

MOS Capacitor – (I) energy band & bias; (II) electrostatics

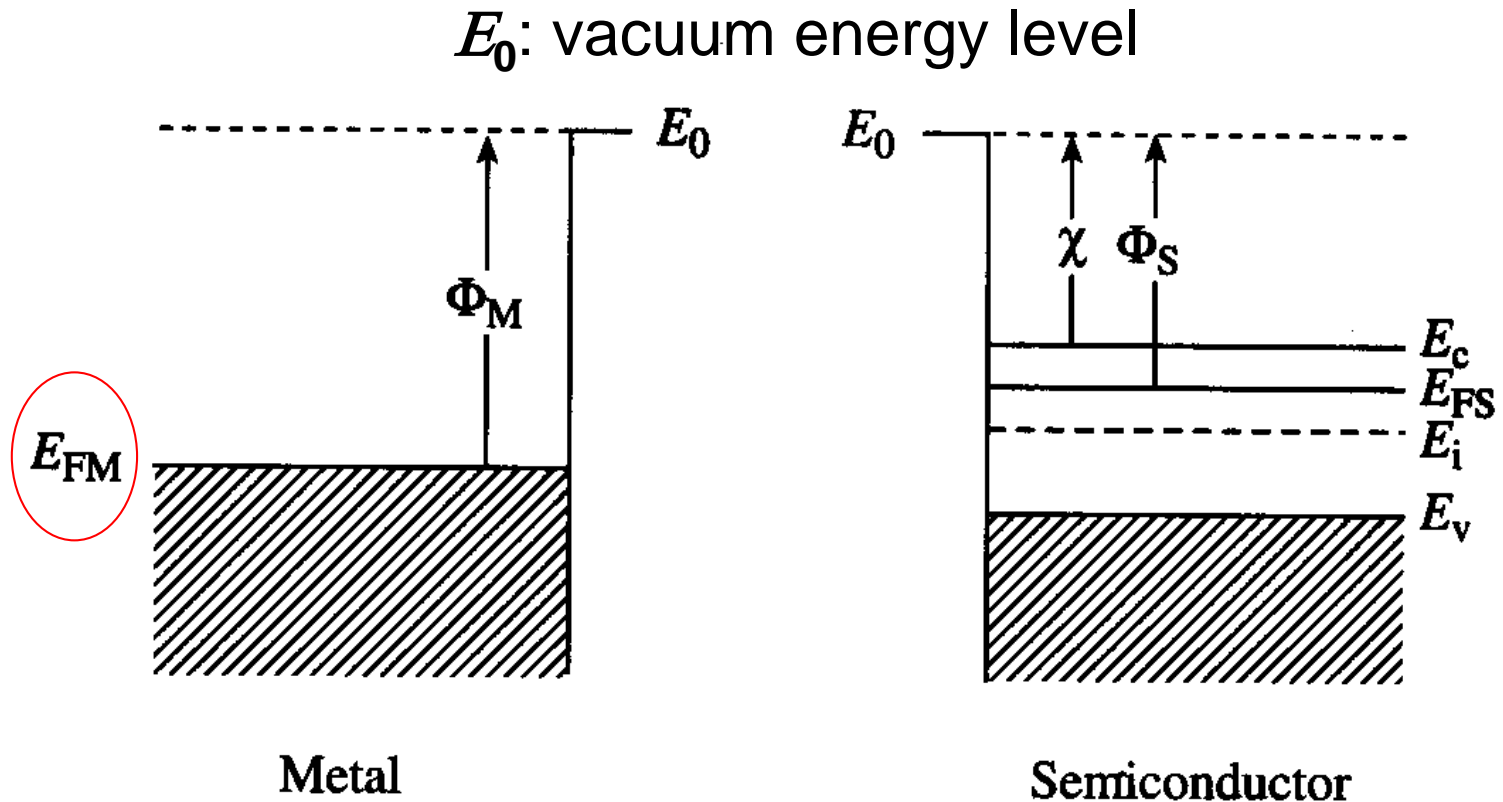
(material originally developed by Professor Cezhou Zhao)

MOS Capacitor

OUTLINE

- Physical Structure of MOS Capacitor
- Energy Band Diagram of MOS Structure
- Effects of Applied Voltage
 - **operation modes & capacitance**
- Non-Ideal MOS Capacitors

Work Function



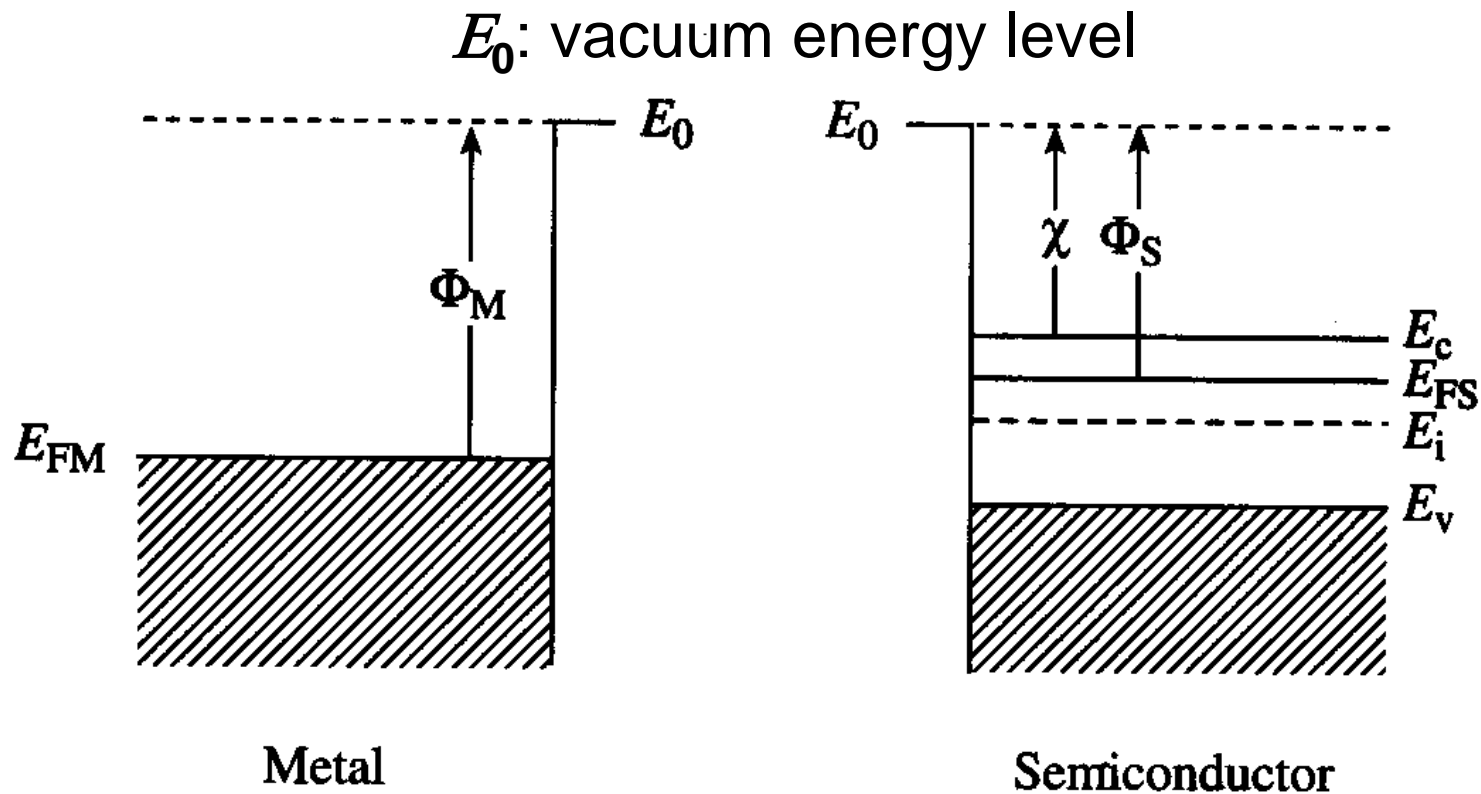
Φ_M : metal work function

Φ_S : semiconductor work function

χ : electron-affinity

chi

Work Function



Φ_M : metal work function

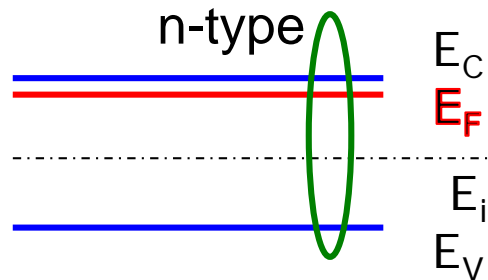
$\Phi_{Al} = 4.28\text{eV}$ $\Phi_{Au} = 5.1\text{eV}$

Φ_S : semiconductor work function

χ : electron-affinity

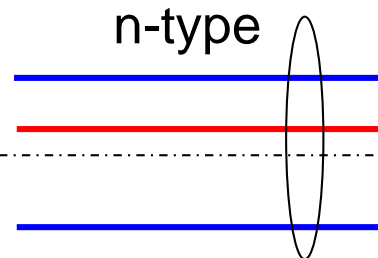
Electron concentration & $E_C - E_F$

Heavily doped



More electrons

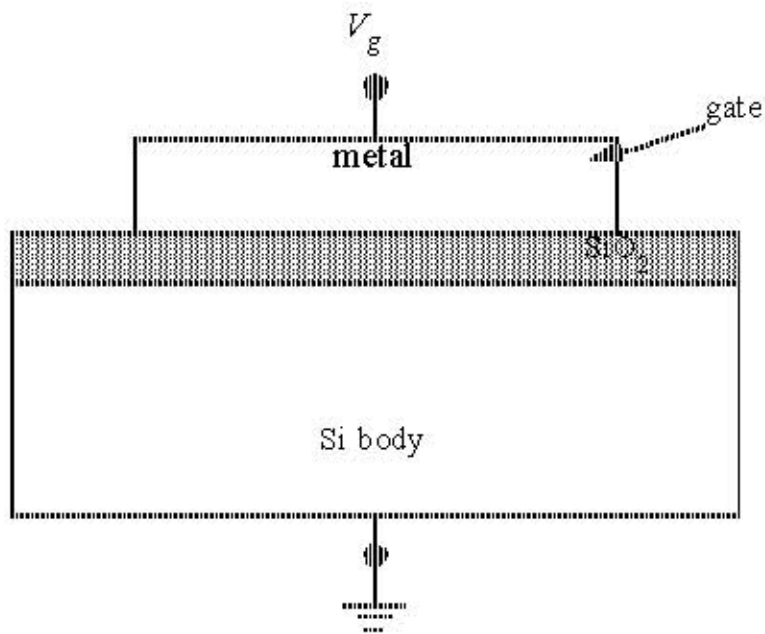
Lightly doped



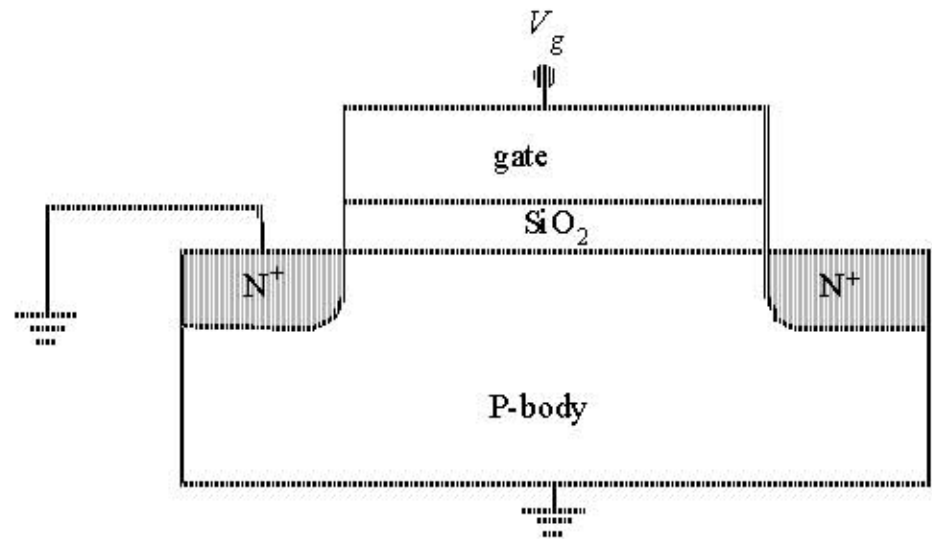
Less electrons

MOS Capacitors

MOS: Metal-Oxide-Semiconductor

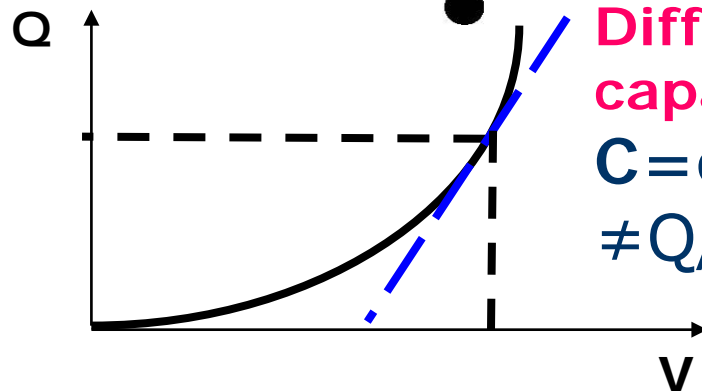
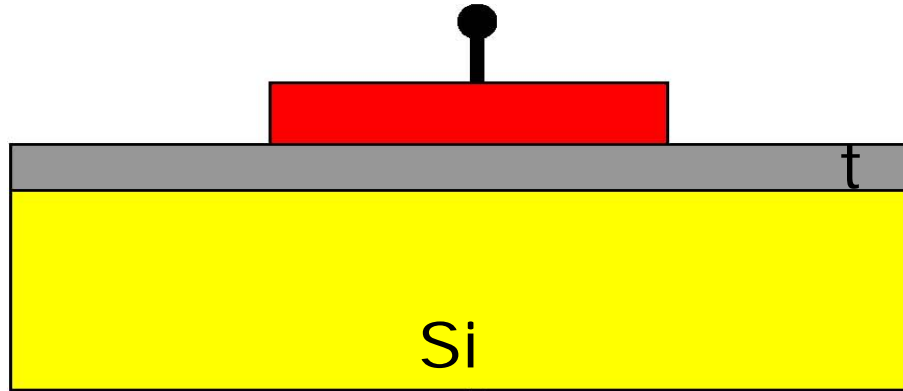


MOS capacitor



MOS transistor

MOS Capacitor



Differential capacitance:
 $C = dQ/dV$
 $\neq Q/V$

● Definition:

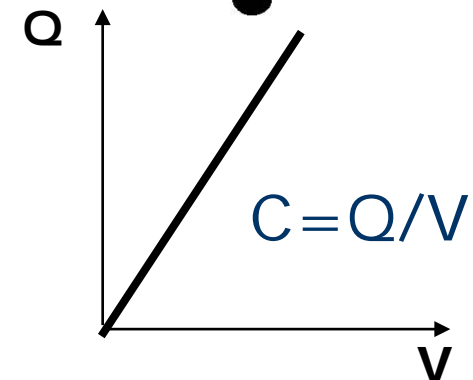
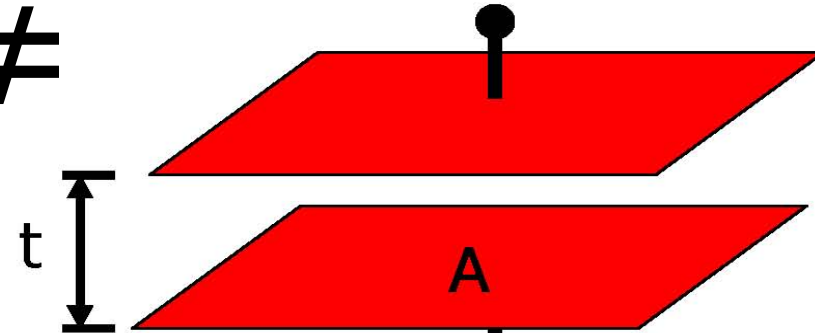
$$C = \frac{\epsilon A}{t_{ox}} = \frac{\epsilon_r \epsilon_0 A}{t_{ox}}$$

$$\epsilon_{r, SiO_2} = 3.9, \quad \epsilon_{r, Si} = 11.9, \quad \epsilon_0 = 8.85 \times 10^{-14} \text{ F/cm}$$

epsilon

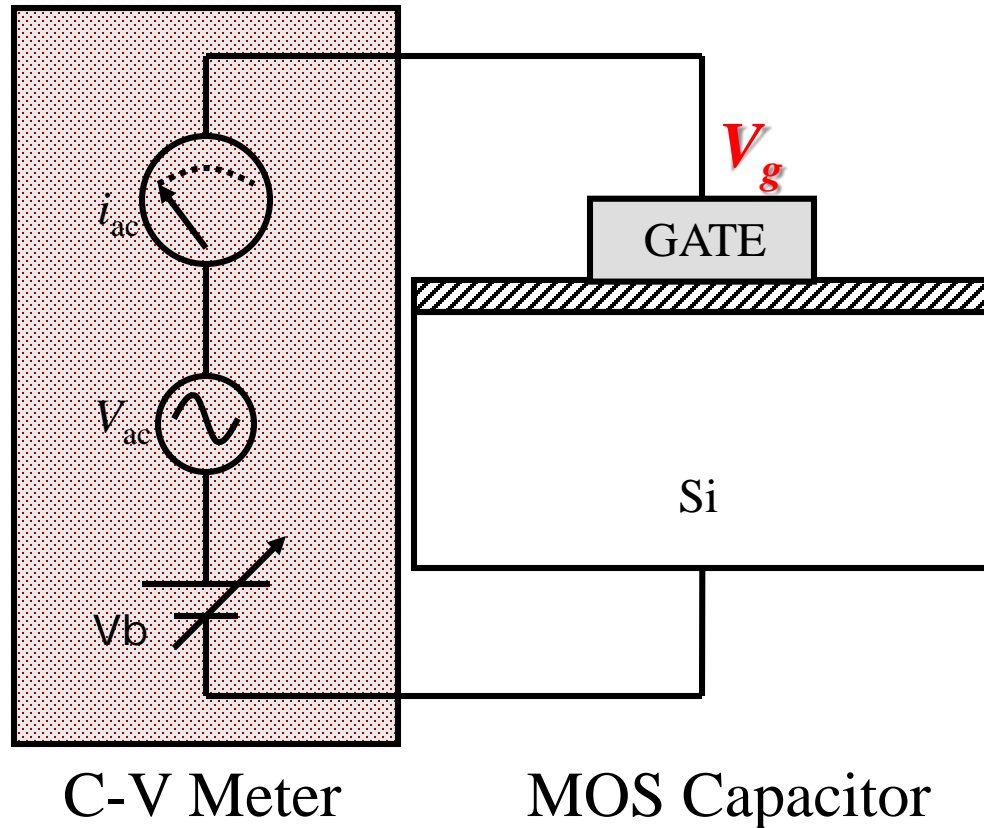
parallel-plate capacitor

\neq



$$C = Q/V$$

MOS Capacitance Measurement



- V_b : dc biasing voltage and scanned slowly
- V_{ac} : small ac signal
- $V_g = V_b + V_{ac}$

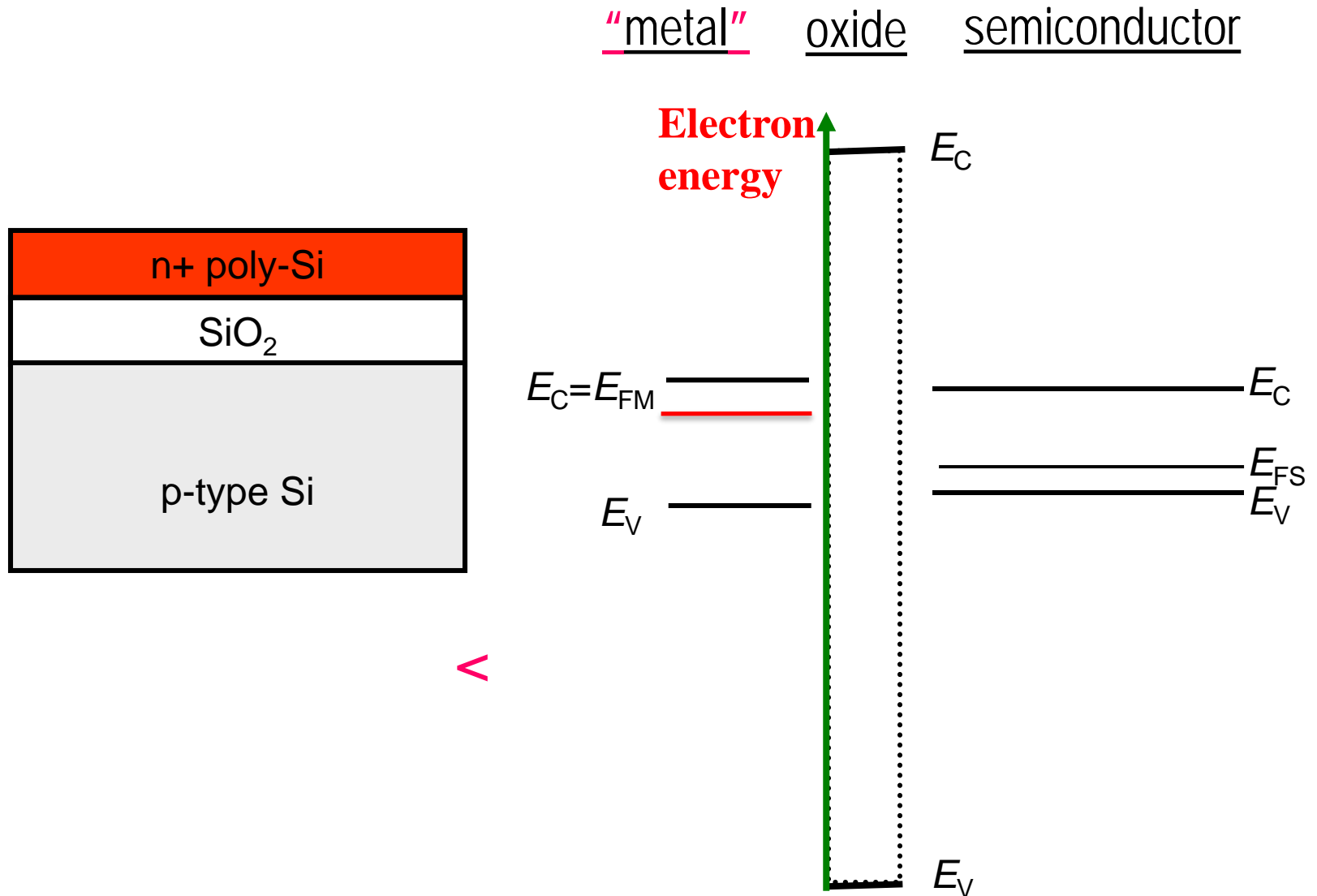
$$C = \left| \frac{dQ_{GATE}}{dV_g} \right| = \left| \frac{dQ_s}{dV_g} \right| = \left| \frac{dQ_s}{dV_{ac}} \right|$$

- Capacitive current due to V_{ac} is measured

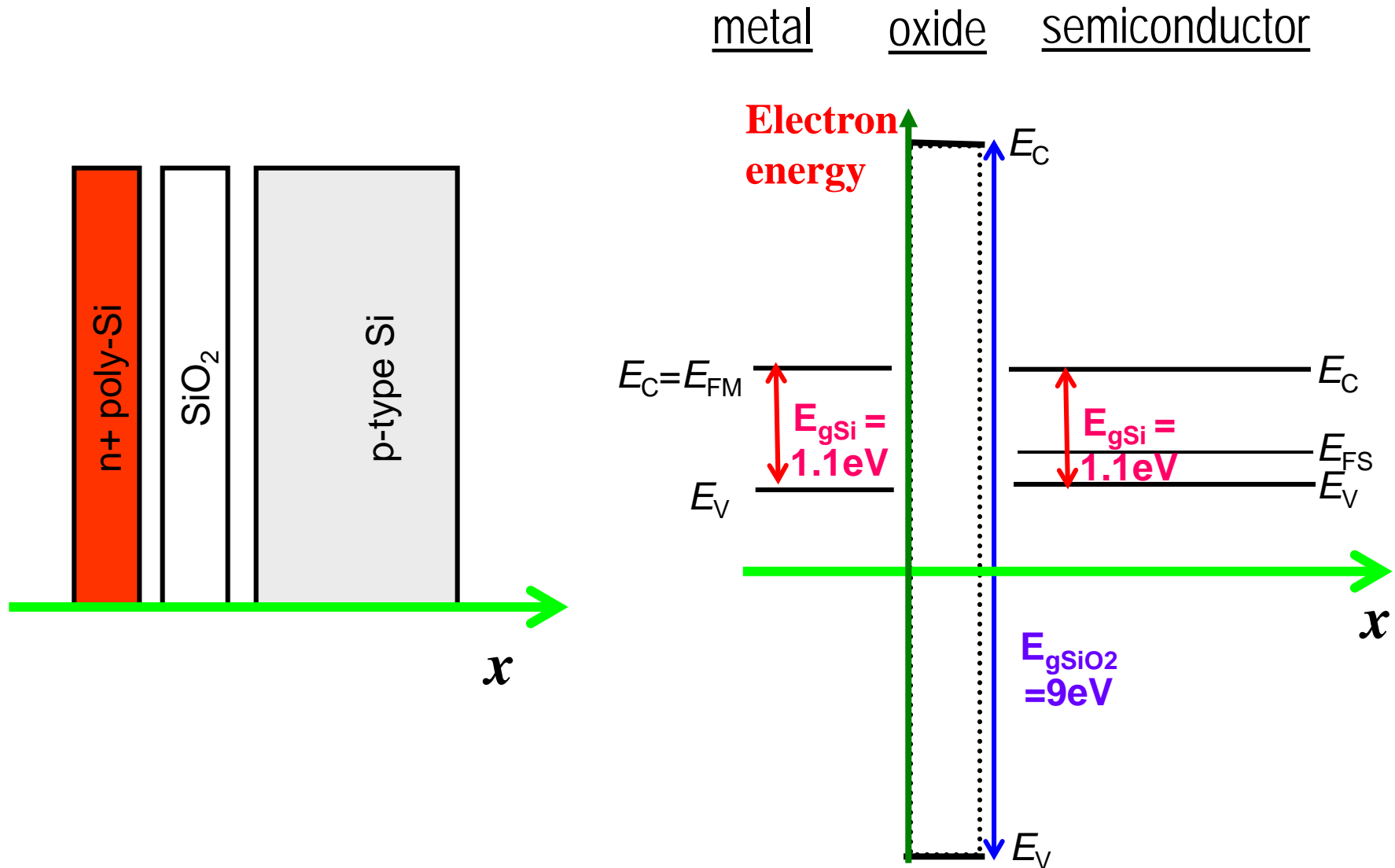
$$i_{ac} = C \frac{dV_{ac}}{dt}$$

$$|V_b| \gg |V_{ac}|$$

Poly-Si gate

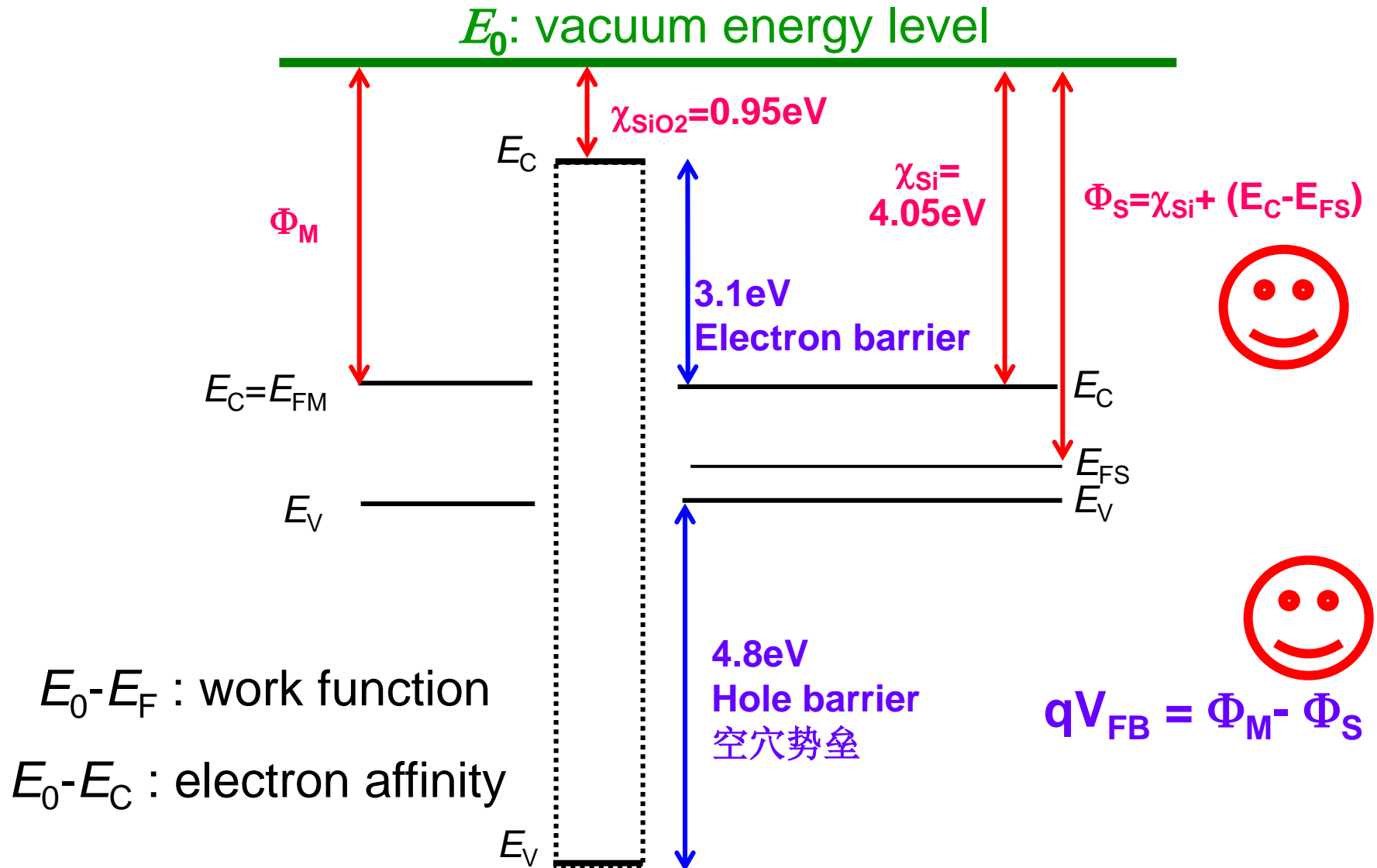


Coordinate system



$$\chi_{\text{Si}} = 4.05 \text{ or } 4.03 \text{ eV}$$


Guidelines for Drawing MOS Band Diagrams



Guidelines for Drawing MOS Band Diagrams

- 1) Fermi level E_F is flat (constant with distance x) in the Si
 - Since no current flows in the x direction, we can assume that equilibrium conditions prevail
- 2) **Band bending is linear in the oxide**
 - No charge in the oxide $\Rightarrow d\mathcal{E}/dx = \rho/\epsilon_{ox} = 0$, so \mathcal{E} is constant
 $\Rightarrow dE_C/dx$ is constant

$$\mathcal{E} = -dV/dx$$
$$E_C = -qV$$

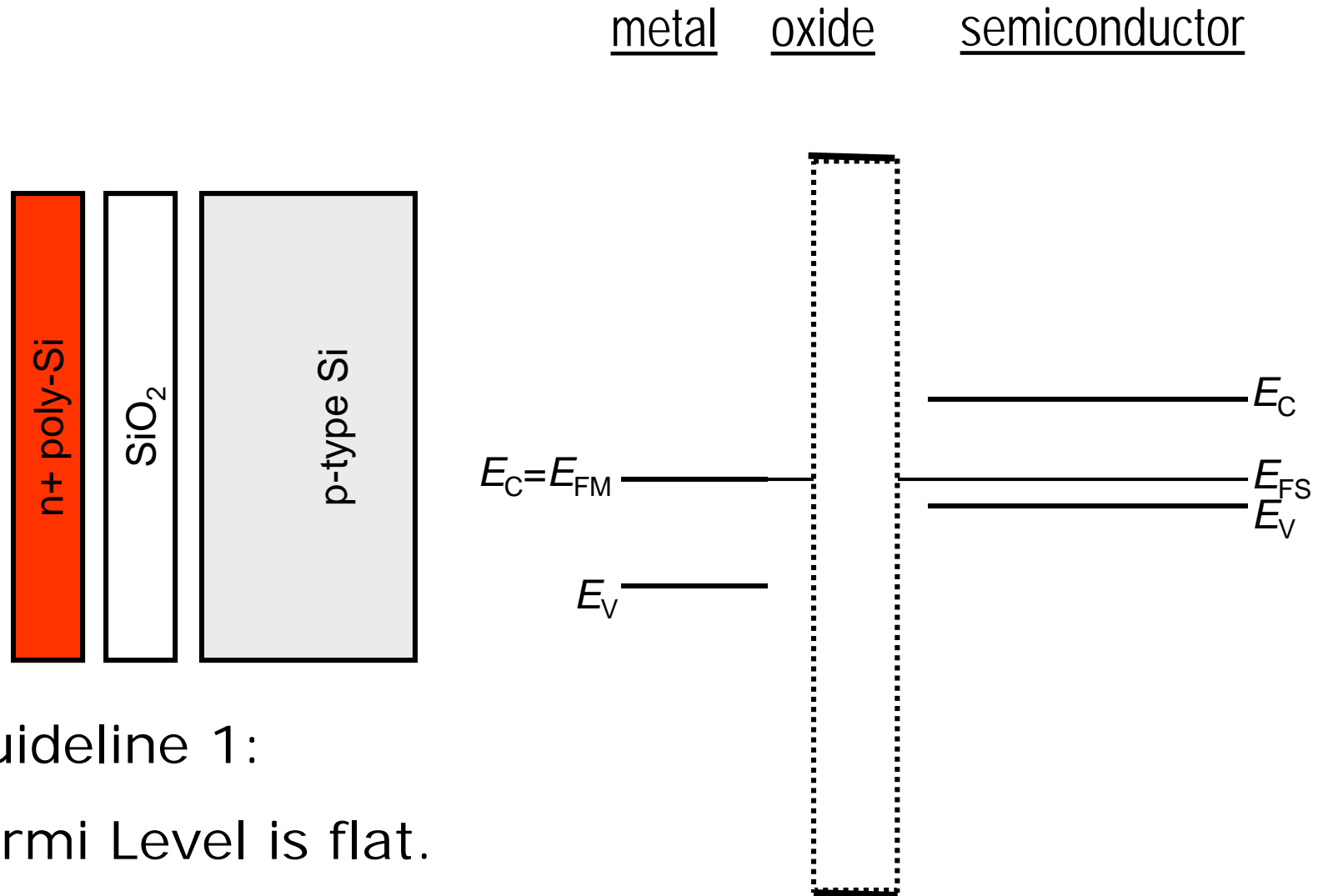


Guidelines for Drawing MOS Band Diagrams

- 3) The barrier height for conduction-band electron flow from the Si into SiO₂ is 3.1 eV
 - This is equal to the electron-affinity difference (χ_{Si} and χ_{SiO_2})
- 4) The barrier height for valence-band hole flow from the Si into SiO₂ is 4.8 eV
- 5) The vertical distance between the Fermi level in the metal, E_{FM} , and the Fermi level in the Si, E_{FS} , is equal to the applied gate voltage:

$$qV_G = E_{\text{FS}} - E_{\text{FM}}$$

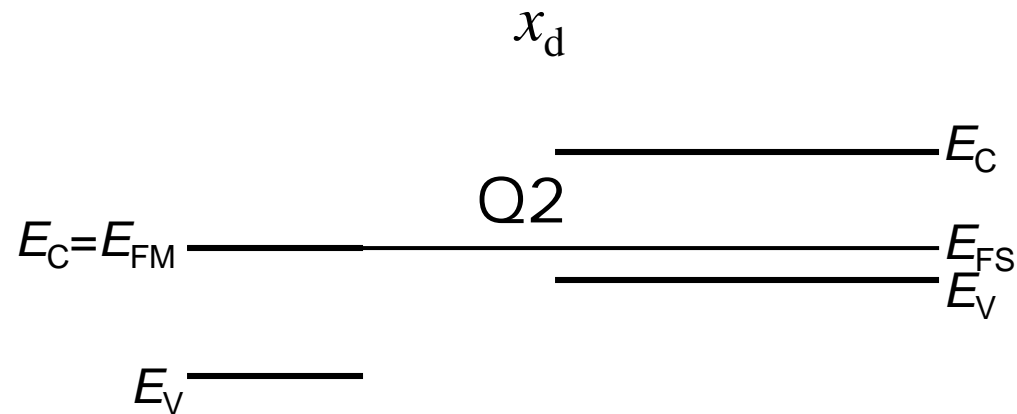
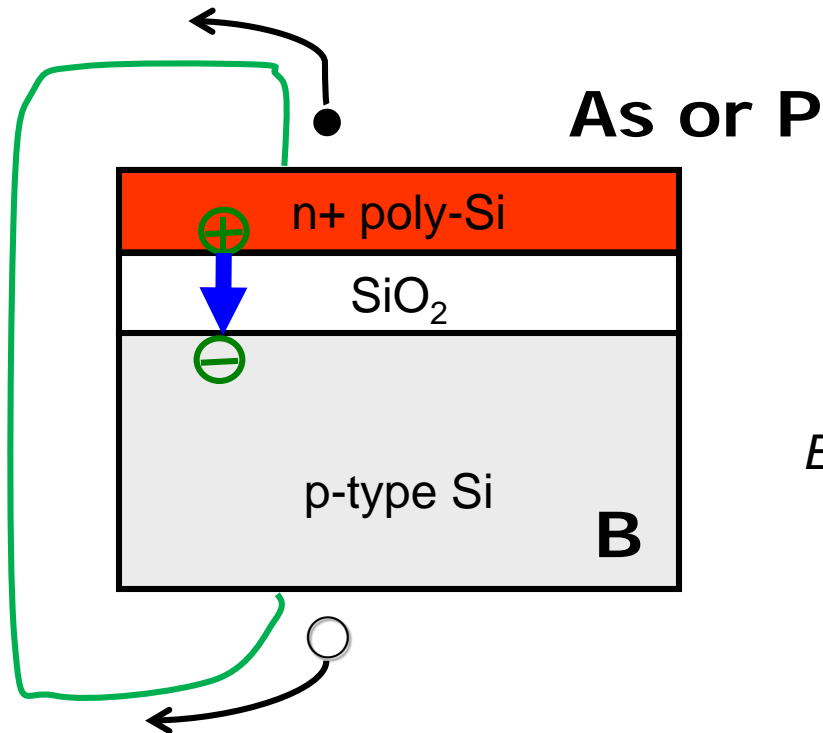
MOS Equilibrium Energy-Band Diagram



MOS Equilibrium Energy-Band Diagram

Fermi Level is flat.

metal oxide semiconductor
Q1



After contact, there are two questions: Q1 & Q2.

Q1: Carrier and ion in silicon

Guideline 2:

$$\mathcal{E} = -dV/dx$$

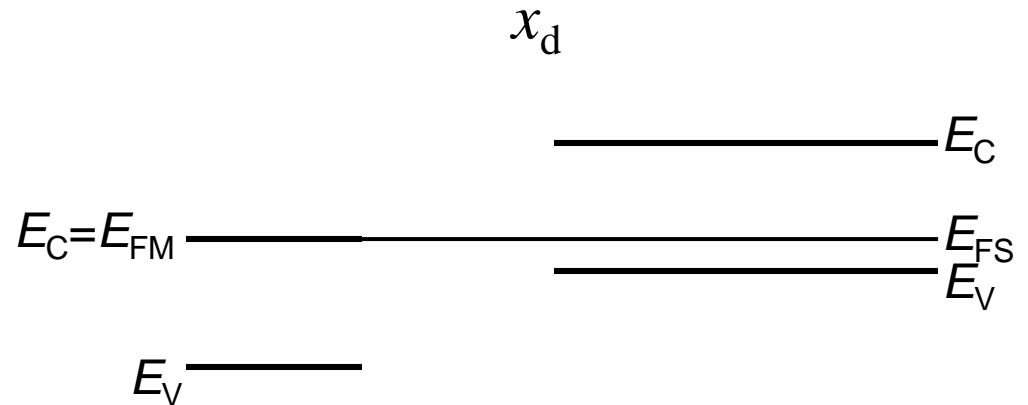
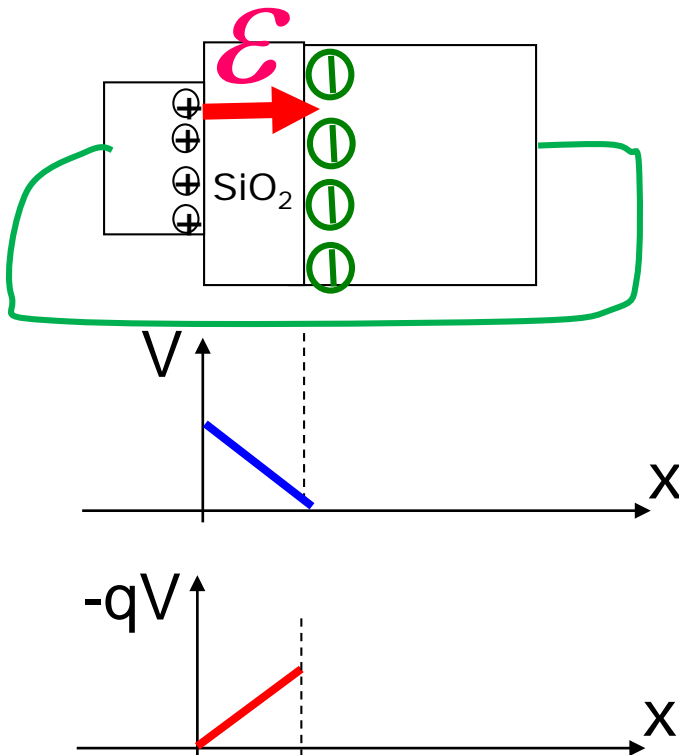
$$E_C = -qV$$

metal

oxide

semiconductor

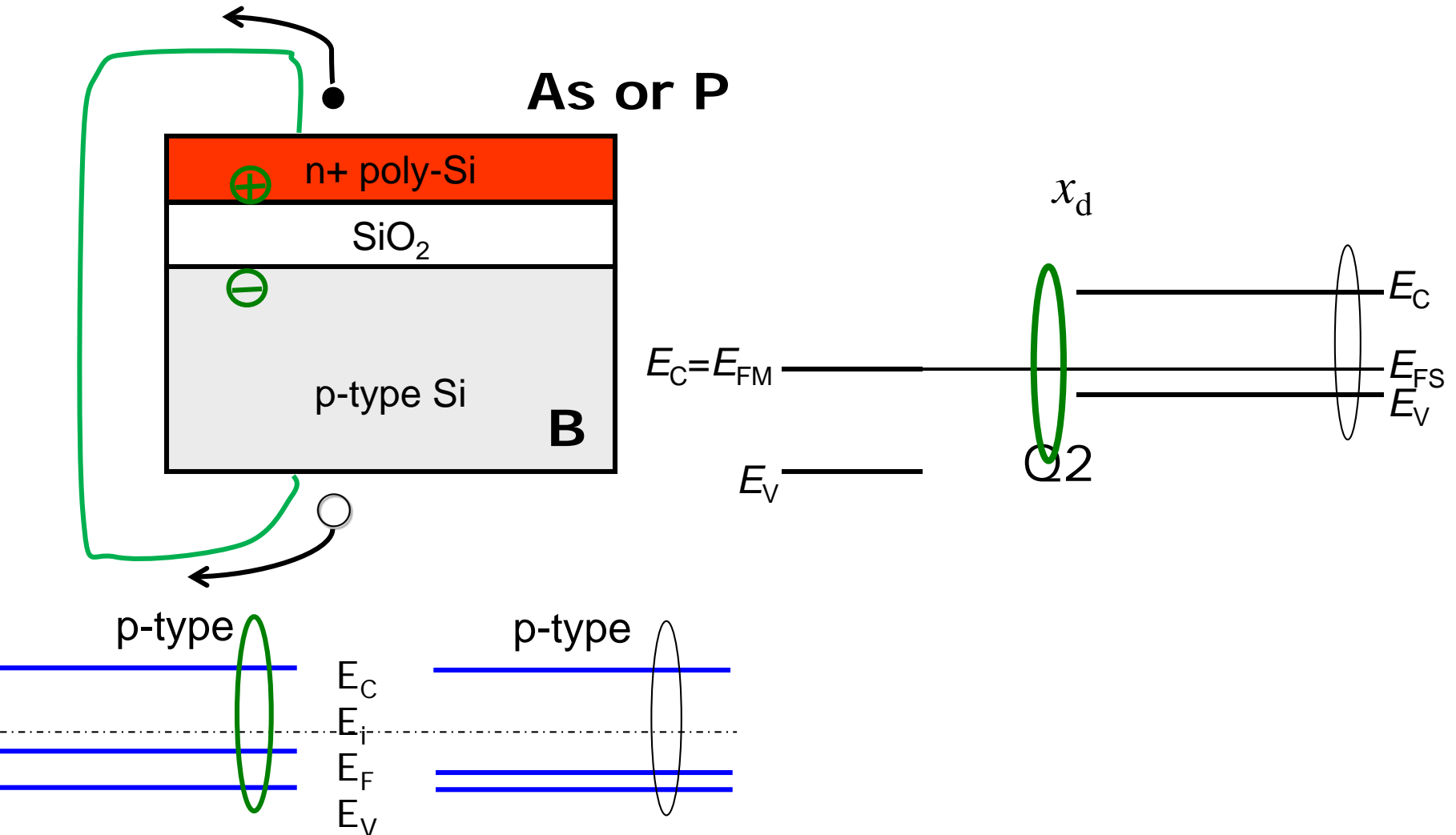
Q1



Q2: Carrier and ion in silicon

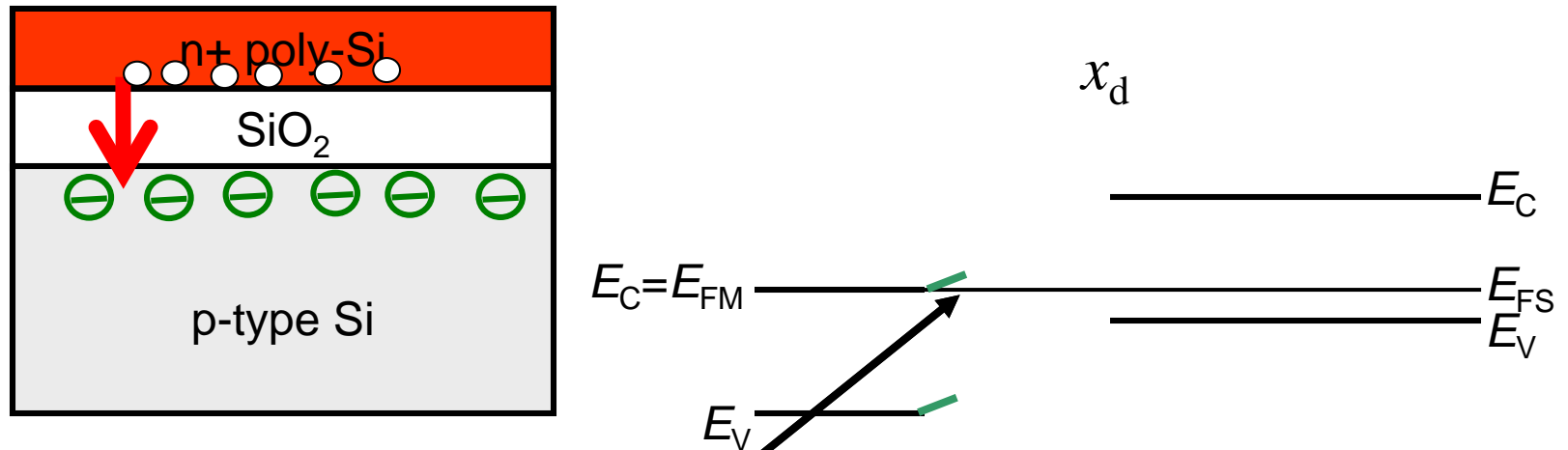
$$n_0 p_0 = n_i^2$$

metal oxide semiconductor



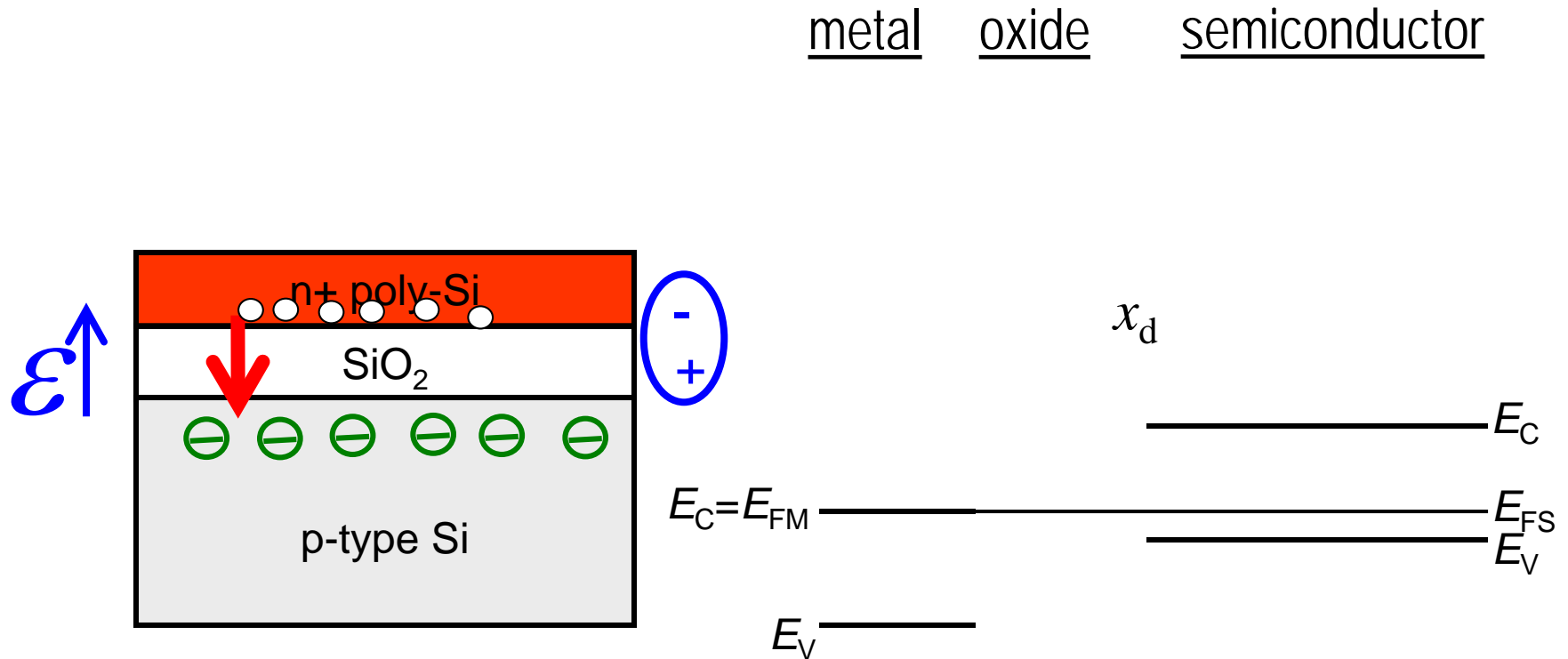
Poly-Si Depletion

metal oxide semiconductor



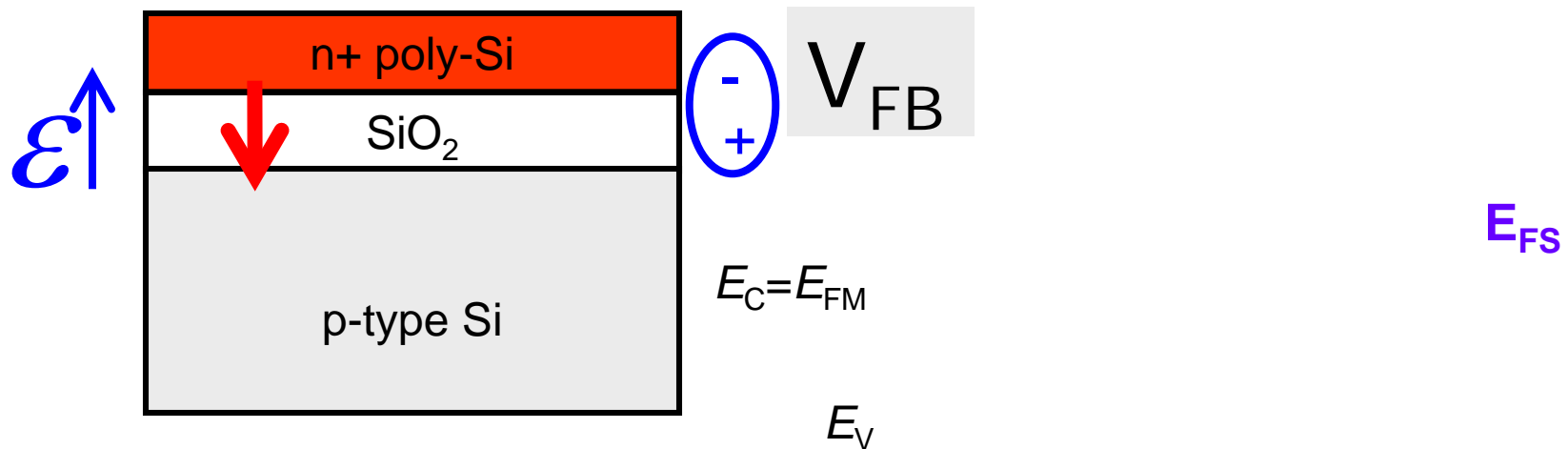
Band bending up is negligible because of heavy doping of poly-Si.

Flat-Band Voltage



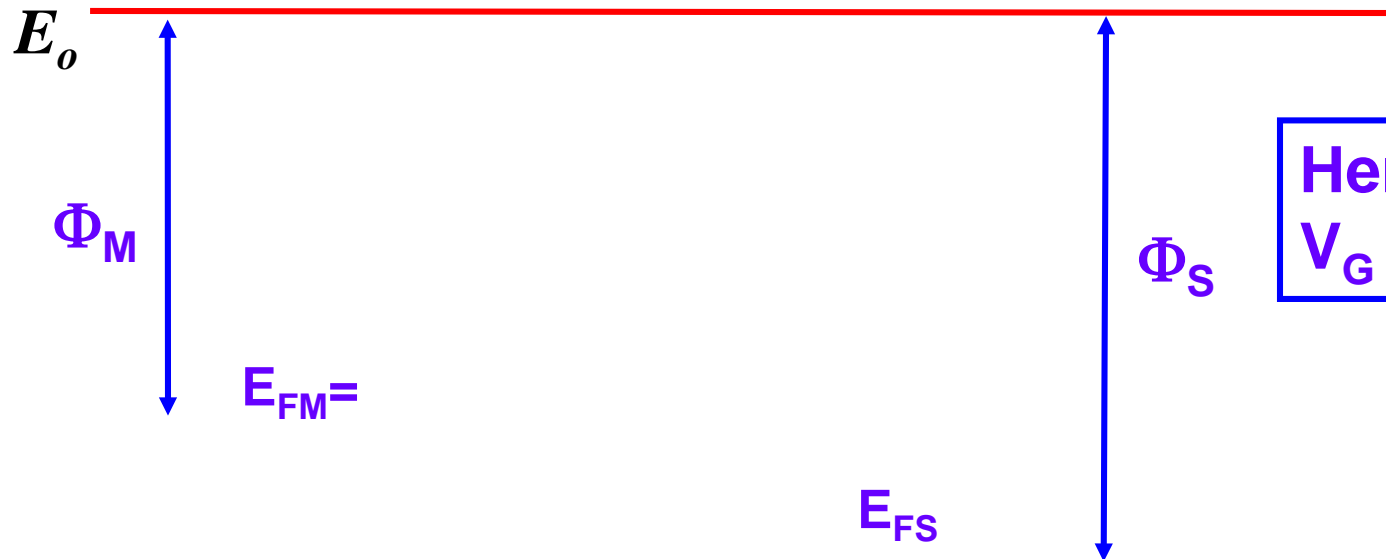
- The built-in potential can be “cancelled out” by applying a gate voltage that is equal in magnitude (but of the opposite polarity) as the built-in potential. This gate voltage is called the **flatband voltage** because the resulting potential profile is flat.

Flat-Band Voltage



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Flat-Band Condition



Here

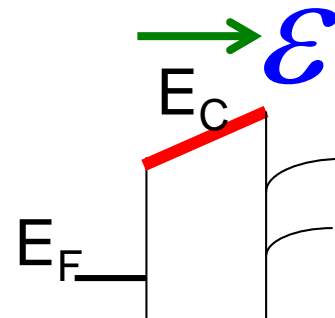
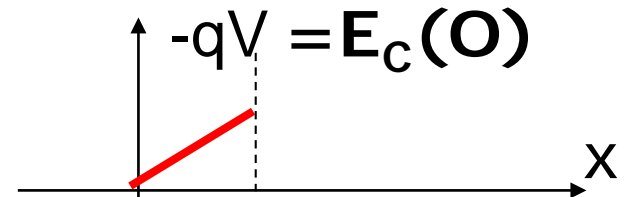
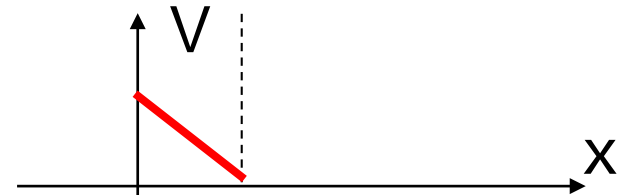
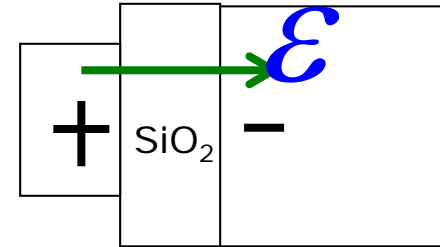
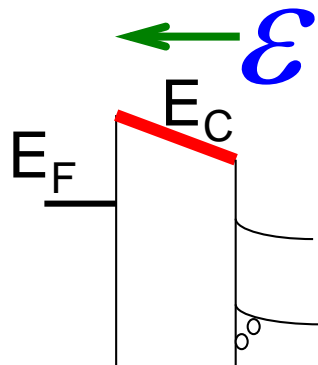
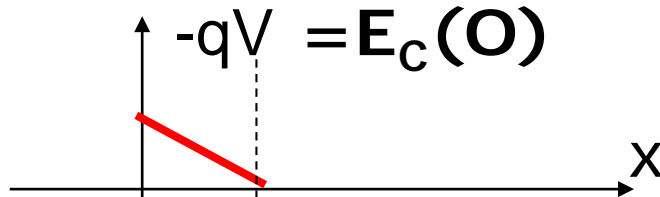
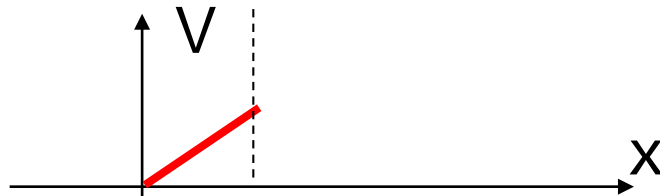
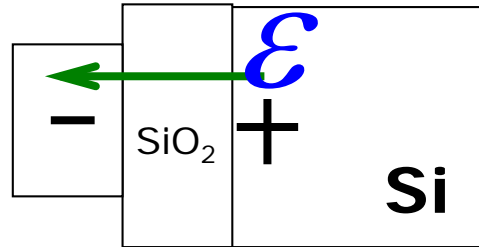
$$V_G = V_{FB} \approx -1V$$

$$\begin{aligned} qV_G &= E_{FS} - E_{FM} \\ &= \Phi_M - \Phi_S \end{aligned}$$

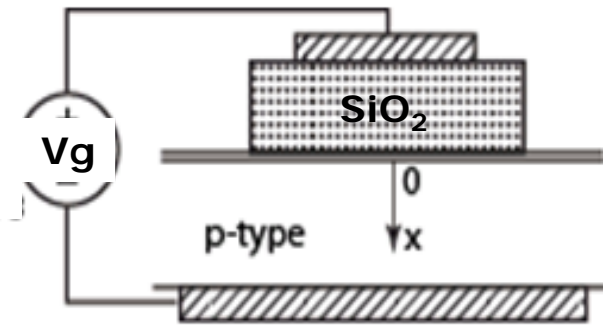
$$V_G = V_{FB}$$

$$qV_{FB} = \Phi_M - \Phi_S$$

$E_c(O)$ and **external** electric field direction



Effects of applied biases



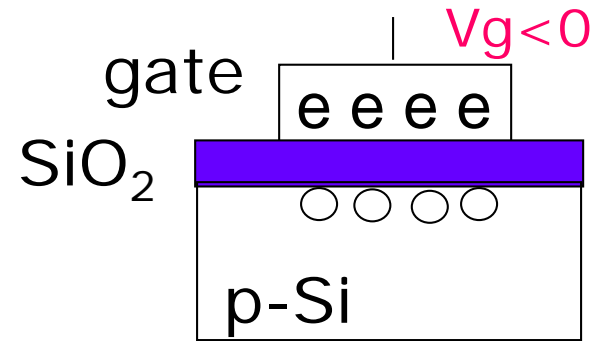
V_g increase from “-” to “+” :

1. $V_g < V_{FB} < 0$	Accumulation: Majority carriers
2. $V_g = V_{FB}$	Flatband
3. $V_m \geq V_g > V_{FB}$ including $V_g = 0$	Depletion: Majority carriers
4.1 $V_T > V_g > V_m$	} } Weak Inversion: Minority carriers
4.2 $V_g \geq V_T$	Strong Inversion: Minority carriers

What **are** V_{FB} , V_m and V_T ?

1. Accumulation (p-type Si) : $V_g < V_{FB} < 0$

Accumulation:
Majority carriers



- Physical process: $V_g < 0$: holes **attracted** to the oxide/Si interface and accumulate there.
- Separation between “-” and “+” charges: oxide

1. Accumulation:

$$qV_g = E_{FS} - E_{FM}$$

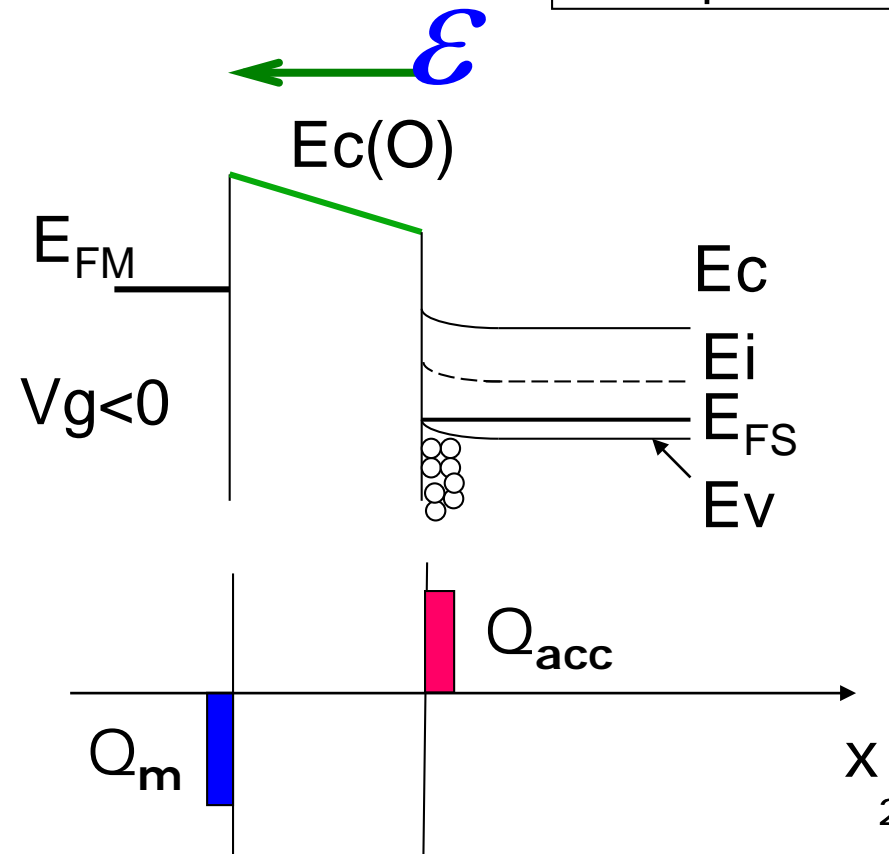
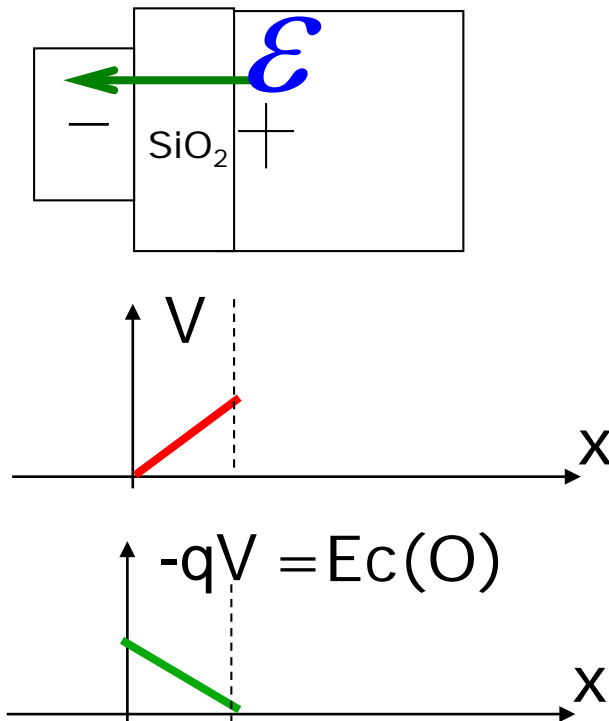
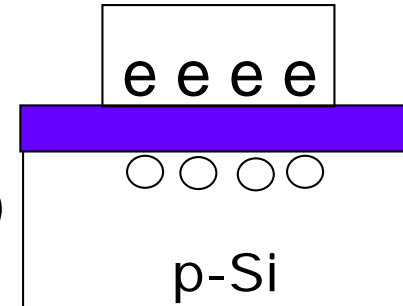
Energy band diagrams: $V_g < 0$

$V_g < 0$

Two diagrams:

block charge density diagram & energy band diagram

More negative: higher electron energy (band bending up)



1. Accumulation:

$$qV_g = E_{FS} - E_{FM}$$

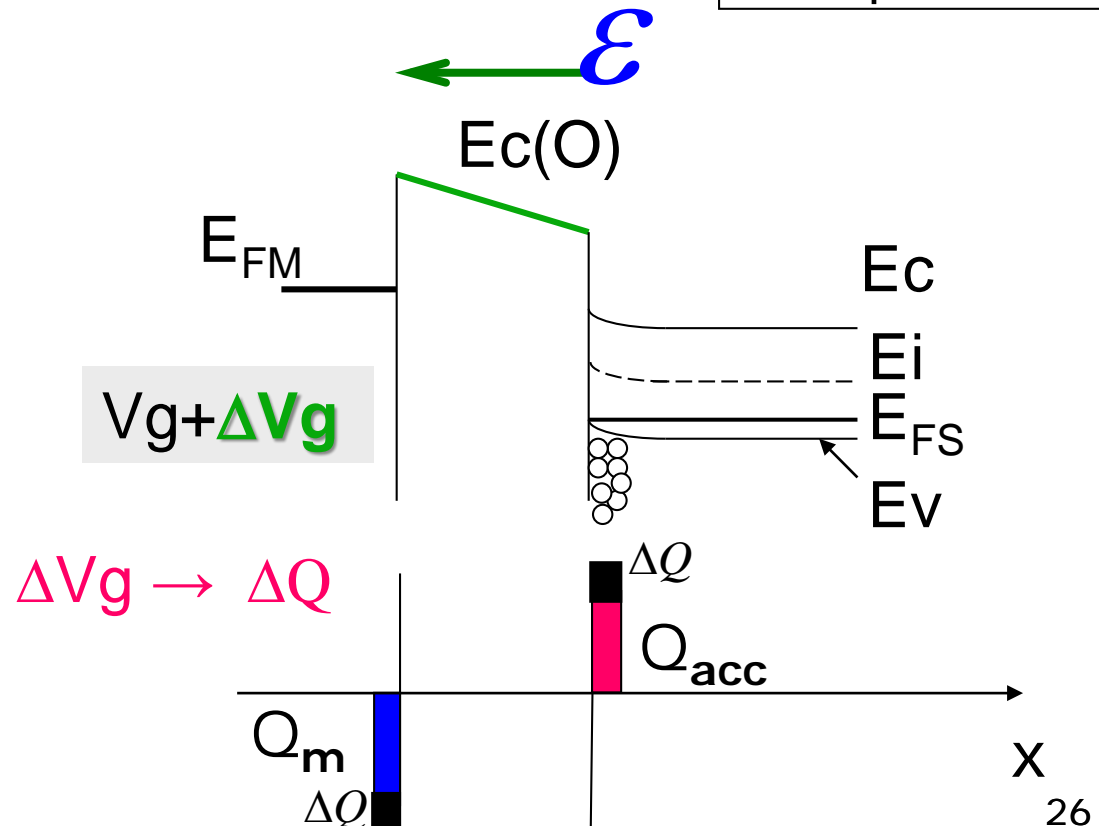
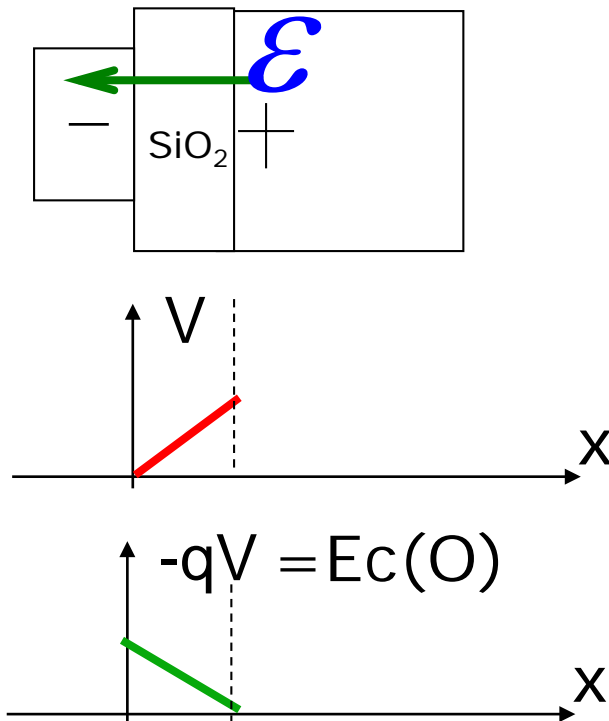
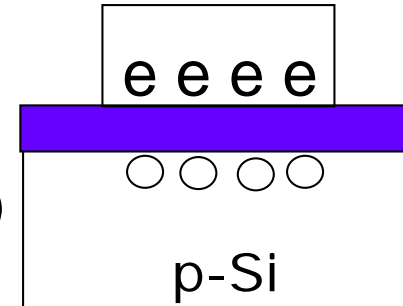
Energy band diagrams: $V_g < 0$

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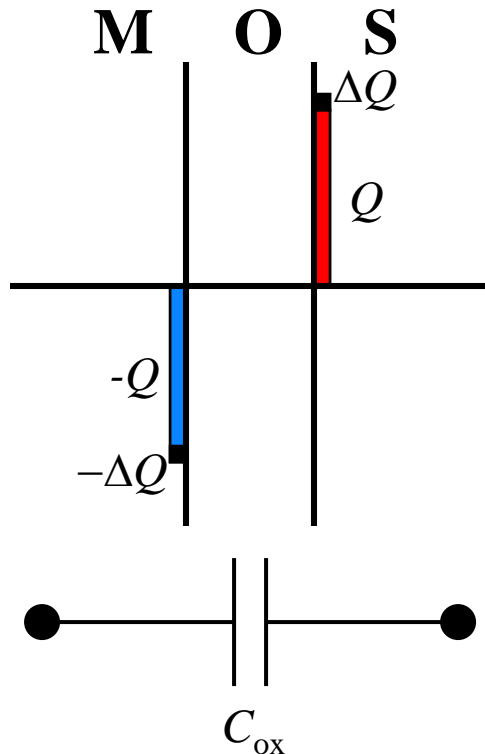
block charge density diagram & energy band diagram

More negative: higher electron energy (band bending up)



1. Capacitance in Accumulation

- As the gate voltage is varied, incremental charge is added/subtracted to/from the gate and substrate.
- The incremental charges are separated by the gate oxide.



$$C = \left| \frac{dQ_{acc}}{dV_g} \right| = C_{ox}$$

$$C = C_{ox} = \frac{\epsilon_{ox} A}{t_{ox}} = \text{constant}$$

$\epsilon_{r'ox} = 3.9$ is the relative dielectric constant of the oxide, $\epsilon_{ox} = \epsilon_{r'ox} \epsilon_0$, $\epsilon_0 = 8.85 \times 10^{-12}$ F/m is the permittivity of free space, t_{ox} is the oxide thickness.

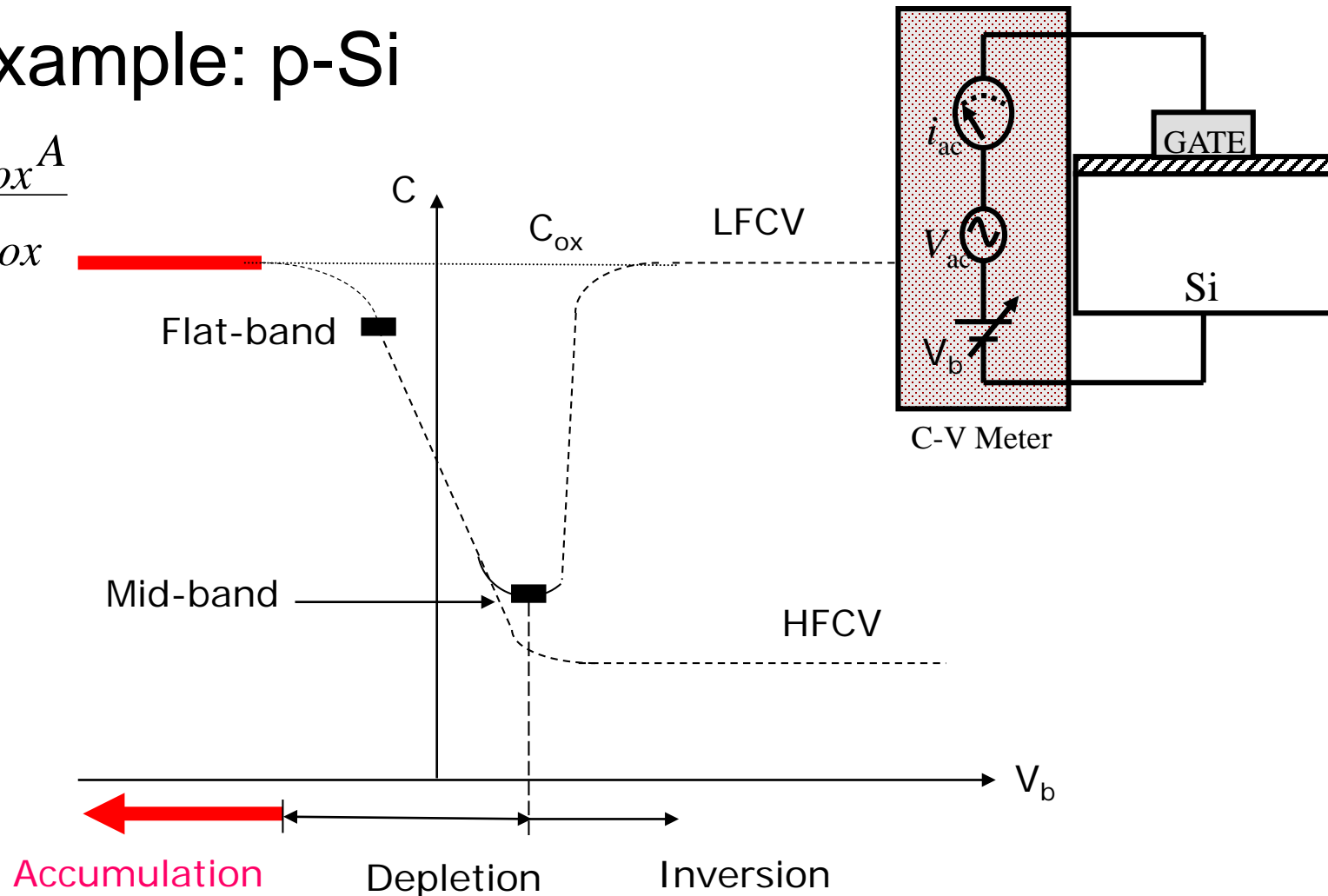
Normally, we consider the capacitance per unit area, so $A=1$

1. Accumulation:

Capacitance-voltage (C-V) characteristics

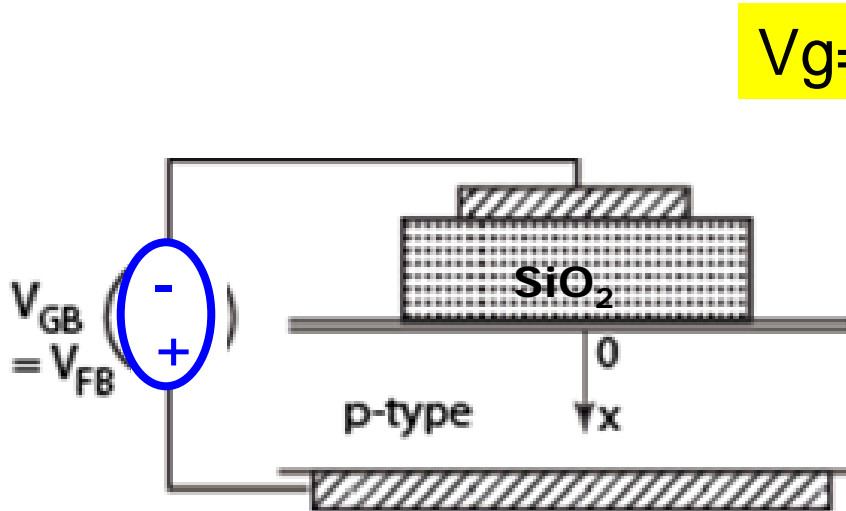
- Example: p-Si

$$C = C_{ox} = \frac{\epsilon_{ox} A}{t_{ox}}$$

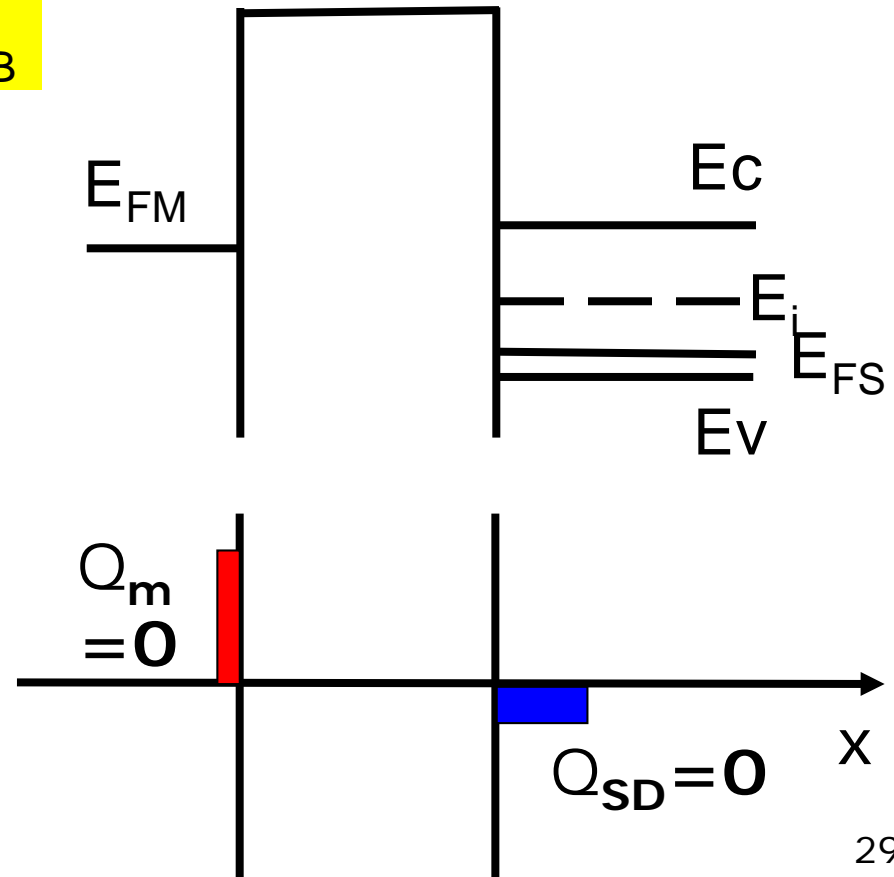


2. Flatband Voltage, V_{FB}

- The built-in potential can be “cancelled out” by applying a gate voltage that is equal in magnitude (but of the opposite polarity) as the built-in potential. This gate voltage is called the **flatband voltage** because the resulting potential profile is flat.



$$V_g = V_{FB}$$



There is no net charge (*i.e.* $\rho(x)=0$) in the semiconductor under $V_{GB} = V_{FB}$.

3. Depletion: $V_g > V_{FB}$

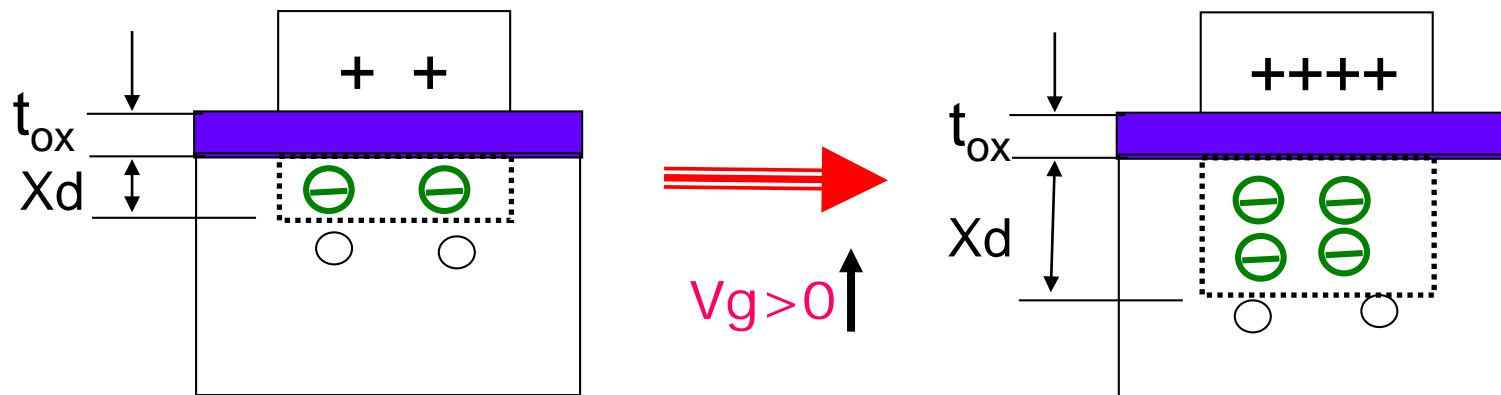
Depletion:
Majority carriers

- Physical process:

- holes **repelled** from the interface
- fixed negative charge left behind
- More “+” charges on the gate, holes are pushed further from the interface, to expose more “-” space charges.

○: Hole

⊖: B⁻

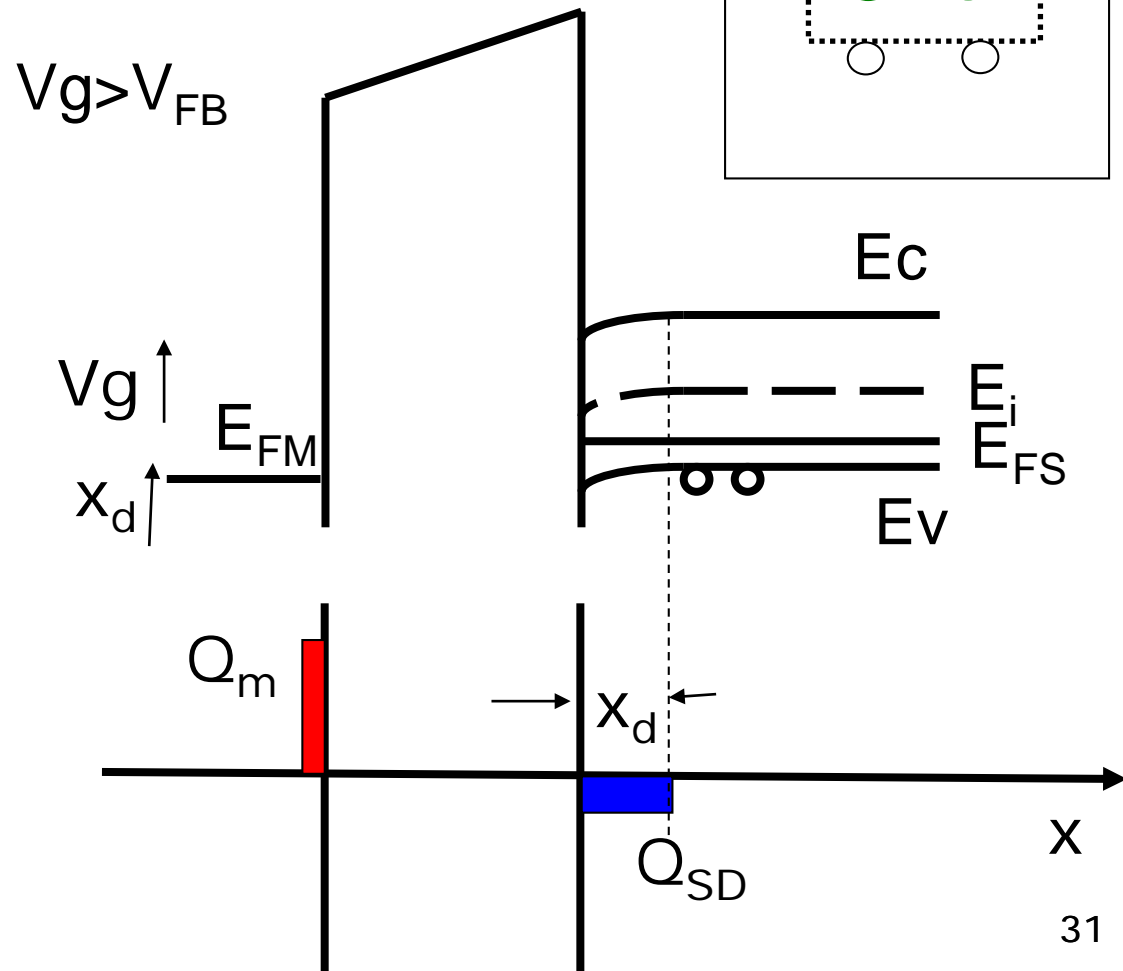
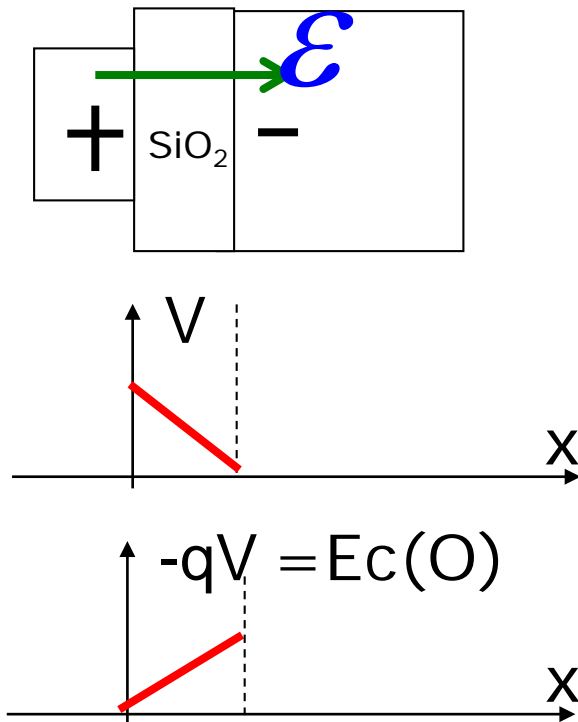


$$qV_g = E_{FS} - E_{FM}$$

3. Depletion:

Energy band diagram: $V_g > V_{FB}$

- The band is bent downward now.
- No mobile charges at the interface.

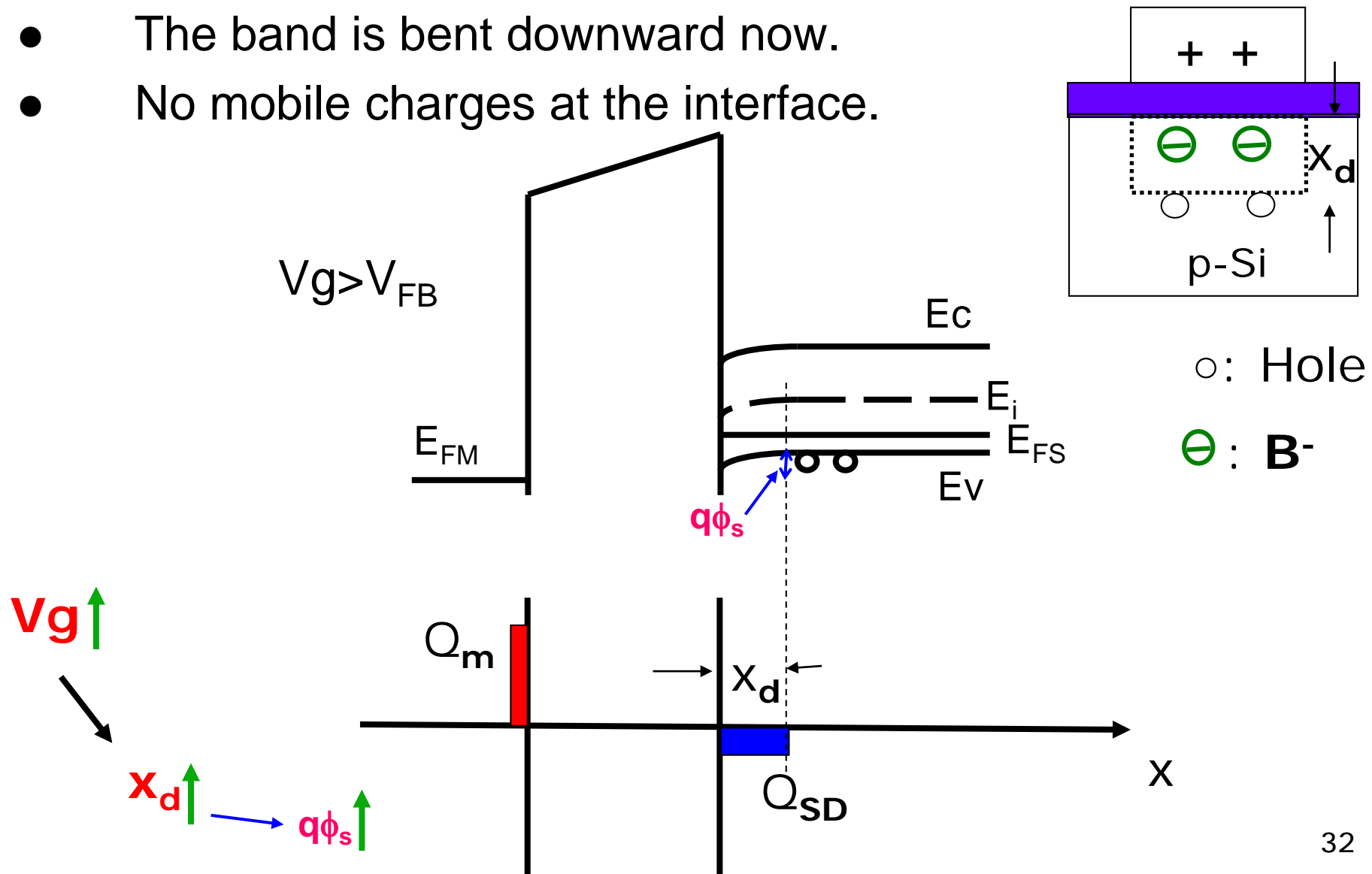


$$qV_g = E_{FS} - E_{FM}$$

3. Depletion:

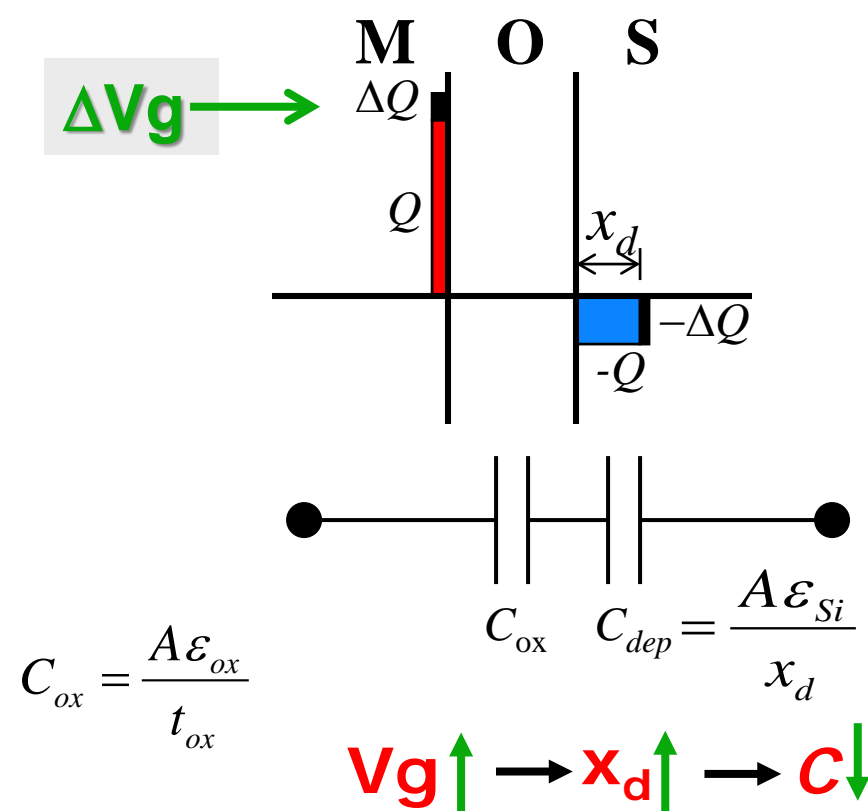
Energy band diagram: $V_g > V_{FB}$

- The band is bent downward now.
- No mobile charges at the interface.



3. Capacitance in Depletion

- As the gate voltage is varied, the width of the depletion region varies.
- Incremental charge is effectively added/subtracted at a depth x_d in the substrate.



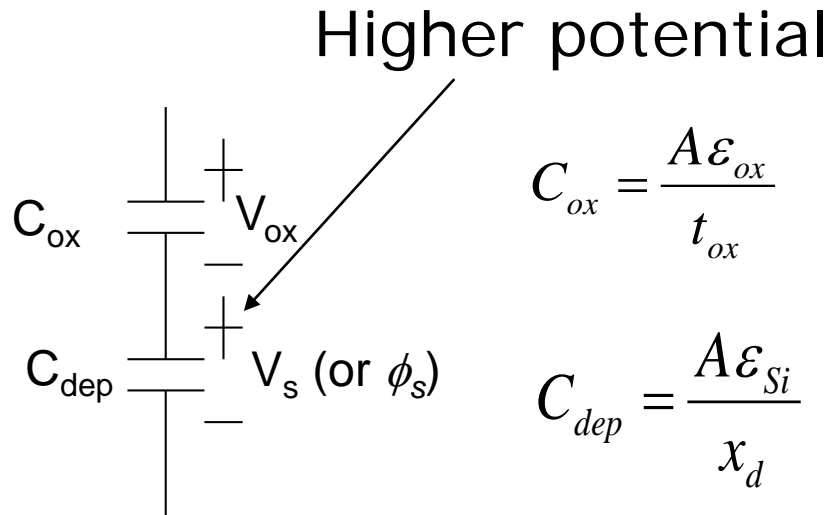
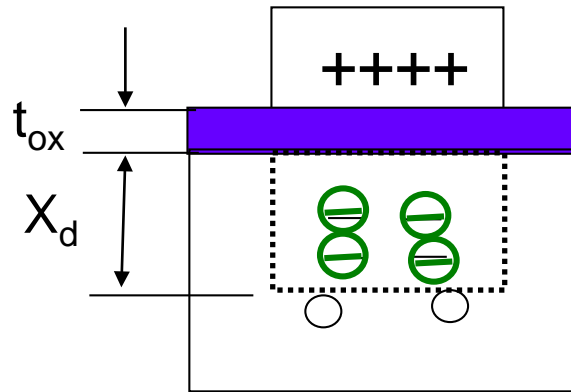
$$C = \left| \frac{dQ}{dV_G} \right|$$

$$\frac{1}{C} = \frac{1}{C_{ox}} + \frac{1}{C_{dep}} = \frac{1}{C_{ox}} + \frac{x_d}{A\epsilon_{Si}}$$

$\epsilon_{r'Si} = 11.9$ is the relative dielectric constant of silicon,
 $\epsilon_{Si} = \epsilon_{r'Si}\epsilon_0$.

3. Depletion Capacitance

- Capacitance



- When V_g increases, X_d increases and C_{dep} reduces. This in turn reduces C .
- Solving Poisson's equation, we have

$$x_d = \left(\frac{2\epsilon_{Si}V_s}{qN_a} \right)^{1/2}$$

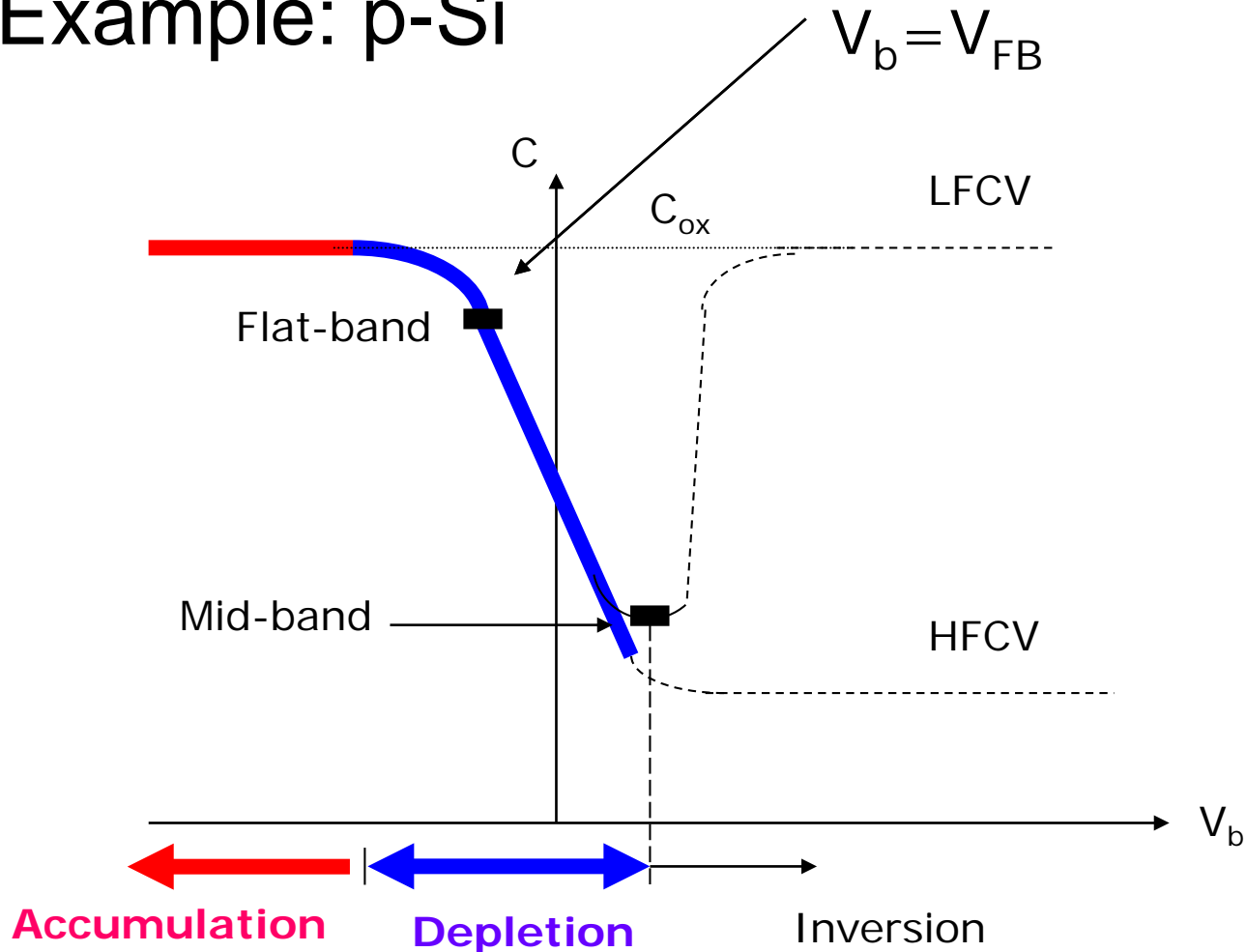
$$V_g \uparrow \rightarrow x_d \uparrow \rightarrow C \downarrow$$

$$\frac{1}{C} = \frac{1}{C_{ox}} + \frac{1}{C_{dep}} = \frac{1}{C_{ox}} + \frac{x_d}{A\epsilon_{Si}}$$

3. Depletion:

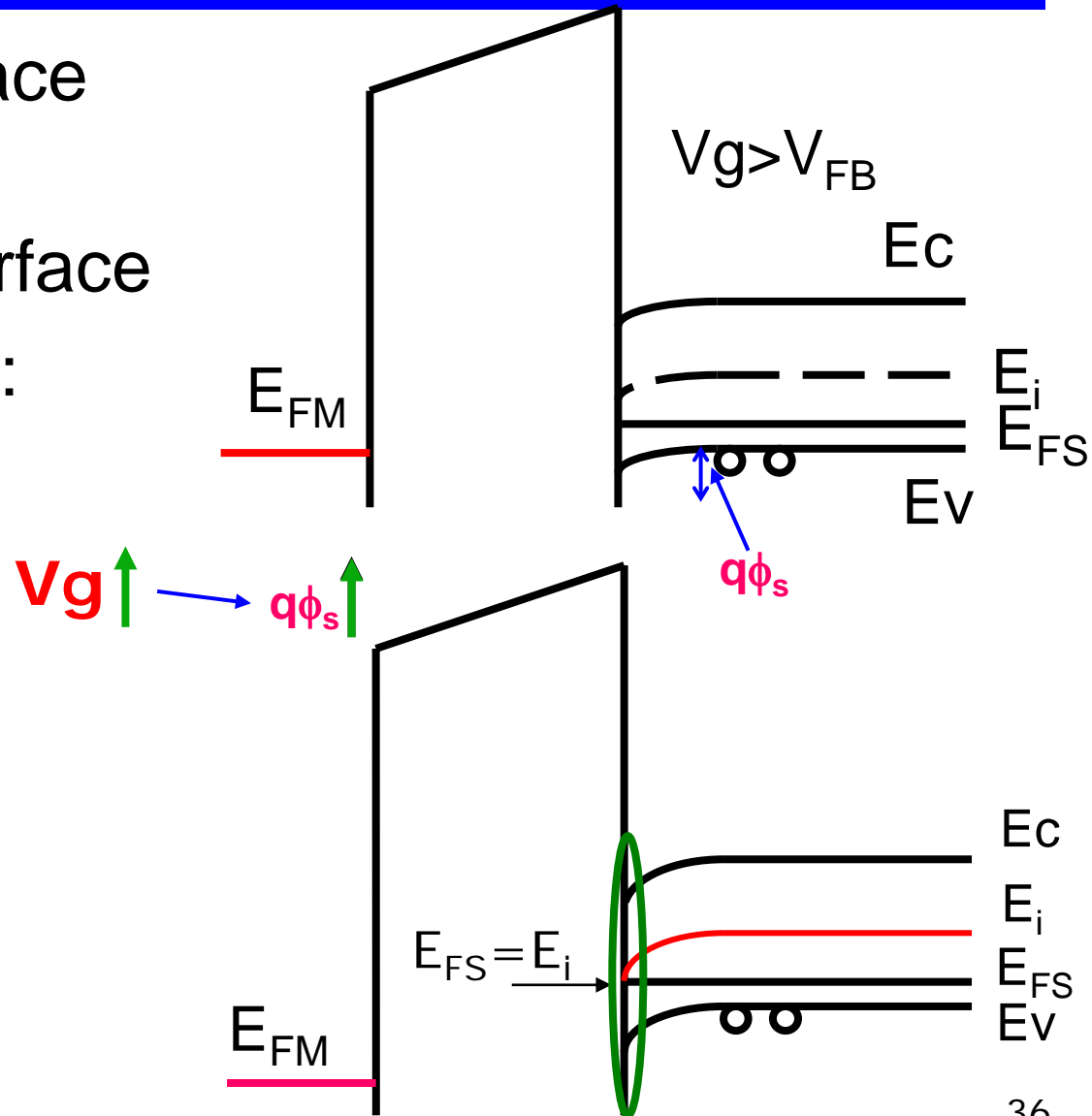
Capacitance-voltage (C-V) characteristics

- Example: p-Si



Midband: further increase V_g

- $E_{FS} = E_i$ at interface
- Silicon becomes **“intrinsic”** at surface
- This is ‘Midband’:
 $V_g = V_m$

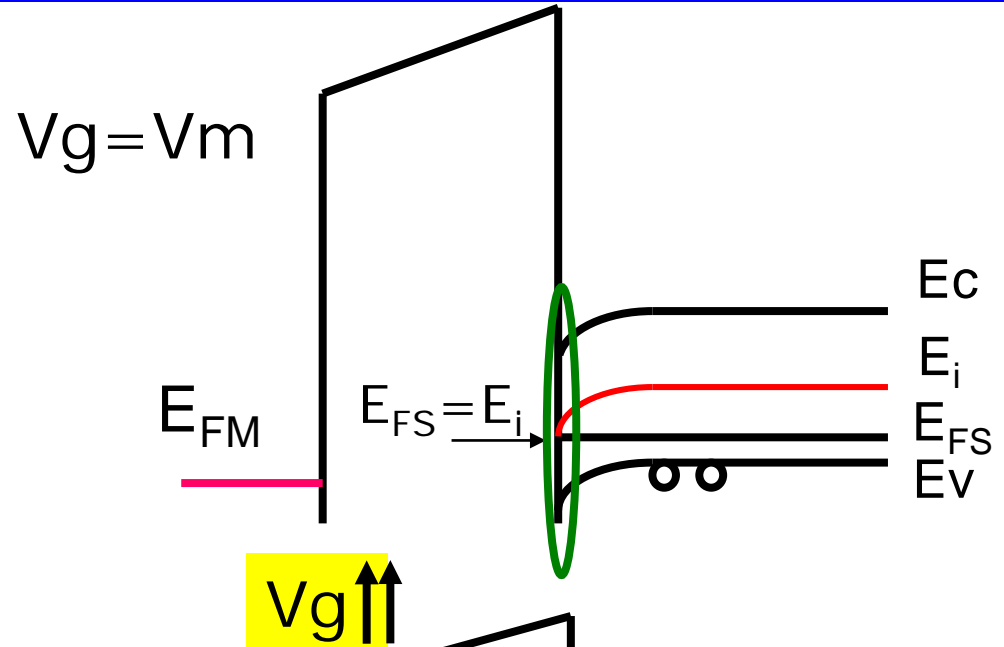


At the interface

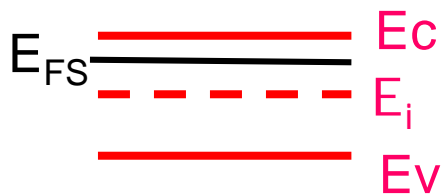


Further increase V_g

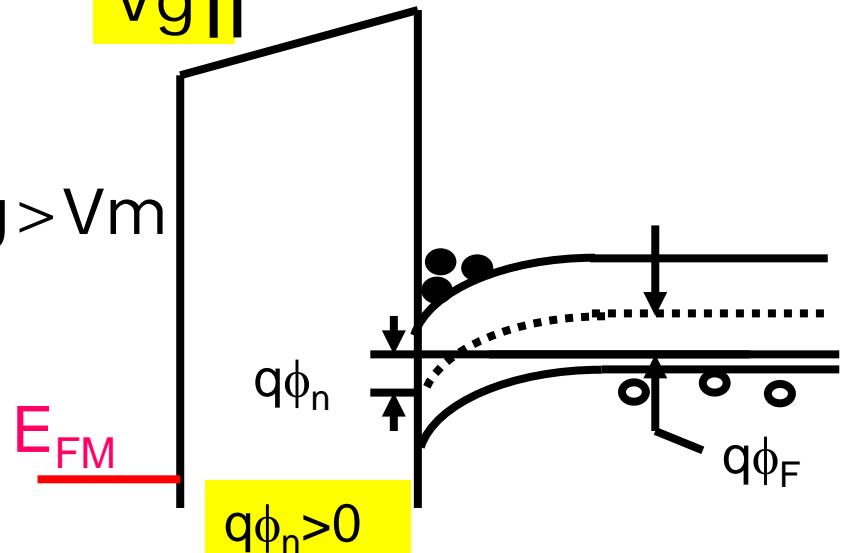
At the interface



At the interface



$V_g > V_m$

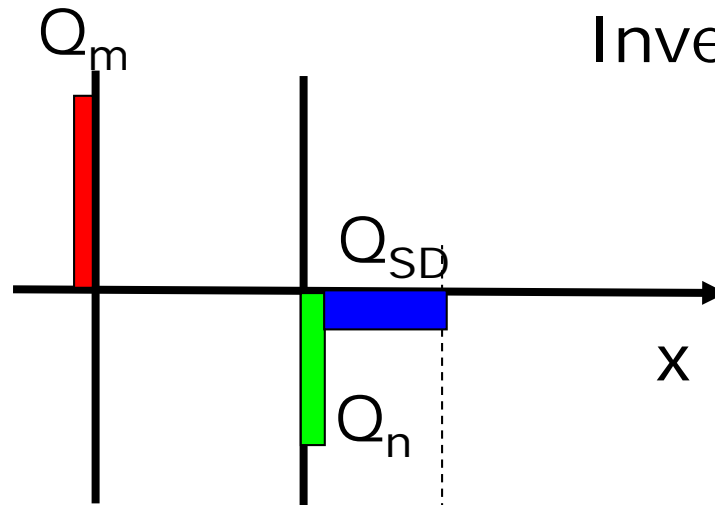


$$qV_g = E_{FS} - E_{FM}$$

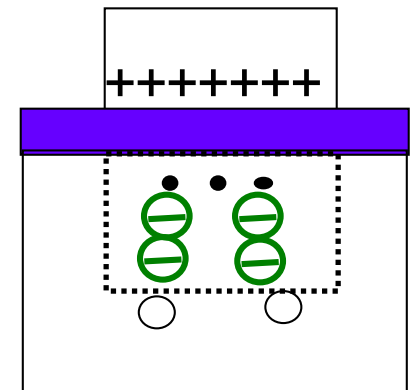
4. Energy band diagram: $V_g > V_m$

$V_g \uparrow \uparrow$

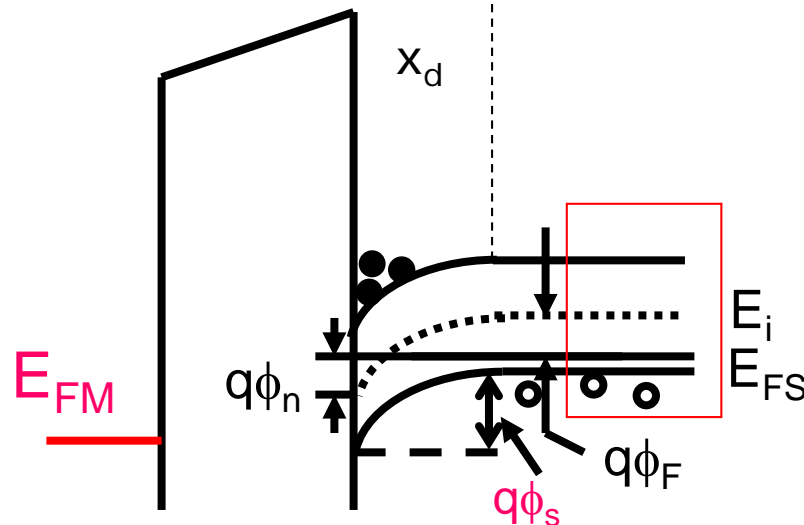
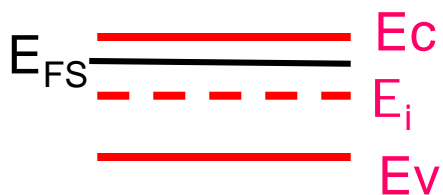
As $V_g > V_m$, E_{FS} is at the up-half of the bandgap



Inversion: **Minority carriers**



At the interface



surface potential

$$\phi_s = \phi_F + \phi_n$$

$$qV_g = E_{FS} - E_{FM}$$

4. Energy band diagram: $V_g > V_m$

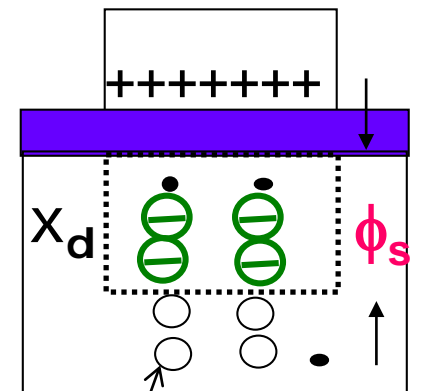
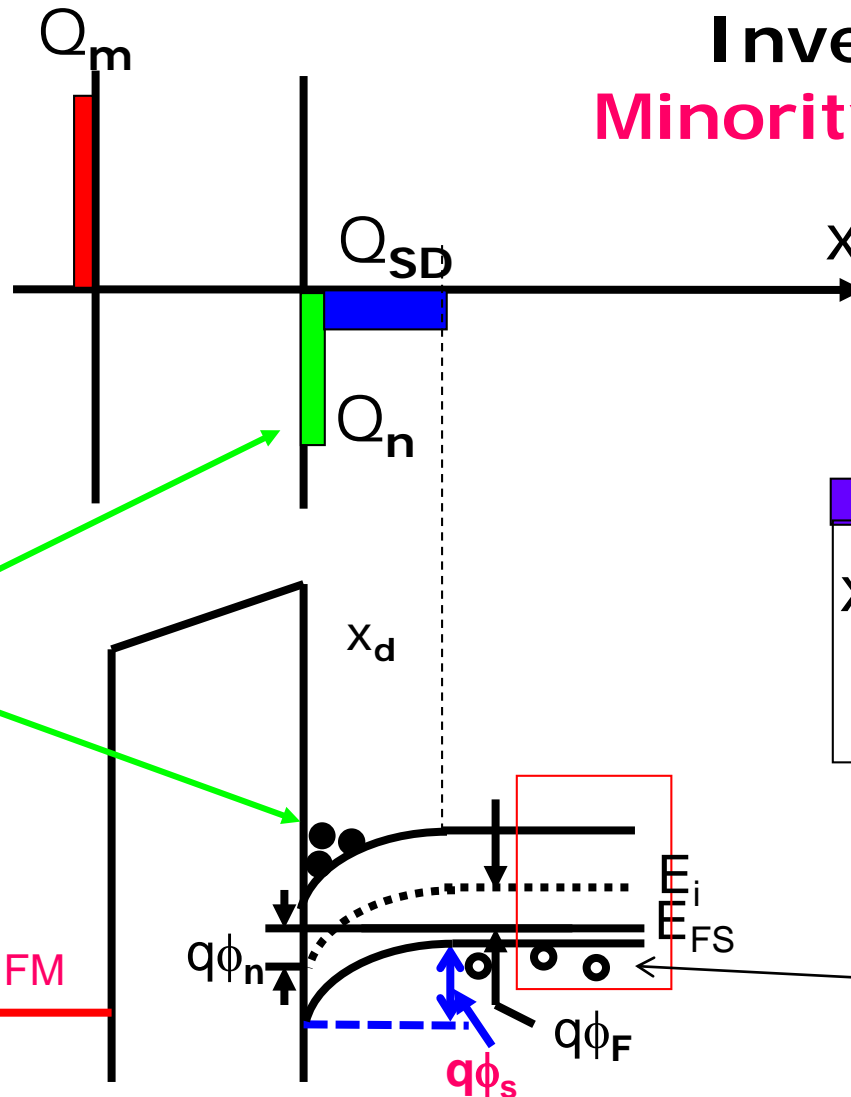
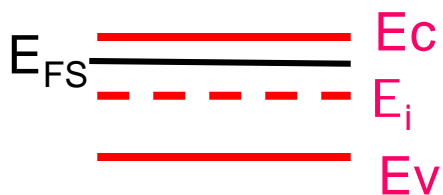
$$V_g \uparrow \uparrow \rightarrow q\phi_s \uparrow \uparrow$$

As $V_g > V_m$, E_{FS} is at the up-half of the bandgap.

$$n = n_i \exp\left[\frac{(E_F - E_i)}{kT}\right]$$

$$\Rightarrow n_s = n_i \exp\left[\frac{q\phi_n}{kT}\right]$$

At the interface



$$np = n_i^2$$

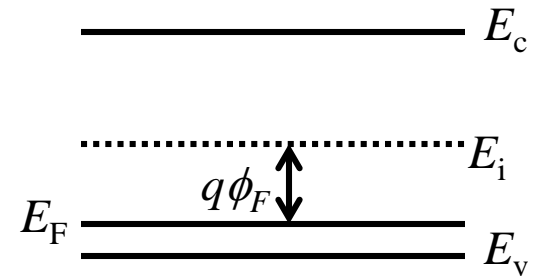
$$p = n_i \exp\left[\frac{q\phi_F}{kT}\right]$$

Bulk Semiconductor Potential, ϕ_F

$$q\phi_F \equiv E_i - E_F$$

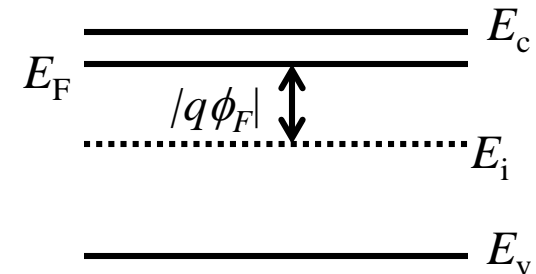
- p-type Si:

$$\phi_F = \frac{kT}{q} \ln(N_A / n_i) > 0$$



- n-type Si:

$$\phi_F = -\frac{kT}{q} \ln(N_D / n_i) < 0$$

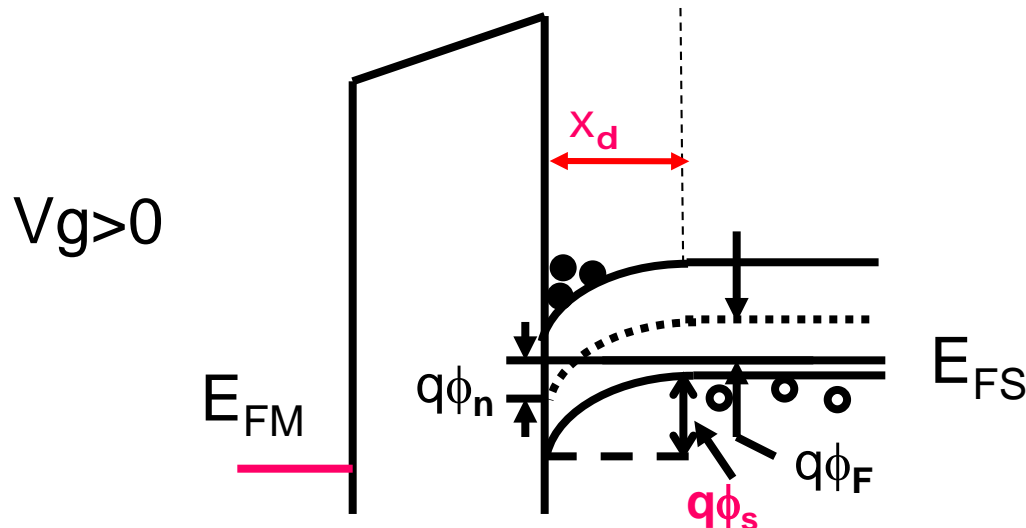


$$\text{or } q\phi_F \equiv -(E_i - E_F)$$

4. Inversion

$$p = n_i \exp\left[\frac{q\phi_F}{kT}\right]; \quad n_s = n_i \exp\left[\frac{q\phi_n}{kT}\right]$$

- Physical process and band diagram
 - As $V_g > V_m$, E_{FS} is at the up-half of the bandgap.
 - Si near interface becomes n-type.
 - Many electrons (now majority carriers).
 - Weak Inversion: $0 < \phi_n < \phi_F$ ($\phi_F < \phi_s < 2\phi_F$)
 - Strong Inversion: $\phi_n \geq \phi_F$ ($\phi_s \geq 2\phi_F$), electron density at the interface \geq hole density in Si bulk.
 - V_g for strong inversion: $\geq V_T$ 'threshold voltage'.



$$\phi_s = \phi_F + \phi_n$$

$$V_g = V_m, \phi_n = 0$$

$$V_g = V_T, \phi_n = \phi_F \\ \rightarrow \phi_s = 2\phi_F$$

$$\phi_s = \phi_F + \phi_n$$

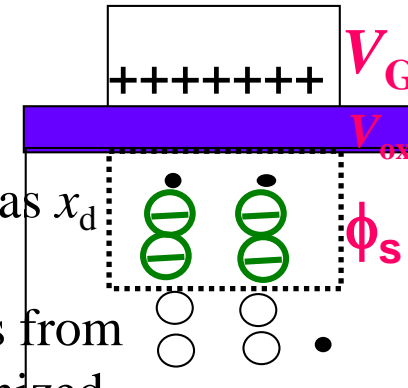
Maximum Depletion Depth, $x_{d,max}$

- As V_G is increased above V_T , ϕ_s and hence the depth of the depletion region (x_d) increases very slowly.

→

$$\phi_s \approx 2\phi_F$$

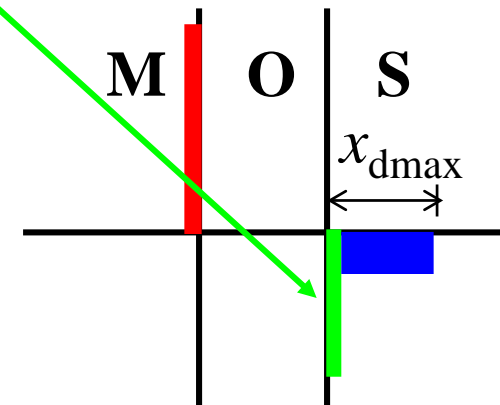
- This is because n increases exponentially with ϕ_s , whereas x_d increases with the square root of ϕ_s . Thus, most of the incremental negative charge in the semiconductor comes from additional conduction electrons rather than additional ionized acceptor atoms, when n exceeds N_A .



$$n_s = n_i \exp\left[\frac{q\phi_n}{kT}\right]; \quad \phi_n = \phi_s - \phi_F$$

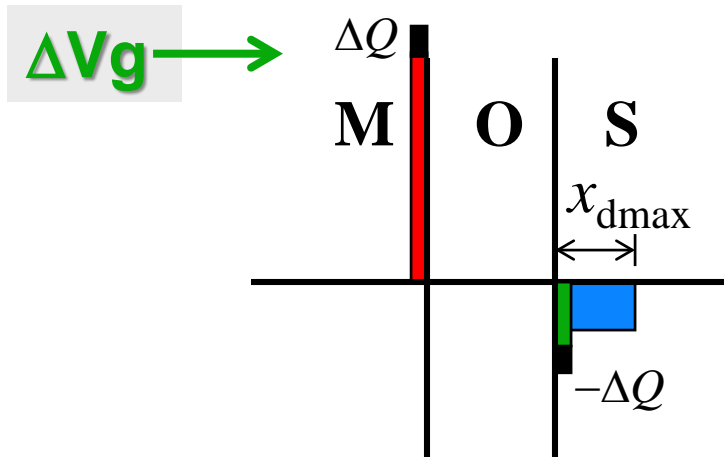
- x_d can be reasonably approximated to reach a maximum value ($x_{d,max}$) for $V_G \geq V_T$.

$$x_{d,max} = \sqrt{\frac{2\epsilon_{Si}\phi_s}{qN_A}} \approx \sqrt{\frac{2\epsilon_{Si}(2\phi_F)}{qN_A}}$$

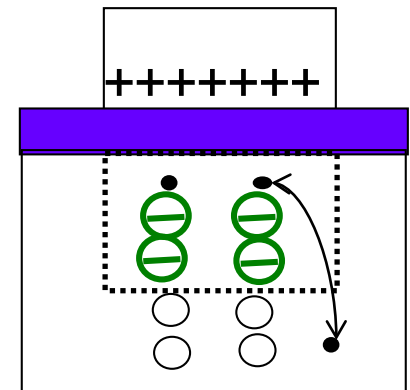
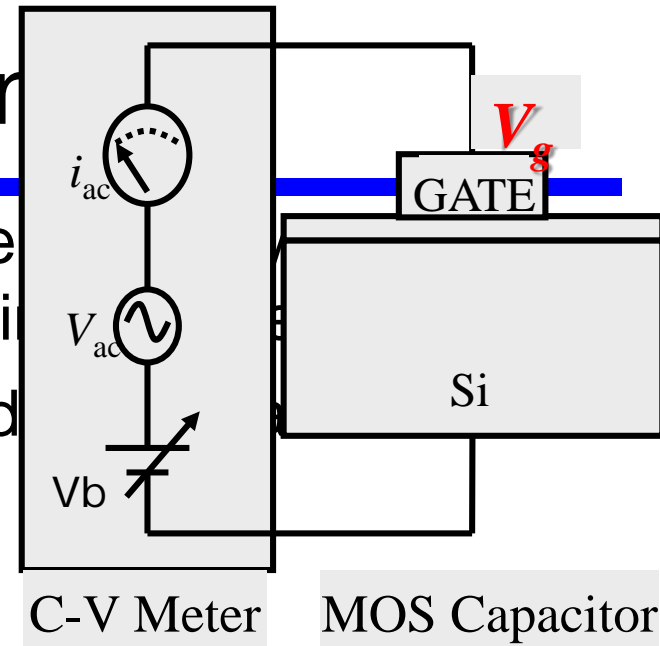


4. Capacitance in Inversion

CASE 1: Inversion-layer charge *can* be quickly enough to respond to changes in V_G
 → Incremental charge is effectively added to surface of the substrate.



$$C = \left| \frac{dQ_{inv}}{dV_G} \right| = C_{ox}$$

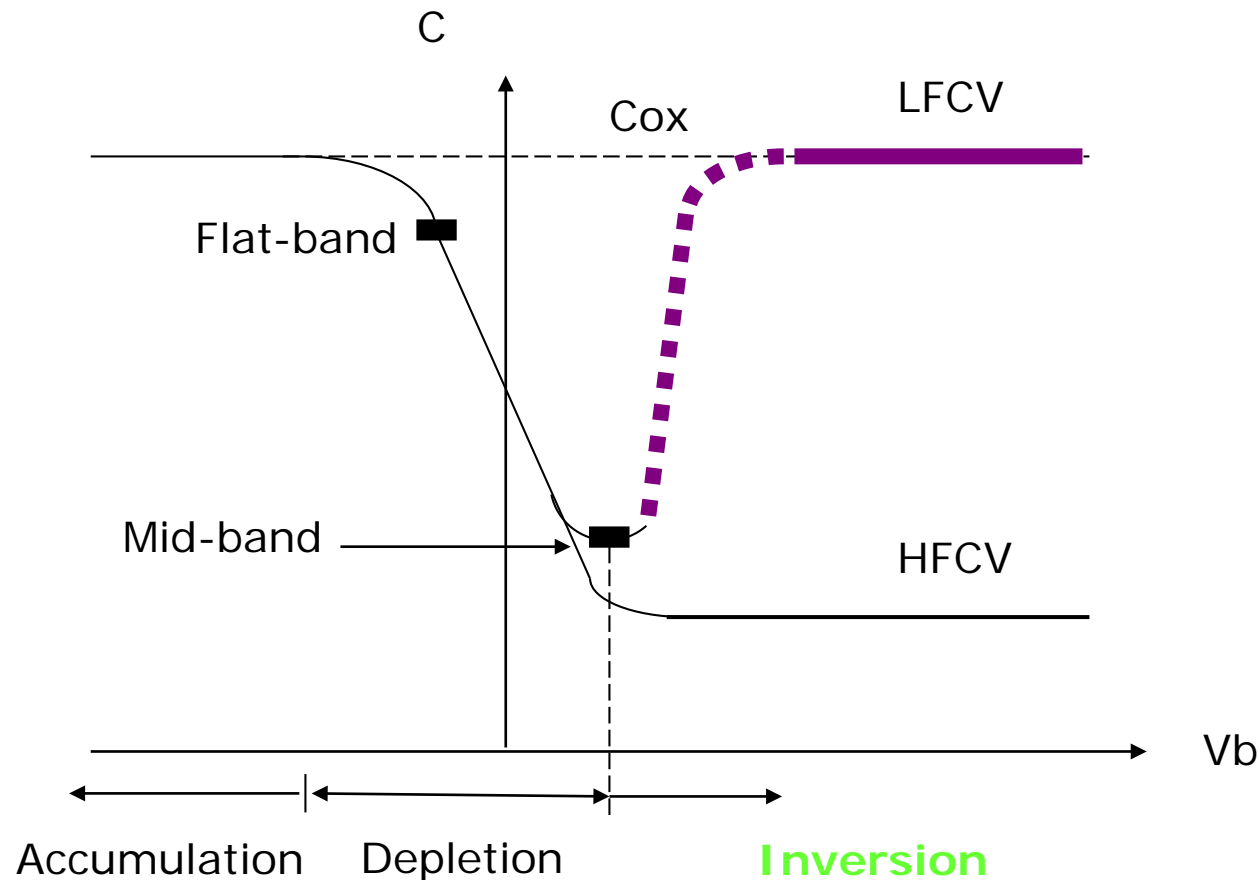


Electrons can respond to the change in V_{ac}

4. Inversion:

CASE 1: Capacitance-voltage (C-V) characteristics

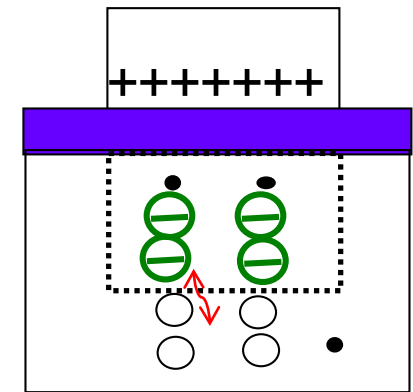
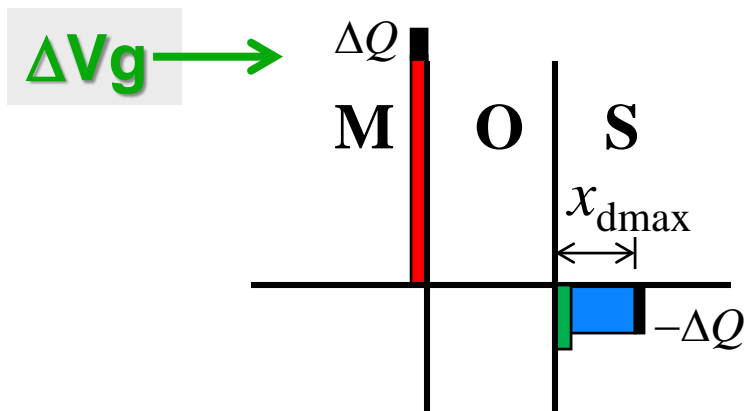
- Example: p-Si



4. Capacitance in Inversion: HF

CASE 2: Inversion-layer charge *cannot* be supplied/removed quickly enough to respond to changes in the gate voltage.

→ Incremental charge is effectively added/subtracted at a depth x_{dmax} in the substrate.

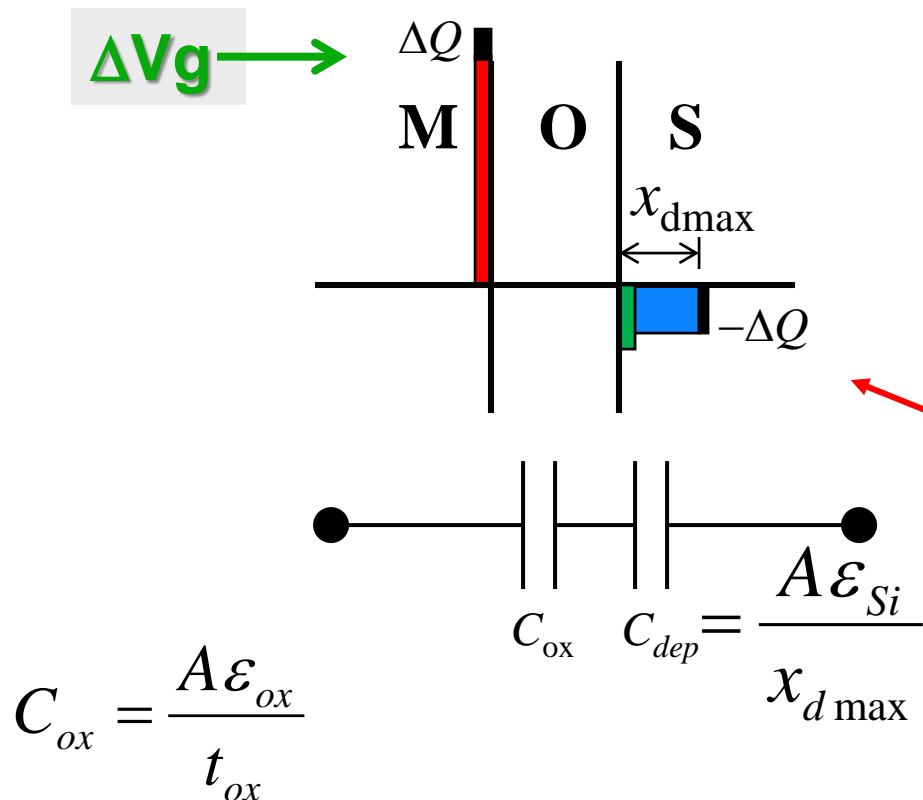


1. When V_{ac} changes rapidly (e.g., 1MHz), electron creation cannot keep up.
2. Negative charges supplied by pushing holes away.

4. Capacitance in Inversion: HF

CASE 2: Inversion-layer charge *cannot* be supplied/removed quickly enough to respond to changes in the gate voltage.

→ Incremental charge is effectively added/subtracted at a depth x_{dmax} in the substrate.



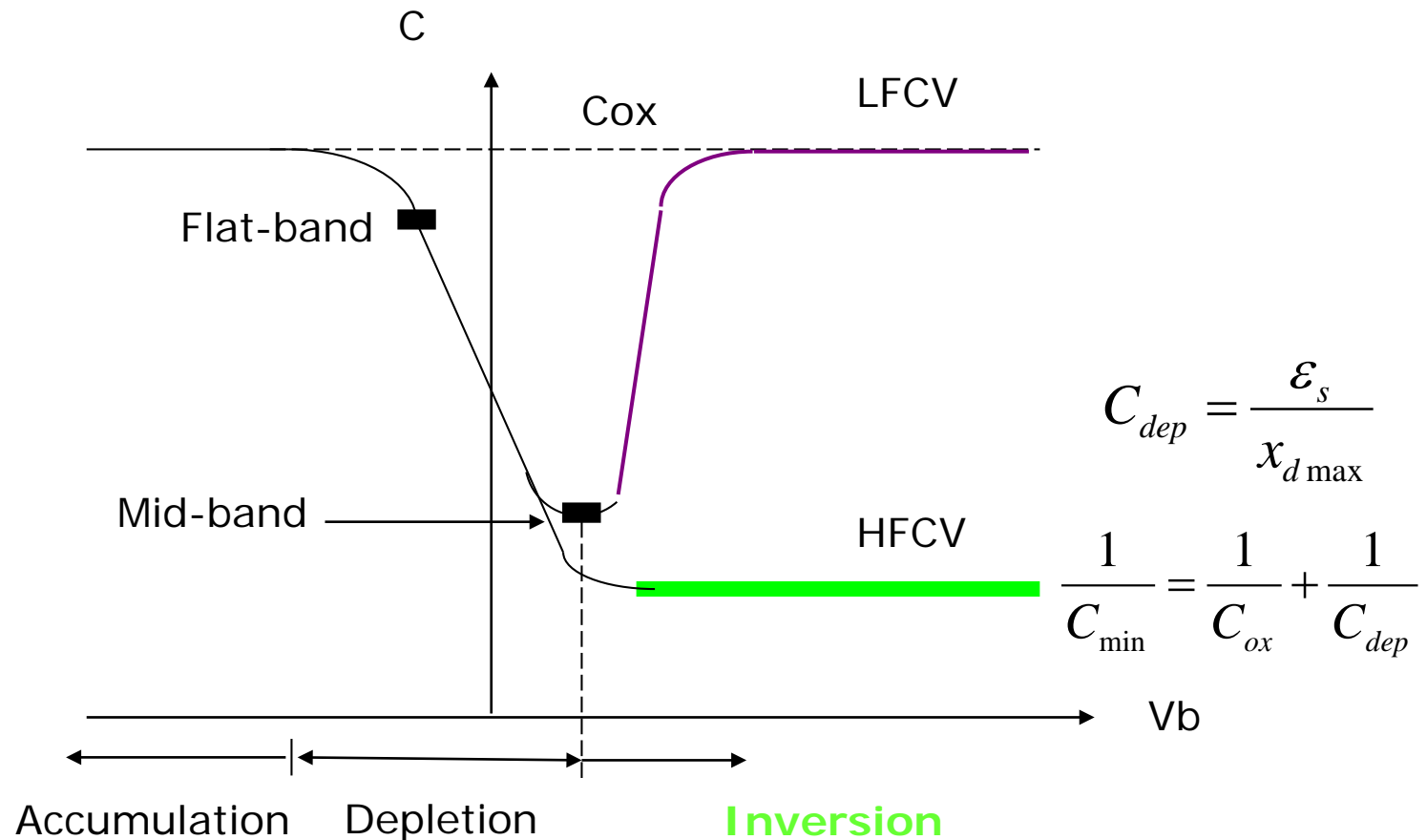
$$\begin{aligned}\frac{1}{C} &= \frac{1}{C_{ox}} + \frac{1}{C_{dep}} \\ &= \frac{1}{C_{ox}} + \frac{x_{dmax}}{A\epsilon_{Si}} \equiv \frac{1}{C_{min}}\end{aligned}$$

1. When V_{ac} changes rapidly (e.g., 1MHz), electron creation cannot keep up.
2. Negative charges supplied by pushing holes away.

4. Inversion

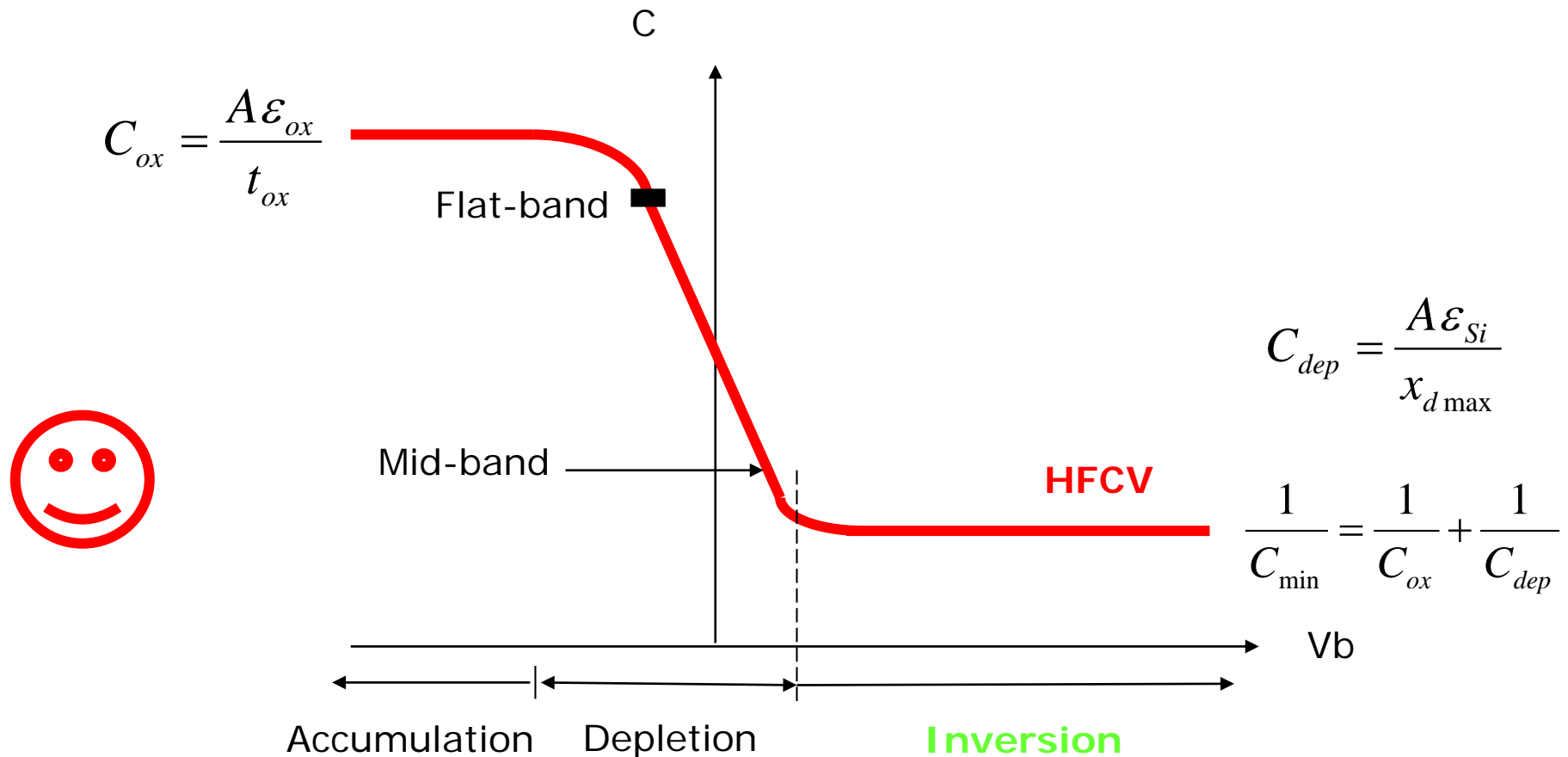
CASE 2: Capacitance-voltage (C-V) characteristics

- Example: p-Si



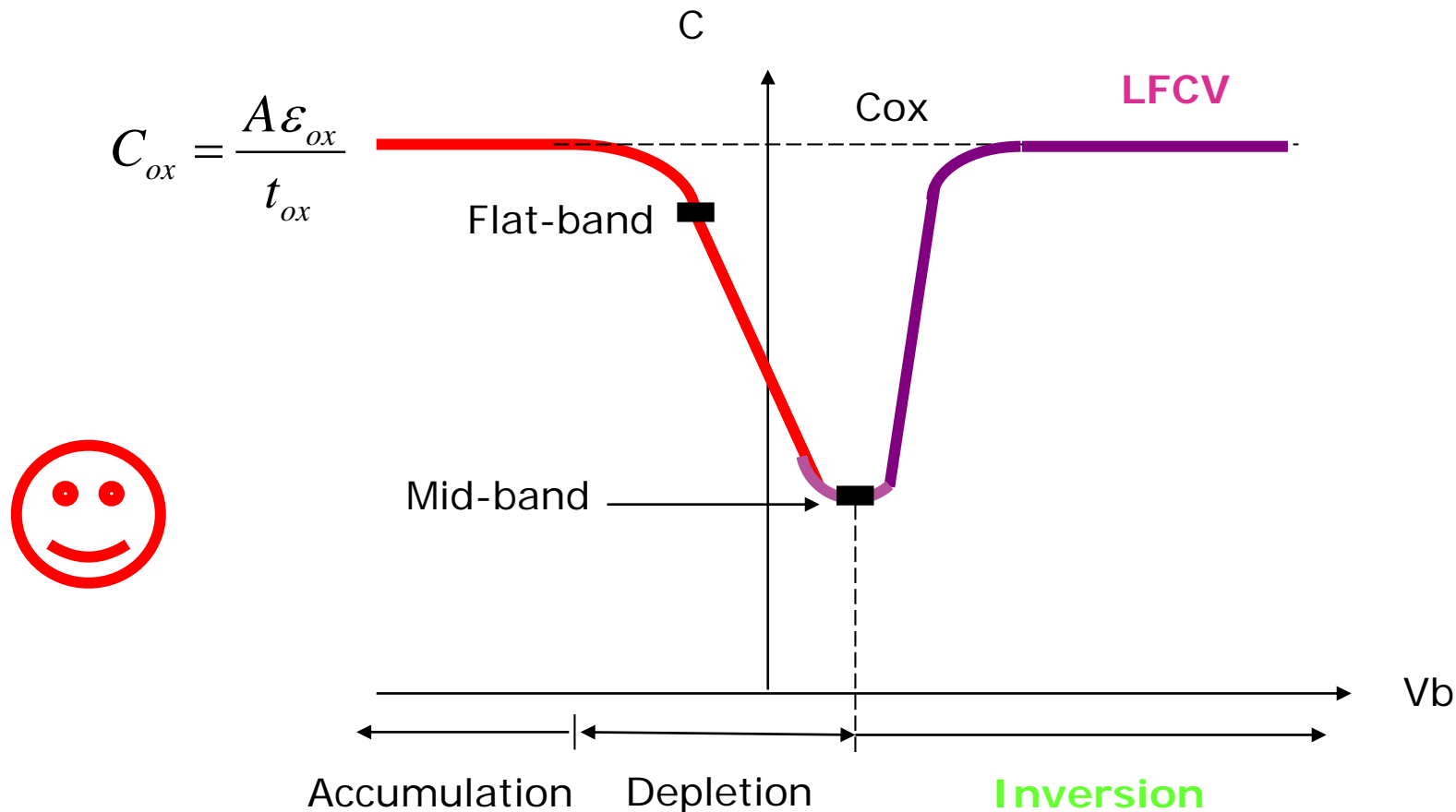
Capacitance-voltage (C-V) characteristics

- Example: p-Si



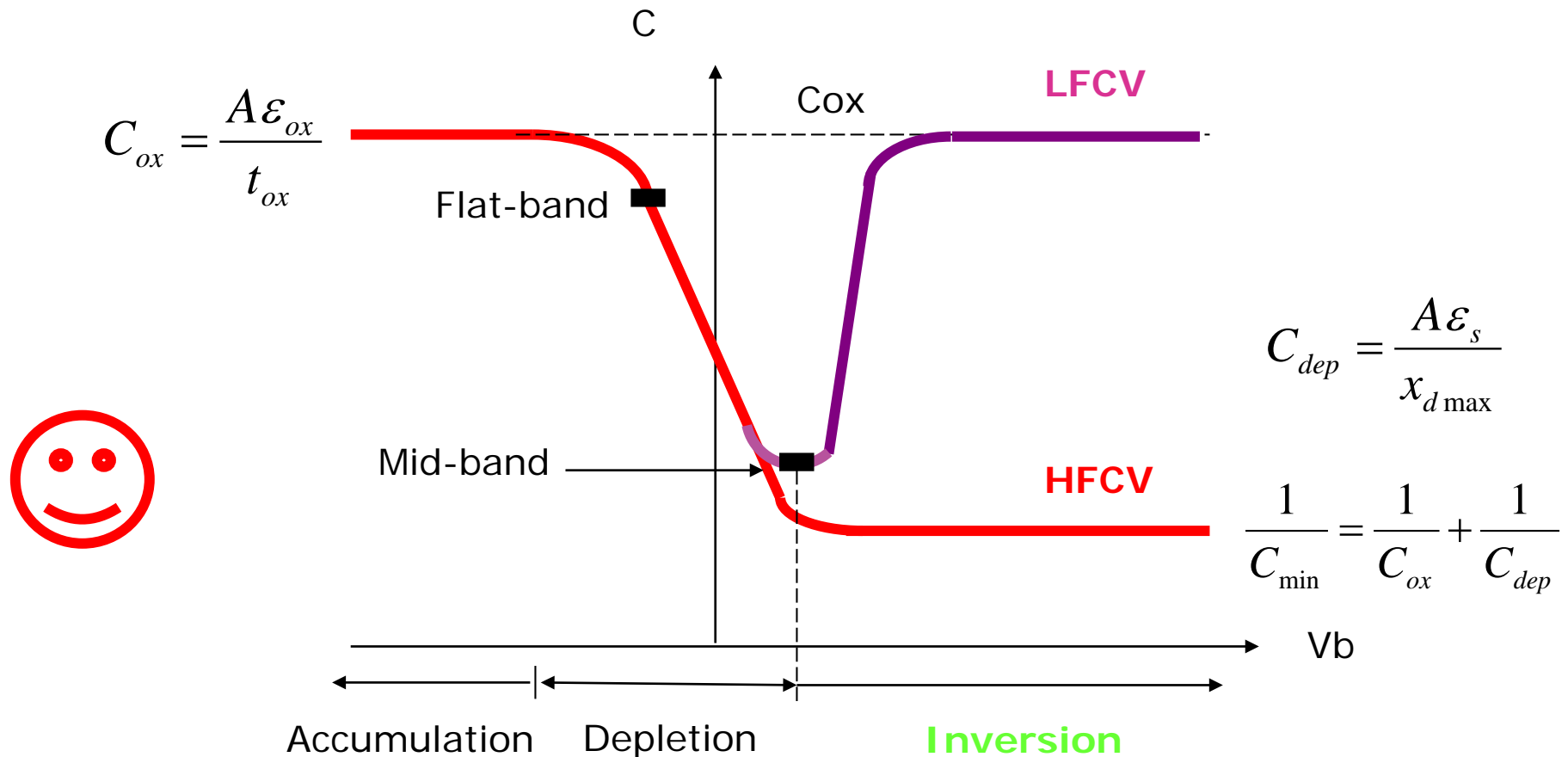
Capacitance-voltage (C-V) characteristics

- Example: p-Si



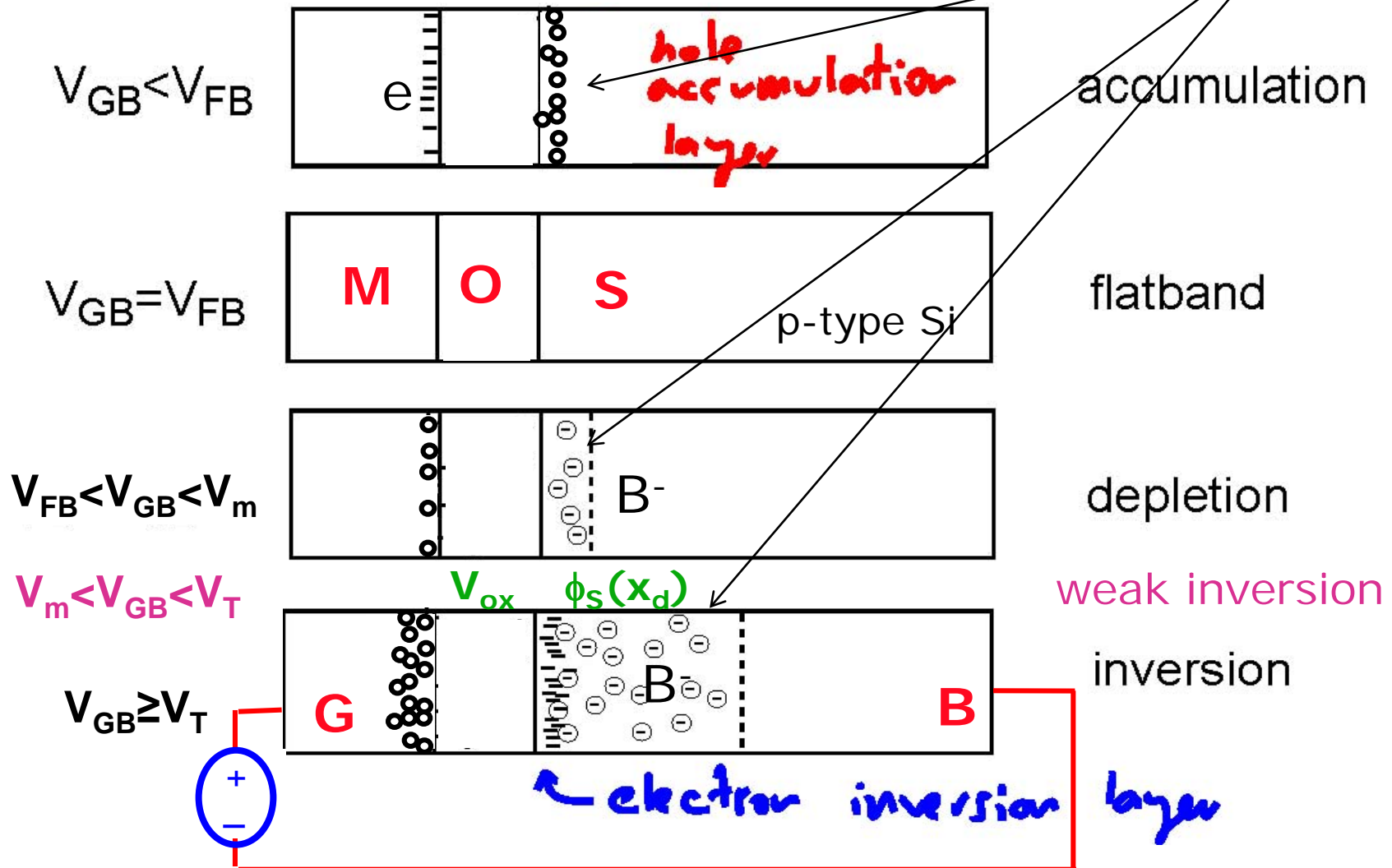
Capacitance-voltage (C-V) characteristics

- Example: p-Si



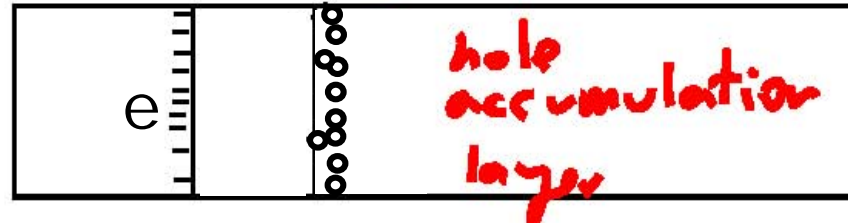


Review: Voltage drops and charges Q_s

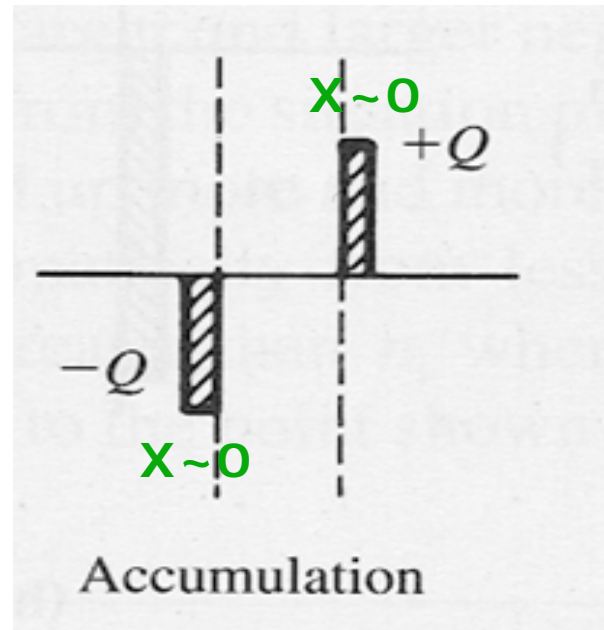


Review: Voltage drops and charges

$$V_{GB} < V_{FB}$$

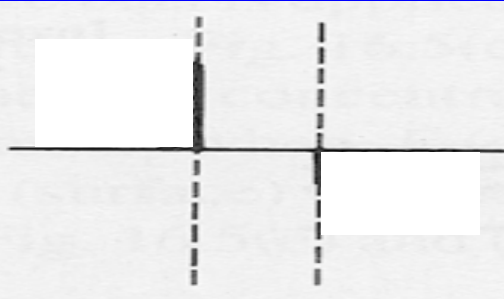


accumulation



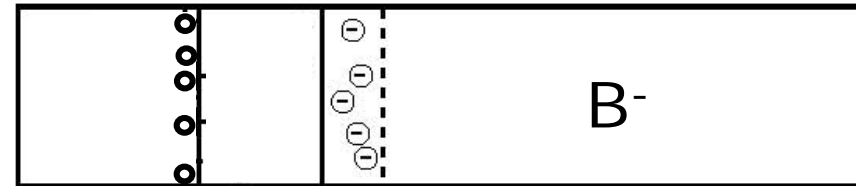
Review: Voltage drops and charges

$$V_{GB} = V_{FB}$$

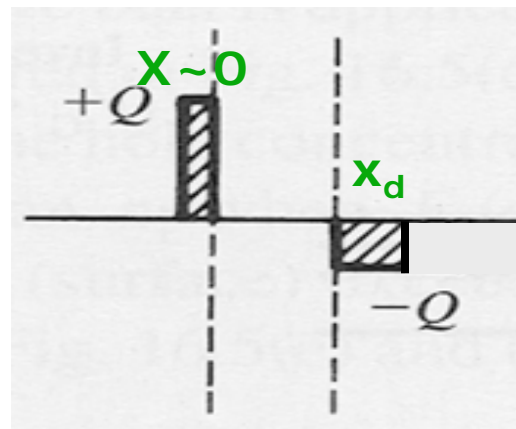


flatband

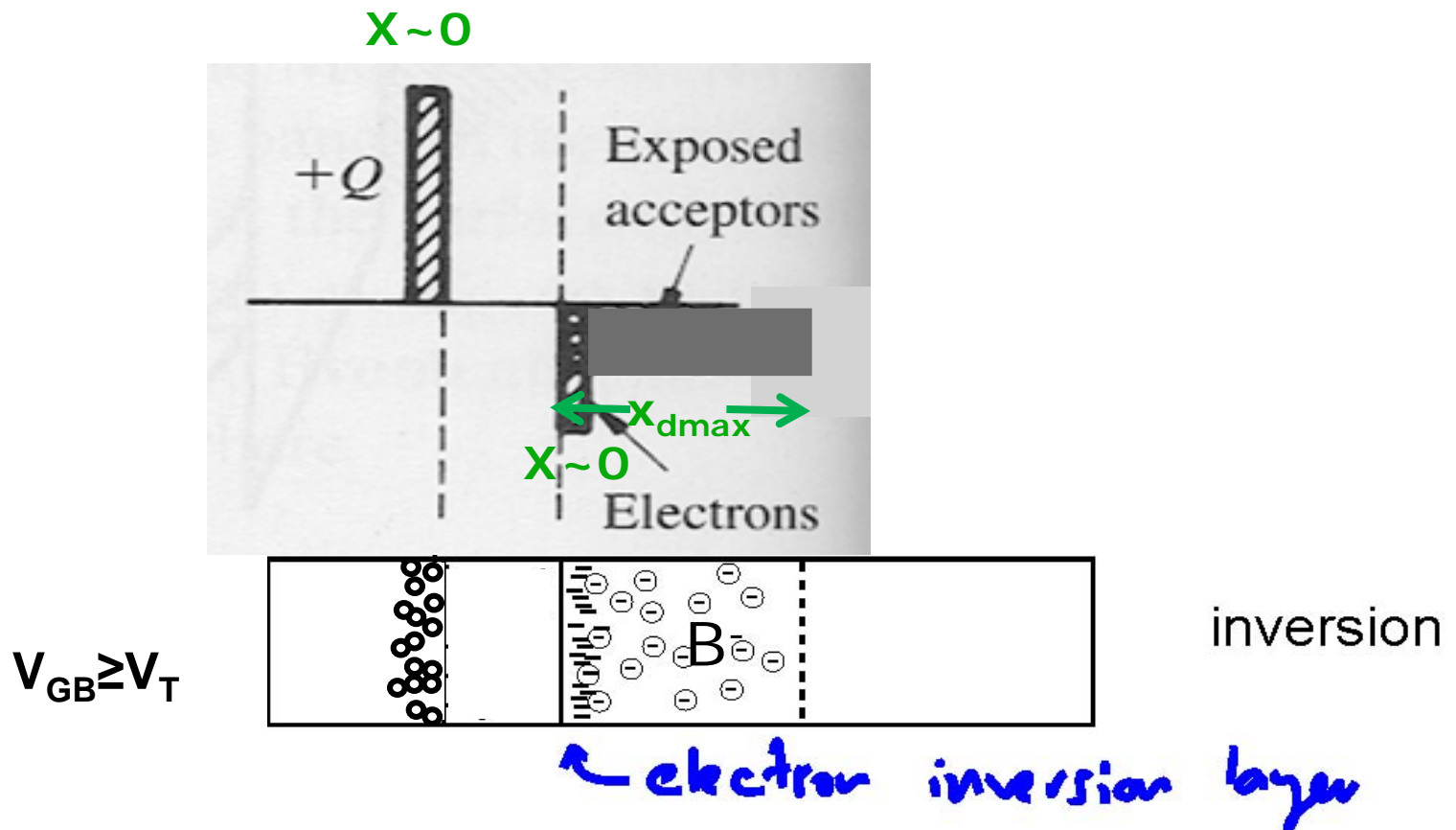
$$V_{FB} < V_{GB} < V_m$$



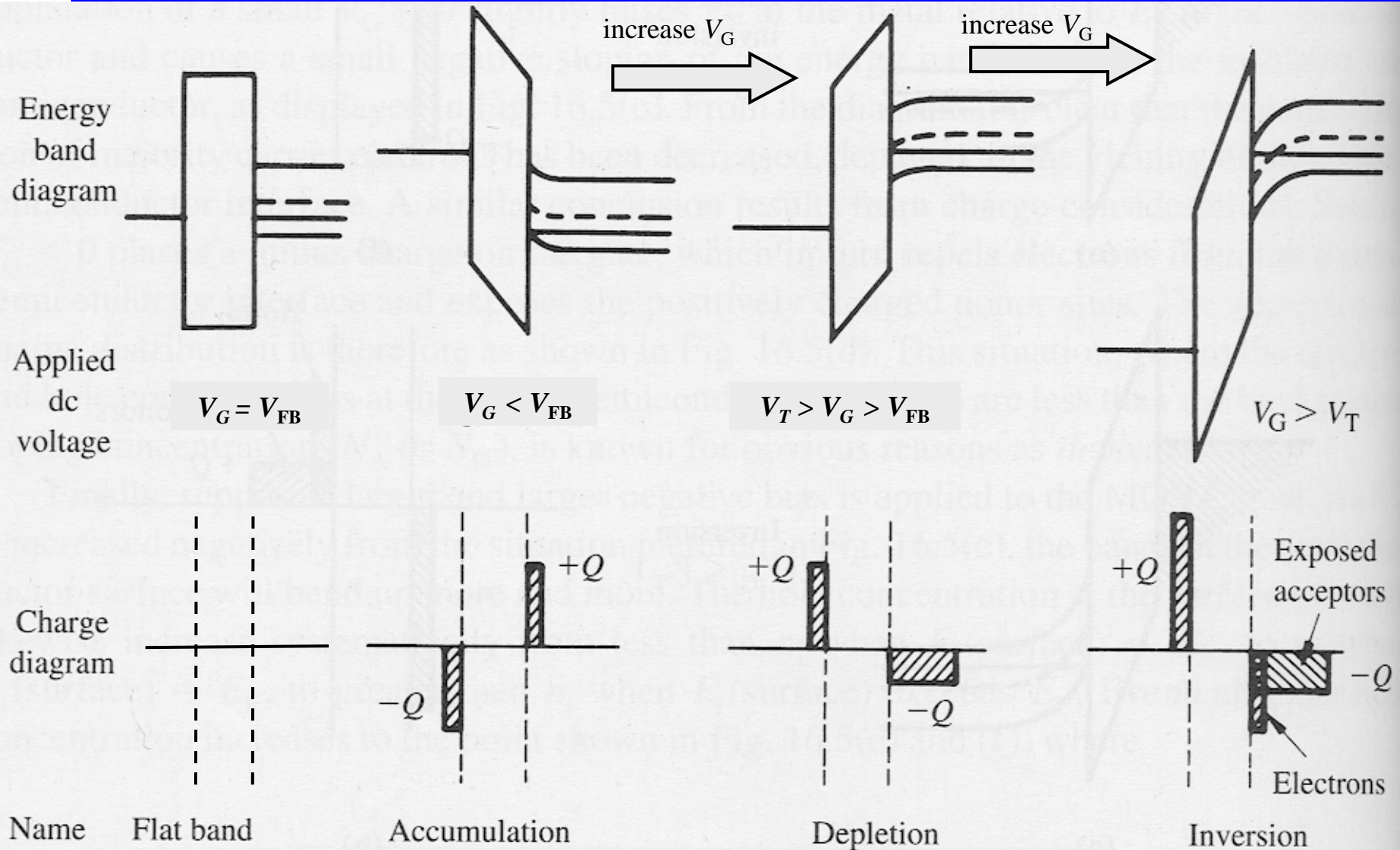
depletion



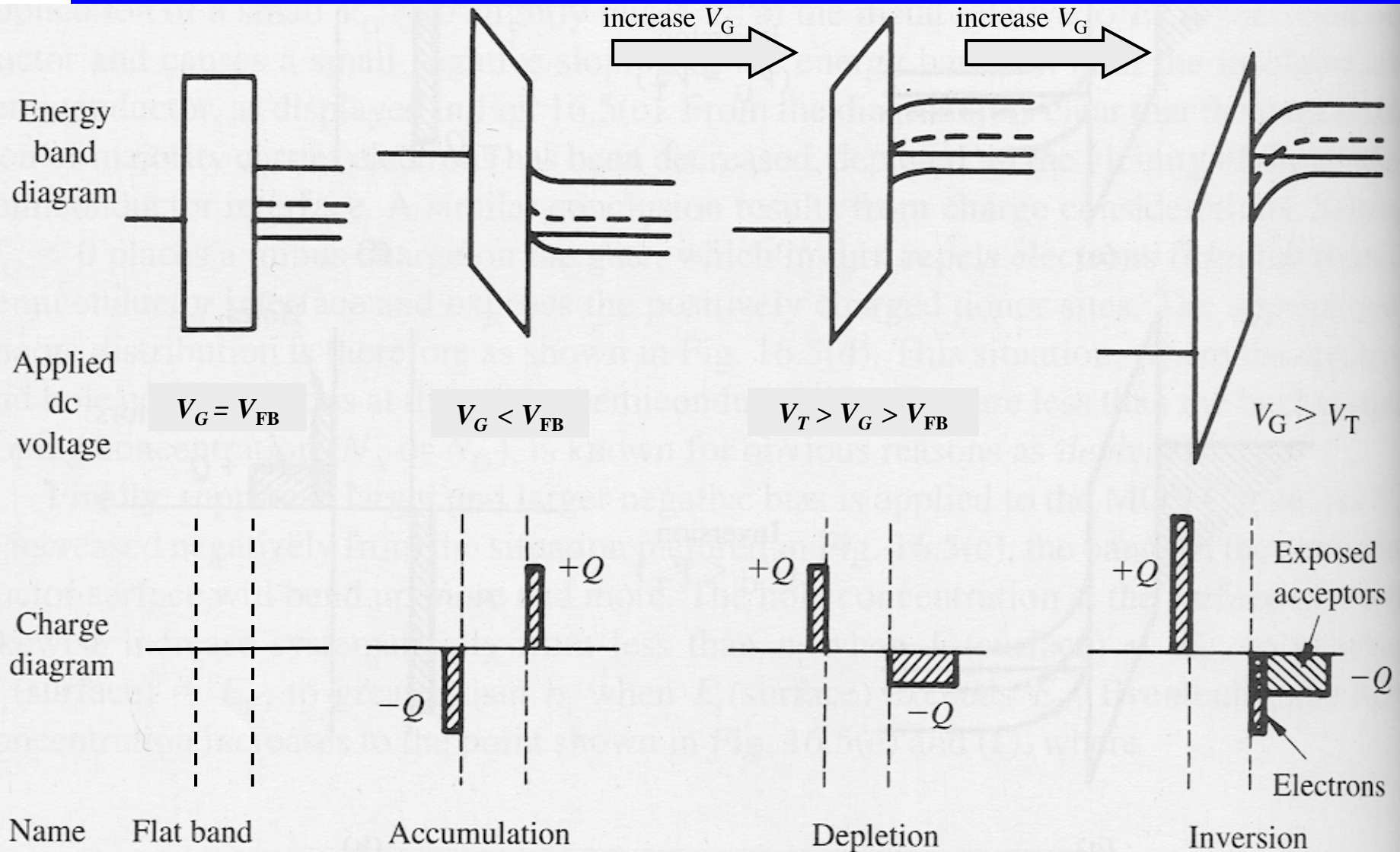
Review: Voltage drops and charges



Review: Biasing Conditions for p-type Si

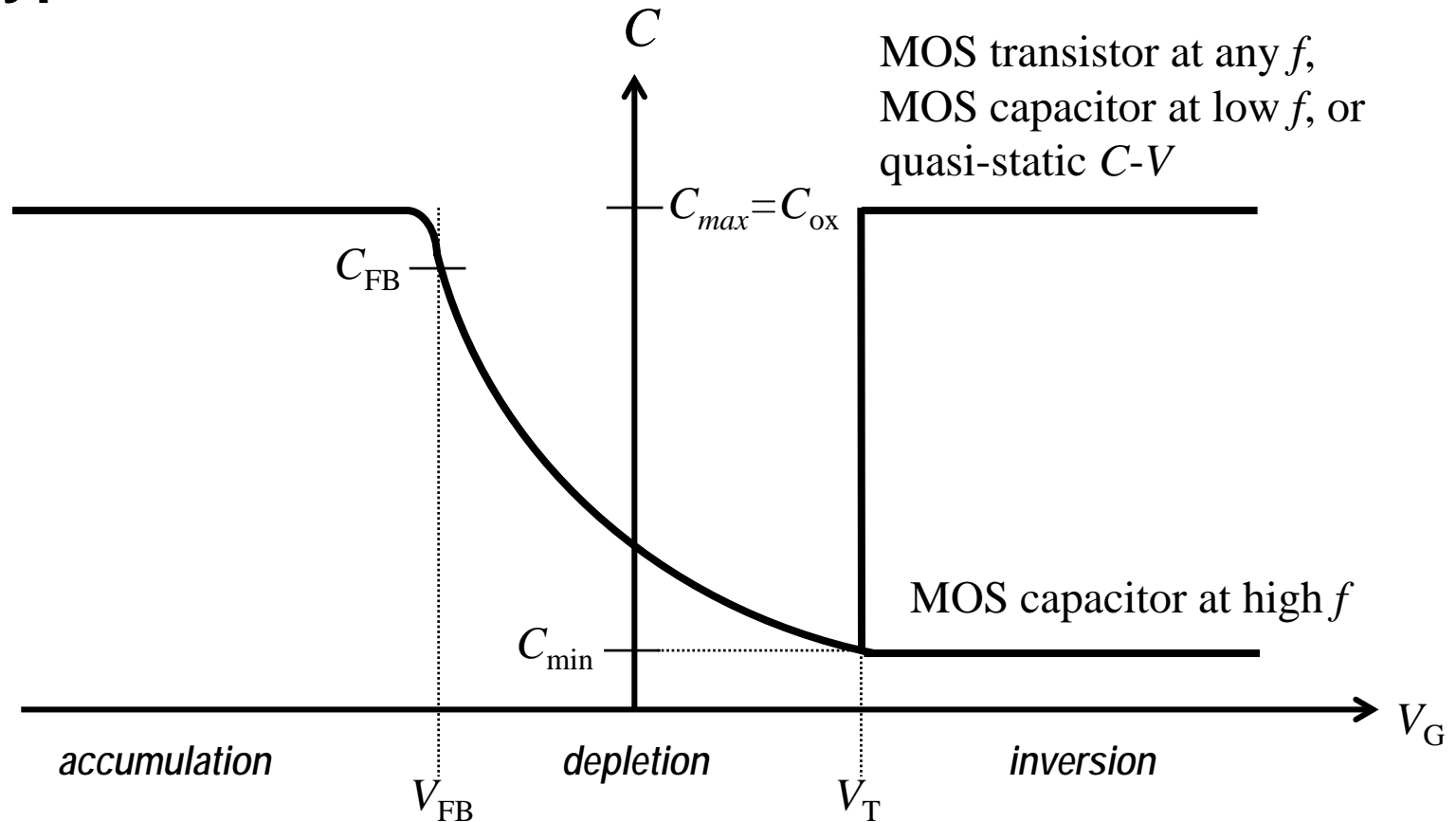


Review: Biasing Conditions for p-type Si



Capacitor vs. Transistor C-V (or LF vs. HF C-V)

p-type Si:



MOS Capacitor – MS contact

OUTLINE

- Metal-semiconductor contacts
- **MOS structure**
- MOS energy band diagram
- Effects of applied biases
- Non-Ideal MOS Capacitors

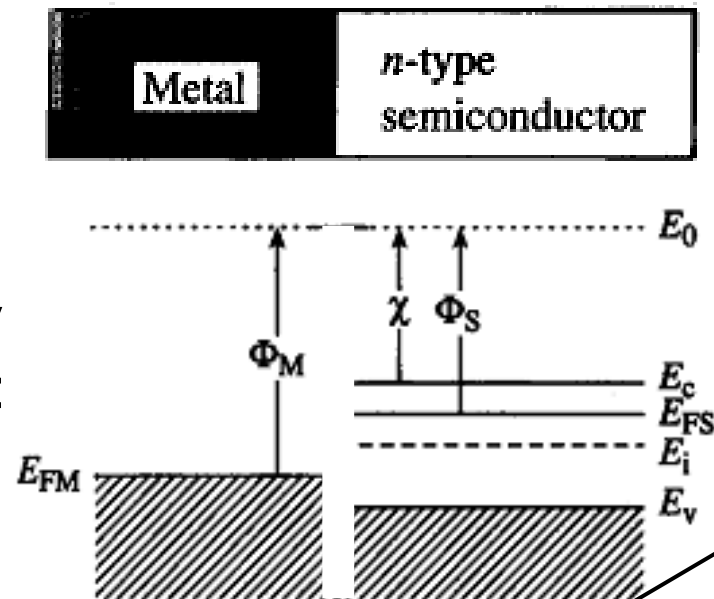
MOS Capacitor – MS contact

OUTLINE

- Physical Structure of MOS Capacitor
- Energy Band Diagram of MOS Structure
- Effects of Applied Voltage Biases
 - **operation modes & capacitance**
- **Appendix: Metal-semiconductor contact**

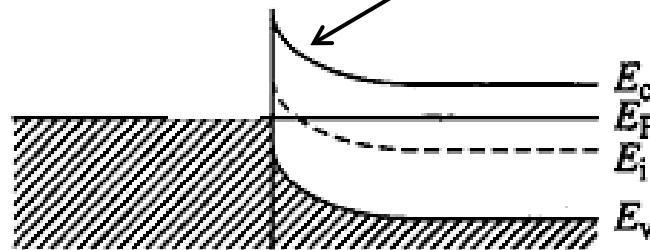
Ideal MS Contact: $\Phi_M > \Phi_S$, n-type

Band diagram instantly after contact formation:



Electron depletion

Equilibrium band diagram:

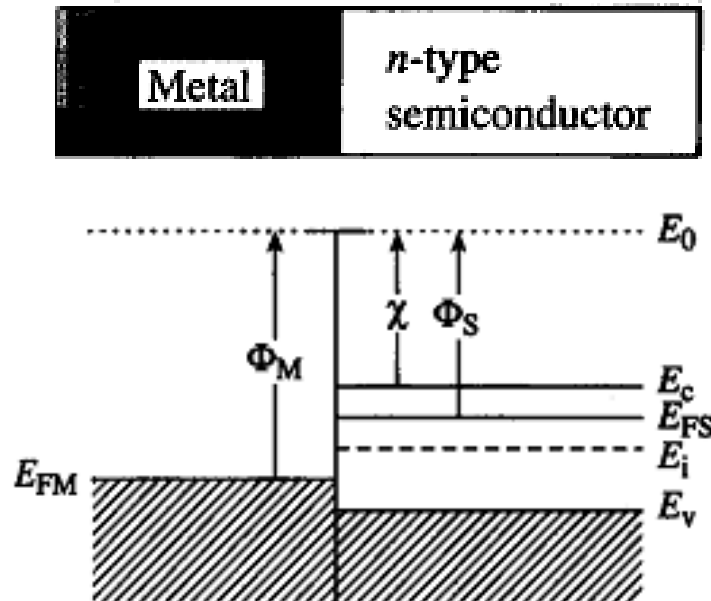


Schottky Barrier :

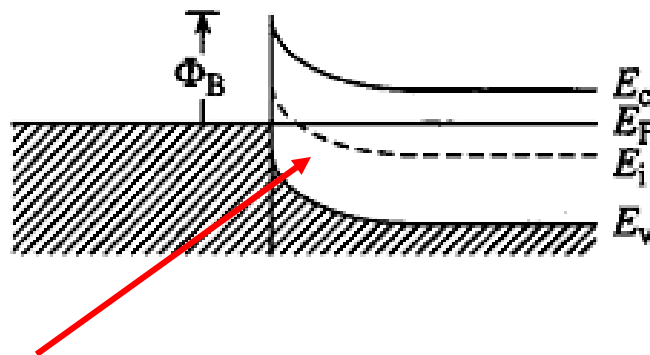
$$\Phi_{Bn} = \Phi_M - \chi$$

Ideal MS Contact: $\Phi_M > \Phi_S$, n-type

Band diagram instantly after contact formation:



Equilibrium band diagram:

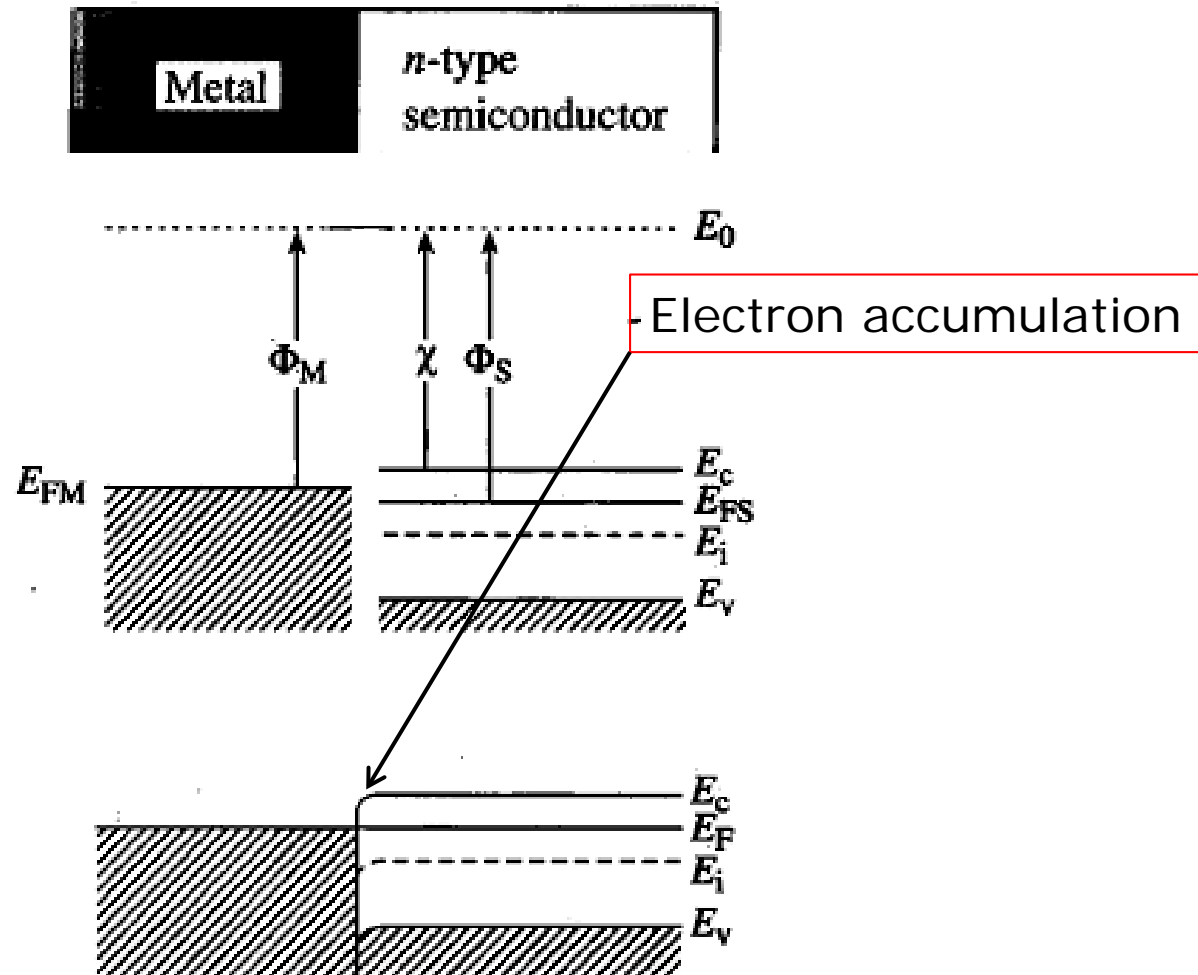


Schottky Barrier:

$$\Phi_{Bn} = \Phi_M - \chi$$

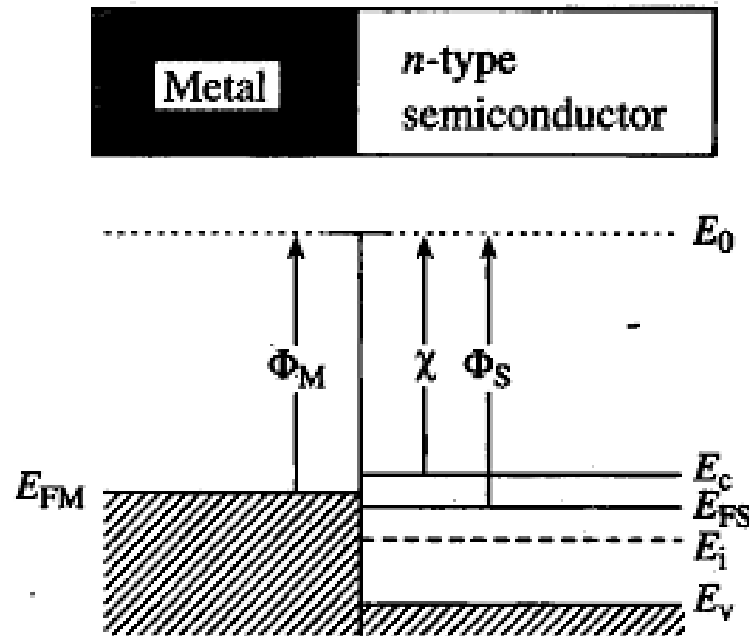
Ideal MS Contact: $\Phi_M < \Phi_S$, n-type

Band diagram instantly after contact formation:

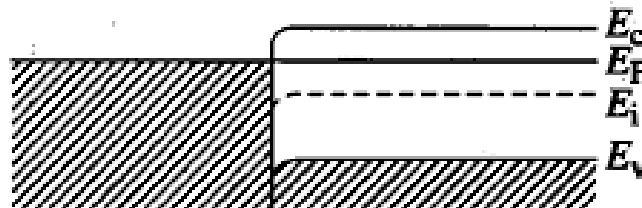


Ideal MS Contact: $\Phi_M < \Phi_S$, n-type

Band diagram instantly after contact formation:



Equilibrium band diagram:

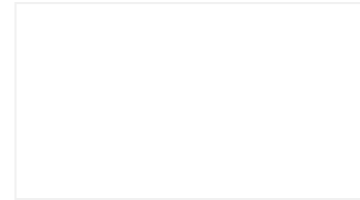


Heavy doping is needed to form ohmic contact.

Metal-Semiconductor Contacts

There are 2 kinds of metal-semiconductor contacts:

- rectifying
“Schottky diode”
- non-rectifying
“ohmic contact”



Metal-Semiconductor Contacts

There are 2 kinds of metal-semiconductor contacts:

- rectifying
“Schottky diode”

Polarity dependence

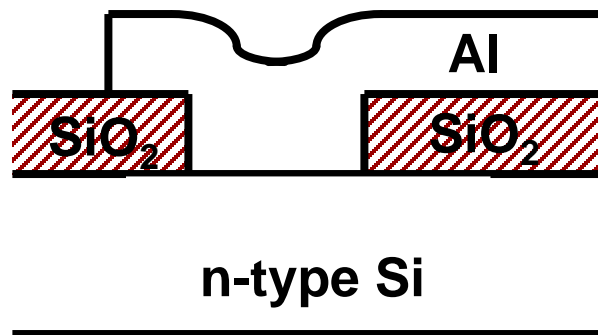
- non-rectifying
“ohmic contact”

Polarity independence

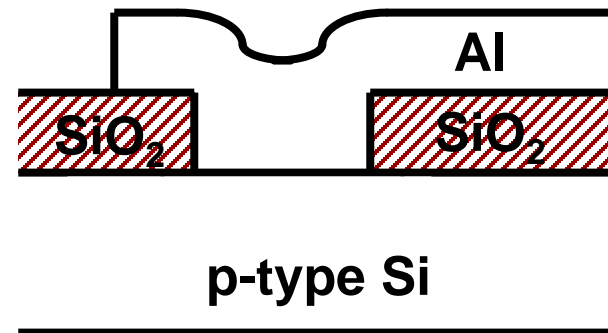
Electrical Contacts to Si

- In order to achieve a low-resistance (“ohmic”) contact between metal and silicon, the silicon must be heavily doped:

Metal contact to n-type Si



Metal contact to p-type Si



→ To contact the body of a MOSFET, locally heavy doping is used.

Forming Body Contacts

Open holes in oxide layer for body contacts after doping these regions

