Tutorial 09 Basic concepts of PDEs and wave equation

1. Verify that the function $u(x,y) = a\ln(x^2+y^2)+b$ satisfies Laplace's equation
$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0.$$
And determine *a*, *b* so that *u* satisfies the boundary conditions *u*=110 on the circle $x^2+y^2=1$ and *u*=0 on the circle $x^2+y^2=100$.

2. Solve the following PDEs using the same method to solve ODEs.
(a) $u_{xx}+16\pi^2 u=0$

(b) $25u_{yy}-4u=0$

(c) $u_y+y^2u=0$

3. Boundary value problem
(a) $\begin{cases} \frac{\partial u}{\partial x}-3\frac{\partial u}{\partial y}=0 \\ u(x,0)=3e^{5x} \end{cases}$

(b) $\begin{cases} 2u_x+u_y=0 \\ u(0,y)=5\mathrm{e}^{-7y} \end{cases}$

(c) $\begin{cases} 5\frac{\partial u}{\partial x}=6\frac{\partial u}{\partial y} \\ u(0,y)=10\mathrm{e}^{2y}+2\mathrm{e}^{y} \end{cases}$

4. Wave equation, solution by separating variables.
Consider the following one-dimensional wave equation problem:

$$\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2}, \text{ for } 0 < x < 1, \qquad t \geq 0$$

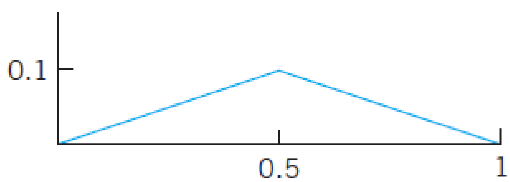
with the initial boundary conditions:

$$y(0,t) = y(1,t) = 0, \text{for } t \geq 0;$$
$$y_t(x,0) = 0 \text{ and } y(x,0) = f(x).$$

Using separating variables, find the solution of the problem for the following given initial deflections *f(x)* and k. [Hint: let $y(x,t) = X(x)T(t)$] and $\frac{X''}{X} = \frac{T''}{T} = -\lambda$, where $\lambda = \lambda_n = n^2\pi^2$, for n=1,2,⋯.

(a) $f(x) = k\sin 3\pi x$

(b) $f(x) = kx(1-x)$



(c)