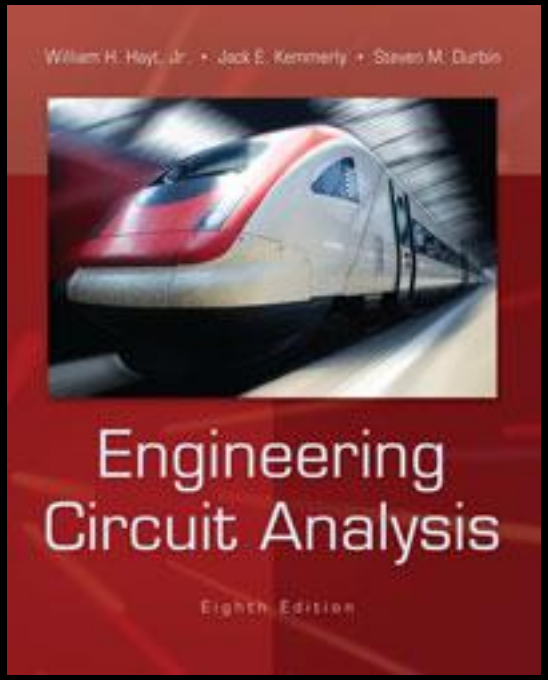
Chapter 4
Basic Nodal and Mesh Analysis

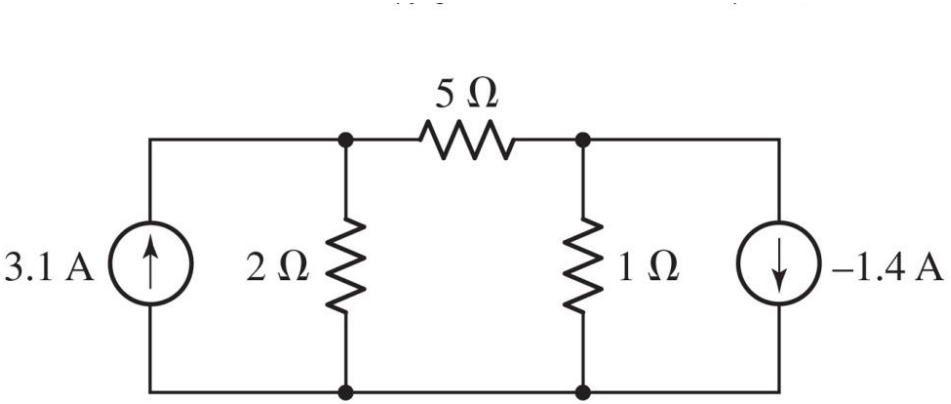


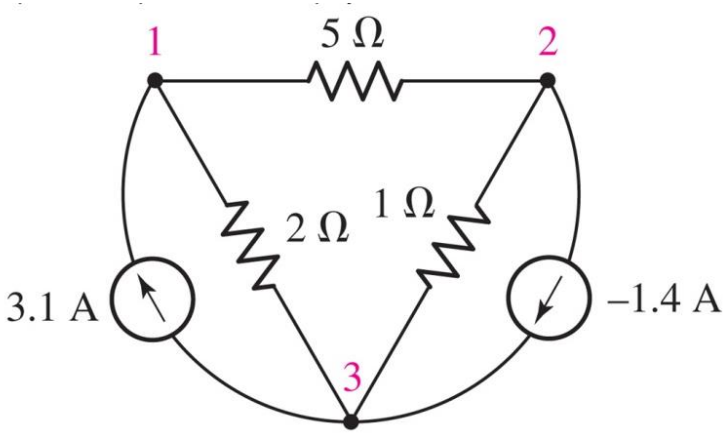
Circuit Analysis

- as circuits get more complicated, we need an organized method of applying KVL, KCL, and Ohm's
- *nodal* analysis assigns *voltages* to each node, and then we apply *KCL*
- *mesh* analysis assigns *currents* to each mesh, and then we apply *KVL*

The Nodal Analysis Method

- assign voltages to every node relative to a reference node

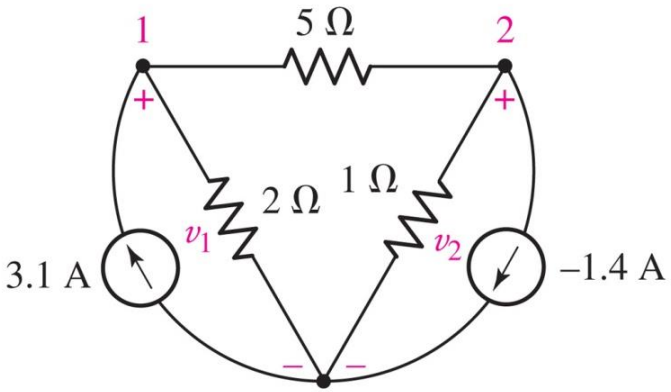




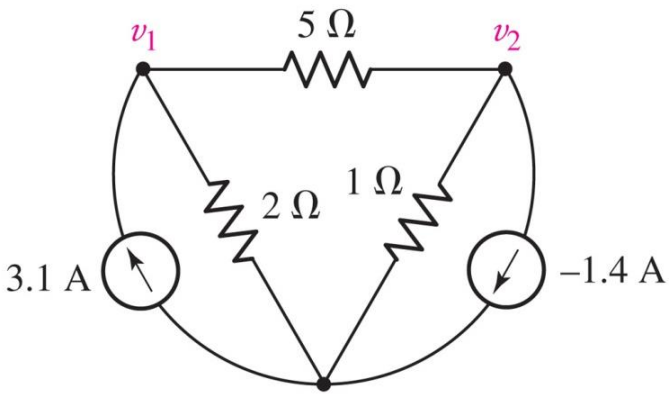
- in this example, there are three nodes

Choosing the Reference Node

- as the bottom node, or
- as the ground connection, if there is one, or
- a node with many connections



Reference node



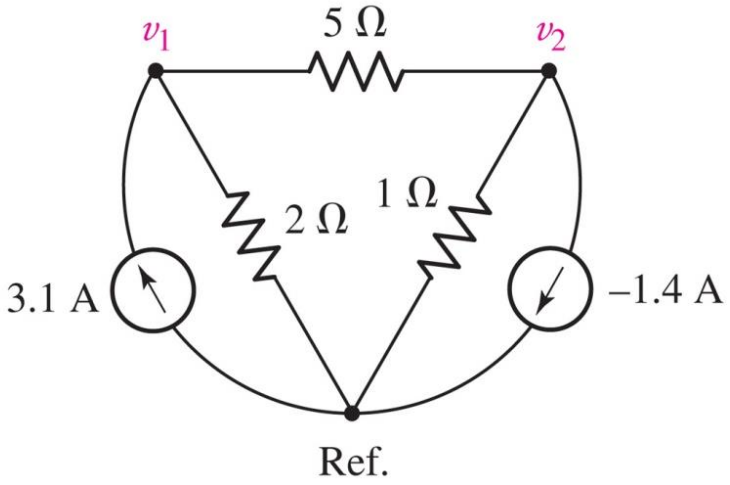
Ref.

- assign voltages relative to reference

Apply KCL to Find Voltages

Apply KCL to node 1 (Σ out = Σ in) and Ohm's law to each resistor:

$$\frac{v_1}{2} + \frac{v_1 - v_2}{5} = 3.1$$



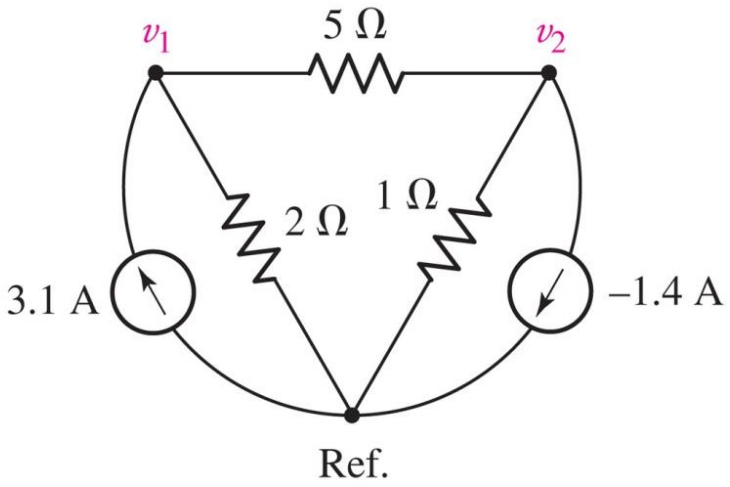
Note: the current flowing out of node 1 through the 5 Ω resistor is

$$v_1 - v_2 = i5$$

Apply KCL to Find Voltages

Apply KCL to node 2 (Σ out = Σ in) and Ohm's law to each resistor:

$$\frac{v_2}{1} + \frac{v_2 - v_1}{5} = -(-1.4)$$

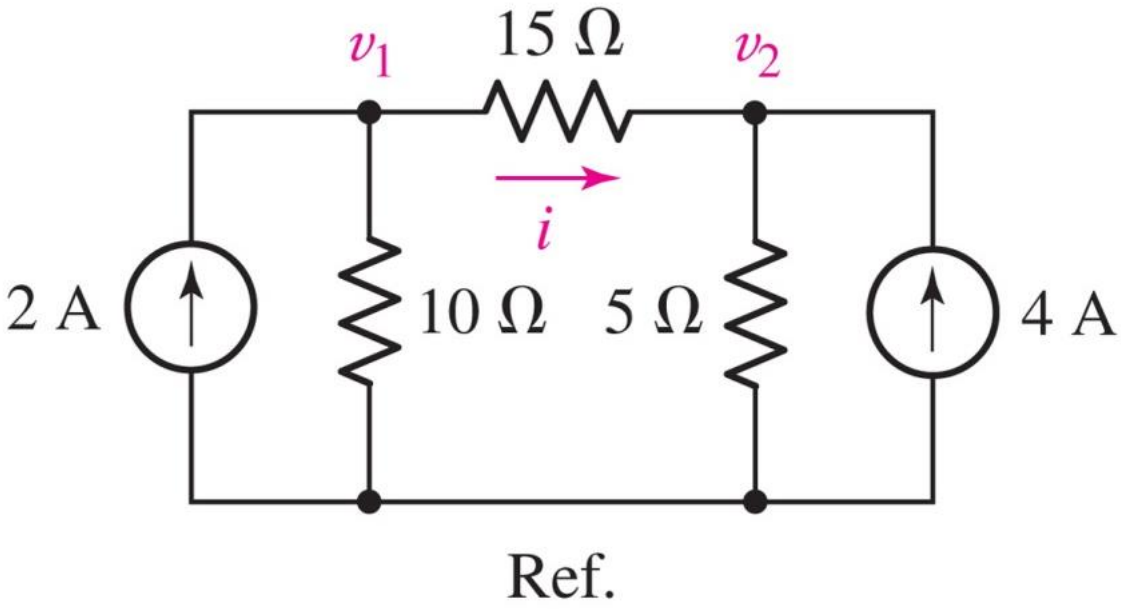


We now have two equations for the two unknowns v_1 and v_2 and can solve.

[v_1 = 5 V and v_2= 2V]

Example: Nodal Analysis

Find the current i in the circuit.

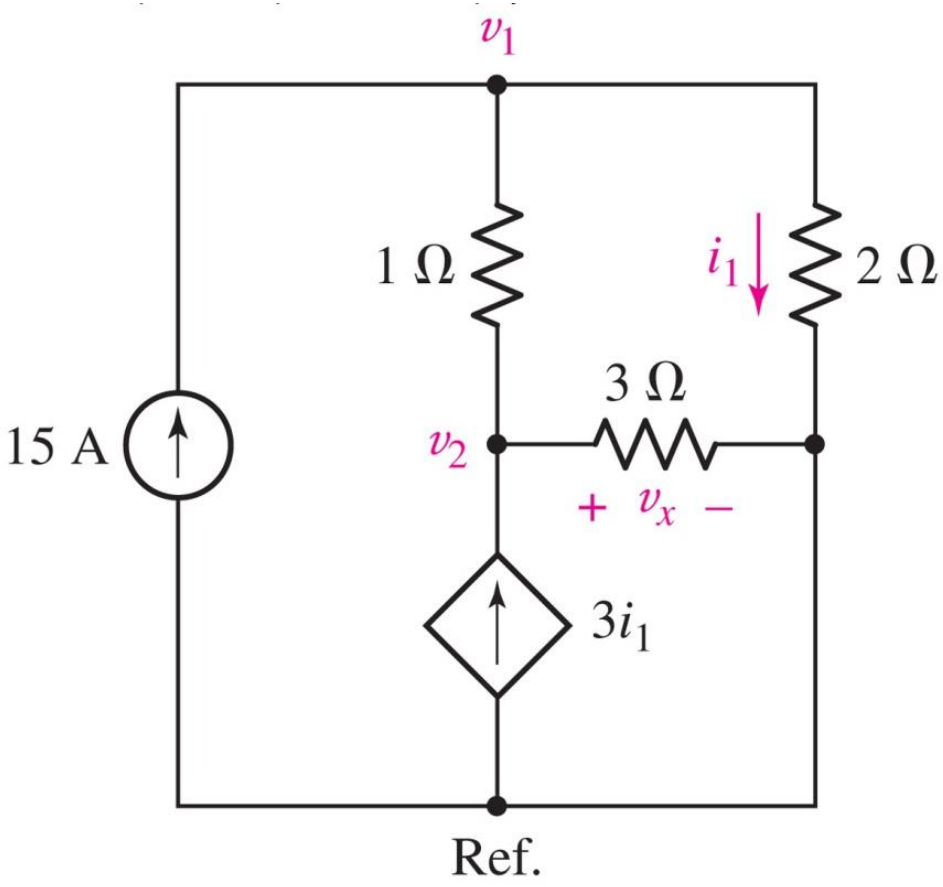


Answer: $i = 0$ (since $v_1 = v_2 = 20$ V)

Nodal Analysis: Dependent Source Example

Determine the power supplied by the dependent source.

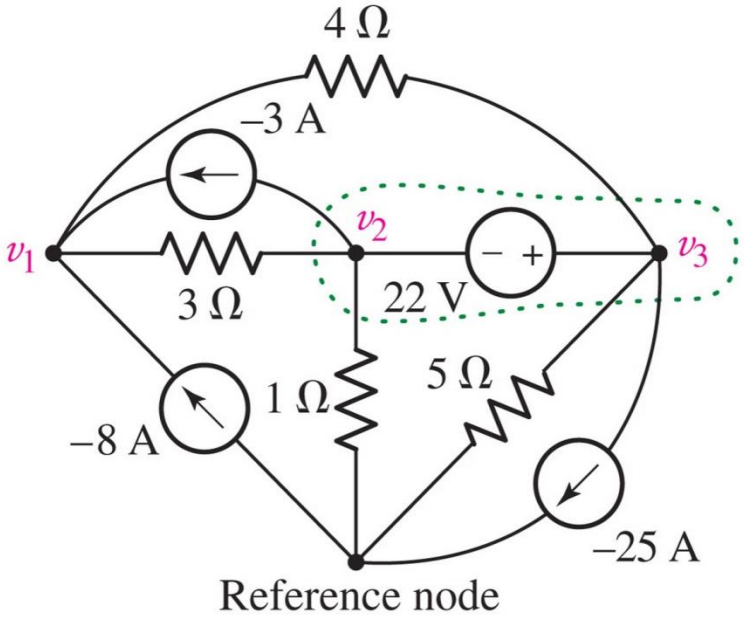
Key step: eliminate i_1 from the equations using $v_1 = 2i_1$



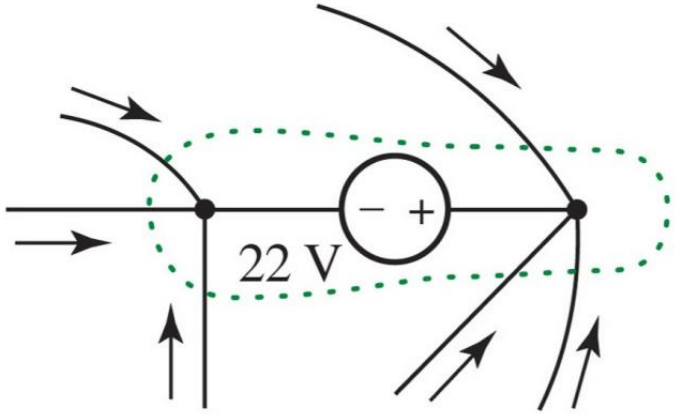
Answer: 4.5 kW

Voltage Sources and the Supernode

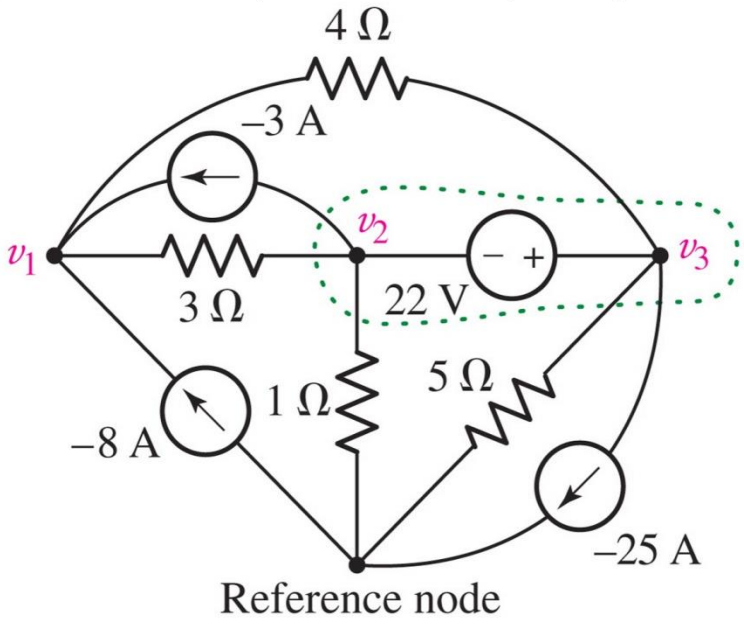
What is the current through a voltage source connected between nodes?



We can eliminate the need for introducing a current variable by applying KCL to the *supernode*.



The Supernode



- Apply KCL at Node 1.
- Apply KCL at the supernode.
- Add the equation for the voltage source inside the supernode.

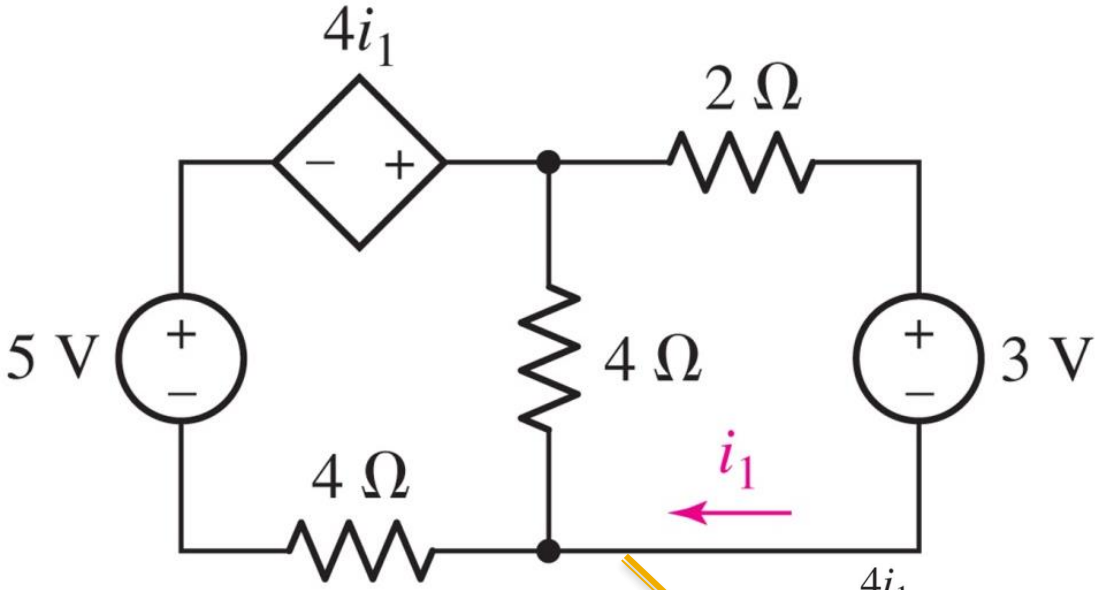
$$\frac{v_1 - v_3}{4} + \frac{v_1 - v_2}{3} = -3 - 8$$

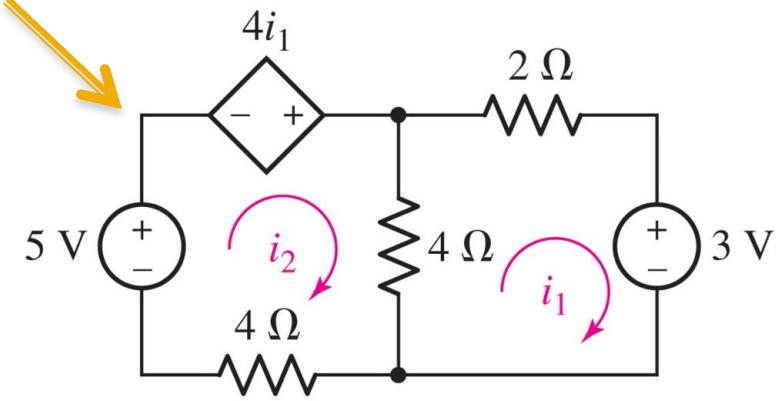
$$\frac{v_2}{1} + \frac{v_2 - v_1}{3} + \frac{v_3}{5} + \frac{v_3 - v_1}{4} = -(-25) - (-3)$$

$$v_3 - v_2 = 22$$

Dependent Source Example

Find i_1

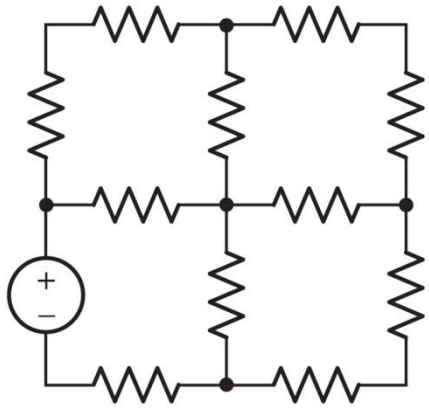




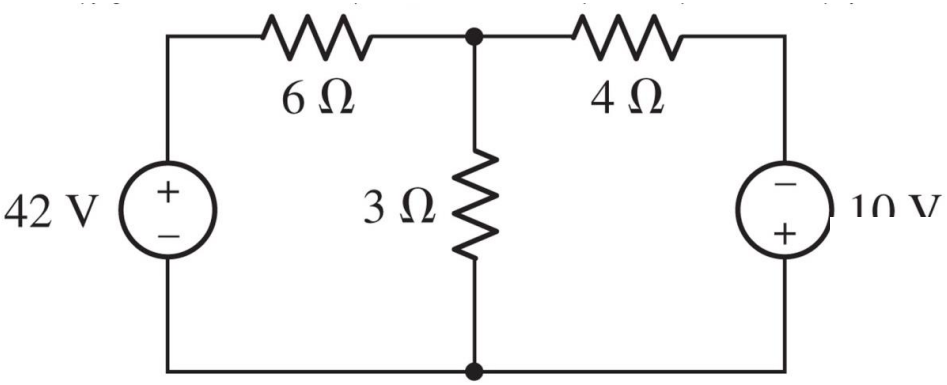
Answer: $i_1 =$ *- 250 mA.*

Mesh Analysis: Nodal Alternative

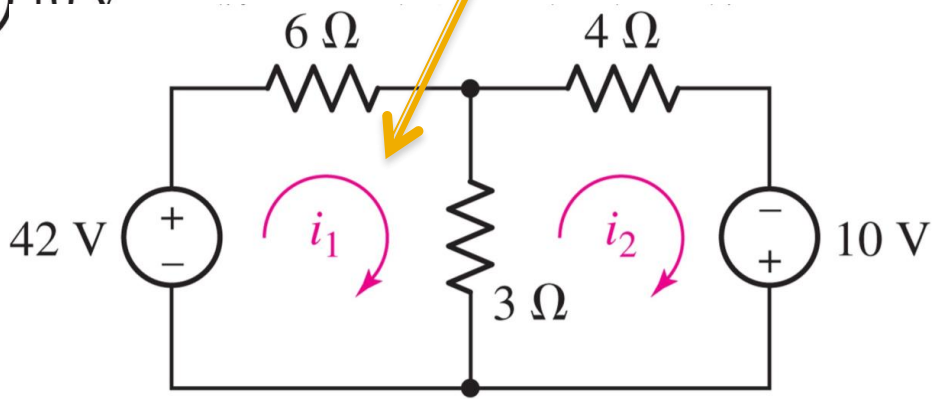
- a mesh is a loop which does not contain any other loops within it
- in mesh analysis, we assign currents and solve using KVL
- assigning mesh currents automatically ensures KCL is followed
- this circuit has four meshes:

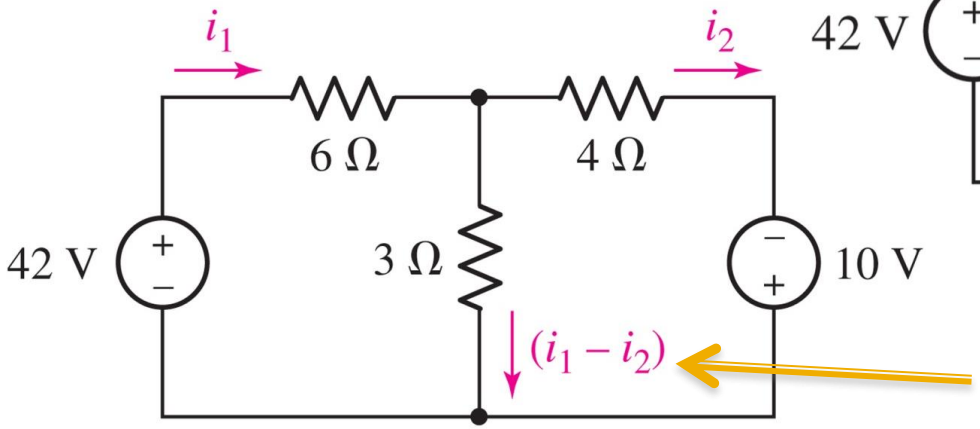


The Mesh Analysis Method



Mesh currents





Branch currents

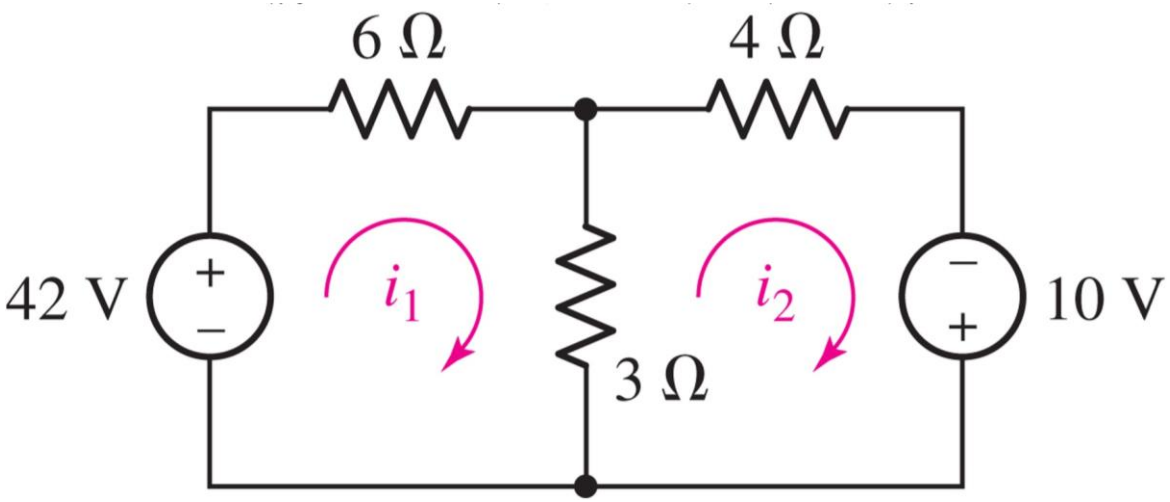
Mesh: Apply KVL

Apply KVL to mesh 1 (Σ drops=0):

$$-42 + 6i_1 + 3(i_1 - i_2) = 0$$

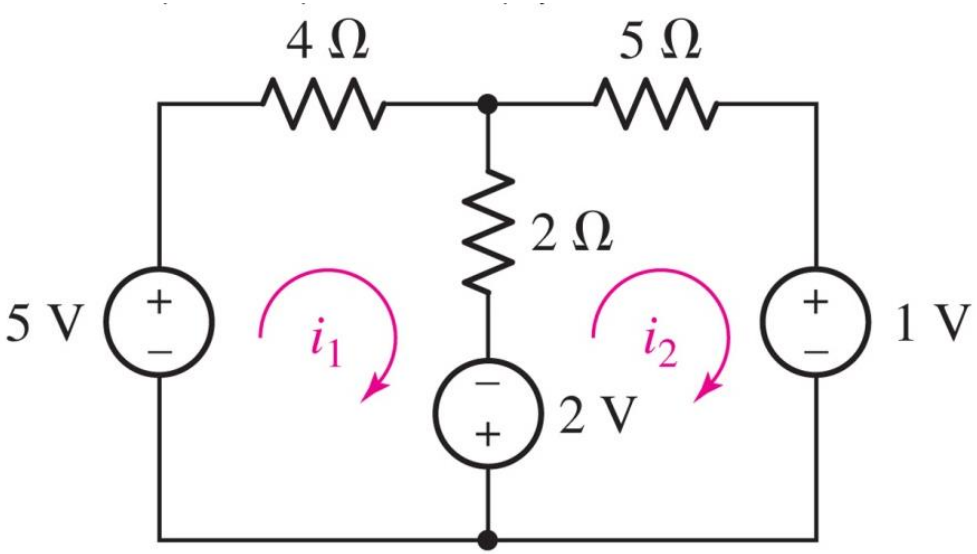
Apply KVL to mesh 2 (Σ drops=0):

$$3(i_2 - i_1) + 4i_2 - 10 = 0$$



Example: Mesh Analysis

Determine the power supplied by the 2 V source.



Applying KVL to the meshes:

$$-5 + 4i_1 + 2(i_1 - i_2) - 2 = 0$$

$$+2 + 2(i_2 - i_1) + 5i_2 + 1 = 0$$

Solve: i_1=1.132 A, $i_2 = -0.1053$ A.

Answer: 2.474 W

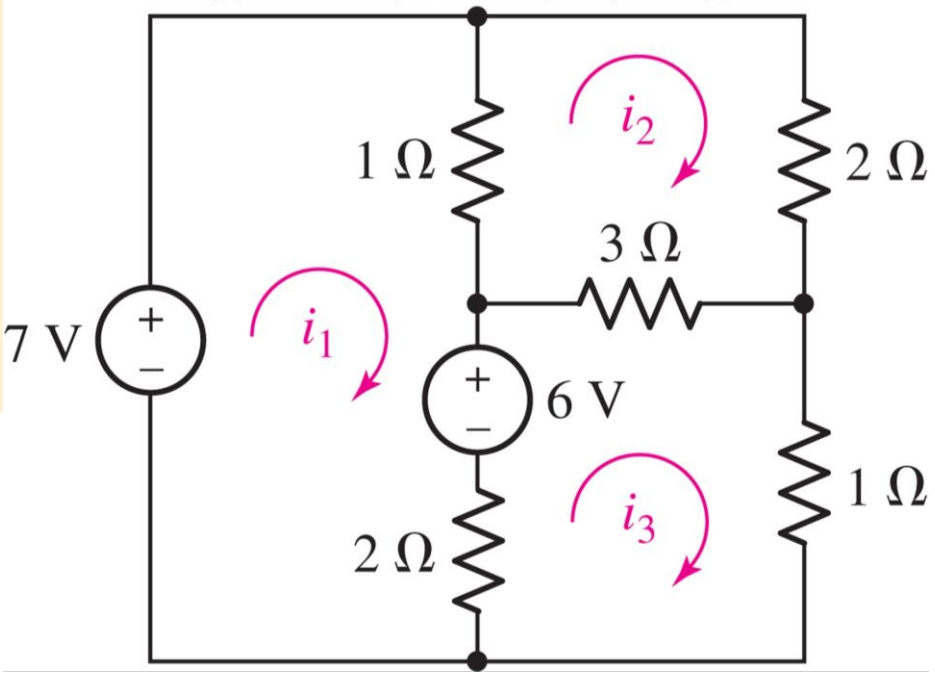
A Three Mesh Example

$$-7 + 1(i_1 - i_2) + 6 + 2(i_1 - i_3) = 0$$
$$1(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) = 0$$
$$2(i_3 - i_1) - 6 + 3(i_3 - i_2) + 1i_3 = 0$$

Follow each mesh clockwise

Simplify

$$3i_1 - i_2 - 2i_3 = 1$$
$$-i_1 + 6i_2 - 3i_3 = 0$$
$$-2i_1 - 3i_2 + 6i_3 = 6$$

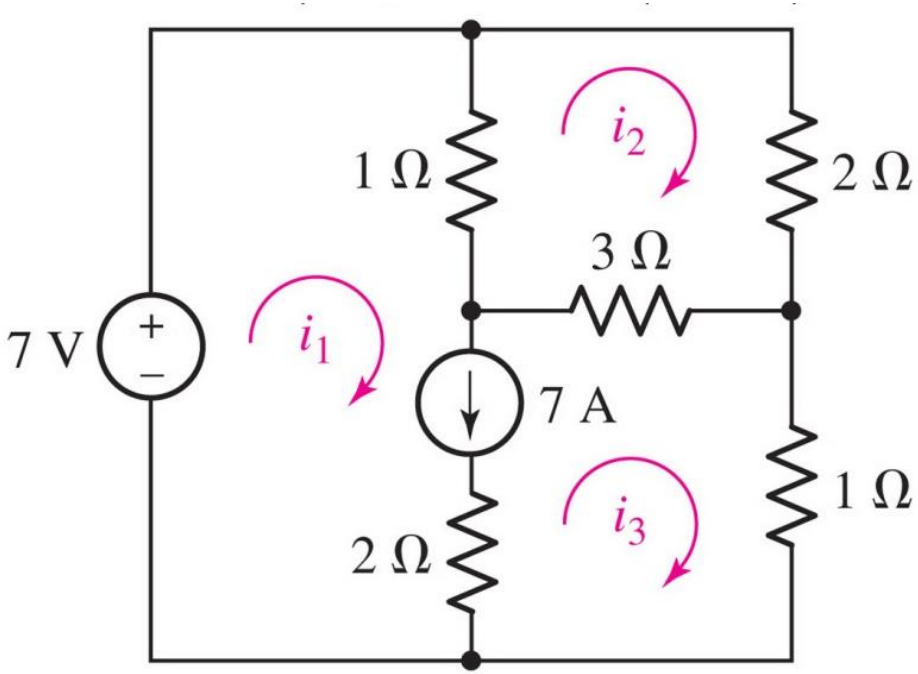


Solve the equations:

$i_1 = 3$ A, $i_2 = 2$ A, and $i_3 = 3$ A.

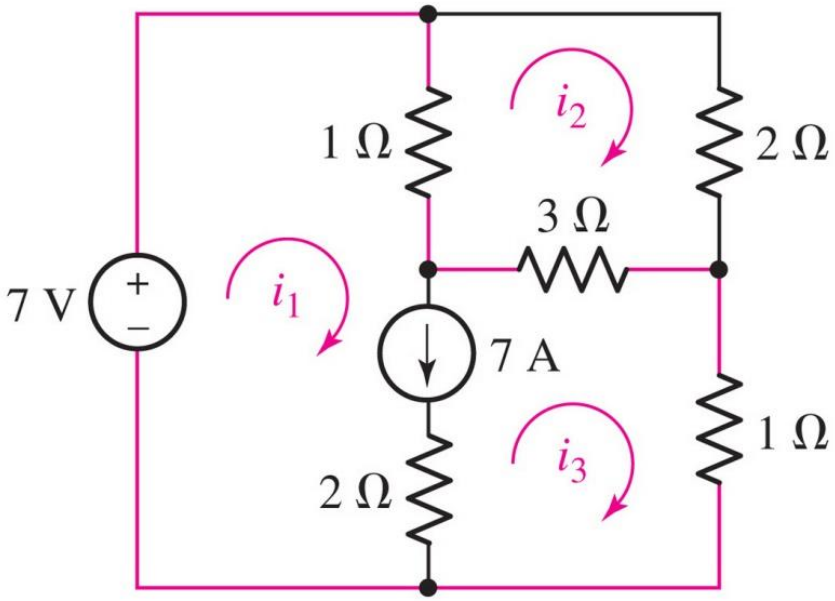
Current Sources and the Supermesh

What is the voltage across a current source in between two meshes?



We can eliminate the need for introducing a voltage variable by applying KVL to the *supermesh* formed by joining mesh 1 and mesh 3.

The Supermesh



Apply KVL to mesh 2:

$$1(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) = 0$$

Apply KVL supermesh 1/3:

$$-7 + 1(i_1 - i_2) + 3(i_3 - i_2) + 1i_3 = 0$$

Add the current source:

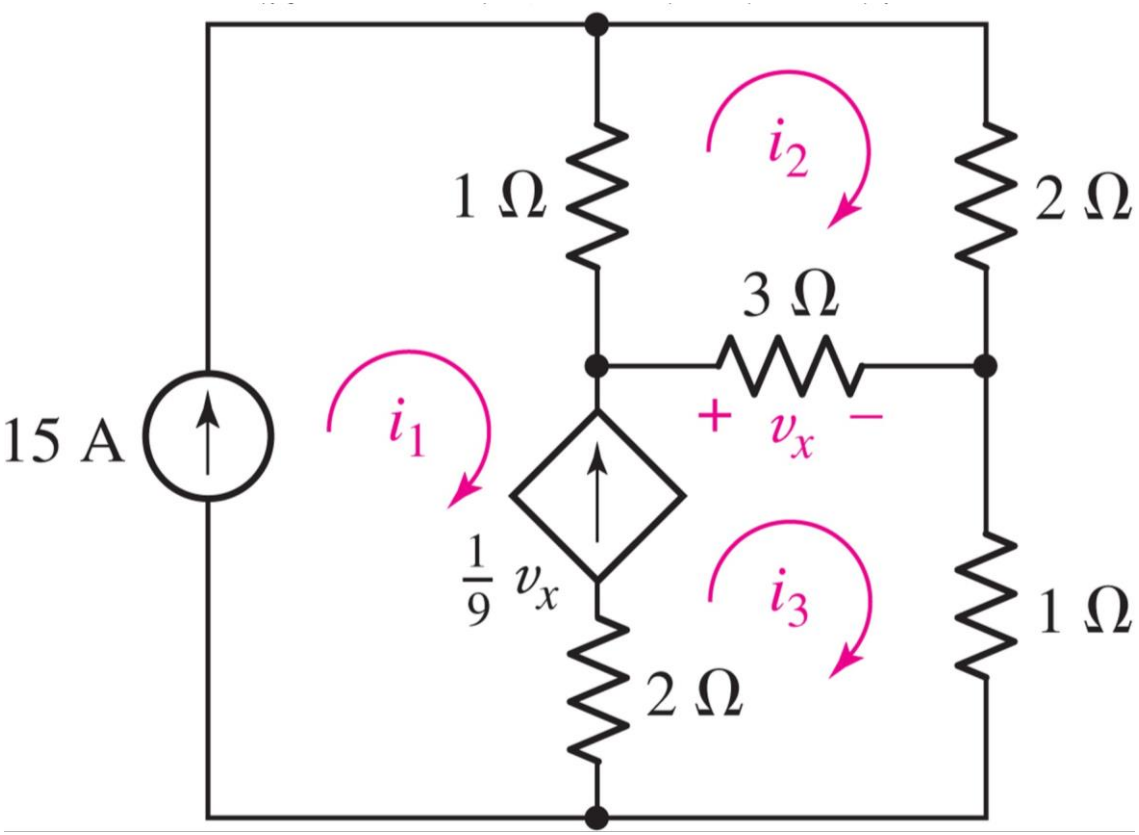
$$7 = i_1 - i_3$$

Dependent Source Example

Find the currents.

Key step:

$$\frac{v_x}{9} = i_3 - i_1$$



Answer: $i_1 = 15\ A$, $i_2 = 11\ A$, *and* $i_3 = 17\ A$

Node or Mesh: How to Choose?

- use the one with fewer equations, or
- use the method you like best, or
- use both (as a check), or
- use circuit simplifying methods from the next chapter