Introduction of Signals

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9:00-11:00 Wednesday

Outline

- Signals
  - ➢ Concept of signals
  - ➢ Signal classification
  - ➢ Energy and power signals
  - ➢ Signal operation
  - ➢ Elementary signals
- Systems
  - ➢ Concept of Systems
  - ➢ Systems classification

*Chapter 1 in the textbook (Oppenheim)

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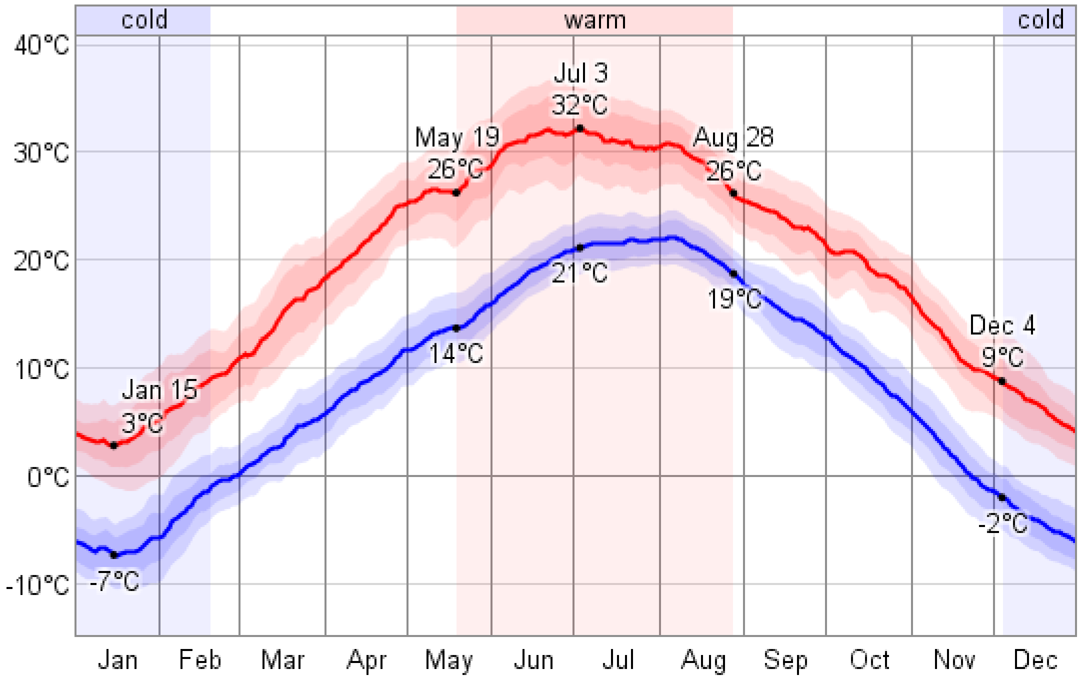
- **Signal:**
  - Can be broadly defined as any quantity that varies as a function of time and/or space and has the ability to convey information about a certain physical phenomenon.

http://news.filehippo.com/2013/10/qardio-ecg-monitor-streams-data-iphone/

The electrocardiogram **(ECG)**

- **Signal:**
  - Any series of measurements of a physical quantity is a signal (temperature measurements for instance).



Temperature in Xi'an, China

Signal ….

- **Signal:**
  - Can be broadly defined as any quantity that varies as a function of time and/or space and has the ability to convey information about a certain physical phenomenon.
  - Any series of measurements of a physical quantity is a signal (temperature measurements for instance).

- **Signal and perception:**
  - Signals: allow us to see, hear, feel and consequently act → information is transmitted and carried in signals.

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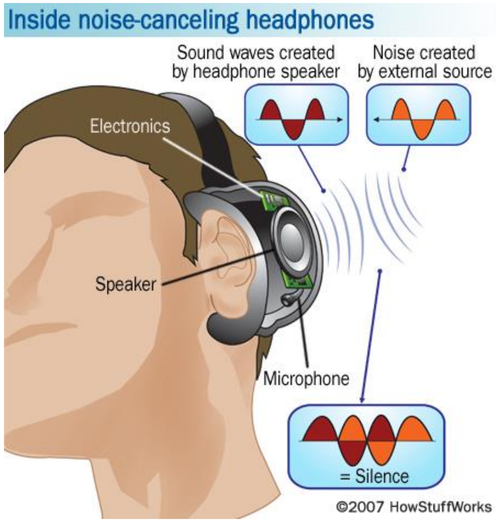
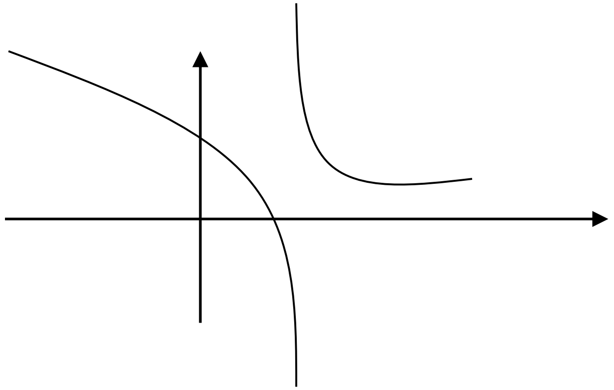
Signal and perception:

– Signals: allow us to see, hear, feel and consequently act → information is transmitted and carried in signals.



Signal representation:

- The most convenient way to represent a signal is via the concept of a function, let us say $x(t)$. In this notation:
  - $x(.)$ represents the dependent variable related to the physical phenomena (e.g., temperature, voltage, pressure, etc.)
  - t represents the independent variable (e.g., time, space, etc.).
- Roughly speaking, any realizable function can be considered as a signal.



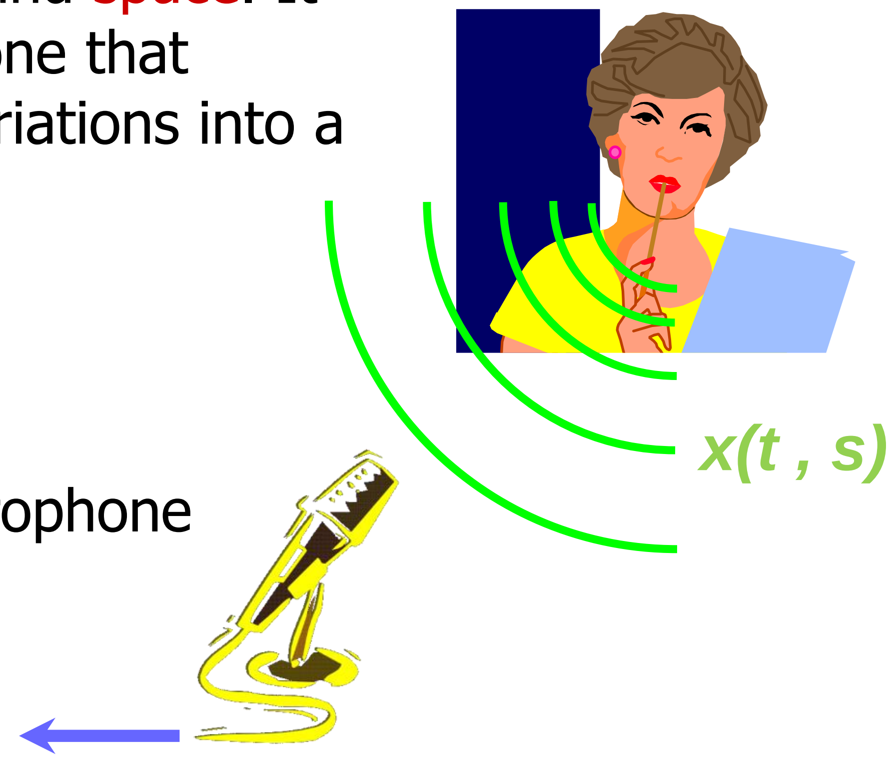
Signal ....

- **Examples of signals include:**
  - Electrical signals: currents and voltages in AC circuits, radio communications signals, audio and video signals.
  - Mechanical signals: sound or pressure waves, vibrations in a structure, earthquakes Seismograph.
  - Biomedical signals: lung and heart monitoring, X-ray and other types of images.
  - Finance: time variations of a stock value or a market index.

Example of signals

Speech signal

A speech signal consists of variations in air pressure as a function of time and space. It can be captured via a microphone that translates the local pressure variations into a voltage signal.



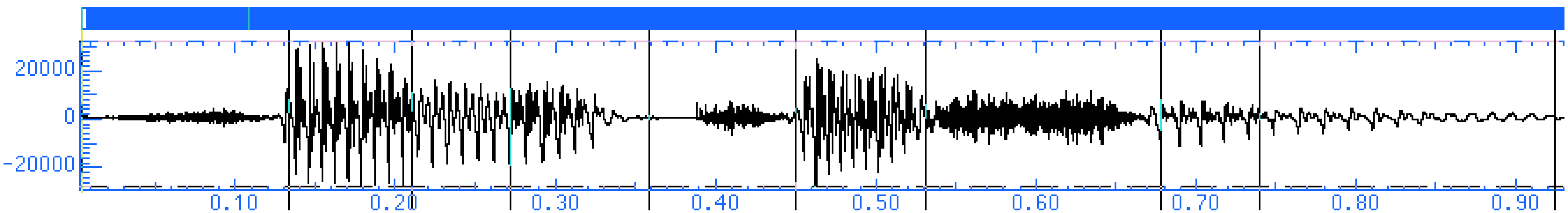
microphone

Speech signal

$x(t) = x(t\ ,\ s0)$

Example of 1D signals

- **Speech signal**
  - At a given space position: speech signals basically represents a continuous-time (CT) signal x(t)



ph - o - n - e - t - i - c - ia - n

Automatic speech recognition

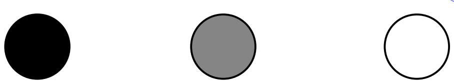
Phonetician: an expert in phonetics.

Example of Multi-Dimensional signals

- **<u>Digital</u> gray images: 2-D signals**
  - The intensity of the image at location (x, y)

pixel
or
pel

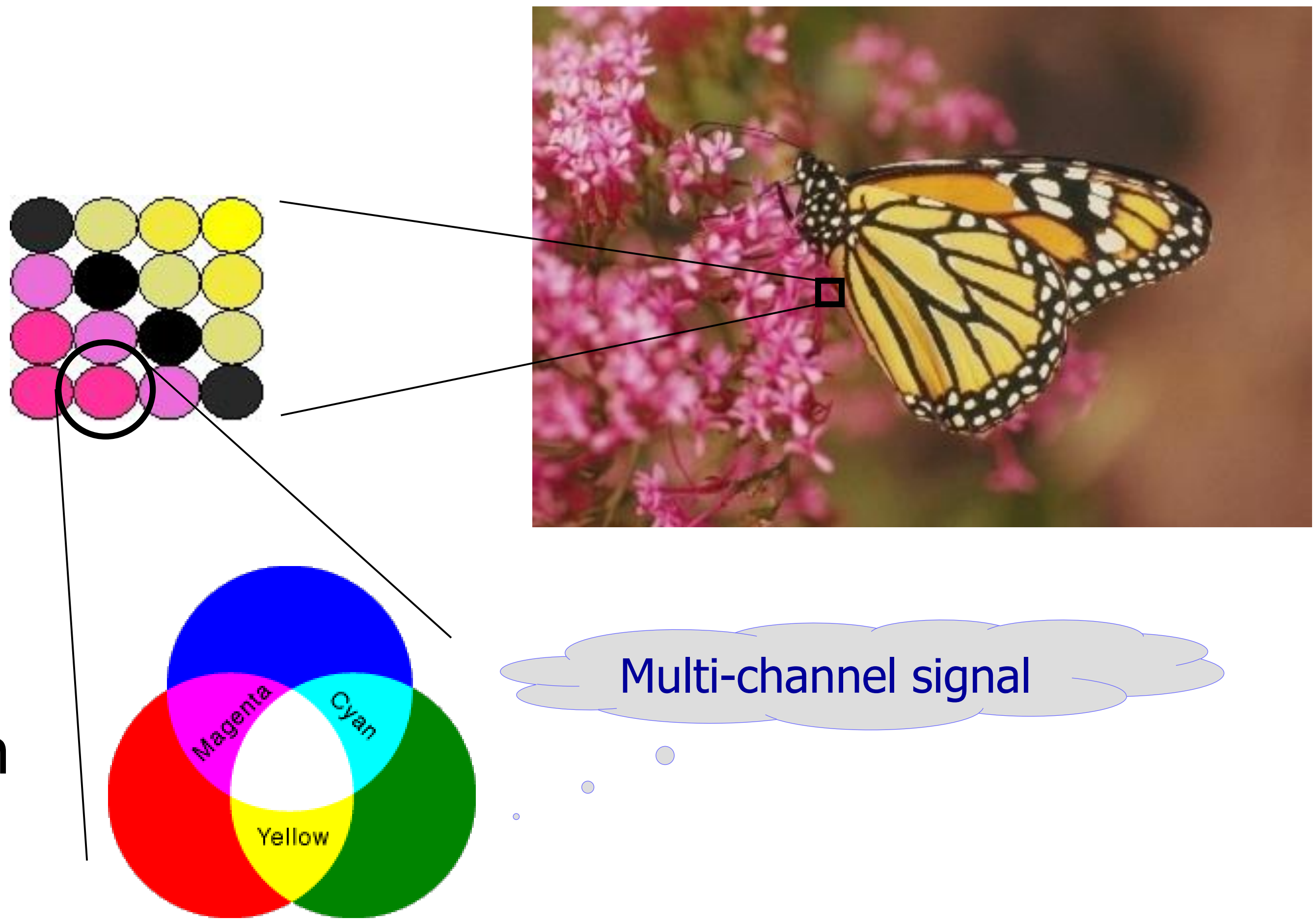
Acquired images are made up of a discrete number of points
→ discrete-space signals



black p=0 | gray <P< | white p=255

Example of Multi-Dimensional signals

- **Colored images: are 2-D signals with respect to spatial variables**

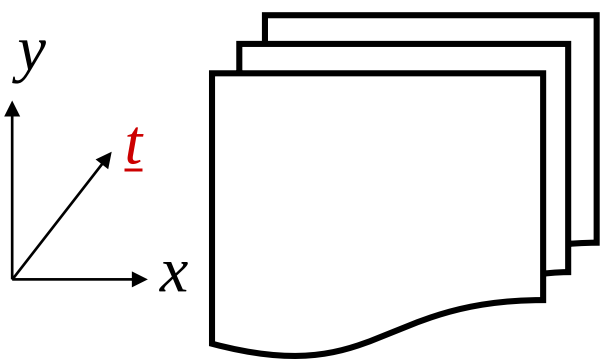


colors:
combination of RGB

Example of Multi-Dimensional signals

- **Video sequences:**
  - Is a collection of 2-D images called frames

Frame 1 51 71 91 111



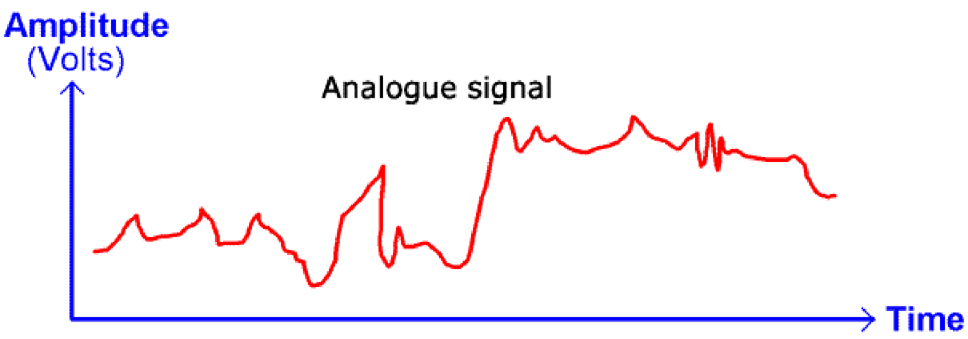
Signal classification

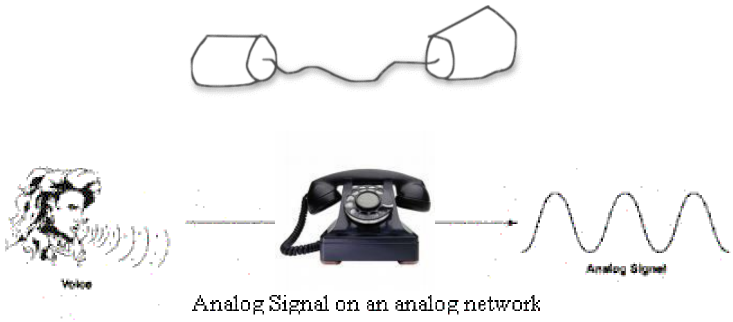
Analogue and Digital signals

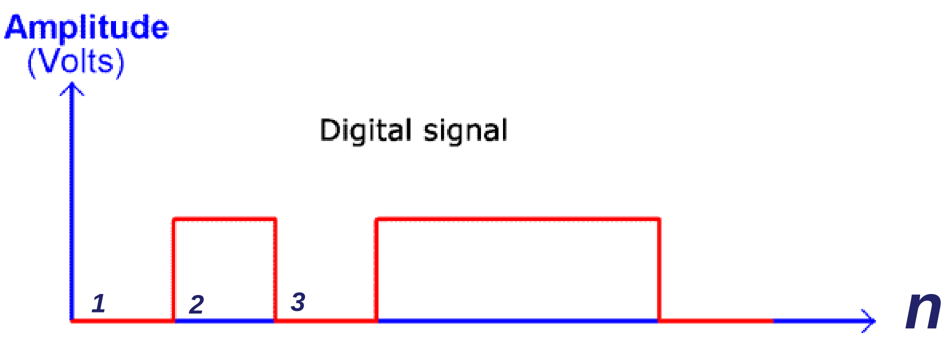
Continuous (Analogue)	$\longleftrightarrow$	Discrete (Digital)
Periodic	$\longleftrightarrow$	Non-periodic
Deterministic	$\longleftrightarrow$	Random
Symmetric (Odd/Even)	$\longleftrightarrow$	Asymmetric
Finite energy	$\longleftrightarrow$	Finite power

Analogue and Digital signals

- **Analogue and Digital**







Types of signals

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- Distinctions can be made at different levels based on their properties :

Continuous (Analogue)	$\longleftrightarrow$	Discrete (Digital)
Periodic	$\longleftrightarrow$	Aperiodic (non Periodic)
Deterministic	$\longleftrightarrow$	Random
Symmetric (Odd/Even)	$\longleftrightarrow$	Asymmetric
Finite energy	$\longleftrightarrow$	Finite power

Periodicity

- **Periodic:**
  - A periodic signal is a function of time that repeat itself every certain period of time $T \neq 0$:

    $$\text{if } x(t+T) = x(t) \text{ for all } t$$

  - The fundamental period is the smallest value of time for which the equation holds true, and it is simply known as the period.

    $$x(t+nT) = x(t),\ n \text{ is an integer}$$

  - The fundamental frequency of the periodic signal, $x(t)$, is $f=1/T$
- **Non-periodic:** if $x(t+T) \neq x(t)$ for whatever $T \neq 0;$

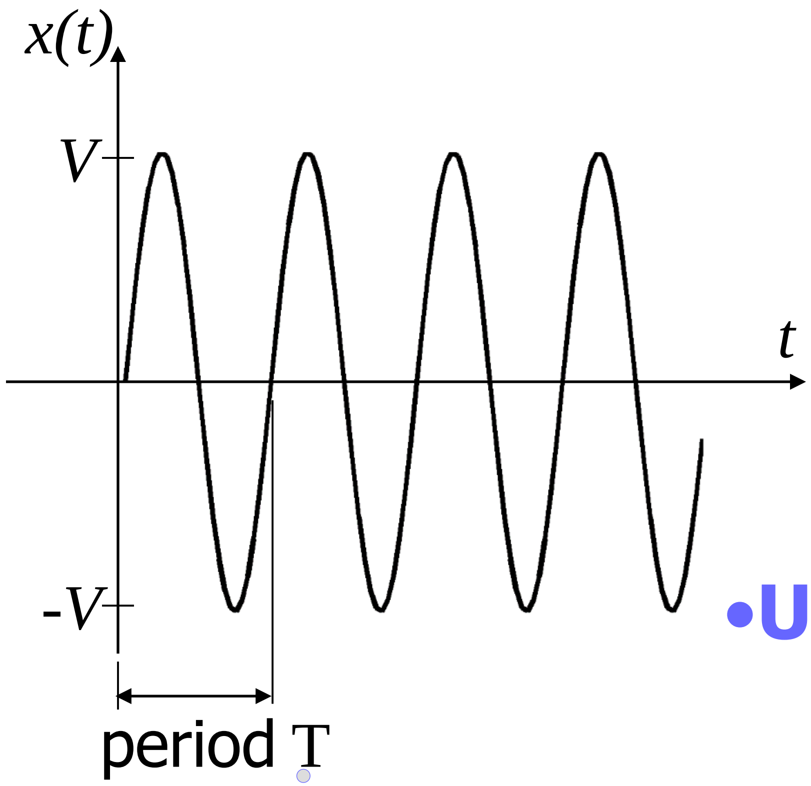
Examples?

- No truly periodic signal exist. (physically)

- **Why periodic signal are important?**
  - The reason for studying periodic functions is because we do not have to (or sometime we are unable to) specify the beginning or end point of a "repeating" signal.

Periodicity

- **Sine Wave (Sinusoid)**



amplitude

phase

$$x(t) = V\sin(\omega t + \theta)$$

sine wave

frequency in radians/second

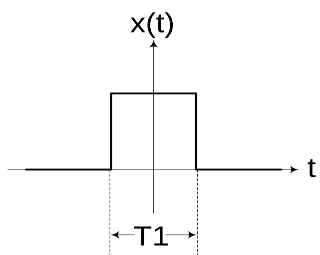
- **Units**
  - –Period: T [s (second)]
  - –Frequency: f [1/second] or [Hz (hertz)]
  - –Phase: [radians]

What is the relationship between T, f and ω ?

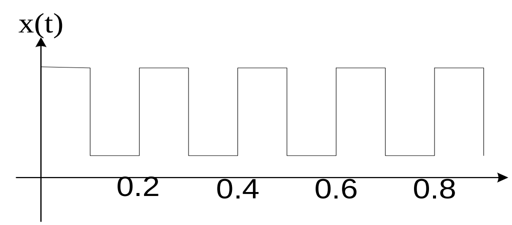
Periodicity

Periodic signals are mathematical abstraction!

Non-periodic signal
(rectangular pulse function)



Periodic signal
(Rectangular waveform)



- **Are the following <u>CT</u> signals periodic or non-periodic?**

1. $\log(|t|)$

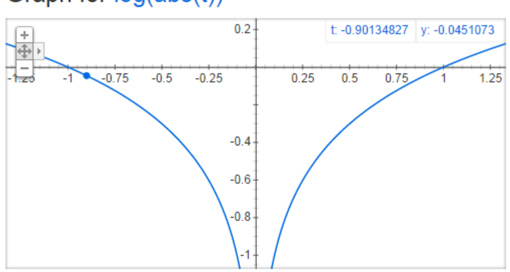
2. $\sin(t-5)$

3. $5\cos(2\pi 5t)$

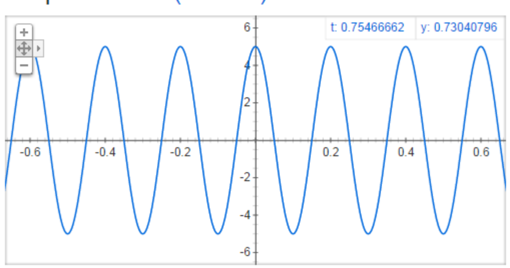
4. $\sin(t^2)$

Test yourself

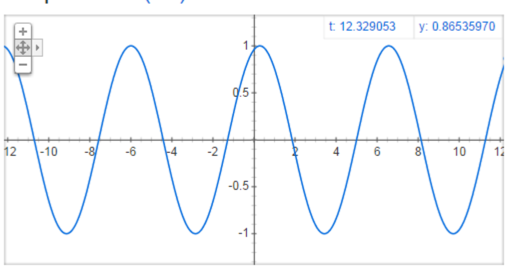
Graph for log(abs(t))



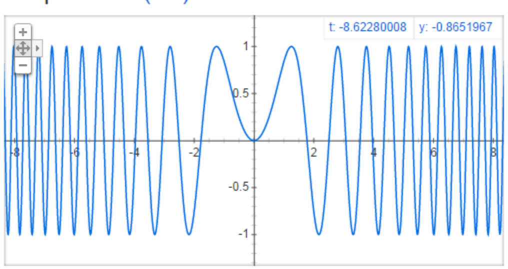
Graph for 5*cos(2*π*5*t)



Graph for sin(t-5)



Graph for sin(t^2)



Types of signals

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– Distinctions can be made at different levels (for example: whether x(t) is considered to be deterministic or random in nature):

Continuous (Analogue)	⟷	Discrete (Digital)
Periodic	⟷	Aperiodic (non Periodic)
Deterministic	⟷	Random
Symmetric (Odd/Even)	⟷	Asymmetric
Finite energy	⟷	Finite power

Deterministic vs random

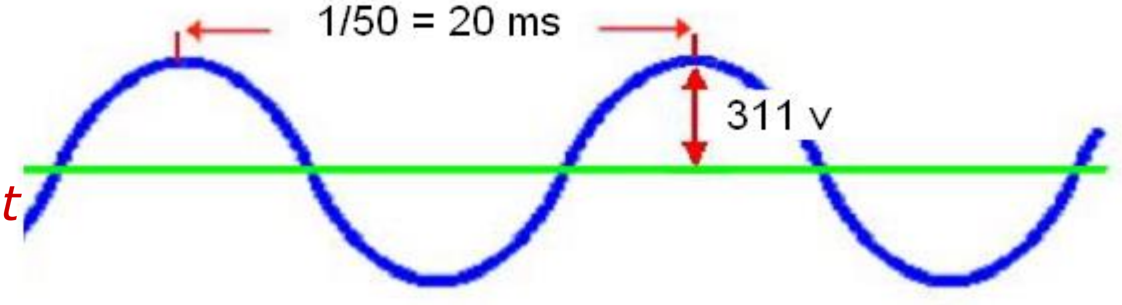
- **Deterministic and random signals :**
  - If the signal can be described by a mathematical equation, it is a deterministic signal
  - If we know how the signal will behave in future then it is deterministic
  - Otherwise it is called a random signal

- **Deterministic and random signals**

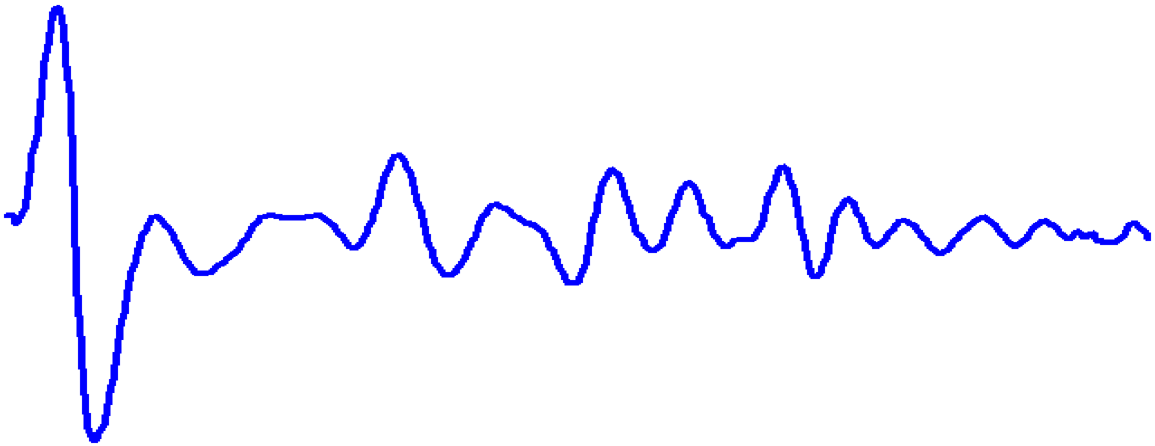
$$x(t) = \sqrt{2}\,220\cos(2\pi 50\, t)$$

RMS = ?
Peak-to-peak voltage?

Deterministic
(and periodic)
AC : Alternating current



Random
Is the temperature random or deterministic signal?



Types of signals

Continuous (Analogue)	$\longleftrightarrow$	Discrete (Digital)
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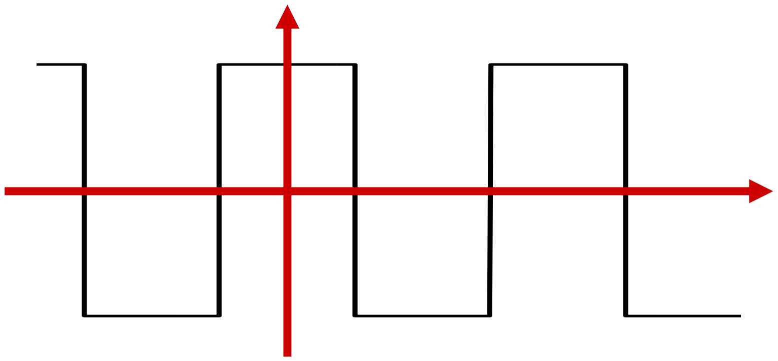
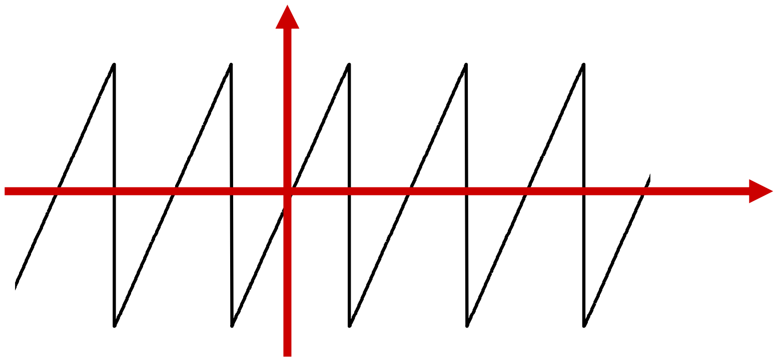
Even and odd signals

- **Even symmetric signal**
  - A CT signal $x(t)$ for which : $x(-t) = x(t)$ and for all t



Even and odd signals

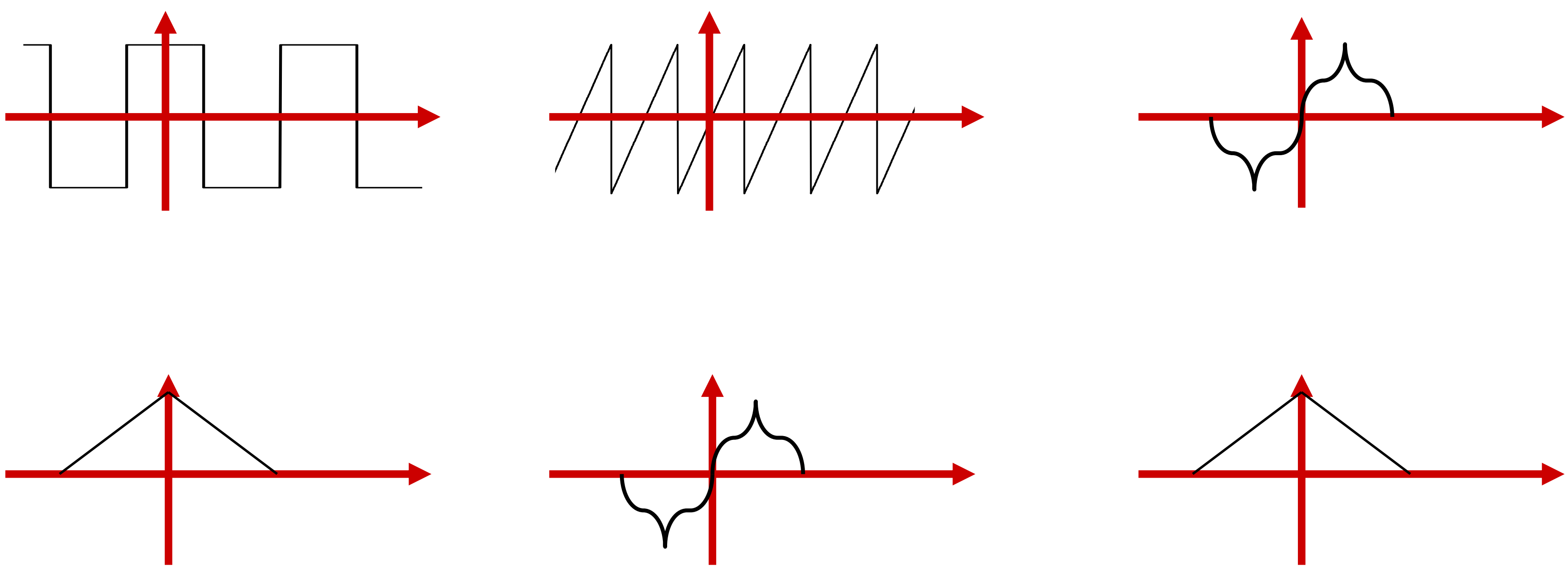
- **Odd symmetric signal**
  - A CT $x(t)$ for which $x(-t) = -x(t)$ for all t



$x(0) = ?$

Even and odd signals

- The product of even functions is …???
- The product of odd functions is …???
- The product of an odd and even function is …???



Energy and power of signals

Energy and power of signals

– Distinctions can be made at different :

Continuous (Analogue)	$\longleftrightarrow$	Discrete (Digital)
Periodic	$\longleftrightarrow$	Aperiodic (non Periodic)
Deterministic	$\longleftrightarrow$	Random
Symmetric (Odd/Even)	$\longleftrightarrow$	Asymmetric
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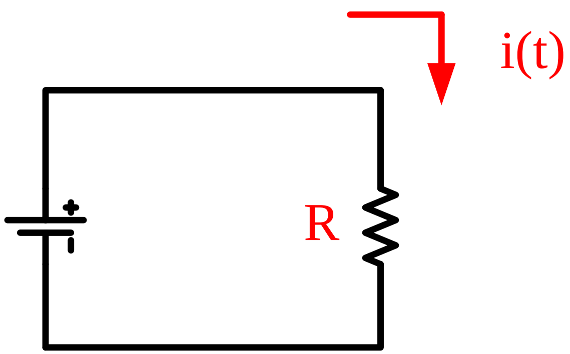
Energy of signals

- **Energy:**
  - The idea of the "size" of a signal is crucial to many applications.
  - The first concept to be introduced is the "energy" of a signal



$$\varepsilon(i) = R\int_{-\infty}^{+\infty} |i(t)|^2 dt$$

$$\varepsilon(u) = \frac{1}{R}\int_{-\infty}^{+\infty} |u(t)|^2 dt$$

R = 1

Energy of signals

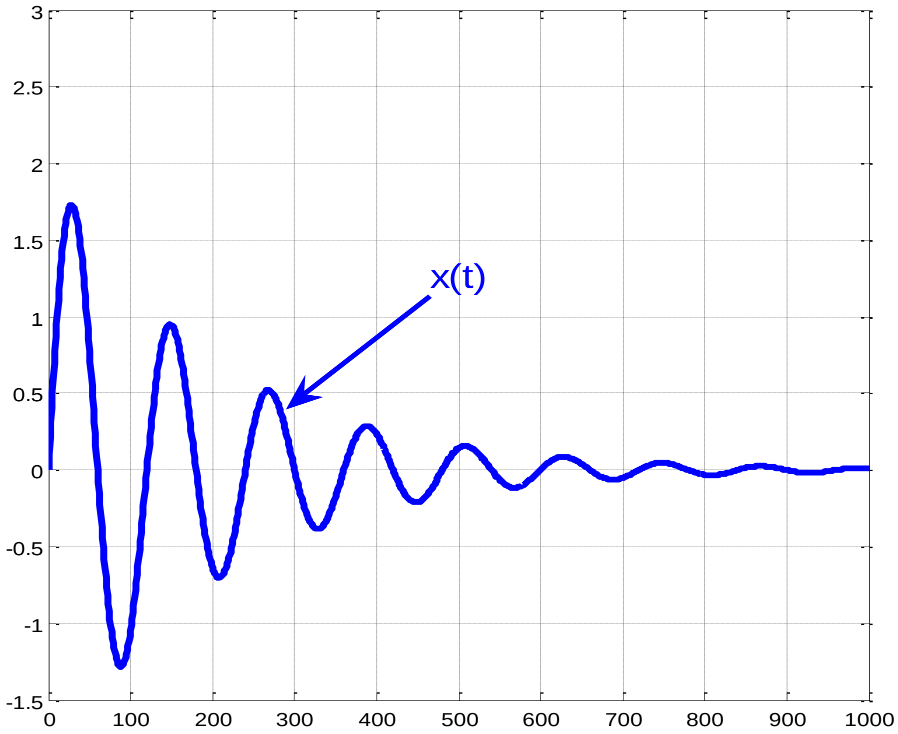
- **Energy:**
  - This concept can be generalized to complex signal

$$\boxed{\varepsilon(x) = \int_{-\infty}^{\infty} |x(t)|^2 dt}$$

Energy of signals

- **Energy**

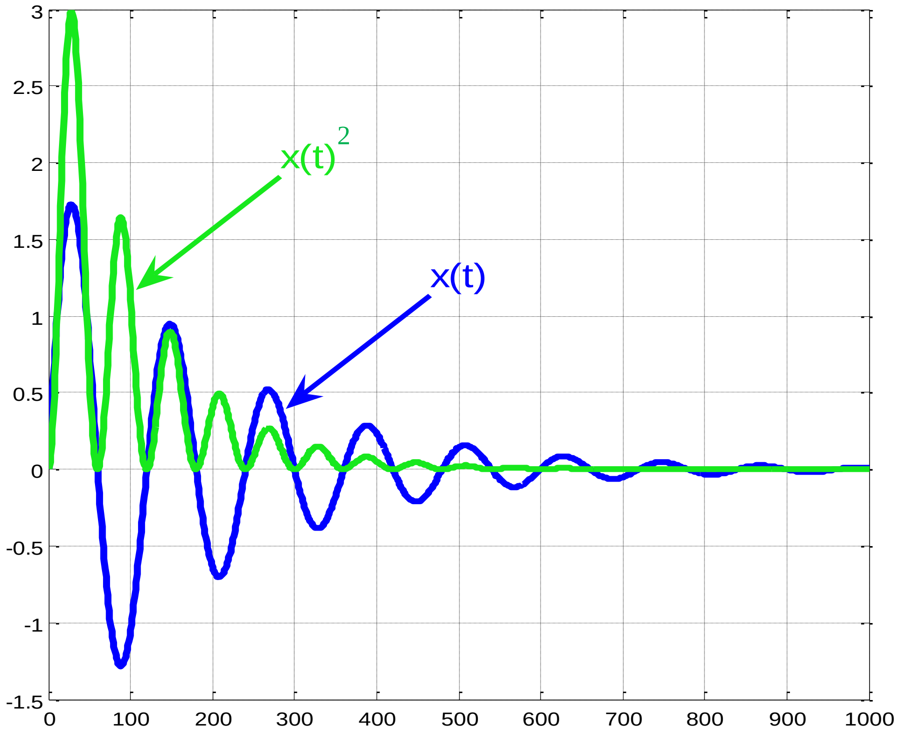
$$\varepsilon(x) = \int_{-\infty}^{\infty} |x(t)|^2 dt$$



Energy of signals

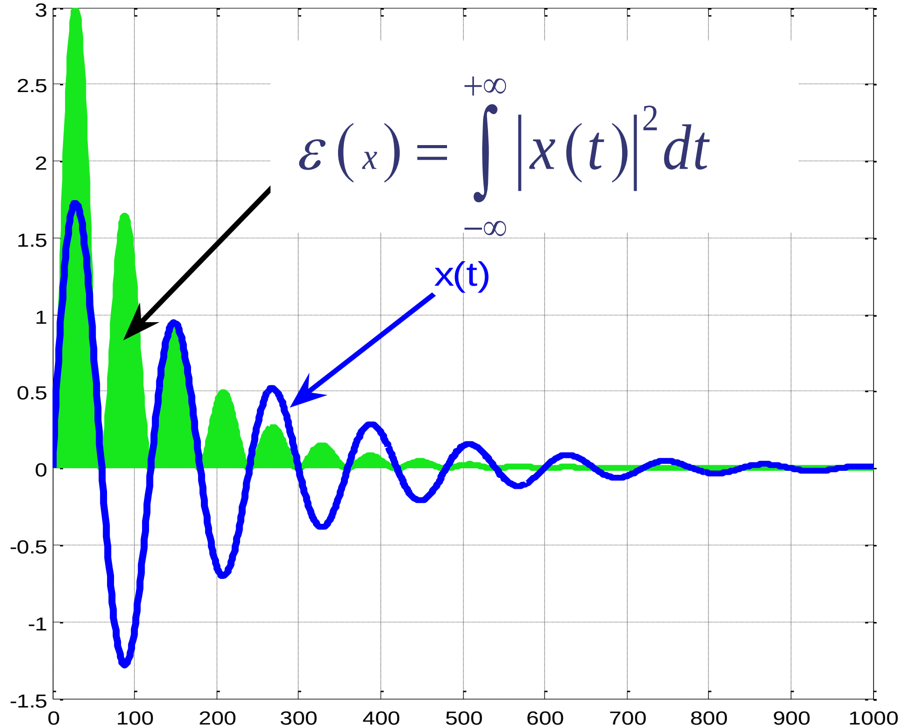
- **Energy**

$$\varepsilon(x) = \int_{-\infty}^{\infty} |x(t)|^2 dt$$



- **Energy**

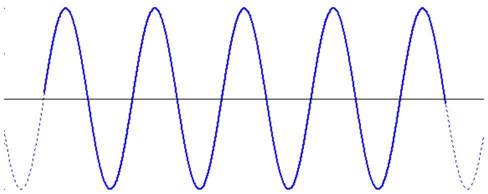
$$\varepsilon(x) = \int_{-\infty}^{\infty} |x(t)|^2 dt$$



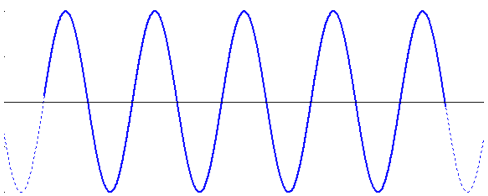
Is ε(x) finite?

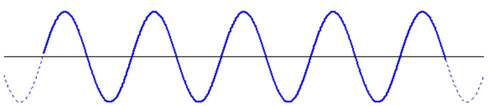
So, it is a
finite energy signal

- **Energy of signals :**
  - What if the signal does not decay?



In this case we have infinite energy for such signal.





Is the left hand signal "stronger" than the right one ?

This leads us to the following concept : signal power

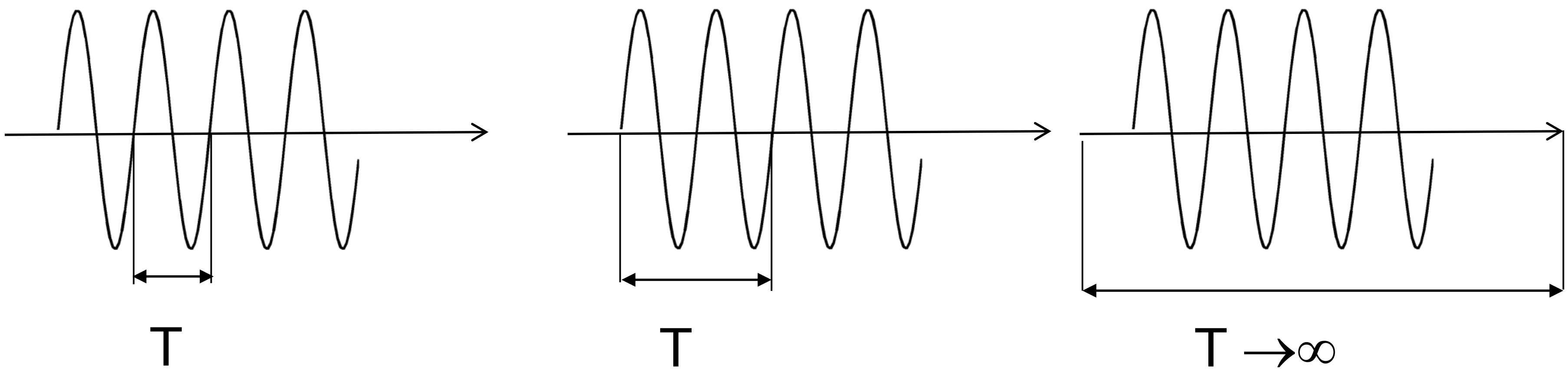
Power of signals

- **Power:**
  - Power is the time average of the energy (energy per unit time).

Continuous-time signal *x(t)*
$$P(x) \overset{\text{def}}{=} \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \lvert x(t)\rvert^2 dt$$

Power of signals

- **Power of a periodic signal (period is T):**
  - Power is a time average of energy …



$$P(x) = \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2 dt$$

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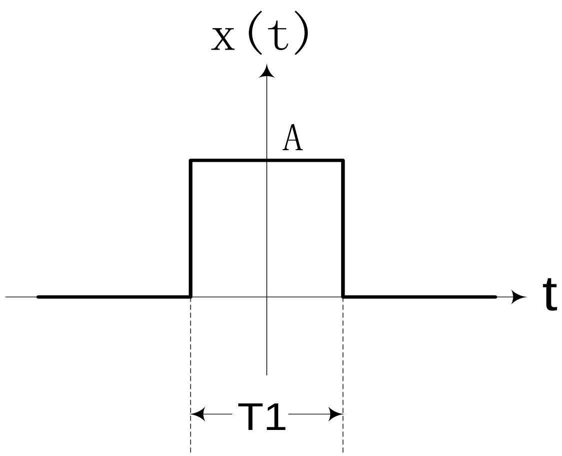
- **Example :**
  - Find the total energy of this rectangular pulse



$$E = \int_{-\infty}^{\infty} |x|^2(t)\,dt$$

$$= \int_{-T_1/2}^{T_1/2} A^2\,dt$$

$$= A^2 T_1$$

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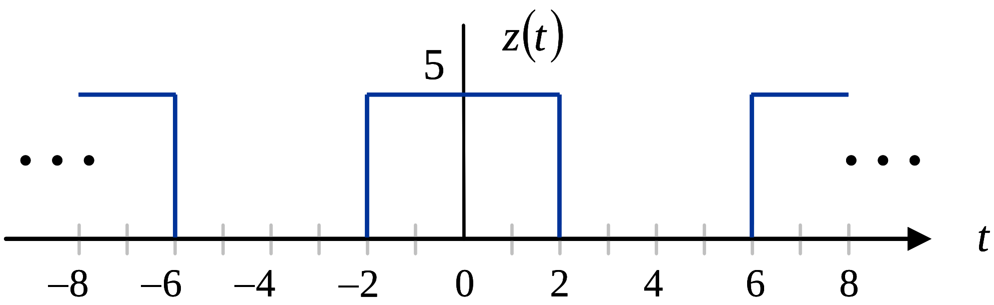
- **Example :**
  - Find the average power of this signal



$$P(z) = \frac{1}{T}\int_{-T/2}^{T/2} |z(t)|^2 dt$$

$$P = \frac{1}{8}\int_{-4}^{4} |z(t)|^2 dt == \frac{1}{8}\int_{-2}^{2} 5^2 dt = \frac{100}{8} = 12.5$$

Energy and Power Signals

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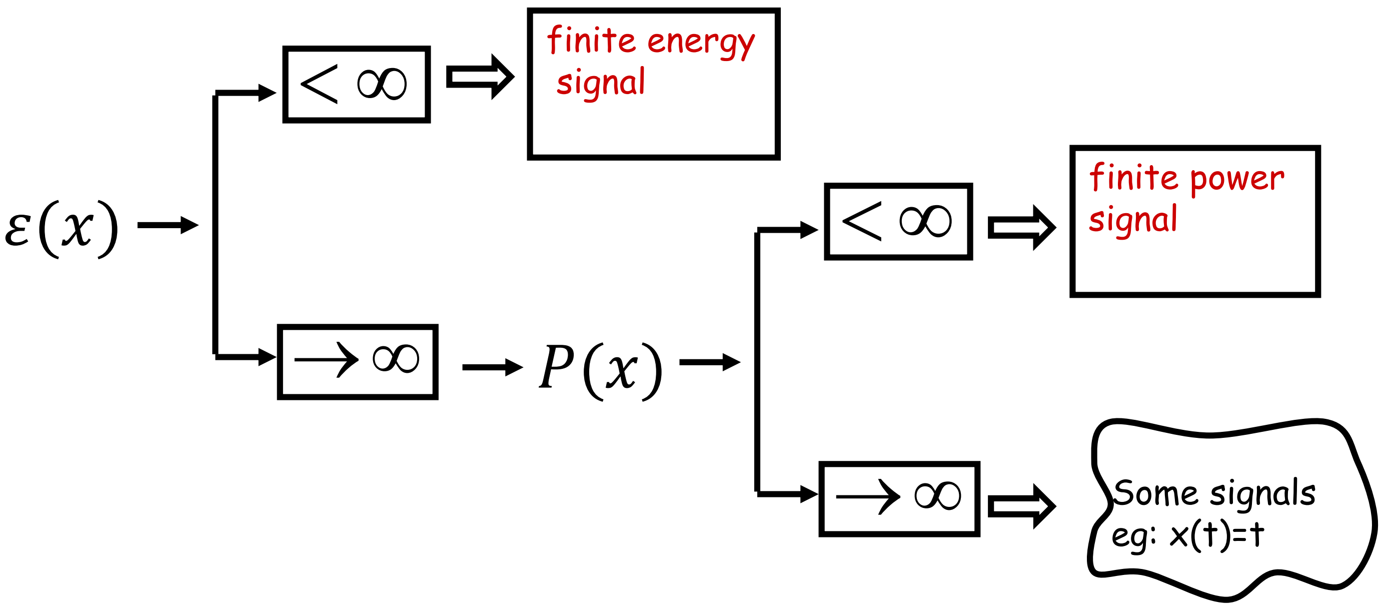
- **Energy vs. Power**
  - "Energy signals" have finite energy → zero average power.
  - "Power signals" have finite and non-zero power → infinite energy.

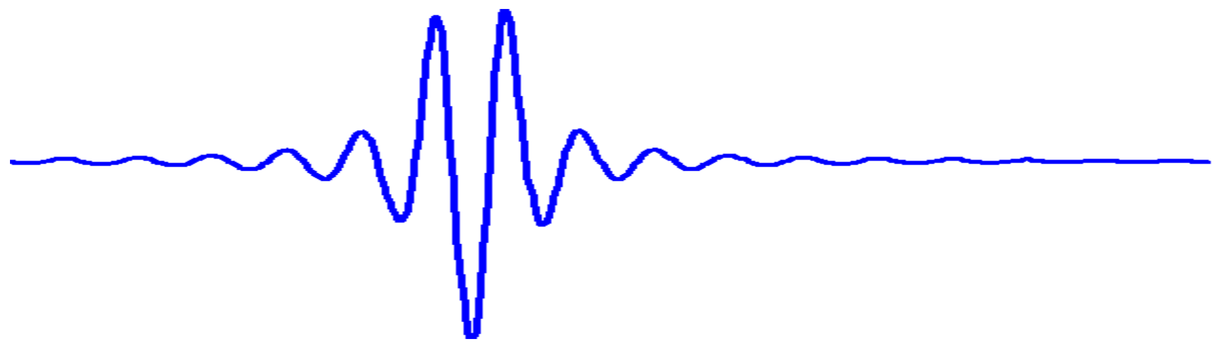
  - In real life all signals have finite energy, because they are related to some physical phenomenon, and as a consequence they have limited amplitude and duration. In fact, when the stimulant phenomenon stops the signal begins to decay.

Energy and Power Signals

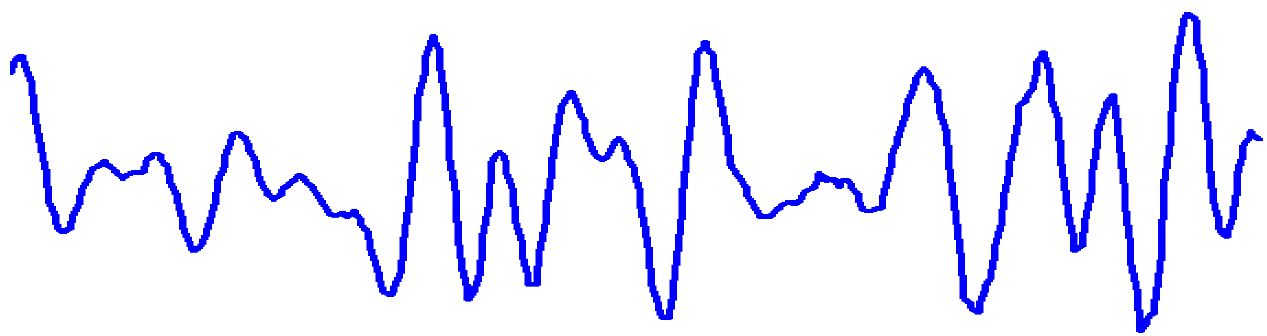


- **Energy and Power Signals**

Energy signal



Power signal



Types of signals

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– A signal can usually be described by one word from each row from the following:

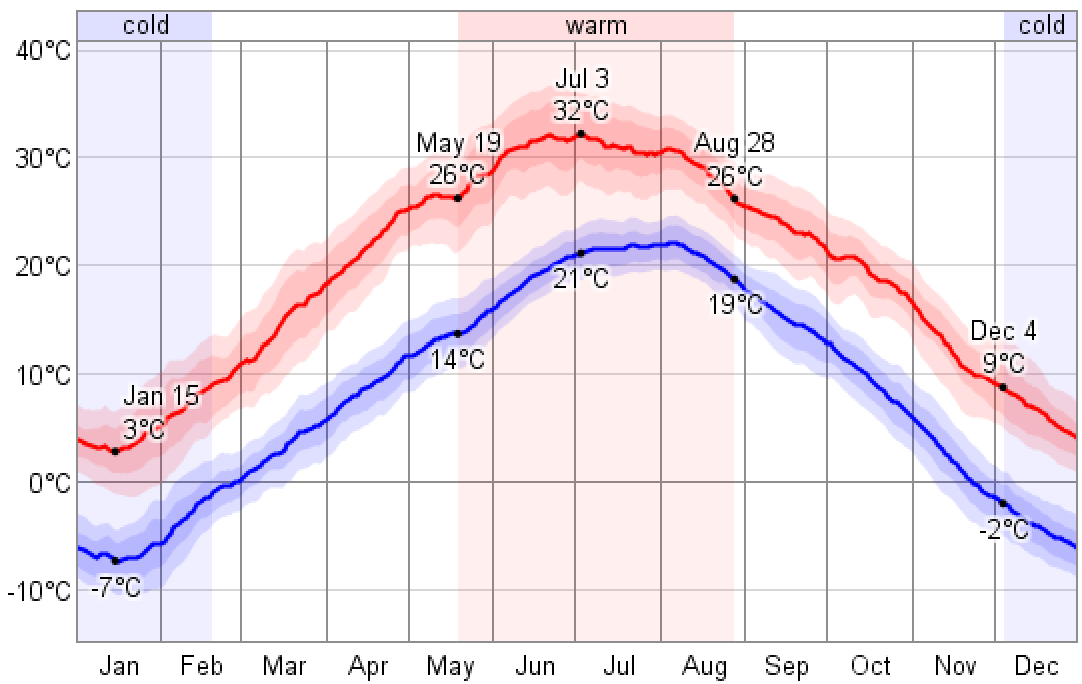
Continuous (Analogue)	$\longleftrightarrow$	Discrete (Digital)
Periodic	$\longleftrightarrow$	Aperiodic
Deterministic	$\longleftrightarrow$	Random
Finite energy	$\longleftrightarrow$	Finite power
Symmetric (Odd/Even)	$\longleftrightarrow$	Asymmetric

- Plot an example of a continuous-time periodic, deterministic, odd symmetric and finite power signal.

Test yourself

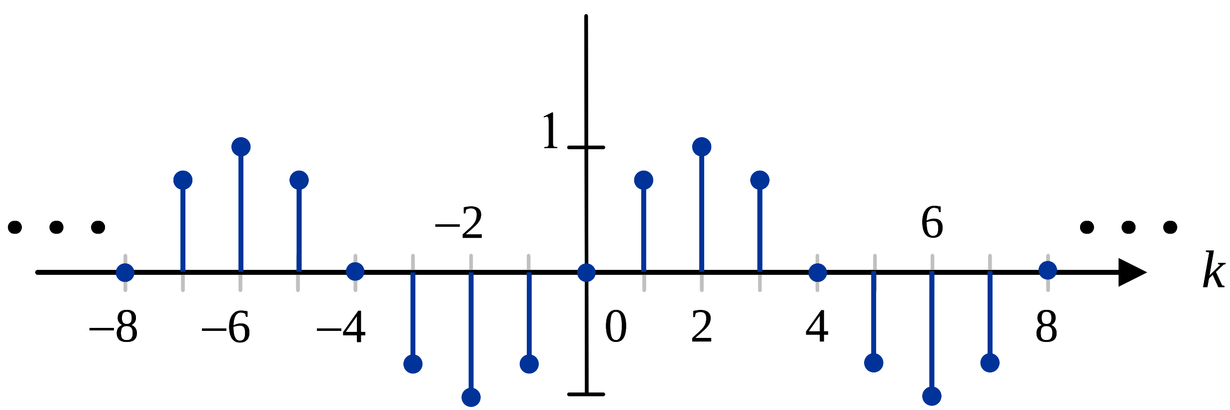
This one is a bit tricky, it depends on how you look at it.
The temperature information exists all the time, so it can be regards as continuous signal.
However, when you use a electronic instrument to measure it, the measured result is a discrete signal.

Continuous (Analogue) √	⟷	Discrete (Digital) √
Periodic	⟷	Aperiodic √
Deterministic	⟷	Random √
Finite energy	⟷	Finite power √
Symmetric (Odd/Even)	⟷	Asymmetric √



Test yourself

Continuous (Analogue)	⟷	Discrete (Digital)	√
Periodic √	⟷	Aperiodic	
Deterministic √	⟷	Random	
Finite energy	⟷	Finite power	√
Symmetric (Odd/Even) √	⟷	Asymmetric	



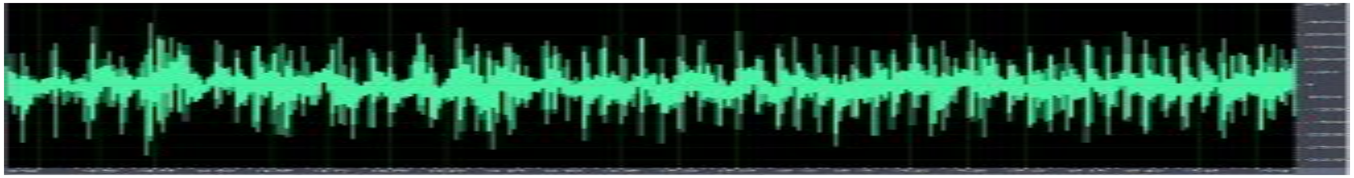
Signal Operations

Transformations of the independent variable

Test yourself

Listen to this sound…

f(t)

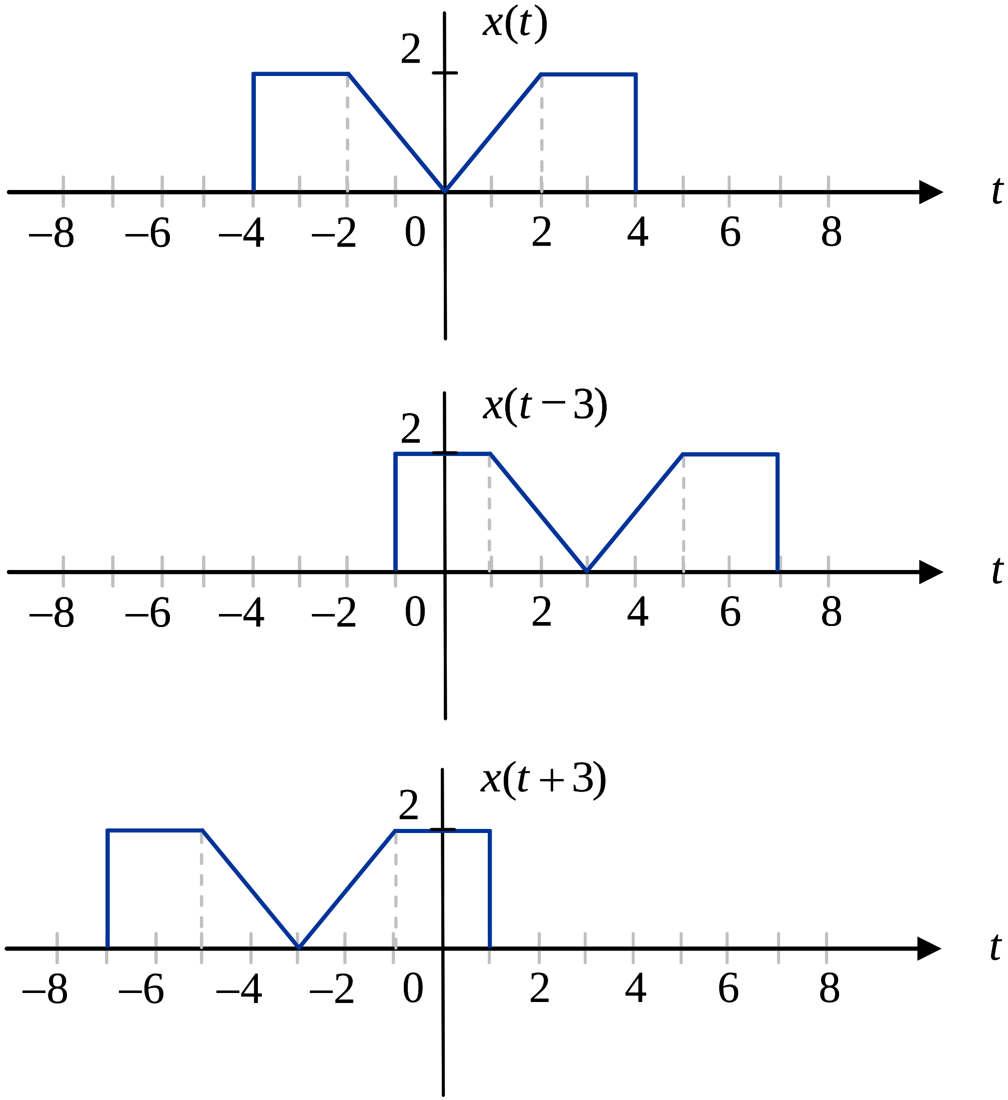


Which of the following describing function is correct?

Sound 1: $f\_2(t) = 3f(t)$

Sound 2: $f\_3(t) = f(0.5\ t)$

Time shifting

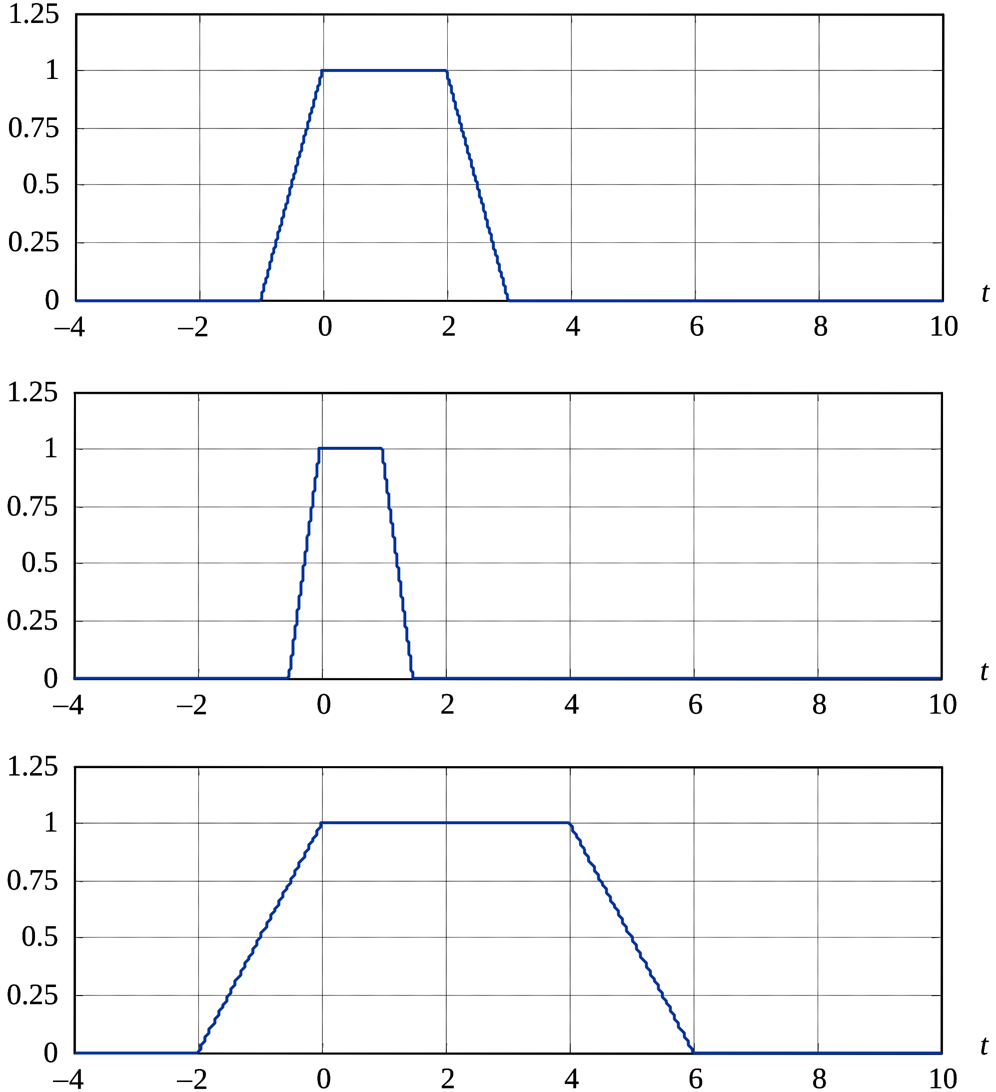


$$\phi(t) = x(t+T)$$

$T < 0$ shift to the right (delayed)

$T > 0$ shift to the left (advanced)

Time scaling



$$\phi(t) = x(ct)$$

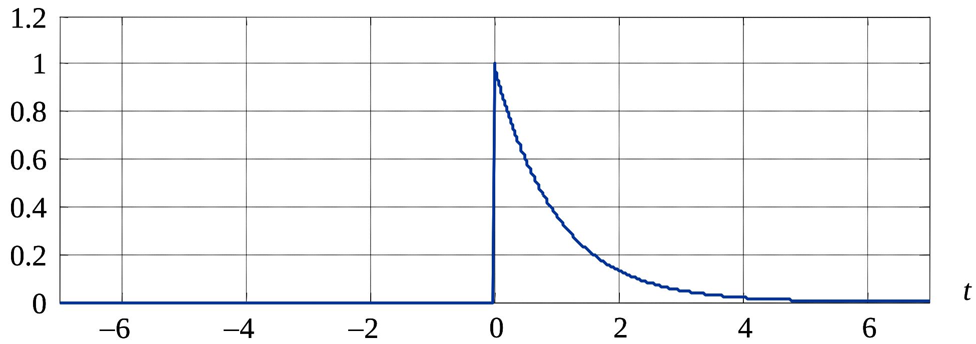
$c > 1$

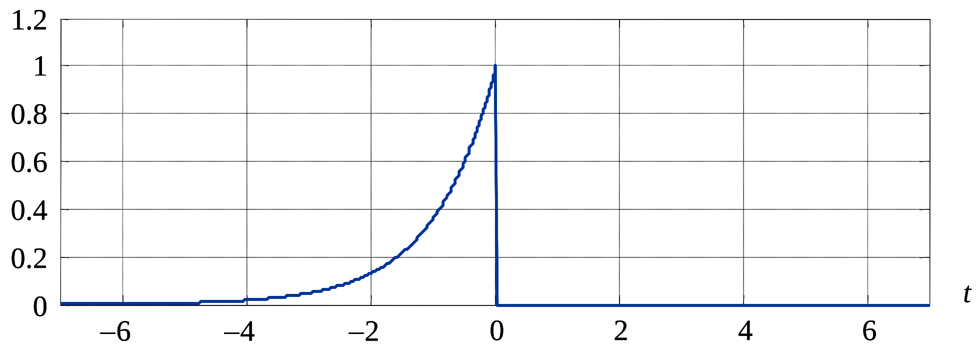
Signal is compressed

$0 < c < 1$

Signal is expanded

Time inversion





$$\phi(t) = x(-t)$$

Time inversion A.K.A.

Time reversal, reflection

Inversion is performed
About vertical axis

Combined operations

$$x(t) \Longrightarrow x(at-b)$$

(1) Time shift $x(t)$ by b to obtain $x(t-b)$
Time scale $x(t-b)$ *by* a *to obtain* $x(at-b)$

(2) Time scale $x(t)$ by a to obtain $x(at)$
Time shift $x(at)$ *by* $\frac{b}{a}$ *to obtain* $x(at-b)$

Example: $x(2t-6)$

Elementary signals

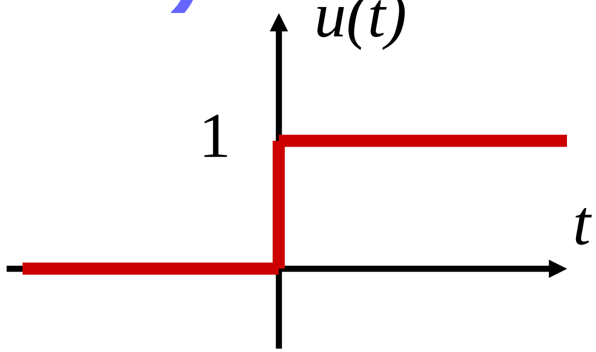
Elementary signals

- **Some elementary signals:**
  - Step Function.
  - Rectangular Pulse Function
  - Impulse Function.
  - Sinc Function.

  - Ramp Function.
  - Exponential Signals.
  - Sinusoidal Signals.

- **Step Function (unit step function)**

$$u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$$



  - This function is useful to evidence signal starting time.
  - Plot the following signals:

$$u(t-1)$$
$$u(t+1)$$
$$u^2(t)$$
$$u(t+0.5)\,u(t-0.5)$$

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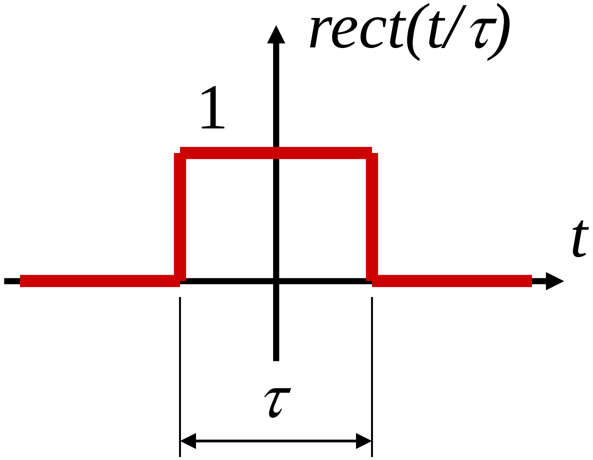
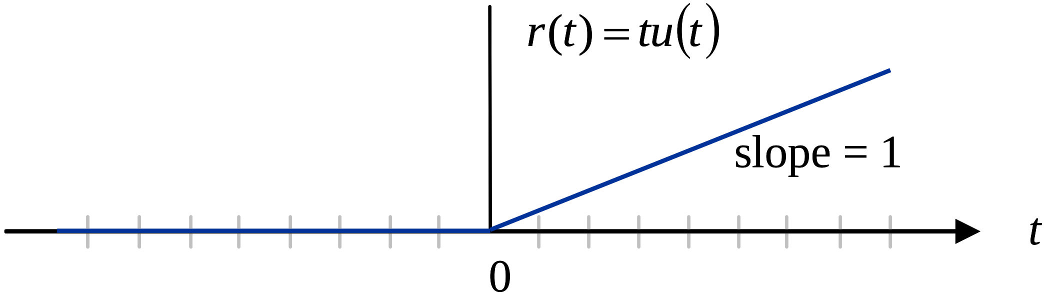
- **Rectangular pulse function (window function)**

$$rect\left(\frac{t}{\tau}\right) = \begin{cases} 1, & |t| \leq -\tau/2 \\ 0, & otherwise \end{cases}$$

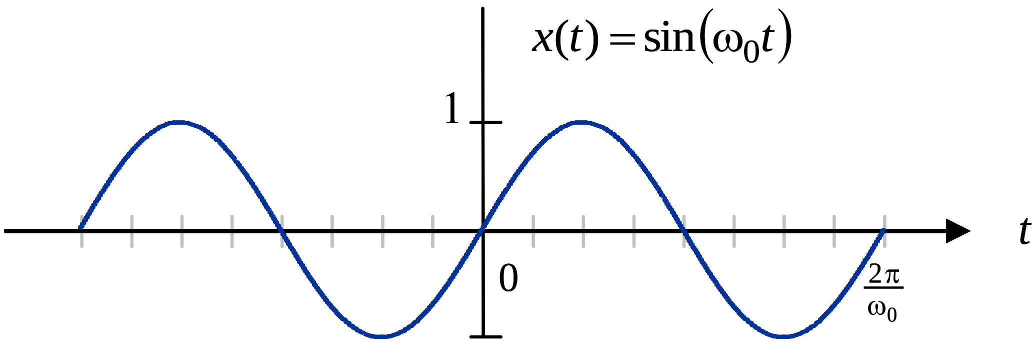


  - It is commonly used as windowing function
  - How it is possible to write *rect(t/τ)* using step function ?

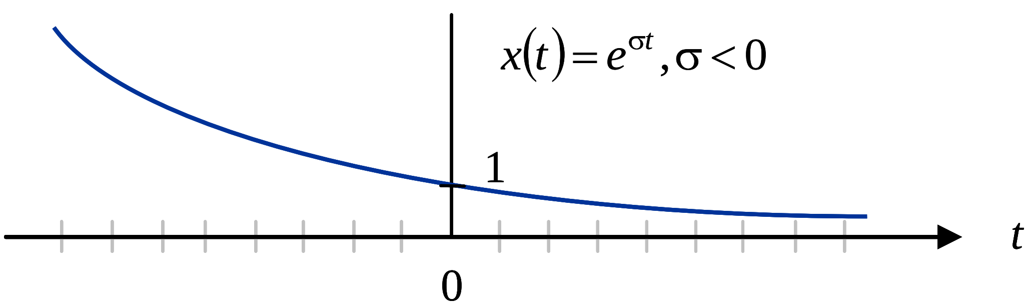
Elementary signals



– Ramp Function.



– Sinusoidal Signals.



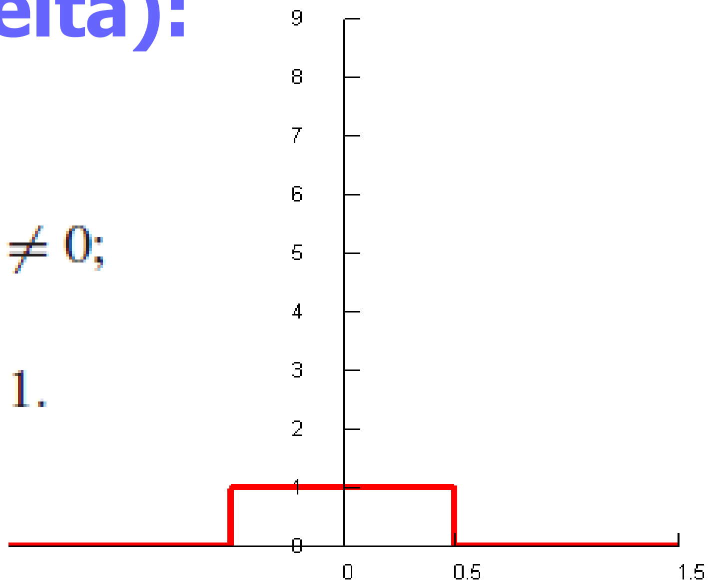
– Exponential Signals.

Elementary signals

- **Impulse Function (Dirac delta):**
  - Describes ideal impulses

(1) amplitude $\delta(t) = 0, \quad t \neq 0;$

(2) area enclosed $\int_{-\infty}^{\infty} \delta(t)\mathrm{d}t = 1.$



$x(t)\delta(t-t_0)=$? $\quad x(t_0)\delta(t-t_0)$

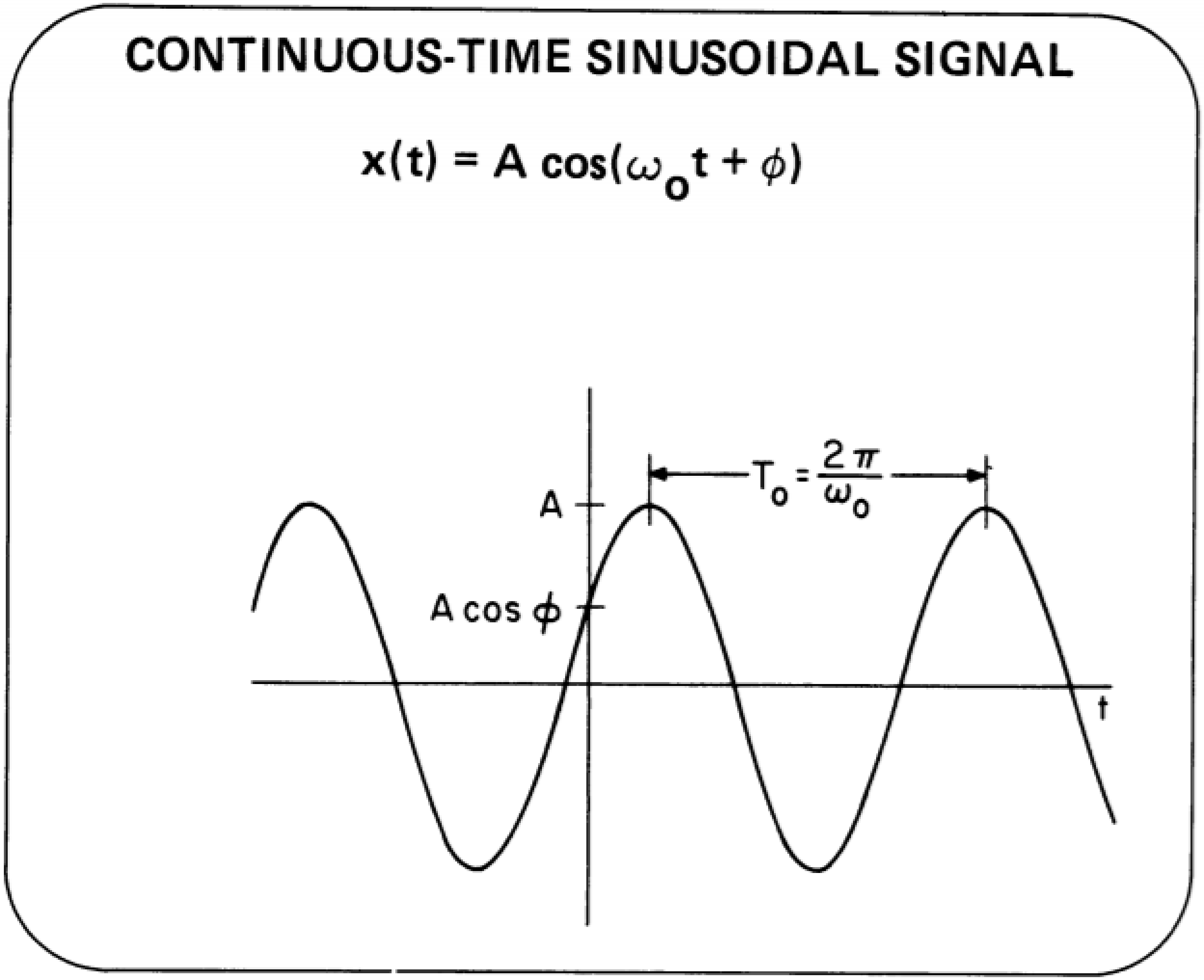
$\int_{-\infty}^{t} \delta(\tau)\ d\tau=$? $\quad u(t)$

$\int_{-\infty}^{\infty} x(t)\delta(t-t_0)\ dt=$? $\quad x(t_0)$

Physically realisable waveforms…

– Have finite time duration
– Are continuous
– Are real-valued
– *Occupy finite frequency spectrum !*

Elementary signals



TRANSPARENCY 2.1
Continuous-time sinusoidal signal indicating the definition of amplitude, frequency, and phase.

Elementary signals

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• Periodic:

$$x(t) = x(t + T_0) \quad \text{period} \triangleq \text{smallest } T_0$$

$$A\cos[\omega_0 t + \phi] = A\cos[\omega_0 t + \underbrace{\omega_0 T_0}_{2\pi m} + \phi]$$

$$T_0 = \frac{2\pi m}{\omega_0} \Rightarrow \text{period} = \frac{2\pi}{\omega_0}$$

• Time Shift <=> Phase Change

$$A\cos[\omega_0 (t + t_0)] = A\cos[\omega_0 t + \omega_0 t_0]$$

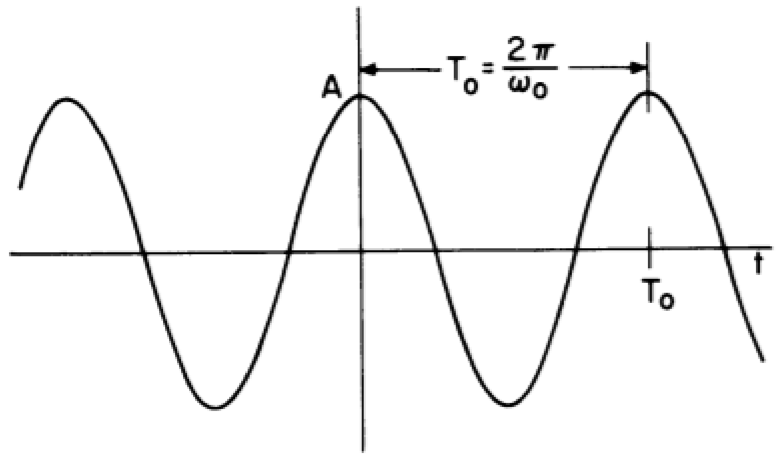
$$A\cos[\omega_0 (t + t_0) + \phi] = A\cos[\omega_0 t + \omega_0 t_0 + \phi]$$

TRANSPARENCY 2.2
Relationship between a time shift and a change in phase for a continuous-time sinusoidal signal.

Elementary signals

TRANSPARENCY 2.3
Illustration of the signal $A \cos \omega_0 t$ as an even signal.

$\phi = 0 \qquad x(t) = A \cos \omega_0 t$



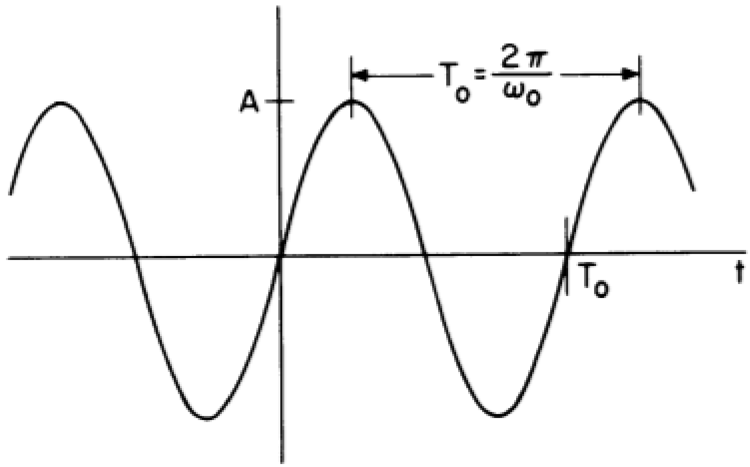
Periodic: $x(t) = x(t + T_0)$

Even: $x(t) = x(-t)$

Elementary signals

TRANSPARENCY 2.4
Illustration of the signal $A \sin \omega_0 t$ as an odd signal.

$$\phi = -\frac{\pi}{2} \qquad x(t) = \begin{cases} A\cos(\omega_0 t - \frac{\pi}{2}) \\ A \sin \omega_0 t \\ A\cos[\omega_0(t - \frac{T_0}{4})] \end{cases}$$



Periodic: $x(t) = x(t + T_0)$

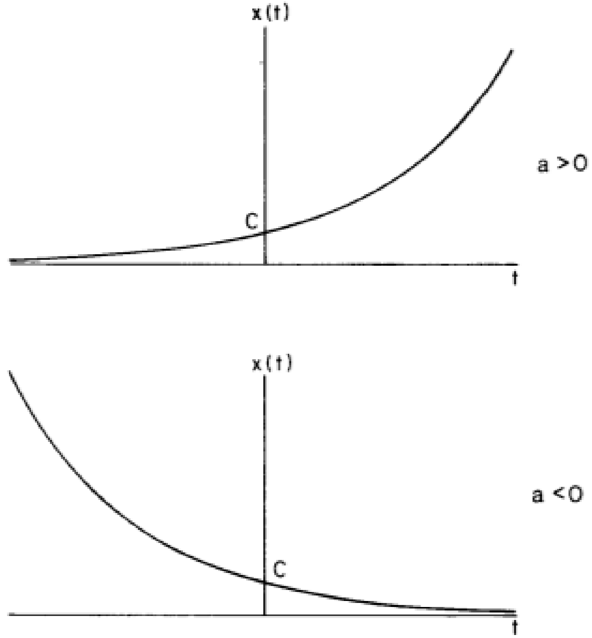
Odd: $x(t) = -x(-t)$

Elementary signals

REAL EXPONENTIAL: CONTINUOUS-TIME

$x(t) = Ce^{at}$

C and a are real numbers



Time Shift <=> Scale Change

$Ce^{a(t+t_0)} = Ce^{at_0}\, e^{at}$

TRANSPARENCY 2.14
Illustration of continuous-time real exponential signals.

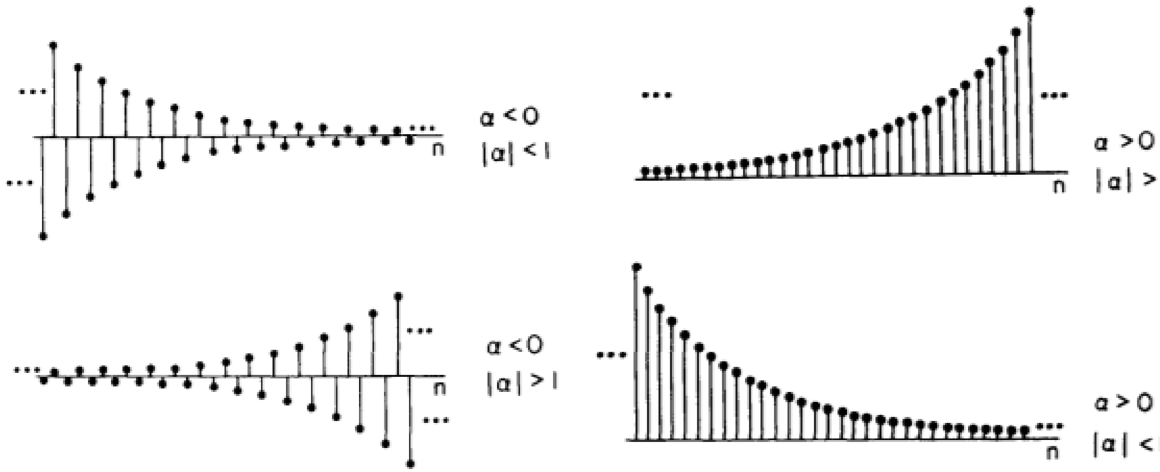
Elementary signals

TRANSPARENCY 2.15
Illustration of discrete-time real exponential sequences.

REAL EXPONENTIAL: DISCRETE-TIME

$$x[n] = Ce^{\beta n} = C\alpha^n$$

C, α are real numbers



Elementary signals

TRANSPARENCY 2.16
Continuous-time complex exponential signals and their relationship to sinusoidal signals.

COMPLEX EXPONENTIAL: CONTINUOUS-TIME

$$x(t) = Ce^{at}$$

C and a are complex numbers

$$C = |C|\, e^{j\theta}$$

$$a = r + j\omega_o$$

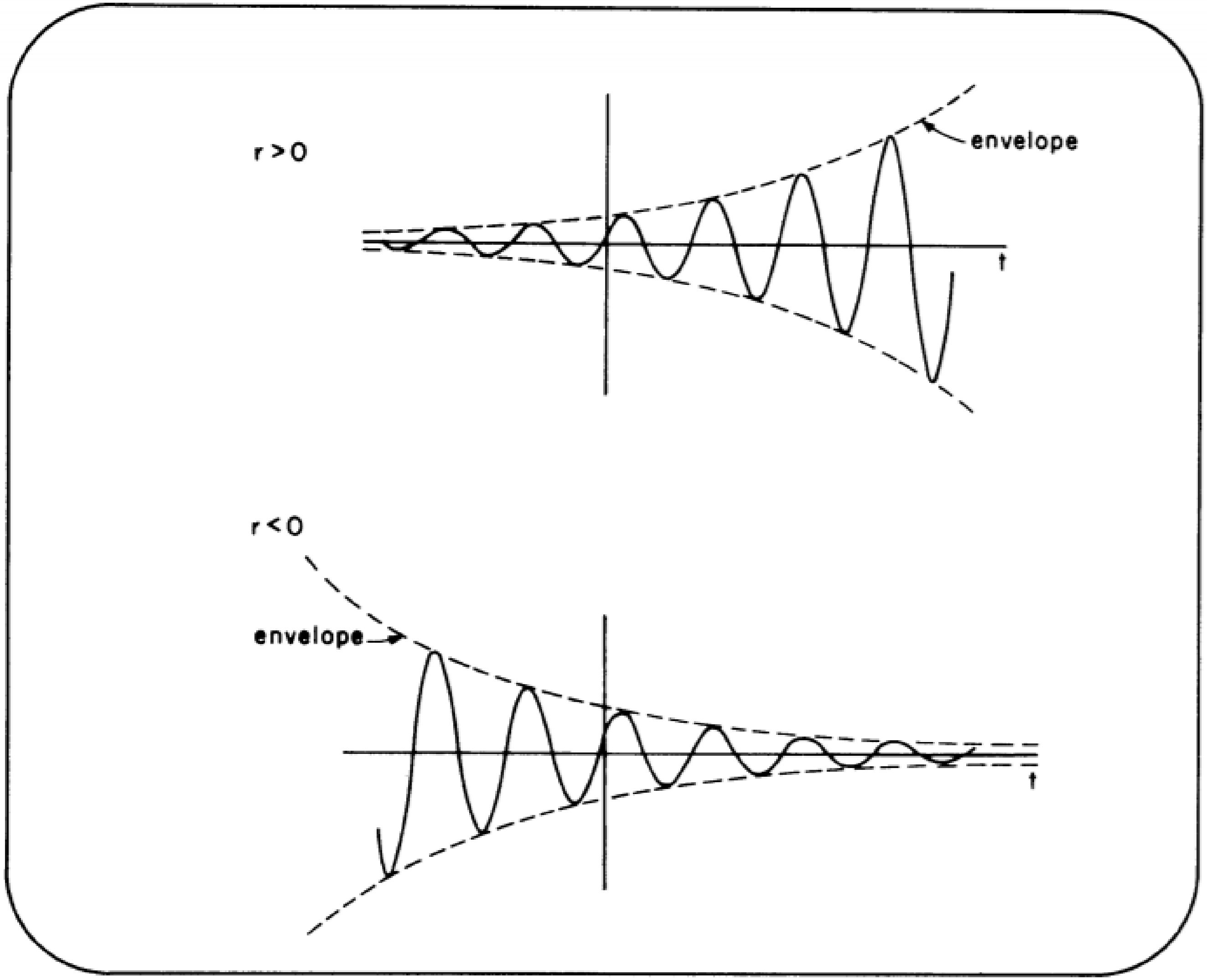
$$x(t) = |C|\, e^{j\theta}\, e^{(r + j\omega_o)t}$$

$$= |C|\, e^{rt}\, \underbrace{e^{j(\omega_o t + \theta)}}$$

Euler's Relation: $\cos(\omega_o t + \theta) + j\sin(\omega_o t + \theta) = e^{j(\omega_o t + \theta)}$

$$x(t) = |C|\, e^{rt}\cos(\omega_o t + \theta) + j\,|C|\, e^{rt}\sin(\omega_o t + \theta)$$

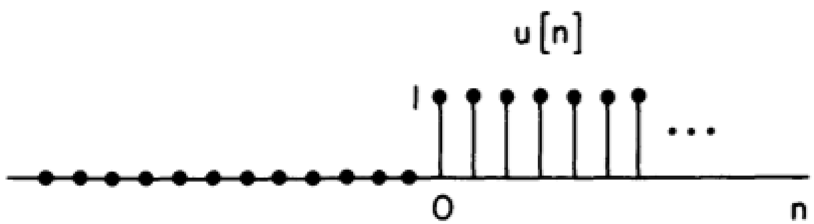
Elementary signals



TRANSPARENCY 2.17
Sinusoidal signals with exponentially growing and exponentially decaying envelopes.

Elementary signals

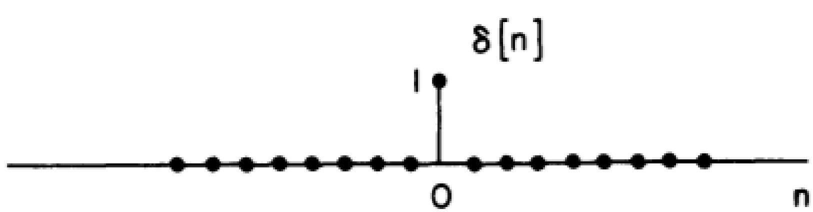
UNIT STEP FUNCTION: DISCRETE-TIME



$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

UNIT IMPULSE FUNCTION: DISCRETE-TIME

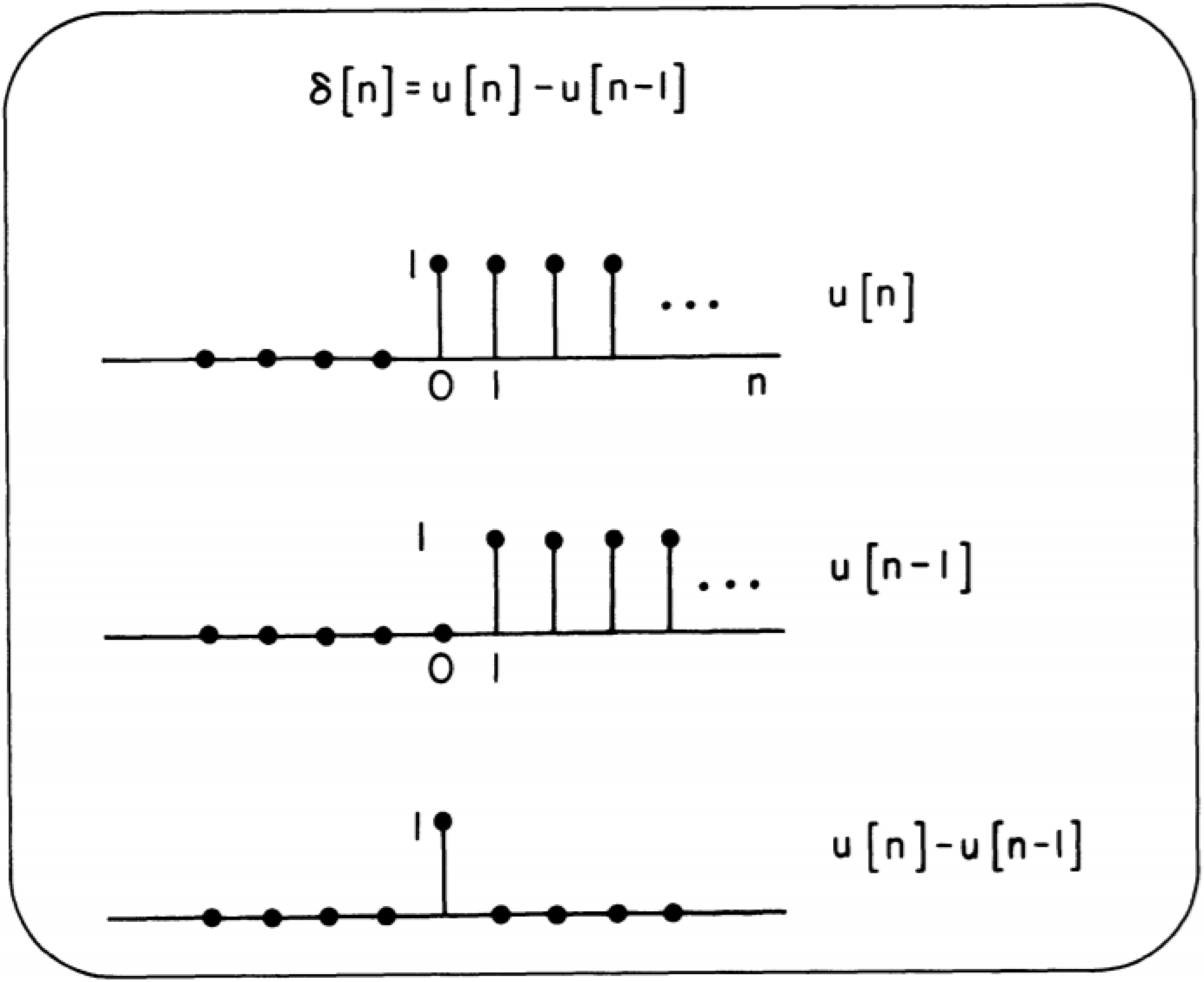
(Unit Sample)



$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

TRANSPARENCY 3.1
Discrete-time unit step and unit impulse sequences.

Elementary signals

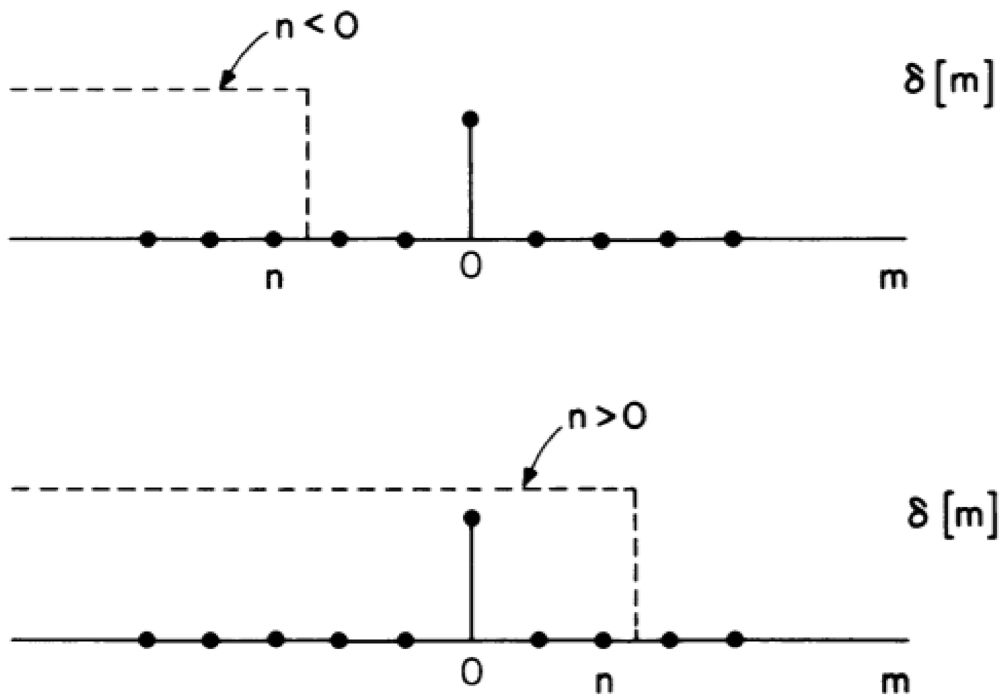


TRANSPARENCY 3.2
The unit impulse sequence as the first backward difference of the unit step sequence.

Elementary signals

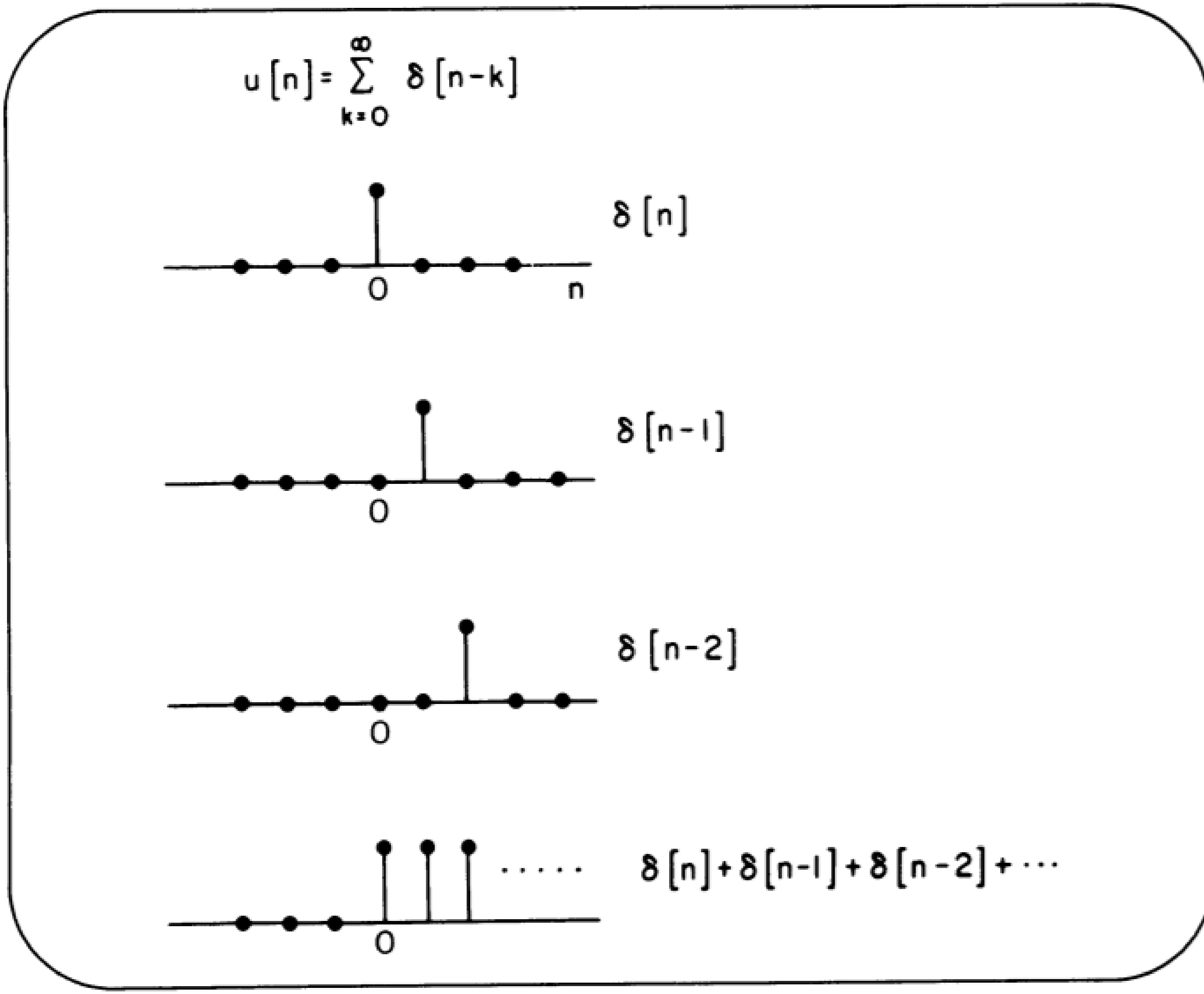
TRANSPARENCY 3.3
The unit step sequence as the running sum of the unit impulse.

$$u[n] = \sum_{m=-\infty}^{n} \delta[m]$$



Elementary signals

TRANSPARENCY 3.4
The unit step sequence expressed as a superposition of delayed unit impulses.



Elementary signals

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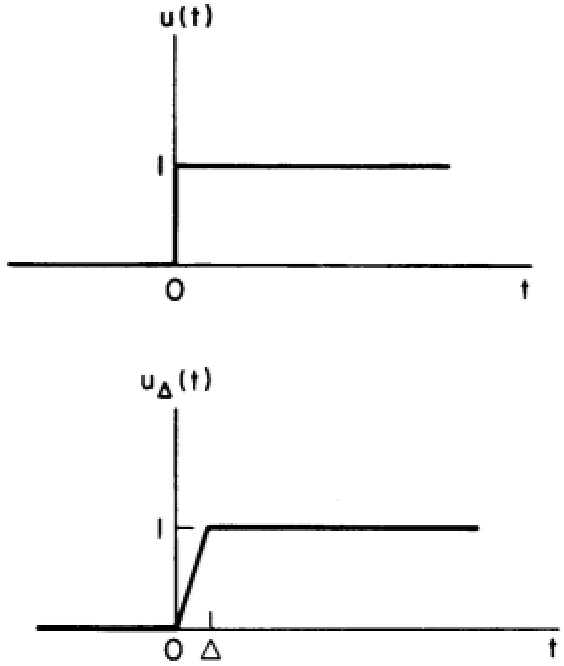
UNIT STEP FUNCTION: CONTINUOUS-TIME

$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}$$



$$u(t) = u_\Delta(t) \text{ as } \Delta \to 0$$

TRANSPARENCY 3.5
The continuous-time unit step function.

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UNIT IMPULSE FUNCTION

$$\delta(t) = \frac{du(t)}{dt}$$

$$\delta_{\triangle}(t) = \frac{du_{\triangle}(t)}{dt}$$

$$\delta(t) = \delta_{\triangle}(t) \text{ as } \triangle \rightarrow 0$$

TRANSPARENCY 3.6
The definition of the unit impulse as the derivative of the unit step.

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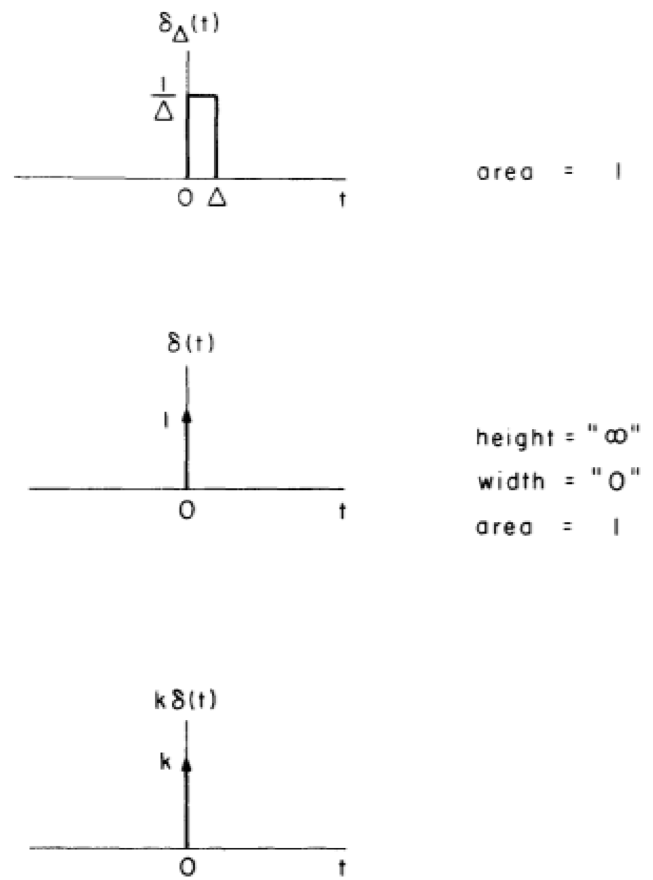
TRANSPARENCY 3.7
Interpretation of the continuous-time unit impulse as the limiting form of a rectangular pulse which has unit area and for which the pulse width approaches zero.

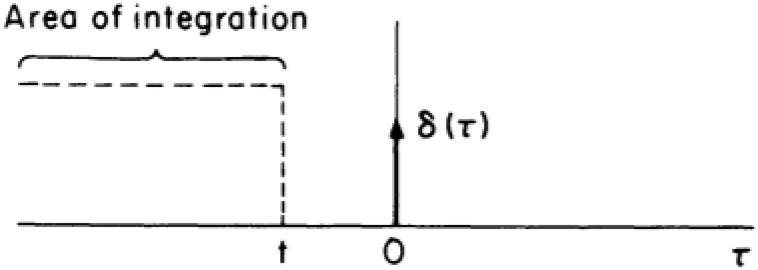


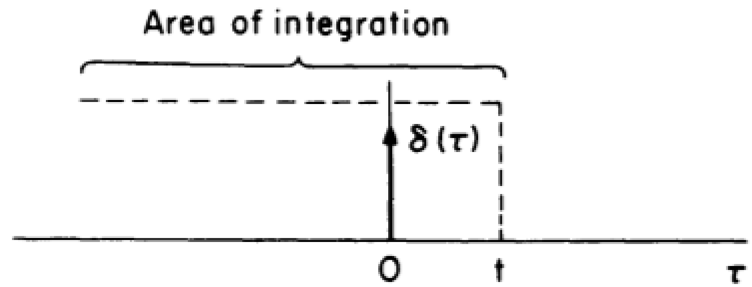
Elementary signals

TRANSPARENCY 3.8
The unit step expressed as the running integral of the unit impulse.

$$\delta(t) = \frac{du(t)}{dt}$$

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau$$





Summary

- Concept of signals
- Signal classification
- Signal operation
- Elementary signals

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Textbook Page 57

- **1.3**
- **1.6**
- **1.21**

1.3. Determine the values of P_∞ and E_∞ for each of the following signals:

(a) $x_1(t) = e^{-2t}u(t)$ **(b)** $x_2(t) = e^{j(2t+\pi/4)}$ **(c)** $x_3(t) = \cos(t)$

(d) $x_1[n] = (\frac{1}{2})^n u[n]$ **(e)** $x_2[n] = e^{j(\pi/2n+\pi/8)}$ **(f)** $x_3[n] = \cos(\frac{\pi}{4}n)$

1.6. Determine whether or not each of the following signals is periodic:

(a) $x_1(t) = 2e^{j(t+\pi/4)}u(t)$ **(b)** $x_2[n] = u[n] + u[-n]$

(c) $x_3[n] = \sum_{k=-\infty}^{\infty}\{\delta[n-4k] - \delta[n-1-4k]\}$

Exercises

Determine if the following CT signals are even, odd, or neither even nor odd. In the latter case, evaluate and sketch the even and odd components of the CT signals:

(i) $x1(t) = 2\sin(2\pi t)[2 + \cos(4\pi t)]$;

(ii) $x2(t) = t^2 + \cos(3t)$;

(iii) $x3(t) = \exp(-3t)\sin(3\pi t)$;

(iv) $x4(t) = t\sin(5t)$;

(v) $x5(t) = tu(t)$;

(vi) $$x6(t) = \begin{cases} 3t & 0 \le t < 2 \\ 6 & 2 \le t < 4 \\ 3(-t+6) & 4 \le t \le 6 \\ 0 & \text{elsewhere.} \end{cases}$$