Xi'an Jiaotong-Liverpool University
西交利物浦大学

EEE220 Instrumentation and Control System

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Dr. Qing Liu

Email: qing.liu@xjtlu.edu.cn

Office: EE516

Department of Electrical and Electronic Engineering

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Lecture 13

Outline

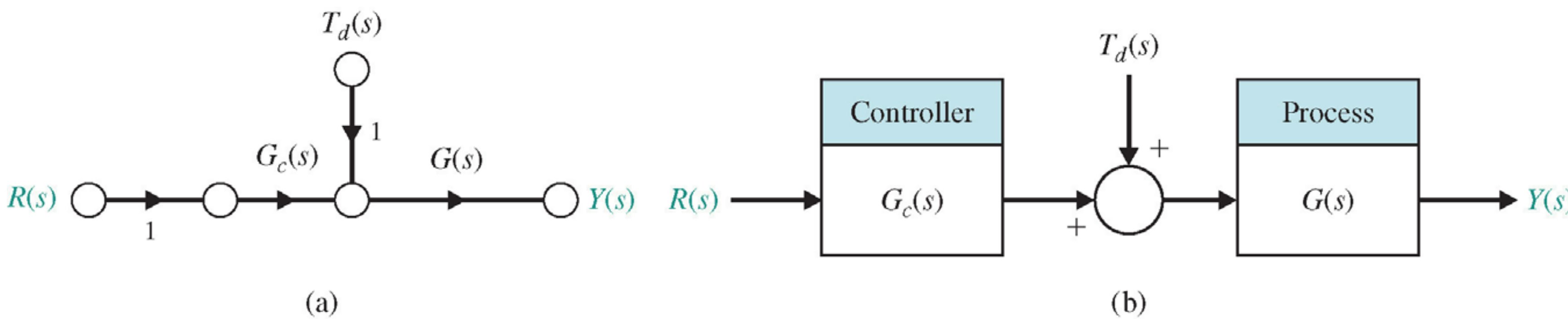
Feedback Control System Characteristics

- ❑ Error Signal Analysis
- ❑ Sensitivity of Control System to Parameter Variations
- ❑ Disturbance Rejection and Measurement Noise Attenuation
- ❑ Control of the Transient Response and Steady-state Error
- ❑ Cost of Feedback

Open-loop Control System

An open-loop control system operates **without feedback** and directly generates the output in response to an input signal.

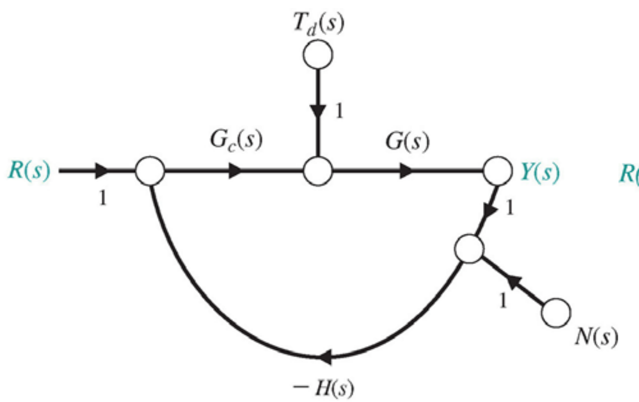
- The disturbance, $T_d(s)$, directly influences the output Y(s). In the absence of feedback, the control system is highly sensitive to disturbances and to both knowledge of and variations in parameters of G(s).



Closed-loop Control System

A closed-loop control system uses a measurement of the output signal and a comparison with the **desired output** to generate **error signal** that is used by the controller to adjust the actuator.

- The introduction of feedback to improve the control system is often necessary;
- It is interesting that feedback is inherent in nature systems such as biological and physiological system (i.e., heart rate control).



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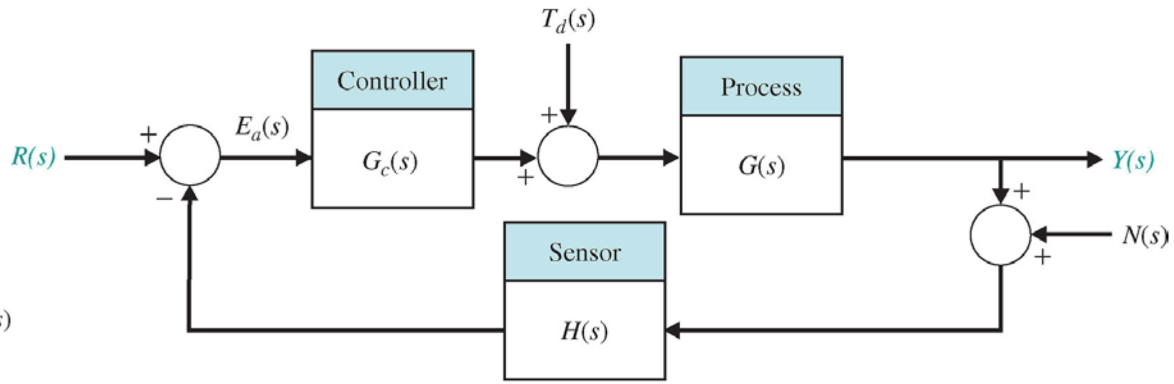

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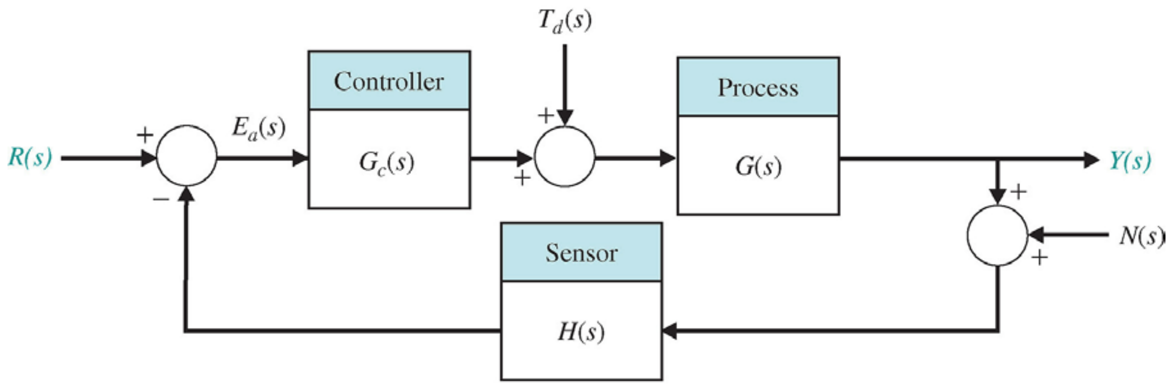
Advantages of Closed-loop Control

- Decreased sensitivity of the system to variations in parameters of the process;
- Improved rejection of the disturbances;
- Improved measurement noise attenuation;
- Improved reduction of the steady-state error of the system;
- Easy control and adjustment of the transient response of the system.

Error Signal Analysis



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For easy discussion, an unity feedback system is considered, i.e., $H(s) = 1$.

Define the **tracking error**: $E(s) = R(s) - Y(s)$

The output can be obtained from the block diagram:

$$Y(s) = \frac{G_c(s)G(s)}{1 + G_c(s)G(s)}R(s) + \frac{G(s)}{1 + G_c(s)G(s)}T_d(s) - \frac{G_c(s)G(s)}{1 + G_c(s)G(s)}N(s)$$

Therefore:

$$E(s) = \frac{1}{1 + G_c(s)G(s)}R(s) - \frac{G(s)}{1 + G_c(s)G(s)}T_d(s) + \frac{G_c(s)G(s)}{1 + G_c(s)G(s)}N(s)$$

Define the **loop gain**: $L(s) = G_c(s)G(s)H(s) = G_c(s)G(s)$

$$E(s) = \frac{1}{1 + L(s)}R(s) - \frac{G(s)}{1 + L(s)}T_d(s) + \frac{L(s)}{1 + L(s)}N(s)$$

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Sensitivity Function

$$E(s) = \frac{1}{1 + L(s)} R(s) - \frac{G(s)}{1 + L(s)} T_d(s) + \frac{L(s)}{1 + L(s)} N(s)$$

Define: $F(s) = 1 + L(s)$

Sensitivity Function

$$\boxed{S(s) = \frac{1}{F(s)} = \frac{1}{1 + L(s)}}$$

Complementary Sensitivity Function

$$\boxed{C(s) = \frac{L(s)}{1 + L(s)}}$$

$$S(s) + C(s) = 1$$

$$E(s) = S(s)R(s) - S(s)G(s)T_d(s) + C(s)N(s)$$

Sensitivity of Control System to Parameter Variations

A process, represented by $G(s)$, is subject to a changing environment, aging, uncertainty in the exact values of the process parameters, and other factors that affect the process.

- In open-loop control system, all these errors and changes result in a changing and inaccurate output;
- However, a closed-loop system senses the change in the output due to the process changes and attempts to correct the output.

The sensitivity of a control system to parameter variations is of prime importance.

A primary advantage of a closed-loop feedback control system is its ability to reduce the system's sensitivity.

How to Reduce Sensitivity?

To analyze influences of changes in G(s), assume $T_d(s) = N(s) = 0$.

Suppose the process (or plant) undergoes a change such that the true plant model is $G(s) + \Delta G(s)$, we then consider the tracking error E(s) due to $\Delta G(s)$.

$$E(s) + \Delta E(s) = \frac{1}{1 + G_c(s)(G(s) + \Delta G(s))} R(s)$$

Then the change in the tracking error is:

$$\Delta E(s) = \frac{-G_c(s)\,\Delta G(s)}{(1 + G_c(s)G(s) + G_c(s)\,\Delta G(s))(1 + G_c(s)G(s))} R(s)$$

Since usually $G_c(s)G(s) \gg G_c(s)$ for all complex frequencies of interest, we have

$$\Delta E(s) \approx \frac{-G_c(s)\,\Delta G(s)}{(1 + L(s))^2} R(s)$$

Therefore, the change in tracking error is reduced by the factor 1+L(s).

For large L(s), we have $1 + L(s) \approx L(s)$, then

$$\Delta E(s) \approx -\frac{1}{L(s)} \frac{\Delta G(s)}{G(s)} R(s)$$ ⟵ Large L(s) implies smaller sensitivity.

Definition of System Sensitivity

$$S = \frac{\Delta T(s)/T(s)}{\Delta G(s)/G(s)}$$ where $T(s) = \frac{Y(s)}{R(s)}$

In the limit, for small incremental changes:

$$\boxed{S = \frac{\partial T/T}{\partial G/G} = \frac{\partial \ln T}{\partial \ln G}.}$$

System sensitivity **is the ratio of the change in the system transfer function T(s) to the change of a process transfer function (G(s)) (or parameter) for a small incremental change.**

Sensitivity for open-loop system: 1.

Sensitivity for closed-loop system: since $T(s) = \frac{G_c(s)G(s)}{1 + G_c(s)G(s)}$

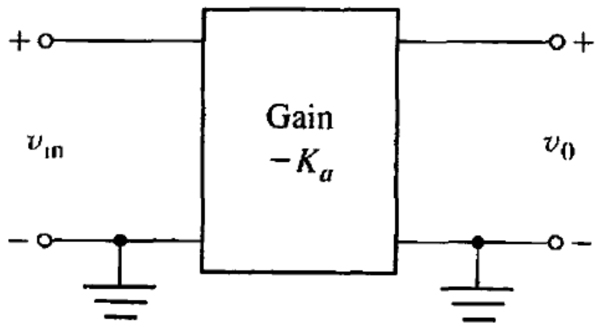
$$S_G^T = \frac{\partial T}{\partial G} \cdot \frac{G}{T} = \frac{G_c}{(1 + G_cG)^2} \cdot \frac{G}{GG_c/(1 + G_cG)}$$ or $$\boxed{S_G^T = \frac{1}{1 + G_c(s)G(s)}.}$$

To determine the influence of process parameter α, can use the chain rule:

$$S_\alpha^T = S_G^T S_\alpha^G.$$

Example 13.1: Feedback Amplifier

Open-loop Amplifier



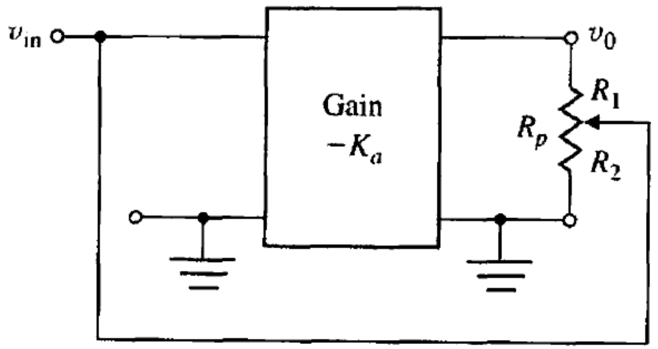
$$v_0 = -K_a v_{in}.$$

$$T = -K_a$$

Sensitivity to the changes in the amplifier gain is:

$$S_{K_a}^T = 1$$

Amplifier with Feedback



Assume: $H(s) = \beta$.

$$T = \frac{-K_a}{1 + K_a\beta}$$

Then:

$$S_{K_a}^T = S_G^T S_{K_a}^G = \frac{1}{1 + K_a\beta}$$

For $K_a = 10^4$ and $\beta = 0.1$:

$$S_{K_a}^T = \frac{1}{1 + 10^3} \approx 0.001$$

Disturbance Rejection

An important effect of feedback in a control system is the control and partial elimination of the effect of disturbance signals.

- A disturbance signal is an unwanted input signal that affects the output signal.
- Many control systems are subject to extraneous disturbance signals that cause the system to provide an inaccurate output. Electronic amplifiers have inherent noise generated within the integrated circuits or transistors; radar antennas are subjected to wind gusts; and many systems generate unwanted distortion signals due to nonlinear elements.
- The benefit of feedback systems is that the effect of distortion, noise, and unwanted disturbances can be effectively reduced.

To analyze rejection of disturbance, assume R(s) = N(s) = 0.

$$E(s) = -S(s)G(s)T_d(s) = -\frac{G(s)}{1 + L(s)}T_d(s)$$

For a fixed G(s) and a given $T_d(s)$, as the loop gain L(s) increases, the effect of Td(s) on the tracking error decreases. For good disturbance rejection, we require a large loop gain over the frequencies of interest associated with the expected disturbance signals.

Measurement Noise Attenuation

A noise signal that is prevalent in many systems is the noise generated by the **measurement sensor.**

To analyze attenuation of measurement noise, assume $R(s) = T_d(s) = 0$.

$$E(s) = C(s)N(s) = \frac{L(s)}{1 + L(s)}N(s)$$

As the loop gain L(s) decreases, the effect of N(s) on the tracking error decreases. For effective measurement noise attenuation, we need a small loop gain over the frequencies associated with the expected noise signals.

How to realize disturbance rejection and measurement noise attenuation at the same time?

❖ In practice, disturbance are often at low frequencies, while measurement noise signals are often high frequency.

------ the controller should be of high gain at low frequencies and low gain at high frequencies.

Control of Transient Response

The **transient response** is the response of a system as a function of time. One of the most important characteristics of control systems is their transient response.

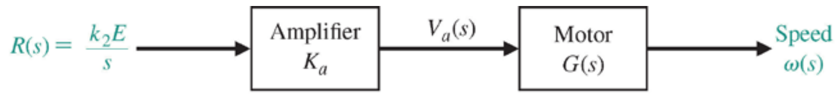
Because the purpose of control systems is to provide a desired response, the transient response of control systems often must be adjusted until it is satisfactory.

- If an open-loop control system does not provide a satisfactory response, then the process, G(s), must be replaced with a more suitable process;
- By contrast, a closed-loop system can often be adjusted to yield the desired response by adjusting the feedback loop parameters.

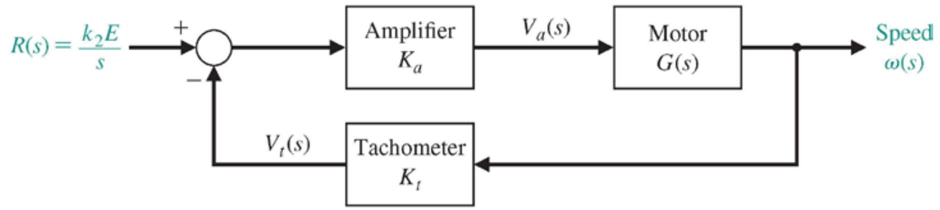
A feedback control system is valuable because it provides the engineer with the ability to adjust the transient response.

Example 13.2: Speed Control System

A speed control system, is often used in industrial processes to move materials and products ((a) open-loop control system; (b) Control system with feedback).



(a)



(b)

Open-loop:

$$\frac{\omega(s)}{V_a(s)} = G(s) = \frac{K_1}{\tau_1 s + 1} \qquad V_a(s) = \frac{k_2 E}{s}.$$

$$\omega(s) = G(s)V_a(s) \longrightarrow \omega(t) = K_1(k_2E)(1 - e^{-t/\tau_1}).$$

Closed-loop:

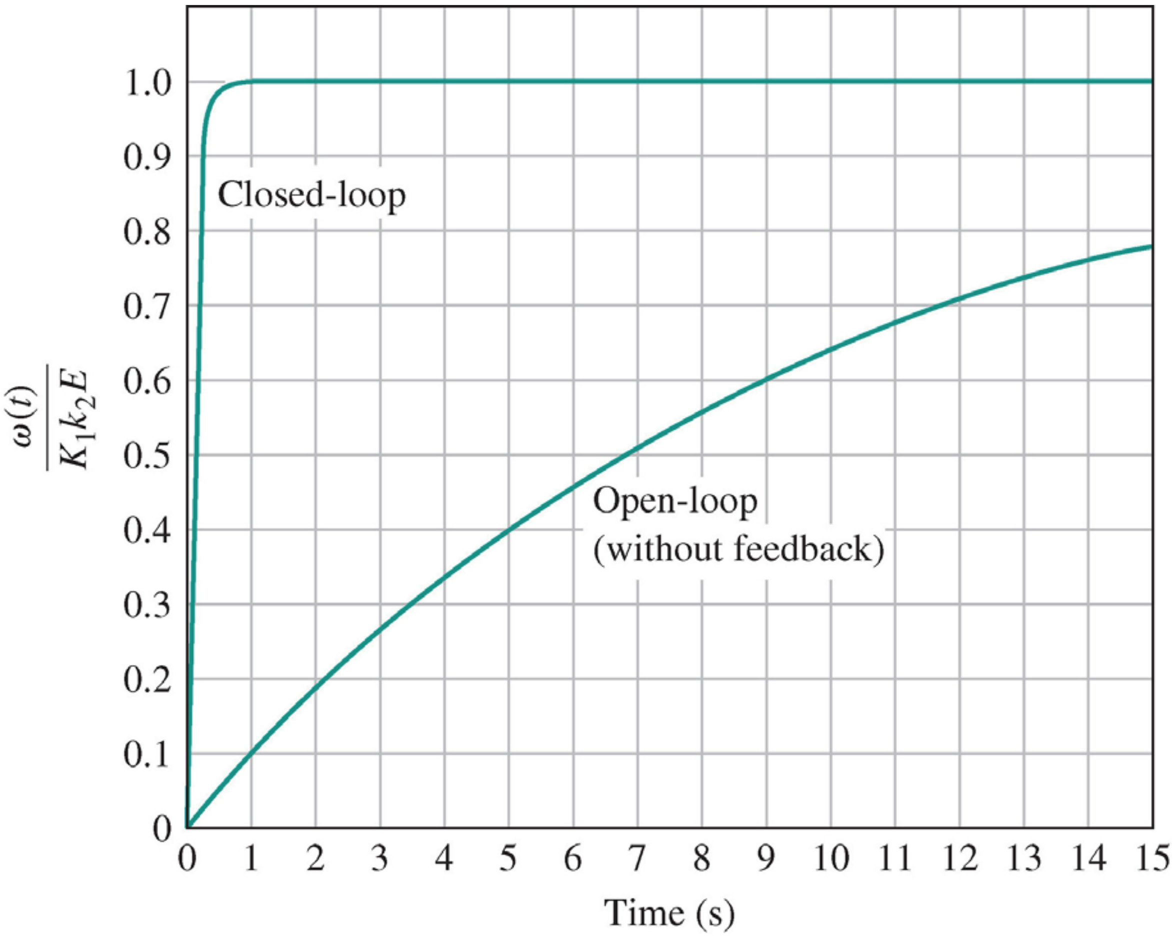
$$\frac{\omega(s)}{R(s)} = \frac{K_a G(s)}{1 + K_a K_t G(s)}$$
$$= \frac{K_a K_1}{\tau_1 s + 1 + K_a K_t K_1} = \frac{K_a K_1/\tau_1}{s + (1 + K_a K_t K_1)/\tau_1}$$

$$\downarrow$$

$$\omega(t) = \frac{K_a K_1}{1 + K_a K_t K_1}(k_2 E)(1 - e^{-pt})$$

Transient Response

The response of the open-loop and closed-loop speed control system when τ = 10 and $K_1K_aK_t$ = 100. The time to reach 98% of the final value for the open-loop and closed-loop system is 40 seconds and 0.4 seconds, respectively.



Steady-state Error

The **steady-state error** is the error after the transient response has decayed, leaving only the continuous response.

Final Value Theorem:

$$\boxed{\lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s).}$$

Assume a unit step input as a comparable input ($r(t) = 1, R(s) = \frac{1}{s}$):

Open-loop:

$$E_o(s) = R(s) - Y(s) = \big(1 - G_c(s)G(s)\big)R(s)$$

$$e_o(\infty) = \lim_{s\to\infty} s\big(1 - G_c(s)G(s)\big)\left(\frac{1}{s}\right) = 1 - G_c(0)G(0)$$

Closed-loop (assume $T_d(s) = N(s) = 0$):

$$\boxed{E_c(s) = \frac{1}{1 + G_c(s)G(s)} R(s).}$$

$$e_c(\infty) = \lim_{s\to 0} s\left(\frac{1}{1 + G_c(s)G(s)}\right)\left(\frac{1}{s}\right) = \frac{1}{1 + G_c(0)G(0)}$$

Large $L(0) = G_c(0)G(0)$ will lead to small steady-state error.

The Cost of Feedback

- **Increased number of components and complexity in the system.**

- To add the feedback, it is necessary to consider several feedback components; the measurement component (sensor) is the key one. The sensor is often the most expensive component in a control system. Furthermore, the sensor introduces noise and inaccuracies into the system.

- **Loss of Gain.**

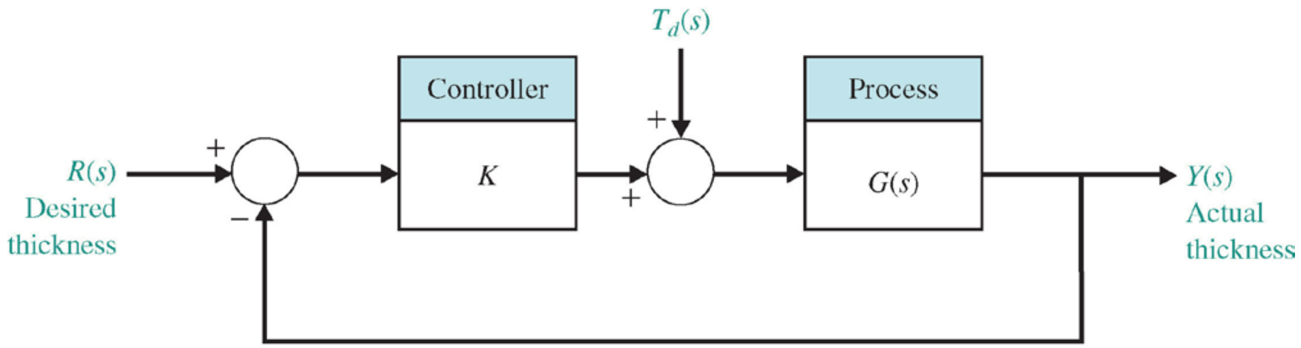
- Open-loop gain: $G_c(s)G(s)$
- Closed-loop gain: $\frac{G_c(s)G(s)}{1+G_c(s)G(s)}$

- **Introduction of the possibility of instability.**

- Whereas the open-loop system is stable, the closed-loop system may not be always stable (will be discussed in later chapter).

Quiz 13.1

Consider the following system, where $G(s) = \frac{1}{s(s+50)}$. Calculate the sensitivity of the system to changes in the controller gain K.

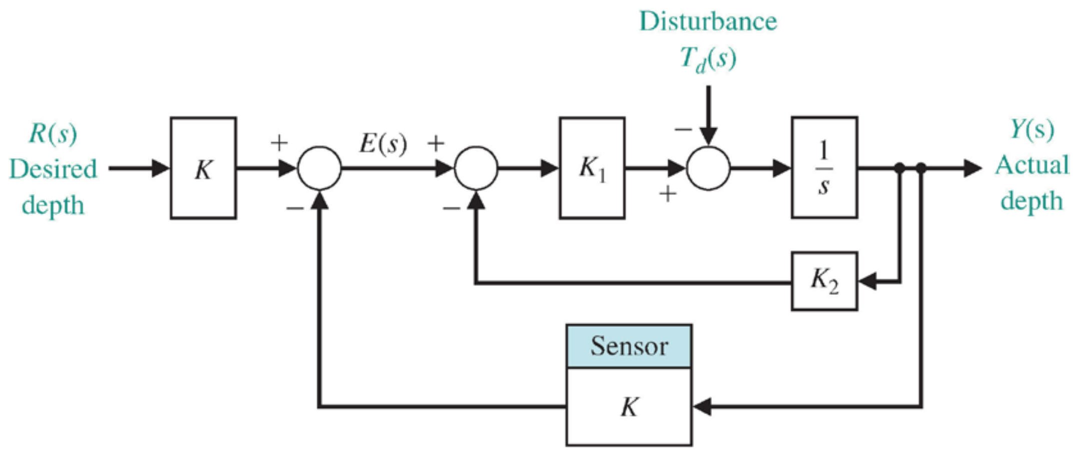


(b)

Quiz 13.2

Consider the following system.

1) Compute the transfer function $T(s) = \frac{Y(s)}{R(s)}$;
2) Determine the sensitivity $S_{K_1}^{T}$ and $S_{K_2}^{T}$;
3) Calculate the steady-state error due to disturbance $T_d = 1/s$.



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