

Chapter 5 Random Variables. Probability Distributions

- 5.1 Random Variables
- 5.2 Discrete Random Variables and Distributions
- 5.3 Continuous Random Variables and Distributions
- 5.4 Summary

A word cloud featuring the words 'Discrete' and 'variable' in large, bold, dark red letters. Other words in various colors and orientations are scattered around them, including 'random' in large purple letters, 'positive' in purple, 'values' in dark red, 'integers' in green, 'example' in brown, 'certain' in green, 'possible' in blue, 'take' in blue, 'set' in brown, and 'specified' in purple.

5.4 Summary

- probability mass function
- probability density function
- distribution functions for discrete and continuous random variables
- Sketching of distribution functions

5.2 Discrete Random Variables: problem

Mr Ali hits a target with probability $p_A = \frac{1}{2}$,

Ms Beatrice hits with probability $p_B = \frac{1}{3}$.

If both shots once, find the range, pmf and cdf for the variable
 $X = \{\textit{number of hits}\}$

5.2 Discrete Random Variables: solution (1)

There can only be: 0, 1 or 2 hits. So, $S = \{0, 1, 2\}$

$X = 0$ if there are two misses, so

$$P(X = 0) = (1 - p_A)(1 - p_B) = \frac{1}{2} \frac{2}{3} = \frac{1}{3}$$

$X = 1$ if Ali or Beatrice hit but not both, so

$$P(X = 1) = P(A \cup B) - P(A \cap B) = p_A + p_B - 2p_A p_B = \frac{1}{2}$$

$X = 2$ if both hit, so

$$P(X = 2) = P(A \cap B) = p_A p_B = \frac{1}{6}$$

5.2 Discrete Random Variables: problem

Mr Ali hits a target with probability $p_A = \frac{1}{2}$,

Ms Beatrice hits with probability $p_B = \frac{1}{3}$.

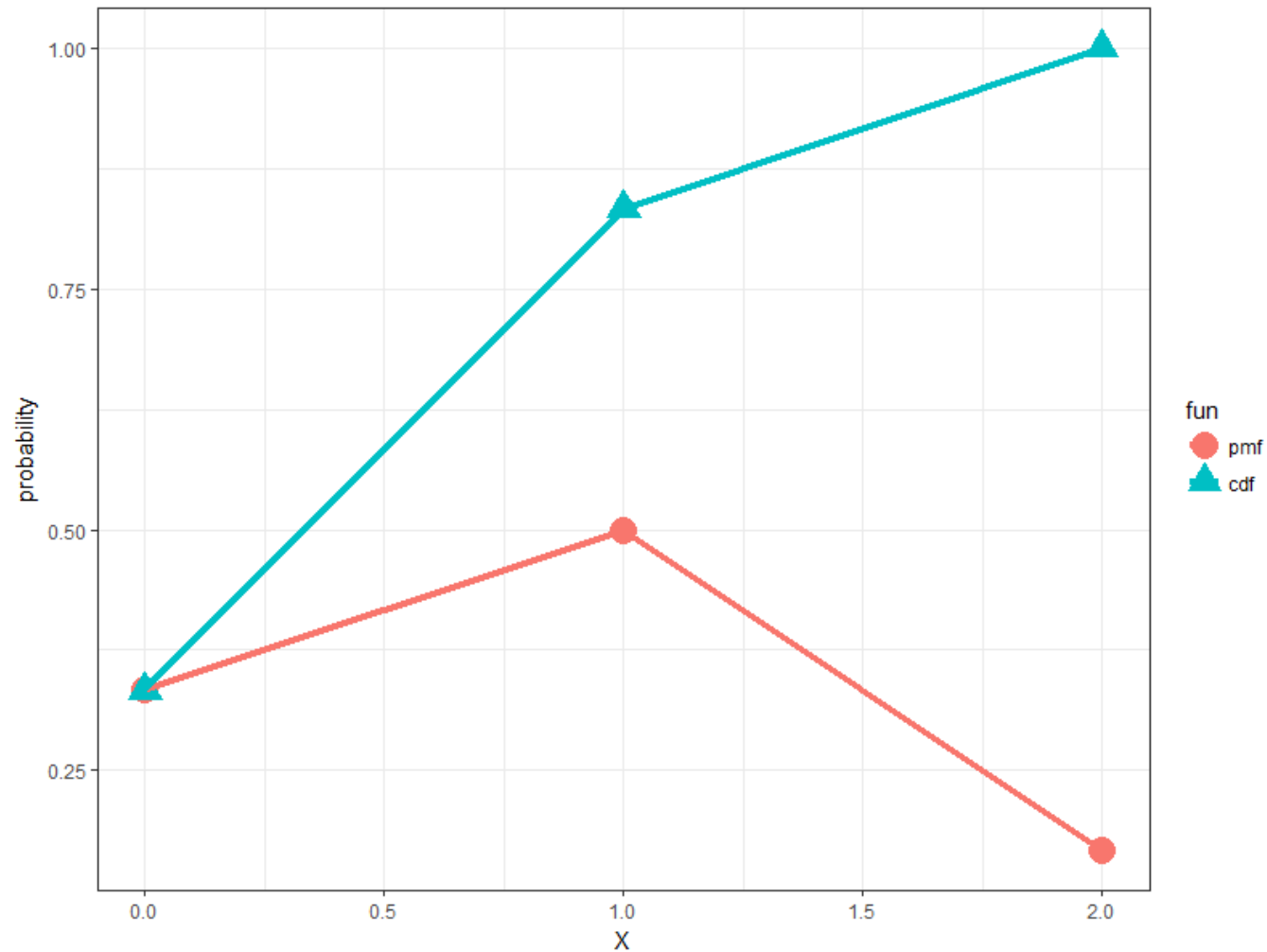
If both shots once, find the range, pmf and cdf for the variable
 $X = \{\textit{number of hits}\}$

5.2 Discrete Random Variables: solution (2)

$$\text{pmf: } P(X = x) = \begin{cases} \frac{1}{3}, & x = 0 \\ \frac{1}{2}, & x = 1, \text{ and } 0 \text{ elsewhere} \\ \frac{1}{6}, & x = 2 \end{cases}$$

$$\text{pdf: } F(x) = P(X \leq x) = \begin{cases} \frac{1}{3}, & x = 0 \\ \frac{5}{6}, & x = 1, \text{ need to add } F(x) = 0, x < 0, F(x) = 1, X > 2 \\ 1, & x = 2 \end{cases}$$

5.2 Discrete Random Variables: solution (3)



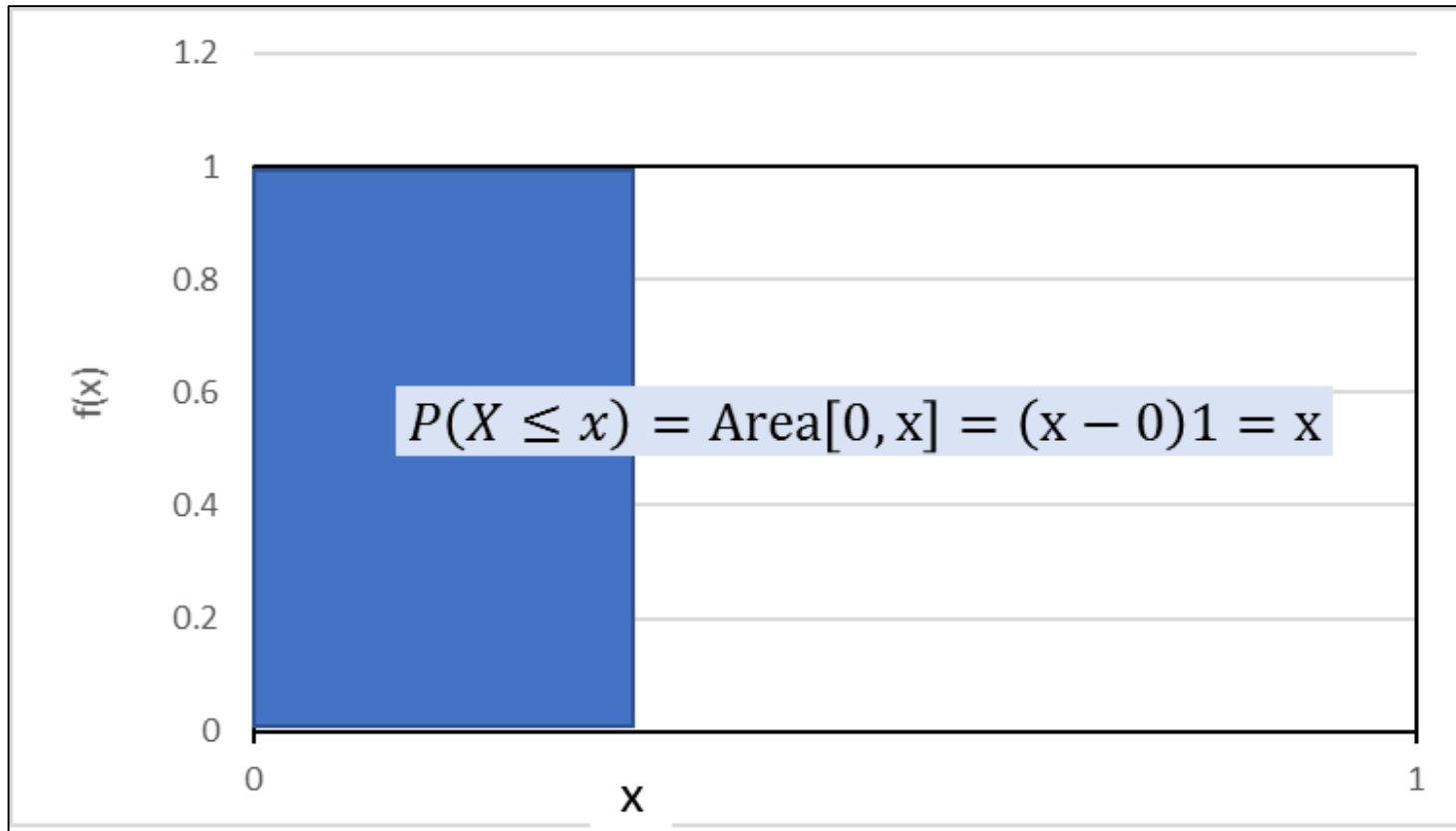
5.3 Continuous Random Variables and Distributions: problem

Plot the pdf

$$f(x) = \begin{cases} 1 & \text{if } 0 < x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Prove that $P(X \leq x) = x$ by geometric arguments.

5.3 Continuous Random Variables and Distributions: Solution



The integral is the area under the pdf. Since it is a rectangle we can do it more easily as base*height!