EEE205 – Digital Electronics (II)

Lecture 9

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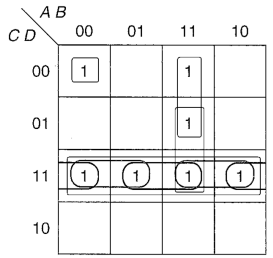
In This Session

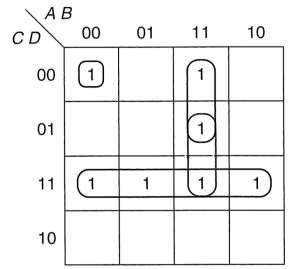
- Quine-McCluskey Method

A Revision of Some Terminology

	Boolean Algebra View	Karnaugh Map View
Minterm	a product term with all variables	a 1 cell
Implicant	a product term	a group of 1's
Prime Implicant	a shortest product term	a group of 1's that is not fully contained in another group of 1's
Essential Prime Implicant	product terms which must stay	has a 1 not included in other group

A Revision of Some Terminology





This K-map contains:

- 7 minterms
- 14 implicants
- 4 prime implicants.
- 3 essential prime implicants.

Quine-McCluskey Method

- The Karnaugh map method is for logic functions with a small number of variables.
- **Quine-McCluskey method** can simplify logic functions with *a large number of variables*.
- The latter is a systematic approach that can be *readily programmed* for a digital computer.

Quine-McCluskey Method

The *procedure* consists of two steps:

1. Finding prime implicants. Eliminate as many as literals as possible from each term by repeatedly applying XY + XY′ = X.
2. Select a minimum set of prime implicants by using a prime implicant chart.

Finding Prime Implicants

- We will start from a standard SOP form consisting of minterms only.
- By using the Theorem XY + XY′ = X, two minterms will be combined if they differ in only one variable.

$$AB'CD' + AB'CD = AB'C$$

$$\underbrace{1\ 0\ 1}_{X}\ 0 + \underbrace{1\ 0\ 1}_{X}\ 1 = \underbrace{1\ 0\ 1}_{X}\ - \quad \text{(the dash indicates a missing variable)}$$

$$A'BC'D + A'BCD' \quad \text{(will not combine)}$$

$$0\ 1\ 0\ 1 + 0\ 1\ 1\ 0 \quad \text{(will not combine)}$$

Finding Prime Implicants

$$f(a, b, c, d) = \Sigma\, m(0, 1, 2, 5, 6, 7, 8, 9, 10, 14)$$

- The binary minterms are sorted into groups according to the number of 1's.
- This is to reduce the unnecessary comparisons.

group 0	0	0000
group 1	1	0001
	2	0010
	8	1000
group 2	5	0101
	6	0110
	9	1001
	10	1010
group 3	7	0111
	14	1110

Finding Prime Implicants

- Only terms in adjacent groups must be compared.
- Terms in non-adjacent groups differ in at least two variables.
- Two terms within a group differ in at least two variables.

group 0	0	0000
group 1	1	0001
	2	0010
	8	1000
group 2	5	0101
	6	0110
	9	1001
	10	1010
group 3	7	0111
	14	1110

Finding Prime Implicants

				Column I	Column II
group 0	0	0000	✓	0, 1	000–
group 1	1	0001	✓	0, 2	00–0
	2	0010	✓	0, 8	–000
	8	1000	✓	1, 5	0–01
group 2	5	0101	✓	1, 9	–001
	6	0110	✓	2, 6	0–10
	9	1001	✓	2, 10	–010
	10	1010	✓	8, 9	100–
group 3	7	0111	✓	8, 10	10–0
	14	1110	✓	5, 7	01–1
				6, 7	011–
				6, 14	–110
				10, 14	1–10

- Terms 0000 and 0001 can be combined to yield 000- (a'b'c'), and so on.
- The outcomes are listed in a new column.
- The terms which can be combined are checked off.
- The remaining ones are prime implicants.

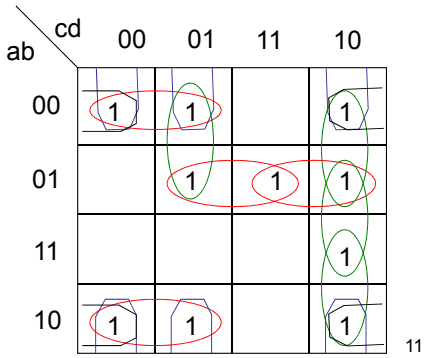
Finding Prime Implicants

Karnaugh Map's View

$f(a, b, c, d) =$
$\Sigma\, m(0, 1, 2, 5, 6, 7, 8, 9, 10, 14)$

- To find groups of 2.
- If a 1 is not in any group of 2, it is a prime implicant.

ab \ cd	00	01	11	10
00	1	1		1
01		1	1	1
11				1
10	1	1		1



Finding Prime Implicants

				Column I	Column II
group 0	0	0000	✓	0, 1	000–
group 1	1	0001	✓	0, 2	00–0
	2	0010	✓	0, 8	–000
	8	1000	✓	1, 5	0–01
group 2	5	0101	✓	1, 9	–001
	6	0110	✓	2, 6	0–10
	9	1001	✓	2, 10	–010
	10	1010	✓	8, 9	100–
group 3	7	0111	✓	8, 10	10–0
	14	1110	✓	5, 7	01–1
				6, 7	011–
				6, 14	–110
				10, 14	1–10

- Terms in the new column are grouped according to the number of 1's.
- Terms which have dashes in the same place and which differ in only one variable can be combined.
- Terms 000- (a'b'c') and 100- (ab'c') yield -00- (b'c').

Finding Prime Implicants

- Find and delete the duplicate terms.

	Column I		Column II		Column III	
group 0	0	0000 ✓	0, 1	000– ✓	0, 1, 8, 9	–00–
group 1	1	0001 ✓	0, 2	00–0 ✓	0, 2, 8, 10	–0–0
	2	0010 ✓	0, 8	–000 ✓	~~0, 8, 1, 9~~	~~–00–~~
	8	1000 ✓	1, 5	0–01	~~0, 8, 2, 10~~	~~–0–0~~
group 2	5	0101 ✓	1, 9	–001 ✓	2, 6, 10, 14	--10
	6	0110 ✓	2, 6	0–10 ✓	~~2, 10, 6, 14~~	~~--10~~
	9	1001 ✓	2, 10	–010 ✓		
	10	1010 ✓	8, 9	100– ✓		
group 3	7	0111 ✓	8, 10	10–0 ✓		
	14	1110 ✓	5, 7	01–1		
			6, 7	011–		
			6, 14	–110 ✓		
			10, 14	1–10 ✓		

Finding Prime Implicants

- Keep comparing terms and forming new groups of terms until no more terms could be combined.
- Terms which have not been checked off are prime implicants.

$$f = a'c'd + a'bd + a'bc + b'c' + b'd' + cd'$$
$$(1, 5)\quad (5, 7)\quad (6,7)\quad (0, 1, 8, 9)\quad (0, 2, 8, 10)\quad (2, 6, 10, 14)$$

- Each term has a minimum number of literals, but *the number of terms is not minimum*. This is left to the **prime implicant chart**.

Finding Prime Implicants

Karnaugh Map's View

- To find groups of 4.
- If a group of 2 is not in any group of 4, it is a prime implicant.