EEE225 Advanced Electrical Circuits and Electromagnetics

Lecture 9 Pre-lecture

Review of Matrix operations

Dr. Zhao Wang

zhao.wang@xjtlu.edu.cn

Room EE322

Content

- Review of matrix operation
  - Addition and Subtraction
  - Scalar multiplication and Transpose
  - Matrix multiplication
  - Square matrix and Identity matrix
  - Determinant
  - ***Inverse***

Matrix – addition and subtraction

$$\mathbf{A}=\begin{bmatrix}3 & 1 & -1\\ 0 & -2 & 4\end{bmatrix} \quad \mathbf{B}=\begin{bmatrix}2 & 6 & 1\\ 10 & -3 & 6\end{bmatrix}$$

1. Addition:

$$\mathbf{A}+\mathbf{B}=\begin{bmatrix}3 & 1 & -1\\ 0 & -2 & 4\end{bmatrix}+\begin{bmatrix}2 & 6 & 1\\ 10 & -3 & 6\end{bmatrix}=\begin{bmatrix}3+2 & 1+6 & -1+1\\ 0+10 & -2+(-3) & 4+6\end{bmatrix}=\begin{bmatrix}5 & 7 & 0\\ 10 & -5 & 10\end{bmatrix}$$

2. Subtraction:

$$\mathbf{B}-\mathbf{A}=\begin{bmatrix}2 & 6 & 1\\ 10 & -3 & 6\end{bmatrix}-\begin{bmatrix}3 & 1 & -1\\ 0 & -2 & 4\end{bmatrix}=\begin{bmatrix}2-3 & 6-1 & 1-(-1)\\ 10-0 & -3-(-2) & 6-4\end{bmatrix}=\begin{bmatrix}-1 & 5 & 2\\ 10 & -1 & 2\end{bmatrix}$$

Matrices must have the same size:

the same number of rows AND the same number of columns

Matrix – Multiplication and transpose

3. Scalar Multiplication

$$2\mathbf{A} = 2\times\begin{bmatrix} 6 & 1 & -3 & 0 \\ 9 & 2 & 3 & 7 \end{bmatrix} = \begin{bmatrix} 2\times 6 & 2\times 1 & 2\times(-3) & 2\times 0 \\ 2\times 9 & 2\times 2 & 2\times 3 & 2\times 7 \end{bmatrix} = \begin{bmatrix} 12 & 2 & -6 & 0 \\ 18 & 4 & 6 & 14 \end{bmatrix}$$

4. Transpose

$$\mathbf{A} = \begin{bmatrix} 6 & 1 & -3 & 0 \\ 9 & 2 & 3 & 7 \end{bmatrix} \quad \Rightarrow \quad \mathbf{A}^T = \begin{bmatrix} 6 & 9 \\ 1 & 2 \\ -3 & 3 \\ 0 & 7 \end{bmatrix}$$

The transpose of a matrix is obtained by writing rows as columns.

Properties:

$$(\mathbf{A}+\mathbf{B})^T = \mathbf{A}^T + \mathbf{B}^T \qquad\qquad (k\mathbf{A})^T = k\mathbf{A}^T$$

Matrix – matrix multiplication

$$\mathbf{A} = \begin{bmatrix} 3 & 1 & 4 \\ 5 & 2 & -2 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 1 & -1 \\ 2 & 0 \\ -6 & 4 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \qquad \mathbf{C} = \mathbf{AB}$$

$$c_{11} = 3\times 1 + 1\times 2 + 4\times(-6) = -19$$
$$c_{12} = 3\times(-1) + 1\times 0 + 4\times 4 = 13$$
$$c_{21} = 5\times 1 + 2\times 2 + (-2)\times(-6) = 21$$
$$c_{22} = 5\times(-1) + 2\times 0 + (-2)\times 4 = -13$$

$$\boxed{\mathbf{AB} = \begin{bmatrix} 3 & 1 & 4 \\ 5 & 2 & -2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & 0 \\ -6 & 4 \end{bmatrix} = \begin{bmatrix} -19 & 13 \\ 21 & -13 \end{bmatrix}}$$

If $\mathbf{C} = \mathbf{AB}$ then the element c_{ij} is found from row i of $\mathbf{A}$ and column j of $\mathbf{B}$.

$$[\mathbf{C}]_{ij} = A_{i1}B_{1j} + A_{i2}B_{2j} + \ldots + A_{in}B_{nj} = \sum_{r=1}^{n} A_{ir}B_{rj}$$

$$\underbrace{\underbrace{\mathbf{A}}_{n\times m} \quad \underbrace{\mathbf{B}}_{p\times q}}_{m=p} = \underbrace{\mathbf{C}}_{n\times q}$$

The number of columns of **A** must equal the number of rows of **B**

Matrix – *matrix multiplication*

Associative

$(AB)C = A(BC)$

$(A+B)C = AC+BC$

$C(A+B) = CA+CB$

But generally

$AB \neq BA$

Matrix multiplication is NOT commutative

Example

$$\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix} \times \begin{bmatrix}0 & 1\\0 & 0\end{bmatrix} = \begin{bmatrix}0 & 1\\0 & 3\end{bmatrix}$$

whereas

$$\begin{bmatrix}0 & 1\\0 & 0\end{bmatrix} \times \begin{bmatrix}1 & 2\\3 & 4\end{bmatrix} = \begin{bmatrix}3 & 4\\0 & 0\end{bmatrix}$$

Matrix – square matrix and identity matrix

Square Matrices

A square matrix: has the same number of rows and columns.

Any two square matrices of the same order can be added and multiplied.

Example

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 3 \end{bmatrix}$$

whereas

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 0 & 0 \end{bmatrix}$$

Identity Matrices

The identity matrix or unit matrix of size n is the n-by-n square matrix with ones on the main diagonal and zeros elsewhere. It is denoted by I_n, or simply by I.

$$I_1 = [1],\ I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ \cdots,\ I_n = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}$$

Matrix – determinants

All square matrices possess a determinant. The determinant of a matrix **A**, is denoted det(**A**), Δ_A or |**A**|:

The determinant of a matri $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ $\quad |A| = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$

The 2×2 matri $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ has determinant det (A) = *ad* − *bc*.

$$A = \begin{bmatrix} 3 & 6 \\ -1 & 1 \end{bmatrix} \quad \Delta_A \text{ or } |A| = 3\times 1 - 6\times(-1) = 9$$

Matrix – inverse

Definition :

Let $\mathbf{A}$ be an $n \times n$ matrix.

Suppose that $\mathbf{B}$ is an $n \times n$ matrix such that :

$$\mathbf{AB} = \mathbf{BA} = \mathbf{I}_n$$

$\mathbf{B}$ is called the inverse of $\mathbf{A}$. Written as $\mathbf{A}^{-1}$.

$\mathbf{A}\mathbf{A}^{-1} = \mathbf{A}^{-1}\mathbf{A} = \mathbf{I}_n$ $\quad \mathbf{A}^{-1}$ does NOT mean $\frac{1}{\mathbf{A}}$.

If $|\mathbf{A}| = 0$, $\mathbf{A}$ does not have an inverse.

If $|\mathbf{A}| \neq 0$, $\mathbf{A}$ does have an inverse.

If $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $|\mathbf{A}| \neq 0$,

then $\mathbf{A}^{-1} = \frac{1}{|\mathbf{A}|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Example

$$\mathbf{A} = \begin{bmatrix} 3 & 4 \\ 2 & 6 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 3 & -6 \\ -6 & 12 \end{bmatrix}$$

Find $\mathbf{A}^{-1}$ and $\mathbf{B}^{-1}$

$$|\mathbf{A}| = 3 \times 6 - 2 \times 4 = 10$$

Then $\mathbf{A}^{-1} = \frac{1}{10}\begin{bmatrix} 6 & -4 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 0.6 & -0.4 \\ -0.2 & 0.3 \end{bmatrix}$

$$|\mathbf{B}| = 3 \times 12 - (-6) \times (-6) = 0$$

As $|\mathbf{B}| = 0$, so matrix $\mathbf{B}$ does not have an inverse.