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Measurements and Circuit Analysis

(Read before coming to the lab)

Module: EEE225

Time: 11:00am – 18:00 pm

Date: Thursday, 25th Oct. 2018

Room: EE215 and EE213, Building 4

Notice: Individual report in pdf. format

(do the experiment as a group but write reports independently)

Submission: 23:55pm, 25th, Nov. 2018; softcopy only, uploaded on ICE

The experiment consists of two parts Part A and Part B. In Part A, you are provided with diagrams of circuits which you are asked to measure how V_1 changes with frequency (if any) for these circuits and plot their graphs. Part A should help you to gain sufficient knowledge to complete part B.

In Part B you are provided with 5 numbered boxes. Each box has four terminals which are labelled as: "**A**", "**B**", "**C**" and "**D**". A network of passive components (i.e. *R*, *C* and *L* only) is connected between these sets of terminals (terminals "**C**" and "**D**" are connected together). You need to use the knowledge gained in Part A to evaluate the components in the unknown boxes and their values. Each box contains a combination of only two of the three possible components. Start with the box that has the lowest number (which is the simplest one) and work through the boxes in ascending numerical order.

1. Equipment

- Signal Generator
- Digital Multimeter
- Unknown Circuit Boxes (5)
- SK-10 Breadboard

You must decide which measurements are required and you may only use the equipment listed above. Before making any measurements read the equipment specifications **VERY CAREFULLY**. Some important notes about the equipment must to be taken into consideration:

a) Digital Multimeters measure RMS values while Oscilloscopes allow peak to peak values to be measured. In essence, this means if a signal has an amplitude of 7 V, the Digital Multimeter reading will show approximately 4.95 V whilst the Oscilloscope will show a peak to peak value of 14 V.

b) Measurements for resistors should be done using DC mode

c) Measurements for capacitors and inductors should be done using AC mode

2. Objectives

The purpose of this experiment is to:

1) Gain some experience in the use of common laboratory instruments and to provide an exercise in circuit theory.

2) Appreciate the behaviour of simple components like resistors, capacitors and inductors.

3) Efficiently use different equipment and tools

4) Learn different analysis techniques

3. Experiments

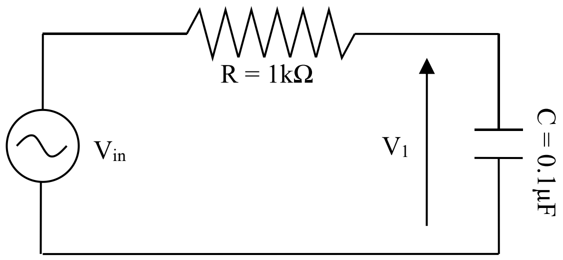
3.1 Part A – AC Circuits

Introduction

The aim of the following exercises is to familiarise yourself with the behaviour of capacitors and inductors when an AC signal is applied across them. The results obtained here will help you with experiments in Part B where you will be presented with boxes with unknown circuits inside them. Therefore, it is important to understand the following experiments and make sure that your graphs and results from Part A are checked by a demonstrator before proceeding to Part B.

Step 1

Construct the following circuit on an SK10 breadboard. Set the amplitude of V_{in} to 5 V and then measure V_1 for the following circuit against frequency. The frequency should be varied over a range covering 20 Hz to 100 kHz. Plot a graph of $20\log(V_1/V_{in})$ against frequency.

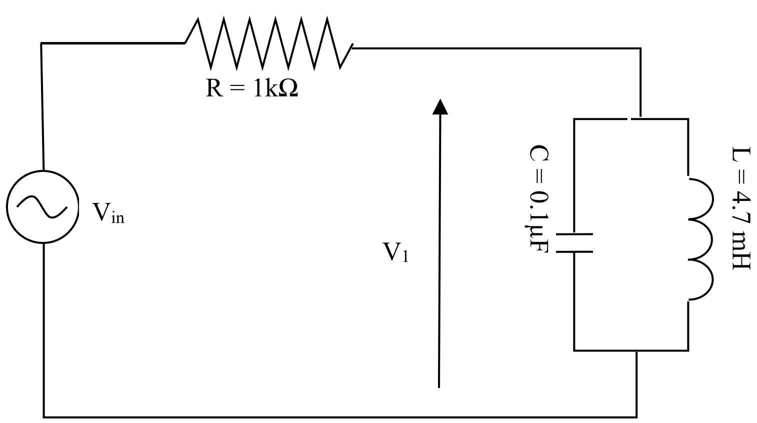


Now using the theoretical analysis in lecture 5 along with your results, predict the capacitance experimental value. During your analysis, always assume that the value of the resistor *R* is known (should be measured by multimeter), but *C* is unknown.

To find the actual value of the capacitor, from your experimental results, find the frequency at the 3 dB point and substitute it into the appropriate equation to obtain the value of *C*. Make sure that you include the plotted graphs, your calculations and your predicted values in your report. Also, compare your experimentally obtained capacitance value (in this case) to the rated value of *C*, i.e. around 0.1 uF.

Step 2

Construct the following circuit on an SK10 breadboard, which can be treated as a modified parallel *RLC* circuit. Set the amplitude of V_{in} to 5 V and then measure V_1 of the following circuit against frequency. The frequency should be varied over the range 20 Hz to100 kHz. Plot a graph of $20\log(V_1/V_{in})$ against frequency.



From your experimental data, draw the graph of V_1 against frequency, find the resonant frequency and the 3 dB bandwidth. Now using the theoretical analysis in lecture 5 along with your results, predict the experimental capacitance and inductance values (C and L). During your analysis, always assume that the value of resistor is known (should be measured by multimeter), but L and C are unknown. In your report, make sure that you include this graph (V_1 vs. frequency), your calculations and your experimental values (for L & C) in your report. Also, compare your predicted reactive values to the actual value of C and L, e.g. around 0.1 uF & 4.7 mH, respectively.

Note, when dealing with unknown circuits as in Part B, the capacitor and inductor values are not given. Therefore, the resonant frequency can only be found from the obtained results of the AC analysis.

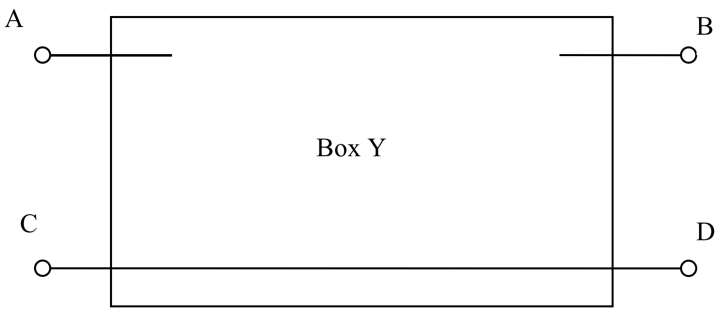
3.2 Part B – Unknown Circuits

There are 5 different unknown boxes and you should start with the box with the lowest number (this is the simplest circuit). Each box has 4 terminals labelled “**A**”, “**B**”, “**C**” and “**D**”. A network of passive components (i.e. *R*, *C*, and *L* only) is connected between these sets of terminals (terminal “**C**” and “**D**” are connected together). Every box contains 2 components.

Step 3

Investigate all 5 boxes to determine the circuit inside each box and the component values to an accuracy of ±10%. The following example may help you in carrying out Part B.

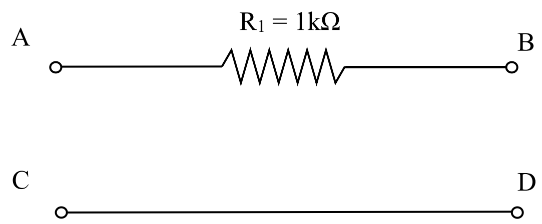
Example



DC measurements

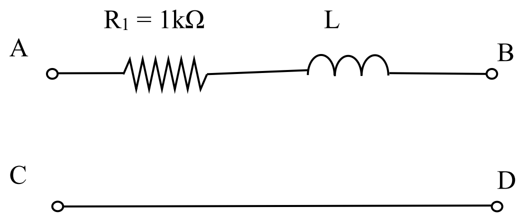
Using the multimeter, measure the resistance between “**A**” & “**B**”,” “**B**” & “**D**” and “**A**” & “**C**”. Capacitors block DC, so if there is a capacitor in parallel with a resistor, the multimeter will read the value of the resistor and the capacitor’s presence will only be determined when carrying out the AC analysis. However, if the capacitor is in series with a resistor then the multimeter will read open circuit. AC analysis will then help you to find the values of both capacitors and inductors.

Inductors allow DC, so if there is an inductor in parallel with a resistor then the multimeter reading will be zero. However, if the inductor is in series with a resistor, then the value of the resistor will be read and the value of the inductor will only be determined by AC analysis. In this case, the resistance between "**A**" and "**B**" was found to be 1 kΩ but both "**A**" and "**C**", and "**B**" and "**D**" were open circuit. Therefore, the DC equivalent circuit of Box Y is;

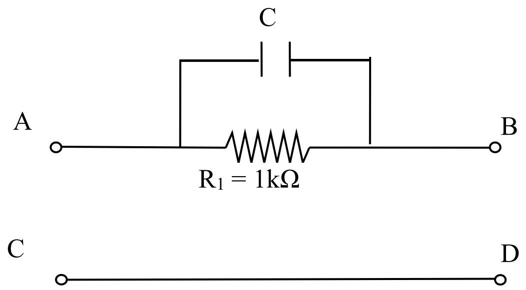


AC measurements

There are many possibilities for the AC equivalent circuit, e.g.

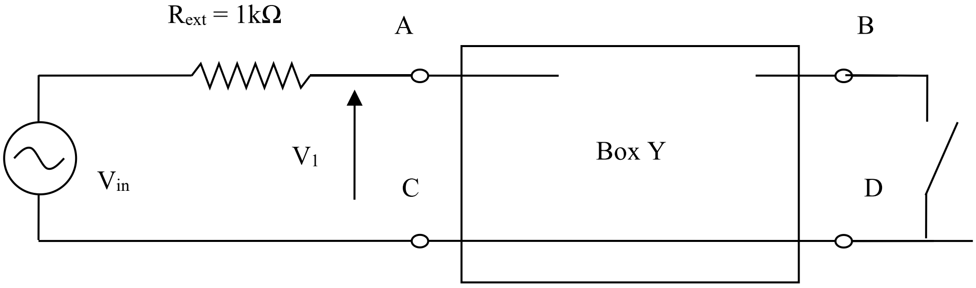


or



,etc.

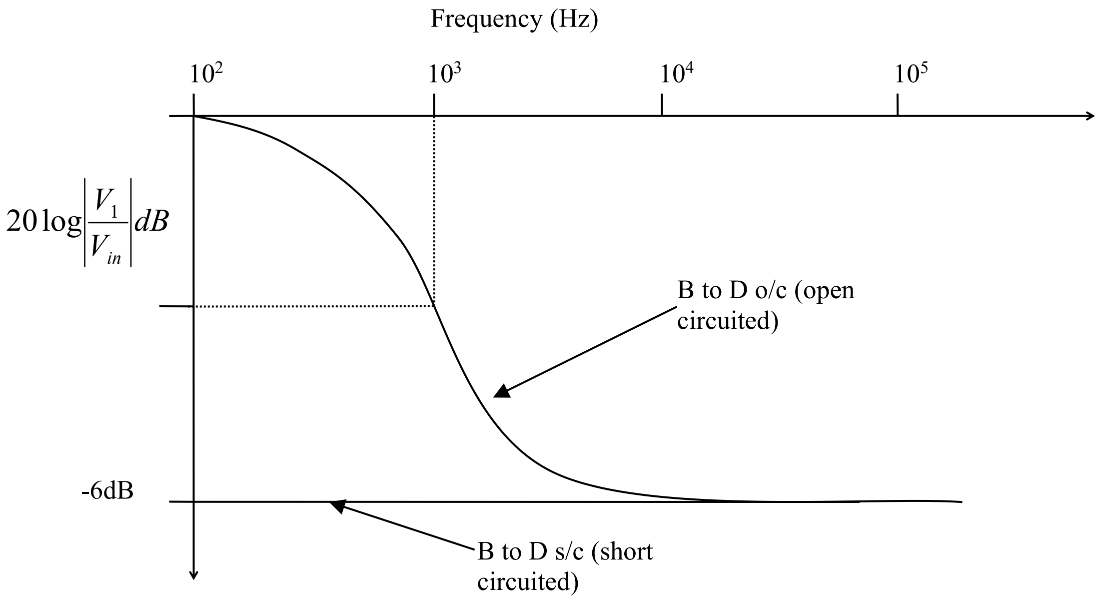
To determine the correct AC equivalent, set up the following;



When "**B**" to "**D**" is short circuited (s/c), this eliminates any components between them. Then increase the frequency gradually and measure V_1. **If the voltage doesn't change significantly, this means that there is no reactive component connected to "A". On the other hand, if the voltage varies with frequency this means that there is a reactive component between "A" and "B".**

Question 1: Explain the reasons for the bold sentences above.

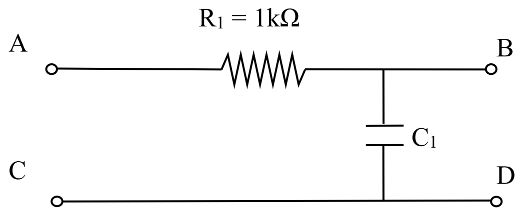
The first graph below shows different results from what was found in Part A as the characteristic is not a representation of the capacitor only, but rather is a representation of R & C.



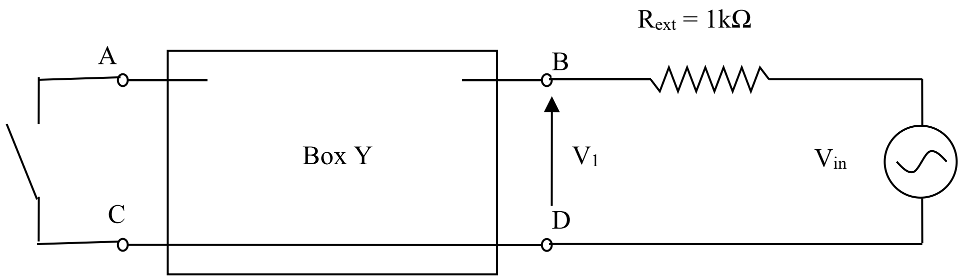
Since $20\log\left|\frac{V_1}{V_{in}}\right| = -6dB$ then;

$$\frac{V_1}{V_{in}} = \frac{R_1}{R_1 + R_{ext}} = 10^{\frac{-6}{20}} = \frac{1}{2} \Rightarrow 1 = \frac{1}{2}\left(\frac{1k\Omega}{R_1} + 1\right) \Rightarrow \therefore R_1 = 1k\Omega$$

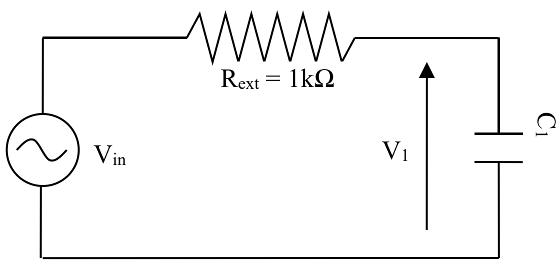
When "**B**" to "**D**" is an open circuit, from the above graph it appears that "**B**" is still s/c to "**D**" at high frequencies (hence the straight line from 10^4 Hz onwards). Therefore, preliminary guess for Box Y is as follows;



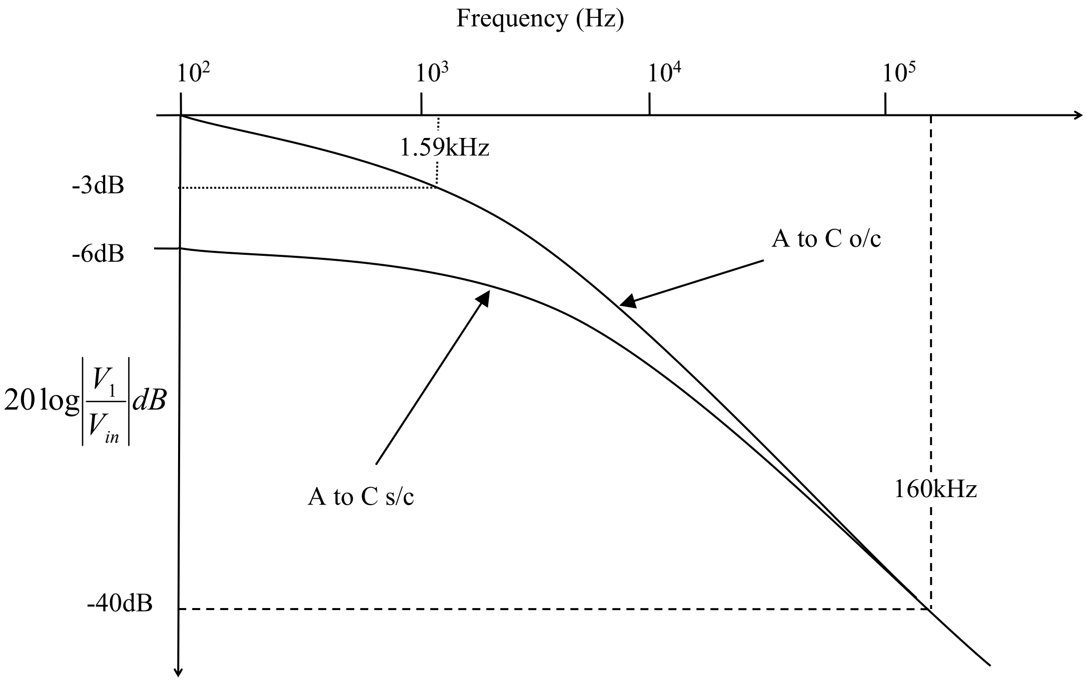
The following was set up to confirm or refute this result.



When "**A**" to "**C**" is o/c, then V_1 is measured across the capacitor only and the circuit reduces to;



If the characteristic of V_1 with varying frequency is as shown below:



the capacitance value can be predicted using the following theoretical analysis:

The modulus of $\frac{V_1}{V_{in}}$ is as follows;

$$\left|\frac{V_1}{V_{in}}\right| = \sqrt{\frac{1}{1+jwC_1R_{ext}} \times \frac{1}{1-jwC_1R_{ext}}} = \sqrt{\frac{1}{1+w^2C_1{}^2R_{ext}{}^2}} = \frac{1}{\sqrt{\left[1+w^2C_1{}^2R_{ext}{}^2\right]}}$$

Given that the 3dB point is at

$$20\log\left|\frac{V_1}{V_{in}}\right| = 20\log\left[\frac{1}{\sqrt{\left[1+w^2C_1{}^2R_{ext}{}^2\right]}}\right] = -3dB$$

From the above graph the 3dB point is at 1.59kHz.

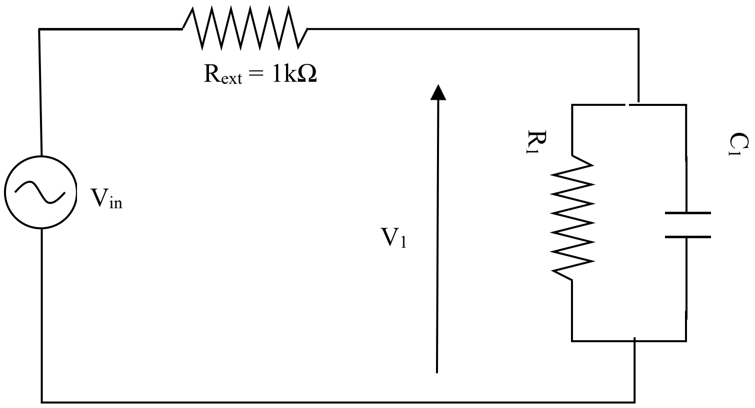
$$\frac{V_1}{V_{in}} = \frac{\frac{1}{jwC_1}}{R_{ext}+\frac{1}{jwC_1}} = \frac{1}{1+jwC_1R_{ext}}$$

Therefore:

$$\frac{1}{\sqrt{1+w^2C_1{}^2R_{ext}{}^2}}=\frac{1}{\sqrt{2}}\Rightarrow w^2C_1{}^2R_{ext}{}^2=1\Rightarrow w=\frac{1}{C_1R_{ext}}\Rightarrow C_1=\frac{1}{2\pi fR_{ext}}$$

$$\therefore C_1=\frac{1}{2\times\pi\times1590\times1000}=0.1uF$$

When "**A**" to "**C**" is short circuit, then C_1 is in parallel with R_1 and the circuit reduces to;



Similar AC analysis can be performed when "**A**" to "**C**" is s/c but this time at high frequency where the impedance of the capacitor becomes very small and therefore becomes more dominant than the resistor. For the above example, -40 dB instead of -3dB is chosen i.e. at 160 kHz

$$20\log\left|\frac{V_1}{V_{in}}\right|=20\log\left[\frac{1}{\sqrt{\left[1+w^2C_1{}^2R_{ext}{}^2\right]}}\right]=-40dB$$

Therefore,

$$\frac{1}{\sqrt{1+w^2C_1{}^2R_{ext}{}^2}}=\frac{1}{\sqrt{10000}}\Rightarrow\therefore w^2C_1{}^2R_{ext}{}^2\approx10000\Rightarrow w=\frac{100}{C_1R_{ext}}\Rightarrow C_1=\frac{100}{2\pi fR_{ext}}$$

$$\therefore C_1=\frac{100}{2\times\pi\times160k\times1k}=0.1uF$$

Note that at low frequencies the impedance of the capacitor is very high and therefore the total impedance is dominated by the resistor.

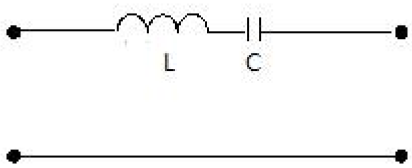
Hence, the resistor value can also be estimated from the AC measurements;

$$20\log\left|\frac{V_1}{V_{in}}\right| = \left(\frac{R_1}{R_1 + R_{ext}}\right) = 20\log\left|\frac{1k\Omega}{1k\Omega + 1k\Omega}\right| = -6dB$$

Question 2: How will the selection of the external resistor influence your analysis?

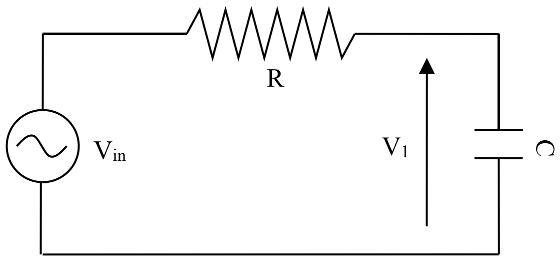
In your report, draw the circuit for each box you have determined clearly indicating all components, their values and discuss the accuracy of these values. Explain how you determined the circuit inside each box and by attaching to your report all of the graphs from which you obtained the values of components. Show this clearly by sketching the circuits and by labelling the graphs to show what the measurements represent.

Question 3: For a circuit with seriesly connected L and C as shown in the figure below, how can we estimate the value of the inductor and capacitor?



Appendix 1

For the following circuit:



The capacitance experimental value can be predicted by:

$$\frac{V_1}{V_{in}} = \frac{\frac{1}{jwC}}{R + \frac{1}{jwC}} = \frac{1}{1 + jwCR}$$

The modulus of $\frac{V_1}{V_{in}}$ is as follows;

$$\left|\frac{V_1}{V_{in}}\right| = \sqrt{\frac{1}{1 + jwCR} \times \frac{1}{1 - jwCR}} = \sqrt{\frac{1}{1 + w^2C^2R^2}} = \frac{1}{\sqrt{[1 + w^2C^2R^2]}}$$

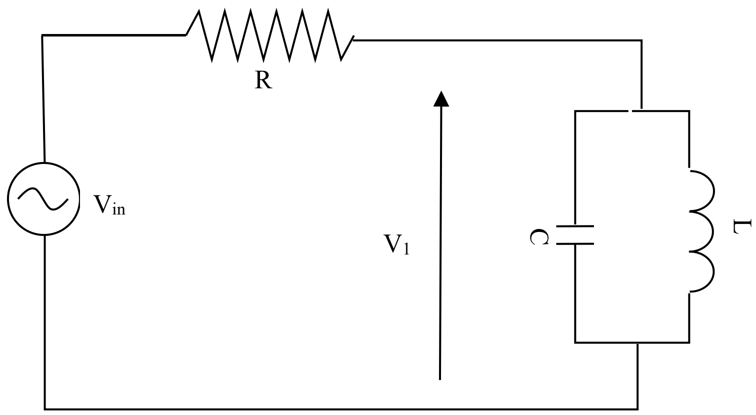
Given that the 3dB point is at

$$20\log\left|\frac{V_1}{V_{in}}\right| = 20\log\left[\frac{1}{\sqrt{[1 + w^2C^2R^2]}}\right] = -3dB$$

Therefore, $\frac{1}{\sqrt{1 + w^2C^2R^2}} = \frac{1}{\sqrt{2}} \Rightarrow w^2C^2R^2 = 1 \Rightarrow w = \frac{1}{CR} \Rightarrow f = \frac{1}{2\pi CR}$

Appendix 2

A condition of resonance will be experienced in a tank circuit (Figure below) when the reactance of the capacitor and the inductor are equal to each other. Because inductive reactance increases with increasing frequency and capacitive reactance decreases with increasing frequency, there will only be one frequency where the two reactances will be equal.



In the above circuit, we have a capacitor and an inductor. Since we know the equations for determining the reactance of each at a given frequency, and we're looking for that point where the two reactances are equal to each other, we can set the two reactance formulae equal to each other and solve for frequency algebraically. When the values of the inductor and the capacitor are known as in this exercise then the resonant frequency can be calculated as shown below. The resonant frequency will then help you to derive the capacitor and inductor values. All mathematical derivations are explained below.

$X_L = 2\pi f L$ and $X_C = \dfrac{1}{2\pi f C}$

Setting the two reactances equal to each other representing a condition of resonance;

$$2\pi f L = \frac{1}{2\pi f C} \Rightarrow f^2 = \frac{1}{4\pi^2 LC} \Rightarrow f = \frac{1}{2\pi\sqrt{LC}}$$

What happens at resonance is quite interesting. With capacitive and inductive reactances equal to each other, the total impedance increases to infinity, meaning that the tank circuit draws no current from the AC power source! We can calculate the individual impedances of the 0.1μF capacitor and the 4.7mH inductor and work through the parallel impedance formula to demonstrate this mathematically. Since the total impedance is infinity, the following equation applies;

Total impedance at resonance is infinity $\Rightarrow \dfrac{\frac{jwL}{jwC}}{\frac{1}{jwC}+jwL} = \dfrac{1}{\frac{1}{jwL}+jwC} = \infty$

Which implies $\Rightarrow \dfrac{1}{jwC} + jwL \approx 0 \Rightarrow \dfrac{1}{jwC} = -jwL \Rightarrow \dfrac{1}{wC} = wL \Rightarrow LC = \dfrac{1}{w^2}$

Appendix 3

Impedance

Impedance (symbol Z) is a measure of the overall opposition of a circuit to current, in other words: how much the circuit impedes the flow of current. It is like resistance, but it also takes into account the effects of capacitance and inductance. Impedance is measured in Ohms, symbol Ω.

Impedance is more complex than resistance because the effects of capacitance and inductance vary with the frequency of the current passing through the circuit. Therefore impedance varies with frequency. The effect of resistance is constant regardless of frequency.

Impedance can be split into two parts:

- Resistance R (the part which is constant regardless of frequency)
- Reactance X (the part which varies with frequency due to capacitance and inductance)

Reactance

Reactance (symbol X) is a measure of the opposition of capacitance and inductance to current. Reactance varies with the frequency of the electrical signal. Reactance is measured in Ohms, symbol 'Ω'. There are two types of reactance: capacitive reactance (X_C) and inductive reactance (X_L).

The total reactance (X) is the difference between the two: $X = X_L - X_C$

- Capacitive reactance, X_C

$$X_C = \frac{1}{2\pi f C}$$

where:
X_C = reactance in Ohms (Ω)
f = frequency in Hertz (Hz)
C = capacitance in Farads (F)

- X_C is large at low frequencies and small at high frequencies. For steady DC which is zero frequency, X_C is infinite (total opposition), hence the rule that capacitors pass AC but block DC.
- For example: a 1μF capacitor has a reactance of 3.2kΩ for a 50Hz signal, but when the frequency is higher at 10kHz its reactance is only 16Ω.
- Inductive reactance, X_L.

$$X_L = 2\pi f L$$

where:
X_L = reactance in Ohms (Ω)
f = frequency in Hertz (Hz)
L = inductance in Henrys (H)

- X_L is small at low frequencies and large at high frequencies. For steady DC (frequency zero), X_L is zero (no opposition), hence the rule that inductors pass DC but block high frequency AC.
- For example: a 1mH inductor has a reactance of only 0.3Ω for a 50Hz signal, but when the frequency is higher at 10kHz its reactance is 63Ω.

The table below summarises the impedance of the different components that will be used in this experiment, i.e. resistors, capacitors and inductors. It is easy to remember that the voltage on the capacitor is behind (lagging) the current, because the charge doesn't build up until after the current has been flowing for a while.

Resistor 	**Capacitor** 	**Inductor** 
Resistance $\frac{V_R}{I} = R$	Capacitive reactance $\frac{V_C}{I} = X_C = \frac{1}{wC}$	Inductive reactance $\frac{V_L}{I} = X_L = wL$
V and I in phase	V lags I by $\pi/2$	V leads I by $\pi/2$

The same information is given graphically below. It is easy to remember the frequency dependence by thinking of the DC (zero frequency) behaviour: at DC, an inductance is a short circuit (a piece of wire) so its impedance is zero. In DC analysis, a capacitor is an open circuit, as its circuit diagram shows, so its impedance goes to infinity.