西交利物浦大学 Xi'an Jiaotong-Liverpool University

EEE108 Electromagnetism and Electromechanics

Lecture 13

Transformers

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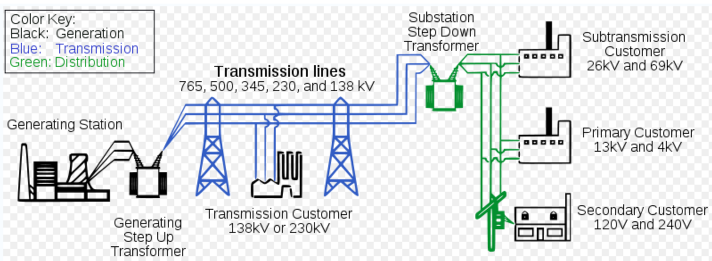
Introduction of Transformers

Electrical Engineering: transfers electric energy, changing the voltage level (or current level), through a magnetic field – **Power transformers/Electrical transformers** (In our study)

Low-power, Low-current Electronic and Control Circuit: matching the impedances of a source, isolating one circuit from another, …

Introduction of Transformers

A primary function of the transformer in the power system is to convert electrical energy at one voltage level to voltage at another level.



Electric power generated at 12 -- 25 kV

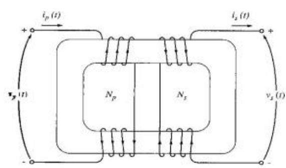
Step up to 110 kV -- 1,000 kV

Step down to 12 -- 34 kV
As low as 120 V

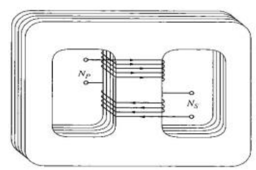
Introduction of Transformers

Two Types of Cores

Core form: transformer windings wrapped on two legs as shown
Shell form: transformer windings wrapped only on center leg



Core form



Shell form

In both cases, the core is constructed of thin laminations electrically isolated from each other in order to minimize **eddy currents**.

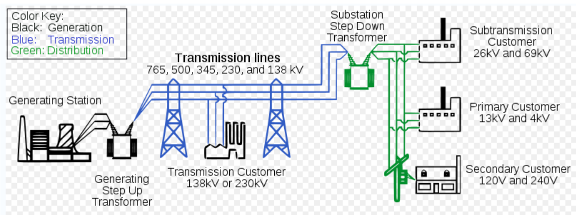
Introduction of Transformers

Different Names

Step up/Unit transformers – located at output of a generator to step up the voltage level to transmit the power

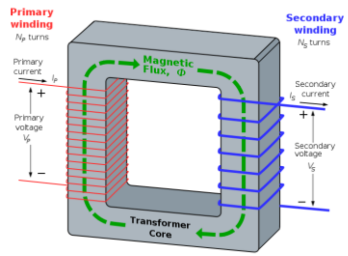
Step down/Substation transformers – located at main distribution or secondary level transmission substations to lower the voltage levels for distribution 1st level purposes

Distribution Transformers – located at small distribution substation. It lowers the voltage levels for 2nd level distribution purposes.

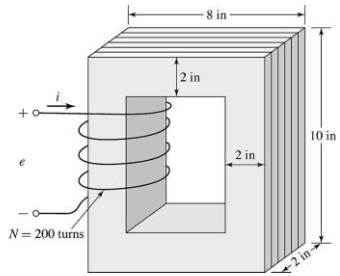


Introduction of Transformers

Principle



The core is constructed of thin laminations electrically isolated from each other in order to minimize **eddy currents**.

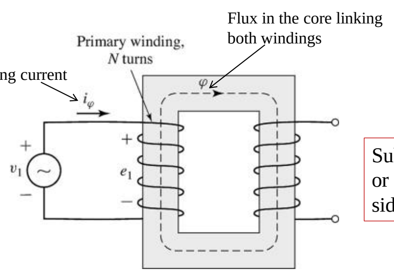


Introduction of Transformers

Principle

$$e_1 = -N_1 \frac{d\varphi}{dt}$$

Subscriptions of 1 or p: primary side



Subscriptions of 2 or s: secondary side

$$v_1 = R_1 i_\varphi + e_1$$

R_1: primary resistance

Changing the current in the primary coil changes the magnetic flux that is developed.

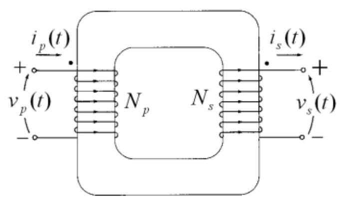
The changing magnetic flux induces a voltage in the secondary coil.

Ideal Transformer

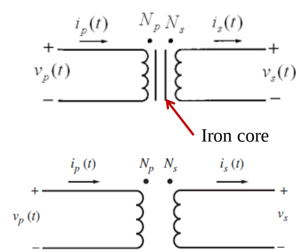
Definition

Definition – a lossless device with an input winding and an output winding:

- the windings have no resistance,
- loss-less magnetic core: no hysteresis or eddy currents,
- reluctance of the core is zero.



Sketch of an ideal transformer



Schematic symbol of a transformer

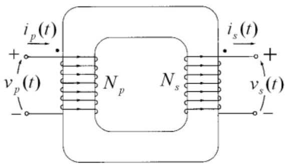
Ideal Transformer

Voltages and Currents

Magnitude of induced voltage

$$v_p(t) = N_p \frac{d\Phi(t)}{dt}, \quad v_s(t) = N_s \frac{d\Phi(t)}{dt} \;\Rightarrow\; \frac{v_p(t)}{v_s(t)} = \frac{N_p}{N_s} = a$$

where $a = \dfrac{N_p}{N_s}$



In terms of RMS quantities:

$$\frac{V_p}{V_s} = \frac{V_{p,\max}/\sqrt{2}}{V_{s,\max}/\sqrt{2}} = \frac{v_p(t)}{v_s(t)} = \frac{N_p}{N_s} = a$$

a (*n*): turns ratio,
voltage ratio,
ratio of transformation

As no loss, so

$$v_p(t) i_p(t) = v_s(t) i_s(t) \;\Rightarrow\; \frac{i_p(t)}{i_s(t)} = \frac{v_s(t)}{v_p(t)} = \frac{1}{a}$$

In terms of RMS quantities: $\dfrac{I_p}{I_s} = \dfrac{i_p(t)}{i_s(t)} = \dfrac{1}{a}$

Ideal Transformer

Power

The power supplied to the transformer by the primay side:

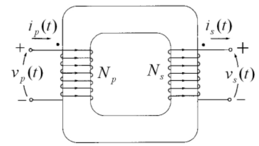
$P_{in} = V_p I_p \cos\theta_p$ θ_p is the angle between V_p and I_p

The power supplied by the second side to it's load:

$P_{out} = V_s I_s \cos\theta_s$ θ_s is the angle between V_s and I_s

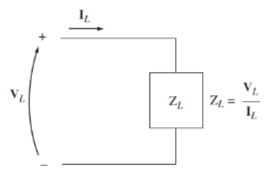
$$V_p I_p \cos\theta_p = V_s I_s \cos\theta_s \quad (1)$$
$$V_p / V_s = a \quad (2)$$
$$I_p / I_s = 1/a \quad (3)$$
$$\cos\theta_p = \cos\theta_s = \cos\theta$$

$$V_p I_p = V_s I_s \;\Rightarrow\; \boxed{P_{in} = V_p I_p \cos\theta = P_{out}}$$

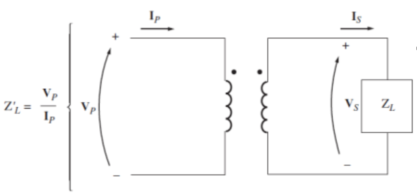


Impedance Transformation through a Transformer

The impedance of a device/element is defined as the ratio of the phasor voltage across it to the phasor current flowing through it: $Z_L = \dfrac{\mathbf{V}_L}{\mathbf{I}_L}$



Phasors are the complex representatives of sinusoidal voltages and currents.



The impedance of the load:

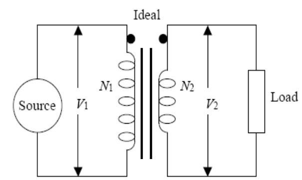
$$Z_L = \frac{\mathbf{V}_S}{\mathbf{I}_S}$$

The load impedance seen from primary side: $Z'_L = \dfrac{\mathbf{V}_P}{\mathbf{I}_P} = \dfrac{a\mathbf{V}_S}{\mathbf{I}_S / a} = a^2 \dfrac{\mathbf{V}_S}{\mathbf{I}_S} = a^2 Z_L$

Ideal Transformer

Example

Consider an ideal, 2400 V - 240 V transformer. The primary is connected to a 2200 V source and the secondary is connected to an impedance of $2\angle 36.9^0\ \Omega$



Find:

a) the secondary output current and voltage
b) the primary input current
c) the input and output reactive powers
d) the load impedance as seen from the primary side

Ideal Transformer

Example

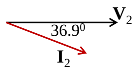
Solution

a) The secondary output current and voltage

$$a=\frac{2400}{240}=10 \Rightarrow V_2=V_1/a=2200/10=220 \quad \text{V}$$

Take V_2 as a reference phase: $\mathbf{V}_2=220\angle 0^0$ V

$$\mathbf{I}_2=\frac{\mathbf{V}_2}{Z_2}=\frac{220\angle 0^0}{2\angle 36.9^0}=110\angle(-36.9^0) \text{ A}$$



$$\boxed{\frac{\mathbf{V}_1}{\mathbf{V}_2}=\frac{N_1}{N_2}=a, \quad \frac{\mathbf{I}_1}{\mathbf{I}_2}=\frac{1}{a} \quad \text{where } \mathbf{V} \text{ and } \mathbf{I}: \text{phasor}}$$

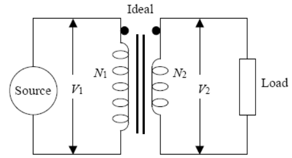
The turns ratio of the ideal transformer affects the magnitudes of the voltages and currents, but not their angles.

Ideal Transformer

Example

b) The primary input current

$$\mathbf{I}_1=\mathbf{I}_2/a=\frac{110\angle(-36.9^0)}{10}=11\angle(-36.9^0) \text{ A}$$



c) The input and output reactive powers

Input:

$Q_1=V_1I_1\sin\theta=2200\times 11\times\sin 36.9^0=14.53\,\text{kVAr}$

Output:

$Q_2=V_2I_2\sin\theta=220\times 110\times\sin 36.9^0=14.53\,\text{kVAr}$

Three kinds of power used by power engineers:

Real power: $P=VI\cos\theta$ Watts (W) due to resistors

Reactive power: $Q=VI\sin\theta$ Volt amperes reactive (Var) due to inductors and capacitors

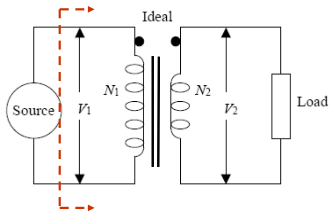
Apparent power: $S=VI$ Volt amperes (V A)

Ideal Transformer

Example

d) The load impedance as seen from the primary side

$$Z_{in}=\frac{\mathbf{V}_1}{\mathbf{I}_1}=\frac{2200\angle 0^0}{11\angle(-36.9^0)}$$
$$=200\angle 36.9^0 \ \Omega$$



The load impedance: $2\angle 36.9^0 \ \Omega$

$$Z_{load}=\frac{\mathbf{V}_2}{\mathbf{I}_2}=\frac{220\angle 0^0}{110\angle(-36.9^0)}=2\angle 36.9^0 \ \Omega$$

$$Z_{load}=\frac{Z_{in}}{a^2}=\frac{200\angle 36.9^0}{10^2}=2\angle 36.9^0 \ \Omega$$

$$\boxed{\begin{aligned}&\frac{V_P}{V_S}=\frac{v_s(t)}{v_p(t)}=\frac{N_P}{N_S}=a\\ &\frac{I_P}{I_S}=\frac{i_p(t)}{i_s(t)}=\frac{1}{a}\\ &P_{in}=P_{out}\\ &\frac{Z_P}{Z_S}=a^2\end{aligned}}$$

Ideal Transformer

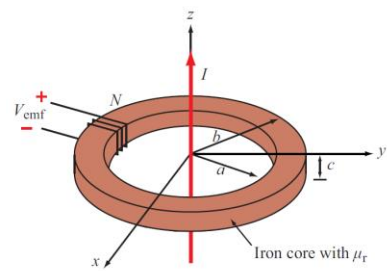
Example

Wire-toroid transformer

A transformer consists of an infinitely long wire on the z-axis and an iron toroid lying in the xy-plane. The wire carries a current $I = I_0 coswt$, and produces a magnetic field that couples to the toroid generating electromagnetic force V_{emf} across the terminals of the toroid coil.

(a) Derive an expression for V_{emf}.

(b) Calculate V_{emf} for f = 60Hz, μ_r = 4000, a = 5cm, b = 6cm, c = 2cm, I_0 = 50A, and number of coil turns N = 100.



Ideal Transformer

Example

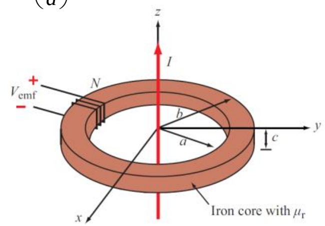
(a) Derive an expression for V_{emf}.

Start by considering the magnetic flux through the coil.

The radius of the toroid is $a \le r \le b$.

$$\Phi = \int_S \mathbf{B} \bullet d\mathbf{S} = \int_a^b \frac{\mu I}{2\pi r}\mathbf{a}_\varphi \bullet cdr\mathbf{a}_\varphi = \frac{\mu Ic}{2\pi}\int_a^b \frac{dr}{r} = \frac{\mu Ic}{2\pi}\ln\left(\frac{b}{a}\right) \quad \text{Wb}$$

$$V_{emf} = -N\frac{d\Phi}{dt} = -\frac{\mu cN}{2\pi}\ln\left(\frac{b}{a}\right)\frac{dI}{dt} = \frac{\mu cN\omega I_0}{2\pi}\ln\left(\frac{b}{a}\right)\sin\omega t \quad \text{volts}$$



Ideal Transformer

Example

(b) Calculate V_{emf} for the specific example:

Substitute in the values $f = 60$ Hz, $\mu_r = 4000$, $a = 5$ cm, $b = 6$ cm, $c = 2$ cm, $I_0 = 50$ A, and number of coil turns $N = 100$.

$$V_{emf} = \frac{\mu cN\omega I_0}{2\pi}\ln\left(\frac{b}{a}\right)\sin\omega t$$

$$= \frac{4000 \times 4\pi \times 10^{-7} \times 2 \times 10^{-2} \times 100 \times 2\pi \times 60 \times 50}{2\pi}\ln\left(\frac{6}{5}\right)\sin(2\pi \times 60)t$$

$$= 5.5\ \sin 377t \quad \text{volts}$$

Ideal Transformer

Summary

Definition – a lossless device with an input winding and an output winding:

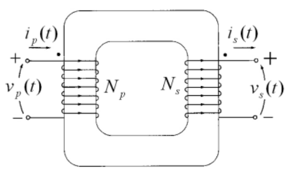
- the windings have no resistance,
- loss-less magnetic core: no hysteresis or eddy currents,
- reluctance of the core is zero.

$$\frac{V_p}{V_s} = \frac{v_p(t)}{v_s(t)} = \frac{N_p}{N_s} = a$$

$$\frac{I_p}{I_s} = \frac{i_p(t)}{i_s(t)} = \frac{1}{a}$$

$$V_p I_p = V_s I_s \qquad P_{in} = V_p I_p \cos\theta = P_{out}$$

$$\frac{Z_p}{Z_s} = a^2$$



Actual Transformers

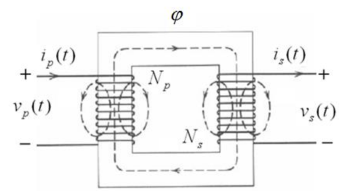
Ideal Transformers

Definition – a lossless device with an input winding and an output winding:

- the windings have no resistance,
- loss-less magnetic core: no hysteresis or eddy currents,
- reluctance of the core is zero.

Real Transformers

- Have resistance in the windings
- Not all of the flux produced by one winding links with the other (flux leakage)
  - Magnetic core has finite permeability
- Core losses
  - Hysteresis
  - Eddy currents



Actual Transformers

Losses

There are several losses in a real transformer, namely:

Copper (I²R) Losses – Resistive heating losses in the primary and secondary windings of the transformer.

Core Losses:

Eddy Current Losses – resistive heating losses in the core of the transformer. They are proportional to the square of the voltage applied to the transformer.

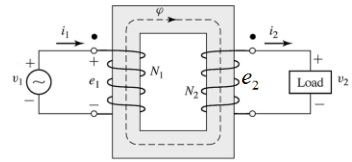
Hysteresis Losses – these are associated with the rearrangement of the magnetic domains in the core during each half-cycle. They are complex, non-linear function of the voltage applied to the transformer.

Leakage Flux – The fluxes in both primary and secondary sides which escape the core and pass through only one of the transformer windings are leakage fluxes.

Actual Transformers

Losses – Copper Losses

Copper (I²R) Losses – Resistive heating losses in the primary and secondary windings of the transformer.



$v_1 = R_1 i_1 + e_1$

$v_2 = e_2 - R_2 i_2$

$v_1 > e_1 \qquad i_1 \uparrow \quad v_1 - e_1 \uparrow$

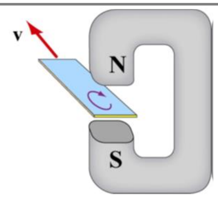
$i_2 \uparrow \Rightarrow v_2 \downarrow$

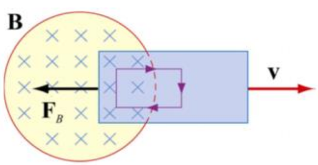
energy : $I_1^2 R_1 + I_2^2 R_2 \Rightarrow$ heat

Actual Transformers

Core Losses – Eddy Current Losses

Current can be induced in a moving solid conductor. The induced current appears to be circulating and is called an ***eddy current***.





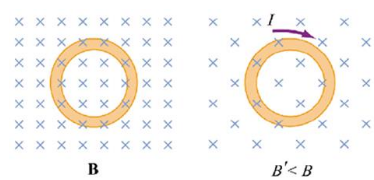
The induced eddy currents also generate a magnetic force that opposes the motion, making it more difficult to move the conductor across the magnetic field.

Joule Heating Loss $\propto I^2$

Actual Transformers

Core Losses – Eddy Current Losses

Eddy current loss in transformers is because of the eddy currents formed in the body of the magnetic core.

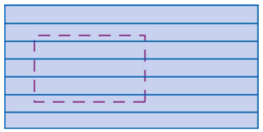


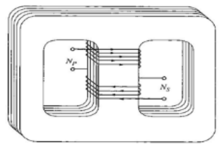
Whenever a conductor (iron core) exposed to a changing magnetic field a magnetic field produced in the body of the magnetic core. That induces a circulating current in it.

Joule Heating Loss $\propto I^2$

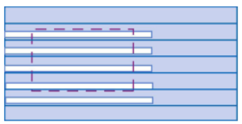
Actual Transformers
Core Losses – Eddy Current Losses

Due to the resistance within the conductor, Joule heating causes a loss of power. By increasing the value of resistance, power loss can be reduced.





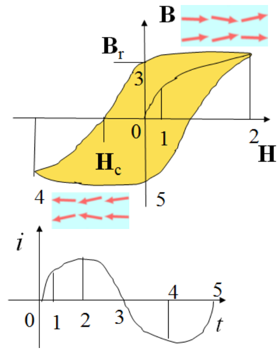
Laminate the conducting slab, or construct the slab by using gluing together thin strips insulated from one another.

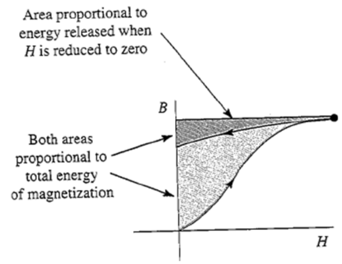


Make cuts in the slab to disrupt the conducting path.

Actual Transformers
Core Losses – Hysteresis Losses

Hysteresis Losses – these are associated with the rearrangement of the magnetic domains in the core during each half-cycle. They are complex, non-linear function of the voltage applied to the transformer.



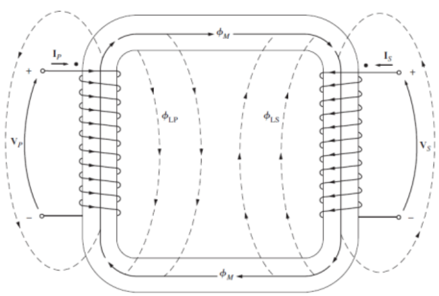


Core losses
= hysteresis losses + eddy current losses

Actual Transformers
Losses: Leakage Flux

The flux that links both windings: mutual flux

The leakage flux contributes only to the self-inductance of each coil.



Leakage flux is that which links one winding only and does not contribute to the energy transfer between windings.

Actual Transformers
Performance Characteristics: Voltage Regulation

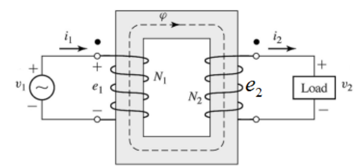
Two important performance characteristics of the transformer:
Voltage regulation, and ***efficiency*** -- to measure a real transformer's imperfections

Voltage regulation: The output voltage of a transformer varies with the load even if the input voltage remains constant. This is because a real transformer has series impedance within it. Full load Voltage Regulation is a quantity that compares the output voltage at no load with the output voltage at full load, defined by :

$$VR = \frac{V_{2,nl} - V_{2,fl}}{V_{2,fl}} \times 100\%$$

Usually it is a good practice to have as small a voltage regulation as possible.

Ideal transformer, VR = 0%.



Performance Characteristics: Efficiency

Efficiency: defined by : $\eta = \frac{P_{out}}{P_{in}} \times 100\% = \frac{P_{out}}{P_{out} + P_{loss}} \times 100\%$

There are three main types of losses incurred in a transformer:

- *Copper I^2R losses, P_{Cu}*
- *Hysteresis losses*
- *Eddy current losses*

$\left.\begin{matrix}\text{Hysteresis losses}\\ \text{Eddy current losses}\end{matrix}\right\} P_{core}$

Therefore, for a transformer, efficiency may be calculated using the following:

$$\eta = \frac{V_2 I_2 \cos\theta_2}{P_{Cu} + P_{core} + V_2 I_2 \cos\theta_2} \times 100\%$$

θ_2 : the angular between the voltage and the current on the secondary side.

$P_{Cu} = I_2^2 R_{eq}$: is a function of the load or the load current for constant voltage operation. It varies as the square of the load current.

P_{core} and V_2 and $\cos\theta_2$ are also assumed to be constant in computation of efficiency, then the load current I_2 is the only variable in the expression of efficiency.

Performance Characteristics: Efficiency

From $\eta = \frac{V_2 I_2 \cos\theta_2}{I_2^2 R_{eq} + P_{core} + V_2 I_2 \cos\theta_2} \times 100\%$

$$\frac{d\eta}{dI_2} = \frac{\left(I_2^2 R_{eq} + P_{core} + V_2 I_2 \cos\theta_2\right) V_2 \cos\theta_2 - V_2 I_2 \cos\theta_2 \left(2 I_2 R_{eq} + V_2 \cos\theta_2\right)}{\left(I_2^2 R_{eq} + P_{core} + V_2 I_2 \cos\theta_2\right)^2}$$

Set the derivative to zero to obtain the condition for maximum efficiency :

$$V_2 \cos\theta_2 + \frac{P_{core}}{I_2} + I_2 R_{eq} - V_2 \cos\theta_2 - 2 I_2 R_{eq} = 0 \;\Rightarrow\; I_2 R_{eq} = \frac{P_{core}}{I_2}$$

or $I_2^2 R_{eq} = P_{core}$ or $I_2 = \sqrt{P_{core}/R_{eq}}$

The efficiency of a transformer is maximum at a load for which the copper loss is equal to the core loss.

The efficiency of the power-system transformers at rated load: **90-99%**

Summary -- Transformer

- A transformer converts AC power at one voltage level to AC power of the same frequency at another voltage level.
- Operation Principles: Faraday's induction law
- Ideal transformers: a lossless device with an input winding and an output winding:
  - the windings have no resistance,
  - loss-less magnetic core,
  - reluctance of the core is zero.

$$\frac{V_p}{V_s} = a \qquad \frac{I_p}{I_s} = \frac{1}{a} \qquad \frac{Z_p}{Z_s} = a^2$$

$$P_{in} = P_{out}$$

- Real Transformers
  - Copper (I^2R) Losses
  - Core Losses:
    - Eddy Current Losses
    - Hysteresis Losses
  - Leakage Flux
- Two important performance characteristics:

  Voltage regulation:

  $$VR = \frac{V_{2,nl} - V_{2,fl}}{V_{2,fl}} \times 100\%$$

  Efficiency

  $$\eta = \frac{P_{out}}{P_{in}} \times 100\% = \frac{P_{out}}{P_{out} + P_{loss}} \times 100\%$$

- The maximum efficiency: when the copper loss is equal to the core loss: $I_2^2 R_{eq} = P_{core}$

Next Lecture

Magnetic Circuits

Thanks for your attendance