Fundamental of Power Systems Part II
EEE210

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Overview

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2.7 Balanced Three-Phase Circuits

- The generation, transmission, and distribution of electric power is accomplished by means of three-phase circuits.
- A power system has Y-connected generators and usually includes both Δ- and Y-connected loads.
- Generators are rarely Δ-connected , because if the voltages are not perfectly balanced, there will be net voltage, and consequently a circulating current, around the Δ.

2.7 Balanced Three-Phase Circuits

- Assuming a positive phase sequence (phase order ABC) the generated voltages are, as shown in Figure 1:

$$
\begin{aligned}
E_{An} &= |E_p|\angle 0^\circ \\
E_{Bn} &= |E_p|\angle -120^\circ \\
E_{Cn} &= |E_p|\angle -240^\circ
\end{aligned}
$$

- In power systems, great care is taken to ensure that the loads of transmission lines are balanced

2.7 Balanced Three-Phase Circuits

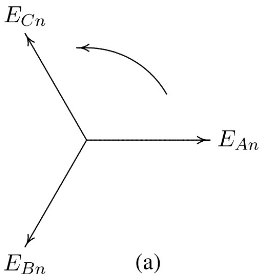

Figure 1: (a) Positive, or ABC, phase sequence. (b) Negative, or ACB, phase sequence.

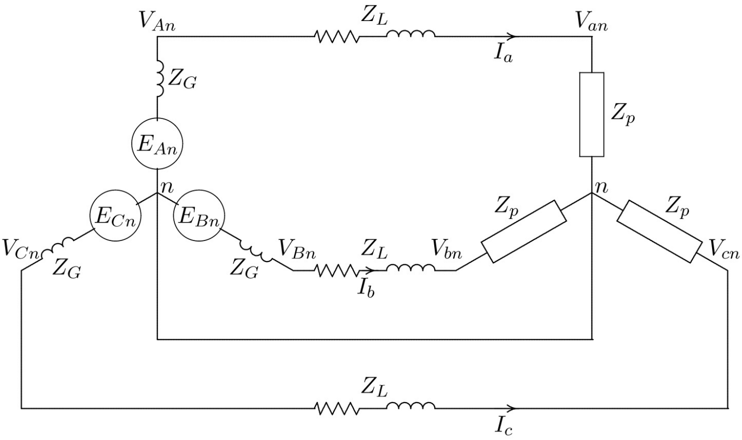

Figure 2: A Y-connected generator supplying a Y-connected load.

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2.8 Y-Connected Loads

- To find the relationship between the line voltages (line-to-line voltages) and the phase voltages (line-to-neutral voltages), we choose the line-to-neutral voltage of the a-phase as the reference, thus

$$\begin{aligned} V_{an} &= |V_p|\angle 0^\circ \\ V_{bn} &= |V_p|\angle -120^\circ \\ V_{cn} &= |V_p|\angle -240^\circ \end{aligned}$$

- The line voltages at the load terminals in terms of phase voltages are found by the application of Kirchhoff's voltage law

$$\begin{aligned} V_{ab} = V_{an} - V_{bn} &= \sqrt{3}|V_p|\angle 30^\circ \\ V_{bc} = V_{bn} - V_{cn} &= \sqrt{3}|V_p|\angle -90^\circ \\ V_{ca} = V_{cn} - V_{an} &= \sqrt{3}|V_p|\angle 150^\circ \end{aligned}$$

Tutorial

Assuming that $V_{an} = |V_p|\angle 0^\circ$ and $V_{bn} = |V_p|\angle -120^\circ$, calculate the value of V_{ab}

The relationship between the line voltages and phase voltages is illustrated in Figure 3.

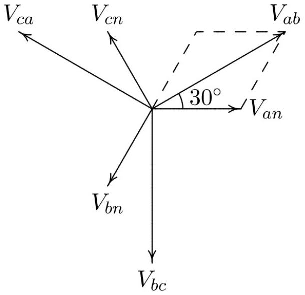

Figure 3: Phasor diagram showing phase and line voltages

2.8 Y-Connected Loads

- If the rms value of any of the line voltages is denoted by V_L, then one of the important characteristics of the Y-connected three-phase load is

$$V_L = \sqrt{3}|V_p|\angle 30^\circ,$$

with three-phase currents as

$$\begin{aligned} I_a &= \frac{V_{an}}{Z_p} = |I_p|\angle -\theta, \\ I_b &= \frac{V_{bn}}{Z_p} = |I_p|\angle -120^\circ - \theta, \\ I_c &= \frac{V_{cn}}{Z_p} = |I_p|\angle -240^\circ - \theta. \end{aligned}$$

where θ is the impedance phase angle.

- Line currents are similar to phase currents. Thus

$$I_L = I_p$$

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2.9 Delta Connected Loads

A balanced Δ-connected load with (with equal phase impedances) is shown in Figure 4.

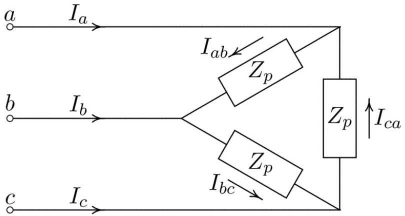

Figure 4: A Δ-connected load.

2.9 Delta Connected Loads

From Figure 4, it is clear that line voltages are the same as phase voltages,

$$V_L = V_p,$$

with the following line currents,

$$\begin{aligned} I_{ab} &= |I_p|\angle 0^\circ, \\ I_{bc} &= |I_p|\angle -120^\circ, \\ I_{ca} &= |I_p|\angle -240^\circ, \end{aligned}$$

where $|I_p|$ represents the magnitude of the phase current.

2.9 Delta Connected Loads

Further, the relationship between phase and line currents can be obtained by applying KCL at the corners of Δ.

$$
\begin{aligned}
I_a &= I_{ab} - I_{ca} = \sqrt{3} I_p \angle -30^\circ, \\
I_b &= I_{bc} - I_{ab} = \sqrt{3} I_p \angle -150^\circ, \\
I_c &= I_{ca} - I_{bc} = \sqrt{3} I_p \angle 90^\circ.
\end{aligned}
$$

In other words, if the rms line current is denoted by I_L, we have

$$I_L = \sqrt{3} |I_p| \angle -30^\circ$$

Relevant phasor diagram is illustrated in Figure 5.

Tutorial

Referring Figure 4, calculate current I_a.

2.9 Delta Connected Loads

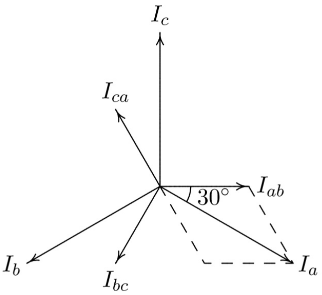

Figure 5: Phasor diagram showing phase and line currents

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2.10 Δ-Y Transformation for Balanced Loads

Consider the fictitious Y-connected circuit of $Z_Y\ \Omega/\text{phase}$ which is equivalent to a balanced Δ-connected circuit of $Z_\Delta\ \Omega/\text{phase}$, as shown in Figure 6.

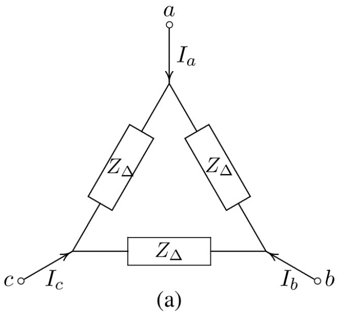

Figure 6: (a) Δ to (b) Y-connection.

2.10 Δ-Y Transformation for Balanced Loads

From circuit analysis, it is found that

$$Z_Y = \frac{Z_\Delta}{3},$$

or,

$$Z_\Delta = 3Z_Y.$$

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2.11 Per-Phase Analysis

1. It is assumed that everything is "balanced."
2. Change all Δ-connected loads/sources to Y connections, this provides a neutral point.
3. All neutral points are at the same potential, hence, all the neutral points may be connected.
4. This breaks up the circuit into three separate circuits, one for each phase.
5. Solve the single-phase circuit. The other phases are the same after a phase shift of $\pm 120°$.

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2.12 Balanced Three-Phase Power

- The three-phase instantaneous power is

$$\begin{aligned} P_{3\phi} &= 3|V_p||I_p|\cos\theta, & (1) \\ S_{3\phi} &= 3|V_p||I_p|\sin\theta. & (2) \end{aligned}$$

- Thus the complex three-phase power is

$$S_{3\phi} = P_{3\phi} + jQ_{3\phi} = 3V_p I_p^*$$

- In a Y-connected load, the phase voltage $|V_p| = |V_L|/\sqrt{3}$ and the phase current $I_p = I_L$.
- And, in a Δ-connected load, $V_p = V_L$ and $|I_p| = |I_L|/\sqrt{3}$.
- Substituting both into (1) and (2), the real and reactive powers for either connection are given by

$$P_{3\phi} = \sqrt{3}|V_L||I_L|\cos\theta$$

and

$$Q_{3\phi} = \sqrt{3}|V_L||I_L|\sin\theta$$

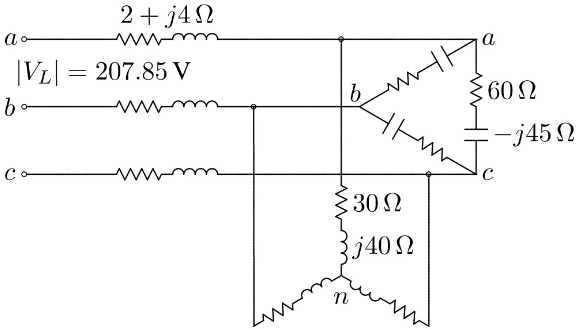

Figure 7: Three-phase circuit diagram for Example 2.7

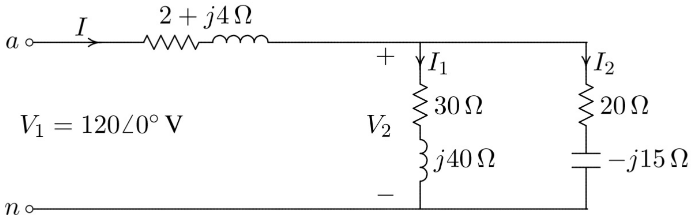

Figure 8: Single-phase circuit diagram for Example 2.7

Examples (2.7)

A three-phase line has an impedance of $2 + j4\ \Omega$ as shown in Figure 7. The line feeds two balanced three-loads that are connected in parallel. The first load is Y-connected and has an impedance of $30 + j40\,\Omega$ per phase. The second load is $\Delta-$connected and has an impedance of $60 - j45\,\Omega$. The line is energized at the sending end from a three-phase balanced supply of line voltage 207.85 V. Taking the phase voltage V_a as reference, determine:

(a) The current, real power, and reactive power drawn from the supply.

(b) The line voltage at the combined loads.

(c) The current per phase in each load.

(d) The total real and reactive powers in each load and the line.

Tutorial: Write down your solution here

Tutorial: Write down your solution here

Important Summary

Y-connected	Δ-connected
$V_L = \sqrt{3}\|V_p\|\angle 30^\circ$	$V_L = V_p$
$I_L = I_p$	$I_L = \sqrt{3}\|I_p\|\angle -30^\circ$

The End