EEE104 – Digital Electronics (I)

Lecture 9

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In This Session

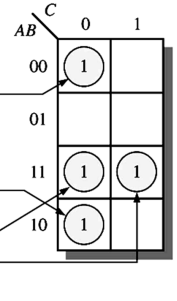
- The Karnaugh Map
- Karnaugh Map SOP Minimization
- Karnaugh Map POS Minimization

Karnaugh Map

- A graphical tool to simplify Boolean expressions.
- It is like a truth table in array form, in which each cell corresponds to a row in the truth table.
- Limited to 5-6 variables.

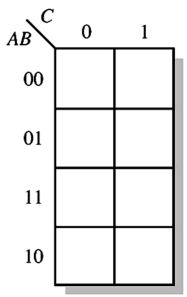
$X = \bar{A}B\bar{C} + A\bar{B}\bar{C} + AB\bar{C} + ABC$

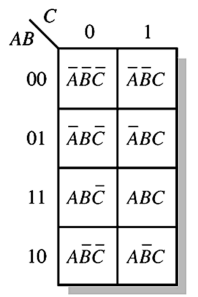
Inputs A	B	C	Output X
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1



Karnaugh Map

- The number of cells is equal to 2^n, where n is the number of variables.
- The cells are not arranged according to the magnitude of binary values, e.g. 00→ 01 → 11 → 10.

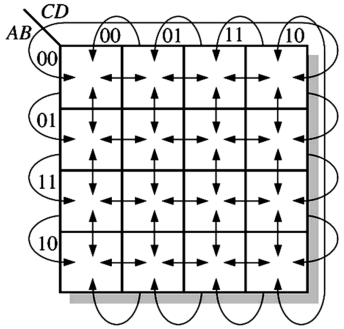




Karnaugh Map

Cell Adjacency

- The cells are arranged so that there is only a **single-variable change** between adjacent cells.
- The binary values of two variables: 00→01→11→10.

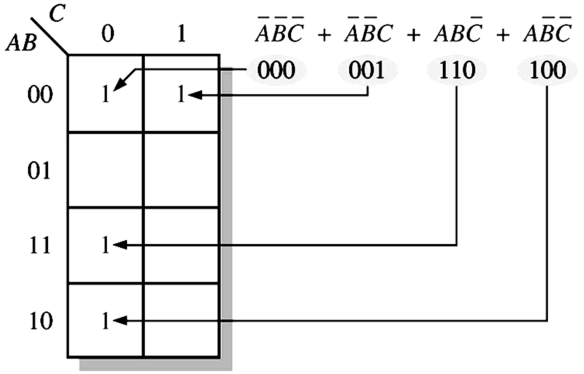


1. Each cell is adjacent to the cells on its four sides.
2. The top row is adjacent to the bottom row.
3. The leftmost column is adjacent to the rightmost column. ("**wrap-around**")

Karnaugh Map SOP Minimization

Mapping a Standard SOP Expression

- For a standard SOP, place a 1 on the Karnaugh map in the cell having the same value as the product term.



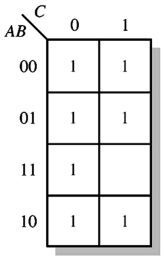
Karnaugh Map SOP Minimization

Mapping a Non-Standard SOP Expression

- Convert it to standard form by **numerical expansion**.
- For each missing variable, the binary value of the product term is split into two by attaching a 1 and 0 respectively.

$$\overline{A} + A\overline{B} + AB\overline{C}$$

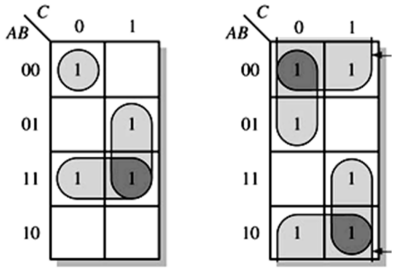
000 100 110
001 101
010
011



Karnaugh Map SOP Minimization

Step 1: Grouping the 1s

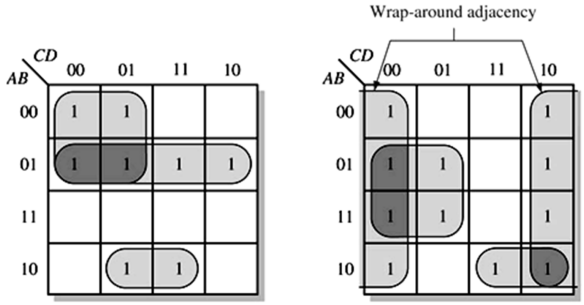
- The goal is to **maximize the size** of the groups (shorter product terms) and to **minimize the number** of groups (less product terms).
- A group may contain 1, 2, 4, 8, or 16 adjacent cells.
- Each 1 must be included in one or **more** groups.



Karnaugh Map SOP Minimization

Step 1: Grouping the 1s

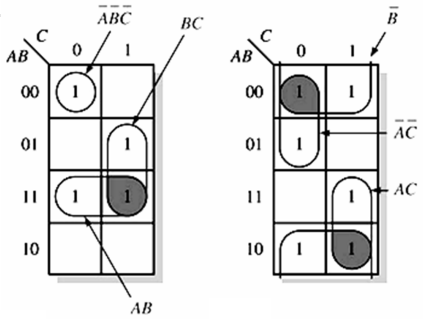
- Alternative grouping will **not** maximize the size or minimize the number of groups.



Karnaugh Map SOP Minimization

Step 2: Determine the Minimum SOP

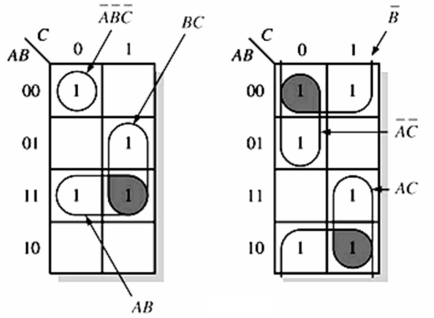
- When a variable appears in both complemented and uncomplemented form in a group, that variable is eliminated.
- Variables that are the same for all cells of the group must appear.



Karnaugh Map SOP Minimization

Step 2: Determine the Minimum SOP

- The variable that is 1 for all cells of the group appear in uncomplemented form.
- The variable that is 0 for all cells of the group appear in complemented form.

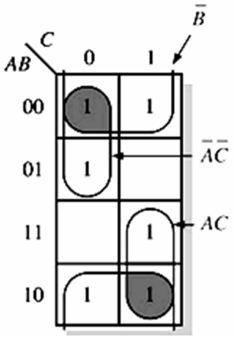


Karnaugh Map SOP Minimization

Step 2: Determine the Minimum SOP

For a 3-variable map:

1. A 4-cell group yields a 1-variable term.
2. A 2-cell group yields a 2-variable product term.
3. A 1-cell group yields a 3-variable product term.

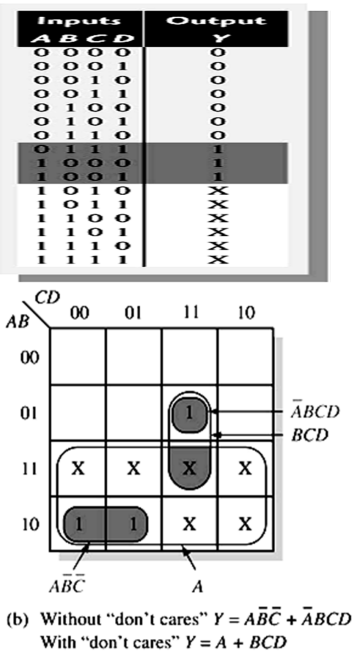


Karnaugh Map SOP Minimization

Inputs A B C D	Output Y
0 0 0 0	0
0 0 0 1	0
0 0 1 0	0
0 0 1 1	0
0 1 0 0	0
0 1 0 1	0
0 1 1 0	0
0 1 1 1	1
1 0 0 0	1
1 0 0 1	1
1 0 1 0	X
1 0 1 1	X
1 1 0 0	X
1 1 0 1	X
1 1 1 0	X
1 1 1 1	X

"Don't Care"

- Sometimes some input variable combinations will never occur, e.g. six invalid numbers in BCD code.
- Either a 1 or a 0 may be assigned to the output. They can be treated as "**don't care**" terms, written as X



(b) Without "don't cares" $Y = A\bar{B}\bar{C} + \bar{A}BCD$
With "don't cares" $Y = A + BCD$

Karnaugh Map SOP Minimization

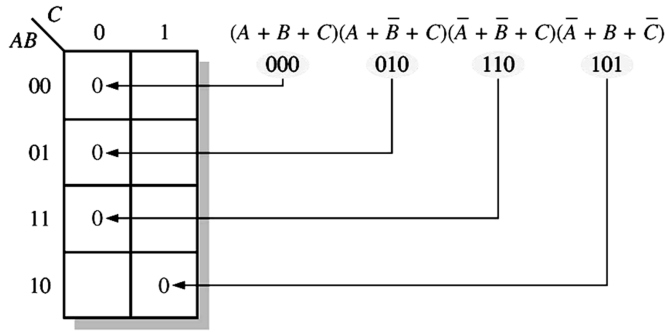
"Don't Care"

- Can be used to simplify Boolean expressions.
- When an X can be grouped with 1s, then it is thought as 1.
- Otherwise, it is thought as 0.

Karnaugh Map POS Minimization

Mapping a Standard POS Expression

- For a standard POS, place a 0 on the Karnaugh map in the cell having the same value as the sum term.



Karnaugh Map POS Minimization

Karnaugh Map Simplification

- Same as for an SOP except grouping 0s.

$$(A + B + C)(A + B + \overline{C})(A + \overline{B} + C)(A + \overline{B} + \overline{C})(\overline{A} + \overline{B} + C)$$

The binary values of the sum terms are 000, 001, 010, 011, 110.

If a variable is always 0, it appears in uncomplemented form; if it is always 1, in complemented form.

The minimum POS is $A(\overline{B} + C)$