EEE225 Advanced Electrical Circuits and Electromagnetics

Lecture 14 Magnetically coupled circuits

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- Self-inductance
- Mutual inductance
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  - Induced voltage
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  - Dot convention
  - Total energy stored
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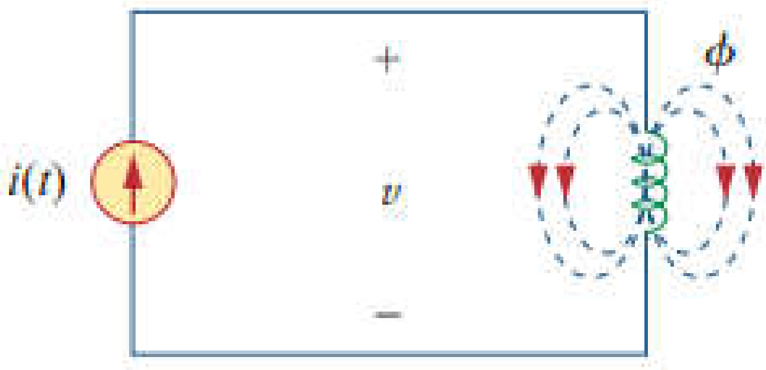
Self-Inductance

- Consider a coil consisting of N turns and carrying current i.
- If current is steady, magnetic flux through the loop remains constant. If i changes with time, then an induced emf arises to **oppose** the change.
- The property of the loop in which its own magnetic field opposes any change in current is called "***self-inductance***," and the emf generated is called the ***self-induced emf*** or ***back emf***.



The self-induced emf:

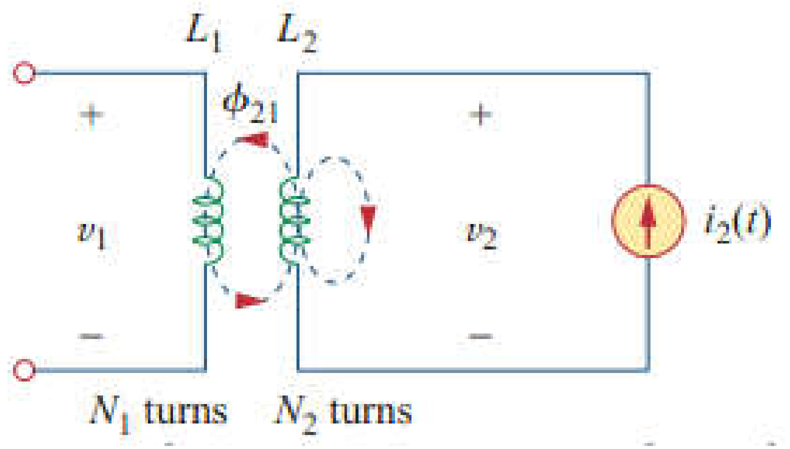
$$v = N\frac{d\Phi}{dt} = L\frac{di}{dt}$$

where the self-inductance:

$$L = N\frac{d\Phi}{di}$$

The SI unit is the Henry (H)

Mutual Inductance



- Suppose two coils are placed near each other. Some of the magnetic field lines through coil 1 will also pass through coil 2.

- Let Φ_{21} denote the magnetic flux through one turn of coil 2 due to I_1. By varying I_1 with time, there will be an induced e.m.f. associated with the changing magnetic flux in coil 2:

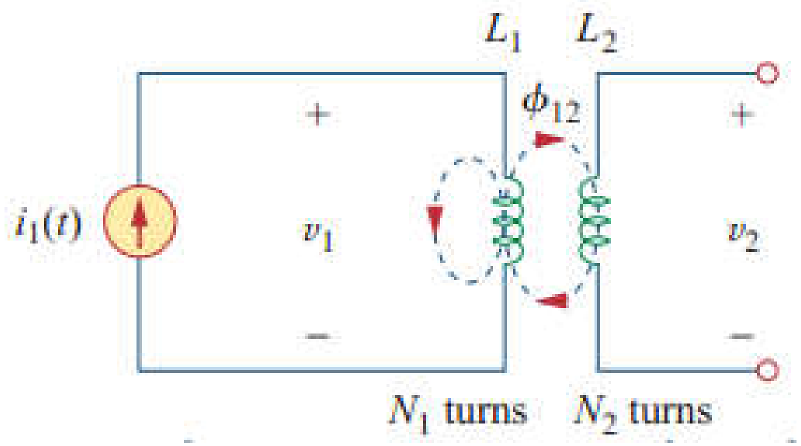
$$v_2 = N_2 \frac{d\Phi_{21}}{dt}$$

- The rate of change of Φ_{21} in coil 2 is proportional to the time rate of the change of the current in coil 1:

$$N_2 \frac{d\Phi_{21}}{dt} = M_{21} \frac{dI_1}{dt}$$

Mutual Inductance

- Similarly, the induced emf in coil 1 due to current change in coil 2: $v_1 = N_1 \dfrac{d\Phi_{12}}{dt}$



- This changing flux in coil 1 is also propor -tional to the changing current in coil 2:

$$N_1 \frac{d\Phi_{12}}{dt} = M_{12} \frac{dI_2}{dt}$$

- The proportionality constant M_{12} and M_{21} are equal:

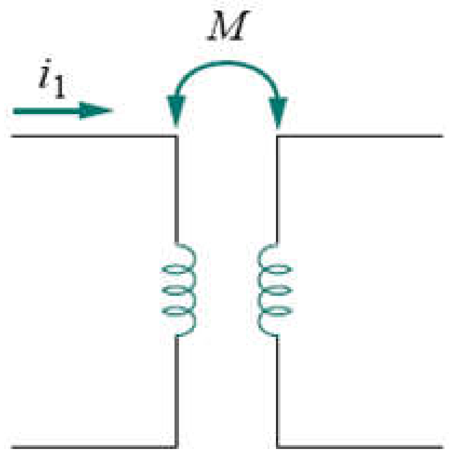
$$\underline{\underline{M_{12} = M_{21} \equiv M}}$$

coefficient of mutual inductance, or simply mutual inductance.

- Mutual inductance is the ability of one inductor to induce a voltage across a neighboring inductor, measured in henrys (H). It is always a positive quantity

Mutual Inductance

The existence of mutual coupling between two coils is indicated by a double-headed arrow.



- Mutual coupling only exists when the inductors or coils are in close proximity, and the circuits are driven by time-varying sources.

Mutual Inductance - Factors Affecting

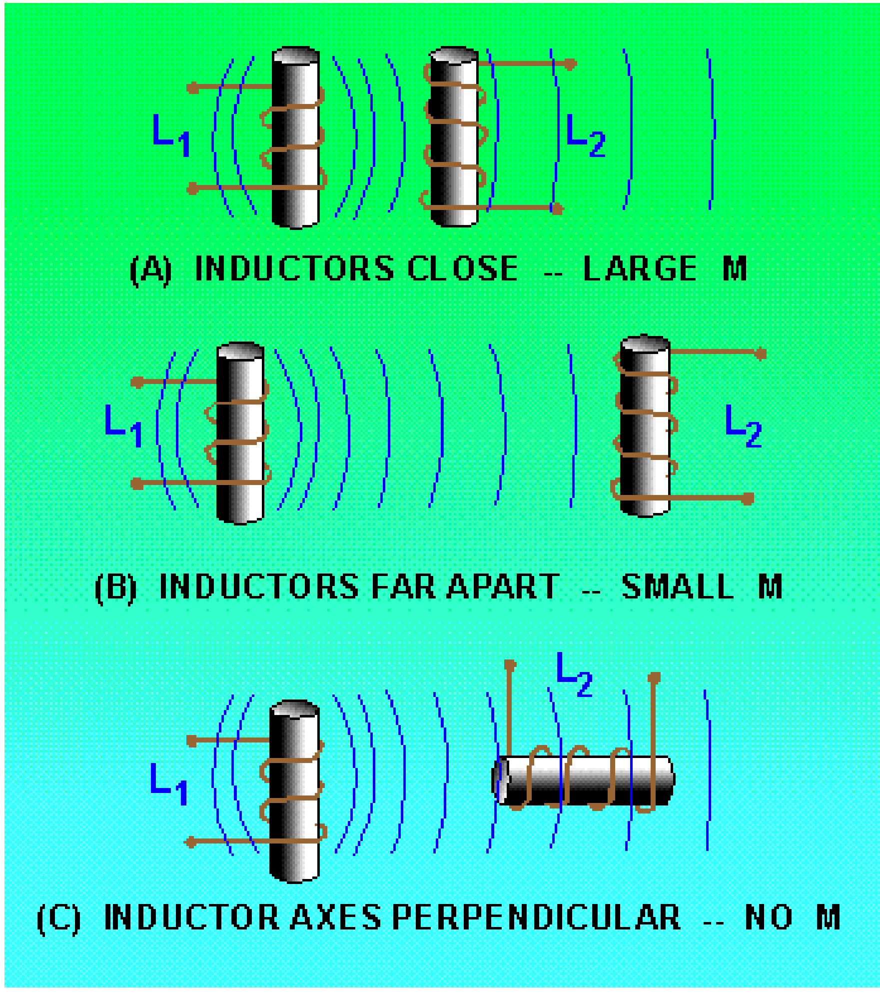
- **Factors affecting the self-inductance:**
  - Number of turns, Diameter of Coil, Coil Length, Core material, Number of layers of windings in the coil.

- **Factors affecting the mutual-inductance:**

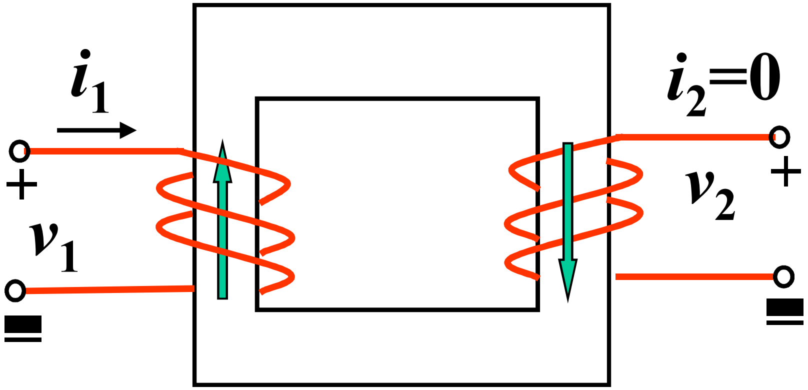
Factors affecting the self-inductance

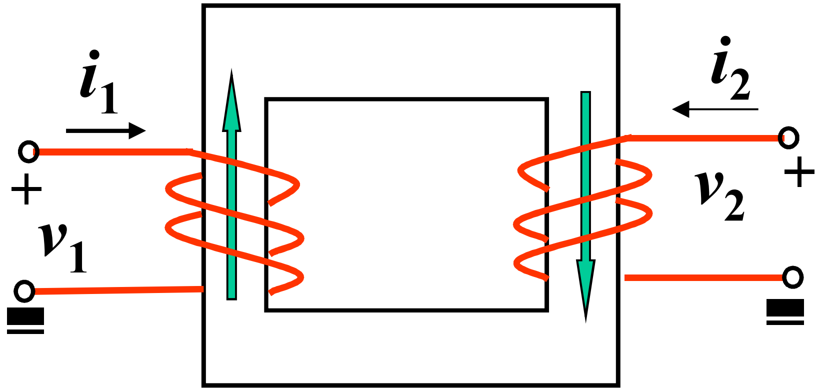
+

the relative positions of the two coils: distance between coils, relative positions of axes of coils, …



Combined Mutual & Self-Induction Voltage





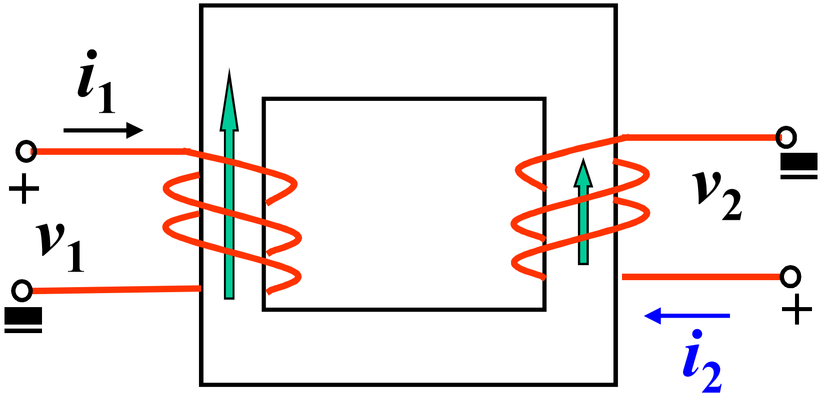
$$v_1(t) = L_1 \frac{di_1(t)}{dt}$$

$$v_2(t) = M \frac{di_1(t)}{dt}$$

$$v_1(t) = L_1 \frac{di_1(t)}{dt} + M \frac{di_2(t)}{dt}$$

$$v_2(t) = L_2 \frac{di_2(t)}{dt} + M \frac{di_1(t)}{dt}$$

Combined Mutual & Self-Induction Voltage

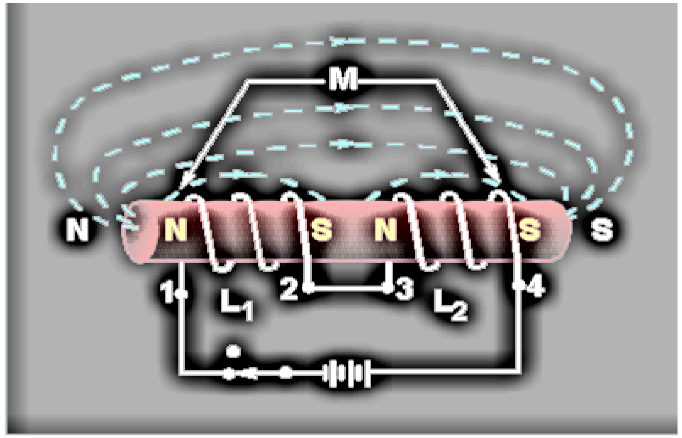


$$v_1(t) = L_1 \frac{di_1(t)}{dt} + M \frac{di_2(t)}{dt}$$

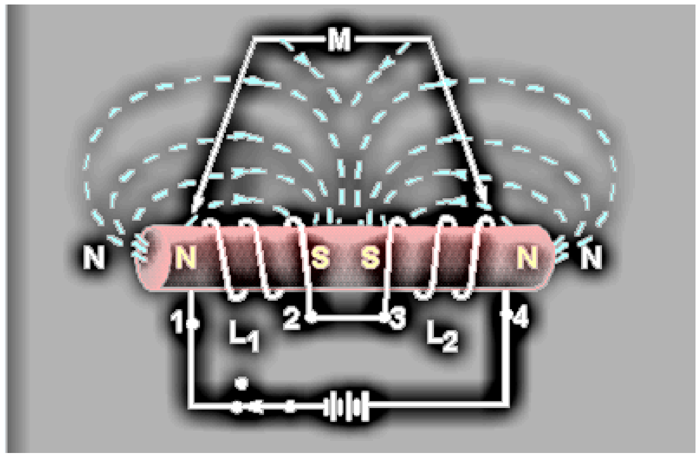
$$v_2(t) = L_2 \frac{di_2(t)}{dt} + M \frac{di_1(t)}{dt}$$

$$v_1(t) = L_1 \frac{di_1(t)}{dt} - M \frac{di_2(t)}{dt}$$

$$v_2(t) = L_2 \frac{di_2(t)}{dt} - M \frac{di_1(t)}{dt}$$



Series inductors with aiding fields



Series inductors with opposing fields

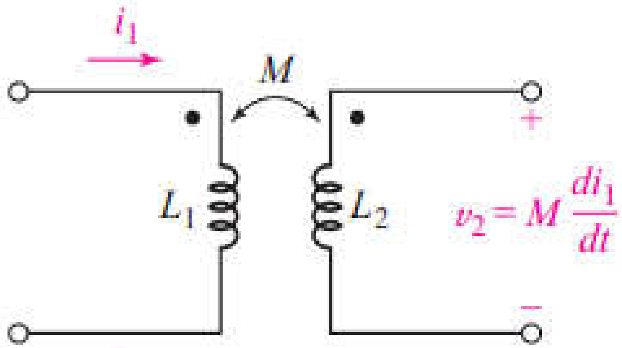
Dot convention

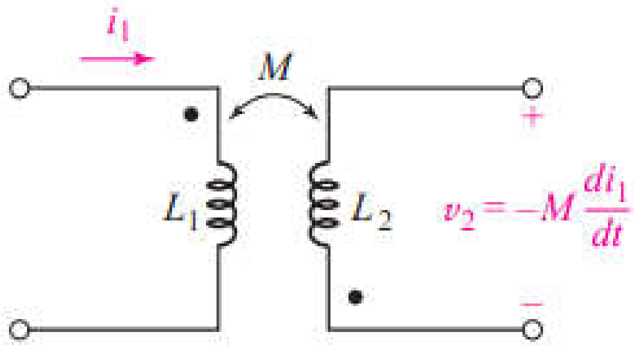
- Practically, we won't draw how the wire is winded around the core, so a simpler way is needed to show the direction of the two coupled windings => Dot convention.
- A dot is placed in the circuit at one end of each of the two magnetically coupled coils to indicate the direction of the magnetic flux if current enters that dotted terminal of the coil.

If a current **enters** the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is **positive** at the dotted terminal of the second coil.	If a current **leaves** the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is **negative** at the dotted terminal of the second coil.

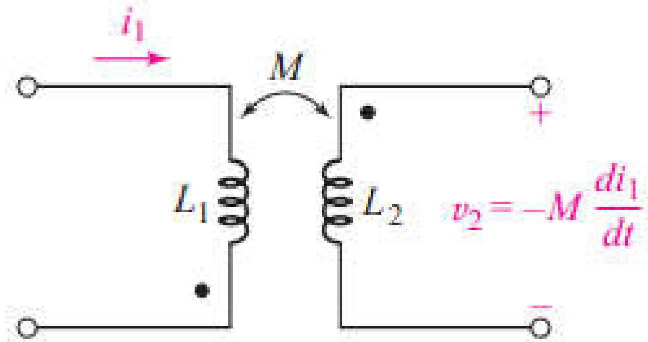
Dot convention

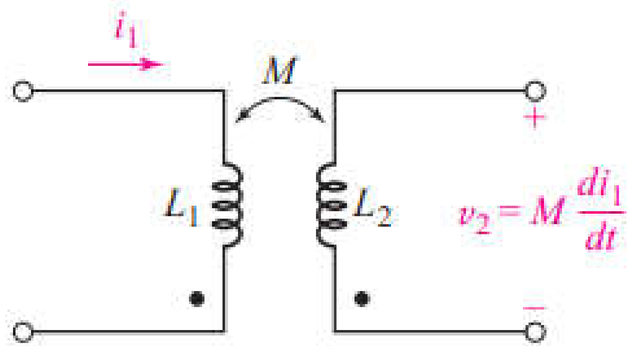
If a current **enters** the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is **positive** at the dotted terminal of the second coil.





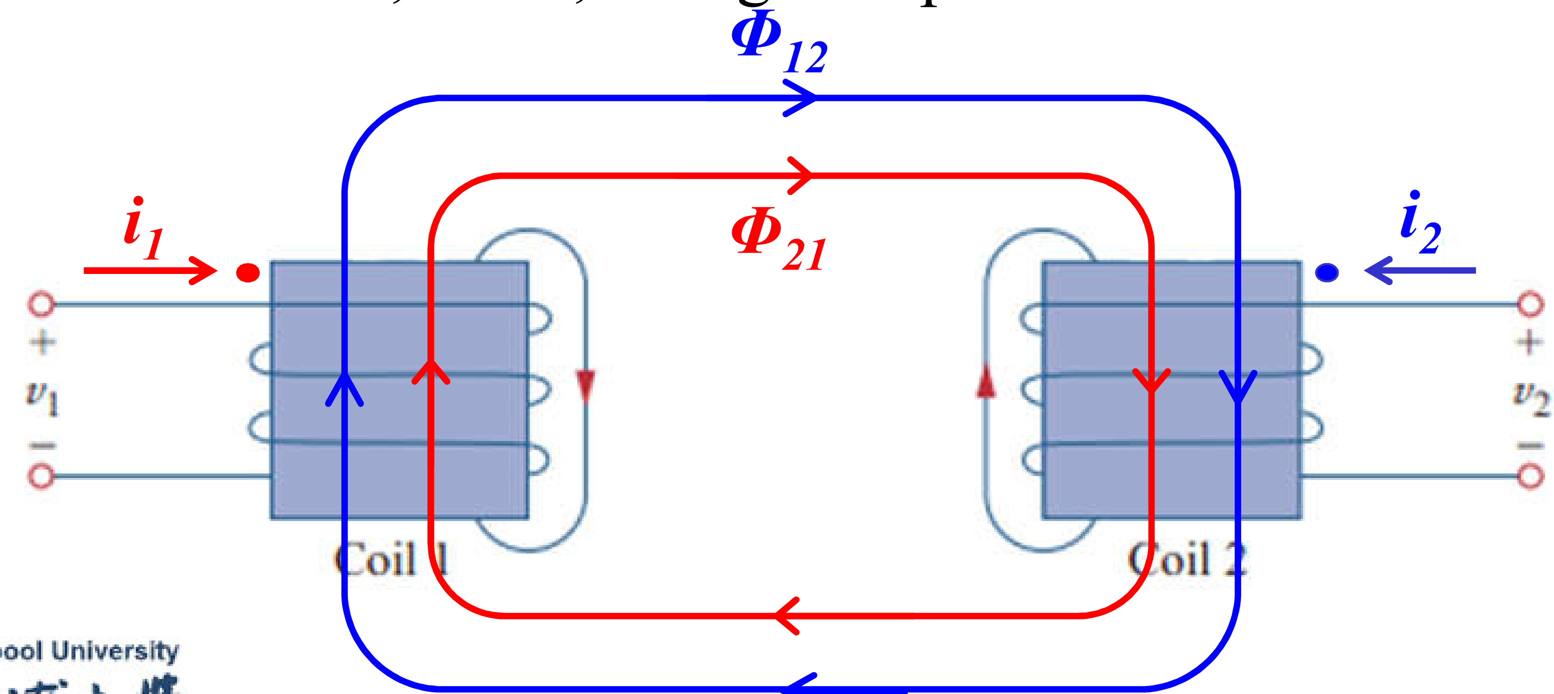
If a current **leaves** the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is **negative** at the dotted terminal of the second coil.





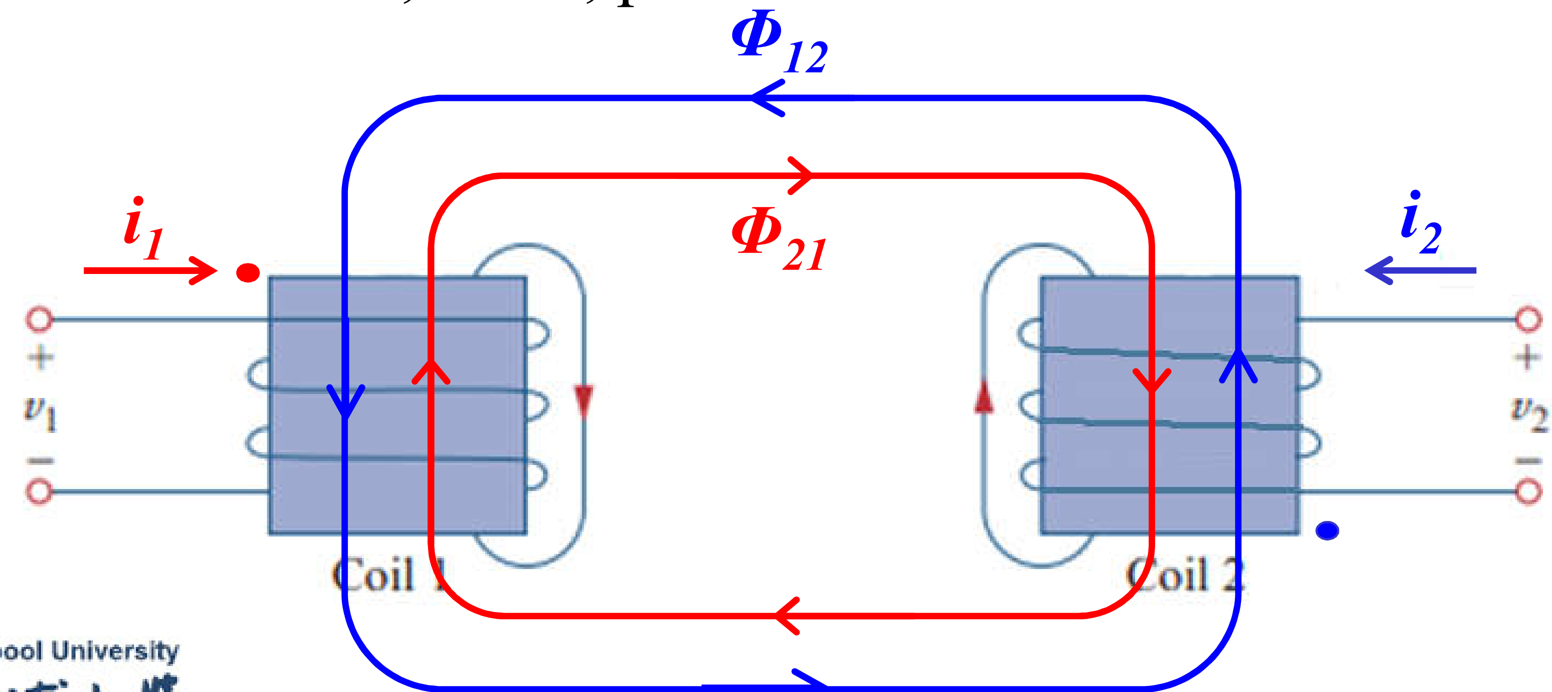
Dot convention – *place the dot*

- 1. Assume a dot at arbitrary terminal of coil 1, currents i_1 enters the dot;
- 2. Select one arbitrary terminal of coil 2, currents i_2 enters the terminal;
- 3. Analyse the direction of Φ_{21} and Φ_{12} based on the direction of i_1 and i_2 and the winding of coil 1 and 2;
- 4. If Φ_{21} and Φ_{12} have the same direction, then the assumed dots' positions are correct; if not, change the position of one of the dots.

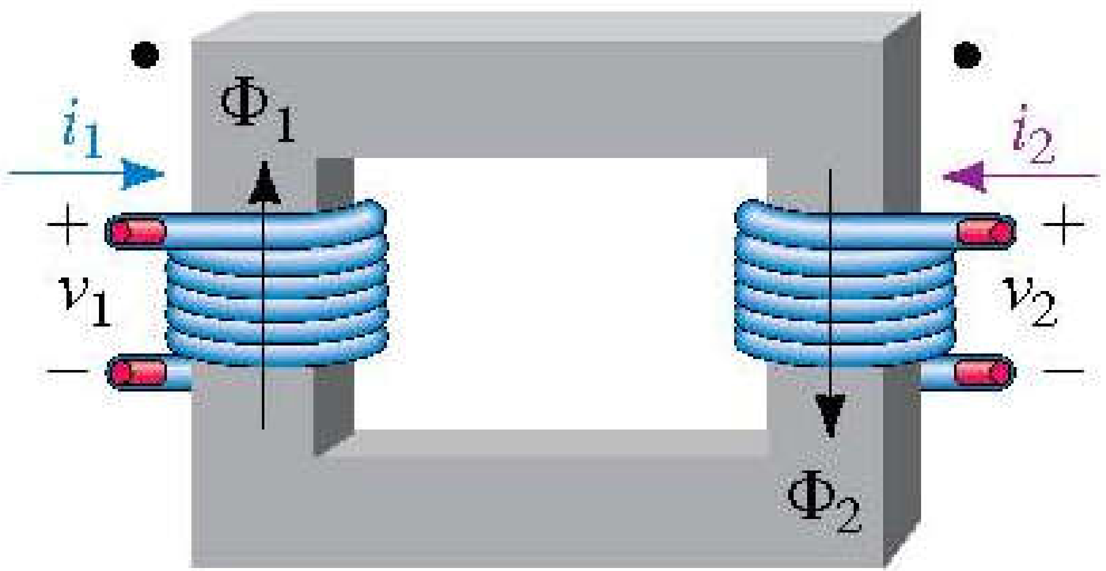


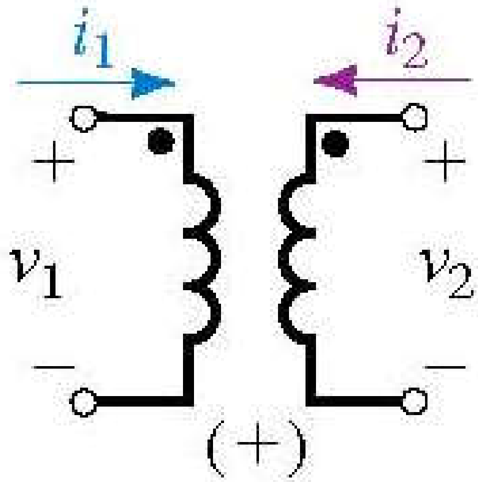
Dot convention – *place the dot*

- 1. Assume a dot at arbitrary terminal of coil 1, currents i_1 enters the dot;
- 2. Select one arbitrary terminal of coil 2, currents i_2 enters the terminal;
- 3. Analyse the direction of Φ_{21} and Φ_{12} based on the direction of i_1 and i_2 and the winding of coil 1 and 2;
- 4. If Φ_{21} and Φ_{12} have the same direction, then place the dot of coil 2 at the selected terminal; if not, place the dot of coil 2 at the other terminal.



Dot convention – Place the dot

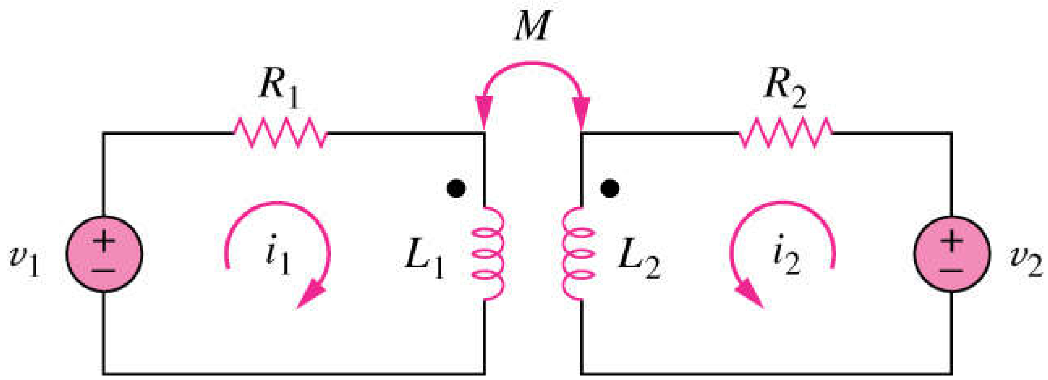




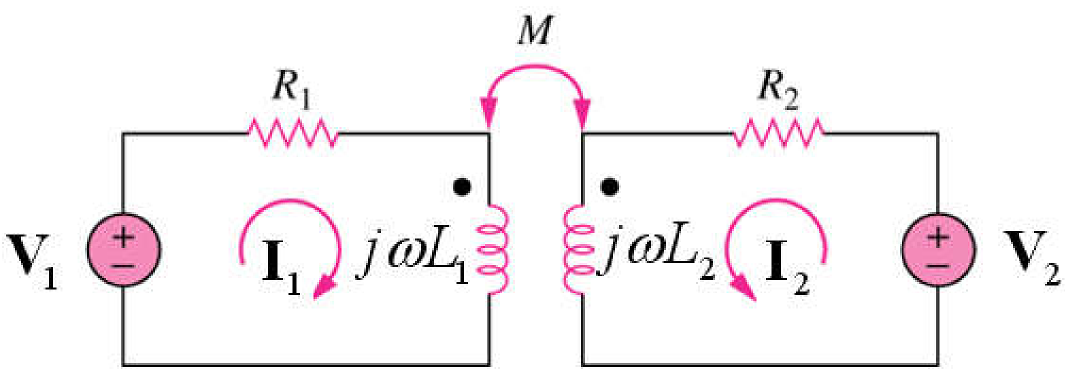
$$v_1 = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$$

$$v_2 = M\frac{di_1}{dt} + L_2\frac{di_2}{dt}$$

Dot convention – Time domain and Frequency domain



a) Time-domain circuit



b) Frequency-domain circuit

Time Domain

$$v_1 = i_1 R_1 + L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

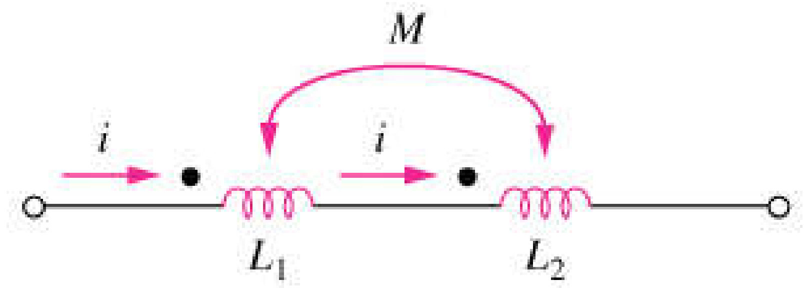
$$v_2 = i_2 R_2 + L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

Frequency Domain

$$\mathbf{V}_1 = (R_1 + j\omega L_1)\mathbf{I}_1 + j\omega M\mathbf{I}_2$$

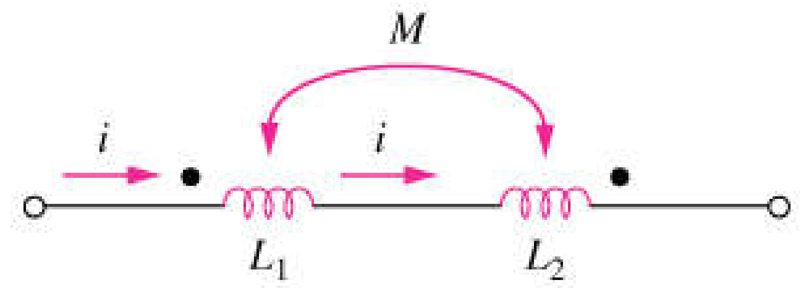
$$\mathbf{V}_2 = (R_2 + j\omega L_2)\mathbf{I}_2 + j\omega M\mathbf{I}_1$$

Dot convention - Two coupled coils in series



a) Series-aiding connection

$L_{aidding} = L_1 + L_2 + 2M$



b) Series-opposing connection

$L_{opposing} = L_1 + L_2 - 2M$

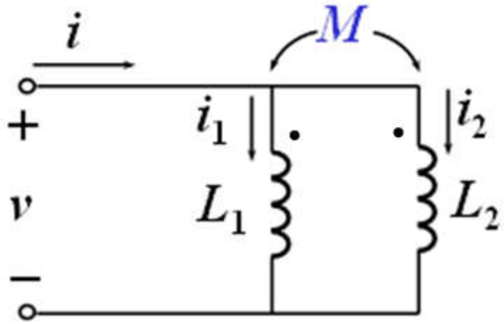
$$\mathbf{V} = 2j\omega M\mathbf{I} + j\omega(L_1 + L_2)\mathbf{I}$$

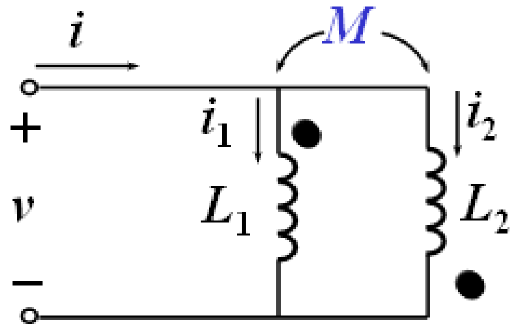
$$\mathbf{V} = -2j\omega M\mathbf{I} + j\omega(L_1 + L_2)\mathbf{I}$$

Measurement of Mutual-inductance:

$$\boxed{M = \frac{L_{aidding} - L_{opposing}}{4}}$$

Dot convention - Two coupled coils in parallel





$$\begin{cases} v = L_1 \dfrac{di_1}{dt} + M\dfrac{di_2}{dt} = L_2\dfrac{di_2}{dt} + M\dfrac{di_1}{dt} \\ i = i_1 + i_2 \end{cases}$$

$$\begin{cases} v = L_1 \dfrac{di_1}{dt} - M\dfrac{di_2}{dt} = L_2\dfrac{di_2}{dt} - M\dfrac{di_1}{dt} \\ i = i_1 + i_2 \end{cases}$$

$$v = \frac{(L_1 L_2 - M^2)}{L_1 + L_2 - 2M}\frac{di}{dt}$$

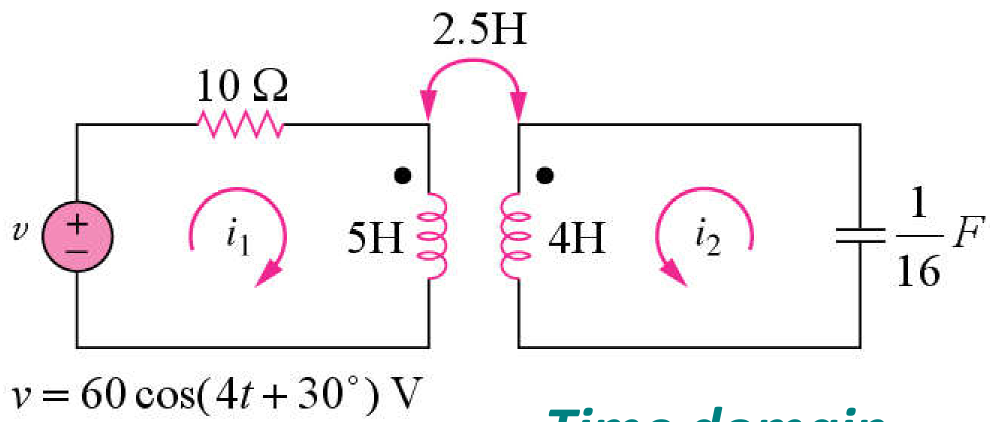
$$L = \frac{(L_1 L_2 - M^2)}{L_1 + L_2 - 2M}$$

$$v = \frac{(L_1 L_2 - M^2)}{L_1 + L_2 + 2M}\frac{di}{dt}$$

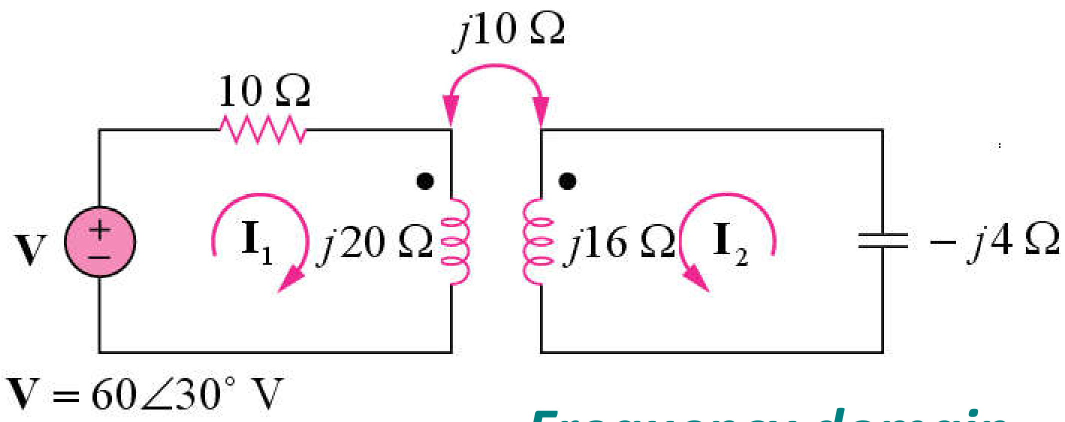
$$L = \frac{(L_1 L_2 - M^2)}{L_1 + L_2 + 2M}$$

Analysing in frequency domain

Time domain ⟺ Frequency domain



Time domain



Frequency domain

$$60\cos(4t + 30^\circ) \Rightarrow 60\angle 30^\circ$$

$$\Rightarrow \omega = 4 \text{ rad/s}$$

$$5\text{H} \Rightarrow j\omega L_1 = j20\ \Omega$$

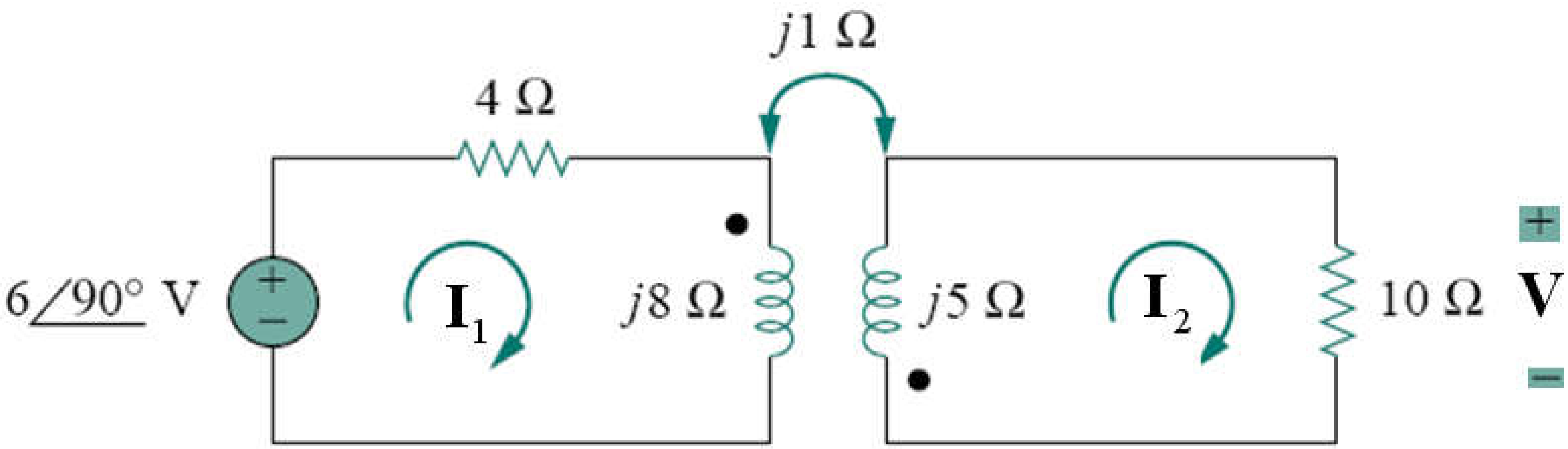
$$2.5\text{H} \Rightarrow j\omega M = j10\ \Omega$$

$$4\text{H} \Rightarrow j\omega L_2 = j16\ \Omega$$

$$\frac{1}{16}\text{F} \Rightarrow \frac{1}{j\omega C} = -j4\ \Omega$$

Dot convention – Example 1

- Determine the voltage **V** in the following figure:



- Solution:

$$\begin{cases} \text{From mesh 1:} \quad j6 = 4\mathbf{I}_1 + j8\mathbf{I}_1 + j\mathbf{I}_2 \\ \text{From mesh 2:} \quad 0 = j\mathbf{I}_1 + j5\mathbf{I}_2 + 10\mathbf{I}_2 \end{cases} \Rightarrow$$

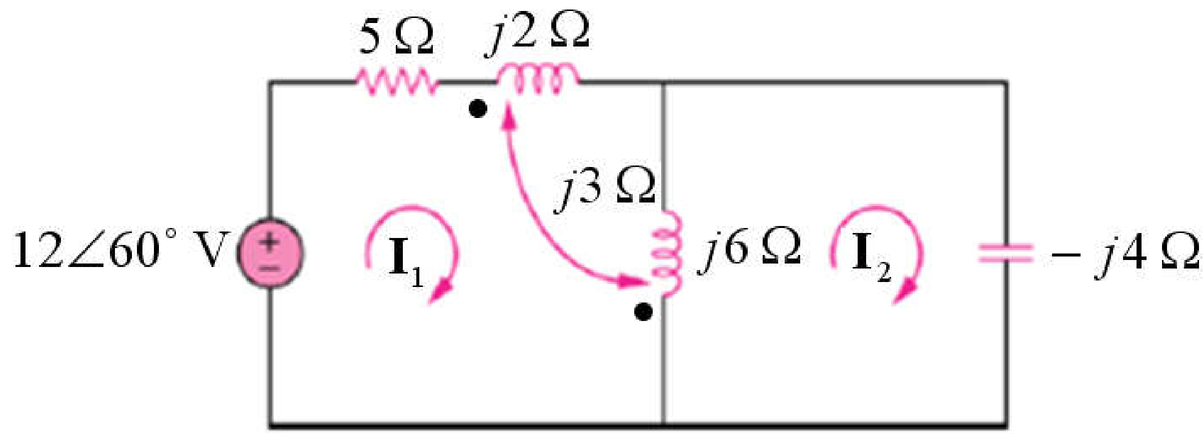
$$\mathbf{I}_2 = 6/(j100)$$

$$\mathbf{V} = 10\mathbf{I}_2 = 60/(j100)$$

$$= 0.6\angle(-90^\circ) \quad \text{V}$$

Dot convention – Example 2

- Find $\mathbf{I}_1$ and $\mathbf{I}_2$



- Solution:

Mesh 1: $12\angle 60^\circ = (5+j2)\mathbf{I}_1 - j3(\mathbf{I}_1-\mathbf{I}_2) + j6(\mathbf{I}_1-\mathbf{I}_2) - j3\mathbf{I}_1 \;\Rightarrow\; 12\angle 60^\circ = (5+j2)\mathbf{I}_1 - j3\mathbf{I}_2$

Mesh 2: $0 = j6(\mathbf{I}_2-\mathbf{I}_1) + j3\mathbf{I}_1 - j4\mathbf{I}_2 \;\Rightarrow\; \mathbf{I}_2 = 1.5\mathbf{I}_1$

$$\Downarrow$$

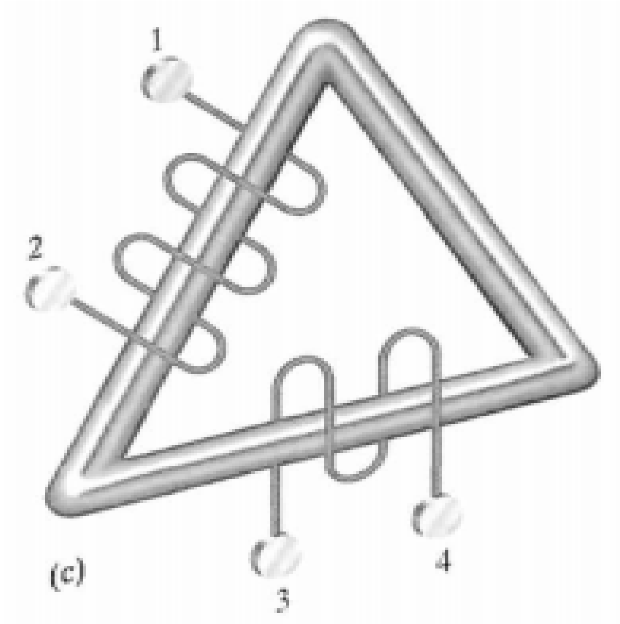
$$12\angle 60^\circ = (5-j2.5)\mathbf{I}_1$$

$$\Leftarrow$$

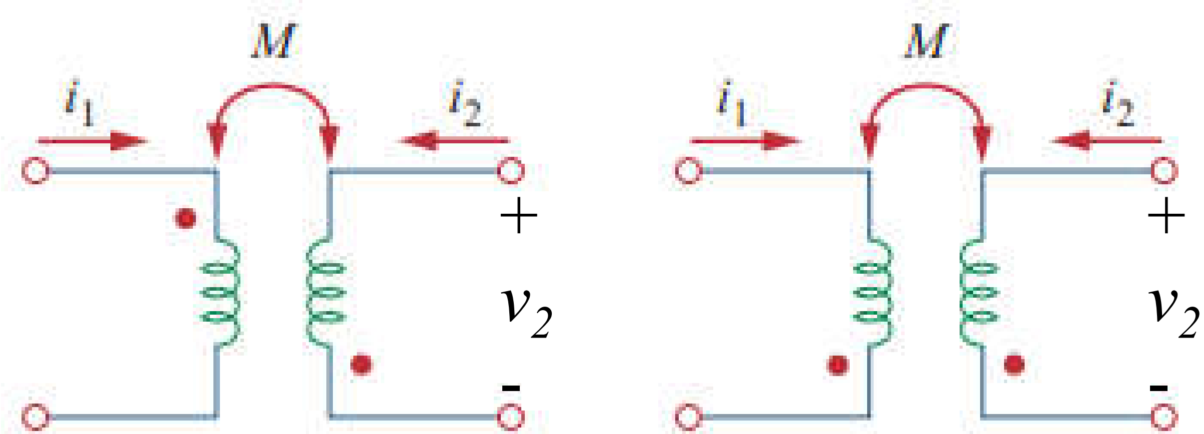
$$\boxed{\begin{aligned} \mathbf{I}_1 &= \frac{12\angle 60^\circ}{5.59\angle(-26.56^\circ)} = 2.147\angle 86.56^\circ \quad \text{A} \\ \mathbf{I}_2 &= 1.5\mathbf{I}_1 = 3.23\angle 86.56^\circ \quad \text{A} \end{aligned}}$$

Quiz

- 1. The physical construction of a pair of magnetically coupled coils is shown as follows. Assume that the magnetic flux is confined to the core material in each structure. Label the dots for the coils.

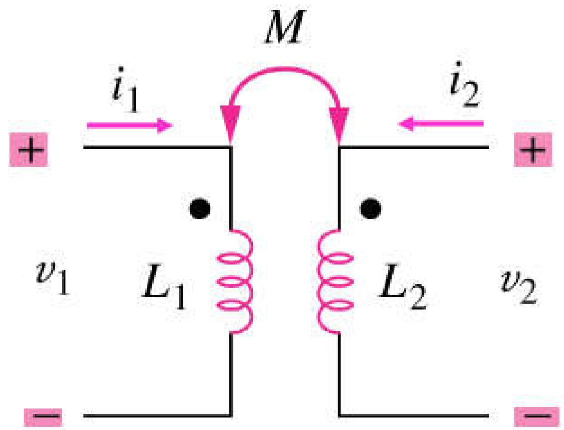


- 2. Refer to the two pairs of magnetically coupled coils. The polarity of the mutual voltages are:
  - (a) positive, negative;
  - (b) positive, positive;
  - (c) negative, negative;
  - (d) negative, positive.



Energy in a coupled circuit

- Assume that current i_1 and i_2 are zero initially, so the energy stored in the coils is zero.
- Increase i_1 from zero to I_1 while i_2 =0, the power stored in the coil 2 is zero as i_2=0, and the power in coil 1 is:



$$p_1(t) = v_1 i_1 = i_1 L_1 \frac{di_1}{dt}$$

- The energy stored in the coils: $w_1 = \int p_1 dt = L_1 \int_0^{I_1} i_1 di_1 = \frac{1}{2} L_1 I_1^2$

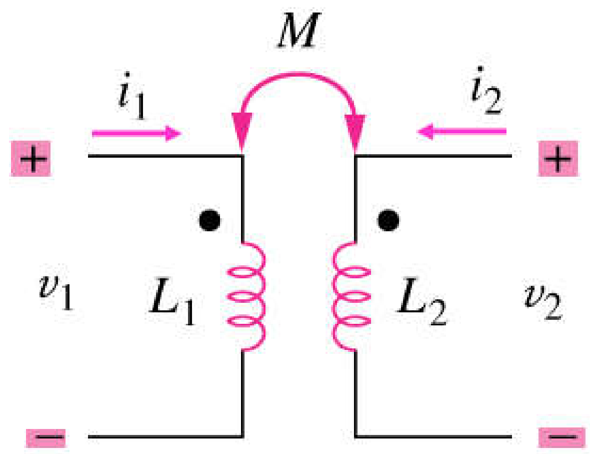
- Maintain i_1=I_1 and increase i_2 from 0 to I_2, the mutual voltage induced in coil 1 is $M_{12} di_2/dt$, while the mutual voltage induced in coil 2 is zero (I_1 = constant). The power in the two coils is now:

$$p_2(t) = i_1 M_{12} \frac{di_2}{dt} + i_2 v_2 = I_1 M_{12} \frac{di_2}{dt} + i_2 L_2 \frac{di_2}{dt}$$

Energy in a coupled circuit

- The energy stored in the coils:

$$W_2 = \int p_2 dt = I_1 M_{12} \int_0^{I_2} di_2 + L_2 \int_0^{I_2} i_2 di_2 = I_1 M_{12} I_2 + \frac{1}{2} L_2 I_2^2$$



- The total energy stored in the coils when both current have reached constant values:

$$W = W_1 + W_2 = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 + I_1 M_{12} I_2 \qquad (1)$$

- If we reverse the order (i_2 increases first, then i_1), the energy stored in the coils:

$$W = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 + I_1 M_{21} I_2 \qquad (2)$$

- The total energy stored should be the same regardless of how we reach the final conditions, so we have:

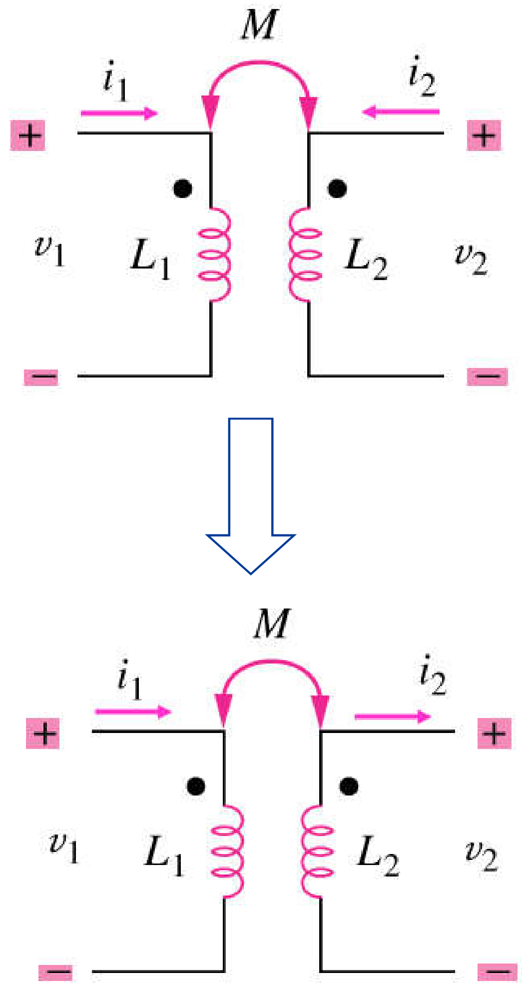
$$M_{12} = M_{21} = M$$

- And the total energy stored in the coils is: $W = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 + I_1 I_2 M$

Energy in a coupled circuit

- If one current enters a dot-marked terminal while the other leaves a dot-marked terminal, the sign of the mutual energy term is reversed:

$$W = \frac{1}{2}L_1 I_1^2 + \frac{1}{2}L_2 I_2^2 - I_1 I_2 M$$



- Since I_1 and I_2 can have any value, they may be replaced by i_1 and i_2, so we have the instantaneous energy stored in the circuit:

$$\boxed{w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 \pm i_1 i_2 M}$$

Mutual Inductance – An upper limit of M

- Since the energy stored in a passive network cannot be negative, so:

$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 - i_1 i_2 M \geq 0$$

- Rewrite the equation by completing the square:

$$w = \frac{1}{2}(i_1\sqrt{L_1} - i_2\sqrt{L_2})^2 + i_1 i_2\sqrt{L_1 L_2} - i_1 i_2 M \geq 0$$

$$w = \frac{1}{2}(i_1\sqrt{L_1} - i_2\sqrt{L_2})^2 + i_1 i_2(\sqrt{L_1 L_2} - M) \geq 0$$

- The squared term is never negative, at its least it is zero, so:

$$\sqrt{L_1 L_2} - M \geq 0 \quad \Rightarrow \quad M \leq \sqrt{L_1 L_2}$$

The mutual inductance cannot be greater than the geometric mean of the self-inductances of the coils.

Mutual Inductance – The coupling coefficient

- Definition: The coupling coefficient is a measure of the magnetic coupling between two coils.

$$\boxed{k = \frac{M}{\sqrt{L_1 L_2}} \qquad 0 \le k \le 1}$$

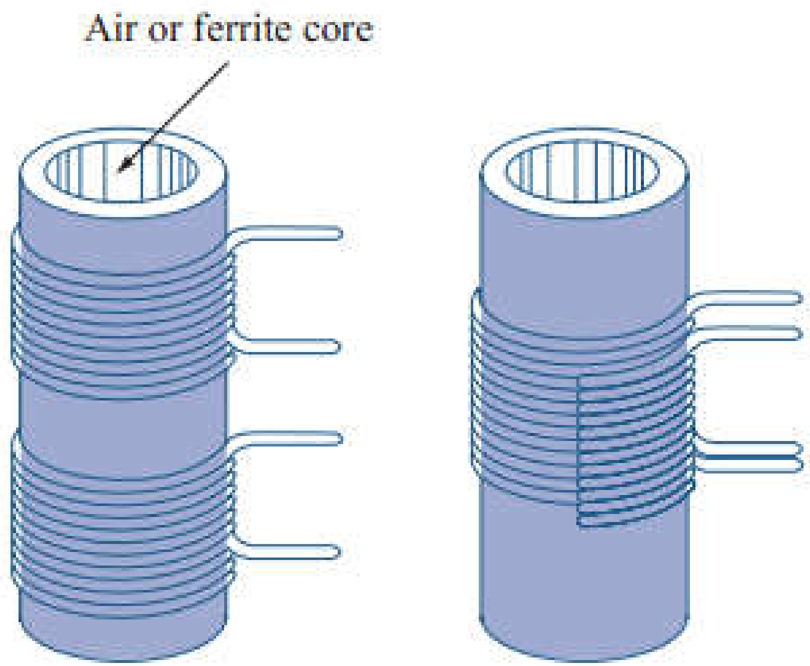
or

$$\boxed{M = k\sqrt{L_1 L_2}}$$

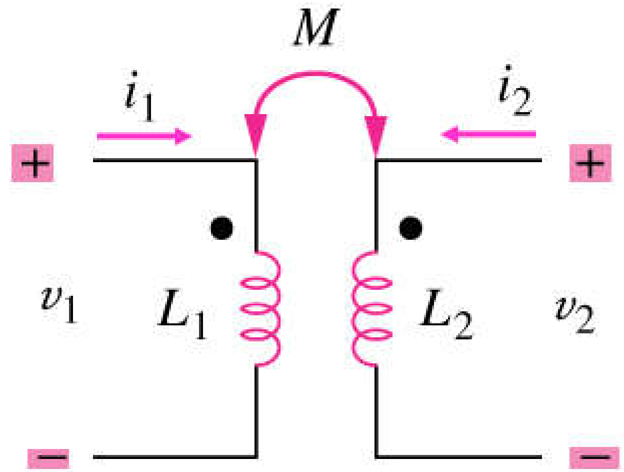
Mutual Inductance – *The coupling coefficient*

- If the entire flux produced by one coil links another coil, then $k = 1$, and we have 100 percent coupling, or the coils are said to be ***perfectly coupled***.
- For $k < 0.5$, coils are said to be ***loosely coupled***; and for $k > 0.5$, they are said to be ***tightly coupled***.

The coupling coefficient k is a measure of the magnetic coupling between two coils; $0 \le k \le 1$.



Energy consideration – *Example 3*



- $L_1 = 0.4$ H, $L_2 = 2.5$H, $k = 0.6$, and $i_1 = 4i_2 = 20\cos(500t - 20^o)$ mA.
- Evaluate the quantities at $t = 0$:
  - (*a*) i_2;   (*b*) v_1;
  - (*c*) the total energy stored in the system.
- Solution: $(a)\ i_2(t) = 5\cos(500t - 20^\circ)$ mA, so $i_2(0) = 5\cos(-20^\circ) = 4.698$ mA

  (*b*) Considering both the self-inductance and the mutual inductance:

$$v_1(t) = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}\ , \quad M = k\sqrt{L_1L_2} = 0.6\sqrt{0.4\times 2.5} = 0.6\ \text{H}$$

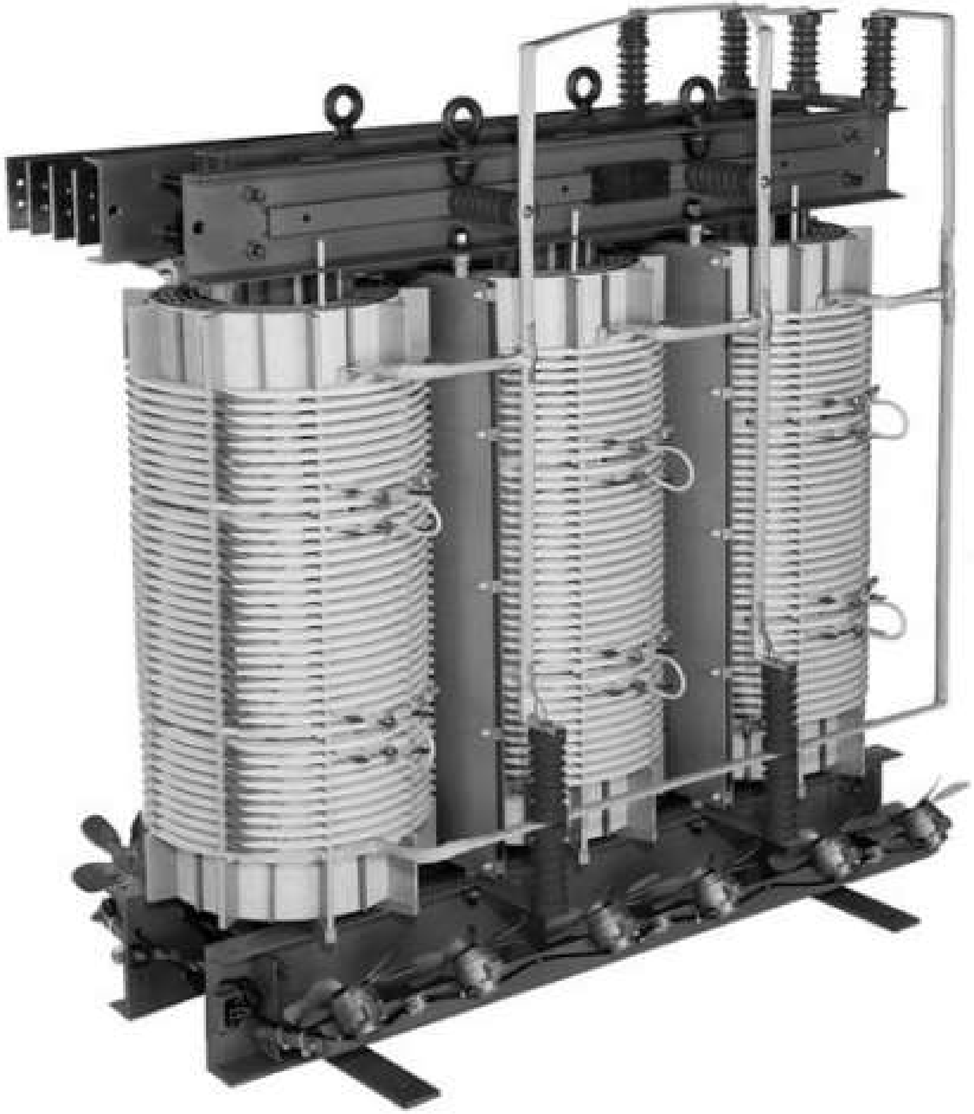
$$\text{Thus, } v_1(0) = 0.4\left[-10\sin(-20^\circ)\right] + 0.6\left[-2.5\sin(-20^\circ)\right] = 1.881\ \text{V}$$

  (*c*) Total energy stored:

$$w(t) = \frac{1}{2}L_1[i_1(t)]^2 + \frac{1}{2}L_2[i_2(t)]^2 \pm i_1(t)i_2(t)M, \text{ with } i_1(0) = 18.79\ \text{mA}$$

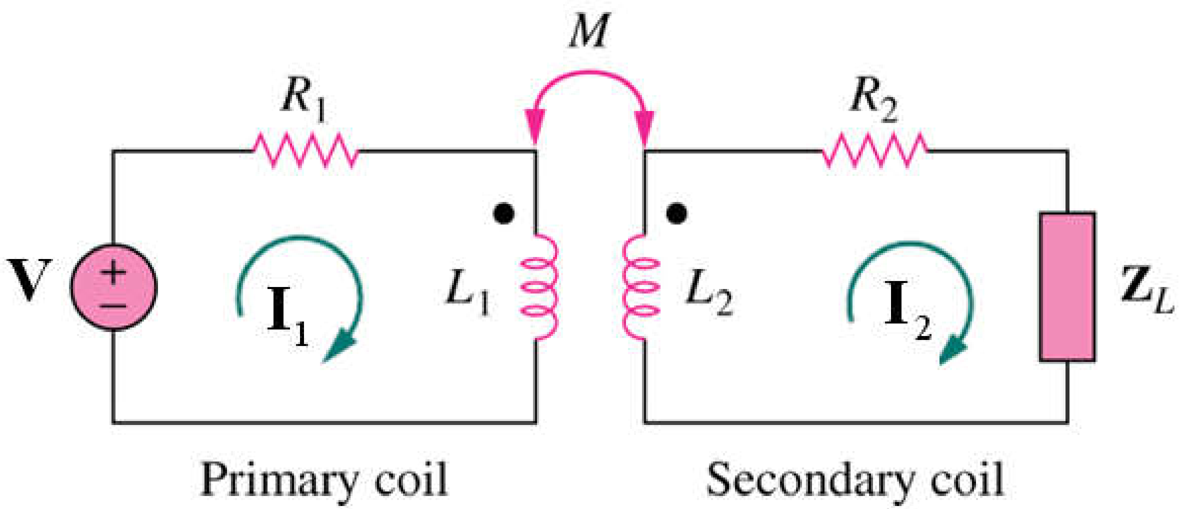
$$\text{Then we have:} \quad w(0) = 151.2\ \mu\text{J}$$

Transformer



Linear Transformer

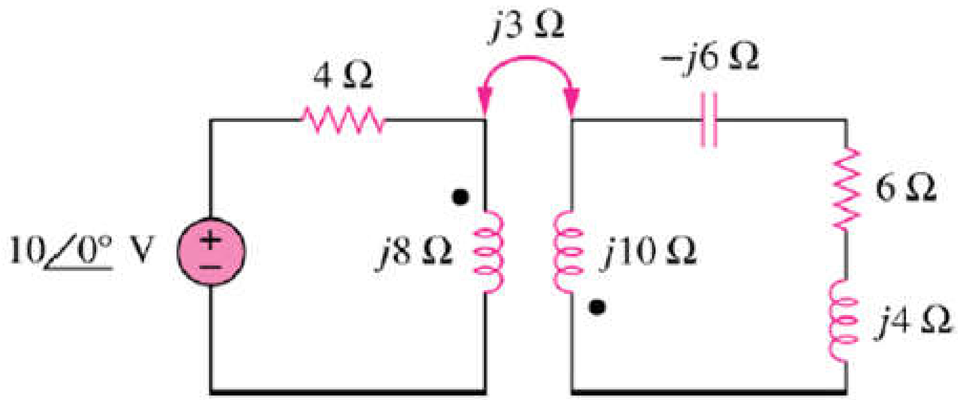
- The transformer is called LINEAR if the coils are wound on magnetically linear material, such as air - Hollow Transformers.
- LINEAR TRANSFORMERS: Flux is proportional to current in the windings.
- Used at radio frequencies, or higher frequencies.



A source in the primary circuit and a load in the secondary circuit.

Linear Transformer – Example 4

- Find the input impedance and the current.



Input impedance:

$$\mathbf{Z}_{in} = \frac{\mathbf{V}}{\mathbf{I}} = \mathbf{Z}_{11} - \frac{(j\omega)^2 M^2}{\mathbf{Z}_{22}}$$

$$= \mathbf{Z}_{11} + \frac{\omega^2 M^2}{R_2 + j\omega L_2 + \mathbf{Z}_L}$$

$$\mathbf{Z}_{in} = 4 + j8 + \frac{3^2}{j10 - j6 + 6 + j4}$$

$$= 4 + j8 + \frac{9}{6 + j8} = 8.58\angle 58.05^\circ$$

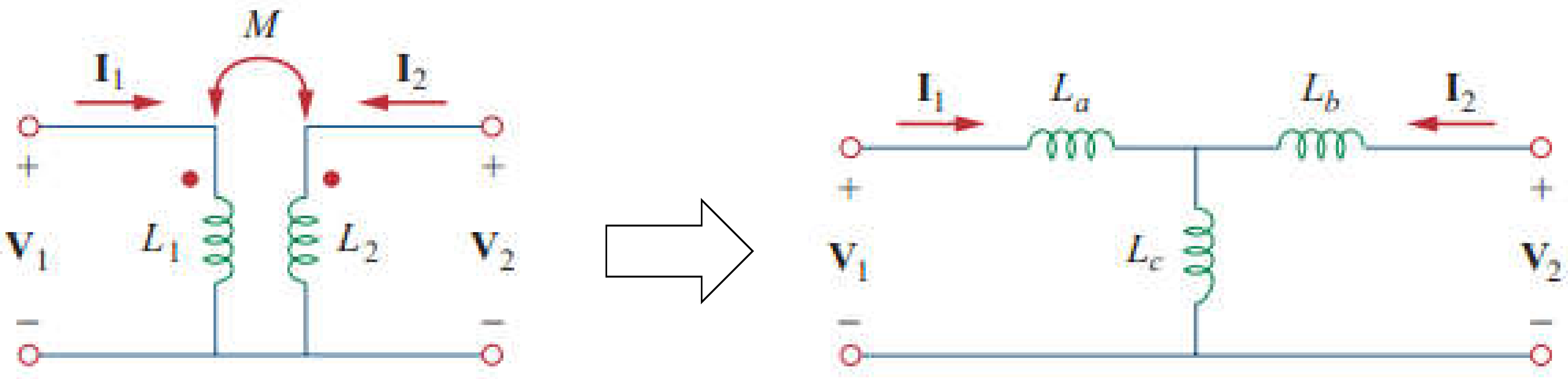
Current:

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_{in}} = \frac{10\angle 0^\circ}{8.58\angle 58.05^\circ}$$

$$= 1.165\angle -58.05^\circ \ \text{A}$$

Linear Transformer – *T equivalent*

- The coupled transformer can equivalently be represented by an equivalent **T** circuit using uncoupled inductors.



a) Transformer circuit

b) Equivalent **T** circuit with uncoupled inductors

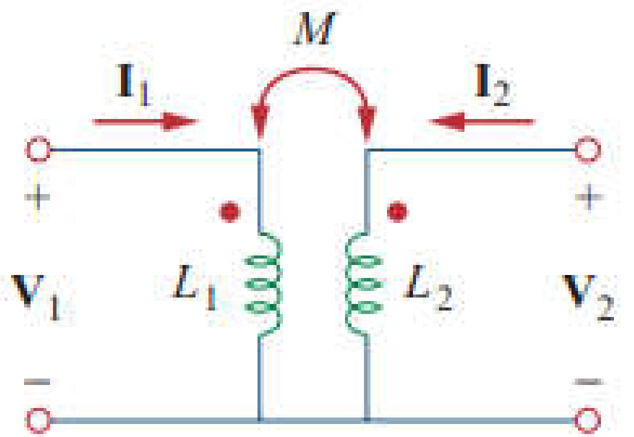
$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} j\omega L_1 & j\omega M \\ j\omega M & j\omega L_2 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} j\omega(L_a + L_c) & j\omega L_c \\ j\omega L_c & j\omega(L_b + L_c) \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

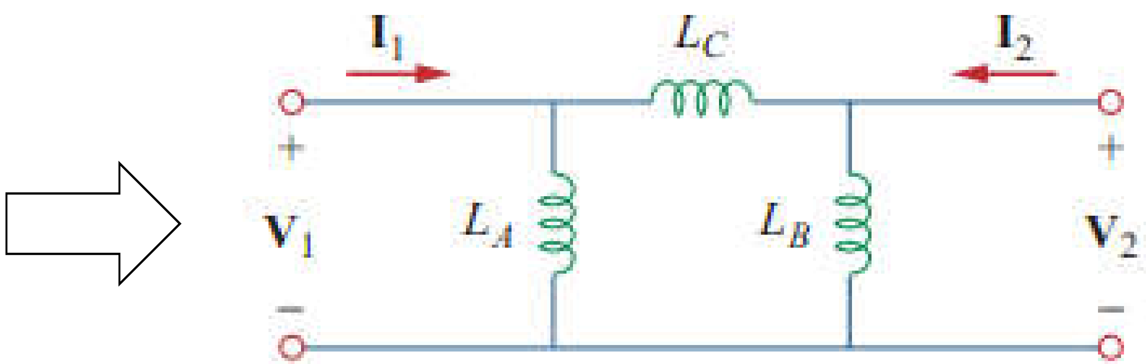
$$\boxed{L_a = L_1 - M, \quad L_b = L_2 - M, \quad L_c = M}$$

Linear Transformer – π equivalent

- The coupled transformer can equivalently be represented by an equivalent **π** circuit using uncoupled inductors.



a) Transformer circuit



b) Equivalent **π** circuit with uncoupled inductors

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} j\omega L_1 & j\omega M \\ j\omega M & j\omega L_2 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

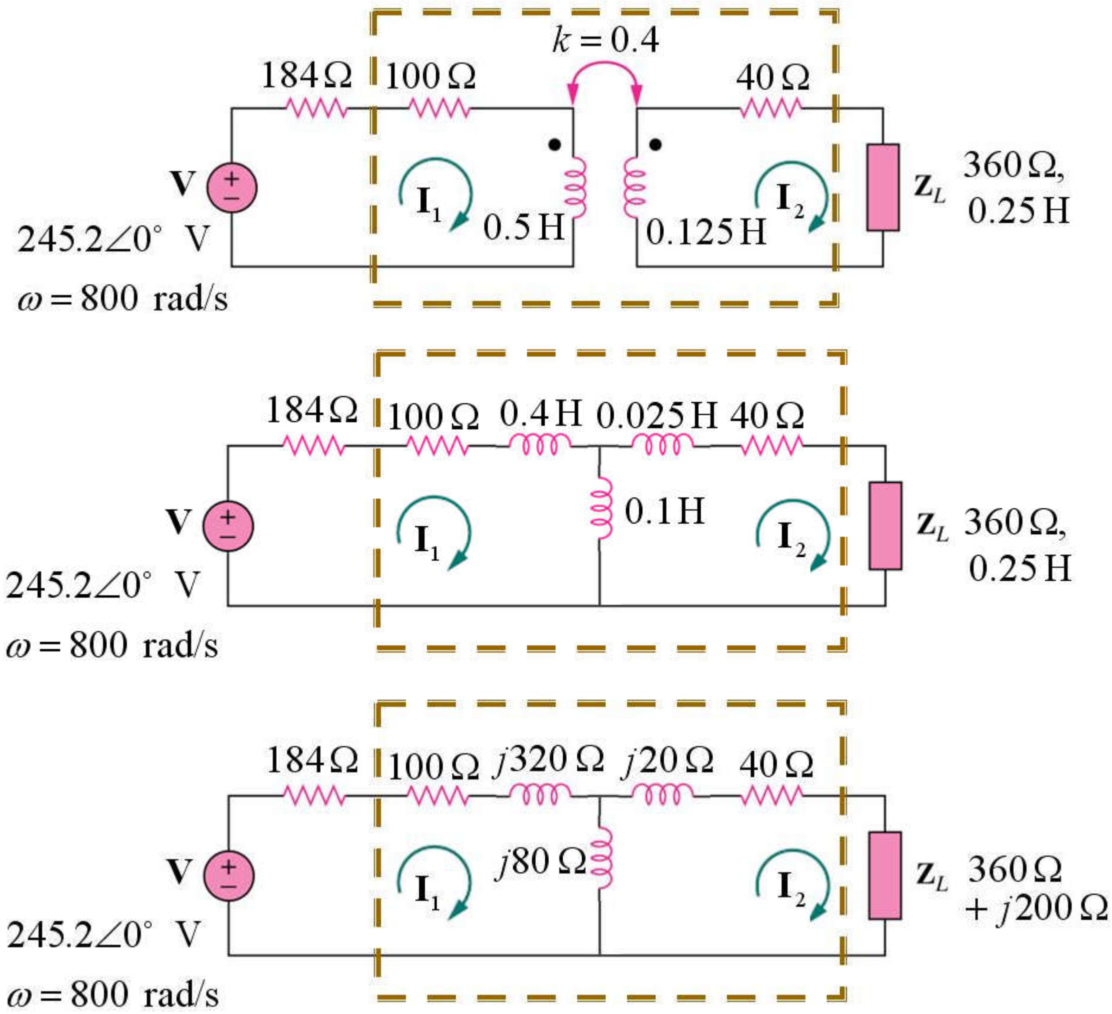
$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \dfrac{1}{j\omega L_A} + \dfrac{1}{j\omega L_C} & -\dfrac{1}{j\omega L_C} \\ -\dfrac{1}{j\omega L_C} & \dfrac{1}{j\omega L_B} + \dfrac{1}{j\omega L_C} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

$$\boxed{L_A = \frac{L_1 L_2 - M^2}{L_2 - M} \qquad L_B = \frac{L_1 L_2 - M^2}{L_1 - M} \qquad L_C = \frac{L_1 L_2 - M^2}{M}}$$

Linear Transformer – *Example 5*

- Find phasor currents $\boldsymbol{I_1}$ and $\boldsymbol{I_2}$

$$\boxed{L_a = L_1 - M, \quad L_b = L_2 - M, \quad L_c = M}$$



Solution

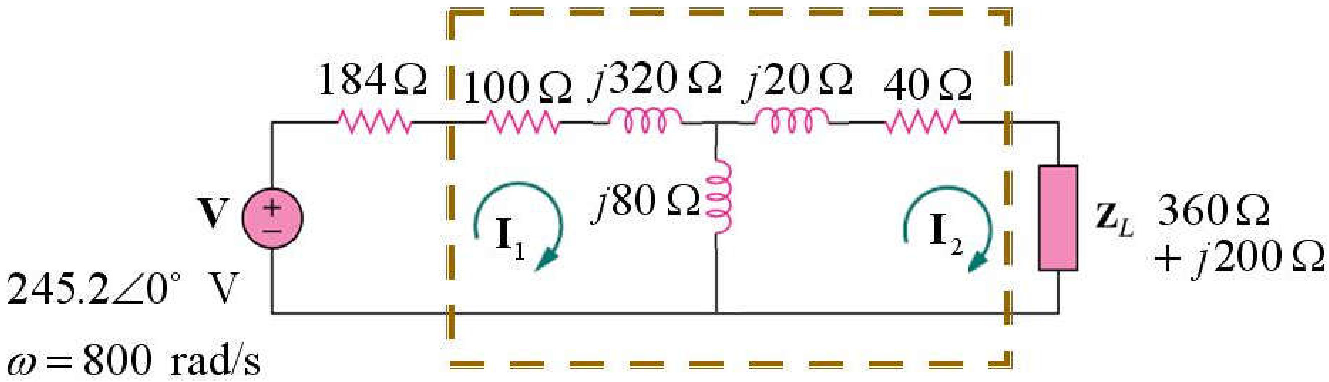
Get the equivalent inductances in the T circuit:

$$M = k\sqrt{L_1 L_2} = 0.4\sqrt{0.5 \times 0.125}$$
$$= 0.1 \ \text{H}$$
$$L_a = L_1 - M = 0.4 \ \text{H}$$
$$L_b = L_2 - M = 0.025 \ \text{H}$$

Transfer the circuit into the phasor domain:

$$X_M = \omega M = 800 \times 0.1 = 80 \ \Omega$$
$$X_{L_a} = \omega L_a = 800 \times 0.4 = 320 \ \Omega$$
$$X_{L_b} = \omega L_b = 800 \times 0.025 = 20 \ \Omega$$

Linear Transformer – Example 5 solution cont.



Two mesh equations:

$$\begin{cases} (284+j400)\mathbf{I}_1 - j80\mathbf{I}_2 = 245.2 & (1) \\ -j80\mathbf{I}_1 + (400+j300)\mathbf{I}_2 = 0 & (2) \end{cases}$$

$$\mathbf{I}_2 = \frac{245.2}{3065} = 0.08\angle 0^\circ \text{ A}$$

$$\mathbf{I}_1 = (3.75-j5)\mathbf{I}_2 = (5.25\angle(-53.13^\circ)) \times 0.08\angle 0^\circ = 0.5\angle(-53.13^\circ) \text{ A}$$

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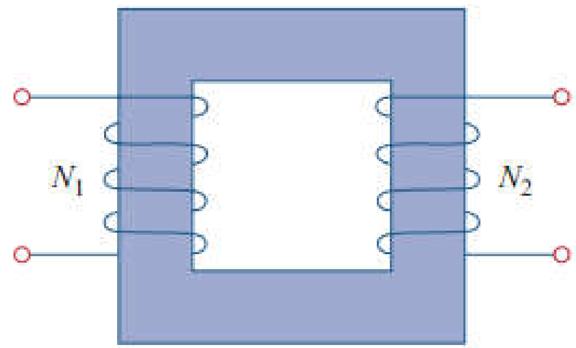
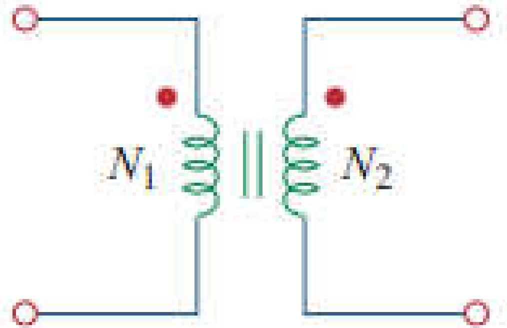
Ideal Transformer

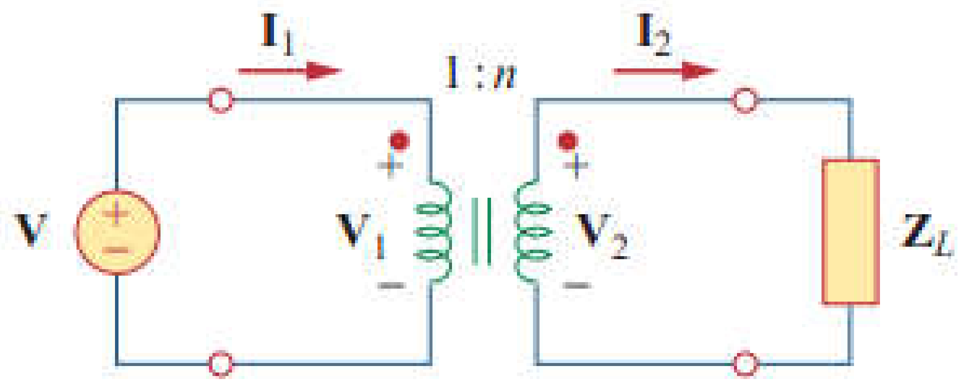
- An ideal transformer is one with perfect coupling (k = 1).
- The first mesh, usually containing the source, is called the ***primary***, while the second mesh, usually containing the load, is known as the ***secondary***.
  - The primary winding has N_1 turns;
  - The secondary winding has N_2 turns.

$$\frac{v_2}{v_1} = \frac{N_2}{N_1} = n$$

  - n is the ***turns ratio*** or ***transformation ratio***.

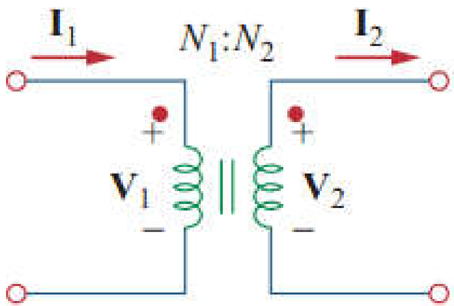




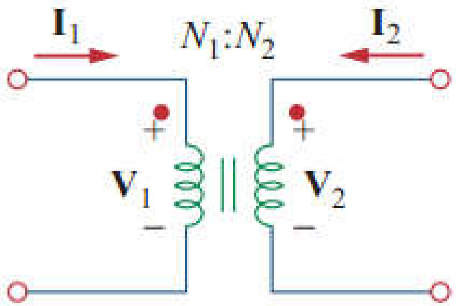


Ideal Transformer - Polarity

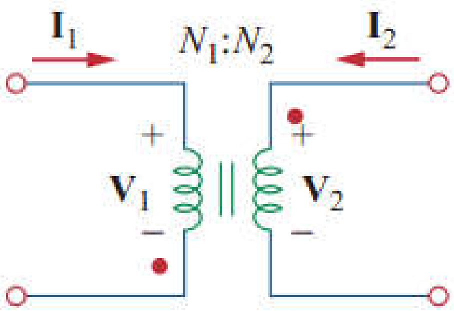
- It is important that we know how to get the proper polarity of the voltages and the direction of the currents for the transformer.



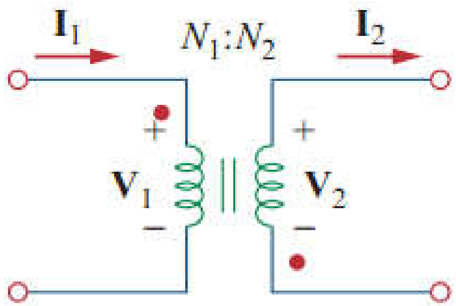
$$\frac{V_2}{V_1} = \frac{N_2}{N_1} \qquad \frac{I_2}{I_1} = \frac{N_1}{N_2}$$



$$\frac{V_2}{V_1} = \frac{N_2}{N_1} \qquad \frac{I_2}{I_1} = -\frac{N_1}{N_2}$$



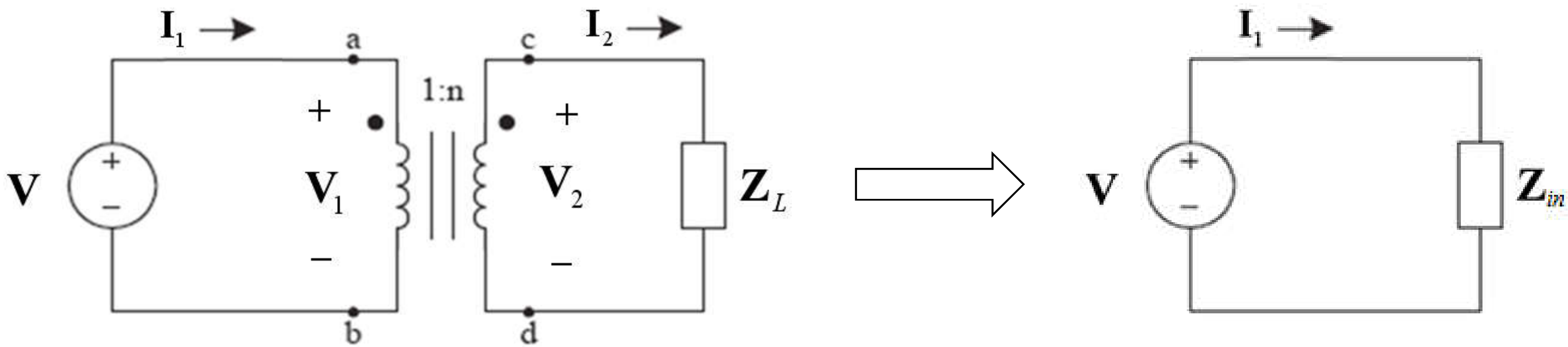
$$\frac{V_2}{V_1} = -\frac{N_2}{N_1} \qquad \frac{I_2}{I_1} = \frac{N_1}{N_2}$$



$$\frac{V_2}{V_1} = -\frac{N_2}{N_1} \qquad \frac{I_2}{I_1} = -\frac{N_1}{N_2}$$

Ideal Transformer – Reflected Impedance

- Ideal transformer:

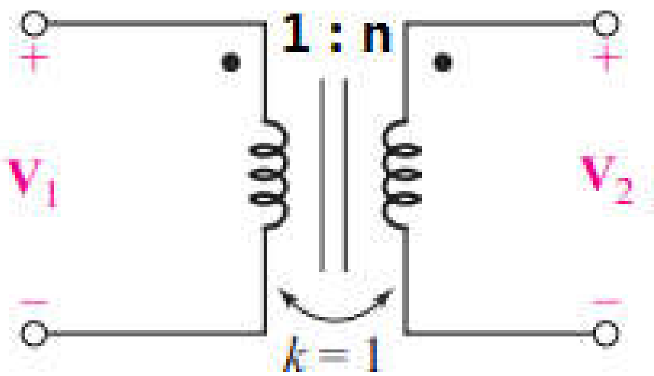


$$\mathbf{Z}_{in} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{\mathbf{V}_2 / n}{\mathbf{I}_2 n} = \frac{\mathbf{V}_2}{\mathbf{I}_2}\frac{1}{n^2} = \frac{\mathbf{Z}_L}{n^2}$$

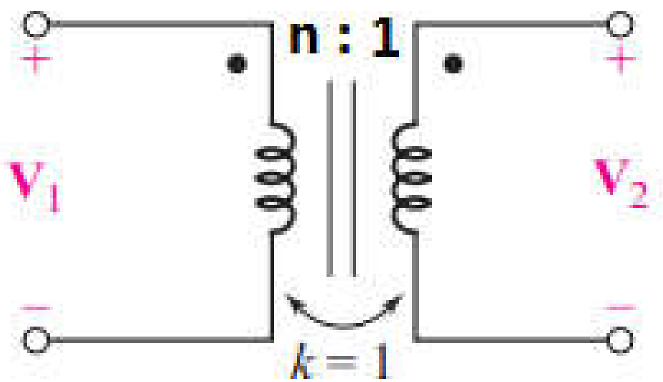
- *Example 6:* A degrade voltage transformer. Voltage ratio is 220/110 V， if the second grade connect with a resistance with 55 Ω.
- Find the input impedance of transformer's primary grade.

Ideal Transformer - *Step-up & step-down transformer*

- An ideal transformer in which the secondary voltage is greater than the primary voltage is stepped up depends on turns ratio.

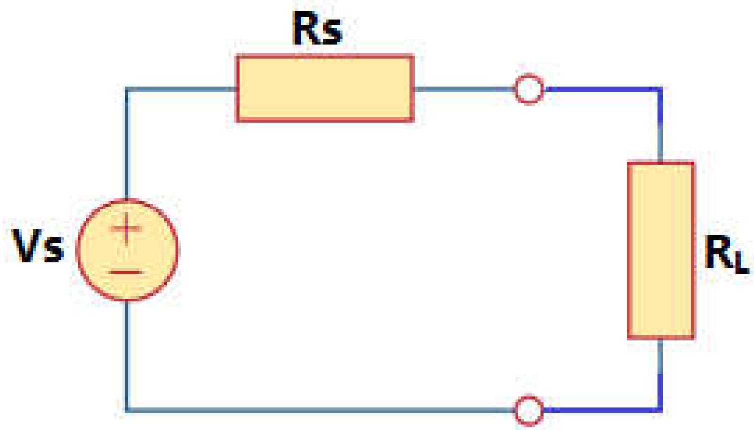


- An ideal transformer in which the secondary voltage is less than the primary voltage is stepped down also depends on the turns ratio.



Ideal Transformer - Maximum power transfer

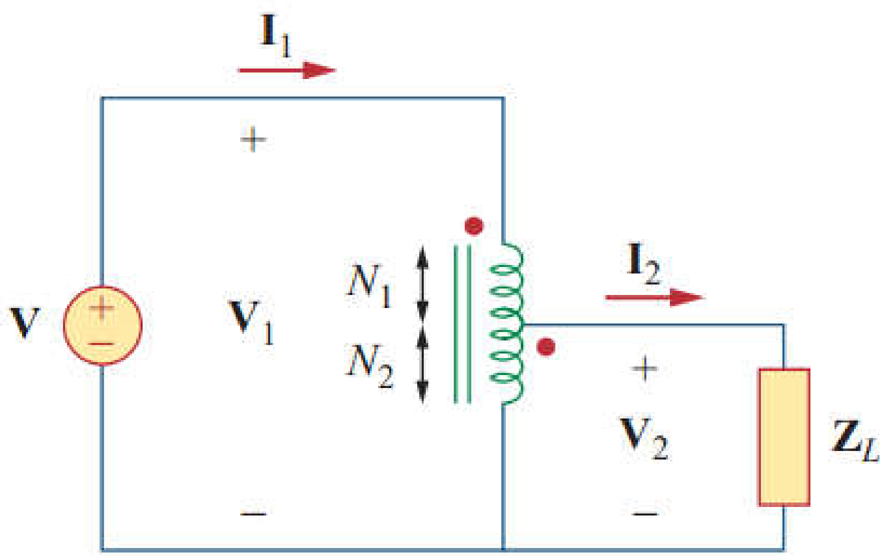
- An AC source V_s with internal resistance R_s is connected to a load.
- For resistive load R_L, max. power delivered to R_L when $R_L=R_s$
  - Use ideal transformer for transfer the maximum power - "impedance matching".



- *Example 7:* An amplifier has an 800Ω internal resistance. In order to provide maximum power to an 8Ω speaker, what turns ratio must be used in the ideal coupling transformer?

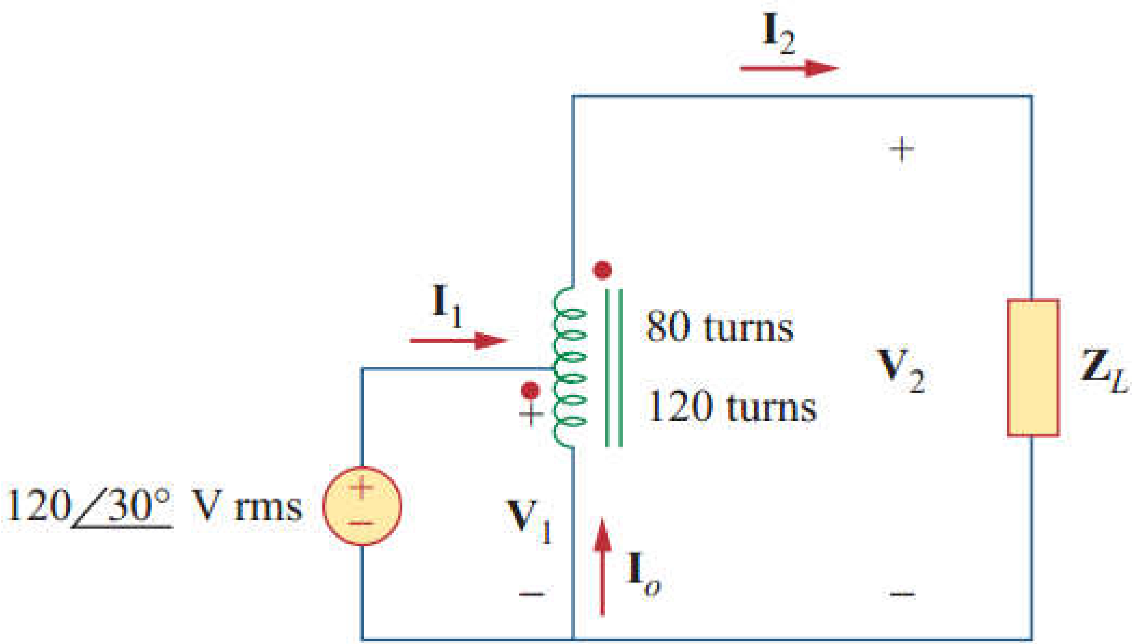
Autotransformer

- An autotransformer is a transformer in which both the primary and the secondary are in a single winding.
- Advantages:
  - It can transfer larger apparent power
  - It is smaller and lighter than an equivalent two-winding transformer.
- Disadvantage:
  - Since both the primary and secondary windings are one winding, electrical isolation (no direct electrical connec -tion) is lost.



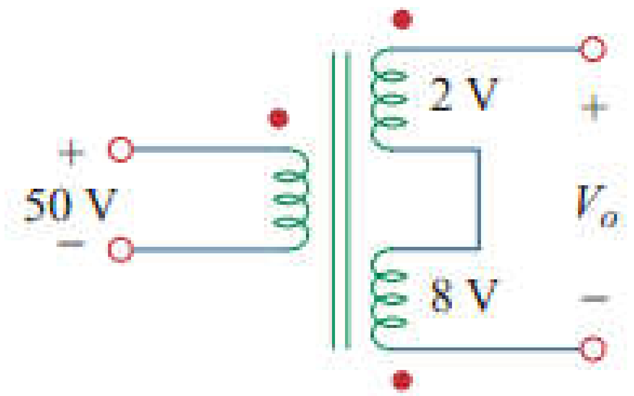
Autotransformer – Example 8

- Refer to the autotransformer circuit in following figure. Calculate:
  - (a) I_1, I_2 and Io if Z_L=8+j6 Ω;
  - (b) the power supplied to the load.



Quiz

- A three-winding transformer is connected as portrayed in the figure below. The value of the output voltage is:
  - (a) 10 V; (b) 6 V;
  - (c) -6 V; (d) 10 V.



- The ideal transformer in figure on right has $N_2 / N_1 = 10$. The ratio V_2 / V_1 is:
  - (a) 10; (b) 0.1;
  - (c) -0.1; (d) -10.