Xi'an Jiaotong-Liverpool University
西交利物浦大学

EEE220 Instrumentation and Control System

2018-19 Semester 2

Dr. Qing Liu

Email: qing.liu@xjtlu.edu.cn

Office: EE516

Department of Electrical and Electronic Engineering

15 April, 2019

Lecture 15

Outline

The Time-Domain Performance of Feedback Systems

- ❑ Test Input Signals
- ❑ Performance of Second-Order System
- ❑ Effects of a Third Pole and a Zero on the Second-Order System Response
- ❑ **The s-Plane Root Location and the Transient Response**
- ❑ **The Steady-State Error of Feedback Control Systems**
- ❑ **System Simulation Using Matlab**

The s-Plane Location and The Transient Response

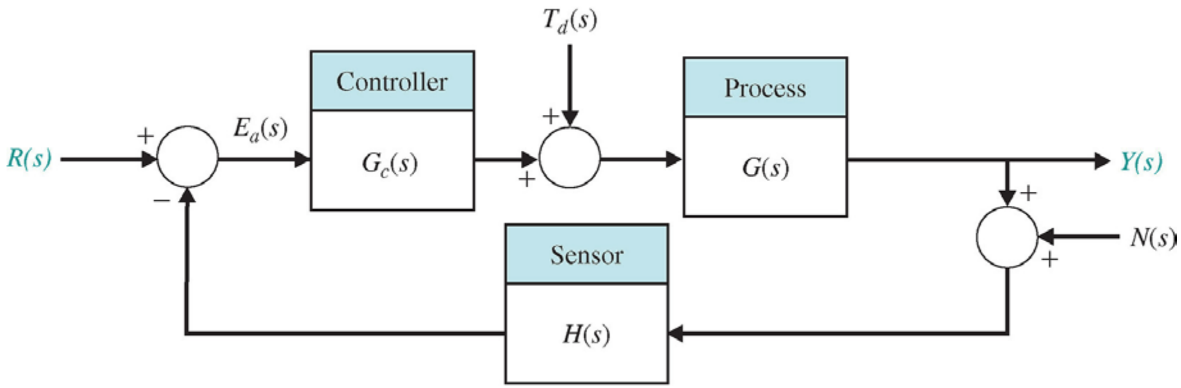
Transfer function for a closed-loop system can be written as:

$$T(s) = \frac{Y(s)}{R(s)} = \frac{\sum P_i(s)\Delta_i(s)}{\Delta(s)}$$

Characteristic equation of the system: $\Delta(s) = 0$

For a unit feedback control system: $\boldsymbol{\Delta(s) = 1 + G_c(s)G(s) = 0}$

Time response of a system depends on the poles and zeros of its transfer function $T(s)$; while for a closed-loop system, the poles of are the roots of the characteristic equation: $\Delta(s)$.



Time Response of System: General Form

If the system (with DC gain = 1) has no repeated roots, its unit step response can be formulated as a partial fraction expansion as:

$$Y(s) = \frac{1}{s} + \sum_{i=1}^{M} \frac{A_i}{s+\sigma_i} + \sum_{K=1}^{N} \frac{B_k s + C_k}{s^2 + 2\alpha_k s + (\alpha_k{}^2 + \omega_k{}^2)}$$

where A_i, B_k and C_k are constants; the roots of the system must be either

$$s = -\sigma_i \qquad \text{or} \qquad s = -\alpha_k \pm j\omega_k$$

The transient response can be obtained by inverse Laplace transform:

$$y(t) = 1 + \sum_{i=1}^{M} A_i e^{-\sigma_i t} + \sum_{k=1}^{N} D_k e^{-\alpha_k t} \sin(\omega_k t + \theta_k)$$

Steady-state output

exponential terms

Damped sinusoidal terms

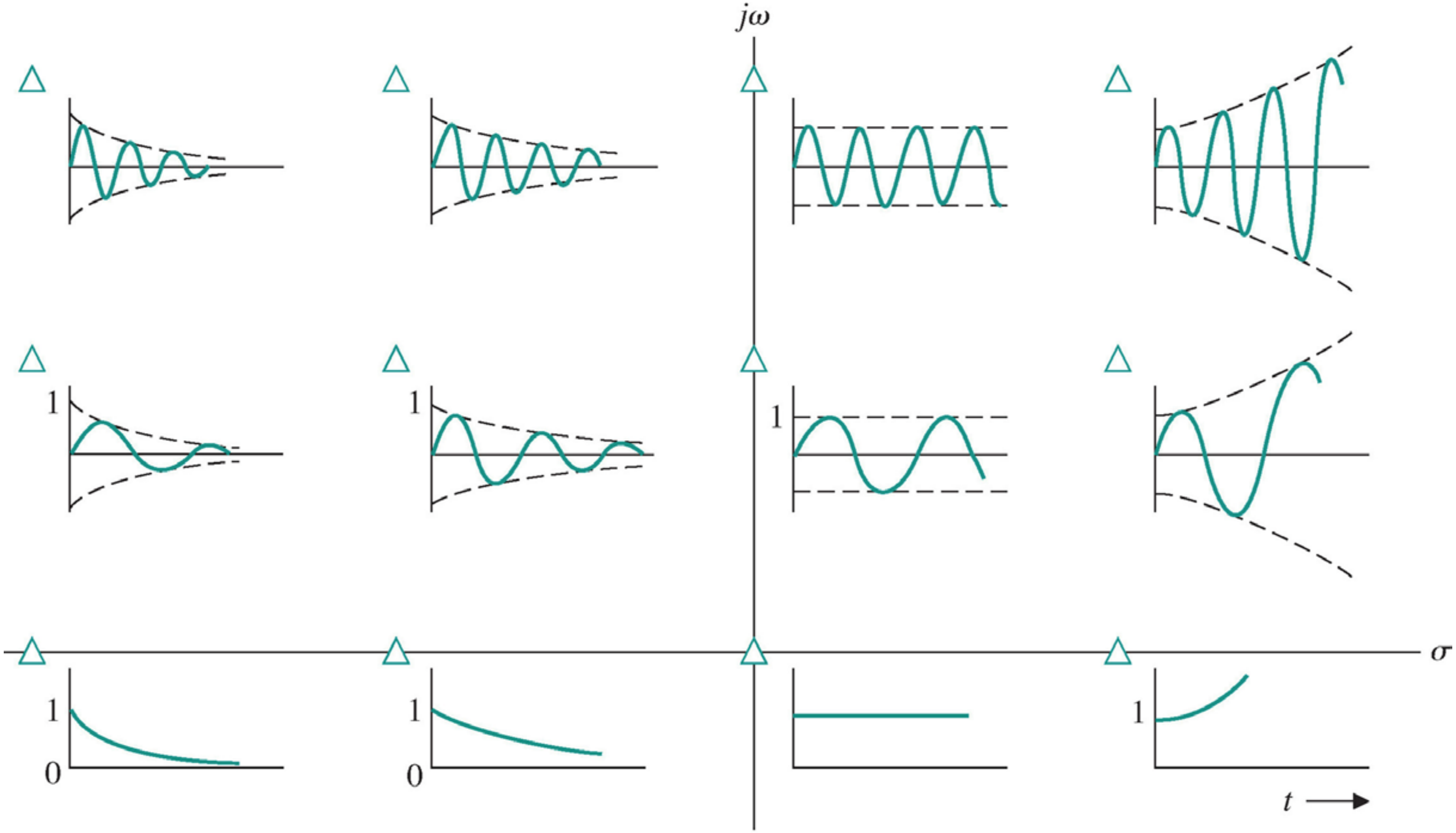
where D_k is a constant depends on B_k, C_k, α_k and ω_k.

For the response to be stable (bounded for a step input) – the **real part of the poles must be in the left-hand portion of the s-plane**.

Impulse Response for Various Root Locations in the s-Plane

$$y(t) = 1 + \sum_{i=1}^{M} A_i e^{-\sigma_i t} + \sum_{k=1}^{N} D_k e^{-\alpha_k t} \sin(\omega_k t + \theta_k)$$

(The conjugate root is not shown in this figure.)



Root Location and System Design

- It is important for the control system designer to understand the complete relationship of the frequency domain representation of a linear system, the poles and zeros of its transfer function, and its time-domain response to step and other inputs;

- In such areas as signal processing and control, many analysis and design calculations are done in the s-plane, where a system model is represented in terms of the poles and zeros of its transfer function;

- The control system designer will envision the effects of the step and impulse response of adding, deleting, or moving poles and zeros of $T(s)$ in the s-plane;

- An experienced designer is aware of the effects of zero locations on system response. For example, moving a zero closer to a specific pole will reduce the relative contribution to the output response. In other words, if there is a zero near the pole at $s = -\sigma_i$, then A_i will be much smaller in magnitude.

The Steady-State Error of Feedback Control System

One of the fundamental reasons for using feedback, despite its cost and increased complexity, it the attendant improvement in the reduction of the stead-state error of the system. Consider a unit negative feedback system ($H(s) = 1$), in the absence of external disturbances ($T_d(s) = 0$) and measurement noise ($N(s) = 0$), tracking error is:

$$E(s) = \frac{1}{1 + G_c(s)G(s)} R(s)$$

Using the final value theorem, the **steady-state error** is:

$$\boxed{\lim_{t \to \infty} e(t) = e_{ss} = \lim_{s \to 0} s \frac{1}{1 + G_c(s)G(s)} R(s)}$$

Steady-State Error to Step Inputs

- **Step Input** of magnitude A:

$$e_{ss} = \lim_{s\to 0} s \frac{A/_{s}}{1 + G_c(s)G(s)} = \frac{A}{1 + \lim\limits_{s\to 0} G_c(s)G(s)}$$

The loop transfer function can be written in general form as

$$G_c(s)G(s) = \frac{K\prod_{i=1}^{M}(s + z_i)}{s^{N}\prod_{k=1}^{Q}(s + p_k)} \qquad \text{where } z_i \neq 0, p_k \neq 0.$$

The number of integration indicates a system with **type number** that is equal to ***N***, which determines the steady-state error of the system.

Given the **position error constant**: $\boxed{K_p = \lim\limits_{s\to 0} G_c(s)G(s)}$

- For a type-zero system ($N = 0$):

$$e_{ss} = \frac{A}{1 + K_p}$$

- For a type-N system with $N \geq 1$:

$$e_{ss} = \lim_{s\to 0} \frac{A}{1 + K\prod z_i/(s^N \prod p_k)} = \lim_{s\to 0} \frac{As^N}{s^N + K\prod z_i/(\prod p_k)} = 0.$$

Steady-State Error to Ramp Inputs

- **Ramp Input** with a slope A:

$$e_{ss} = \lim_{s\to 0} s \frac{A/_{s^2}}{1 + G_c(s)G(s)} = \frac{A}{s + \lim_{s\to 0} sG_c(s)G(s)} = \frac{A}{\lim_{s\to 0} sG_c(s)G(s)}$$

Denote the **velocity error constant**: $\boxed{K_v = \lim_{s\to 0} sG_c(s)G(s)}$

- For a type-zero system ($N = 0$):

$$e_{ss} = \infty$$

- For a type-one system ($N = 1$):

$$e_{ss} = \lim_{s\to 0} \frac{A}{sK\prod(s + z_i)/[s\prod(s + p_k)]} = \frac{A}{K\prod z_i/\prod p_k} = \frac{A}{K_v}$$

- For a type-N system with $N > 1$:

$$e_{ss} = 0$$

Steady-State Error to Acceleration Inputs

- **Acceleration Input** $R(s) = A/s^3$ $(r(t) = At^2/2)$:

$$e_{ss} = \lim_{s\to 0} s\frac{A/_{s^3}}{1+G_c(s)G(s)} = \frac{A}{s^2 + \lim_{s\to 0} s^2 G_c(s)G(s)} = \frac{A}{\lim_{s\to 0} s^2 G_c(s)G(s)}$$

Denote the **acceleration error constant**: $\boxed{K_a = \lim_{s\to 0} s^2 G_c(s)G(s)}$

- For a type-N system with $N < 2$:

$$e_{ss} = \infty$$

- For a type-two system ($N = 2$):

$$e_{ss} = \lim_{s\to 0} \frac{A}{s^2 K\prod(s+z_i)/[s^2\prod(s+p_k)]} = \frac{A}{K\prod z_i/\prod p_k} = \frac{A}{K_a}$$

- For a type-N system with $N > 2$:

$$e_{ss} = 0$$

Summary Table

Table 5.2 Summary of Steady-State Errors

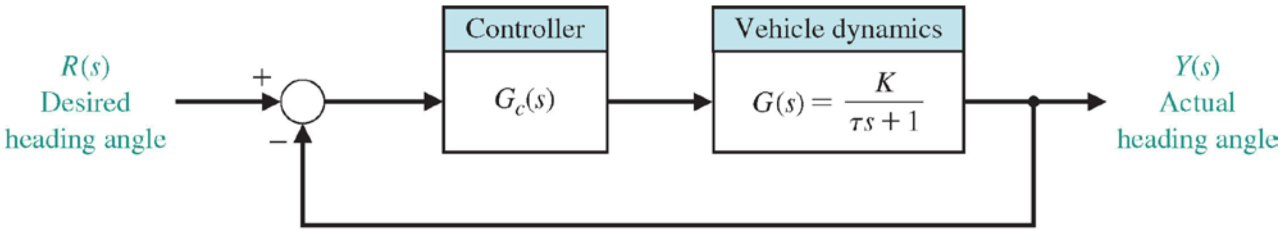
Number of Integrations in $G_c(s)G(s)$, Type Number	Input		
	Step, $r(t) = A$, $R(s) = A/s$	Ramp, $r(t) = At$, $R(s) = A/s^2$	Parabola, $r(t) = At^2/2$, $R(s) = A/s^3$
0	$e_{ss} = \dfrac{A}{1 + K_p}$	∞	∞
1	$e_{ss} = 0$	$\dfrac{A}{K_v}$	∞
2	$e_{ss} = 0$	0	$\dfrac{A}{K_a}$

❖ The control system **error constants** K_p, K_v and K_a , describe the ability of a system to reduce or eliminate the steady-state error. Therefore, they are utilized as numerical measure of the steady-state performance. The designer determines the error constants for a given system and attempts to determine methods of increasing the error constants while maintaining an acceptable transient response.

Example 15.1: Mobile Robot Steering Control

Consider the following system of mobile robot. Transfer function of controller is

$$G_c(s) = K_1 + K_2/s$$



Loop transfer function: $G_c(s)G(s) = \frac{K(K_1 s + K_2)}{\tau s^2 + s}$

- When $K_2 = 0$ -> type-0 system:

  For step input: $e_{ss} = \frac{A}{1 + K_p}$ where

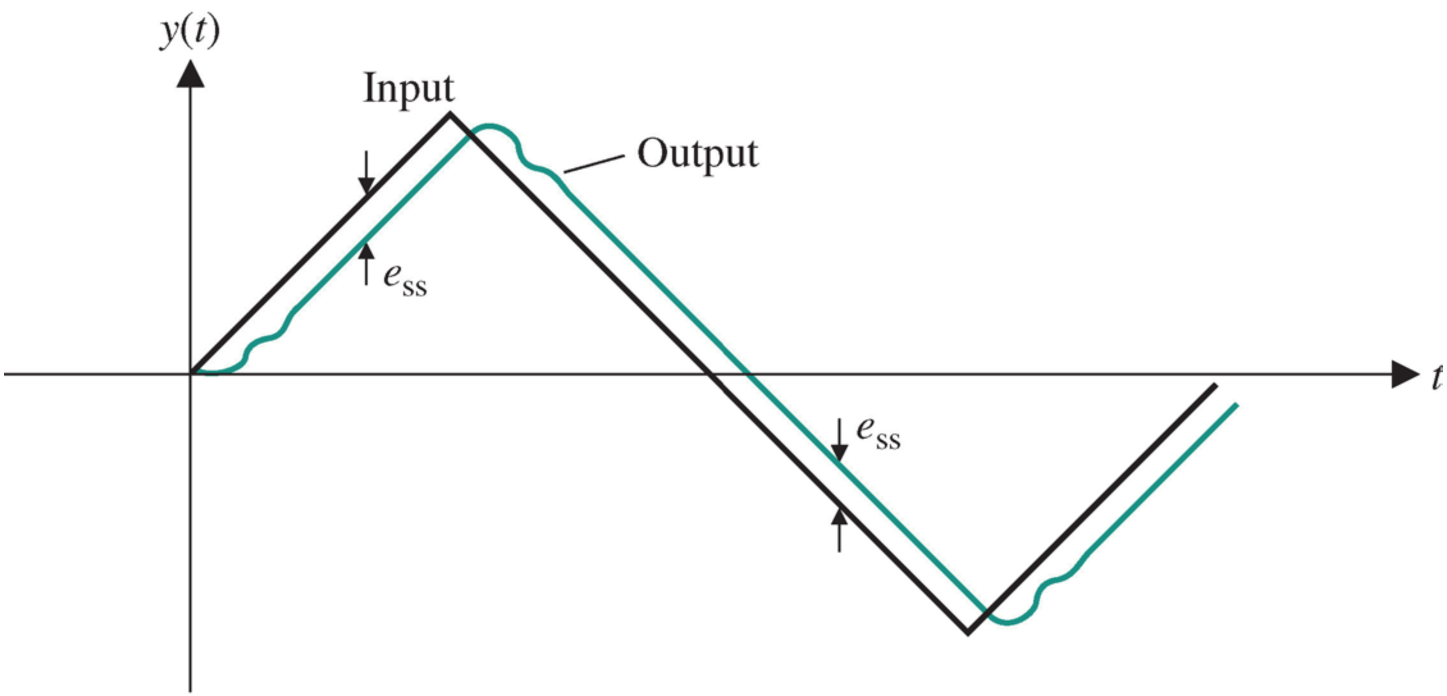
  For ramp input: $e_{ss} = \infty$

- When $K_2 > 0$ -> type-1 system:

  For step input: $e_{ss} = 0$

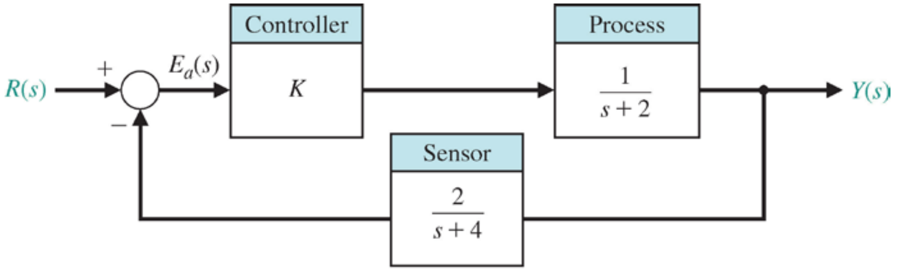
  For ramp input: $e_{ss} = \frac{A}{K_v}$ where $K_v = \lim_{s \to 0} sG_c(s)G(s) = K_2 K$

Transient response of the system to a triangular wave input when $K_2 > 0$



Example 15.2:Steady-State Error for A Nonunity Negative Feedback System

Consider the following system, determine K so that the ESS for a unit step input is minimized.



Solutions:

$$E(s) = R(s) - Y(s) = [1 - T(s)]R(s)$$

$$T(s) = \frac{G_c(s)G(s)}{1 + G_c(s)G(s)H(s)} = \frac{K(s+4)}{(s+2)(s+4)+2K}$$

For a unit step input: $e_{ss} = \lim_{s \to 0} sE(s) = 1 - T(0)$

To minimize ESS, it requires: $T(0) = \frac{4K}{8+2K} = 1$

Therefore: $K = 4$ will yield a zero steady-state error.

Performance Index

A performance index is a quantitative measure of the performance of a system and is chosen so that emphasis is given to the important system specifications.

-- a system is considered an optimum control system when the system parameters are adjusted so that the index reaches an extremum, commonly a minimum value.

Common performance index:

$$\mathrm{ISE} = \int_0^T e^2(t)\, dt.$$

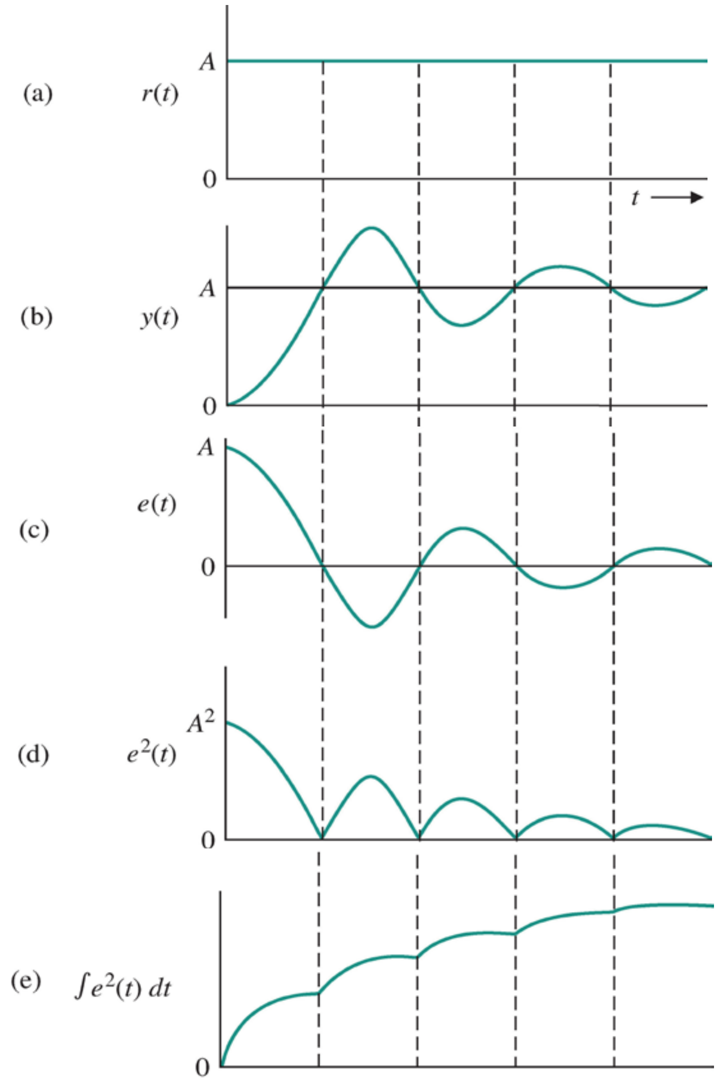
$$\mathrm{IAE} = \int_0^T |e(t)|\, dt.$$

$$\mathrm{ITAE} = \int_0^T t|e(t)|\, dt$$

$$\mathrm{ITSE} = \int_0^T te^2(t)\, dt.$$

General form:

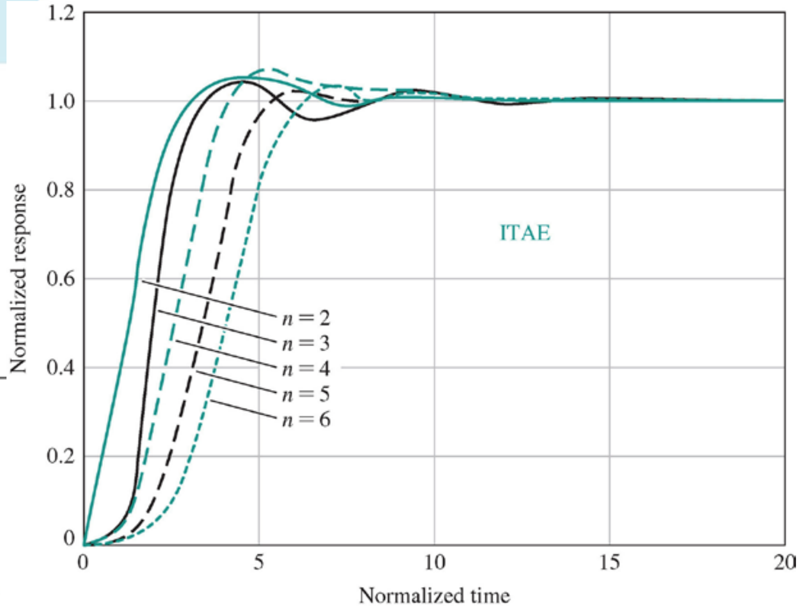
$$I = \int_0^T f(e(t), r(t), y(t), t)\, dt.$$


(a)
r(t)
A
0
t
(b)
y(t)
A
0
(c)
e(t)
A
0
(d)
e²(t)
A²
0
(e)
∫e²(t) dt
0

Optimum Coefficients for T(s) based on ITAE Criterion

Table 5.3 The Optimum Coefficients of $T(s)$ Based on the ITAE Criterion for a Step Input

$$s + \omega_n$$
$$s^2 + 1.4\omega_n s + \omega_n^2$$
$$s^3 + 1.75\omega_n s^2 + 2.15\omega_n^2 s + \omega_n^3$$
$$s^4 + 2.1\omega_n s^3 + 3.4\,\omega_n^2 s^2 + 2.7\omega_n^3 s + \omega_n^4$$
$$s^5 + 2.8\omega_n s^4 + 5.0\omega_n^2 s^3 + 5.5\omega_n^3 s^2 + 3.4\omega_n^4 s + \omega_n^5$$
$$s^6 + 3.25\omega_n s^5 + 6.60\omega_n^2 s^4 + 8.60\omega_n^3 s^3 + 7.45\omega_n^4 s^2 + 3.95\omega_n^5 s + \omega_n^6$$



(c)

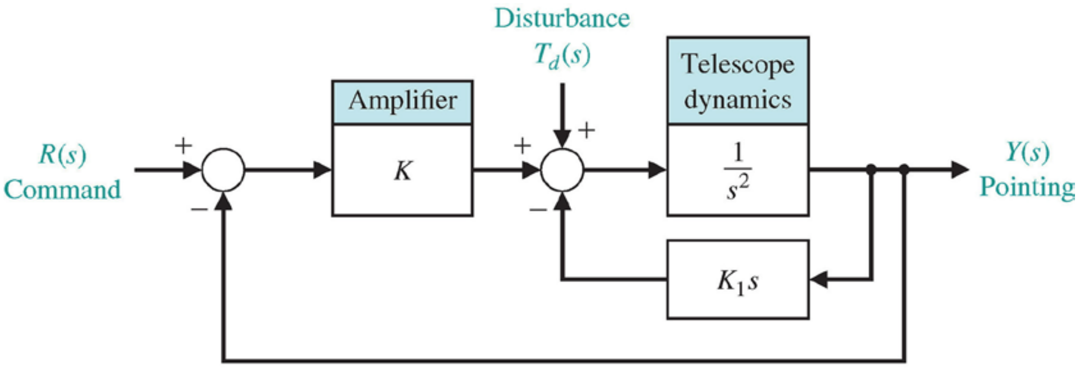
Table 5.4 The Optimum Coefficients of $T(s)$ Based on the ITAE Criterion for a Ramp Input

$$s^2 + 3.2\omega_n s + \omega_n^2$$
$$s^3 + 1.75\omega_n s^2 + 3.25\omega_n^2 s + \omega_n^3$$
$$s^4 + 2.41\omega_n s^3 + 4.93\omega_n^2 s^2 + 5.14\omega_n^3 s + \omega_n^4$$
$$s^5 + 2.19\omega_n s^4 + 6.50\omega_n^2 s^3 + 6.30\omega_n^3 s^2 + 5.24\omega_n^4 s + \omega_n^5$$

Design Example 15.3: Hubble Space Telescope Control

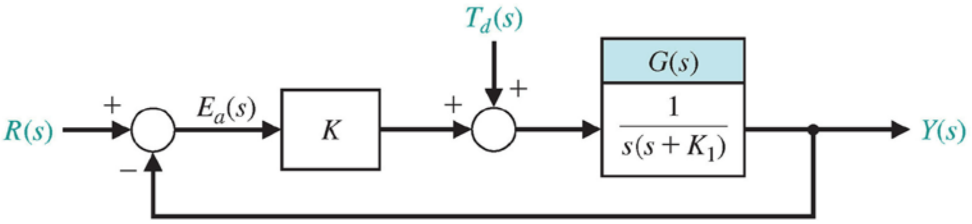
For the following control system, choose K_1 and K, to satisfy:

(1) Percent overshoot of the output to a step command r(t) is $P.O. \leq 10\%$
(2) Steady-state error to a ramp command is minimized;
(3) Effect of a step disturbance is reduced.



(a)

Step 1. Re-arrange the block diagram to achieve a standard form.



(b)

Step 2. Obtain Y(s) and E(s) in terms of R(s), T_d(s) and system parameters.

$$Y(s) = \frac{KG(s)}{1+KG(s)}R(s) + \frac{G(s)}{1+KG(s)}T_d(s)$$

$$E(s) = R(s) - Y(s) = \frac{1}{1+KG(s)}R(s) - \frac{G(s)}{1+KG(s)}T_d(s)$$

Step 3. Consider requirement (1): $P.O. \leq 10\%$ *for a step input.*

Characteristic equation of the system is

$$1 + KG(s) = 0 \longrightarrow s^2 + K_1 s + K = 0$$

Standard form: $s^2 + 2\zeta\omega_n s + \omega_n{}^2$

$$2\zeta\omega_n = K_1, \qquad \omega_n{}^2 = K.$$

For $P.O. \leq 10\%$, it must be satisfied that $\zeta \geq 0.6$. we choose ζ=0.6, therefore

$$\frac{K_1}{1.2} = \sqrt{K}$$

Step 4. Consider requirement (2): minimize ESS to a ramp input.

$$E(s) = \frac{1}{1 + KG(s)} R(s) = \frac{s^2 + K_1 s}{s^2 + K_1 s + K} \frac{A}{s^2}$$

$$e_{ss} = \lim_{s \to 0} sE(s) = \frac{A}{K/K_1}$$

To minimize ESS, we need large value of K/K_1.

Step 5. Consider requirement (3): minimize ESS to a step disturbance.

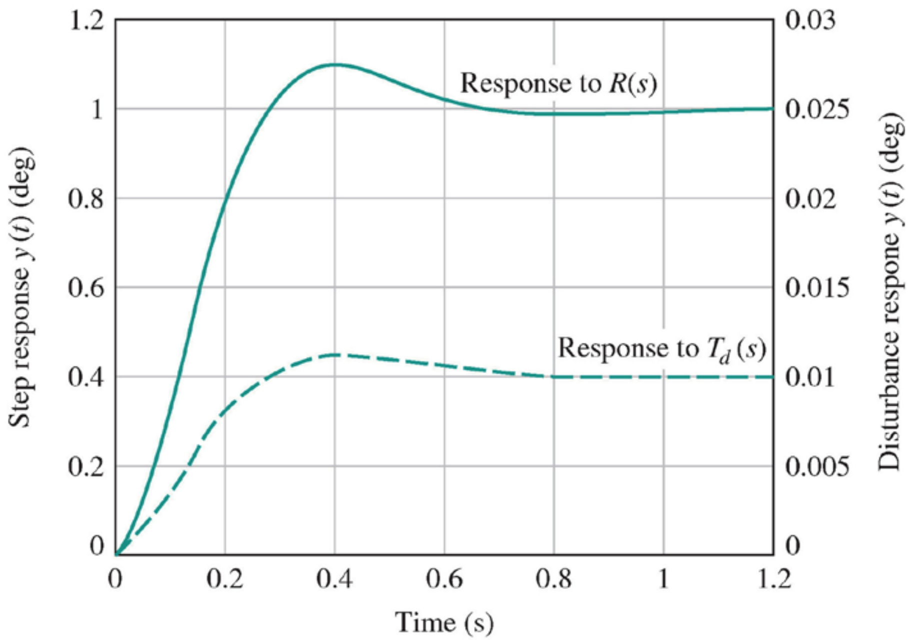
$$E(s) = -\frac{G(s)}{1 + KG(s)} T_d(s) = -\frac{1}{s^2 + K_1 s + K} \frac{B}{s}$$

$$e_{ss} = \lim_{s \to 0} sE(s) = -\frac{B}{K}$$

To minimize ESS, we need large value of K.

Step 6. Choose suitable values.

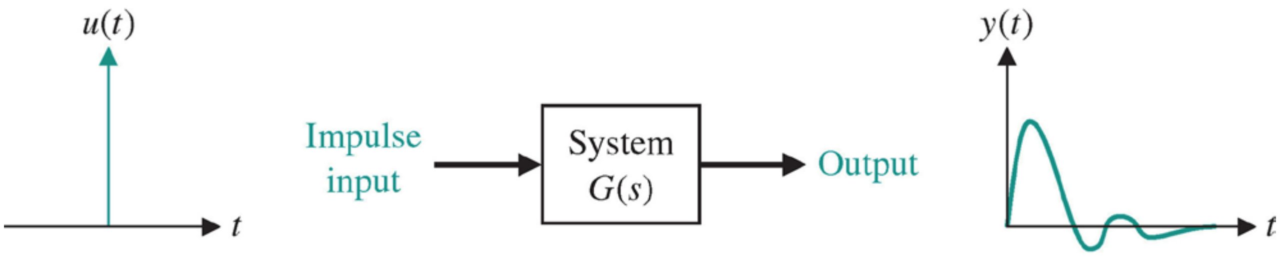
Can choose $K = 100$, then according to $\frac{K_1}{1.2} = \sqrt{K}$, $K_1 = 12$, and $K/K_1 = 8.33$.

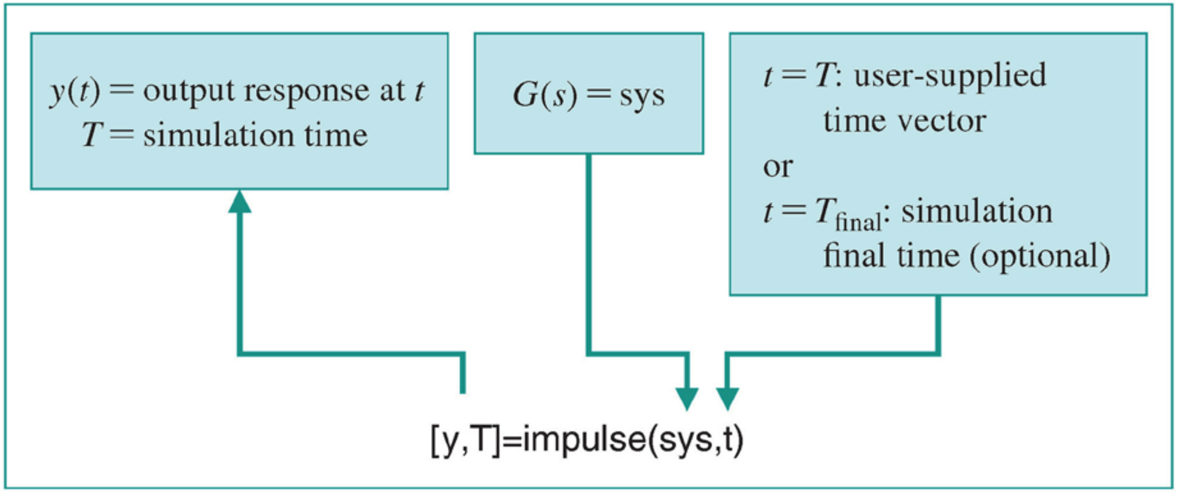


Therefore,
ESS for a ramp input is $\frac{A}{8.33} \approx 0.12A$, ESS for a step disturbance is $-\frac{B}{100} = -0.01B$.
All the requirements have been satisfied.

System Performance Simulation Using Matlab

The **impulse** and **Step** function.



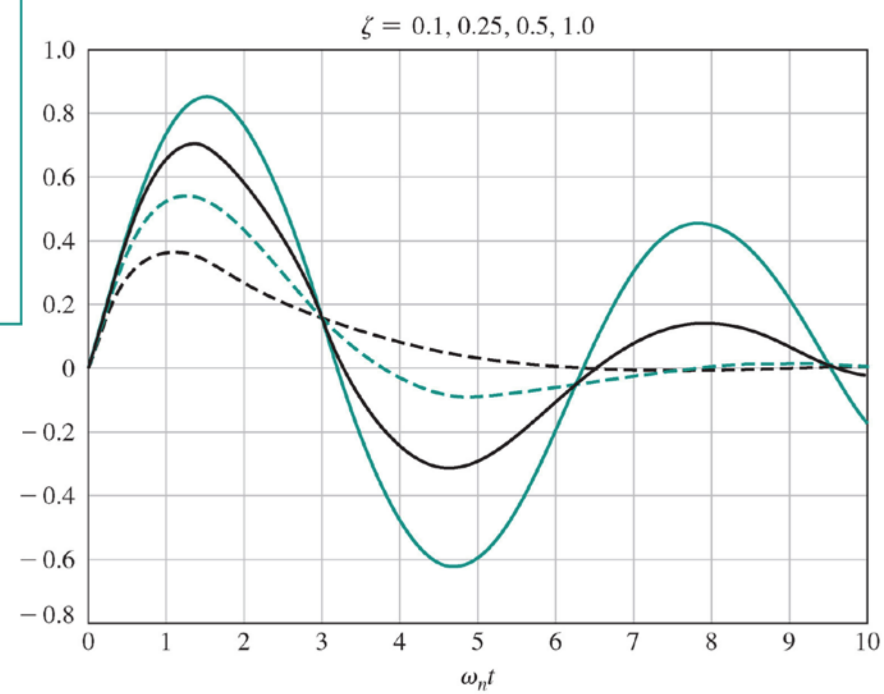


```
%Compute impulse response for a second-order system
%Duplicate Figure 5.5
%
t=[0:0.1:10]; num=[1];
zeta1=0.1; den1=[1 2*zeta1 1]; sys1=tf(num,den1);
zeta2=0.25; den2=[1 2*zeta2 1]; sys2=tf(num,den2);
zeta3=0.5; den3=[1 2*zeta3 1]; sys3=tf(num,den3);
zeta4=1.0; den4=[1 2*zeta4 1]; sys4=tf(num,den4);
%
[y1,T1]=impulse(sys1,t);
[y2,T2]=impulse(sys2,t);
[y3,T3]=impulse(sys3,t);
[y4,T4]=impulse(sys4,t);
%
plot(t,y1,t,y2,t,y3,t,y4)
xlabel(' \omega _nt'), ylabel('y(t)/\omega_n')
title('\zeta = 0.1, 0.25, 0.5, 1.0'), grid
```

Compute impulse response.

Generate plot and labels.

(b)



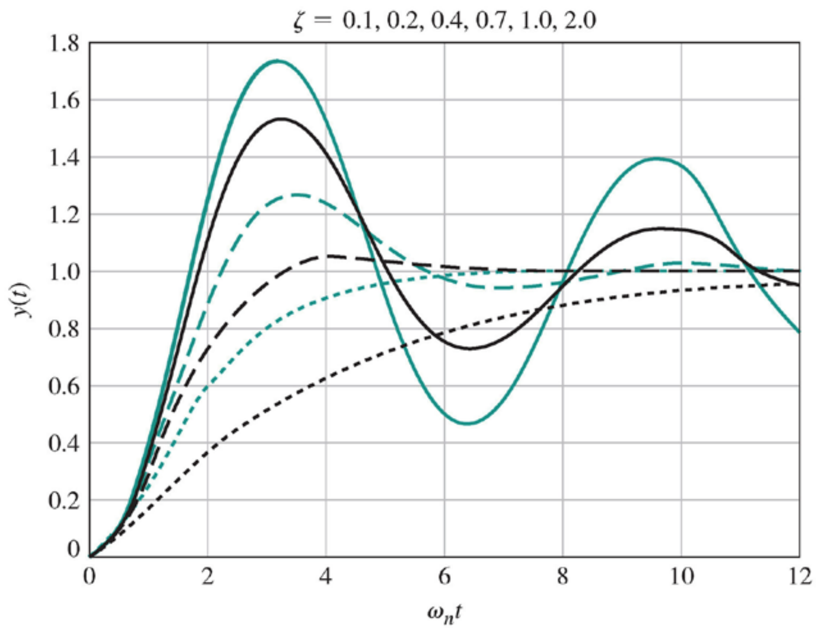
(a)

```
%Compute step response for a second-order system
%Duplicate Figure 5.4
%
t=[0:0.1:12]; num=[1];
zeta1=0.1; den1=[1 2*zeta1 1]; sys1=tf(num,den1);
zeta2=0.2; den2=[1 2*zeta2 1]; sys2=tf(num,den2);
zeta3=0.4; den3=[1 2*zeta3 1]; sys3=tf(num,den3);
zeta4=0.7; den4=[1 2*zeta4 1]; sys4=tf(num,den4);
zeta5=1.0; den5=[1 2*zeta5 1]; sys5=tf(num,den5);
zeta6=2.0; den6=[1 2*zeta6 1]; sys6=tf(num,den6);
%
[y1,T1]=step(sys1,t); [y2,T2]=step(sys2,t);
[y3,T3]=step(sys3,t); [y4,T4]=step(sys4,t);
[y5,T5]=step(sys5,t); [y6,T6]=step(sys6,t);
%
plot(T1,y1,T2,y2,T3,y3,T4,y4,T5,y5,T6,y6)
xlabel(' \omega_n t'), ylabel('y(t)')
title('\zeta = 0.1, 0.2, 0.4, 0.7, 1.0, 2.0'), grid
```

Compute step response.

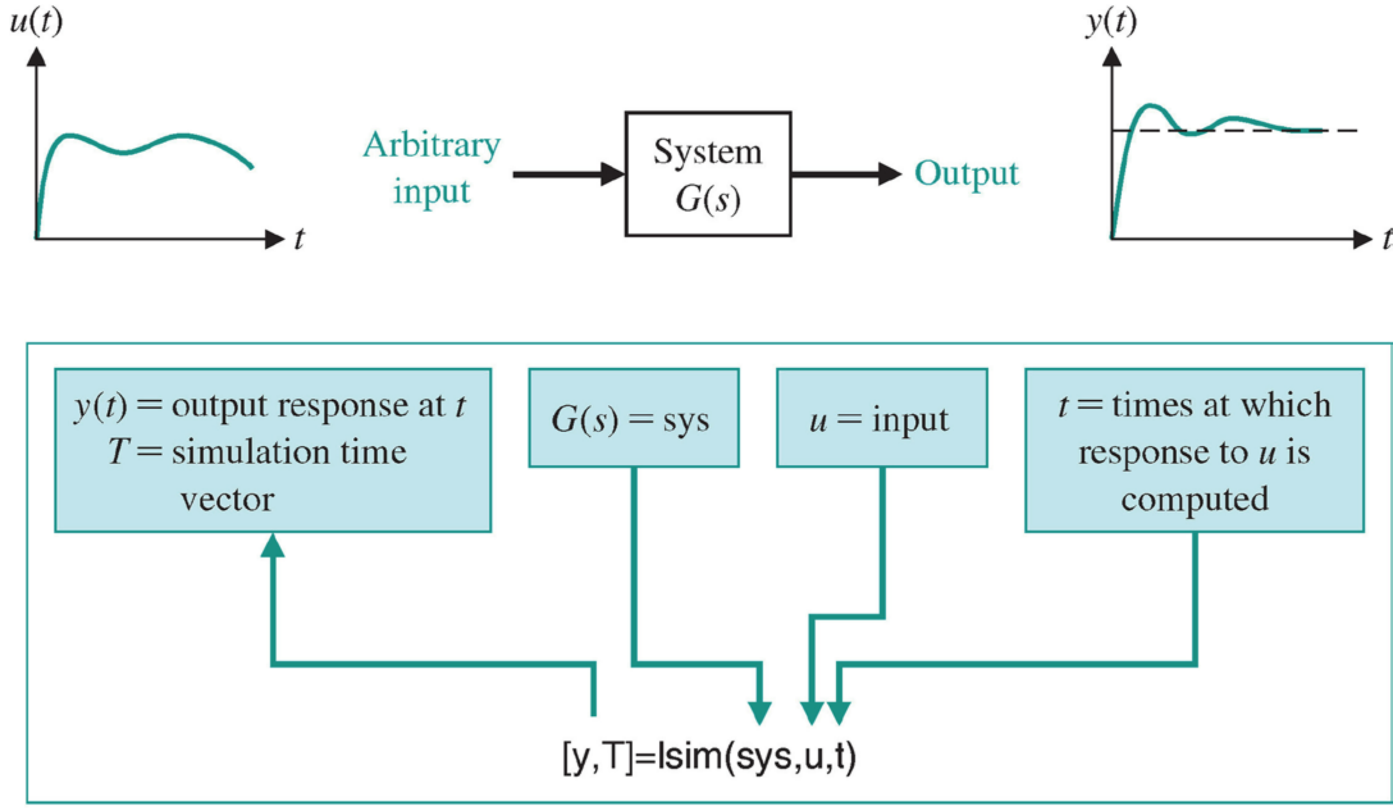
Generate plot and labels.

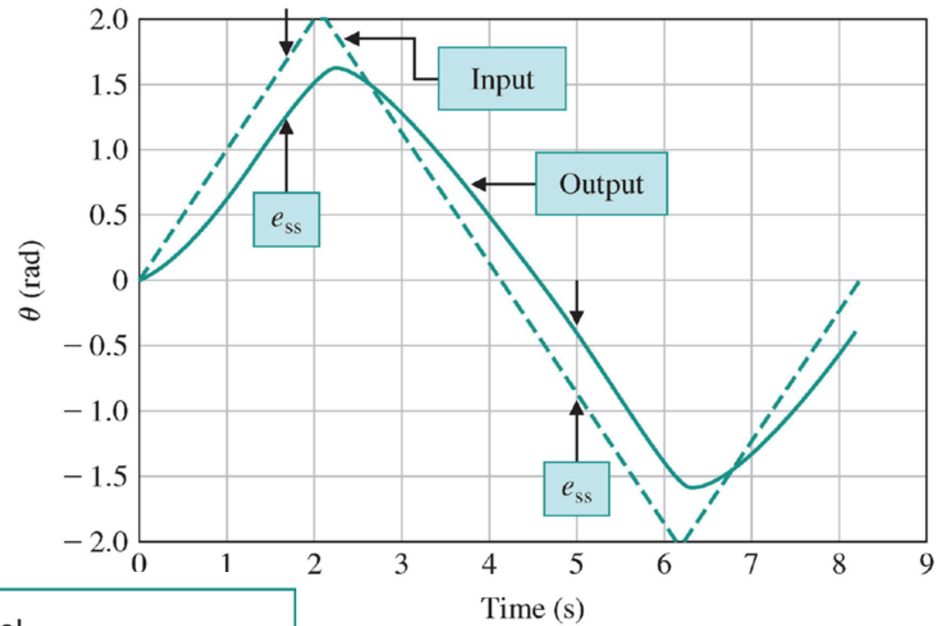
(b)



(a)

The **lsim** function.





(a)


```
%Compute the response of the Mobile Robot Control
%System to a triangular wave input
%
numg=[10 20]; deng=[1 10 0]; sysg=tf(numg,deng);
[sys]=feedback(sysg, [1]);
t=[0:0.1:8.2]';
v1=[0:0.1:2]';v2=[2:-0.1:-2]';v3=[-2:0.1:0]';
u=[v1;v2;v3];
[y,T]=lsim(sys,u,t);
plot(T,y,t,u,'--'),
xlabel('Time (s)'), ylabel('\theta (rad)'), grid
```

$G(s)G_c(s)$

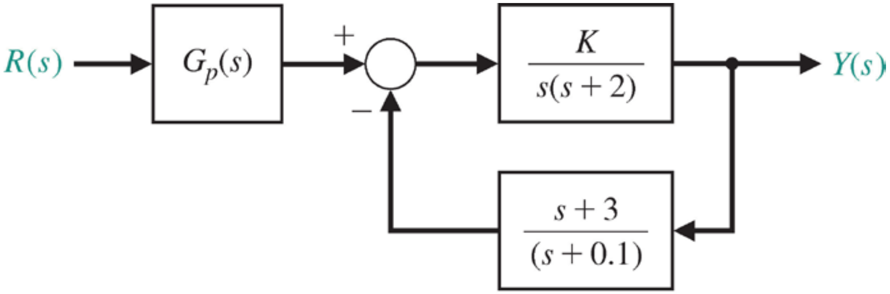
Compute triangular wave input.

Linear simulation.

(b)

Quiz 15.1

Consider the following system, choose suitable G_p to minimize the ESS to a step input.

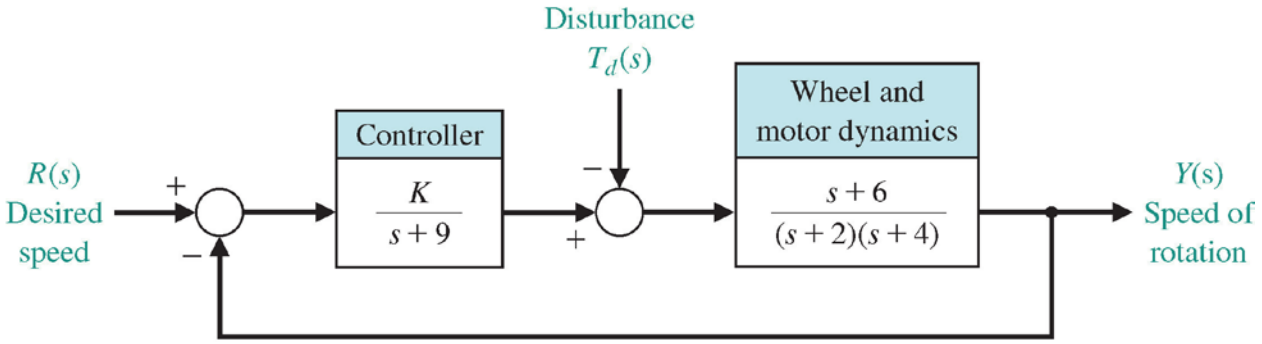


Quiz 15.2

Consider the following system,

(1) Determine K to satisfy: ESS to a unit step input < 0.05;

(2) Calculate ESS due to the unit step disturbance.



Thank You！