EEE108 Electromagnetism and Electromechanics

Lecture 8

Mini-review of Electrostatics

Electric Current

Dr. Jinling Zhang

Dept. of Electrical and Electronic Engineering
University of Xi'an Jiaotong-Liverpool
Email: jinling.zhang@xjtlu.edu.cn

Today

- Mini-review of Electrostatics
- Electric Current

The very basic: Electric Force -- Coulomb's Law

The electric force between charges Q_1 and Q_2 :

Q_1 on Q_2 :

$$\mathbf{F}_{12} = k_e \frac{Q_1 Q_2}{r^2} \mathbf{a}_r$$

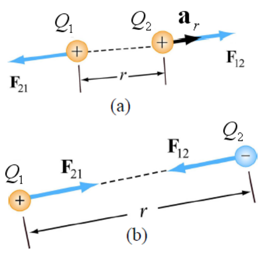
where $k_e = \dfrac{1}{4\pi\varepsilon_0} = 8.9875\times10^9 \quad \mathrm{N\cdot m^2}/C^2$

k_e : called Coulomb constant

ε_0 : permittivity of free space,

$= 8.85\times10^{-12} \quad C^2/\mathrm{N\cdot m^2}$

$\mathbf{a}_r$: unit vector from Q_1 to Q_2



interaction between two charges

Q_2 on Q_1 :

$\mathbf{F}_{21} = -\mathbf{F}_{12}$

NB: this is in principle, all the rest follows from this and the superposition principle

The very basic: Principle of Superposition

When more than two charges are present, the net force on any one charge is the ***vector sum*** of the forces from other charges.

$$\mathbf{F}_j = \sum_{\substack{i=1 \\ j\neq i}}^{N} \mathbf{F}_{ij}$$

Extremely important because it allows us to transform complicated problems into sum of small, simple problems that we know how to solve.

Electric Force, Electric Field and Electric Potential

•Solving problems in terms of $F_{coulomb}$ is not always convenient
F depends on probe charge q
•We get rid of this dependence introducing the Electric Field

$$\mathbf{E} = \mathbf{F}/q_0\,,\quad \mathbf{E} = k_e \frac{q}{r^2}\mathbf{a}_r \quad \text{for a point charge}$$

•Advantages and disadvantages of **E**
E describes the properties of space due to the presence of charge q
It's a vector → hard integrals when applying superposition
•Introduce Electric Potential φ
$\varphi(P)$ is the work done to move a unit positive charge from infinity to $P(x,y,z)$

$$\varphi = -\int_{\infty}^{P} \mathbf{E} \bullet d\mathbf{l} \quad \text{when } \varphi(\infty) = 0$$

Advantages: superposition still holds but simpler calculation (scalar)

Electrical Potential Energy and Electrical Potential

• Similar to gravitation, the electrostatic force $\mathbf{F}_e$ is also conservative.
• The electric potential difference between points A and B is defined :

$$\Delta\varphi = -\int_B^A \mathbf{E} \bullet d\mathbf{l} = -\int_B^A (\mathbf{F}_e / q_0) \bullet d\mathbf{l}$$

The amount of work done to move an unit positive charge from B to A
• The electric potential energy to move a charge q_0 from B to A is

$$\Delta U = q_0\,\Delta\varphi$$

❑ Potential difference depends only on the source charge distribution,
❑ Potential energy difference exists only if a test charge is moved between the points.

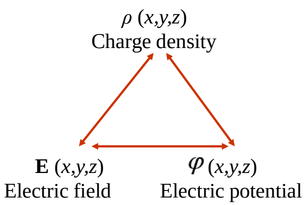
Electrostatics Problems

In electrostatics there are 3 different ways of describing a problem

$\rho \rightarrow \mathbf{E}$
•General case:
• $\mathbf{E} = k_e \frac{Q}{r^2}\mathbf{a}_r$ for a point charge Q
• Superposition principle:

$$d\mathbf{E} = k_e \frac{dq}{r^2}\mathbf{a}_r \Rightarrow \mathbf{E} = \int d\mathbf{E}$$

•Special cases:
•For symmetry systems:
Gauss's law: $\oint\!\!\oint_S \mathbf{E} \bullet d\mathbf{s} = \frac{Q_{enc}}{\varepsilon}$
➢Gauss's law is always true but not always useful:
■Symmetry is needed!
➢Main step: choose the "right" Gaussian surface so that E is constant on the surface of integration



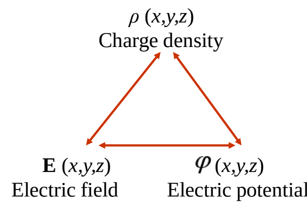
Electrostatics Problems

$\rho \rightarrow \varphi$
•General finite charge sources:
• $\varphi = \frac{Q}{4\pi\varepsilon_0 r}$ For point charge Q

•Superposition principle:

$$\varphi_L = \frac{1}{4\pi\varepsilon_0}\int_L \frac{\rho_L}{r}\,dl$$

$$\varphi_S = \frac{1}{4\pi\varepsilon_0}\iint_S \frac{\rho_S}{r}\,dS \qquad \varphi_V = \frac{1}{4\pi\varepsilon_0}\iiint_V \frac{\rho_V}{r}\,dV$$



•Special cases:
•For symmetry systems: use Gauss's law to extract **E** and then integrate **E** to get φ:

$$\Delta\varphi = -\int_{P_1}^{P_2} \mathbf{E} \bullet d\mathbf{l}$$

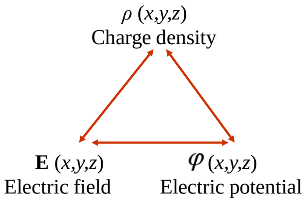
Electrostatics Problems

$\varphi \rightarrow \rho$ and $\mathbf{E}$

$\varphi \rightarrow \mathbf{E}$

$$\mathbf{E} = -\nabla \varphi$$

•make sure you choose the best coordinate



$\varphi \rightarrow \rho$

Poisson's equation

$$\nabla^2 \varphi = -\rho / \varepsilon$$

$$\rho = -\varepsilon_0 \nabla^2 \varphi \qquad \Rightarrow \rho = -\varepsilon_r \varepsilon_0 \nabla^2 \varphi$$

Conductors

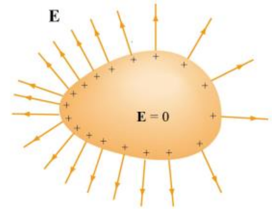
The basic properties of a conductor:

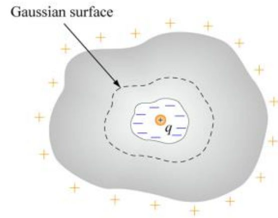
(1) The electric field inside a conductor is zero.

(2) The surface of a conductor is an equipotential surface.

(3) Just outside the conductor, the electric field is normal to the surface, the tangential component of the electric field on the surface is zero.

(4) Any net charge must reside on the surface of the conductor.





Capacitors and Capacitance

• Capacitors and capacitance:

A capacitor is a device to store electric charge and potential energy.

The capacitance: $C = Q / |\Delta\varphi|$

• Energy in a capacitor:

$$U = \frac{1}{2}CV^2 = \frac{1}{2}QV \text{ (Energy stored)} \qquad u_E = \frac{1}{2}\varepsilon_0 E^2 \text{ (Energy density)}$$

• Dielectrics:

When a dielectric material is filled into a capacitor, the capacitance increases by a factor ε_r: $C = \varepsilon_r C_0$

What should you remember?

•Parallel plate capacitor: very well

•Be able to derive the other standard geometries:

cylindrical and spherical

Electric Current

Like a review but with more information in microscopic level

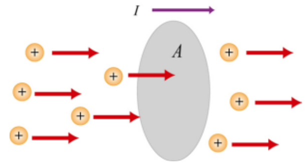
Overview

Phenomenon : a flow of free electric charges

Quantity: the rate of flow of free electric charges

Flowing charge typically:
- moving electrons, in a conductor such as a wire
- ions, in an electrolyte
- both (electrons and ions), in a plasma

SI unit: ampere, A



Average current I_{av} : Charge ΔQ flowing across area A in time interval Δt : $I_{av} = \dfrac{\Delta Q}{\Delta t}$

Instantaneous current :

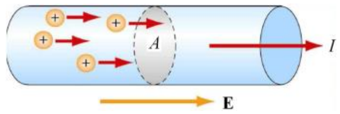
differential limit of I_{av}: $I = \dfrac{dQ}{dt}$

Unit of current : $\text{Ampere} = \dfrac{\text{Coulombs}}{\text{second}}$

More FAQ

Why do charges flow?

If an electric field is set up in a conductor, charge will move (making a current in direction of the electric field).



Are the properties of conductors in electrostatic right?

No. When there is a current, the conductor is not an equipotential surface, and the electric field inside is not zero!

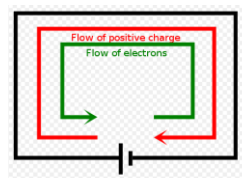
What's the direction of the current?

Direction of current is direction of flow of positive charge or, opposite direction of flow of negative charge

It is a ***convention***.

Is current a vector?

Current is a scalar not a vector! It flows always along a current-carrying wire.



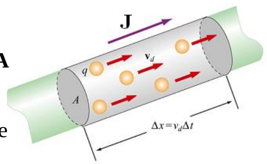
Current Density

It is defined as a **vector** whose magnitude is the electric current per cross-sectional area: $I = \mathbf{J} \bullet \mathbf{A}$

where I is current, $\mathbf{J}$ the current density, and $\mathbf{A}$ the cross-sectional area.

SI units: amperes per square meter, A/m^2.

The total current through a surface : $I = \iint \mathbf{J} \bullet d\mathbf{A}$



If q is the charge of each carrier, n is the number of charge carriers per unit volume the total charge in the section : $\Delta Q = q(nA\Delta x)$

and if the charge carriers move with a speed v_d , the displacement in a time interval Δt is : $\Delta x = v_d \Delta t$

then the average current is : $I_{av} = \dfrac{\Delta Q}{\Delta t} = nqv_d A$

the current density : $\boxed{\mathbf{J} = nq\mathbf{v}_d}$

Current Density

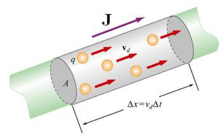
Current density : $\mathbf{J} = nq\mathbf{v}_d$

If q is positive, $\mathbf{v}_d$ is in the same direction as $\mathbf{E}$,

If q is negative, $\mathbf{v}_d$ is opposite to $\mathbf{E}$.

In either case, $\mathbf{J}$ is in the same direction as $\mathbf{E}$.

NB: $\mathbf{J}$ is always in the same direction as $\mathbf{E}$, NOT $\mathbf{v}_d$!



Current Density vs. Current

Current density	Current
$\mathbf{J} = nq\mathbf{v}_d$	$I = n\lvert q\rvert v_d A$
Vector, same direction as $\mathbf{E}$	Scalar
How charges flow at a certain point The magnitude varys around a circuit	Through an extended object, wire The same value at all section of the circuit

Drift Speed/Velocity

The current density: $\mathbf{J} = nq\mathbf{v}_d$

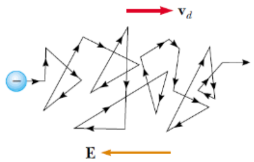
$\mathbf{v}_d$: Drift Speed : average velocity forced by applied electric field in the presence of collisions

A charge e in an electric field experiences an electric force: $\mathbf{F}_e = -e\mathbf{E}$

If no collision, there is an acceleration: $\mathbf{a} = \mathbf{F}/m$ where m: mass of the charge

In gaseous, liquid and solid conductors: ***collision***

If $\mathbf{E}$ is constant and the medium is homogeneous:
the net effect: constant average velocity $\mathbf{v}_d$



Mobility

If $\mathbf{E}$ is constant and the medium is homogeneous:
$\mathbf{v}_d$ is related to $\mathbf{E}$ by a constant called ***mobility*** μ_m :

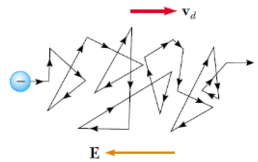
$$\mathbf{v}_d = \mu_m \mathbf{E} \quad \text{(m/s)} \qquad \text{SI unit of } \mu_m : \frac{\text{m}^2}{\text{V}\cdot\text{s}}$$

$$\mathbf{J} = nq\mathbf{v}_d = nq\mu_m \mathbf{E}$$

In a conductor, the free charges are electrons: $\mu_m \rightarrow \mu_e$

μ_m and μ_e both are positive.

Typical values of μ_e (SI unit):
Aluminum: 0.0012
Copper: 0.0032
Silver: 0.0056



Ohm's Law

$$\boxed{\mathbf{J} = \sigma \mathbf{E}} \qquad \boxed{\mathbf{J} = nq\mathbf{v}_d = nq\mu_m \mathbf{E}}$$

σ : conductivity, SI units: $(\Omega\cdot\text{m})^{-1}$

For many materials (including most metals), the ratio of the current density to the electric field is a constant that is independent of the electric field producing the current.

- The conductivity of a material is a measure of how easily free charges can travel through the material under the influence of an externally applied electric field.
  - A *perfect dielectric* is a material with σ = 0, and
  - a *perfect conductor* is a material with σ = ∞.
- The conductivity depends only on the microscopic properties of the material, not on its shape.

Resistivity

$\rho \equiv \dfrac{1}{\sigma}$ where ρ : resistivity

ρ depends only on the microscopic properties of the material, not on its shape.

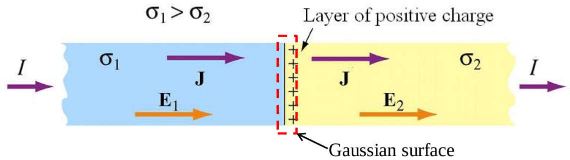
The greater the resistivity, the greater the field needed to cause a given current density.

SI units: $\dfrac{\text{V}\cdot\text{m}}{\text{A}}$ or ohm-meter $(\Omega\cdot\text{m})$, $1\ \dfrac{\text{V}}{A}$ is called one ohm $(1\,\Omega)$.

The resistivity of a material actually varies with temperature.

Example -- Charge at a Junction

Find the total amount of charge at the junction of the two conducting materials.



Solution

In a steady state of current flow, the normal component of the current density **J** must be the same on both sides of the junction.

Since $J = \sigma E$, we have $\sigma_1 E_1 = \sigma_2 E_2$ or $E_2 = (\sigma_1/\sigma_2)\, E_1$

Let the charge on the interface be q_{in}, from the Gauss's law:

$$\oiint_S \mathbf{E} \bullet d\mathbf{s} = (E_2 - E_1)A = \frac{q_{in}}{\varepsilon} \;\Rightarrow\; E_2 - E_1 = \frac{q_{in}}{A\varepsilon}$$

Example -- Charge at a Junction

Solution Cont.

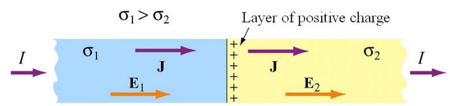
Substituting the expression for E_2 from $E_2 - E_1 = \dfrac{q_{\text{in}}}{A\varepsilon}$

$$q_{\text{in}} = \varepsilon A E_1\left(\frac{\sigma_1}{\sigma_2} - 1\right) = \varepsilon A \sigma_1 E_1\left(\frac{1}{\sigma_2} - \frac{1}{\sigma_1}\right)$$

$\boxed{E_2 = (\sigma_1/\sigma_2)\, E_1}$

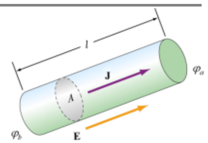
Since the current is $I = JA = (\sigma_1 E_1)\, A$,
the amount of charge on the interface becomes

$$q_{\text{in}} = \varepsilon\, I\left(\frac{1}{\sigma_2} - \frac{1}{\sigma_1}\right)$$



Resistance

Suppose a potential difference is applied between the ends of the wire, creating a uniform electric field **E**:



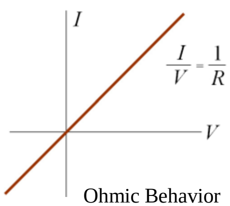
$$\Delta\varphi = \varphi_b - \varphi_a = -\int_a^b \mathbf{E} \bullet d\mathbf{l} = El$$

The current density: $J = \sigma E = \sigma\left(\dfrac{\Delta\varphi}{l}\right) = \sigma\left(\dfrac{V}{l}\right)$

also $J = \dfrac{I}{A}$, then $V = \dfrac{l}{\sigma} J = \left(\dfrac{l}{\sigma A}\right) I = RI$

$\boxed{V = RI}$

Very useful in practical applications

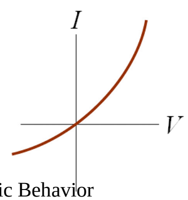


$R = \dfrac{l}{\sigma A} \;\Rightarrow\;$ ***Resistance***

or $R = \dfrac{V}{I}$ SI unit: ohm (Ω)

$1\Omega \equiv \dfrac{1\text{V}}{1\text{A}}$

Ohmic Behavior



Nonohmic Behavior

Resistance and Resistivity

Resistivity: $\rho = \dfrac{1}{\sigma} = \dfrac{E}{J}$ SI: $\dfrac{V/l}{I/A} = \dfrac{RA}{l}$ $\Omega \bullet \text{m}$

Resistance: $R = \dfrac{\rho l}{A}$ SI: Ω

Resistivity is property of a substance. Every ohmic material has a characteristic resistivity that depends on the properties of the material and on temperature, not on its shape or size.

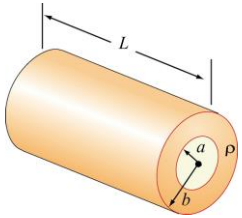
Resistance is property of an object, depends on geometry (shape and size) as well as resistivity.

Resistance of a Hollow Cylinder

Consider a hollow cylinder of length L and inner radius a and outer radius b. The material has resistivity ρ.

(a) Suppose a potential difference is applied between the ends of the cylinder and produces a current flowing parallel to the axis. What is the resistance measured?

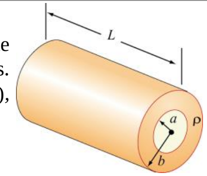
(b) If instead the potential difference is applied between the inner and outer surfaces so that current flows radially outward, what is the resistance measured?



Resistance of a Hollow Cylinder

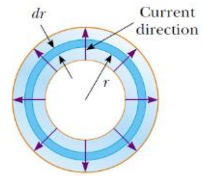
Solution

(a) When a potential difference is applied between the ends of the cylinder, current flows parallel to the axis. In this case, the cross-sectional area is $A=\pi(b^2-a^2)$, and the resistance is given by:



$$R = \frac{\rho L}{A} = \frac{\rho L}{\pi(b^2 - a^2)}$$

(b) Consider a differential element which is made up of a thin cylinder of inner radius r and outer radius $r + dr$ and length L. Its contribution to the resistance of the system is given by:



$$dR = \frac{\rho\, dl}{A} = \frac{\rho\, dr}{2\pi r L}$$

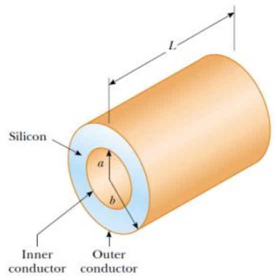
The total resistance of the system: $R = \int_a^b \frac{\rho\, dr}{2\pi r L} = \frac{\rho}{2\pi L}\ln\left(\frac{b}{a}\right)$

Resistance of a Coaxial Cable

A coaxial cable consists of two concentric cylindrical conductors. The region between the conductors is completely filled with silicon.

a = 0.5 cm, b = 1.75 cm, L = 15.0 cm.
The resistivity of silicon: 640 Ω-m

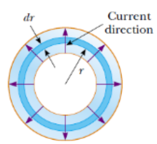
Calculate the resistance of the silicon between the two conductors.



Solution

$$R_{silicon} = \int_a^b \frac{\rho dr}{2\pi r L} = \frac{\rho}{2\pi L}\ln\left(\frac{b}{a}\right)$$

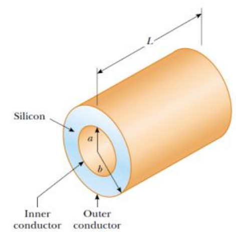
$$= \frac{640}{2\pi \times 0.15}\ln\left(\frac{1.75}{0.5}\right) = 851\ \Omega$$



Resistance of a Coaxial Cable

The current leakage through the silicon, in the radial direction, is unwanted.

Let's compare this resistance to that of the inner conductor. Assuming that the conductor is made of copper with $\rho = 1.7\times10^{-8}\ \ \Omega\cdot\text{m}$



$$R_{copper} = \rho\frac{l}{A} = 1.7\times10^{-8}\frac{0.15}{\pi\left(5\times10^{-3}\right)^2} = 3.2\times10^{-5}\ \ \Omega$$

$$\frac{I_{silicon}}{I_{copper}} = \frac{R_{copper}}{R_{silicon}} = \frac{3.2\times10^{-5}}{851}$$

Almost all of the current corresponds to charge moving along the length of the cable

Electrical Energy and Power

Electric energy of charge Δq increased by through the battery: $\Delta U = \Delta q V$

Neglect the internal resistance of the battery and the connecting wires

The rate of electric energy loss through the resistor: $P = \frac{\Delta U}{\Delta t} = \left(\frac{\Delta q}{\Delta t}\right)V = IV$

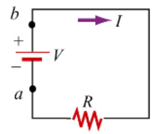
With $V = IR$, the power supplied by the battery: $\underline{P = I^2 R = \frac{V^2}{R}}$

In a time t, the energy consumed by the device:

$\underline{W = Pt = I^2 Rt}$ $\Leftarrow$ Joule's law

W: energy, SI unit: J, 1J = 1W × 1sec

P: power, W, R: resistance, Ω, t: time, sec



If power is not constant over the time, then $W = R\int_0^t I^2 dt$

Electrical Energy and Power

Example

What is the required resistance of an immersion heater that will increase the temperature of 1.5 kg of water from 10 ^{0}C to 50 ^{0}C in 10 min while operating at 110 V?

Solution

Ignore the initial period during which the temperature of the resistor increases,

Ignore any variation of resistance with temperature.

The specific heat of water: C_p = 4186 J/(kg - ^{0}C)

Assume: a constant rate of energy transfer for the entire 10 min, and the rate of energy delivered to the resistor equal to the rate of energy entering the water by heat

We have: The total amount of energy required

$$P = \frac{V^2}{R} = \frac{Q}{\Delta t} \qquad R = \frac{V^2 \Delta t}{m C_p \Delta T}$$

t: time

T: temperature

Electrical Energy and Power

Example

Solution Cont.

$$R = \frac{V^2 \Delta t}{m C_p \Delta T} = \frac{110^2 (10 \times 60)}{1.5 \times 4186 \times (50 - 10)} = 28.9 \quad \Omega$$

The amount of energy transferred:

$$P\Delta t = \frac{V^2 \Delta t}{R} = \frac{110^2 \times 10}{28.9}\frac{1}{60} = 69.8 \text{ Wh} = 0.0698 \text{ kWh}$$

Fuel	Pounds of CO2 per million Btu	Heat rate (Btu per kWh)	Pounds of CO2 per kWh	g of CO2 per kWh
Coal				1 Pounds = 0.4356 Kg
Bituminous	205.691	10,080	2.07	902
Subbituminous	214.289	10,080	2.16	941
Lignite	215.392	10,080	2.17	945
Natural gas	116.999	10,408	1.22	531
Distillate oil (No. 2)	161.290	10,156	1.64	714
Residual oil (No. 6)	173.702	10,156	1.76	767

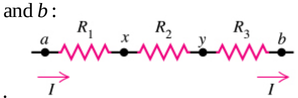
Last updated: February 29, 2016 https://www.eia.gov/tools/faqs/faq.cfm?id=74&t=11

Resistors in a Circuit

The three resistors R_1, R_2 and R_3 in series,

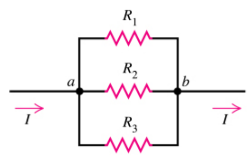
the equivalent resistance R_{eq} between points a and b:

$$R_{eq} = \frac{V_{ab}}{I} = R_1 + R_2 + R_3$$



Generally, to any number of resistors in series:

$$R_{eq} = R_1 + R_2 + R_3 + ... + R_N = \sum_{i=1}^{N} R_i$$



If they are in parallel, the equivalent resistance R_{eq} between points a and b:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Generally, to any number of resistors in parallel:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ... + \frac{1}{R_N} = \sum_{i=1}^{N} \frac{1}{R_i}$$

Resistors and Capacitors

	Resistors	Capacitors
Series	$R_{eq} = \sum_{i=1}^{N} R_i$	$\frac{1}{C_{eq}} = \sum_{i=1}^{N} \frac{1}{C_i}$
Parallel	$\frac{1}{R_{eq}} = \sum_{i=1}^{N} \frac{1}{R_i}$	$C_{eq} = \sum_{i=1}^{N} C_i$

Electromotive Force

Electrical energy must be supplied to maintain a constant current in a closed circuit. The source of energy is commonly referred to as the electromotive force, or emf (symbol ε).

Examples: Batteries, solar cells, thermocouples, …

Mathematically emf is defined as: $\varepsilon \equiv dW/dq$

Physical meaning: the work done to move a unit charge in the direction of higher potential. The SI unit: the volts (V).

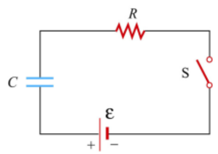
In analyzing circuits, there are two fundamental (Kirchhoff's) rules:
Junction rule and **Loop rule**

1. Junction Rule
At any point where there is a junction between various current carrying branches, by current conservation the sum of the currents into the node must equal the sum of the currents out of the node.

2. Loop Rule
The sum of the voltage drops ΔV, across any circuit elements that form a closed circuit is zero.

RC Circuits
Charging a Capacitor

The capacitor is connected to a DC voltage source of emf ε. When $t < 0$, there is no voltage across the capacitor. We say that the capacitor initially is uncharged. At time $t = 0$, the switch is closed and current, $I_0 = \varepsilon/R$, begins to flow.



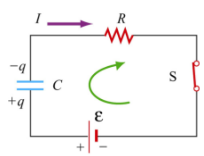
$t < 0$

As the capacitor starts to charge, the voltage across the capacitor increases in time, $V_C(t) = q(t)/C$. Using Kirchhoff's loop rule for capacitors and traversing the loop clockwise, we obtain:

$$\varepsilon - I(t)R - V_C(t) = \varepsilon - \frac{dq}{dt}R - V_C(t) = 0$$

The charging capacitor then satisfies a first order differential equation :

$$\frac{dq}{dt} = \frac{1}{R}\left(\varepsilon - \frac{q}{C}\right)$$



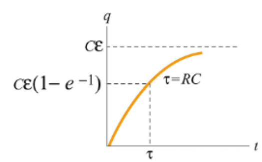
$t \geq 0$

RC Circuits
Charging a Capacitor

By using the method of separation of variables :

$$\frac{dq}{q - C\varepsilon} = -\frac{1}{RC}dt \quad \Rightarrow \quad \int_0^q \frac{dq'}{q' - C\varepsilon} = -\frac{1}{RC}\int_0^t dt'$$

Then : $q(t) = C\varepsilon\left(1 - e^{-t/(RC)}\right) = Q\left(1 - e^{-t/(RC)}\right)$
where $Q = C\varepsilon$ is the maximum amount of charge stored on the plates.



Charge as a function of time

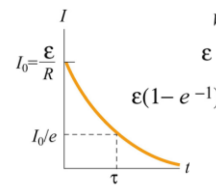
The voltage across the capacitor :

$$V_C(t) = \frac{q(t)}{C} = \varepsilon\left(1 - e^{-t/(RC)}\right)$$

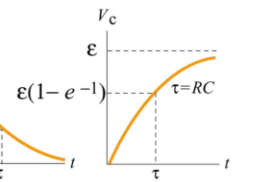
The current flowing in the circuit :

$$I(t) = \frac{dq}{dt} = \left(\frac{\varepsilon}{R}\right)e^{-t/(RC)} = I_0 e^{-t/(RC)}$$

where I_0 is the initial current at $t = 0$



Current as a function of time



Voltage across capacitor as a function of time

RC Circuits

Charging a Capacitor

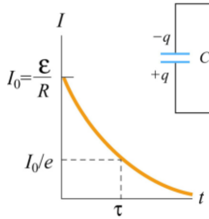
$\tau = RC$. τ is called the ***time constant*** of the system.

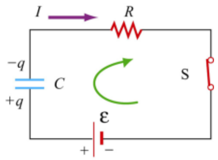
The SI units of τ are seconds:

$$[\Omega][\text{F}] = ([\text{V}]/[\text{A}])([\text{C}]/[\text{V}]) = [\text{C}]/[\text{A}] = [\text{C}]/([\text{C}]/[\text{s}]) = [\text{s}]$$

In term of τ, for example:

$$I(t) = I_0 e^{-t/(RC)} = I_0 e^{-t/\tau}$$





t	0	τ	2τ	3τ	…	5τ
$I = I_0 e^{-\frac{t}{\tau}}$	I_0	$I_0 e^{-1}$	$I_0 e^{-2}$	$I_0 e^{-3}$		$I_0 e^{-5}$
	I_0	$0.368\ I_0$	$0.135\ I_0$	$0.05\ I_0$		$0.007\ I_0$

RC Circuits

Discharging a Capacitor

Suppose initially the capacitor has been charged to some value Q.
The potential difference across the capacitor is given by $V_C = Q/C$. At $t = 0$ the switch is closed and the capacitor will begin to discharge.

Applying the Kirchhoff's loop rule by traversing the loop counterclockwise, the equation that describes the discharging process is: $q/C - IR = 0$

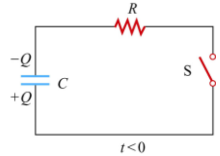
The current flowing away from the positive plate is proportional to the charge on the plate: $I = -dq/dt$

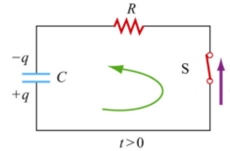
Thus, charge satisfies a first order differential equation:

$$\frac{q}{C} + R\frac{dq}{dt} = 0$$

Similarly, the equation can also be integrated by the method of separation of variables, and we have:

$$\int_Q^q \frac{dq'}{q'} = -\frac{1}{RC}\int_0^t dt' \Rightarrow \ln\left(\frac{q}{Q}\right) = -\frac{t}{RC} \Rightarrow q(t) = Qe^{-t/\tau}$$





RC Circuits

$$\boxed{q(t) = Qe^{-t/\tau}}$$

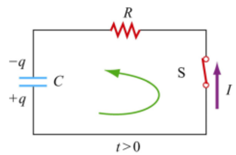
Discharging a Capacitor

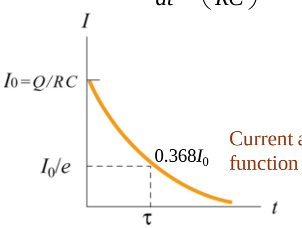
The voltage across the capacitor:

$$V_C(t) = \frac{q(t)}{C} = \left(\frac{Q}{C}\right)e^{-t/\tau}$$

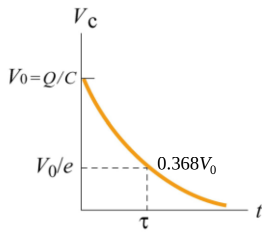
The current also exponentially decays:

$$I(t) = -\frac{dq}{dt} = \left(\frac{Q}{RC}\right)e^{-t/\tau}$$





Current as a function of time



Voltage across the capacitor as a function of time

RC Circuits

Quiz 1

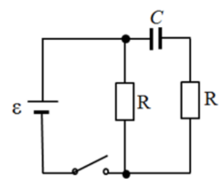
Consider the circuit in the Figure and assume that the battery has no internal resistance. Just after the switch is closed, the current in the battery is:

a) $\frac{2\varepsilon}{R}$.

b) $\frac{\varepsilon}{2R}$.

c) zero.

d) $\frac{\varepsilon}{R}$.



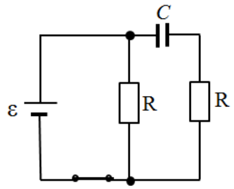
RC Circuits

Quiz 2

Consider the circuit in the Figure and assume that the battery has no internal resistance. The switch has been closed for a very long time, then the current in the battery is:

a) $\frac{\varepsilon}{R}$. b) $\frac{\varepsilon}{2R}$.

c) zero. d) $\frac{2\varepsilon}{R}$.



Summary of Current

- The electric current defined :

$$I = \frac{dQ}{dt}$$

- The average current in a conductor :

$$I_{av} = nqv_d A$$

- The current density :

$$\mathbf{J} = nq\mathbf{v}_d$$

- The Ohm's law :

$$\mathbf{J} = \sigma\mathbf{E}$$

- Resistance R and resistivity ρ :

$$R = \frac{\rho l}{A}$$

- Drift velocity :

$$\mathbf{v}_d = \mu_m \mathbf{E}$$

- Electric power :

$$P = IV = I^2 R$$

- Electric energy --Joule's law :

$$W = Pt = I^2 Rt$$

Summary of Current

- The equivalent resistance of a set of resistors connected in series:

$$\boxed{R_{eq} = R_1 + R_2 + R_3 + \ldots = \sum_{i=1}^{N} R_i}$$

- The equivalent resistance of a set of resistors connected in parallel:

$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots = \sum_{i=1}^{N} \frac{1}{R_i}}$$

- In a charging capacitor, the charges and the current as a function of time:

$$\boxed{q(t) = Q\left(1 - e^{-t/\tau}\right), \quad I(t) = \left(\frac{\varepsilon}{R}\right) e^{-t/\tau} \quad t \text{ is time and } \tau \text{ is time constant}, \tau = \mathrm{RC}}$$

- In a discharging capacitor, the charges and the current as a function of time:

$$\boxed{q(t) = Qe^{-t/\tau}, \quad I(t) = \left(\frac{Q}{RC}\right) e^{-t/\tau} \quad t \text{ is time and } \tau \text{ is time constant}, \tau = \mathrm{RC}}$$

Next

Electromagnetism:

- Biot-Savart Law
- Ampere’s Law
- Gauss’s Law for Magnetism

Thanks for your attendance