Lecture 3

of

EEE307

Electronics for Communications

Department of Electrical & Electronic Engineering

Xi'an Jiaotong-Liverpool University (XJTLU)

Friday, 20th September 2019

- **LC Resonant Circuits**
  - resonant frequency
- LC **Filters**
  - band-pass filter
  - band-stop filter
  - SAW filter

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Radio-Frequency Circuits

(more passive components)

- In electronic circuits for communications using radio waves, passive components are used quite often.
  - This is in sharp contrast with digital VLSI circuits which consist of almost exclusively transistors.
  - The relative large ratio of passive components to transistors is in fact one obvious characteristics of RF circuits.
- Among the passive components, inductors and capacitors are used more often.
  - Why not resistors in RFICs?

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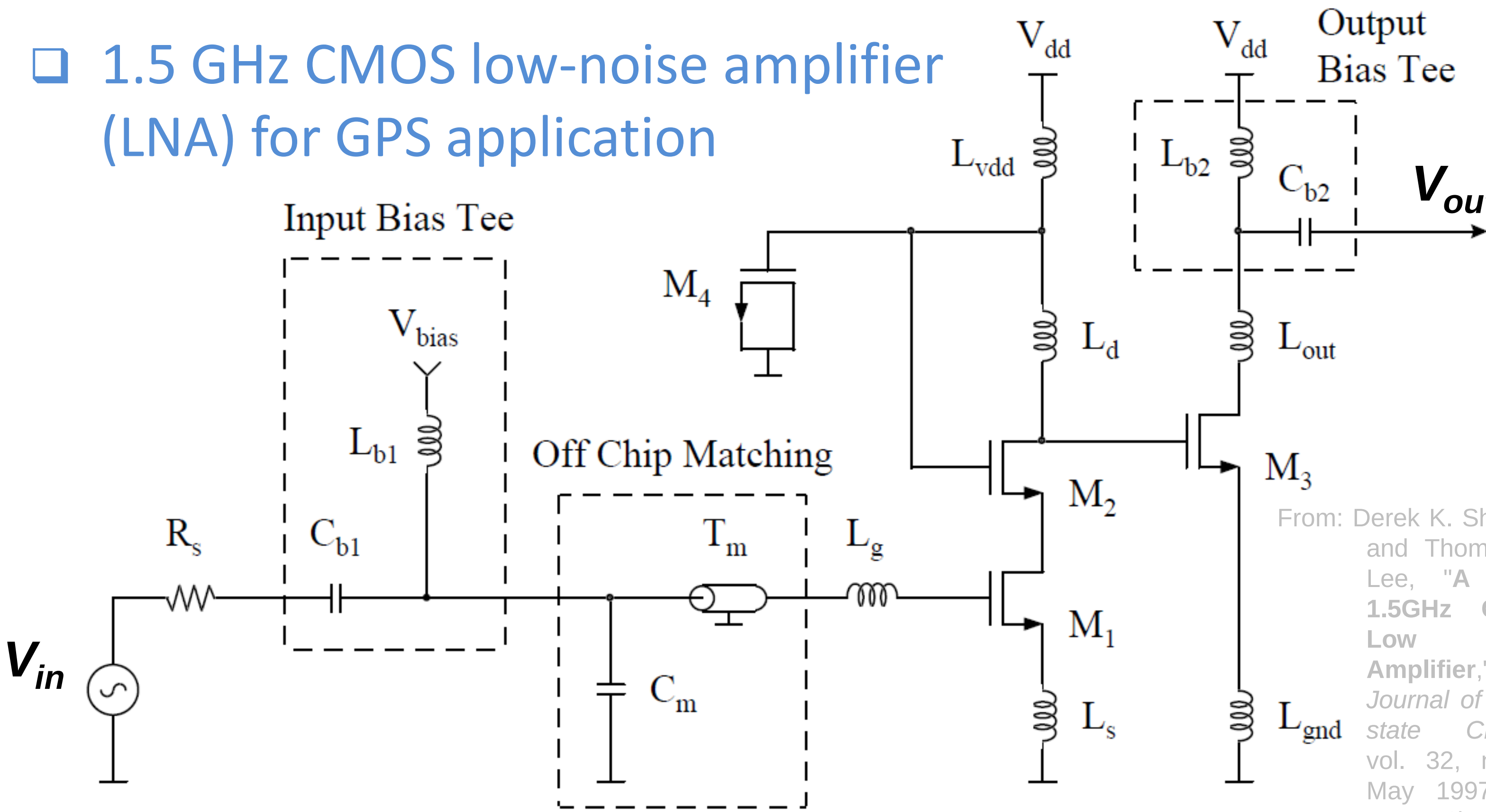
Use of Passive Components

(LNA example)

- 1.5 GHz CMOS low-noise amplifier (LNA) for GPS application



From: Derek K. Shaeffer and Thomas H. Lee, "**A 1.5V, 1.5GHz CMOS Low Noise Amplifier**," *IEEE Journal of Solid-state Circuits*, vol. 32, no. 5, May 1997 (pp. 745-759).

- More inductors than transistors are used in a typical RF circuit.

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LC Resonant Circuits

(parallel resonance)

- Inductors and capacitors are often used in RF circuits in the form of an **LC-resonant circuit** or called an **LC tank**.
  - The word "tank" conveys the idea that energy is stored in the resonant circuit like the storage of water in a tank.
  - It is a resonator with energy resonantly circulated from the inductor to the capacitor and vice versa.
  - The **LC-resonant circuit** is commonly seen in the design of LC voltage controlled oscillator (VCO).
- Inductors and capacitors are also used in RF circuits for **impedance transformation**.

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Use of LC-Resonant Circuits

(VCO example)

- Two LC-tanks are used a typical CMOS voltage controlled oscillator (VCO).

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LC Resonant Circuits

(parallel resonance)

- In the LC resonant circuits, the inductor and capacitor can be connected either in series or in parallel.
- Consider an inductor and a capacitor connected together in parallel to ground, with their corresponding reactance $\boldsymbol{X_C}$ and $\boldsymbol{X_L}$, $\boldsymbol{R_S}$ as the source resistance of the input signal.

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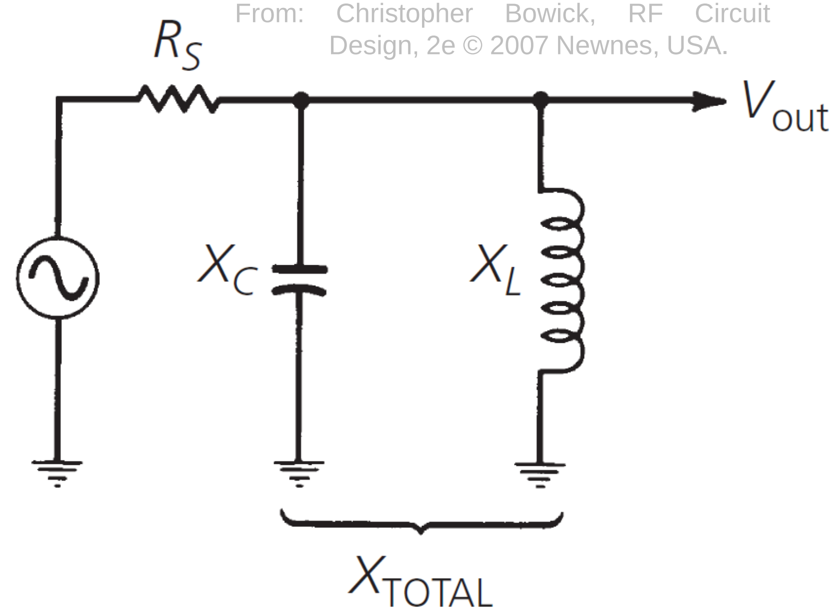


$$X_C = \frac{1}{j\omega C}$$

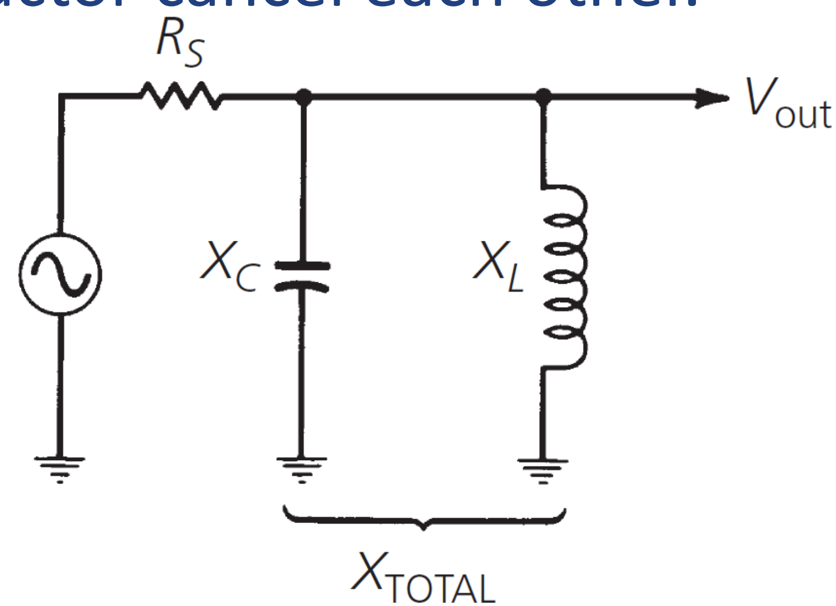
$$X_L = j\omega L$$

LC Resonant Circuits

(parallel resonance)

- In the parallel LC circuit, it is intuitive to think that there exists a frequency at which the reactances of the capacitor and the inductor cancel each other.
- That frequency is called the resonant frequency:

$$\omega_0 = \frac{1}{\sqrt{LC}}$$



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- What happens at ω_0?

$$\left.\frac{V_{out}}{V_{in}}\right|_{pLC} = \frac{j\omega L}{R_S\left(1-\omega^2 LC\right)+j\omega L}$$

Parallel LC Resonant Circuit

(simple band-pass filter)

- At ω_0, the inductor and capacitor are almost like non-existent, with a zero admittance which means infinitely large impedance (i.e. open circuit) ideally.

$$Y_{parallLC} = X_C // X_L = \frac{1}{X_C} + \frac{1}{X_L}$$

$$= 1 \Big/ \left(\frac{1}{j\omega C} \right) + \frac{1}{j\omega L} = j\omega C + \frac{1}{j\omega L}$$

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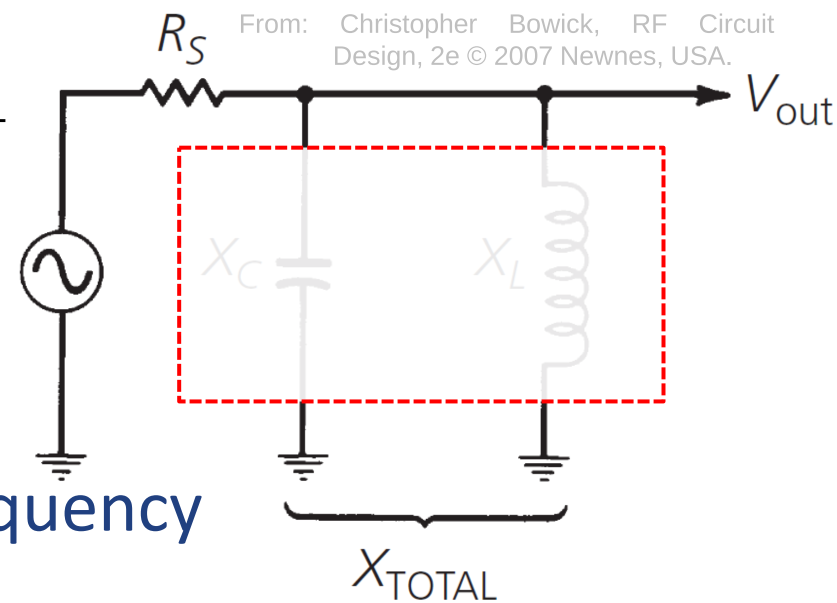
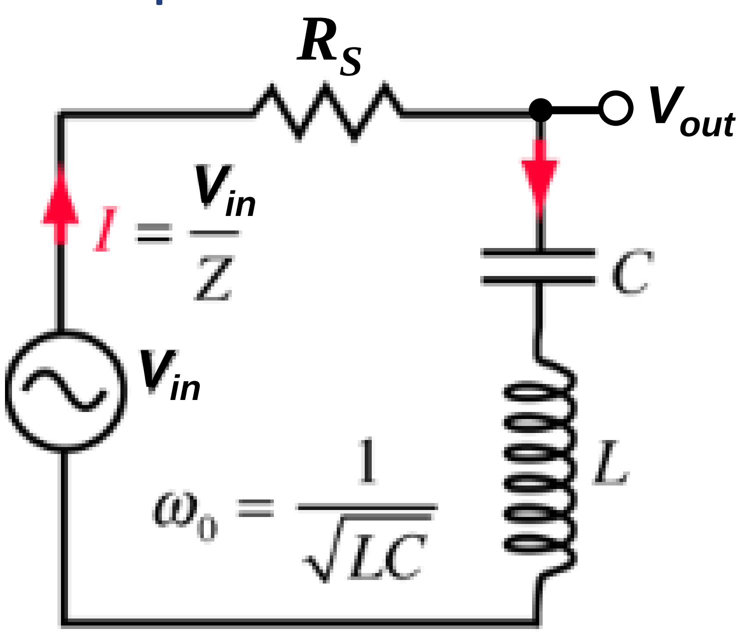


- As a result, a signal of frequency ω_0 will pass to the output.
- It is a simple **band-pass filter**.

LC Resonant Circuits

(series resonance)

- When the inductor and capacitor are connected in series to ground, there also exists the same resonant frequency at which the reactances of the capacitor and the inductor cancel each other.



$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\left.\frac{V_{out}}{V_{in}}\right|_{sLC} = \frac{1-\omega^2 LC}{\left(1-\omega^2 LC\right)+jR_S C}$$

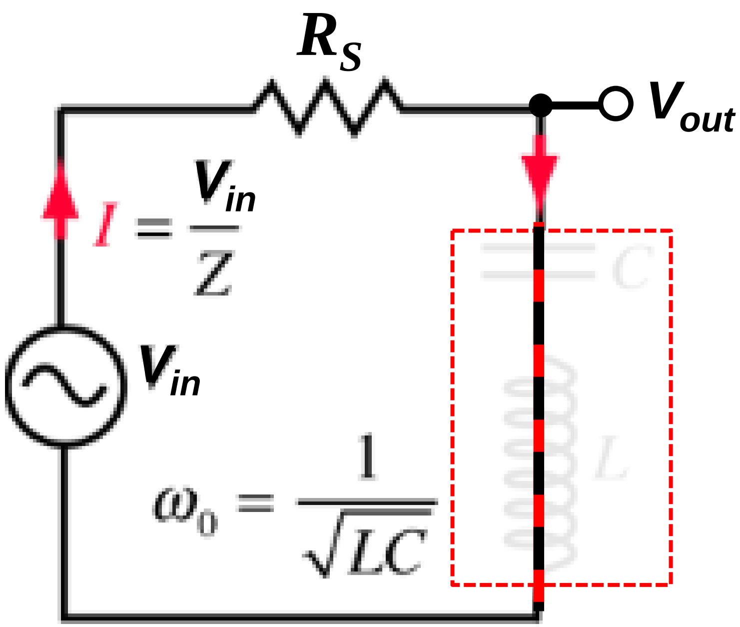
Image taken from HyperPhysics: available at: http://hyperphysics.phy-astr.gsu.edu/hbase/electric/serres.html

Series LC Resonant Circuit

(simple band-stop filter)

- At ω_0, the inductor and capacitor in series are almost like a short circuit, with a zero impedance ideally.

$$Z_{seriesLC} = X_C + X_L = \frac{1}{j\omega C} + j\omega L$$



- As a result, an RF signal of frequency ω_0 will pass to the ground, instead of the output.
- It is a simple **band-stop filter**.

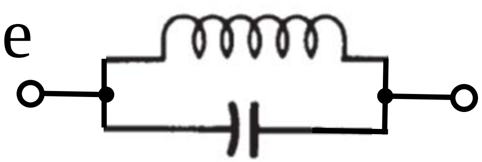
Image taken from HyperPhysics: available at: http://hyperphysics.phy-astr.gsu.edu/hbase/electric/serres.html

LC Resonant Circuit - summary

(series vs. parallel LC)

- In the parallel-connected *LC*, it is *equivalent* to ***open-circuit*** at resonance.
  - low impedance at either low or high frequencies

$$Y_{parallelLC} = j\omega C + \frac{1}{j\omega L} = \to 0 \text{ at resonance}$$



$$\Rightarrow Z_{parallelLC} = 1/Y_{parallelLC} = \to \infty \text{ at resonance}$$

- In the series-connected LC, it is *equivalent* to ***short-circuit*** at resonance.

  - high impedance at either low or high frequencies

$$Z_{seriesLC} = \frac{1}{j\omega C} + j\omega L \to 0 \text{ at resonance}$$

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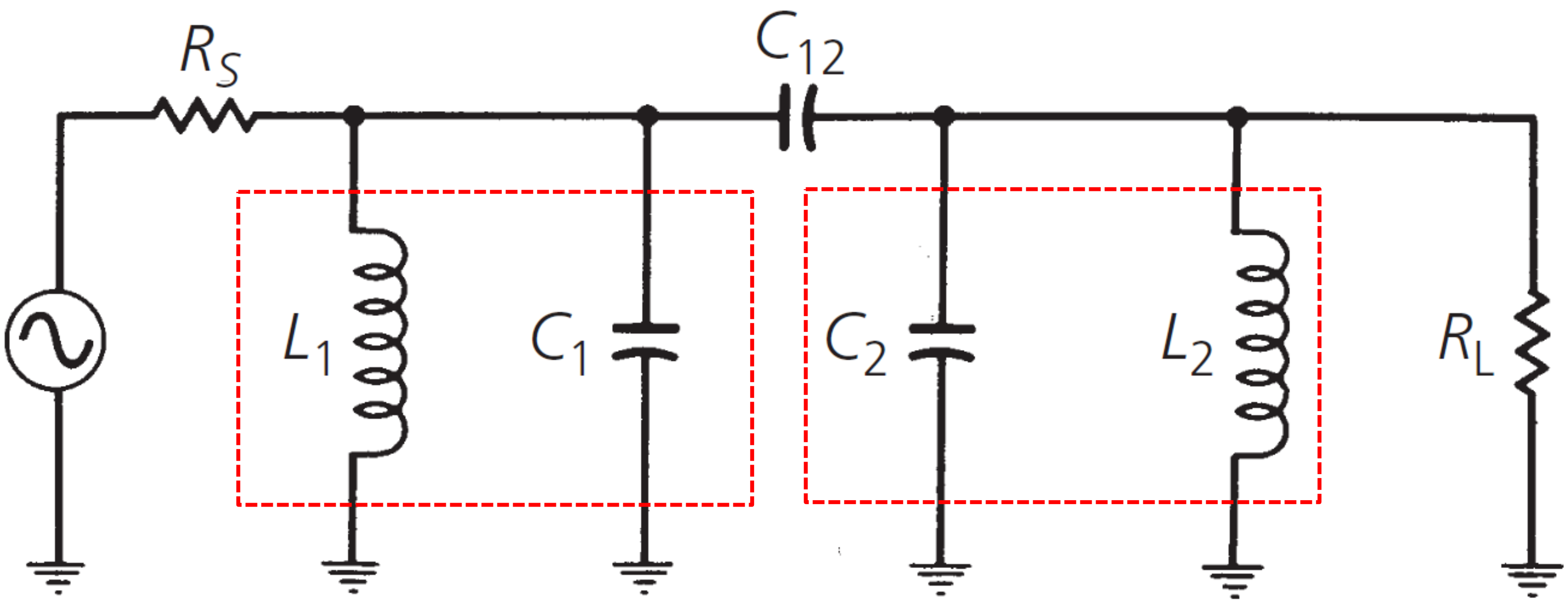
RF Filters of LC Resonant Circuits

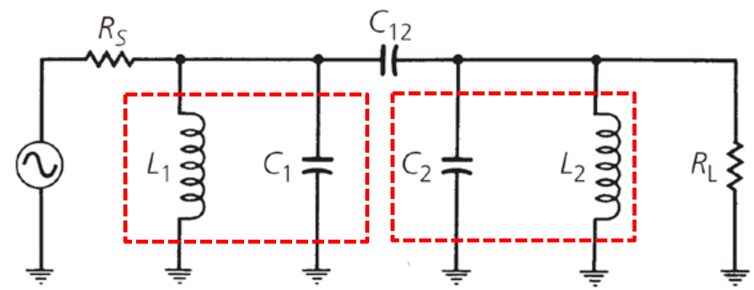
(capacitive coupled LC tanks)

- The simple **band-pass filter** can be improved by using two parallel LC resonant circuits which are coupled to each other, through a capacitor $\boldsymbol{C_{12}}$.



Band-Pass Filters – 2 parallel LC tanks
(below resonance)

- At frequency below the resonance, the circuit is equivalent to the following:



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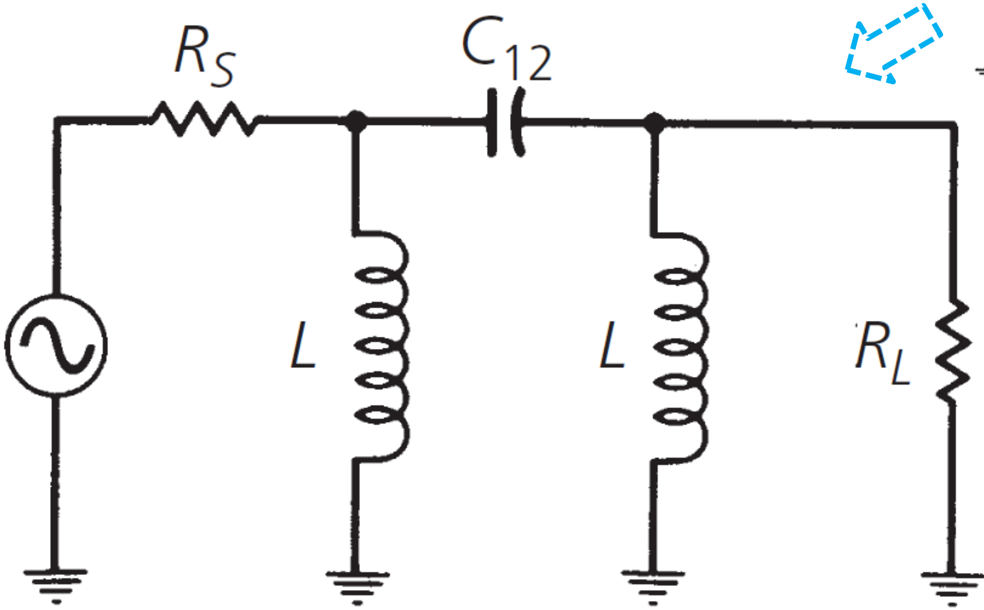

(A) *Below resonance*

- RF signals of very low frequencies will pass to ground through the inductors which have small impedance values ($|Z| = \omega L$).

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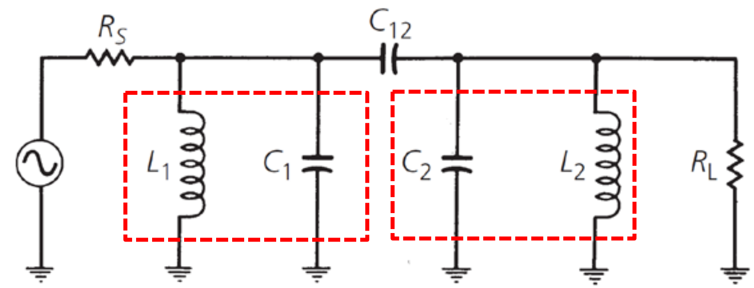
Band-Pass Filters – 2 parallel LC tanks (above resonance)

- At frequency above the resonance, the circuit is equivalent to the following:



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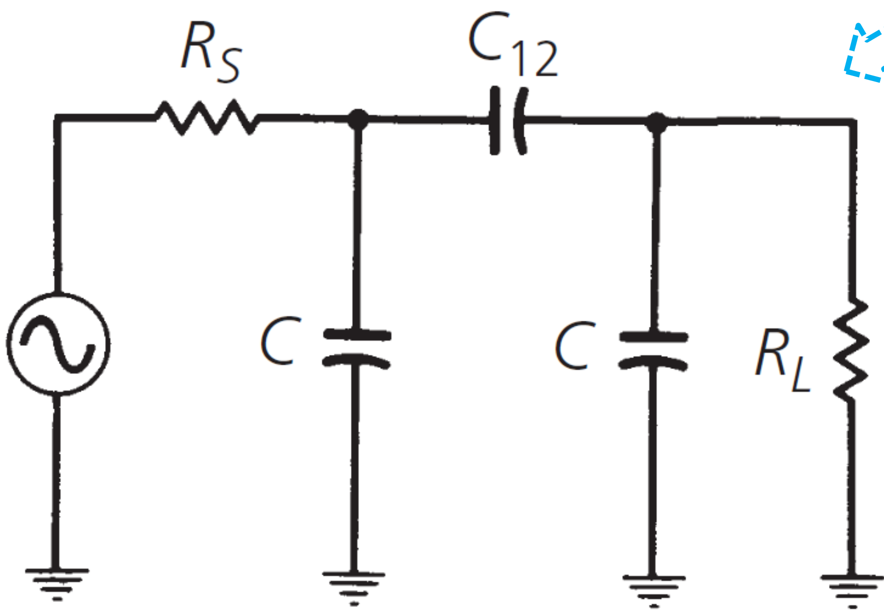

(B) *Above resonance*

- RF signals of very high frequencies will also pass to ground through the capacitors which have small impedance values ($|Z| = 1/(\omega C)$).

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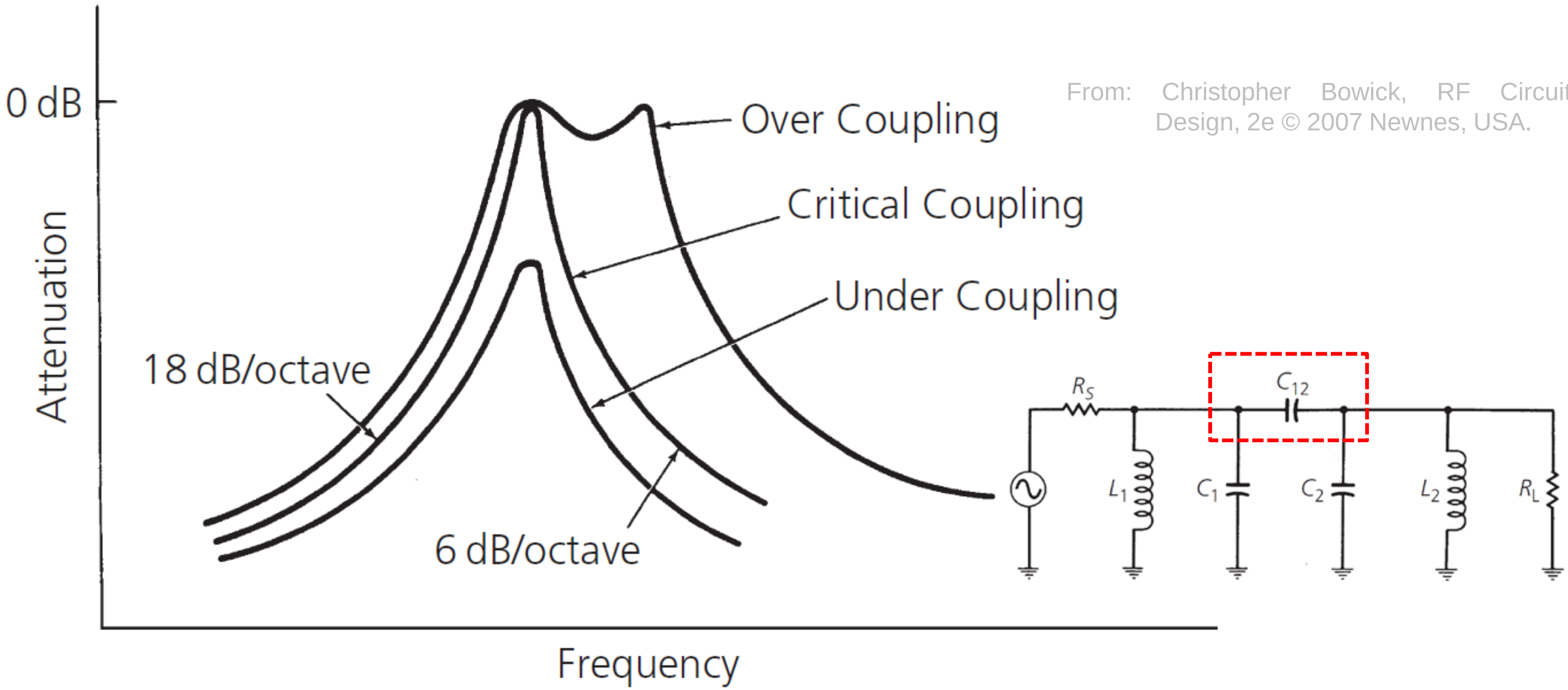
Band-Pass Filters – 2 parallel LC tanks (frequency response)



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- Depending on the capacitance $\boldsymbol{C_{12}}$ of the coupling capacitor, the **capacitive coupling** between the two LC tanks can be either too strong or too weak.
  - This affects the filtering performance.

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Attenuation of Unwanted Signals

(dB per octave vs dB per decade)

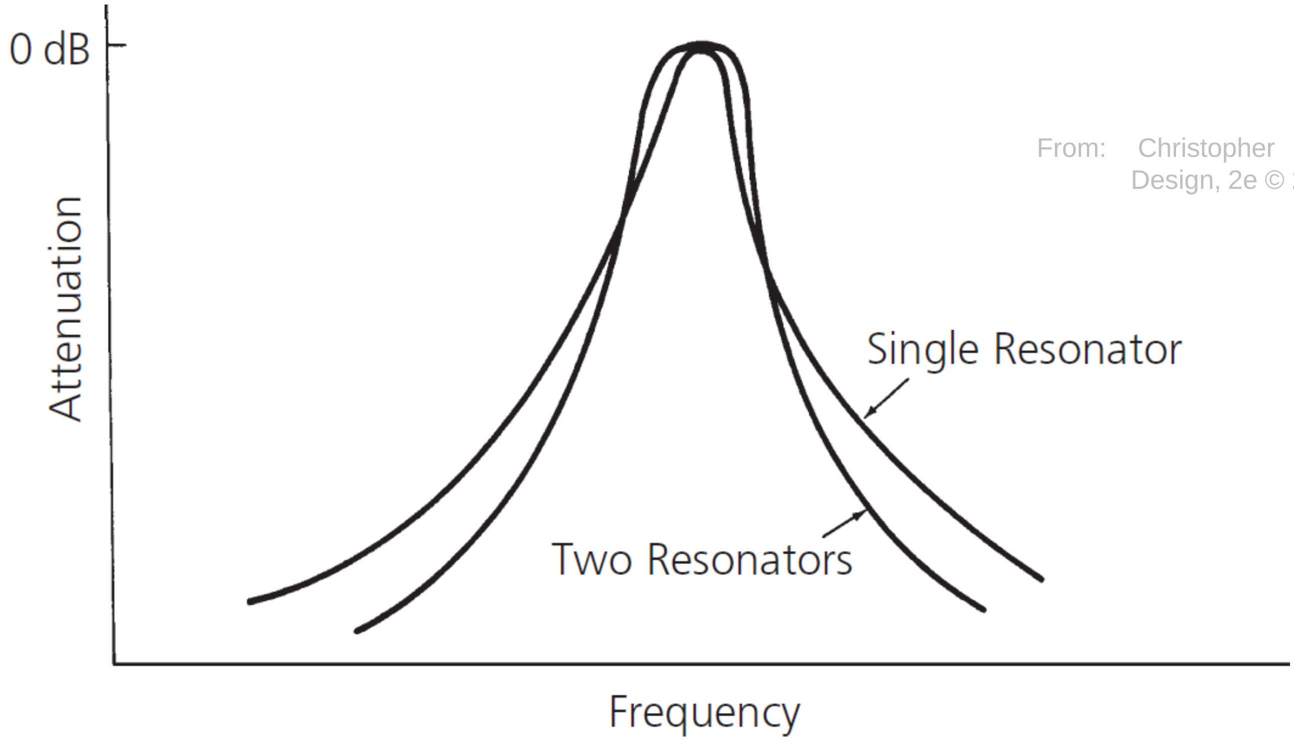
- In gauging the performance of a filter, one aspect is how steep it attenuates signals of unwanted frequencies. Such steepness is called **roll-off**.
- It is usual to measure roll-off as a function of logarithmic frequency. For this reason, the units of roll-off are either decibels per decade (**dB/decade**), where a decade is a 10-time increase in frequency, or **dB per octave**, where an octave is a 2-time increase in frequency.

$$\Delta L = 20\log_{10}(10) = 20\,\text{dB}/\text{decade}$$

$$\Delta L = 20\log_{10}(2) \approx 20 \times 0.3 = 6\,\text{dB}/\text{octave}$$

RF Filters of LC Resonant Circuits

(sharper filter with more LC tanks)



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- Quite obviously, a filter can have better roll-off (i.e. steeper attenuation of signals of unwanted frequencies) if more LC tanks are used.

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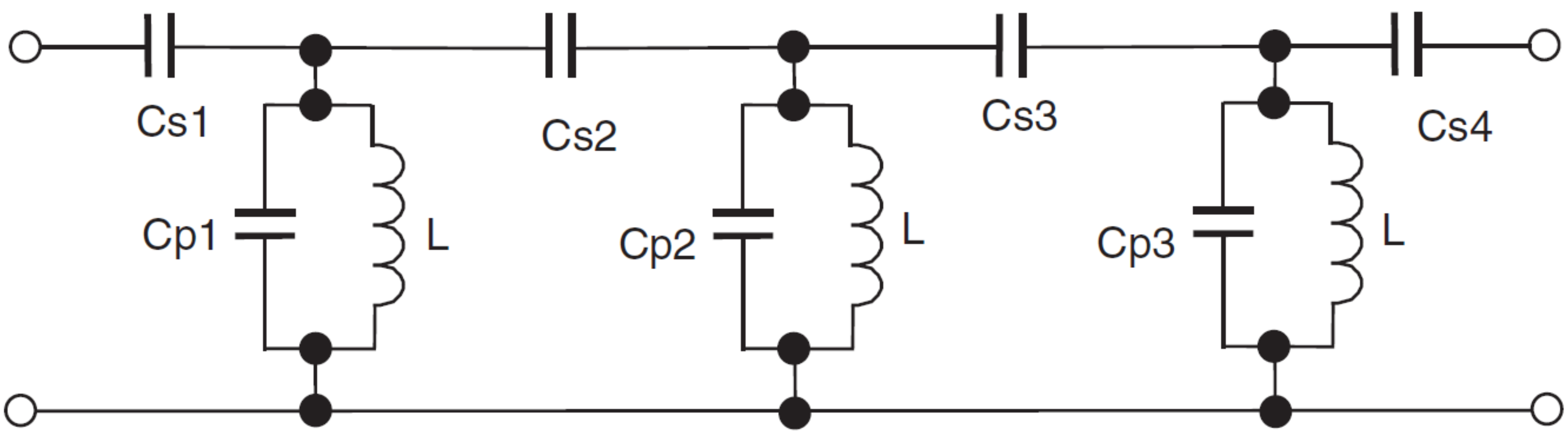
Band-Pass Filters – 3 parallel LC tanks (higher order)

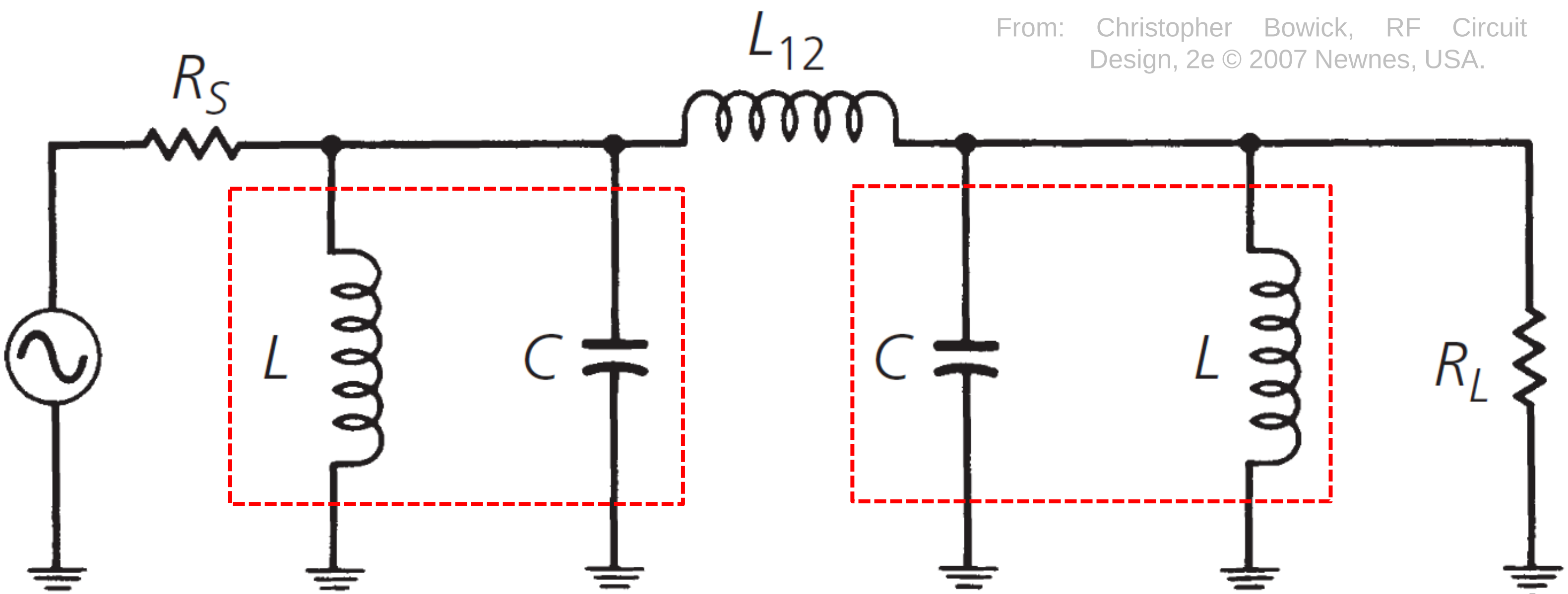
- ❑ More parallel LC tanks with capacitively coupling can be used for better filter performance.
  - ➢ What is the trade-off in such filters of higher order?
  - ➢ What would be the number of components? How about the signal propagation?



RF Filters of LC Resonant Circuits

(inductive coupled LC tanks)

- Apart from capacitive coupling, the parallel LC resonant circuits can also be coupled to each other, through an inductor $\boldsymbol{L_{12}}$.



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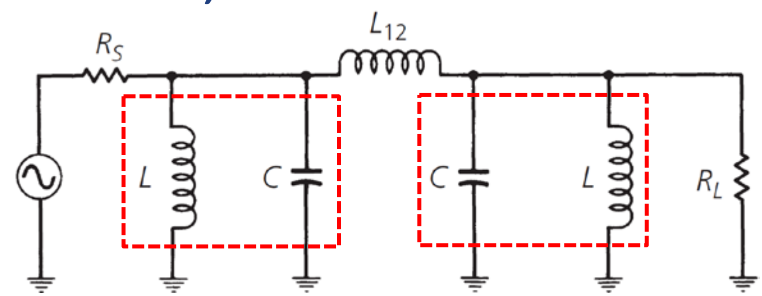
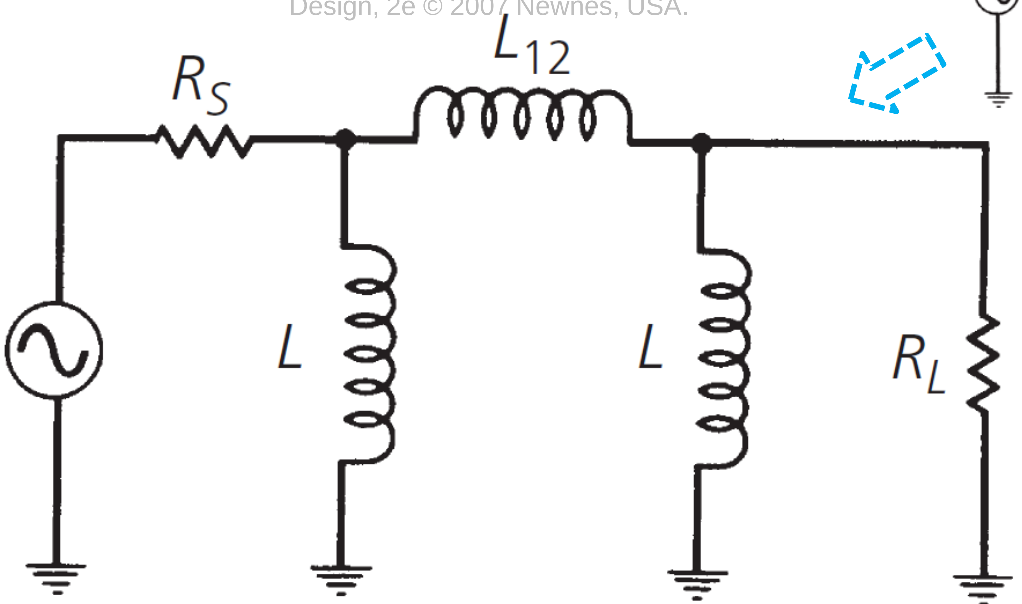
Inductively Coupled LC tanks

(below resonance)

- As frequency below the resonance, the circuit is equivalent to the following:





(A) *Below resonance*

- RF signals of very low frequencies will pass to ground through the inductors which have small impedance values ($|Z| = \omega L$).

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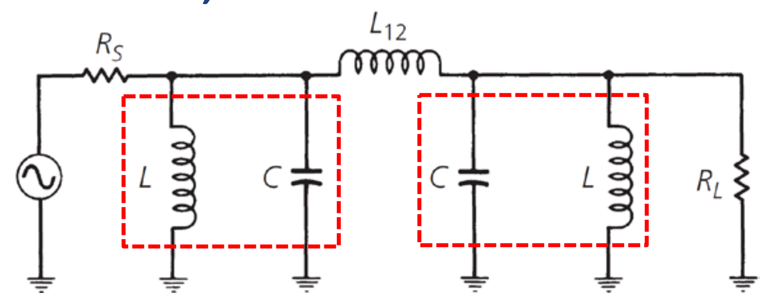
Inductively Coupled LC tanks

(above resonance)

- As frequency above the resonance, the circuit is equivalent to the following:



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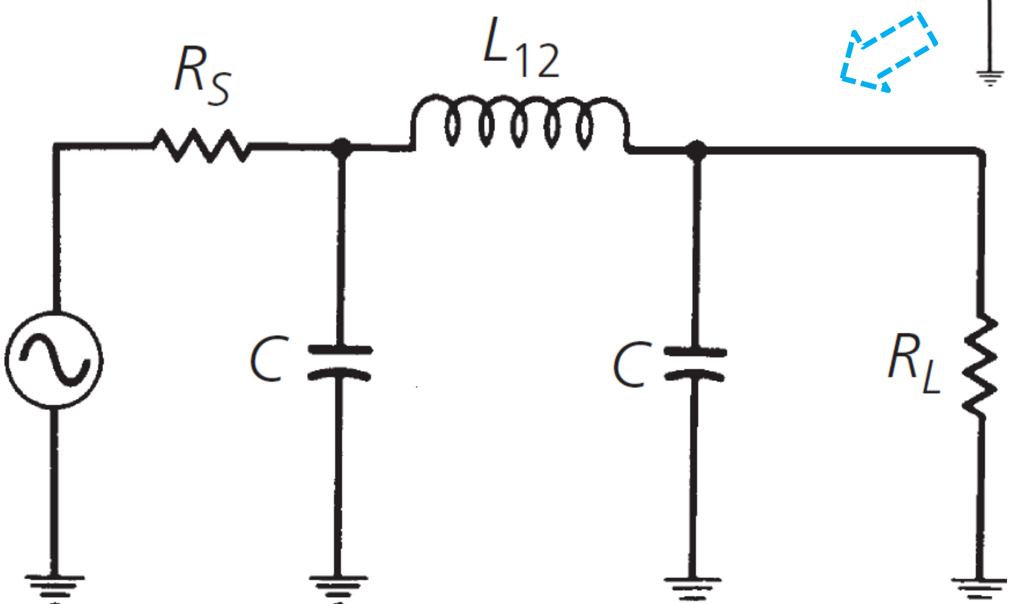
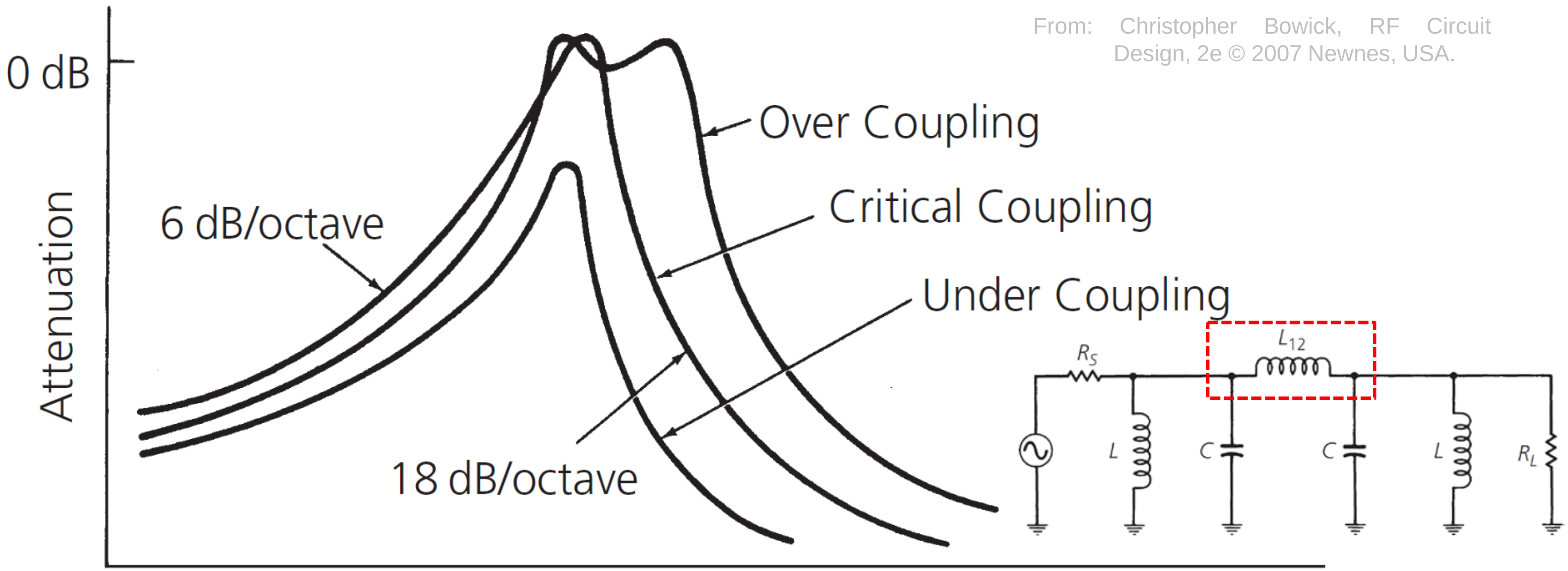


(B) *Above resonance*

- RF signals of very high frequencies will pass to ground through the capacitors which have small impedance values ($|Z| = 1/(\omega C)$).

Inductively Coupled LC tanks

(frequency response)



- ❑ Similar to the capacitive coupling case, the filtering performance depends on the inductance $\boldsymbol{L_{12}}$ of the coupling inductor.
  - ➢ It can be over- or under-coupling.

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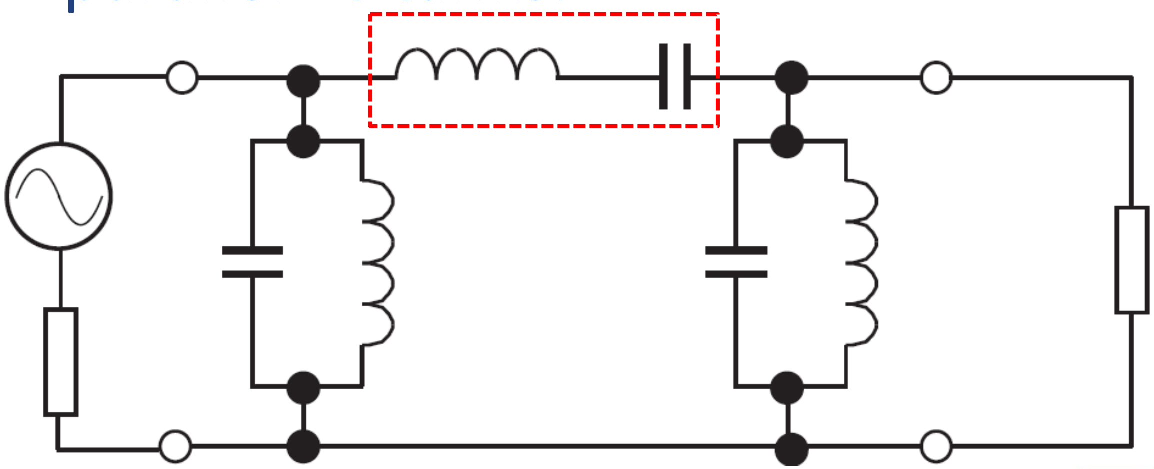
RF Band-Pass Filters

(LC coupling)

- To enjoy advantages of the better roll-off at frequencies below and above the resonance in capacitive and inductive coupling respectively, a series LC tank can be used for coupling the two parallel LC tanks.

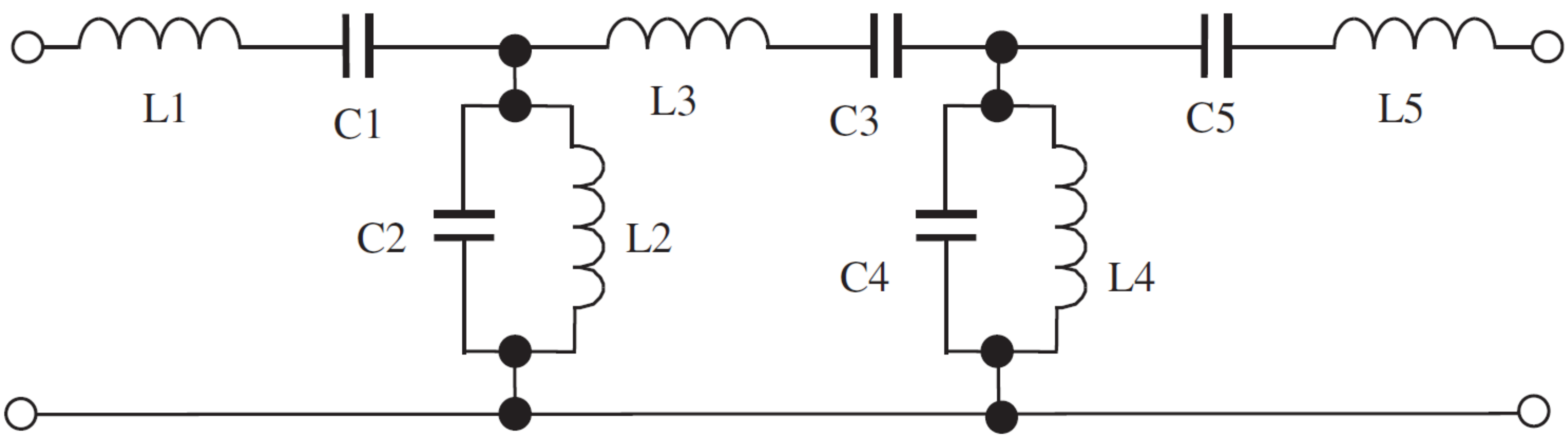


From: Ian Robertson *et al.*, Microwave and Millimetre-Wave Design for Wireless Communications. © 2016 John Wiley & Sons, USA.

RF Band-Pass Filters

(more series LC tanks)

- The band-pass filter can be somewhat improved more by using two more series LC tanks.
  - What is the trade-off for better filter performance?



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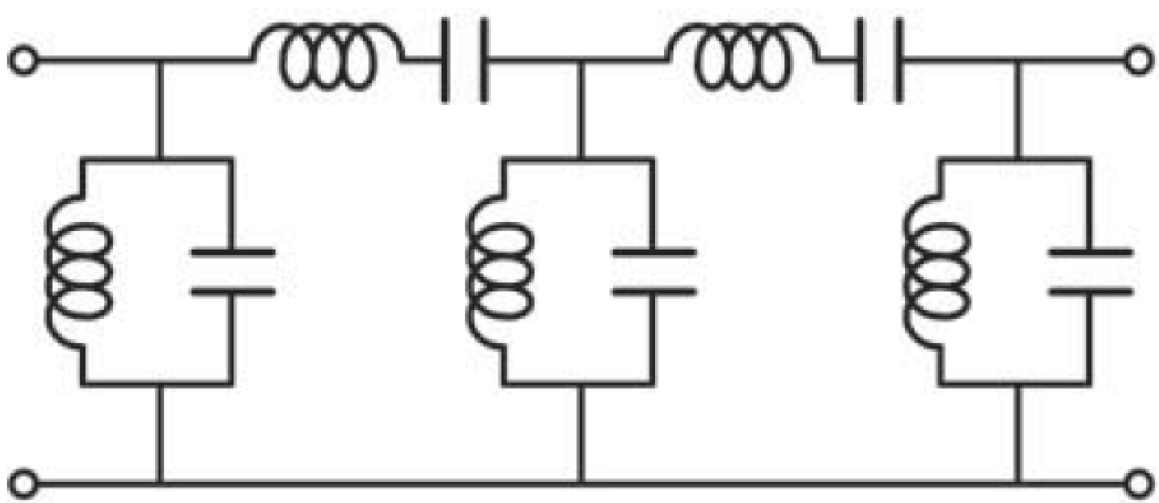
RF Band-Pass Filters

(higher order)

- The band-pass filter performance in terms of the roll-off can be improved in higher order filters (i.e. using more parallel LC tanks.



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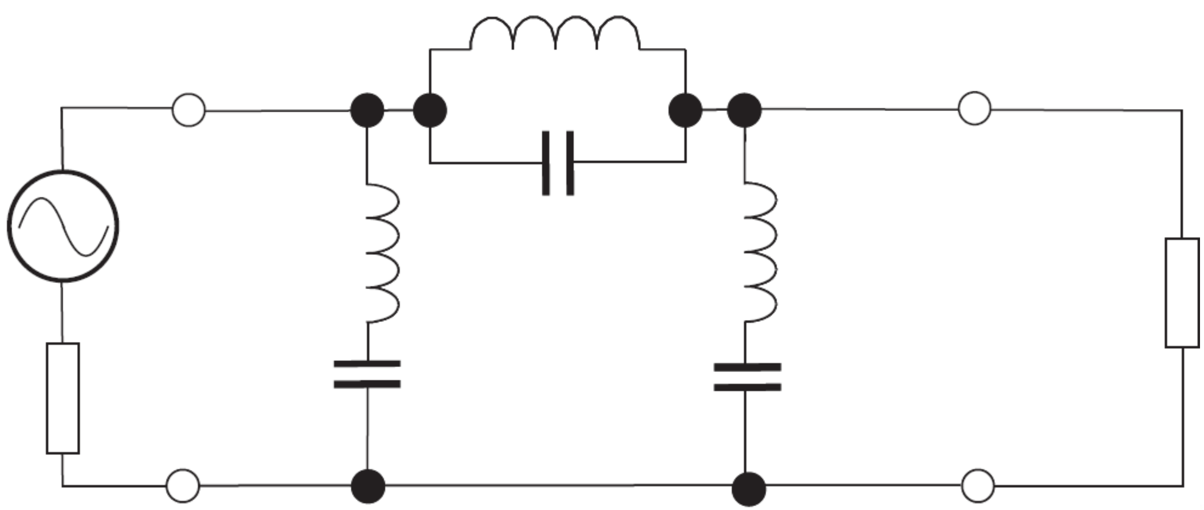
LC Resonant Circuit

(band-stop filter)

- The principles of using LC resonant circuits can be used to construct a band-stop filter.
- Instead of using parallel LC tanks connected to the ground, series LC tanks are used.



- The coupling can be through similarly a capacitor, inductor, or an LC tank.

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LC Filter Elements

(summary)

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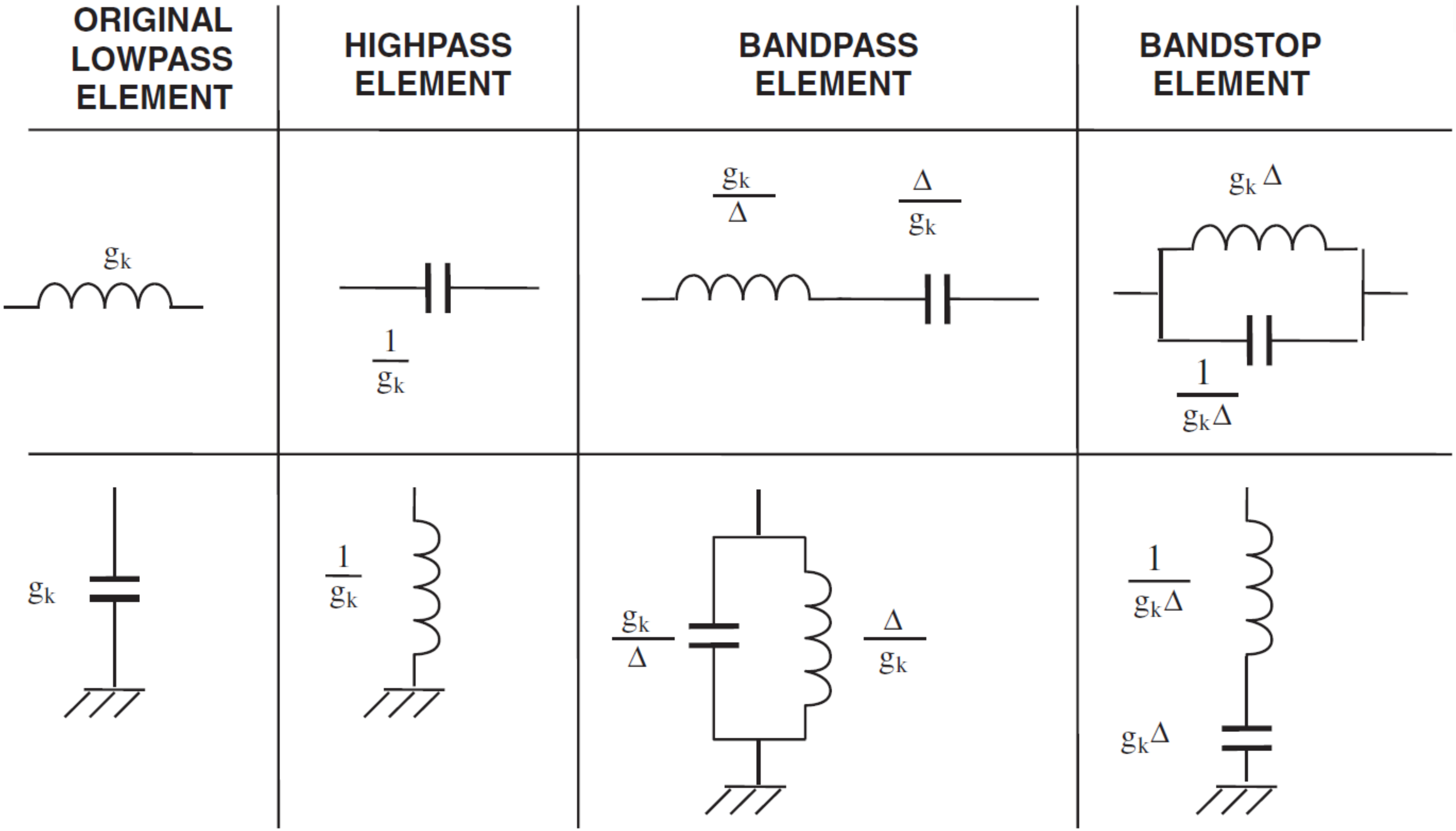

$$g_k = 2\sin\left[\frac{(2k-1)\pi}{2n}\right], \quad k = 1, 2, \ldots, n \qquad \Delta = \frac{\omega_2 - \omega_1}{\omega_c}$$

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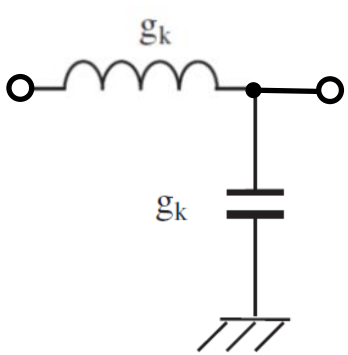
LC Filter Design

(determination of L & C values)

- To design an LC filter, inductors and capacitors are placed either along the signal path or across to ground.

- The ***L*** and ***C*** values can then be determined from the normalised component values (g_k or $1/g_k$ etc.) and the fixed corner or centre frequency ω_c.
  - a low-pass filter example with one inductor & one capacitor:



$$L_k = \frac{g_k R_0}{\omega_c} \qquad g_k = 2\sin\left[\frac{(2k-1)\pi}{2n}\right], \quad k = 1, 2, \ldots, n$$

n: number of *L* or *C* (***n*** = 2 in this case)

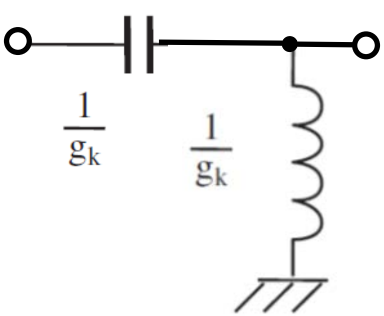
$$C_k = \frac{1}{\omega_c g_k R_0}$$

R_0: source/load resistance

LC Filter Design

(high-pass filter)

- The high-pass filter design is very similar to that of the low-pass filter counterpart, except that the normalised component values are inversed.
  - The ***L*** and ***C*** values are also determined from the normalised component values ($1/g_k$ in this case) and the fixed corner or centre frequency ω_c.
  - When $n = 2$, $g_1 = g_2 = 1.414$; $1/g_1 = 0.876$.



$$C_k = \frac{g_k}{\omega_c R_0}$$

$$L_k = \frac{R_0}{\omega_c g_k}$$

$$g_k = 2\sin\left[\frac{(2k-1)\pi}{2n}\right], \quad k = 1, 2, \dots, n$$

$$g_1 = 2\sin\left(\frac{\pi}{4}\right) = \sqrt{2} \qquad g_2 = 2\sin\left(\frac{3\pi}{4}\right) = \frac{2}{\sqrt{2}}$$

when $n = 2$

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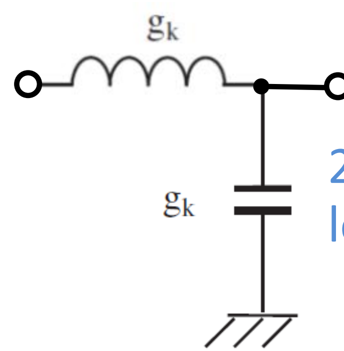
Normalised Component Values
(relation to reactance)

- Note that the normalised component values (***g**$_k$* or 1/***g**$_k$* etc.) are relative to ***R**$_0$*.
  - The reactance of the inductor or capacitor would simply be the normalised component values multiplied by ***R**$_0$*.

$$X_L = g_k R_0 \Rightarrow \omega_c L_k = g_k R_0$$

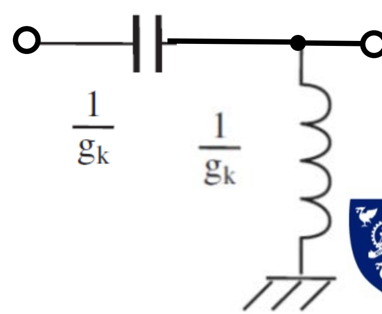
$$X_C = g_k R_0 \Rightarrow \frac{1}{\omega_c C_k} = g_k R_0$$



2-element
low-pass filter

$$X_L = \left(\frac{1}{g_k}\right) R_0 \Rightarrow \omega_c L_k = \frac{R_0}{g_k}$$

$$X_C = \left(\frac{1}{g_k}\right) R_0 \Rightarrow \frac{1}{\omega_c C_k} = \frac{R_0}{g_k}$$



2-element
high-pass filter

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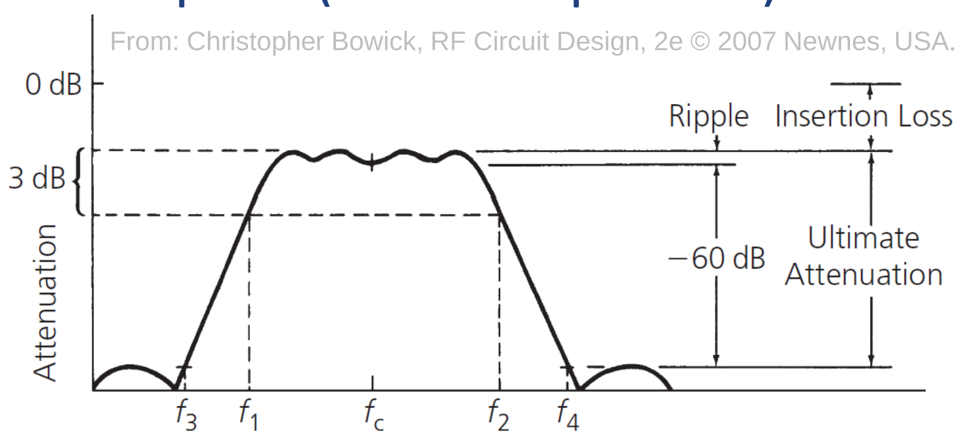
LC Filter Design

(band-pass & band stop)

- To design the band-pass and band-stop LC filters, the determination of the ***L*** and ***C*** values is also by de-normalising the normalised component values.
  - The normalised component values in these two cases contain the bandwidth fraction Δ.
  - ***n*** in these cases would be the number of ***LC*** resonant pairs (series or parallel).



$$\Delta = \frac{\omega_2 - \omega_1}{\omega_c} = \frac{f_2 - f_1}{f_c}$$

$$\omega_c = \sqrt{\omega_2 \omega_1} = 2\pi\sqrt{f_2 f_1} = 2\pi f_c$$

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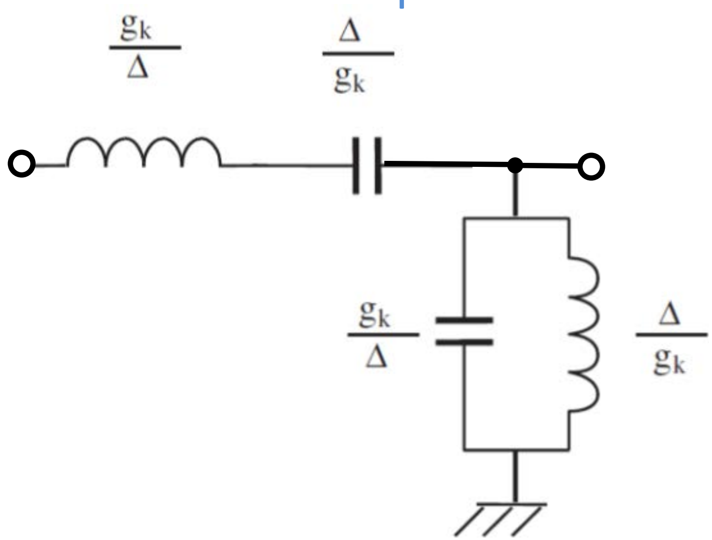
LC Filter Design

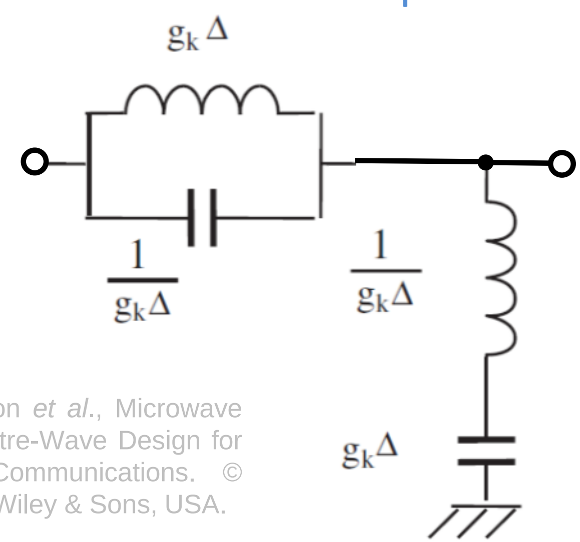
(band-pass & band stop)

- ❑ A two-element band-pass LC filter and band-stop filters are shown here with the corresponding normalised component values.

2-element band-pass filter



2-element band-stop filter



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Limitations of LC Filters

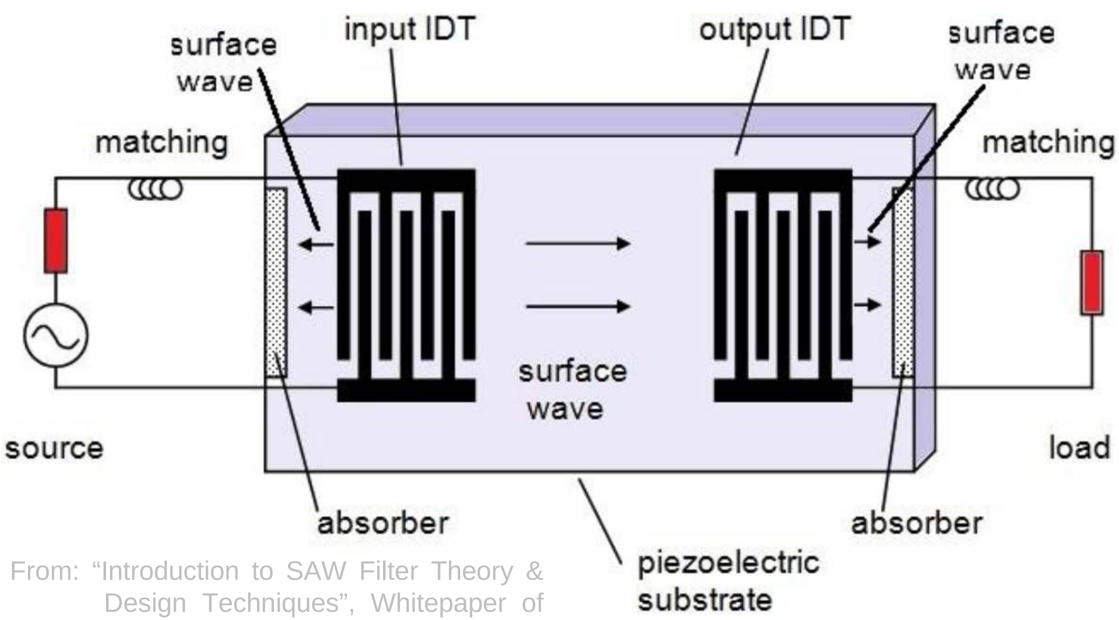
(not feasible in multi-GHz frequencies)

- Due to the parasitic circuit elements in real inductors and capacitors, LC filters are usually not feasible for operation at multi-GHz frequencies (e.g. above 2 GHz).
  - The **self-resonance** frequencies of especially inductors would set the maximum operation frequency of the LC filters.
  - The quality factor of mainly inductors would limit the filter performance of the LC filters.
- The **surface acoustic wave** (**SAW**) filters are used in modern communication circuits.
  - 30 MHz – 2.6 GHz

SAW Filter Structure

(use of piezoelectric material)

- ❑ SAW filters make use of a piezoelectric crystal substrate with deposited metal electrodes.
  - ➢ Acoustic waves of only the designed frequencies can propagate at the surface from the input to output ports.



From: "Introduction to SAW Filter Theory & Design Techniques", Whitepaper of API Technologies Corp.; available at: http://micro.apitech.com/pdf/whitepapers/SAW-Filter-WhitePaper.pdf

- ➢ SAW filters have far better filter performance than LC filters yet with fairly low cost.

IDT: **i**nter-**d**igitated **t**ransducer

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