MTH101: Tutorial 11

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Exercise 1.1

Find the power series solution to the following questions.

1. $xy' - 4y = k, \quad k$ *constant.*
2. $(1 - x^2)y'' - 2xy' + 2y = 0.$

Exercise 2.1

We know that the Legendre's differential equation admit the general solution

$$y(x) = a_0 y_1(x) + a_1 y_2(x),$$

where

$$y_1(x) = 1 - \frac{n(n+1)}{2}x^2 + \frac{(n-2)n(n+1)(n+3)}{4!}x^4 - + \cdots$$
$$y_2(x) = x - \frac{(n-1)(n+2)}{3!}x^3 + \frac{(n-3)(n-1)(n+2)(n+4)}{5!}x^5 \cdots.$$

For $n = 1$*, show that* $y_2(x) = P_1(x) = x$ *and*

$$y_1(x) = 1 - \frac{1}{2}x \ln\left(\frac{1+x}{1-x}\right).$$

Exercise 3.1

Find a basis of solutions by the Frobenius method for the following question.

1. $x^2y'' + 2x^3y' + (x^2 - 2)y = 0.$
2. $x^2y'' + 6xy' + (4x^2 + 6)y = 0.$

Exercise 4.1

Find a general solution to the following Bessel's equation in terms of J_ν, $J_{-\nu}$, or indicate when this is not possible.

$$x^2y'' + xy' + \left(x^2 - \frac{1}{9}\right) y = 0.$$