

EEE205 – Digital Electronics (II)

Lecture 8

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In This Session

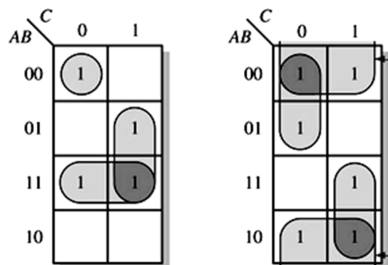
- A Revision of Karnaugh Maps
- The Terminology
- Five-Variable Karnaugh Maps

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A Revision of Karnaugh Maps

Grouping the 1s

- The goal is to **maximize the size** of the groups (shorter product terms) and to **minimize the number** of groups (less product terms).
- A group may contain 1, 2, 4, 8, or 16 adjacent cells.
- Each 1 must be included in one or **more** groups.

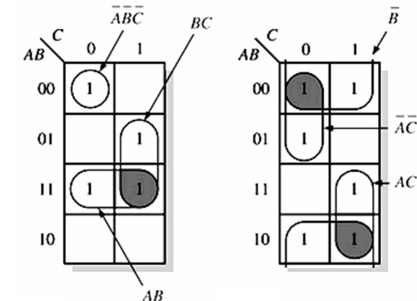


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A Revision of Karnaugh Maps

Determine the Minimum SOP

1. When a variable appears in both complemented and uncomplemented form in a group, that variable is eliminated.
2. Variables that are the same for all cells of the group must appear — 1 for uncomplemented form and 0 for complemented form.



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The Terminology

- A **minterm** is a product term that includes all the variables in a Boolean function.
- It corresponds a 1 cell in a K-map.
- Here 7 minterms in total.

CD \ AB	AB			
	00	01	11	10
00	1		1	
01			1	
11	1	1	1	1
10				

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The Terminology

- A minterm is often referred to by the decimal number converted from its binary value.

ABC	Minterm	Number
000	$A'B'C'$	0
001	$A'B'C$	1
010	$A'BC'$	2
011	$A'BC$	3
100	$AB'C'$	4
101	$AB'C$	5
110	ABC'	6
111	ABC	7

$$m_1 = A'B'C$$

$$m_6 = ABC'$$

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The Terminology

- Minterms are often used to express a specific Boolean function or a truth table in a compact way.

ABC	f
000	0
001	1
010	1
011	1
100	1
101	1
110	0
111	0

$$f(A,B,C) = A'B'C + A'BC' + A'BC + AB'C' + AB'C$$

$$f(A,B,C) = m_1 + m_2 + m_3 + m_4 + m_5 = \Sigma m(1,2,3,4,5)$$

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The Terminology

- If the function includes **don't cares**, those terms are included in a separate sum.

abc	g
000	X
001	1
010	1
011	X
100	0
101	1
110	0
111	0

$$g(a,b,c) = \Sigma m(1,2,5) + \Sigma d(0,3)$$

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The Terminology

- An **implicant** is a product form that can be used in the sum-of-products expression for a function.
- The function is 1 whenever the implicant is 1.
- An implicant corresponds to a rectangle of 1, 2, 4, 8, ... (any power of 2) 1's in a K-map.
- Here 14 implicants: seven 1's, six groups of 2 and one group of 4.

$\backslash AB$	00	01	11	10
CD 00	1		1	
01			1	
11	1	1	1	1
10				

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The Terminology

- A **prime implicant** is an implicant which can not be merged with another implicant to remove a variable.
- It corresponds to a 1's group which is not fully contained in another group.
- Here 4 prime implicants: $A'B'C'D'$, ABC' , ABD and CD .

$\backslash AB$	00	01	11	10
CD 00	1		1	
01			1	
11	1	1	1	1
10				

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The Terminology

- An **essential prime implicant** is a prime implicant that includes at least one 1 that is not included in any other prime implicant.
- Essential prime implicants must appear in a fully minimized SOP expression.
- Here 3 essential prime implicants: $A'B'C'D'$, ABC' and CD .

$\backslash AB$	00	01	11	10
CD 00	1		1	
01			1	
11	1	1	1	1
10				

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Karnaugh Maps

Find prime implicants

$$f(a, b, c, d) =$$

$$\sum m(0, 1, 2, 5, 6, 7, 8, 9, 10, 14)$$

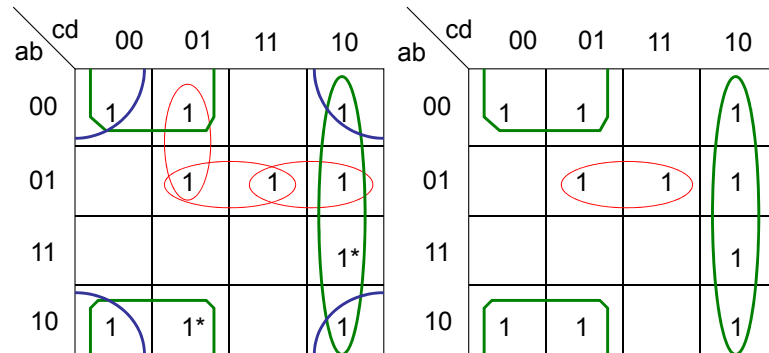
$\backslash cd$	00	01	11	10
ab 00	1	1		1
01		1	1	1
11				1
10	1	1		1

$\backslash cd$	00	01	11	10
ab 00	1	1		1
01		1	1	1
11				1*
10	1	1		1

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Karnaugh Maps

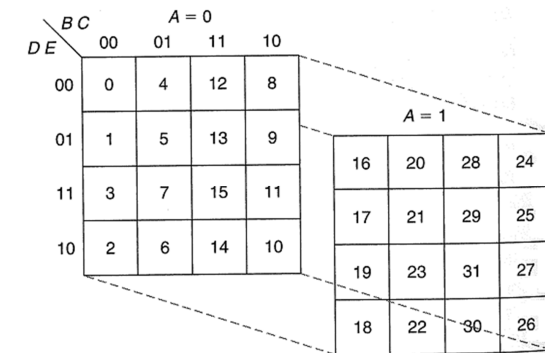
Find essential prime implicants



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Five-Variable Karnaugh Maps

- Boolean functions with five variables can be simplified using **two** 4-variable maps.
- Squares directly above or below each other are adjacent.

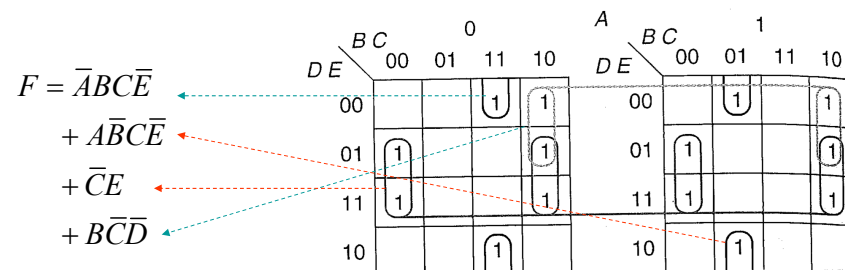
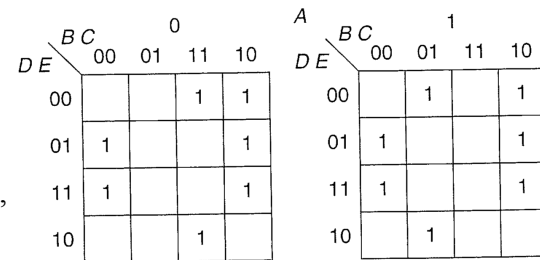


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Five-Variable Karnaugh Maps

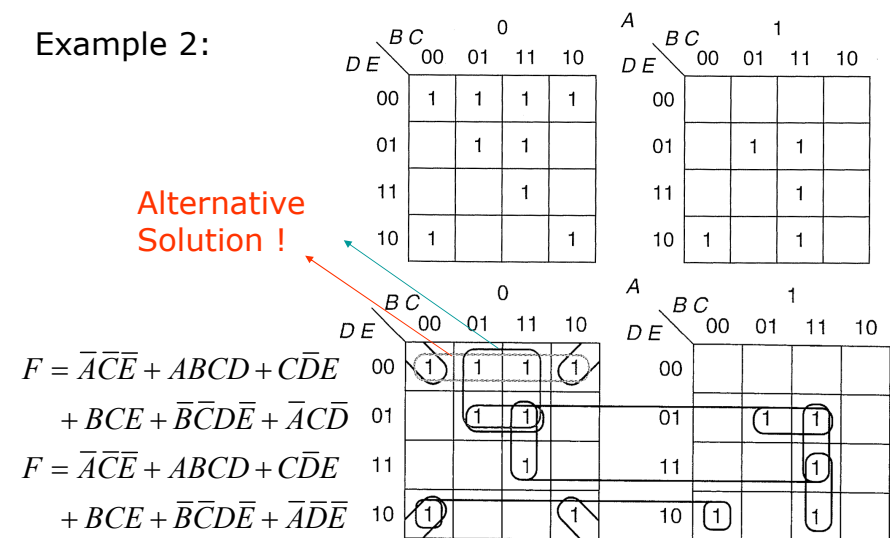
Example 1:

$$F(A, B, C, D, E) = \sum m(1, 3, 8, 9, 11, 12, 14, 17, 19, 20, 22, 24, 25, 27)$$



Five-Variable Karnaugh Maps

Example 2:



Alternative Solution !

$$F = \bar{A}\bar{C}\bar{E} + ABCD + C\bar{D}E + BCE + \bar{B}\bar{C}\bar{D}\bar{E} + \bar{A}\bar{C}\bar{D}$$

$$F = \bar{A}\bar{C}\bar{E} + ABCD + C\bar{D}E + BCE + \bar{B}\bar{C}\bar{D}\bar{E} + \bar{A}\bar{D}\bar{E}$$