EEE108 Electromagnetism and Electromechanics

Lecture 4

Gauss's Law Applications

Energy and Electric Potential

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Last Lecture

Electric Fields Produced by Continuous Charge Distributions: Examples

Electric Flux :

- In general, the electric flux through a surface S is

$$\Phi_E = \iint_S \mathbf{E} \bullet d\mathbf{s}$$

where $\mathbf{s}$ is the area vector.

Gauss' s Law :

- The electric flux through any closed Gaussian surface is proportional to the charge enclosed by the surface :

$$\Phi_E = \oint\!\!\oint_S \mathbf{E} \bullet d\mathbf{s} = \frac{Q_{enc}}{\varepsilon}$$

$$\oint\!\!\oint_S \mathbf{D} \bullet d\mathbf{s} = Q_{enc}$$

- Gauss's law can be used for a system that possesses planar, cylindrical or spherical symmetry.

Today

Gauss's Law
- Application (2)

Divergence Theorem/Gauss's Theorem

Energy and Electric Potential
- Work Done By Moving A Charge In An Electric Field
- Electric Potential: Definition; Potential Energy; …
- Point Charges

Gauss's Law Applications

Example 4: Cylindrical Symmetry

A coaxial transmission line consists of two concentric cylinders. This is a common type of structure used to guide EM waves from one point to another. The inner cylinder has a radius *a* and the outer cylinder has radius *b*.

Determine the electric field between the two cylinders.

Solution

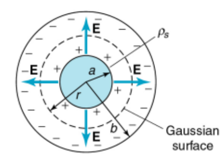
System: cylindrical symmetry

Gaussian surface : a coaxial cylinder : length l and radius $a < r < b$

Using Gauss's law :

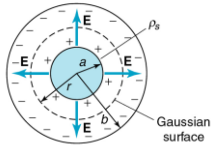
$$\oint\!\!\oint_S \mathbf{E} \bullet d\mathbf{s} = EA = E(2\pi r l) = \frac{\rho_l l}{\varepsilon_0} \Rightarrow E = \frac{\rho_l}{2\pi\varepsilon_0 r}$$

where $\rho_l = Q/L$ is the charge per unit length.



Gauss's Law Applications
Example 4: Cylindrical Symmetry

If net charge is on the surface of the inner cylinder,
then when $r < a, Q_{enc} = 0$, so $\mathbf{E} = 0$
When $r > b, Q_{enc} = \rho_l l - \rho_l l = 0$
$\rightarrow$ Gaussian surface encloses equal but opposite charges, so also : $\mathbf{E} = 0$



$$\mathbf{E} = \begin{cases} 0 & r < a \\ \dfrac{\rho_l}{2\pi\varepsilon_0 r}\mathbf{a}_r & a < r < b \\ 0 & r > b \end{cases}$$

A "shielded cable": the exterior is "shielded" from the fields inside the cable.

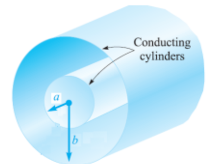
Gauss's Law Applications
Example 4: Cylindrical Symmetry

Let us select a 50-cm length of coaxial cable having an inner radius of 1 mm and an outer radius of 4 mm. The space between conductors is assumed to be filled with air. The total charge on the inner conductor is 30 nC. We wish to know the charge density on each conductor, and the **E** and **D** fields.

Solution

the surface charge density on the inner cylinder

$$\rho_{S,\text{inner cyl}} = \frac{Q_{\text{inner cyl}}}{2\pi a L} = \frac{30 \times 10^{-9}}{2\pi(10^{-3})(0.5)} = 9.55\ \mu\text{C/m}^2$$



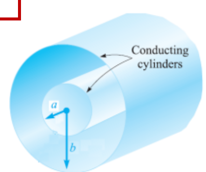
The negative charge density on the inner surface of the outer cylinder

$$\rho_{S,\text{outer cyl}} = \frac{Q_{\text{outer cyl}}}{2\pi b L} = \frac{-30 \times 10^{-9}}{2\pi(4 \times 10^{-3})(0.5)} = -2.39\ \mu\text{C/m}^2$$

Gauss's Law Applications
Example 4: Cylindrical Symmetry

Solution

$$\boxed{\mathbf{E} = \frac{\rho_l}{2\pi\varepsilon_0 r}\mathbf{a}_r}$$



The internal fields : where $1 < r < 4$ mm

$$D = \frac{a\rho_S}{r} = \frac{10^{-3}(9.55 \times 10^{-6})}{r} = \frac{9.55}{r}\ \text{nC/m}^2$$

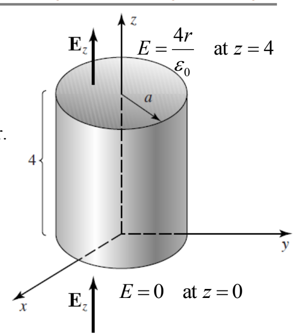
$$E = \frac{D}{\epsilon_0} = \frac{9.55 \times 10^{-9}}{8.854 \times 10^{-12} r} = \frac{1079}{r}\ \text{V/m}$$

For $r < 1$ mm or $r > 4$ mm, **E** and **D** are zero.

Gauss's Law Applications
Example 5: Cylindrical Symmetry

A cylindrical volume, $0 \le z \le 4$ m and $0 \le r \le 2$ m, encloses charge. If the electric field is $\mathbf{E} = \dfrac{zr}{\varepsilon_0}\mathbf{a}_z$

determine the total charge enclosed by the cylinder.



Solution

Since **E** is directed in the z direction, there is no flux through the sides. Hence Gauss' law gives

$$Q_{\text{enc}} = \varepsilon_o\Big(\underbrace{\int \mathbf{E} \bullet d\mathbf{s}}_{\text{top}} + \underbrace{\int \mathbf{E} \bullet d\mathbf{s}}_{\text{bottom}}\Big)$$

$$= \int_{\phi=0}^{2\pi}\int_{r=0}^{a}(z=4)r\underbrace{rdrd\phi}_{ds} - \int_{\phi=0}^{2\pi}\int_{r=0}^{a}(z=0)r\underbrace{rdrd\phi}_{ds} = \frac{8\pi a^3}{3} = \frac{64\pi}{3}\text{C}$$

Gauss's Law Applications

Example 6: Planar Symmetry

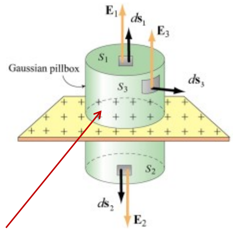
Infinite slab with uniform charge density ρ_S

Find $\mathbf{E}$ outside the plane

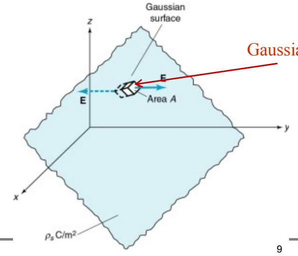
Solution

In the case, Symmetry is Planar

Gaussian Surface: Circular cylinder with faces parallel to the plane of charge。



Gaussian Pillbox.



A is arbitrary (its size and shape) and should divide out.

Gauss's Law Applications

Example 6: Planar Symmetry

Solution Cont.

Gaussian surface : $S_1 + S_2 + S_3$

Total charge enclosed $Q_{in} = \rho_S A$

The flux through the Gaussian surface :

$$\Phi_E = \oint\!\!\oint_S \mathbf{E} \bullet d\mathbf{s} = E \oint\!\!\oint_S ds = E(2A)$$

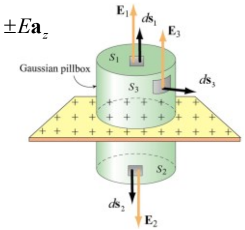
No flux through side of cylinder

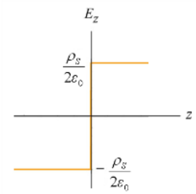
By using Gauss's Law :

$$\Phi_E = E(2A) = \frac{Q_{in}}{\varepsilon_0} = \frac{\rho_S A}{\varepsilon_0}$$

$$\text{Then} : E = \frac{\rho_S}{2\varepsilon_0} \Rightarrow \begin{cases} \mathbf{E} = \dfrac{\rho_S}{2\varepsilon_0}\mathbf{a}_z & z \ge 0 \\ \mathbf{E} = -\dfrac{\rho_S}{2\varepsilon_0}\mathbf{a}_z & z \le 0 \end{cases}$$

$\mathbf{E} = \pm E\mathbf{a}_z$





Gauss's Law Applications

Procedures Summary

Gauss's law provides a convenient tool for evaluating electric field. However, its application is limited only to systems that possess certain symmetry: cylindrical, planar and spherical symmetry.

The following steps may be useful when applying Gauss's law:

(1) Identify the symmetry associated with the charge distribution.
(2) Determine the direction of **E**-field (**D**-field), and a "Gaussian surface".
(3) Divide the space into different regions associated with the charge distribution.
(4) Calculate the electric flux Φ_E through the Gaussian surface for each region.
(5) Equate Φ_E with Q_{enc}/ε, and deduce the magnitude of the electric field.

Divergence Theorem/Gauss's Theorem

Operators -- Gradient Operator

Gradient **Divergence** **Laplacian**

$$\nabla = \mathbf{a}_x \frac{\partial}{\partial x} + \mathbf{a}_y \frac{\partial}{\partial y} + \mathbf{a}_z \frac{\partial}{\partial z} \quad \text{Cartesian}$$

$$\nabla = \mathbf{a}_r \frac{\partial}{\partial r} + \mathbf{a}_\phi \frac{1}{r}\frac{\partial}{\partial \phi} + \mathbf{a}_z \frac{\partial}{\partial z} \quad \text{Cylindrical}$$

$$\nabla = \mathbf{a}_R \frac{\partial}{\partial R} + \mathbf{a}_\theta \frac{1}{R}\frac{\partial}{\partial \theta} + \mathbf{a}_\phi \frac{1}{R\sin\theta}\frac{\partial}{\partial \phi} \quad \text{Spherical}$$

Example

$$\nabla T = \mathbf{a}_x \frac{\partial T}{\partial x} + \mathbf{a}_y \frac{\partial T}{\partial y} + \mathbf{a}_z \frac{\partial T}{\partial z}$$

T is a scalar, but ∇T (gradient of T) is a vector

Divergence Theorem/Gauss's Theorem
Operators -- Divergence Operator

$\boxed{\nabla \bullet}$

$$\boxed{\begin{aligned}\nabla &= \mathbf{a}_x \frac{\partial}{\partial x} + \mathbf{a}_y \frac{\partial}{\partial y} + \mathbf{a}_z \frac{\partial}{\partial z}\\ \nabla &= \mathbf{a}_r \frac{\partial}{\partial r} + \mathbf{a}_\phi \frac{1}{r}\frac{\partial}{\partial \phi} + \mathbf{a}_z \frac{\partial}{\partial z}\\ \nabla &= \mathbf{a}_R \frac{\partial}{\partial R} + \mathbf{a}_\theta \frac{1}{R}\frac{\partial}{\partial \theta} + \mathbf{a}_\phi \frac{1}{R\sin\theta}\frac{\partial}{\partial \phi}\end{aligned}}$$

If $\mathbf{A}$ is a vector

$$\nabla \bullet \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \quad \text{Cartesian}$$

$$\nabla \bullet \mathbf{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z} \quad \text{Cylindrical}$$

$$\nabla \bullet \mathbf{A} = \frac{1}{R^2}\frac{\partial}{\partial R}(R^2 A_R) + \frac{1}{R\sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{R\sin\theta}\frac{\partial A_\phi}{\partial \phi} \quad \text{Spherical}$$

$\mathbf{A}$ is a vector, $\nabla \bullet \mathbf{A}$ (divergence of $\mathbf{A}$) is a scalar

Divergence Theorem/Gauss's Theorem
Operators -- Laplacian Operator

Definition : The divergence $(\nabla \bullet)$ of the gradient (∇T) :

$\Delta T = \nabla^2 T = \nabla \bullet \nabla T$ $\quad \boxed{\nabla \bullet \nabla}$

Gradient Operator:
act on a scalar, result a vector
Divergence Operator:
act on a vector, result a scalar
Laplacian Operator:
act on a scalar, result a scalar

$$\Delta T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}$$

Cartesian

$$\Delta T = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2}$$

Cylindrical

$$\Delta T = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial T}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial \theta}\left(\frac{\partial T}{\partial \theta}\sin\theta\right) + \frac{1}{R^2\sin^2\theta}\frac{\partial^2 T}{\partial \phi^2} \quad \text{Spherical}$$

Divergence Theorem/Gauss's Theorem

Suppose V is a volume which is compact and has a piecewise smooth boundary.

If $\mathbf{F}$ is a continously differentiable vector field defined on a neighborhood of V, then :

$$\oint\!\!\oint_S \mathbf{F} \bullet d\mathbf{s} = \iiint_V (\nabla \bullet \mathbf{F}) dV$$

Where the right side is a volume integral over the volume V, the left side is the surface integral over the boundary of the volume V.

The integral of the normal component of ANY vector field over a closed surface is equal to the integral of the divergence of this vector field throughout the volume enclosed by the closed surface.

Divergence Theorem/Gauss's Theorem
Gauss's Law in Differential Form

$\varepsilon = \text{Constant}$

$$\begin{cases}\oint\!\!\oint_S \mathbf{E} \bullet d\mathbf{s} = \iiint_V (\nabla \bullet \mathbf{E}) dV\\ \oint\!\!\oint_S \mathbf{E} \bullet d\mathbf{s} = \dfrac{Q}{\varepsilon} = \iiint_V (\nabla \bullet \mathbf{E}) dV\end{cases} \Rightarrow \iiint_V (\nabla \bullet \mathbf{E}) dV = \iiint_V \frac{\rho_v}{\varepsilon} dV$$

$$\downarrow \qquad\qquad \Downarrow$$

$$Q = \iiint_V \rho_v dV \qquad \iiint_V \left(\nabla \bullet \mathbf{E} - \frac{\rho_v}{\varepsilon}\right) dV = 0$$

The electric field
$\updownarrow$
the charge density

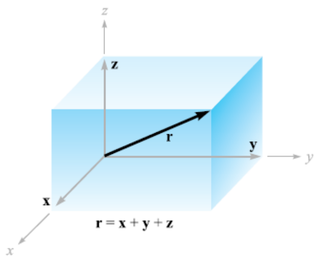
$\boxed{\nabla \bullet \mathbf{E} = \dfrac{\rho_v}{\varepsilon}}$ $\quad \boxed{\nabla \bullet \mathbf{D} = \rho_v}$

Gauss's law in differential form

Divergence Theorem/Gauss's Theorem

Example

Evaluate both sides of the divergence theorem for the field $\mathbf{D} = 2xy\mathbf{a}_x + x^2\mathbf{a}_y$ C/m^2 and the rectangular parellelepiped formed by the planes $x = 0$ and 1, $y = 0$ and 2, and $z = 0$ and 3.



$$\oiint_S \mathbf{E} \bullet d\mathbf{s} = \iiint_V (\nabla \bullet \mathbf{E}) dV$$

$\varepsilon = \text{Constant}$

$$\oiint_S \mathbf{D} \bullet d\mathbf{s} = \iiint_V (\nabla \bullet \mathbf{D}) dV$$

Divergence Theorem/Gauss's Theorem

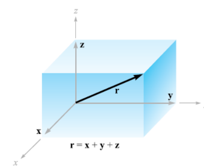
Example

Solution. Evaluating the surface integral first, we note that $\mathbf{D}$ is parallel to the surfaces at $z = 0$ and $z = 3$, so $\mathbf{D} \cdot d\mathbf{S} = 0$ there. For the remaining four surfaces we have

$$\boxed{\mathbf{D} = 2xy\mathbf{a}_x + x^2\mathbf{a}_y \text{ C/m}^2}$$

Surface integral

$$\begin{aligned}\oint_S \mathbf{D} \cdot d\mathbf{S} &= \int_0^3 \int_0^2 (\mathbf{D})_{x=0} \cdot (-dy\,dz\,\mathbf{a}_x) + \int_0^3 \int_0^2 (\mathbf{D})_{x=1} \cdot (dy\,dz\,\mathbf{a}_x) \\ &\quad + \int_0^3 \int_0^1 (\mathbf{D})_{y=0} \cdot (-dx\,dz\,\mathbf{a}_y) + \int_0^3 \int_0^1 (\mathbf{D})_{y=2} \cdot (dx\,dz\,\mathbf{a}_y) \\ &= -\int_0^3 \int_0^2 (D_x)_{x=0} dy\,dz + \int_0^3 \int_0^2 (D_x)_{x=1} dy\,dz \\ &\quad - \int_0^3 \int_0^1 (D_y)_{y=0} dx\,dz + \int_0^3 \int_0^1 (D_y)_{y=2} dx\,dz\end{aligned}$$



Divergence Theorem/Gauss's Theorem

Example

$$\boxed{\mathbf{D} = 2xy\mathbf{a}_x + x^2\mathbf{a}_y \text{ C/m}^2}$$

Surface integral

$$\begin{aligned}\oint_S \mathbf{D} \cdot d\mathbf{S} &= -\int_0^3 \int_0^2 (D_x)_{x=0} dy\,dz + \int_0^3 \int_0^2 (D_x)_{x=1} dy\,dz \\ &\quad - \int_0^3 \int_0^1 (D_y)_{y=0} dx\,dz + \int_0^3 \int_0^1 (D_y)_{y=2} dx\,dz\end{aligned}$$

$$(D_x)_{x=0} = 0$$

$$(D_y)_{y=0} = (D_y)_{y=2}$$

$$= \int_0^3 \int_0^2 (D_x)_{x=1} dy\,dz = \int_0^3 \int_0^2 2y\,dy\,dz \quad = \int_0^3 4\,dz = \underline{12}$$

Divergence Theorem/Gauss's Theorem

Example

$$\boxed{\mathbf{D} = 2xy\mathbf{a}_x + x^2\mathbf{a}_y \text{ C/m}^2}$$

Volume integral

$$\oiint_S \mathbf{D} \bullet d\mathbf{s} = \iiint_V (\nabla \bullet \mathbf{D}) dV$$

$$\nabla \cdot \mathbf{D} = \frac{\partial}{\partial x}(2xy) + \frac{\partial}{\partial y}(x^2) = 2y$$

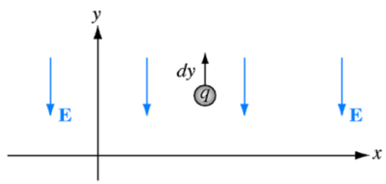
$$\int_{\text{vol}} \nabla \cdot \mathbf{D}\,dv = \int_0^3 \int_0^2 \int_0^1 2y\,dx\,dy\,dz = \int_0^3 \int_0^2 2y\,dy\,dz$$

$$= \int_0^3 4\,dz = \underline{12}$$

Work Done By Moving A Charge In An Electric Field

- A simple case : a positive charge q in a uniform electric field $\mathbf{E} = -\mathbf{a}_y E$
- The field $\mathbf{E}$ exerts a force $\mathbf{F}_e = q\mathbf{E}$ on the charge (negative y-direction)
- To move the charge along the positive y-direction, even without any acceleration, an external force $\mathbf{F}_{ext} = -\mathbf{F}_e = -q\mathbf{E}$ is needed.
- The work done by moving the charge a vector differential distance $d\mathbf{L}$ under the force $\mathbf{F}_{ext}$:

$$dW = \mathbf{F}_{ext} \bullet d\mathbf{L} = -q\mathbf{E} \bullet d\mathbf{L} = -q(-\mathbf{a}_y E) \bullet \mathbf{a}_y dy = qEdy$$



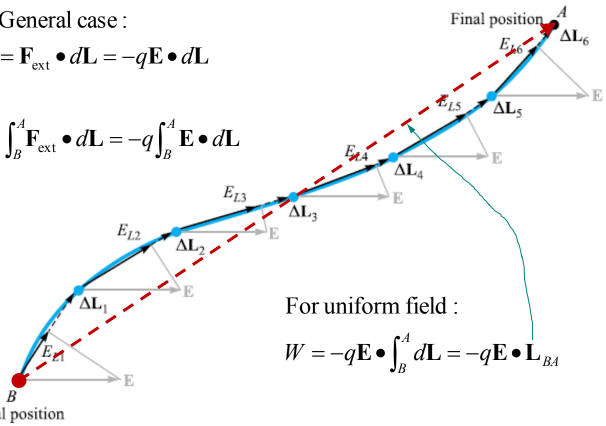
Work Done By Moving A Charge In An Electric Field

For General case :

$$dW = \mathbf{F}_{ext} \bullet d\mathbf{L} = -q\mathbf{E} \bullet d\mathbf{L}$$

and

$$W = \int_B^A \mathbf{F}_{ext} \bullet d\mathbf{L} = -q\int_B^A \mathbf{E} \bullet d\mathbf{L}$$



For uniform field :

$$W = -q\mathbf{E} \bullet \int_B^A d\mathbf{L} = -q\mathbf{E} \bullet \mathbf{L}_{BA}$$

Work Done By Moving A Charge In An Electric Field
Example 1

We are given the nonuniform field $\mathbf{E} = y\mathbf{a}_x + x\mathbf{a}_y + 2\mathbf{a}_z$ and we are asked to determine the work expended in carrying 2C from $B(1, 0, 1)$ to $A(0.8, 0.6, 1)$ along the shorter arc of the circle $x^2 + y^2 = 1 \quad z = 1$

Solution

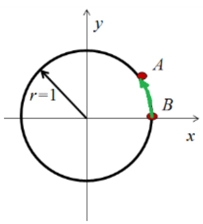
SI units

$W = -Q\int_B^A \mathbf{E} \cdot d\mathbf{L}$

$\mathbf{E}$ is not constant.

rectangular coordinates

$d\mathbf{L} = dx\mathbf{a}_x + dy\mathbf{a}_y + dz\mathbf{a}_z$



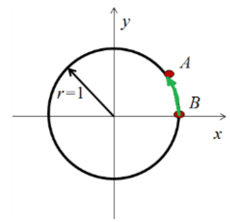
$$W = -Q\int_B^A \mathbf{E} \cdot d\mathbf{L}$$

$$= -2\int_B^A (y\mathbf{a}_x + x\mathbf{a}_y + 2\mathbf{a}_z) \cdot (dx\,\mathbf{a}_x + dy\,\mathbf{a}_y + dz\,\mathbf{a}_z)$$

$$= -2\int_1^{0.8} y\,dx - 2\int_0^{0.6} x\,dy - 4\cancelto{0}{\int_1^1 dz}$$

Work Done By Moving A Charge In An Electric Field
Example 1

Solution Cont.



$$W = -2\int_1^{0.8} y\,dx - 2\int_0^{0.6} x\,dy - 4\cancelto{0}{\int_1^1 dz}$$

$$= -2\int_1^{0.8} \sqrt{1-x^2}\,dx - 2\int_0^{0.6} \sqrt{1-y^2}\,dy$$

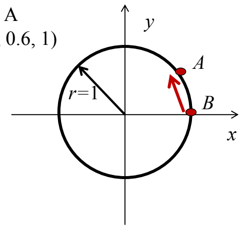
$$= -\left[x\sqrt{1-x^2} + \sin^{-1} x\right]_1^{0.8} - \left[y\sqrt{1-y^2} + \sin^{-1} y\right]_0^{0.6}$$

$$= -(0.48 + 0.927 - 0 - 1.571) - (0.48 + 0.644 - 0 - 0)$$

$$= \boxed{-0.96\,\text{J}}$$

Work Done By Moving A Charge In An Electric Field
Example 1

The work required to move 2C from point B to point A along the straight line path from B (1, 0, 1) to A (0.8, 0.6, 1)



The equation of the straight line path from B to A:

$$y = -3(x - 1)$$

$$W = -2\int_{1}^{0.8} y\,dx - 2\int_{0}^{0.6} x\,dy - 4\int_{1}^{1} dz$$

$$= 6\int_{1}^{0.8} (x-1)\,dx - 2\int_{0}^{0.6}\left(1 - \frac{y}{3}\right)dy$$

$$= \boxed{-0.96\text{ J}}$$

The work done is independent of the path taken in any electrostatic field.

Electrostatic field is a conservative field.

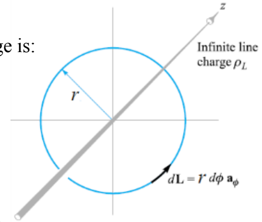
Work Done By Moving A Charge In An Electric Field
Example 2

The electric field is created by an infinite line charge.

Find the work required to move a positive charge q around a circular path of radius r centered at the line charge.

The electric field is created by an infinite line charge is:

$$\mathbf{E} = E_r\mathbf{a}_r = \frac{\rho_L}{2\pi\varepsilon_0 r}\mathbf{a}_r$$



The work done:

$$W = -q\int_0^{2\pi} \frac{\rho_L}{2\pi\varepsilon_0 r}\mathbf{a}_r \bullet d\mathbf{L} = -q\int_0^{2\pi} \frac{\rho_L}{2\pi\varepsilon_0 r}\mathbf{a}_r \bullet r\,d\varphi\,\mathbf{a}_\varphi$$

$$= -q\int_0^{2\pi} \frac{\rho_L}{2\pi\varepsilon_0} \overset{0}{\cancel{\mathbf{a}_r \bullet \mathbf{a}_\varphi}}\, d\varphi$$

Then: $W = 0$

Conservative Force and Potential Energy

- Generally, if a force $\mathbf{F}$ satisfies $\oint \mathbf{F} \bullet d\mathbf{l} = 0$, it is said to be *Conservative / Conservative force.*
- Potential energy ⇔ Conservative force
- The change in potential energy associated with a conservative force $\mathbf{F}$ acting on an object as it moves from A to B is defined :

$$\Delta U = U_B - U_A = -\int_A^B \mathbf{F} \bullet d\mathbf{l} = -W \qquad W \text{ is the work done by the force.}$$

> - Potential energy is energy stored within a physical system as a result of the position or configuration of the different parts of that system.
> - It has the potential to be converted into other forms of energy, such as kinetic energy, and to do work in the process.

Conservative Force and Potential Energy
Examples

- Gravitational potential energy:
  stored in a gravitational field
- Elastic potential energy:
  stored in a stretched spring
- Chemical potential energy:
  related to the structural arrangement of atoms or molecules
- Nuclear potential energy:
  is the potential energy of the particles inside an atomic nucleus
- ……

For electric force :

$$\oint \mathbf{F}_{ext} \bullet d\mathbf{L} = -q\oint \mathbf{E} \bullet d\mathbf{L} = 0$$

Electric force is conservative.

⇕

Potential energy : Electric Potential

Electric Potential and Voltage

- Electric potential (electrostatic potential)
- Denoted by φ, φ_E, or V
- A scalar quantity
- SI units : volts (V), kV

● **Voltage** is commonly used as a short name for **electrical potential difference**.

● Its corresponding SI units: volts (V) or kV

Definition of Electric Potential

- Potential difference between any two points P_2 and P_1

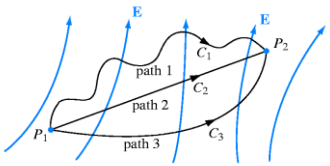
$$\varphi_{21} = \varphi_2 - \varphi_1 = -\int_{P_1}^{P_2} \mathbf{E} \bullet \mathrm{d}\mathbf{L}$$

$$W = \int_B^A \mathbf{F}_{\text{ext}} \bullet d\mathbf{L} = -q \int_B^A \mathbf{E} \bullet d\mathbf{L}$$

- The differential form

$$d\varphi = \frac{dW}{dq} = -\mathbf{E} \bullet \mathrm{d}\mathbf{L}$$

- Usually, assume $\varphi_1 = 0$ when P_1 is at infinity, then electric potential at any point P is



It is a scalar!

$$\boxed{\varphi = -\int_{\infty}^{P} \mathbf{E} \bullet \mathrm{d}\mathbf{L} \quad \text{(V)}}$$

The potential at a point is the work done in bringing a unit positive charge from the zero reference (infinity) to the point.

Electric Potential

$$\varphi = -\int_{\infty}^{P} \mathbf{E} \bullet \mathrm{d}\mathbf{L} \quad \text{(V)}$$

The potential of a system of charges has a value at any point which is independent of the path taken in carrying the test charge to that point.

Or

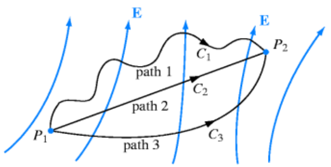
The intergration is independent of the path:

$$\oint_l \mathbf{E} \bullet \mathrm{d}\mathbf{L} = 0$$

$$\Downarrow$$

$$\varphi_{21} = \varphi_2 - \varphi_1 = -\int_{P_1}^{P_2} \mathbf{E} \bullet \mathrm{d}\mathbf{L}$$

The integral is independent of the path; superposition is applicable.



Electric Potential and Electric Energy

- The electric potential difference between points A and B is :

$$\Delta\varphi = -\int_B^A (\mathbf{F}_e / q_0) \bullet d\mathbf{l} = -\int_B^A \mathbf{E} \bullet d\mathbf{l}$$

The work done (by an external source) in moving a unit positive charge from point B to point A in an electric field.

- The electric energy is : $\Delta U = q_0 \, \Delta\varphi$

SI unit of electric potential: volt (V)

1 volt = 1 joule/coulomb (1 V = 1 J/C)

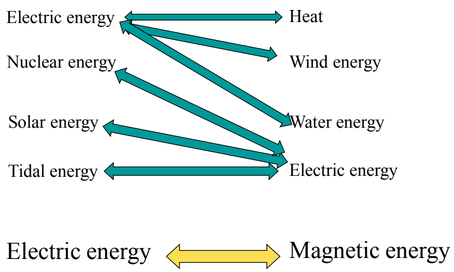
Electron volt (eV): $1\,\text{eV} = (1.6\times10^{-19}\,\text{C})(1\,\text{V}) = 1.6\times10^{-19}\,\text{J}$

eV : the energy an electron gains/loses when moving through a potential difference of one volt.

Conservation of Energy

Energy is the amount of work that can be performed by a force. It can also never be created or destroyed.

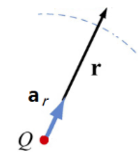
The only thing that can happen to energy in a closed/ isolated system is that it can change form. ---**The law of conservation of energy**



Electric Potential due to a Point Charge

- A point charge Q located at the origin of a coordinate system, creates the electric field at a distance r:

$$\mathbf{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\mathbf{a}_r$$



- The potential difference at distances r_A and r_B:

$$\varphi_{AB} = -\int_B^A \mathbf{E} \bullet d\mathbf{L} = -\int_{r_B}^{r_A} \frac{Q}{4\pi\varepsilon_0 r^2}\mathbf{a}_r \bullet \mathbf{a}_r dr = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_A} - \frac{1}{r_B}\right)$$

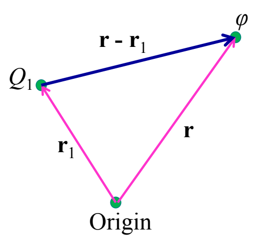
If distance r_B is infinity:

$$\varphi_{AB} = \boxed{\varphi = \frac{Q}{4\pi\varepsilon_0 r}}$$

Electric Potential due to a Point Charge

A point charge Q_1 located at $\mathbf{r}_1$, find the potential created by Q_1 at point $\mathbf{r}$:

$$\varphi(\mathbf{r}) = \frac{Q_1}{4\pi\varepsilon_0 |\mathbf{r} - \mathbf{r}_1|}$$

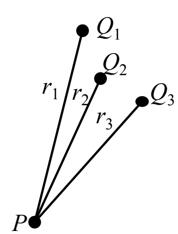


Electric Potential due to Point Charges

Superposition of Potential

- The total electric potential at a point is the ***algebraic sum*** of the individual potentials at the point.
- For example: for three point charges Q_1, Q_2, and Q_3, the total electric potential at the point P is:

$$\varphi_P = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1}{r_1} + \frac{Q_2}{r_2} + \frac{Q_3}{r_3}\right) \quad \text{(V)}$$



where r_1 = distance from Q_1 to P

r_2 = distance from Q_2 to P

r_3 = distance from Q_3 to P

Generally:

$$\boxed{\varphi_P = \frac{1}{4\pi\varepsilon_0}\sum_{n=1}^{N}\frac{Q_n}{r_n}} \quad \text{(V)}$$

It is much easier than the vector operation!

Electric Potential
Superposition – Zero reference

For a zero reference at infinity, then:

1. The potential arising from a single point charge is the work done in carrying a unit positive charge from infinity to the point at which we desire the potential, and the work is independent of the path chosen between those two points.

2. The potential field in the presence of a number of point charges is the sum of the individual potential fields arising from each charge.

EEE108 Homework 1

Homework 1 and the reference solutions can be found from ICE.

No submission, but it's better to do the homework independently!

Next

- Electric Potential due to Continuous Distributions
- Poisson's Equation
- Laplace Equation
- Electric Dipole

Thanks for your attendance