EEE213 Power Electronics and Electromechanism

1a. Review of Three-phase system

Rectification
Controllability
Uncontrolled and Controlled
vs
Input power (source)
Single phase and Three phase

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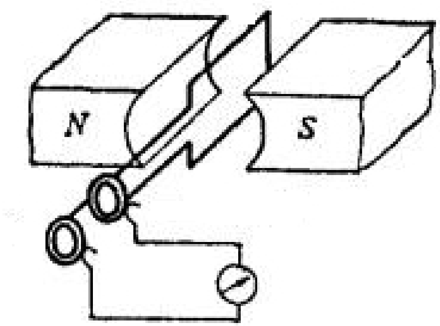
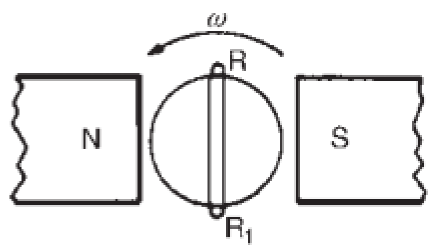
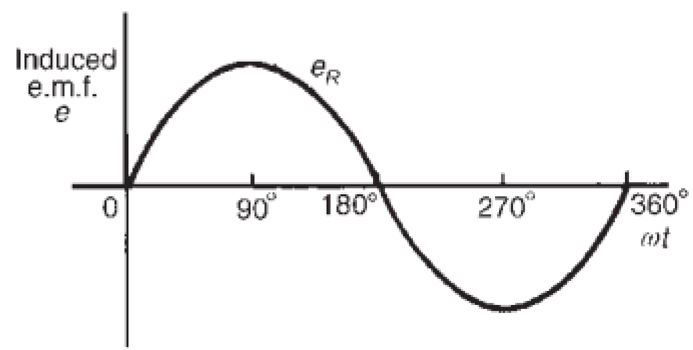
Single-phase voltage

- The voltage induced by a single coil when rotated in a uniform magnetic field is known as a **single-phase voltage**.

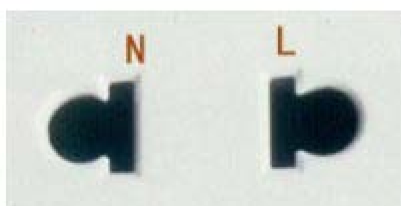


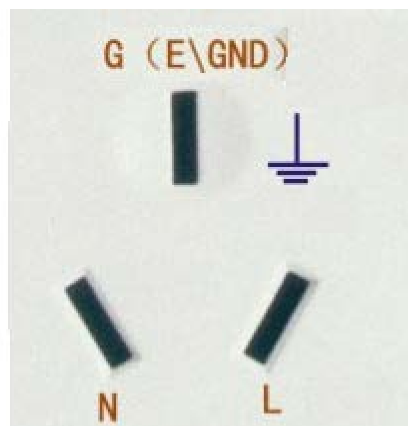




Single-phase voltage

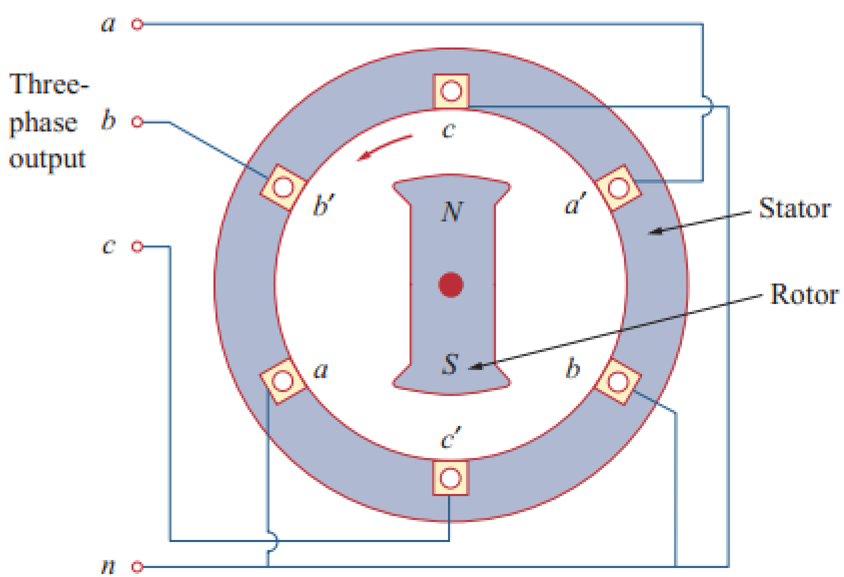
- The standard voltage for a single-phase a.c. supply is:
  - 220V (r.m.s.) in Chinese system;
  - 240V (r.m.s.) in UK system.
- When electric power is supplied to consumers, two wires are used, one called *the live conductor (usually coloured red, L)* and the other is called *the neutral conductor (usually coloured black, N)*. The neutral is usually connected via protective gear to earth, *the earth wire being coloured green, E.*

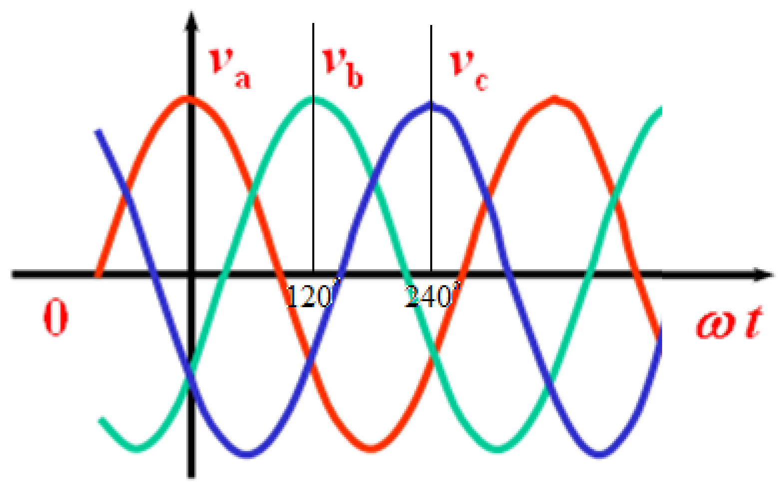




Three-phase voltage

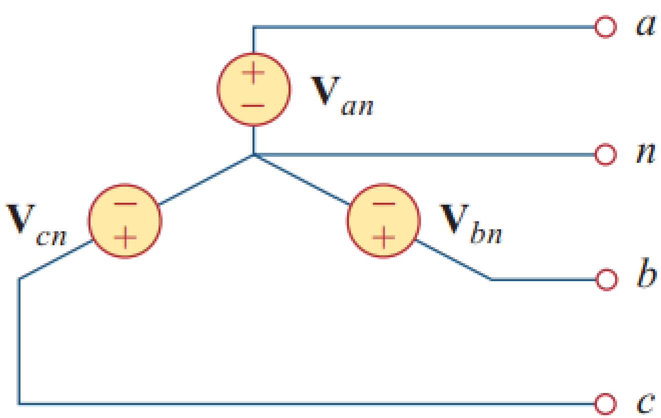
- A three-phase supply is generated when three coils are placed 120° apart and the whole rotated in a uniform magnetic field.
- The result is three independent supplies of equal volt-ages which are each displaced by 120° from each other.



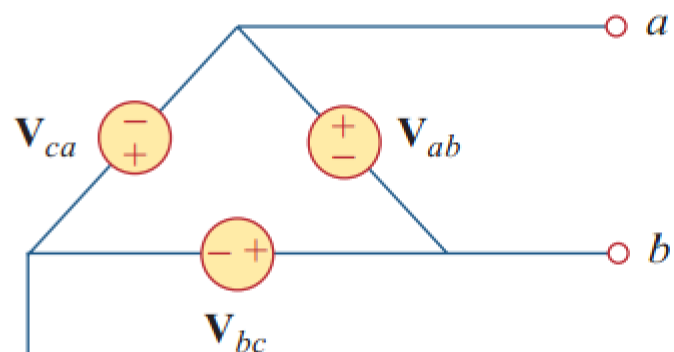


Connections of three-phase supply

- A typical three-phase system consists of three voltage sources connected to loads by three or four wires.
- A three-phase system is equivalent to three single-phase circuits.
- The voltage sources can be either Y-connected or Δ-connected.



Y-connection



Δ-connection
(not used in this module)

Y-connection

- The voltages and are respectively between lines a, b, and c, and the neutral line n.
- These voltages are called ***phase voltages***.

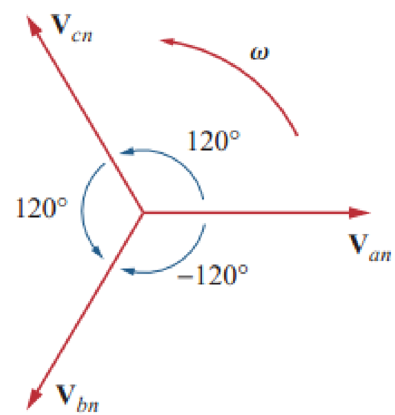
Time domain

$$v_{an}(t) = V\cos(\omega t)$$
$$v_{bn}(t) = V\cos(\omega t - 120^\circ)$$
$$v_{cn}(t) = V\cos(\omega t - 240^\circ)$$
$$= V\cos(\omega t + 120^\circ)$$

Phasor domain

$$\mathbf{V}_{an} = V\angle 0^\circ$$
$$\mathbf{V}_{bn} = V\angle(-120^\circ)$$
$$\mathbf{V}_{cn} = V\angle(-240^\circ) = V\angle 120^\circ$$

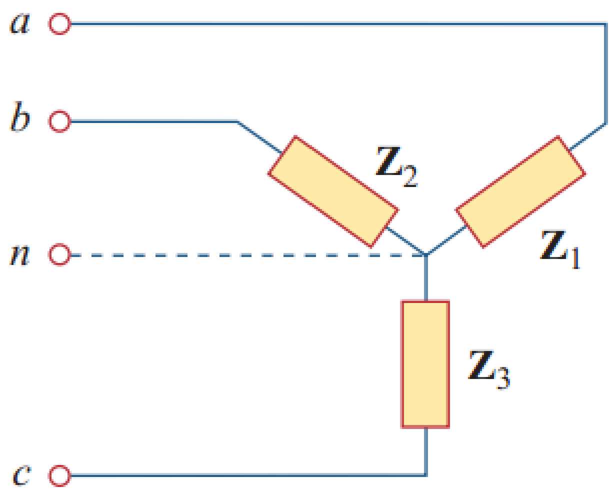
Phasor plane



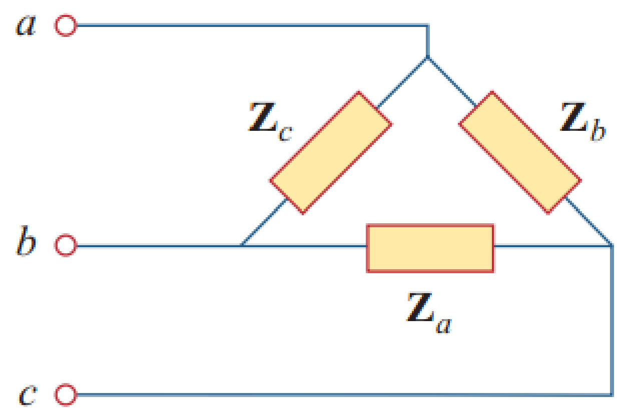
- This is known as the *abc sequence* or *positive sequence* . In this phase sequence, $\mathbf{V}_{an}$ leads $\mathbf{V}_{bn}$, which in turn leads $\mathbf{V}_{cn}$.

Connections of three-phase load

- Like the generator connections, a three-phase load can be either Y-connected or Δ-connected, depending on the end application.



Y-connection



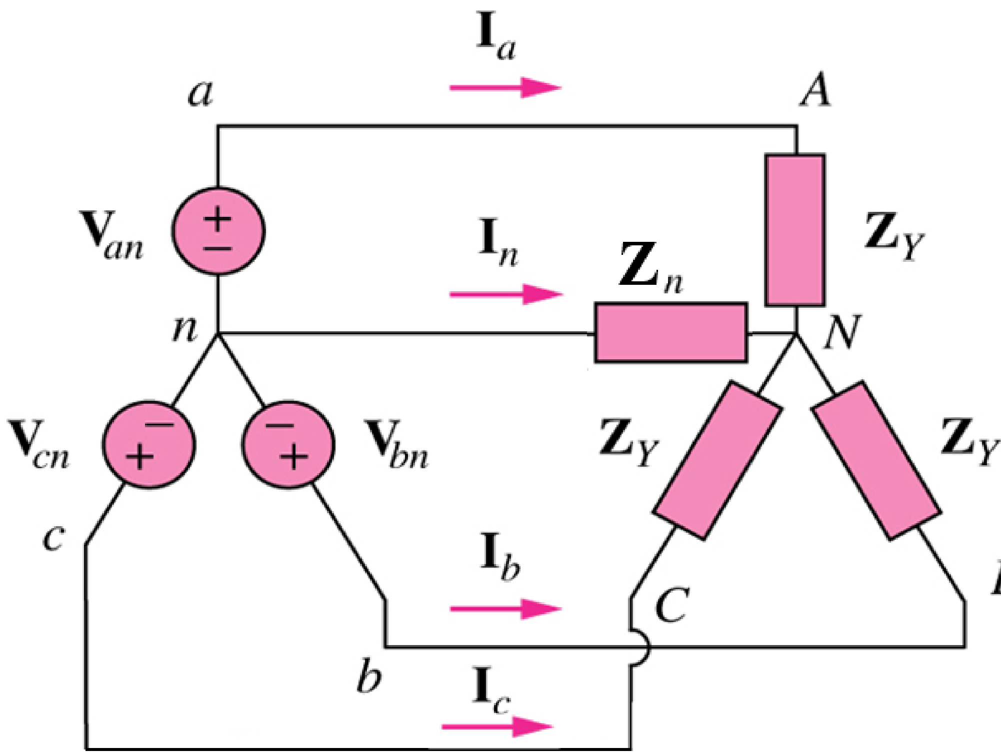
Δ-connection

(not used in this module)

- A balanced load is one in which the phase impedances are equal in magnitude and in phase, i.e. $Z_1=Z_2=Z_3=Z_Y$.

A balanced Y-Y connected circuit

- A balanced Y-Y system is a three-phase system with a balanced Y-connected source and a balanced Y-connected load.



Phase voltages V_p:

V_{an}, V_{bn} and V_{cn};

Line voltages V_L:

V_{ab}, V_{bc} and V_{ca};

Phase currents I_p:

I_a, I_b and I_c;

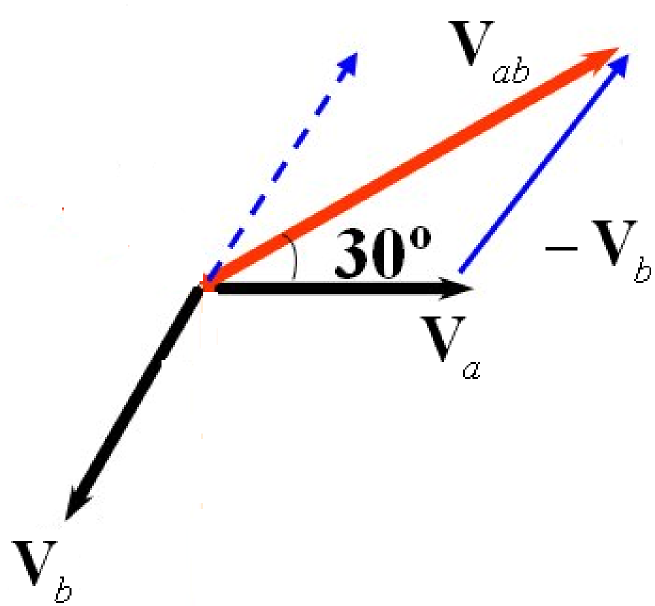
Line currents I_L=I_p.

Voltages in Y-Y connected circuit

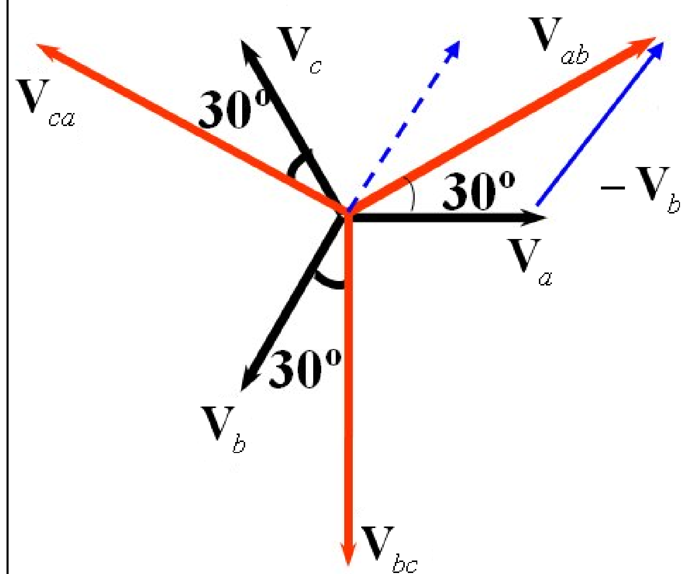
- Phase voltage V_p: The voltage across the ends of each coil
- Line voltage V_L: The voltage between any two lines

$$\mathbf{V}_a = V\angle 0^\circ \ \ \text{V}$$
$$\mathbf{V}_b = V\angle -120^\circ \ \ \text{V}$$
$$\mathbf{V}_c = V\angle -240^\circ \ \ \text{V}$$

$$\mathbf{V}_{ab} = \mathbf{V}_a - \mathbf{V}_b$$
$$= V\angle 0^\circ - V\angle -120^\circ = \sqrt{3}V\angle 30^\circ$$
$$= \sqrt{3}V(\cos 30^\circ + j\sin 30^\circ)$$



$$\mathbf{V}_{ab} = \mathbf{V}_a - \mathbf{V}_b = \sqrt{3}V_p\angle 30^\circ$$
$$\mathbf{V}_{bc} = \mathbf{V}_b - \mathbf{V}_c = \sqrt{3}V_p\angle(-90^\circ)$$
$$\mathbf{V}_{ca} = \mathbf{V}_c - \mathbf{V}_a = \sqrt{3}V_p\angle(-210^\circ)$$



Currents in Y-Y connected circuit

- In the balanced Y-Y circuit, Line currents are the same as phase currents, $I_p = I_L$.

$$\mathbf{I}_a = \frac{\mathbf{V}_a}{\mathbf{Z}_Y}$$

$$\mathbf{I}_b = \frac{\mathbf{V}_b}{\mathbf{Z}_Y} = \frac{\mathbf{V}_a\angle(-120^\circ)}{\mathbf{Z}_Y} = \mathbf{I}_a\angle(-120^\circ)$$

$$\mathbf{I}_c = \frac{\mathbf{V}_c}{\mathbf{Z}_Y} = \frac{\mathbf{V}_a\angle(-240^\circ)}{\mathbf{Z}_Y} = \mathbf{I}_a\angle(-240^\circ)$$

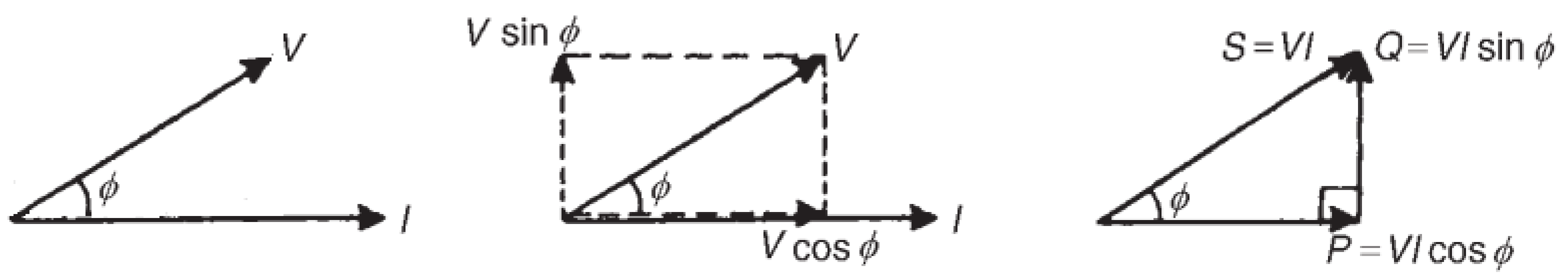
$$\mathbf{I}_n = \mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c = 0$$

$$\boxed{\mathbf{V}_{nN} = \mathbf{Z}_n\mathbf{I}_n = 0} \Rightarrow$$

The neutral line could be removed without any influence to the circuit.
(The source and the load must be balanced!!!)

Power in AC circuit

- The power at any instant is given by the product of the voltage and current at that instant, i.e. the instantaneous power: $p(t) = v(t)i(t)$.
- In the case where the current $\boldsymbol{I}$ lags the applied voltage $\boldsymbol{V}$ by angle φ.



Phasor diagram **Power triangle**

- Average power dissipated in the circuit (the real power) is calculated by: $P = VI \cos \varphi$.

Real Power in a Balanced 3-phase System

- The power in a three-phase system is the sum of the power in each phase.
- Real power in phase A:

If $v_A(t) = \sqrt{2}V_p \sin \omega t$

$$i_A(t) = \sqrt{2}I_p \sin(\omega t - \varphi)$$

So

$$p_{\mathrm{A}} = v_{\mathrm{A}} i_{\mathrm{A}} = 2V_p I_p \sin \omega t \sin(\omega t - \varphi)$$

$$= V_p I_p \cos\varphi - V_p I_p \cos(2\omega t - \varphi) \text{ (W)}$$

- Similarly for phase B and C:

$$p_{\mathrm{B}} = v_{\mathrm{B}} i_{\mathrm{B}} = V_p I_p \cos\varphi - V_p I_p \cos[2(\omega t - 120^{\circ}) - \varphi]$$

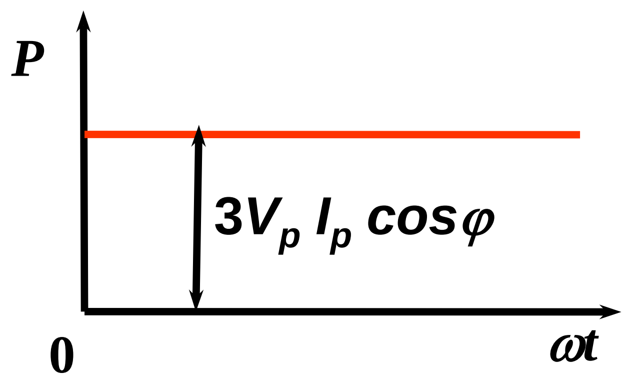
$$p_{\mathrm{C}} = v_{\mathrm{C}} i_{\mathrm{C}} = V_p I_p \cos\varphi - \mathrm{V}_p I_p \cos[2(\omega t + 120^{\circ}) - \varphi]$$

Real Power in a Balanced 3-phase System

- Total power:

$$p = p_{\mathrm{A}} + p_{\mathrm{B}} + p_{\mathrm{C}} = 3V_p I_p \cos\varphi = P$$

- The instantaneous total power is not function of time



- Expressed in line voltages and currents:

$$V_L = \sqrt{3} V_p \qquad I_L = I_p$$

$$P = 3\frac{V_L}{\sqrt{3}} I_L \cos\varphi = \sqrt{3} V_L I_L \cos\varphi \qquad \boxed{P = 3V_p I_p \cos\varphi}$$

Virtual power and Apparent power

- Virtual power (Reactive power): Reactive power is the product of the voltage, current and the sine of the phase angle between them:

$$Q = \sqrt{3} V_L I_L \sin\varphi$$

Volt amperes reactive (Var)

  - Inductive reactive power is defined as positive power
  - Capacitive reactive power is defined as negative reactive power.

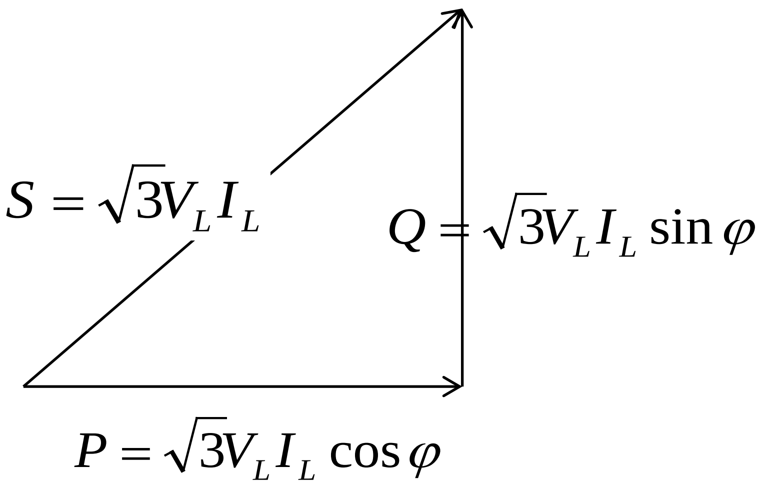
- Apparent power: Apparent power is the product of the voltage and current without accounting of the phase angle:

$$S = \sqrt{3} V_L I_L$$

Volt amperes (V A)

Power Triangle & Power Factor

Power triangle



Power complex form

$$\mathbf{S} = \boldsymbol{P} + \boldsymbol{jQ}$$

$$\mathbf{S} = \mathbf{3}\boldsymbol{V_p}\boldsymbol{I_p}^*$$

Power factor

$$\text{Power Factor} = \frac{\text{Real Power } P}{\text{Apparent Power } S}$$

$$= \cos\varphi$$