EEE225 Advanced Electrical Circuits and Electromagnetics

Lecture 15 Three-phase Systems

Dr. Zhao Wang

zhao.wang@xjtlu.edu.cn

Room EE322

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- Balanced three-phase circuits:
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- Unbalanced three-phase circuits
- Three-phase Power
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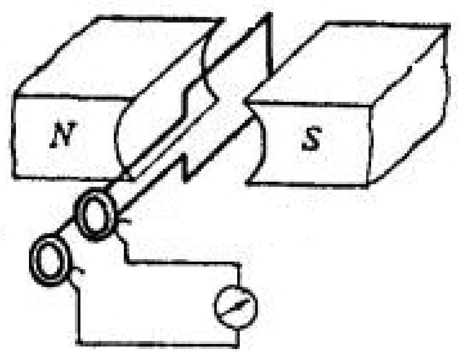
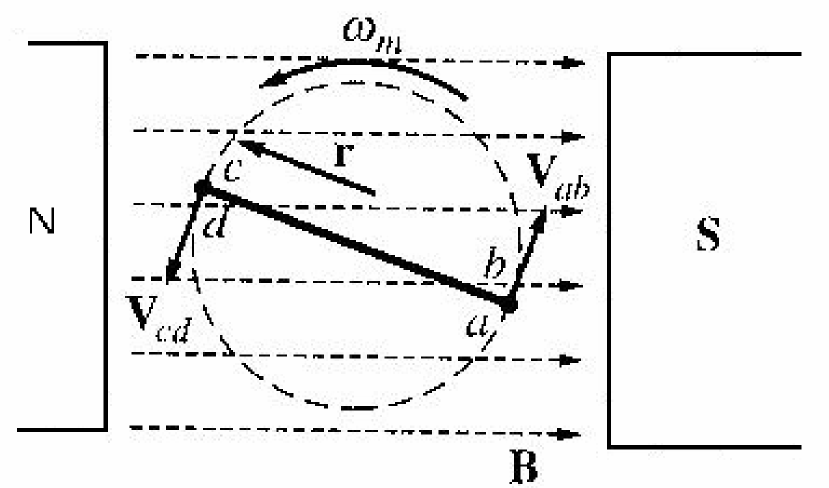
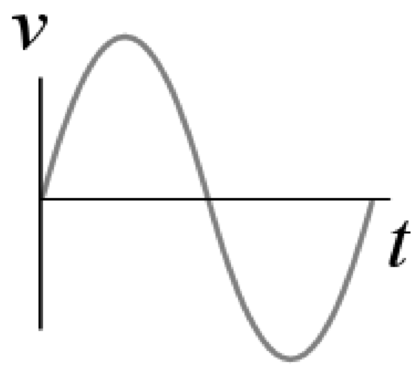
A Single-phase Supply

- The voltage induced by a single coil when it rotates in a magnetic field.

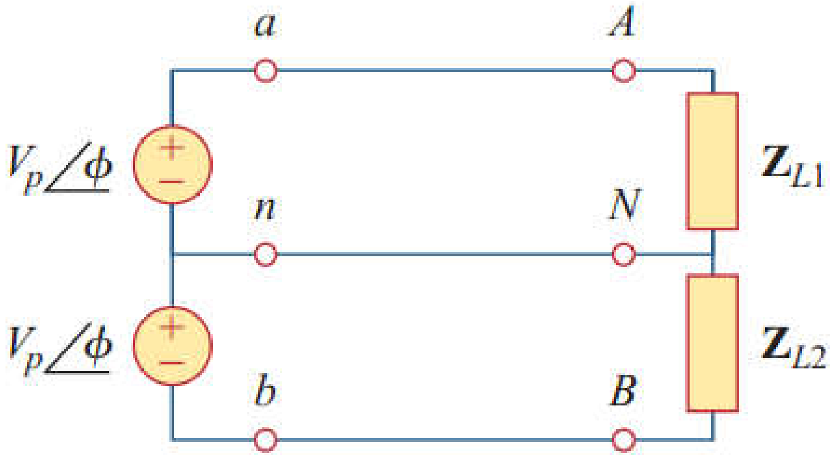


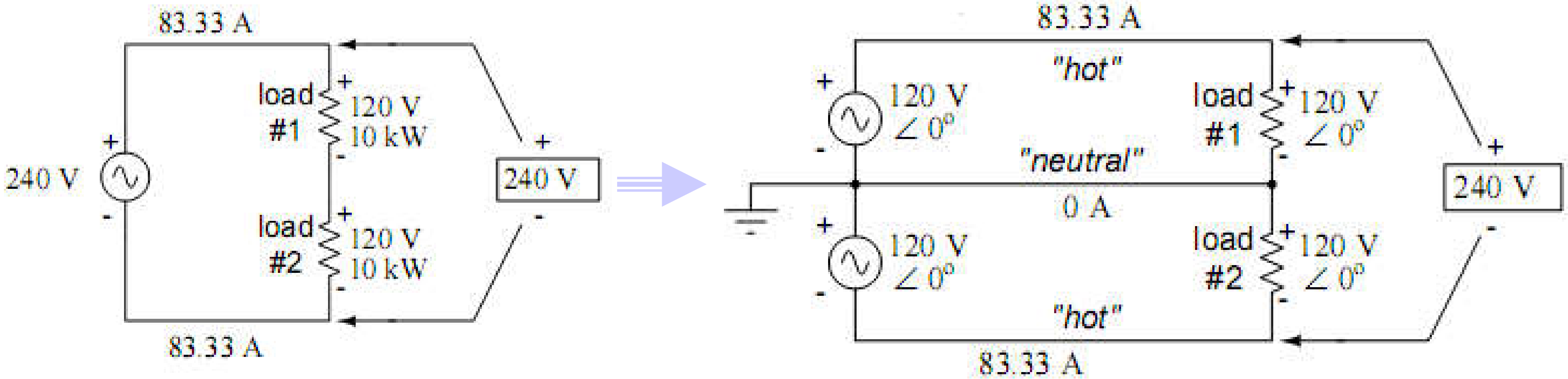




  - All the voltages of the supply vary in unison.
  - Used mostly for lighting and heating, no above 10 or 20 kW

Single phase circuits – Three-wire systems

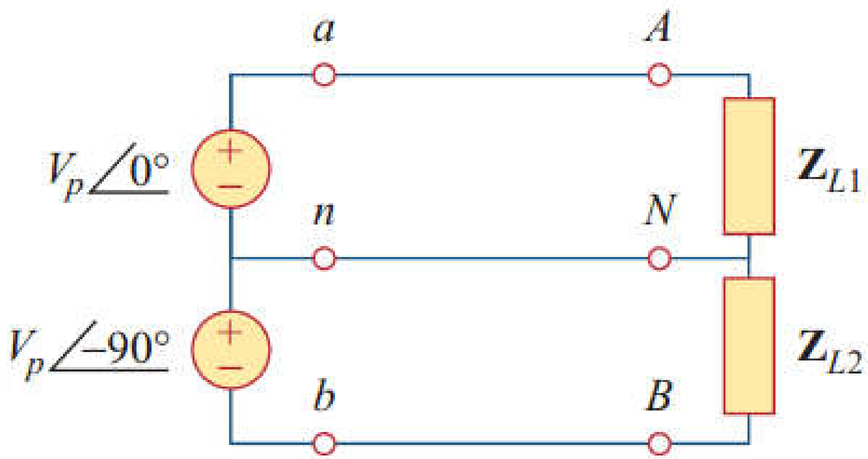




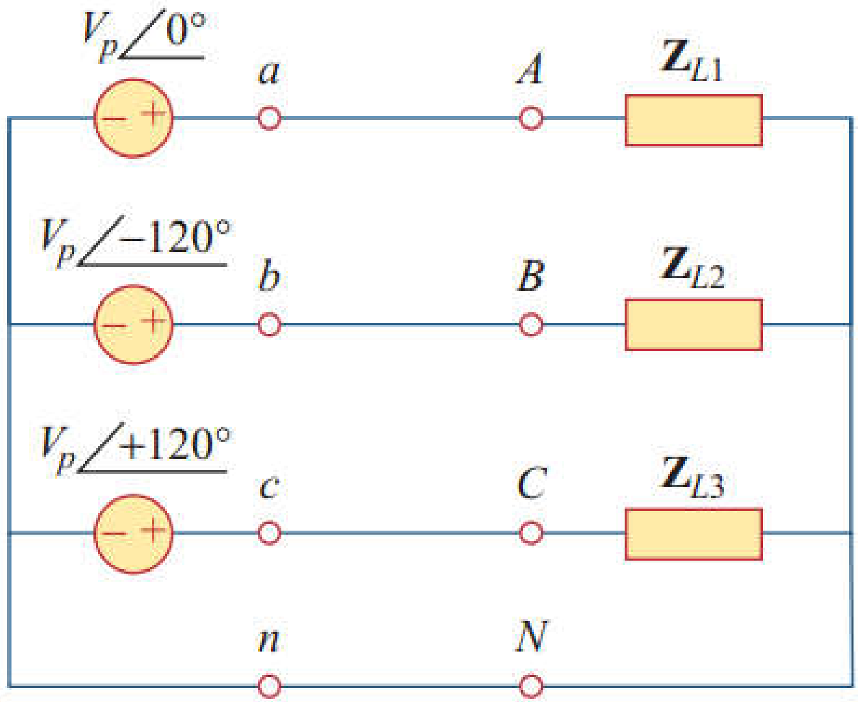
Three-wire systems

Poly-phase System

- Circuits or systems in which the ac sources operate at the same frequency but different phases are known as ***polyphase***.



Two-phase Three-wire



Three-phase Four-wire

Three-phase Circuits

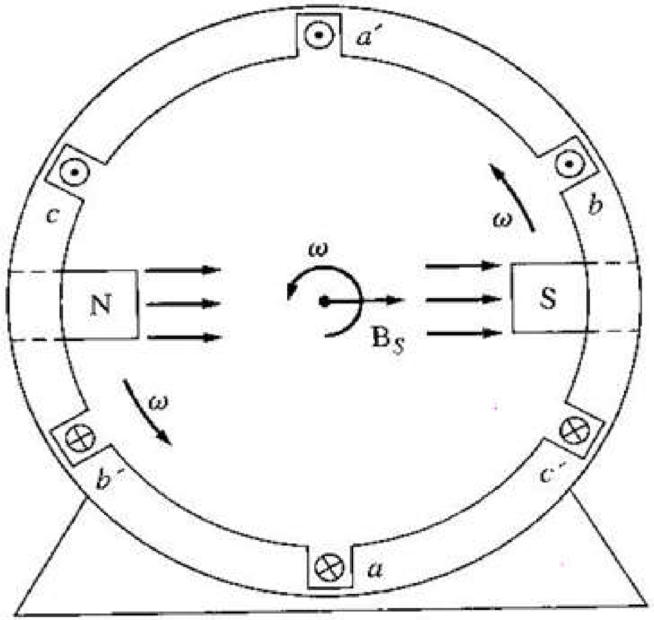
- Three-phase systems are important for at least three reasons.
  - First, nearly all electric power is generated and distributed in three-phase;

Generation, transmission and distribution of electricity via the grid system is accomplished by three-phase alternating voltages.

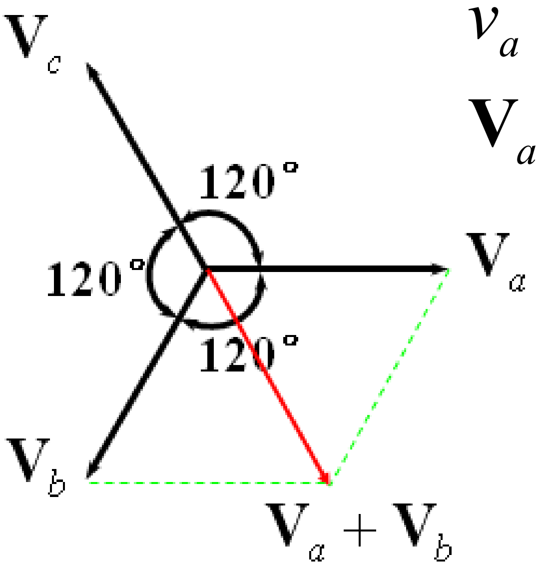
  - Second, the instantaneous power in a three-phase system can be constant;
  - Third, for the same amount of power, the three-phase system is more economical than the single-phase.

Balanced Three-phase Supply – Three-phase Generators

Instead of just one coil, there are three identical coils, each makes an angle of 120^0 with each other two, rotating in a uniform magnetic field.

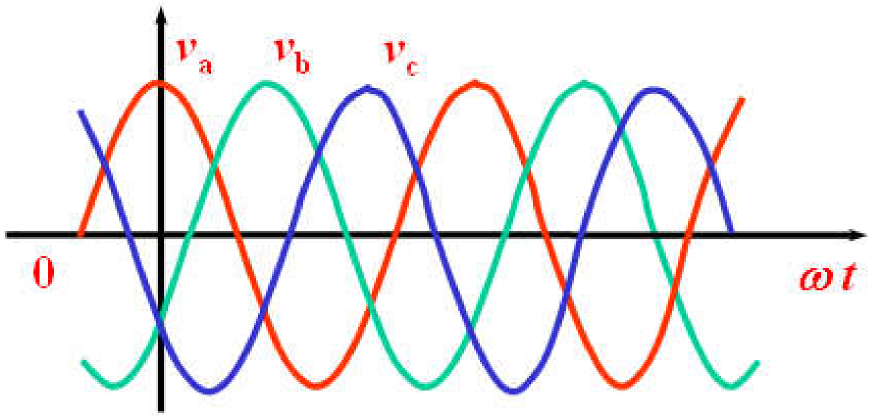


- All 3 variables have the same amplitude
- All 3 variables have the same frequency
- All 3 variables are 120^0 in phase



$$v_a + v_b + v_c = 0$$

$$\mathbf{V}_a + \mathbf{V}_b + \mathbf{V}_c = 0$$



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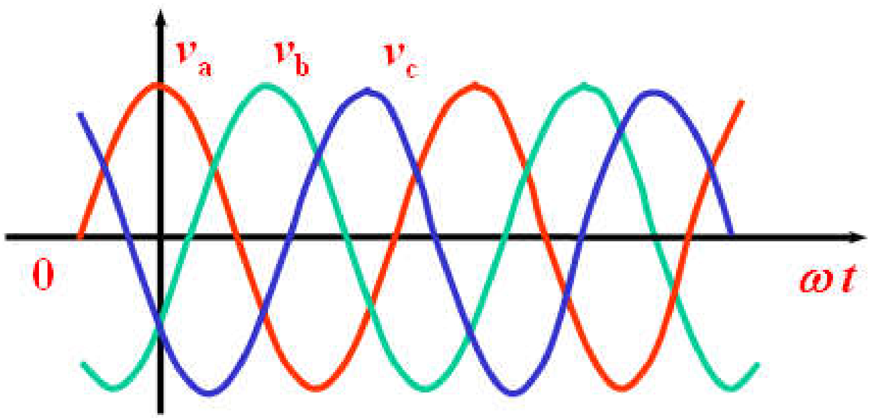
Balanced Three-phase Supply – Three-phase Expression

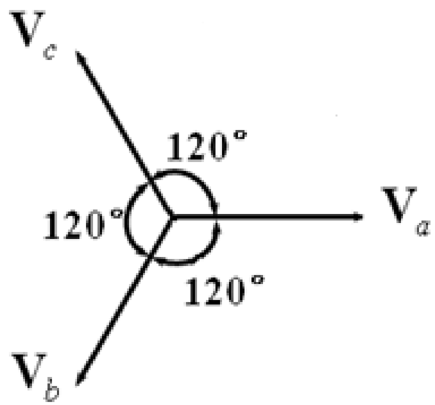
Time domain

$$v_{an}(t) = V_M \cos(\omega t)$$
$$v_{bn}(t) = V_M \cos(\omega t - 120°)$$
$$v_{cn}(t) = V_M \cos(\omega t - 240°)$$
$$= V_M \cos(\omega t + 120°)$$

Phasor domain

$$\mathbf{V}_a = V\angle 0^\circ$$
$$\mathbf{V}_b = V\angle(-120^\circ)$$
$$\mathbf{V}_c = V\angle(-240^\circ) = V\angle 120^\circ$$





Balanced Three-phase Supply – Phase sequence

The phase sequence is the time order in which the voltages pass through their respective maximum values.

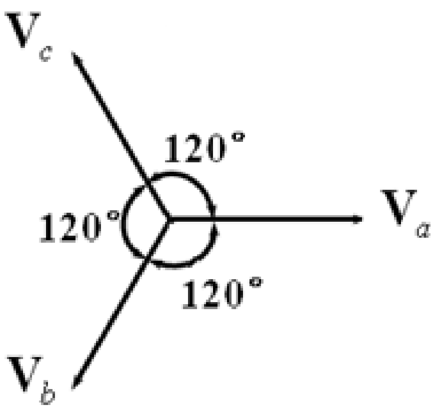
Two possible combinations:

abc sequence (positive sequence)

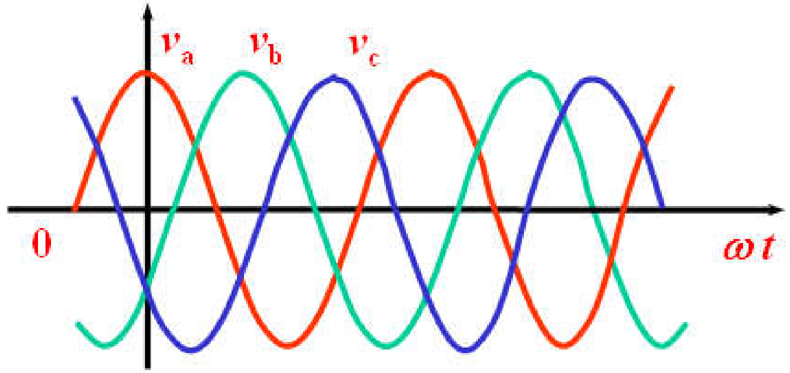
$$\begin{cases} \mathbf{V}_a = V\angle 0^\circ \\ \mathbf{V}_b = V\angle(-120^\circ) \\ \mathbf{V}_c = V\angle(-240^\circ) = V\angle 120^\circ \end{cases}$$

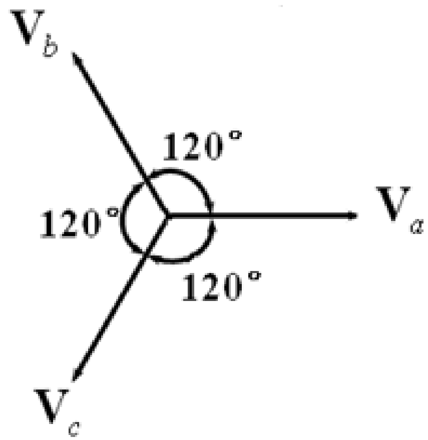
acb sequence (negative sequence)

$$\begin{cases} \mathbf{V}_a = V\angle 0^\circ \\ \mathbf{V}_c = V\angle(-120^\circ) \\ \mathbf{V}_b = V\angle(-240^\circ) = V\angle 120^\circ \end{cases}$$



abc sequence





Balanced three-phase Circuits – Advantages

- Advantages of three-phase systems:
  - Power and torque are constant in a three-phase motor or generator, which means that the machines will run more smoothly than single-phase machines.
  - For a given size, i.e. for a given amount of steel and copper, a three-phase machine has a greater output power.
  - A three-phase transmission system will carry a greater power than a single-phase system with other things being equal.

Balanced Three-phase Supply – Three-phase connections

- A three phase system consists of a **three-phase voltage source** that is used to supply a **three-phase load**. Both the three-phase voltage source and the three-phase load can be connected in two different ways:

  1. wye (or star) (Y) , and
  2. delta (Δ)

Four possible connections:

1. Y-Y connection (Y-connected source with a Y-connected load)
2. Y-Δ connection (Y-connected source with a Δ-connected load)
3. Δ-Δ connection (Δ -connected source with a Δ -connected load)
4. Δ-Y connection (Δ -connected source with a Y -connected load)

Balanced Three-phase Supply – Y connection

- The similar ends of the three coils connected together to form the star point. The three remaining ends brought out to form the three terminals.

Voltages in Y connections

Phase voltage V_p: The voltage across the ends of each coil

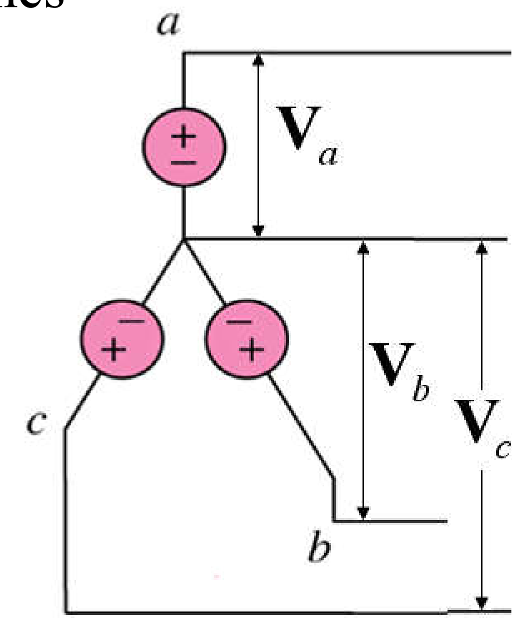
Line voltage V_L: The voltage between any two lines

For a balanced three-phase voltages:

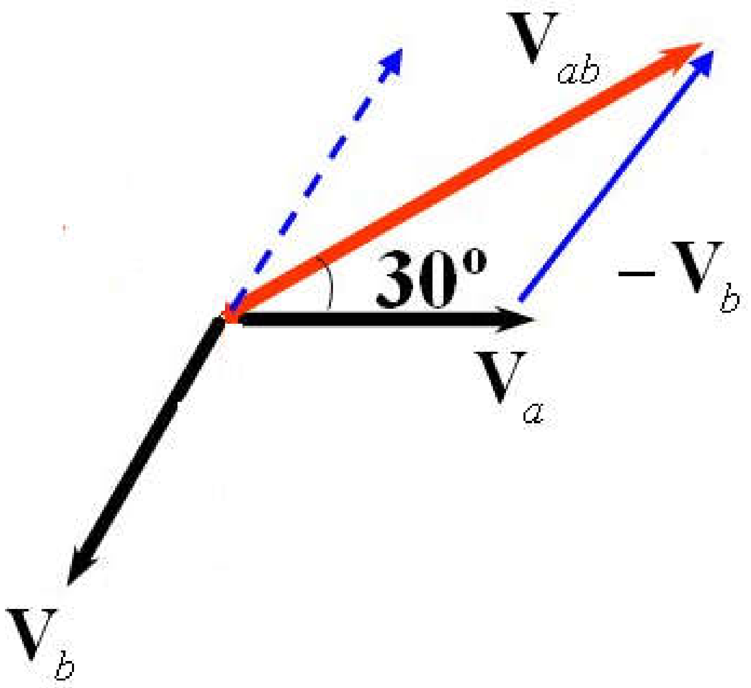
$$\boxed{V_L = \sqrt{3} V_p}$$

where $V_p = |\mathbf{V}_a| = |\mathbf{V}_b| = |\mathbf{V}_c|$

$V_L = |\mathbf{V}_{ab}| = |\mathbf{V}_{bc}| = |\mathbf{V}_{ca}|$



Balanced Three-phase Supply – Voltages in Y connection



$$\mathbf{V}_a = V\angle 0^\circ \quad \text{V}$$
$$\mathbf{V}_b = V\angle -120^\circ \quad \text{V}$$
$$\mathbf{V}_c = V\angle -240^\circ \quad \text{V}$$

$$\begin{aligned}
\mathbf{V}_{ab} &= \mathbf{V}_a - \mathbf{V}_b \\
&= V\angle 0^\circ - V\angle -120^\circ \\
&= V - [V\cos(-120^\circ) + jV\sin(-120^\circ)] \\
&= V(1+\sin 30^\circ) + jV\cos 30^\circ \\
&= V\left(\frac{3}{2} + j\frac{\sqrt{3}}{2}\right) = \sqrt{3}V\left(\frac{\sqrt{3}}{2} + j\frac{1}{2}\right) \\
&= \sqrt{3}V(\cos 30^\circ + j\sin 30^\circ) \\
&= \sqrt{3}V\angle 30^\circ
\end{aligned}$$

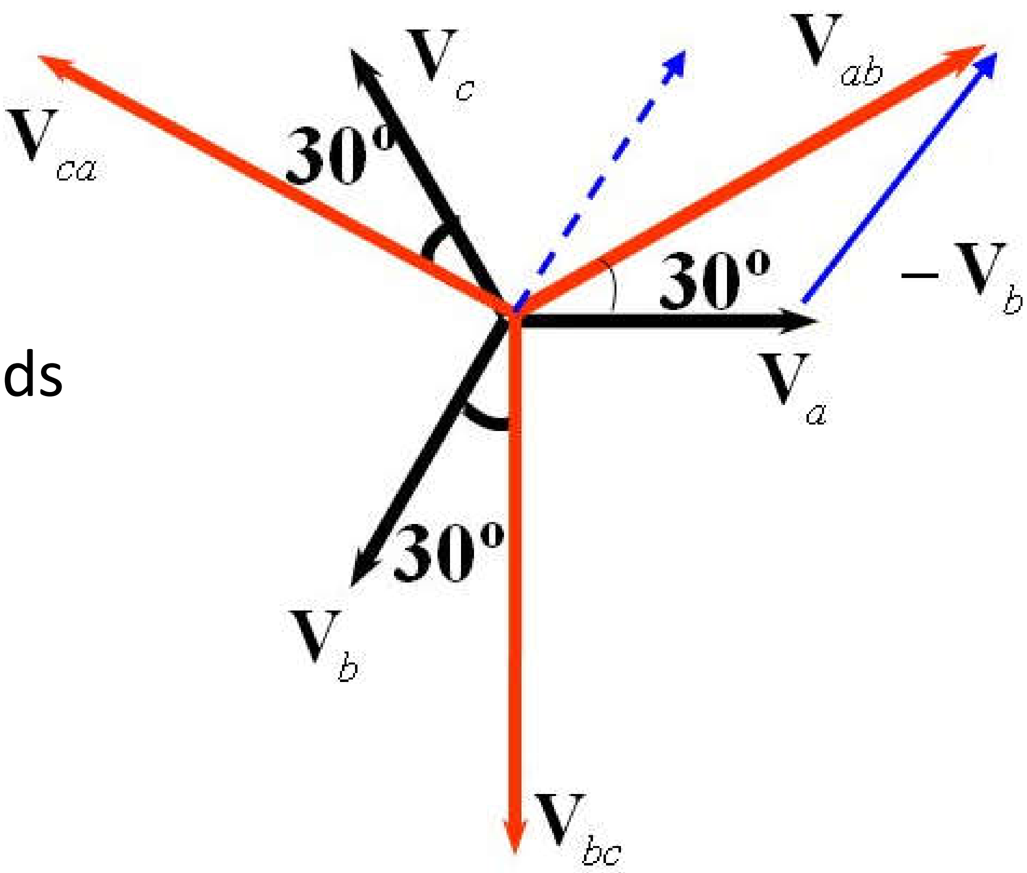
$$\boxed{V_L = \sqrt{3}V_p}$$

Balanced Three-phase Supply – *Voltages in Y connection*

- Phase Relationships of Voltages in Y Connections

For a balanced three-phase voltages:

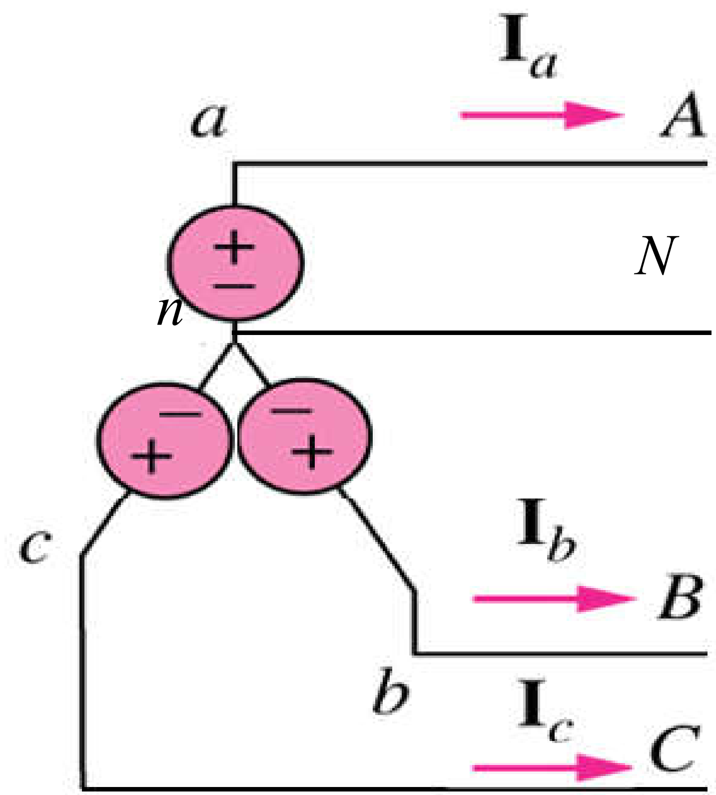
$$V_L = \sqrt{3} V_p$$



Each line-to-line voltage leads its corresponding phase voltage by 30°.

Balanced Three-phase Supply – Currents in Y connection

- Line currents are the same as phase currents, $I_p = I_L$.
- In a balanced or 4-wire system $\mathbf{I}_a = \mathbf{V}_{an}/\mathbf{Z}_{an}$
- If the load is balanced line currents form a balanced set and if one current is known, the other five currents can be determined by inspection.



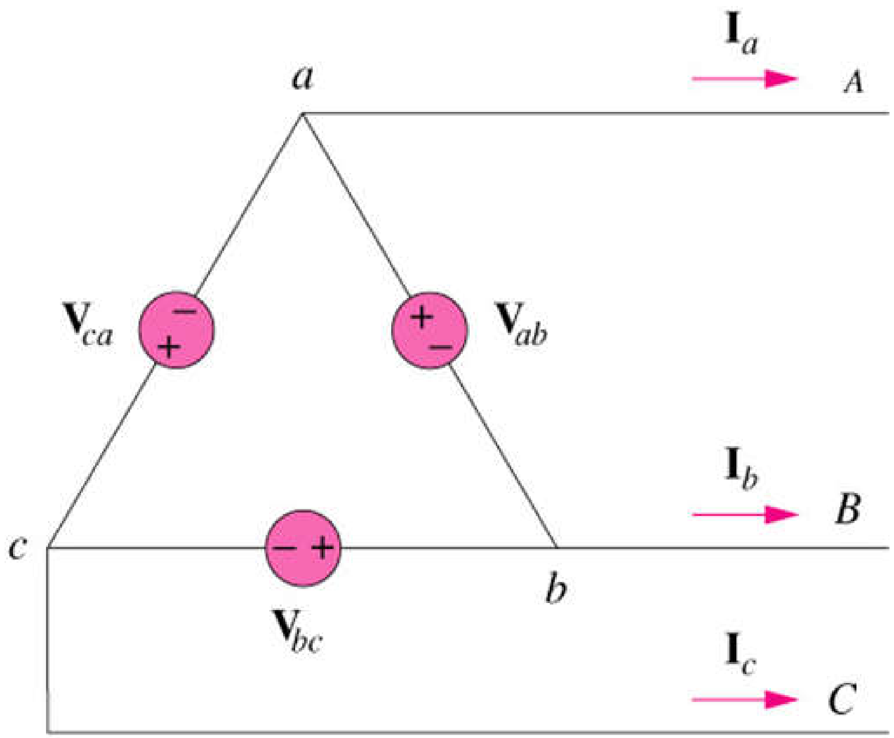
Balanced Three-phase Supply – Summary of Y connection

- **Voltages:**
  1. For a balanced Y system, the magnitude of line-to-line voltage is $\sqrt{3}$ times the magnitude of the phase voltage.
  2. Each line-to-line voltage leads its corresponding phase voltage by 30°.
  3. The line voltages and phase voltages both are symmetrical.

- **Currents:**
  - Line current is equal to phase current $I_L = I_P$, and symmetrical.

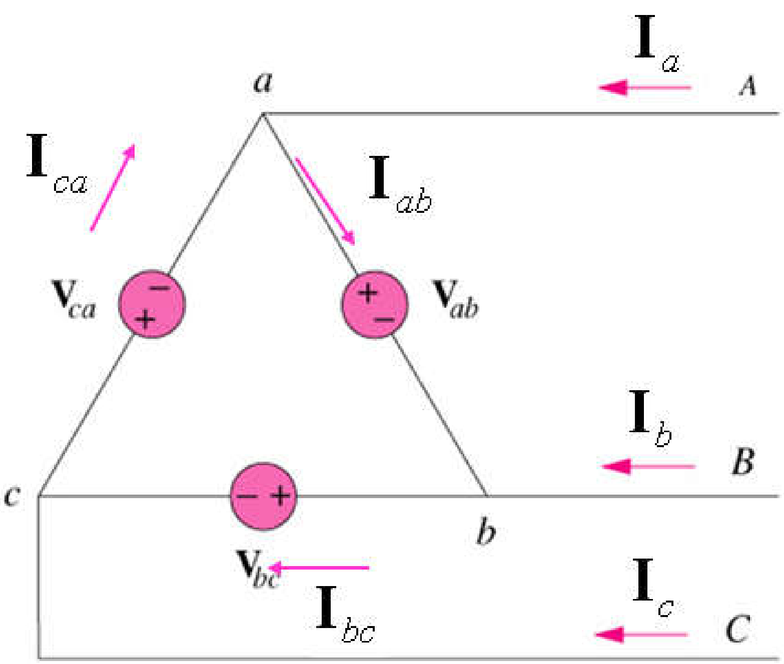
Balanced Three-phase Supply – Δ connection

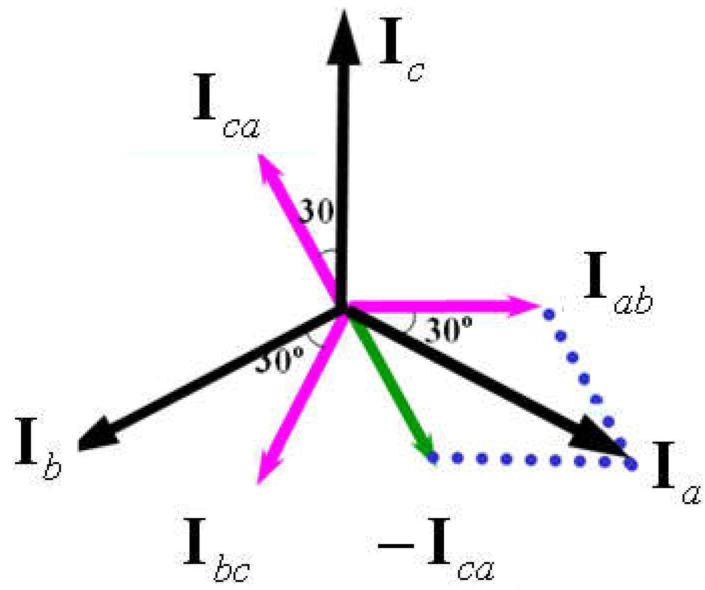
- The end of one coil is connected to the start of the next coil to form the loop.



Line voltage is identical to the corresponding phase voltage $V_L = V_P$.

Balanced Three-phase Supply – Currents in Δ connection





$$\mathbf{I}_a = \mathbf{I}_{ab} - \mathbf{I}_{ca} = \sqrt{3}\mathbf{I}_{ab}\angle -30^\circ$$

$$\mathbf{I}_b = \mathbf{I}_{bc} - \mathbf{I}_{ab} = \sqrt{3}\mathbf{I}_{bc}\angle -30^\circ$$

$$\mathbf{I}_c = \mathbf{I}_{ca} - \mathbf{I}_{bc} = \sqrt{3}\mathbf{I}_{ca}\angle -30^\circ$$

$$I_L = \sqrt{3} I_p, \text{ where}$$

$$I_L = |\mathbf{I}_a| = |\mathbf{I}_b| = |\mathbf{I}_c|$$

$$I_p = |\mathbf{I}_{ab}| = |\mathbf{I}_{bc}| = |\mathbf{I}_{ca}|$$

Balanced Three-phase Supply – Summary of Δ connection

- **Currents:**
  1. The magnitude of line current is $\sqrt{3}$ times as that of phase current
  2. Line current lags the corresponding phase current by 30°.
  3. Line currents form a balanced set.

- **Voltages:**
  – Line voltage is identical to the corresponding phase voltage $V_L = V_P$

Quiz

- 1. What is the phase sequence of a three-phase motor for which $V_{AN} = 220\angle -100°$ V and $V_{BN} = 220\angle 140°$ V?
  - (a) abc
  - (b) acb

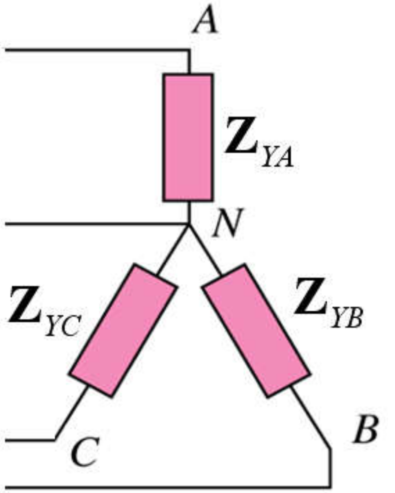
- 2. For a three-phase supply with the "abc" phase sequence, if $V_{BN} = 220\angle 140°$ V, which is the expression of V_{CA}?
  - (a) $380\angle -130°$ V;
  - (b) $380\angle 50°$ V;
  - (c) $380\angle 130°$ V;
  - (d) $380\angle 170°$ V;

Balanced Three-phase Supply – Balanced load

- A balanced three-phase circuit: a balanced three-phase supply,
  a balanced three-phase load.
- A balanced three-phase load is one in which the phase impedances are equal in magnitude and in phase.

Y connection

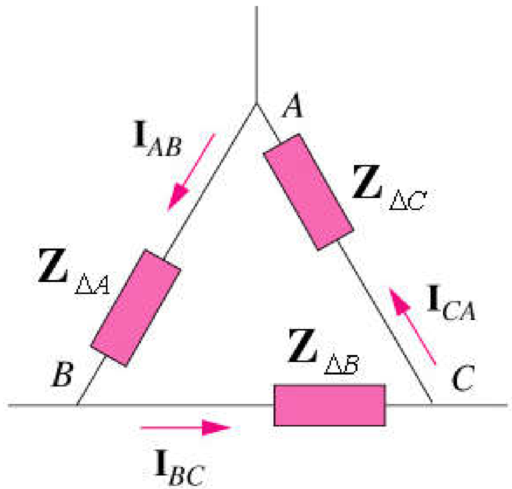
$$\mathbf{Z}_Y = \mathbf{Z}_{YA} = \mathbf{Z}_{YB} = \mathbf{Z}_{YC}$$



$$\boxed{\begin{aligned} Z_Y &= Z_\Delta/3 \\ Z_\Delta &= 3Z_Y \end{aligned}}$$

Δ connection

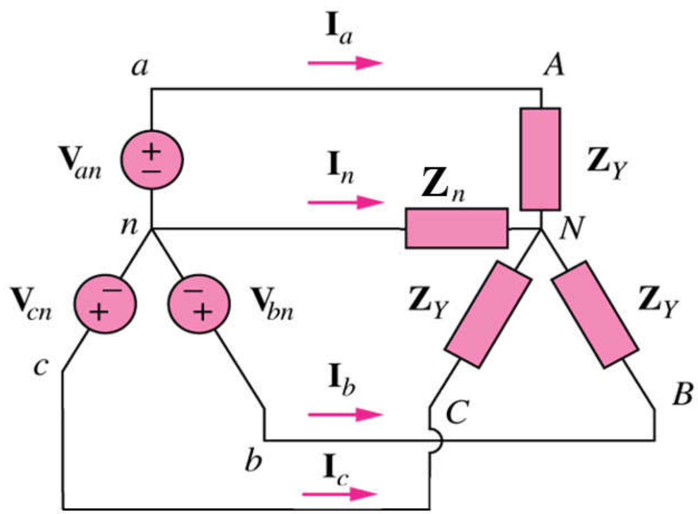
$$\mathbf{Z}_\Delta = \mathbf{Z}_{\Delta A} = \mathbf{Z}_{\Delta B} = \mathbf{Z}_{\Delta C}$$



Balanced Three-phase Supply – Balanced Y-Y connection

- A balanced Y-Y system is a three-phase system with a balanced Y-connected source and a balanced Y-connected load.

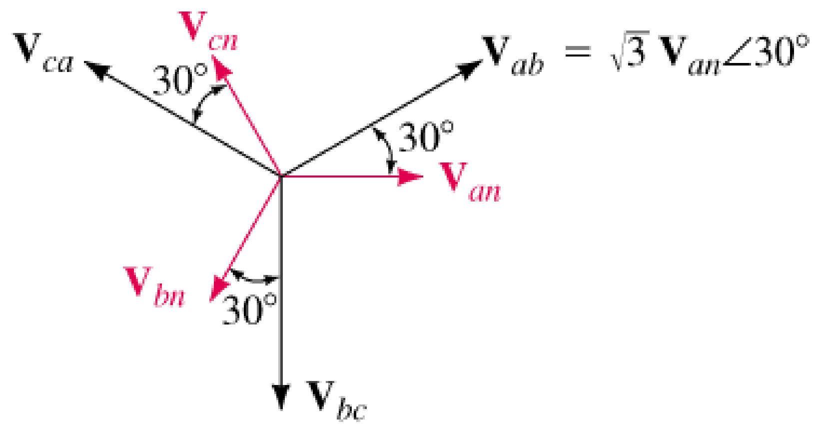
$$\begin{cases} \mathbf{V}_a = V_p\angle 0^\circ \\ \mathbf{V}_b = V_p\angle(-120^\circ) \\ \mathbf{V}_c = V_p\angle(-240^\circ) \end{cases} \qquad \begin{cases} \mathbf{V}_{ab} = \mathbf{V}_a - \mathbf{V}_b = \sqrt{3}V_p\angle 30^\circ \\ \mathbf{V}_{bc} = \mathbf{V}_b - \mathbf{V}_c = \sqrt{3}V_p\angle(-90^\circ) \\ \mathbf{V}_{ca} = \mathbf{V}_c - \mathbf{V}_a = \sqrt{3}V_p\angle(-210^\circ) \end{cases}$$



$$\boxed{V_L = \sqrt{3}V_p}$$

where $\quad V_p = |\mathbf{V}_a| = |\mathbf{V}_b| = |\mathbf{V}_c|$

$$V_L = |\mathbf{V}_{ab}| = |\mathbf{V}_{bc}| = |\mathbf{V}_{ca}| = \sqrt{3}V_p$$



Balanced Three-phase Supply – Balanced Y-Y connection

- A balanced Y-Y system is a three-phase system with a balanced Y-connected source and a balanced Y-connected load.

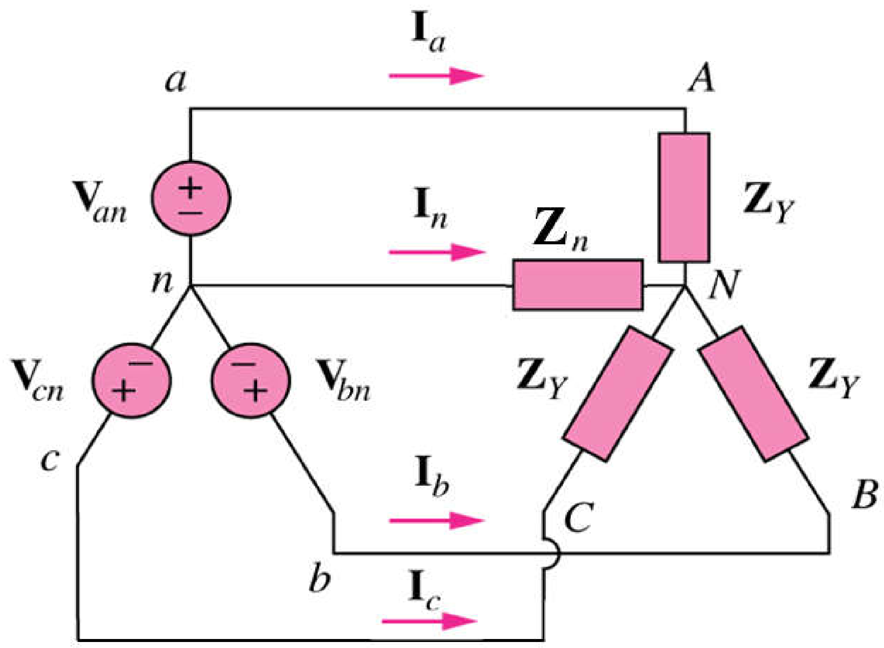
$$\mathbf{I}_a = \frac{\mathbf{V}_a}{\mathbf{Z}_Y}$$

$$\mathbf{I}_b = \frac{\mathbf{V}_b}{\mathbf{Z}_Y} = \frac{\mathbf{V}_a\angle(-120^\circ)}{\mathbf{Z}_Y} = \mathbf{I}_a\angle(-120^\circ)$$

$$\mathbf{I}_c = \frac{\mathbf{V}_c}{\mathbf{Z}_Y} = \frac{\mathbf{V}_a\angle(-240^\circ)}{\mathbf{Z}_Y} = \mathbf{I}_a\angle(-240^\circ)$$

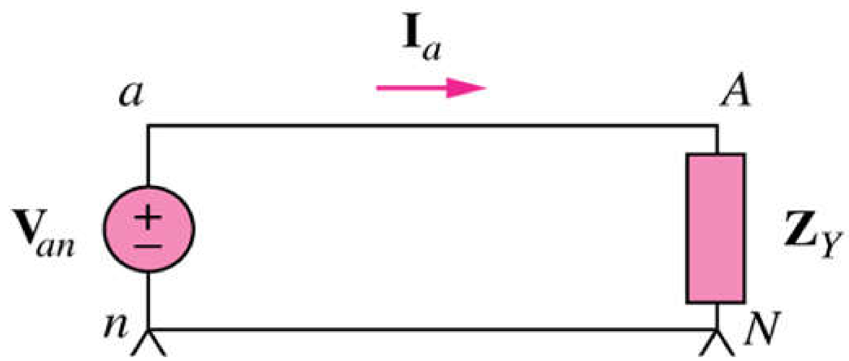
$$\mathbf{I}_n = \mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c = 0$$

$$\mathbf{V}_{nN} = \mathbf{Z}_n\mathbf{I}_n = 0$$



Balanced Three-phase Supply – Balanced Y-Y connection

- Example: Line voltage 380 $\angle 30°$ V and load $100\angle 30°\Omega$. Find the line currents.



- Solution:

Choose phase A :

Since $\mathbf{V}_{ab} = 380\angle 30^\circ$,

so the phase voltage: $\mathbf{V}_a = \dfrac{380}{\sqrt{3}}\angle 0^\circ = 220\angle 0^\circ \quad \text{V}$

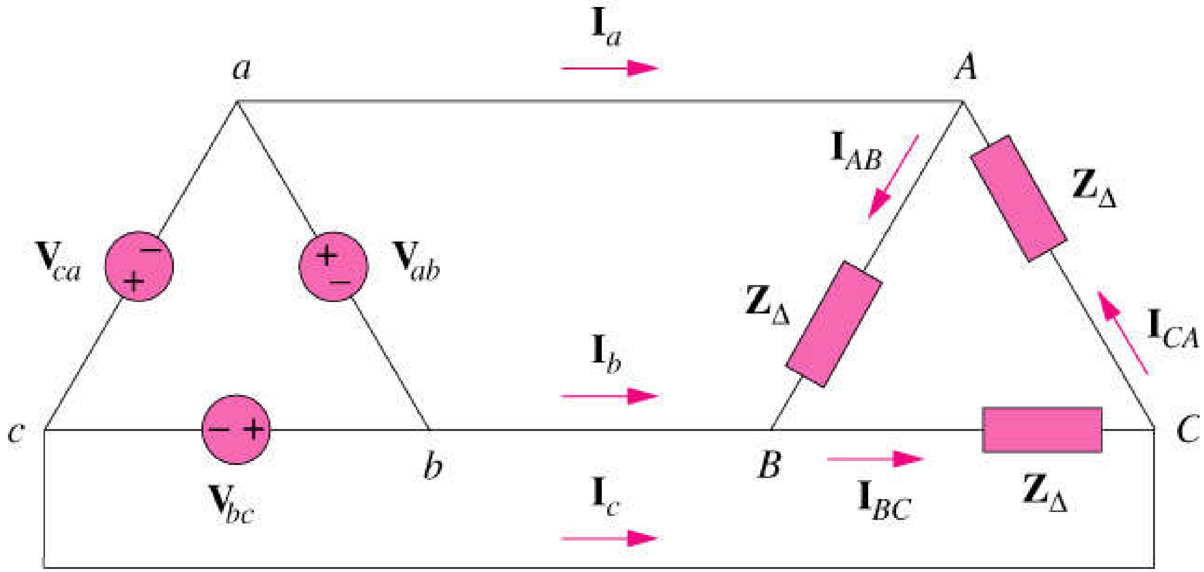
Then the current: $\mathbf{I}_a = \dfrac{\mathbf{V}_a}{\mathbf{Z}} = \dfrac{220\angle 0^\circ}{100\angle 30^\circ} = 2.2\angle(-30^\circ) \quad \text{A}$

By being symmetrical: $\mathbf{I}_b = 2.2\angle(-150^\circ) \quad \text{A}$

$\mathbf{I}_c = 2.2\angle 90^\circ \quad \text{A}$

Balanced Three-phase Supply – Balanced Δ-Δ connection

- A balanced Δ-Δ system is a three-phase system with a balanced Δ-connected source and a balanced Δ-connected load.



$$\mathbf{V}_{ab} = V_p\angle 0^\circ \qquad \mathbf{V}_{ab} = \mathbf{V}_{AB}$$

$$\mathbf{V}_{bc} = V_p\angle(-120^\circ) \qquad \mathbf{V}_{bc} = \mathbf{V}_{BC}$$

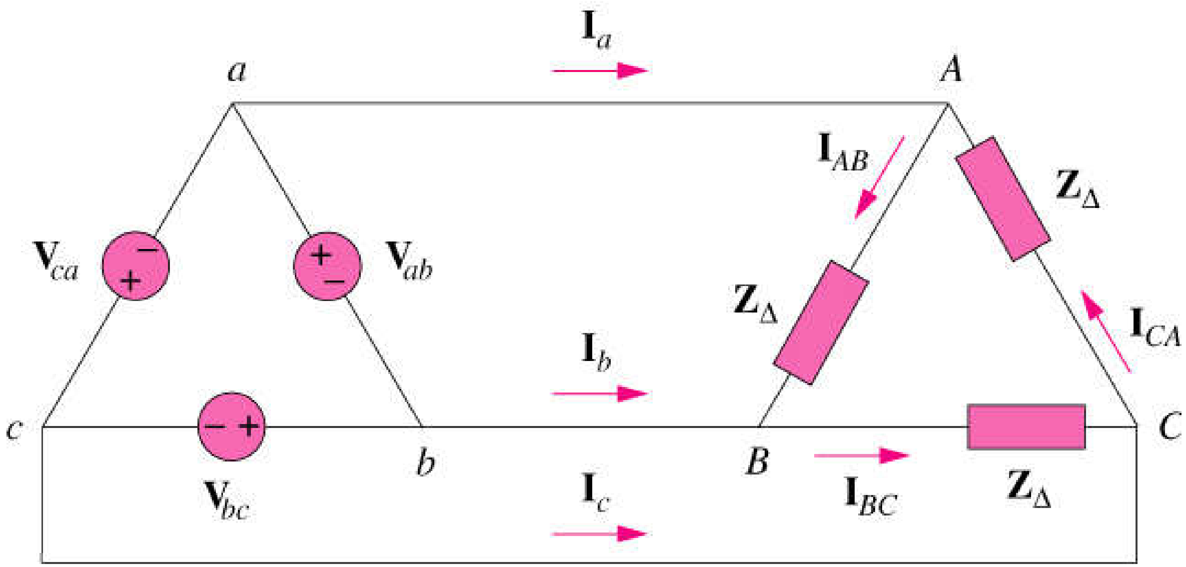
$$\mathbf{V}_{ca} = V_p\angle(-240^\circ) \qquad \mathbf{V}_{ca} = \mathbf{V}_{CA}$$

Balanced Three-phase Supply – Balanced Δ-Δ connection

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta} = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_\Delta}$$

$$\mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{\mathbf{Z}_\Delta} = \frac{\mathbf{V}_{bc}}{\mathbf{Z}_\Delta}$$

$$\mathbf{I}_{CA} = \frac{\mathbf{V}_{CA}}{\mathbf{Z}_\Delta} = \frac{\mathbf{V}_{ca}}{\mathbf{Z}_\Delta}$$



$$\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA} = \sqrt{3}\mathbf{I}_{AB}\angle(-30^\circ)$$

$$\mathbf{I}_b = \mathbf{I}_{BC} - \mathbf{I}_{AB} = \sqrt{3}\mathbf{I}_{AB}\angle(-150^\circ)$$

$$\mathbf{I}_c = \mathbf{I}_{CA} - \mathbf{I}_{BC} = \sqrt{3}\mathbf{I}_{AB}\angle(-270^\circ)$$

$$I_L = \sqrt{3}I_p \qquad \text{where} \qquad I_L = |\mathbf{I}_a| = |\mathbf{I}_b| = |\mathbf{I}_c|$$

$$I_p = |\mathbf{I}_{AB}| = |\mathbf{I}_{BC}| = |\mathbf{I}_{CA}|$$

Balanced Three-phase Supply – Balanced Δ-Δ connection

- Example: A balanced Δ-connected load having an impedance 20 - j15 Ω is connected to a balanced Δ-connected positive-sequence generator having Vab = 330∠0^o V. Calculate the phase currents of the load and the line currents.
- Solution:

$$\mathbf{Z}_\Delta = 20\text{-}j15 = \sqrt{20^2+15^2}\,arctg\frac{-15}{20} = 25\angle(-36.87°)$$

The phase currents

$$\begin{cases} \mathbf{I}_{AB} = \dfrac{\mathbf{V}_{ab}}{\mathbf{Z}_\Delta} = \dfrac{330\angle 0°}{25\angle(-36.87°)} = 13.2\angle 36.87° \quad \text{A} \\ \mathbf{I}_{BC} = 13.2\angle(36.87°-120°) = 13.2\angle(-83.13°) \ \text{A} \\ \mathbf{I}_{AB} = 13.2\angle(36.87°-240°) = 13.2\angle(-201.13°) \ \text{A} \end{cases}$$

The line currents

$$\begin{cases} \mathbf{I}_a = \sqrt{3}\mathbf{I}_{AB}\angle(-30°) = 22.86\angle 6.87° \quad \text{A} \\ \mathbf{I}_b = 22.86\angle(6.87°-120°) = 22.86\angle(-113.13°) \ \text{A} \\ \mathbf{I}_c = 22.86\angle(6.87°-240°) = 22.86\angle(-233.13°) \ \text{A} \end{cases}$$

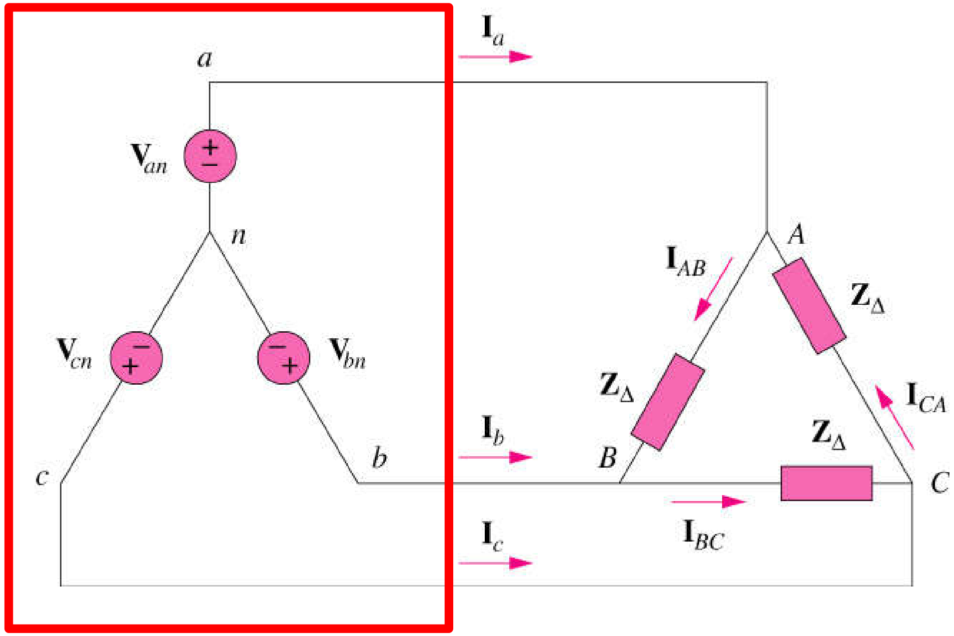
Balanced Three-phase Supply – Balanced Y-Δ connection

- A balanced Y-Δ system is a three-phase system with a balanced Y-connected source and a balanced Δ-connected load.

$$\mathbf{V}_{an} = V_p \angle 0^\circ$$

$$\mathbf{V}_{bn} = V_p \angle (-120^\circ)$$

$$\mathbf{V}_{cn} = V_p \angle (-240^\circ)$$



$$\mathbf{V}_{ab} = \mathbf{V}_{an} - \mathbf{V}_{bn} = \sqrt{3} V_p \angle 30^\circ = \mathbf{V}_{AB}$$

$$\mathbf{V}_{bc} = \mathbf{V}_{bn} - \mathbf{V}_{cn} = \sqrt{3} V_p \angle (-90^\circ) = \mathbf{V}_{BC}$$

$$\mathbf{V}_{ca} = \mathbf{V}_{cn} - \mathbf{V}_{an} = \sqrt{3} V_p \angle (-210^\circ) = \mathbf{V}_{CA}$$

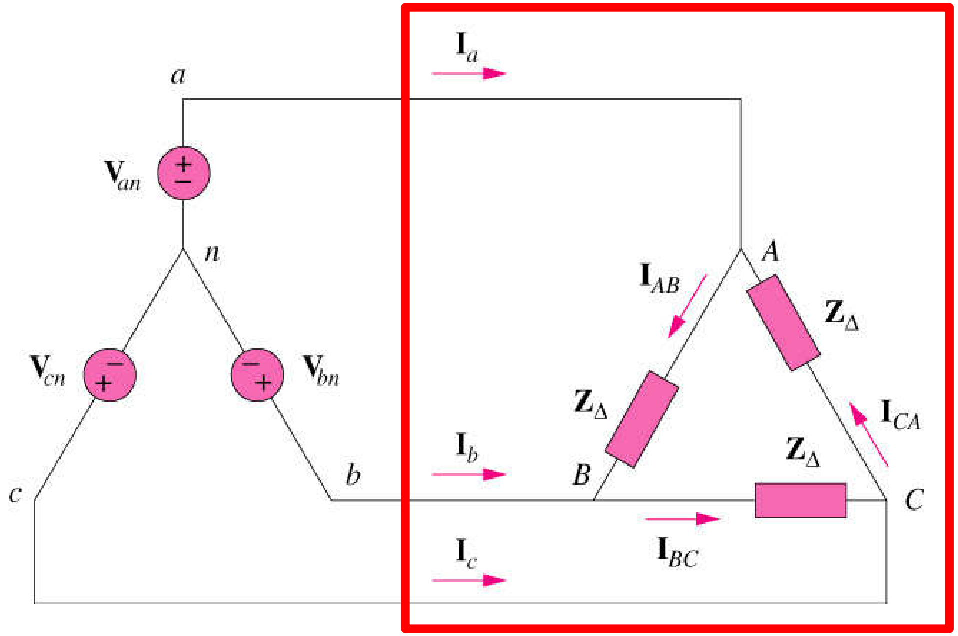
Balanced Three-phase Supply – *Balanced Y-Δ connection*

$$\mathbf{I}_{AB}=\frac{\mathbf{V}_{AB}}{\mathbf{Z}_{\Delta}},\qquad \mathbf{I}_{BC}=\frac{\mathbf{V}_{BC}}{\mathbf{Z}_{\Delta}},\qquad \mathbf{I}_{CA}=\frac{\mathbf{V}_{CA}}{\mathbf{Z}_{\Delta}}$$

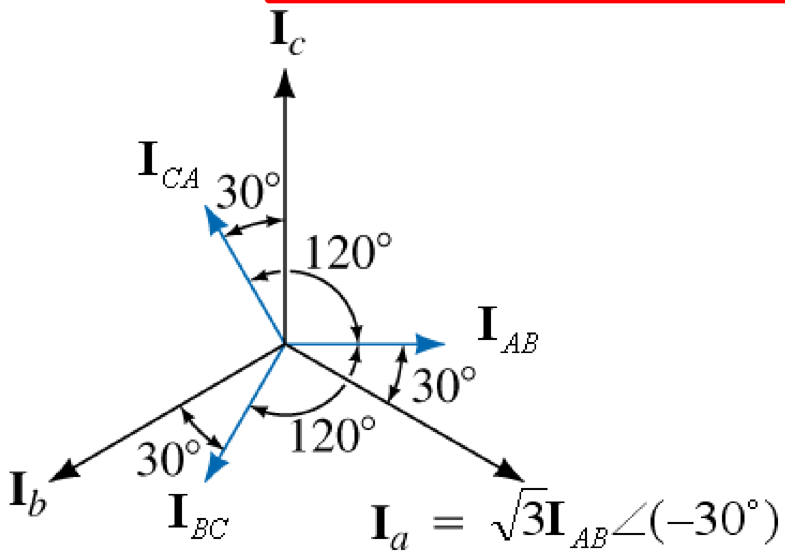
$$\mathbf{I}_a=\mathbf{I}_{AB}-\mathbf{I}_{CA}=\sqrt{3}\mathbf{I}_{AB}\angle(-30^\circ)$$

$$\mathbf{I}_b=\mathbf{I}_{BC}-\mathbf{I}_{AB}=\sqrt{3}\mathbf{I}_{AB}\angle(-150^\circ)$$

$$\mathbf{I}_c=\mathbf{I}_{CA}-\mathbf{I}_{BC}=\sqrt{3}\mathbf{I}_{AB}\angle(-270^\circ)$$



$$\boxed{I_L=\sqrt{3}I_p}$$



where $\quad I_L=|\mathbf{I}_a|=|\mathbf{I}_b|=|\mathbf{I}_c|$

$\qquad\quad I_p=|\mathbf{I}_{AB}|=|\mathbf{I}_{BC}|=|\mathbf{I}_{CA}|$

Balanced Three-phase Supply – Balanced Y-Δ connection

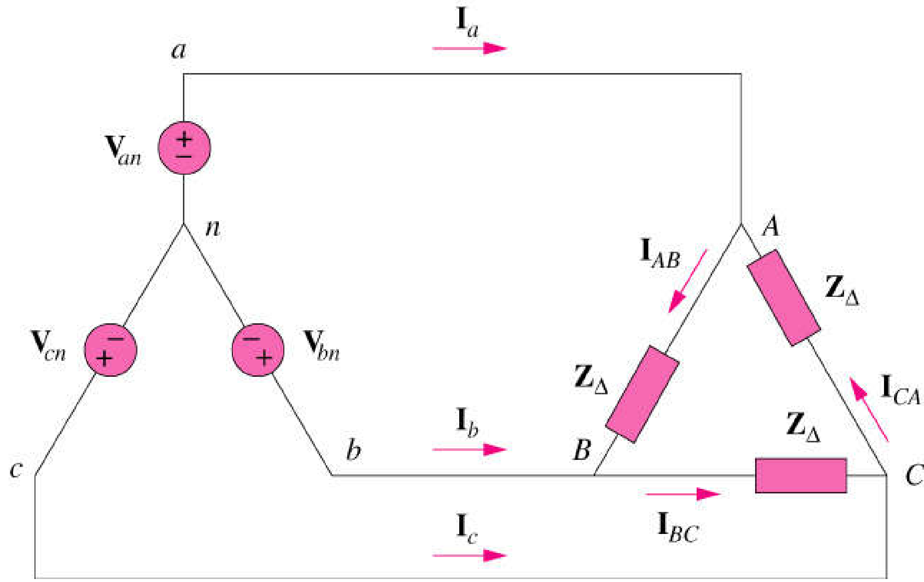
- Example: A three-phase system with a balanced Y-connected source of line voltage 415 V supplies a balanced delta connected load. The load is purely resistive, of 10 Ω resistance per phase. Calculate the currents in the system.
- Solution:

Line voltage of the system is 415 V. As load is delta-connected, this voltage appears across each of the three resistances. Phase current in the load is then:

415 / 10 = 41.5 A

Line current in the system is:

√3 x 41.5 =71.88 A



$$\mathbf{I}_{AB} = 41.5\angle 0^\circ \text{ A}, \qquad \mathbf{I}_{BC} = 41.5\angle(-120^\circ) \text{ A}, \qquad \mathbf{I}_{CA} = 41.5\angle 120^\circ \text{ A}$$

$$\mathbf{I}_a = 71.88\angle(-30^\circ) \text{ A}, \quad \mathbf{I}_b = 71.88\angle(-150^\circ) \text{ A}, \qquad \mathbf{I}_c = 71.88\angle(-270^\circ) \text{ A}$$

Balanced Three-phase Supply – Balanced Δ-Y connection

- A balanced Δ-Y system is a three-phase system with a balanced Δ-connected source and a balanced Y-connected load.

$$\mathbf{V}_{ab} = V_p \angle 0^\circ$$

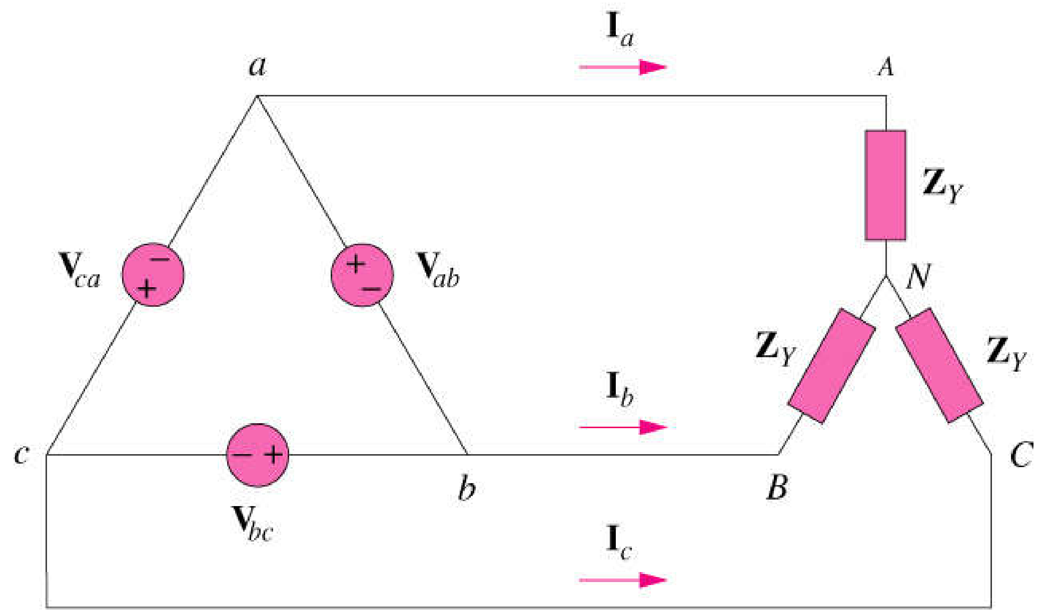
$$\mathbf{V}_{bc} = V_p \angle(-120^\circ)$$

$$\mathbf{V}_{ca} = V_p \angle(-240^\circ)$$

$$\mathbf{V}_{AN} = \frac{V_p}{\sqrt{3}} \angle(-30^\circ)$$

$$\mathbf{V}_{BN} = \frac{V_p}{\sqrt{3}} \angle(-150^\circ)$$

$$\mathbf{V}_{CN} = \frac{V_p}{\sqrt{3}} \angle(-270^\circ)$$

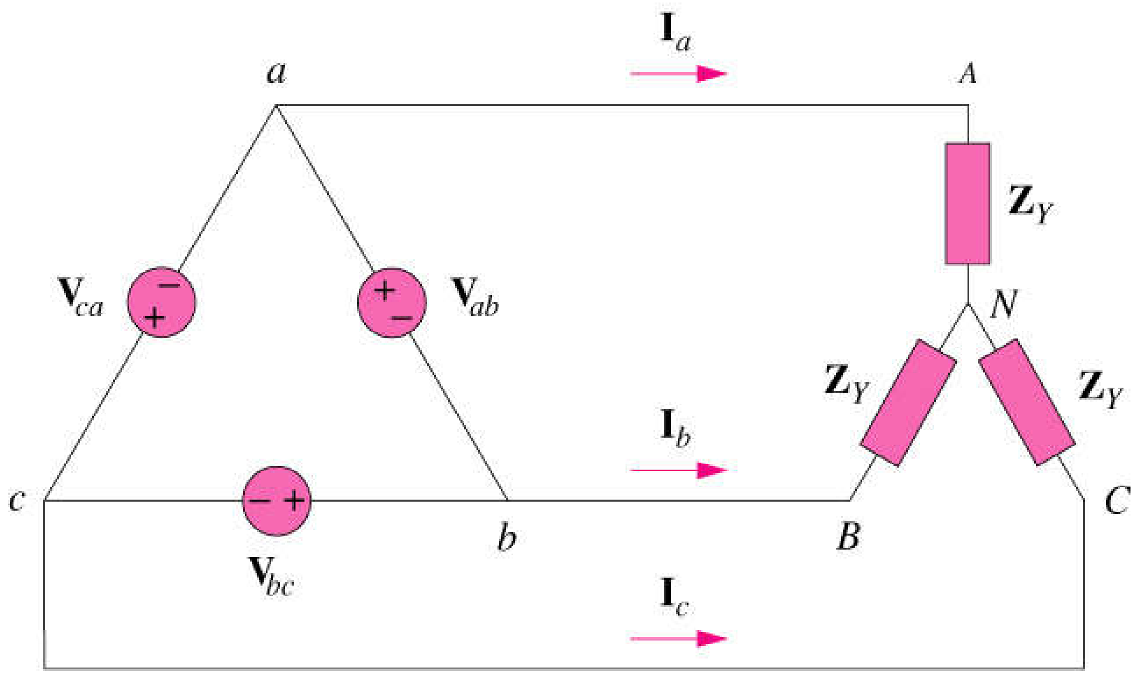


Balanced Three-phase Supply – Balanced Δ-Y connection

$$\mathbf{I}_a = \frac{\dfrac{V_p}{\sqrt{3}}\angle(-30^\circ)}{\mathbf{Z}_Y}$$

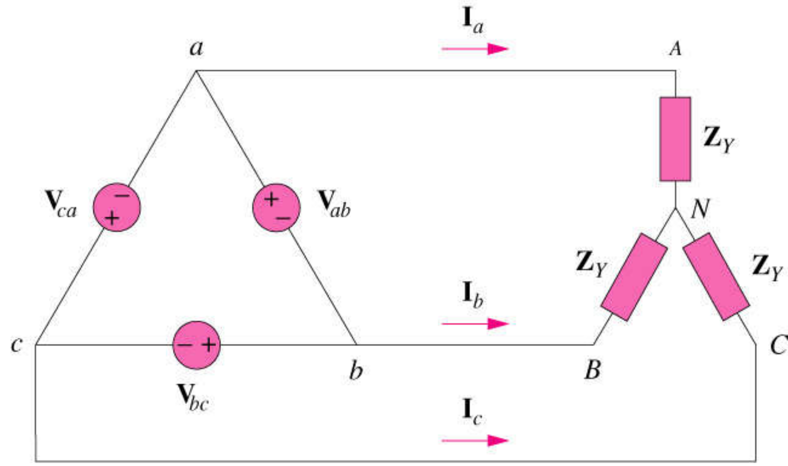
$$\mathbf{I}_b = \mathbf{I}_a\angle(-120^\circ)$$

$$\mathbf{I}_c = \mathbf{I}_a\angle(-240^\circ)$$



Balanced Three-phase Supply – Balanced Δ-Y connection

- Example: A balanced Y-connected load with a phase impedance 40 + *j*25 Ω is supplied by a balanced, positive-sequence Δ-connected source with a line voltage of 210 V. Calculate the phase currents. Use $\mathbf{V}_{ab}$ as reference.
- Solution:



The load impedance: $\mathbf{Z}_Y = 40 + j25 = 47.17\angle 32^\circ\ \Omega$

Source voltage: $\mathbf{V}_{ab} = 210\angle 0^\circ\ \text{V}$

The phase currents are the same as the line currents:

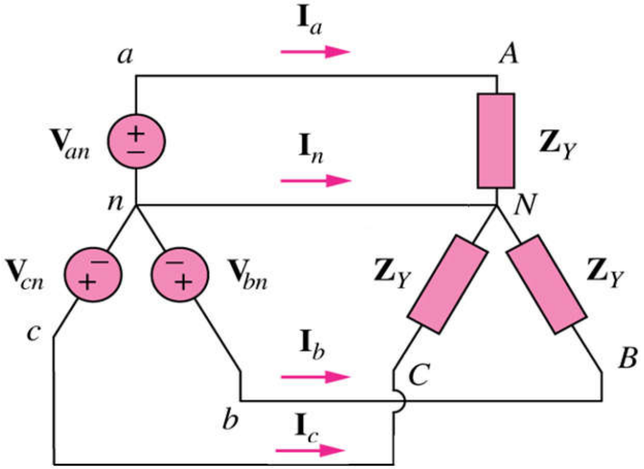
$$\mathbf{I}_{AN} = \mathbf{I}_a = \frac{\frac{\mathbf{V}_{ab}}{\sqrt{3}}\angle(-30^\circ)}{\mathbf{Z}_Y} = \frac{121.2\angle(-30^\circ)}{47.17\angle 32^\circ} = 2.57\angle(-62^\circ)\ \text{A}$$

$$\mathbf{I}_{BN} = \mathbf{I}_a\angle(-120^\circ) = 2.57\angle(-182^\circ)\ \text{A}$$

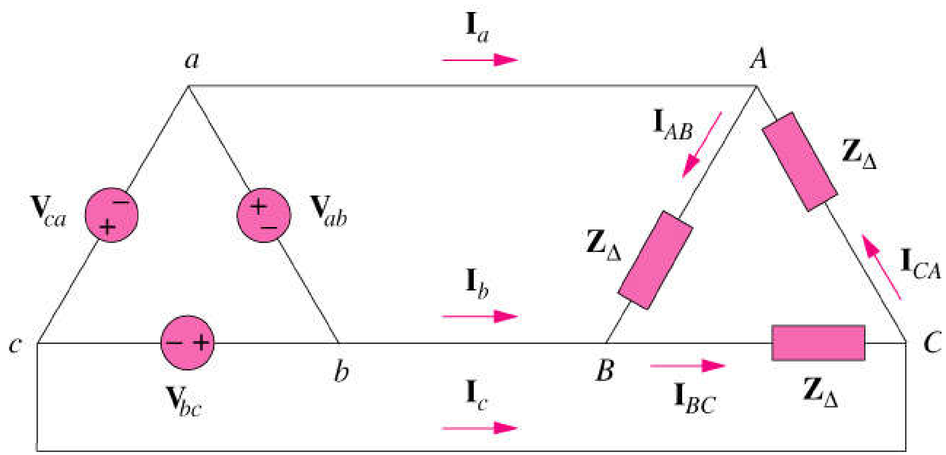
$$\mathbf{I}_{CN} = \mathbf{I}_a\angle 120^\circ = 2.57\angle 58^\circ\ \text{A}$$

Balanced Three-phase Supply – Summary

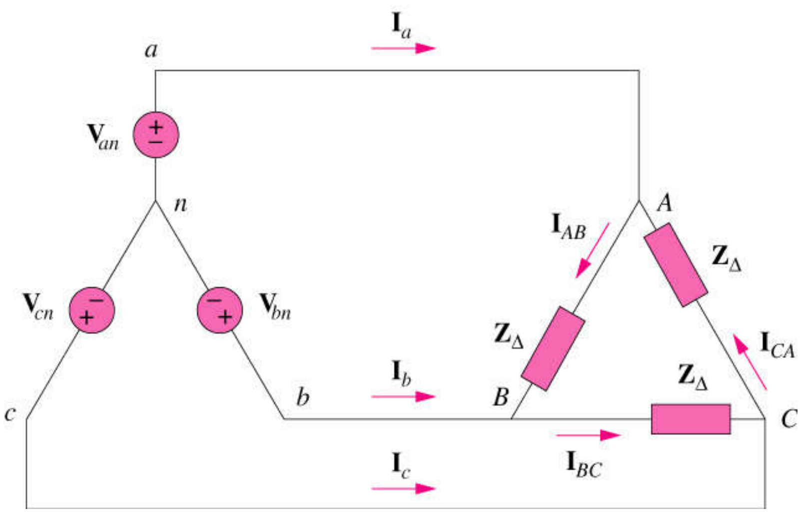
Y-Y connections



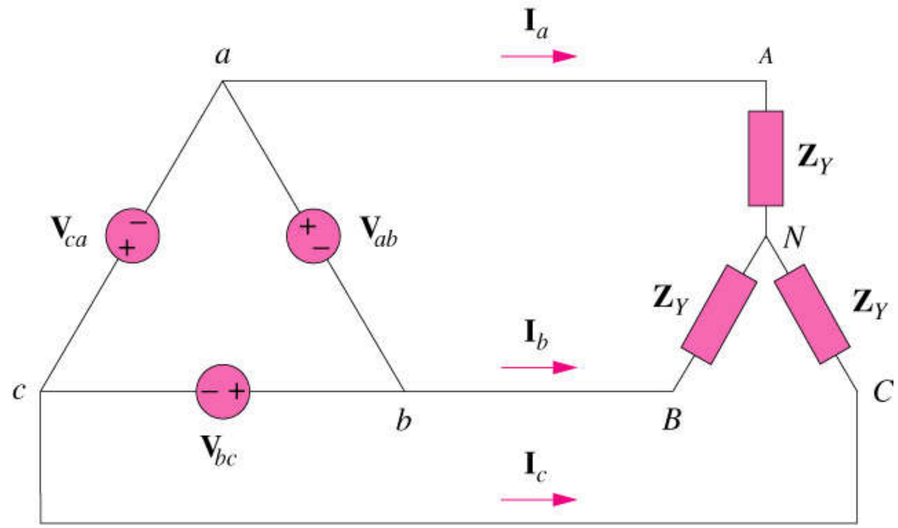
Δ - Δ connections



Y- Δ connections



Δ - Y connections



Balanced Three-phase Supply – Summary

Connection	Phase voltages	Phase currents	Line voltages	Line currents
$\mathrm{Y-Y}$	$\mathbf{V}_{an}=V_p\angle 0^\circ$ $\mathbf{V}_{bn}=V_p\angle -120^\circ$ $\mathbf{V}_{cn}=V_p\angle 120^\circ$	Same as line currents	$\mathbf{V}_{ab}=\sqrt{3}V_p\angle 30^\circ$ $\mathbf{V}_{bc}=\mathbf{V}_{ab}\angle -120^\circ$ $\mathbf{V}_{ca}=\mathbf{V}_{ab}\angle 120^\circ$	$\mathbf{I}_a=\mathbf{V}_{an}/\mathbf{Z}_Y$ $\mathbf{I}_b=\mathbf{I}_a\angle -120^\circ$ $\mathbf{I}_c=\mathbf{I}_a\angle 120^\circ$
$\mathrm{Y}-\Delta$	$\mathbf{V}_{an}=V_p\angle 0^\circ$ $\mathbf{V}_{bn}=V_p\angle -120^\circ$ $\mathbf{V}_{cn}=V_p\angle 120^\circ$	$\mathbf{I}_{AB}=\mathbf{V}_{AB}/\mathbf{Z}_\Delta$ $\mathbf{I}_{BC}=\mathbf{V}_{BC}/\mathbf{Z}_\Delta$ $\mathbf{I}_{CA}=\mathbf{V}_{CA}/\mathbf{Z}_\Delta$	$\mathbf{V}_{ab}=\mathbf{V}_{AB}=\sqrt{3}V_p\angle 30^\circ$ $\mathbf{V}_{bc}=\mathbf{V}_{BC}=\mathbf{V}_{ab}\angle -120^\circ$ $\mathbf{V}_{ca}=\mathbf{V}_{CA}=\mathbf{V}_{ab}\angle 120^\circ$	$\mathbf{I}_a=\sqrt{3}\mathbf{I}_{AB}\angle -30^\circ$ $\mathbf{I}_b=\mathbf{I}_a\angle -120^\circ$ $\mathbf{I}_c=\mathbf{I}_a\angle 120^\circ$
$\Delta-\Delta$	$\mathbf{V}_{ab}=V_p\angle 0^\circ$ $\mathbf{V}_{bc}=V_p\angle -120^\circ$ $\mathbf{V}_{ca}=V_p\angle 120^\circ$	$\mathbf{I}_{AB}=\mathbf{V}_{ab}/\mathbf{Z}_\Delta$ $\mathbf{I}_{BC}=\mathbf{V}_{bc}/\mathbf{Z}_\Delta$ $\mathbf{I}_{CA}=\mathbf{V}_{ca}/\mathbf{Z}_\Delta$	Same as phase voltages	$\mathbf{I}_a=\mathbf{I}_{AB}\sqrt{3}\angle -30^\circ$ $\mathbf{I}_b=\mathbf{I}_a\angle -120^\circ$ $\mathbf{I}_c=\mathbf{I}_a\angle 120^\circ$
$\Delta-\mathrm{Y}$	$\mathbf{V}_{ab}=V_p\angle 0^\circ$ $\mathbf{V}_{bc}=V_p\angle -120^\circ$ $\mathbf{V}_{ca}=V_p\angle 120^\circ$	Same as line currents	Same as phase voltages	$\mathbf{I}_a=V_p\angle 0^\circ/(\sqrt{3}\mathbf{Z}_Y)$ $\mathbf{I}_b=\mathbf{I}_a\angle -120^\circ$ $\mathbf{I}_c=\mathbf{I}_a\angle 120^\circ$

Quiz

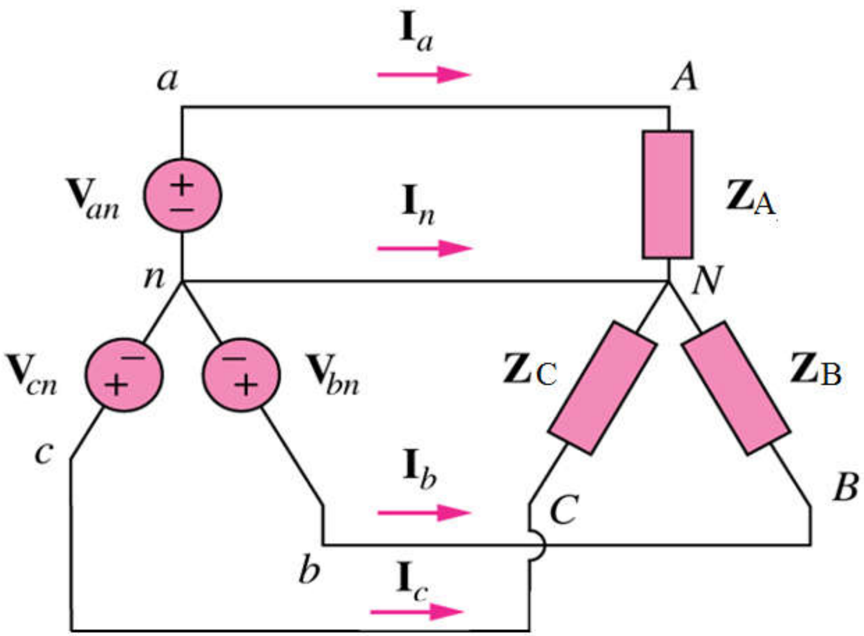
- 1. In a Y-connected load, the line current and phase current are equal.
  - (a) True
  - (b) False

- 2. In a Δ-Δ system, a phase voltage of 100 V produces a line voltage of:
  - (a) 71 V
  - (b) 141 V
  - (c) 100 V
  - (d) 173 V

Unbalanced Three-phase Circuits

An unbalanced system $\begin{cases} \text{unbalanced source – voltage} \\ \text{unbalanced load} \end{cases}$

In a unbalanced system the neutral current is NOT zero.

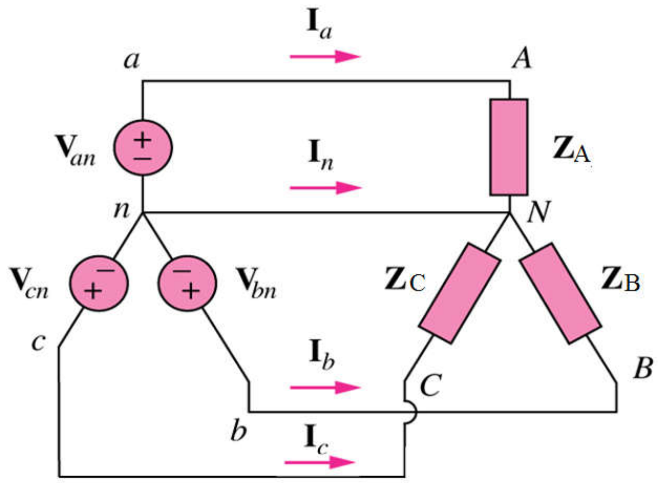


$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_A} \qquad \mathbf{I}_b = \frac{\mathbf{V}_{bn}}{\mathbf{Z}_B} \qquad \mathbf{I}_c = \frac{\mathbf{V}_{cn}}{\mathbf{Z}_C}$$

$$\mathbf{I}_n = -(\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c)$$
$$= -\left(\frac{\mathbf{V}_{an}}{\mathbf{Z}_A} + \frac{\mathbf{V}_{bn}}{\mathbf{Z}_B} + \frac{\mathbf{V}_{cn}}{\mathbf{Z}_C}\right) \neq 0$$

Unbalanced Three-phase Circuits – Example 1

- The unbalanced Y-load with each resistance respectively $R_A = 5\Omega$, $R_B = 10\Omega$, and $R_C = 20\Omega$ is supplied by a balanced Y-source with phase voltage V_p=220V.
- Find: load phase voltages, load currents and neutral current.
- Solution:



$$\mathbf{I}_a = \frac{\mathbf{V}_{AN}}{R_A} = \frac{220\angle 0^\circ}{5} = 44\angle 0^\circ \text{ A}$$

$$\mathbf{I}_b = \frac{\mathbf{V}_{BN}}{R_B} = \frac{220\angle(-120^\circ)}{10} = 22\angle(-120^\circ) \text{ A}$$

$$\mathbf{I}_c = \frac{\mathbf{V}_{CN}}{R_c} = \frac{220\angle 120^\circ}{20} = 11\angle 120^\circ \text{ A}$$

$$\begin{aligned}\mathbf{I}_n &= \mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c \\ &= 44\angle 0^\circ + 22\angle(-120^\circ) + 11\angle 120^\circ \\ &= 44 + (-11 - j18.9) + (-5.5 + j9.45) \\ &= 27.5 - j9.45 = 29.1\angle(-19^\circ) \text{ A}\end{aligned}$$

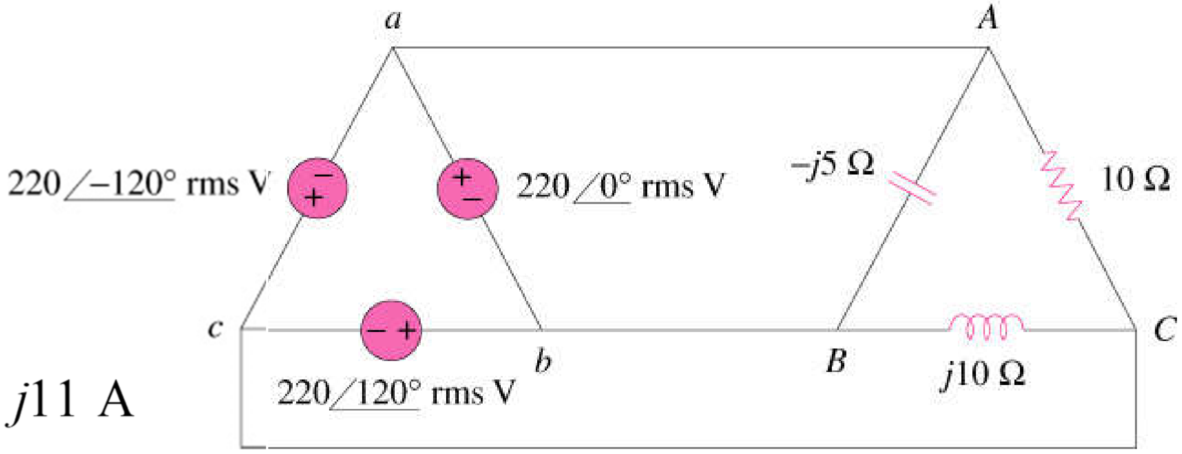
$$\mathbf{V}_{an} = \mathbf{V}_{AN} = V_p\angle 0^\circ \text{ V}$$

$$\mathbf{V}_{bn} = \mathbf{V}_{BN} = V_p\angle(-120^\circ) \text{ V}$$

$$\mathbf{V}_{cn} = \mathbf{V}_{CN} = V_p\angle(+120^\circ) \text{ V}$$

Unbalanced Three-phase Circuits – Example 2

- Find the line currents in the unbalanced three-phase circuit.
- Solution:



The phase currents:

$$\mathbf{I}_{AB} = \frac{220\angle 0^\circ}{-j5} = j44 \text{ A}$$

$$\mathbf{I}_{BC} = \frac{220\angle(+120^\circ)}{j10} = 22\angle 30^\circ = 19.05 + j11 \text{ A}$$

$$\mathbf{I}_{CA} = \frac{220\angle(-120^\circ)}{10} = 22\angle(-120^\circ) = -11 - j19.05 \text{ A}$$

The line currents

$$\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA} = (j44) - (-11 - j19.05) = 11 + j63.05 = 64\angle 80^\circ \text{ A}$$

$$\mathbf{I}_b = \mathbf{I}_{BC} - \mathbf{I}_{AB} = (19.05 + j11) - (j44) = 19.05 - j33 = 38\angle(-60^\circ) \text{ A}$$

$$\mathbf{I}_c = \mathbf{I}_{CA} - \mathbf{I}_{BC} = (-11 - j19.05) - (19.05 + j11) = -30.05 - j30.05 = 43\angle 225^\circ \text{ A}$$

Unbalanced Three-phase Circuits - Summary

- How to solve
  - Unbalanced four-wire Y systems without line impedance, use Ohm's law.
  - Three-wire and four-wire systems with line and neutral impedance require the use of mesh equations.
  - Voltages across each phase of the load may be different
  - Current between neutral points is not zero
  - Δ-Load with line impedances
    - Including line impedances with a Δ load makes the analysis more difficult. A good way to approach the problem is to use a Δ-Y conversion to change the Δ load into a Y load.

Unbalanced Three-phase Circuits – Δ-Y Conversion

Δ - Y Conversion Equations:

$$\mathbf{Z}_1 = \frac{\mathbf{Z}_b \cdot \mathbf{Z}_c}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c}$$

$$\mathbf{Z}_2 = \frac{\mathbf{Z}_c \cdot \mathbf{Z}_a}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c}$$

$$\mathbf{Z}_3 = \frac{\mathbf{Z}_a \cdot \mathbf{Z}_b}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c}$$

For a balanced load

$$\mathbf{Z}_Y = \frac{\mathbf{Z}_\Delta}{3}$$

Y - Δ Conversion Equations:

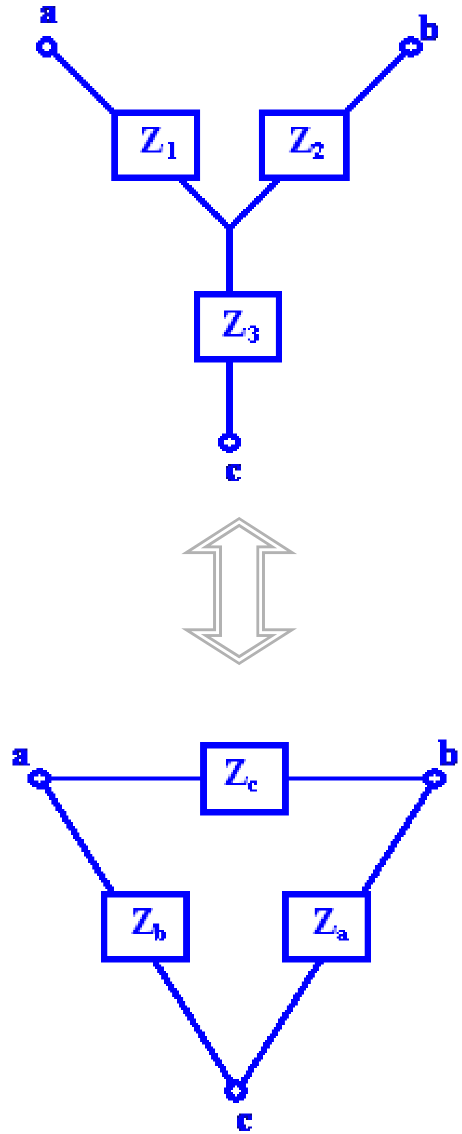
$$\mathbf{Z}_a = \frac{\mathbf{Z}_1 \cdot \mathbf{Z}_2 + \mathbf{Z}_2 \cdot \mathbf{Z}_3 + \mathbf{Z}_3 \cdot \mathbf{Z}_1}{\mathbf{Z}_1}$$

$$\mathbf{Z}_b = \frac{\mathbf{Z}_1 \cdot \mathbf{Z}_2 + \mathbf{Z}_2 \cdot \mathbf{Z}_3 + \mathbf{Z}_3 \cdot \mathbf{Z}_1}{\mathbf{Z}_2}$$

$$\mathbf{Z}_c = \frac{\mathbf{Z}_1 \cdot \mathbf{Z}_2 + \mathbf{Z}_2 \cdot \mathbf{Z}_3 + \mathbf{Z}_3 \cdot \mathbf{Z}_1}{\mathbf{Z}_3}$$

For a balanced load

$$\mathbf{Z}_\Delta = 3 \cdot \mathbf{Z}_Y$$

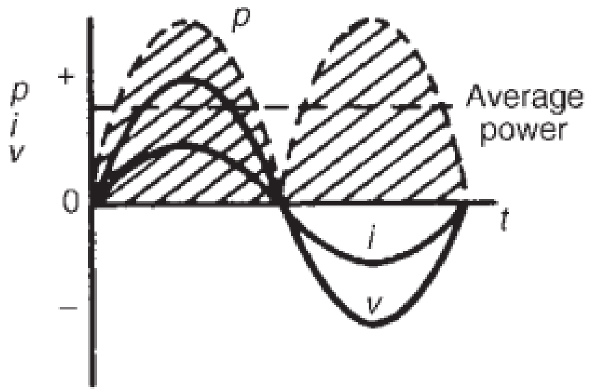


Review – Power in AC Circuit

- The power at any instant is given by the product of the voltage and current at that instant, i.e. the instantaneous power: $p = vi$

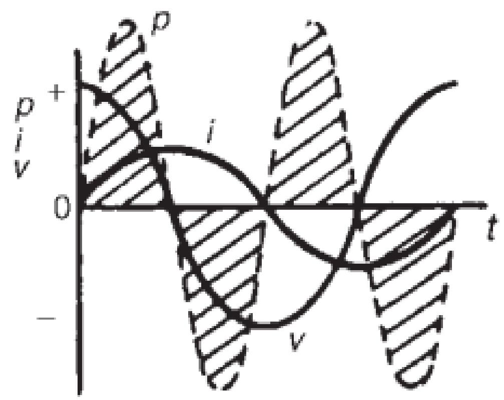
Average power dissipated $P = VI\cos\phi$ watts

Pure resistive



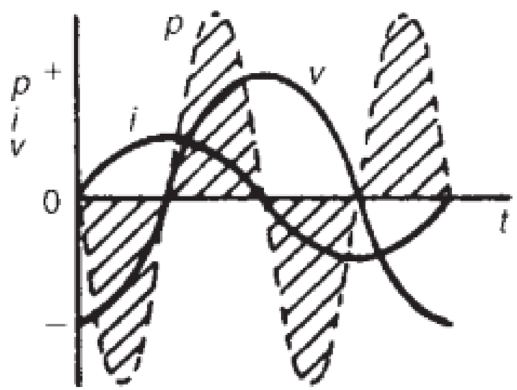
$$P = VI = I^2R = \frac{V^2}{R} \text{ watts}$$

Pure inductive



$P = 0$ watts

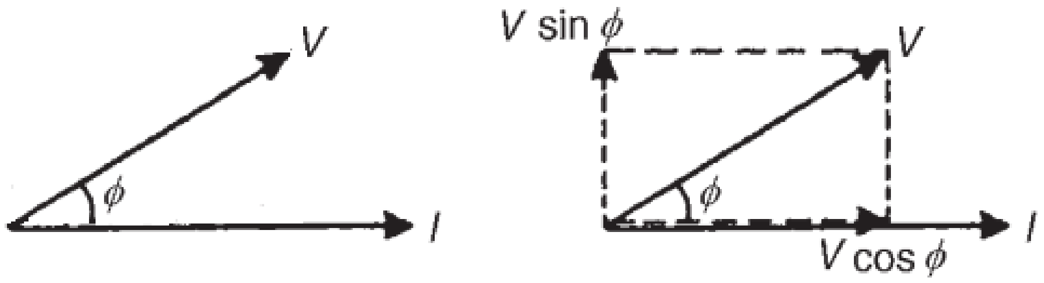
Pure capacitive



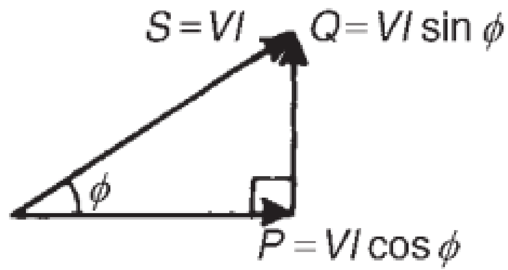
$P = 0$ watts

Review – Power triangle

- In the case where the current ***I*** lags the applied voltage ***V*** by angle φ.



Phasor diagram



Power triangle

- Three kinds of powers in AC circuit:

Apparent power,	$S = VI$ **voltamperes (VA)**
True or active power,	$P = VI \cos\phi$ **watts (W)** → *Average power dissipated*
Reactive power,	$Q = VI \sin\phi$ **reactive voltamperes (var)**

$$\text{Power factor} = \frac{\text{True power } P}{\text{Apparent power } S} \quad \rightarrow \quad \text{p.f.} = \cos\phi = \frac{R}{Z}$$

Review – Power evaluation - Example

- A series circuit of resistance 60 Ω and inductance 75 mH is connected to a 110 V, 60 Hz supply.
  - Calculate the power dissipated.
  - Calculate the power factor.

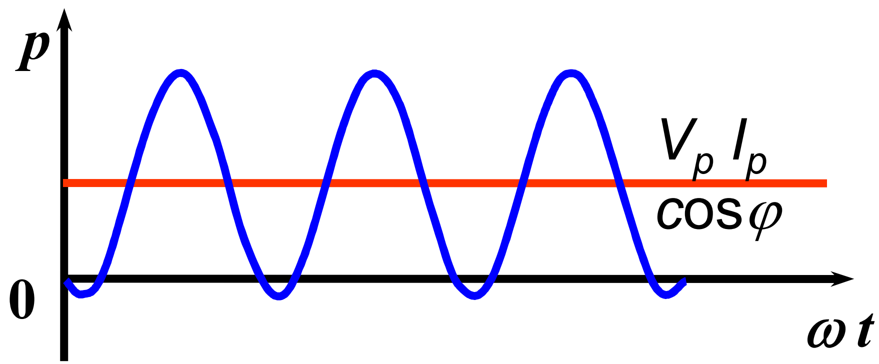
Three-phase Power - Real Power in a Balanced System

- The power in a three-phase system is the sum of the power in each phase.
- Three kinds of power used by power engineers: Real power, Apparent power and Reactive power.

Real power in phase A

If $v_A(t) = \sqrt{2}V_p \sin \omega t$

$i_A(t) = \sqrt{2}I_p \sin(\omega t - \varphi)$



$$p_A = v_A i_A = 2V_p I_p \sin \omega t \sin(\omega t - \varphi)$$

$$= V_p I_p \cos\varphi - V_p I_p \cos(2\omega t - \varphi)$$

Watts (W)

Xi'an Jiaotong-Liverpool University 西交利物浦大学

Three-phase Power - Real Power in a Balanced System

Similar:

$$p_B = v_B i_B = V_p I_p \cos\varphi - V_p I_p \cos[2(\omega t - 120^\circ) - \varphi]$$

$$p_C = v_C i_C = V_p I_p \cos\varphi - V_p I_p \cos[2(\omega t + 120^\circ) - \varphi]$$

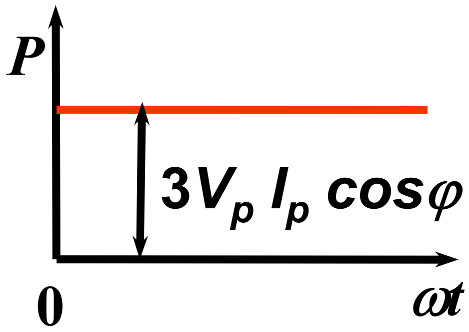
Watts (W)

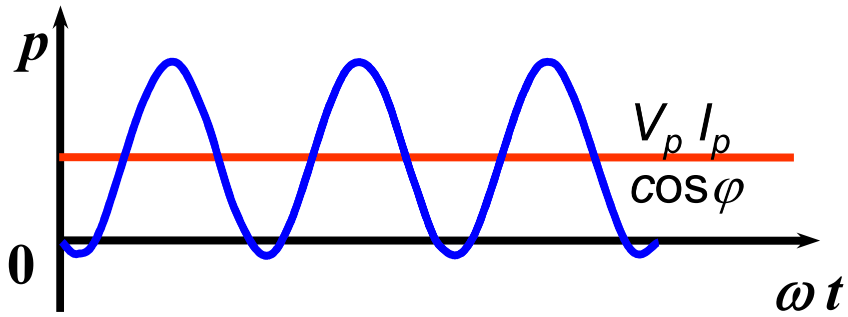
Total Power:

$$p = p_A + p_B + p_C = 3V_p I_p \cos\varphi = P$$ **Watts (W)**

The instantaneous total power is not function of time

The instantaneous phase power is impulse (function of time).





Three-phase Power - Real Power in a Balanced System

Y connected $V_L = \sqrt{3}\,V_p \quad I_L = I_p$

$$\boxed{P = 3V_p I_p \cos\varphi}$$

$$P = 3\frac{V_L}{\sqrt{3}} I_L \cos\varphi = \sqrt{3} V_L I_L \cos\varphi$$ **Watts (W)**

Δ connected $V_L = V_p \quad I_L = \sqrt{3}\,I_p$

$$P = 3V_L \frac{I_L}{\sqrt{3}} \cos\varphi = \sqrt{3} V_L I_L \cos\varphi$$ **Watts (W)**

Independent on the connection methods

Phase angle between V_P and I_P

Three-phase Power - *Reactive Power & Apparent Power*

- Reactive power
  - Reactive power is the product of the voltage, current and the sine of the phase angle between them:
  - Inductive reactive power is defined as positive power and capacitive reactive power is defined as negative reactive power.

$$Q = \sqrt{3} V_L I_L \sin\varphi$$

Volt amperes reactive (Var)

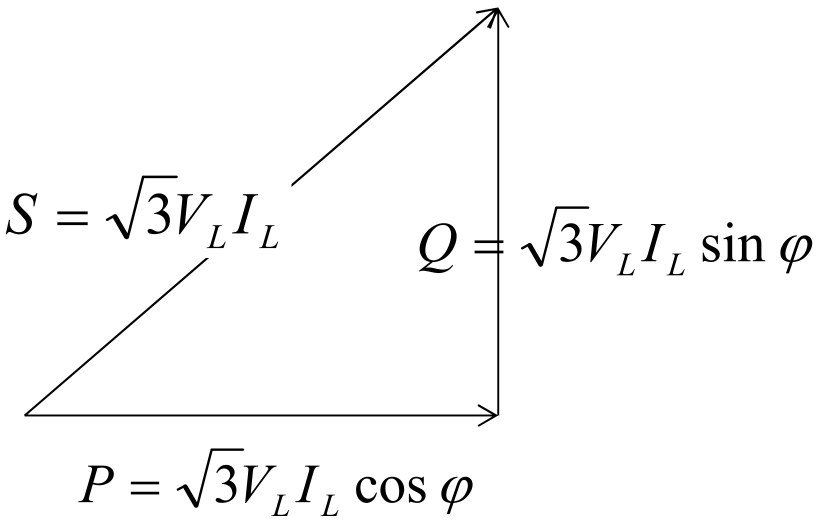
- Apparent power
  - Apparent power is the product of the voltage and current without accounting of the phase angle:

$$S = \sqrt{3} V_L I_L$$

Volt amperes (V A)

Three-phase Power - *Power Triangle & Power Factor*

Power triangle



Power complex form

$$\mathbf{S} = \boldsymbol{P} + \boldsymbol{jQ}$$

$$\mathbf{S} = \mathbf{3}\boldsymbol{V_p I_p^*}$$

Power factor

$$\text{Power Factor} = \frac{\text{Real Power } P}{\text{Apparent Power } S}$$

$$= \cos\varphi$$

Three-phase Power – Example 1 & 2

- How much reactive power is consumed by a perfect inductor?

- Solution

  Voltage : V, Current : I

  Reactance : X_L

  Reactive Power : $Q = VI \sin\varphi$

  Inductor : i lags v by 90°, $\sin\varphi = 1$

  By Ohm's law : $V = IX_L$

  so $Q = I^2 X_L$

- How much reactive power is consumed by a perfect capacitor?

- Solution

  Voltage : V, Current : I

  Reactance : X_C

  Reactive Power : $Q = VI \sin\varphi$

  Capacitor : i leads v by 90°, $\sin\varphi = 1$

  By Ohm's law : $V = IX_C$

  so $Q = I^2 X_C$ $\qquad Q = \dfrac{V^2}{X_C}$

Three-phase Power – Example 3

- A three-phase motor can be regarded as a balanced Y-load. A three-phase motor draws 5.6 kW when the line voltage is 220 V and the line current is 18.2 A. Find the power factor of the motor.
- Solution

The apparent power :

$$S = \sqrt{3} V_L I_L = \sqrt{3}(220)(18.2) = 6935.13 \text{ VA}$$

As the real power is :

$$P = S\cos\theta = 5600 \text{ W}$$

Then the power factor is :

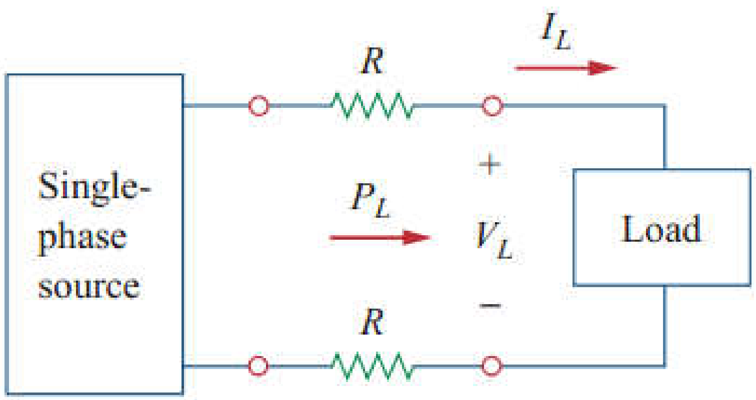
$$pf = \cos\theta = \frac{P}{S} = \frac{5600}{6935.13} = 0.81$$

Three-phase system is more economical (Optional)

Compare 3-phase and 1-phase system

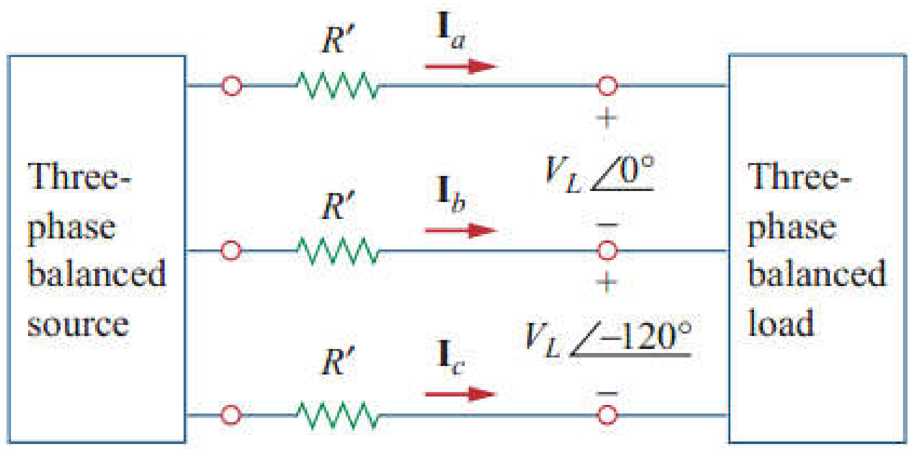
- The third advantage: the three-phase system uses a less amount of wire than the single-phase system for the same line voltage and the same absorbed power.
  - We will compare these cases and assume in both that the wires are of the same material (e.g., copper with resistivity ρ), of the same length and that the loads are resistive (i.e., unity power factor $\cos\varphi=1$).

two-wire single-phase system



$$P_{\text{loss}} = 2I_L^2 R = 2R\frac{P_L^2}{V_L^2}$$

three-wire three-phase system



$$P'_{\text{loss}} = 3(I'_L)^2 R' = 3R'\frac{P_L^2}{3V_L^2} = R'\frac{P_L^2}{V_L^2}$$

Three-phase system is more economical (Optional)

Compare 3-phase and 1-phase system

- For the same total power delivered P_L and same line voltage V_L

$$\left.\begin{array}{l} \dfrac{P_{\text{loss}}}{P'_{\text{loss}}} = \dfrac{2R}{R'} \\ R = \rho\ell/\pi r^2 \text{ and } R' = \rho\ell/\pi r'^2 \end{array}\right\} \quad \frac{P_{\text{loss}}}{P'_{\text{loss}}} = \frac{2r'^2}{r^2}$$

- If the same power loss is tolerated in both systems, then $r^2 = 2r'^2$.
- Therefore, the ratio of material required is determined by the number of wires and their volumes, so

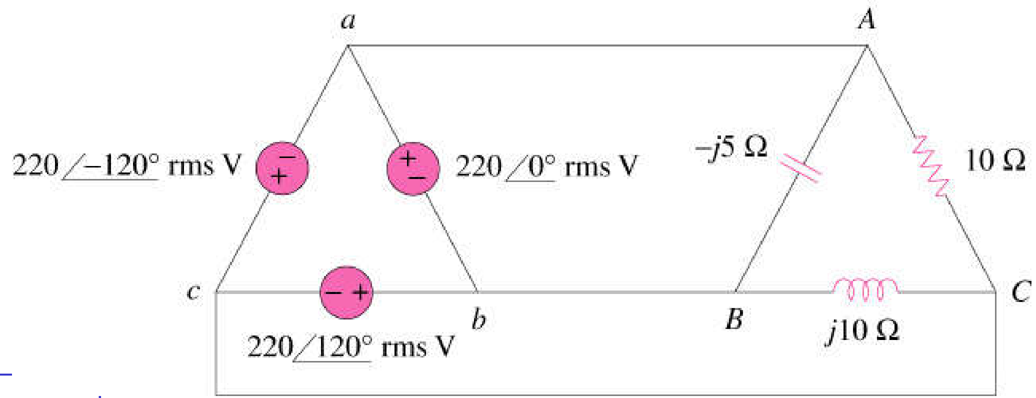
$$\frac{\text{Material for single-phase}}{\text{Material for three-phase}} = \frac{2(\pi r^2 \ell)}{3(\pi r'^2 \ell)} = \frac{2r^2}{3r'^2} = \frac{2}{3}(2) = 1.333$$

- This means that the single-phase system uses 33 percent more material than the three-phase system or that the three-phase system uses only 75 percent of the material used in the equivalent single-phase system.

Three-phase Power - Power in an unbalanced System

- Total power delivered = Σ(power delivered to each phase)
- Power factor $\cos\varphi$ is different from each phase.

Example: Find the real power absorbed by the load.



Recall

The phase currents :

$$\mathbf{I}_{AB} = \frac{220\angle 0^\circ}{-j5} = j44 \text{ A}$$

$$\mathbf{I}_{BC} = \frac{220\angle(+120^\circ)}{j10} = 22\angle 30^\circ = 19.05 + j11 \text{ A}$$

$$\mathbf{I}_{CA} = \frac{220\angle(-120^\circ)}{10} = 22\angle(-120^\circ) = -11 - j19.05 \text{ A}$$

The real power absorbed by the resistive load :

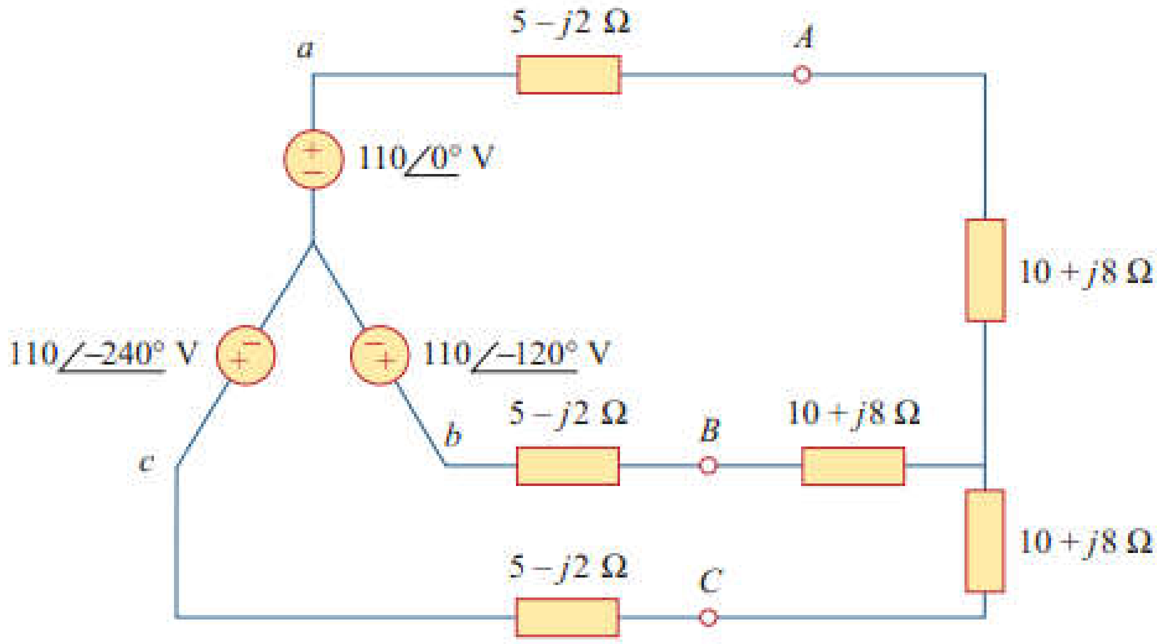
$$P = |\mathbf{I}_{CA}|^2 \times 10 = 4.84 \text{ kW}$$

Three-phase Power - Energy Conservation (Optional)

- Reactive power:
  - Loads with inductance will consume real power and consume reactive power. Capacitors will supply reactive power to the system.
  - Total of the reactive power generated equals the total of the reactive power consumed.
- Real power:
  - The total of the real power generated equals the total of the real power consumed.
  - Electrical energy cannot be stored in the system.

Three-phase Power - Energy Conservation (Optional)

- Example: Determine the total average power, reactive power, and complex power at the source and at the load.



Three-phase Power - Summary

- The total instantaneous power in a balanced three-phase system is constant.
- The total complex power absorbed by a balanced three-phase Y-connected or Δ-connected load is $\mathbf{S} = P + jQ$, where

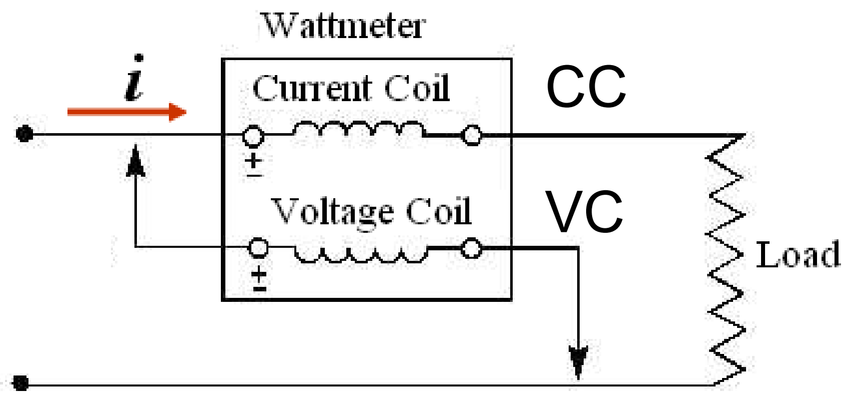
Real power	$P = \sqrt{3} V_L I_L \cos \varphi$
Apparent power	$S = \sqrt{3} V_L I_L$
Reactive power	$Q = \sqrt{3} V_L I_L \sin \varphi$

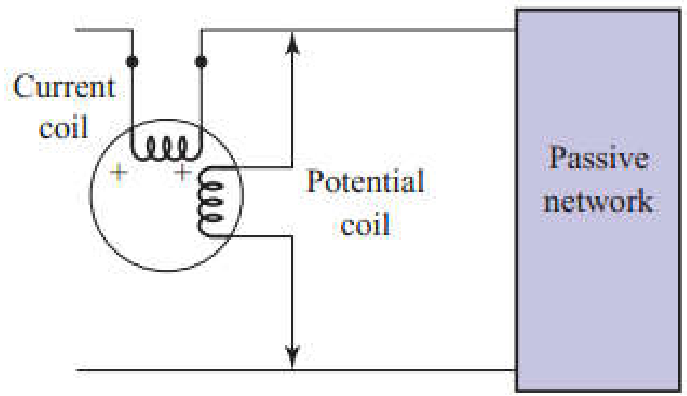
- In an unbalanced three-phase, total power delivered = Σ(power delivered to each phase).

Measurement of Power in Three-phase Circuits

- The average power absorbed by a load is measured by an instrument called the **wattmeter**.

- A wattmeter has connections for both current and voltage. The positive side of the current coil and the positive side of the voltage coil are normally labeled + or $\pm$.

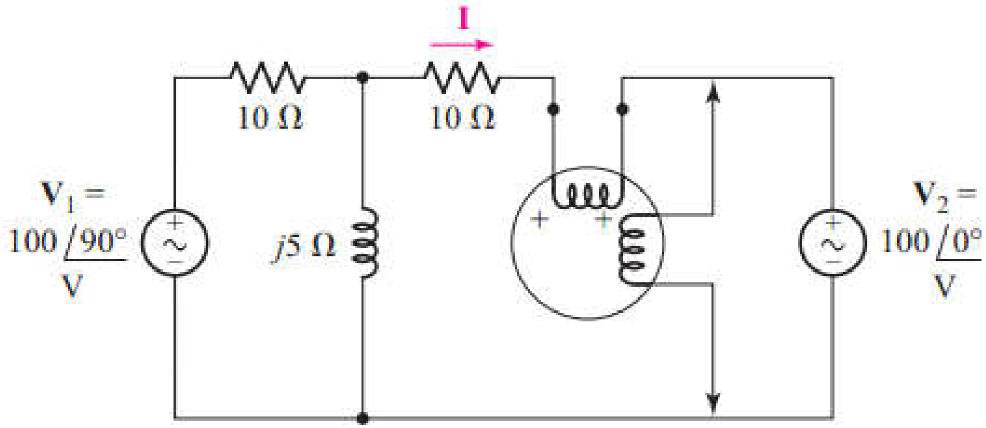




The wattmeter connected to a load

Example of the use of wattmeter

- An example in which the wattmeter is installed to give an upscale indication of the power absorbed by the right source.
- Solution:



The power absorbed by this source is given by

$$P = |\mathbf{V}_2|\,|\mathbf{I}|\cos(ang\,\mathbf{V}_2 - ang\,\mathbf{I})$$

Using mesh analysis, get:

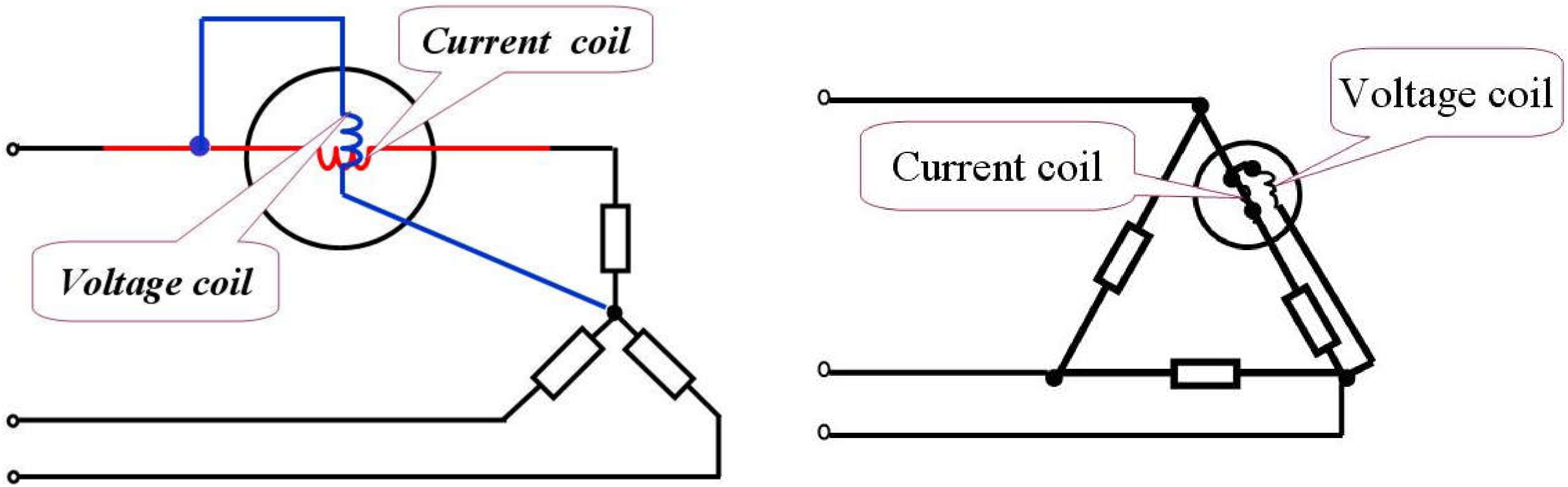
$$\mathbf{I} = 11.18\underline{/153.4^\circ}\text{ A}$$

So the absorbed power is:

$$P = (100)(11.18)\cos(0^\circ - 153.4^\circ) = -1000\text{ W}$$

Measurement of Power in Three Phase Circuits

One-wattmeter Method



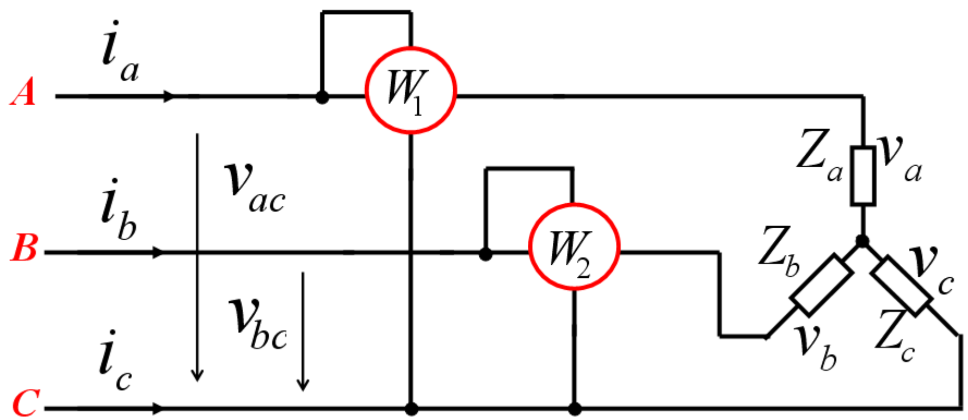
Wattmeter connections for both star and delta

$$\boxed{\text{Total power} = 3 \times \text{wattmeter reading}}$$

One-wattmeter method can be used for a balanced load

Measurement of Power in Three Phase Circuits

Two-wattmeter method



In the 2-wattmeter method, the positive voltage terminals on two wattmeters are connected to any two of the lines and both negative terminals are connected to the third line.

If we have: $i_c = -i_a - i_b$

$$
\begin{aligned}
p &= v_a i_a + v_b i_b + v_c i_c \\
&= i_a (v_a - v_c) + i_b (v_b - v_c) \\
&= i_a v_{ac} + i_b v_{bc}
\end{aligned}
$$

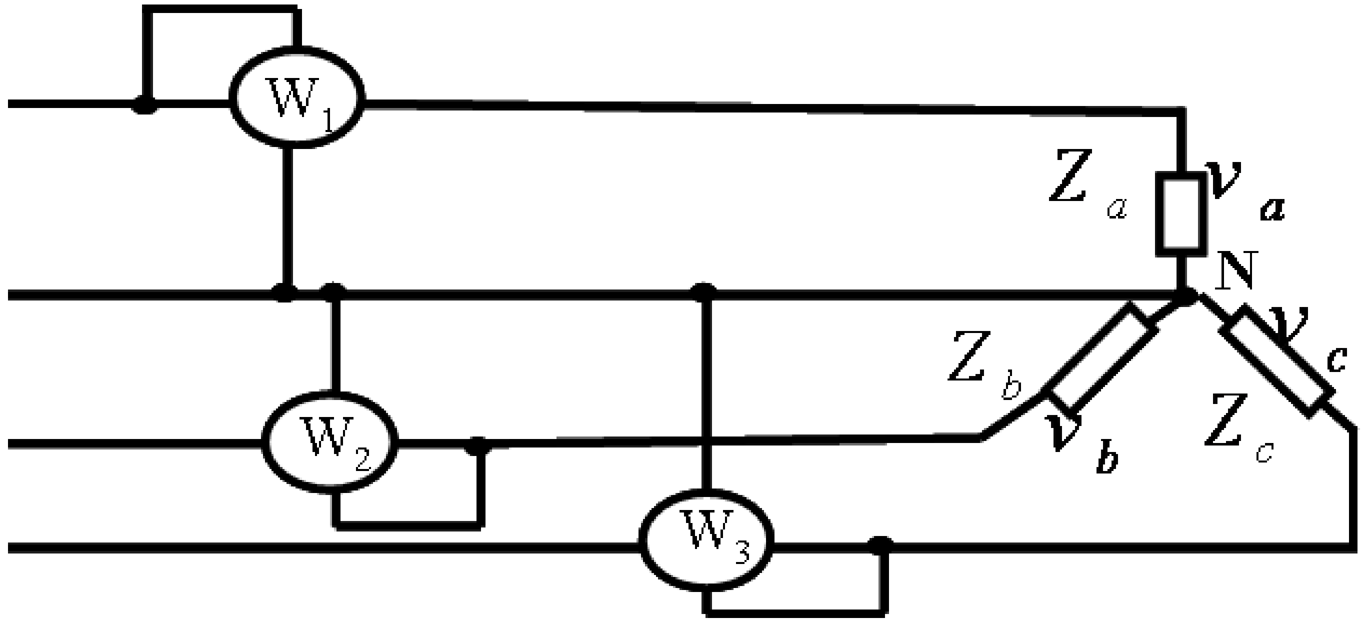
Total power : $p = p_1 + p_2$

Two-wattmeter method can be used for:

- **Balanced or unbalanced 3-phase, 3-wire**
- **Balanced 3-phase, 4-wire**

Measurement of Power in Three Phase Circuits

Three-wattmeter



Total power : $p = p_1 + p_2 + p_3$

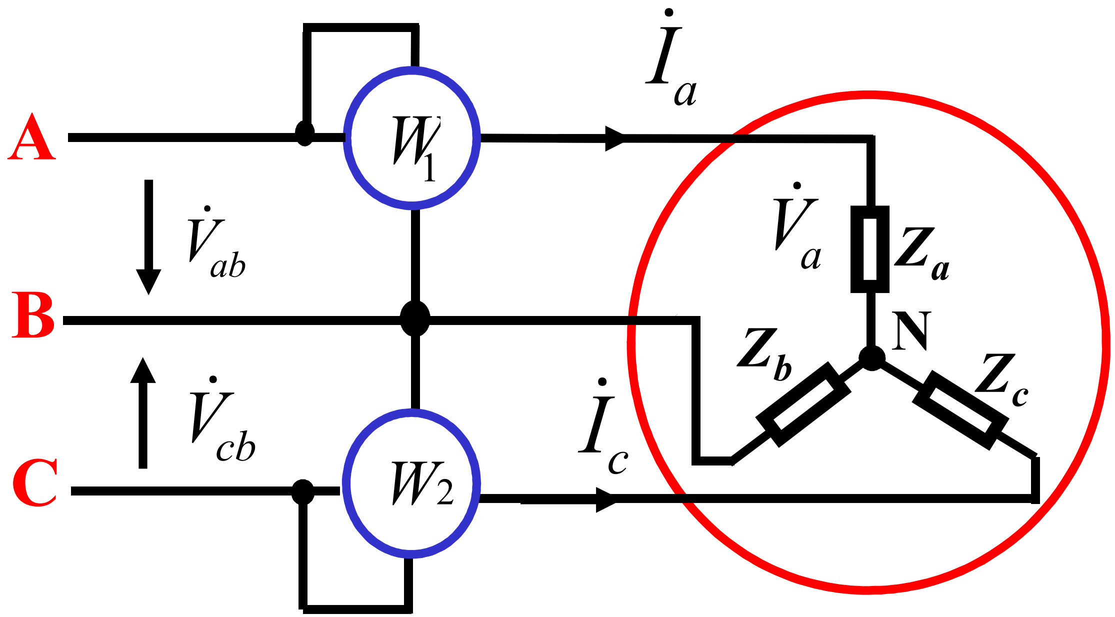
Three-wattmeter method for a 3-phase, 4-wire system for balanced or unbalanced loads

In the 3-wattmeter method, all negative voltage connection on each of the wattmeters is common (typically on the neutral line).

Measurement of Power in Three Phase Circuits

Example

- A balanced 3-phase, Y-connected system with the 380 V of the line voltage, 2.5 kW load and 0.866 of the power factor.
- Find the readings of the watt meters.



- Solution:

1. Find the line current:

As $P = \sqrt{3}\,V_l I_l \cos\varphi$, so we have:

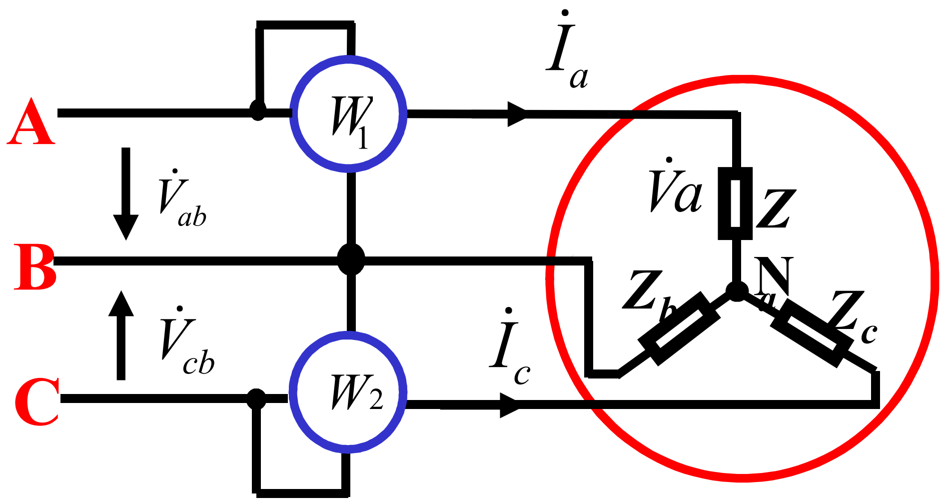
$$I_l = \frac{P}{\sqrt{3}V_l \cos\varphi} = \frac{2.5\times 10^3}{\sqrt{3}\times 380\times 0.866} = 4.386\ \text{A}$$

Then $I_a = I_b = I_c = I_l = 4.386\ \text{A}$

Measurement of Power in Three Phase Circuits

Example cont.

$$\mathbf{V}_{aN} = 220\angle 0^\circ \text{ V}$$
$$\mathbf{V}_{ab} = 380\angle 30^\circ \text{ V}$$
$$\mathbf{V}_{bc} = 380\angle -90^\circ \text{ V}$$
$$\mathbf{V}_{cb} = 380\angle 90^\circ \text{ V}$$

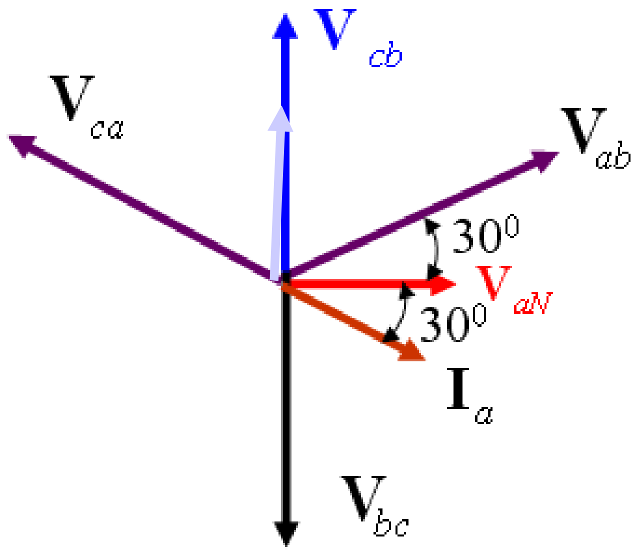


2. Find the angle between the phase current and the line voltage

From $\varphi = \cos^{-1} 0.866 = 30^\circ$, we have :

$$\mathbf{I}_a = 4.386\angle -30^\circ \text{ A}$$

Then the angle between $\mathbf{I}_a$ and $\mathbf{V}_{ab}$ is 60°



Measurement of Power in Three Phase Circuits

Example cont.

3. Reading of W_1 : $P_1 = V_{ab} I_a \cos 60^\circ$

$$= 380 \times 4.386 \times \cos 60^\circ$$

$$= 833.3 \text{ W}$$

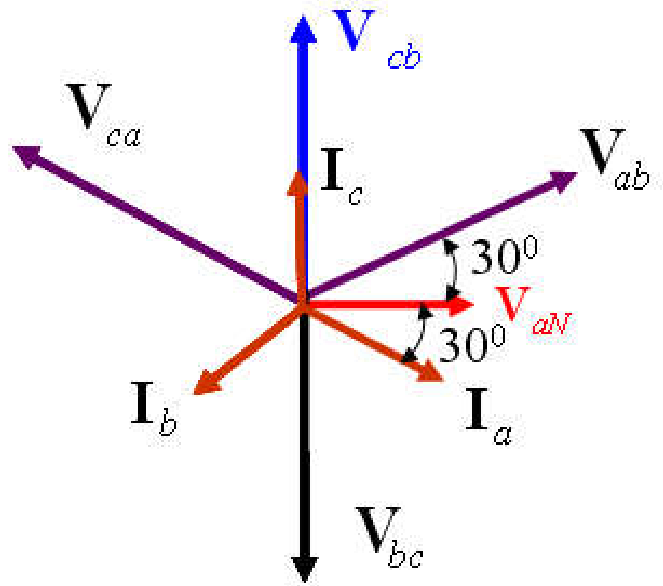
4. $\mathbf{I}_c = 4.386\angle 90^\circ$ A

Angle between I_c and V_{cb} : 0°

Reading of W_2 : $P_2 = V_{cb} I_c \cos 0^\circ$

$$= 380 \times 4.386 \times \cos 0^\circ$$

$$= 1666.7 \text{ W}$$



Quiz

- 1. In a balanced three-phase circuit, the total instantaneous power is equal to the average power.
  - (a) True (b) False

- 2. The total power supplied to a balanced Δ-load is found in the same way as for a balanced Y-load.
  - (a) True (b) False