Tutorial 3 Green's theorem and surface

1. Line integrals: evaluation by Green's theorem (page 438).
   Evaluate $\int_C \mathbf{F}(\mathbf{r})\bullet d\mathbf{r}$ counterclockwise around the boundary C of the region R by Green's theorem.
   (1) $\mathbf{F} = \langle y, -x \rangle$, C is the circle $x^2 + y^2 = \frac{1}{4}$.
   (2) $\mathbf{F} = \langle 6y^2, 2x - 2y^4 \rangle$, R is the square with vertices $\pm(2,2), \pm(2,-2)$.
   (3) $\mathbf{F} = \langle x^2 e^y, \mathrm{y}^2 e^x \rangle$, R is the rectangle with vertices (0,0), (2,0), (2,3), (0,3).
   (4) $\mathbf{F} = \langle x^2 + y^2, x^2 - y^2 \rangle$, R: $1 \le y \le 2 - x^2$
   (5) $\mathbf{F} = \langle -e^{-x}\cos y, -e^{-x}\sin y \rangle$, R is the semidisk $x^2 + y^2 \le 16, x \ge 0$.
2. Parametric surface representation (page 442). Familiarize yourself with parametric representations of important surfaces by deriving a representation as $z = f(x, y)$ or $g(x, y, z) = 0$, then find a normal vector $\boldsymbol{N} = \boldsymbol{r}_u \times \boldsymbol{r}_v$ of the surface. Show the details of your work.
   (1) xy-plane $\boldsymbol{r}(u, v) = (u, v)$= $u\mathbf{i} + \mathrm{v}\mathbf{j}$.
   (2) xy-plane in polar coordinates $\mathbf{r}(u,v) = [u\cos v, u\sin v]$ (thus $u = r, v = \theta$).
   (3) Cone $\mathbf{r}(u,v) = [u\cos v, u\sin v, \mathrm{cu}]$.
   (4) Elliptic cylinder $\mathbf{r}(u,v) = [a\cos v, b\sin v, u]$.
   (5) Paraboloid of revolution $\mathbf{r}(u,v) = [u\cos v, u\sin v, u^2]$.
   (6) Hyperbolic paraboloid $\mathbf{r}(u,v) = [au\cosh v, bu\sinh v, u^2]$