EEE213 Power Electronics and Electromechanism

Lab arrangement (Room EE411)

11th April Thursday (9:00-12:00 & 2:00 -5:00)

Deadline: May 5th, 23:55pm

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EEE213 Power Electronics and Electromechanism

8. DC-DC Converters

Outline

- Step-down operation
  - Duty cycle generation
- Types of DC-DC converter
  - Buck converters
  - Boost converters
  - Buck-Boost converter
- Closed-loop control of DC-DC converters

Types of DC-DC converters

- Step-down: Buck converters
  - the output voltage is less than the input voltage

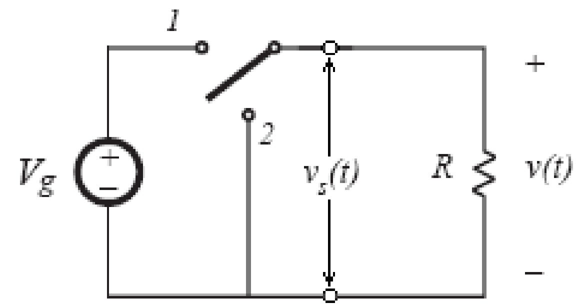
- Step-up: Boost converters
  - the output voltage is higher than the input voltage

- Step-up/down: Buck-boost converters
  - the output voltage can be higher or less than the input voltage

There are different circuit topologies, having different names.

1.1 Step-down operation

- Operation principle:
  - Switch on (position 1) for t_1, $v_o=v_s=v_g$
  - Switch off (position 2) for t_2, $v_o=v_s=0$.



- Average output voltage:

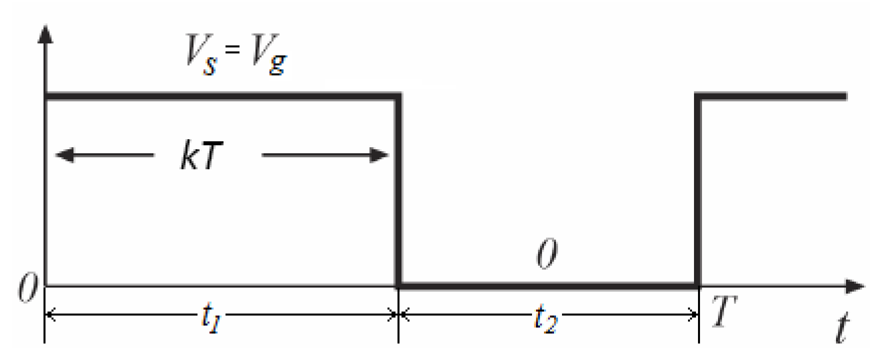
$$V_o = \frac{1}{T}\int_0^T v_o dt = \frac{1}{T}\int_0^{t_1} V_g dt = \frac{t_1}{T} V_g = kV_g$$

where k is the duty cycle:

$$D = k = \frac{t_1}{T}$$

- RMS output voltage:

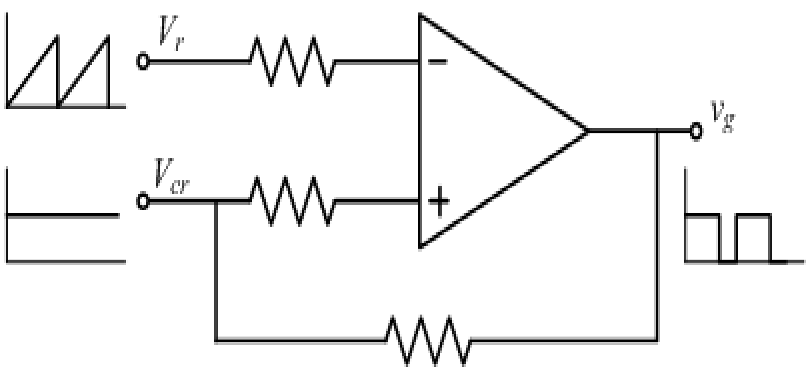
$$V_{RMS} = \sqrt{\frac{1}{T}\int_0^T v_o^2 dt} = \sqrt{k} V_g$$

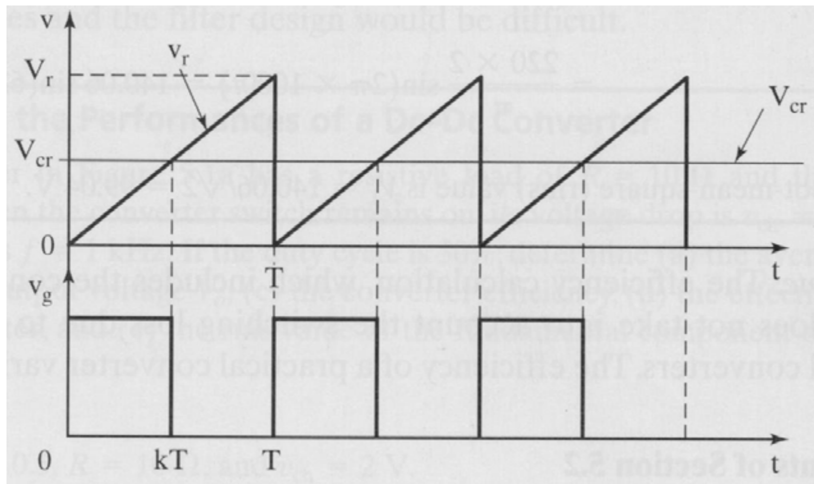


k is called the duty cycle.
Sometimes using D instead of k.

1.2 Generation of duty cycle

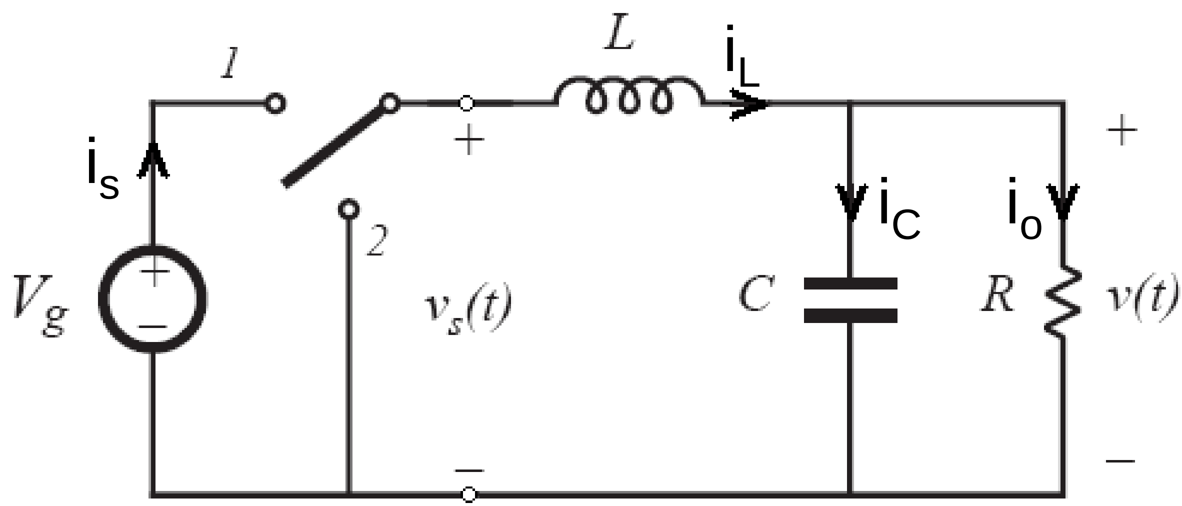
- If a saw-tooth signal V_r and a DC signal V_{cr} are supplied to a comparator, then the output of the comparator can be shown as v_g.
- The duty cycle of v_g will be changed if V_{cr} changes.
- This is how we control the voltage of a DC-DC converter.





2.1 Buck converter

- Devices: one switch and one diode
- Filters: LC filter to remove the switching harmonics and to pass only the DC component so that the output voltage v is nearly a constant.



Small ripple approximation

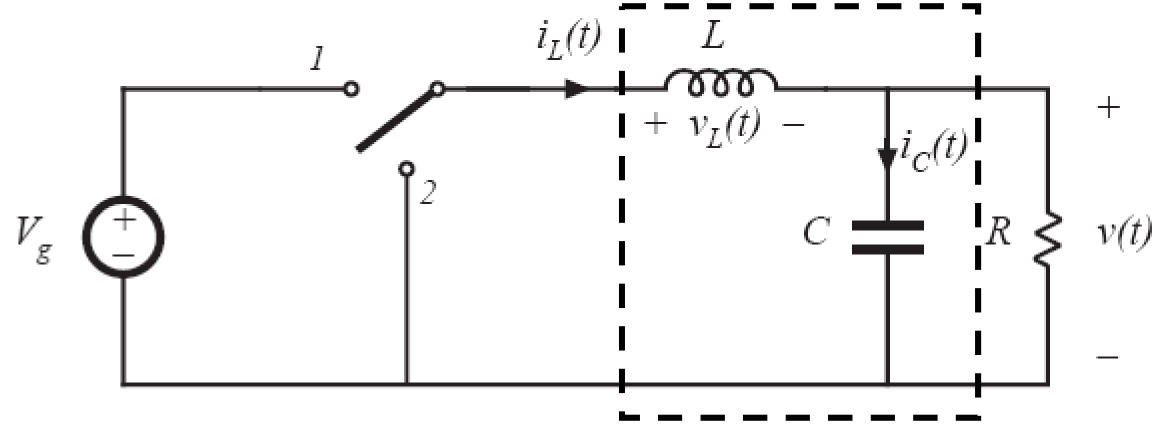
- The L-C form a practical low-pass filter
- Actual output voltage waveform:

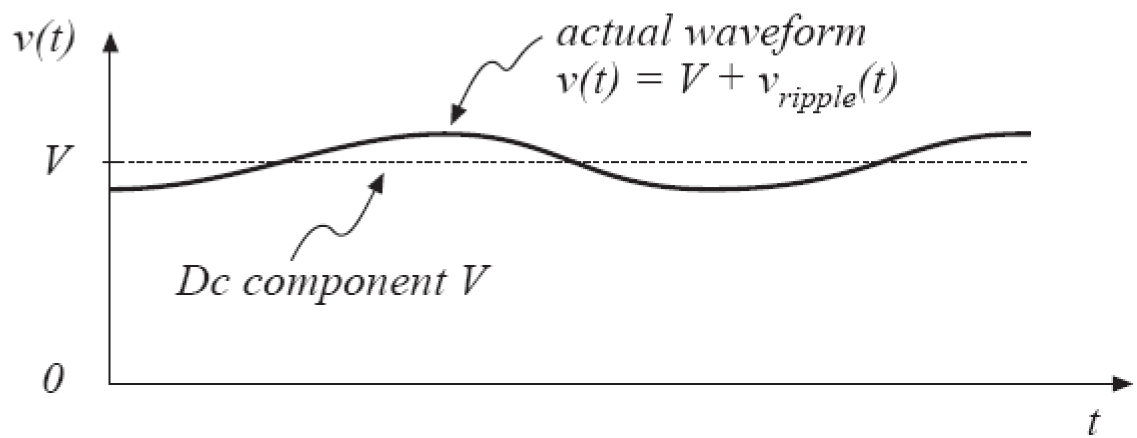
  $$v(t) = V + v_{ripple}(t)$$

- In a well-designed converter, the output voltage ripple is small.
- Hence, the waveforms can be easily determined by ignoring the ripple:

  $$\| v_{ripple} \| << V$$

  $$v(t) \approx V$$





Operation modes

- Mode 1
  - Switch is on (position 1)
  - Inductor voltage

    $$v_L = V_g - v(t)$$

  - Small ripple approximation

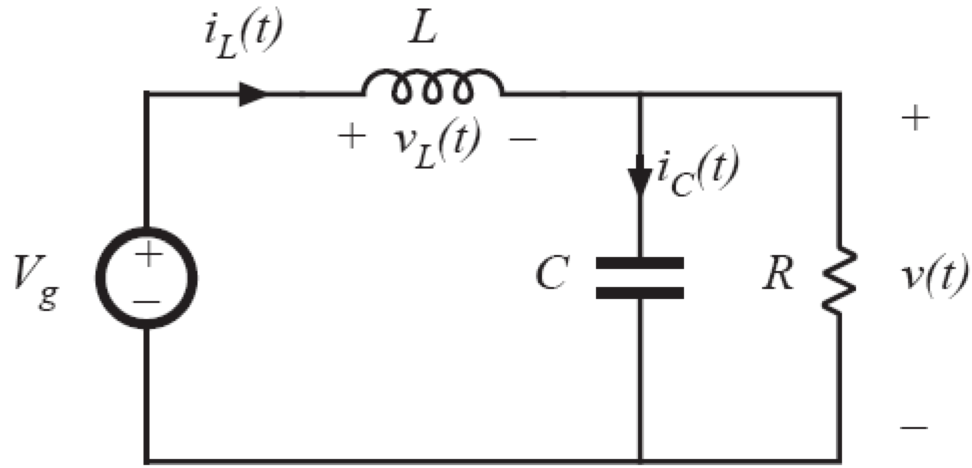
    $$v_L \approx V_g - V$$

  - Inductor current

    $$v_L(t) = L\frac{di_L(t)}{dt}$$

  - Solve for the slope:

    $$\frac{di_L(t)}{dt} = \frac{v_L(t)}{L} \approx \frac{V_g - V}{L}$$



Energy is transferred to L and the load;

In the first half period, C discharges and then is charged when the inductor current is bigger than the load current

⇩

The inductor current changes with an essentially constant slope

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Operation modes

- Mode 2
  - Switch is off (position 2)
  - Inductor voltage

$$v_L(t) = -v(t)$$

  - Small ripple approximation

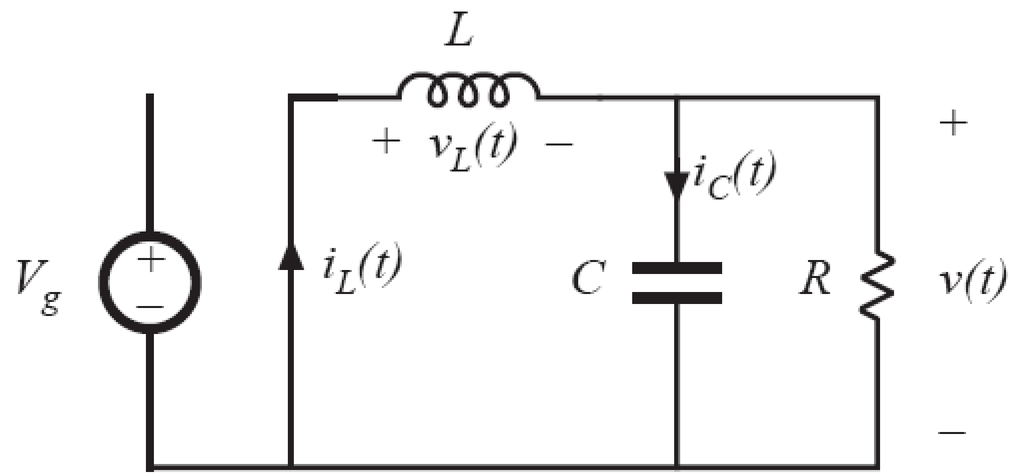
$$v_L(t) \approx -V$$

  - Inductor current

$$v_L(t) = L\frac{di_L(t)}{dt}$$

  - Solve for the slope:

$$\frac{di_L(t)}{dt} \approx -\frac{V}{L}$$



Energy is transferred from L to the load

In the first half period, C remains being charged and then discharges when the inductor current is smaller than the load current

The inductor current changes with an essentially constant slope

Diode voltage

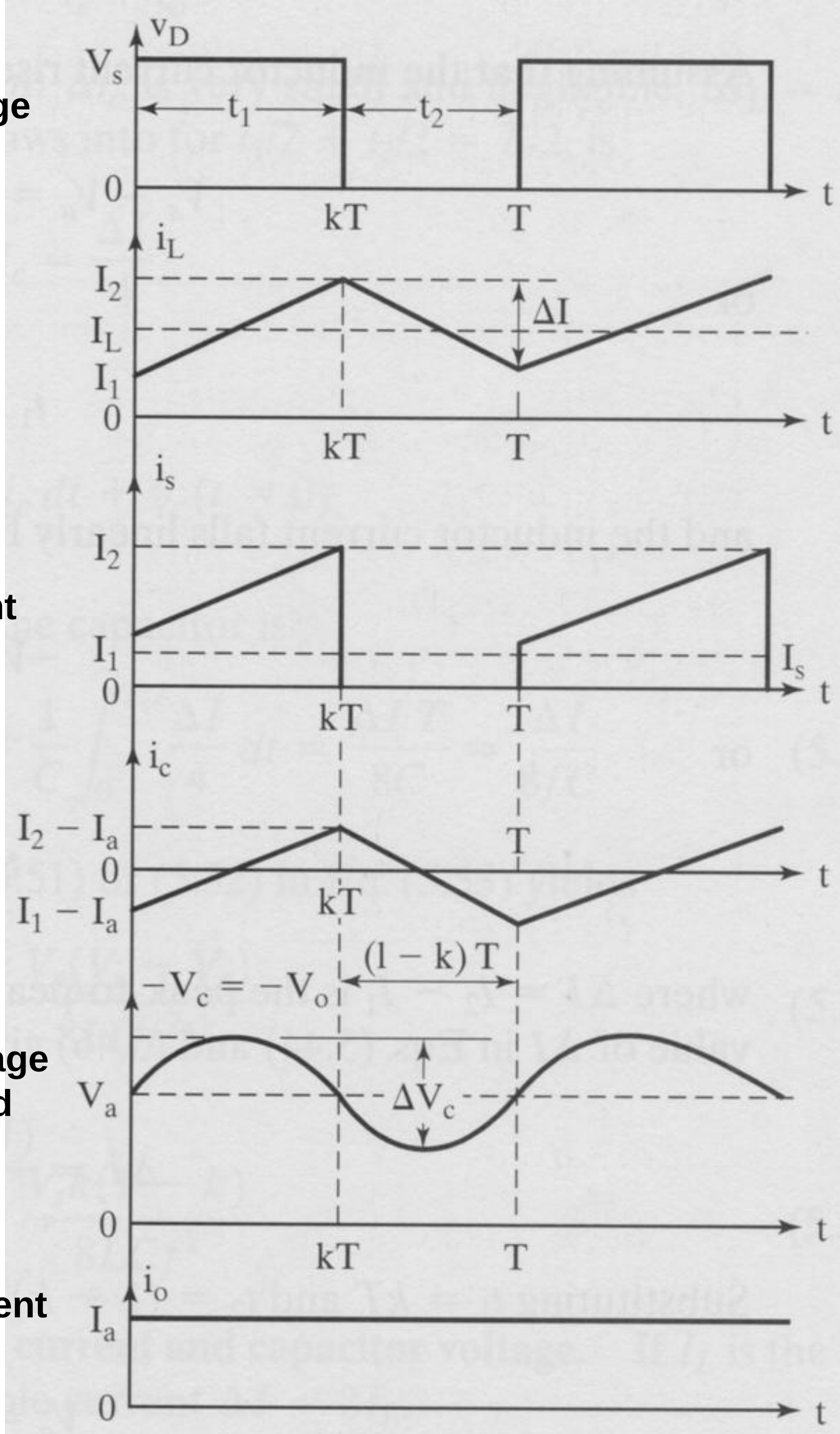
Inductor current

Input current

Capacitor current

Output voltage with zoomed ripples

Output current



Assumption for steady-state analysis:

- *The components are all ideal.*
- *The ripple in the output voltage is negligible.*
- *The current in the inductor is continuous.*
- *Ripples in the load current is negligible.*

Basic principles in the steady state

- ***The inductor volt-second balance:*** *the average inductor voltage is zero in the steady state.*
- ***The capacitor ampere-second (charge) balance:*** *the average capacitor current is zero in the steady state.*

Inductor volt-second balance: Derivation

- Inductor defining relation:

$$v_L(t) = L\,\frac{di_L(t)}{dt}$$

- Integrate over one complete switching period:

$$i_L(T_s) - i_L(0) = \frac{1}{L}\int_0^{T_s} v_L(t)\,dt$$

- In periodic steady state, the net change in inductor current is zero:

$$0 = \int_0^{T_s} v_L(t)\,dt$$

- Hence, the total area (or volt-second) under the inductor voltage waveform is zero whenever the converter operates in steady state.

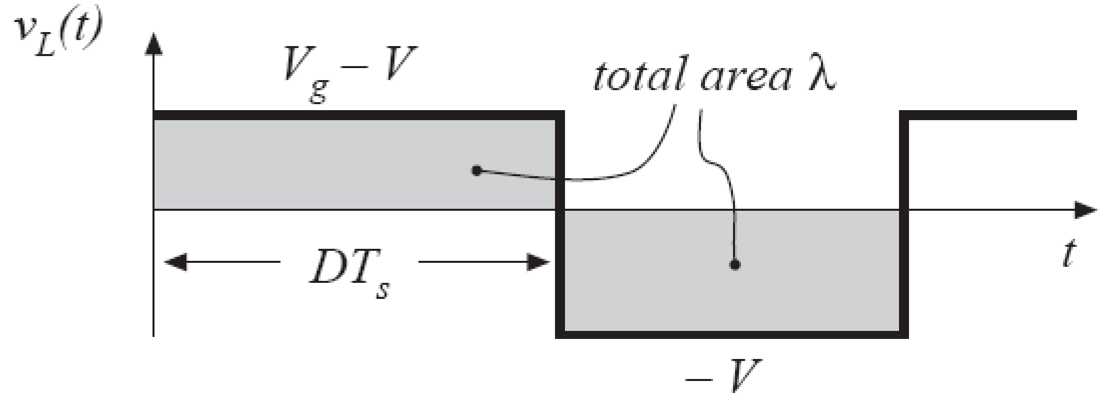
  An equivalent form:

$$0 = \frac{1}{T_s}\int_0^{T_s} v_L(t)\,dt = \langle v_L \rangle$$

The average inductor voltage is zero in steady state.

Inductor volt-second balance: Example

Inductor voltage waveform previously derived:



$$0 = \frac{1}{T_s}\int_0^{T_s} v_L(t)\,dt = \langle v_L \rangle$$

- Integral of voltage waveform is area of rectangles:

$$\lambda = \int_0^{T_s} v_L(t)\,dt = (V_g - V)(DT_s) + (-V)(D'T_s)$$

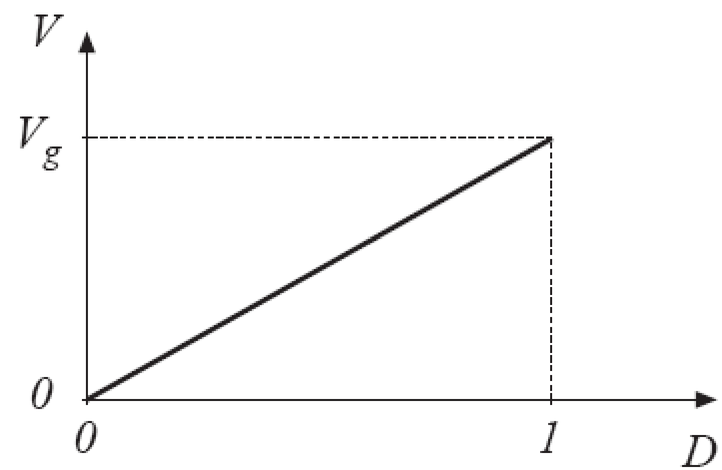
- Average voltage is:

$$\langle v_L \rangle = \frac{\lambda}{T_s} = D(V_g - V) + D'(-V)$$

- Equate to zero and solve for V:

$$0 = DV_g - (D + D')V = DV_g - V$$

$$\Rightarrow \qquad V = DV_g$$



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Capacitor charge balance: Derivation

- Capacitor defining relation:

$$i_C(t) = C\,\frac{dv_C(t)}{dt}$$

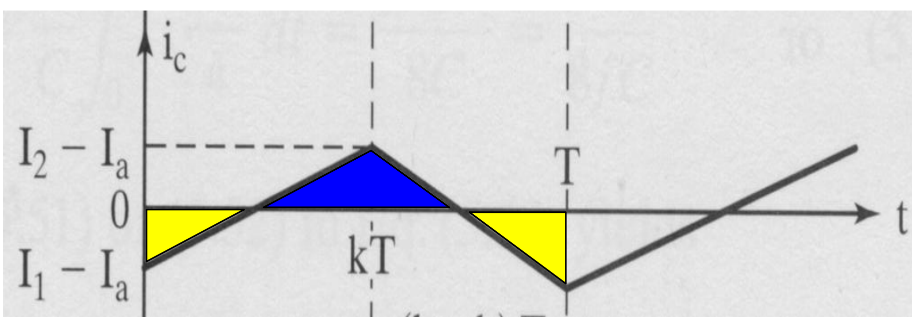
- Integrate over one complete switching period:

$$v_C(T_s) - v_C(0) = \frac{1}{C}\int_0^{T_s} i_C(t)\,dt$$

- In periodic steady state, the net change in capacitor voltage is zero:

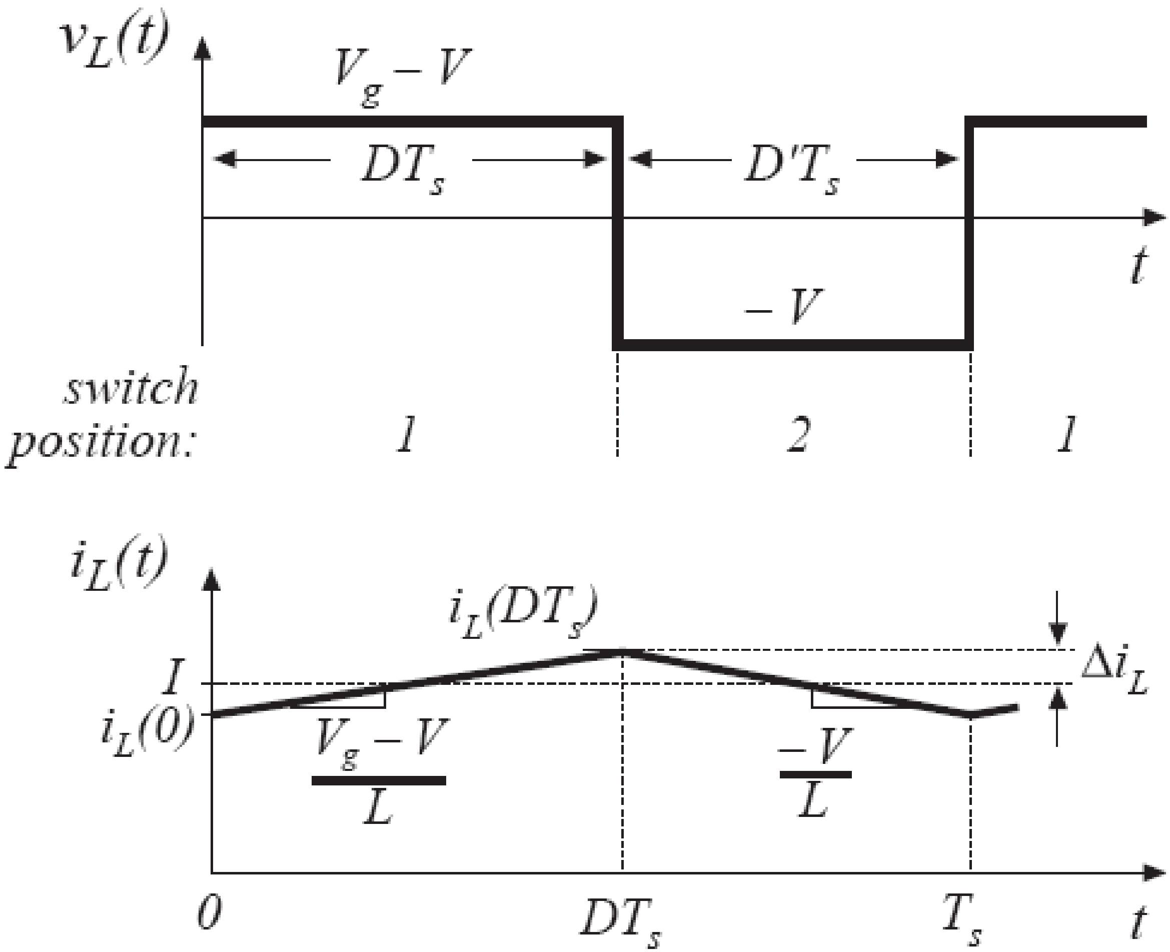
$$0 = \frac{1}{T_s}\int_0^{T_s} i_C(t)\,dt = \langle i_C \rangle$$

- Hence, the total area (or charge) under the capacitor current waveform is zero whenever the converter operates in steady state.



The average capacitor current is zero in steady state.

Inductor current ripples



$$\frac{di_L(t)}{dt} = \frac{v_L(t)}{L} \approx \frac{V_g - V}{L}$$

$$\Downarrow$$

$$(2\Delta i_L) = \left(\frac{V_g - V}{L}\right)(DT_s)$$

$$\Downarrow$$

$$\Delta i_L = \frac{V_g - V}{2L} DT_s$$

$$L = \frac{V_g - V}{2\Delta i_L} DT_s$$

$$\Downarrow \quad \boxed{V = DV_g}$$

$$\Delta i_L = \frac{(1-D)VT_s}{2L} = \frac{(1-D)DV_g}{2 f_s L}$$

The higher the switching frequency and the bigger the inductor, the smaller the ripple

Capacitor voltage ripples

- Assume that the load ripple current is negligible (value I_a), which means all the ripple (inductor) current flows through the capacitor. Then the capacitor current can be shown as:

$$i_c = i_L - I_a$$

- The total charge q is the area of the triangle, as shown:

$$q = \frac{1}{2}\frac{\Delta i_L}{2}\frac{T_s}{2}$$

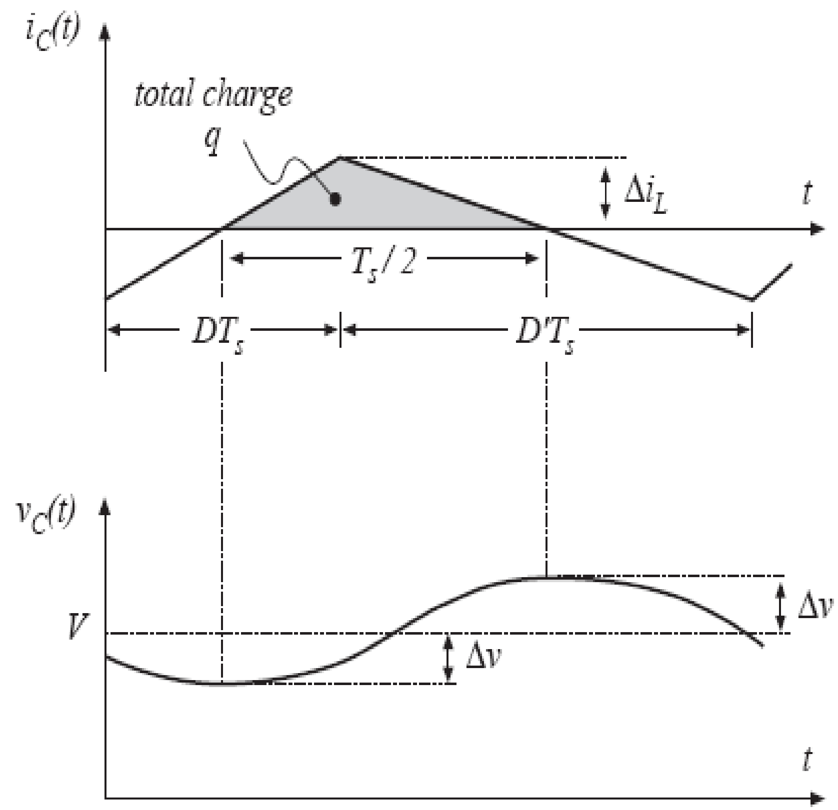
- Current $i_C(t)$ is positive for half of the switching period which is charging the capacitor C. The total charge q deposited on the capacitor (the grey area) is:

$$q = C\,(2\Delta v)$$

$$\Delta v = \frac{\Delta i_L T_s}{8\,C}$$

$$\Delta i_L = \frac{(1-D)VT_s}{2L} = \frac{(1-D)DV_g}{2f_sL}$$

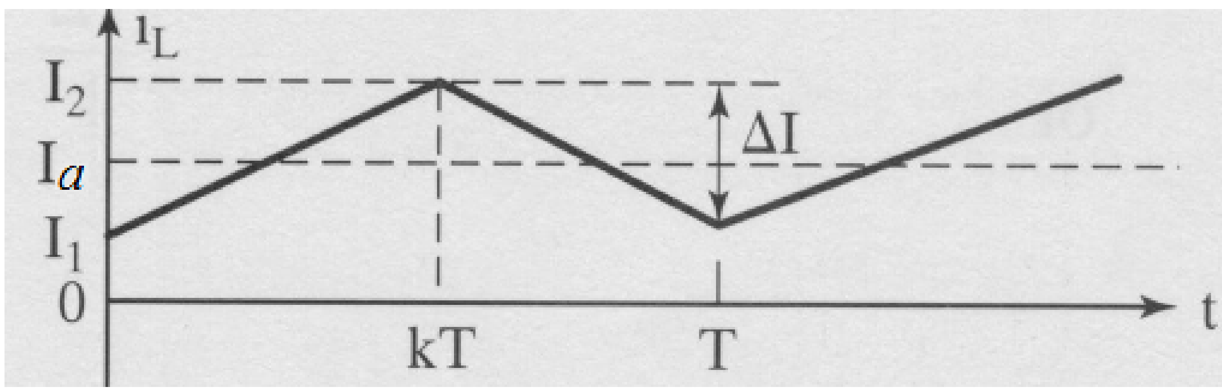
$$\Delta v_C = \frac{\Delta i_L}{8f_sC} = \frac{(1-D)DV_g}{16f_s^2LC}$$



Discontinuous inductor current

- If the average load (inductor) current I_a is less than $\Delta I/2$, then the inductor current will become discontinuous. Assume that the load is a resistor R, in order to guarantee a continuous inductor current the following condition needs to be satisfied

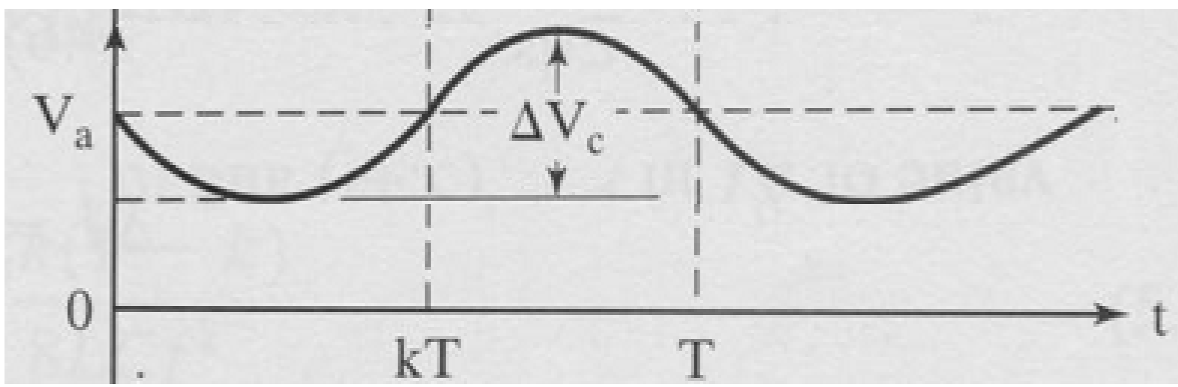
$$I_a = \frac{V_a}{R} > \frac{\Delta I}{2} = \frac{(1-k)V_a}{2fL} \Longrightarrow L > \frac{1-k}{2f}R$$



Discontinuous capacitor voltage

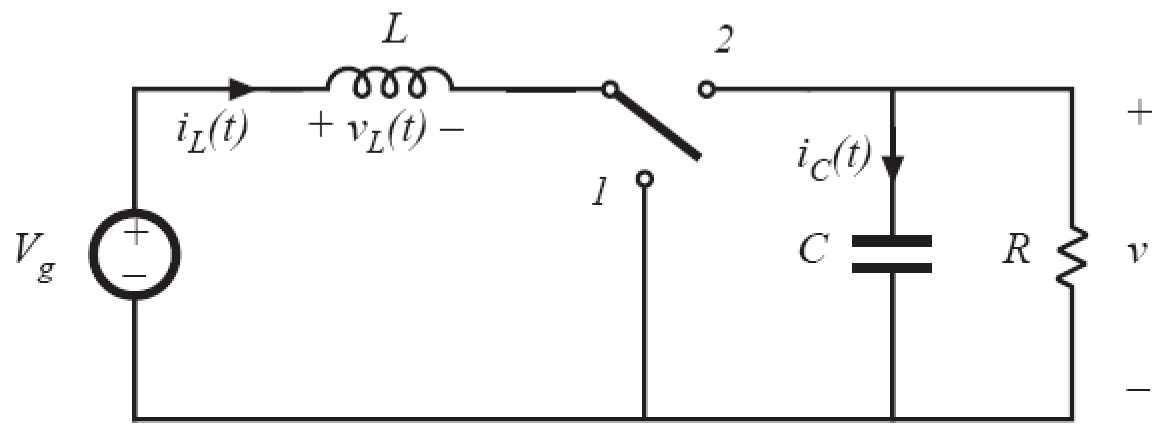
- If the average output (capacitor) voltage V_a is less than $\Delta V_c/2$, then the capacitor voltage will become discontinuous. In order to guarantee a continuous capacitor voltage the following condition needs to be satisfied

$$V_a > \frac{1}{2}\Delta V_c \quad \text{or} \quad kV_s > \frac{(1-k)k}{16 f^2 LC} V_s \Rightarrow C > \frac{1-k}{16 f^2 L}$$

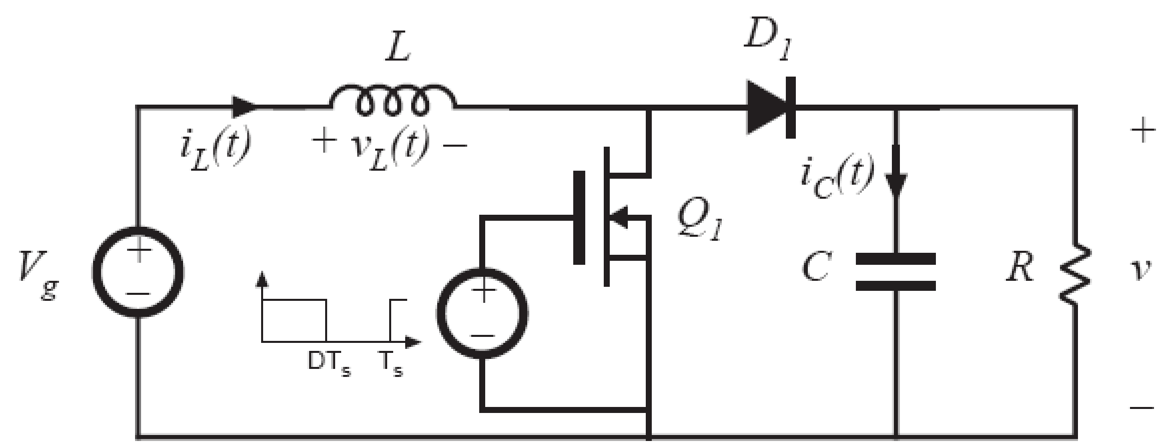


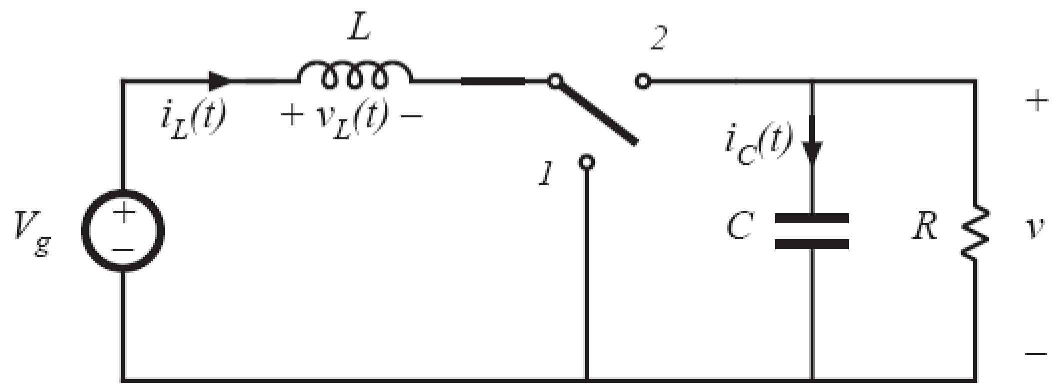
2.2 Boost converter

- Boost converter with ideal switch



- Realization using power MOSFET and diode

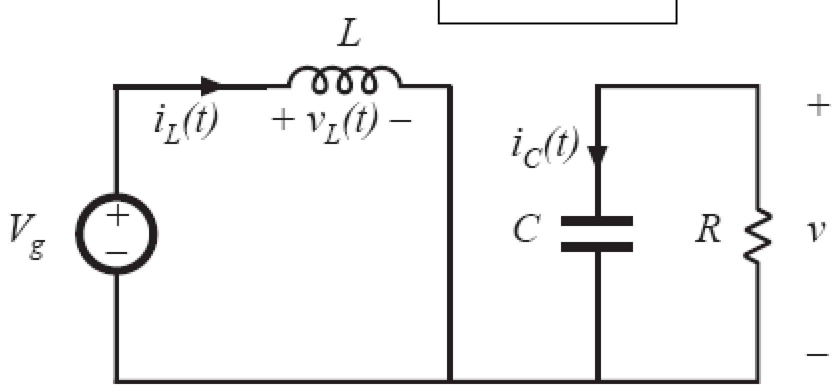




switch in position 1

switch in position 2

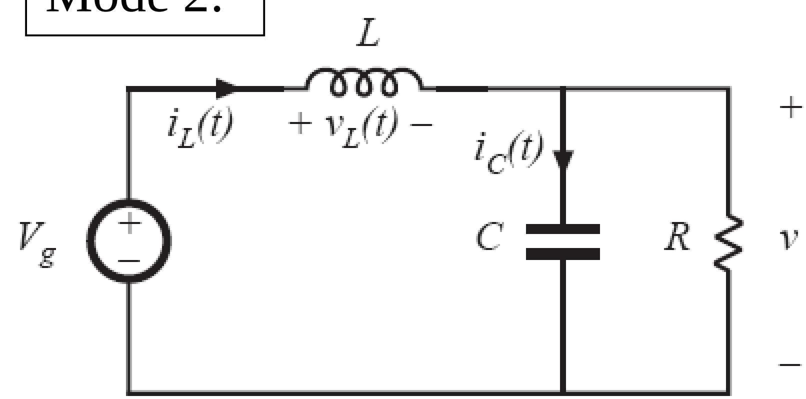
Mode 1:



– Energy is being stored in L and the inductor current increases

– C discharges to supply the load

Mode 2:



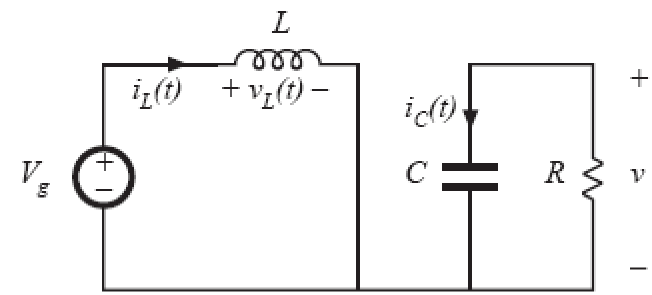
– Energy is transferred from L and V_s to the load

– C is being charged

Mode 1:

Inductor voltage and capacitor current

$$v_L = V_g$$
$$i_C = -v / R$$



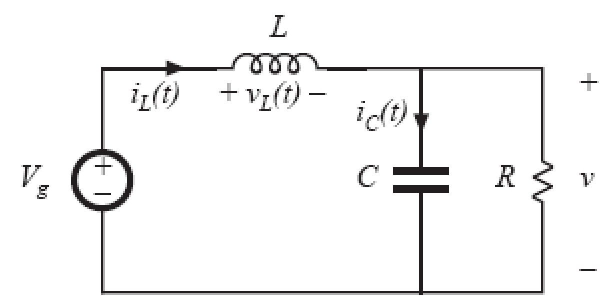
Small ripple approximation:

$$v_L = V_g$$
$$i_C = -V / R$$

Mode 2:

Inductor voltage and capacitor current

$$v_L = V_g - v$$
$$i_C = i_L - v / R$$



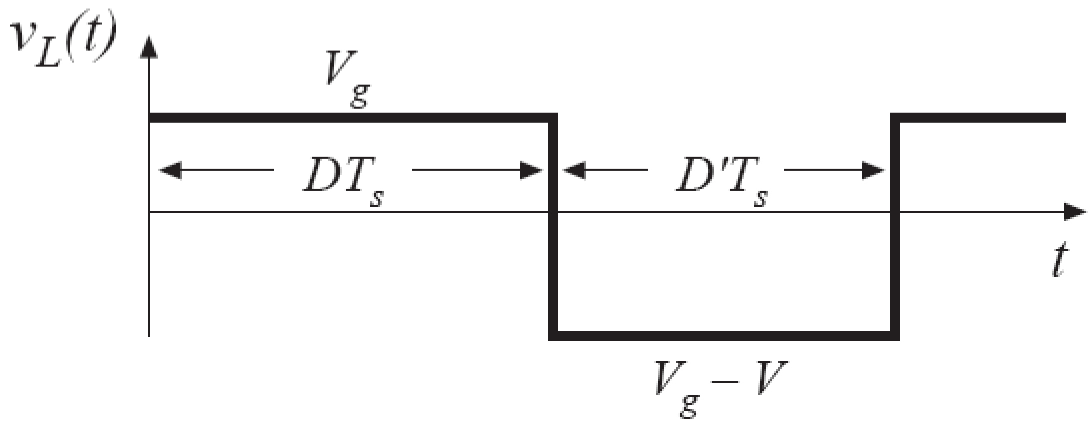
Small ripple approximation:

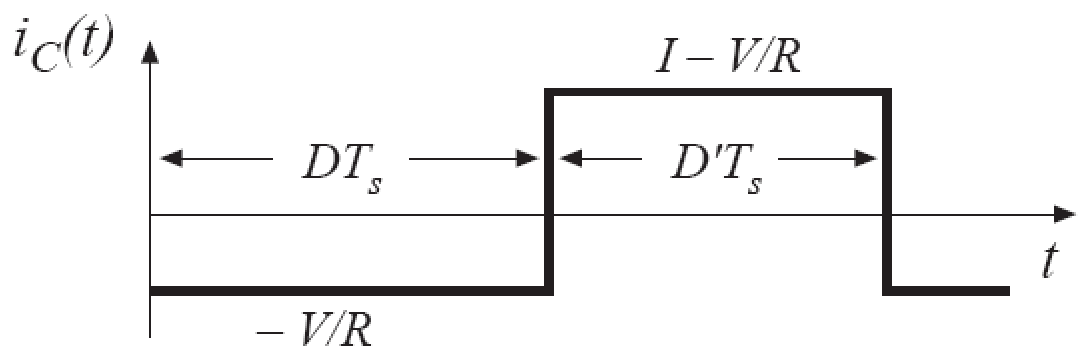
$$v_L = V_g - V$$
$$i_C = I - V / R$$

I: average inductor current

V: average output voltage

Inductor voltage and capacitor current

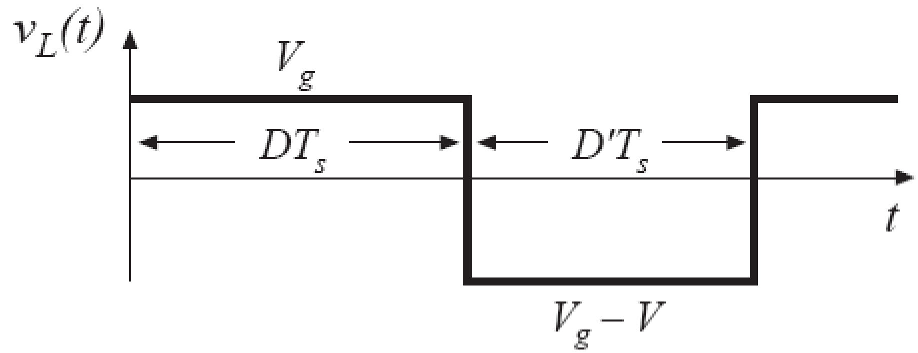




Inductor volt-second balance: average voltage

- Net volt-seconds applied to inductor over one switching period:

$$\int_0^{T_s} v_L(t)\,dt = (V_g)\,DT_s + (V_g - V)\,D'T_s$$



- Equate to zero and collect terms:

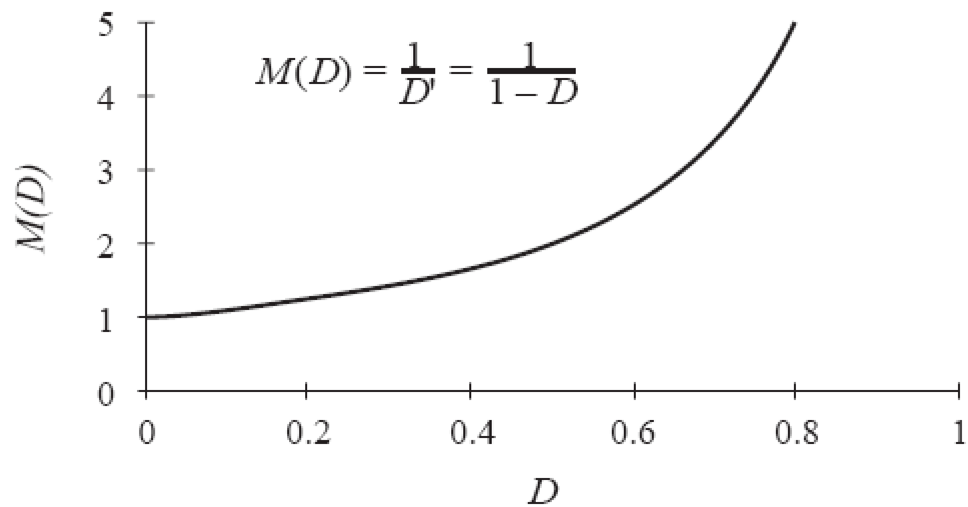
$$V_g\,(D + D') - V\,D' = 0$$

- Solve for V:

$$V = \frac{V_g}{D'}$$

- The voltage conversion ratio is:

$$M(D) = \frac{V}{V_g} = \frac{1}{D'} = \frac{1}{1-D}$$



Capacitor charge balance: average current

- Capacitor charge balance:

$$\int_0^{T_s} i_C(t)\,dt = (-\frac{V}{R})\,DT_s + (I - \frac{V}{R})\,D'T_s$$

- Collect terms and equate to zero:

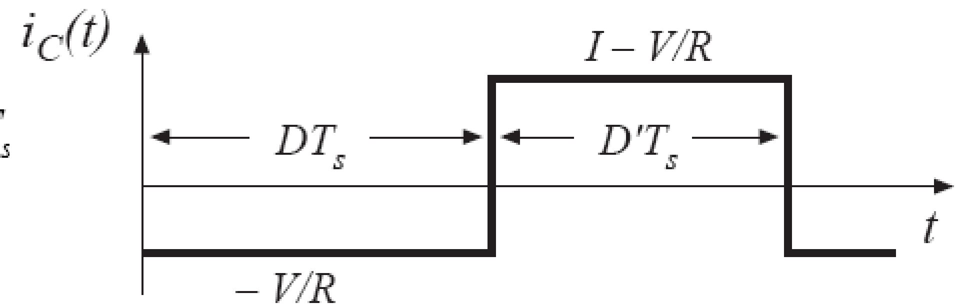
$$-\frac{V}{R}(D + D') + I\,D' = 0$$

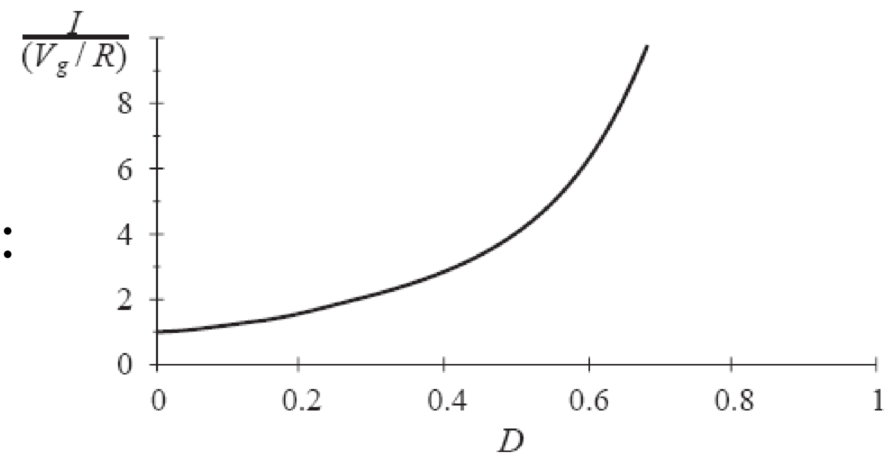
- Solve for I:

$$I = \frac{V}{D'\,R}$$

- Eliminate V to express in terms of V_g:

$$I = \frac{V_g}{D'^2\,R}$$





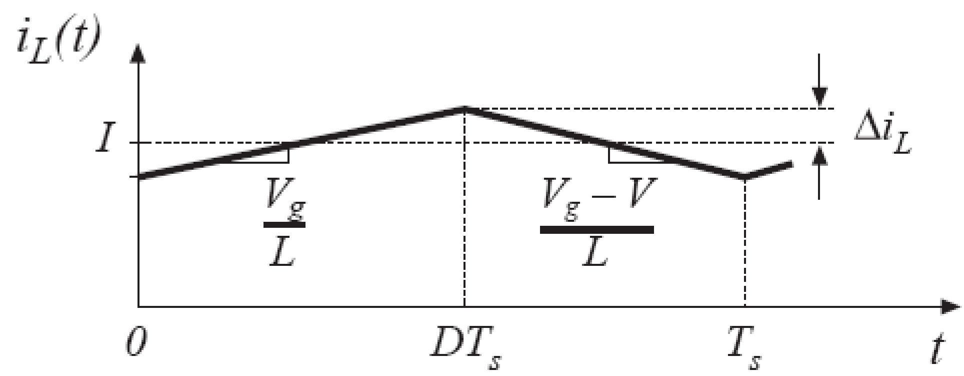
Determination of inductor current ripple

Inductor current slope during subinterval 1:

$$\frac{di_L(t)}{dt} = \frac{v_L(t)}{L} = \frac{V_g}{L}$$

Inductor current slope during subinterval 2:

$$\frac{di_L(t)}{dt} = \frac{v_L(t)}{L} = \frac{V_g - V}{L}$$



Change in inductor current during subinterval 1 is *(slope) (length of subinterval)*:

$$2\Delta i_L = \frac{V_g}{L} DT_s$$

Solve for peak ripple:

$$\Delta i_L = \frac{V_g}{2L} DT_s$$

- Choose *L* such that desired ripple magnitude is obtained

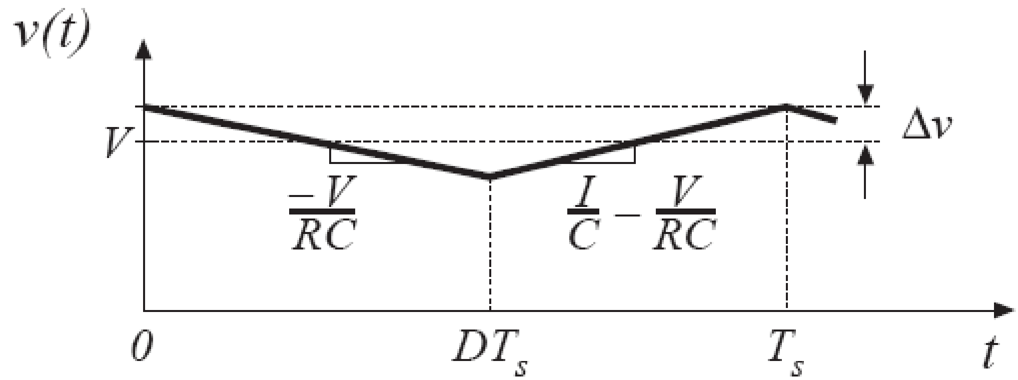
Determination of capacitor voltage ripple

Capacitor voltage slope during subinterval 1:

$$\frac{dv_C(t)}{dt} = \frac{i_C(t)}{C} = \frac{-V}{RC}$$

Capacitor voltage slope during subinterval 2:

$$\frac{dv_C(t)}{dt} = \frac{i_C(t)}{C} = \frac{I}{C} - \frac{V}{RC}$$



Change in capacitor voltage during subinterval 1 is *(slope) (length of subinterval)*:

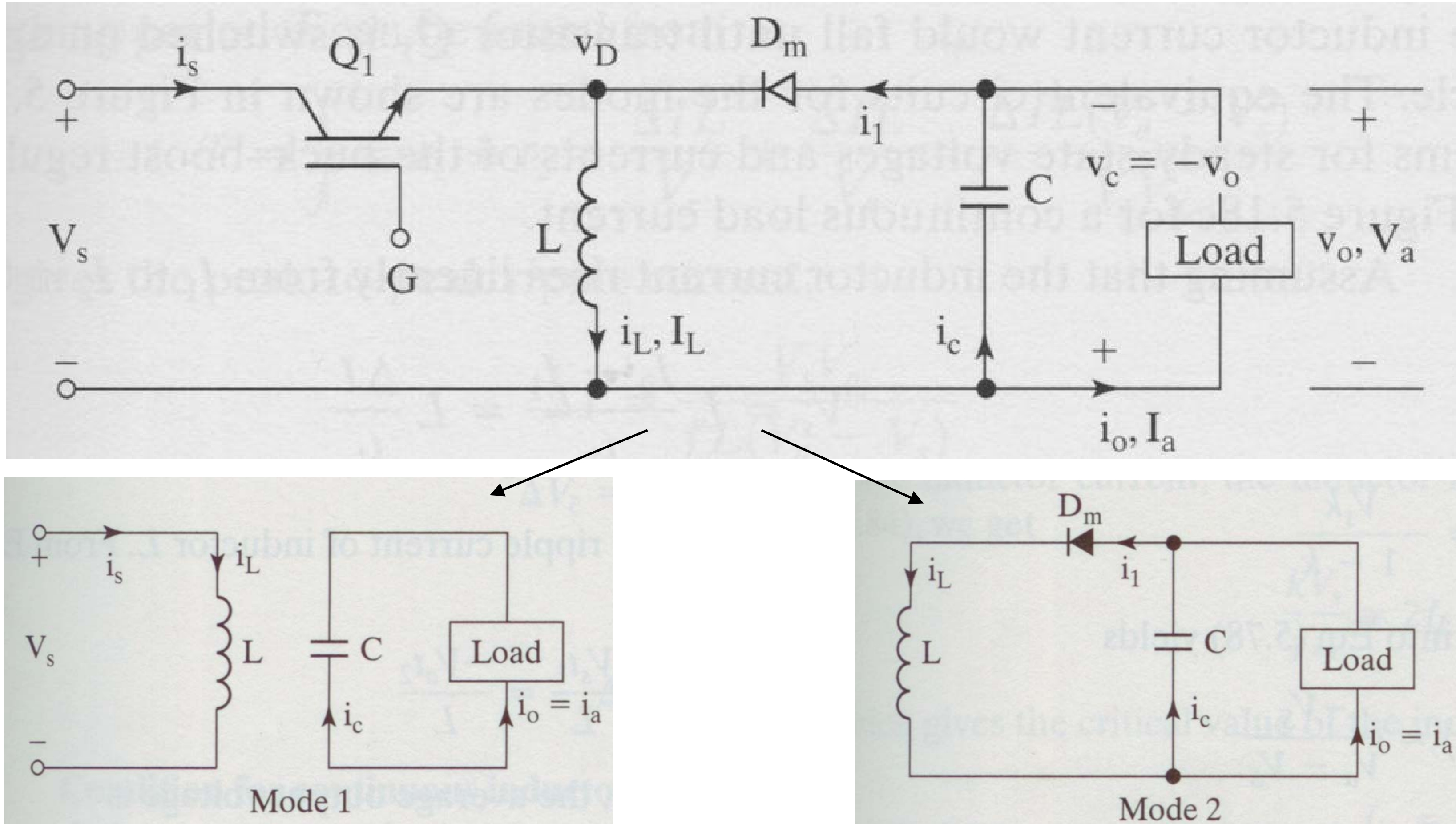
$$-2\Delta v = \frac{-V}{RC} DT_s$$

Solve for peak ripple:

$$\Delta v = \frac{V}{2RC} DT_s$$

- Choose *C* such that desired voltage ripple magnitude is obtained
- In practice, capacitor *equivalent series resistance* (esr) leads to increased voltage ripple

2.3 Buck-Boost converter

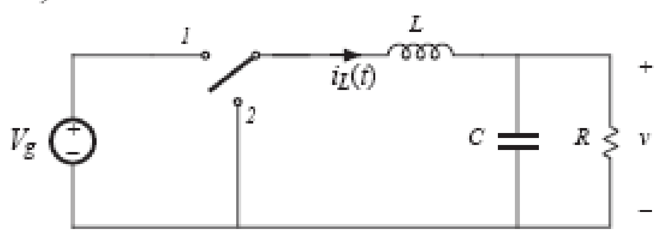


$$V_a = -\frac{k}{1-k} V_s \longrightarrow \text{Voltage polarity is changed}$$

Comparison of basic converters

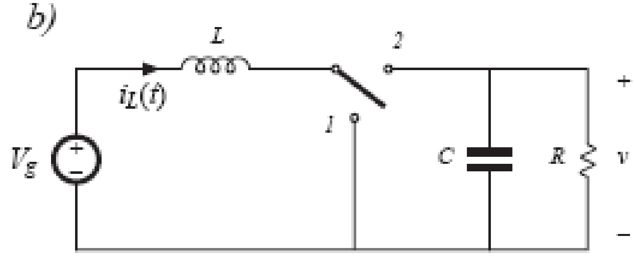
Buck

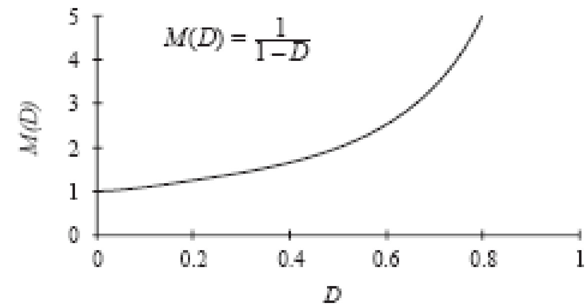
a)



Boost

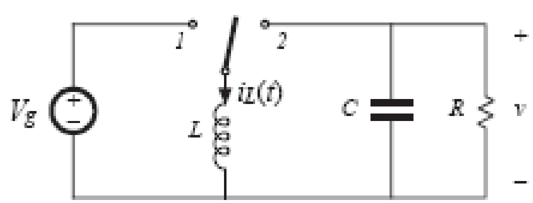
b)

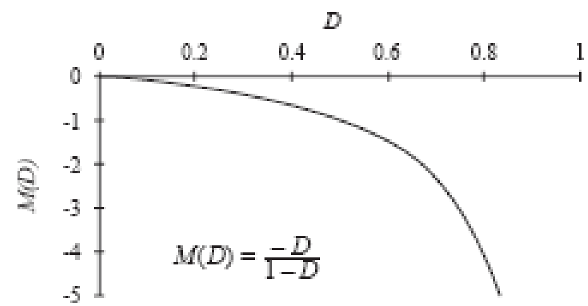




Buck-boost

c)





3. Closed-loop control of DC-DC converters

- As have been seen, the output voltage of a DC-DC converter is related to the duty cycle k (or D). In practice, one will never get an accurate output voltage if the duty cycle is fixed at the calculated value because
  - The components are not ideal: switching losses, diode voltage drop, inductor resistance etc
  - The load might change
  - There are fluctuations in the supply voltage Vs
  - …
- A closed-loop controller to regulate the output voltage is needed!