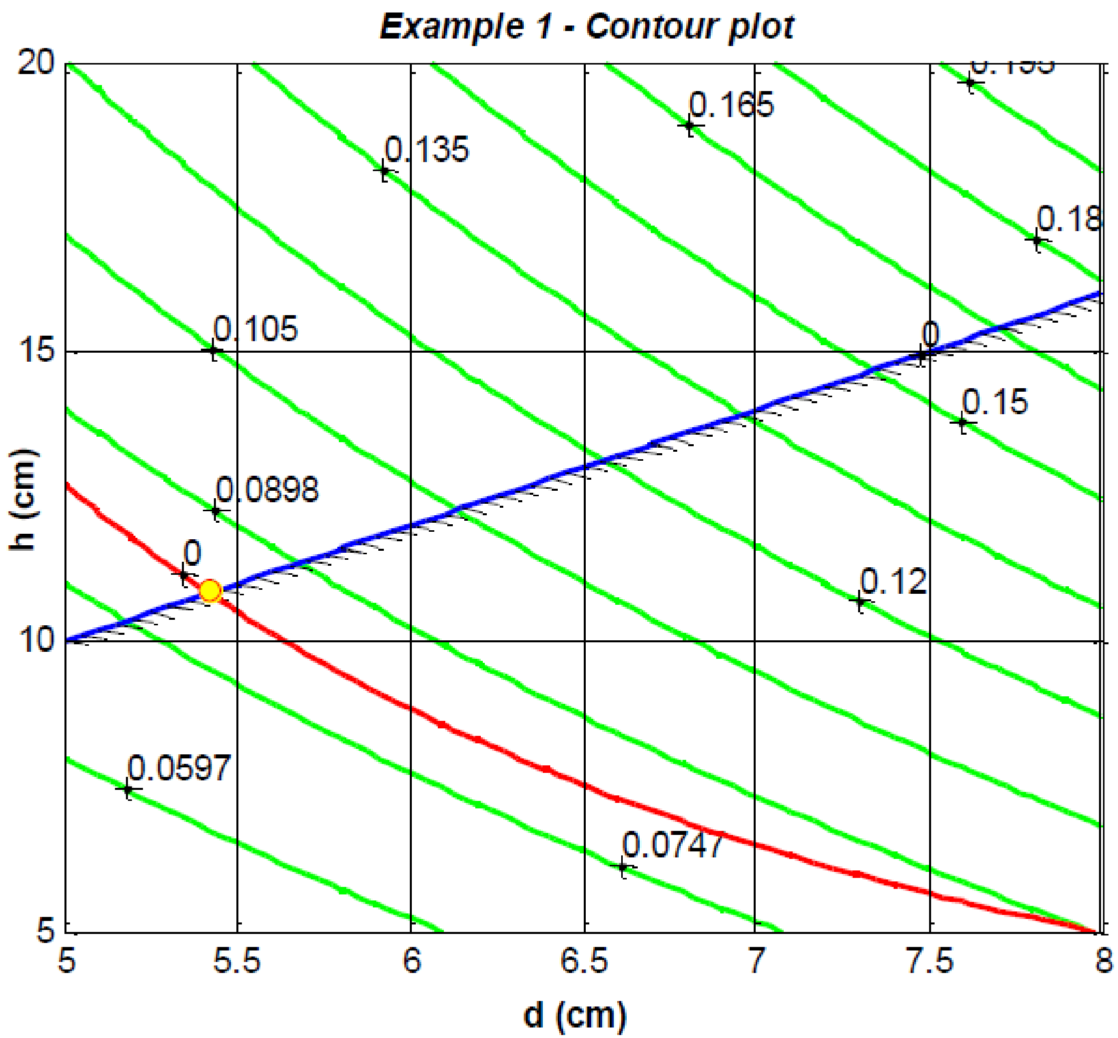
$$\text{Subject to: } h_1(d,h) = \pi d^2 h/4 - 250 = 0$$

$$g_1(d,h) = 2d - h \leq 0$$

$$5 \leq \mathrm{d} \leq 8; \quad 4 \leq \mathrm{h} \leq 20$$

5. LAB-3

Result:



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6. LAB-4

Utilizing of nlib toolbox

- Please go to the ICE page on EEE319, and download the zipped file **NLIB**. Then, unzip the file to a local folder, for example

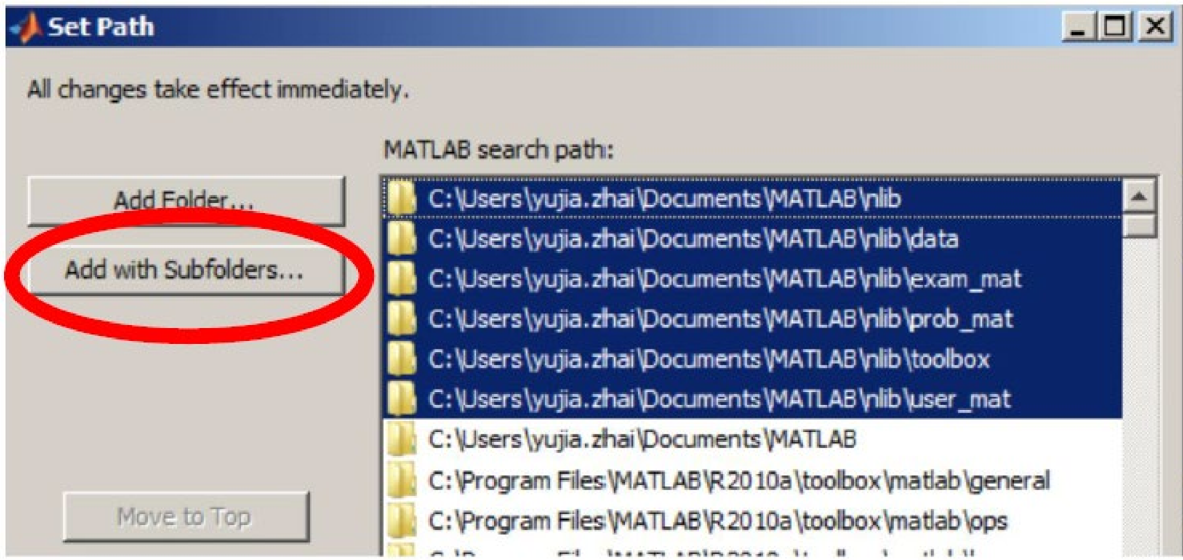
  C:\Users\YourName\Documents\MATLAB\nlib

- The NLIB folder should have five sub folders as listed:

  (1) .\nlib\data
  (2) .\nlib\exam_mat
  (3) .\nlib\prob_mat
  (4) .\nlib\toolbox
  (5) .\nlib\user_mat

6. LAB-4

- Please run MATLAB, and set the path as following:



- Set Path --> Add with Subfolders --> nlib

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6. LAB-4

◆ **Example**

(1)On Command Window, enter the directory

.\nlib\exam_matOpen m-file –e631.m

(2) On Command Window, enter the directory

.\nlib\toolboxOpen m-file –**GOLDEN.m**

(3) The two files above show that how to use Golden Section Method to solve a one dimensional optimization problem.

Minimize

$$F(\alpha) = -10\alpha^3\exp(-2\alpha)$$

with the range $\alpha \in [0\ 4]$

6. LAB-4

◆ Example

(4) Match the format of functions' name in the toolbox and code

```matlab
% Find a three-point pattern

    fprintf ('Example 6.3.1: Golden Section Search\n');
    [a,b,c,err,k] = bracket(h,x,d,mu,fun631,'','');
    fprintf ('\nFunction evaluations = %g',k);
    fprintf ('\nThree-point pattern = (%.7f,%.7f,%.7f).',a,b,c);
```

6. LAB-4

◆ **Your work**

Please write a MATLAB program to find the minimum of

$$f = x\,(x - 1.5)$$

in the interval**(0.0, 1.00),** by using the **NLIB** &function - GOLDEN

7. LAB-5

Steepest Descent Method and Conjugate Gradient Method

◆ Study the m files -e641.m and e651.m in the path .nlib\exam_mat\

```
%------------------------------------------------------------------
% Example 6.4.1: Steepest Descent Method
%------------------------------------------------------------------
```

```
%------------------------------------------------------------------
% Example 6.5.1: Conjugate Gradient Method
%------------------------------------------------------------------
```

7. LAB-5

- In toolbox. v=1/n for steepest descent/full conjugate gradient method

```
function [x,ev,j] = conjgrad (x0,tol,v,m,f)
%------------------------------------------------------------------
% Usage:       [x,ev,j] = conjgrad (x0,tol,v,m,f)
%
% Description: Use the Fletcher-Reeves version of the conjugate
%              gradient method to solve the following n-dimensional
%              unconstrained optimization problem:
%
%                  minimize: f(x)
%
% Inputs:       x0  = n by 1 vector containing initial guess
%               tol = error tolerance used to terminate search (tol >= 0)
%               v   = order of method:
%
%                           v       method
%                           ---------------------------
%                           1   steepest descent
%                           n   full conjugate gradient
%                           ---------------------------
```

7. LAB-5

- **Your work**

  Finish solving Question (a) for at least one method

  For interest, finish the rest questions

  No need to plot

Thank You

Question?

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