EEE340 Protective Relaying

Lecture 2 – Symmetrical faults & Symmetrical components

Short Circuit

❖ Short circuit is the most common and serious fault of power system.

❖ Shot circuits can be caused by different reasons:

- Damage of components: such as natural aging of insulation, defects of equipment by improper design, installation and maintenance;
- Weather conditions: such as lightning strikes, high wind and ice;
- Operating against regulations: such as to connect with high voltage after maintenance without removing the grounded wire.
- Other reasons: such as damage on cables by digging trench, birds or animals connected between conductors.

Short Circuit

❖ Results of short circuits:

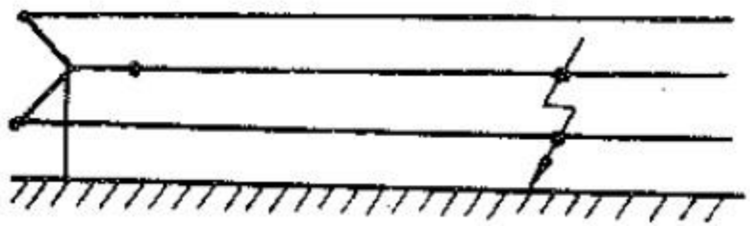
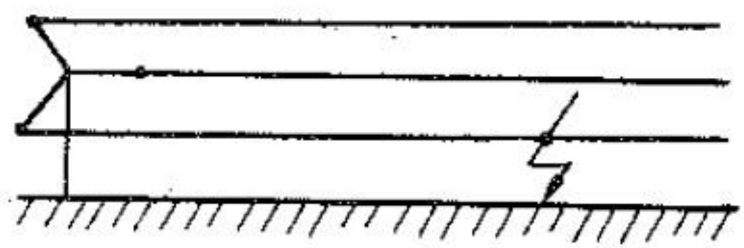
- Force induced by large fault current may damage components;
- Heat generated by large fault current with long duration may damage equipment;
- Serious impacts on customer usage due to low voltage, especially for induction motors;
- Generators may loose synchronism if the fault location is close with long duration and the system stability may be broken with large scale blackout.
- Impacts on neighboring information systems.

Short Circuit

- Symmetrical Faults
  - Three-Phase Short Circuit
- Unsymmetrical Faults
  - Single Line-to-Ground Fault
  - Line-to-Line Fault
  - Double Line-to-Ground Fault

Short Circuit

- Three-Phase Short Circuit
- Double Line-to-Ground Fault
- Line-to-Line Fault
- Single Line-to-Ground Fault

短路种类	示意图	短路代表符号
三相短路		$f^{(3)}$
两相短路接地		$f^{(1,1)}$
两相短路		$f^{(2)}$
单相短路		$f^{(1)}$

Three-Phase Short Circuit

❖ In order to calculate the fault current:

- Transformers are only represented by their leakage reactance;

- Transmission lines are represented by their equivalent series reactance. Series resistance and shunt admittances are neglected;

- Synchronous machines are represented by constant-voltage sources behind subtransient reactance. Armature resistance, saliency, and saturation are neglected.

- Loads are generally represented by grounded branches with constant impedances whose values are calculated by load power before the fault and the actual bus voltage:

$$z_{LD.k} = V_k^2 / \overset{*}{S}_{LD.k}$$

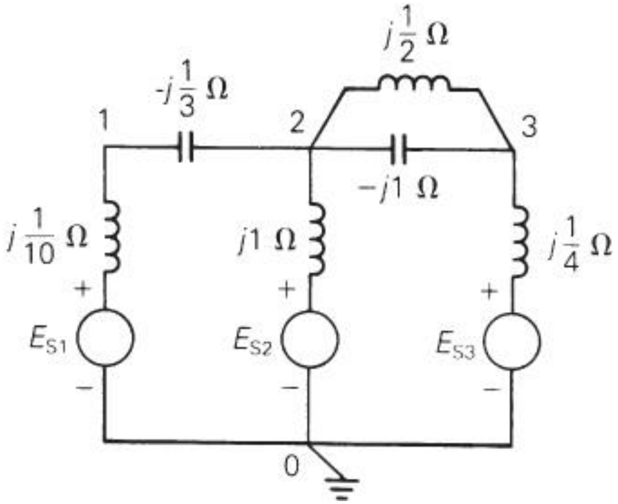
Network Equations

Nodal equation can be written in the following steps:

- Step 1: For a circuit with (N+1) nodes (also called buses), select one bus as the reference bus and define the voltages at the remaining buses with respect to the reference bus.

FIGURE 2.8

Circuit diagram for reviewing nodal analysis



N=3, Bus 0 is selected as the reference bus. Bus voltage V_{10}, V_{20} and V_{30} are then defined with respect to bus 0.

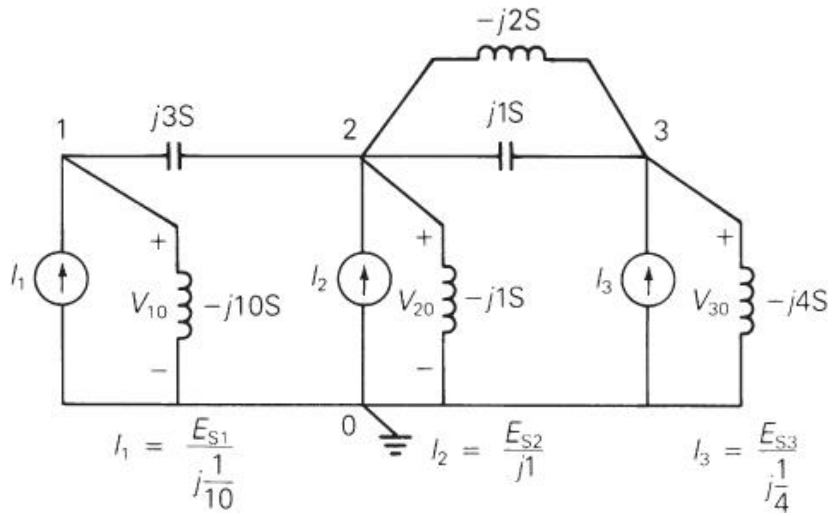
Network Equations

Nodal equation can be written in the following steps:

- Step 2: Transform each voltage sources in series with an impedance to an equivalent current sources in parallel with that impedance. Also show admittance values instead of impedance values. Each current source is equal to the voltage source divided by the source impedance.

FIGURE 2.9

Circuit of Figure 2.8 with equivalent current sources replacing voltage sources. Admittance values are also shown



Network Equations

Nodal equation can be written in the following steps:

- Step 3: Write nodal equations in matrix format as:

$$\begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & \cdots & Y_{1N} \\ Y_{21} & Y_{22} & Y_{23} & \cdots & Y_{2N} \\ Y_{31} & Y_{32} & Y_{33} & \cdots & Y_{3N} \\ \cdots & \cdots & \cdots & & \cdots \\ Y_{N1} & Y_{N2} & Y_{N3} & \cdots & Y_{NN} \end{bmatrix} \begin{bmatrix} V_{10} \\ V_{20} \\ V_{30} \\ \cdots \\ V_{N0} \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ \cdots \\ I_N \end{bmatrix}$$

$$\boldsymbol{Y}_{\text{bus}} \boldsymbol{V} = \boldsymbol{I}$$

Admittance Matrix

$\boldsymbol{Y}_{\text{bus}}$ is the admittance matrix, $\boldsymbol{V}$ is the column vector of N nus voltages, and $\boldsymbol{I}$ is the column vector of N current sources.

The elements Y_{kn} of the bus admittance matrix are formed as follows:

Diagonal elements:
Y_{kk}=sum of admittances connected to bus k (k=1,2,…,N)

Off-diagonal elements:
Y_{kn}= -(sum of admittances connected between buses k and n)

Impedance Matrix

$$\begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} & Z_{23} & \cdots & Z_{2N} \\ Z_{31} & Z_{32} & Z_{33} & \cdots & Z_{3N} \\ \cdots & \cdots & \cdots & & \cdots \\ Z_{N1} & Z_{N2} & Z_{N3} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ \cdots \\ I_N \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ \cdots \\ V_N \end{bmatrix}$$

$$\boldsymbol{Z}_{\text{bus}}\boldsymbol{I} = \boldsymbol{V}$$

Impedance Matrix

$$\boldsymbol{Z}_{\text{bus}} = \boldsymbol{Y}_{\text{bus}}^{-1}$$

The physical meaning of elements Z_{kn} in the bus impedance matrix is :

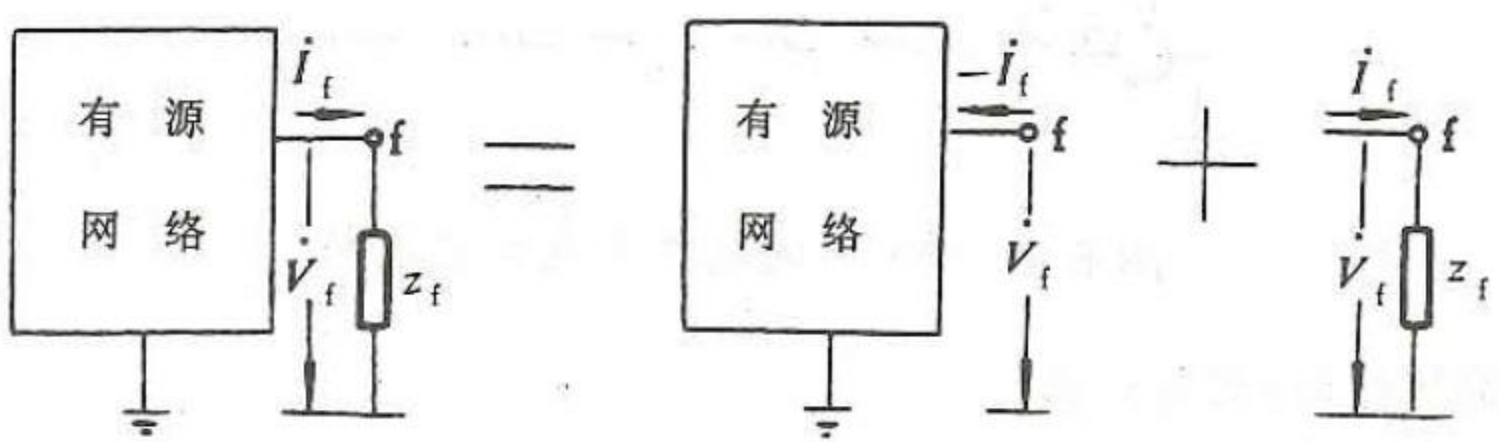
Diagonal elements is also called self-impedance.
If we input current source at bus *k*, and the input currents for other buses are all equal to zero. The ratio between the corresponding voltage at bus *k* and the input current is equal to the self-impedance Z_{kk} at bus *k*.

Off-diagonal elements is also called mutual-impedance.
If we input current source at bus *k*, and the input currents for other buses are all equal to zero. The ratio between corresponding voltage at bus *i* and the input current at bus *k* is equal to the mutual-impedance Z_{ik} between bus *i* and *k*.

Calculation of Short Circuit

❖Calculation based on the impedance matrix:

- A three-phase short circuit at bus f can be considered as connection through a transition impedance z_f to the ground;
- The faulted network can be decomposed as two parts;



Calculation of Short Circuit

❖Calculation based on the impedance matrix:

- By keeping the boundary conditions invariant, the initial network can be separated from the faulted branch;
- For the initial network, the fault condition is equivalent to add a current source with $-\boldsymbol{I_f}$ at bus f.
- Therefore, the voltage of any bus can be calculated as:

$$\dot{V}_i = \sum_{j \in G} Z_{ij} \dot{I}_j - Z_{if} \dot{I}_f$$

Calculation of Short Circuit

❖ Calculation based on the impedance matrix:

$$\dot{V}_i = \sum_{j \in G} Z_{ij} \dot{I}_j - Z_{if} \dot{I}_f$$

- The voltage of any bus can be considered as the sum of two components.
- The first component represents the voltage of bus *i* when $\dot{I}_f = 0$, which is just corresponding to the voltage $\dot{V}_i^{(0)}$ before the fault happens.
- The second component represents the voltage of bus *i* only caused by the fault current $\dot{I}_f$ (all other current sources are open and all other voltage sources are short circuited).

Calculation of Short Circuit

❖ Calculation based on the impedance matrix:

- Therefore, the actual voltage of bus *i* after fault can be calculated as:

$$\dot{V}_i = \dot{V}_i^{(0)} - Z_{if}\dot{I}_f$$

- The voltage of the faulted bus *f* can be calculated as:

$$\dot{V}_f = \dot{V}_f^{(0)} - Z_{ff}\dot{I}_f$$

$$\dot{V}_f^{(0)} = \sum_{j \in G} Z_{fj}\dot{I}_j$$

- $\dot{V}_f^{(0)}$ is the voltage of bus *f* before the fault; Z_{ff} is the self-impedance of bus *f*.

Calculation of Short Circuit

❖ Calculation based on the impedance matrix:

$$\dot{V}_f = \dot{V}_f^{(0)} - Z_{ff} \dot{I}_f$$

- Both the voltage and current of the faulted bus are unknown, further boundary condition is needed:

$$\dot{V}_f - z_f \dot{I}_f = 0$$

- So the fault current can be calculated as:

$$\dot{I}_f = \frac{\dot{V}_f^{(0)}}{Z_{ff} + z_f}$$

- The voltage of any bus can be calculated as:

$$\dot{V}_i = \dot{V}_i^{(0)} - \frac{Z_{if}}{Z_{ff} + z_f} \dot{V}_f^{(0)}$$

Calculation of Short Circuit

❖ Calculation based on the impedance matrix:

- In case of low accuracy required, all loads can be neglected.
- The per unit values for all bus voltages before fault can be all considered as 1.

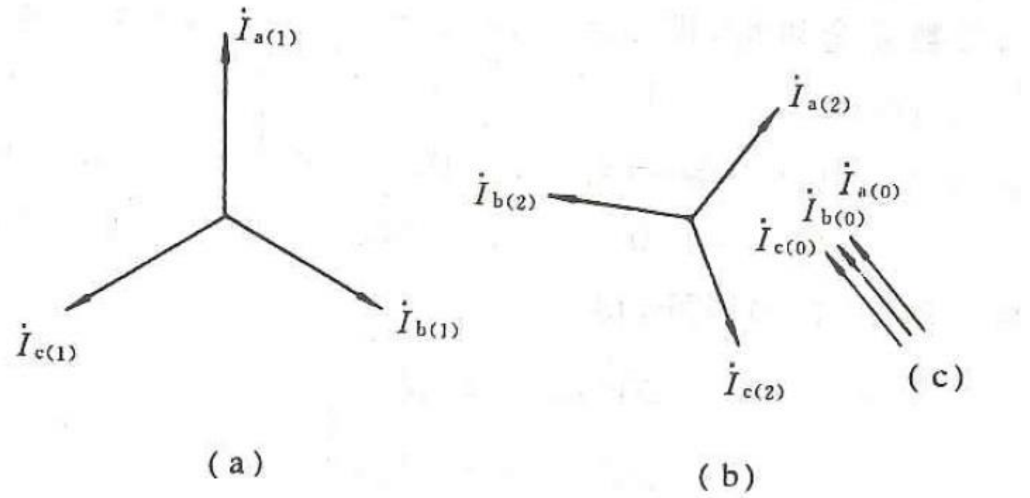
$$\dot{I}_f = \frac{1}{Z_{ff} + z_f} \qquad \dot{V}_i = 1 - \frac{Z_{if}}{Z_{ff} + z_f}$$

- If z_f is small enough to be considered as 0, the short circuit calculation can be directly implemented by the elements from the impedance matrix.

Symmetrical Components

❖ Assume that a set of unsymmetrical phasors is given, these unsymmetrical phasors can be resolved into following three sets of symmetrical sequence components:

- ***Zero-sequence*** components, consisting of three phasors with equal magnitudes and with zero phase displacement.
- ***Positive-sequence*** components, consisting of three phasors with equal magnitudes, 120 degree phase displacement, and positive sequence.
- ***Negative-sequence*** components, consisting of three phasors with equal magnitudes, 120 degree phase displacement, and negative sequence.



Symmetrical Components

❖ If we consider *phase a* as a reference, the relation between the three phasors and their symmetrical components is:

$$\begin{bmatrix} \dot{I}_{a(1)} \\ \dot{I}_{a(2)} \\ \dot{I}_{a(0)} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & a & a^2 \\ 1 & a^2 & a \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} \dot{I}_a \\ \dot{I}_b \\ \dot{I}_c \end{bmatrix}$$

$$a = e^{j120^o}, \quad a^2 = e^{j240^o}, \quad 1 + a + a^2 = 0, \quad a^3 = 1$$

$\dot{I}_{a(1)}, \dot{I}_{a(2)}, \dot{I}_{a(0)}$ are the positive, negative and zero sequence components for phase a respectively.

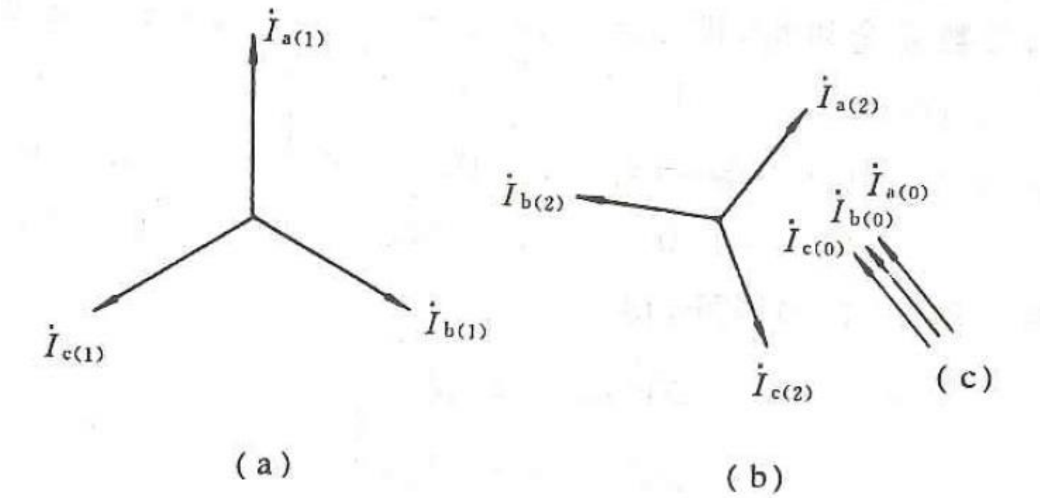
Symmetrical Components

❖ With the positive, negative and zero sequence components of phase *a* given, the symmetrical components for phase b and c can be calculated as:

$$\dot{I}_{b(1)} = a^2\dot{I}_{a(1)},\ \dot{I}_{c(1)} = a\dot{I}_{a(1)}$$

$$\dot{I}_{b(2)} = a\dot{I}_{a(2)},\ \dot{I}_{c(2)} = a^2\dot{I}_{a(2)}$$

$$\dot{I}_{b(0)} = \dot{I}_{c(0)} = \dot{I}_{a(0)}$$



Symmetrical Components

- To decompose an unsymmetrical set of three phasors into symmetrical components, the transform can be written as:

$$I_{120} = SI_{abc}$$

- If the symmetrical components are given, the original unsymmetrical three phasors can be calculated as:

$$I_{abc} = S^{-1}I_{120}$$

$$S^{-1} = \begin{bmatrix} 1 & 1 & 1 \\ a^2 & a & 1 \\ a & a^2 & 1 \end{bmatrix}$$

Symmetrical Components

$$I_{abc} = S^{-1} I_{120} \qquad S^{-1} = \begin{bmatrix} 1 & 1 & 1 \\ a^2 & a & 1 \\ a & a^2 & 1 \end{bmatrix}$$

- This equation can be further extended as:

$$\dot{I}_a = \dot{I}_{a(1)} + \dot{I}_{a(2)} + \dot{I}_{a(0)}$$

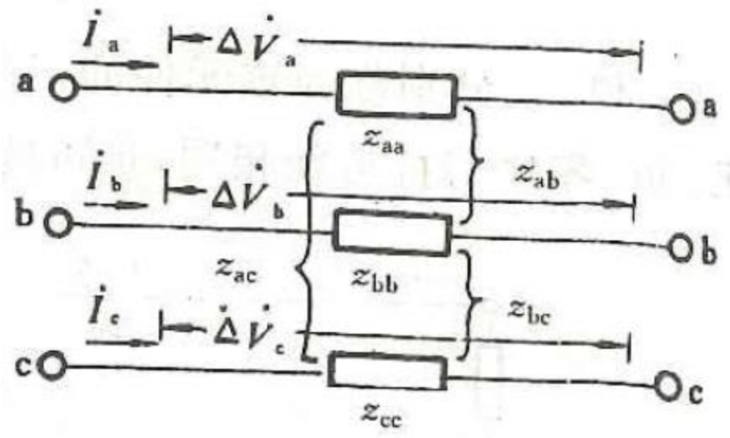
$$\dot{I}_b = \dot{I}_{b(1)} + \dot{I}_{b(2)} + \dot{I}_{b(0)} = a^2 \dot{I}_{a(1)} + a\dot{I}_{a(2)} + \dot{I}_{a(0)}$$

$$\dot{I}_c = \dot{I}_{c(1)} + \dot{I}_{c(2)} + \dot{I}_{c(0)} = a\dot{I}_{a(1)} + a^2 \dot{I}_{a(2)} + \dot{I}_{a(0)}$$

- This relation can be applied to both current and voltage phasors.

Sequence Impedance

- We can consider a static three phase circuit as an example for sequence impedance.



- The self-impedances are z_{aa}, z_{bb}, z_{cc}, and the mutual impedances are $z_{ab}=z_{ba}$, $z_{bc}=z_{cb}$, $z_{ca}=z_{ac}$. With unsymmetrical three phase currents, the voltage drop for each phase can be calculated as:

$$\begin{bmatrix} \Delta\dot{V}_a \\ \Delta\dot{V}_b \\ \Delta\dot{V}_c \end{bmatrix} = \begin{bmatrix} z_{aa} & z_{ab} & z_{ac} \\ z_{ba} & z_{bb} & z_{bc} \\ z_{ca} & z_{cb} & z_{cc} \end{bmatrix} \begin{bmatrix} \dot{I}_a \\ \dot{I}_b \\ \dot{I}_c \end{bmatrix} \Rightarrow \Delta \mathbf{V}_{abc} = Z\,\mathbf{I}_{abc}$$

Sequence Impedance

$$\Delta V_{abc} = S^{-1}\Delta V_{120}$$

$$\Delta V_{abc} = ZI_{abc}$$

$$I_{abc} = S^{-1}I_{120}$$

$$\Delta V_{120} = SZS^{-1}I_{120} = Z_{sc}I_{120}$$

$Z_{sc} = SZS^{-1}$ is called as Sequence Impedance Matrix.

Sequence Impedance

❖ When the parameters of components are completely symmetrical, $z_{aa}=z_{bb}=z_{cc}=z_s$, $z_{ab}=z_{bc}=z_{ca}=z_m$:

$$Z_{sc}=\begin{bmatrix} z_s-z_m & 0 & 0 \\ 0 & z_s-z_m & 0 \\ 0 & 0 & z_s+2z_m \end{bmatrix}=\begin{bmatrix} z_{(1)} & 0 & 0 \\ 0 & z_{(2)} & 0 \\ 0 & 0 & z_{(0)} \end{bmatrix}$$

$$\Downarrow$$

$$\Delta V_{120}=SZS^{-1}I_{120}=Z_{sc}I_{120}$$

$$\Downarrow$$

$$\Delta\dot{V}_{a(1)}=z_{(1)}\dot{I}_{a(1)}$$

$$\Delta\dot{V}_{a(2)}=z_{(2)}\dot{I}_{a(2)}$$

$$\Delta\dot{V}_{a(0)}=z_{(0)}\dot{I}_{a(0)}$$

Sequence Impedance

- In linear circuits with three phase symmetrical parameters, each sequence component is independent.
- If the circuit is supplied with any sequence of current, only voltage of the same sequence will be produced.
- If voltage of only one sequence is applied, only the current of the same sequence will be generated.
- Therefore, the positive, negative and zero sequence components can be analyzed independently.
- If the three phase parameters are not symmetrical, the non-diagonal elements of Z_{sc} may not all be zero. Sequence analysis can not be performed independently.

Sequence Impedance

- When the three phase parameters of components are symmetrical, the sequence impedance of one component is defined as the ratio between the voltage drop on this component of that sequence and the current of the same sequence through this component.

Positive-sequence impedance $$z_{(1)} = \Delta\dot{V}_{a(1)} / \dot{I}_{a(1)}$$

Negative-sequence impedance $$z_{(2)} = \Delta\dot{V}_{a(2)} / \dot{I}_{a(2)}$$

Zero-sequence impedance $$z_{(0)} = \Delta\dot{V}_{a(0)} / \dot{I}_{a(0)}$$

- For each component of power system, the positive, negative and zero sequence impedance may be equal or not equal, that depends on the structure of the component.

Next Lecture

Unsymmetrical Faults

Thanks for your attendance