Chapter 5   Random Variables. Probability Distributions

5.4   Summary

- probability mass function
- probability density function
- distribution functions for discrete and continuous random variables
- Sketching of distribution functions

5.2 Discrete Random Variables: problem

Mr Ali hits a target with probability $p_A = \frac{1}{2}$,

Ms Beatrice hits with probability $p_B = \frac{1}{3}$.

If both shots once, find the range, pmf and cdf for the variable $X = \{number\ of\ hits\}$

5.2 Discrete Random Variables: solution (1)

There can only be: 0, 1 or 2 hits. So, $S = \{0, 1, 2\}$
$X = 0$ if there are two misses, so

$$P(X = 0) = (1 - p_A)(1 - p_B) = \frac{1}{2}\frac{2}{3} = \frac{1}{3}$$

$X = 1$ if Ali or Beatrice hit but not both, so

$$P(X = 1) = P(A \cup B) - P(A \cap B) = p_A + p_B - 2p_A p_B = \frac{1}{2}$$

X = 2 if both hit, so

$$P(X = 2) = P(A \cap B) = p_A p_B = \frac{1}{6}$$

5.2 Discrete Random Variables: problem

Mr Ali hits a target with probability $p_A = \frac{1}{2}$,

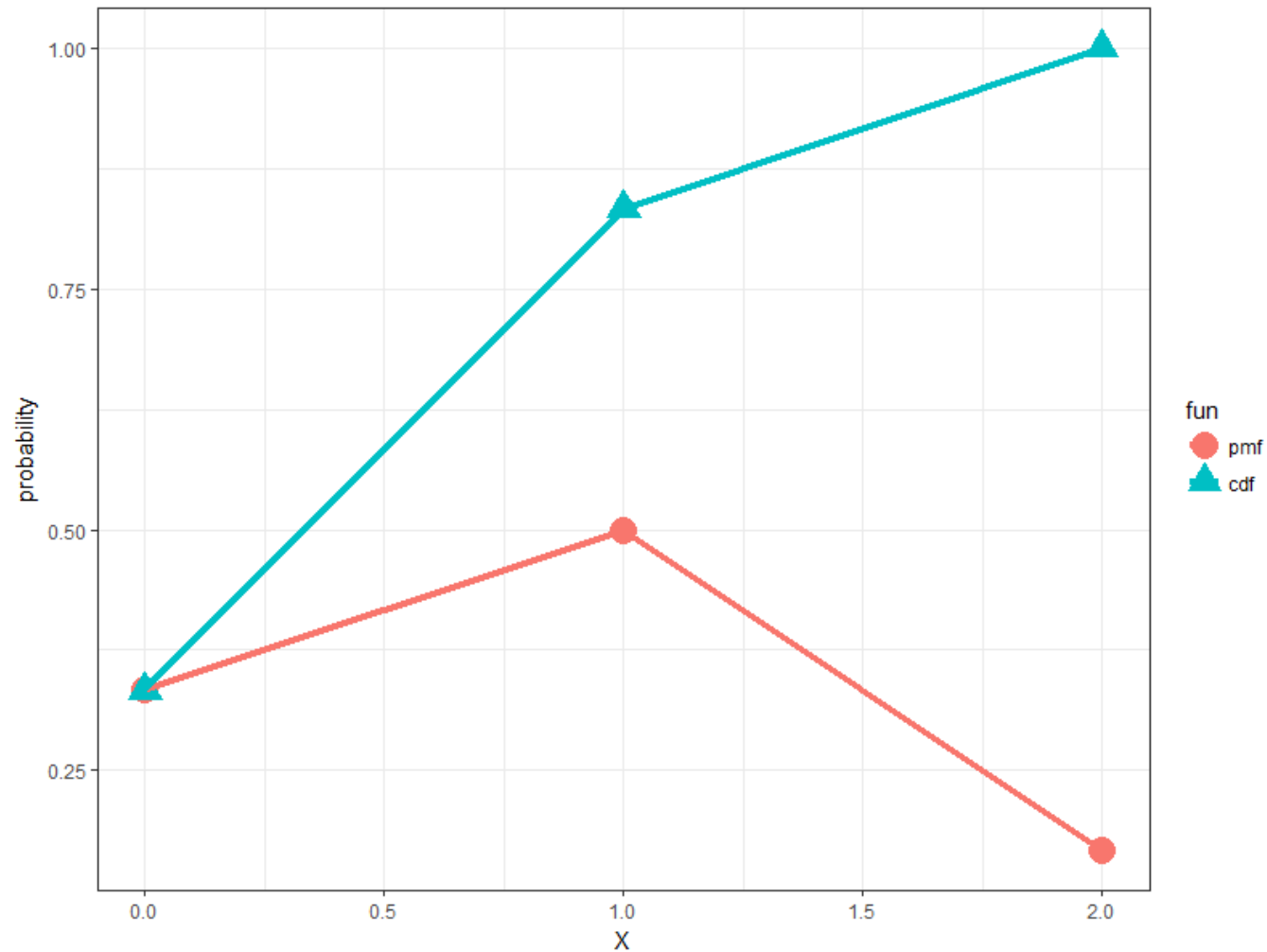Ms Beatrice hits with probability $p_B = \frac{1}{3}$.

If both shots once, find the range, pmf and cdf for the variable $X = \{number\ of\ hits\}$

5.2 Discrete Random Variables: solution (2)

pmf: $P(X = x) = \begin{cases} \frac{1}{3}, x = 0 \\ \frac{1}{2}, x = 1, \\ \frac{1}{6}, x = 2 \end{cases}$ and 0 *elsewhere*

pdf: $F(x) = P(X \leq x) = \begin{cases} \frac{1}{3}, x = 0 \\ \frac{5}{6}, x = 1, \\ 1, x = 2 \end{cases}$ need to add $F(x) = 0, x < 0, F(x) = 1, X > 2$
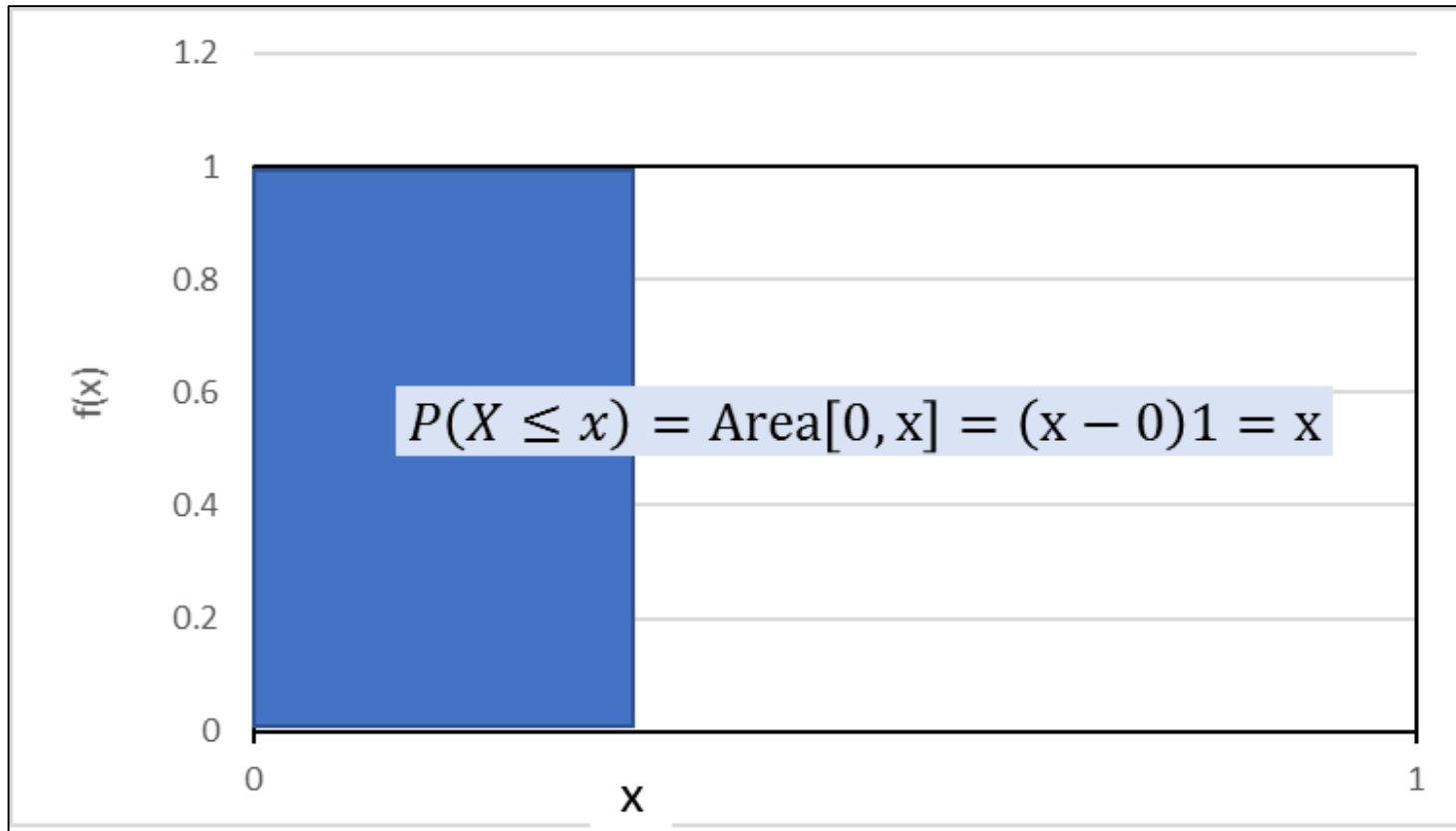
5.2 Discrete Random Variables: solution (3)

5.3   Continuous Random Variables and Distributions: problem

Plot the pdf

$$f(x) = \begin{cases} 1 \; if \; 0 < x \leq 1 \\ 0 \;\; otherwise \end{cases}$$

Prove that $P(X \leq x) = x$ by geometric arguments.

5.3   Continuous Random Variables and Distributions: Solution

$$P(X \leq x) = \text{Area}[0, x] = (x - 0)1 = x$$

The integral is the area under the pdf. Since it is a rectangle we can do it more easily as base*height!