

# Semiconductor Fundamentals – (II)

---

2.3 Energy Bands

2.4 The doping of semiconductors

**Material developed  
by Prof. C. Z. Zhao**

# HW-1: solution

- Si atomic density:

$$\frac{\# \text{Atoms}}{\text{Volume}} = \frac{8 \times (1/8) + 6 \times (1/2) + 4}{a_0^3} = \frac{8}{(5.43 \times 10^{-8} \text{ cm})^3} = 5 \times 10^{22} \text{ cm}^{-3}$$

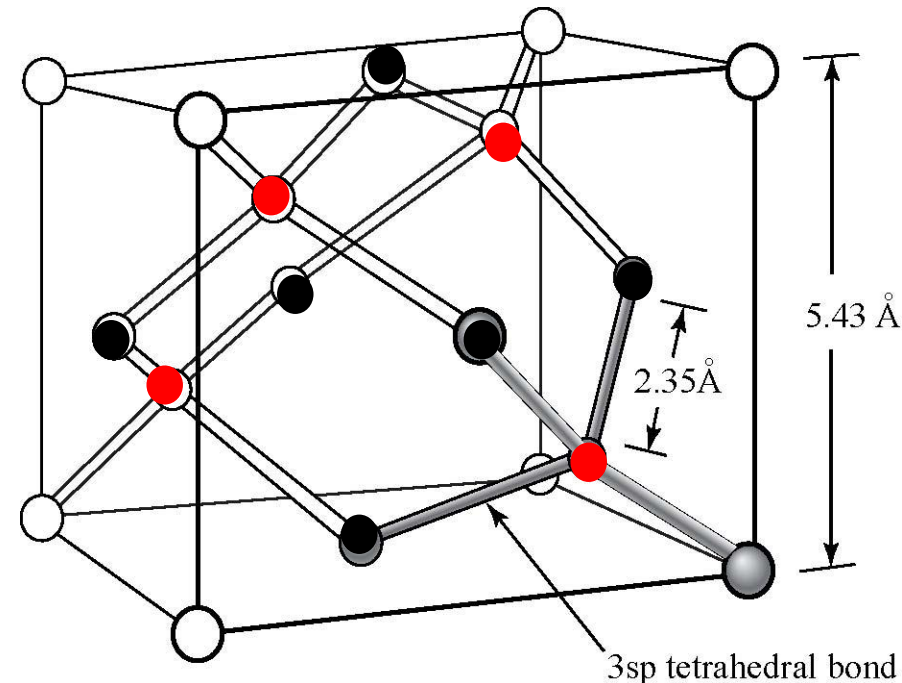
- Number of atoms in a unit cell:

- 4 atoms completely inside cell
- Each of the 8 atoms on corners are shared among cells → count as 1 atom inside cell
- Each of the 6 atoms on the faces are shared among 2 cells → count as 3 atoms inside cell

Total number inside the cell = 4 + 1 + 3 = 8

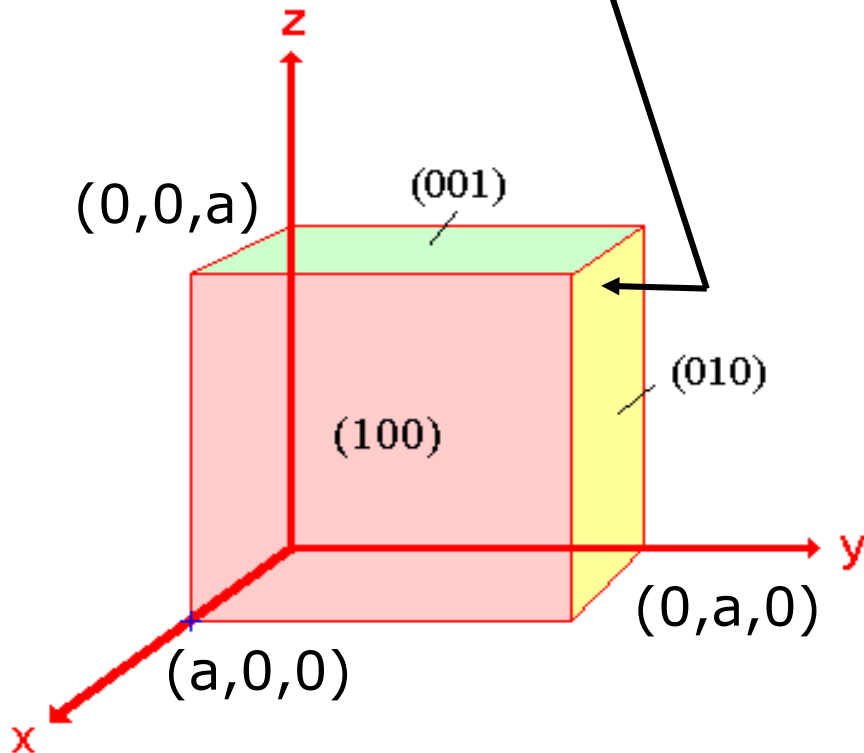
- Cell volume:

$$(.543 \text{ nm})^3 = 1.6 \times 10^{-22} \text{ cm}^3$$



# HW-2: solution

Why the Miller indices of this plane is (010)?



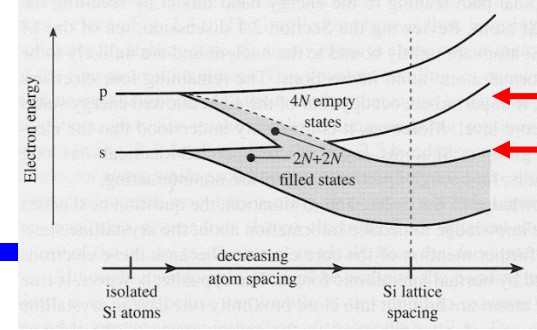
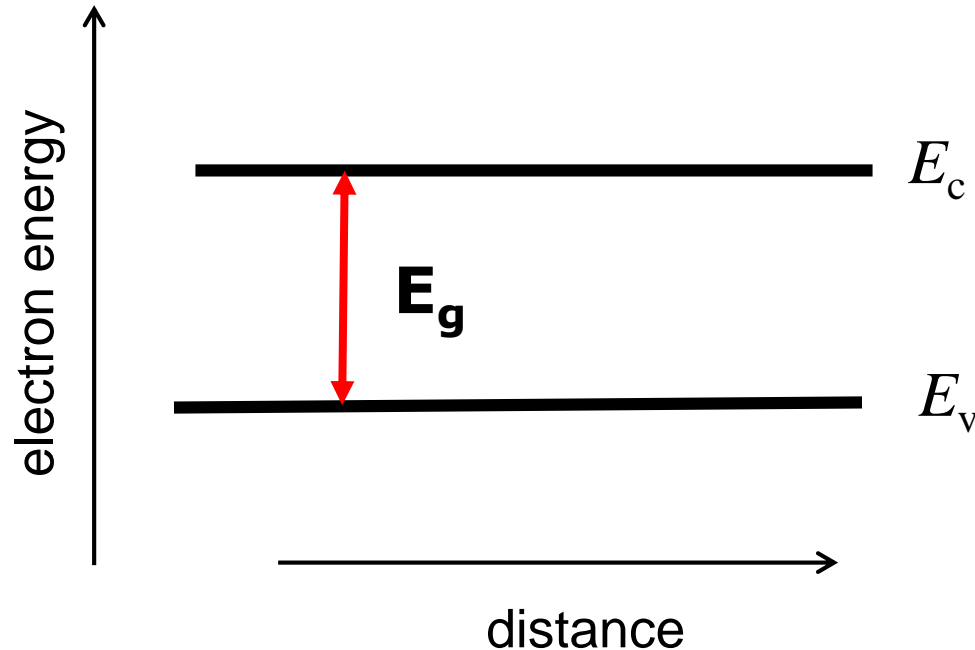
*x*-intercept of plane  
*y*-intercept of plane  
*z*-intercept of plane

*h*: inverse *x*-intercept of plane  
*k*: inverse *y*-intercept of plane  
*l*: inverse *z*-intercept of plane

*h*, *k* and *l* are reduced to 3  
integers having the same ratio.

**(010)**

# Last lecture:



Simplified version of energy band model, indicating

- bottom edge of the conduction band ( $E_c$ )
- top edge of the valence band ( $E_v$ )
- $E_c$  and  $E_v$  are separated by the **band gap energy  $E_g$**

## 2.3 Energy Bands

---

- Band theory
  - What's a Semiconductor
  - Fermi Level
  - Band model of  $e$  &  $h$
  - Bond model of  $e$  &  $h$
- Generation and recombination
- Intrinsic semiconductor

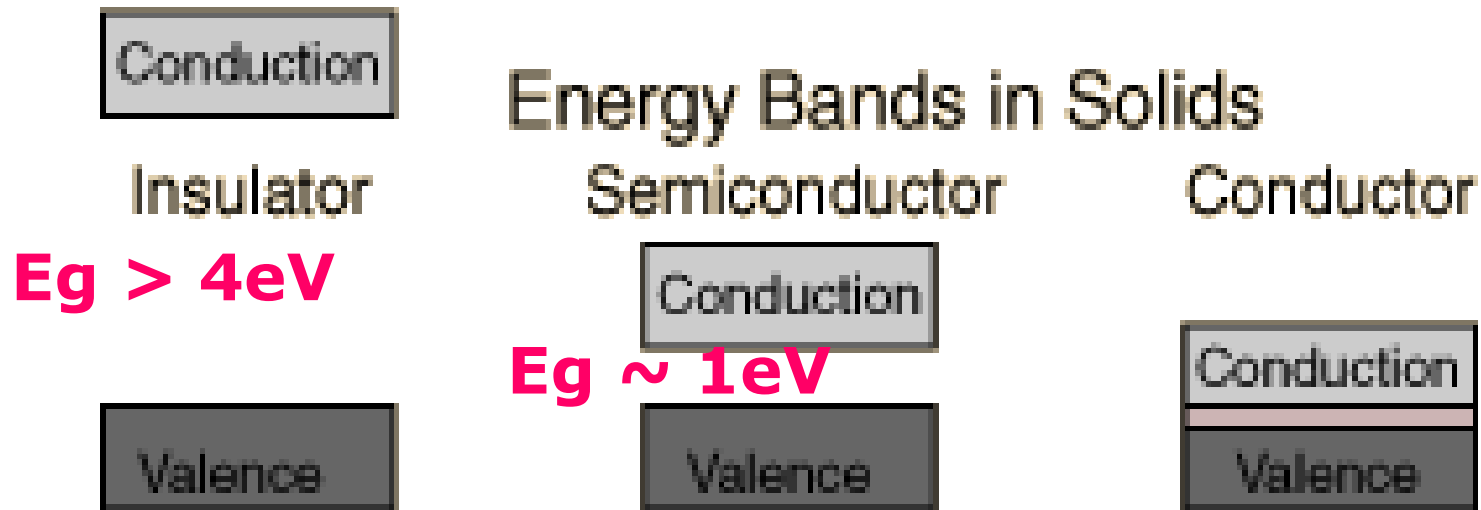
# What is a Semiconductor?

---

- Low resistivity => “conductor” e.g. Al, Cu
- High resistivity => “insulator” e.g.  $\text{SiO}_2$
- Intermediate resistivity => “semiconductor”
  - conductivity lies between that of conductors and insulators

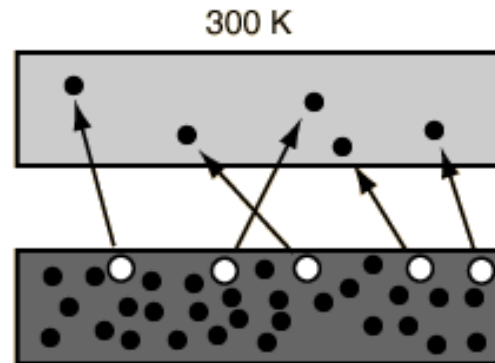
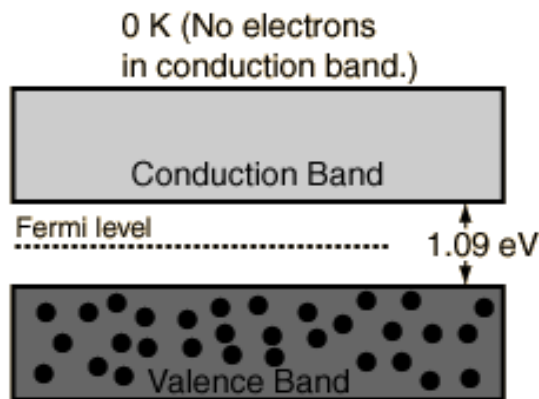
# Band Theory of Solids

- A useful way to visualize the difference between [conductors](#), [insulators](#) and [semiconductors](#) is to plot the available energies for electrons in the materials. In conductors the valence band overlaps the conduction band, and in semiconductors/insulator there is a small/big gap between the valence and conduction bands.
- An important parameter in the band theory is the [Fermi level](#), the top of the available electron energy levels at low temperatures.



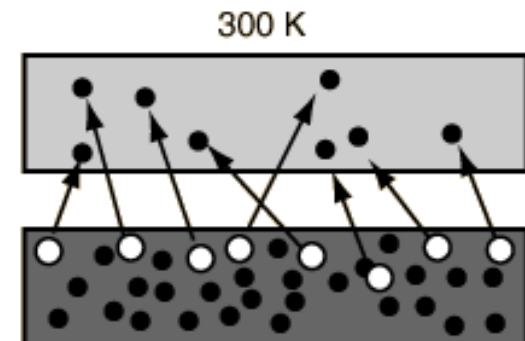
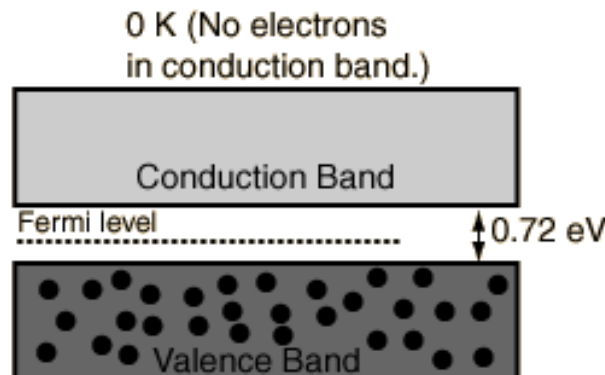
# Energy Bands of Silicon & Germanium

- At finite temperatures, the number of electrons which reach the conduction band and contribute to current can be modeled by the Fermi function.



1) Fermi level?

2) "holes"?





# Fermi function and Fermi level

---

概率 Probability **that** a **state** at energy level,  $E$ , is occupied by one electron is,

$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$

Students  $\rightarrow$  Electrons

Seat row  $\rightarrow$  energy level,  $E$ .

Seat  $\rightarrow$  state

**Example:** Students in a theatre class room.

Every row has different potential,  $E$ . For example, for row 7, its potential is  $E_7$ .

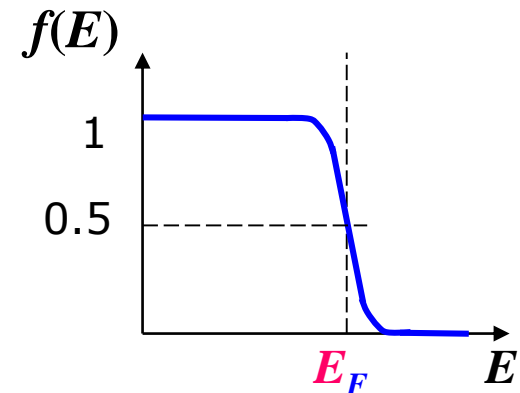
The probability for one **student** to occupy a **seat** on **row 7** can be calculated by  $f(E_7)$ .

$E_F$  is a energy level at which  $f(E)$  is 50%.

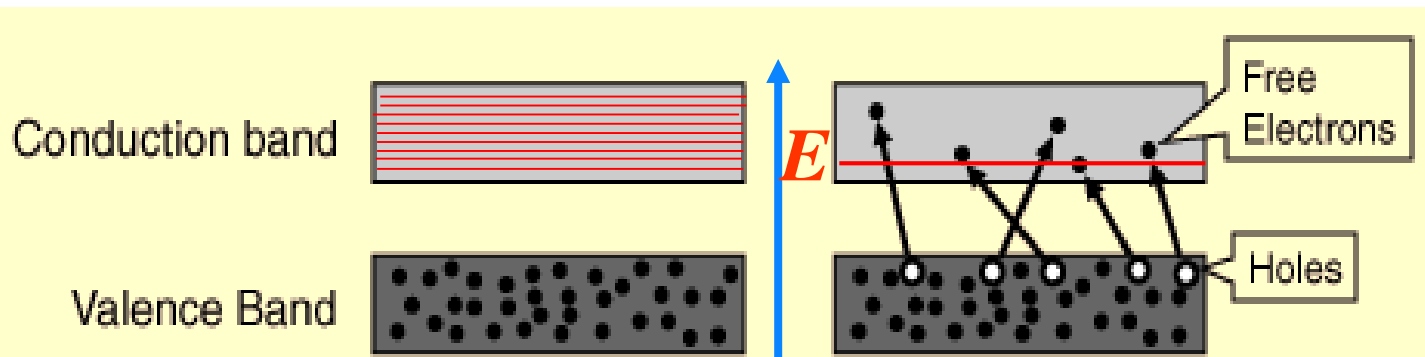
# Fermi function and Fermi level

- Probability **that** a **state** at energy level,  $E$ , is occupied by one electron is,

$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$



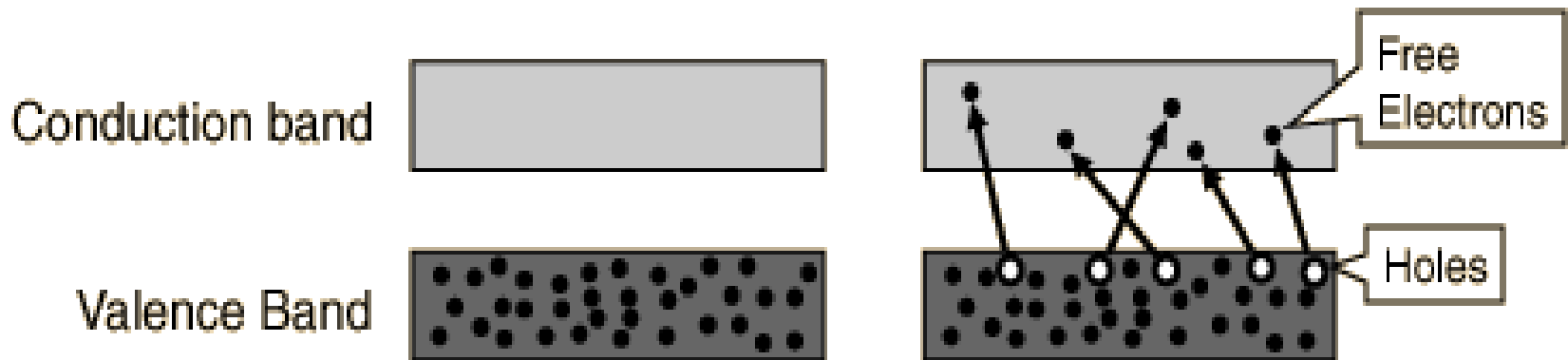
- $f(E)$ : Fermi-Dirac function
- An increase in  $E$  will reduce  $f(E)$
- **$E_F$  --- Fermi-level**
  - **When  $E = E_F$ ,  $f(E=E_F) = 0.5$ .**



**textbook**  
**P.66**

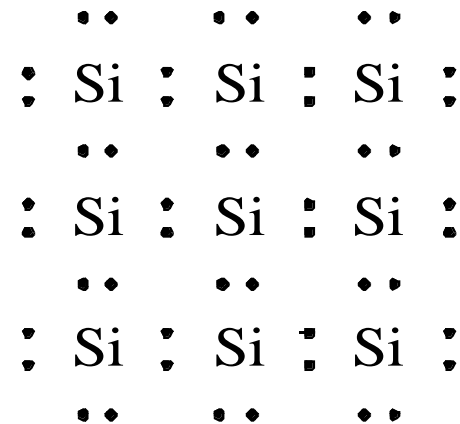
# Band Model of Electrons and Holes

- In an pure semiconductor like silicon at temperatures above absolute zero, there will be some electrons which are excited across the band gap into the conduction band and which can produce current.
- When the electron in pure silicon crosses the gap, it leaves behind an **electron vacancy** or "**hole**" in the regular silicon lattice.
- Under the influence of an external voltage, both the electron and the hole can move across the material.

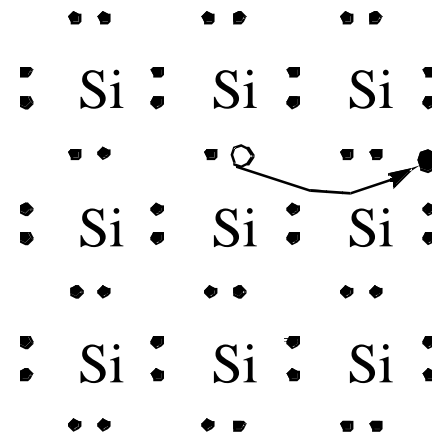


# Bond Model of Electrons and Holes

2-D representation:  
**Covalent Bonds**



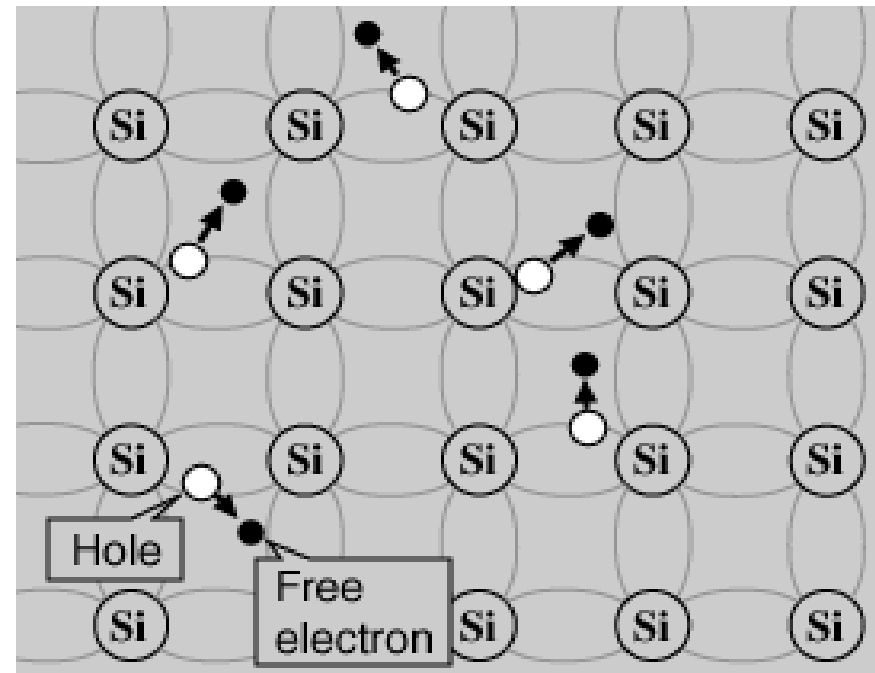
When an electron **breaks loose** and becomes a **conduction electron**, a **hole** is also created.



# Bond Model of Electrons and Holes

- When an electron breaks loose and becomes a **conduction electron**, a **hole** is also created.
- A hole (along with its associated positive charge) is mobile!
- Hole density = electron density in a pure Si.

2-D representation:



**Pure Si**

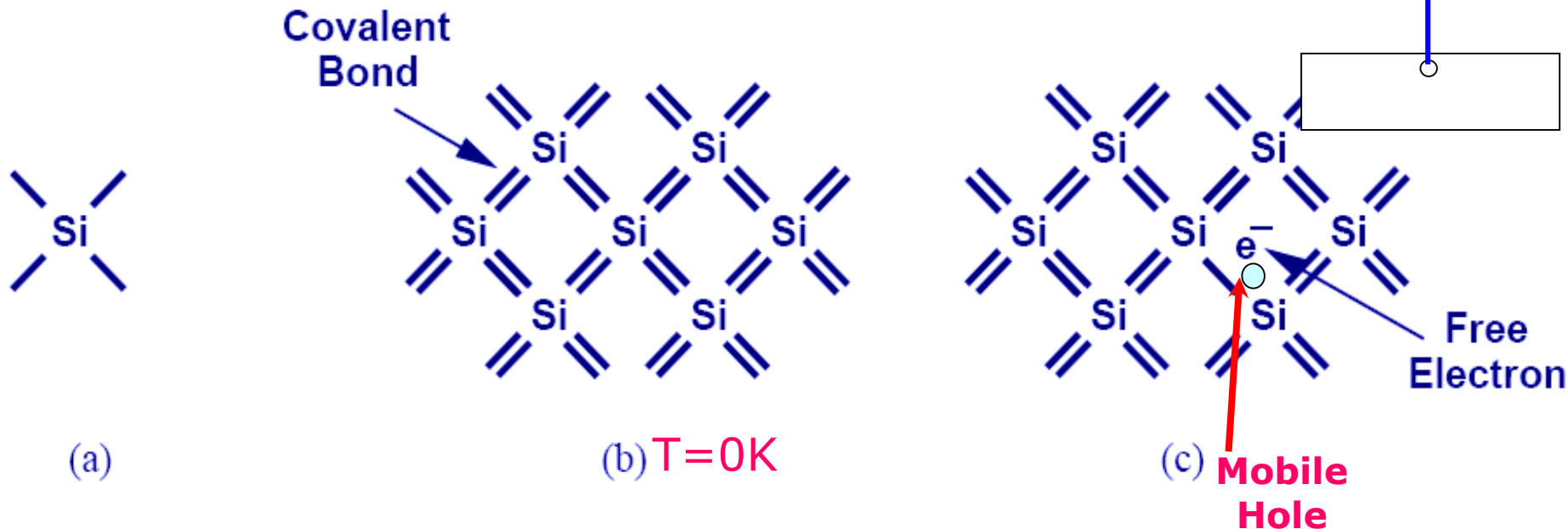
## 2.3 Energy Bands

---

- Band theory
  - What's a Semiconductor
  - Fermi Level
  - Band model of  $e$  &  $h$
  - Bond model of  $e$  &  $h$
- **Generation and recombination**
- Intrinsic semiconductor

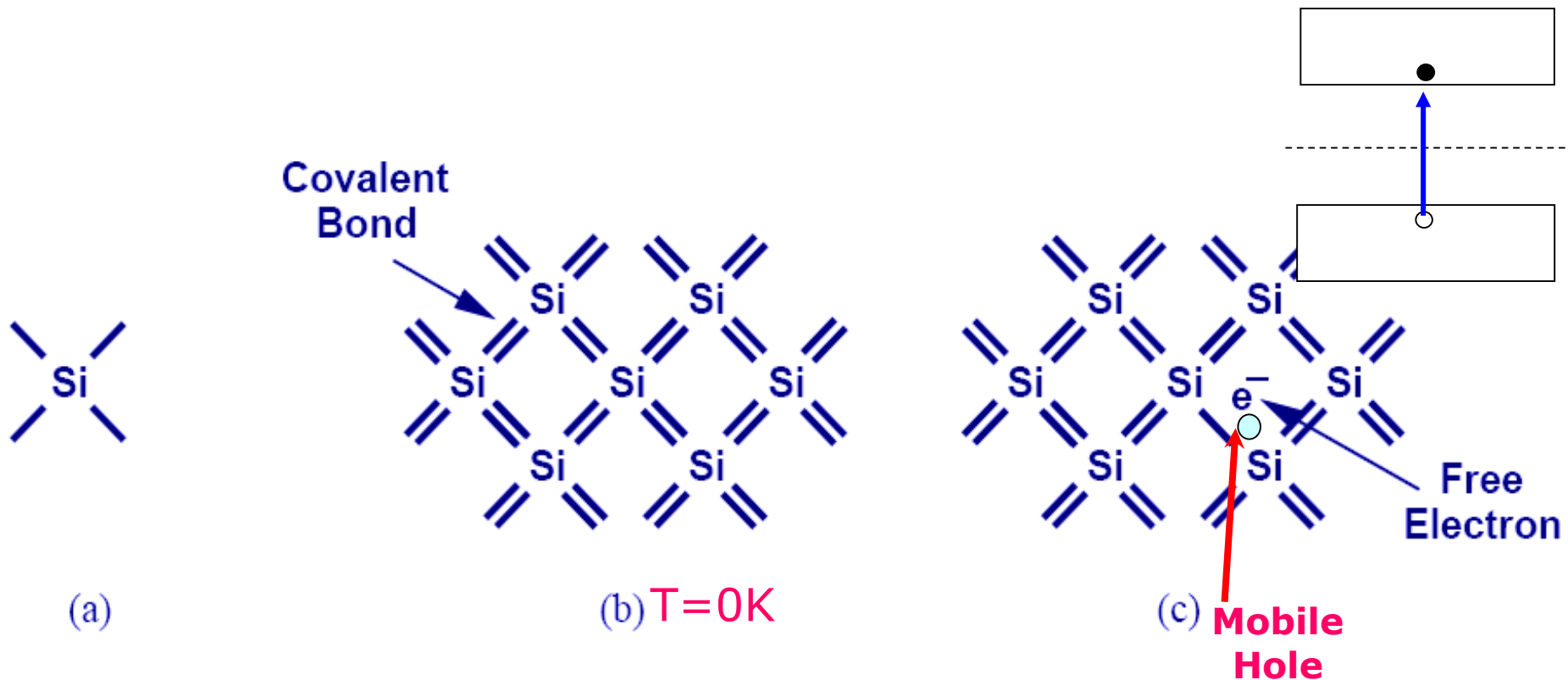
# Thermal Generation

- **Inverse process: recombination**
- Si has four valence electrons. Therefore, it can form covalent bonds with four of its nearest neighbors.
- When temperature goes up, electrons can become free to move about the Si lattice.



# Generation

- **Generation: A process to create electron-hole pairs.**

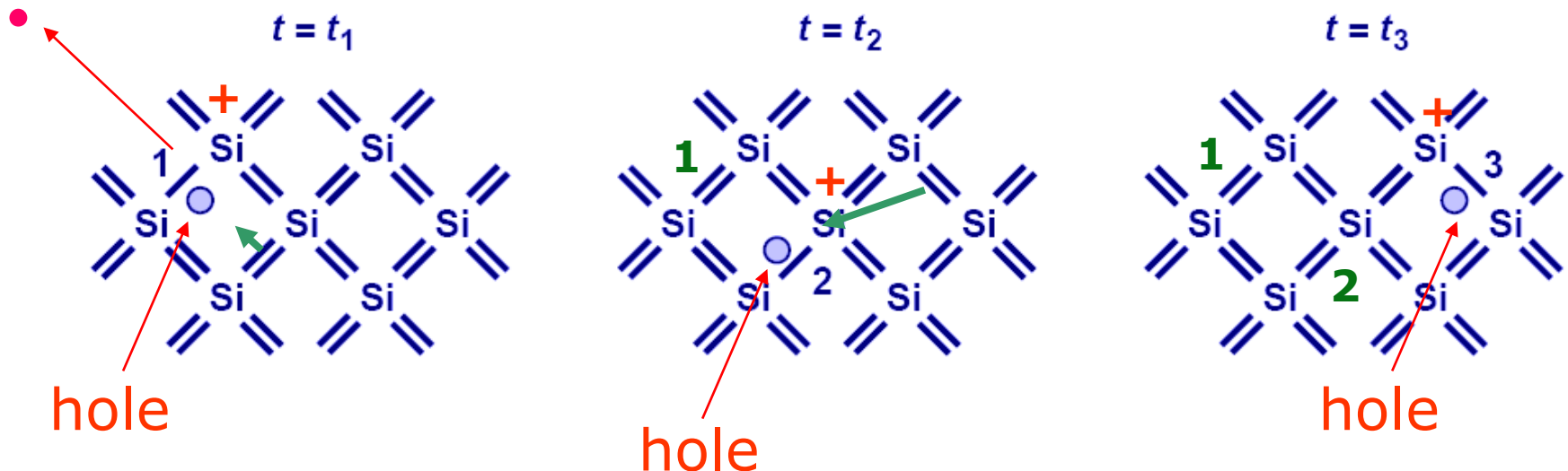




# Electron-Hole Pair Generation

- When a conduction electron is thermally generated, a “hole” is also generated.
- A hole is associated with a positive charge, and is free to move about the Si lattice as well.

A hole is mobile!



# Generation

---

- We have seen that conduction (mobile) electrons and holes can be created in pure (intrinsic) silicon by **thermal generation**.
  - Thermal generation rate increases exponentially with temperature  $T$
- Another type of generation process which can occur is **optical generation**
  - The energy absorbed from a **photon** frees an electron from covalent bond
    - In Si, the minimum energy required is **1.1eV**, which corresponds to  $\sim 1 \mu\text{m}$  wavelength (infrared region).  $1 \text{ eV} = \text{energy gained by an electron falling through } 1 \text{ V potential} = q_e V = 1.6 \times 10^{-19} \text{ C} \times 1 \text{ V} = 1.6 \times 10^{-19} \text{ J}$ .
- Note that conduction electrons and holes are continuously generated, if  $T > 0$

# Light interactions with Semiconductors

## Absorption of light in a semiconductor:

- For energies of light greater than the bandgap ( $h\nu > E_g$ ), light is absorbed
- For energies less than the bandgap ( $h\nu < E_g$ ), light is out
- Silicon is transparent to IR (can see through it with an IR camera)
- Diamond is a wide bandgap semiconductor (transparent to visible light)

Solar cell

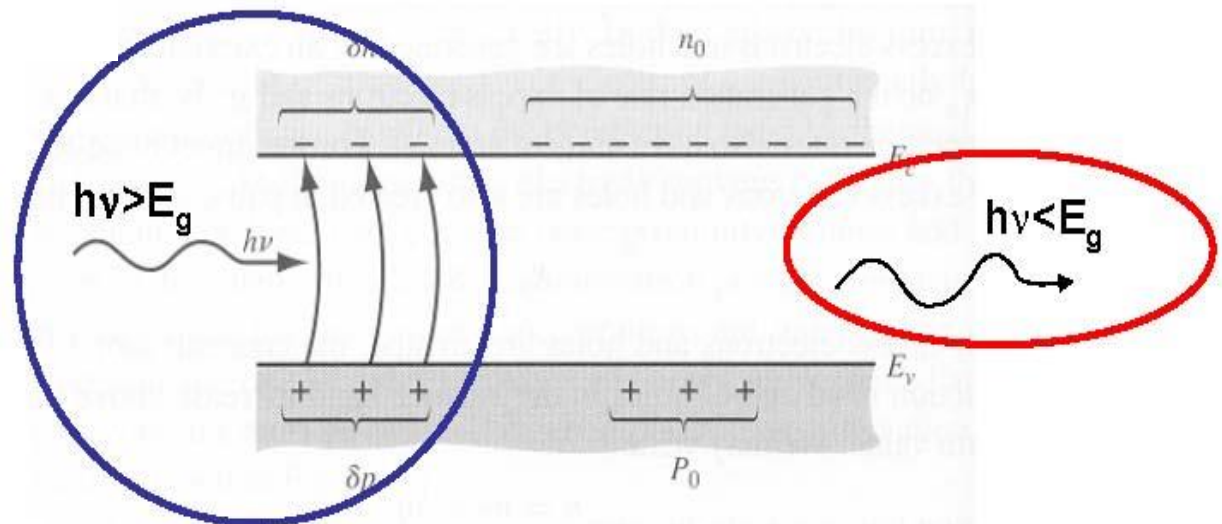


Figure 6.2 | Creation of excess electron and hole densities by photons.

# Recombination

- When a conduction electron and hole meet, each one is eliminated, a process called “recombination”. The energy lost by the conduction electron (when it “falls” back into the covalent bond) can be released in two ways:

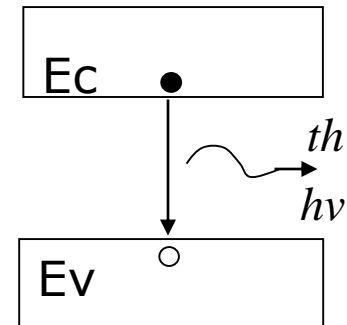
1. to the semiconductor lattice (vibrations)

“thermal recombination” → semiconductor is heated

2. to photon emission

“optical recombination” → light is emitted

- It is the basis for light-emitting diodes and laser diodes.



LED

## 2.3

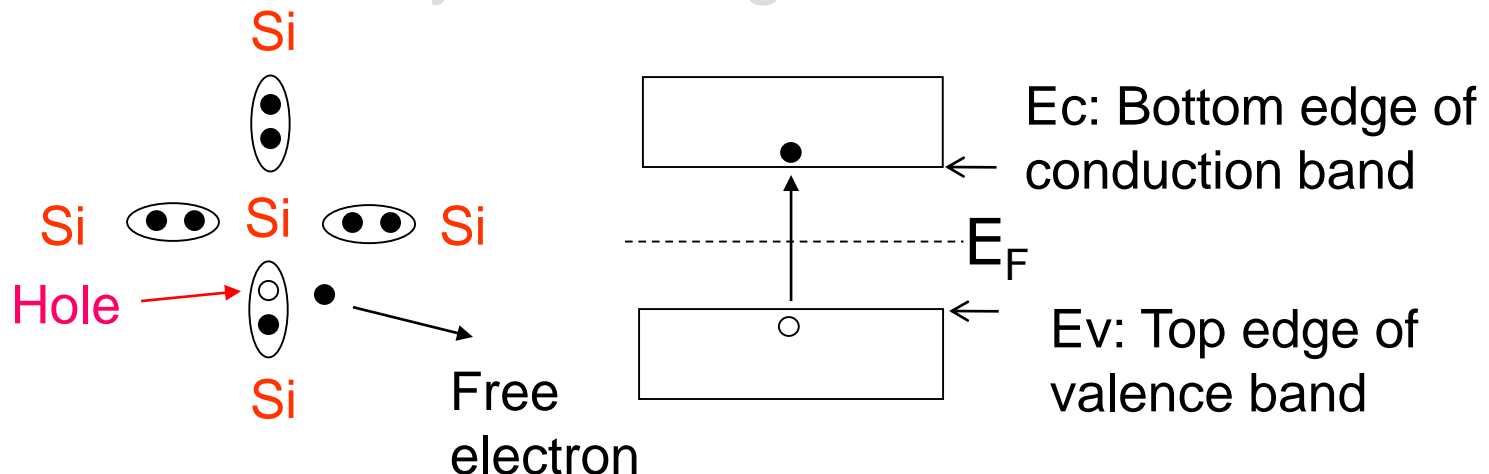
# Energy Bands

---

- Band theory
  - What's a Semiconductor
  - Fermi Level
  - Band model of  $e$  &  $h$
  - Bond model of  $e$  &  $h$
- Generation and recombination
- **Intrinsic semiconductor**

# Intrinsic semiconductors

- **Intrinsic:** pure semiconductor
- A hole is created simultaneously with a free electron
  - $n$ (free electron density) =  $p$ (hole density)
- $E_F$  is in the middle of the bandgap
- Its resistivity is too high for most of devices



# Carrier Concentrations in Intrinsic Si

- The “band-gap energy”  $E_g$  is the amount of energy needed to remove an electron from a covalent bond.
- The concentration of conduction electrons in intrinsic silicon,  $n_i$ , depends exponentially on  $E_g$  and the absolute temperature ( $T$ ):

$$n_i = 5.2 \times 10^{15} T^{3/2} \exp \frac{-E_g}{2kT} \text{ electrons/cm}^3$$

$E_g = 1.12 \text{ eV}$

Boltzmann constant  
8.62E-5 eV/K

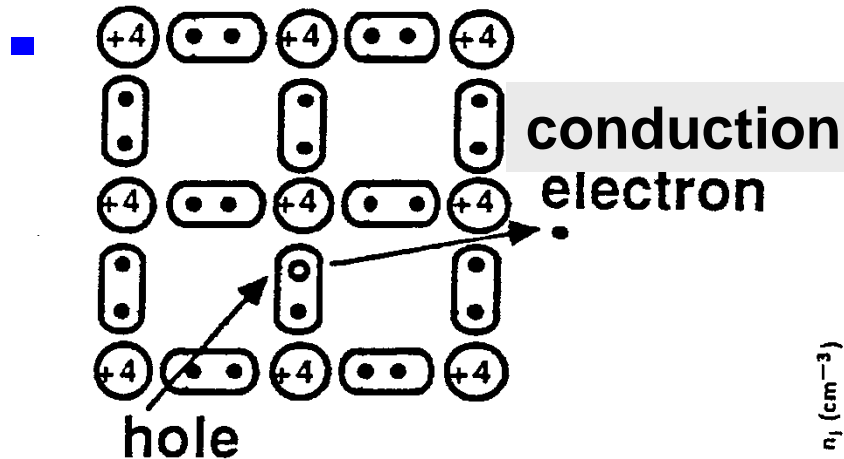
$$n_i \cong 1 \times 10^{10} \text{ electrons/cm}^3 \text{ at } 300\text{K}$$

$$n_i \cong 1 \times 10^{15} \text{ electrons/cm}^3 \text{ at } 600\text{K}$$

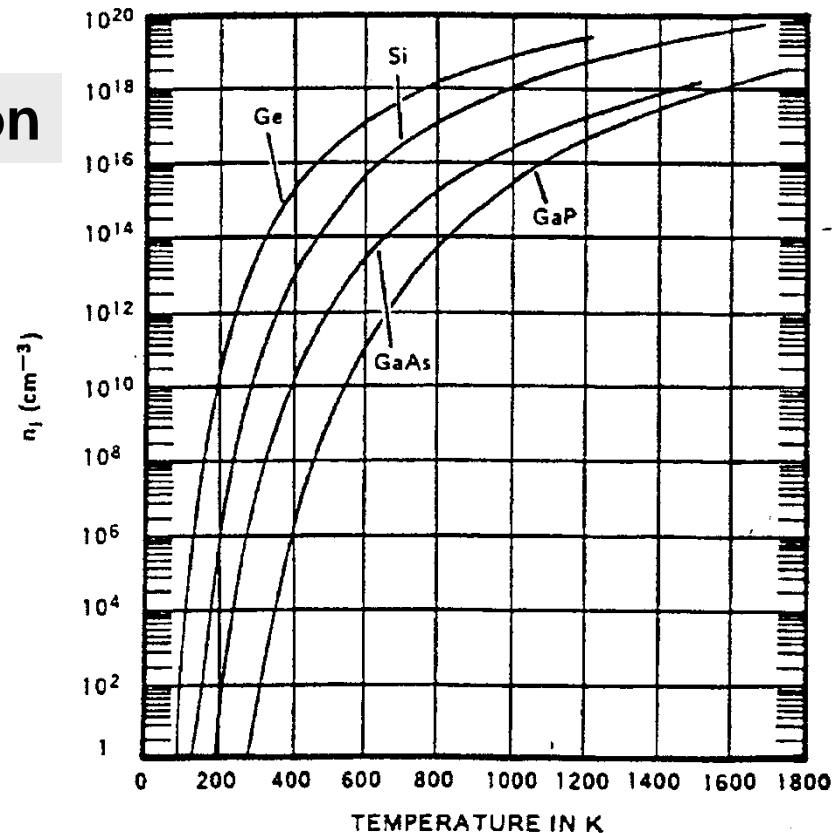
RT

# Pure Si

$$n_i = 5.2 \times 10^{15} T^{3/2} \exp \frac{-E_g}{2kT} \text{ electrons/cm}^3$$



Covalent (shared  $e^-$ ) bonds exist between Si atoms in a crystal. Since the  $e^-$  are loosely bound, some will be free at any  $T$ , creating hole electron pairs.



Si:

$$n_i = 3.9 \times 10^{16} T^{3/2} e^{-\frac{0.605\text{eV}}{kT}} / \text{cm}^3$$

$$n_i \cong 1.5 \times 10^{10} \text{ cm}^{-3} \text{ at room temperature}$$

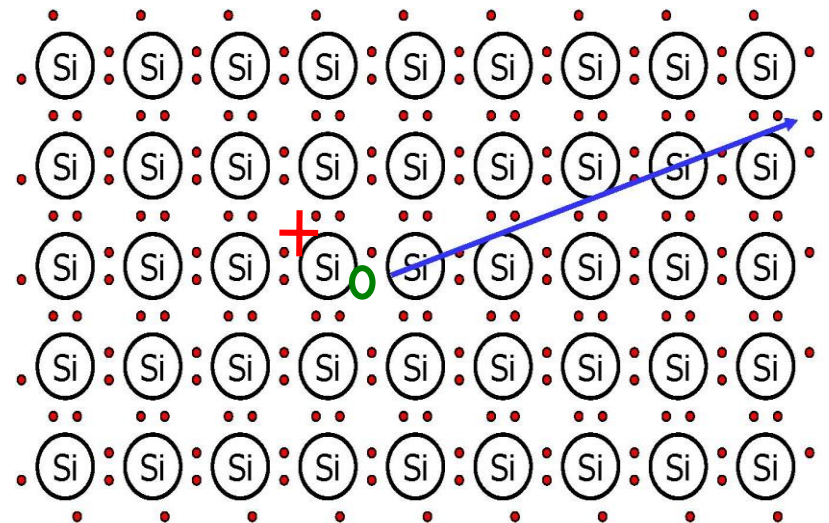
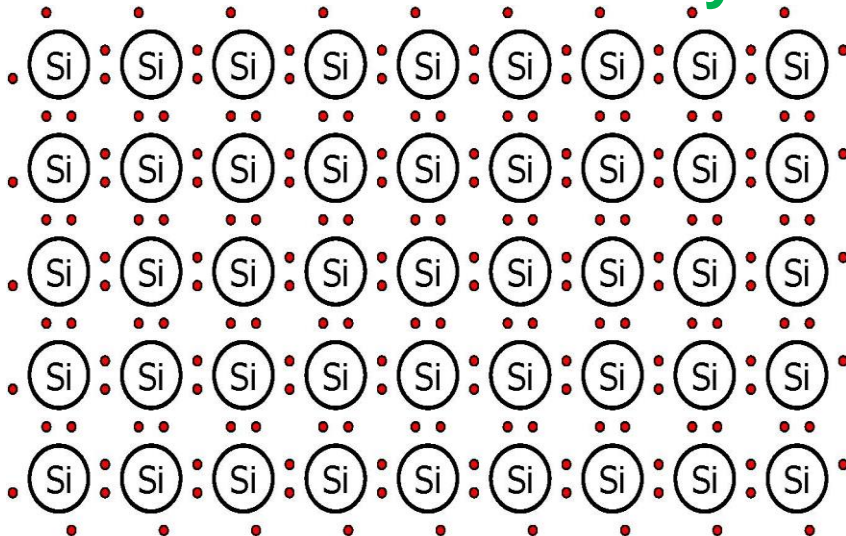
p.75



# Intrinsic Semiconductor

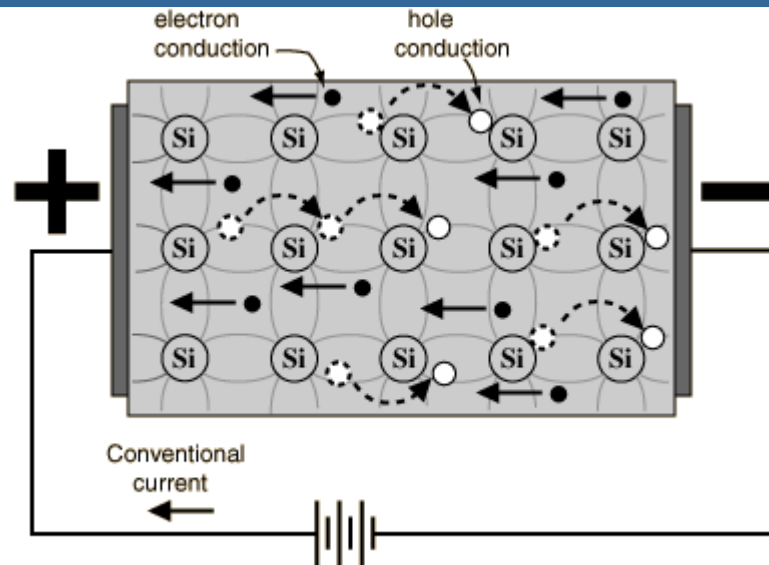
Silicon has four valence electrons

- It covalently bonds with 4 adjacent atoms in the crystal lattice
- Increasing Temperature Causes Creation of Free Carriers.  $10^{10}\text{cm}^{-3}$  free carriers at  $23^\circ\text{C}$  (out of  $2 \times 10^{23}\text{cm}^{-3}$ ): Intrinsic Conductivity.
- **Si atomic density:  $5 \times 10^{22} \text{ cm}^{-3}$**



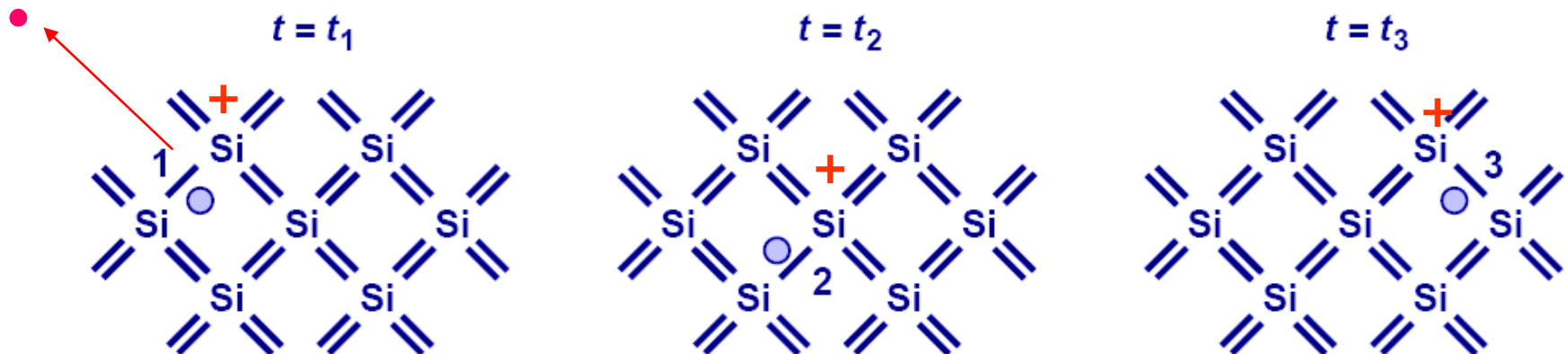
# Semiconductor Current

- Both electrons and holes contribute to current flow in an intrinsic semiconductor.



# Electron-Hole Pair Generation

- When a conduction electron is thermally generated, a “hole” is also generated.
- A hole is associated with a positive charge, and is free to move about the Si lattice as well.



# Summary of Section 2.3

---

- In a pure Si crystal, conduction electrons and holes are formed in pairs.
  - Holes can be considered as positively charged mobile particles which exist inside a semiconductor.
  - Both holes and electrons can conduct current.
- Splitting of allowed atomic energy levels occurs in a crystal
  - Separation between energy levels is small, so we can consider them as bands of continuous energy levels
    - Highest nearly-filled band is the valence band
    - Lowest nearly-empty band is the conduction band
- The band gap energy is the energy required to free an electron from a covalent bond.
  - $E_g$  for Si at 300K = 1.12eV

# 2.4 The doping of semiconductors

---

掺杂

- **Doping elements**
- Doping: N type
- Doping: P type
- Counter doping

补偿掺杂

# The Doping

---

- The addition of a **small** percentage of foreign atoms in the regular crystal lattice of silicon or germanium produces dramatic changes in their electrical properties, producing n-type and p-type semiconductors.
- Definition of Terms:

**$n$  = number of electrons/cm<sup>3</sup>**

**$p$  = number of holes/cm<sup>3</sup>**

**$n_i$  = intrinsic carrier concentration**

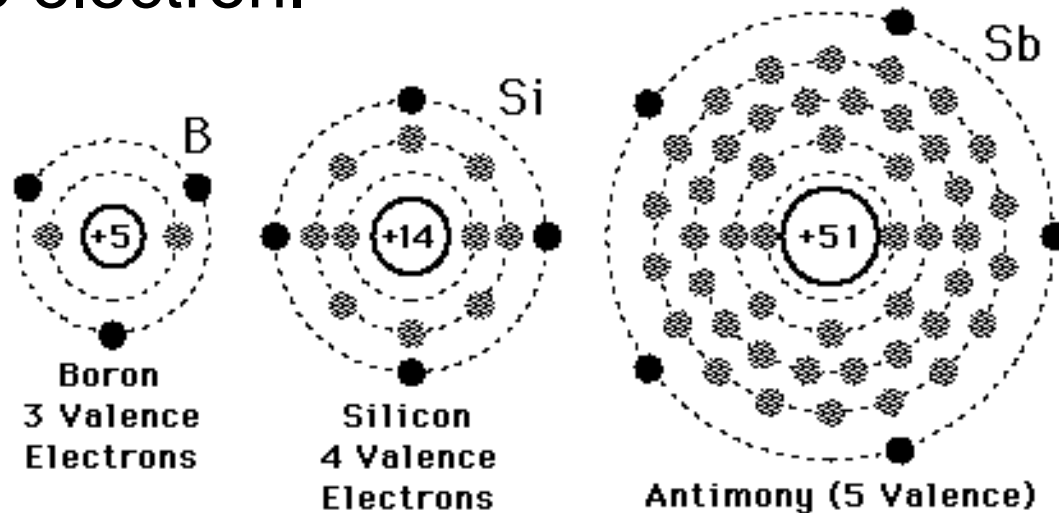
**In a pure semiconductor,**

$$n = p = n_i$$

# Valence Electrons

PL

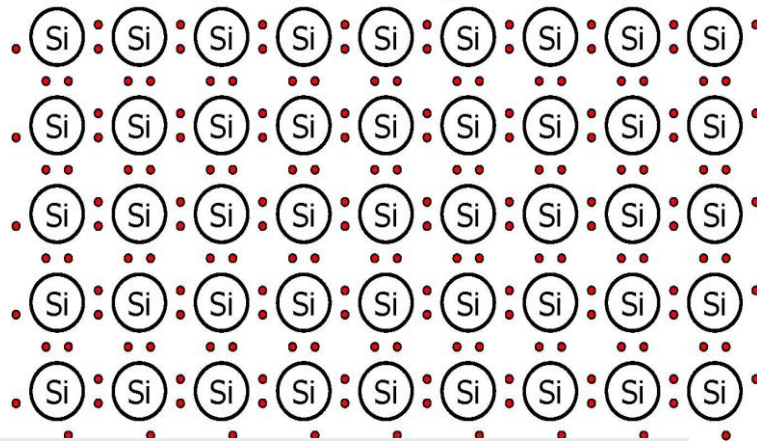
- The electrons in the outermost shell of an atom are called valence electrons; they dictate the nature of the chemical reactions of the atom and largely determine the electrical nature of solid matter. The electrical properties of matter are pictured in the [band theory of solids](#) in terms of how much energy it takes to free a valence electron.



# The Doping of Semiconductors

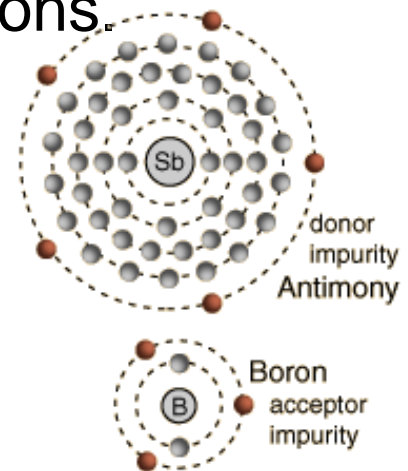
## 五价的杂质（施主杂质）

**Pentavalent impurities** (donor impurities = donors)  
Impurity atom with 5 valence electrons produce **n-type** semiconductors by contributing **extra** electrons.



Antimony  
Arsenic  
Phosphorous

Boron  
Aluminum  
Gallium



## 三价的杂质（受主杂质）

**Trivalent impurities** (acceptor impurities = acceptors)  
Impurity atoms with 3 valence electrons produce **p-type** semiconductors by producing a "hole" or electron deficiency.



# 2.4 The doping of semiconductors

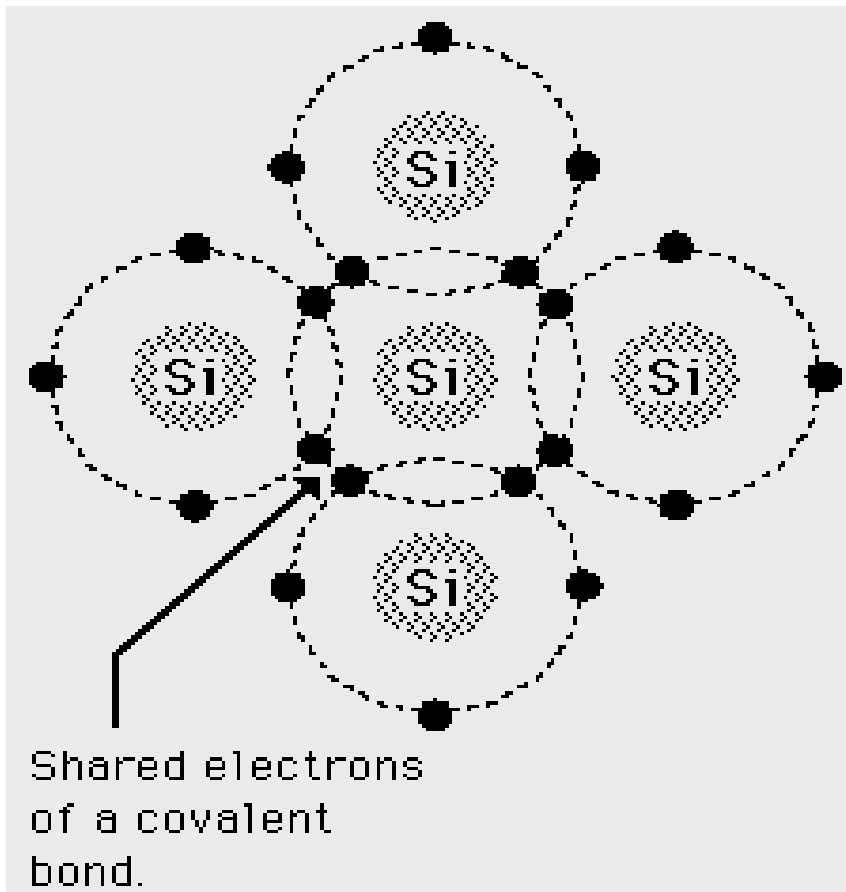
---

- Doping elements
- **Doping: N type**
- **Doping: P type**
- Counter doping

# Doping (N type)

---

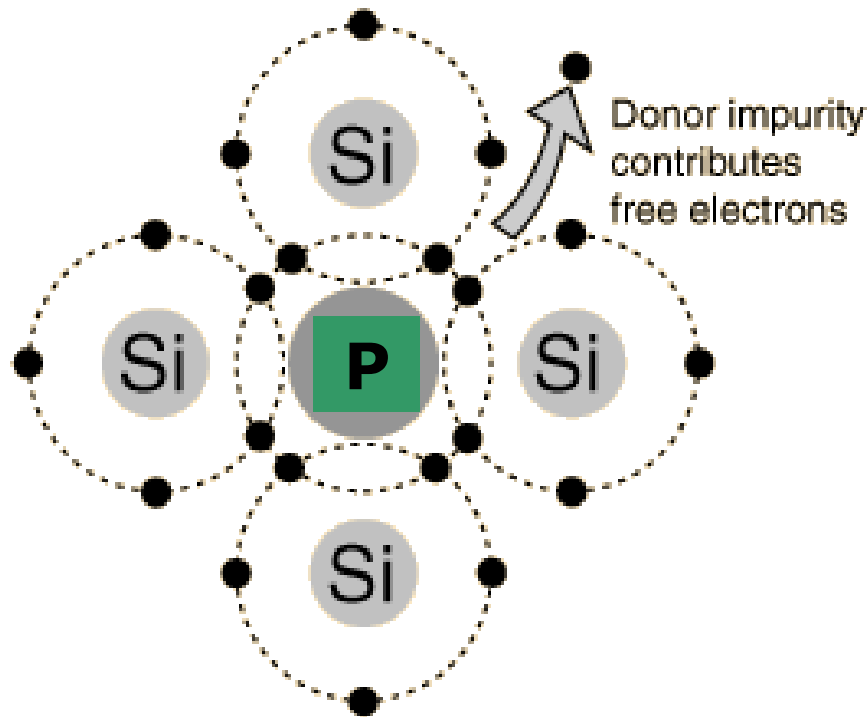
Column V elements are donors, e.g. P, As, Sb



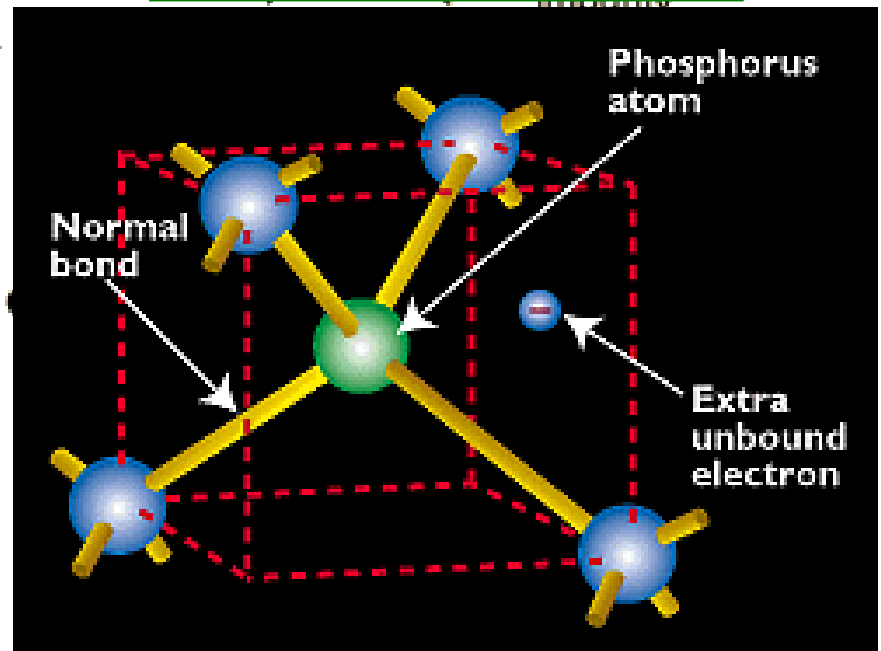
# Doping (N type)

Column V elements are donors, e.g. P, As, Sb

By substituting a Si atom with a special impurity atom (Column V element), a conduction electron is created.



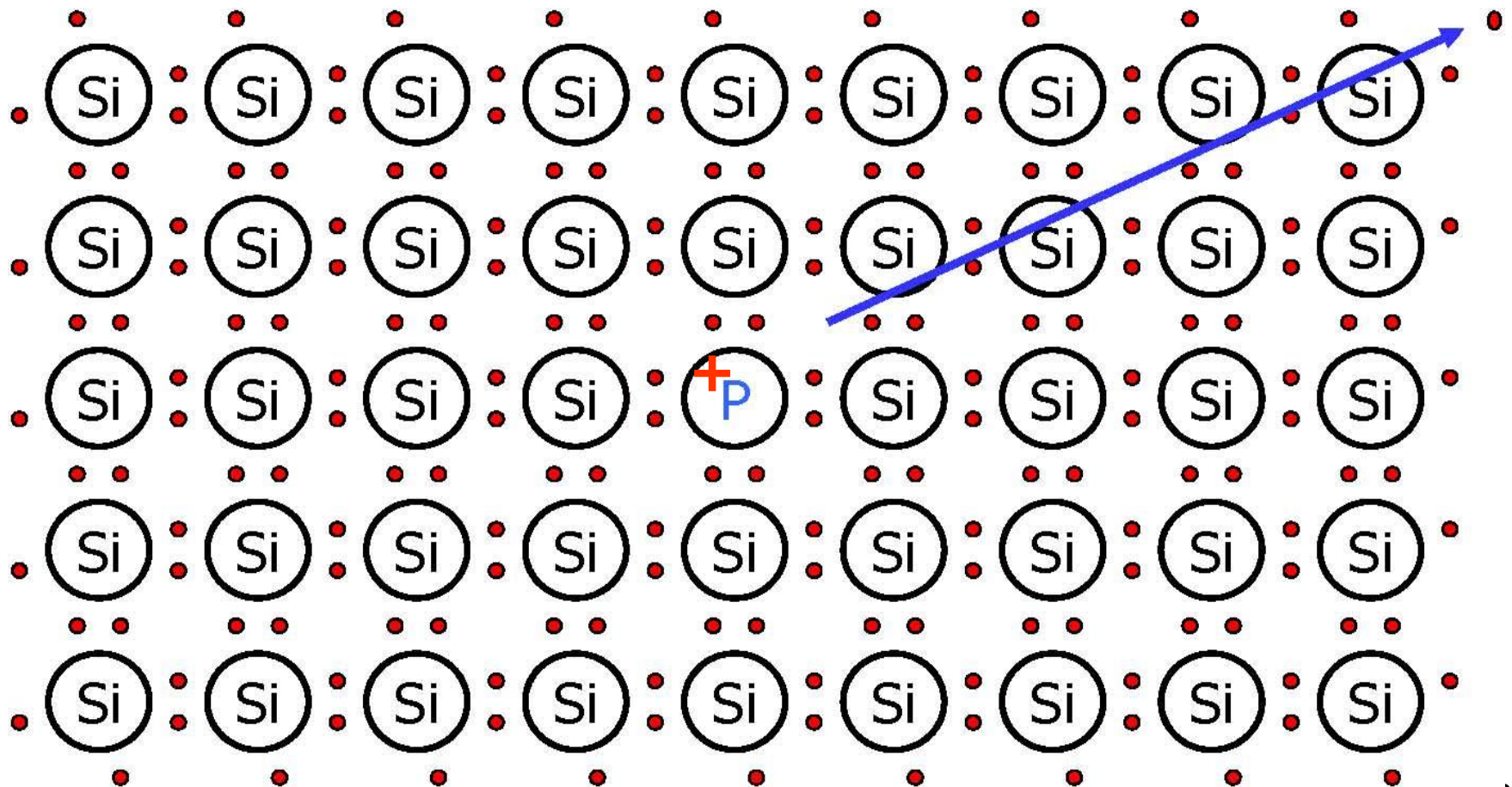
Donors: P, As, Sb



# Phosphorus has 5 valence electrons

- 'Donates' one conduction electron to lattice
- Our substrate has  $10^{15}\text{cm}^{-3}$  phosphorus (1 in  $10^8$ )

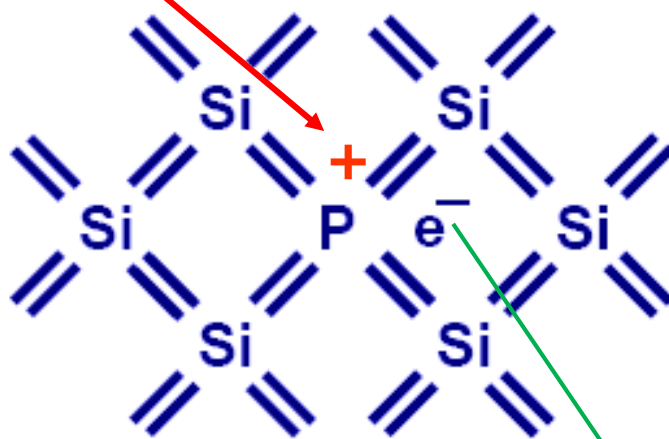
**Free**



# Doping (N type)

- If Si is doped with phosphorus (P), each P atom can contribute a conduction electron, so that the Si lattice has more electrons than holes, *i.e.* it becomes “**N type**”:

Immobile ion:  $N_D^+$   
Ionized donor



$n_{iD}$  ?

$$n = N_D^+ + n_{iD}$$

## Notation:

$N_D$  = Concentration of donors

$n$  = electron concentration

$N_D^+$  = Concentration of ionized donors

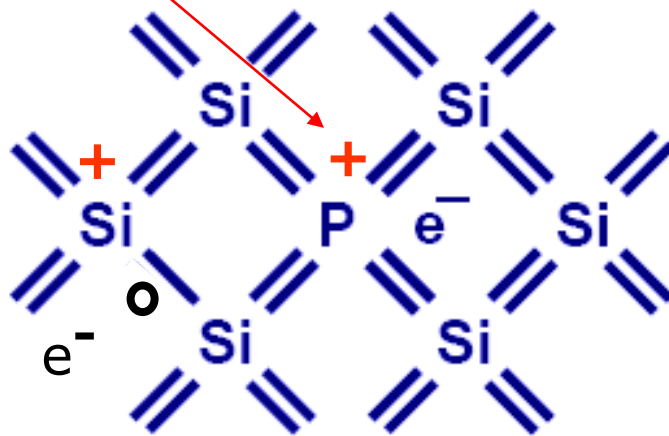
Ionization energy < 50meV:

At RT,  $N_D \approx N_D^+ \gg n_{iD}$

# Doping (N type)

- If Si is doped with phosphorus (P), each P atom can contribute a conduction electron, so that the Si lattice has more electrons than holes, *i.e.* it becomes “**N type**”:

Immobile ion:  $N_D^+$   
Ionized donor



$n_{iD} ?$

$$n = N_D^+ + n_{iD}$$

$$p = n_{iD} = ?$$

## Notation:

$N_D$  = Concentration of donors

$n$  = electron concentration

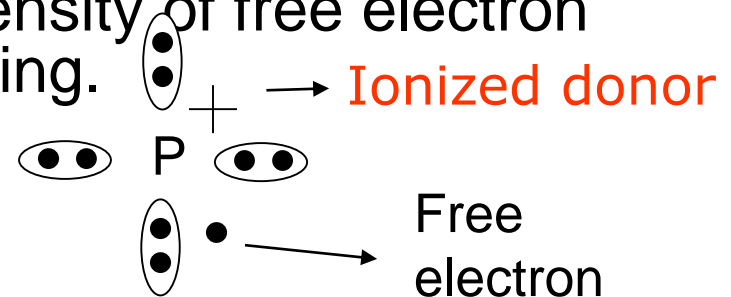
$N_D^+$  = Concentration of ionized donors

Ionization energy < 50meV:

At RT,  $N_D \approx N_D^+ \gg n_{iD}$

# N-type

- Doped by impurities of 5 valence electrons (donors)
- At room temperature, one donor will create one free electron.
- Holes are not created
- $n = n_{iD} + N_D$ 
  - $n_{iD} \ll n_i$  = density of free electrons in the intrinsic semiconductor
  - $N_D$  = density of donors
- Normally,  $N_D > 10^{14} \text{ cm}^{-3}$  and  $n_i \approx 10^{10} \text{ cm}^{-3}$
- Since  $N_D \gg n_{iD}$ ,  **$n \approx N_D$** . The density of free electron can be controlled through doping.



# Electron and Hole Concentrations

No E field, no B field, no light

- Under thermal equilibrium conditions, the product of the conduction-electron density and the hole density is ALWAYS equal to the square of  $n_i$ :

$$np = n_i^2 = (10^{10})^2/\text{cm}^3 \text{ at RT}$$

N-type material at RT

$$n \approx N_D$$

$$p \approx \frac{n_i^2}{N_D}$$

Example: at RT

$$N_D = 10^{15}/\text{cm}^3$$

$$n = 10^{15}/\text{cm}^3$$

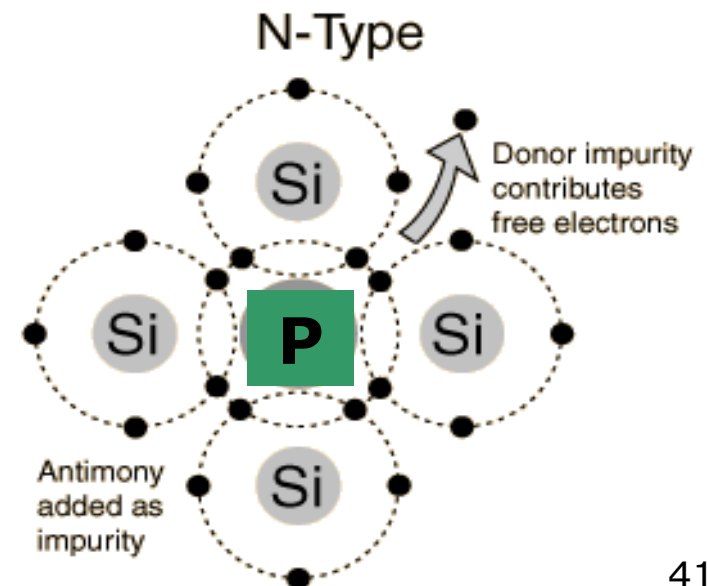
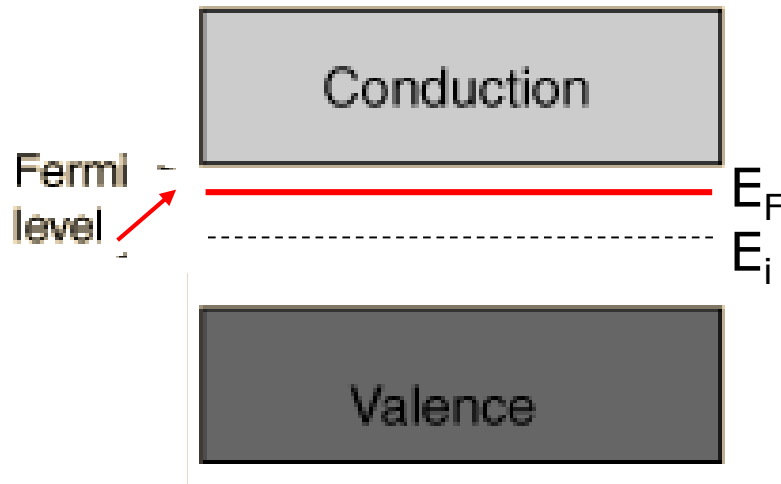
$$N_D^+ = 10^{15}/\text{cm}^3$$

$$p = 10^5/\text{cm}^3$$



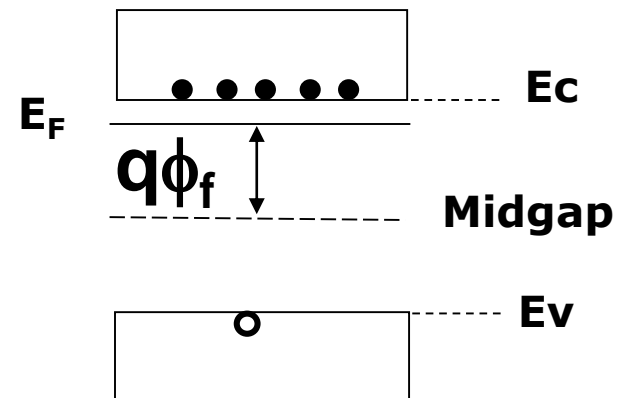
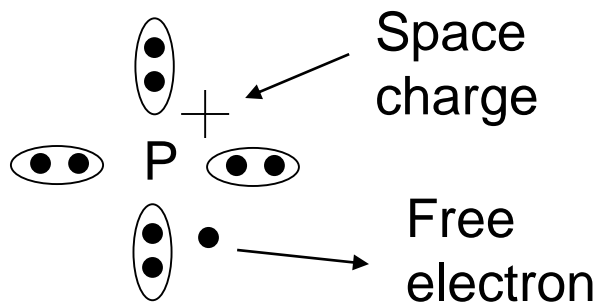
# N-Type Semiconductor

- The addition of pentavalent impurities such as Sb, As or P contributes free electrons, greatly increasing the conductivity of the intrinsic semiconductor.
- Phosphorus may be added by diffusion of phosphine gas ( $\text{PH}_3$ ).
- $E_F$  is shifted to the up-half of the bandgap for n-type.



# Properties of n-type

- $n \gg p$ , so “**n-type**”.
- Electrons are ‘**majority**’ charge carriers and holes are ‘**minority**’ charge carriers.
- Space charge: when an electron is freed, it left a positively charged atom behind, which is fixed in space
- Fermi potential:  $\phi_f$ 
  - How ‘strong’ the n-type is

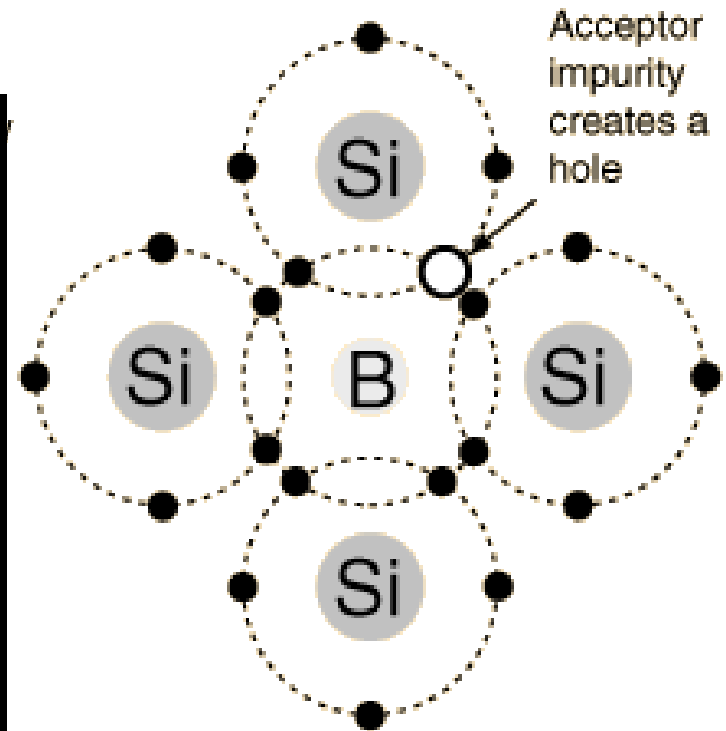
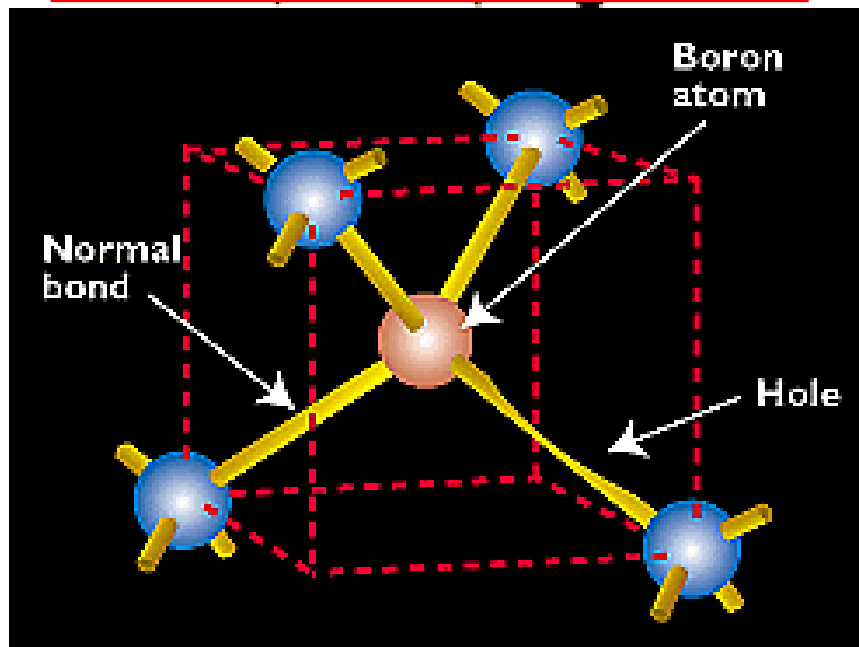


# Doping (P type)

Column III elements are acceptors, e.g. B, Al, Ga

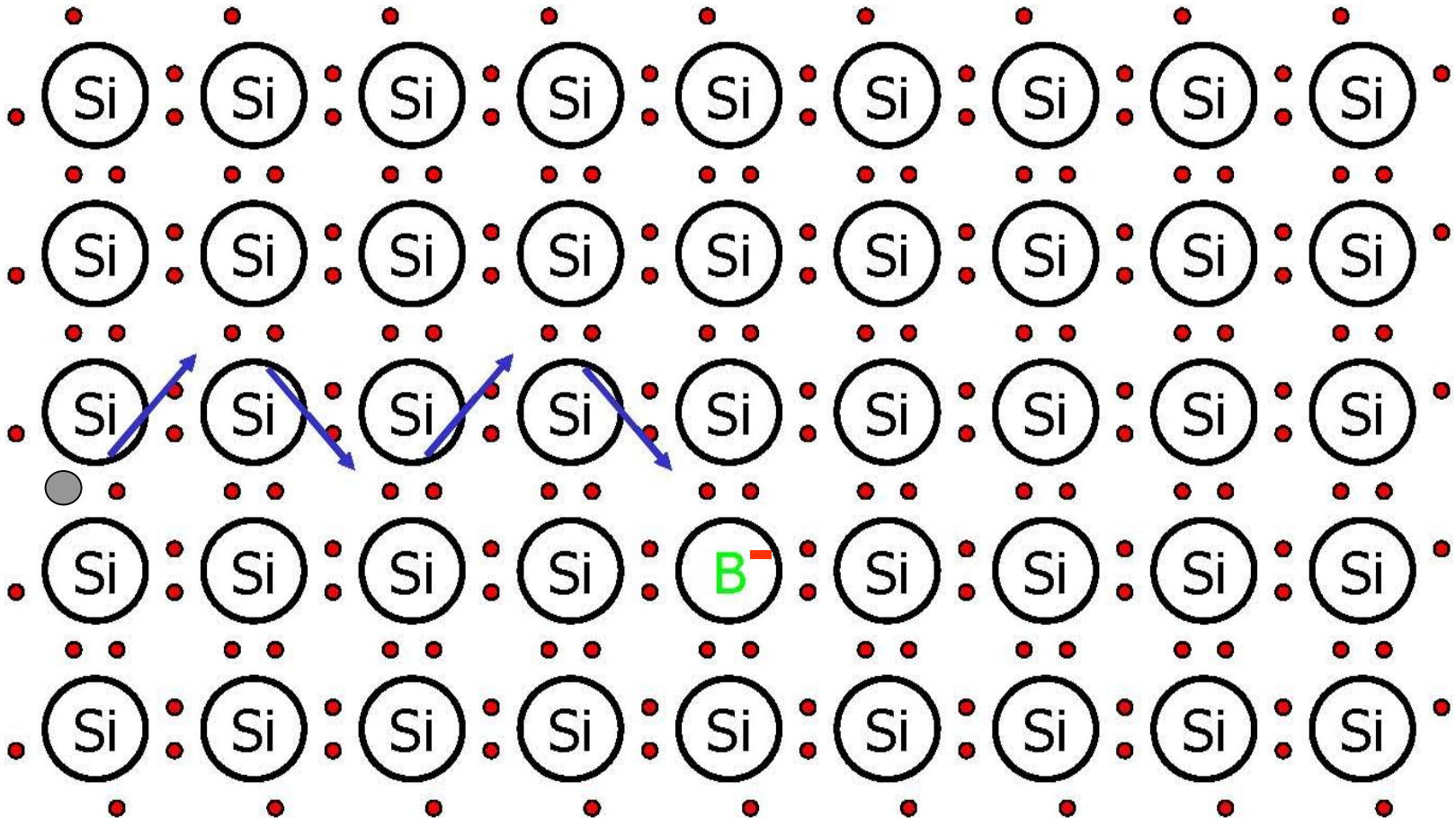
By substituting a Si atom with a special impurity atom (**Column III element**), a conduction hole is created.

**Acceptors: B, Al, Ga, In**



# Boron has 3 valence electrons

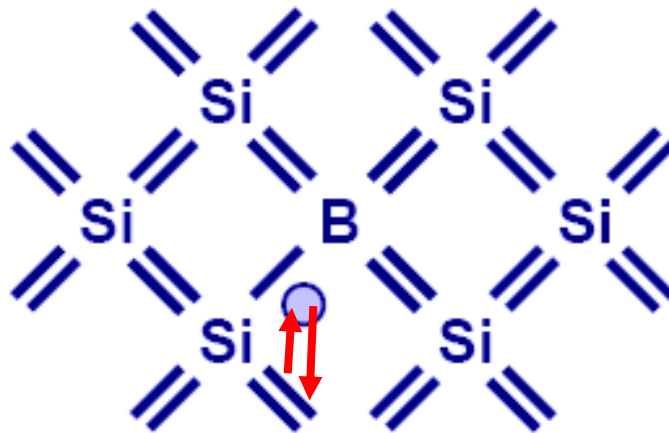
- 'Accepts' one electron from lattice
- Creates a 'hole'



# Doping (P type)

Column III elements  
are acceptors, e.g. B

- If Si is doped with Boron (B), each B atom can contribute a hole, so that the Si lattice has more holes than electrons, *i.e.* it becomes “P type”:



## Notation:

$N_A$  = concentration of acceptors

$p$  = hole concentration

$N_A^-$  = concentration of ionized acceptors

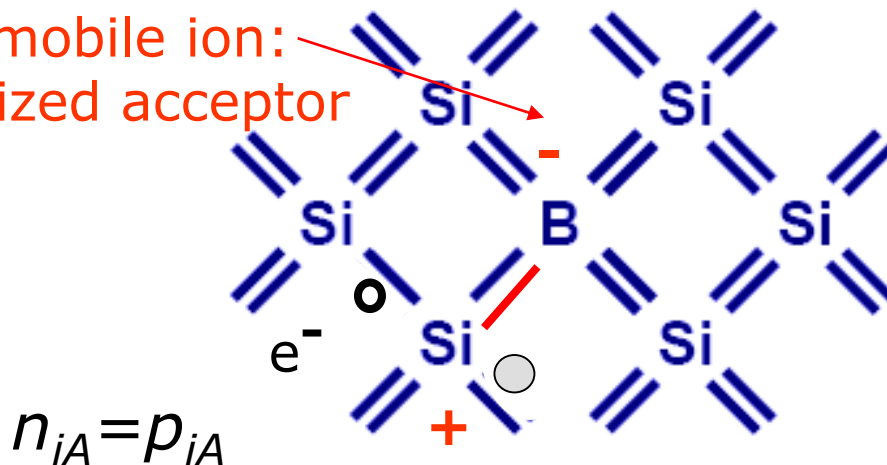
Hole is created when a neighboring valence electron moves to the B atom.

# Doping (P type)

Column III elements  
are acceptors, e.g. B

- If Si is doped with Boron (B), each B atom can contribute a hole, so that the Si lattice has more holes than electrons, *i.e.* it becomes “P type”:

Immobile ion:  
ionized acceptor



## Notation:

$N_A$  = concentration of acceptors

$p$  = hole concentration

$N_A^-$  = concentration of ionized acceptors

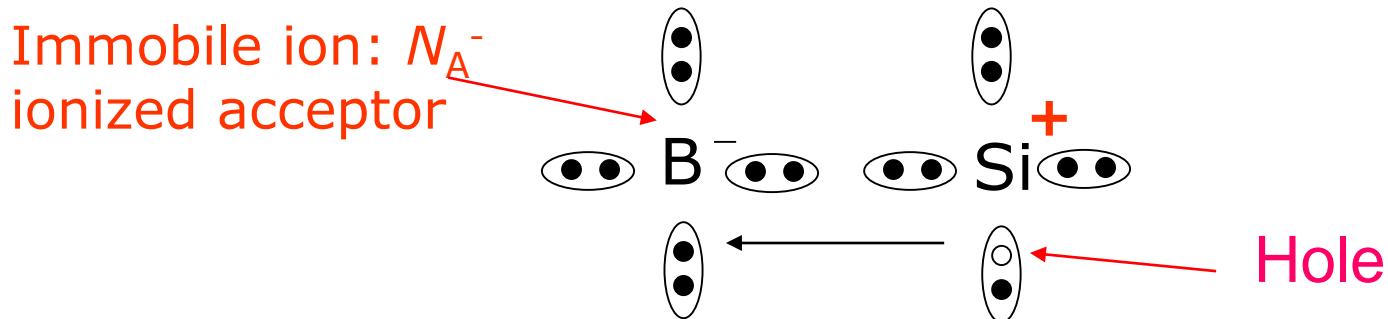
Ionization energy < 50meV:

At RT,  $N_A \approx N_A^- \gg p_{iA}$

$$p = N_A^- + p_{iA}$$

# P-type

- Doped by impurities of 3 valence electrons (acceptors)
- At room temperature, one acceptor will create one hole.
- Free electrons are not created
- $p = p_{iA} + N_A \approx N_A$ .
  - $p_{iA} \ll p_i$  = density of holes in the intrinsic semiconductor
  - $N_A$  = density of acceptors



# Electron and Hole Concentrations

- Under thermal equilibrium conditions, the product of the conduction-electron density and the hole density is ALWAYS equal to the square of  $n_i$ :

$$np = n_i^2 = (10^{10})^2/\text{cm}^3 \text{ at RT}$$

P-type material at RT

$$p \approx N_A$$

$$n \approx \frac{n_i^2}{N_A}$$

Example: at RT

$$N_A = 10^{15}/\text{cm}^3$$

$$p = 10^{15}/\text{cm}^3$$

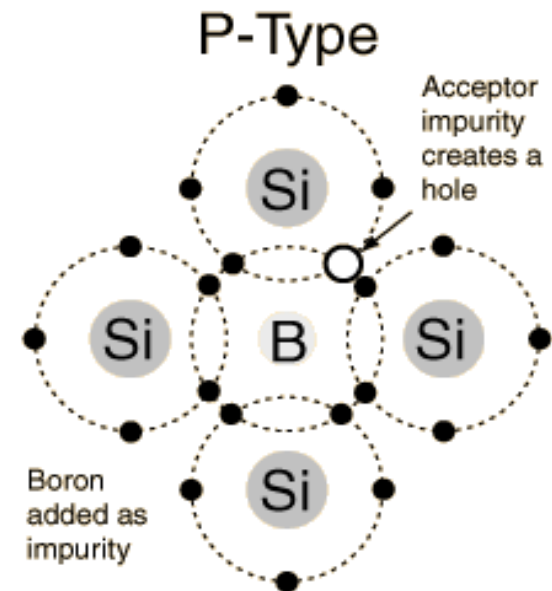
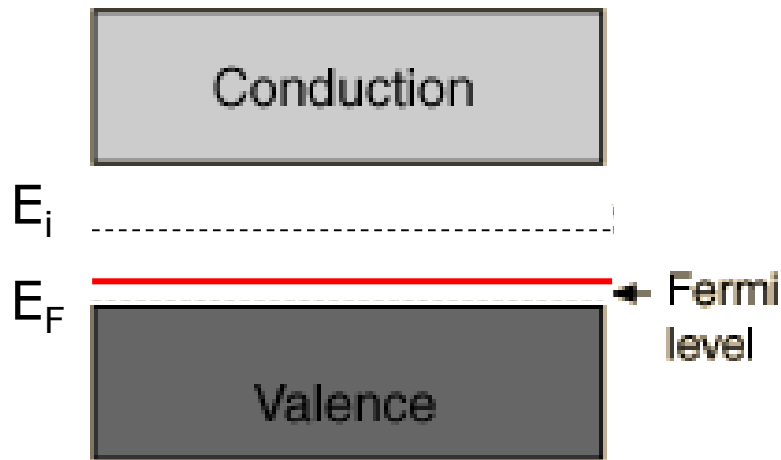
$$N_A^- = 10^{15}/\text{cm}^3$$

$$n = 10^5/\text{cm}^3$$



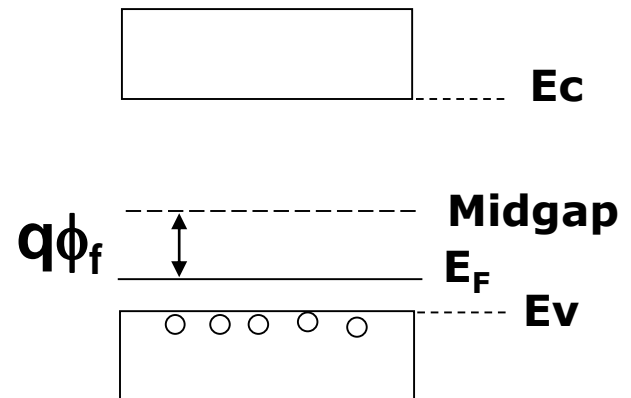
# P-Type Semiconductor

- The addition of trivalent impurities such as B, Al, or Ga to an intrinsic semiconductor creates deficiencies of valence electrons, called "holes".
- It is typical to use  $B_2H_6$  diborane gas to diffuse boron into the silicon material.
- $E_F$  is shifted to the down-half of the bandgap for p-type.



# Properties of p-type

- $p \gg n$ , so “p-type”.
- **Holes are ‘majority’** charge carriers and **electrons are ‘minority’** charge carriers.
- Space charge: negative charges bonded to Boron atoms
- Fermi potential:  $\phi_f$ 
  - How ‘strong’ the p-type is

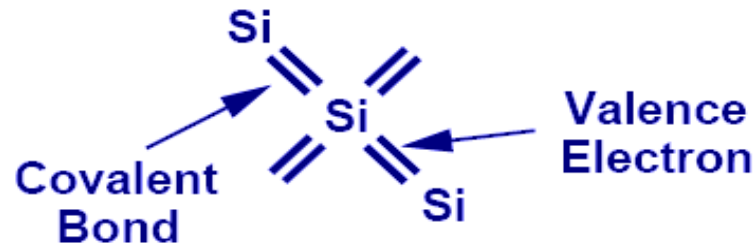


# Summary of doping

Column-V  
elements  
contribute  
conduction  
electrons,  
and are  
called  
**donors**.

Column-III  
elements  
contribute  
holes, and  
are called  
**acceptors**

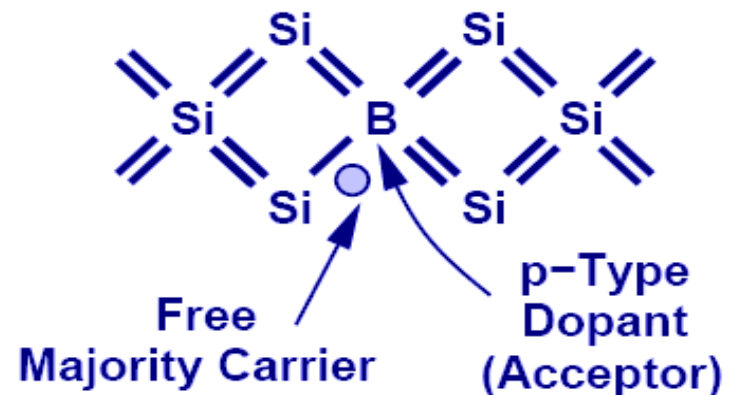
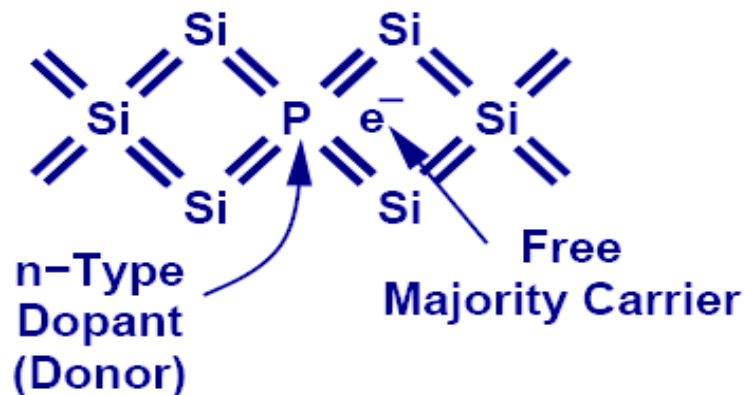
## Intrinsic Semiconductor



## Extrinsic Semiconductor

Silicon Crystal  
 $N_D$  Donors/cm<sup>3</sup>

Silicon Crystal  
 $N_A$  Acceptors/cm<sup>3</sup>



# *Types of charges in semiconductors*



Hole

$p$



Electron

$n$

***Mobile Charge Carriers***

**they contribute to current flow  
with electric field is applied.**



Ionized  
Donor

$N_D^+$



Ionized  
Acceptor

$N_A^-$

***Immobile Charges***

**they DO NOT  
contribute to current flow  
with electric field is applied.  
However, they affect the  
local electric field**

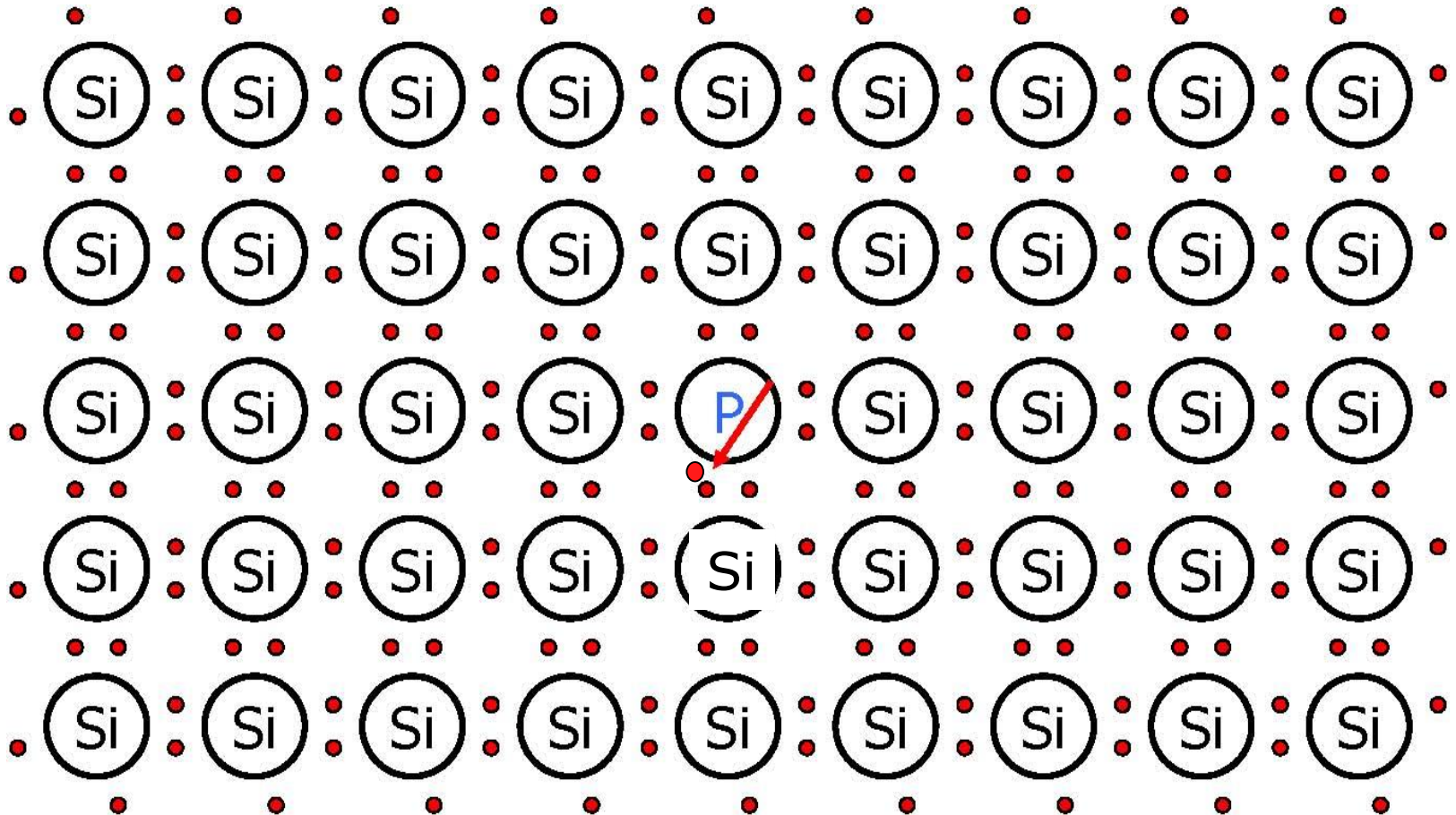
# 2.4 The doping of semiconductors

---

- Doping elements
- Doping: N type
- Doping: P type
- **Counter doping**  
**补偿掺杂**

# Counter Doping

This is a n-type Si,  $n = N_D + n_i$ . Normally  $N_D \gg n_i$ , so  $n = N_D$

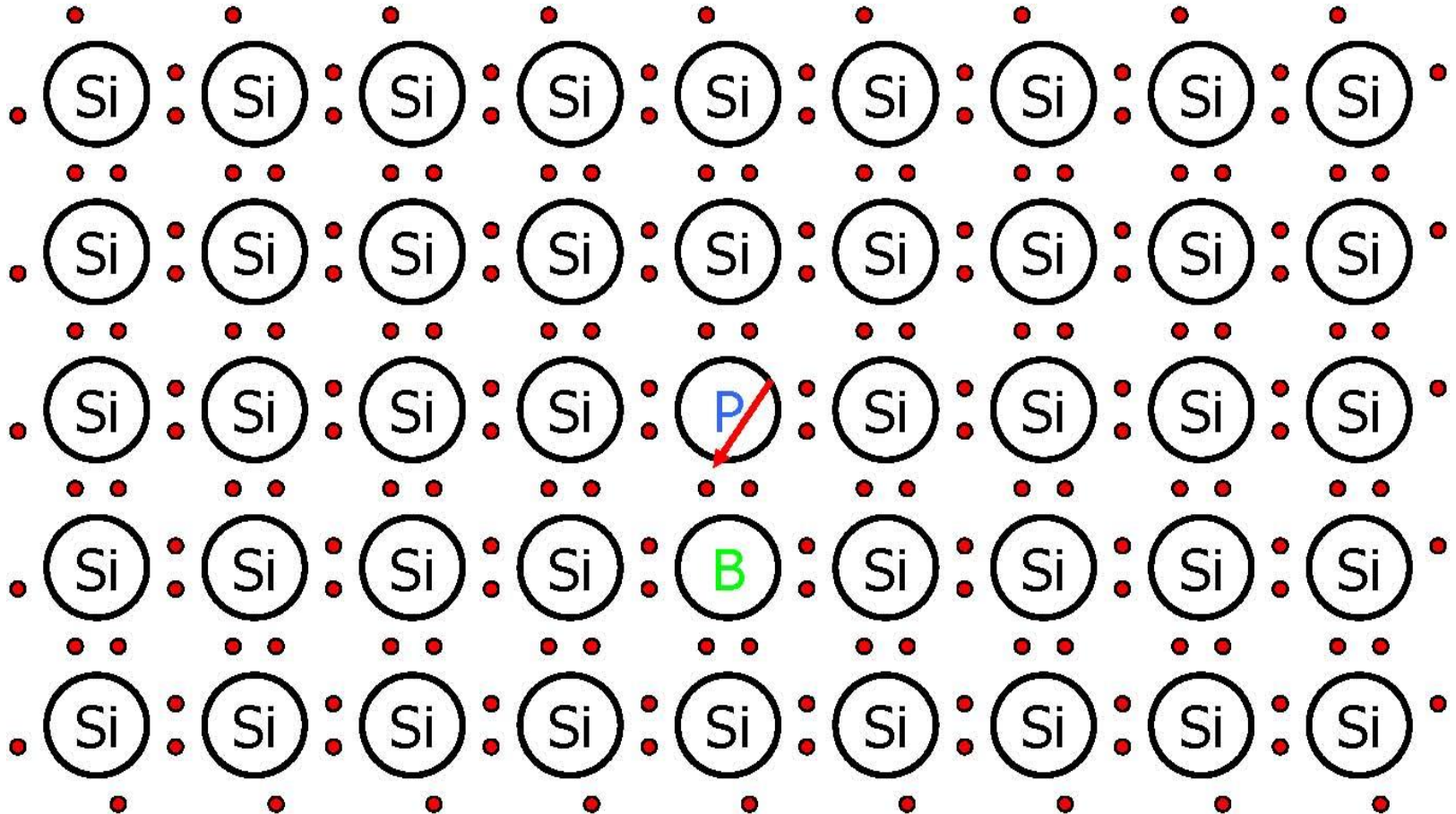




# Counter Doping

Adding the same **B** as **P** causes the doping type to change.

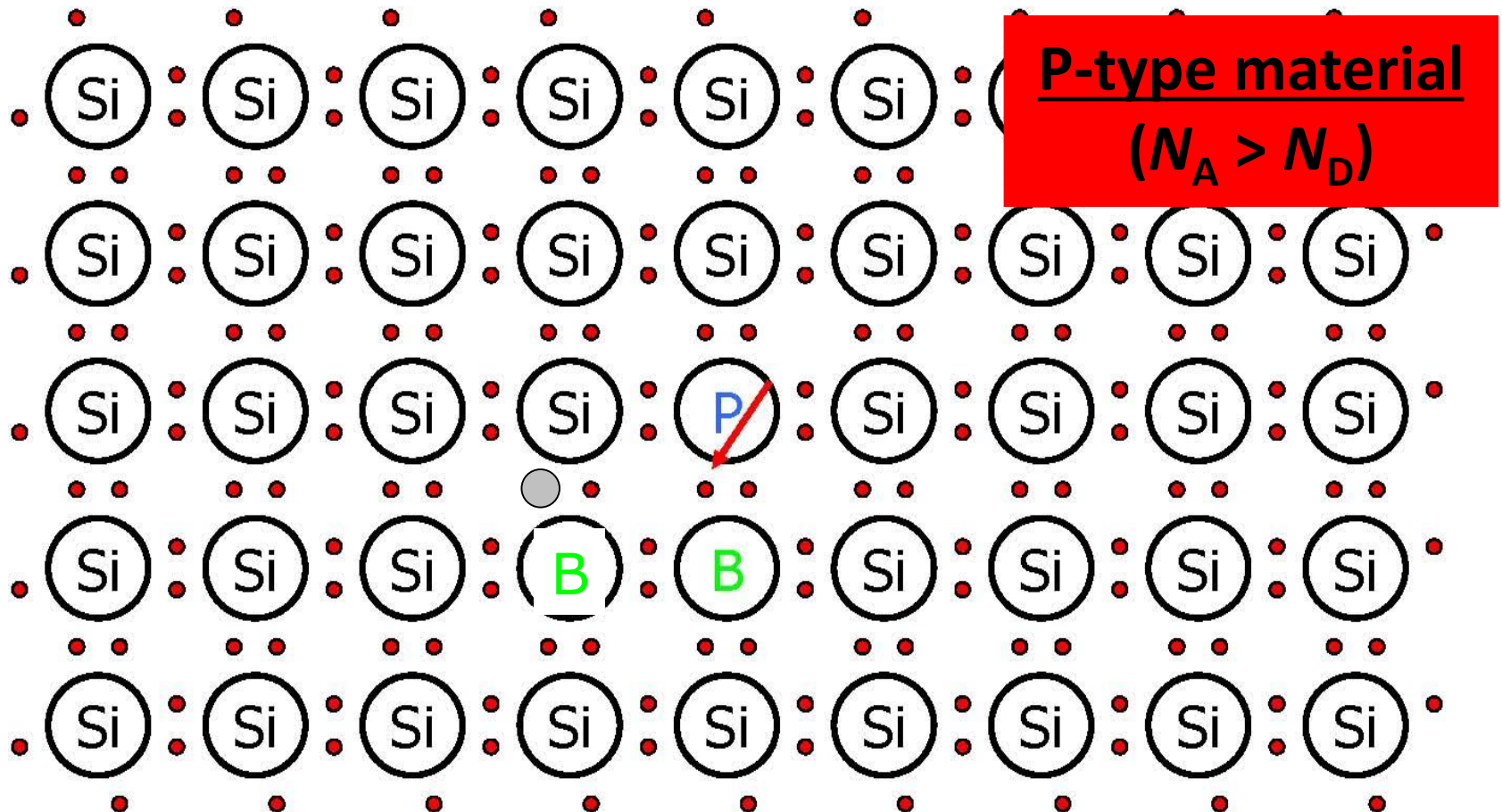
$$n = p = N_D - N_A + n_i = n_i.$$



# Counter Doping

$$p \approx N_A - N_D, \quad n \approx \frac{n_i^2}{N_A - N_D}$$

The addition of one more B than P causes the doping type to change from n-type to p-type

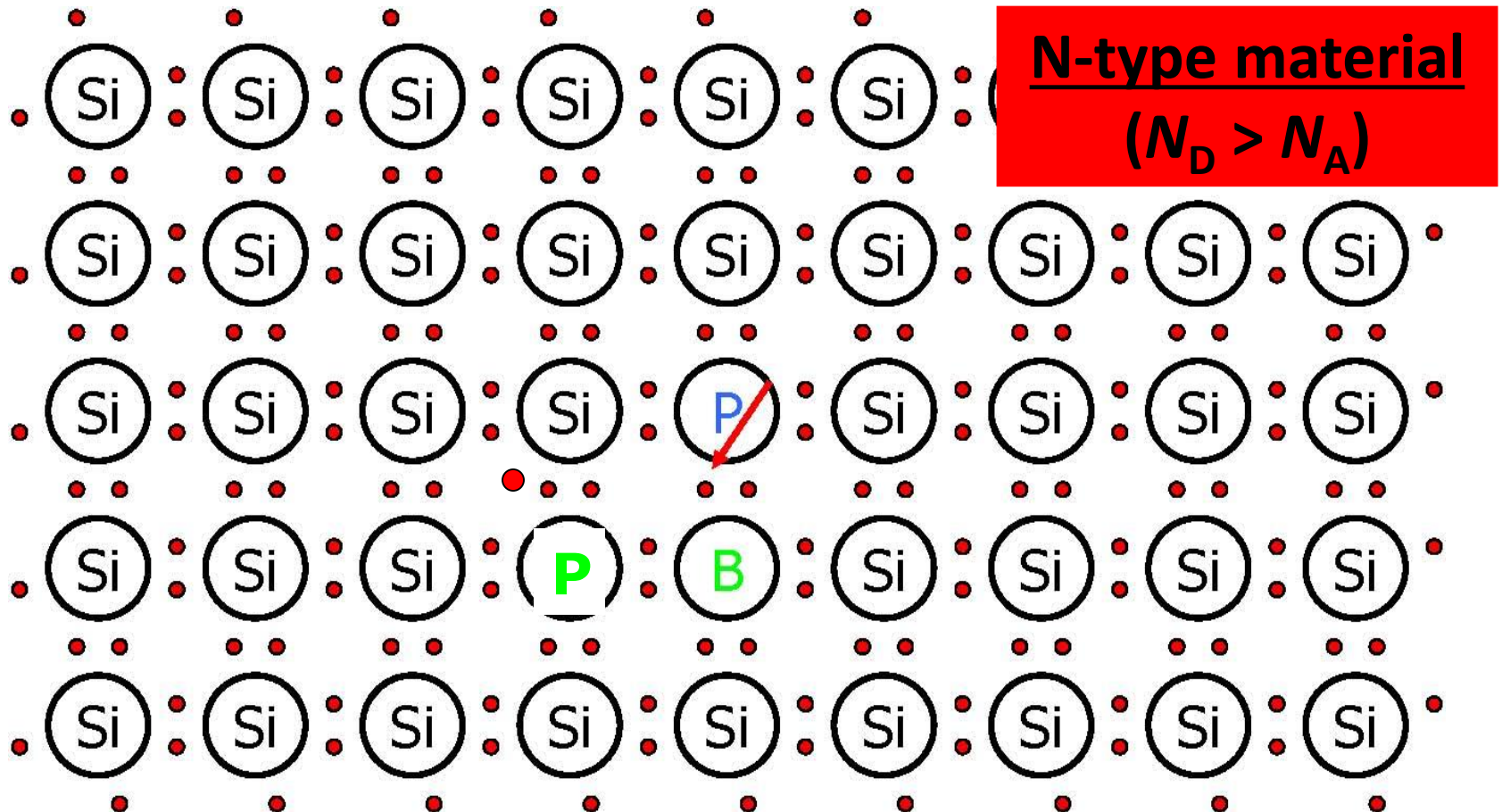




# Counter Doping

$$n \approx N_D - N_A, \quad p \approx \frac{n_i^2}{N_D - N_A}$$

The addition of one more P than B causes the doping type to change from p-type to n-type



# Dopant Compensation

- An N-type semiconductor can be converted into P-type material by counter-doping it with acceptors such that  $N_A > N_D$ .
- A ***compensated semiconductor material*** has both acceptors and donors.

N-type material

$$(N_D > N_A)$$

$$n \approx N_D - N_A$$

$$p \approx \frac{n_i^2}{N_D - N_A}$$

“net  
doping”

P-type material

$$(N_A > N_D)$$

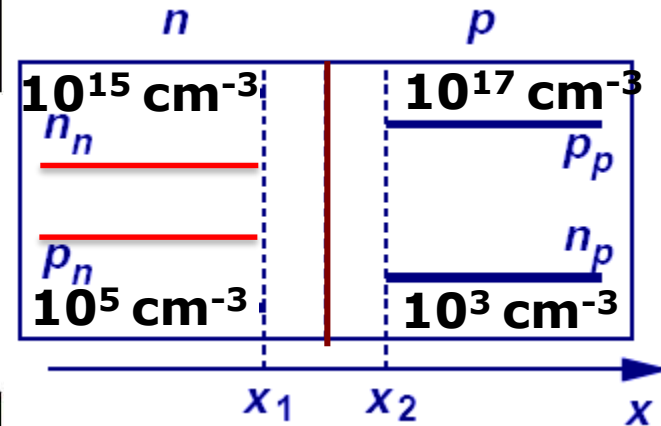
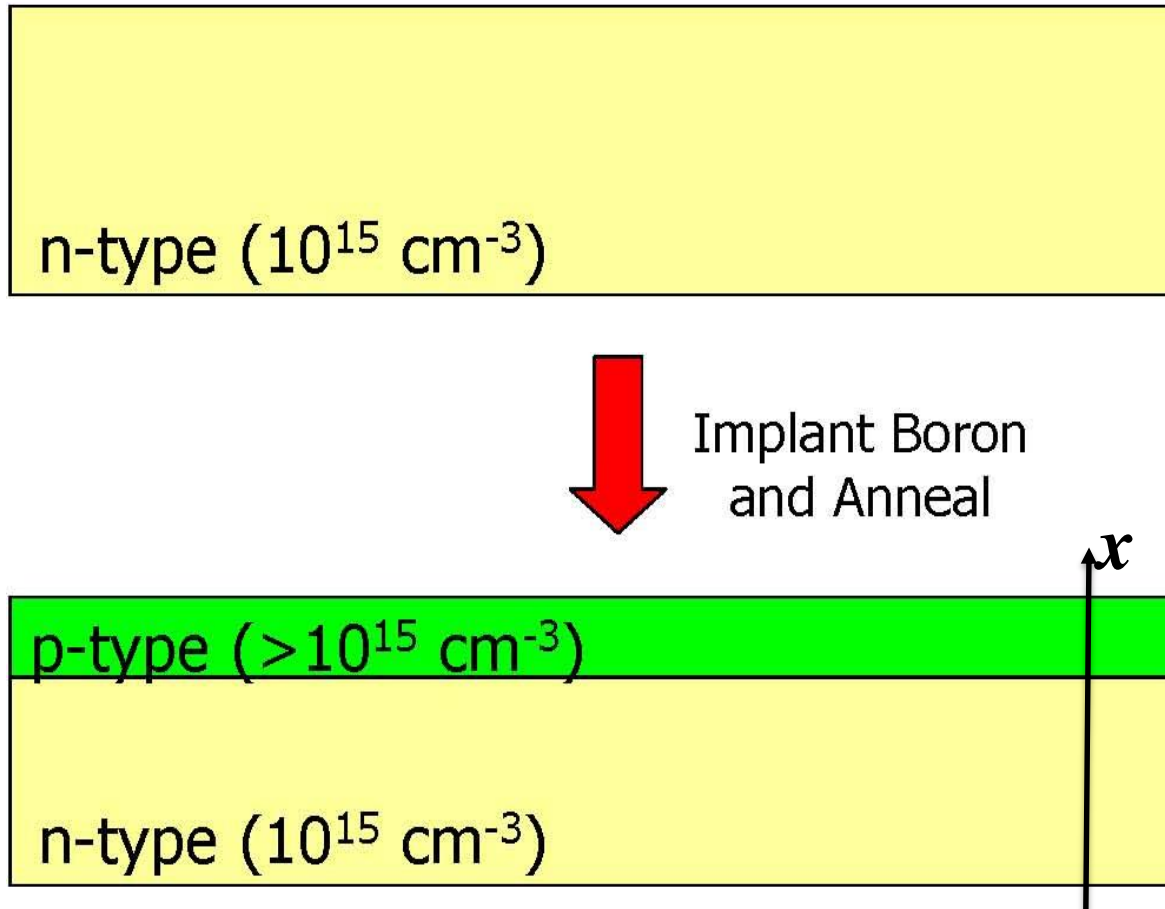
$$p \approx N_A - N_D$$

$$n \approx \frac{n_i^2}{N_A - N_D}$$

What is the relationship between  $E_F$  and  $n/p$ ?

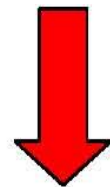
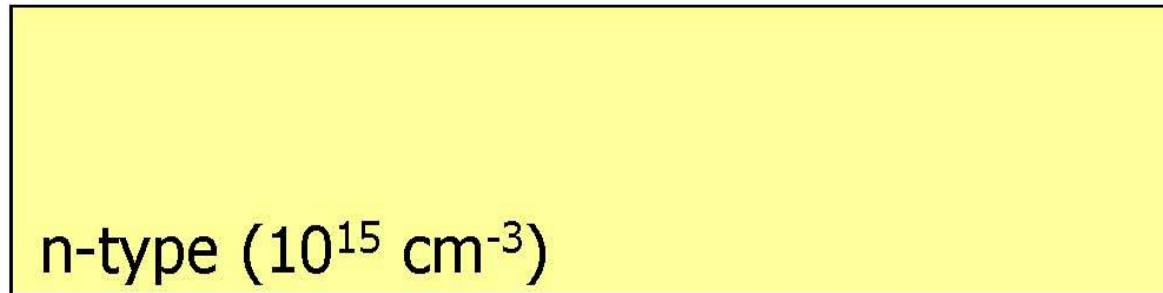
# Counter Doping Process

- To form pn junction

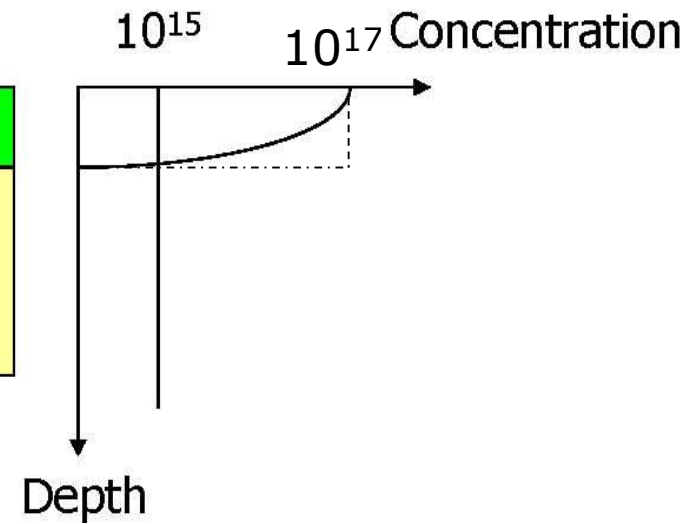
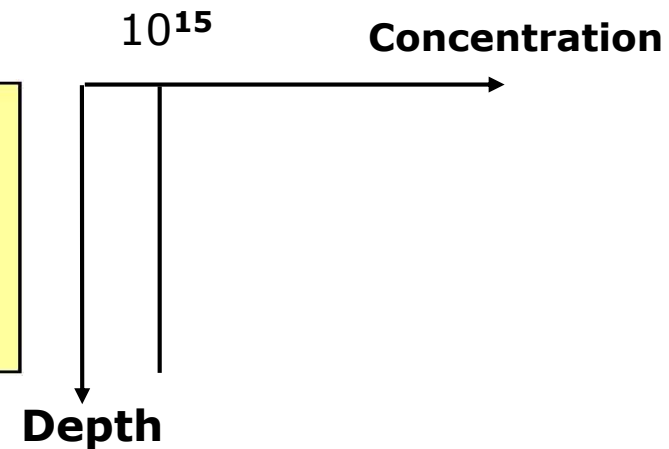
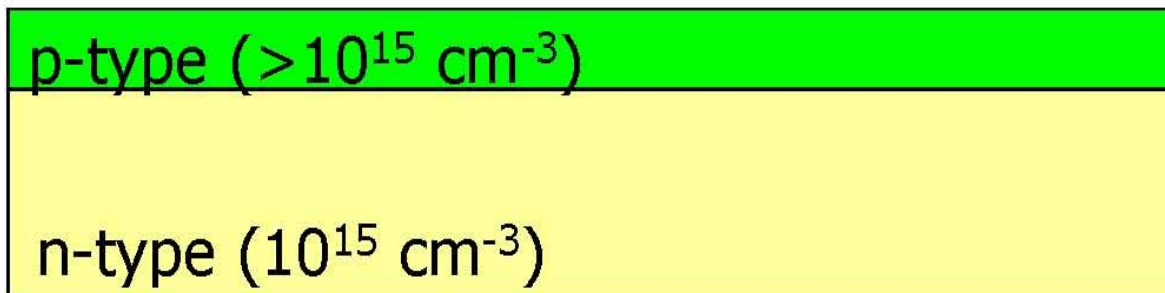


# Counter Doping Process

- To form pn junction



Implant Boron  
and Anneal



**Next week:**

# Semiconductor Fundamentals – (III)

---

2.5 Boltzmann approximation &  $E_F$ ,  $n$ ,  $p$

2.6 Carrier drift and diffusion