Tutorial 5 Surface integrals and divergence theorem

1. Flux integrals $\iint_S \boldsymbol{F} \cdot \boldsymbol{n} dA$ (page 450). Evaluate the second type surface integral for the given data. Describe the kind of surface. Show the details of your work.

   (1) $\boldsymbol{F} =< -x^2, y^2, 0 >$, S: $\boldsymbol{r} =< u, v, 3u - 2v >$, $0 \leq u \leq \frac{3}{2}, -2 \leq v \leq 2$.

   (2) $\boldsymbol{F} =< \mathrm{e}^y, \mathrm{e}^x, 1 >$, S: $x + y + z = 1, x \geq 0, y \geq 0, z \geq 0$.

2. Surface integrals $\iint_S G(\boldsymbol{r})d\boldsymbol{A}$ (page 450). Evaluate the first type surface integral for the following data. Indicate the kind of surface. Show the details of your work.
(1) $G = \cos x + \sin x$, S is the portion of $x + y + z = 1$ in the first octant.

(2) $G = x + y + z, z = x + 2y, 0 \leq x \leq \pi, 0 \leq y \leq x$.

3. Evaluate the integral $\iint_S \boldsymbol{F} \cdot \boldsymbol{n} dA$ where $\boldsymbol{F} =< 0,0, z >$, and S is the oriented surface parametrized by $\boldsymbol{r}(u, v) =< u\cos v, u\sin v, v >, 0 \leq u \leq 1, 0 \leq v \leq 2\pi$.

4. Evaluate the surface integral $\iint_S \boldsymbol{F} \cdot \boldsymbol{n} dA$ where $\boldsymbol{F} =< x, y, 2z >$, and S is the part of the paraboloid $z = 4 - x^2 - y^2$ that lies above the unit square $[0,1] \times [0,1]$ with the downward orientation.

5. Evaluate the surface integral $\iint_S \boldsymbol{F} \cdot \boldsymbol{n} dA$ by the divergence theorem. Show the details.
   (1) $\boldsymbol{F} =< x^2,\ 0,\ z^2 >$, S is the surface of the box $|x| \leq 1, |y| \leq 3, 0 \leq z \leq 2$.

   (2) $\boldsymbol{F} =< \sin y, \cos x, \cos z >$, S is the surface of $x^2 + y^2 \leq 4$, $|z| \leq 2$ (a cylinder and two disks).

6. Evaluate the integral $\iint_S \boldsymbol{F} \cdot \boldsymbol{n} dA$ directly or, if possible, by the divergence theorem. Show the details.

(1) $\boldsymbol{F} = < ax, by, cz >$, S is the sphere $x^2 + y^2 + z^2 = 36$.

(2) $\boldsymbol{F} = < y + z, 20y, 2z^3 >$, S is the surface of $0 \leq x \leq 2, 0 \leq y \leq 1, 0 \leq z \leq y$.

(3) $\boldsymbol{F} = < y^2, x^2, z^2 >$, S is $\boldsymbol{r} = < u, u^2, v >, 0 \leq u \leq 2, -2 \leq v \leq 2$.