MTH102
Engineering Mathematics II
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Lecture 1.2.1 Set theory

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Chapter 1.2 Experiments, Outcomes and Events

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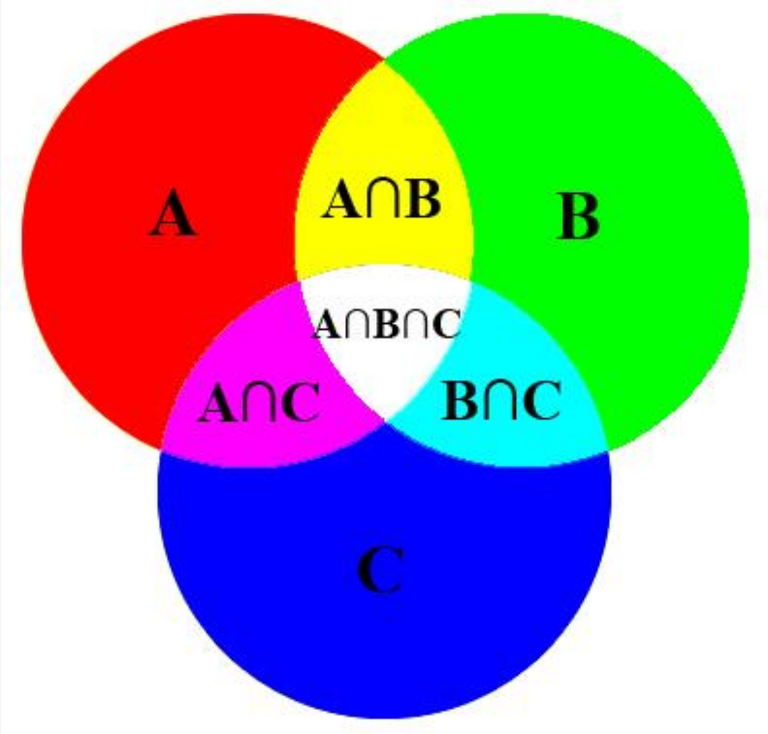


1.2.1 Basic Definition

- In probability, any process of observation is an *experiment*.
- The results of an observation are the *outcomes* of the experiment.

Example 1

1) Roll of a die
2) Toss of a coin

are examples of an experiment.

1.2.1 Basic Definition

- The set of all possible outcomes of an experiment is called the *sample space* S.
- An element in S is a *sample point*.

Example 2

Find the sample space for the experiment of tossing a coin

i. once, and

ii. twice.

1.2.1 Basic Definition: sample space

Solution

Let H and T represent head and tail respectively.

i. 1 toss: There are two possible outcomes, head or tail. $S = \{H, T\}$

ii. 2 tosses: There are four possible outcomes, i.e. pairs of head and tail.

$$S = \{HH, HT, TH, TT\} \qquad \blacksquare$$

Example 3

Find the sample space for the experiment of tossing a coin repeatedly and of **counting the number of tosses** required until the first head appears.

Solution

$$S = \{1,2,3, \cdots\}$$

There are an infinite number of outcomes. $\blacksquare$

1.2.1 Basic Definition

- A ***trial*** is a single occurrence of an experiment.
- If there are n trials, then we have a ***sample*** of size n consisting of n sample points.

Example 4

Where you are required to differentiate between a trial and an experiment, consider the experiment to be a larger entity formed by the combination of a number of trials.

i. In the experiment of tossing 4 coins, we may consider tossing each coin as a trial and therefore say that there are 4 trials in the experiment.

ii. In the experiment of picking 3 balls from a bag containing 10 balls 4 of which are red and 6 blue, we can consider picking a ball as a trial and so there are 3 trials in the experiment. ■

1.2.1 Basic Definition: events

- Any subset of the sample space S is called an *event*.
- If in a trial an outcome a and $a \in A$, we say that event A happens.

Example 5

When rolling a die, let A be the event of getting an odd number; let S be the sample space for rolling a die.

If a die turns up a 3, we say that event A happens. Note that S always happens in the experiment. ■

1.2.1 Basic Definition: events

Example 5

When rolling a die, let A be the event of getting an odd number; let S be the sample space for rolling a die.

Roll a die (1 trial) | event A = odd number = {1, 3, 5}

sample space S = {1, 2, 3, 4, 5, 6} | A happens if face up is 1 or 3 or 5

Roll a dice 2 times (2 trials) | event A = 2 equal numbers

Sample Space = {(1,1), (1,2),…,(5,6), (6,6)}

A happens if {(1,1) or (2,2) or (3,3) or (4,4) or (5,5) or (6,6)}

1.2.1 Events: problem

- Experiment: pick driver at random and check car brand:

  Trial: Honda (H), Toyota (T), Fiat (F), BMW (B)

  1) define the sample space

- Experiment 2: car brand nationality: check if the brand is Japanese

  Trial2: Japan(Honda), Japan(Japan), Italy(Fiat), Germany(BMW)

  2) define the sample space

1.2.2 Set Theory

Consider the events (subsets) $A, B, C, \cdots$ of a given sample space S.

- The *union* (*OR*) $A \cup B = \{x : x \in A \text{ or } x \in B\}$
- The *intersection* (AND) $A \cap B = \{x : x \in A \text{ and } x \in B\}$

We can generalize to

$\bigcup_{j=1}^{m} A_j = A_1 \cup A_2 \cup \cdots \cup A_m$ (A_1 or A_2 or $\cdots$ or A_m)

$\bigcap_{j=1}^{m} A_j = A_1 \cap A_2 \cap \cdots \cap A_m$ (A_1 and A_2 and $\cdots$ and A_m)

1.2.2 Set Theory

Example Union and intersection Roll a die

Event E = face up is even = {2, 4, 6}

Event G = face up is > 3 = {4, 5, 6}

$E \cup G = \{2, 4, 5, 6\}$ (either in E or in G or in both)

note: 4 and 6 are in both events but show only once

$E \cap G = \{4, 6\}$ (in E and in G)

1.2.2 Set Theory

- If A and B are such that

$$A \cap B = \emptyset$$

we call A and B *mutually exclusive*. (Cannot happen together.)

- If $A_i \cap A_j = \emptyset$ for $i \neq j$ and $\bigcup_{i=1}^{k} A_i = S$ then the collection $\{A_i : 1 \leq i \leq k\}$ forms a *partition* of S.
- The *complement* of A, denoted $\bar{A}$, (one or the other must happen)

$$\bar{A} = \{x : x \in S \text{ and } x \notin A\} \text{ NOT A}$$

Example: Head and Tails, the outcome of a coin flip can only be one not Head and Tails at the same time. $P(H \cap T) = \emptyset$

1.2.2 Set Theory

Example partition and complement Roll a die

Event E = face up is even = {2, 4, 6}

Event G = face up is odd = {1, 3, 5}

i) E and G are mutually exclusive: $E \cap G = \emptyset$

ii) E and G are a partition of S: (i) + $E \cup G = S$

iii) E is the complement of G: if G doesn't happen -> E happens

$E = \bar{G}$ (and vice versa, $\bar{E} = G$). $\bar{E} \cup E = S$.

1.2.2 Set Theory: multiple set operations

- Consider $A = \{1, 2, 3\}$, $B = \{3, 4, 5\}$ and $C = \{1, 6\}$

- $A \cup B = \{1,2,3,4,5\}$, $(A \cup B) \cap C = \{1,2,3,4,5\} \cap \{1\} = \{1\}$
- Now Consider $(A \cap C) \cup (B \cap C) = \{1\} \cup \emptyset = \{1\}$

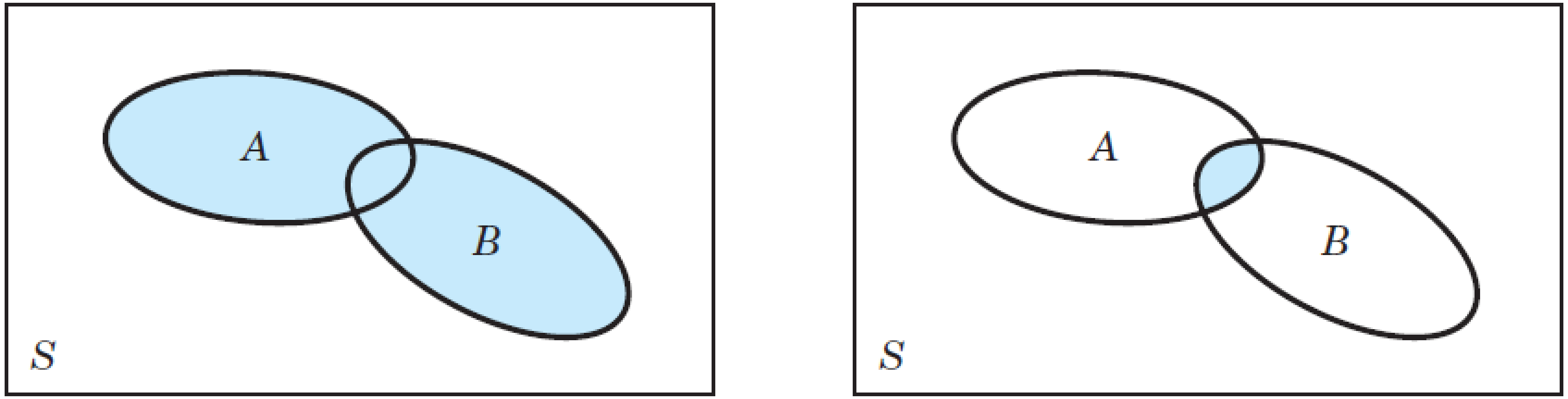
- Union and intersection are <u>distributive</u>:
- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ and $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
- and <u>associative:</u>
  $A \cup (B \cup C) = (A \cup B) \cup C$ and $A \cap (B \cap C) = (A \cap B) \cap C$

1.2.3 Venn Diagram

It is a graphical representation useful for illustrating the set operations.

Example 6

For events A, B such that $A \cap B \neq \emptyset$,



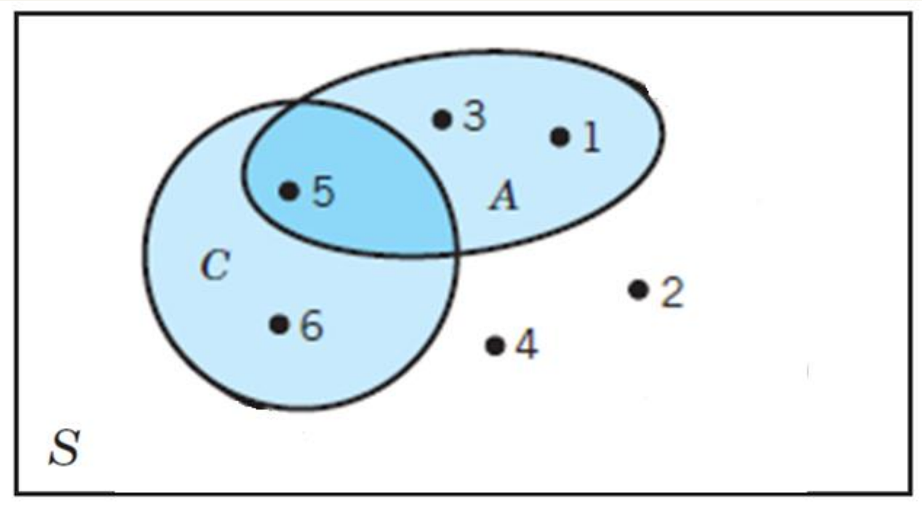
Union $A \cup B$ Intersection $A \cap B$

1.2.3 Venn Diagram: problem

Example 7

In the experiment of rolling a die, the events $A = \{1,3,5\}, C = \{5,6\}, A \cup C = \{1,3,5,6\}$,
$A \cap C = \{5\}$. The corresponding Venn diagram is given below.

What is the event $\{2,4\}$?



Summary

- Basic definitions and their usage in a given problem
  - Sample space, event, experiment, trial
- Set theory
  - Union, intersection, complement
- Representation of sets using Venn diagrams