Generator and Transformer Models: The Per-unit System (Part III) EEE210

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May 16, 2019

Overview

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3.10 Autotransformers

Transformers can be constructed so that the primary and secondary coils are electrically connected. This type of transformer is called an autotransformer.

A conventional two-winding transformer can be changed into an autotransformer by connecting the primary and secondary windings in series.

The two-winding transformer is converted to an autotransformer arrangement as shown in Figure 1 by connecting the two windings electrically in series so that the polarities are additive.

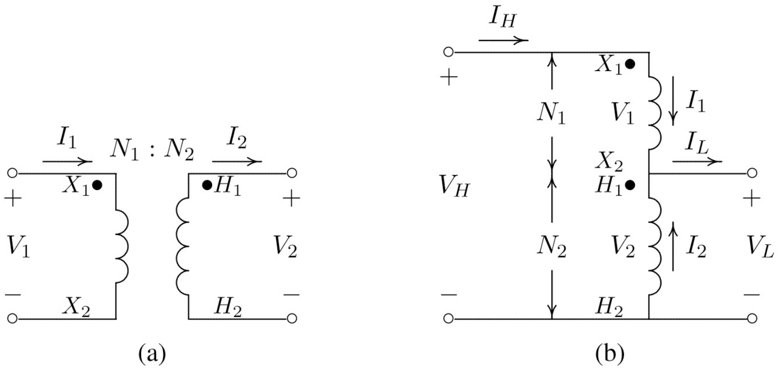

Figure 1: (a) Two-winding transformer, (b) reconnected as an autotransformer

3.10 Autotransformers

The winding from X_1 to X_2 is called the series winding, and the winding from H_1 to H_2 is called the common winding.

From an inspection of this figure it follows that an autotransformer can operate as a step-up as well as a step-down transformer.

In both cases, winding part H_1H_2 is common to the primary as well as the secondary side of the transformer.

To determine the power rating of an autotransformer, the ideal relations are normally applied, which provides and adequate approximation to the actual transformer values.

3.10 Autotransformers

From Figure 1, the two-winding voltages and currents are related by

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} = a \tag{1}$$

and

$$\frac{I_2}{I_1} = \frac{N_1}{N_2} = a \tag{2}$$

where a is the turns ratio of the two-winding transformer. From Figure 1, we have

$$V_H = V_2 + V_1 \tag{3}$$

Substituting V_1 from (1) into (3) yields

$$V_H = V_2 + \frac{N_1}{N_2} V_2 \tag{4}$$

Since $V_2 = V_L$, the voltage relationship between the two sides of an autotransformer becomes

$$\begin{aligned} V_H &= V_L + \frac{N_1}{N_2} V_L \\ &= (1 + a) V_L \end{aligned} \tag{5}$$

or

$$\frac{V_H}{V_L} = 1 + a \tag{6}$$

Since the transformer is ideal, the mmf due to I_1 must be equal and opposite to the mmf produced by I_2. As a result, we have

$$N_2 I_2 = N_1 I_1 \tag{7}$$

From Kirchoff's law, $I_2 = I_L - I_1$, the above equation becomes

$$N_2(I_L - I_1) = N_1 I_1 \tag{8}$$

or

$$I_L = \frac{N_1 + N_2}{N_2} I_1 \tag{9}$$

Since $I_1 = I_H$, the current relationship between the two sides of an autotransformer becomes

$$\frac{I_L}{I_H} = 1 + a \tag{10}$$

3.10 Autotransformers

The ratio of the apparent power rating of an autotransformer to a two-winding transformer, known as the power rating advantage, is found from

$$\frac{S_{auto}}{S_{2-w}} = \frac{(V_1 + V_2)I_1}{V_1 I_1} = 1 + \frac{N_2}{N_1} = 1 + \frac{1}{a} \tag{11}$$

From (11), we can see that a higher rating is obtained as an autotransformer with a higher number of turns of the common winding (N_2).

Example

A two-winding transformer is rated at 60-kVA, 240/1200-V, 60-Hz. When operated as a conventional two-winding transformer at rated load, 0.8 power factor, its efficiency is 0.96. This transformer is to be used as a 1440/1200-V step-down autotransformer in a power distribution system.

(a) Assuming that the transformer is ideal, find the transformer kVA rating when used as an autotransformer.

(b) Find the efficiency with the kVA loading of part (a) and 0.8 power factor.

The equivalent impedance of the autotranformer in per-unit is much smaller compared to the two-winding connection.

It is a common practice to consider an autotransformer as a two-winding transformer with its two windings connected in series as shown in Figure 2.

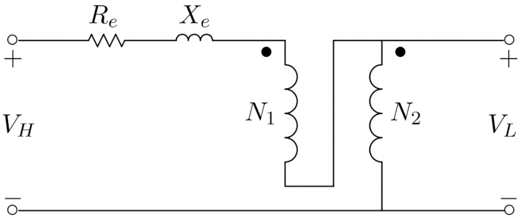

Figure 2: Autotransformer equivalent circuit.

Three-winding transformers (Figure 3 connects three independent circuits with different voltages.

Relevant winding is called primary, secondary, and tertiary windings.

Usually the tertiary windings provide voltage for auxiliary power purposes in the substation, or to supply a local distribution system.

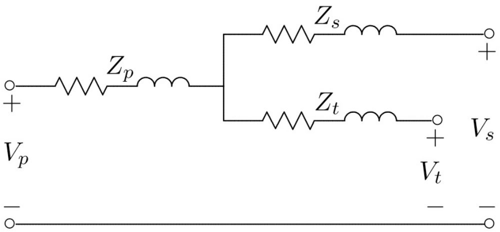

Figure 3: Equivalent circuit of three-winding transformer

Example

A two-winding transformer rated at 9-kVA, 120/90-V, 60-Hz has a core loss of 200 W and a full-load copper loss of 500 W.

a. The above transformer is to be connected as an auto transformer to supply a load at 120 V from 210 V source. What kVA load can be supplied without exceeding the current rating of the windings? (For this part assume an ideal transformer.)
b. Find the efficiency with the kVA loading of part (a) and 0.8 power factor.

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3.13 The Per-Unit System

The solution of an interconnected power having several different voltage levels requires the cumbersome transformation of all impedances to a single voltage level.

However, power system engineers have devised the per-unit system such that the various physical quantities such as power, voltage, current, and impedance are expressed as a decimal fraction or multiples of base quantities.

In this system, the different voltage levels disappear, and a power network involving generators, transformers, and lines (of different voltage levels) reduces to a system of simple impedances. The per-unit value of any quantity is defined as

$$\text{Quantity in per-unit} = \frac{\text{actual quantity}}{\text{base value of quantity}} \tag{12}$$

The Per-Unit System

For example,

$$
\begin{aligned}
S_{pu} &= \frac{S}{S_B} \\
V_{pu} &= \frac{V}{V_B} \\
I_{pu} &= \frac{I}{I_B} \\
Z_{pu} &= \frac{Z}{Z_B}
\end{aligned}
$$

where the numerators (actual values) are phasor quantities or complex values and the denominators (base values) are always real numbers.

The Per-Unit System

A minimum of four base quantities are required to completely define a per-unit system:

1. volt-ampere,
2. voltage,
3. current, and
4. impedance.

Usually, the three-phase base volt-ampere S_B and line-to-line base voltage V_B are selected. Base current and base impedance are then dependent on S_B and V_B and must obey the circuit laws. These are given by

$$I_B = \frac{S_B}{\sqrt{3}V_B} \tag{13}$$

and

$$Z_B = \frac{V_B/\sqrt{3}}{I_B} \tag{14}$$

Proof for Eqns. (13) and (14)

Adopted from *Power System Analysis & Design* by J.D. Glover, pg. 114:

$$S_{base1\phi} = \frac{S_{base3\phi}}{3} \tag{15}$$

$$V_{baseLN} = \frac{V_{baseLL}}{\sqrt{3}} \tag{16}$$

The base current is given by

$$I_{base} = \frac{S_{base1\phi}}{V_{baseLN}} = \frac{\frac{S_{base3\phi}}{3}}{\frac{V_{baseLL}}{\sqrt{3}}} = \frac{S_{base3\phi}}{\sqrt{3}V_{baseLL}}$$

and

$$Z_{base} = \frac{V_{baseLN}}{I_{base}} = \frac{V_{baseLL}/\sqrt{3}}{I_{base}}$$

The Per-Unit System

Substituting I_B from (13) into (14), the base impedance becomes

$$\begin{aligned} Z_B &= \frac{(V_B)^2}{S_B} \\ Z_B &= \frac{(kV_B)^2}{MVA_B} \end{aligned} \tag{17}$$

The phase and line quantities expressed in per-unit are the same, and the circuit laws are valid, i.e.,

$$S_{pu} = V_{pu} I_{pu}^* \tag{18}$$

$$V_{pu} = Z_{pu} I_{pu} \tag{19}$$

$$Z_{pu} = \frac{|V_{pu}|^2}{S_{L(pu)}^*} \tag{20}$$

Advantages of the per-unit system

1. It gives clear magnitudes of V, I, P, and Z,
2. The per-unit impedance of equipments falls in a narrow range,
3. The per-unit values of Z, V, and I of a transformer are the same regardless of primary or secondary side. This is a great advantage since the different voltage levels disappear and the entire system reduces to a system of simple impedance,
4. The per-unit systems are ideal for the computerized analysis and simulation of complex power system problems, and
5. The circuit laws are valid in per-unit systems, and the power and voltage equations are simplified since the factors of $\sqrt{3}$ and 3 are eliminated in the per-unit system.

Example (3.7)

The one-line diagram of a three-phase power system is shown in Figure 4. Select a common base of 100-MVA and 22-kV on the generator side. Draw an impedance diagram with all impedances including the load impedance marked in per-unit. The manufacturer's data for each device is given as follow:

G:	90 MVA	22 kV	X=18%
T_1:	50 MVA	22/220 kV	X=10%
T_2:	40 MVA	220/11 kV	X=6.0%
T_3:	40 MVA	22/110 kV	X=6.4%
T_4:	40 MVA	110/11 kV	X=8.0%
M:	66.5 MVA	10.45 kV	X=18.5%

The three-phase load at bus 4 absorbs 57 MVA, 0.6 power factor lagging at 10.45 kV. Line 1 and line 2 have reactances of 48.4 and 65.43 Ω, respectively.

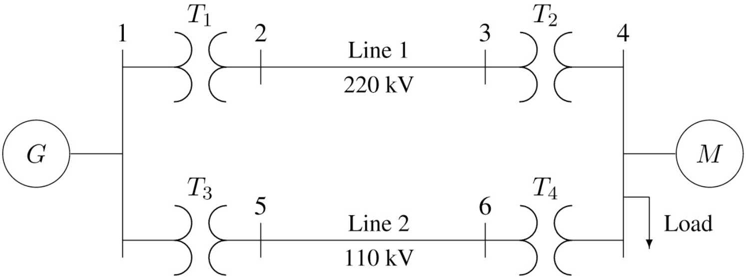
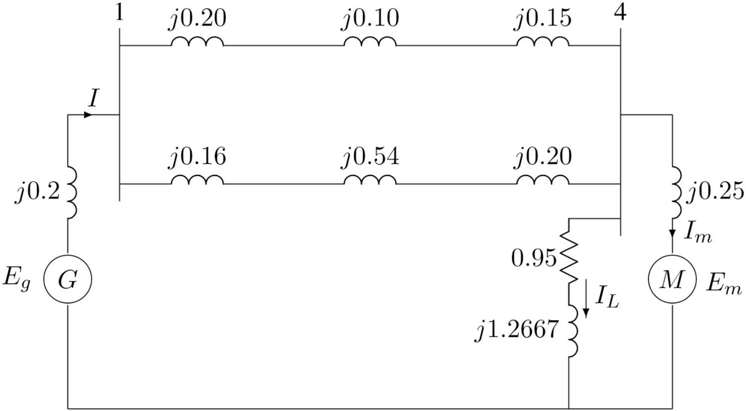


Figure 4: One-line diagram for Example 3.7


1
$j0.20$
$j0.10$
$j0.15$
4
I
$j0.16$
$j0.54$
$j0.20$
$j0.2$
$j0.25$
I_m
0.95
I_L
E_g
G
M
E_m
$j1.2667$

Example (1)

Draw an impedance diagram for the electric power system shown in figure below, showing all impedances in per unit on a 100-MVA base. Choose 20 kV as the voltage base for generator. The three-phase power and line-line ratings are given below.

G_1:	90 MVA	20 kV	X=9%
T_1:	80 MVA	20/200 kV	X=16%
T_2:	80 MVA	200/20 kV	X=20%
G_2:	90 MVA	18 kV	X=9%
Line:		200 kV	X=120 Ω
Load:		200 kV	S=48 MW+$j64$ Mvar

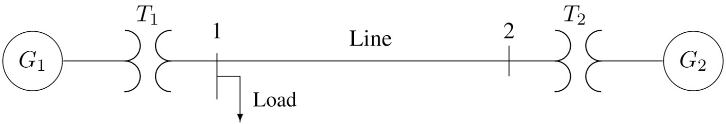


Figure 5: On-line diagram for Example 1

Example (2)

The three-phase power and line-line ratings of the electric power system shown in Figure 6 are given below.

G_1:	60 MVA	20 kV	X=9%
T_1:	50 MVA	20/200 kV	X=10%
T_2:	50 MVA	200/20 kV	X=10%
M:	43.2 MVA	18 kV	X=8%
Line:		200 kV	Z=120+j200 Ω

1. Draw an impedance diagram showing all impedances in per unit on a 100-MVA base. Choose 20 kV as the voltage base for generator.
2. The motor is drawing 45 MVA, 0.8 power factor lagging at a line-to-line terminal voltage of 18 kV. Determine the terminal voltage and the internal emf of the generator in per unit and in kV.

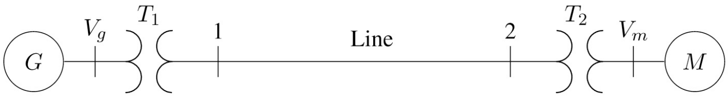


Figure 6: On-line diagram for Example 2

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