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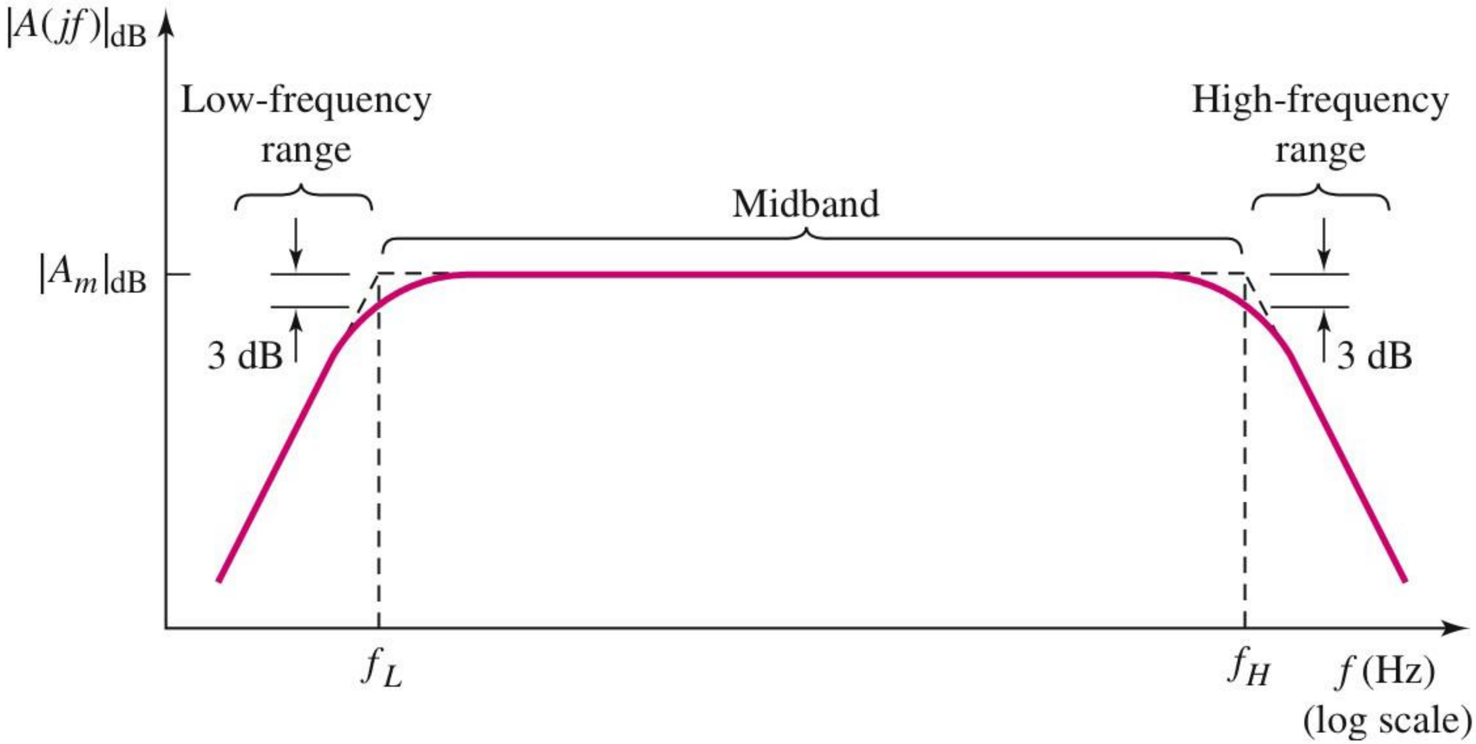
EEE109: Electronic Circuits

Frequency Response

- Capacitor Effect and Examples

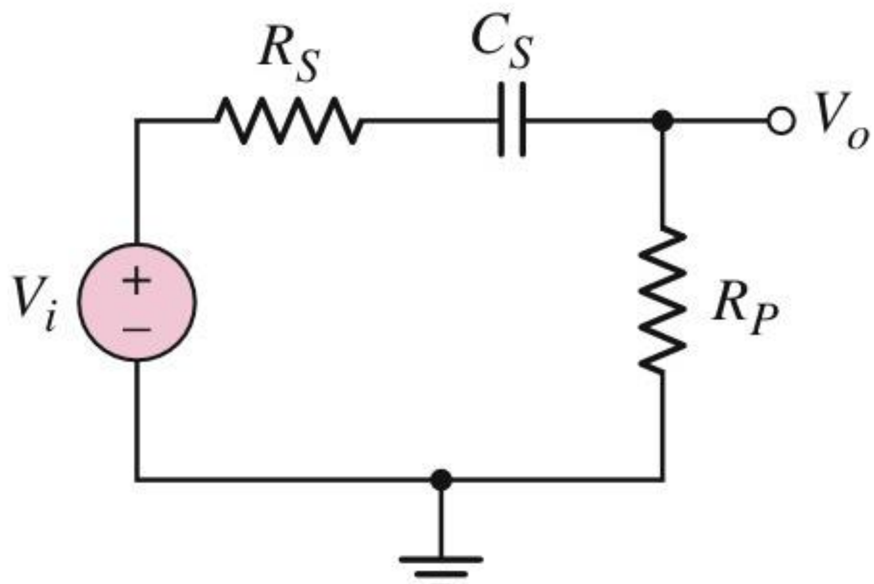
Amplifier Gain Versus Frequency



Transfer Functions of the Complex Frequency

Name of Function	Expression
Voltage Transfer Function	$T(s) = V_o(s)/V_i(s)$
Current Transfer Function	$I_o(s)/I_i(s)$
Transresistance Function	$V_o(s)/I_i(s)$
Transconductance Function	$I_o(s)/V_i(s)$

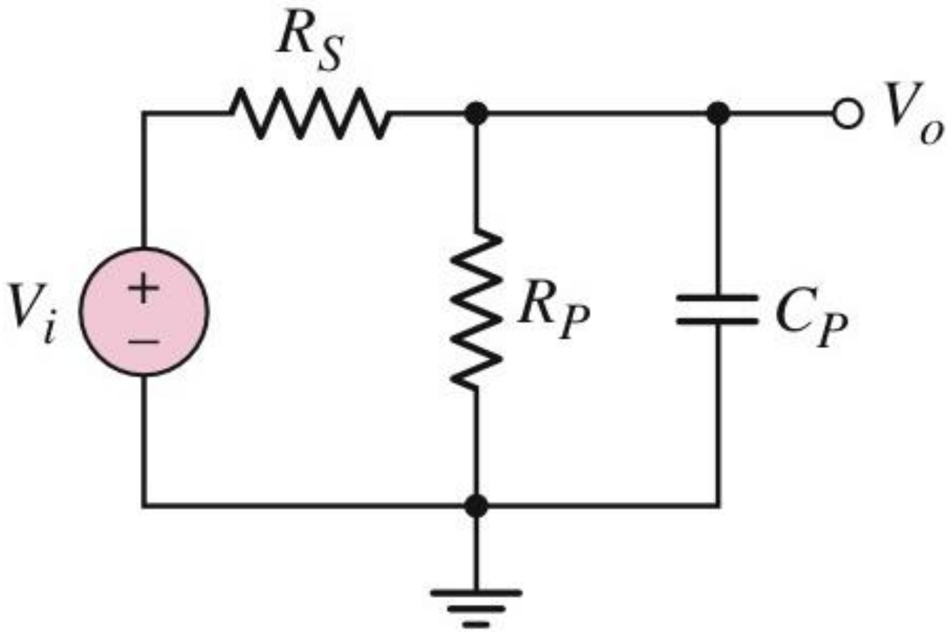
Series Coupling Capacitor Circuit



$$T(s) = K_2\left(\frac{s\tau}{1+s\tau}\right)$$

$$\tau = (R_S + R_P)C_S$$

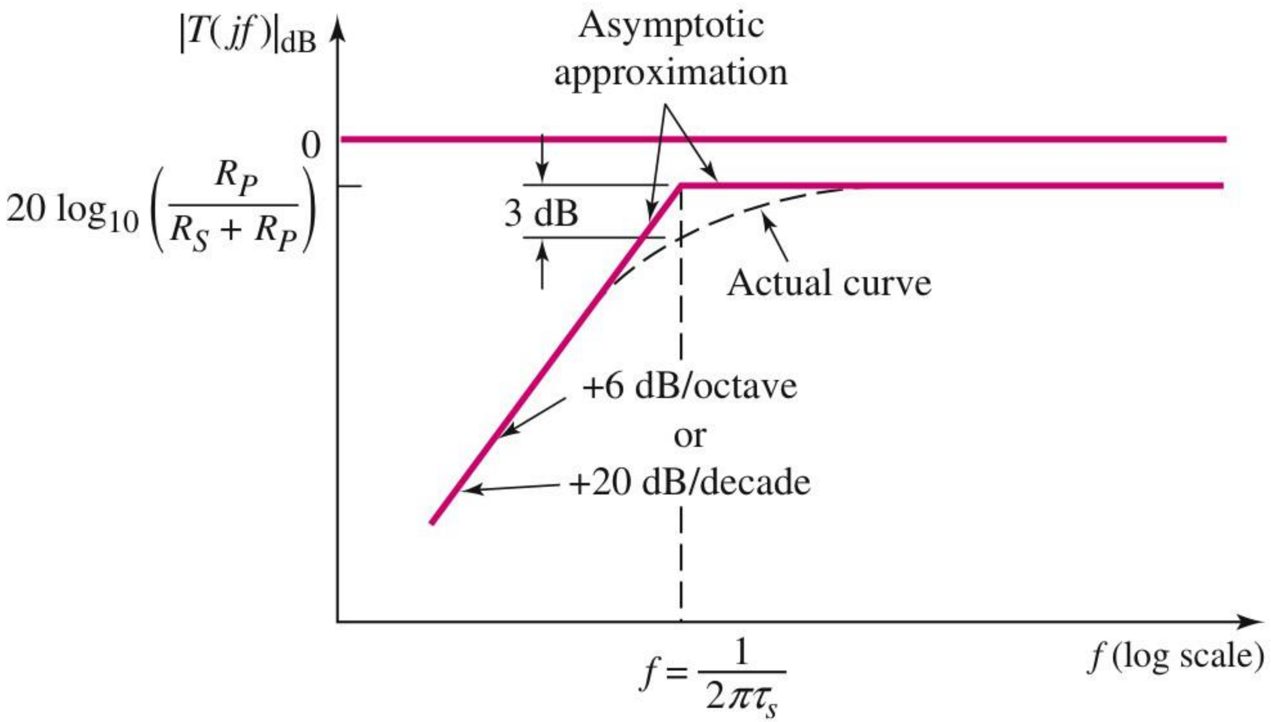
Parallel Load Capacitor Circuit



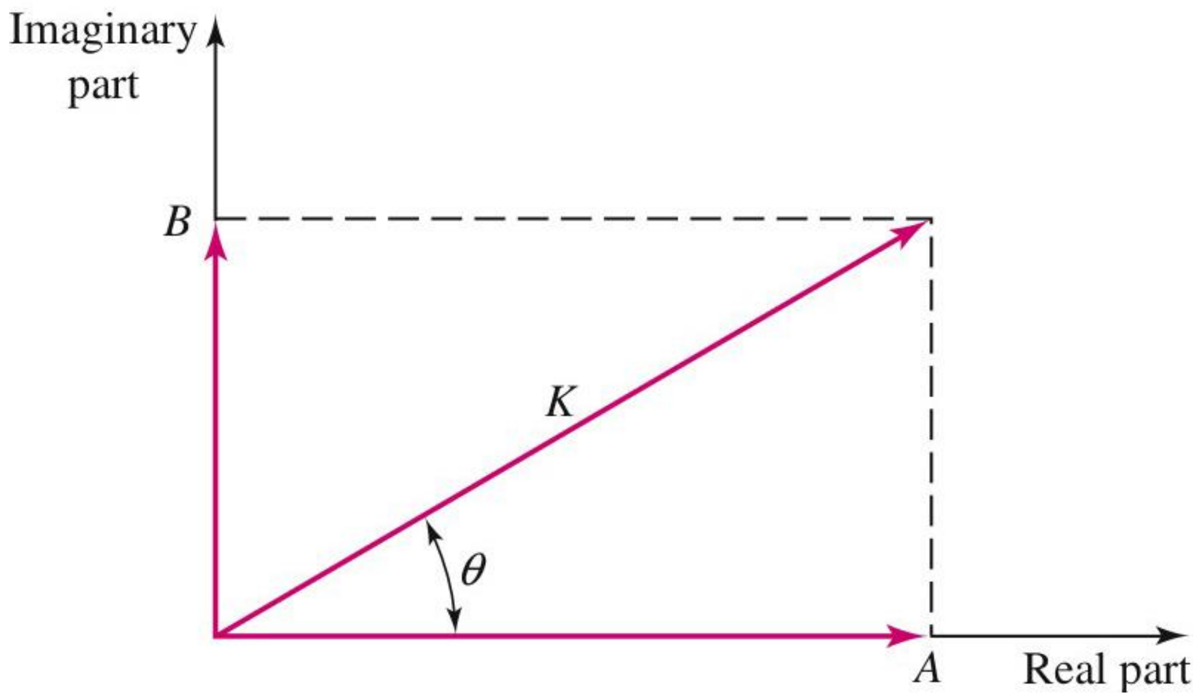
$$T(s) = K_1\left(\frac{1}{1+s\tau}\right)$$

$$\tau = (R_S \| R_P)C_P$$

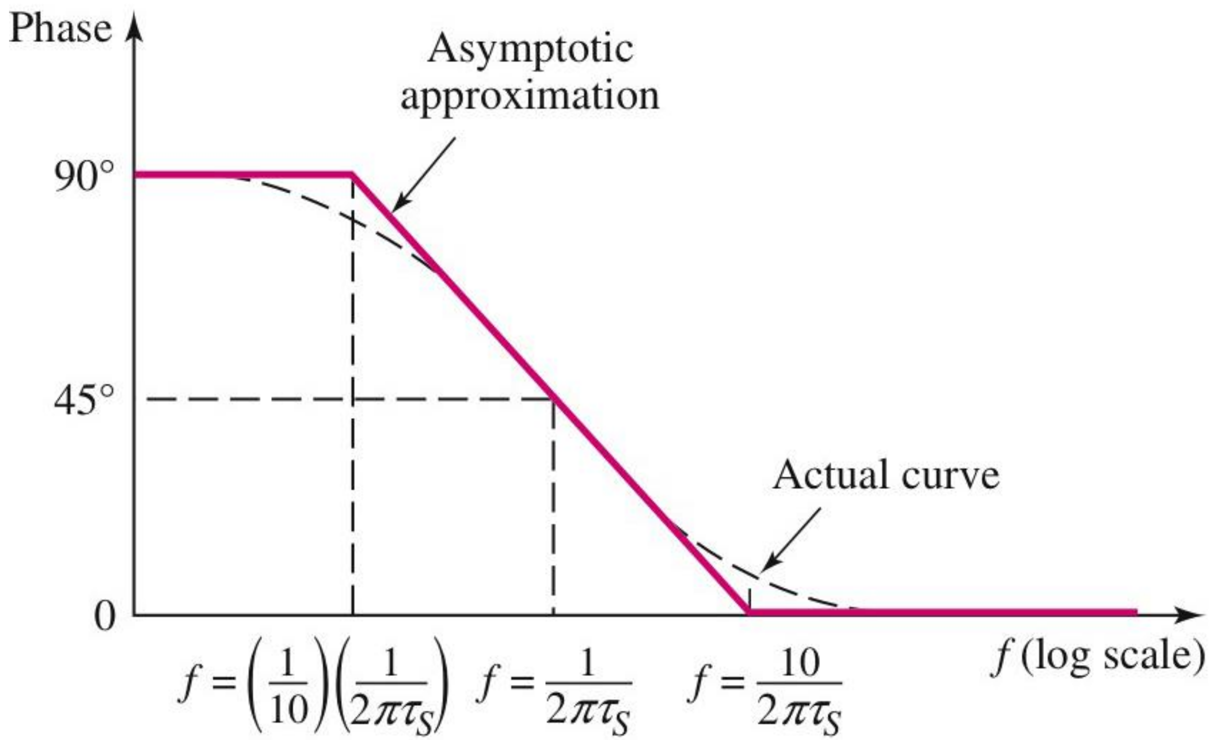
Bode Plot of Voltage Transfer Function Magnitude: Series Coupling Capacitor Circuit



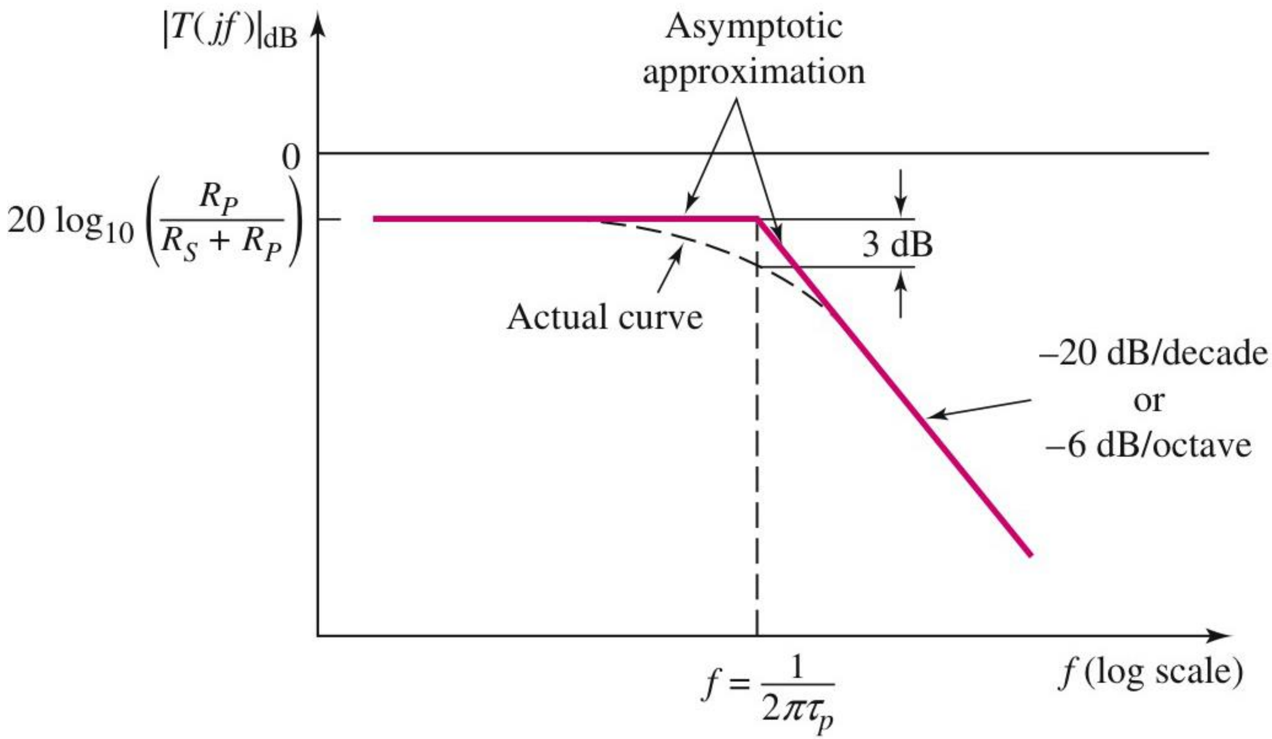
Relationship Between Rectangular and Polar Coordinates



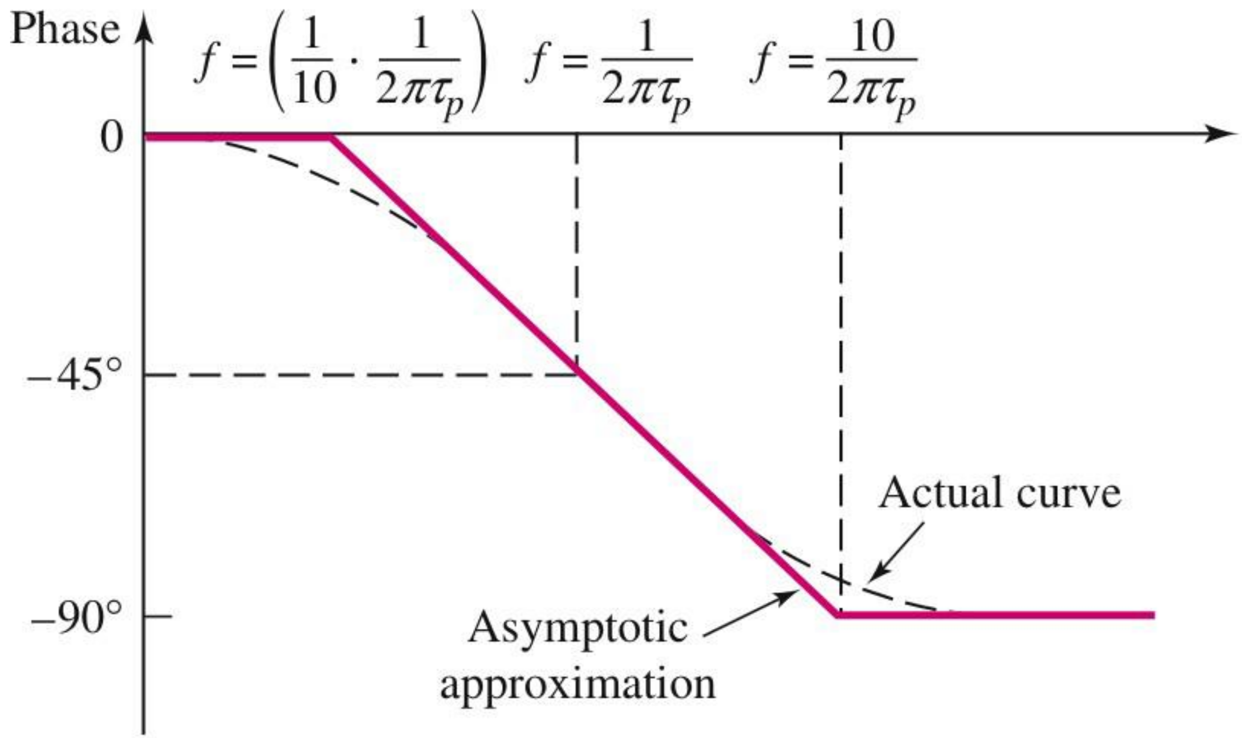
Bode Plot of Voltage Transfer Function Phase: Series Coupling Capacitor Circuit



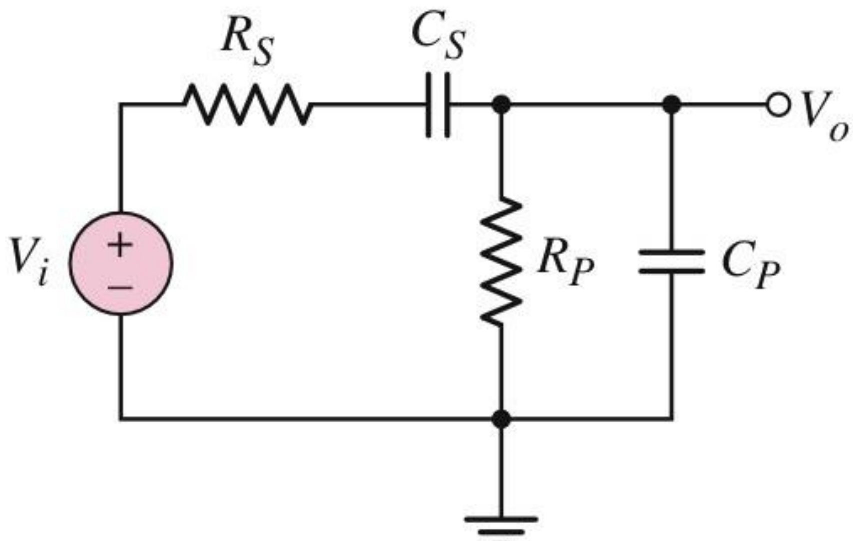
Bode Plot of Voltage Transfer Function Magnitude: Parallel Load Capacitor Circuit



Bode Plot of Voltage Transfer Function Phase: Parallel Load Capacitor Circuit



Circuit with Series Coupling and Parallel Load Capacitor



$$\tau_S = (R_S + R_P)C_S$$

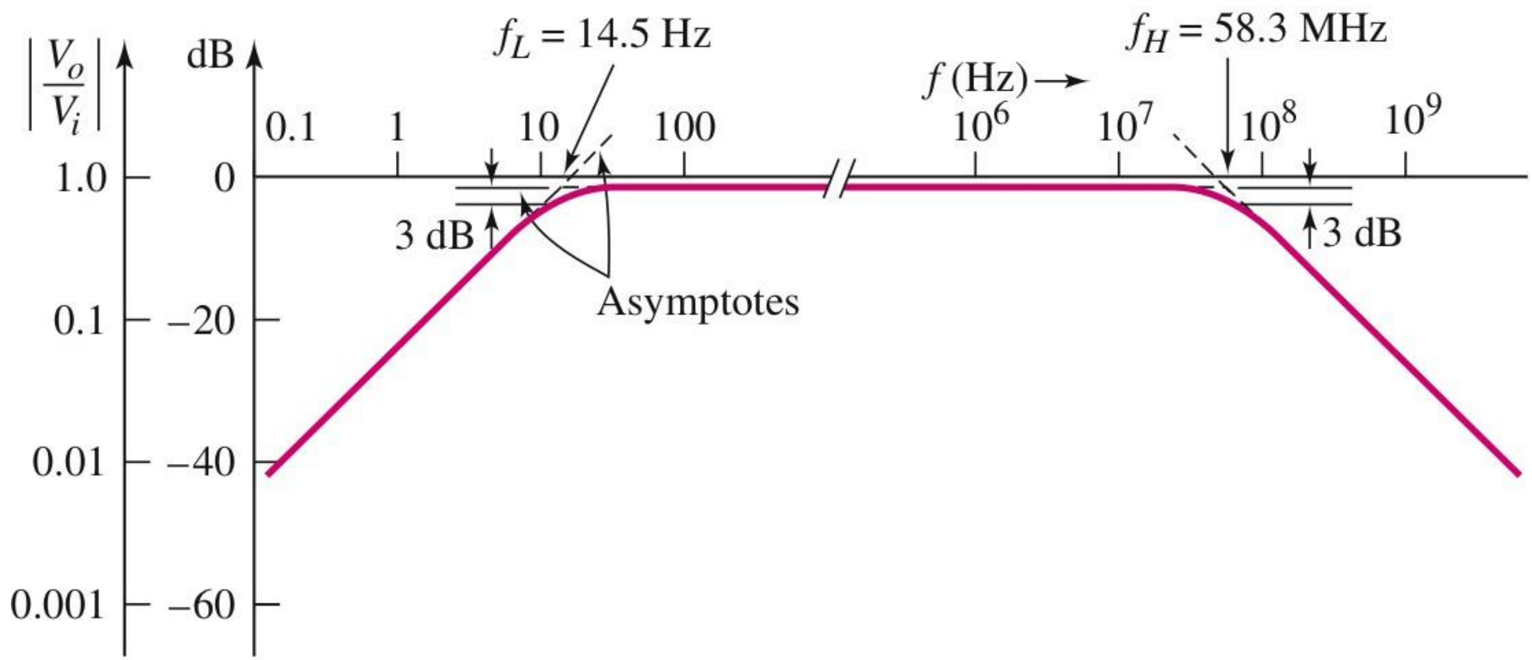
$$\tau_P = (R_S \| R_P)C_P$$

$$f_L = \frac{1}{2\pi\tau_S}$$

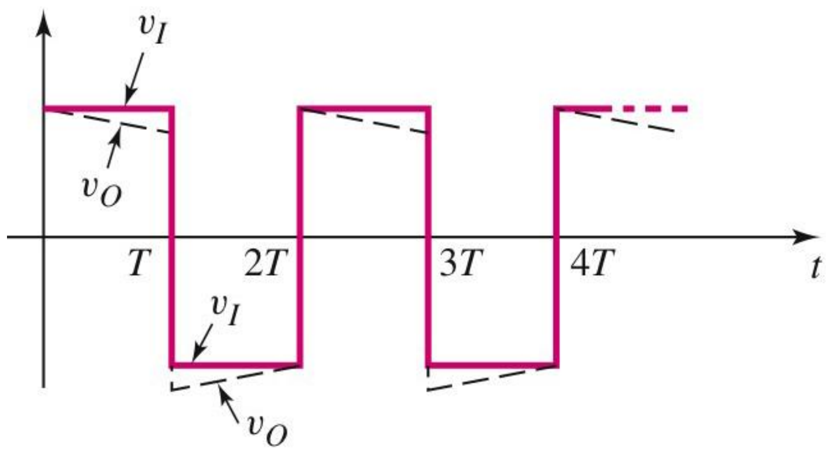
$$f_H = \frac{1}{2\pi\tau_P}$$

Bode Plot of Magnitude of Voltage Transfer Function:

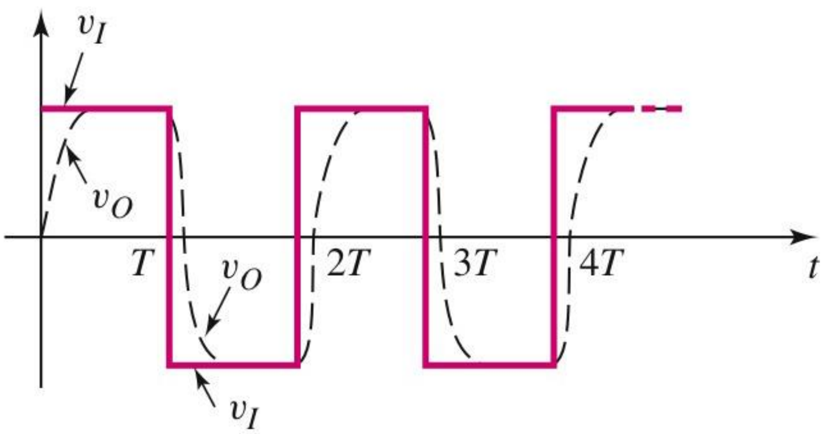
Series Coupling and Parallel Load Capacitor



Steady-State Output Response



(a)

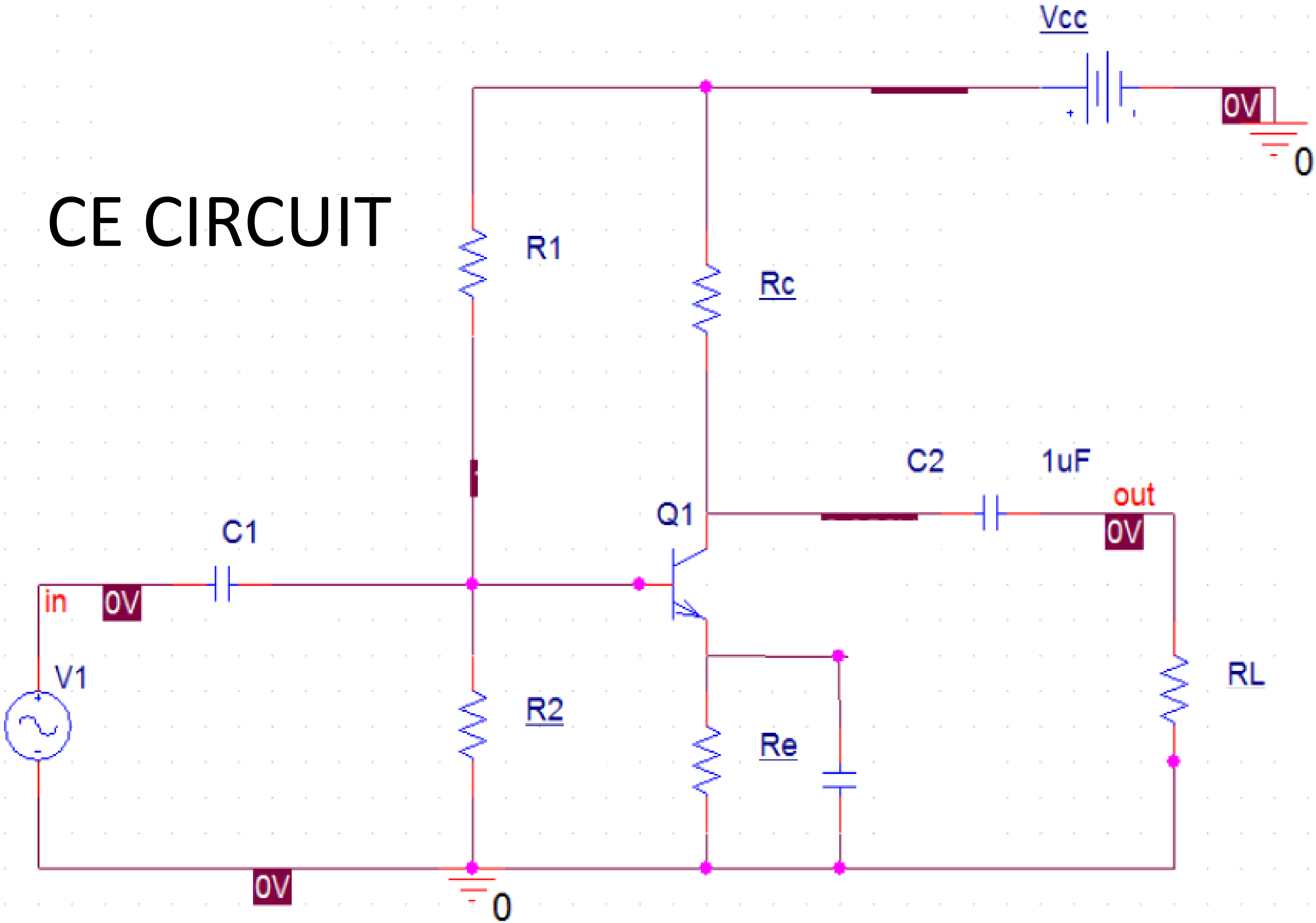


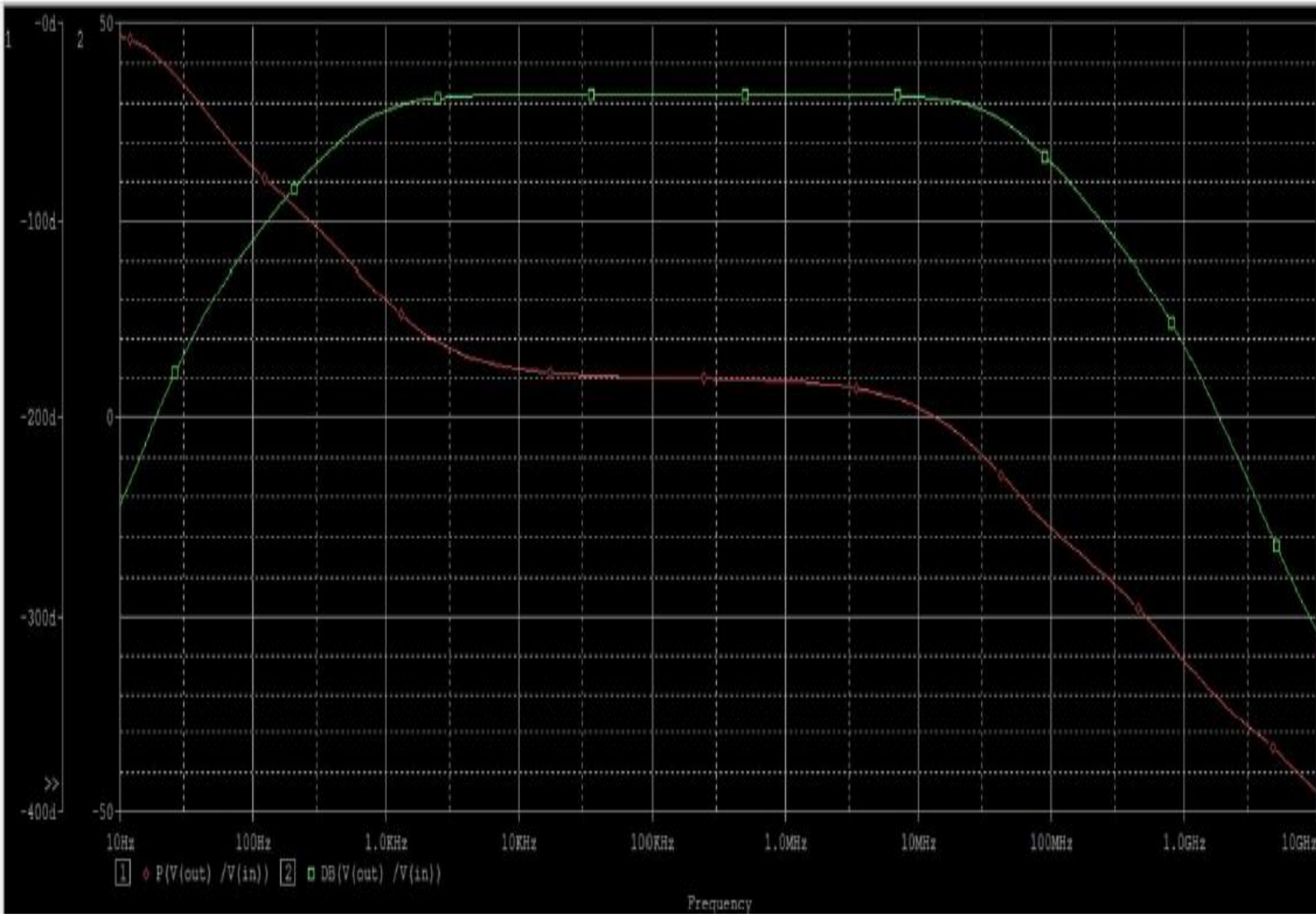
(b)

Coupling Capacitor

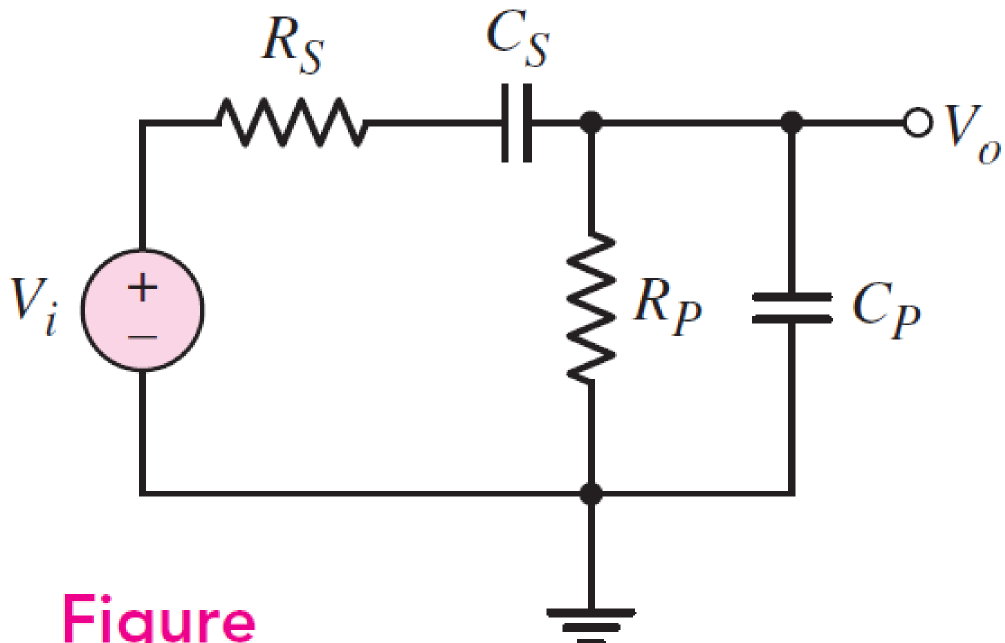
Load Capacitor

CE CIRCUIT




1
2
-0d
50
-100d
-200d
0
-300d
>>
-400d
-50
10Hz
100Hz
1.0KHz
10KHz
100KHz
1.0MHz
10MHz
100MHz
1.0GHz
10GHz
1 P(V(out) /V(in))
2 DB(V(out) /V(in))
Frequency

RC Circuits for CE



Figure

Circuit with both a series coupling and a parallel load capacitor

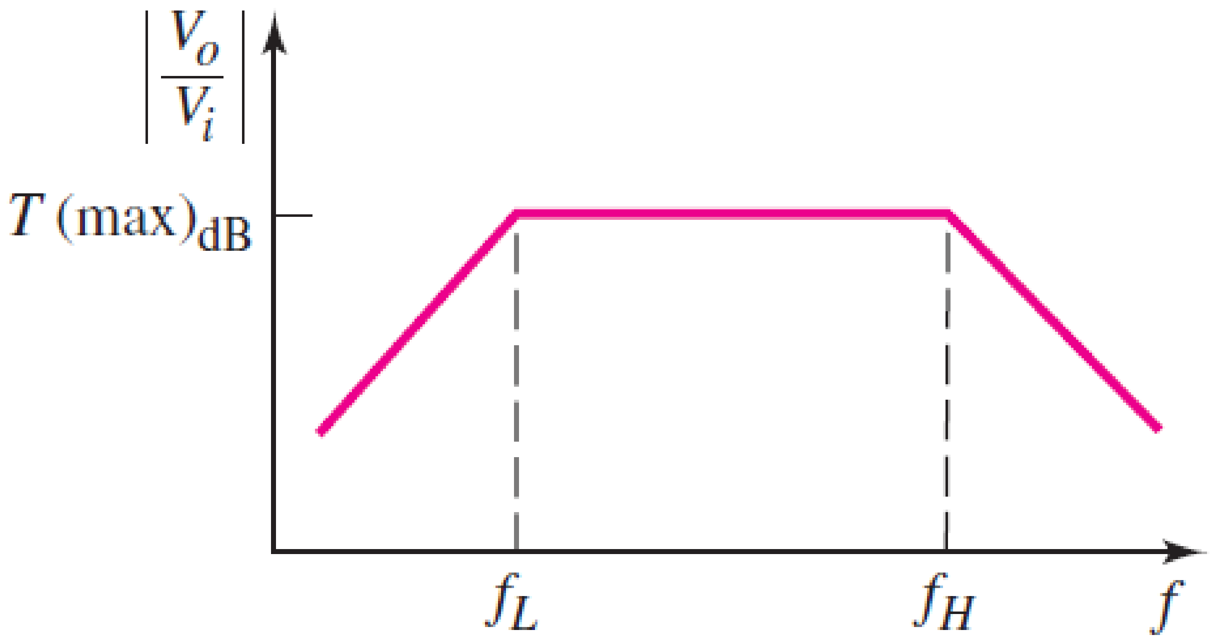
TRANSFER FUNCTION

$$\frac{V_o(s)}{V_i(s)} = \left(\frac{R_P}{R_S + R_P}\right) \times \frac{1}{\left[1 + \left(\frac{R_P}{R_S + R_P}\right)\left(\frac{C_P}{C_S}\right) + \frac{1}{s\tau_S} + s\tau_P\right]}$$

$\tau_S = (R_S + R_P)C_S$ called an **open-circuit time constant.**

$\tau_P = (R_S \| R_P)C_P$ called the **short-circuit time constant.**

BODE PLOT



Bode plot of the voltage transfer function magnitude for the circuit

Contents

Input Coupling Capacitor Common-Emitter (1)

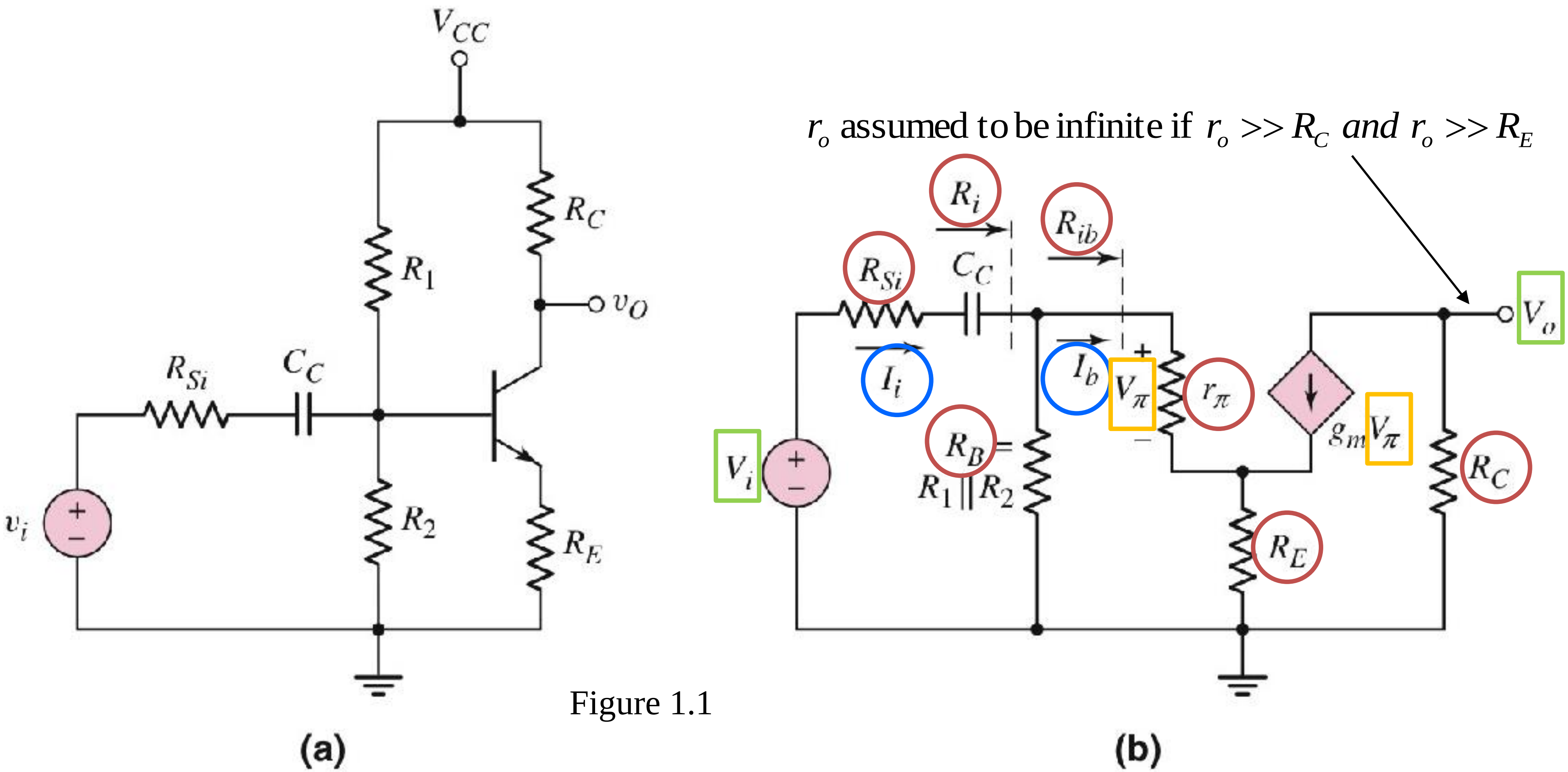
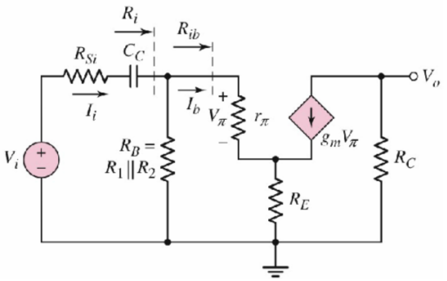


Figure 1.1

a) Common-emitter circuit with input coupling capacitor

b) Small-signal equivalent circuit

Input Coupling Capacitor Common-Emitter (2)



The input current can be written as

$$I_i = \frac{V_i}{R_{Si} + \dfrac{1}{s\,C_C} + R_i} \qquad (1.1)$$

Where the input resistance s and R_i is given by

$$s = j\omega$$

$$R_i = R_B \,//\left[r_\pi + (1+\beta)R_E\right] = R_B \,//\, R_{ib} \qquad (1.2)$$

Using a current divider, we determine the base current to be

$$I_b = \left(\frac{R_B}{R_B + R_{ib}}\right) I_i \qquad (1.3)$$

and then

$$V_\pi = I_b\, r_\pi \qquad (1.4)$$

Input Coupling Capacitor Common-Emitter (3)

The output voltage is given by

$$V_o = -g_m V_\pi R_C \tag{1.5}$$

Combining Equation (1.1) through (1.5)

$$V_o = -g_m r_\pi R_C \left(\frac{R_B}{R_B + R_{ib}} \right) \left(\frac{V_i}{R_{si} + \frac{1}{s C_C} + R_i} \right) \tag{1.6}$$

Therefore, the small-signal voltage gain is

$$A_v(s) = \frac{V_o(s)}{V_i(s)} = -g_m r_\pi R_C \left(\frac{R_B}{R_B + R_{ib}} \right) \left(\frac{s C_C}{1 + s(R_{si} + R_i) C_C} \right) \tag{1.7}$$

Input Coupling Capacitor Common-Emitter (4)

Which can be written in the form

$$A_v(s) = \frac{V_o(s)}{V_i(s)} = \frac{-g_m\ r_\pi\ R_C}{(R_{si} + R_i)}\left(\frac{R_B}{R_B + R_{ib}}\right)\left(\frac{s\,\tau_s}{1 + s\,\tau_s}\right) \qquad (1.8)$$

Where the time constant is

$$\tau_s = (R_{si} + R_i)\,C_C \qquad (1.9)$$

The corner frequency is

$$f_L = \frac{1}{2\pi\,\tau_s} = \frac{1}{2\pi(R_{si} + R_i)\,C_C} \qquad (1.10)$$

and the maximum magnitude, in decibels, is

$$\left|A_v(\max)\right|_{\mathrm{DB}} = 20\log_{10}\left(\frac{g_m\,r_\pi\,R_C}{R_{si} + R_i}\right)\left(\frac{R_B}{R_B + R_{ib}}\right) \qquad (1.11)$$

Example 1.1:

Calculate the corner frequency and maximum gain of a bipolar common-emitter circuit with a coupling capacitor.

For the circuit shown in figure 1.1, the parameters are: R_1=51.2 Kohms, R_2=9.6Kohms, R_C=2 Kohms, R_E=0.4Kohms, R_{Si}=0.1 Kohms, C_C=1uF, and Vcc=10V. The transistor parameter are $V_{BE}(on)$ =0.7V, β =100, $V_A = \infty$,

$I_{CQ} = 1.81\text{mA}$

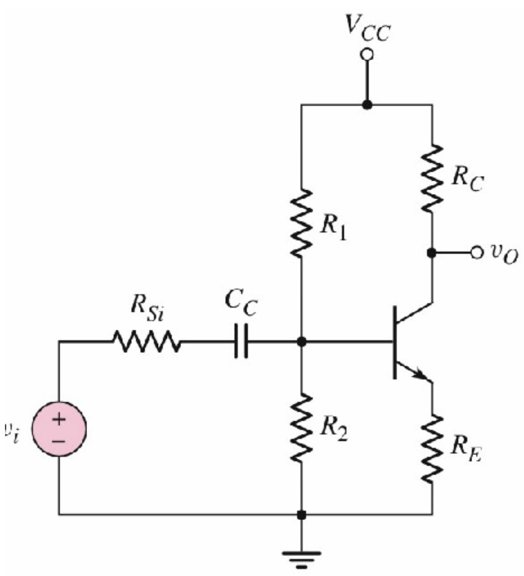


Figure 1.1

Example 1.1 :

Given :

$R_1 = 51.2\ K\Omega$

$R_2 = 9.6\ k\Omega$

$R_C = 2\ k\Omega$

$R_E = 0.4\ K\Omega$

$R_{S_i} = 0.1\ K\Omega$

$C_c = 1\ \mu F$

$I_{CQ} = 1.81\ mA$

$\beta = 100$

$V_{cc} = 10\ V$

$V_{BE(on)} = 0.7\ V$

$V_A = \infty$

Calculate :

① f_L ; ② Maximum gain $|A_v(max)|$

Solution :

step1 : Calcute π model parameter g_m & r_π .

$$g_m = \frac{I_{CQ}}{V_T} = \frac{1.81}{0.026} = 69.6\ mA/V$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{100 \cdot 0.026}{1.81} = 1.44\ K\Omega$$

step2 : Calculate corner frequency f_L

① input resistance R_i

$$R_i = R_1 \| R_2 \| [r_\pi + (1+\beta) R_E]$$
$$= 51.2 \| 9.6 \| [1.44 + 101 \cdot 0.4] = 6.77\ K\Omega$$

② time constant τ_s

$$\tau_s = (R_{S_i} + R_i) C_c = (0.1 + 6.77) \cdot 10^3 \cdot 1 \cdot 10^{-6} = 6.87\ ms$$

③ corner frequency f_L

$$f_L = \frac{1}{2\pi \tau_s} = \frac{10^3}{2\pi \cdot 6.87} = 23.2\ HZ$$

No.

Date

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Step 3: Calculate maximum gain (magnitude) $|A_v(max)|$

① Calculate input resistance at the base R_{ib}

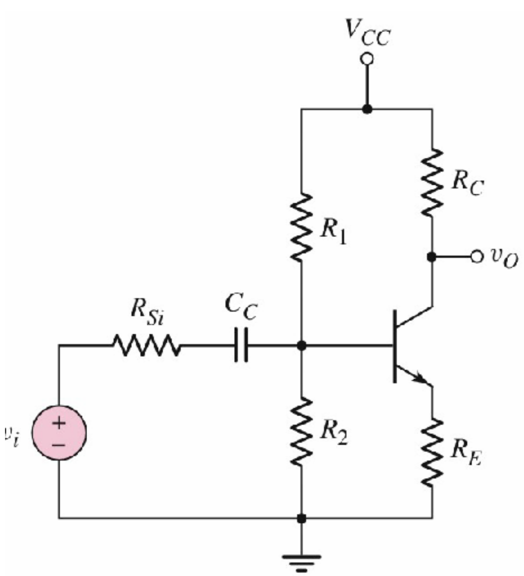
$$R_{ib} = r_\pi + (1+\beta) R_E = 41.8\ k\Omega$$

② $|A_v(max)|$

$$= \left(\frac{g_m r_\pi R_C}{R_{Si} + R_i}\right)\left(\frac{R_B}{R_B + R_{ib}}\right) = 4.72$$

Example 1.2:

Calculate the τ_s , **corner frequency** and **maximum gain** of a bipolar common-emitter circuit with a coupling capacitor.
For the circuit shown in figure 1.1, the parameters are: R_1=20 Kohms, R_2=2.2Kohms, R_C=2 Kohms, R_E=0.1Kohms, R_{si}=0.1 Kohms, C_C=47uF, and V_{CC}=10V. The transistor parameter are V_{BE}(on) =0.7V, β=200, $V_A=\infty$



Date

Example 1.2:

Given:

$R_1 = 20\ k\Omega$ $\quad C_c = 47\ \mu F$ $\quad V_{cc} = 10\ v$

$R_2 = 2.2\ k\Omega$ $\quad V_{BE(on)} = 0.7\ v$

$R_C = 2\ k\Omega$ $\quad \beta = 200$ $\quad V_A = \infty$

$R_E = 0.1\ k\Omega$

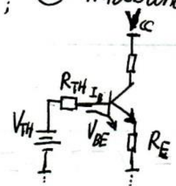
$R_{S_i} = 0.1\ k\Omega$

Calculate:

① τ_s ; ② f_L ; ③ midband voltage gain. A_v

Solution:

step1: DC analysis



① $R_{TH} = R_1 \| R_2 = 20 \| 2.2 = 1.98\ k\Omega$

$$V_{TH} = \frac{R_2}{R_1+R_2} \cdot V_{ce} = \frac{2.2}{20+2.2} \cdot 10 = 0.991\ V$$

② $$I_{BQ} = \frac{V_{TH} - V_{BE(on)}}{R_{TH} + (1+\beta) R_E} = \frac{0.991 - 0.7}{1.98 + 201 \cdot 0.1} = 0.0132\ mA$$

$I_{CQ} = \beta I_{BQ} = 200 \times 0.0132 = 2.64\ mA$

step2: calcute π-paramters.

$$g_m = \frac{I_{CQ}}{V_T} = \frac{2.64}{0.026} = 101.4\ mA/V$$

$$r_\pi = \frac{\beta}{g_m} = \frac{200}{101.4} = 1.97\ k\Omega$$

step3: Calculate τ_s

① resistors R_B, R_i, R_{iB} involved

$R_B = R_1 \| R_2 = R_{TH} = 1.98\ k\Omega$

$$R_{ib} = r_\pi + (1+\beta) R_E = 1.97 + 201 \cdot 0.1 = 22.1\ k\Omega$$

$$\Rightarrow R_i = R_{ib} \parallel R_B$$

$$= 22.1 \parallel 1.98 = 1.817\ k\Omega$$

$$\Rightarrow \tau_s = (R_i + R_{si}) \cdot C_c = (1.817 + 0.1) \cdot 10^3 \times 47 \times 10^{-6}$$

$$= 90.1\ ms$$

step 4: Calculate corner frequency

$$f_L = \frac{1}{2\pi \tau_s} = \frac{10^3}{2\pi \cdot 901.1} = 1.77\ HZ$$

step5:

$$V_o = -g_m V_\pi \cdot R_C$$

$$KVL \Rightarrow \frac{R_i}{R_{si}+R_i} \cdot V_i = I_b \cdot R_{ib} = I_b \cdot [r_\pi + (1+\beta) R_E]$$

$$= \frac{V_\pi}{r_\pi} \cdot [r_\pi + (1+\beta) R_E]$$

$$\Rightarrow V_i = \frac{R_{si}+R_i}{R_i} \cdot \frac{V_\pi}{r_\pi} \cdot [r_\pi + (1+\beta) R_E]$$

$$\Rightarrow A_V = \frac{V_o}{V_i} = \frac{-g_m \cancel{V_\pi} \cdot R_c}{\frac{R_{si}+R_i}{R_i} \cdot \cancel{V_\pi}/r_\pi \cdot [r_\pi + (1+\beta) R_E]} = \frac{-g_m R_c \cdot r_\pi}{r_\pi + (1+\beta) R_E} \cdot \frac{R_i}{R_i + R_{si}}$$

Output Coupling Capacitor – Emitter-follower Circuit (1)

An emitter follower with a coupling capacitor in the output portion of the circuit is shown in figure 3.1(a). We assume that coupling capacitor C_{C1} . , which is part of original emitter follower, is very large and that it acts as a short circuit to input signal

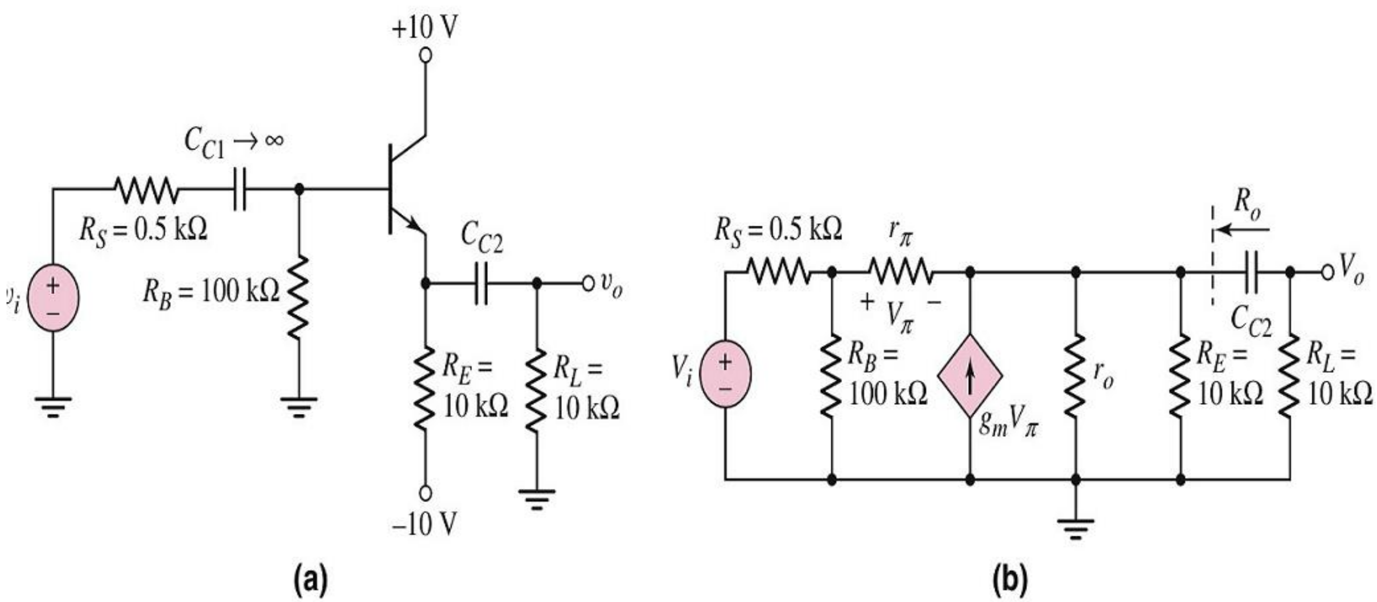


Figure 3.1: (a) Emitter-follower circuit with output coupling capacitor and (b) small-signal equivalent circuit.

Output Coupling Capacitor – Emitter-follower Circuit (2)

The equivalent resistance (r_o) seen by coupling capacitor C_{C2} is $\left[R_o + R_L\right]$, and the time constant is

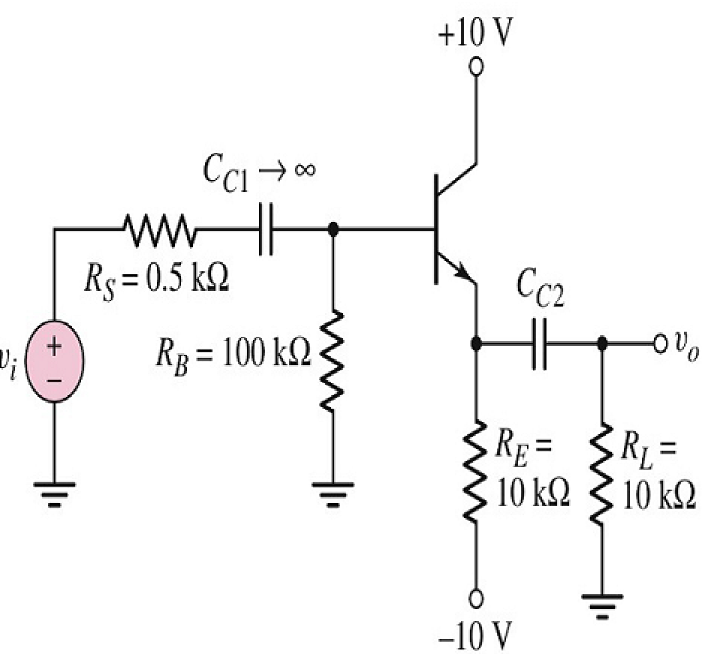
$$\tau_S = \left[R_o + R_L\right] C_{C2} \qquad (3.1)$$

Where R_o is the output resistance as defined in figure 11pi-3(b) and

$$R_o = R_E \,//\, r_o \,//\left\{ \frac{\left[r_\pi + \left(R_S \,//\, R_B\right)\right]}{1+\beta} \right\} \qquad (3.2)$$

Example 2.1:

Determine the 3 dB frequency of an emitter-follower amplifier circuit with an output coupling capacitor.
Consider the circuit shown in figure below with transistor parameters .
$\beta = 100, V_{BE}(on) = 0.7$, and, $V_A = 120V$. The output coupling capacitance is
$C_{C2} = 1\mu F$



Solution:

A DC analysis shows that $I_{CQ} = 0.838$ mA. Therefore the small signal parameters are:

The output resistance R_o of the emitter follower is

Emitter-follower Amplifier

Example 3.1

Given: $R_S = 0.5\ K\Omega$, $V_+ = 10\ V$, $V_- = -10\ V$, $C_{C2} = 1\ \mu F$

$R_B = 100\ K\Omega$, $V_{BE(on)} = 0.7\ V$, $C_{C1} \to \infty$

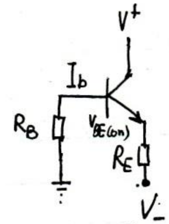
$R_E = 10\ K\Omega$, $V_A = 120\ V$

$R_L = 10\ K\Omega$, $\beta = 100$

Calculate: f_L

Solution:

step 1: DC analysis



KVL

$$\Rightarrow 0 - V_- = I_{BQ} \cdot \left(R_B + (1+\beta) R_E \right) + V_{BE(on)}$$

$$I_{BQ} = \frac{-V_{BE(on)} - V_-}{R_B + (1+\beta) R_E} = \frac{10 - 0.7}{100 + 101 \cdot 10} = 8.38 \times 10^{-6}\ A$$

$$I_{CQ} = \beta I_{BQ} = 0.838\ mA$$

step 2: Calculate π-model parameters

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.838}{0.026} = 32.2\ mA/V$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{32.2} = 3.1\ K\Omega$$

$$r_o = \frac{V_A}{I_{CQ}} = \frac{120}{0.838} = 143\ K\Omega$$

step 3: Calculate output resistance R_o

$$R_o = R_E \parallel r_o \parallel \frac{r_\pi + R_B \parallel R_S}{1+\beta} = 35.5\ \Omega$$

step4: Time constant τ_s

$$\tau_s = (R_o + R_L)\cdot C_{C2}$$

$$= (35.5 + 10^4)\cdot 1\times10^{-6} = 1\times10^{-2}\ s$$

step 5: Corner frequency

$$f_L = \frac{1}{2\pi\tau_s} = 15.9\ HZ$$

Bypass Capacitor Effects (1)

The bypass capacitors are assumed to act as short circuits at the signal frequency. However, to guide us in choosing a bypass capacitor, we must determine the circuit response in the frequency range where these capacitors are neither open or short circuits.

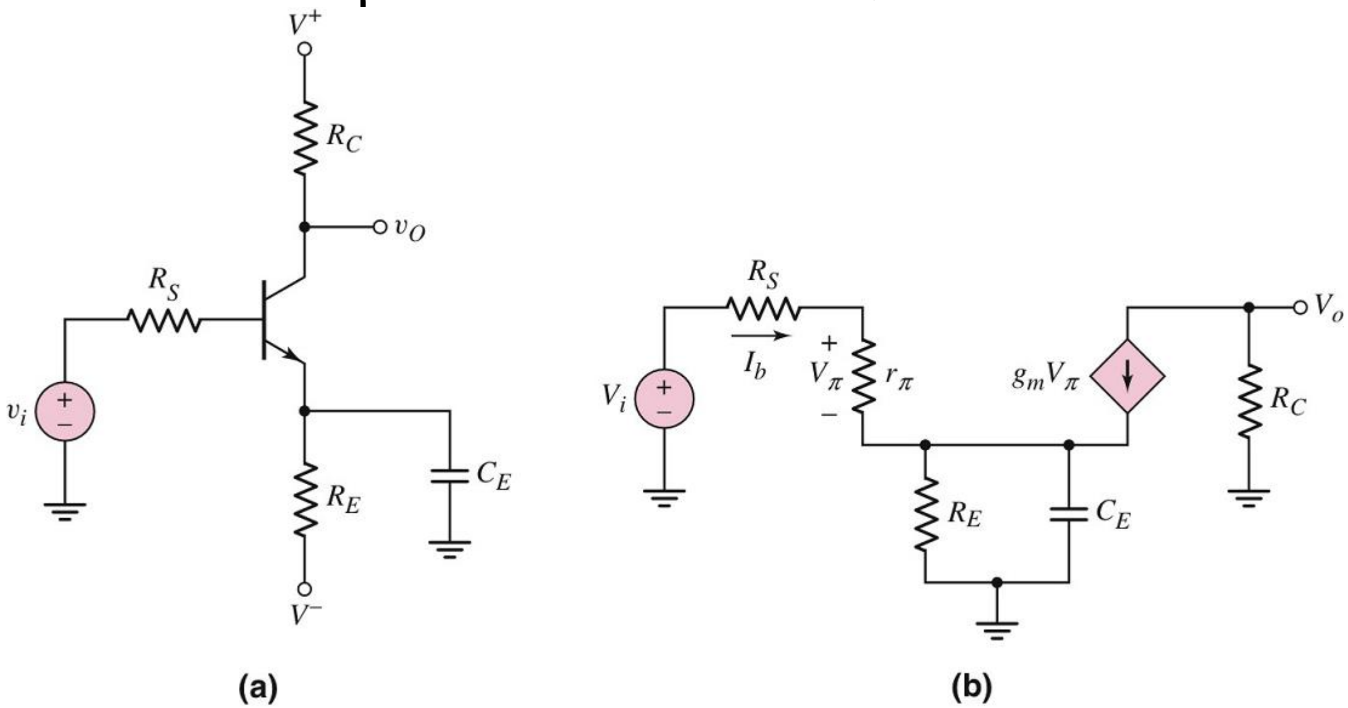


Figure 4.1: (a) Circuit with emitter bypass capacitor
(b) small-signal equivalent circuit.

Bypass Capacitor Effects (2)

We can find the small-signal voltage gain as a function of frequency. Using the impedance reflection rule, the small-signal input current is

$$I_b = \frac{V_i}{R_S + r_\pi + (1+\beta)\left(R_E \,//\, \frac{1}{s\, C_E} \right)} \qquad (4.1)$$

The total impedance in the emitter is multiplied by the factor $(1+\beta)$. The control voltage is

$$V_\pi = I_b \; r_\pi \qquad (4.2)$$

and the output voltage is

$$V_o = -g_m \, V_\pi \; R_C \qquad (4.3)$$

Combining equations produces the small-signal voltage gain, as follows:

$$A_v(s) = \frac{V_o(s)}{V_i(s)} = \frac{-g_m \, r_\pi \; R_C}{R_S + r_\pi + (1+\beta)\left(R_E \,//\, \frac{1}{s\, C_E} \right)} \qquad (4.4)$$

Bypass Capacitor Effects (3)

Expanding the parallel combination of R_E and $1/sC_E$ and rearranging terms, we find

$$A_v = \frac{-g_m r_\pi R_C}{\left[R_S + r_\pi + (1+\beta)R_E\right]} \text{ x } \frac{(1 + s R_E C_E)}{\left[1 + \dfrac{s R_E (R_S + r_\pi) C_E}{\left[R_S + r_\pi + (1+\beta)R_E\right]}\right]} \qquad (4.5)$$

Equation 11pi.31 can be written in terms of time constant as

$$A_v = \frac{-g_m r_\pi R_C}{\left[R_S + r_\pi + (1+\beta)R_E\right]} \text{ x } \frac{1 + s\tau_A}{1 + s\tau_B} \qquad (4.6)$$

The Bode plot of the voltage gain magnitude has two limiting horizontal asymptotes. If we set $s = j\omega$, we can then consider the limit as $\omega \to 0$ and the limit as $\omega \to \infty$. For $\omega \to 0$, C_E acts as an open circuit; for $\omega \to \infty$, C_E acts as a short circuit. From Equation (4.5), we have

$$\left|A_v\right|_{\omega \to 0} = \frac{g_m r_\pi R_C}{\left[R_S + r_\pi + (1+\beta)R_E\right]} \qquad (4.7)$$

Bypass Capacitor Effects (4)

and $$|A_v|_{\omega\to\infty} = \frac{g_m\, r_\pi\, R_C}{R_S + r_\pi} \tag{4.8}$$

From these results, we see that for $\omega \to 0$, R_E is included in the gain expression , and for $\omega \to \infty$, R_E is not part of the gain expression, since it has been effectively shorted out by C_E.

If we assume that the time constants τ_A and τ_B in equation (4.6) differ substantially in magnitude, then the corner frequencies due to τ_A and are τ_B

$$f_B = 1/2\pi\tau_B \tag{4.9(a)}$$

$$f_A = 1/2\pi\tau_A \tag{4.9(b)}$$

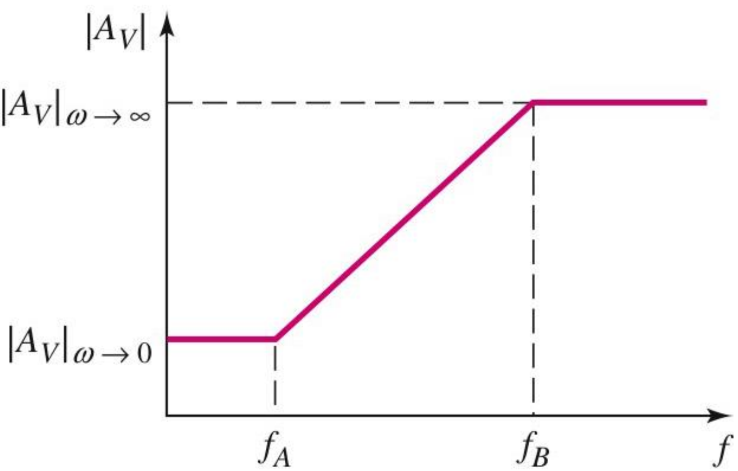


Figure 4.2: Bode plot of the voltage gain magnitude for the circuit with an emitter bypass capacitor.

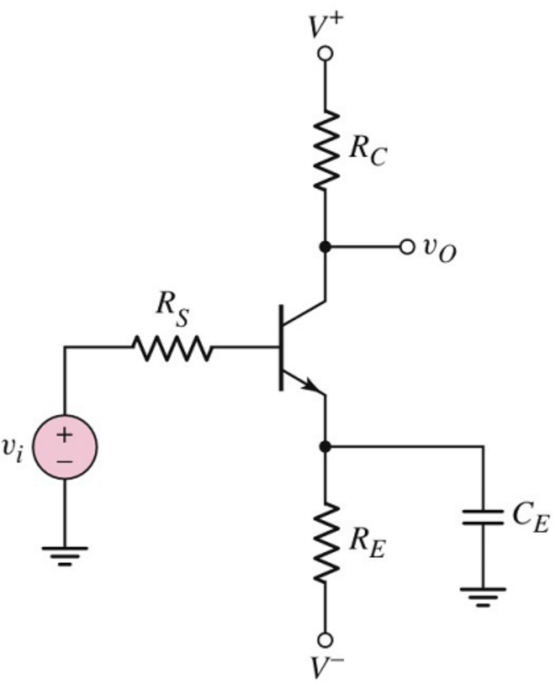
Example 3.1:

Determine the **corner frequencies** and **limiting horizontal asymptotes** of a common-emitter circuit with an emitter bypass capacitor.

Consider the circuit in figure below with parameters $R_E = 4K\Omega, R_C = 2K\Omega$

$R_S = 0.5K\Omega, C_E = 1\mu\text{F}, \text{V}^+ = 5\text{V}, \text{and } \text{V}^- = -5\text{V}$

The transistor parameters are $\beta = 100, V_{BE}(on) = 0.7, \text{and}, r_o = \infty$



Solution:

A DC analysis shows that $I_{CQ} = 1.06\text{ mA}$. Therefore the transconductance is :

Example 4.1 (Common - Emitter)

Given: $R_E = 4\ k\Omega$ $\quad C_E = 1\ \mu F$ $\quad V+ = 5V$

$R_C = 2\ k\Omega$ $\quad V- = -5\ V$

$R_S = 0.5\ k\Omega$ $\quad \beta = 100$ $\quad V_{BE(on)} = 0.7$

$r_o = \infty$

Calculate: ① f_A, f_B ; ② $|A_v|_{\omega \to 0}$, $|A_v|_{\omega \to \infty}$

Solution:

Step 1: DC analysis

$$KVL \Rightarrow -V^- - V_{BE(on)} = I_{BQ} (R_S + (1+\beta) R_E)$$

$$\Rightarrow I_{BQ} = \frac{4.3}{0.5 + 101 \cdot 4} = 0.0106\ mA$$

$$\Rightarrow I_{CQ} = \beta I_{BQ} = 1.06\ mA$$

step 2: π-model

$$g_m = \frac{I_{CQ}}{V_T} = \frac{1.06}{0.026} = 40.77\ mA/V$$

$$r_\pi = \beta / g_m = 2.45\ k\Omega$$

Step 3: Calculate time constants

$$\tau_A = R_E \cdot C_E = 4 \times 1 = 4\ ms$$

$$\tau_B = \frac{R_E (R_S + r_\pi)}{R_S + r_\pi + (1+\beta) R_E} = 2.9 \times 10^{-5}\ s$$

step4: Calculate corner frequency

$$f_A = \frac{1}{2\pi \tau_A} = 39.8\ HZ$$

$$f_B = \frac{1}{2\pi \tau_B} = 5.49\ KHZ$$

step5: Voltage gains.

① $\omega \to 0 \Rightarrow \frac{1}{sC_E} \to \infty$

$$|A_v|_{\omega \to 0} = \frac{g_m r_\pi \cdot R_C}{R_S + r_\pi + (1+\beta) R_E} = 0.491$$

② $\omega \to \infty \Rightarrow \frac{1}{sC_E} = 0$

$$|A_v|_{\omega \to \infty} = \frac{g_m r_\pi R_C}{R_S + r_\pi} = 67.7$$

EEE109: Electronic Circuits

Frequency Response

General Purposes

- Discuss the general frequency response characteristics of amplifiers.
- Derive the system transfer functions
  - Develop the Bode diagrams of the magnitude and phase of the transfer functions.
- Analyze the frequency response of transistor circuits with capacitors.

Contents

- Capacitor Involved in Previous Circuits
- RC Circuits as Lowpass and Highpass filters
- Frequency Response Characteristics of Amplifiers
- Frequency Response of the Common Emitter Amplifiers

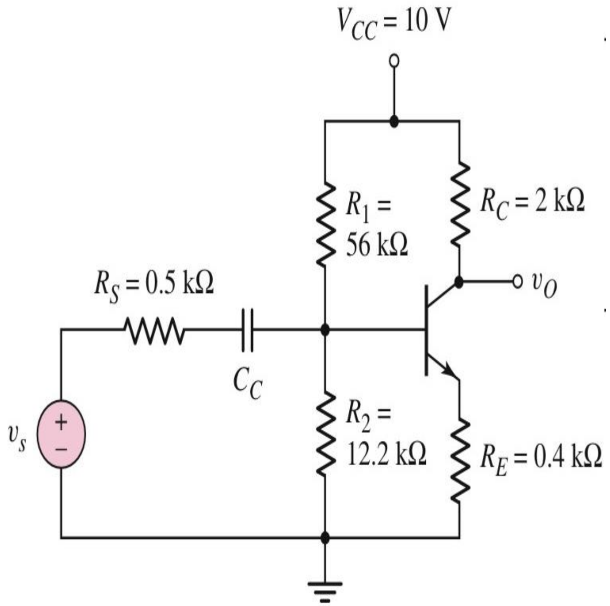
Capacitors Involved in Precious Circuits

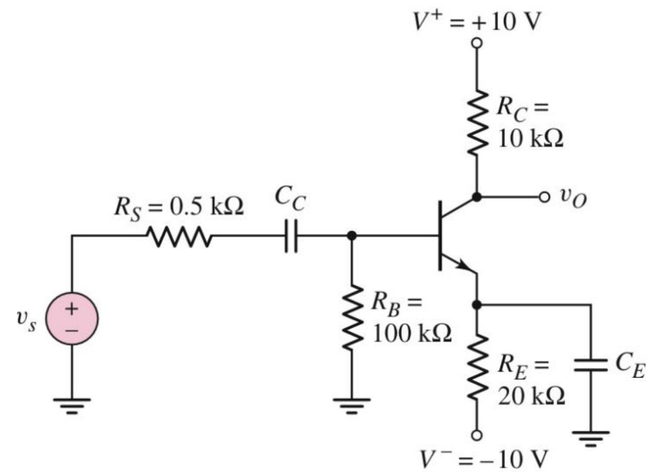
Previously it was assumed that the ***input coupling capacitor***, ***output coupling capacitor*** and ***emitter or source bypass capacitor*** are **short** circuits at **a.c.** signal frequencies of interest and **open** circuits for **d.c.**

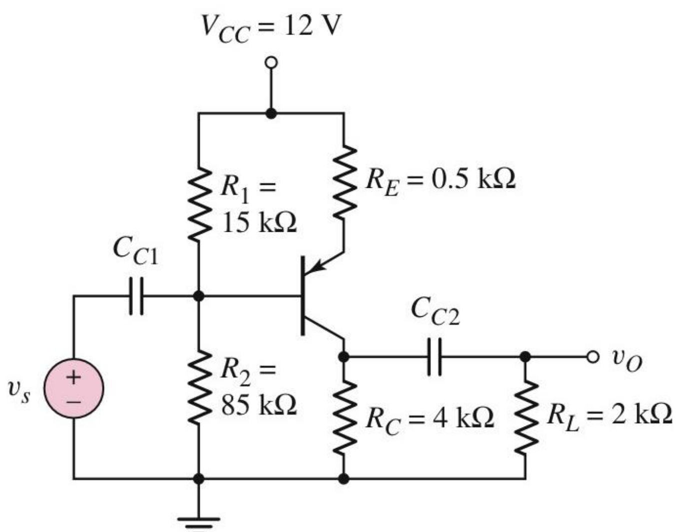
At frequencies of interest the impedances of the capacitors are so small compared with the impedances of other circuit elements that they may be regarded as **zero**

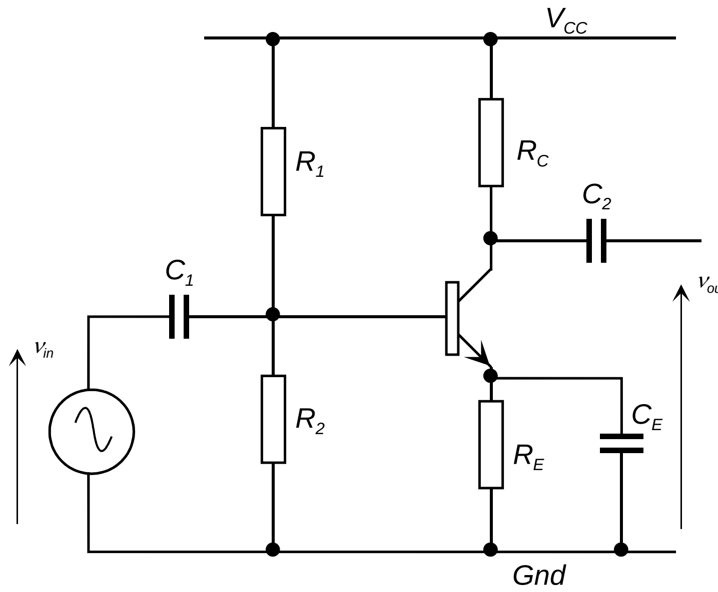
$$\frac{1}{j\omega C_1} \to 0 \qquad \frac{1}{j\omega C_2} \to 0$$

$$\frac{1}{j\omega C_X} \to 0$$ (X is E or S) at the operating frequency $f = \frac{\omega}{2\pi}$


$V_{CC} = 10$ V
$R_1 =$ 56 kΩ
$R_C = 2$ kΩ
$R_S = 0.5$ kΩ
v_O
C_C
v_s
$R_2 =$ 12.2 kΩ
$R_E = 0.4$ kΩ


$V^+ = +10$ V
$R_C =$ 10 kΩ
$R_S = 0.5$ kΩ
C_C
v_O
v_s
$R_B =$ 100 kΩ
$R_E =$ 20 kΩ
C_E
$V^- = -10$ V


$V_{CC} = 12$ V
$R_1 =$ 15 kΩ
$R_E = 0.5$ kΩ
C_{C1}
C_{C2}
v_O
v_s
$R_2 =$ 85 kΩ
$R_C = 4$ kΩ
$R_L = 2$ kΩ

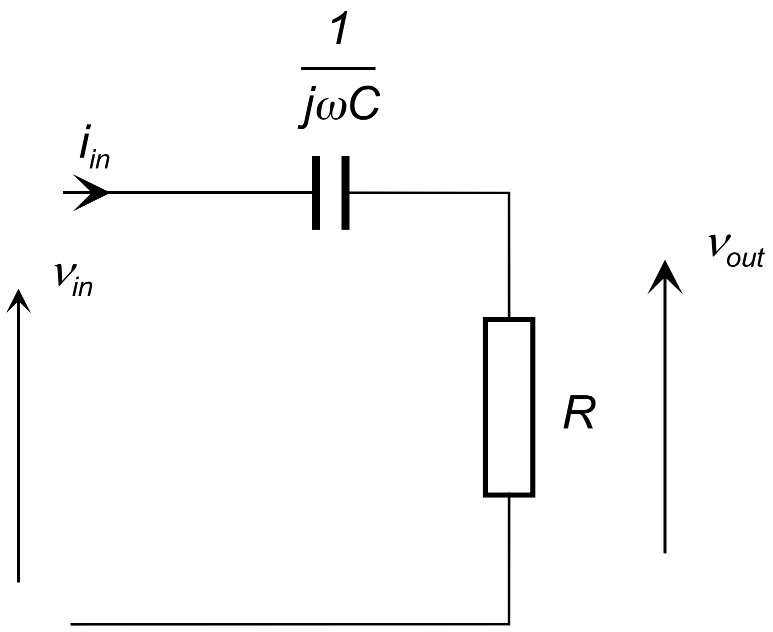

V_{CC}
R_1
R_C
C_2
C_1
v_{out}
v_{in}
R_2
C_E
R_E
Gnd

Contents (Cont')

- Capacitor Involved in Previous Circuits
- RC Circuits as Lowpass and Highpass filters
- Frequency Response Characteristics of Amplifiers
- Frequency Response of the Common Emitter Amplifiers

RC Circuit as a Highpass Filter (1)

Consider the simple *RC* circuit

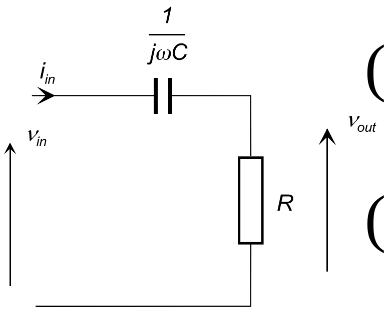


Resistor impedance is *R*, **capacitor** impedance is $\dfrac{1}{j\omega C}$

and **inductor** impedance is $j\omega L$.

RC Circuit as a Highpass Filter (2)

Kirchoff's Law gives $$v_{in} = i_{in}R - j\frac{i_{in}}{\omega C} \quad (1.1)$$



Ohm's Law $$v_{out} = i_{in}R \quad (1.2)$$

Hence $$v_{out} = \frac{\omega^2 R^2 C^2 + j\omega RC}{\omega^2 R^2 C^2 + 1} v_{in} \quad (1.3)$$

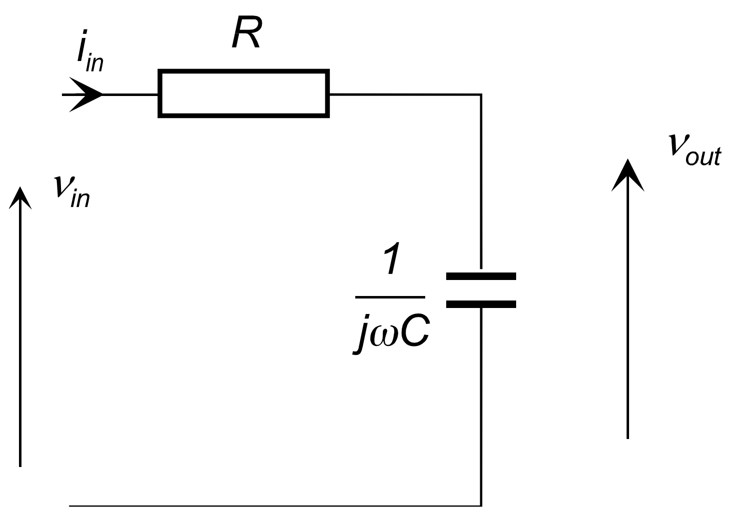
If $\omega \to 0$ then $v_{out} \to \frac{0}{1} \to 0$ and if $\omega \to \infty$ then $\omega^2 R^2 C^2 >> 1$

and $v_{out} \to v_{in}$

An output voltage appears across the resistor only if the **input frequency** is **sufficiently high**, this *RC* circuit configuration is a **high pass** filter (a high pass stage).

RC Circuit as a Lowpass Filter (1)

For the same *RC* circuit, but with the **voltage output** taken across the **capacitor**.



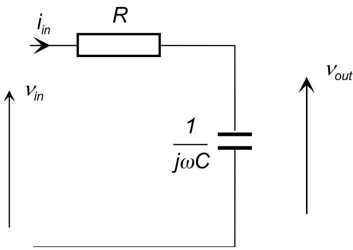
Kirchoff's Law still gives

$$v_{in} = i_{in}R - j\frac{i_{in}}{\omega C} \qquad (1.1)$$

but now

$$v_{out} = -i_{in}\frac{j}{\omega C} \qquad (1.4)$$

RC Circuit as a Lowpass Filter (2)



Hence $$v_{out} = \frac{1 - j\omega RC}{\omega^2 R^2 C^2 + 1} v_{in} \qquad (1.5)$$

If $\omega \to 0$ then $v_{out} \to \frac{1}{1} v_{in} \to v_{in}$ and if $\omega \to \infty$ then $\omega^2 R^2 C^2 >> 1$ and $v_{out} \to 0$

In this case, $v_{out} \to v_{in}$ if $\omega << \frac{1}{RC}$, i.e. frequency must be **below** some characteristic value for the **output to be close to the input**. This circuit is a **low pass filter**

At $\omega = \frac{1}{RC}$ (or $R = \frac{1}{\omega C}$) the impedances of R and C have **equal magnitude**

ω is the cut-off angular frequency, the cut-off point of an *RC* filter (often ω_0 is used for this value of ω).

RC Circuit as Lowpass/Highpass Filters

From 1.3 and 1.5

$$\left|\frac{v_{out}}{v_{in}}\right|_{\substack{high\\pass}} = \frac{\omega RC}{\sqrt{\omega^2 R^2 C^2 + 1}} \qquad (1.6)$$

$$\left|\frac{v_{out}}{v_{in}}\right|_{\substack{low\\pass}} = \frac{1}{\sqrt{\omega^2 R^2 C^2 + 1}} \qquad (1.7)$$

Contents (Cont')

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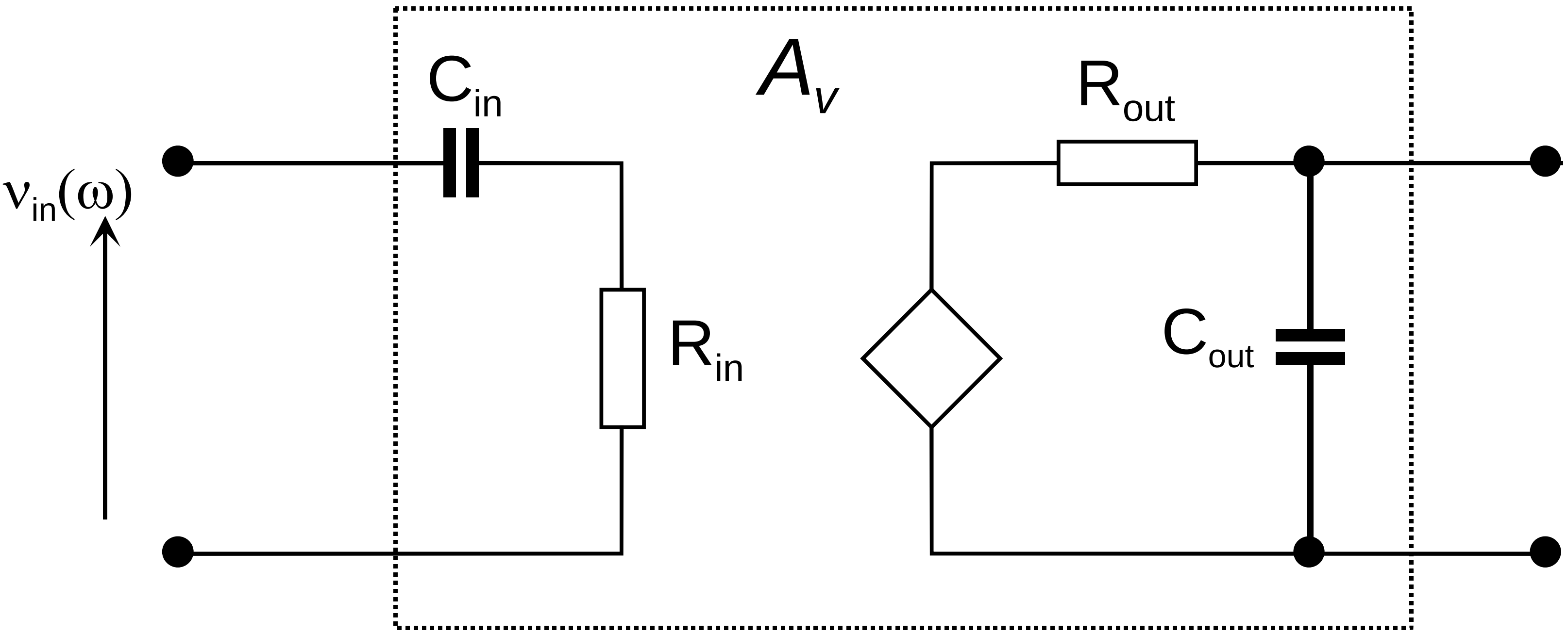
Frequency Response Characteristics of Amplifiers (1)

Frequency response characteristics may be included in the **generic four terminal amplifier** as



Frequency Response Characteristics of Amplifiers (2)

The input is a **high pass stage**, C_{in} and R_{in}. The output is a **low pass stage**, C_{out} and R_{out}. The cut-off frequency of the **high pass filter** at a much lower frequency than that of the low pass filter

The input signal $v_{in}(\omega)$ only appears across the input resistance R_{in} when $\omega >> \dfrac{1}{R_{in}C_{in}}$. For the output voltage to be $v_{out}(\omega) = A_V v_{in}(\omega)$ requires that in addition $\omega << \dfrac{1}{R_{out}C_{out}}$

The **voltage gain** of the amplifier is

$$|A(\omega)| = \left|\frac{v_{out}(\omega)}{v_{in}(\omega)}\right|$$

or in decibels

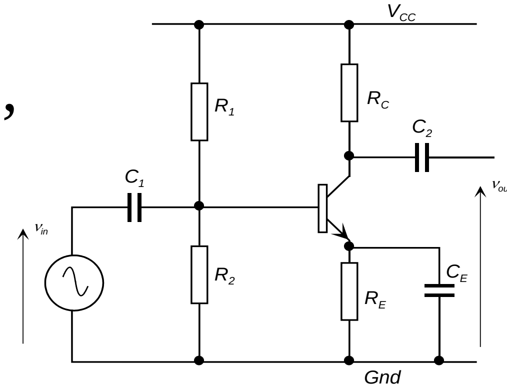
$$|A(\omega)|_{dB} = 20\,log_{10}\left(\left|\frac{v_{out}(\omega)}{v_{in}(\omega)}\right|\right)$$

Frequency Response Characteristics of Amplifiers (3)

For an amplifier $R_{out} << R_{in}$ and $C_{out} << C_{in}$,

hence

$$\frac{1}{R_{out}C_{out}} >> \frac{1}{R_{in}C_{in}}$$



For the single transistor common emitter and common source amplifiers examined, C_{in} is an **equivalent capacitor** representing the effects of **coupling capacitors** C_1 and C_2 combined with the emitter/source **bypass capacitor**.

Important - C_1 is not the input capacitor C_{in} and C_2 is not the output capacitor C_{out} . C_{out} represents **residual capacitance** between the ***output terminal*** and ***earth*** plus effects of ***internal capacitances*** within the transistor. The internal capacitances of transistors are not examined in detail in year 2.

Frequency Response Characteristics of Amplifiers (4)

Five frequency ranges can be identified.

Very low frequency $\omega << \frac{1}{R_{in}C_{in}}$

Low frequency $\omega \sim \frac{1}{R_{in}C_{in}}$

Mid-range $\omega >> \frac{1}{R_{in}C_{in}}$ and $\omega << \frac{1}{R_{out}C_{out}}$

High frequency $\omega \sim \frac{1}{R_{out}C_{out}}$

Very high frequency $\omega >> \frac{1}{R_{out}C_{out}}$

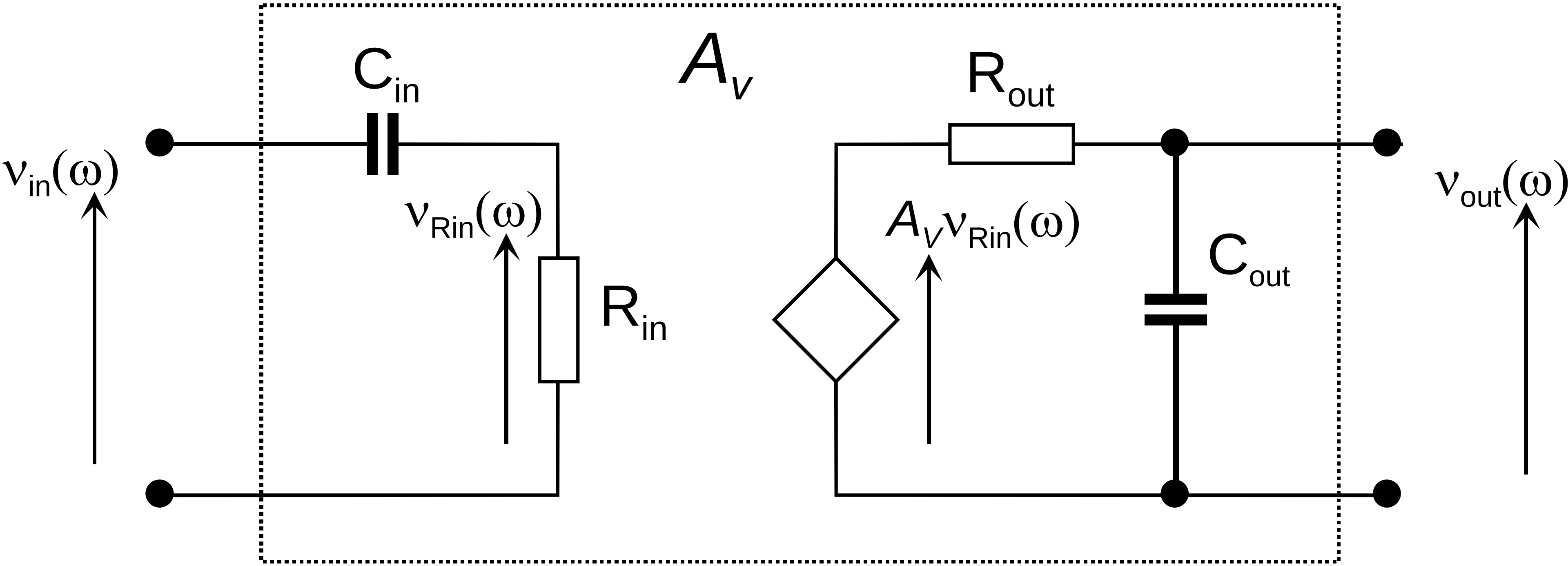
Frequency Response Characteristics of Amplifiers (5)

Mid-range –already examined, neither filter has any significant effect, the output voltage is A_V times the input voltage, that is

$$|A(\omega)| = \left| \frac{v_{out}(\omega)}{v_{in}(\omega)} \right| = A_V$$

Below mid-range the low pass filter in the output has no effect but the high pass filter in the input circuit does affect the circuit. To examine this it is convenient to consider the input circuit as a **voltage divider** with a reduced voltage defined as v_{Rin} developed across R_{in} and then amplified

Frequency Response Characteristics of Amplifiers (6)



Frequency Response Characteristics of Amplifiers (7)

$$\left|\frac{v_{out}}{v_{in}}\right|_{\substack{high\\pass}} = \frac{\omega RC}{\sqrt{\omega^2 R^2 C^2 + 1}}$$

Very low frequency - Equation 1.6 describes the input circuit, if

$\omega << \dfrac{1}{R_{in}C_{in}}$ then, 1 is much larger than $\omega^2 R_{in}^2 C_{in}^2$ so the voltage

v_{Rin} across R_{in} is given by 1.6 simplified to $\left|\dfrac{v_{Rin}}{v_{in}}\right| = \omega R_{in} C_{in}$

As a result the **overall behaviour** in this frequency region is

$$|A(\omega)| = \left|\frac{v_{out}(\omega)}{v_{in}(\omega)}\right| = A_V \omega R_{in} C_{in}$$

or in dBs $|A(\omega)|_{dB} = 20\log_{10}\left(\left|\dfrac{v_{out}(\omega)}{v_{in}(\omega)}\right|\right) = 20\log_{10}(A_V \omega R_{in} C_{in})$

$$= 20\log_{10}(A_V R_{in} C_{in}) + 20\log_{10}(\omega)$$

Frequency Response Characteristics of Amplifiers (8)

A plot of gain in dBs against log of frequency at very low frequency is a **straight line rising** as frequency increases. There is enough information to plot gain against frequency at very low frequency and mid frequency (next page).

Note gain at angular frequency 2ω is

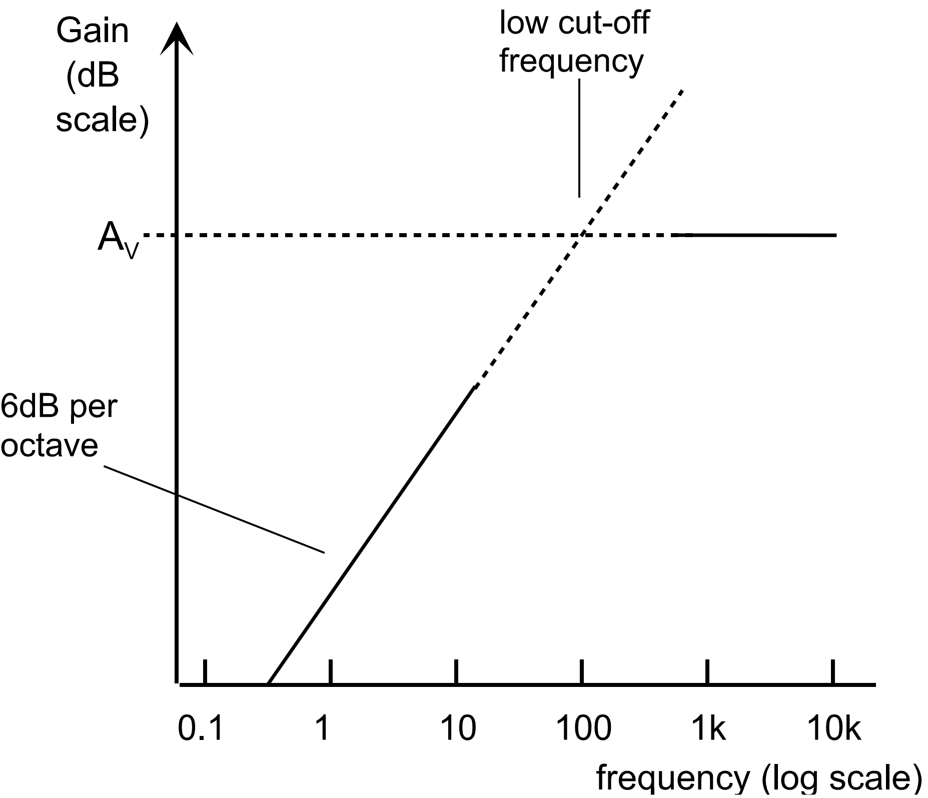
$$\left|A(\omega)\right|_{dB} = 20\,log_{10}\left(A_V R_{in} C_{in}\right) + 20\,log_{10}(2\omega)$$

and the **difference in gain** from ω to 2ω is

$$20\,log_{10}(2) = 6 \text{ dBs.}$$

Gain **increases by 6dBs** when the **frequency doubles** (6dBs **per octave**), calculation for frequency increase of **ten** gives **20dBs** increase, 20dBs **per decade**.

Frequency Response Characteristics of Amplifiers (9)



Increasing the straight line to meet the horizontal line of constant gain, A_V, shows that they cross when

$$0 = log_{10}\left(\omega R_{in} C_{in}\right)$$

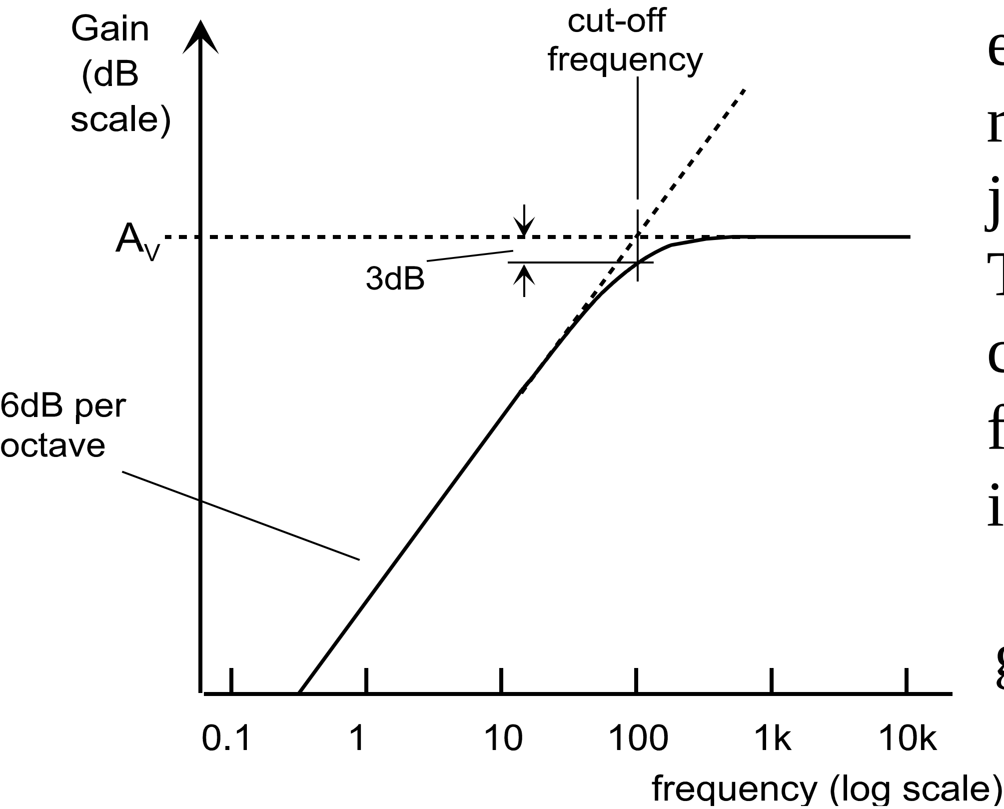
or $$1 = \omega R_{in} C_{in}$$,

i.e. at the low cut-off frequency

$$\omega = \frac{1}{R_{in} C_{in}}$$

Low frequency Say about **0.2 times to 5 times** the cut off.

Frequency Response Characteristics of Amplifiers (10)



In this region the exact form of equation 1.6 must be used – the results will form a smooth curve joining the two straight lines. The exact form can be used to calculate the gain at the cut-off frequency, the result is that the gain is $\frac{1}{\sqrt{2}}$ **lower** than the **mid-frequency** gain.

Therefore the cut-off is also known as the lower -3dB frequency or **3dB point**.

Frequency Response Characteristics of Amplifiers (11)

Above mid-range the situation is the **reverse** of the **low frequency one**. The high pass filter in the input has no effect but the low pass filter in the output does. The treatment is similar to the low pass case. Use the low pass result in equation 1.7 at the output of the amplifier. Now there is a **high frequency cut-off point** – also with gain 3dB below mid frequency - given by

$$\omega = \frac{1}{R_{out}C_{out}}$$

Very high frequency is when and $\omega \gg \dfrac{1}{R_{out}C_{out}}$ and the gain falls at 6dB per octave (20dB per decade). Equation 1.7 gives the approximate result

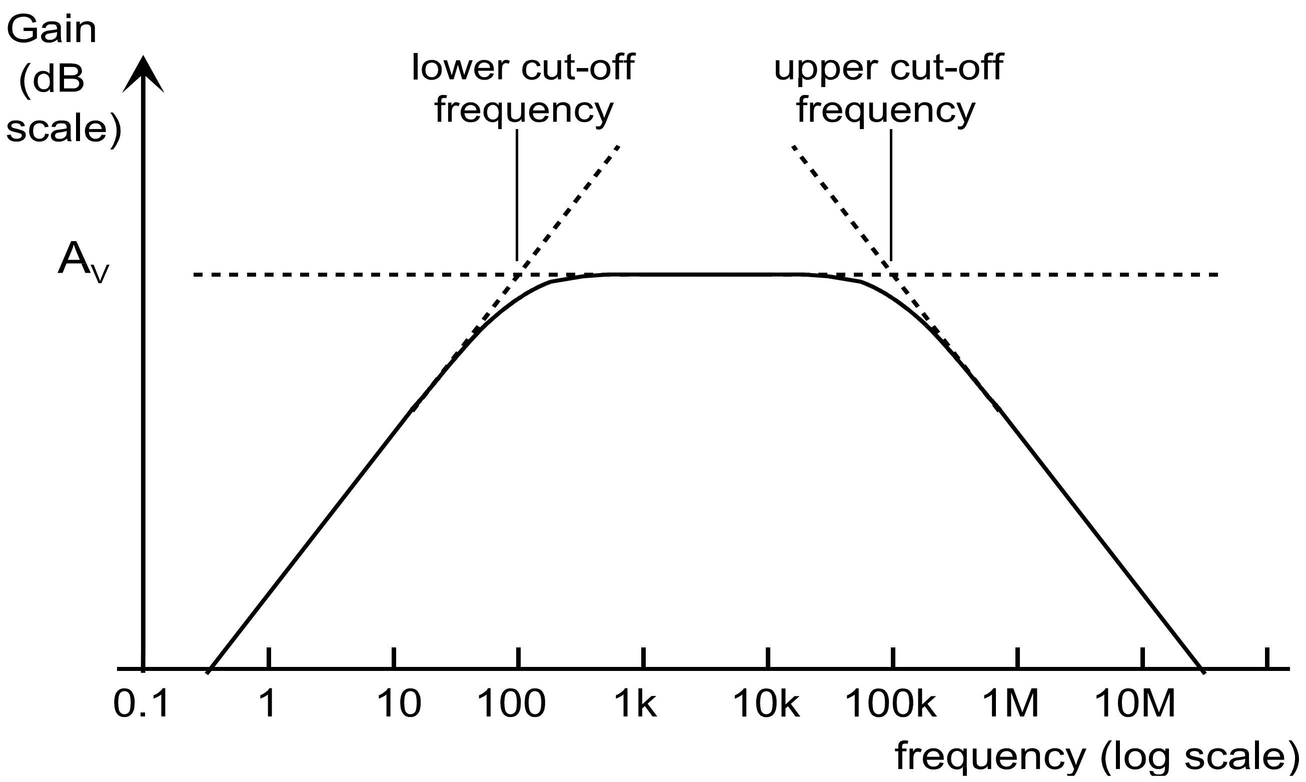
$$|A(\omega)| = \left|\frac{v_{out}(\omega)}{v_{in}(\omega)}\right| = \frac{A_V}{\omega R_{out}C_{out}}$$

or in dBs

$$|A(\omega)|_{dB} = 20\log_{10}\left(\frac{A_V}{R_{out}C_{out}}\right) - 20\log_{10}(\omega)$$

Frequency Response Characteristics of Amplifiers (12)

High frequency region - the exact form of equation 1.7 must be used.



The complete plot of gain in decibels against frequency on a log scale is known as a **Bode plot**.

Frequency Response of the Common Emitter Amplifier (10)

Detailed consideration of **high frequency** behaviour is outside the second year course as it depends on transistor properties not yet examined. This is a very brief introduction. At high frequencies **internal capacitances** of the transistor have a significant effect, the revised small signal equivalent circuit is