EEE336 Signal Processing and Digital Filtering

Lecture 5 Discrete-Time Signals in Time Domain

5_1 Introduction

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Room EE322

Content: Discrete-time Signals and Systems

Discrete-time Signals in Time Domain	Discrete-time Signals in Frequency Domain
Lecture 5	Lecture 7
Discrete-time Systems in Time Domain	Discrete-time Systems in Frequency Domain
Lecture 6	Lecture 8

Content: Lecture 5

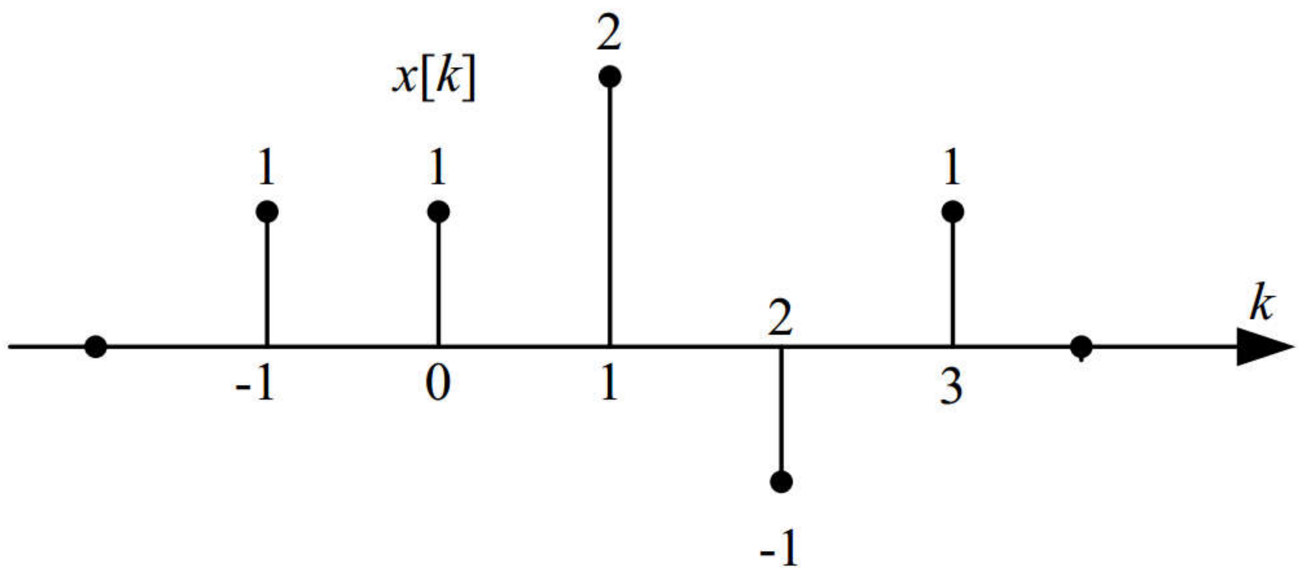
Signals (Sequences)	Operations	Properties
• Unit Impulse	• Elementary (addition, multiplication, production)	• Symmetry
• Unit Step	• Time Shifting	• Periodicity
• Rectangular	• Time Reversal (folding)	• Energy
• Real Exponential	• Branching	• Power
• Complex Exponential	• Decimation	• Bound
• Sinusoidal	• Interpolation	• Summable (absolutely, square)
	• Convolution	
	• Correlation	

Discrete-time Signal Generation

- Method 1: Periodically sampling a continuous time signal x(t) with uniform sampling rate F_s
  - Eg: sensor signals, audio signals
- Method 2: Naturally discrete in time
  - Eg: population data, financial data
- Method 3: Sequences generated by digital devices
  - Eg: synthesize music, file/data stored in computer

Discrete-time Signal Representation

- Graphical:



- Sequence: $x[k]=\{1,\overset{\downarrow}{1},2,-1,1\}$

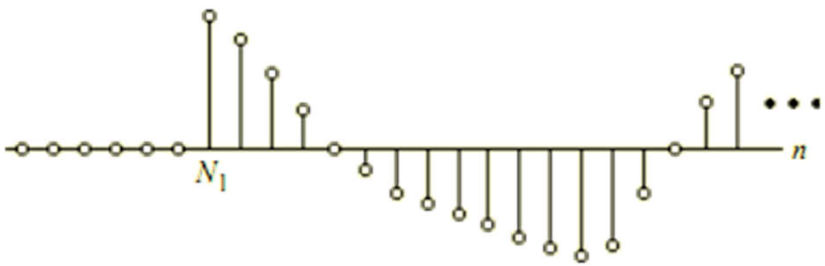
Vector (array) $x[k]=\{1,1,2,-1,1;k=-1,0,2,3\}$

- Functional: $x[k]=2^{k}u[k]$

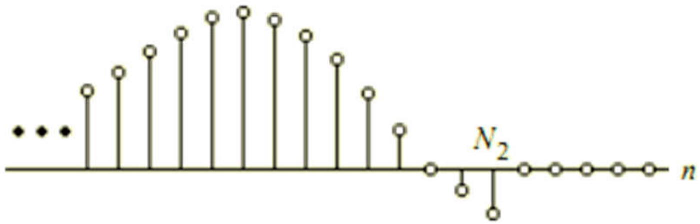
Length of sequences

- Finite-duration or finite-length sequences:
  - Defined in the interval $N_1 \le n \le N_2$
  - Length (duration): **L=N2-N1+1**
  - A length-N sequence is often referred to as an N-point sequence

- Infinite-duration or infinite-length sequences:
  - Right-side sequence: x[n]=0, n>=N_1
  - Left-side sequence: x[n]=0, n<=N_2



A right-sided sequence



A left-sided sequence

  - Double-sides sequence

5_1 Wrap up

- Content of the following 4 lectures
  - Discrete Signal in Time D
  - Discrete System in Time D
  - Discrete Signal in Frequency D (Transform D)
  - Discrete System in Frequency D (Transform D)
- Content of lecture 5:
  - Basic sequences (signals)
  - Fundamental operations
  - Properties of signals (Classification)
- About the discrete-time signals
  - Representation
  - Length

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5_2 Basic DT Sequences

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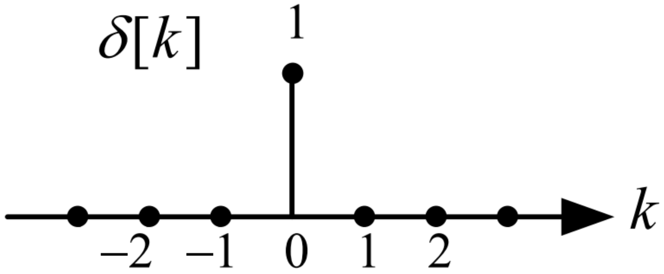
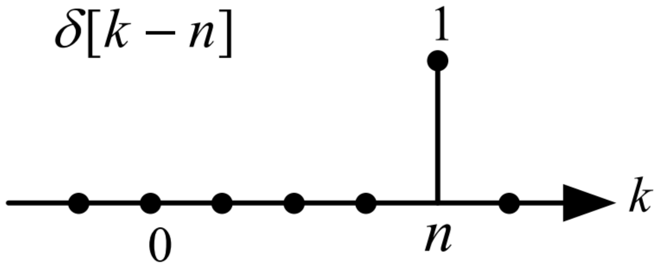
Basic Sequence

- Unit Impulse

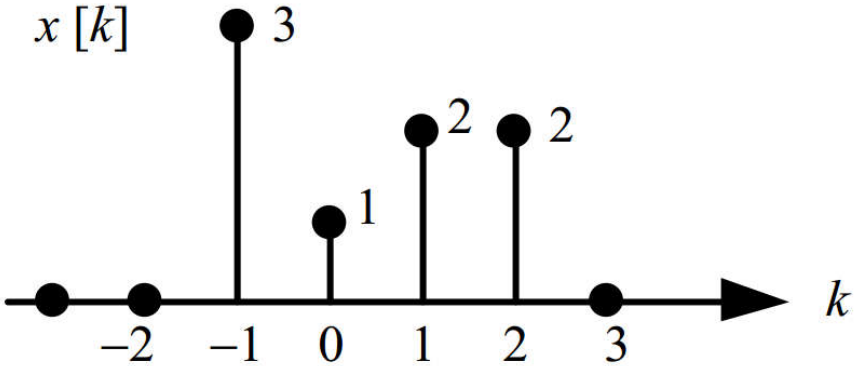
$$\delta[k] = \begin{cases} 1 & k = 0 \\ 0 & k \neq 0 \end{cases}$$



$$\delta[k-n] = \begin{cases} 1 & k = n \\ 0 & k \neq n \end{cases}$$



- Application:

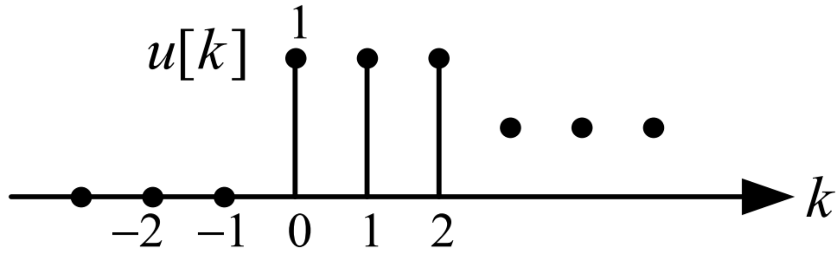


$$x[k] = 3\delta[k+1] + \delta[k] + 2\delta[k-1] + 2\delta[k-2]$$

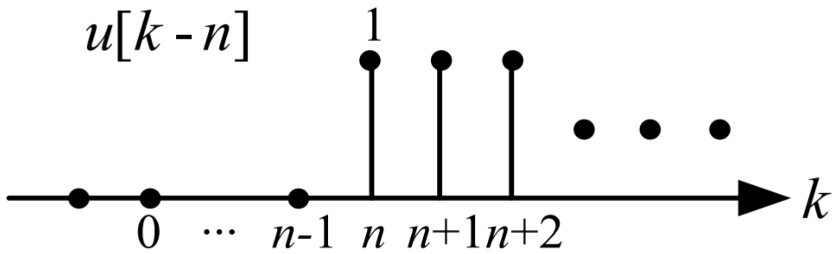
Basic Sequence

- Unit Step

$$u[k]=\begin{cases}1 & k\geq 0\\ 0 & k<0\end{cases}$$

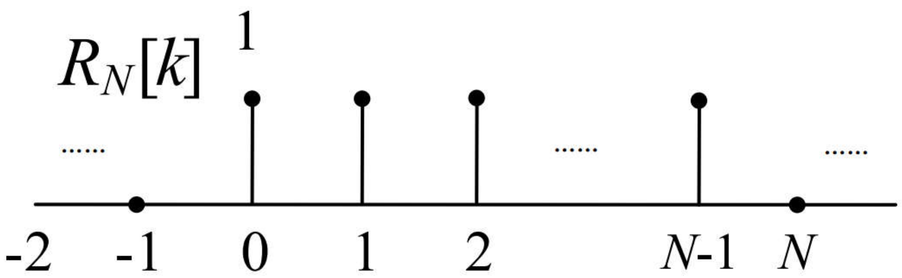


$$u[k-n]=\begin{cases}1 & k\geq n\\ 0 & k<n\end{cases}$$



- Application: rectangular sequence

$$R_N[k]=\begin{cases}1 & 0\leq k\leq N-1\\ 0 & 其他\end{cases}$$

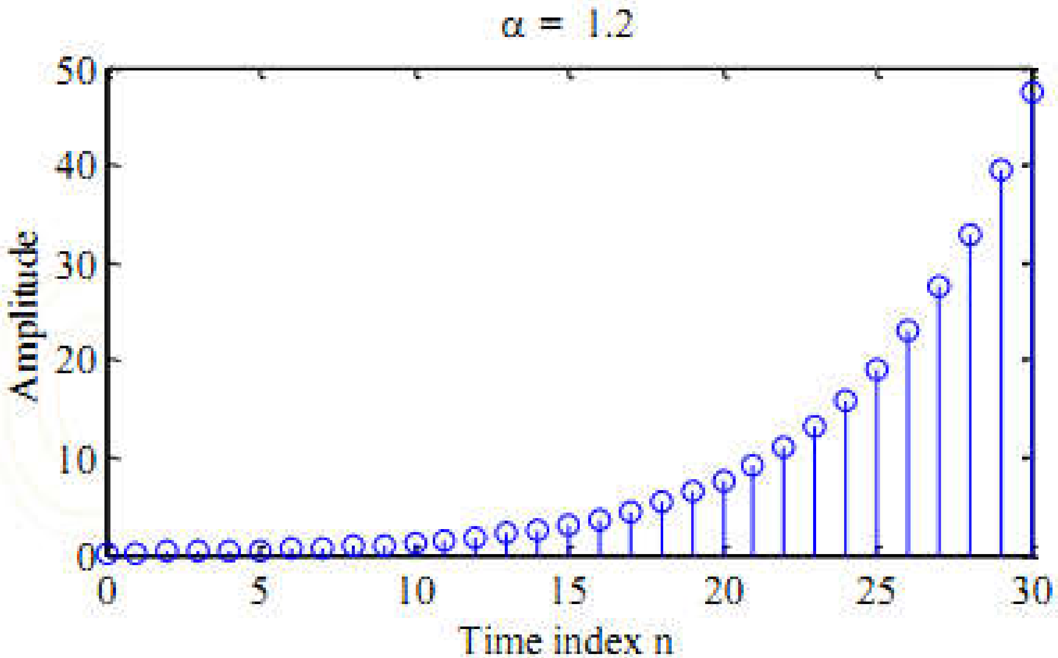


Basic Sequence

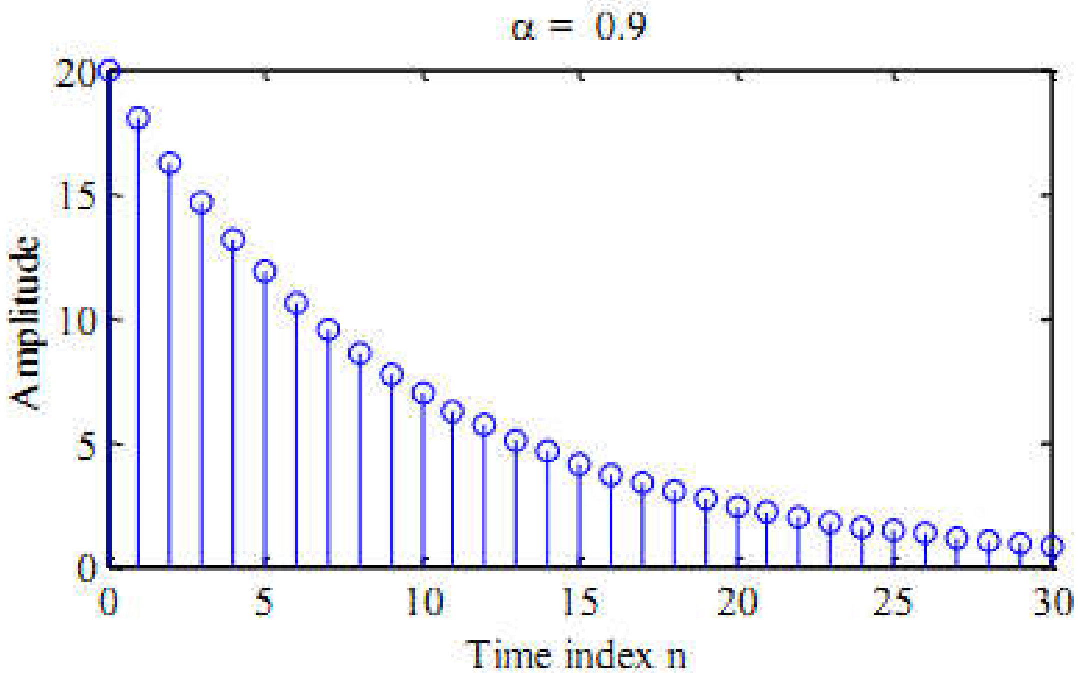
- Real Exponential:

$$x[n] = A\alpha^n, -\infty < n < \infty$$

  – If both A and α are real:



Unbounded



Bounded

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Basic Sequence

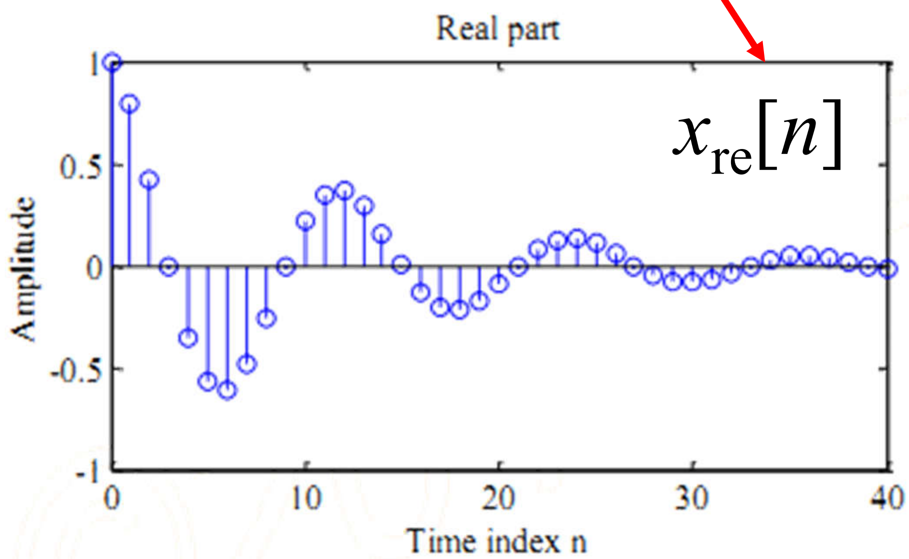
- Complex Exponential:

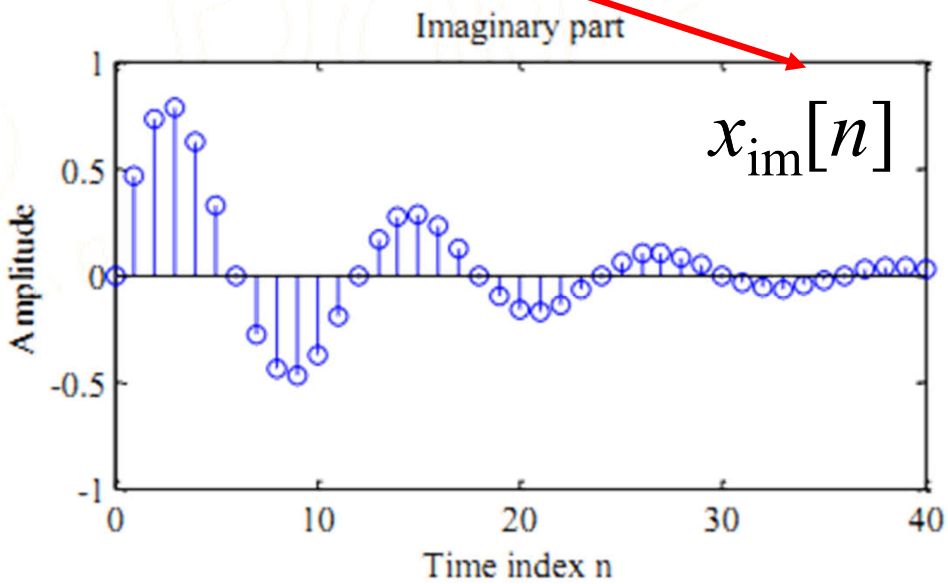
$$x[n] = A\alpha^n, -\infty < n < \infty$$

  - If both $A = |A|e^{j\varphi}$ and $\alpha = e^{\sigma + j\omega}$ are complex, then

$$x[n] = A\alpha^n = |A|e^{j\varphi}e^{(\sigma+j\omega)n} = |A|e^{\sigma}e^{j(\omega n+\varphi)}$$

- Sinusoidal $= \underline{|A|e^{\sigma}\cos(\omega n + \varphi)} + \underline{j|A|e^{\sigma}\sin(\omega n + \varphi)}$





5_2 Wrap up

- Important and useful basic sequences are introduced
  - Unit impulse -> sequence representation
  - Unit step -> functional representation
  - Exponential sequences:
    - Real
    - Complex
    - Euler’s formula -> sinusoidal

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5_3 Fundamental Operations

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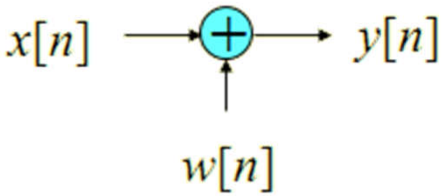
Operations

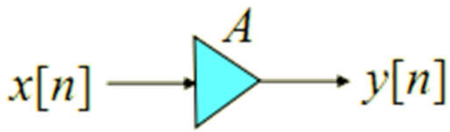
Elementary operations

- Addition
  - Adder



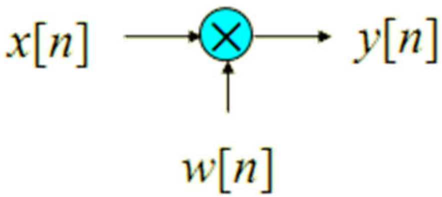
$$y[n] = x[n] + w[n]$$

- Multiplication
  - Multiplier



$$y[n] = A \cdot x[n]$$

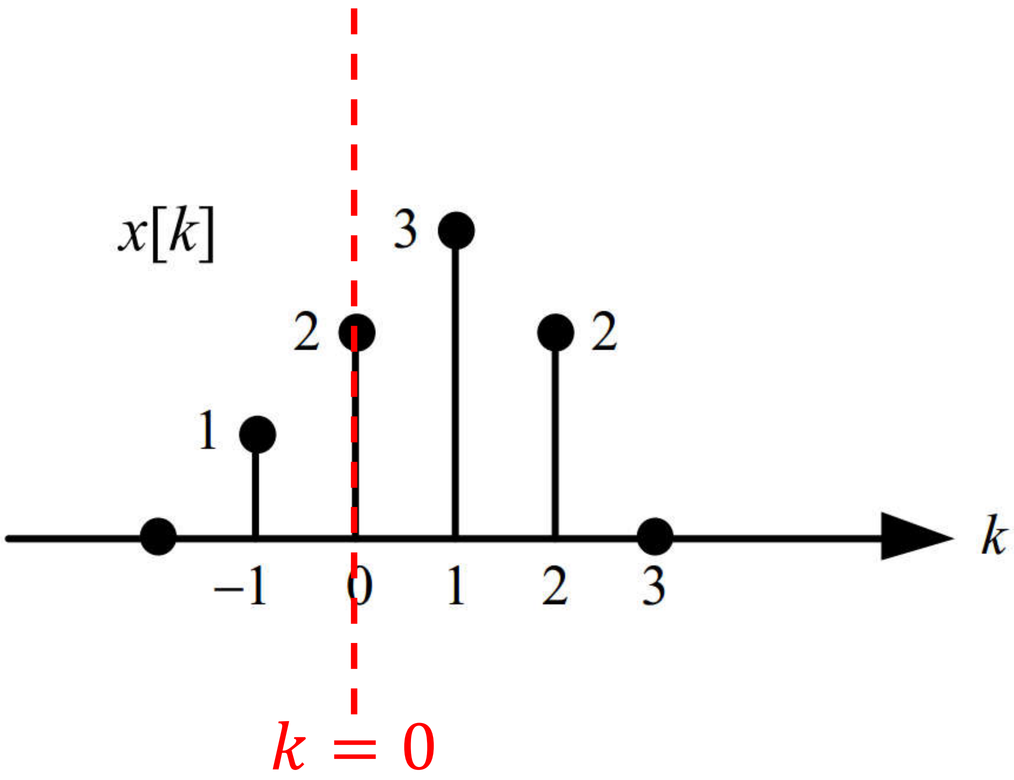
- Production
  - Productor

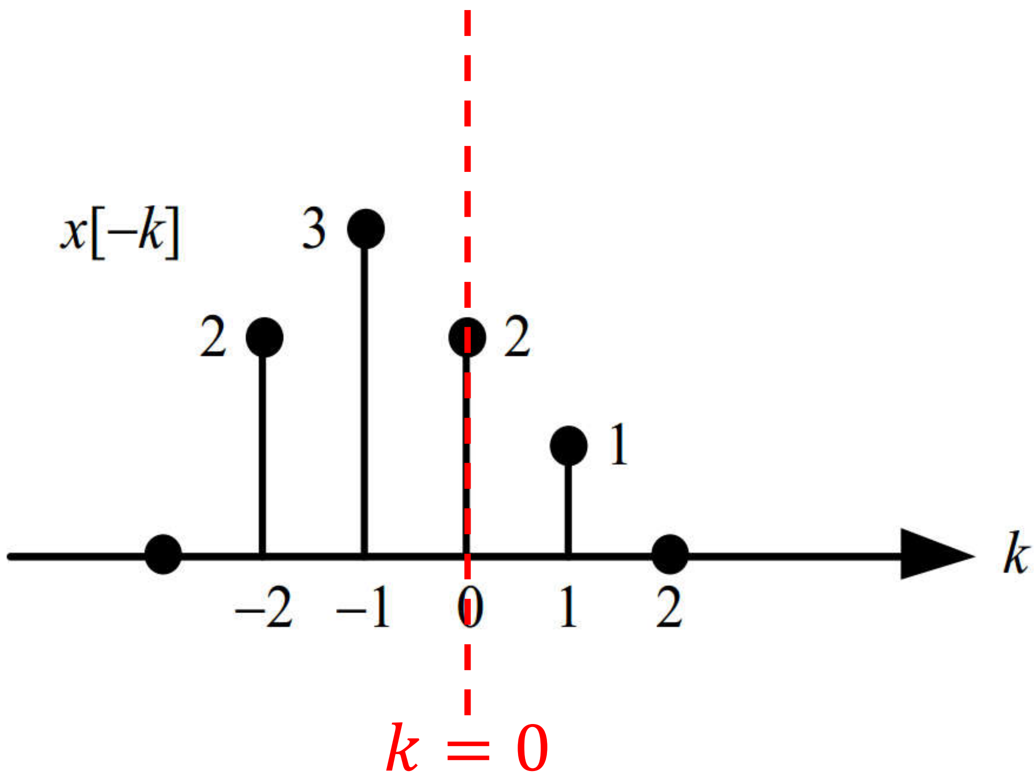


$$y[n] = x[n] \cdot w[n]$$

- Time-reversal (folding)

$$x[k] \rightarrow x[-k]$$

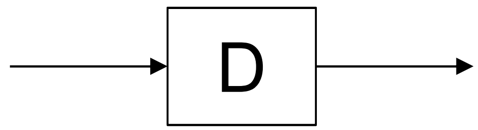


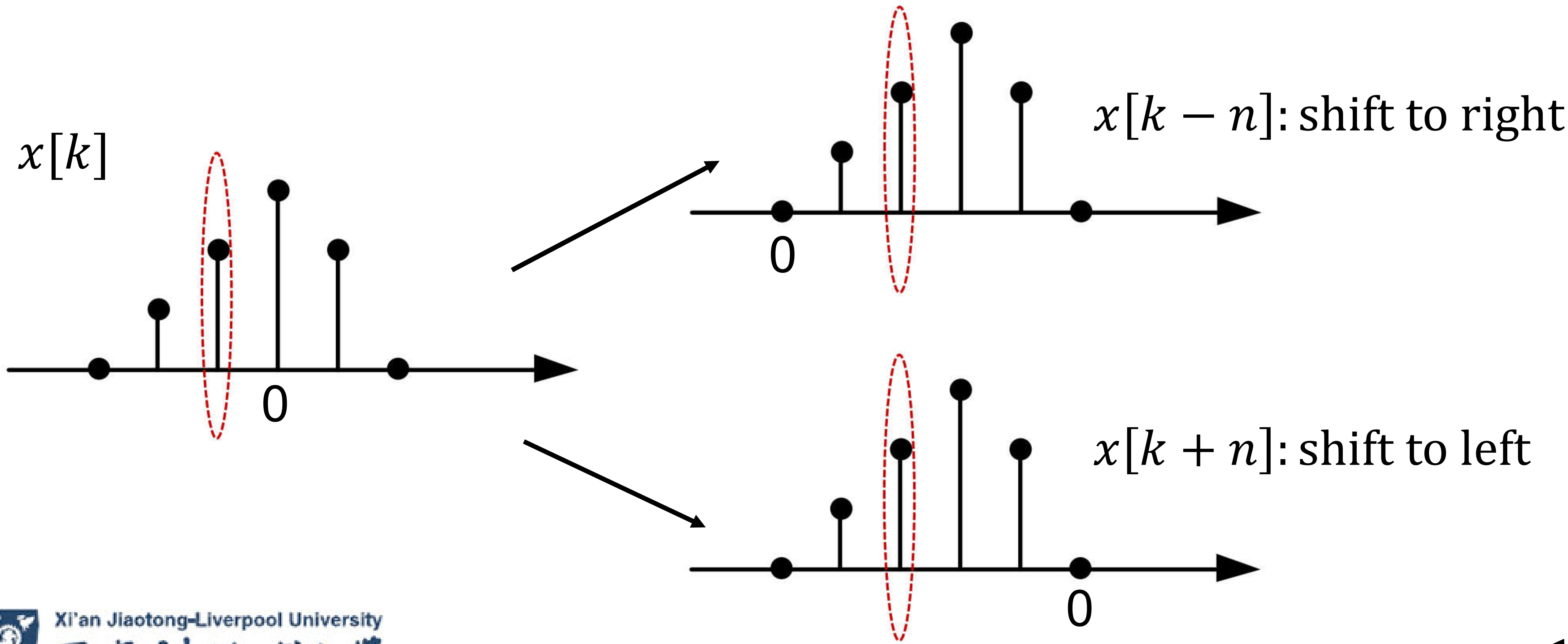


Operations

- Time-shifting

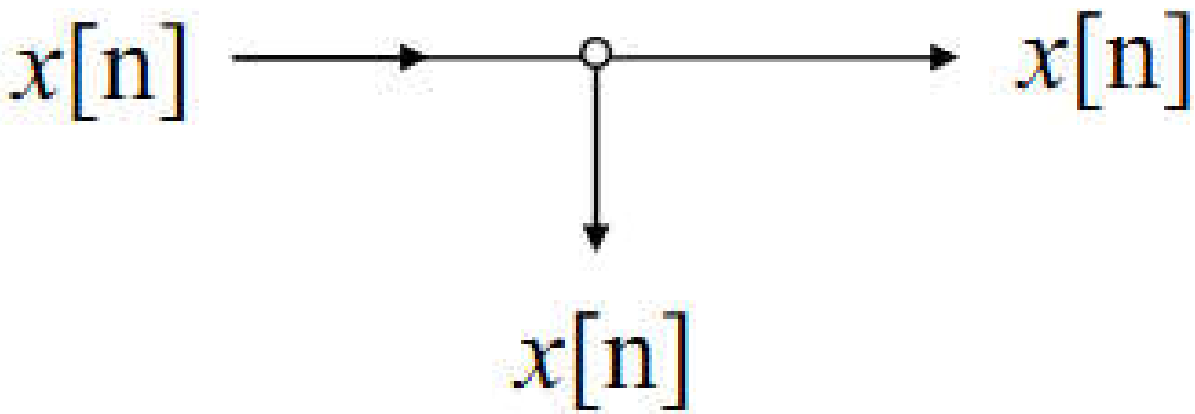
$$x[k] \rightarrow x[k \pm n]$$





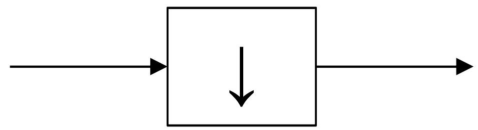
Operations

- Branching: used to provide multiple copies of a sequence



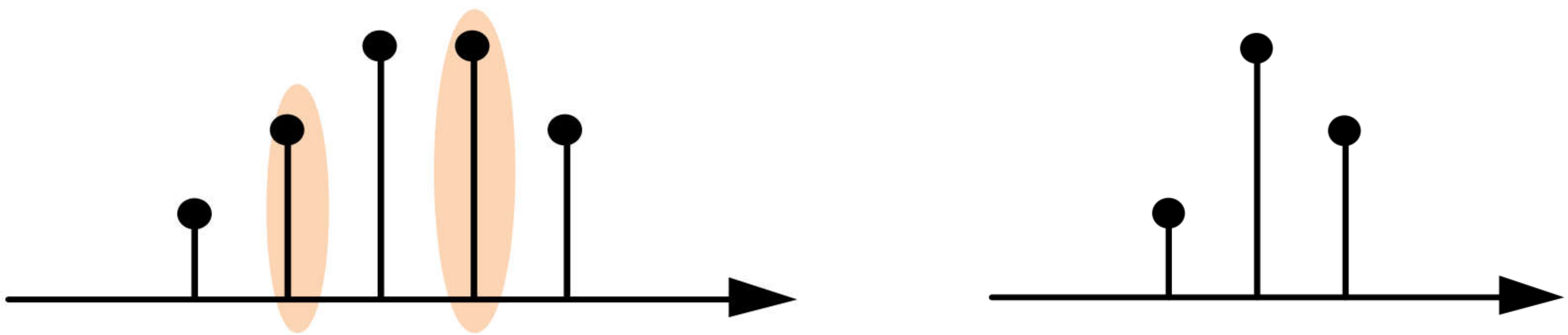
Operations

- Decimation



$$x_D[k] = x[Mk]$$

  – Take one point for every M point from original sequence

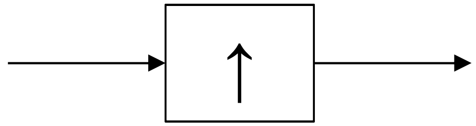


Example for M = 2

  – Down sampling

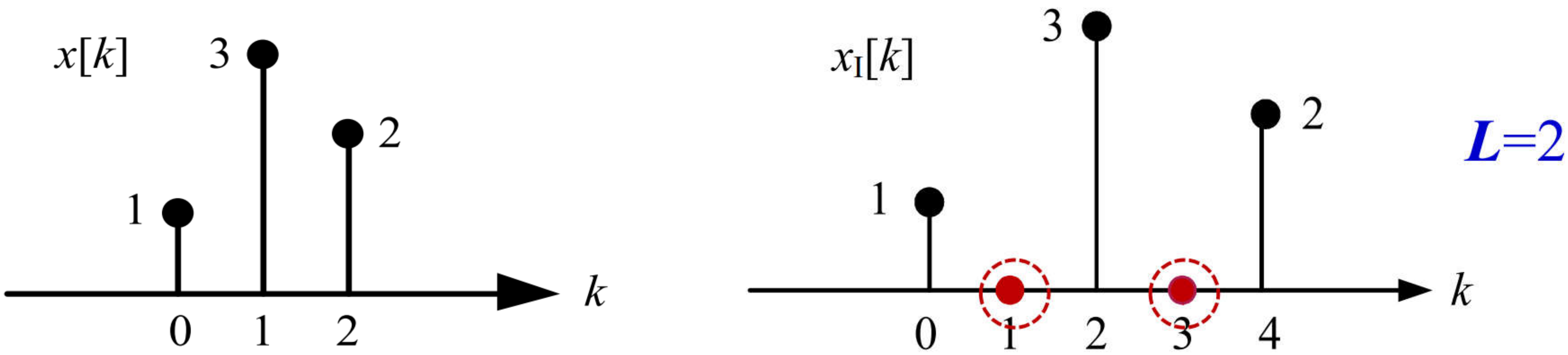
Operations

- Interpolation



$$x_I[k] = \begin{cases} x[k/L], & k = nL \\ 0, & others \end{cases}$$

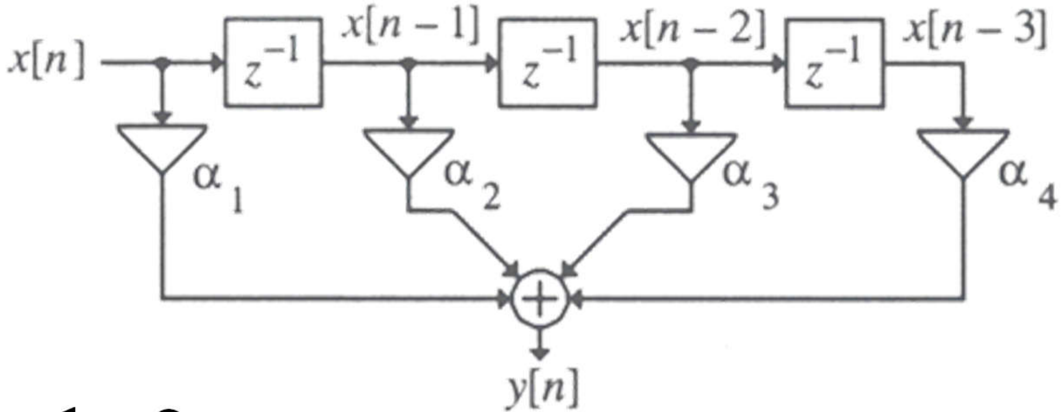
  – Inserting L-1 points between two points of the original sequence



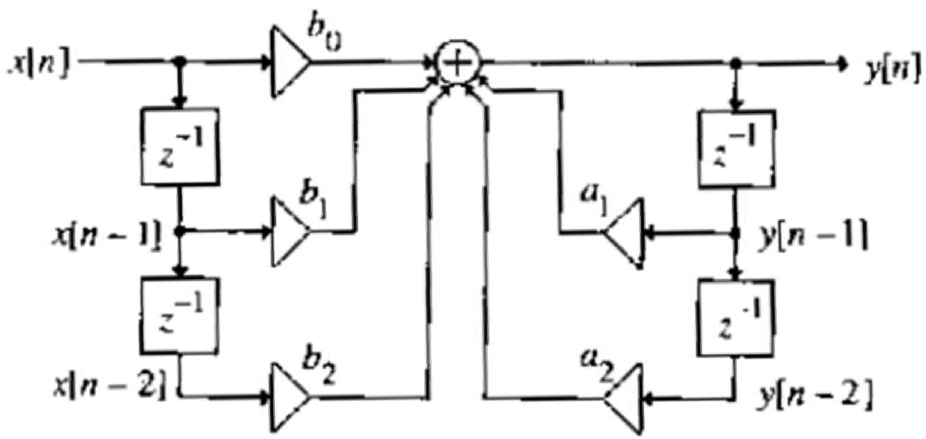
Example for L = 2

Combination of fundamental operations

- Example 1:

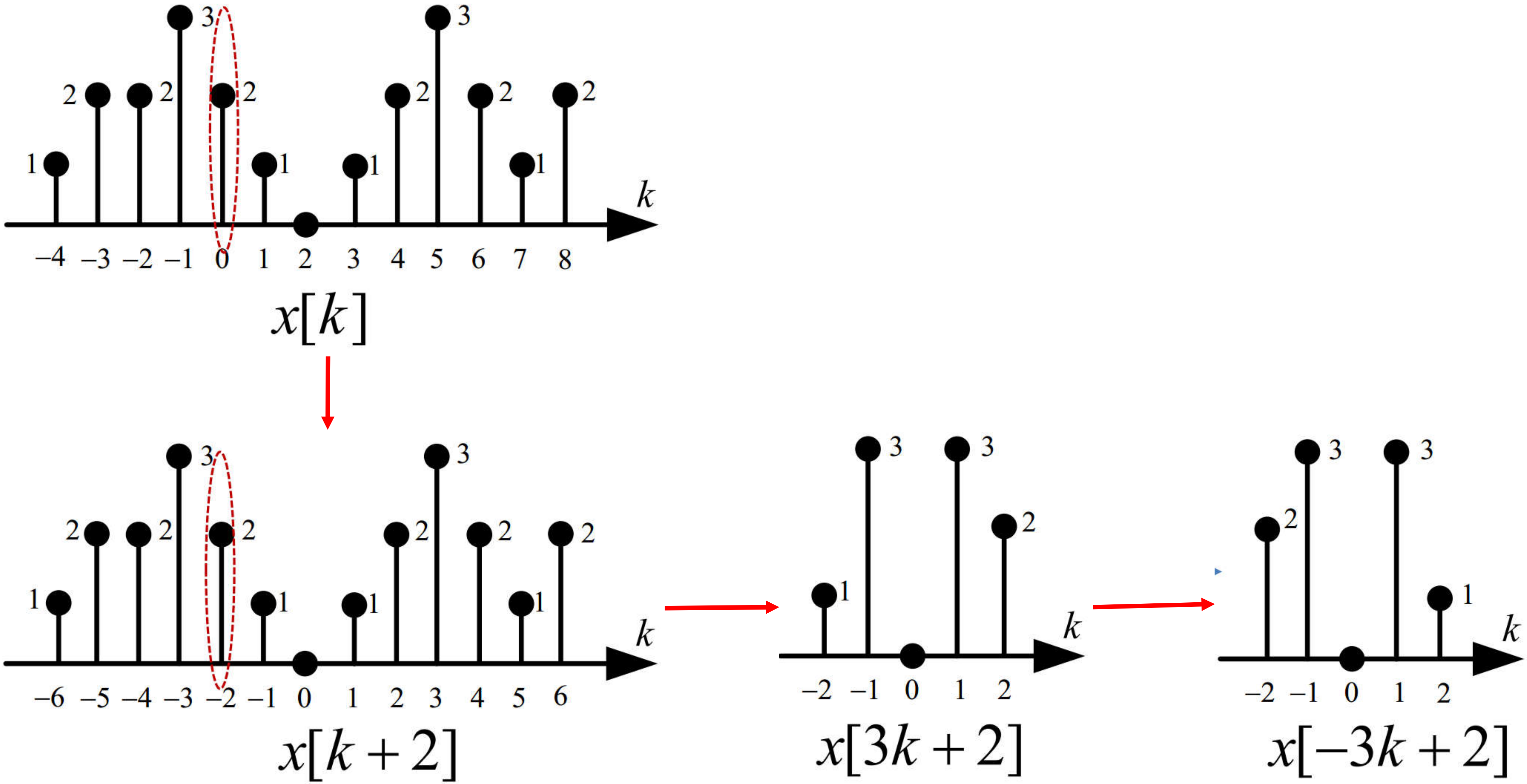


- Example 2:



Example

- x[k] is given as follows. Draw x[-3k+2].



5_3 Wrap up

- Be familiar with the fundamental operations of signals
- Be able to combine them to obtain a complex operation, or separate a complex operation into fundamental building blocks

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5_4 Properties of DT Signals

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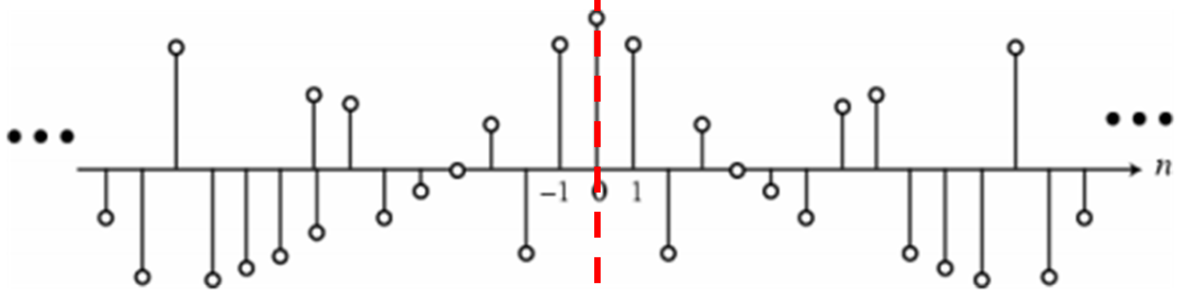
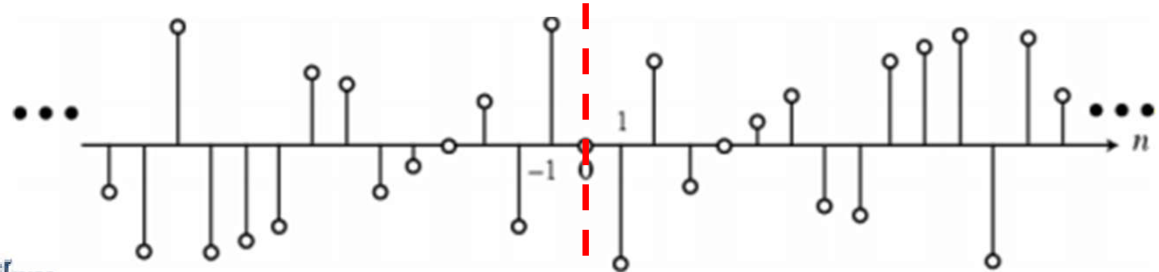
Properties of Signals

- Symmetric: if x[n]=x[-n];
- Conjugate-symmetric sequence: $x[n]=x^*[-n]$;
  - If x[n] is real, then the symmetric is the same as conjugate-symmetric, and the signal is an even sequence



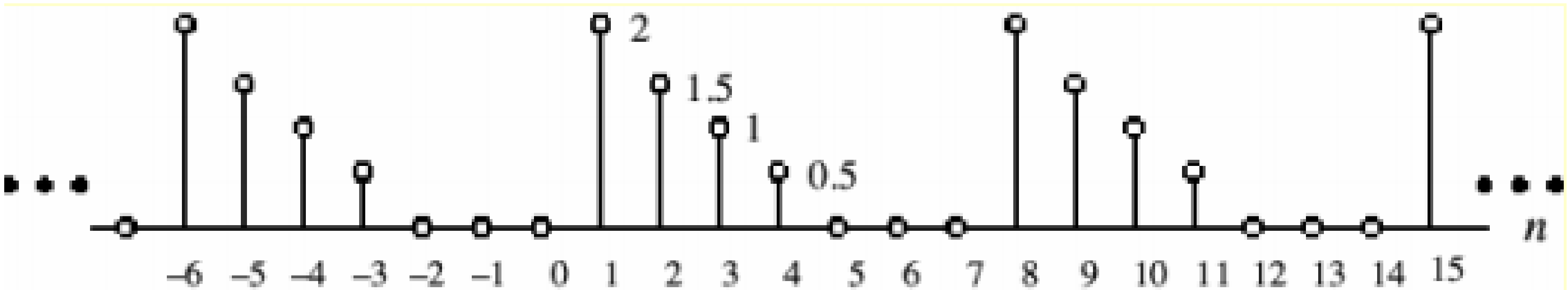
- Conjugate-antisymmetric sequence: $x[n]=-x^*[-n]$
  - If x[n] is real, the signal is called anti-symmetric or odd sequence



- Periodic signals and aperiodic signals
  - A signal is periodic with period N (N > 0) if and only if

$$x_p[n]=x_p[n+kN], \text{ for all } n$$

  - The smallest value of N for which the above condition holds is called the (fundamental) period



  - A signal not satisfying the periodicity condition is called nonperiodic or aperiodic

Example: Sinusoidal sequence

$$x[n] = A\cos(\omega_0 n + \varphi)$$

- Note any continuous sinusoidal /exponential signal is periodic
- However, not all discrete sinusoidal sequences are periodic.
  - A discrete time sequence $\mathbf{cos(\omega_0 n+\varphi)}$ is periodic with period N, if and only if, there exists an integer m, such that mT_0 is an integer, where $T_0=2\pi/\omega_0$;
  - In the other word, $\omega_0 N = 2\pi r$ must be satisfied with two integers N and r, or N/r must be rational number.
- Verify $x_1[n] = \cos(\omega_o n + \varphi)$ $\qquad x_2[n] = \cos(\omega_o(n+N)+\varphi)$

$$x_2[n] = \cos(\omega_o n + \phi)\cos\omega_o N - \sin(\omega_o n + \phi)\sin\omega_o N$$

$$= \cos(\omega_o n + \phi) = x_1[n] \quad \textbf{iff} \quad \sin\omega_o N = 0 \ \text{ and } \ \cos\omega_o N = 1$$

  - These two conditions are met if and only if $\omega_0 N = 2\pi r$.

Properties of Signals

- The total energy of a signal x[n] is defined by

$$E \equiv \sum_{n=-\infty}^{\infty} \left| x(n) \right|^2$$

- The average power of a discrete-time signal [x] is defined by

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left| x[n] \right|^2$$

  – where the signal energy of x[n] over the finite interval $-N \le n \le N$:

$$E_N = \sum_{n=-N}^{N} \left| x(n) \right|^2$$

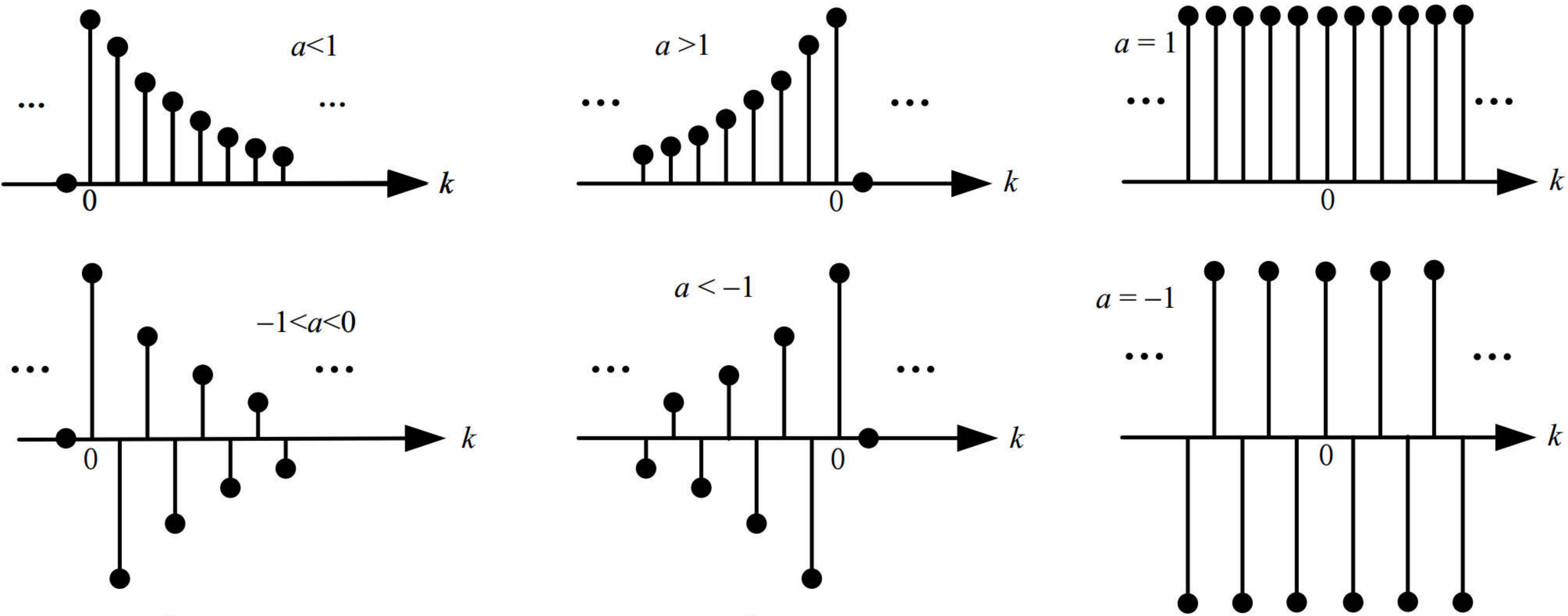
Properties of Signals

- Signal energy is: $E \equiv \lim_{N\to\infty} E_N$
- Average signal power is: $P = \lim_{N\to\infty} \frac{1}{2N+1} E_N$

- Power Signals and Energy Signals:
  - An infinite signal with finite average power is called a **power signal**
    - Eg.: A periodic sequence which has a finite average power but infinite energy
    - Eg.: A unit step signal
  - A finite signal with zero average power is called an **energy signal**
    - Eg.: A finite-length sequence which has finite energy but zero average power

Properties of Signals

- A sequence is bounded if $|x[n]| \leq B_x < \infty$
  - For example: $x[k] = a^k$



$a^k u[k]$ $a^k u[-k]$ a^k

Properties of Signals

- A sequence is absolutely summable if

$$\boxed{\sum_{n=-\infty}^{\infty} |x[n]| < \infty}$$

  - Eg: $y[n] = \begin{cases} 0.3^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$ => $\sum_{n=-\infty}^{\infty} |0.3^n| = \frac{1}{1-0.3} = 1.43 < \infty$

- A sequence is square-summable if

$$\boxed{\sum_{n=-\infty}^{\infty} |x[n]|^2 < \infty}$$

  - Eg: $h[n] = \frac{\sin 0.4\pi n}{\pi n}$ => $\sum_{n=-\infty}^{\infty} \left|\frac{\sin 0.4\pi n}{\pi n}\right| = \infty$ not absolutely summable

    but $\sum_{n=-\infty}^{\infty} \left|\frac{\sin 0.4\pi n}{\pi n}\right|^2 = 0.24 < \infty$ square summable

5_4 Wrap up

- Signal properties are introduced
  - Symmetry
  - Periodicity
  - Energy
  - Power
  - Boundness
  - Summability
- Signals can be classified according to their properties
- Be able to analyse and determine their classification.

Chapter 5 Summary

Signals (Sequences)	Operations	Properties
• Unit Impulse	• Elementary (addition, multiplication, production)	• Symmetry
• Unit Step	• Time Shifting	• Periodicity
• Rectangular	• Time Reversal (folding)	• Energy
• Real Exponential	• Branching	• Power
• Complex Exponential	• Decimation	• Bound
• Sinusoidal	• Interpolation	• Summable (absolutely, square)