EEE108 Electromagnetism and Electromechanics

Lecture 20

AC Machinery Fundamentals

Dr. Jinling Zhang

Dept. of Electrical and Electronic Engineering
University of Xi'an Jiaotong-Liverpool
Email: jinling.zhang@xjtlu.edu.cn

AC Machines

- Synchronous Machines
  - Magnetic field current is supplied by a separate DC power source
- Induction Machines
  - Field current is supplied by magnetic induction into their field windings.

The field circuits are located on their rotors.

Today

AC Machinery Fundamentals

-- A simple loop in a uniform magnetic field

- Induced voltage
- Induced torque

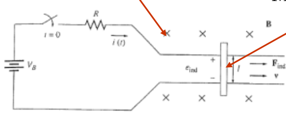
-- Rotating Magnetic Field

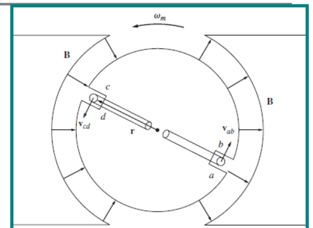
- The rotating magnetic field concept
- Reversing the direction of magnetic field rotation
- The relationship between electrical frequency and the speed of magnetic field rotation

A Simple Loop in a Uniform Magnetic Field

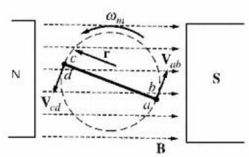
Stationary part

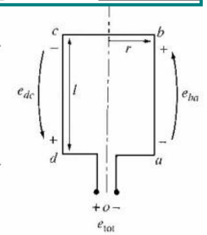
Moving part





The figure shows a simple rotating loop in a uniform magnetic field. The rotating part is called the rotor, and the stationary part is called the stator.

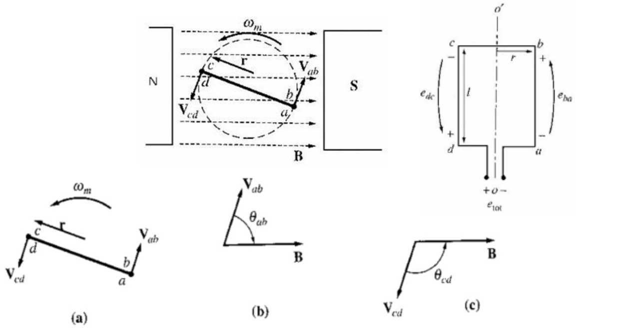




The factors that control the voltage and torque on the loop are the same as the factors that control the voltage and torque in real AC machines.

A Simple Loop in a Uniform Magnetic Field
The Voltage Induced in the Loop

If the rotor (loop) is rotated, a voltage will be induced in the wire loop.

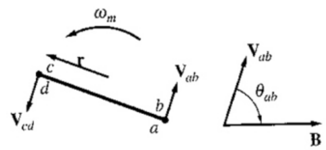


To determine the total voltage induced e_{tot} on the loop, examine each segment of the loop separately and sum all the resulting voltages. The voltage on each segment is given by equation

$$e_{ind} = (\mathbf{v} \times \mathbf{B}) \bullet \mathbf{l}$$

A Simple Loop in a Uniform Magnetic Field
The Voltage Induced in the Loop

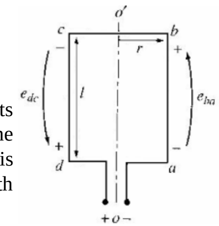
1. Segment *ab*



The velocity of the wire is tangential to the path of rotation, while the magnetic field **B** points to the right. The quantity **v** x **B** points into the page, which is the same direction as segment *ab*. Thus, the induced voltage on this segment is:

$$e_{ba} = (\mathbf{v} \times \mathbf{B}) \bullet \mathbf{l} = vBl \sin \theta_{ab} \quad \text{into the page}$$

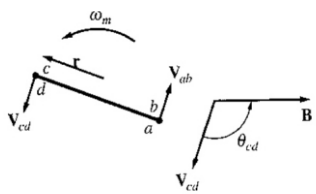
2. Segment *bc*



In the first half of this segment, the quantity **v** x **B** points into the page, and in the second half of this segment, the quantity **v** x **B** points out of the page. Since the length *l* is in the plane of the page, **v** x **B** is perpendicular to **l** for both portions of the segment. Thus: $e_{cb} = 0$

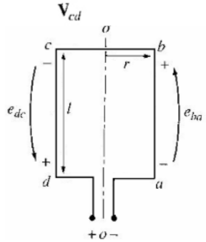
A Simple Loop in a Uniform Magnetic Field
The Voltage Induced in the Loop

3. Segment *cd*



The velocity of the wire is tangential to the path of rotation, while **B** points to the right. The quantity **v** x **B** points into the page, which is the same direction as segment *cd*. Thus,

$$e_{dc} = (\mathbf{v} \times \mathbf{B}) \bullet \mathbf{l} = vBl \sin \theta_{cd} \quad \text{out of the page}$$



4. Segment *da*

same as segment *bc*, **v** x **B** is perpendicular to *l*. Thus, $e_{da} = 0$

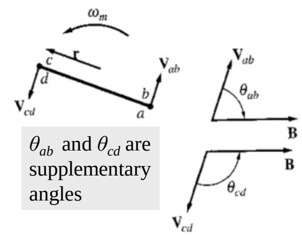
A Simple Loop in a Uniform Magnetic Field
The Voltage Induced in the Loop

Total induced voltage on the loop

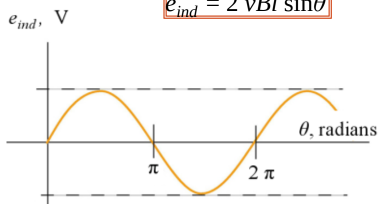
$e_{ind} = e_{ba} + e_{cb} + e_{dc} + e_{ad}$
$\quad = vBl \sin \theta_{ab} + vBl \sin \theta_{cd}$

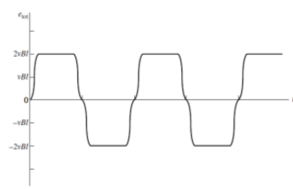
since $\theta_{ab} = 180^o - \theta_{cd}$ and
$\sin \theta = \sin(180^o - \theta)$, the total induced voltage on the loop becomes:

$$\boxed{e_{ind} = 2\, vBl \sin\theta}$$



θ_{ab} and θ_{cd} are supplementary angles

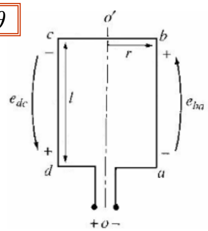




The induced voltage is shown as a function of angle.

A Simple Loop in a Uniform Magnetic Field
The Voltage Induced in the Loop

Alternative way to express e_{ind}: $\boxed{e_{ind} = 2\,vBl\,\sin\theta}$



If the loop is rotating at a constant angular velocity ω, then the angle θ of the loop will increase linearly with time.

$$\theta = \omega t$$

also, the tangential velocity v of the edges of the loop is:

$$v = r\omega$$

where r is the radius from axis of rotation out to the edge of the loop and ω is the angular velocity of the loop. Hence,

$$e_{ind} = 2r\omega Bl\,\sin\omega t$$

since area, $A = 2rl$,

$$e_{ind} = AB\omega\,\sin\omega t$$

A Simple Loop in a Uniform Magnetic Field
The Voltage Induced in the Loop

Finally, since maximum flux through the loop occurs when the loop is perpendicular to the magnetic flux density lines, so

$$\Phi_{\max} = AB$$

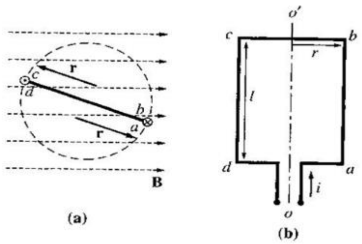
Thus $\boxed{e_{ind} = \Phi_{\max}\,\omega \sin \omega t}$

The voltage generated in the loop is a sinusoid. This also true of real AC machines. In general, the voltage in a real machine will depend on three factors:

1. Flux level (the **B** component)
2. Speed of Rotation (the **v** component)
3. Machine Constants (such as: the number of loops, machine materials, …)

A Simple Loop in a Uniform Magnetic Field
The Torque Induced in a Current-Carrying Loop

Assume that the rotor loop is at some arbitrary angle θ with the magnetic field, and that current is flowing in the loop.



To determine the magnitude and direction of the torque, we examine the force and torque on each segment of the loop.

They are given by:

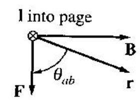
$$\mathbf{F} = i(\mathbf{l}\times\mathbf{B}) \quad \text{and}$$

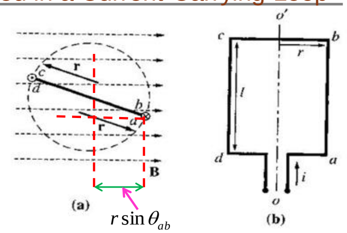
$$\tau = (\text{force})(\text{perpendicular distance})$$

$$= (F)(r\sin\theta) = rF\sin\theta$$

A Simple Loop in a Uniform Magnetic Field
The Torque Induced in a Current-Carrying Loop

Segment *ab*:



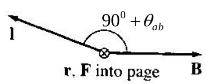


The direction of the current is into the page, while the magnetic field **B** points to the right. (**l** x **B**) points down.

$$\mathbf{F} = i(\mathbf{l}\times\mathbf{B}) = ilB$$

$$\tau_{ab} = F(r\sin\theta_{ab}) = rilB\sin\theta_{ab} \text{ Clockwise}$$

Segment *bc*:

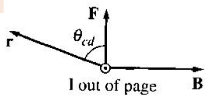


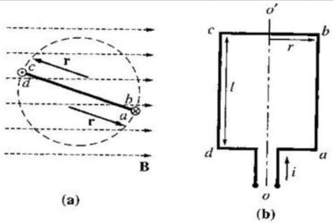
The direction of the current is in the plane of the page, while the magnetic field **B** points to the right. (**l** x **B**) points into the page. $\mathbf{F} = i(\mathbf{l}\times\mathbf{B}) = ilB\,\sin(90^0+\theta_{ab})$
The force and the shaft are parallel and the angle $\theta_{bc} = 0$.
Then:

$$\tau_{bc} = F(r\sin\theta_{bc}) = 0$$

A Simple Loop in a Uniform Magnetic Field
The Torque Induced in a Current-Carrying Loop

Segment *cd*:





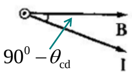
The direction of the current is out of the page, while the magnetic field **B** points to the right. (**l** x **B**) points up.

$$\mathbf{F} = i(\mathbf{l}\times\mathbf{B}) = ilB$$

$$\tau_{cd} = F(r\sin\theta_{cd}) = rilB\sin\theta_{cd} \quad \text{Clockwise}$$

Segment *da*:

r, **F** out of page

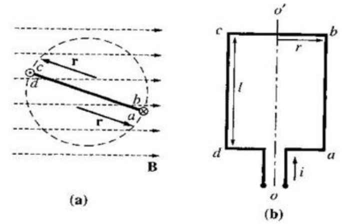


The direction of the current is in the plane of the page, while the magnetic field **B** points to the right. (**l** x **B**) points out of the page. $\mathbf{F} = i(\mathbf{l}\times\mathbf{B}) = ilB\sin(90^0-\theta_{cd})$
The force and the shaft are parallel and the angle $\theta_{da} = 0$.
Then:

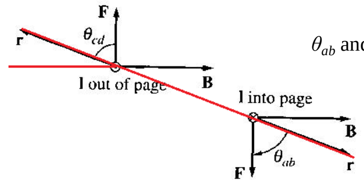
$$\tau_{da} = F(r\sin\theta_{da}) = 0$$

A Simple Loop in a Uniform Magnetic Field
The Torque Induced in a Current-Carrying Loop

The total induced torque on the loop:



$$\tau_{total} = \tau_{ab} + \tau_{bc} + \tau_{cd} + \tau_{da}$$
$$= rilB\sin\theta_{ab} + rilB\sin\theta_{cd}$$
$$= 2rilB\sin\theta$$

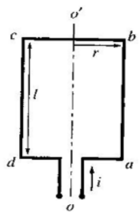


θ_{ab} and θ_{cd} are vertical angles.

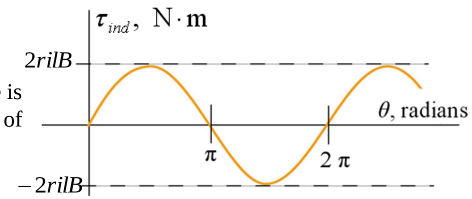
A Simple Loop in a Uniform Magnetic Field
The Torque Induced in a Current-Carrying Loop

$$\mathbf{T} = i\mathbf{A}\times\mathbf{B} = (i)(2rl)(B)\sin\theta$$

$$\boxed{\tau_{ind} = 2rilB\sin\theta}$$



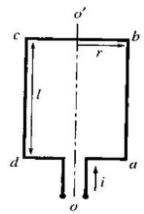
The resulting torque is shown as a function of angle.

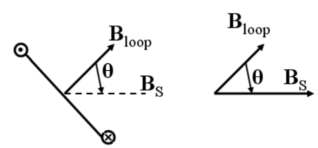


A Simple Loop in a Uniform Magnetic Field
The Torque Induced in a Current-Carrying Loop

$$\tau_{ind} = 2rilB\sin\theta$$

An alternative way to express the equation to relate the behaviour of the single loop to the behaviour of a real machine:





If the current in the loop is as shown, that current will generate a magnetic flux density $\mathbf{B}_{loop}$ with the direction shown. The magnitude of $\mathbf{B}_{loop}$ is:

$$B_{loop} = \frac{\mu i}{G}$$

Where G is a factor that depends on the geometry of the loop.

A Simple Loop in a Uniform Magnetic Field
The Torque Induced in a Current-Carrying Loop

The area of the loop A is $2rl$

$$B_{loop} = \frac{\mu i}{G}$$

$$\tau_{ind} = 2rilB\sin\theta$$

$$\tau_{ind} = \frac{AG}{\mu} B_{loop} B_S \sin\theta$$
$$= kB_{loop} B_S \sin\theta$$

Where $k=AG/\mu$ is a factor depending on the construction of the machine, B_S is used for the stator magnetic field to distinguish it from the magnetic field generated by the rotor, and θ is the angle between $\mathbf{B}_{loop}$ and $\mathbf{B}_S$.

$$\boxed{\boldsymbol{\tau}_{ind} = k\mathbf{B}_{loop} \times \mathbf{B}_S}$$

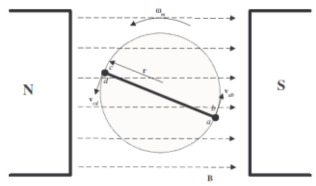
From here, we may conclude that torque is dependent upon:

- Strength of rotor magnetic field
- Strength of stator magnetic field
- Angle between the 2 fields
- Machine constants – represent the construction of the machine

A Simple Loop in a Uniform Magnetic Field
Example

The simple loop is rotating in a uniform magnetic field with the following characteristics:

$\mathbf{B}$ = 0.5 T to the right r = 0.1 m
l = 0.5 m ω = 103 rad/s



Calculate:

(a) the voltage $e_{tot}(t)$ induced in the loop.
(b) suppose that a 5 Ω resistor is connected as a load across the terminals of the loop: find the current that flows through the resistor.
(c) the magnitude and direction of the induced torque on the loop for the conditions in (b).
(d) the electric power being generated by the loop for the conditions in (b).
(e) the mechanical power being consumed by the loop for the conditions in (b). How does this number compare to the amount of electric power being generated by the loop?

A Simple Loop in a Uniform Magnetic Field
Example

$\mathbf{B}$ = 0.5 T to the right r = 0.1 m
l = 0.5 m ω = 103 rad/s

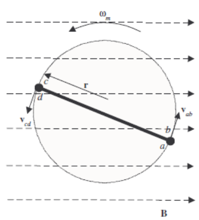
(a) the voltage $e_{tot}(t)$ induced in the loop.

$$e_{ind}(t) = 2r\omega Bl \sin\omega t$$
$$e_{ind}(t) = 2(0.1\text{ m})(103\text{ rad/s})(0.5\text{ T})(0.5\text{ m})\sin 103t$$
$$e_{ind}(t) = 5.15\ \sin 103t\text{ V}$$

(b) Suppose that a 5 Ω resistor is connected as a load across the terminals of the loop: the current that flows through the resistor.

$$i(t) = \frac{e_{ind}}{R} = \frac{5.15\sin 103t\text{ V}}{5\ \Omega} = 1.03\sin 103t\text{ A}$$

(c) the magnitude and direction of the induced torque on the loop for the conditions in (b).



$$\tau_{ind}(t) = 2rilB\sin\theta$$
$$\tau_{ind}(t) = 2(0.1\text{ m})(1.03\sin\omega t\text{ A})(0.5\text{ m})(0.5\text{ T})\sin\omega t$$
$$\tau_{ind}(t) = 0.0515\sin^2\omega t\ \text{N}\cdot\text{m},$$ ~~counterclockwise~~

clockwise

A Simple Loop in a Uniform Magnetic Field
Example

(d) the electric power being generated by the loop for the conditions in (b).

The instantaneous power generated by the loop is:

$$P(t) = e_{ind}i = (5.15\sin\omega t\text{ V})(1.03\sin\omega t\text{ A}) = 5.30\sin^2\omega t\ \text{W}$$

The average power generated by the loop is:

$$P_{ave} = \frac{1}{T}\int_0^T 5.30\sin^2(\omega t)dt = 2.65 \quad \text{W}$$

(e) the mechanical power being consumed by the loop for the conditions in (b). How does this number compare to the amount of electric power being generated by the loop?

$$P = \tau_{ind}\omega = (0.0515\sin^2\omega t\text{ V})(103\text{ rad/s}) = 5.30\sin^2\omega t\ \text{W}$$

The amount of mechanical power consumed by the loop is equal to the amount of electrical power created by the loop.

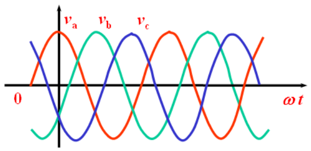
This machine is acting as a generator, converting mechanical power into electrical power.

The Rotating Magnetic Field

$i_{aa'}(t) = I_M \sin \omega t \; A$

$i_{bb'}(t) = I_M \sin(\omega t - 120°) A$

$i_{cc'}(t) = I_M \sin(\omega t - 240°) A$



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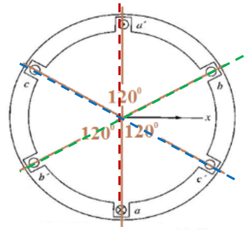
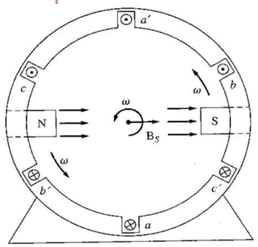


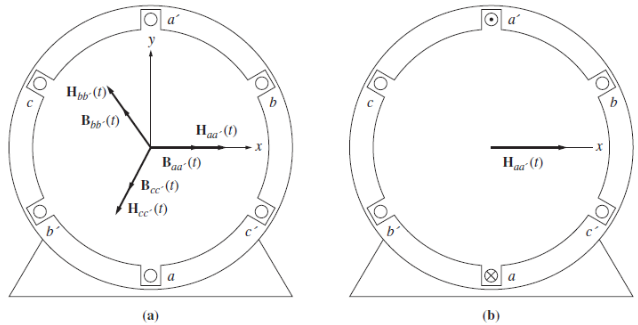


Fundamental principle:

a 3-phase set of currents, each of equal magnitude and differing in phase by 120°, flows in a 3-phase winding 120° apart spatially in the stator then it will produce a rotating magnetic field of constant magnitude.

The Rotating Magnetic Field

The rotating magnetic field concept is illustrated below – empty stator containing 3 coils 120° apart. It is a 2-pole winding (one north and one south).



(a) A simple three phase stator. Currents in this stator are assumed positive if they flow into the unprimed end and out the primed end of the coils.

(b) The magnetizing intensity vector $H_{aa'}(t)$ produced by a current flowing in coil aa'.

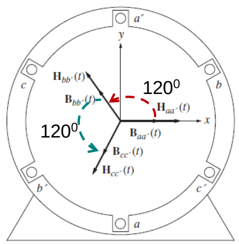
The Rotating Magnetic Field

Assume currents in the 3 coils are:

$i_{aa'}(t) = I_M \sin \omega t \; A$

$i_{bb'}(t) = I_M \sin(\omega t - 120°) A$

$i_{cc'}(t) = I_M \sin(\omega t - 240°) A$



Then the magnetic field intensity created by the three coils:

$\mathbf{H}_{aa'}(t) = H_M \sin \omega t \angle 0°$ A • turns / m

$\mathbf{H}_{bb'}(t) = H_M \sin (\omega t - 120°) \angle 120°$ A • turns / m

$\mathbf{H}_{cc'}(t) = H_M \sin (\omega t - 240°) \angle 240°$ A • turns / m

With $\mathbf{B} = \mu \mathbf{H}$

$\mathbf{B}_{aa'}(t) = B_M \sin \omega t \angle 0°$ T

$\mathbf{B}_{bb'}(t) = B_M \sin (\omega t - 120°) \angle 120°$ T

$\mathbf{B}_{cc'}(t) = B_M \sin (\omega t - 240°) \angle 240°$ T

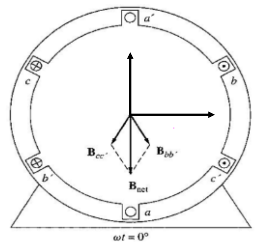
Here $B_M = \mu H_M$

The Rotating Magnetic Field

$\mathbf{B}_{aa'}(t) = B_M \sin \omega t \angle 0°$ T

$\mathbf{B}_{bb'}(t) = B_M \sin (\omega t - 120°) \angle 120°$ T

$\mathbf{B}_{cc'}(t) = B_M \sin (\omega t - 240°) \angle 240°$ T



At $\omega t = 0$:

$\mathbf{B}_{aa'} = 0$

$\mathbf{B}_{bb'} = B_M \sin(-120°) \angle 120°$

$\mathbf{B}_{cc'} = B_M \sin(-240°) \angle 240°$

The total magnetic field:

$$\mathbf{B}_{net} = \mathbf{B}_{aa'} + \mathbf{B}_{bb'} + \mathbf{B}_{cc'}$$

$$= 0 + \left(-\frac{\sqrt{3}}{2} B_M\right) \angle 120° + \left(\frac{\sqrt{3}}{2} B_M\right) \angle 240°$$

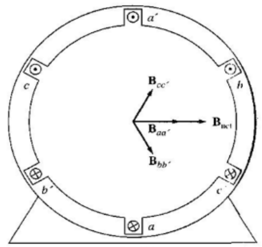
$$= 1.5 B_M \angle -90°$$

The Rotating Magnetic Field

$\mathbf{B}_{aa'}(t) = B_M \sin \omega t \angle 0° \quad \text{T}$

$\mathbf{B}_{bb'}(t) = B_M \sin (\omega t - 120°) \angle 120° \quad \text{T}$

$\mathbf{B}_{cc'}(t) = B_M \sin (\omega t - 240°) \angle 240° \quad \text{T}$



$\omega t = 90°$

At $\omega t = 90^0$:

$\mathbf{B}_{aa'} = B_M \angle 0°$

$\mathbf{B}_{bb'} = -0.5\ B_M \angle 120°$

$\mathbf{B}_{cc'} = -0.5\ B_M \angle 240°$

The total magnetic field:

$$\mathbf{B}_{\text{net}} = \mathbf{B}_{aa'} + \mathbf{B}_{bb'} + \mathbf{B}_{cc'}$$

$$= B_M \angle 0 + (-0.5 B_M)\angle 120° + (-0.5 B_M)\angle 240°$$

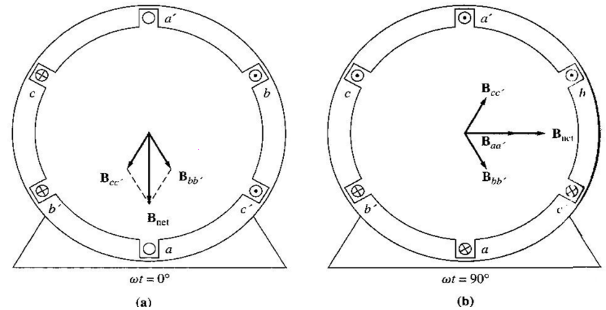
$$= 1.5 B_M \angle 0°$$

$\sin(90^\circ - 240^\circ) = -\sin(150^\circ) = -\sin(180^\circ - 30^\circ) = -\sin 30^\circ$

$\sin(90^\circ - 120^\circ) = \sin(-30^\circ) = -\sin 30^\circ$

The Rotating Magnetic Field

The resulting magnetic flux is:



$\omega t = 0°$ (a) $\qquad$ $\omega t = 90°$ (b)

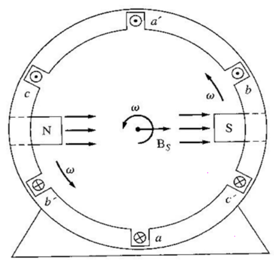
The vector magnetic field in a stator at time $\omega t = 0$

The vector magnetic field in a stator at time $\omega t = 90^o$

The Rotating Magnetic Field

At any time t, the magnetic field will have the same magnitude 1.5 B_M and it will continue to rotate at angular velocity ω.

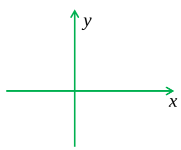
$$\mathbf{B}_{\text{net}}(t) = B_M \sin \omega t \angle 0° + B_M \sin(\omega t - 120°)\angle 120° + B_M \sin(\omega t - 240°)\angle 240°\ T$$



We may convert the total flux density into unit vector forms to give:

$$\mathbf{B}_{\text{net}}(t) = (1.5 B_M \sin \omega t)\hat{x} - (1.5 B_M \cos \omega t)\hat{y}$$

The magnitude of the field is a constant $1.5B_M$ and the angle changes continually in a counterclockwise direction at angular velocity ω.



The Rotating Magnetic Field

Reversing the direction of Magnetic Field Rotation

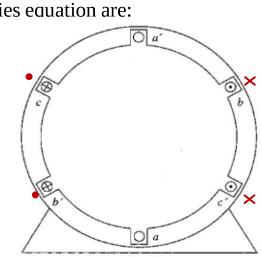
If the current in any two of the 3 coils is swapped, the direction of the magnetic field's rotation will be reversed.

Phases B and C are switched. Now the flux densities equation are:

$\mathbf{B}_{aa'}(t) = B_M \sin \omega t \angle 0° \quad \text{T}$

$\mathbf{B}_{bb'}(t) = B_M \sin (\omega t - 240°) \angle 120° \quad \text{T}$

$\mathbf{B}_{cc'}(t) = B_M \sin (\omega t - 120°) \angle 240° \quad \text{T}$



$\omega t = 0°$

At $\omega t = 0$:

$\mathbf{B}_{aa'} = 0$

$\mathbf{B}_{bb'} = B_M \sin(-240°)\angle 120°\ T$

$\mathbf{B}_{cc'} = B_M \sin(-120°)\angle 240°\ T$

The total magnetic field: $\mathbf{B}_{\text{net}} = \mathbf{B}_{aa'} + \mathbf{B}_{bb'} + \mathbf{B}_{cc'}$

$$= 0 + \left(\frac{\sqrt{3}}{2} B_M\right)\angle 120° + \left(-\frac{\sqrt{3}}{2} B_M\right)\angle 240°$$

$$= 1.5 B_M \angle 90°$$

The Rotating Magnetic Field

Reversing the direction of Magnetic Field Rotation

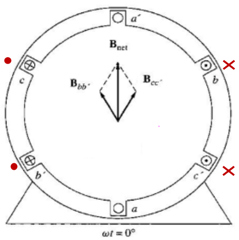
At $\omega t = 90^0$:

$$\mathbf{B}_{aa'} = B_M \angle 0°$$
$$\mathbf{B}_{bb'} = -0.5B_M \angle 120° \ T$$
$$\mathbf{B}_{cc'} = -0.5B_M \angle 240° \ T$$

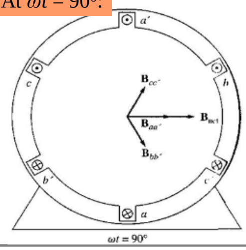
The total magnetic field: $\mathbf{B}_{\text{net}} = \mathbf{B}_{aa'} + \mathbf{B}_{bb'} + \mathbf{B}_{cc'}$

$$= B_M \angle 0 + (-0.5B_M)\angle 120° + (-0.5B_M)\angle 240°$$
$$= 1.5B_M \angle 0°$$

At $\omega t = 0$:



At $\omega t = 90^0$:

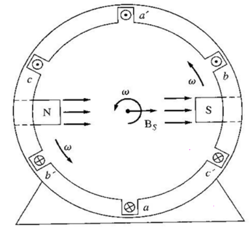


Now, the magnetic field rotates in a clockwise direction.

The Rotating Magnetic Field

Relationship between Electrical Frequency and the Speed of Magnetic Field Rotation

$$\mathbf{B}_{\text{net}}(t) = (1.5B_M \sin \omega t)\hat{x} - (1.5B_M \cos \omega t)\hat{y}$$



The rotating magnetic field in the stator can be represented as a north pole (the flux leaves the stator) and a south pole (flux enters the stator).

Two poles

These magnetic poles complete one mechanical rotation around the stator surface for each electrical cycle of the applied current. The mechanical speed of rotation of the magnetic field in revolutions per second is equal to electric frequency in hertz:

f_e (hertz) = f_m (revolutions per second) two poles
ω_e (radians per second) = ω_m (radians per second) two poles

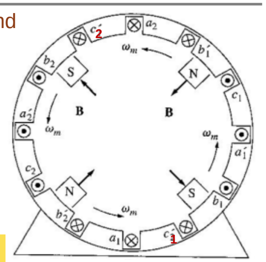
The windings occur in the order ***a – c' – b – a' – c – b'***

The Rotating Magnetic Field

Relationship between Electrical Frequency and the Speed of Magnetic Field Rotation

If we double the amount of windings with the sequence of windings as follows:

$a_1 - c_1' - b_1 - a_1' - c_1 - b_1' - a_2 - c_2' - b_2 - a_2' - c_2 - b_2'$ ***a – c' – b – a' – c – b'***



For a three-phase set of currents, this stator will have 2 north poles and 2 south poles produced in the stator winding.

Four poles

In this winding, a pole moves only halfway around the stator surface in one electrical cycle.

Since one electrical cycle is 360 electrical degrees, and mechanical motion is 180 mechanical degrees, the relationship between the electrical angle θ_e and the mechanical θ_m in this stator is: $\theta_e = 2\,\theta_m$

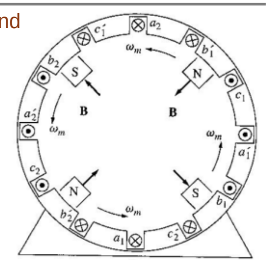
The electrical frequency of the current is twice the mechanical frequency of rotation: $f_e = 2\,f_m$ $\omega_e = 2\,\omega_m$

The Rotating Magnetic Field

Relationship between Electrical Frequency and the Speed of Magnetic Field Rotation

General format:

$$\theta_e = \frac{P}{2}\theta_m$$
$$f_e = \frac{P}{2}f_m$$
$$\omega_e = \frac{P}{2}\omega_m$$



Since $f_m = \dfrac{n_m}{60} \Rightarrow f_e = \dfrac{n_m P}{120}$

where n_m is the number of rotation per minute.

P is the number of magnetic poles.

The repetitions of the winding sequence is *a-c'-b-a'-c-b'*

The Rotating Magnetic Field

Example

The rotor of a six-pole synchronous generator is rotating at a mechanical speed of 1200 r/min.

(a)Express this mechanical speed in radians per second.

(b)What is the frequency of the generated voltage in hertz and in radians per second?

(c)What mechanical speed in revolutions per minute would be required to generate voltage at a frequency of 50 Hz?

The Rotating Magnetic Field

Example

(a) Mechanical speed in radians per second.

$$\omega_m = \frac{n_m \times 2\pi}{60} = \frac{1200 \times 2\pi}{60} = 125.6 \ \text{rad/sec}$$

(b) The frequency of the generated voltage in hertz and in radians per second:

$$f_e = \frac{n_m P}{120} = \frac{1200 \times 6}{120} = 60 \ \ \text{Hz}$$

$$\omega_e = 2\pi f_e = 2\pi \times 60 = 376.8 \ \text{rad/sec}$$

(c) Mechanical speed in revolutions per minute would be required to generate voltage at a frequency of 50 Hz:

$$n_m = \frac{120 f_e}{P} = \frac{120 \times 50}{6} = 1000 \ \text{r/min}$$

Summary

1. A simple loop in a uniform magnetic field

- The voltage induced in a simple rotating loop
- The Torque induced in a current-carrying loop

2. The Rotating Magnetic Field

- The rotating magnetic field concept
- Reversing the direction of magnetic field rotation
- The relationship between electrical frequency and the speed of magnetic field rotation

Next Lecture

AC Machinery Fundamentals （2）

Synchronous Machines

Thanks for your attendance