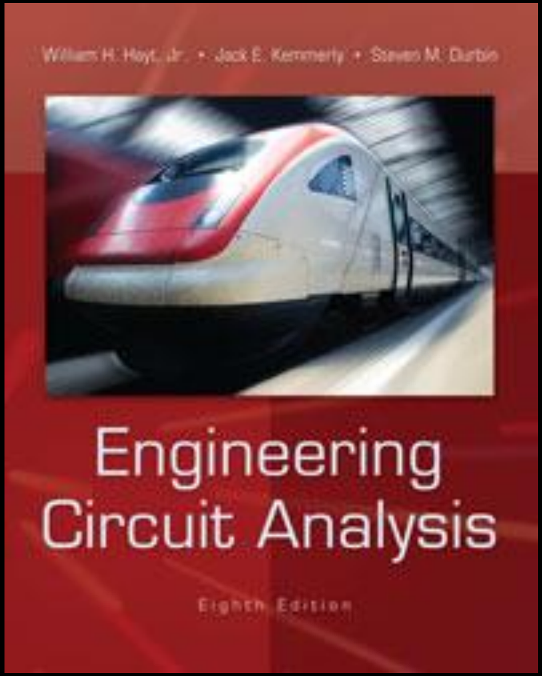
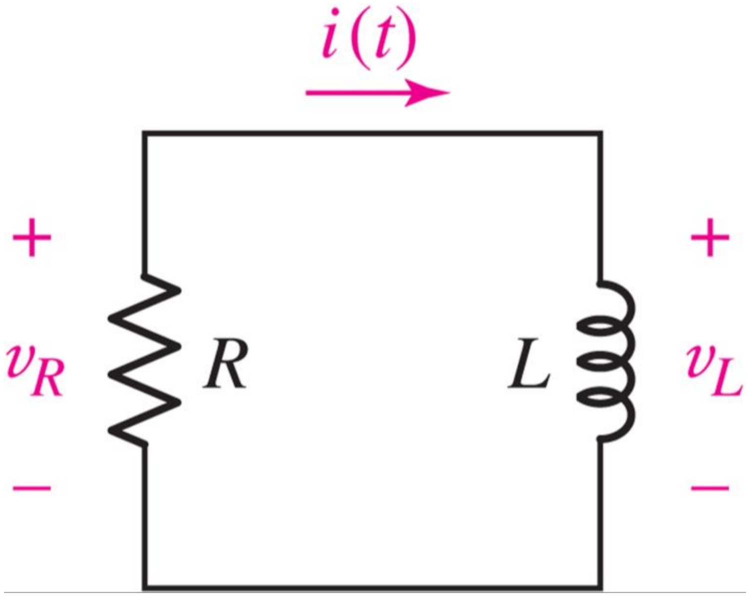
# Chapter 8
# Basic RL and RC Circuits



# The Source-Free RL Circuit

Applying KVL:

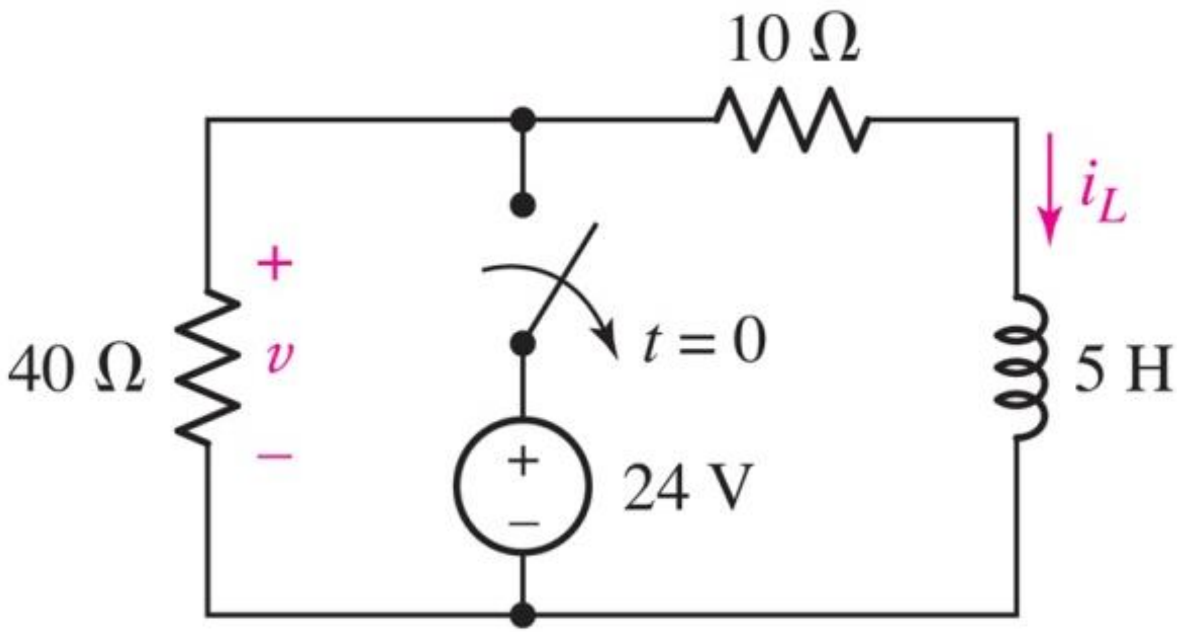
$$\frac{di}{dt} + \frac{R}{L} i = 0$$




We can solve for the *natural response* if we know the *initial condition i(0)=$I_0$*:

$$i(t) = I_0 e^{-Rt/L} \quad \text{for} \quad t>0$$

# Example: RL with a Switch




Show that the voltage *v(t)* will be -12.99 volts at *t*=200 ms.

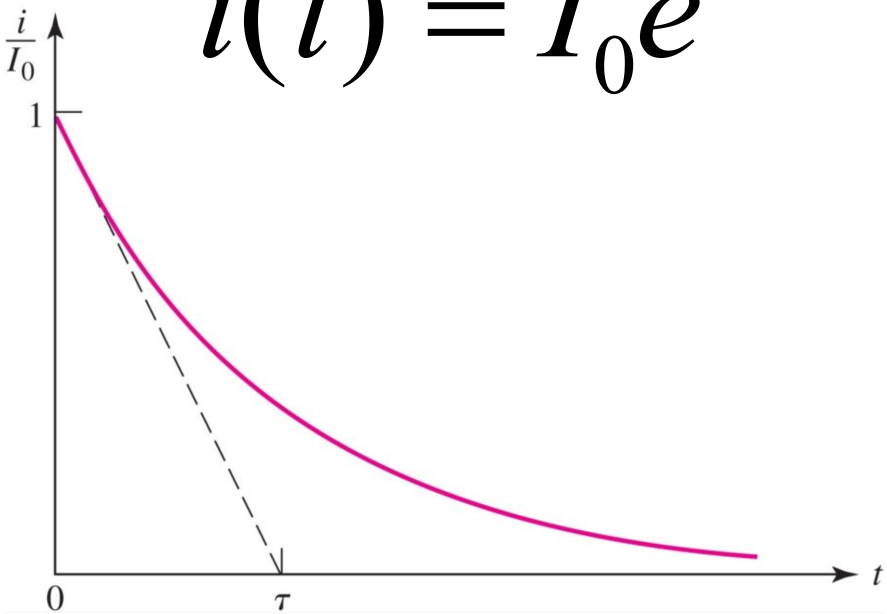
# The Exponential Response

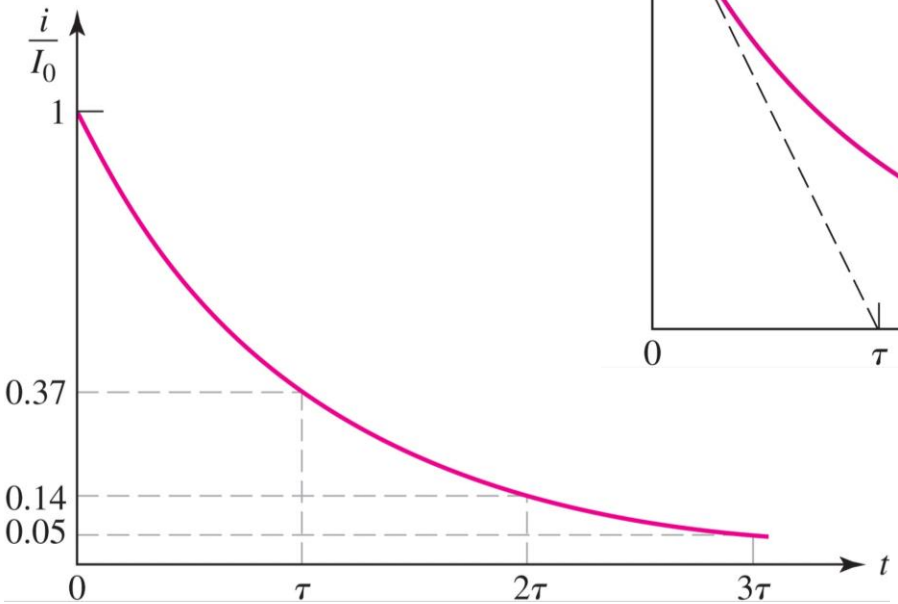
The time constant

$\tau = L/R$

determines the rate of decay.

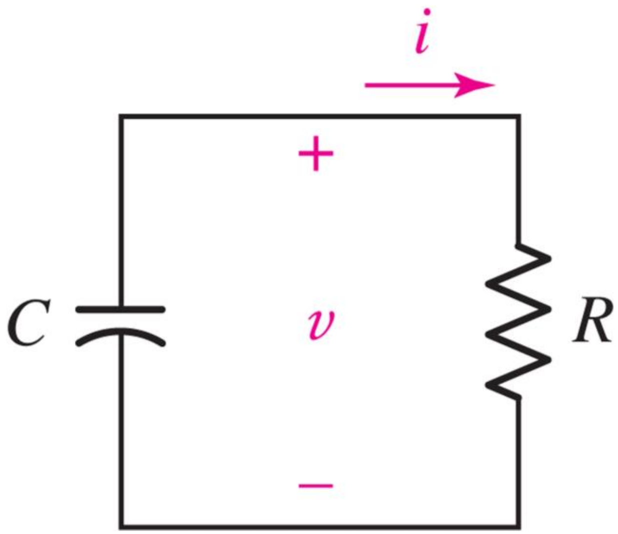
$$i(t) = I_0 e^{-t/\tau}$$



# The Source-Free RC Circuit

Applying KCL:

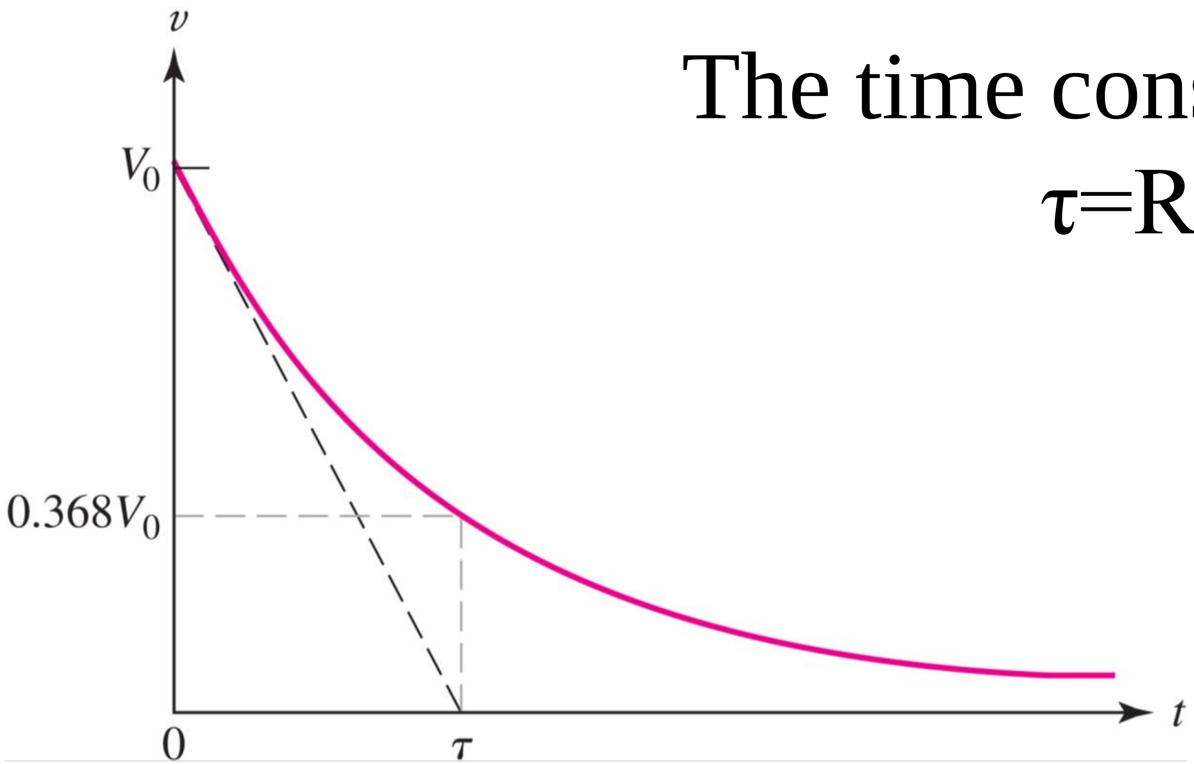



$$\frac{dv}{dt} + \frac{1}{RC} v = 0$$

We can solve for the *natural response* if we know the *initial condition* $v(0)=V_0$

$$v(t)=V_0 e^{-t/RC} \quad \text{for} \quad t>0$$

# RC Natural Response

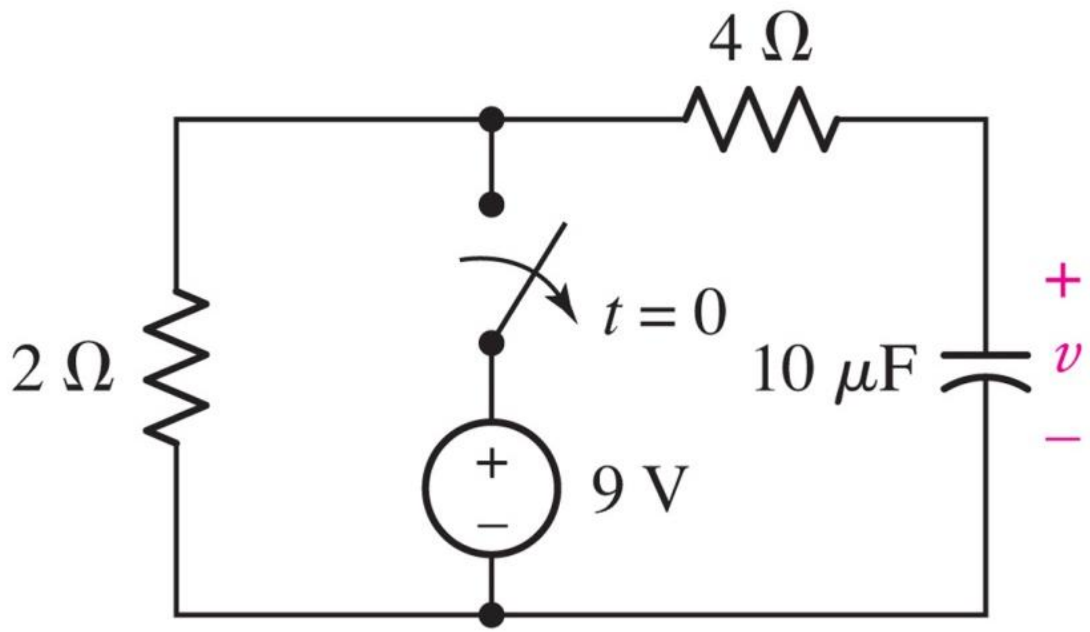



The time constant is

$\tau = RC$

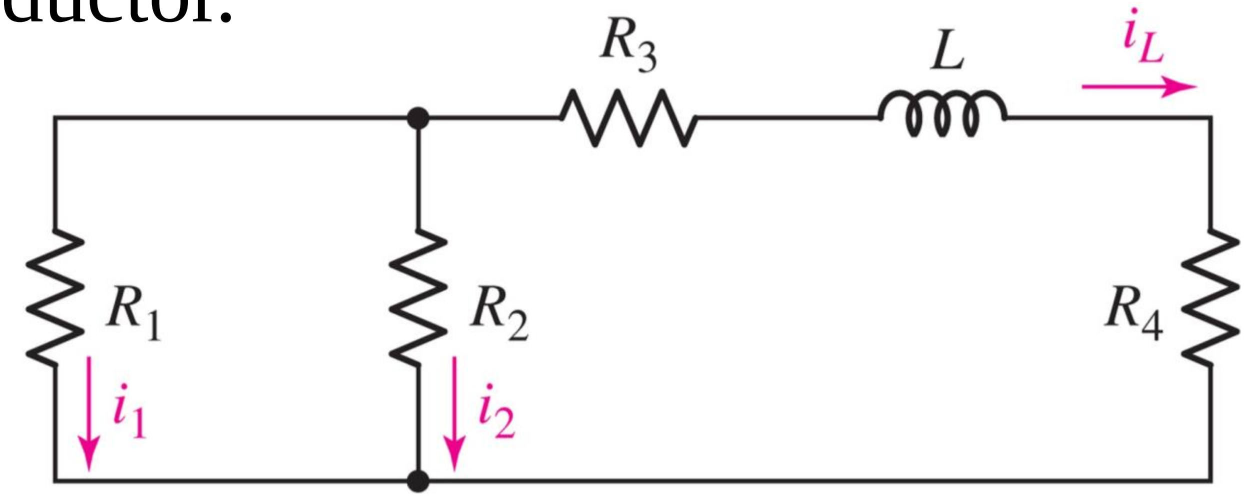
# The Source Free RC Circuit

Show that the voltage *v(t)* is 321 mV at t=200 μs.



# General RL Circuits

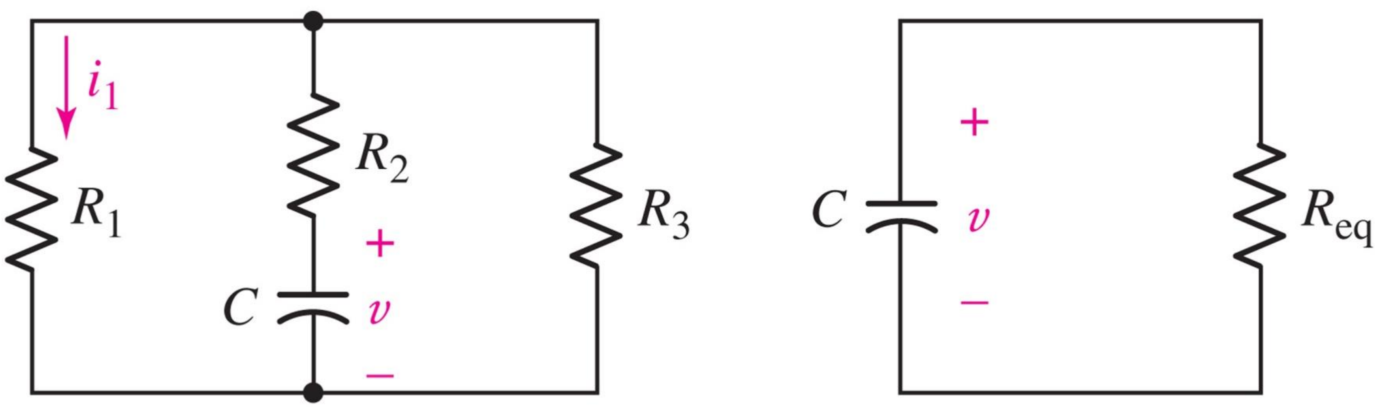
The time constant of a single-inductor circuit will be $\tau = L/R_{eq}$ where $R_{eq}$ is the resistance seen by the inductor.




*Example:* $R_{eq} = R_3 + R_4 + R_1 R_2 / (R_1 + R_2)$

# General RC Circuits

The time constant of a single-capacitor circuit will be $\tau = R_{eq}C$ where $R_{eq}$ is the resistance seen by the capacitor.



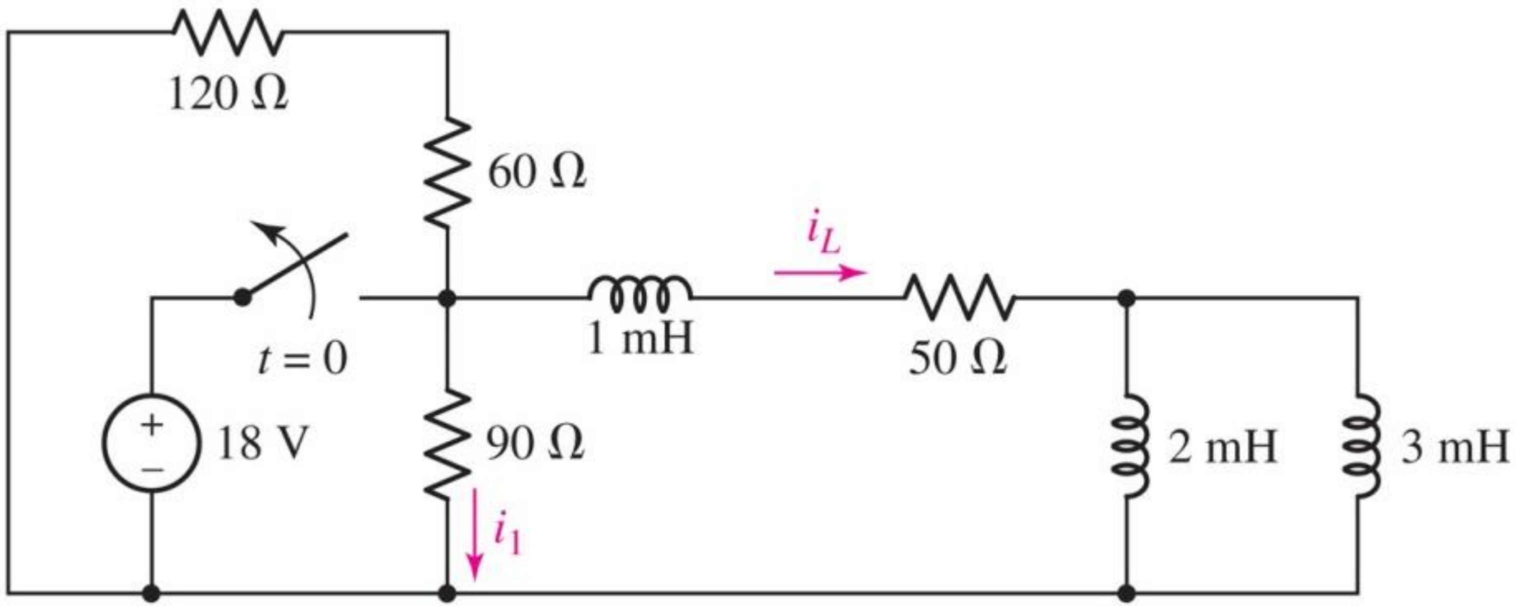

*Example:* $R_{eq} = R_2 + R_1R_3 / (R_1 + R_3)$

# $1^{st}$ Order Response Observations

- The voltage on a capacitor or the current through a inductor is the same *prior to* and *after* a switch at $t=0$.
- Resistor voltage (or current) prior to the switch $v(0^-)$ can be different from the voltage after the switch $v(0^+)$.
- All voltages and currents in an RC or RL circuit follow the same natural response $e^{-t/\tau}$.

# Example: L and R Current

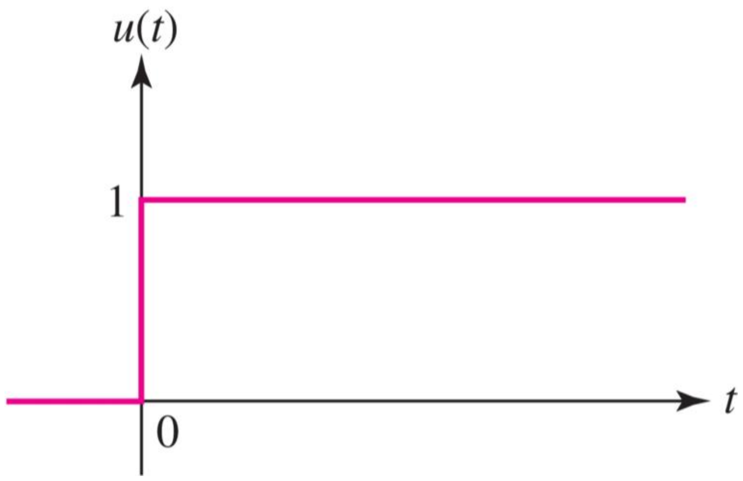
Find $i_1(t)$ and $i_L(t)$ for $t>0$.

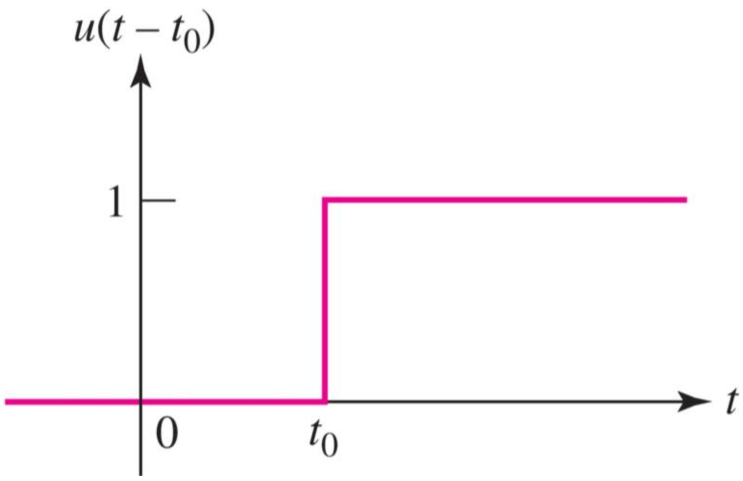



*Answer:* $\tau$=20 µs; $i_1=-0.24e^{-t/\tau}, i_L=0.36e^{-t/\tau}$ for $t>0$

# The Unit Step Function

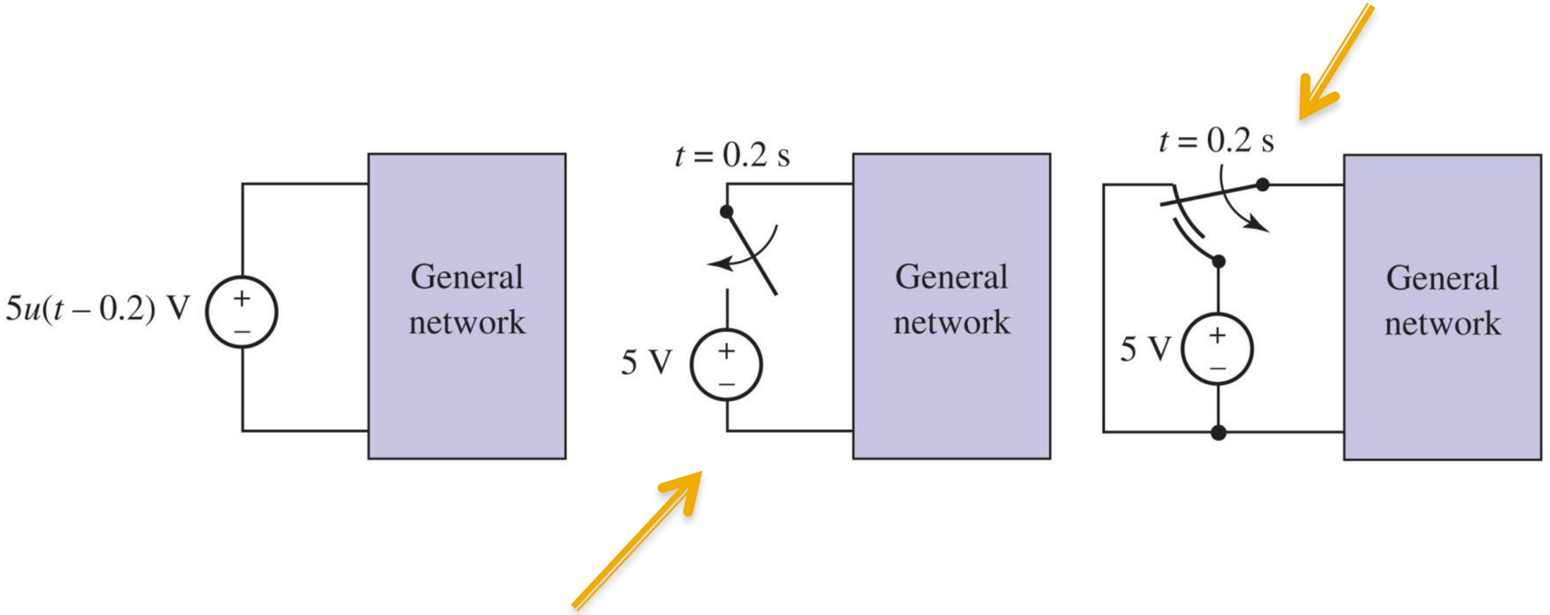
The unit-step function *u(t)* is a convenient notation to respresent change:



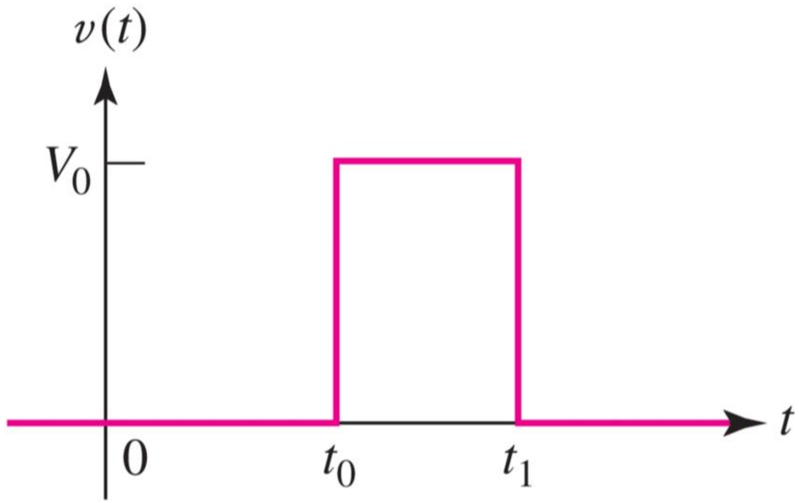
# Switches and Steps

- The unit step models a double-throw switch.



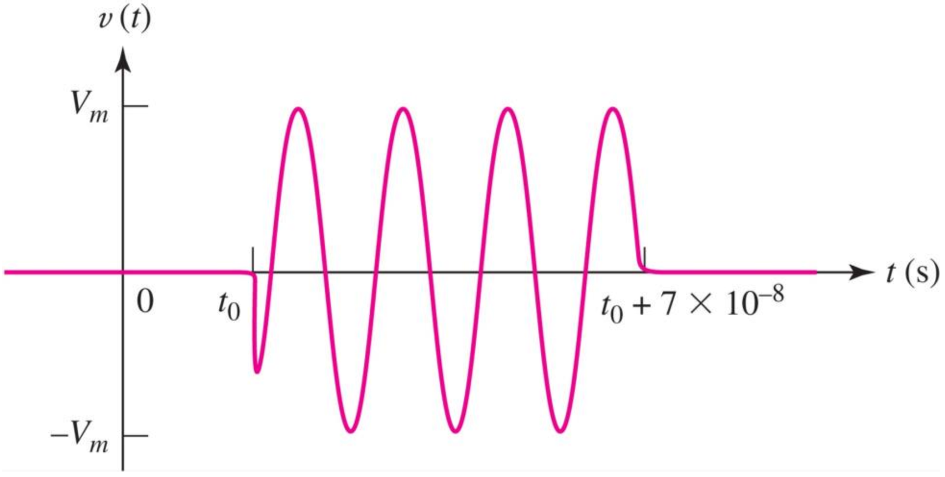

- A single-throw switch is open circuit for t<0, not short circuit.

# Modeling Pulses using u(t)




Rectangular pulse

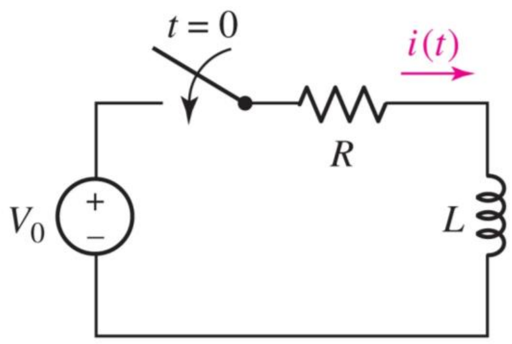
Pulsed sinewave:



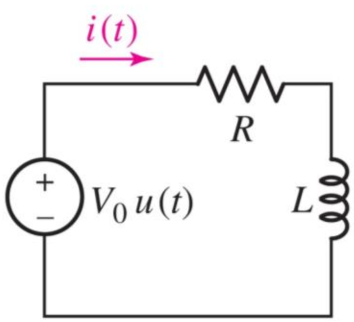
# Driven RL Circuits

- The two circuits shown both have $i(t)=0$ for $t<0$ and are also the same for $t>0$. Both are similar

- We now have to find both the *natural response* and and the *forced response* due to the source $V_0$



(a)

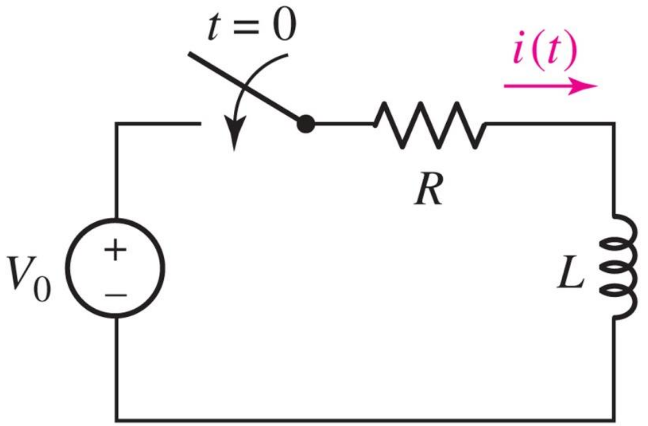


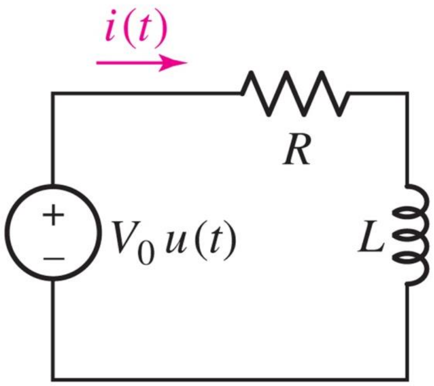
(b)

# Driven RL Circuits

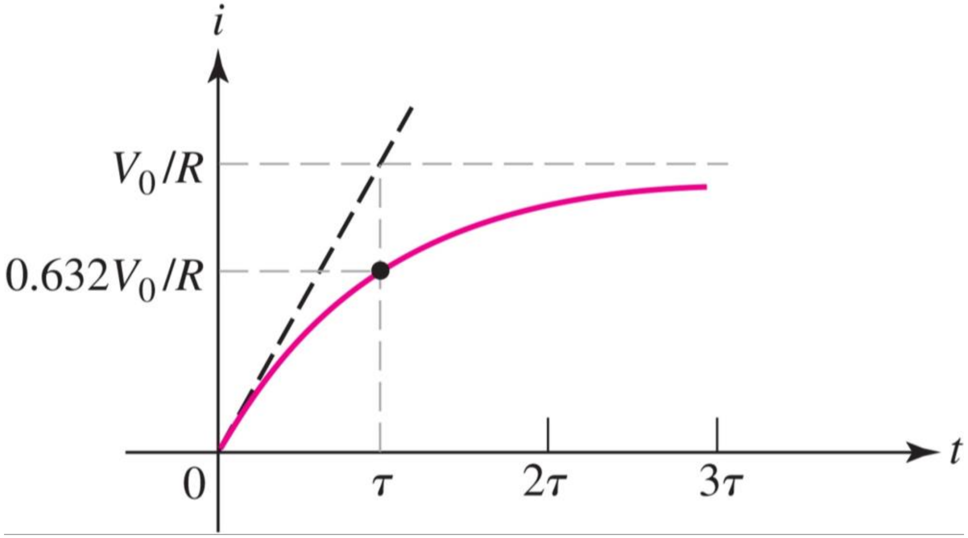
The total response is the combination of the transient/natural response and the forced response:

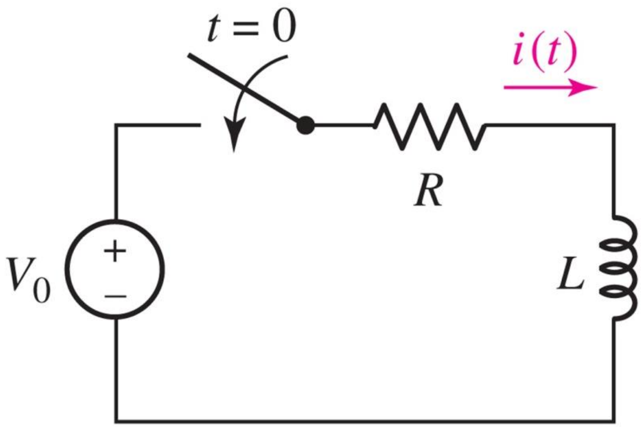
$$i(t) = \frac{V_0}{R}\left(1 - e^{-Rt/L}\right)u(t)$$

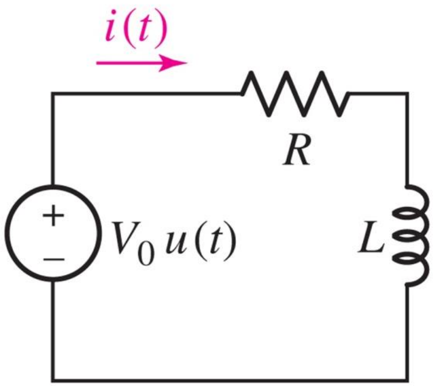


# Driven RL Circuits

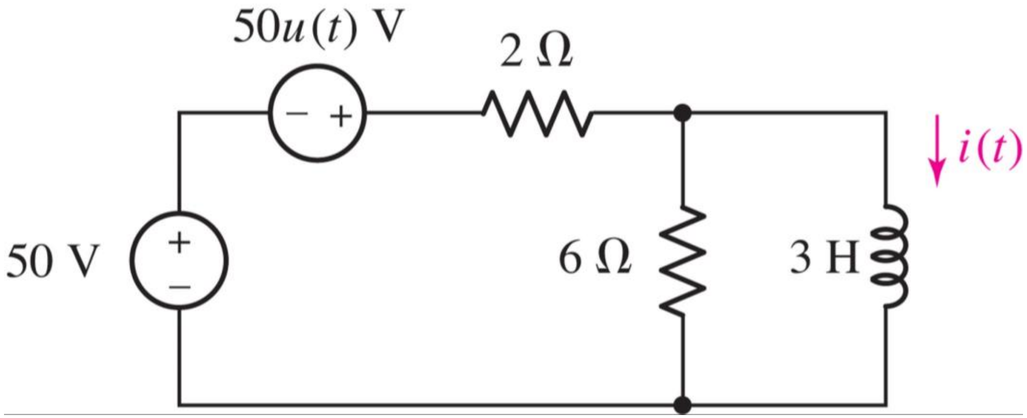


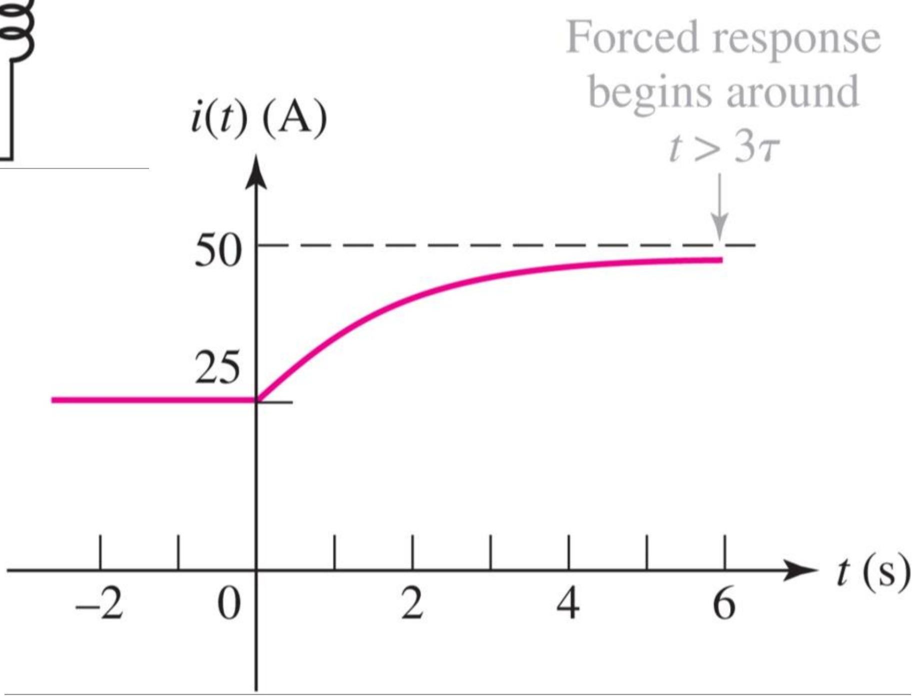




$$i(t) = \frac{V_0}{R}\left(1 - e^{-Rt/L}\right)u(t)$$

# Example: RL Circuit with Step

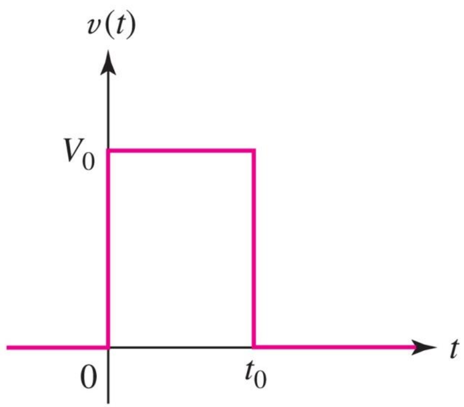


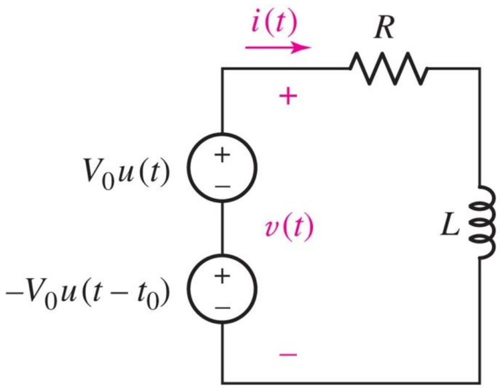

Show that

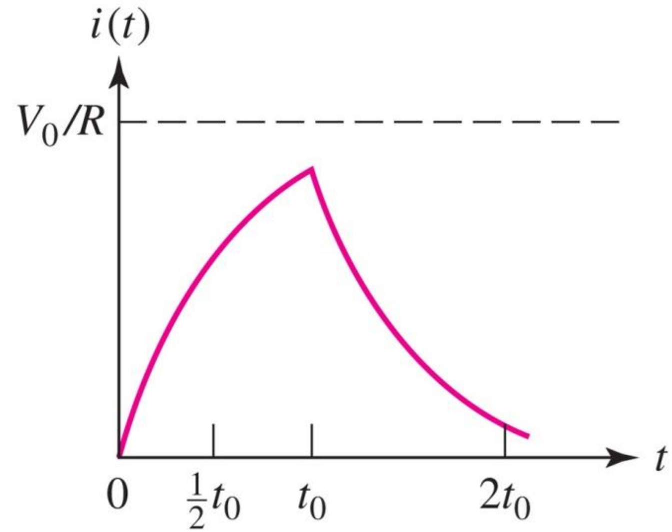
$i(t) = 25 + 25(1 - e^{-t/2})u(t)$ A

# Example: Voltage Pulse

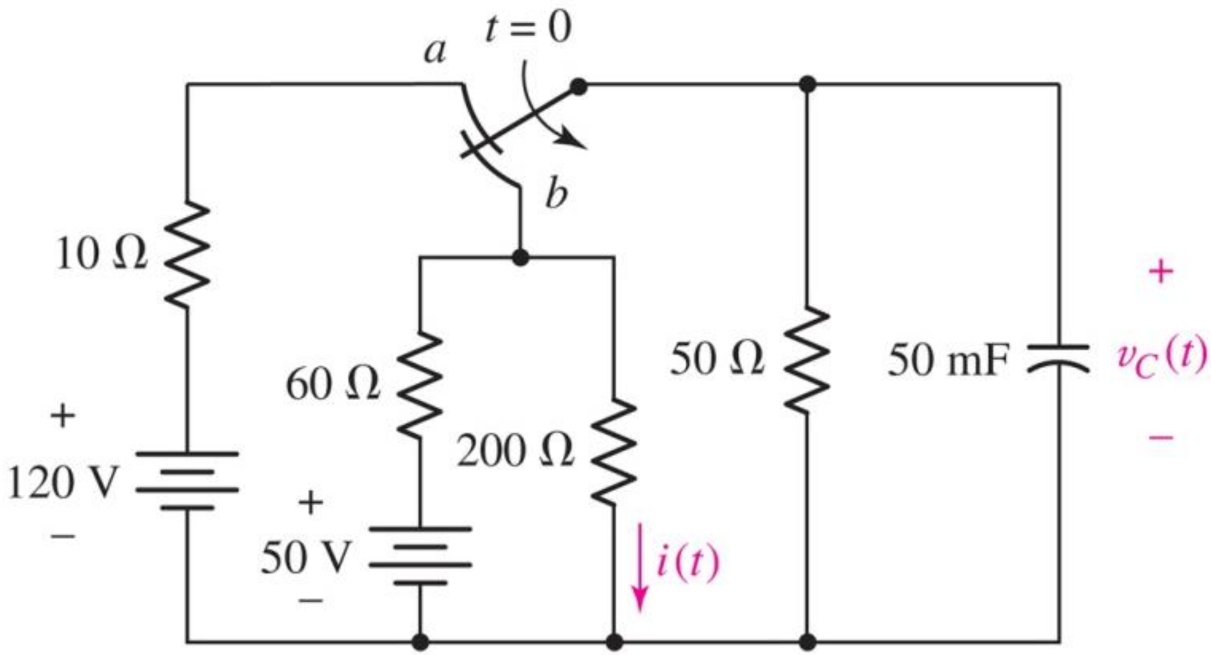



(a)



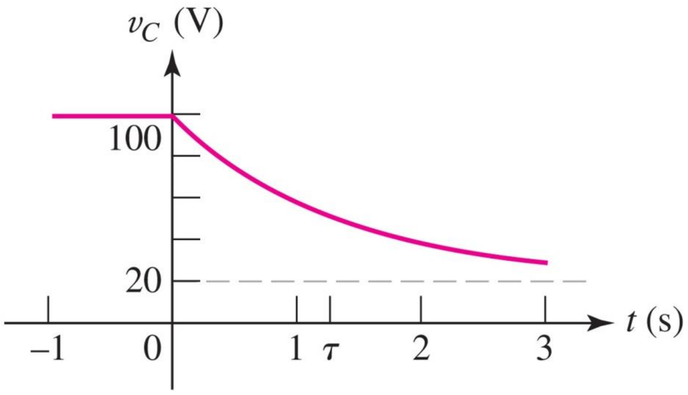



# Driven RC Circuits (part 1 of 2)



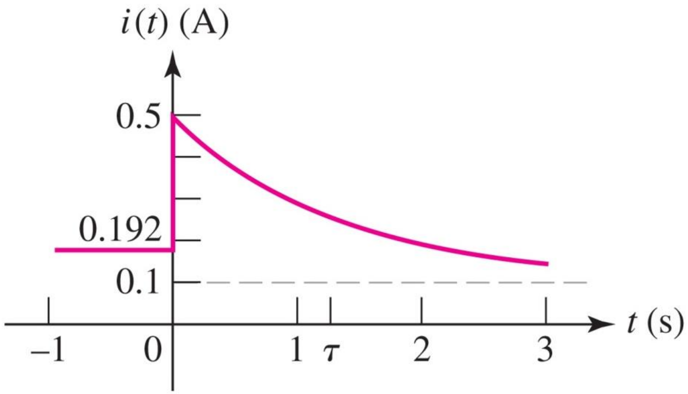

$v_C = 20 + 80e^{-t/1.2}$ V and $i = 0.1 + 0.4e^{-t/1.2}$ A

# Driven RC Circuits (part 2 of 2)




(a)

$$v_C = 20 + 80e^{-t/1.2}\ \text{V}$$




(b)

$$i = 0.1 + 0.4e^{-t/1.2}\ \text{A}$$

# RL Driven Circuit

1. With all independent sources zeroed out, simplify the circuit to determine $R_{eq}$, $L_{eq}$, and the time constant $\tau = L_{eq}/R_{eq}$.
2. Viewing $L_{eq}$ as a short circuit, use dc analysis methods to find $i_L(0^-)$, the inductor current just prior to the discontinuity.
3. Again viewing $L_{eq}$ as a short circuit, use dc analysis methods to find the forced response. This is the value approached by $f(t)$ as $t \to \infty$; we represent it by $f(\infty)$.
4. Write the total response as the sum of the forced and natural responses: $f(t) = f(\infty) + Ae^{-t/\tau}$.
5. Find $f(0^+)$ by using the condition that $i_L(0^+) = i_L(0^-)$. If desired, $L_{eq}$ may be replaced by a current source $i_L(0^+)$ [an open circuit if $i_L(0^+) = 0$] for this calculation. With the exception of inductor currents (and capacitor voltages), other currents and voltages in the circuit may change abruptly.
6. $f(0^+) = f(\infty) + A$ and $f(t) = f(\infty) + [f(0^+) - f(\infty)]\, e^{-t/\tau}$, or total response = final value + (initial value − final value) $e^{-t/\tau}$.

# RC Driven Circuit

1. With all independent sources zeroed out, simplify the circuit to determine $R_{eq}$, $C_{eq}$, and the time constant $\tau = R_{eq}C_{eq}$.
2. Viewing $C_{eq}$ as an open circuit, use dc analysis methods to find $v_C(0^-)$, the capacitor voltage just prior to the discontinuity.
3. Again viewing $C_{eq}$ as an open circuit, use dc analysis methods to find the forced response. This is the value approached by $f(t)$ as $t \to \infty$; we represent it by $f(\infty)$.
4. Write the total response as the sum of the forced and natural responses: $f(t) = f(\infty) + Ae^{-t/\tau}$.
5. Find $f(0^+)$ by using the condition that $v_C(0^+) = v_C(0^-)$. If desired, $C_{eq}$ may be replaced by a voltage source $v_C(0^+)$ [a short circuit if $v_C(0^+) = 0$] for this calculation. With the exception of capacitor voltages (and inductor currents), other voltages and currents in the circuit may change abruptly.
6. $f(0^+) = f(\infty) + A$ and $f(t) = f(\infty) + [f(0^+) - f(\infty)]e^{-t/\tau}$, or total response = final value + (initial value − final value) $e^{-t/\tau}$.