EEE108 Electromagnetism and Electromechanics

Lecture 10

Magnetic Field

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Last

- Biot-Savart Law

$$d\mathbf{B} = \frac{\mu}{4\pi}\frac{Id\mathbf{L}\times\mathbf{a}_r}{r^2} \Rightarrow \mathbf{B} = \int_l d\mathbf{B} = \frac{\mu I}{4\pi}\int_l \frac{d\mathbf{L}\times\mathbf{a}_r}{r^2}$$

- Ampere's Law

$$\oint \mathbf{B}\bullet d\mathbf{L} = \mu I_{enc} \qquad \nabla\times\mathbf{B} = \mu\mathbf{J}$$

Applicable to the following current configurations:

1. Infinitely long straight wires carrying a steady current I
2. Infinitely large sheet of thickness b with a current density J
3. Infinite solenoid
4. Toroid

- Gauss's Law for Magnetism   Magnetic monopoles do not exist.

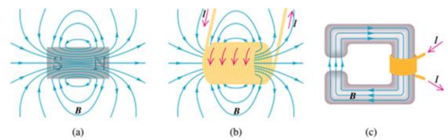
$$\oiint_S \mathbf{B}\bullet d\mathbf{s} = 0 \qquad \nabla\bullet\mathbf{B} = 0$$

Today

- Magnetic Flux and Magnetic Flux Density
- Magnetic Force and Magnetic Torque

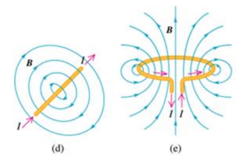
Magnetic Field Lines

Magnetic field lines of a permanent magnet (a), cylindrical coil (b), iron-core electromagnet (c), straight current-carrying wire (d), and a circular current-carrying loop (e)



◆At each point, the field line is tangent to the magnetic flux density.

◆The more densely the field lines are packed, the stronger the field is at that point.



◆At each point, the field lines point in the same direction a compass would, therefore, magnetic field lines point away from N poles and toward S poles.

◆Because the direction of magnetic flux density at each point is unique, field lines never intersect.

Magnetic Field Lines

The properties of electric field lines:

- The direction of the electric field at any point is tangent to the field lines at that point.
- The number of lines per unit area through a surface perpendicular to the line is devised to be proportional to the magnitude of the electric field in a given region.
- The field lines must begin on positive charges (or at infinity) and then terminate on negative charges (or at infinity).
- No two field lines can cross each other.

The properties of magnetic field lines:

- At each point, the field line is tangent to the magnetic flux density.
- The more densely the field lines are packed, the stronger the field is at that point.
- At each point, the field lines point in the same direction a compass would, therefore, magnetic field lines point away from *N* poles and toward *S* poles.
- Because the direction of magnetic flux density at each point is unique, field lines never intersect.

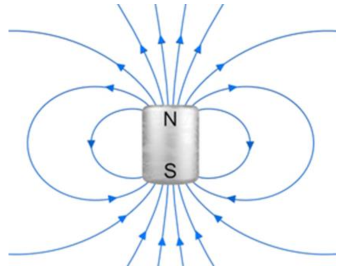
Magnetic Field Lines

Quiz 1

The picture shows the field lines outside a permanent magnet.

The field lines inside the magnet point:

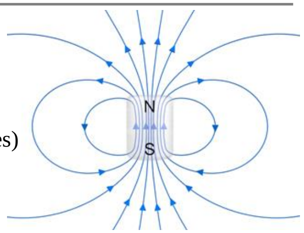
1. Up
2. Down
3. Left to right
4. Right to left
5. The field inside is zero



Magnetic Field Lines

Quiz 1 Cont.

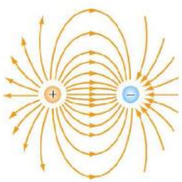
Answer: They point up inside the magnet



Magnetic field lines are continuous.

There are no magnetic charges (monopoles) so **B** field lines ***never*** begin or end!

E field lines begin and end on charges.



The electric field lines must begin on positive charges (or at infinity) and then terminate on negative charges (or at infinity).

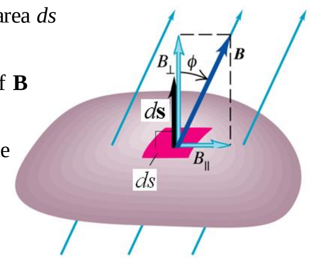
Magnetic Flux

Magnetic flux $d\Phi_B$ through a element of area ds is defined as: $d\Phi_B = B\cos\phi ds = \mathbf{B} \bullet d\mathbf{s}$

where is the angle between the direction of **B** and normal of the element.

The total magnetic flux through the surface is the sum of the contributions from the individual area elements:

$$\Phi_B = \iint B\cos\phi ds = \iint \mathbf{B} \bullet d\mathbf{s}$$



The SI unit of magnetic flux is: Unit of magnetic flux density (1 T) times unit of area (1 m^2). This unit is called weber (Wb).

$$1\,\text{Wb} = 1\,\text{T} \cdot \text{m}^2 = 1\frac{\text{N} \cdot \text{m}}{\text{A}} \quad \text{with } 1\,\text{T} = 1\frac{\text{N}}{\text{A} \cdot \text{m}}$$

Wilhelm Eduard Weber
1804 – 1891
Physicist Germany

Electric Field vs. Magnetic Field

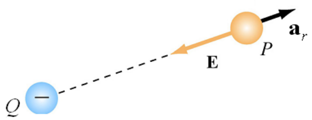
Electric Field of Point Charge

An electric charge produces an electric field

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\mathbf{a}_r$$

ε_0 : permittivity of free space,

$= 8.85\times10^{-12}\ \ \mathrm{C^2/N\cdot m^2}$ (F/m)



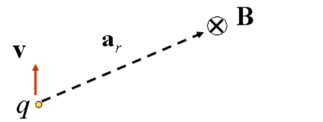
Magnetic Field of a Moving Charge

Moving charge with velocity **v** produces magnetic field

$$\mathbf{B} = \frac{\mu_0}{4\pi}\frac{q\mathbf{v}\times\mathbf{a}_r}{r^2} \Leftarrow \frac{\mu_0}{4\pi}\frac{q}{r^2}\mathbf{v}\times\mathbf{a}_r$$

μ_0 : permeability of free space,

$= 4\pi\times10^{-7}\ \ \mathrm{T\cdot m/A}$ (H/m)



Electric Field vs. Magnetic Field

Electric Field

- A distribution of electric charge at ***rest/moving*** creates an electric field in the surrounding space
- The electric field exerts a force $\mathbf{F} = q\mathbf{E}$ on any other charge q that is present in the field

Magnetic Field

- A ***moving*** charge or a current creates a magnetic field in the surrounding space (in addition to its *electric field*).
- The magnetic field exerts a force **F** on any other ***moving*** charge or current that is present in the field.

Magnetic Force on a Moving Charge

Four key characteristics:

1. The magnitude of the force is proportional to the magnitude of the charge
   The bigger the charge, the greater the magnetic force on it
2. The magnitude of the force is proportional to the magnitude, or "density' of the field
   Double the magnitude of the field, the force doubles
3. The magnitude of the force is also depends on the particle's velocity
   A rest charge experiences no magnetic force. Different from the electric force
4. The direction of the magnetic force is perpendicular to both magnetic field and the charge velocity.

By experiment observation

Magnetic Force on a Moving Charge
Mathematic Expression

The observations can be summarized by the equation:

$$\mathbf{F}_B = q\mathbf{v}\times\mathbf{B}$$

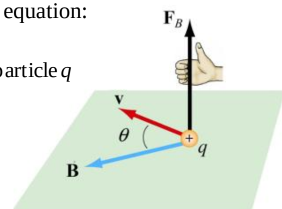
where $\mathbf{F}_B$ is the magnetic force on the charged particle q

v is the velocity of the charge

B is the magnetic flux density

The magnitude of $\mathbf{F}_B$ is given by

$$F_B = |q| vB\sin\theta$$

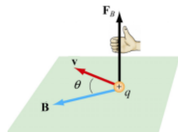


Right-hand rule for the direction of magnetic force on a positive charge moving in a magnetic field:

1. Place the vectors tail to tail
2. Imagine turning toward in the plane (through the smaller angle)
3. The force acts along a line perpendicular to the plane. Do right-hand as shown. The thumb points the direction of the force.

Magnetic Force on a Moving Charge

Four key characteristics: $F_B = |q| vB\sin\theta$



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   Double the magnitude of the field, the force doubles
3. The magnitude of the force is also depends on the particle's velocity
   A rest charge experiences no magnetic force. Different from the electric force
4. The direction of the magnetic force is perpendicular to both magnetic field and the charge velocity.

By experiment observation

Magnetic Force on a Moving Charge

Right-Hand-Rule

Right-hand rule for the direction of magnetic force on a negative charge moving in a magnetic field:

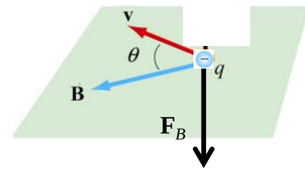
The direction of the force is opposite to that given by the right-hand rule:

$$\mathbf{F}_B = q\mathbf{v}\times\mathbf{B} = -|q|\mathbf{v}\times\mathbf{B}$$

Magnetic force $\mathbf{F}_B$:

- is always perpendicular to $\mathbf{v}$ and $\mathbf{B}$.
- cannot change the particle's speed $\mathbf{v}$
- cannot do work on the particle:

$$dW = \mathbf{F}_B \bullet d\mathbf{L} = q(\mathbf{v}\times\mathbf{B})\bullet \mathbf{v}dt$$
$$= q(\mathbf{v}\times\mathbf{v})\bullet \mathbf{B}dt = 0$$

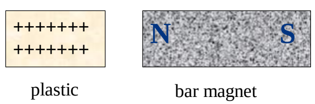


Magnetic Force

Quiz 1

The north-pole end of a bar magnet is held near a positively charged piece of plastic.

Is the plastic:
a) attracted,
b) repelled,
c) unaffected by the magnet?

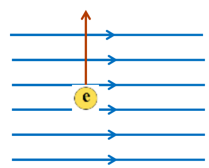


Answer

Magnetic Force

Quiz 2

An electron moves in the plane of this paper toward the top of the page. A magnetic field is also in the plane of the page and directed toward the right.
The direction of the magnetic force on the electron is
a) toward the top of the page
b) toward the bottom of the page
c) toward the left edge of the page
d) toward the right edge of the page
e) out of the page
f) into the page



Answer

Lorentz Force on a Charged Particle

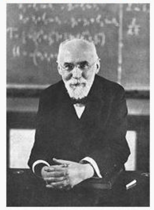
When a charged particle moves in electric ($\mathbf{E}$) and magnetic ($\mathbf{B}$) fields, it feels a total force $\mathbf{F}_{\text{Lorentz}}$ exerted by both fields:

$$\mathbf{F}_{\text{Lorentz}} = q(\mathbf{E} + \mathbf{v}\times\mathbf{B})$$

$\mathbf{F}_{\text{Lorentz}}$ is called Lorentz force.

It was introduced by Lorentz in 1892.

Hendrik Antoon Lorentz



Born	18 July 1853 Arnhem, Netherlands
Died	4 February 1928 (aged 74) Haarlem, Netherlands
Nationality	Netherlands
Fields	Physics

Units of B

From $\mathbf{F}_{\text{B}} = q\mathbf{v}\times\mathbf{B}$

SI unit of $\mathbf{B} = \dfrac{\text{N}}{\text{C}\cdot\text{m/s}} = \dfrac{\text{N}}{\text{A}\cdot\text{m}}$

$1\,\text{N/(A}\cdot\text{m)}$ is called 1 Tesla (T)

T here is not for temperature!!

$1\,\text{T} = 10^4$ Gauss (G)

The magnetic flux density of the Earth is of the order of 1 G.

Lorentz Force on a Charged Particle

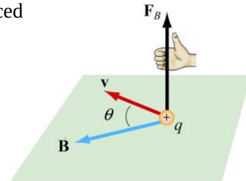
$$\mathbf{F}_{\text{Lorentz}} = \mathbf{F}_E + \mathbf{F}_B = q\mathbf{E} + q(\mathbf{v}\times\mathbf{B})$$

Electric Force

- always parallel with the direction of the electric field
- acts on a charged particle whether or not it is moving
- expends energy in displacing a charged particle

Magnetic Force

- always perpendicular to the magnetic field.
- acts on it only when it is in motion.
- does no work when a particle is displaced



Vector / Cross Product (Again)

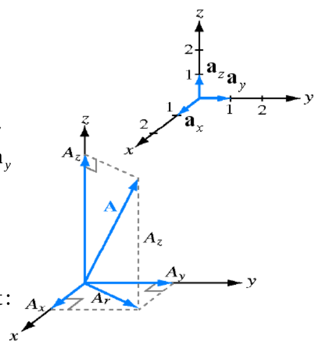
$$\mathbf{F}_{\text{Lorentz}} = \mathbf{F}_E + \mathbf{F}_B = q\mathbf{E} + q(\mathbf{v}\times\mathbf{B})$$

In cartesian coordinates:

$\mathbf{a}_x\times\mathbf{a}_y = \mathbf{a}_z,\quad \mathbf{a}_y\times\mathbf{a}_z = \mathbf{a}_x,\quad \mathbf{a}_z\times\mathbf{a}_x = \mathbf{a}_y$

$\mathbf{a}_y\times\mathbf{a}_x = -\mathbf{a}_z,\quad \mathbf{a}_z\times\mathbf{a}_y = -\mathbf{a}_x,\quad \mathbf{a}_x\times\mathbf{a}_z = -\mathbf{a}_y$

$\mathbf{a}_x\times\mathbf{a}_x = 0,\quad \mathbf{a}_y\times\mathbf{a}_y = 0,\quad \mathbf{a}_z\times\mathbf{a}_z = 0$



For $\mathbf{v} = v_x\mathbf{a}_x + v_y\mathbf{a}_y + v_z\mathbf{a}_z$ and

$\mathbf{B} = B_x\mathbf{a}_x + B_y\mathbf{a}_y + B_z\mathbf{a}_z$, the cross product:

$$\mathbf{v}\times\mathbf{B} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ v_x & v_y & v_z \\ B_x & B_y & B_z \end{vmatrix} = (v_yB_z - v_zB_y)\mathbf{a}_x + (v_zB_x - v_xB_z)\mathbf{a}_y + (v_xB_y - v_yB_x)\mathbf{a}_z$$

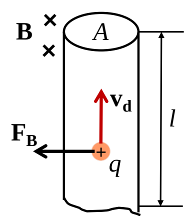
Magnetic Force on a Current-Carrying Wire

The total amount of charge in the segment $Q_{tot} = q(nAl)$, n is the number of charges per unit volume, so

$\mathbf{F}_{\text{B}} = Q_{tot}\mathbf{v}_d\times\mathbf{B} = qnAl(\mathbf{v}_d\times\mathbf{B})$

$\boxed{\mathbf{F}_{\text{B}} = I(\mathbf{L}\times\mathbf{B})}$

where $I = nqv_dA$, and $\mathbf{L}$ is a length vector with a magnitude l and directed along the direction of the current.

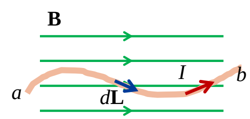


For a wire of arbitrary shape, the force acting on the small segment $d\mathbf{L}$: $\quad d\mathbf{F}_B = Id\mathbf{L}\times\mathbf{B}$

The total force is:

$$\boxed{\mathbf{F}_{\text{B}} = I\int_a^b (d\mathbf{L}\times\mathbf{B})}$$

where a and b represent the end points of the wire.



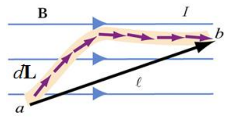
Magnetic Force on a Current-Carrying Wire

Curved Wire in a Uniform B Field

A curved wire carrying a current I in a uniform magnetic field

$$\mathbf{F}_B = I\left(\int_a^b d\mathbf{L}\right)\times\mathbf{B} = I\mathbf{L}\times\mathbf{B}$$

Where $\mathbf{L}$ is the length vector directed from a to b. The integral of $d\mathbf{L}$ from a to b has the same value irrespective of the path taken between a and b.



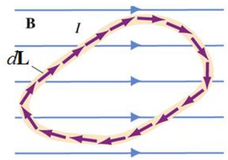
Closed Wire in a Uniform B Field

If the wire forms a closed loop, the force acting on the loop: $\mathbf{F}_\mathrm{B} = I\left(\oint d\mathbf{L}\right)\times\mathbf{B}$

Since the vector sum of the displacement vectors $d\mathbf{L}$ over a closed path is equal to zero, so

$$\mathbf{F}_\mathrm{B} = I\left(\oint d\mathbf{L}\right)\times\mathbf{B} = 0$$

The total megnetic force on any closed current loop in a uniform magnetic field is zero.



Magnetic Force on a Current-Carrying Wire

Example

A horizontal wire with a mass per unit length of 0.2 kg/m carries a current of 4 A in the $+x$-direction. If the wire is place in a uniform magnetic flux density $\mathbf{B}$, what should the direction and minimum magnitude of $\mathbf{B}$ be in order to magnetical ly lift the wire vertically upward? The accelerati on due to gravity is $\mathbf{g} = -\mathbf{a}_z 9.8\ \mathrm{m/s^2}$

Solution

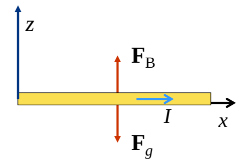
For a length l

$\mathbf{F}_\mathrm{g} = -\mathbf{a}_z 0.2l \times 9.8 = -\mathbf{a}_z 1.96l \quad (\mathrm{N})$

$\mathbf{F}_\mathrm{B} = I\mathbf{L}\times\mathbf{B} \quad (\mathrm{N})$

For $\mathbf{F}_\mathrm{g} + \mathbf{F}_\mathrm{B} = 0$, $\mathbf{F}_\mathrm{B}$ should be along $+\mathbf{a}_z$,

then $\mathbf{B}$ has to be along $+\mathbf{a}_y$



Hence: $1.96l = IlB \;\Rightarrow\; B = \dfrac{1.96}{I} = 0.49 \quad (\mathrm{T})$

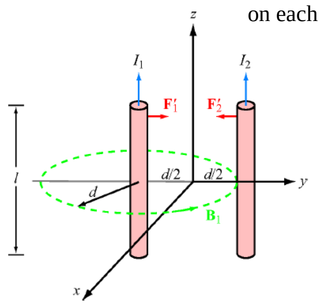
$$\boxed{\mathbf{B} = \mathbf{a}_y 0.49 \quad (\mathrm{T})}$$

Magnetic Force Between Two Parallel Wires

A current-carrying wire produces a magnetic field.

In a magnetic field, a wire carrying a current experiences a net force.

Two current-carrying wires to exert force on each other.

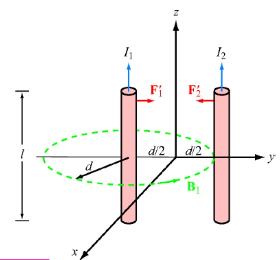


Consider two long parallel wires separated by a distance d and carrying currents I_1 and I_2 in the $+z$-direction at $y = -d/2$ and $y = d/2$, respectively.

Magnetic Force Between Two Parallel Wires

$\mathbf{B}_1$: the magnetic field due to current I_1 at the location of the wire carrying current I_2.

$\mathbf{B}_2$: the magnetic field due to current I_2 at the location of the wire carrying current I_1.



With the field $\mathbf{B}_1$ pointing in the tangential direction.

Thus, at an arbitrary point P on wire 2, we have

$$\mathbf{B}_1 = -\left(\frac{\mu_0 I_1}{2\pi d}\right)\mathbf{a}_x, \qquad \boxed{\mathbf{B} = \frac{\mu_0 I}{2\pi r}\mathbf{a}_\phi}$$

The force $\mathbf{F}_2$ exerted on a length l of wire 2 due to its presence in field $\mathbf{B}_1$

$$\mathbf{F}_2 = I_2\mathbf{L}\times\mathbf{B}_1 = I_2(l\mathbf{a}_z)\times\left(-\frac{\mu_0 I_1}{2\pi d}\mathbf{a}_x\right) = -\frac{\mu_0 I_1 I_2 l}{2\pi d}\mathbf{a}_y$$

Clearly $\mathbf{F}_2$ points toward wire 1.

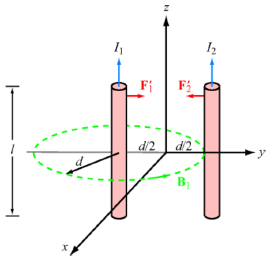
Magnetic Force Between Two Parallel Wires

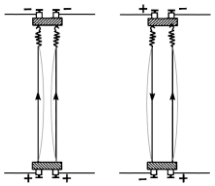
The corresponding force per unit length is

$$\mathbf{F}_2' = \frac{\mathbf{F}_2}{l} = -\frac{\mu_0 I_1 I_2}{2\pi d}\mathbf{a}_y$$

Similar, the force per unit length exerted on the wire carrying I_1 leads to:

$$\mathbf{F}_1' = \frac{\mu_0 I_1 I_2}{2\pi d}\mathbf{a}_y$$



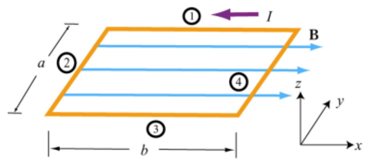


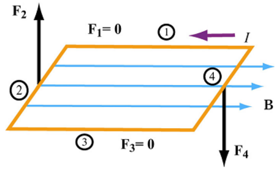
The conclusion we can draw from the calculation is that two parallel wires carrying currents in the same direction will attract each other.
On the other hand, if the currents flow in opposite directions, the resultant force will be repulsive.

Magnetic Force on a Closed Current Loop

A rectangular current loop placed in a uniform magnetic field.

What happens?





No magnetic forces act on sides 1 and 3 because the length vector $\mathbf{L}_1 = -b\mathbf{a}_x$ and $\mathbf{L}_3 = b\mathbf{a}_x$ are parallel and anti-parallel to $\mathbf{B}$ and their cross products vanish.

The magnetic forces act on sides 2 and 4:

$$\begin{cases} \mathbf{F}_2 = I(-a\mathbf{a}_y)\times(B\mathbf{a}_x) = IaB\mathbf{a}_z \\ \mathbf{F}_4 = I(a\mathbf{a}_y)\times(B\mathbf{a}_x) = -IaB\mathbf{a}_z \end{cases}$$

$\mathbf{F}_2$ points up

$\mathbf{F}_4$ points down

$\boxed{\mathbf{F}_B = I(\mathbf{L}\times\mathbf{B})}$

The net force on the loop:

$$\mathbf{F}_{net} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \mathbf{F}_4 = 0$$

Torque on a Closed Current Loop

$\mathbf{F}_2$ and $\mathbf{F}_4$ produce a torque $\Rightarrow$ causes the loop to rotate about the y-axis

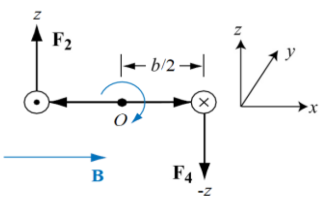
The torque with respect to the centre of the loop:

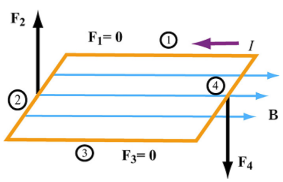
$$\mathbf{T} = \left(-\frac{b}{2}\mathbf{a}_x\right)\times\mathbf{F}_2 + \left(\frac{b}{2}\mathbf{a}_x\right)\times\mathbf{F}_4 = \left(-\frac{b}{2}\mathbf{a}_x\right)\times(IaB\mathbf{a}_z) + \left(\frac{b}{2}\mathbf{a}_x\right)\times(-IaB\mathbf{a}_z)$$

$$= \left(\frac{IabB}{2} + \frac{IabB}{2}\right)\mathbf{a}_y = IabB\mathbf{a}_y = IAB\mathbf{a}_y$$

$A = ab$ represents the area of the loop

Rotation is clockwise about the y-axis.





Torque on a Closed Current Loop

It is convenient to introduce the area vector: $\mathbf{A} = A\mathbf{a}_n$ $\boxed{\mathbf{T} = IAB\mathbf{a}_y}$

where $\mathbf{a}_n$ is a unit vector in the direction normal to the plane of the loop by following ***right-hand rule***: when the four fingers of the right hand advance in the direction of the current I, the direction of the thumb specifies the direction of $\mathbf{a}_n$. In the case, $\mathbf{a}_n = +\mathbf{a}_z$

Then the expression for torque can then be rewritten as:

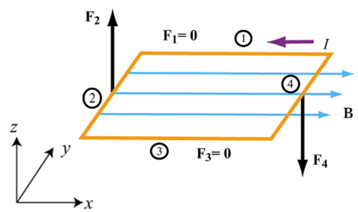
$$\mathbf{T} = IAB\mathbf{a}_y = I\mathbf{A}\times\mathbf{B}$$

The net torque:

$$\mathbf{T} = I\mathbf{A}\times\mathbf{B} = IAB\sin\theta\mathbf{a}_y$$

* The torque is maximum when the magnetic field is parallel to the plane of the loop ($\theta = 90^\circ$)

* The torque is zero when the field is perpendicular to the plane of the loop ($\theta = 0^\circ$)



Torque on a Closed Current Loop

For a loop consisting of N turns, the magnitude of the torque is :

$$T = NIAB\sin\theta$$

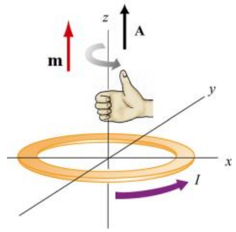
The NIA is called ***magnetic dipole moment*** and the vector form is :

$$\mathbf{m} = NI\mathbf{A}$$

The direction of the magnetic dipole moment is the same as the area vector $\mathbf{A}$.

In terms of $\mathbf{m}$, the torque vector $\mathbf{T}$ can be written as :

$$\mathbf{T} = \mathbf{m}\times\mathbf{B}$$



For one turn loop, the torque :

$$\mathbf{T} = I\mathbf{A}\times\mathbf{B} = \mathbf{m}\times\mathbf{B}$$

The magnitude of the torque is :

$$T = mB\sin\theta$$

A current loop, or any other body, such as bar magnets, that experiences a magnetic torque given by the above equation is called a ***magnetic dipole***.

Summary

∗ Magnetic flux

$$\Phi_B = \iint B\cos\phi ds = \iint \mathbf{B}\bullet d\mathbf{s}$$

∗ The magnetic force acting on a moving charge q

$$\mathbf{F}_B = q\mathbf{v}\times\mathbf{B}$$

∗ The magnetic force acting on a wire

$$\mathbf{F}_B = I\mathbf{L}\times\mathbf{B}$$

∗ The magnitude of the magnetic force between two straight wires of carrying steady current of I_1 and I_2 and separated by r in free space

$$F_B = \frac{\mu_0 I_1 I_2 l}{2\pi r}$$

∗ The torque acting on a close loop of wire

$$\mathbf{T} = I\mathbf{A}\times\mathbf{B}$$

∗ The magnetic dipole moment of a closed loop

$\mathbf{m} = I\mathbf{A}$ one turn

$\mathbf{m} = NI\mathbf{A}$ N turns

Next Lecture

- Faraday's Law of Induction
- Magnetic Materials

Thanks for your attendance