EEE108 Electromagnetism and Electromechanics

Lecture 17

DC Machinery Fundamentals

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Last

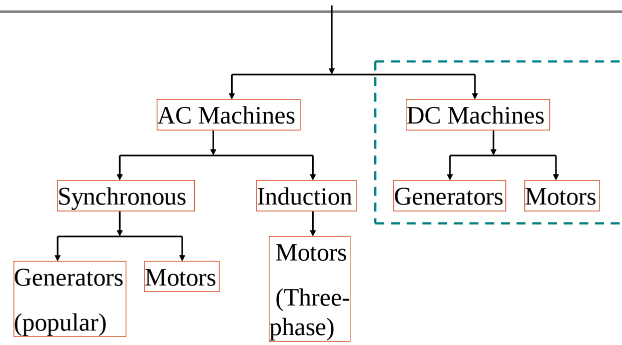
Linear DC machines

- Four basic equations
- Starting behavior
- The Linear DC Machine as a motor
- The Linear DC Machine as a Generator
- The Linear DC Machine Starting Problems

Today

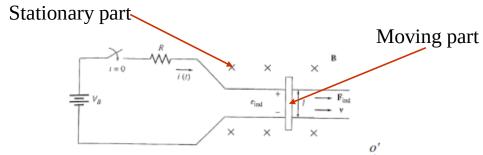
DC Machinery Fundamentals

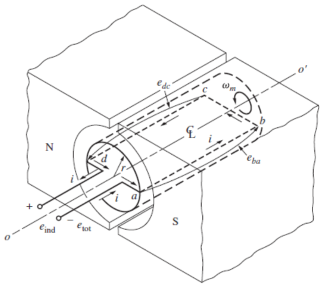
Electrical Machines

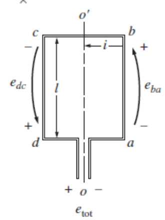


DC Machinery Fundamentals $\begin{cases} \text{Induced voltage} \\ \text{Induced torque} \end{cases}$

The Voltage Induced in a Rotating Loop

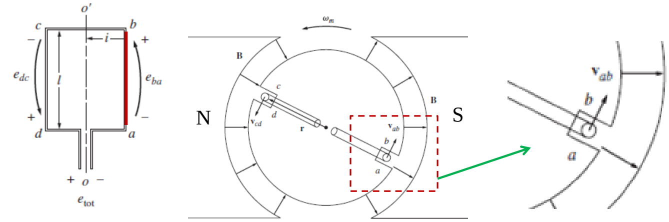






The figure shows a simple rotating loop between curved faces. The rotating part is called the **rotor**, and the stationary part is called the **stator**.

The Voltage Induced in a Rotating Loop



$e_{ind} = (\mathbf{v} \times \mathbf{B}) \bullet \mathbf{l}$

•Segment *ab:*

$$e_{ba} = (\mathbf{v} \times \mathbf{B}) \bullet \mathbf{l} = \begin{cases} vBl & \text{Positive into page} \quad \text{under the pole face} \\ 0 & \text{beyond the pole edges} \end{cases}$$

The Voltage Induced in a Rotating Loop

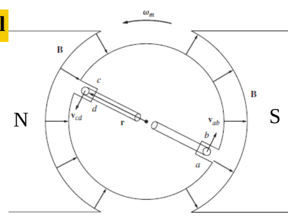
•Segment *bc*:

$e_{ind} = (\mathbf{v} \times \mathbf{B}) \bullet \mathbf{l}$

v x **B** is either into the page (*o'b*) or out of the page (*o'c*).

The length **l** is in the plane of the page.

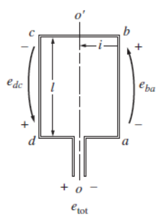
So **v** x **B** is perpendicular to **l**. Then: $e_{bc} = 0$



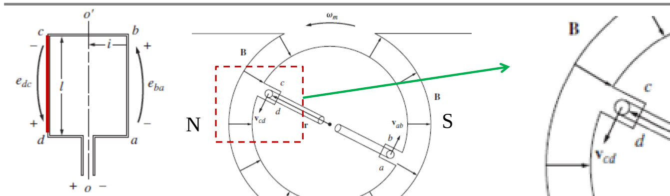
•Segment *da*:

Similar with *bc*:

v x **B** is perpendicular to **l**. So: $e_{da} = 0$



The Voltage Induced in a Rotating Loop



$e_{ind} = (\mathbf{v} \times \mathbf{B}) \bullet \mathbf{l}$

•Segment *cd:*

$$e_{cd} = (\mathbf{v} \times \mathbf{B}) \bullet \mathbf{l} = \begin{cases} vBl & \text{Positive out of page} \quad \text{under the pole face} \\ 0 & \text{beyond the pole edges} \end{cases}$$

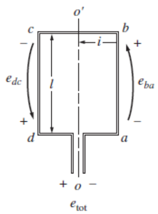
The total induced voltage : $e_{ind} = e_{ba} + e_{cb} + e_{dc} + e_{ad}$

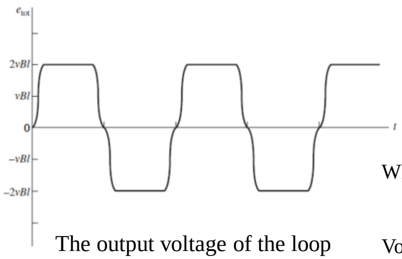
$$e_{ind} = \begin{cases} 2vBl & \text{under the pole face} \\ 0 & \text{beyond the pole edges} \end{cases}$$

The Voltage Induced in a Rotating Loop

The total induced voltage :

$$e_{ind} = \begin{cases} 2vBl & \text{under the pole face} \\ 0 & \text{beyond the pole edges} \end{cases}$$





The output voltage of the loop

When the loop rotates 180^0:

ab from S to N

dc from N to S

Voltage:

Direction: reverses

Magnitude: same

The Voltage Induced in a Rotating Loop

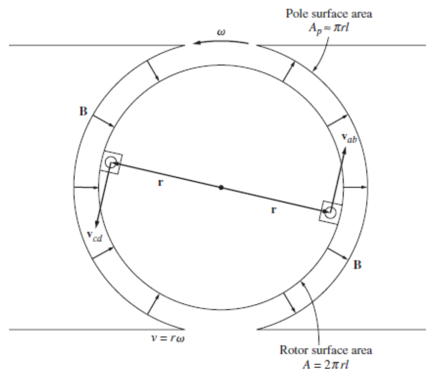
An alternative way to express the e_{ind} equation, which clearly relates the behaviour of the single loop to the behaviour of larger, real DC machines.

Substituting $v = r\omega$ into the e_{ind} equation:

$e_{ind} = 2r\omega Bl$

The rotor surface is a cylinder, so the area of the rotor surface: $A = 2\pi rl$.

Since there are 2 poles, the area under each pole is $A_p = \pi rl$.



$$e_{ind} = 2vBl = 2(r\omega)Bl = 2rlB\omega = \frac{2}{\pi} A_p B\omega$$

The Voltage Induced in a Rotating Loop

$$e_{ind} = 2vBl = 2(r\omega)Bl = 2rlB\omega = \frac{2}{\pi} A_p B\omega$$

The magnitude of the flux density **B** is constant everywhere in the air gap under the pole faces, thus, the final form of the voltage equation is:

$$e_{ind} = \begin{cases} \frac{2}{\pi}\Phi\omega & \text{under the pole faces} \\ 0 & \text{beyond the pole edges} \end{cases}$$

where $\Phi = A_p B$

In general, the voltage will depend on the same 3 factors:

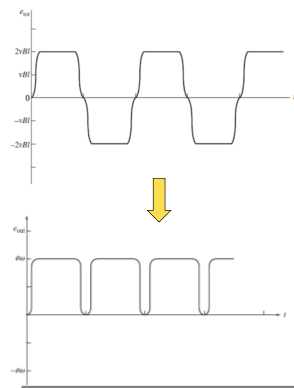
1. the flux in the machine
2. the speed of rotation
3. a constant representing the construction of the machine.

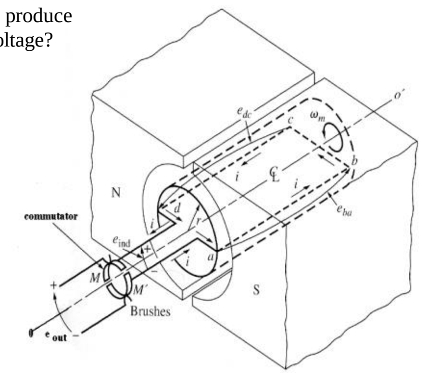
$$e_{ind} = K\Phi\omega$$

The Voltage Induced in a Rotating Loop

Getting DC Voltage out of the Loop

How can this machine be made to produce a DC voltage instead of the AC voltage?

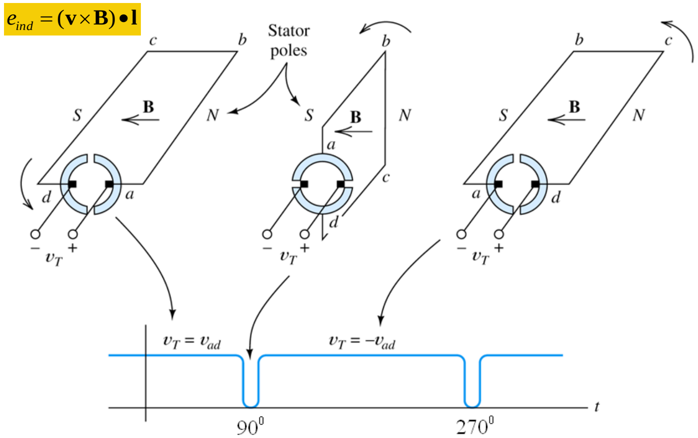




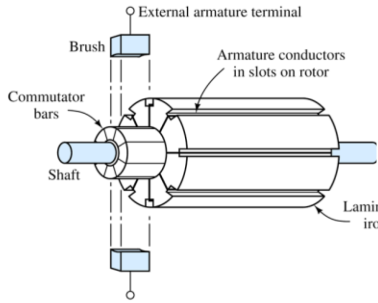
This can be done by using a mechanism called commutator and brushes.

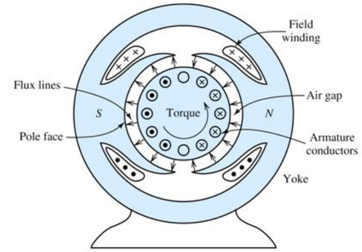
The Voltage Induced in a Rotating Loop

$e_{ind} = (\mathbf{v} \times \mathbf{B}) \bullet \mathbf{l}$

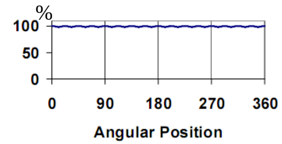


The Voltage Induced in a Rotating Loop

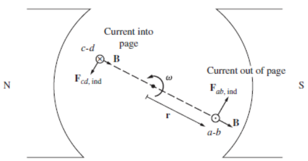


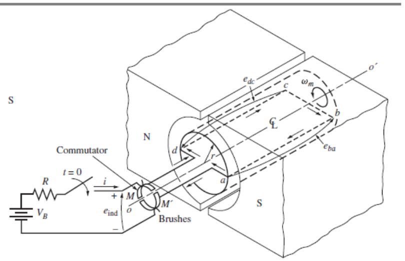


Cross section of a two-pole dc machine



The Torque Induced in a Rotating Loop





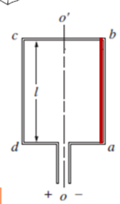
•Segment *ab*:

$\mathbf{F}_{ab} = i(\mathbf{l} \times \mathbf{B}) = ilB$ Tangent to direction of motion

$\tau_{ab} = rF \sin\theta = r(ilB)\sin 90^0 = rilB$ CCW

The magnitude of the induced torque:

$$\tau_{ab} = \begin{cases} rilB & \text{under pole faces} \\ 0 & \text{beyond pole edges} \end{cases}$$



$\mathbf{F} = i(\mathbf{l} \times \mathbf{B})$

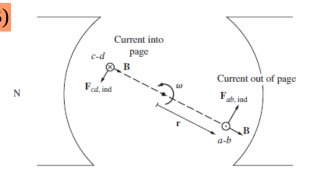
The Torque Induced in a Rotating Loop

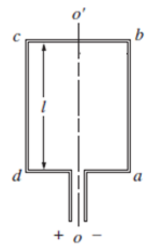
•Segment *bc*:

$\mathbf{F} = i(\mathbf{l} \times \mathbf{B})$

The length **l** is parallel to **B**. Then:

$\mathbf{F}_{bc} = i(\mathbf{l} \times \mathbf{B}) = 0 \Rightarrow \tau_{bc} = 0$





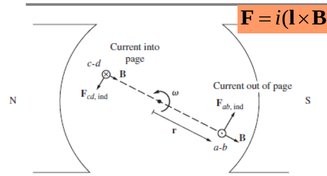
•Segment *da*:

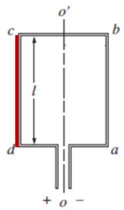
The length **l** is parallel to **B**. Then:

$\mathbf{F}_{da} = i(\mathbf{l} \times \mathbf{B}) = 0 \Rightarrow \tau_{da} = 0$

The Torque Induced in a Rotating Loop

$\mathbf{F} = i(\mathbf{l} \times \mathbf{B})$





•Segment *cd*:

$\mathbf{F}_{cd} = i(\mathbf{l} \times \mathbf{B}) = ilB$

Tangent to direction of motion

$$\begin{aligned}\tau_{cd} &= rF\sin\theta \\ &= r(ilB)\sin 90^0 \\ &= rilB\end{aligned}$$

CCW

The magnitude of the induced torque:

$$\tau_{cd} = \begin{cases} rilB & \text{under pole faces} \\ 0 & \text{beyond pole edges} \end{cases}$$

The total induced torque on the loop:

$$\tau_{\text{ind}} = \tau_{\text{ab}} + \tau_{\text{bc}} + \tau_{\text{cd}} + \tau_{\text{da}}$$

$$\tau_{ind} = \begin{cases} 2rilB & \text{under pole faces} \\ 0 & \text{beyond pole edges} \end{cases}$$

The Torque Induced in a Rotating Loop

The magnitude of the total induced torque:

$$\tau_{ind} = \begin{cases} \dfrac{2}{\pi}\Phi i & \text{under pole faces} \\ 0 & \text{beyond pole edges} \end{cases}$$

$$\tau_{ind} = \begin{cases} 2rilB \\ 0 \end{cases}$$

where $\Phi = \pi r l B$

A_p

In general, the torque in any real machine will depend on the same 3 factors:

1. The flux in the machine
2. The current in the machine
3. A constant representing the construction of the machine.

$$\tau_{ind} = K\Phi i$$

DC Machinery Fundamentals

Example

Example 7–1. Figure 7–6 shows a simple rotating loop between curved pole faces connected to a battery and a resistor through a switch. The resistor shown models the total resistance of the battery and the wire in the machine. The physical dimensions and characteristics of this machine are

$$r = 0.5\text{ m} \qquad l = 1.0\text{ m}$$
$$R = 0.3\ \Omega \qquad B = 0.25\text{ T}$$
$$V_B = 120\text{ V}$$

(a) What happens when the switch is closed?
(b) What is the machine's maximum starting current? What is its steady-state angular velocity at no load?
(c) Suppose a load is attached to the loop, and the resulting load torque is 10 N • m. What would the new steady-state speed be? How much power is supplied to the shaft of the machine? How much power is being supplied by the battery? Is this machine a motor or a generator?
(d) Suppose the machine is again unloaded, and a torque of 7.5 N • m is applied to the shaft in the direction of rotation. What is the new steady-state speed? Is this machine now a motor or a generator?
(e) Suppose the machine is running unloaded. What would the final steady-state speed of the rotor be if the flux density were reduced to 0.20 T?

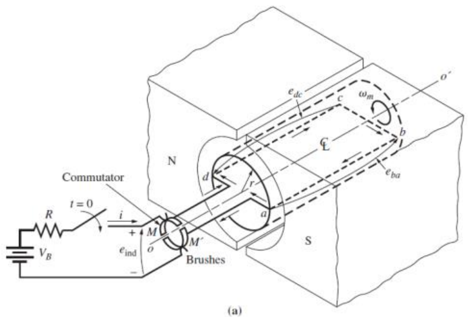
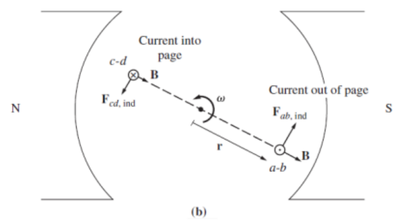


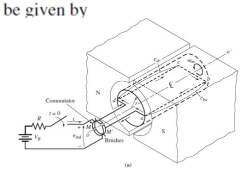
Figure 7-6



(a) What happens when the switch is closed?

When the switch in Figure 7–6 is closed, a current will flow in the loop. Since the loop is initially stationary, $e_{ind} = 0$. Therefore, the current will be given by

$$i = \frac{V_B - e_{\text{ind}}}{R} = \frac{V_B}{R}$$



This current flows through the rotor loop, producing a torque

$$\tau_{\text{ind}} = \frac{2}{\pi}\phi i \qquad \text{CCW}$$

This induced torque produces an angular acceleration in a counterclockwise direction, so the rotor of the machine begins to turn. But as the rotor begins to turn, an induced voltage is produced in the motor, given by

$$e_{\text{ind}} = \frac{2}{\pi}\phi\omega_m$$

so the current i falls. As the current falls, $\tau_{\text{ind}} = (2/\pi)\phi i\downarrow$ decreases, and the machine winds up in steady state with $\tau_{\text{ind}} = 0$, and the battery voltage $V_B = e_{\text{ind}}$.

(b) What is the machine's maximum starting current? What is its steady-state angular velocity at no load?

At *starting conditions,* the machine's current is

$$i = \frac{V_B}{R} = \frac{120\text{ V}}{0.3\ \Omega} = 400\text{ A}$$

At *no-load steady-state conditions,* the induced torque τ_{ind} must be zero. But $\tau_{\text{ind}} = 0$ implies that current i must equal zero, since $\tau_{\text{ind}} = (2/\pi)\phi i$, and the flux is nonzero. The fact that $i = 0$ A means that the battery voltage $V_B = e_{\text{ind}}$. Therefore, the speed of the rotor is

$$V_B = e_{\text{ind}} = \frac{2}{\pi}\phi\omega_m$$

$$\omega = \frac{V_B}{(2/\pi)\phi} = \frac{V_B}{2rlB}$$

$$= \frac{120\text{ V}}{2(0.5\text{ m})(1.0\text{ m})(0.25\text{ T})} = 480\text{ rad/s}$$

(c) Suppose a load is attached to the loop, and the resulting load torque is 10 N • m. What would the new steady-state speed be? How much power is supplied to the shaft of the machine? How much power is being supplied by the battery? Is this machine a motor or a generator?

If a load torque of 10 N • m is applied to the shaft of the machine, it will begin to slow down. But as ω decreases, $e_{\text{ind}} = (2/\pi)\phi\,\omega\downarrow$ decreases and the rotor current increases $[i = (V_B - e_{\text{ind}}\downarrow)/R]$. As the rotor current increases, $|\tau_{\text{ind}}|$ increases too, until $|\tau_{\text{ind}}| = |\tau_{\text{load}}|$ at a lower speed ω.

At steady state, $|\tau_{\text{load}}| = |\tau_{\text{ind}}| = (2/\pi)\phi i$. Therefore,

$$i = \frac{\tau_{\text{ind}}}{(2/\pi)\phi} = \frac{\tau_{\text{ind}}}{2rlB} = \frac{10\text{ N}\bullet\text{m}}{(2)(0.5\text{ m})(1.0\text{ m})(0.25\text{ T})} = 40\text{ A}$$

By Kirchhoff's voltage law, $e_{\text{ind}} = V_B - iR$, so

$$e_{\text{ind}} = 120\text{ V} - (40\text{ A})(0.3\ \Omega) = 108\text{ V}$$

Finally, the speed of the shaft is

$$\omega = \frac{e_{\text{ind}}}{(2/\pi)\phi} = \frac{e_{\text{ind}}}{2rlB} = \frac{108\text{ V}}{(2)(0.5\text{ m})(1.0\text{ m})(0.25\text{ T})} = 432\text{ rad/s}$$

The power supplied to the shaft is

$$P = \tau\omega_m = (10\text{ N}\bullet\text{m})(432\text{ rad/s}) = 4320\text{ W}$$

The power out of the battery is

$$P = V_B i = (120\text{ V})(40\text{ A}) = 4800\text{ W}$$

This machine is operating as a *motor,* converting electric power to mechanical power.

(d) Suppose the machine is again unloaded, and a torque of 7.5 N • m is applied to the shaft in the direction of rotation. What is the new steady-state speed? Is this machine now a motor or a generator?

If a torque is applied in the direction of motion, the rotor accelerates. As the speed increases, the internal voltage e_{ind} increases and exceeds V_B, so the current flows out of the top of the bar and into the battery. This machine is now a *generator*. This current causes an induced torque opposite to the direction of motion. The induced torque opposes the external applied torque, and eventually $|\tau_{\text{load}}| = |\tau_{\text{ind}}|$ at a higher speed ω_m.

The current in the rotor will be

$$i = \frac{\tau_{\text{ind}}}{(2/\pi)\phi} = \frac{\tau_{\text{ind}}}{2rlB}$$

$$= \frac{7.5\text{ N}\cdot\text{m}}{(2)(0.5\text{ m})(1.0\text{ m})(0.25\text{ T})} = 30\text{ A}$$

The induced voltage e_{ind} is

$$e_{\text{ind}} = V_B + iR = 120\text{ V} + (30\text{ A})(0.3\ \Omega) = 129\text{ V}$$

Finally, the speed of the shaft is

$$\omega = \frac{e_{\text{ind}}}{(2/\pi)\phi} = \frac{e_{\text{ind}}}{2rlB}$$

$$= \frac{129\text{ V}}{(2)(0.5\text{ m})(1.0\text{ m})(0.25\text{ T})} = 516\text{ rad/s}$$

DC Machinery Fundamentals

Example

(e) Suppose the machine is running unloaded. What would the final steady-state speed of the rotor be if the flux density were reduced to 0.20 T?

Since the machine is initially unloaded at the original conditions, the speed ω_m = 480 rad/s. If the flux decreases, there is a transient. However, after the transient is over, the machine must again have zero torque, since there is still no load on its shaft. If $\tau_{\text{ind}} = 0$, then the current in the rotor must be zero, and $V_B = e_{\text{ind}}$. The shaft speed is thus

$$\omega = \frac{e_{\text{ind}}}{(2/\pi)\phi} = \frac{e_{\text{ind}}}{2rlB}$$

$$= \frac{120\ \text{V}}{(2)(0.5\ \text{m})(1.0\ \text{m})(0.20\ \text{T})} = 600\ \text{rad/s}$$

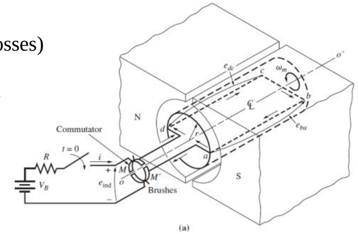
Notice that when the flux in the machine is decreased, its speed increases. This is the same behavior seen in the linear machine and the same way that real dc motors behave.

ISBN	Books Title	Author	Publisher
0073529540	Electric Machinery Fundamentals	Stephen J. Chapman	Cengage Learning; 3 edition (November 11, 2008)

Losses in DC Machines

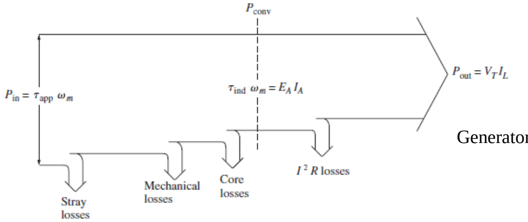
The Losses in DC Machine:

1. Electrical or Copper Losses (I^2R Losses)
   Armature loss: $P_A = I^2_A R_A$
   Field loss: $P_F = I^2_F R_F$
2. Brush Losses
   $P_{BD} = V_{BD} I_A$
3. Core Losses
   Hysteresis and Eddy Current Loss
4. Mechanical Losses
   Friction and windage loss
5. Stray Losses

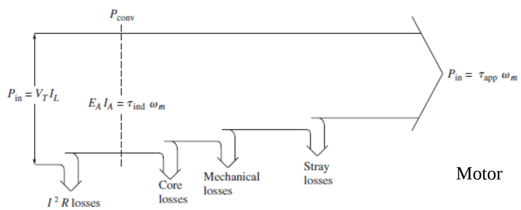


$$\text{Efficiency}: \eta = \frac{P_{in} - P_{loss}}{P_{in}} \times 100\%$$

Power Flow in DC Machines



Generator



Motor

Next

DC Motors

Thanks for your attendance