EEE108 Electromagnetism and Electromechanics

Lecture 14

Magnetic Circuits

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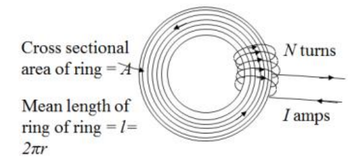
Magnetic Circuits

Consider a coil wound onto a ring of material with high relative permeability μ_r.

The flux created in the ring by a current I through the coil is:

$$\Phi_B = \frac{\mu NIA}{l} = \frac{\mu_0\mu_r A}{l} NI = \frac{NI}{\Re}$$

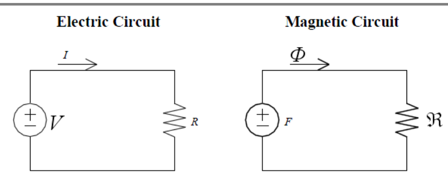
Where $\boxed{\Re = \frac{l}{\mu_0\mu_r A}}$ is the reluctance of the magnetic circuit



$\boxed{F = NI}$ is the magneto - motive force (MMF/mmf)

- any physical force that produces magnetic flux, SI Unit: Ampere-turns
- distinct from mechanical force measured in Newton
- name came because in magnetic circuits it plays a role analogous to the role electromotive force EMF/emf (voltage) plays in electric circuits

Analogy with Electric Circuits



	Electric Circuit	Magnetic Circuit
Ohm's law	$I = \frac{V}{R}$	$\Phi = \frac{F}{\Re}$
	$V = IR$	$F = \Phi\Re$
	$\mathbf{J} = \sigma\mathbf{E}$	$\mathbf{B} = \mu\mathbf{H}$
Resistance	$R = \frac{l}{\sigma A} = \frac{1}{G}$	$\Re = \frac{l}{\mu A} = \frac{1}{\wp}$

Conductance

permeance

It is a magnetic analog of conductance

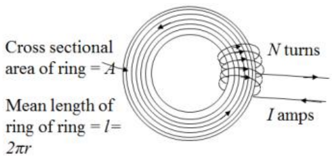
Comparison of Electrical and Magnetic Quantities

Electrical			*Magnetic*		
Voltage	V	(V)	mmf	F	(A-t)
Current	I	(A)	Flux	ϕ	(Wb)
Resistance	R	(Ω)	Reluctance	$\mathcal{R}$	(H^{-1})
Electric field intensity	E	(V/m)	Magnetic field intensity	H	(A-t/m)
Current density	J	(A/m^2)	Flux density	B	(T)
Conductivity	σ	(S/m)	Permeability	μ	(H/m)
$V = IR$			$\mathcal{F} = \phi\mathcal{R}$		
$J = \sigma E$			$B = \mu H$		
$R = \frac{\ell}{\sigma A} = \frac{1}{G}$			$\mathcal{R} = \frac{\ell}{\mu A} = \frac{1}{\mathcal{P}}$		

Magnetic Circuits

Example

Consider a ring of cross-sectional area 5 x 10^{-4} m^2 and length 0.5m, made of steel with relative permeability of 3,500. It is wound with a coil of 250 turns that carries 2 amps. Find the flux in the ring.



Solution using first principles:

Ampere's Law: $H \cdot l = N \cdot I$

$$H = \frac{250 \times 2}{0.5} = 1000 \quad \text{A/m}$$

$$B = \mu_0 \mu_r H = 4\pi \times 10^{-7} \times 3500 \times 1000$$
$$= 4.4 \quad \text{Wb/m}^2$$

So the flux in the circuit is

$$\Phi = B \times A = 4.4 \times 5 \times 10^{-4}$$
$$= 2.2 \times 10^{-3} \quad \text{Wb}$$

Magnetic Circuits

Example

Consider a ring of cross-sectional area 5 x 10^{-4} m^2 and length 0.5m, made of steel with relative permeability of 3,500. It is wound with a coil of 250 turns that carries 2 amps. Find the flux in the ring.

Solution using circuit reluctance

$F = NI = 250 \times 2 = 500$ ampere-turns

The reluctance of the circuit: $\boxed{\Re = \frac{l}{\mu_0 \mu_r A}}$

$$\frac{0.5}{4\pi \times 10^{-7} \times 3500 \times 5 \times 10^{-4}} = 2.3 \times 10^5 \quad \text{H}$$

So the flux in the circuit:

$$\Phi = \frac{F}{\Re} = \frac{500}{2.3 \times 10^5} = 2.2 \times 10^{-3} \quad \text{Wb}$$

Solution using first principles:

Ampere's Law: $H \cdot l = N \cdot I$

$$H = \frac{250 \times 2}{0.5} = 1000 \quad \text{A/m}$$

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Magnetic Circuit with Air Gap

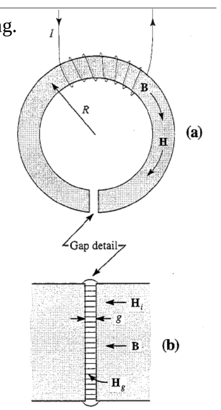
Let a narrow air gap of thickness g be cut in the iron ring.

$$H = \frac{N \cdot I}{l}$$

MMF

Per unit length MMF

$\boxed{\begin{array}{l} \text{air}: \; H_g, \; H_a \\ \text{core}: H_i, \; H_c \end{array}}$



The field in the gap: $H_g = B_g/\mu_0$

The field in the iron: $H_i = B_i/\mu = B_i/(\mu_r{*}\mu_0)$

By continuity of the magnetic flux density:

$B_i = B_g = B$ with assuming of the gap being small, and then the fringing may be neglected. So:

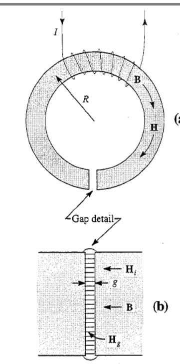
$$H_i = \frac{B}{\mu} = \frac{B}{\mu_r \mu_0} = \frac{H_g}{\mu_r} \Rightarrow \frac{H_g}{H_i} = \mu_r$$

Magnetic Circuit with Air Gap

Example

If the iron ring has a cross-sectional area A = 1000 mm^2, an air gap of width g = 2 mm, and a mean length L = $2\pi R$ = 600 mm, including the air gap.

Find the number of ampere-turn required to produce a flux density B = 1 T.



Solution

$$NI = \oint_L \mathbf{H} \bullet d\mathbf{l} = H_i(L-g) + H_g g$$

From a BH curve for the iron, when $B = 1$T,

$H_i = 1000$ A/m and $\mu_r = 795$

So, $NI = H_i[(L-g) + \mu_r g]$

$= 1000[(0.6 - 0.002) + 759 \times 0.002]$

$= 2188$ A Turns

If there is no gap, then:

$NI = H_i L = 1000 \times 0.6 = 600$ A turns

The narrow air gap makes it necessary to increase the ampere-turns from **600 to 2188** to maintain the flux density at 1 T.

Magnetic Circuit with Air Gap

Energy Stored in the Gap

The density of energy stored in a magnetic field :

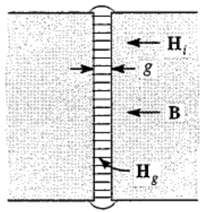
$$w_m = \int_0^B H dB = \int_0^B \frac{B}{\mu} dB = \frac{1}{2}\frac{B^2}{\mu}$$

By assuming the gap is small and a uniform field in the air gap, the total energy W_m stored in the gap :

$$W_m = w_m A g = \frac{B^2 Ag}{2\mu_0}$$

Volume of the air gap

A: area of the gap, m^2, and g : width of the gap, m



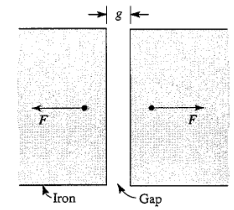
Magnetic Circuit with Air Gap

Magnetic Gap Force

The magnetic poles of opposite polarity at the sides of the gap are attracted to each other, so that the gap must be held open by a force *F*.

If the force increased to make the gap increase an small amount *dg*, the energy stored in the gap is increased by:

$$dW_m = \frac{B^2 A}{2\mu_0} dg$$



From other side, energy may also be expressed as force times distance: *F dg* . Thus:

$$Fdg = \frac{B^2 A}{2\mu_0} dg \quad \text{or the gap force : } F = \frac{B^2 A}{2\mu_0} \ \mathrm{N}$$

F : gap force, N

B : flux density, T

A : area of the gap, m^2

μ_0 : permeability of air, H/m

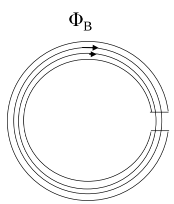
The gap pressure, P

$$P = \frac{F}{A} = \frac{B^2}{2\mu_0} \quad (\mathrm{N/m^2})$$

Permanent Magnet with Gap

If a permanent magnet creates the flux in a magnetic circuit, the flux exists without the need for a coil, so how does Amperes Law apply to this case?

In this case the flux is driven round the magnetic circuit by the internal magnetism of the material.



Here $NI = 0$ and applying Ampere's Law results in:

$$H_g l_g + H_c l_c = NI = 0$$

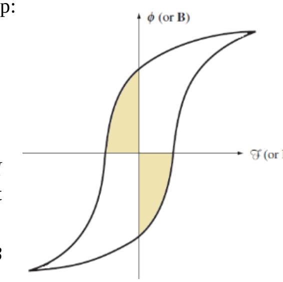
where l_g is the length of the air gap and l_c is the length of the magnet.

So $$B_c = B_g = \mu_0 H_g = -\frac{l_c}{l_g}\mu_0 H_c$$

Permanent Magnet with Gap

So we have the relationship between *B* and *H* in the core set by the geometry of the ring and air gap:

$$B_c = -\frac{l_c}{l_g}\mu_0 H_c$$



B_c and H_c are also related by the *B/H* characteristics of the permanent magnet material.

Note that the above equation indicates that *B* and *H* have opposite signs.

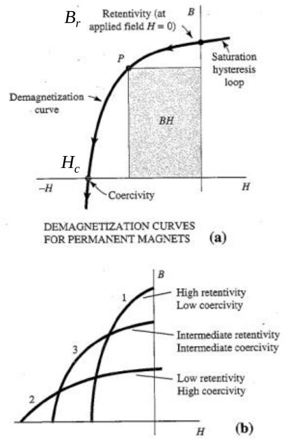
So the relevant part of the *B/H* loop for a permanent magnet is either the second or fourth quadrant.

Materials for Permanent Magnets

Materials with strong residual magnetism are useful for making ***permanent magnets***

For permanent magnets, the followings are desirable:

1. big B_r
2. large H_c so that magnet will not be easily demagnetized
3. The maximum BH product, most important – the maximum energy density stored in the magnet and a big BH magnet delivers a given flux with a minimum of magnetic material.



Materials for Permanent Magnets

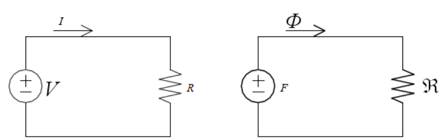
Permanent magnetic materials

Material	Retentivity T	Coercivity A/m	BH_{max} J/m^3
Chrome steel (98 Fe)	1.0	4 000	1 600
Oxide (57 Fe)	0.2	72 000	4 800
Alnico 12 (33 Fe)	0.6	76 000	12 000
Alnico 2 (55 Fe)	0.7	44 800	13 600
Alnico 5 (51 Fe)	1.25	44 000	36 000
Platinum cobalt (77 Pt)	0.6	290 000	52 000

Summary – Magnetic Circuits

Analogy with Electric Circuits

Electric Circuit **Magnetic Circuit**



Ohm's law $I = \frac{V}{R}$ $\Phi = \frac{F}{\Re}$

$V=IR$ $F = \Phi\Re$

$\mathbf{J} = \sigma\mathbf{E}$ $\mathbf{B} = \mu\mathbf{H}$

Resistance $R = \frac{l}{\sigma A} = \frac{1}{G}$ $\Re = \frac{l}{\mu A} = \frac{1}{\wp}$

Conductance

Permeance

It is a magnetic analog of conductance

Summary – Magnetic Circuits

Magnetic Circuit with Air Gap:

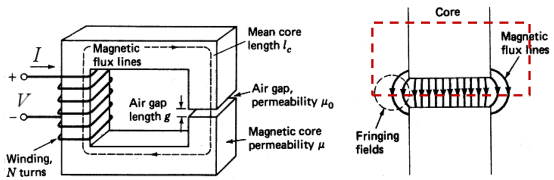
Energy Stored in the Gap:

$$W_m = w_m A g = \frac{B^2 A g}{2\mu_0}$$

Magnetic Gap Pressure/Force:

$$P = \frac{F}{A} = \frac{B^2}{2\mu_0}$$

Magnetic Circuital Laws



A simple magnetic circuit with an air gap

By Ampere's law

$$F = NI = H_c l_c + H_g l_g$$

$$\text{where } H_c l_c = \frac{B_c}{\mu_c} l_c = \frac{\Phi_c}{\mu_c A_c} l_c = \Phi_c \Re_c$$

$$\text{and } \quad H_g l_g = \frac{B_g}{\mu_0} l_g = \frac{\Phi_g}{\mu_0 A_g} l_g = \Phi_g \Re_g$$

According to Gauss's law in magnetics :

$$\oint_S \mathbf{B} \bullet d\mathbf{A} = 0$$

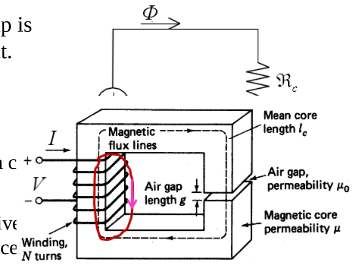
We have : $\Phi_c = \Phi_g = \Phi$

Therefore $\boxed{F = (\Re_c + \Re_g)\Phi}$

Magnetic Circuital Laws

The magnetic circuit with an air gap is analogous to a series electric circuit.

$$\boxed{F = (\Re_c + \Re_g)\Phi}$$



In an electric circuit, a voltage drives a c[urrent]
I through each resistor.

In a magnetic circuit, the magnetomotive [force]
drives a flux through the each reluctance

A magnetic circuit with a single flux path, if **leakage flux** were neglected, is commonly called a series magnetic circuit.

The equivalent reluctance of a number of reluctances in series is just the sum of the individual reluctance:

$$\Re_{eq} = \Re_1 + \Re_2 + \ldots + \Re_n = \sum_{i=1}^{n} \Re_i$$

Magnetic Circuital Laws

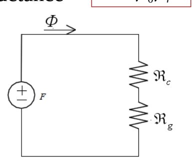
Reluctance in Series

$$\boxed{\Re = \frac{l}{\mu_0 \mu_r A}}$$

High material permeability ⇨ low core reluctance

2000 ~ 6000 times of air

2000 ~ 80,000 times of air


Series magnetic circuit

This can often make: $\Re_c << \Re_g$

Not always!

If $\Re_c << \Re_g$, then the reluctance of the core can be neglected.

The flux (also B) can be found from equation of $F = \Phi(\Re_c + \Re_g)$:

$$\Phi \approx \frac{F}{\Re_g} = \frac{F\mu_0 A_g}{l_g} = NI\frac{\mu_0 A_g}{l_g}$$

Magnetic Circuital Laws

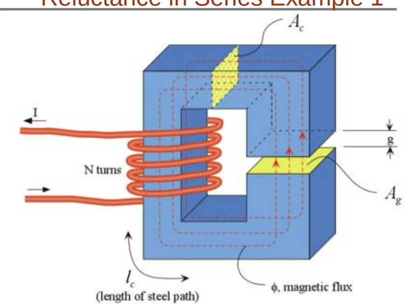
Reluctance in Series Example 1

Dimensions:

$A_c = A_g = 9\ \text{cm}^2$, $g = 0.05\text{cm}$

$l_c = 30$ cm, $N = 500$ turns

$\mu_r = 70{,}000$ for the core and 1 for the gas. The circuit is operating with $B_c = 1.0$ T



Find:

(a) reluctances in the core and in the gap

(b) the flux, and

(c) the current I

Magnetic Circuital Laws

Reluctance in Series Example 1

(a) The reluctances: $\Re_c = \dfrac{l_c}{\mu_r \mu_0 A_c} = \dfrac{0.3}{70000(4\pi\times10^{-7})(9\times10^{-4})} = 3.79\times10^3 \quad \text{A/Wb}$

$$\Re_g = \frac{l_g}{\mu_r \mu_0 A_c} = \frac{5\times10^{-4}}{1(4\pi\times10^{-7})(9\times10^{-4})} = 4.42\times10^5 \quad \text{A/Wb}$$

(b) The magnetic flux: $\Phi = B_c A_c = 1.0(9\times10^{-4}) = 9\times10^{-4} \quad \text{Wb}$

(c) The current: $I = \dfrac{F}{N} = \dfrac{\Phi(\Re_c + \Re_g)}{N} = \dfrac{(9\times10^{-4})(4.46\times10^5)}{500} = 0.8028\text{ A} \approx 0.80 \text{ A}$

If the reluctance in the core is ignored, then the current

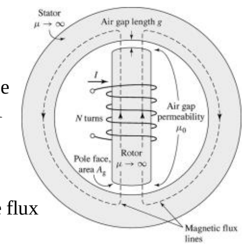
$$I = \frac{F}{N} = \frac{\Phi(\Re_c + \Re_g)}{N} \approx \frac{\Phi\Re_g}{N} = \frac{(9\times10^{-4})(4.42\times10^5)}{500} = 0.7956 \text{ A} \approx 0.80 \text{ A}$$

$$\Delta I = \frac{0.8028 - 0.7956}{0.8028}\times100\% = 0.87\%$$

Magnetic Circuital Laws

Reluctance in Series Example 2

A synchronous machine is shown schematically in the Fig.



Assuming that rotor and stator iron have infinite permeability ($\mu \to \infty$), find the air gap flux Φ and the flux density B_g, when I = 10 A, N = 1000 turns, g = 1 cm, and A_g = 2000 cm^2.

Solution

There are two air gaps in series and by symmetry the flux density in each is equal.

As the iron permeability is infinite, its reluctance is negligible.

$$\Phi = \frac{NI\mu_0 A_g}{2g} = \frac{1000\times10\times(4\pi\times10^{-7})(2000\times10^{-4})}{(1+1)\times10^{-2}} = 0.13 \text{ Wb}$$

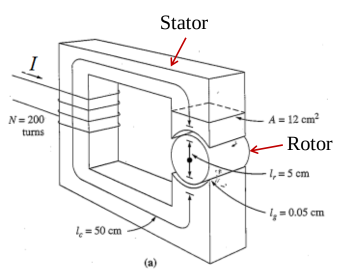
$$B_g = \frac{\Phi}{A_g} = \frac{0.13}{(2000\times10^{-4})} = 0.65 \text{ T}$$

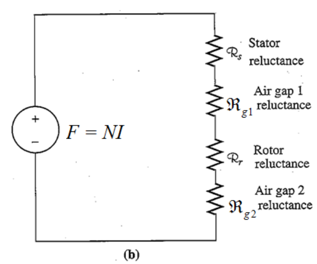
Magnetic Circuital Laws

Reluctance in Series Example 3

The cross-sectional area of each air gap (including fringing is 14 cm^2) and the iron of the core has a relative permeability of 2000.

If the current in the wire is adjusted to be 1 A, what will the resulting flux density in the air gaps be?





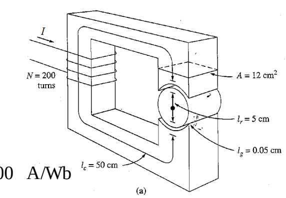
Magnetic Circuital Laws

Reluctance in Series Example 3

$B_g = \Phi/A_g$ $\quad\quad A_g = 14 \text{ cm}^2$

$\Downarrow$

$\Phi = F/\Re$



The reluctance of the stator:

$$\Re_s = \frac{l_s}{\mu_r\mu_0 A_s} = \frac{0.5}{(2000)(4\pi\times10^{-7})(0.0012)} = 166000 \quad \text{A/Wb}$$

The reluctance of the rotor:

$$\Re_r = \frac{l_r}{\mu_r\mu_0 A_r} = \frac{0.05}{(2000)(4\pi\times10^{-7})(0.0012)} = 16600 \quad \text{A/Wb (A}\cdot\text{turns/Wb)}$$

The reluctance of the air gap:

$$\Re_g = \frac{l_g}{\mu_r\mu_0 A_g} = \frac{0.0005}{(1)(4\pi\times10^{-7})(0.0014)} = 284000 \quad \text{A/Wb}$$

Proper units

The total reluctance of the motor:

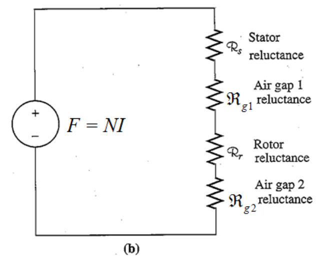
$$\Re_{total} = \Re_s + \Re_{g1} + \Re_r + \Re_{g2} = 166000 + 284000 + 16600 + 284000 = 751000 \quad \text{A/Wb}$$

Magnetic Circuital Laws

Reluctance in Series Example 3

$B_g = \Phi / A_g$ $\quad A_g = 14\ \text{cm}^2$

$\Phi = F/\Re$



The total reluctance:

$\Re_{total} = 751000\ \text{A/Wb}$

The net magnetomotive force applied to the core:

$F = NI = (200)(1.0) = 200\ \ \text{A(A}\cdot\text{turns)}$

The total flux:

$$\Phi = \frac{F}{\Re_{total}} = \frac{200}{751000} = 0.00266\ \text{Wb}$$

Then the magnetic flux density in the gap: $B = \frac{\Phi}{A_g} = \frac{0.00266}{0.0014} = 0.19\ \ \text{T (Wb/m}^2\text{)}$

Magnetic Circuital Laws

Reluctance in Series Example 3

$$\frac{\Re_s}{\Re_g} = \frac{166000}{284000} = 0.5845$$

$\Rightarrow\ \Re_s\ (\Re_r)$ not $<< \Re_g$

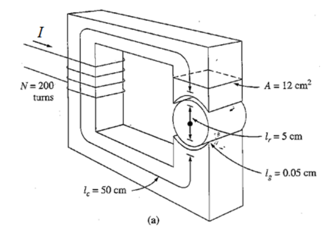
$$\frac{\Re_r}{\Re_g} = \frac{16600}{284000} = 0.05845$$

especially $\Re_s$

The core has a relative permeability of 2000.

Two reasons:

1. μ_r not big enough
2. Length of core is much longer than that of gap



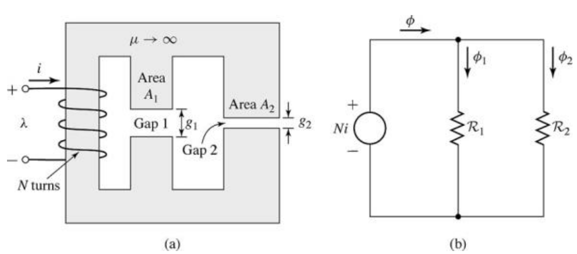
Magnetic Circuital Laws

Reluctance in Parallel

The magnetic circuits with two or more flux paths, neglecting leakage flux, are classified as parallel magnetic circuits.

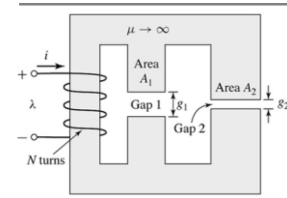
The equivalent reluctance of a number of reluctances in parallel:

$$\frac{1}{\Re_{eq}} = \frac{1}{\Re_1} + \frac{1}{\Re_2} + \ldots + \frac{1}{\Re_n} = \sum_{i=1}^{n} \frac{1}{\Re_i}$$



Permeances in series and parallel obey the same rules as electrical conductances.

Flux Linkage



$F = Ni$ $\quad$ *MMF/mmf*

$N\Phi = ??$

$$\varepsilon = N\frac{d\Phi}{dt}$$

$\Phi = f(F, \Re)$

parallel magnetic circuits

series magnetic circuits

The self-induced emf:

$$N\frac{d\Phi_B}{dt} = L\frac{di}{dt}$$

where the self-inductance:

$$L = \frac{N\Phi_B}{i}$$

The SI unit is the henry (H)

Flux Linkage

For a coil with N turns, the magnitude of the total induced emf :

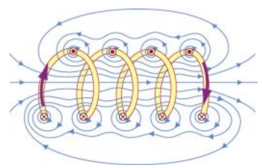
$$\varepsilon = N\frac{d\Phi}{dt} = \frac{d\lambda}{dt}$$ The flux passing each turn is same.

where λ is called the ***flux linkage*** of the winding, and is defined as

$$\lambda = N\Phi$$ The units of λ are webers (weber - turns)

Flux linkage in general form:

$$\lambda = \sum_{i=1}^{N} \Phi_i$$



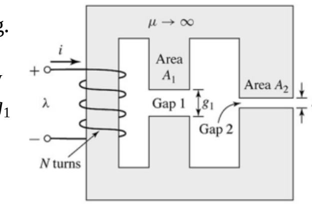
The magnitude of the total induced emf:

$$\varepsilon_i = \sum_{i=1}^{N} \frac{d\Phi_i}{dt} = \frac{d}{dt}\sum_{i=1}^{N} \Phi_i = \frac{d\lambda}{dt}$$

Magnetic Circuital Laws

Reluctance in Parallel Example

The magnetic circuit shown in the Fig. consists of an N-turn winding on a magnetic core of infinite permeability with two parallel air gaps of lengths g_1 and g_2 and areas A_1 and A_2, respectively.

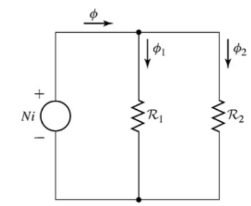


Find:

(a) the inductance of the winding

(b) The flux density B_1 in gap 1 when the winding is carrying a current i.

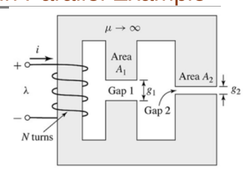
Neglect fringing effects at the air gap.



Magnetic Circuital Laws

Reluctance in Parallel Example

(a) From the equivalent circuit:



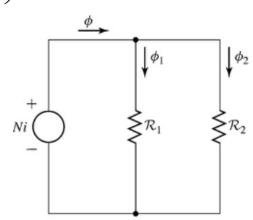
$$\Phi = \frac{Ni}{\dfrac{\Re_1\Re_2}{\Re_1+\Re_2}} \quad \text{where } \Re_1 = \frac{g_1}{\mu_0 A_1}, \text{ and } \Re_2 = \frac{g_2}{\mu_0 A_2}$$

$$\boxed{L = \frac{\lambda}{i}} = \frac{N\Phi}{i} = \frac{N^2(\Re_1+\Re_2)}{\Re_1\Re_2} = \mu_0 N^2\left(\frac{A_1}{g_1}+\frac{A_2}{g_2}\right)$$

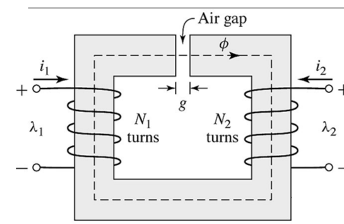
(b) From the equivalent circuit:

$$\Phi_1 = \frac{Ni}{\Re_1} = \frac{\mu_0 A_1 Ni}{g_1}$$

so we have: $B_1 = \dfrac{\Phi_1}{A_1} = \dfrac{\mu_0 Ni}{g_1}$



Magnetic Circuit with Two Windings



Magnetic core permeability μ, mean core length l_c, cross-sectional area A_c

In this case, the MMF acting on the magnetic circuit is given by the total ampere-turns acting on the circuit. For the current directions shown by the Fig, the flux produced by the two windings is in the same direction. The total MMF:

$$F = N_1 i_1 + N_2 i_2$$

The total resultant core flux produced by the total MMF of the two windings with assumption of $A_c = A_g$:

$$\Phi = (N_1 i_1 + N_2 i_2)\frac{\mu_0 A_c}{g} \quad \text{if } \mu_r \text{ is large}$$

Magnetic Circuits Approaches

Assumptions :

- Constant magnetic permeability: linear relationship between B-H– gives results of acceptable engineering accuracy
- No leakage
- Air gap is small and the fringing can be ignored: $A_c = A_g$

Under the conditions provided, the magnetic circuit model introduced is true.

Reluctance in Series $$\Re_{eq} = \Re_1 + \Re_2 + \ldots + \Re_n = \sum_{i=1}^{n} \Re_i$$

Reluctance in Parallel $$\frac{1}{\Re_{eq}} = \frac{1}{\Re_1} + \frac{1}{\Re_2} + \ldots + \frac{1}{\Re_n} = \sum_{i=1}^{n} \frac{1}{\Re_i}$$

Next

- Transducers
- Linear Actuators

Thanks for your attendance