MTH101:   Tutorial 3

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September 26, 2017

Exercise 1.1

Show that $\cosh z = \cosh x \cos y + i \sinh x \sin y$.

Exercise 1.2

Find the function value in the form $u + iv$.

$$\cosh(-1 + 2i), \quad \cos(-2 - i)$$

Exercise 1.3

Verify that $\cos x \sinh y$ is a harmonic function.

Exercise 2.1

Find the path and sketch it.

1 $z(t) = (1 + 2i)t, \quad (2 \le t \le 5);$

2 $z(t) = 2 + 4e^{\pi it/2}, \quad (0 \le t \le 2)$

Exercise 2.2

Find a parametrization representation and sketch the path.

1. *Upper half of $|z - 2 + i| = 2$ from $(4, -1)$ to $(0, -1)$.*
2. *Parabola $y = 1 - \frac{1}{4}x^2, \quad (-2 \leq x \leq 2)$*

Exercise 2.3

Find a parametrization for the **Counterclockwise** *oriented path*
$\gamma = \gamma_1 \cup \gamma_2 \cup \gamma_3$ *where*

γ_1 *is the segment joining z_1 to z_2,*
γ_2 *is the segment joining z_2 to z_3,*
γ_3 *is the upper semicircle with center $z_0 = 0$ and radius $R = 2$,*

and

$$z_1 = -2, \quad z_2 = -3i, \quad z_3 = 2.$$
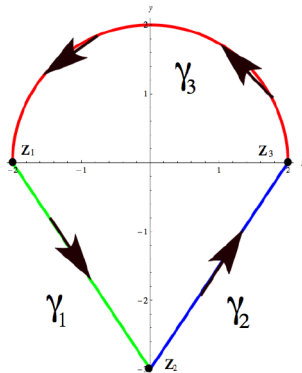
Compute the Integral

$$\oint_\gamma z dz.$$

Figure: The path $\gamma = \gamma_1 \cup \gamma_2 \cup \gamma_3$

Exercise 2.4

Integrate the following complex functions using appropriate method.

1

$$\int_{\gamma} \operatorname{Re} z \, dz$$

γ is the shortest path from $1 + i$ to $3 + 3i$.

2

$$\int_{\gamma} e^z \, dz$$

γ is the shortest path from πi to $2\pi i$.

3

$$\int_{\gamma} \sec^2 z \, dz$$

γ is any path from $\pi/4$ to $\pi i/4$.

Exercise 2.5

4

$$\oint_\gamma \frac{\tan \frac{1}{2} z}{z^4 - 16}$$

γ is the boundary of the square with vertices ± 1, $\pm i$ clockwise.