

MTH101: Tutorial 12

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Exercise 1.1

Find a general solution to the following Bessel's equation in terms of J_ν , Y_ν . Indicate whether you could use $J_{-\nu}$ instead of Y_ν . Use the indicated substitution.

$$1. \quad x^2 y'' + \left(\frac{3}{16} + x \right) y = 0, \quad (y = 2u\sqrt{x}, \sqrt{x} = z).$$

$$2. \quad xy'' + 5y' + xy = 0, \quad (y = u/x^2).$$

$$3. \quad y'' + xy = 0, \quad (y = u\sqrt{x}, z = \frac{2}{3}x^{\frac{3}{2}}).$$

Exercise 1.2

Derive the Bessel's equation

$$x^2 y'' + xy' + (x^2 - \nu^2)y = 0,$$

by the following equations

$$(a) [x^\nu J_\nu(x)]' = x^\nu J_{\nu-1}(x),$$

$$(b) [x^{-\nu} J_\nu(x)]' = -x^{-\nu} J_{\nu+1}(x),$$

$$(c) J_{\nu-1}(x) + J_{\nu+1}(x) = \frac{2\nu}{x} J_\nu(x),$$

$$(d) J_{\nu-1}(x) - J_{\nu+1}(x) = 2J'_\nu(x).$$