EEE336 Signal Processing and Digital Filtering

Lecture 14 FIR Filters Design

Lect_14_1 Simple FIR Filters

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Simple FIR filter

- Two-point (M=1 → 1st order) moving average filter

  h[n]= (½) [1 1] = [½ ½] -> h[n]= ½ (δ[n]+δ[n-1])

$$H_0(z) = \frac{1}{2}\left(1 + z^{-1}\right) = \frac{z+1}{2z}$$

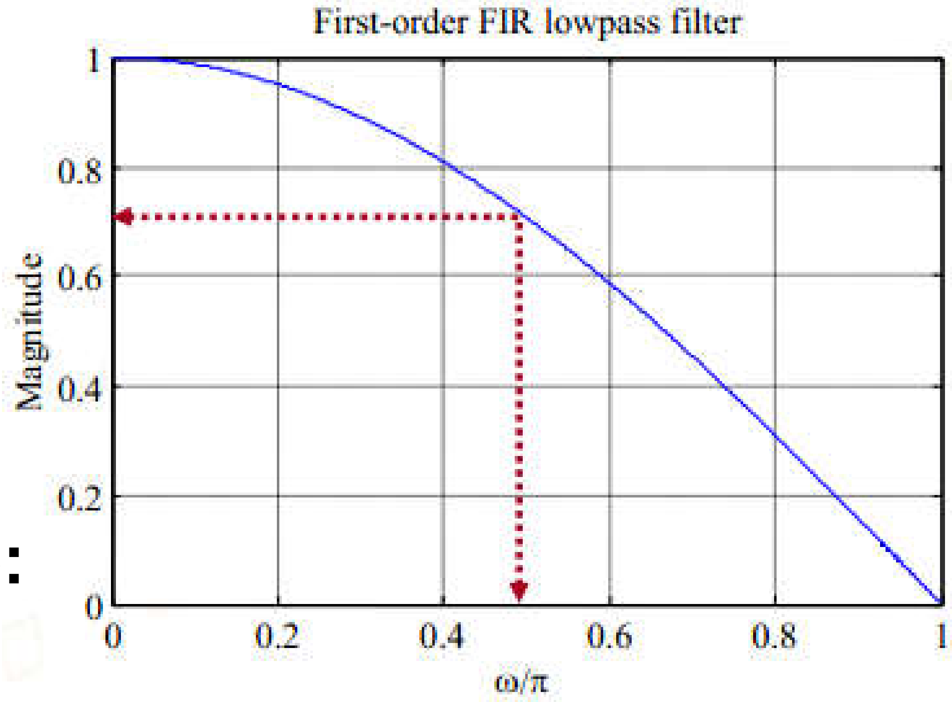
- Notice that H(z) has a zero at z=-1, and a pole at z=0.
  - Remember: for stable systems (and FIR filters are always stable), the frequency response can be obtained by substituting $z=e^{j\omega}$

$$H(\omega) = H(e^{j\omega}) = H(z)\big|_{z=e^{j\omega}} = \frac{z+1}{2z}\bigg|_{z=e^{j\omega}} = \frac{e^{j\omega}+1}{2e^{j\omega}}$$

  - The zero at ω=π (suppress high frequency components π), coupled with the pole at z=0, makes this a lowpass filter

Simple FIR filter

$$H(e^{j\omega}) = \frac{e^{j\omega}+1}{2e^{j\omega}} = e^{-j\omega/2}\cos(\omega/2)$$



Monotonically decreasing
=> a low pass filter

System Gain at some frequency ω:

$$\left|H(\omega)\right| = \cos(\omega/2)$$

The frequency ω_c at which $\left|H(\omega_c)\right| = \frac{1}{\sqrt{2}}\left|H(0)\right|$ is of special interest:
3-dB cutoff frequency

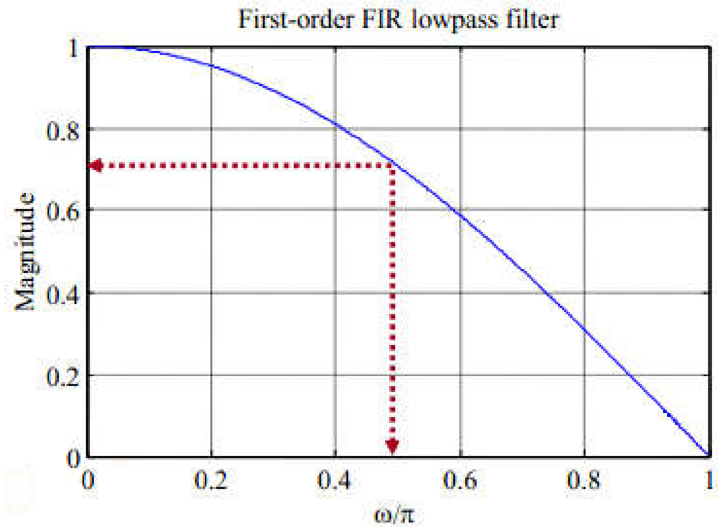
$$20\log_{10}\left|H(\omega)\right|\Big|_{\omega=\omega_c} = 20\log_{10}\left|H(0)\right| - 20\log_{10}\left(\sqrt{2}\right) = 0 - 0.30103 \cong -3.0\text{dB}$$

Simple FIR filter

- For realizable filters, the cut-off frequency is the frequency at which the system gain reaches its 0.707 multiple of its peak value.
- This gain represents the frequency at which the signal power is half of its peak power!
- For a lowpass filter, the gain at the cut-off frequency is 3dB less than its gain at zero frequency (or 0.707 of its zero frequency amplitude, or half the power of its power at zero frequency).
- For the first order filter, this occurs at $\omega_c = \pi/2$

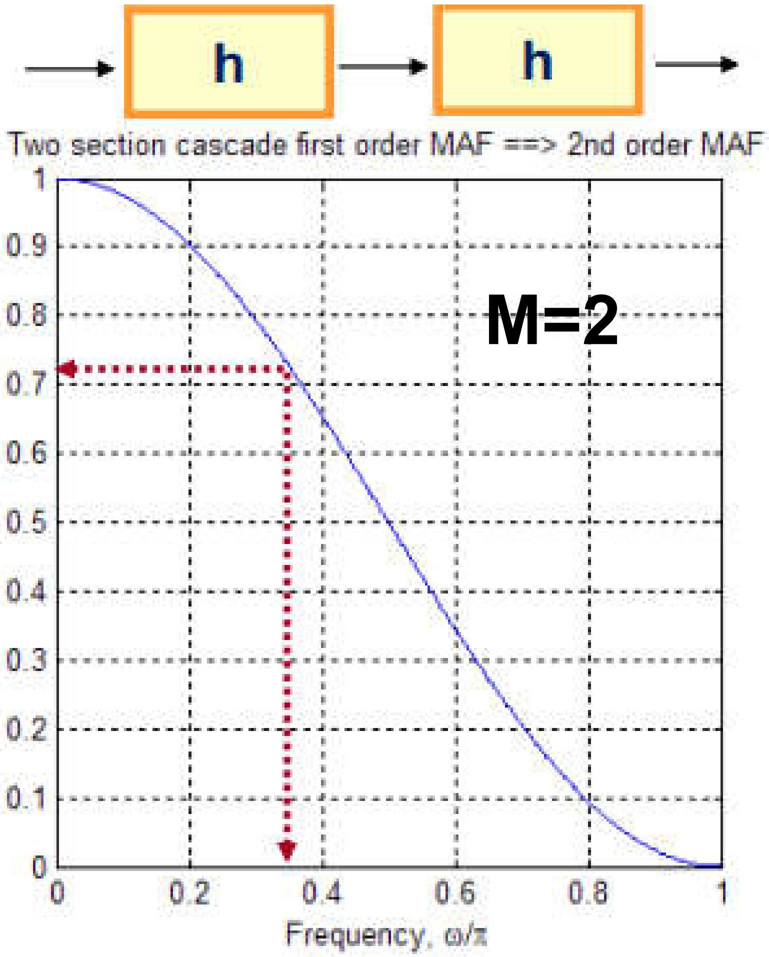
$$|H(\omega_c)|^2 = \cos^2(\omega_c/2) = 1/2$$

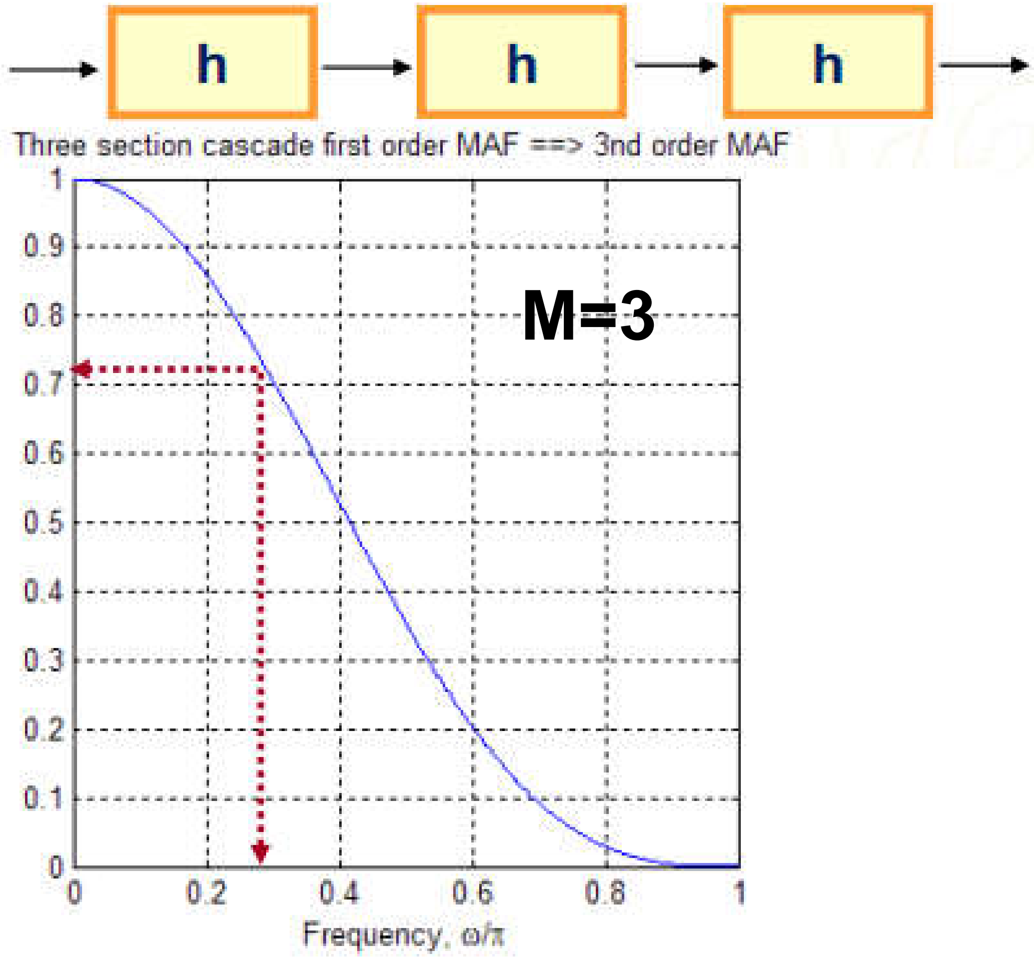
$$\Rightarrow \cos(\omega_c/2) = 1/\sqrt{2} \quad \Rightarrow \omega_c = \pi/2$$



Cascaded FIR Filters

$$H(z)=\left(\frac{1}{2}\left(1+z^{-1}\right)\right)^{M} \Rightarrow \left|H(\omega)\right| = \cos^{M}(\omega/2) \;\Rightarrow \omega_c = 2\cos^{-1}\left(2^{-1/2M}\right)$$





$$\omega_c = 2\cos^{-1}\left(2^{-1/4}\right) = 1.144 = 0.364\pi \qquad \omega_c = 2\cos^{-1}\left(2^{-1/6}\right) = 0.943 = 0.3\pi$$

Example

- If we want a LPF with a cutoff frequency of 0.2π, what order filter do we need? What linear frequency does this correspond to (with the sampling frequency of 3kHz)?

Simple FIR Highpass Filters

- The simplest highpass filter is obtained from the lowpass filter by replacing z with -z resulting in a transfer function

$$H_1(z) = \frac{1}{2}\left(1 - z^{-1}\right) = \frac{z-1}{2z}$$

- Notice that $H_1(z)$ has a zero at z=1, and a pole at z=0.
  - The frequency response is

$$H_1(\omega) = je^{-j\omega/2}\sin(\omega/2) = e^{-(j\omega/2-\pi/2)}\sin(\omega/2)$$

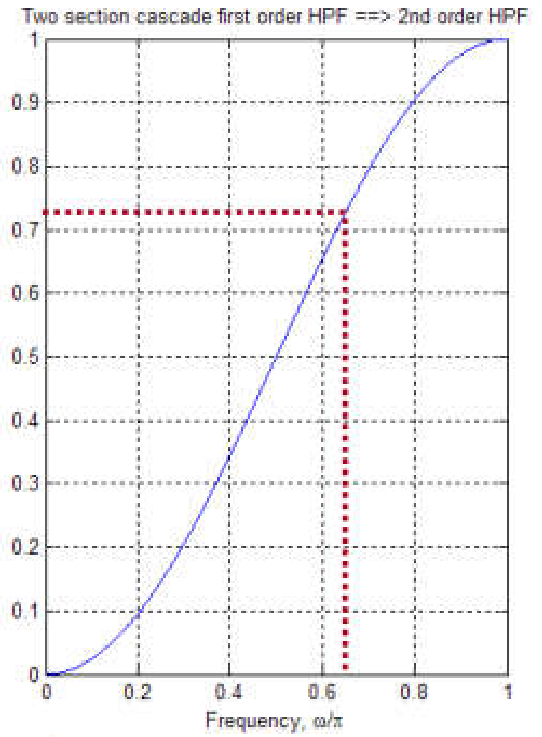
  - Therefore, the frequency response has a zero at ω=0, corresponding to z=1.
  - The zero at ω=0 (suppress low frequency components 0), makes this a highpass filter

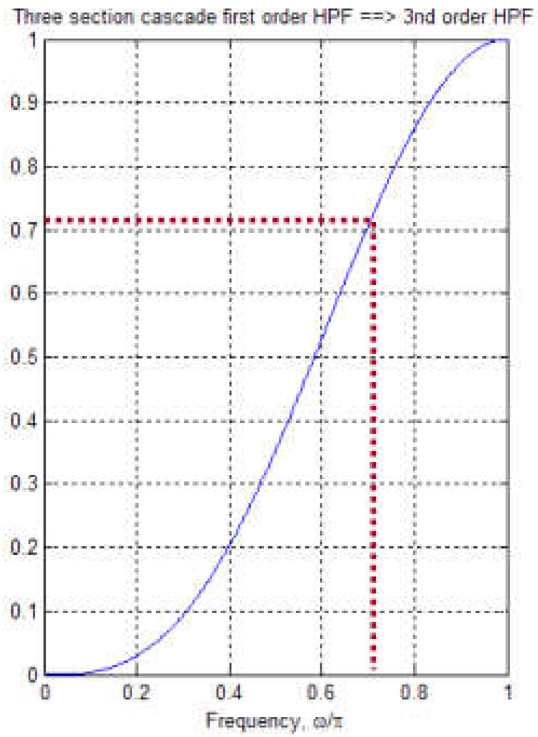
Cascaded FIR Highpass Filters

- A higher-order highpass filter

$$H_1(z) = \frac{1}{M}\sum_{n=0}^{M-1}(-1)^n z^{-n}$$

is obtained by replacing z with -z in the transfer function of a moving average filter





- When M is odd number

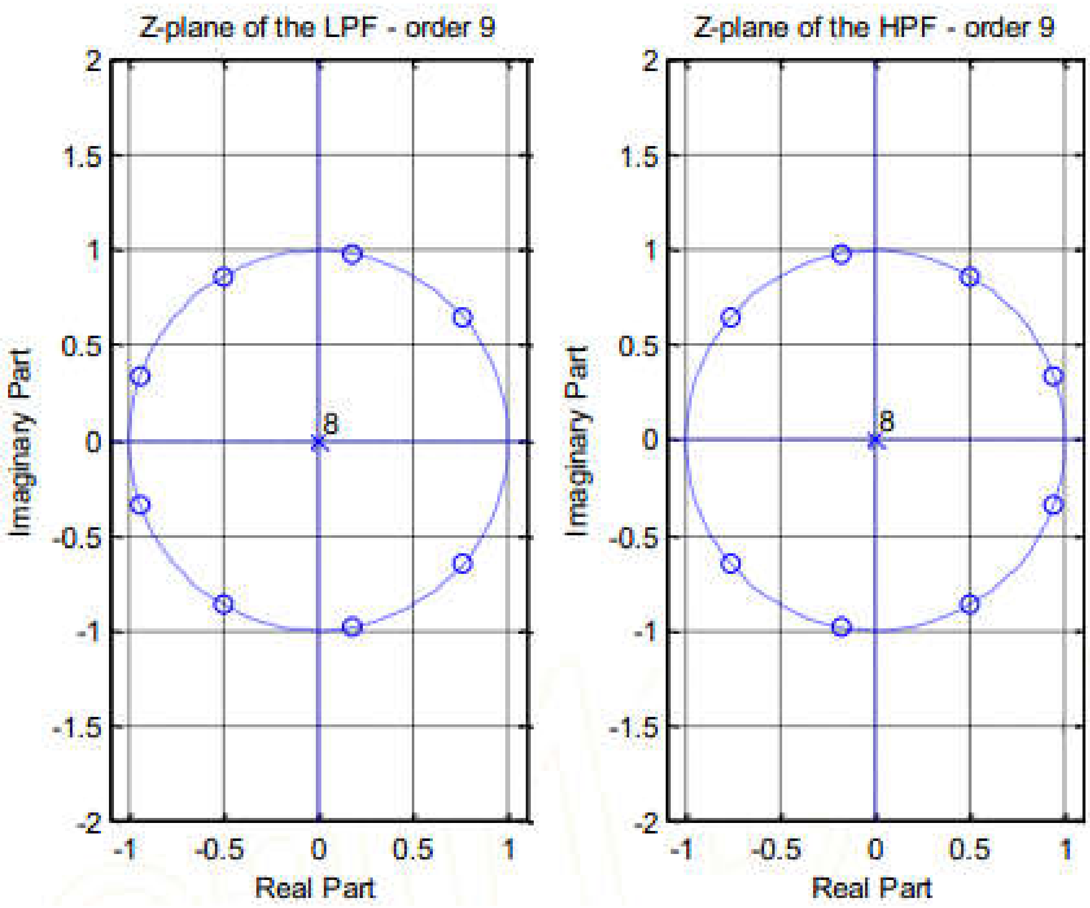
$$H_1(z) = -\frac{z^M + 1}{M[z^{M-1}(z+1)]}$$

- When M is even number

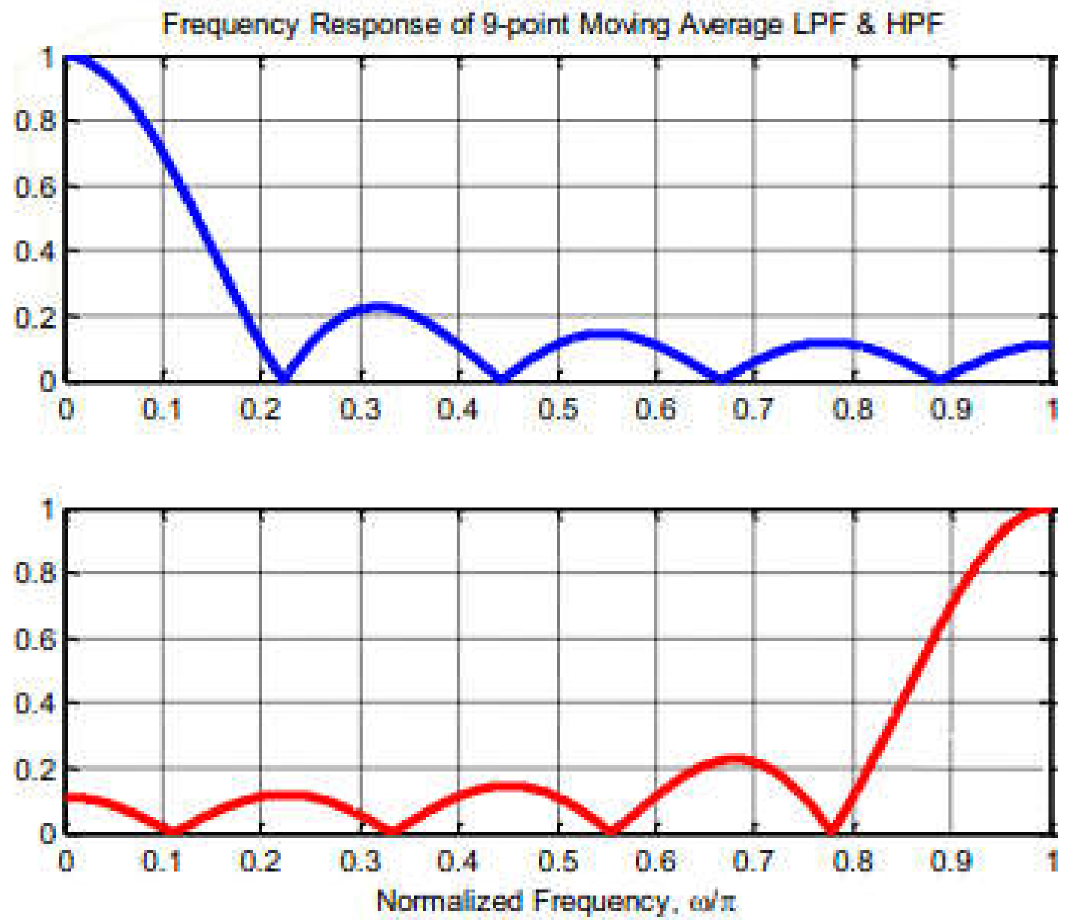
$$H_1(z) = -\frac{z^M - 1}{M[z^{M-1}(z+1)]}$$

Cascaded FIR Highpass Filters

Zero and poles in z-plane



Frequency responses



14_1 Wrap Up

- So far we have seen several basic filter architectures:
  - Impulse response based: FIR & IIR
  - Magnitude response based: LPF, HPF, BPF and BSF, allpass
  - Phase response based: zero-phase, linear phase, non-linear phase

- FIR examples
  - 1^{st} order Moving Average Filter (MAF) – lowpass
  - 1^{st} order highpass by replacing "z" with "-z" in MAF
  - High order FIR by cascading

- How to design a filter with specific desired passband and stopband characteristics?

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Lecture 14 FIR Filters Design

Lect_14_2 Specification and Design by Truncation

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Fundamental Requirements

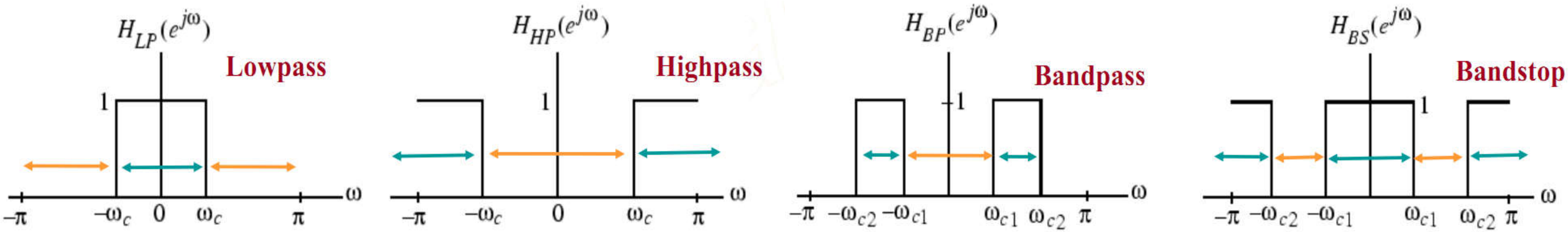
- For a practical system:
  - Real coefficients;
  - Stable
  - Causal
  - Lowest order M;
  - Linear phase

Fundamental Requirements

- The following must be taken into consideration in making filter selection:
  - H(z) satisfying the frequency response specifications must be causal and stable (*poles inside the unit circle, ROC includes the unit circle, h[n] is right-sided*).
  - If the filter is FIR, then H(z) is a polynomial in z^{-1} with real coefficients:
    - If linear phase is desired, the filter coefficients h[n] must satisfy symmetry constraints: $h[n] = \pm h[M-n]$;
    - For computational efficiency, the minimum filter order M that satisfies design criteria must be used.
  - If the filter is IIR, the H(z) is a real rational function of z^{-1}, so:
    - Stability must be ensured;
    - Minimum (M, N) that satisfies the design criteria must be used.

FIR Filter Design

- Objective: Obtain a **<u>realizable</u>** transfer function H(z) approximating a desired frequency response.
  - Typically magnitude (and sometimes phase) response of the desired filter is specified



  - Since the ideal filters cannot be realized, we need to relax (i.e., smooth) the sharp filter characteristics in the passband and stopband by providing ***acceptable tolerances***

Filter Specifications

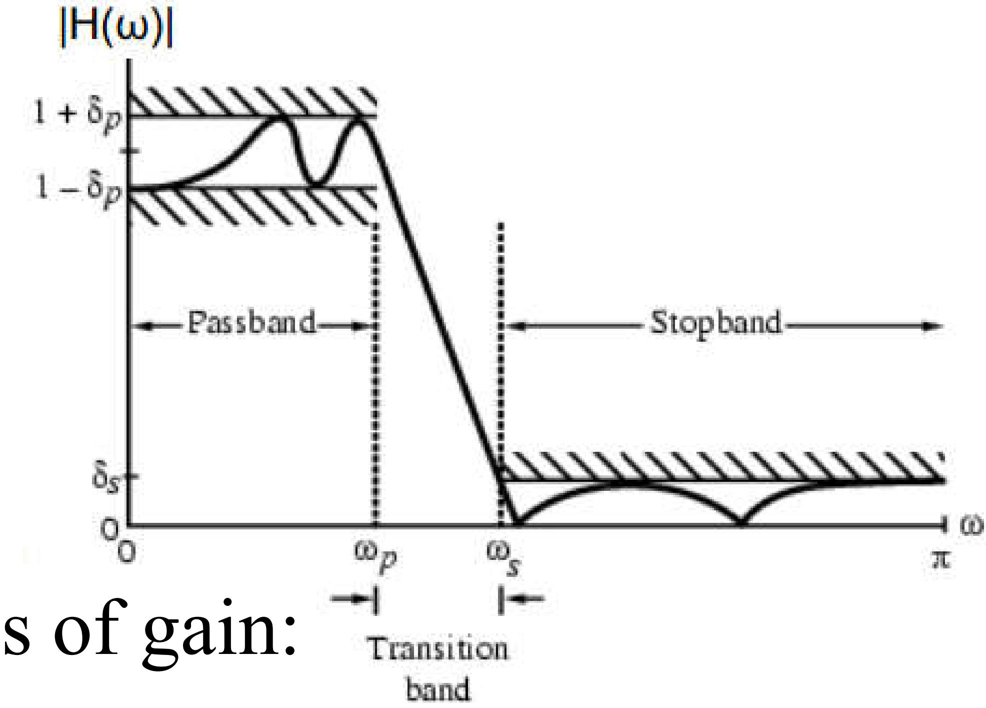
- $|H(e^{j\omega})| \approx 1$, with an error $\pm\delta_p$ in the passband:

$$1-\delta_p \le |H(\omega)| \le 1+\delta_p, \ |\omega| \le \omega_p$$

- $|H(e^{j\omega})| \approx 0$, with an error δ_s in the stopband:

$$|H(\omega)| \le \delta_s, \quad \omega_s \le |\omega| \le \pi$$

ω_p – passband edge frequency
ω_s – stopband edge frequency
δ_p - peak ripple value in the passband
δ_s - peak ripple value in the stopband



- Filter specifications are often given in decibels, in terms of loss of gain:

$$G(\omega) = -20\log_{10}\left|H(e^{j\omega})\right|$$

Cut-off frequency in Digital Domain

- In general, filter specifications are given in Hz, but most digital filters are designed in normalized frequencies, where $2\pi <=> f_t$.
  - Then, the normalized passband and stopband edge frequencies can be obtained from linear frequencies as follows:
    - f_p: linear pass band edge frequency
    - ω_p: normalized (angular) pass band edge frequency
    - f_s : linear stop band edge frequency
    - ω_s: normalized (angular) stop band edge frequency
    - f_t: sampling frequency
    - T: sampling period

$$\boxed{\omega_p = \frac{2\pi f_p}{f_t} = 2\pi f_p T}$$

$$\boxed{\omega_s = \frac{2\pi f_s}{f_t} = 2\pi f_s T}$$

  - E.g. If the sampling frequency is $f_t = 10$kHz, and we want $f_p = 3$kHz and $f_s = 4$kHz, find ω_p and ω_s.

FIR Filter Design

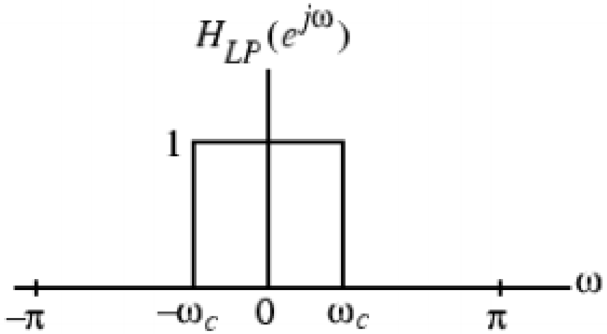
- Start with the ideal lowpass filter

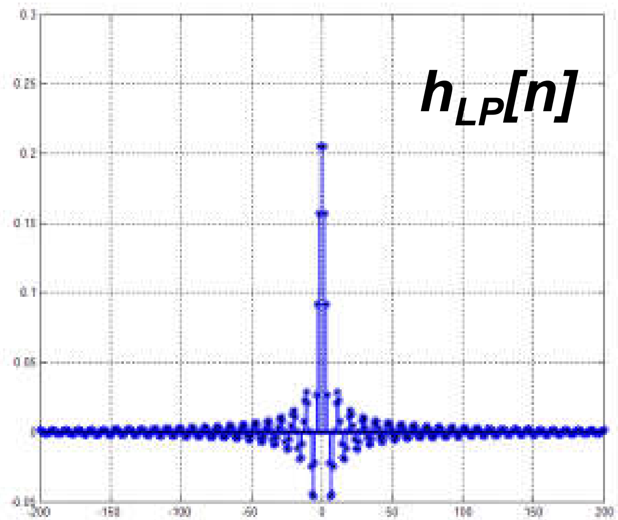
$$H_{LP}(\omega)=\begin{cases}1, & 0\le|\omega|\le\omega_c\\ 0, & \omega_c\le|\omega|\le\pi\end{cases}$$

$$h_{LP[n]}=\frac{1}{2\pi}\int_{-\omega_c}^{\omega_c}1\cdot e^{j\omega n}d\omega$$

$$=\frac{\sin\ \omega_c n}{\pi n}=\frac{\omega_c}{\pi}\mathrm{sinc}\ \ \omega_c n\ \ ,-\infty<n<\infty$$

- Two problems with this filter:
  - infinitely long
  - non-causal





Least Integral-squared Error Design of FIR Filters

- Let $H_d(e^{j\omega})$ denote the desired (ideal) frequency response, then $h_d[n]$ is the corresponding impulse response.
  - Two problems of the desired filter: infinitely long and non-causal;
- The objective of filter design is to find a finite-duration impulse response sequence $h_t[n]$ of length 2M+1 whose DTFT $H_t(e^{j\omega})$ approximates the desired DTFT $H_d(e^{j\omega})$ in some sense.

$H_d(\omega)$	$H_t(\omega)$
$h_d(\omega)$	$h_t(\omega)$

- One commonly used approximation criterion is to minimize the integral-squared error Φ:

$$\Phi = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left|H_t(e^{j\omega}) - H_d(e^{j\omega})\right|^2 d\omega$$

Least Integral-squared Error Design of FIR Filters

- Using Parseval's relation, we can get:

$$\Phi = \sum_{n=-\infty}^{\infty} \left| h_t[n] - h_d[n] \right|^2$$

$$= \sum_{n=-M}^{M} \left| h_t[n] - h_d[n] \right|^2 + \sum_{n=-\infty}^{-M-1} h_d^2[n] + \sum_{n=M+1}^{\infty} h_d^2[n]$$

  - It is evident that Φ is minimum when

$$h_t[n] = h_d[n] \;\text{ for }\; -M \le n \le M$$

  - Or, in other words, the best finite-length approximation to the ideal infinite-length impulse response in the mean-square error sense is simply obtained by **truncation**.

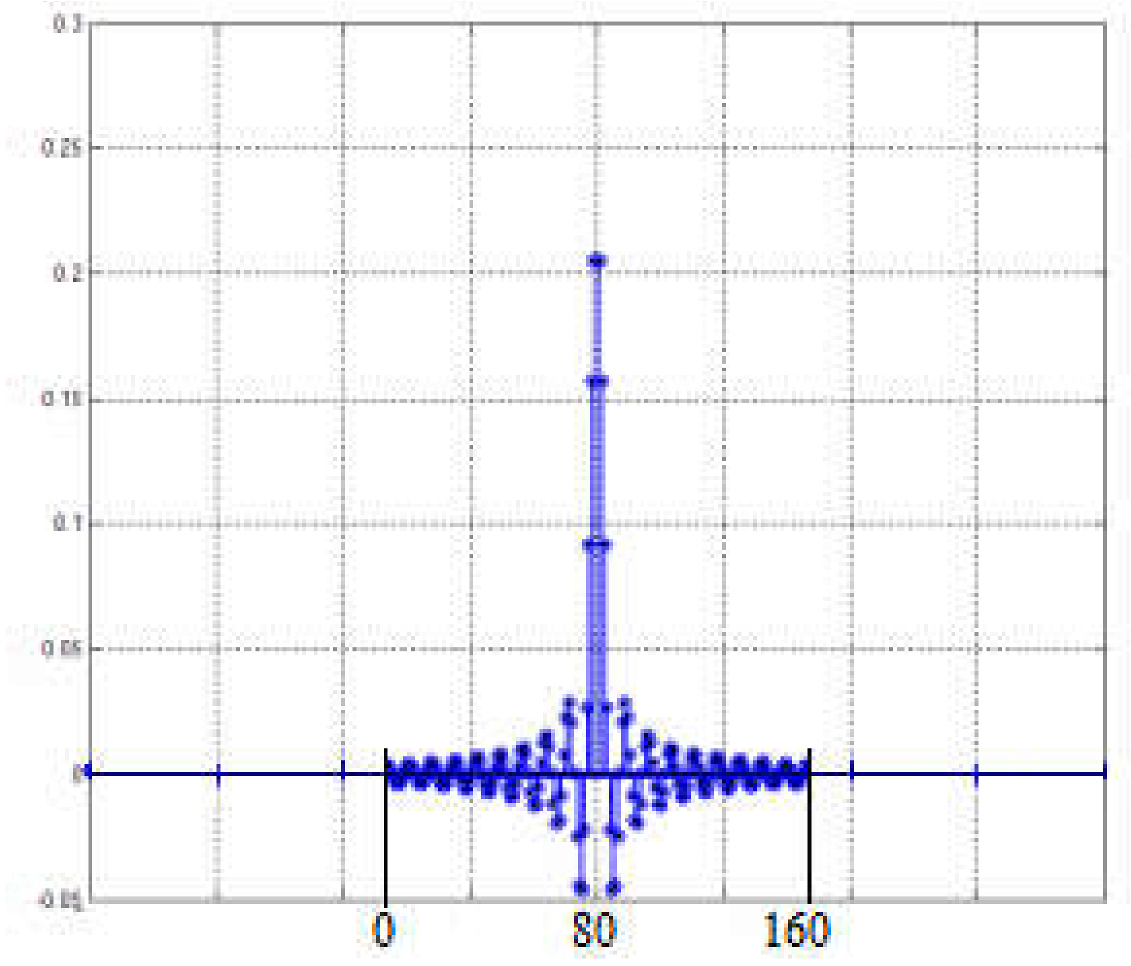
Least Integral-squared Error Design of FIR Filters

Unrealizable!

$$h_d[n] = \frac{\sin(\omega_c n)}{\pi n}, \ -\infty < n < \infty$$

$$h_t[n] = \begin{cases} \dfrac{\sin\left(\omega_c \left(n - M/2\right)\right)}{\pi\left(n - M/2\right)}, 0 \le n \le 2M, n \ne M \\ \dfrac{\omega_c}{\pi}, \quad n = \dfrac{M}{2} \end{cases}$$

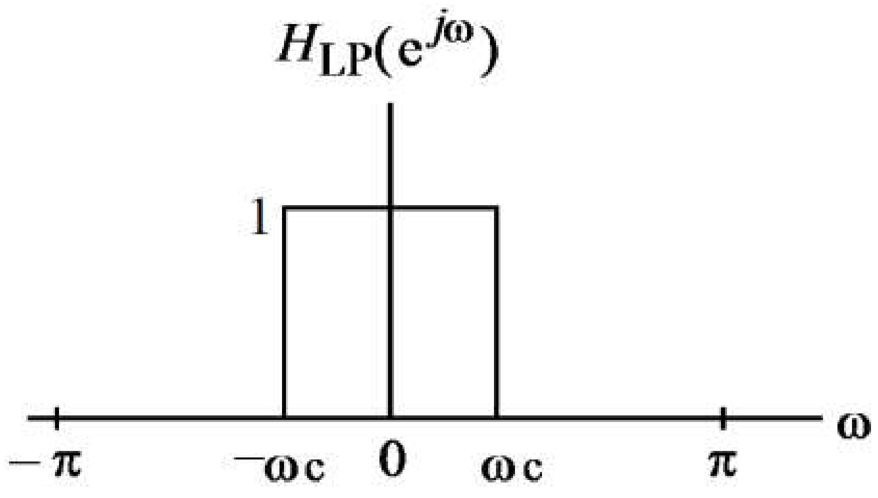
Realizable!



FIR Filter Design

- This is the basic, straightforward approach to FIR filter design:
  - Step 1: Start with an ideal filter that meets the design criteria, say a filter $H_d(\omega)$
  - Step 2: Take the inverse DTFT of this $H_d(\omega)$ to obtain $h_d[n]$.
    - This $h_d[n]$ will be double infinitely long, and non-causal -> unrealizable
  - Step 3: Truncate using a window, say a rectangle, so that 2M+1 coefficients of $h_d[n]$ are retained, and all the others are discarded.
    - We now have a finite length (order 2M) filter, $h_t[n]$, however, it is still non-causal
  - Step 4: Shift the truncated $h_t[n]$ to the right (i.e., delay) by M samples, so that the first sample now occurs at n=0.
    - The resulting impulse response, $h_t[n-M]$ is a causal, stable, FIR filter, which has an almost identical magnitude response and a phase factor or $e^{-j\omega M}$ compared to the original filter, due to delay introduced.

LPF



Unrealizable!

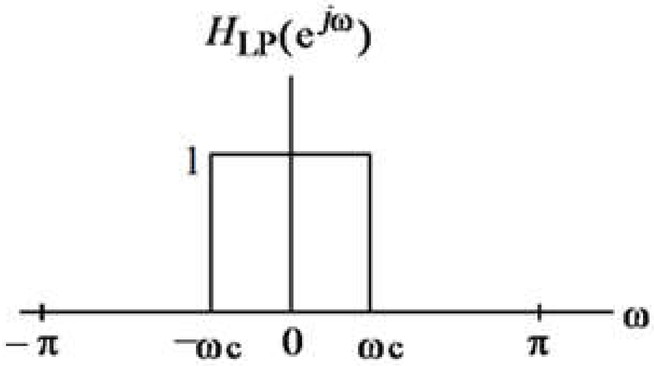
$$h_{LP[n]} = \frac{\sin(\omega_c n)}{\pi n}, \quad -\infty < n < \infty$$

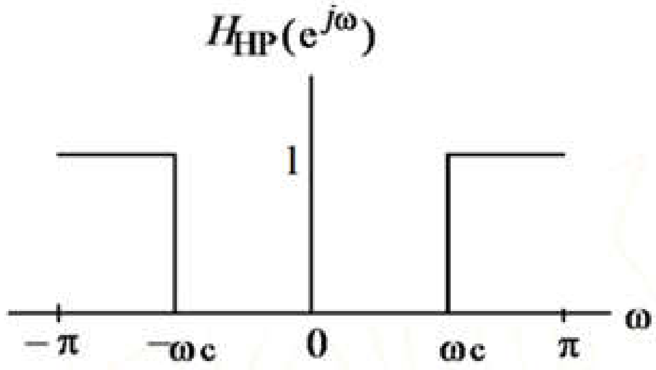
Realizable!

$$h_{LP[n]} = \begin{cases} \dfrac{\sin(\omega_C (n - M/2))}{\pi (n - M/2)}, & 0 \le n \le M, \ n \ne \dfrac{M}{2} \\ \dfrac{\omega_C}{\pi}, & n = \dfrac{M}{2} \end{cases}$$

Notice: lengths of the impulse responses in the following 3 pages are M+1.

From LPF to HPF





$$H_{HP}(\omega) = 1 - H_{LP}(\omega)$$

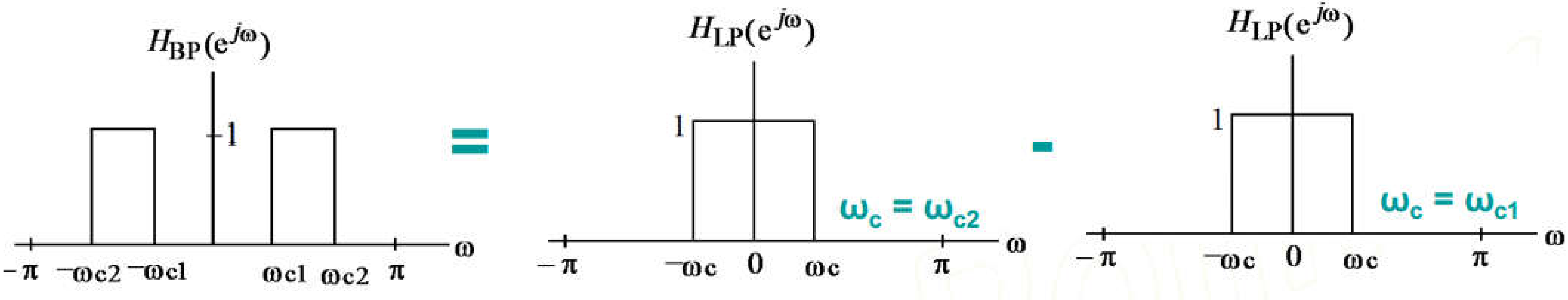
$$\Updownarrow$$

$$h_{HP}[n] = \delta[n] - h_{LP}[n]$$

$$h_{LP}[n] = \begin{cases} \dfrac{\sin(\omega_c(n - M/2))}{\pi(n - M/2)}, & 0 < n < M,\ n \neq \dfrac{M}{2} \\ \dfrac{\omega_c}{\pi}, & n = \dfrac{M}{2} \end{cases}$$

$$h_{HP}[n] = \begin{cases} -\dfrac{\sin(\omega_c(n - M/2))}{\pi(n - M/2)}, & 0 < n < M,\ n \neq \dfrac{M}{2} \\ 1 - \dfrac{\omega_c}{\pi}, & n = \dfrac{M}{2} \end{cases}$$

BPF / BSF



$$H_{BP}(\omega) = H_{LP1}(\omega) - H_{LP2}(\omega) \qquad h_{BP}[n] = h_{LP1}[n] - h_{LP2}[n]$$

$$h_{BP[n]} = \begin{cases} \dfrac{\sin\left(\omega_{c_2}(n-M/2)\right)}{\pi(n-M/2)} - \dfrac{\sin\left(\omega_{c_1}(n-M/2)\right)}{\pi(n-M/2)}, & 0<n<M,\ n\neq \dfrac{M}{2} \\ \dfrac{\omega_{c_2}}{\pi} - \dfrac{\omega_{c_1}}{\pi}, & n=\dfrac{M}{2} \end{cases}$$

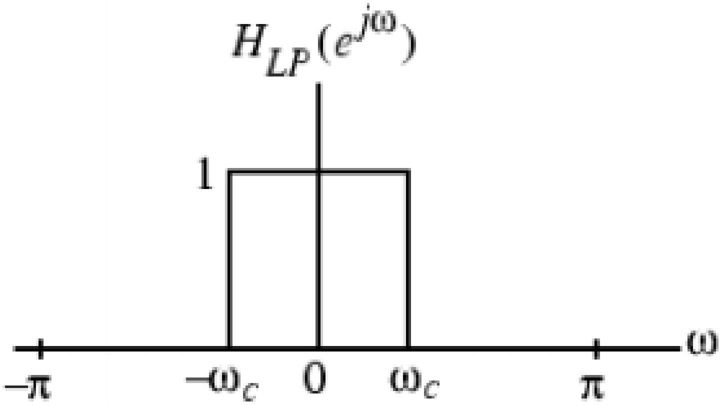
Similarly,

$$H_{BS}(\omega) = 1 - H_{BP}(\omega) \quad \Longleftrightarrow \quad h_{BS}[n] = \delta[n] - h_{BP}[n]$$

Gibbs Phenomenon

However…

Truncating the impulse response of an ideal filter to obtain a realizable filter, creates oscillatory behaviour in the frequency domain.



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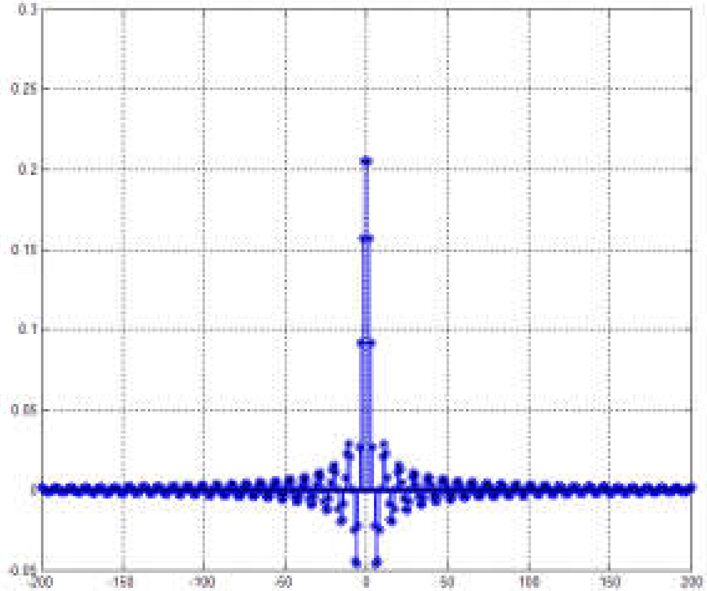

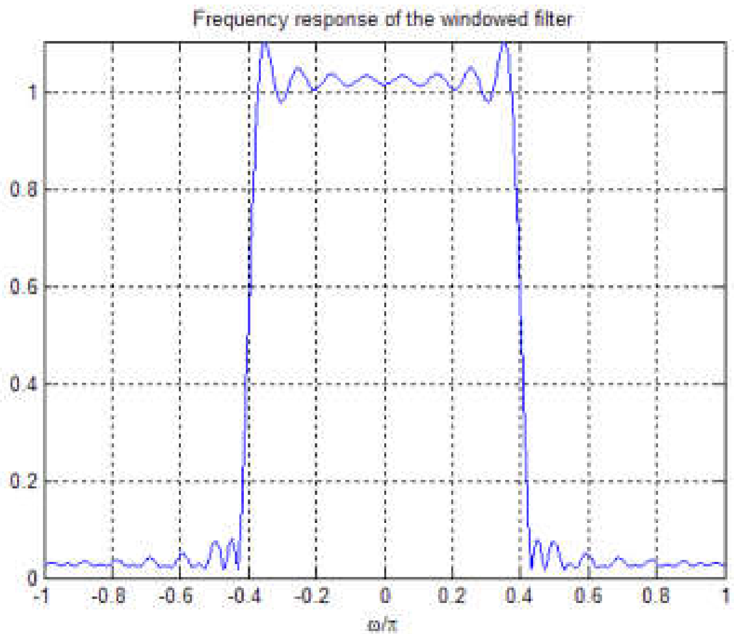

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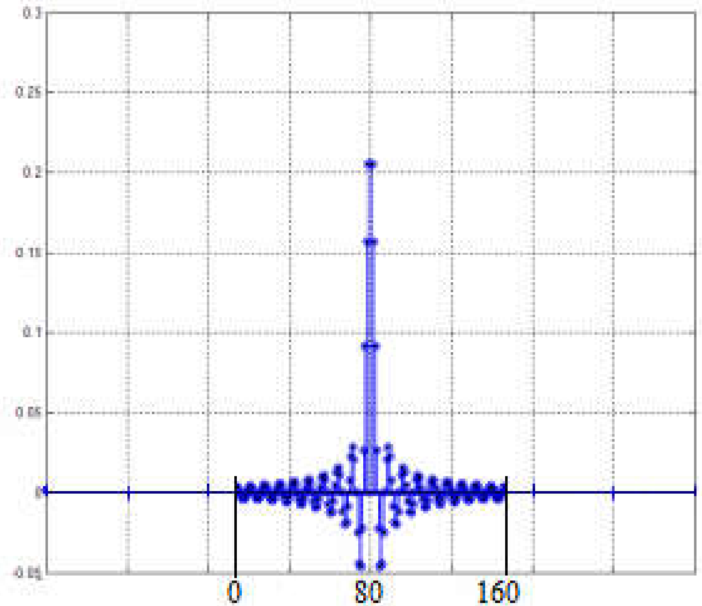
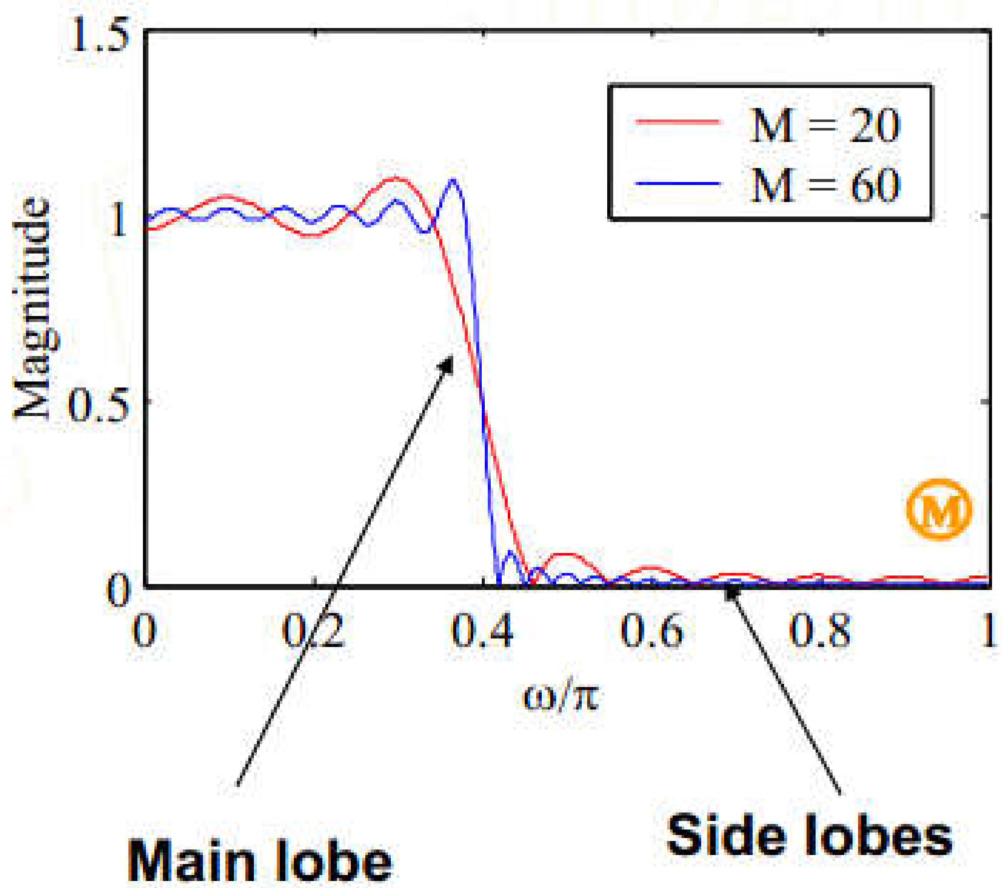


Gibbs Phenomenon

- Observe the following:
  - As M↑, the number of ripples↑. However, ripple widths↓
  - The height of the largest ripples remain constant, regardless of the filter length
  - As M↑, the height of all other ripples ↓
  - The transition band gets narrower as M↑, that is, the drop-off becomes sharper
  - Similar oscillatory behaviour can be seen in all types of truncated filters



Gibbs Phenomenon

- The Gibbs phenomenon is a result of windowing operation.
  - Multiplying the ideal filter's impulse response with a rectangular window function is equivalent to convolving the underlying frequency response with a sinc

$$h_t[n] = h_d[n] \cdot w[n] \Leftrightarrow H_t(\omega) = H_d(\omega) * W(\omega)$$

Truncated filter impulse response

Desired filter impulse response

Windowing function

  - We want $H_t(\omega)$ to be as close as possible to $H_d(\omega)$, which can only be possible if the $W(\omega)=\delta(\omega)$ -> $\delta[n]=1$, an infinite window.
- Examining the windowing process in the frequency domain:
  - To find the frequency response after windowing we need to perform the periodic continuous convolution:

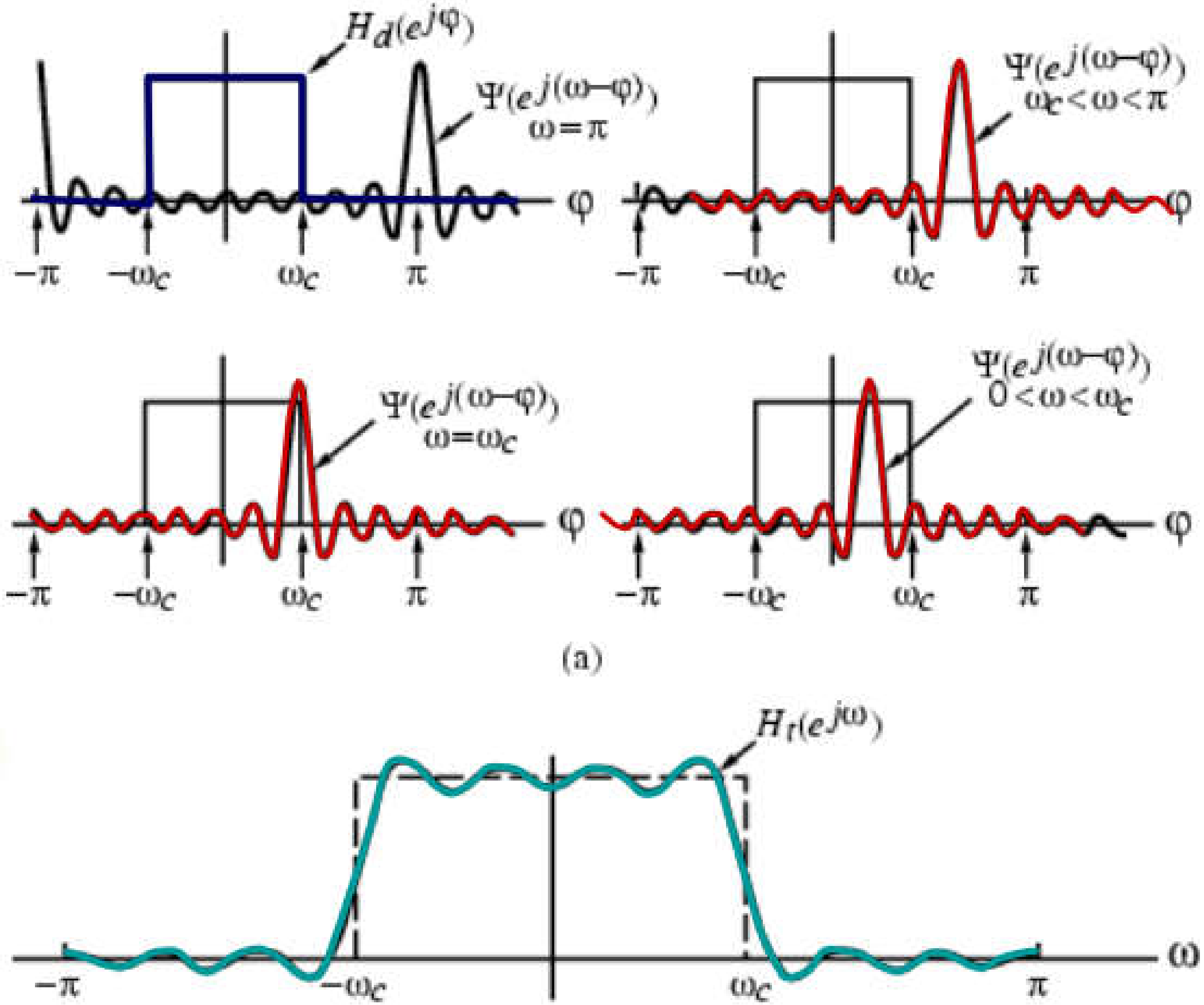
$$H_w(e^{j\omega}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\phi})W\left(e^{j(\omega-\phi)}\right)d\phi$$

Gibbs Phenomenon

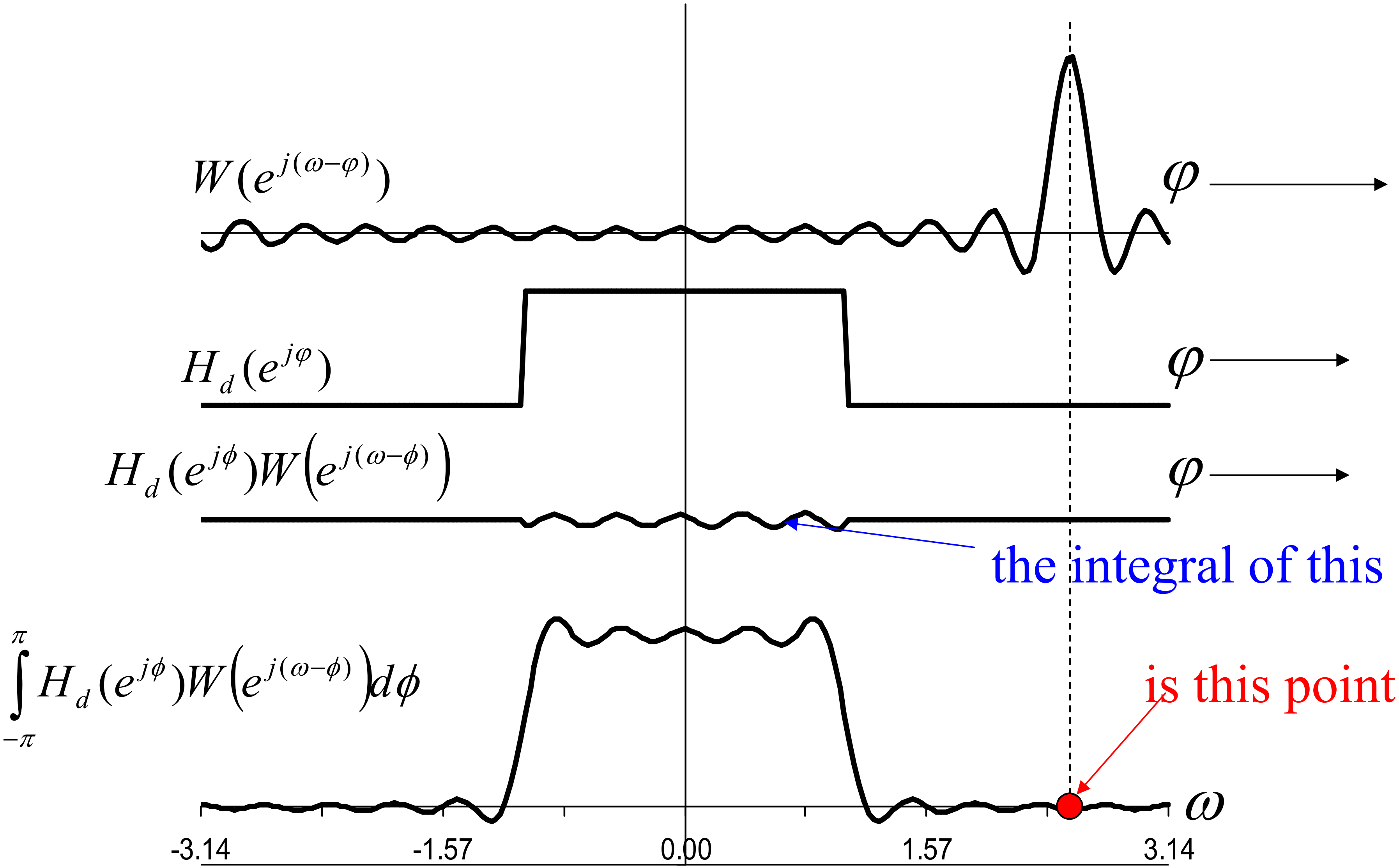
H_d: Ideal filter frequency response

Ψ: Rectangular window frequency response

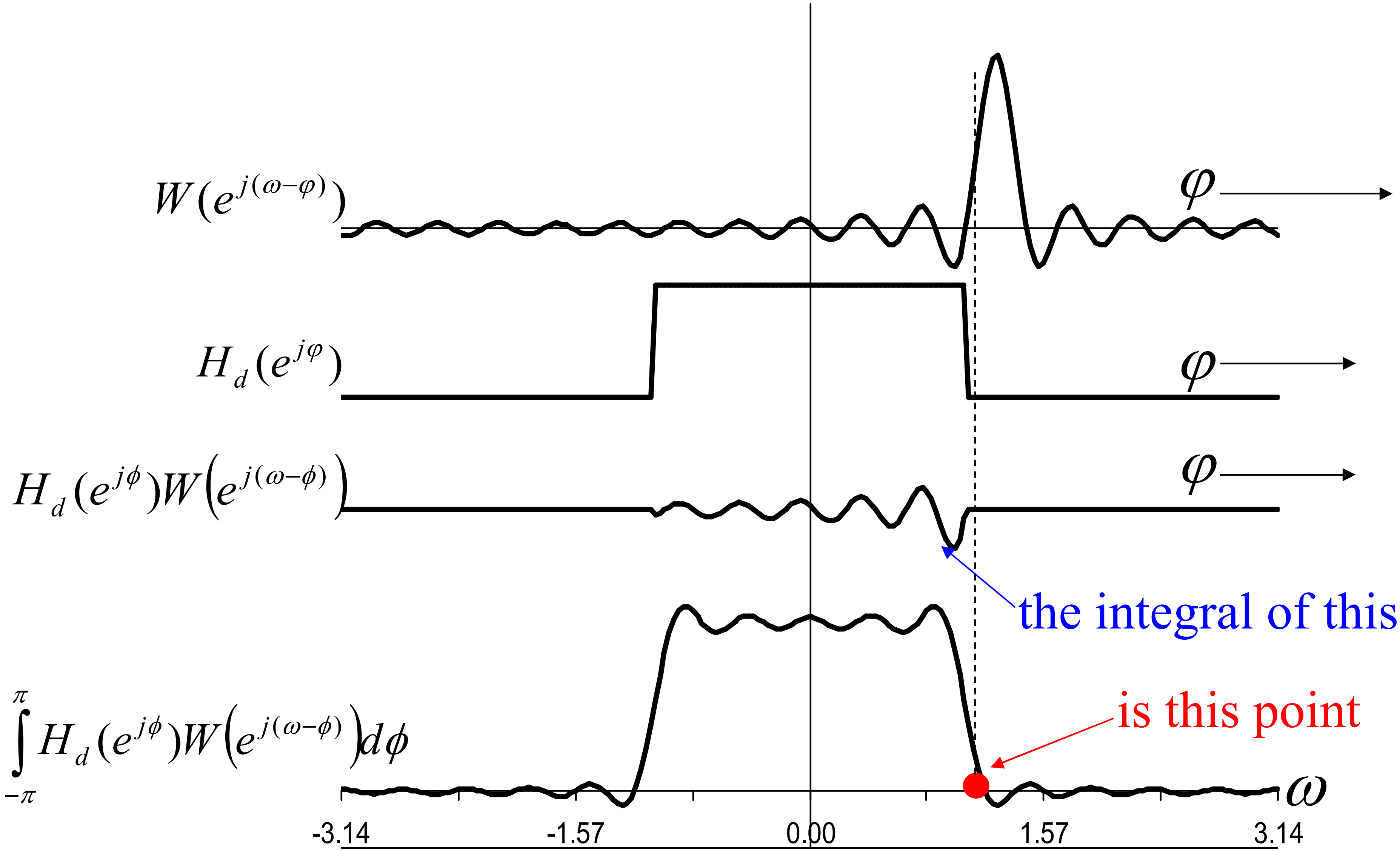
H_t: Truncated filter's frequency response



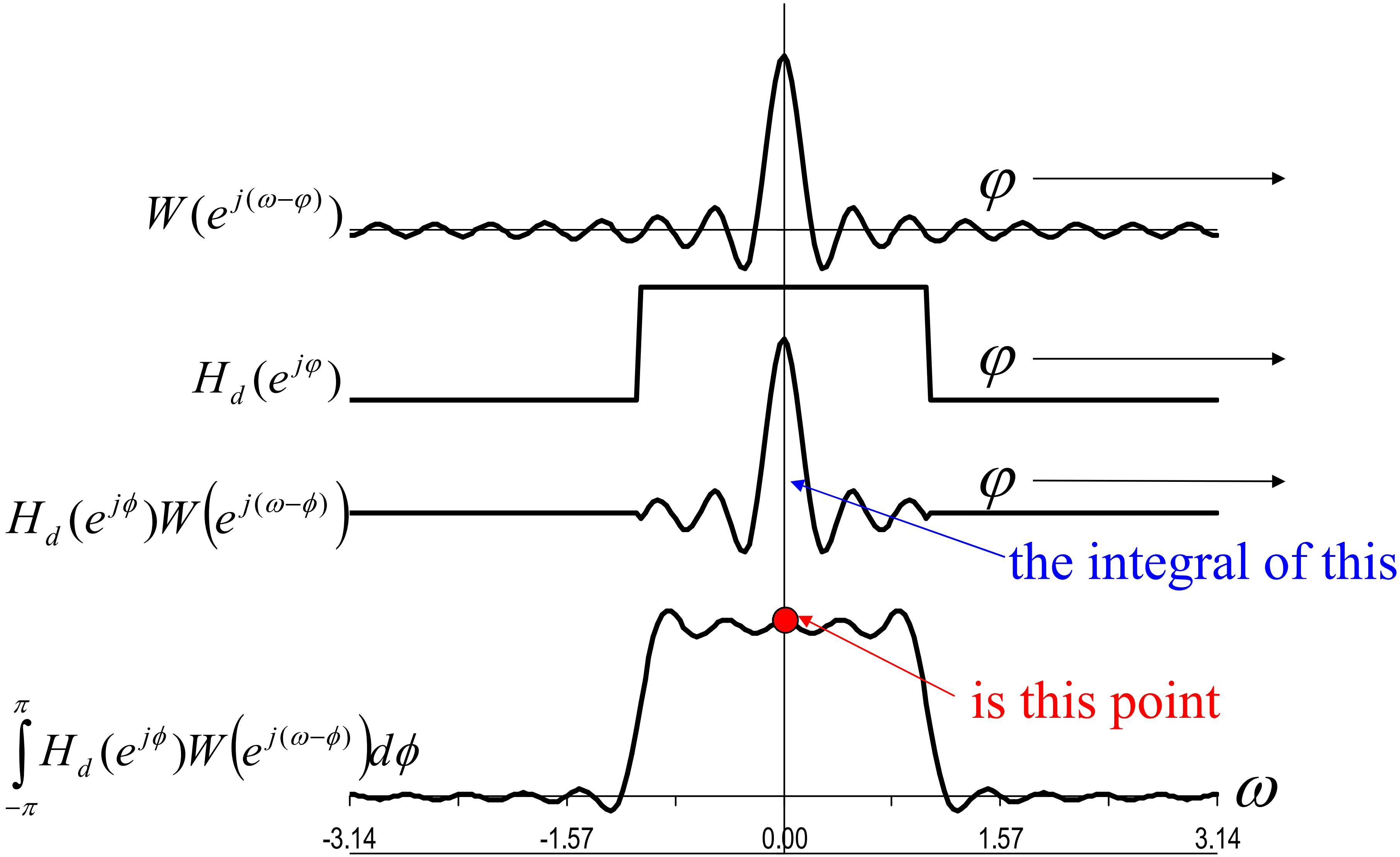
The integral for ω=2.5



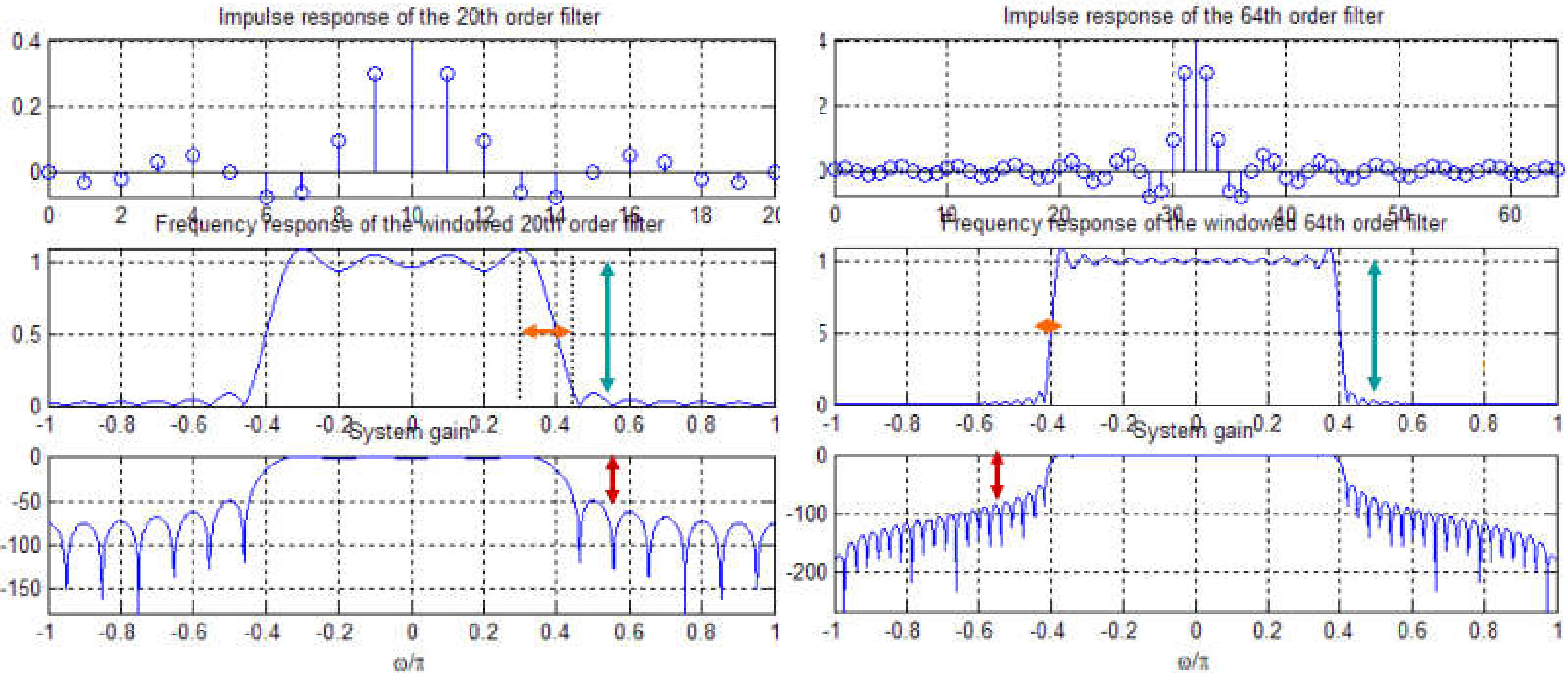
The integral for ω=1.25



The integral for ω=0



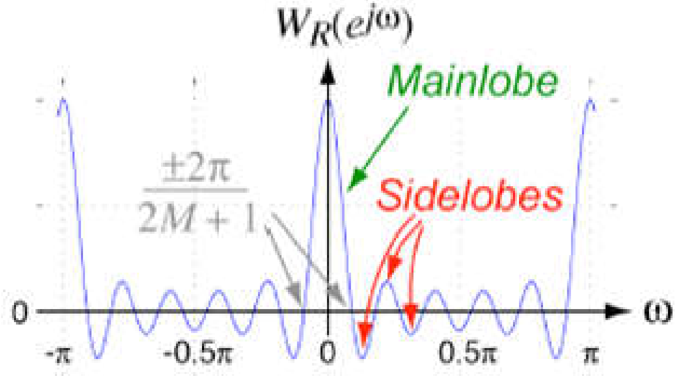
M=20 M=64


Impulse response of the 20th order filter
Frequency response of the windowed 20th order filter
System gain
ω/π
Impulse response of the 64th order filter
Frequency response of the windowed 64th order filter
System gain
ω/π

Transition band / main lobe width
Stopband attenuation
In dB

Gibbs Phenomenon Summary

- Rectangular window has an abrupt transition to zero outside the range $-M \le n \le M$, which results in Gibbs phenomenon;
  - Mainlobe width determines transition band width ($\propto 1/M$)
  - Sidelobe height determines ripples near transition (constant, about 11%)



"periodic sinc"

- Gibbs phenomenon can be reduced either:
  - Using a window that tapers smoothly to zero at each end, or
  - Providing a smooth transition from passband to stopband in the magnitude specifications.

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14_2 Wrap up

- Fundamental requirements
- Specifications
- Least Integral-Square Error Design
- LPF Design by Truncation (rectangular window)
- From LPF to HPF, BPF and BSF
- Gibbs Phenomenon

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Lecture 14 FIR Filters Design

Lect_14_3 Window Method

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FIR Filter Design using window

- Here's what we want:
  - Quick drop off -> Narrow transition band
    - Narrow main lobe
    - Increased stopband attenuation

    Conflicting requirements
  - Reduce the height of the side-lobe which causes the ripples
  - Reduce Gibb's phenomenon (ringing effects, all ripples)
  - Minimize the order of the filter.
- Gibb's phenomenon can be reduced (but not eliminated) by using a ***smoother window*** that gently tapers off to zero, rather than the brick wall behaviour of the rectangular filter.
  - Several window functions are available, which usually trade-off main-lobe width and stopband attenuation.

Commonly Used Windows

- Several window functions are available, which usually trade-off main-lobe width and stopband attenuation.
  - Rectangular window has the narrowest main-lobe width, but poor side-lobe attenuation.
  - Tapered window causes the height of the sidelobes to diminish, with a corresponding increase in the main lobe width resulting in a wider transition at the cutoff frequency.

Based on length 2M+1

Bartlett window: $w[n] = 1 - \dfrac{|n|}{M+1}, \qquad -M \le n \le M$

Hanning window: $w[n] = \dfrac{1}{2}\left[1 + \cos\left(\dfrac{2\pi n}{2M+1}\right)\right], \qquad -M \le n \le M$

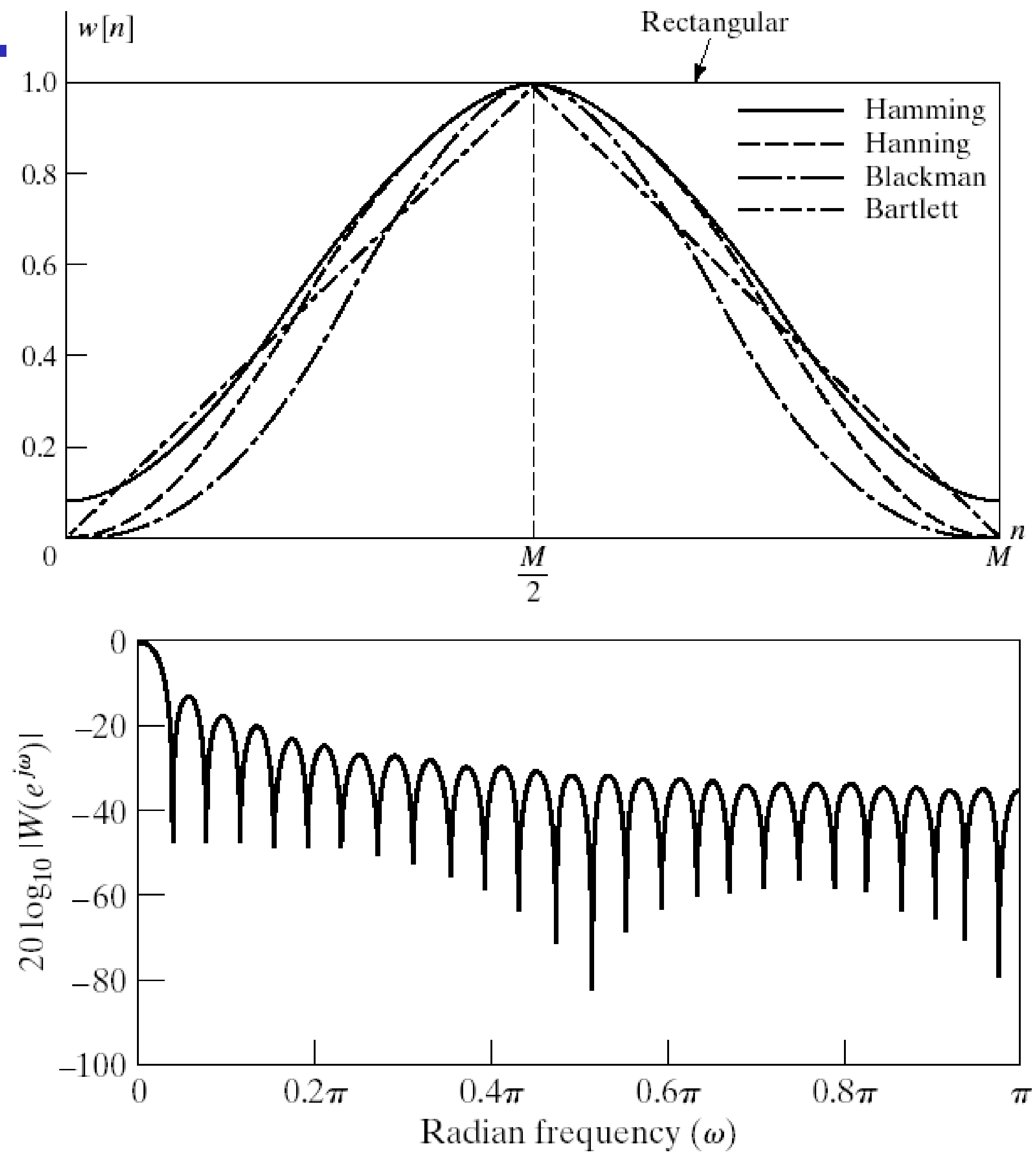
Hamming window: $w[n] = 0.54 + 0.46\cos\left(\dfrac{2\pi n}{2M+1}\right), \qquad -M \le n \le M$

Blackman window: $w[n] = 0.42 + 0.5\cos\left(\dfrac{2\pi n}{2M+1}\right) + 0.08\cos\left(\dfrac{4\pi n}{2M+1}\right), \qquad -M \le n \le M$

Rectangular Window

- Narrowest main lob
  - 4π/(M+1)
  - Sharpest transitions at discontinuities in frequency
- Large side lobs
  - -13 dB
  - Large oscillation around discontinuities
- Simplest window possible

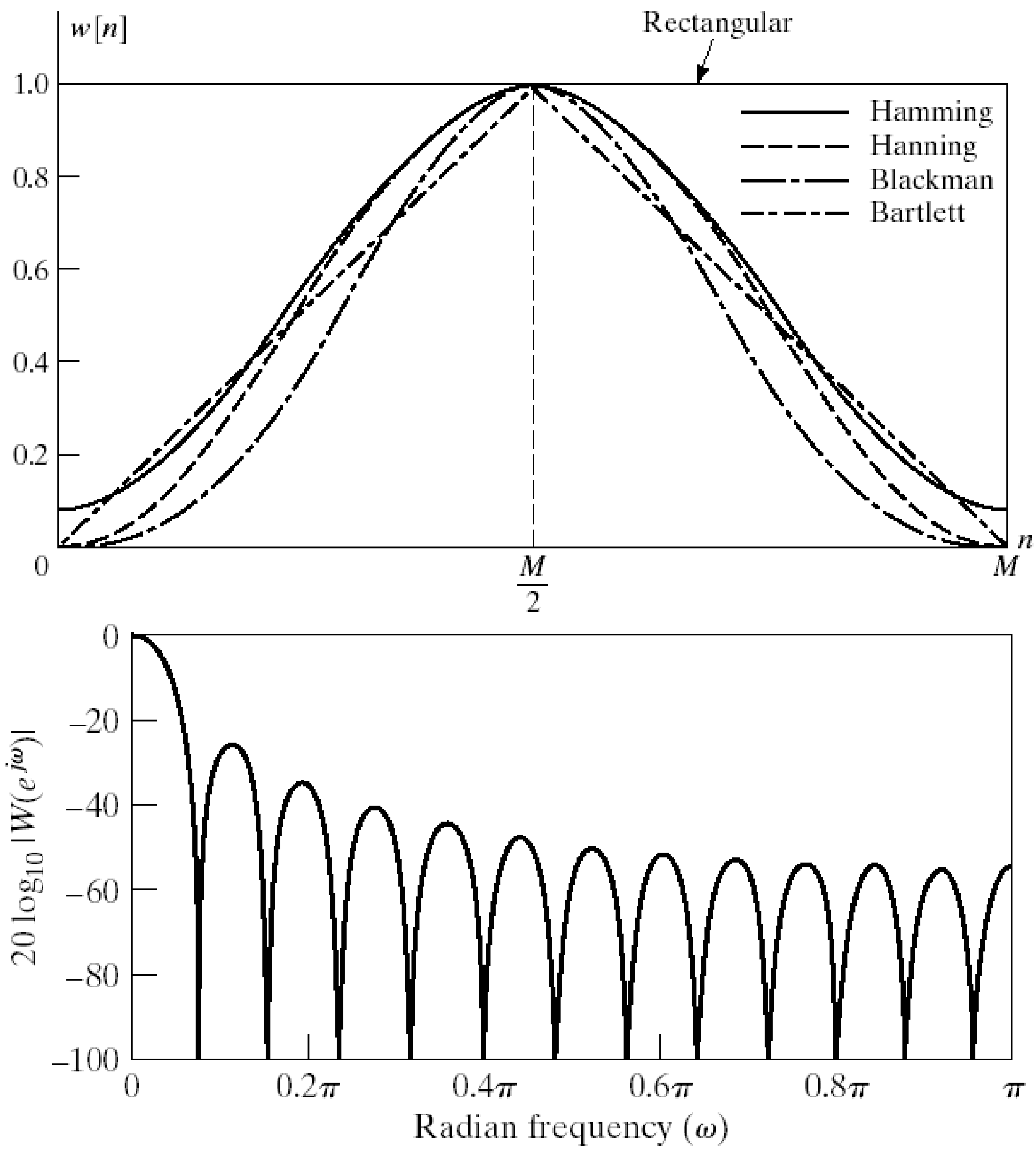
$$w[n] = \begin{cases} 1 & 0 \le n \le M \\ 0 & \text{else} \end{cases}$$



Bartlett (Triangular) Window

- Medium main lob
  - $8\pi/M$
- Side lobs
  - -25 dB
- Hamming window performs better
- Simple equation

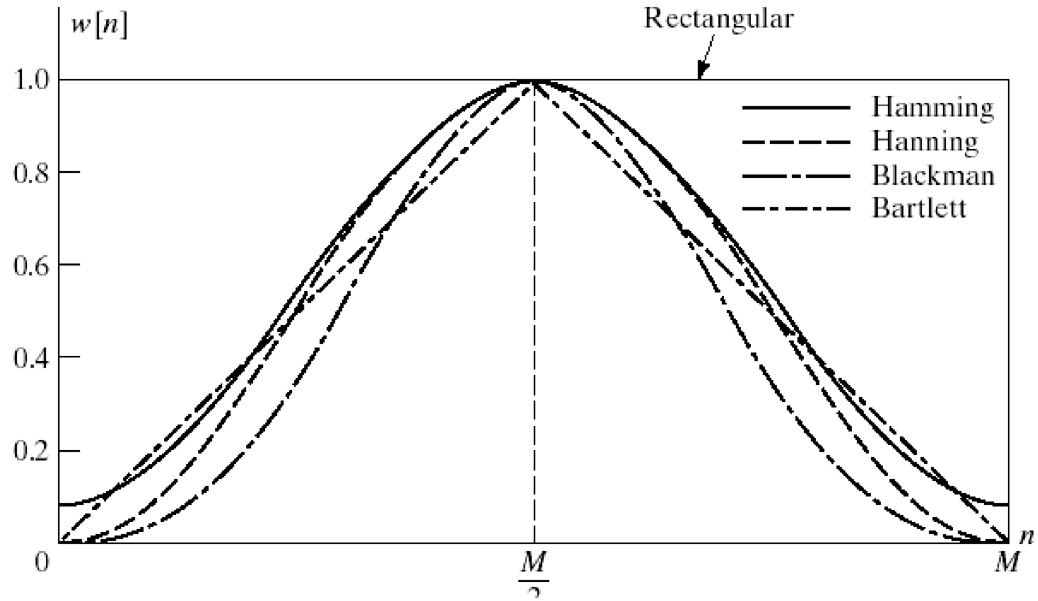
$$w[n] = \begin{cases} 2n/M & 0 \le n \le M/2 \\ 2 - 2n/M & M/2 \le n \le M \\ 0 & \text{else} \end{cases}$$

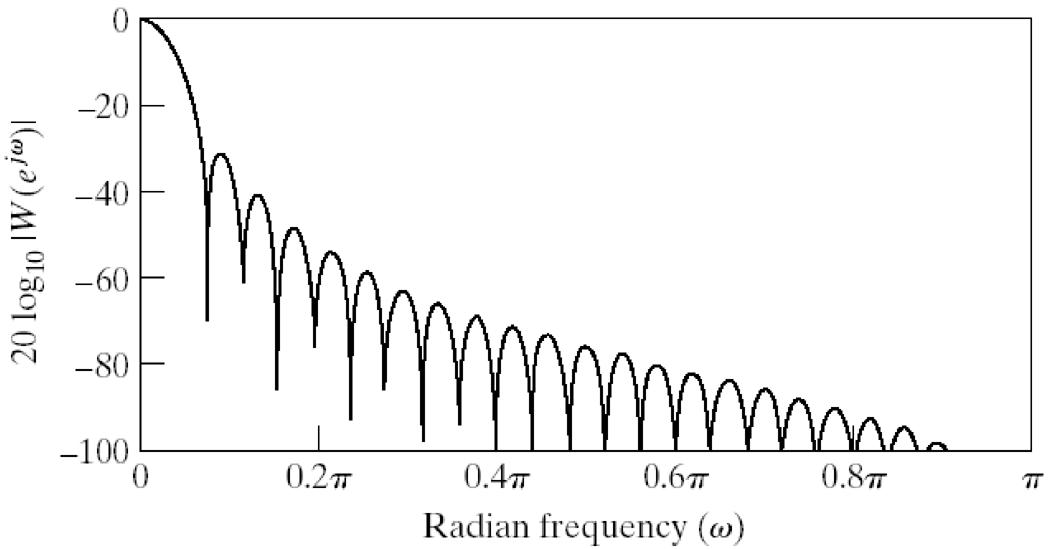


Hanning Window

- Medium main lob
  - $8\pi/M$
- Side lobs
  - -31 dB
- Hamming window performs better
- Same complexity as Hamming

$$w[n] = \begin{cases} \frac{1}{2}\left[1 - \cos\left(\frac{2\pi n}{M}\right)\right] & 0 \le n \le M \\ 0 & \text{else} \end{cases}$$

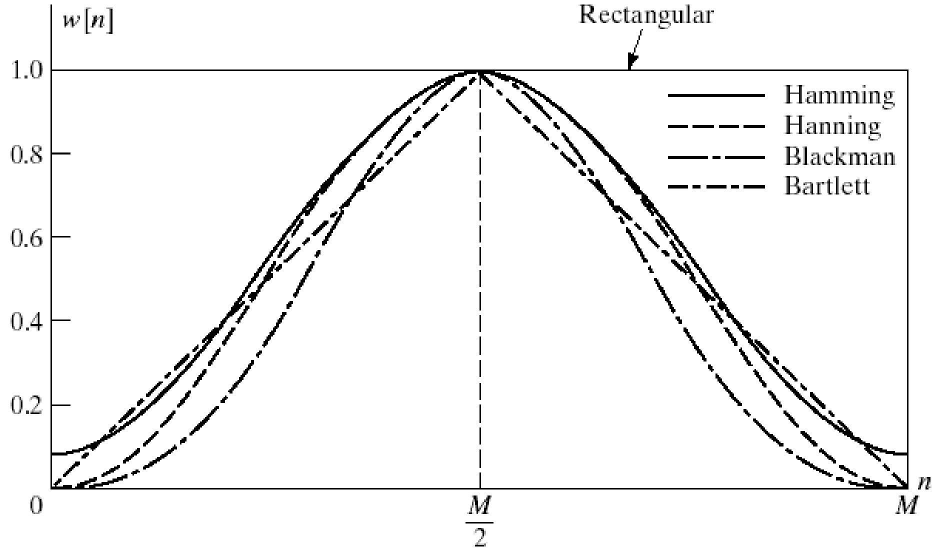


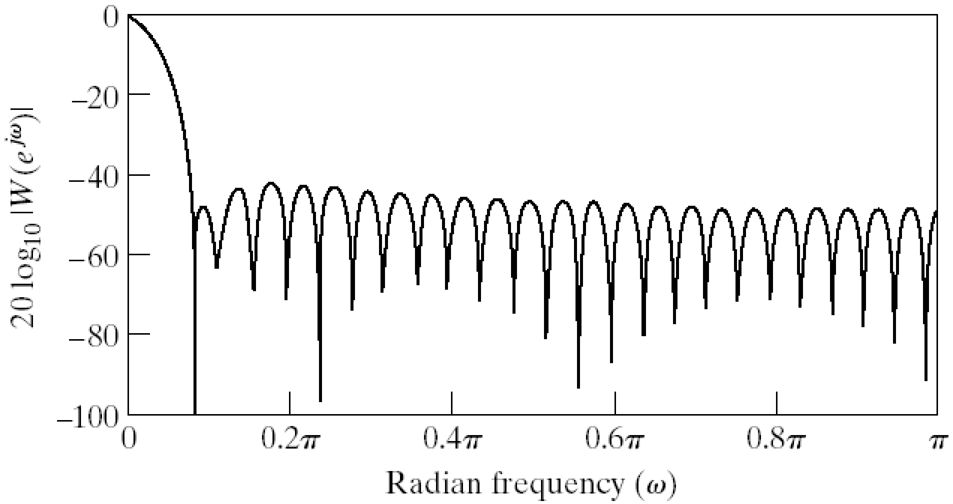


Hamming Window

- Medium main lob
  - $8\pi/M$
- Good side lobs
  - -41 dB
- Simpler than Blackman

$$w[n] = \begin{cases} 0.54 - 0.46\cos\left(\dfrac{2\pi n}{M}\right) & 0 \le n \le M \\ 0 & \text{else} \end{cases}$$

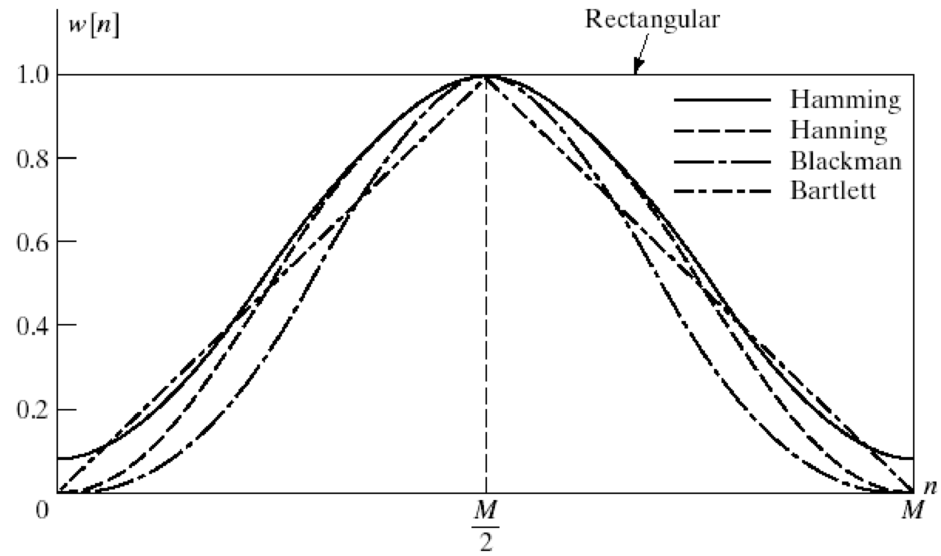


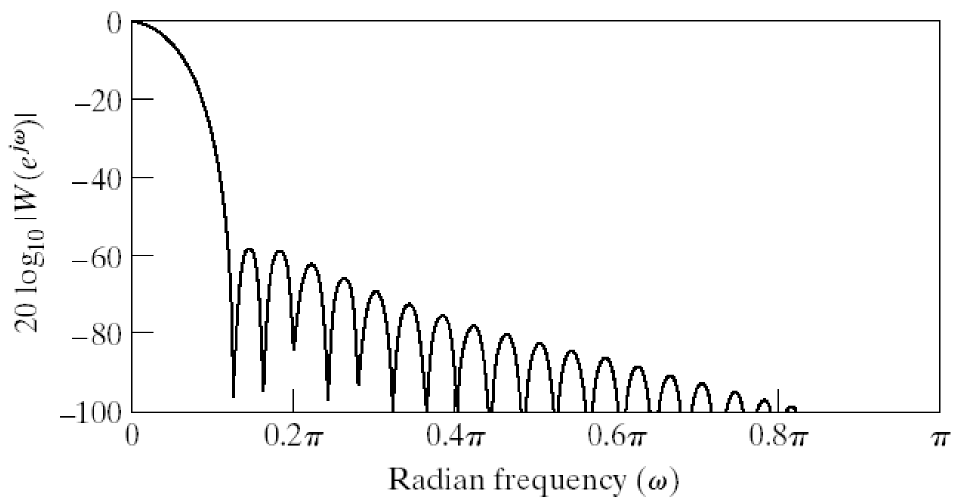


Blackman Window

- Large main lob
  - $12\pi/M$
- Very good side lobs
  - -57 dB
- Complex equation

$$w[n] = \begin{cases} 0.42 - 0.5\cos\left(\frac{2\pi n}{M}\right) + 0.08\cos\left(\frac{4\pi n}{M}\right) & 0 \le n \le M \\ 0 & \text{else} \end{cases}$$

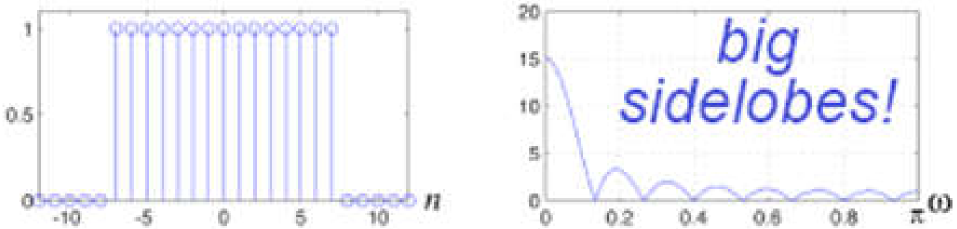




Window Shapes of classic FIR filters

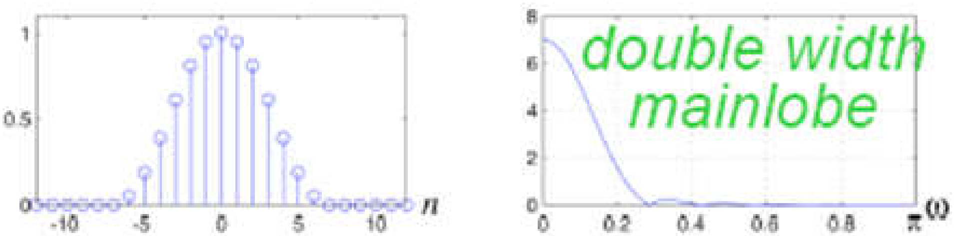
- **Rectangular:**

$$w[n]=1 \quad -M \le n \le M$$



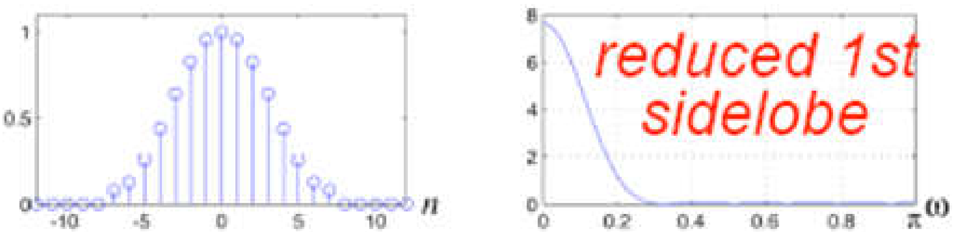
- **Hann:**

$$0.5+0.5\cos(2\pi \tfrac{n}{2M+1})$$



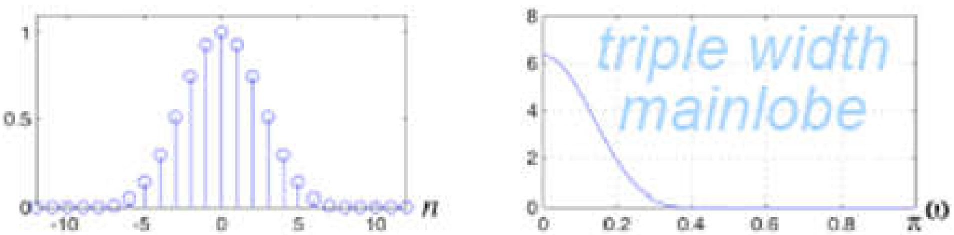
- **Hamming:**

$$0.54+0.46\cos(2\pi \tfrac{n}{2M+1})$$

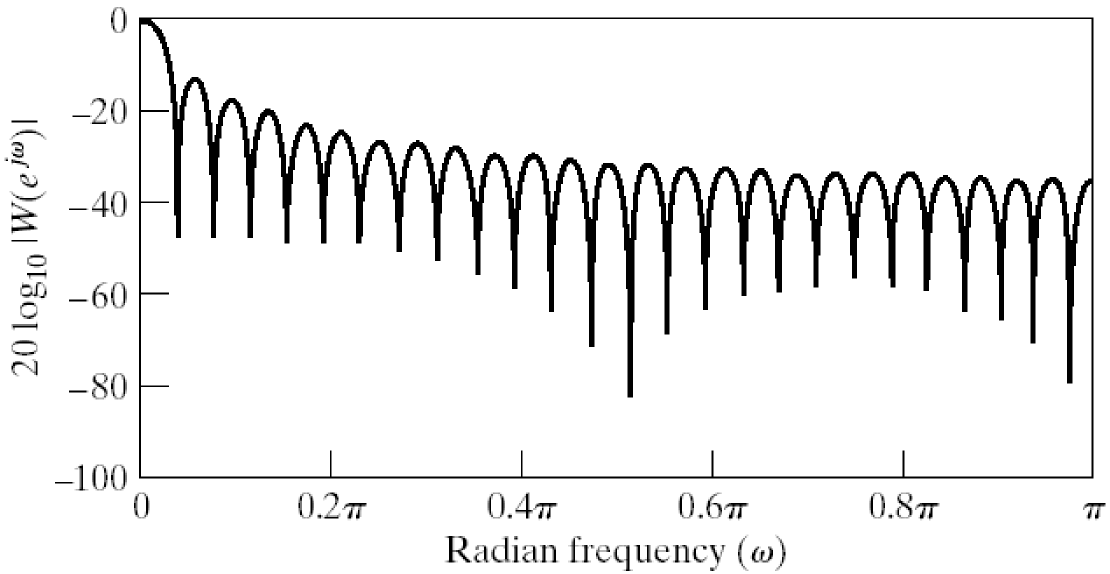


- **Blackman:**

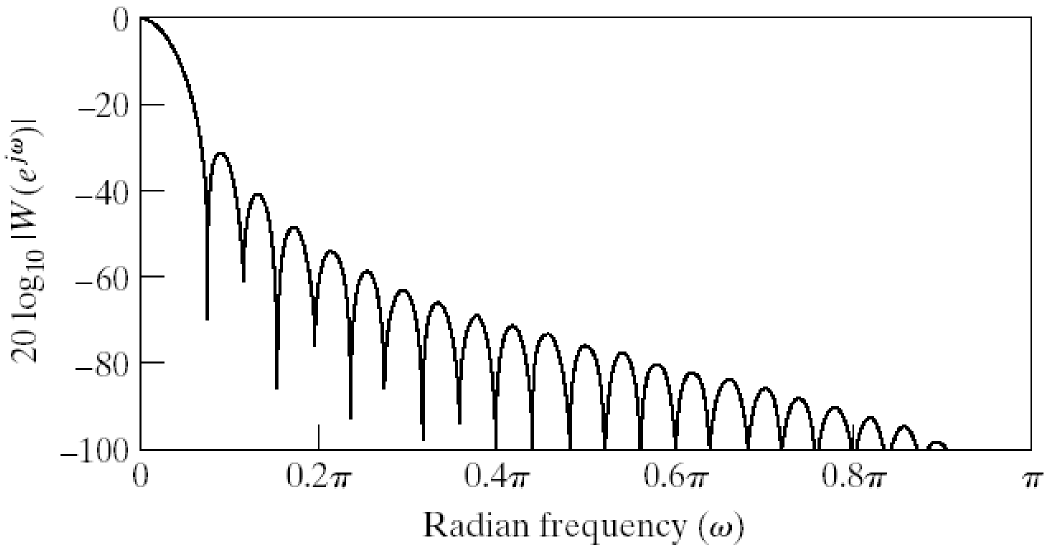
$$0.42+0.46\cos(2\pi \tfrac{n}{2M+1}) +0.08\cos(2\pi \tfrac{2n}{2M+1})$$



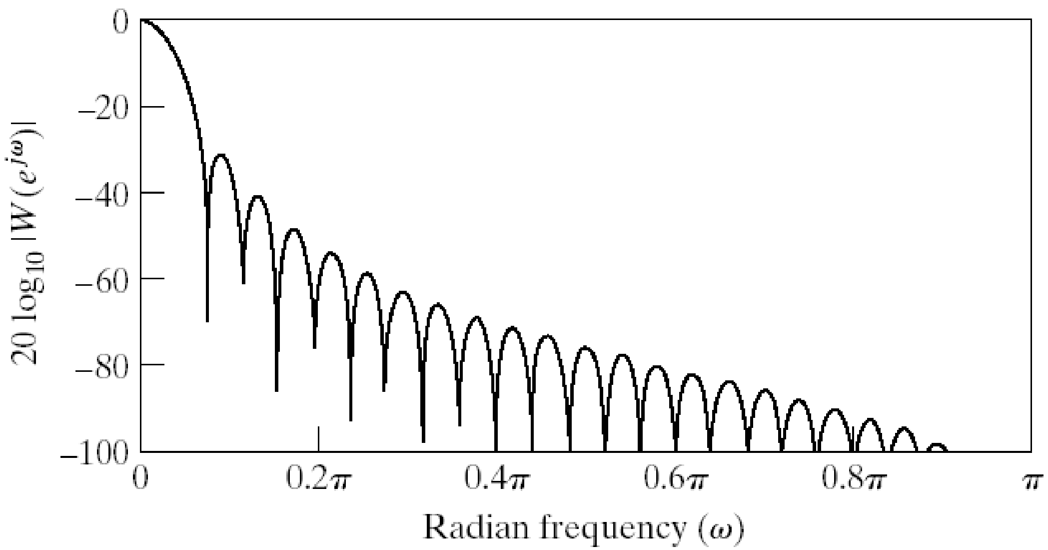
Rectangular window



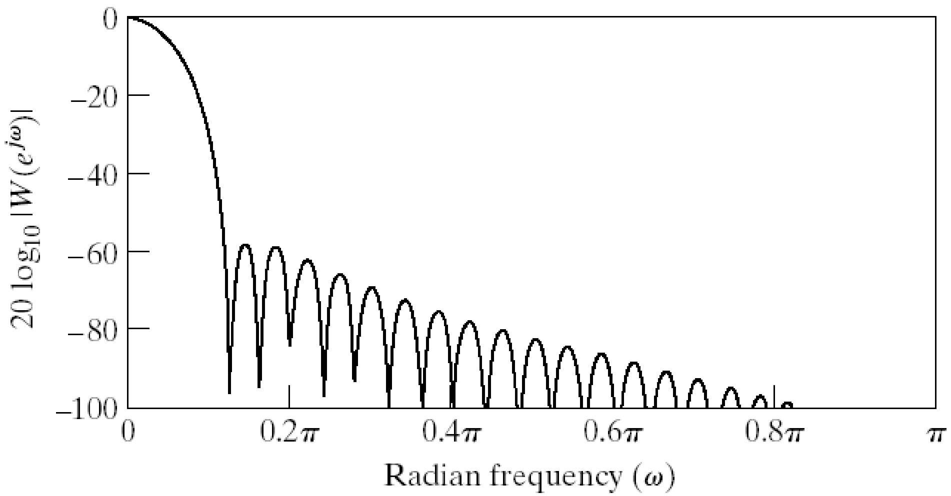
Bartlett window



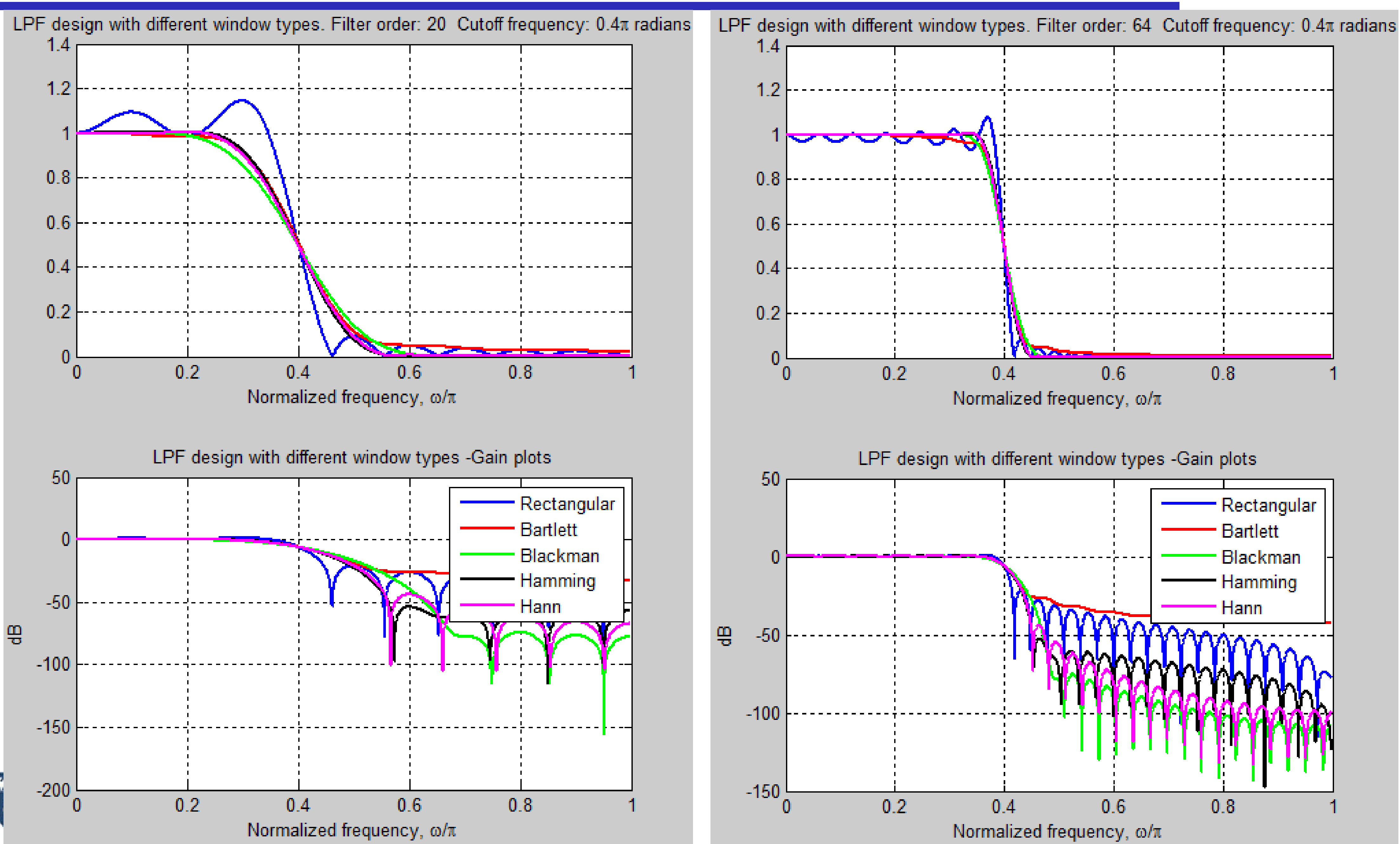
Hanning window



Blackman window



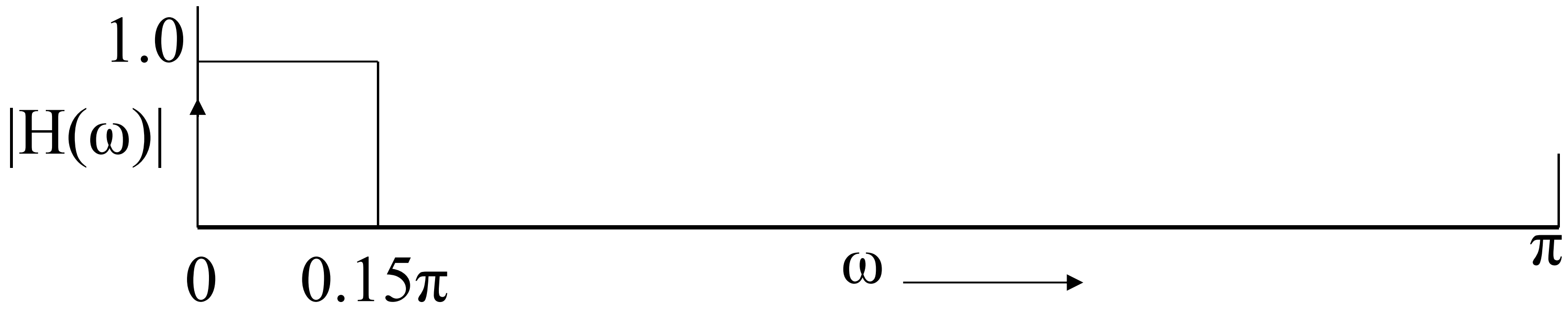
A LPF using the commonly used windows



Example - Windowed Filter Design

- Design a 25-point low pass filter with cut-off at 600 Hz for a sampling rate of 8000 Hz. Stop band ripples to be less than -40 dB.
- Solve:
  - A Hamming Window will meet this specification.
  - First calculate ω_c in radians/sample:

$$\omega_c = \frac{600}{4000}\pi = 0.15\pi$$



Example - Windowed Filter Design (cont.)

- Step 1: Find unwindowed impulse response:

$$h_d[n] = \frac{\omega_c}{\pi}\operatorname{sinc}(n\omega_c) = \frac{1}{n\pi}\sin(0.15n\pi)$$

- Step 2: Find window samples for -12$\leq n \leq$ 12:

$$w[n] = 0.54 + 0.46\cos\frac{2n\pi}{24} = 0.54 + 0.46\cos\frac{n\pi}{12}$$

- Step 3: Apply window (multiply in TD):

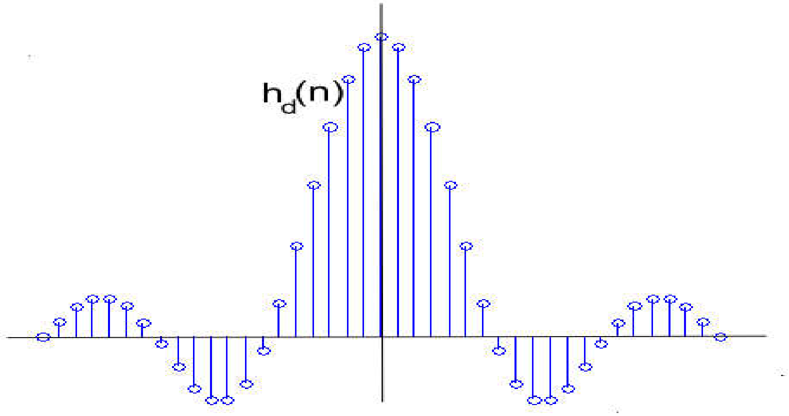
$$h_w[n] = \frac{1}{n\pi}\sin(0.15n\pi)\left[0.54 + 0.46\cos\frac{n\pi}{12}\right]$$

- Step 4: Make causal (shift to right):

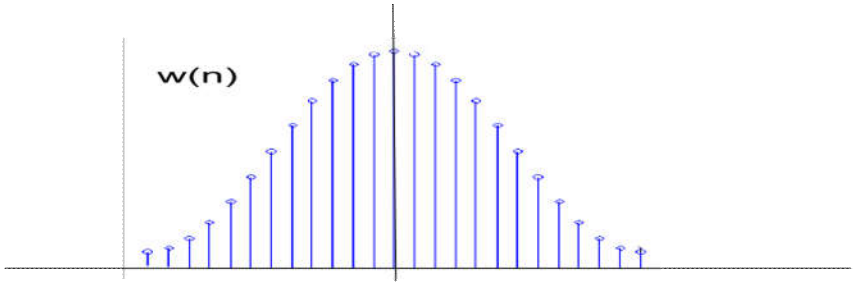
$$h[n] = h_w[n-12] \qquad n = 0,2,3,\ldots.24.$$

Example - Windowed Filter Design (cont.)

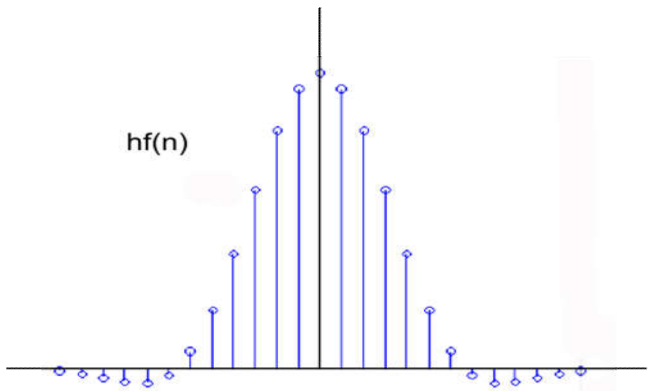
- Unwindowed



- Window

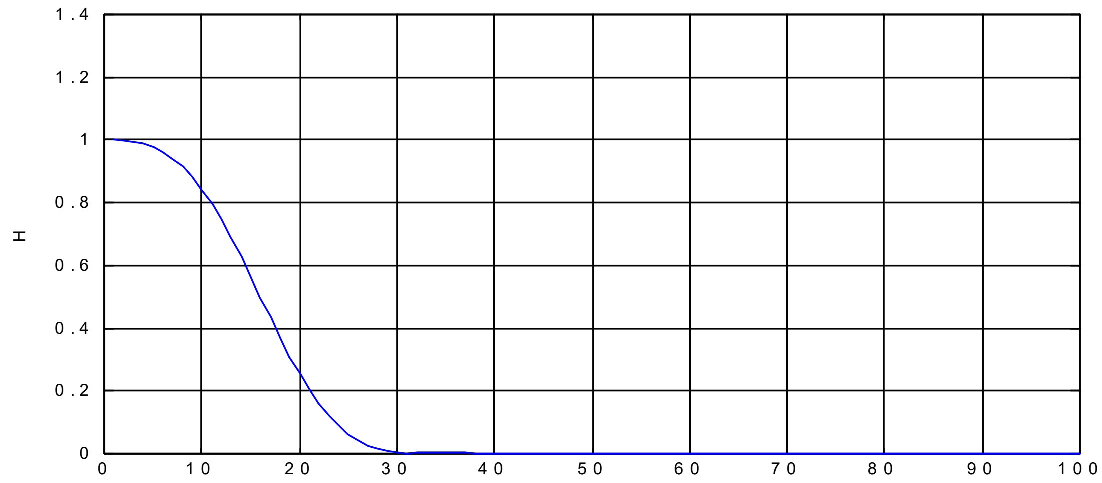


- Final tap weights (before time-shifting)

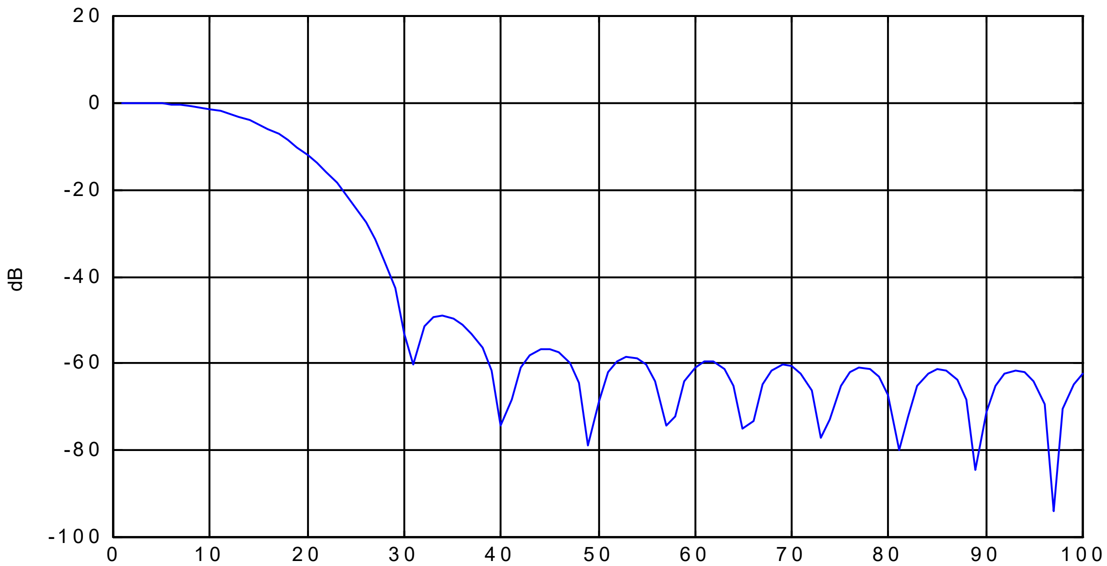


Example - Windowed Filter Design (cont.)

- Gain

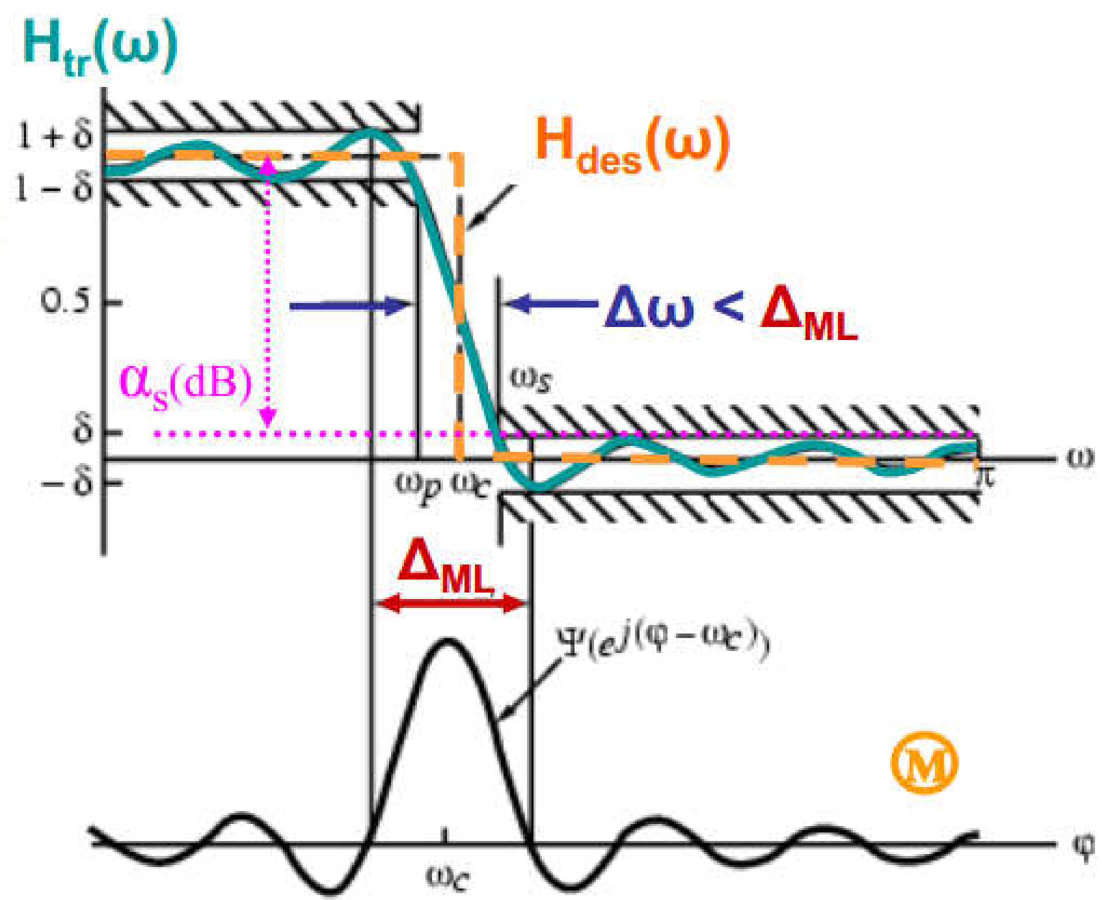


- dB Gain



Fixed window functions

- All windows shown so far are ***fixed window*** functions
  - Magnitude spectrum of each window characterized by a main lobe centered at ω = 0 followed by a series of sidelobes with decreasing amplitudes
  - Parameters predicting the performance of a window in filter design are:
    - Main lobe width (Δ_{ML}) : the distance b/w nearest zero-crossings on both sides or transition bandwidth ($\Delta\omega = \omega_s - \omega_p$)
    - Relative sidelobe level (A_{sl}): difference in dB between the amp. of the largest sidelobe and the main lobe (or sidelobe attenuation (α_s))



  - For a given window, both parameters are completely determined once the filter order M is set.

Fixed window functions

- For these windows, the value of ripples does not depend on filter length or cut-off frequency ωc, and is essentially constant.
- In addition $\Delta\omega = c/M$, where c is a constant for most practical purposes.

	Δ_{ML}	Δ_M=C/M	A_{sl}(dB)	α_s(dB)
Rectangular	4π/(2M+1)	0.92π	13.3	20.9
Hanning	8π/(2M+1)	3.11π	31.5	43.9
Hamming	8π/(2M+1)	3.32π	42.7	54.5
Blackman	12π/(2M+1)	5.56π	58.1	75.3

Example

- Design a lowpass filter with $\omega_p = 0.3\pi$, $\omega_s = 0.5\pi$ and $\alpha_s = 40dB$.
  - Determine which type of window is desired, and the length of it.

Fixed window functions

- How to design:
  - Set $\omega_c = (\omega_p + \omega_s)/2$
  - Choose window type based on the specified sidelobe attenuation (A_{sl}) or minimum stopband attenuation (α_s)
  - Estimate M according to the transition band width ($\Delta\omega = c/M$) and/or mainlobe width (Δ_{ML}).
    - Note that this is the only parameter that can be adjusted for fixed window functions. Once a window type and M is selected, so are A_{sl}, α_s , and Δ_{ML}
    - Ripple amplitudes cannot be custom designed.
  - Adjustable windows have a parameter that can be varied to trade-off between main-lobe width and side-lobe attenuation.
    - Such as : Kaiser window, Dolph-Chebyshev window, etc.

Complete design procedure:

- Depending on your specs, determine what kind of window you would like to use.
  - For fixed windows, once you choose the window, the only other parameter to choose is filter length M;
- Compute the window coefficients w[n] for the chosen window.
- Compute filter coefficients (taps)
  - Determine the ideal impulse response $h_I[n]$ from the given equations for the type of magnitude response you need (lowpass, highpass, etc.)
  - Multiply window and ideal filter coefficients to obtain the realizable filter coefficients (also called **taps** or **weights**): $h[n]=h_I[n].w[n]$

FIR or IIR

- For FIR filters, the transfer function is a polynomial in z^{-1}, so the stability is always guaranteed.
- Advantages of FIR filters:
  - Can be designed with exact linear phase
  - Filter structure always stable with quantized coefficients
  - The filter startup transients have finite duration
- Disadvantages of FIR filters
  - Order of an FIR filter is usually much higher than the order of an equivalent IIR filter meeting the same specifications -> higher computational complexity

14_3 Wrap up

- Design an FIR LPF Filter using window method:
  - Step 1: determine your filter spec (in digital domain);
  - Step 2: design the ideal (desired) filter
    - According to the spec in FD: $H_d(\omega)$
    - Convert to TD: $h_d[n]$ (infinitely long and noncausal)
  - Step 3: design the window w[n] to make the ideal filter practical
    - Choose window type (based on the specified sidelobe attenuation (A_{sl}) or minimum stopband attenuation (α_s));
    - Estimate the window order / length 2M+1 (according to the transition band width ($\Delta\omega = c/M$) and/or mainlobe width (Δ_{ML}))
  - Step 4: multiply the window function and desired filter in TD:

$$h_w[n] = h_d[n] \cdot w[n]$$

  - Step 5: shift it to right by M

$$h[n] = h_w[n-M], \qquad \text{for } n = 0,1,\ldots 2M$$

Chapter 14 Summary

- Simple FIR filters:
  - 1st order LP and HP filters;
  - High order (cascaded 1st order) system
- Specification of practical filters:
  - Transition band width
  - Ripples in passband and stopband
- Window method:
  - Truncation = rectangular window
    - Gibb's phenomenon
  - Other windows
    - How to design a LPF/HPF based on window method?