Generator and Transformer Models: The Per-unit System (Part II) EEE210

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Overview

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- Due to the non-uniform airgap of the salient poles, the flux is stronger in the direct axis than in the quadrature axis.
- Thus the reactance X_d in the direct axis direction is larger than X_q in the quadrature axis direction.

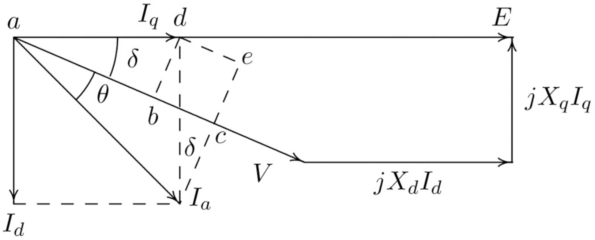

Figure 1: Phasor diagram for a salient-pole generator.

- The common way of dealing with this is to decompose the armature current into two components
  1. Along the direct axis, and
  2. Along the quadrature axis
- Now the voltage can be found by adding the components along these directions.
- It is noted that an Ampere in the direct axis direction will produce more voltage than an Ampere in the quadrature direction (due to the difference in magnitude between X_d and X_q, illustrated in Figure 1.

3.4 Salient-Pole Synchronous Generators

The excitation voltage E is given by

$$|E| = |V|\cos\delta + X_d I_d \tag{1}$$

The three-phase real power at the generator terminal is

$$P_{3\phi} = 3|V||I_a|\cos\theta \tag{2}$$

The power component of the armature current can be expressed in terms of I_d and I_q as follows.

$$\begin{aligned} |I_a|\cos\theta^{1} &= ab + de \\ &= I_q\cos\delta + I_d\sin\delta \end{aligned} \tag{3}$$

Substituting (3) into (2), we have

$$P_{3\phi} = 3|V|(I_q\cos\delta + I_d\sin\delta) \tag{4}$$

[1] not δ

Now from the phasor diagram in Figure 1,

$$|V|\sin\delta = X_q I_q \tag{5}$$

or

$$I_q = \frac{|V|\sin\delta}{X_q} \tag{6}$$

Also from (1), I_d is given by

$$I_d = \frac{|E| - |V|\cos\delta}{X_d} \tag{7}$$

Substituting for I_d and I_q from (7) and (6) into (4), the real power with armature resistance neglected becomes

$$P_{3\phi} = 3\frac{|E||V|}{X_d}\sin\delta + 3|V|^2\frac{X_d - X_q}{2X_dX_q}\sin 2\delta \tag{8}$$

Note that the new term known as reluctance power is at frequency (in space) twice that of the normal power. Thus the power-angle relationship is changed from a perfect sinusoid.

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3.5 Power Transformer

- Transformers are essential elements in any power system.
- They allow the relatively low voltages from generators to be raised to a very high level for efficient power transmission.
- In modern utility systems, the energy may undergo four or five transformations between generator and ultimate user.
- Hence, a system is likely to have five times more kVA of installed capacity of transformers than of generators.

3.6 Equivalent Circuit of a Transformer

Considering an ideal transformer, one with no leakage and infinite permeability. Assume a transformer has

- N_1 turns on the left side,
- N_2 on the right side,
- I_2' is the input current on the left and, and
- I_2 is the output current on the right.

In such a case, we have (assuming sinusoidal flux $\phi = \Phi \cos wt$)

$$
\begin{aligned}
e_1 &= N_1 \frac{d\phi}{dt} \\
&= -wN_1 \Phi \sin wt \\
&= E_{1max} \cos(wt + 90^\circ)
\end{aligned}
$$

where $E_{max} = 2\pi f N_1 \Phi$. Thus the RMS value of E_1 is

$$E_1 = 4.44 f N_1 \Phi$$

Since the same flux links both windings, we have on the right

$$E_2 = 4.44 f N_2 \Phi$$

3.6 Equivalent Circuit of a Transformer

Also, since the core has no losses, we must have $I_2' N_1 = I_2 N_2$. This leads to the relationships

$$\frac{E_1}{E_2} = \frac{I_2}{I_2'} = \frac{N_1}{N_2} \tag{9}$$

Therefore, in the ideal transformer, the ratio of currents and voltages is the turns ratio (though the current ratio is inversely so).

3.6 Equivalent Circuit of a Transformer

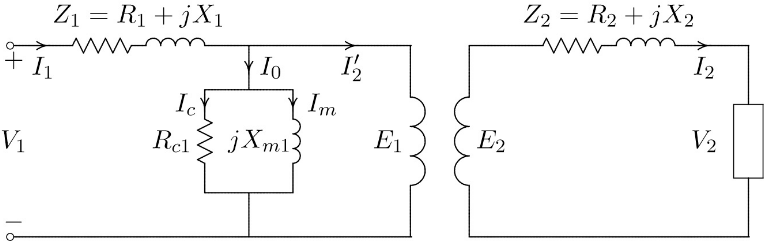

Figure 2: Equivalent circuit of a transformer.

3.6 Equivalent Circuit of a Transformer

- In a real transformer, there would be some leakage flux, also some losses. There is also a magnetizing current. These are simulated by adding components to the ideal model
- Thus a real transformer would be as shown in Figure 2.
- This is the T-equivalent model or equivalent circuit.

3.6 Equivalent Circuit of a Transformer

To obtain the performance characteristics of a transformer, it is convenient to use an equivalent circuit model referred to one side of the transformer.

From Kirchhoff's voltage law (KVL), the voltage equation of the secondary side is

$$E_2 = V_2 + Z_2 I_2 \tag{10}$$

From the relationship (9) developed for the ideal transformer, the secondary induced voltage and current are $E_2 = (N_2/N_1)E_1$ and $I_2 = (N_1/N_2)I_2'$, respectively.

Upon substitution, (10) reduces to

$$\begin{aligned} E_1 &= \frac{N_1}{N_2}V_2 + \left(\frac{N_1}{N_2}\right)^2 Z_2 I_2' \\ &= V_2' + Z_2' I_2' \end{aligned} \qquad (11)$$

where

$$Z_2' = R_2' + jX_2' = \left(\frac{N_1}{N_2}\right)^2 R_2 + j\left(\frac{N_1}{N_2}\right)^2 X_2$$

Relation (11) is the KVL equation of the secondary side referred to the primary, and the equivalent circuit of Figure 2 can be redrawn as shown in Figure 3.

3.6 Equivalent Circuit of a Transformer

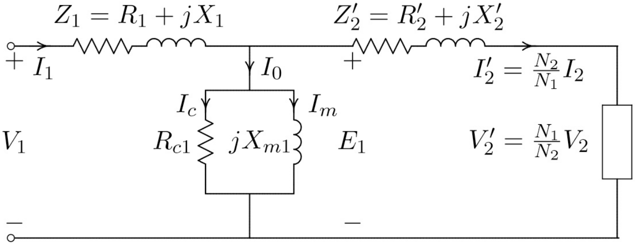

Figure 3: Exact equivalent circuit referred to the primary side.

3.6 Equivalent Circuit of a Transformer

On no-load, the primary voltage drop is very small, and V_1 can be used in place of E_1 for computing the no-load current I_0

The primary quantities R_1 and X_1 can be combined with the referred secondary quantities R_2' and X_2' to obtain the equivalent primary quantities R_{e1} and X_{e1}, and thus

$$Z_{e1} = Z_1 + Z_2' \tag{12}$$

This equivalent circuit is shown in Figure 4

3.6 Equivalent Circuit of a Transformer

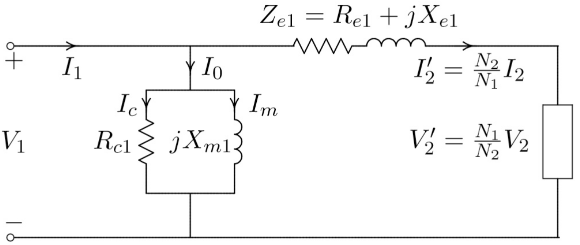

Figure 4: Approximate equivalent circuit referred to the primary.

From Figure 4,

$$V_1 = V_2' + (R_{e1} + jX_{e1})I_2' \tag{13}$$

where

$$\begin{aligned}
R_{e1} &= R_1 + \left(\frac{N_1}{N_2}\right)^2 R_2, \\
X_{e1} &= X_1 + \left(\frac{N_1}{N_2}\right)^2 {\color{red}X_2}, \quad \text{and} \\
I_2' &= \frac{S_L^*}{3V_2'^*}
\end{aligned}$$

The equivalent circuit referred to the secondary is shown in Figure 5. From Figure 5, the referred primary voltage V_1' is given by

$$V_1' = V_2 + (R_{e2} + jX_{e2})I_2 \tag{14}$$

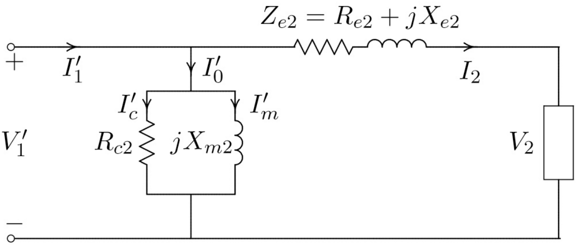

Figure 5: Approximate equivalent circuit referred to the secondary.

3.6 Equivalent Circuit of a Transformer

Power transformers are generally designed with very high permeability core and very small core loss.

Consequently, a further approximation of the equivalent circuit can be made by omitting the shunt branch, as shown in Figure 6.

The equivalent circuit referred to secondary is also shown in Figure 6.

3.6 Equivalent Circuit of a Transformer

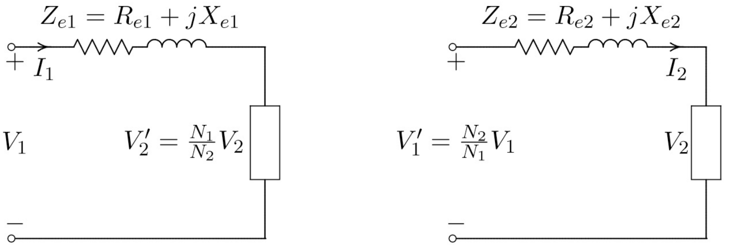

Figure 6: Simplified circuits referred to primary side (left figure) and secondary side (right figure).

Determination of Equivalent Circuit Parameters

The parameters of the approximate equivalent circuit are readily obtained from open circuit and short circuit tests.

In the open-circuit test, rated voltage is applied at the terminals of one winding while the other winding terminals are open-circuit.

The primary voltage drop $(R_1 + jX_1)I_0$ can be neglected, and the equivalent circuit reduces to the form shown in Figure 7.

Determination of Equivalent Circuit Parameters

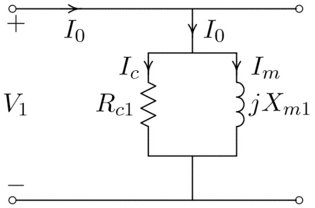

Figure 7: Equivalent circuit for the open-circuit test.

Determination of Equivalent Circuit Parameters

In this open-circuit test, the shunt elements R_c and X_m can be determined from the relations

$$R_{c1} = \frac{V_1^2}{P_0} \tag{15}$$

Note that the no-load input power P_0 represents the transformer core loss commonly referred to as *iron loss*. The two components of the no-load current are

$$I_c = \frac{V_1}{R_{c1}} \tag{16}$$

and

$$I_m = \sqrt{I_0^2 - I_c^2} \tag{17}$$

Therefore, the magnetizing reactance is

$$X_{m1} = \frac{V_1}{I_m} \tag{18}$$

In the short-circuit test, a reduced voltage V_{sc} is applied at the terminals of one winding while the other terminals are short-circuited.

Instruments are connected to measure the input voltage V_{sc}, the input current I_{sc}, and the input power P_{sc}.

The transformer appears as a short when viewed from the primary with the equivalent leakage impedance Z_{e1} consisting of primary leakage impedance and the referred secondary leakage impedance as shown in Figure 8

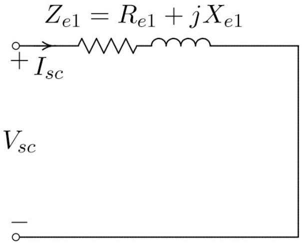

Figure 8: Equivalent circuit for the short-circuit test.

From Figure 8, the series elements R_{e1} and X_{e1} may then be determined from the relations

$$Z_{e1} = \frac{V_{sc}}{I_{sc}} \tag{19}$$

and

$$R_{e1} = \frac{P_{sc}}{(I_{sc})^2} \tag{20}$$

Therefore, the equivalent leakage reactance is

$$X_{e1} = \sqrt{Z_{e1}^2 - R_{e1}^2} \tag{21}$$

Transformer Performance

The equivalent circuit can now be used to predict the performance characteristics of the transformer. An important aspect is the transformer efficiency.

Power transformer efficiencies vary from 95% to 99%, the higher efficiencies being obtained from transformers with greater ratings.

The actual efficiency of a transformer in percent is given by

$$\eta = \frac{\text{output power}}{\text{input power}} \tag{22}$$

and the conventional efficiency of a transformer at n fraction of the full-load power is given by

$$\eta = \frac{n \times S \times PF}{(n \times S \times PF) + n^2 \times P_{cu} + P_c} \tag{23}$$

where

- S is the full-load rated volt-ampere,
- P_{cu} is the full-load copper loss, and for a three-phase transformer, they are given by

$$\begin{aligned} S &= 3|V_2||I_2| \\ P_{cu} &= 3R_{e2}|I_2|^2 \end{aligned}$$

Transformer Performance

Another important performance characteristic of a transformer is the change in secondary voltage from no-load to full load. Voltage regulation is defined as the change in the magnitude of the secondary terminal voltage from no-load to full-load expressed as a percentage of the full-load value.

$$\text{Regulation} = \frac{|V_{2nl}| - |V_2|}{|V_2|} \times 100\%$$

where V_2 is the full-load rated voltage, V_{2nl} is calculated from the equivalent circuits referred to either primary or secondary. When the equivalent circuit is referred to the primary side, the voltage regulation becomes

$$\text{Regulation} = \frac{|V_1| - |V_2'|}{|V_2'|} \times 100\%$$

And when the equivalent circuit is referred to the secondary side, the voltage regulation is

$$\text{Regulation} = \frac{|V_1'| - |V_2|}{|V_2|} \times 100\%$$

3.9 Three-phase Transformer Connections

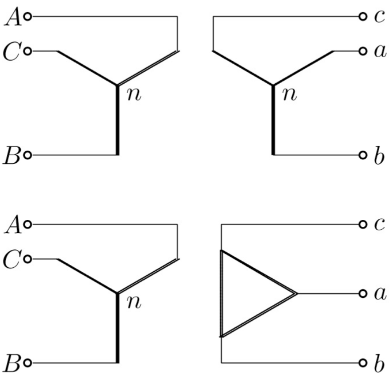

Figure 9: Three-phase transformer connections.

$Y - Y$ This is rarely used due to third harmonics. Often a third "tertiary" winding is added connected in Δ to provide a path for the third harmonic current thus allowing the voltage to remain sinusoidal and almost distortion free. Voltage out is in phase with voltage in.

$\Delta - \Delta$ This allows for the third harmonic, but it lacks the neutral connection. It has the advantage that one bank (assuming three banks are used) of transformers can be removed for repairs, and it will continue to operate (at reduced power) till the repair is completed. It is then connected as a $V - V$ or simply $V-$connection.

$\Delta - Y$ Is used frequently to raise voltage. Note that there is a connection voltage gain (in addition to the inherent turns ratio gain). There is also a phase shift of 30°.

$Y - \Delta$ Is used to lower the voltage from high transmission values. Note that it also compensates for phase shifts on a $\Delta - Y$ connection thus bringing the line back in phase.

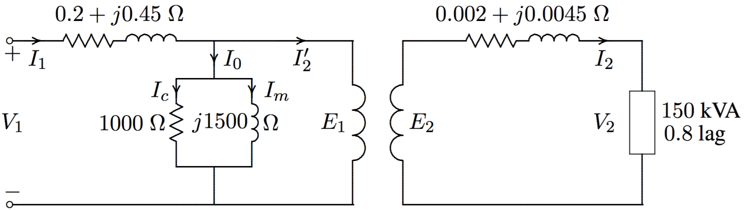

Figure 10: Transformer circuit.

Example

A 150-kVA, 2400/240-V single-phase transformer has the parameters as shown in Figure 10.

a. Determine the equivalent circuit referred to the high-voltage side.
b. Find the primary voltage when the transformer is operating at full load 0.8 power factor lagging and 240 V.
c. Find the primary voltage when the transformer is operating at full-load 0.8 power factor leading.

The End