CHAPTER
2
BASIC PRINCIPLES

2.1 INTRODUCTION

The concept of power is of central importance in electrical power systems and is the main topic of this chapter. The typical student will already have studied much of this material, and the review here will serve to reinforce the power concepts encountered in the electric circuit theory.

In this chapter, the flow of energy in an ac circuit is investigated. By using various trigonometric identities, the instantaneous power $p(t)$ is resolved into two components. A plot of these components is obtained using *MATLAB* to observe that ac networks not only consume energy at an average rate, but also borrow and return energy to its sources. This leads to the basic definitions of average power P and reactive power Q. The volt-ampere S, which is a mathematical formulation based on the phasor forms of voltage and current, is introduced. Then the complex power balance is demonstrated, and the transmission inefficiencies caused by loads with low power factors are discussed and demonstrated by means of several examples.

Next, the transmission of complex power between two voltage sources is considered, and the dependency of real power on the voltage phase angle and the dependency of reactive power on voltage magnitude is established. *MATLAB* is used conveniently to demonstrate this idea graphically.

Finally, the balanced three-phase circuit is examined. An important property of a balanced three-phase system is that it delivers constant power. That is, the power delivered does not fluctuate with time as in a single-phase system. For the purpose of analysis and modeling, the per-phase equivalent circuit is developed for the three-phase system under balanced condition.

2.2 POWER IN SINGLE-PHASE AC CIRCUITS

Figure 2.1 shows a single-phase sinusoidal voltage supplying a load.

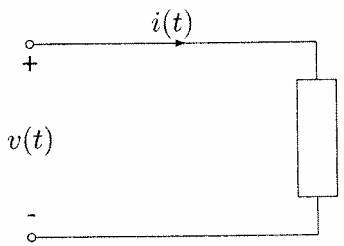

FIGURE 2.1
Sinusoidal source supplying a load.

Let the instantaneous voltage be

$$v(t) = V_m \cos(\omega t + \theta_v) \tag{2.1}$$

and the instantaneous current be given by

$$i(t) = I_m \cos(\omega t + \theta_i) \tag{2.2}$$

The instantaneous power $p(t)$ delivered to the load is the product of voltage $v(t)$ and current $i(t)$ given by

$$p(t) = v(t)\, i(t) = V_m I_m \cos(\omega t + \theta_v)\cos(\omega t + \theta_i) \tag{2.3}$$

In Example 2.1, *MATLAB* is used to plot the instantaneous power $p(t)$, and the result is shown in Figure 2.2. In studying Figure 2.2, we note that the frequency of the instantaneous power is twice the source frequency. Also, note that it is possible for the instantaneous power to be negative for a portion of each cycle. In a passive network, negative power implies that energy that has been stored in inductors or capacitors is now being extracted.

It is informative to write (2.3) in another form using the trigonometric identity

$$\cos A \cos B = \frac{1}{2}\cos(A - B) + \frac{1}{2}\cos(A + B) \tag{2.4}$$

which results in

$$\begin{aligned} p(t) &= \frac{1}{2} V_m I_m [\cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i)] \\ &= \frac{1}{2} V_m I_m \{\cos(\theta_v - \theta_i) + \cos[2(\omega t + \theta_v) - (\theta_v - \theta_i)]\} \\ &= \frac{1}{2} V_m I_m [\cos(\theta_v - \theta_i) + \cos 2(\omega t + \theta_v)\cos(\theta_v - \theta_i) \\ &\quad + \sin 2(\omega t + \theta_v)\sin(\theta_v - \theta_i)] \end{aligned}$$

The *root-mean-square* (rms) value of $v(t)$ is $|V| = V_m/\sqrt{2}$ and the rms value of $i(t)$ is $|I| = I_m/\sqrt{2}$. Let $\theta = (\theta_v - \theta_i)$. The above equation, in terms of the rms values, is reduced to

$$p(t) = \underbrace{|V||I|\cos\theta[1 + \cos 2(\omega t + \theta_v)]}_{\substack{p_R(t) \\ \text{Energy flow into} \\ \text{the circuit}}} + \underbrace{|V||I|\sin\theta\sin 2(\omega t + \theta_v)}_{\substack{p_X(t) \\ \text{Energy borrowed and} \\ \text{returned by the circuit}}} \tag{2.5}$$

where θ is the angle between voltage and current, or the impedance angle. θ is positive if the load is inductive, (i.e., current is lagging the voltage) and θ is negative if the load is capacitive (i.e., current is leading the voltage).

The instantaneous power has been decomposed into two components. The first component of (2.5) is

$$p_R(t) = |V||I|\cos\theta + |V||I|\cos\theta\cos 2(\omega t + \theta_v)] \tag{2.6}$$

The second term in (2.6), which has a frequency twice that of the source, accounts for the sinusoidal variation in the absorption of power by the resistive portion of the load. Since the average value of this sinusoidal function is zero, the average power delivered to the load is given by

$$P = |V||I|\cos\theta \tag{2.7}$$

This is the power absorbed by the resistive component of the load and is also referred to as the *active power* or *real power*. The product of the rms voltage value and the rms current value $|V||I|$ is called the *apparent power* and is measured in units of volt ampere. The product of the apparent power and the cosine of the angle between voltage and current yields the real power. Because $\cos\theta$ plays a key role in the determination of the average power, it is called *power factor*. When the current lags the voltage, the power factor is considered lagging. When the current leads the voltage, the power factor is considered leading.

The second component of (2.5)

$$p_X(t) = |V||I|\sin\theta\sin 2(\omega t + \theta_v) \tag{2.8}$$

pulsates with twice the frequency and has an average value of zero. This component accounts for power oscillating into and out of the load because of its reactive element (inductive or capacitive). The amplitude of this pulsating power is called *reactive power* and is designated by Q.

$$Q = |V||I| \sin\theta \tag{2.9}$$

Both P and Q have the same dimension. However, in order to distinguish between the real and the reactive power, the term "var" is used for the reactive power (var is an acronym for the phrase "volt-ampere reactive"). For an inductive load, current is lagging the voltage, $\theta = (\theta_v - \theta_i) > 0$ and Q is positive; whereas, for a capacitive load, current is leading the voltage, $\theta = (\theta_v - \theta_i) < 0$ and Q is negative.

A careful study of Equations (2.6) and (2.8) reveals the following characteristics of the instantaneous power.

- For a pure resistor, the impedance angle is zero and the power factor is unity (UPF), so that the apparent and real power are equal. The electric energy is transformed into thermal energy.
- If the circuit is purely inductive, the current lags the voltage by 90° and the average power is zero. Therefore, in a purely inductive circuit, there is no transformation of energy from electrical to nonelectrical form. The instantaneous power at the terminal of a purely inductive circuit oscillates between the circuit and the source. When $p(t)$ is positive, energy is being stored in the magnetic field associated with the inductive elements, and when $p(t)$ is negative, energy is being extracted from the magnetic fields of the inductive elements.
- If the load is purely capacitive, the current leads the voltage by 90°, and the average power is zero, so there is no transformation of energy from electrical to nonelectrical form. In a purely capacitive circuit, the power oscillates between the source and the electric field associated with the capacitive elements.

Example 2.1 (chp2ex1, chp2ex1gui)

The supply voltage in Figure 2.1 is given by $v(t) = 100\cos\omega t$ and the load is inductive with impedance $Z = 1.25\angle 60°\ \Omega$. Determine the expression for the instantaneous current $i(t)$ and the instantaneous power $p(t)$. Use *MATLAB* to plot $i(t)$, $v(t)$, $p(t)$, $p_R(t)$, and $p_X(t)$ over an interval of 0 to 2π.

$$I_{max} = \frac{100\angle 0°}{1.25\angle 60°} = 80\angle -60° \text{ A}$$

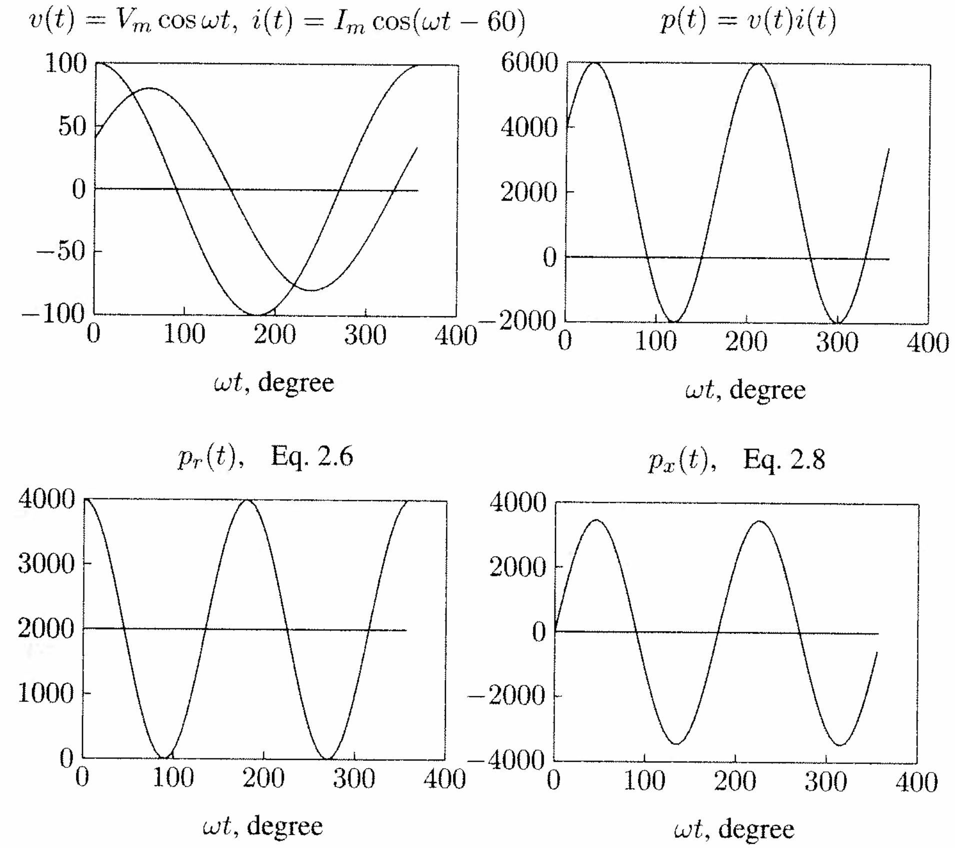

FIGURE 2.2
Instantaneous current, voltage, power, Eqs. 2.6 and 2.8.

therefore

$$i(t) = 80\cos(\omega t - 60°) \text{ A}$$
$$p(t) = v(t)\,i(t) = 8000\cos\omega t\cos(\omega t - 60°) \text{ W}$$

The following statements are used to plot the above instantaneous quantities and the instantaneous terms given by (2.6) and (2.8).

```
Vm = 100; thetav = 0;       % Voltage amplitude and phase angle
Z = 1.25;  gama = 60;     % Impedance magnitude and phase angle
thetai = thetav - gama;          % Current phase angle in degree
theta = (thetav - thetai)*pi/180;            % Degree to radian
Im = Vm/Z;                                  % Current amplitude
wt = 0:.05:2*pi;                            % wt from 0 to 2*pi
v = Vm*cos(wt);                          % Instantaneous voltage
```

```
i = Im*cos(wt + thetai*pi/180);          % Instantaneous current
p = v.*i;                                  % Instantaneous power
V = Vm/sqrt(2); I=Im/sqrt(2);          % rms voltage and current
P = V*I*cos(theta);                            % Average power
Q = V*I*sin(theta);                           % Reactive power
S = P + j*Q                                    % Complex power
pr = P*(1 + cos(2*(wt + thetav)));                 % Eq. (2.6)
px = Q*sin(2*(wt + thetav));                       % Eq. (2.8)
PP = P*ones(1, length(wt));%Average power of length w for plot
xline = zeros(1, length(wt));              %generates a zero vector
wt=180/pi*wt;                         % converting radian to degree
subplot(2,2,1), plot(wt, v, wt,i,wt, xline), grid
title(['v(t)=Vm coswt, i(t)=Im cos(wt+',num2str(thetai), ')'])
xlabel('wt, degree'), subplot(2,2,2), plot(wt, p, wt, xline)
title('p(t)=v(t) i(t)'),xlabel('wt, degree'), grid
subplot(2,2,3), plot(wt, pr, wt, PP,wt,xline), grid
title('pr(t)   Eq. 2.6'), xlabel('wt, degree')
subplot(2,2,4), plot(wt, px, wt, xline), grid
title('px(t)  Eq. 2.8'), xlabel('wt, degree'), subplot(111)
```

Run the new GUI program (**chp2ex1gui**). This allows you to see instantly the effect of changing load from inductive to resistive and capacitive on the instantaneous power $p(t)$, $p_R(t)$, and $p_X(t)$.

2.3 COMPLEX POWER

The rms voltage phasor of (2.1) and the rms current phasor of (2.2) shown in Figure 2.3 are

$$V = |V|\angle\theta_v \text{ and } I = |I|\angle\theta_i$$

The term VI^* results in

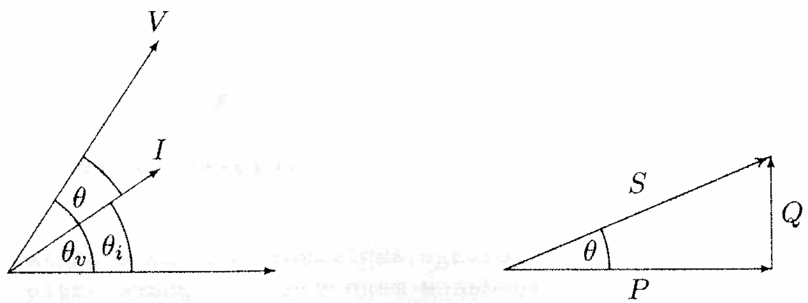

FIGURE 2.3
Phasor diagram and power triangle for an inductive load (lagging PF).

$$VI^* = |V||I|\angle\theta_v - \theta_i = |V||I|\angle\theta$$

$$= |V||I|\cos\theta + j|V||I|\sin\theta$$

The above equation defines a complex quantity where its real part is the average (real) power P and its imaginary part is the reactive power Q. Thus, the complex power designated by S is given by

$$S = VI^* = P + jQ \tag{2.10}$$

The magnitude of S, $|S| = \sqrt{P^2 + Q^2}$, is the apparent power; its unit is volt-amperes and the larger units are kVA or MVA. Apparent power gives a direct indication of heating and is used as a rating unit of power equipment. Apparent power has practical significance for an electric utility company since a utility company must supply both average and apparent power to consumers.

The reactive power Q is positive when the phase angle θ between voltage and current (impedance angle) is positive (i.e., when the load impedance is inductive, and I lags V). Q is negative when θ is negative (i.e., when the load impedance is capacitive and I leads V) as shown in Figure 2.4.

In working with Equation (2.10) it is convenient to think of P, Q, and S as forming the sides of a right triangle as shown in Figures 2.3 and 2.4.

FIGURE 2.4
Phasor diagram and power triangle for a capacitive load (leading PF).

If the load impedance is Z then

$$V = ZI \tag{2.11}$$

substituting for V into (2.10) yields

$$S = VI^* = ZII^* = R|I|^2 + jX|I|^2 \tag{2.12}$$

From (2.12) it is evident that complex power S and impedance Z have the same angle. Because the power triangle and the impedance triangle are similar triangles, the impedance angle is sometimes called the *power angle*.

Similarly, substituting for I from (2.11) into (2.10) yields

$$S = VI^* = \frac{VV^*}{Z^*} = \frac{|V|^2}{Z^*} \tag{2.13}$$

From (2.13), the impedance of the complex power S is given by

$$Z = \frac{|V|^2}{S^*} \tag{2.14}$$

2.4 THE COMPLEX POWER BALANCE

From the conservation of energy, it is clear that real power supplied by the source is equal to the sum of real powers absorbed by the load. At the same time, a balance between the reactive power must be maintained. Thus the total complex power delivered to the loads in parallel is the sum of the complex powers delivered to each. Proof of this is as follows:

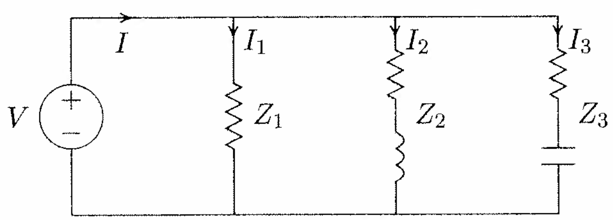

FIGURE 2.5
Three loads in parallel.

For the three loads shown in Figure 2.5, the total complex power is given by

$$S = VI^* = V[I_1 + I_2 + I_3]^* = VI_1^* + VI_2^* + VI_3^* \tag{2.15}$$

Example 2.2 (chp2ex2)

In the above circuit $V = 1200\angle 0°$ V, $Z_1 = 60 + j0\ \Omega$, $Z_2 = 6 + j12\ \Omega$ and $Z_3 = 30 - j30\ \Omega$. Find the power absorbed by each load and the total complex power.

$$I_1 = \frac{1200\angle 0°}{60\angle 0} = 20 + j0 \text{ A}$$
$$I_2 = \frac{1200\angle 0°}{6 + j12} = 40 - j80 \text{ A}$$
$$I_3 = \frac{1200\angle 0°}{30 - j30} = 20 + j20 \text{ A}$$

$$S_1 = VI_1^* = 1200\angle 0°(20 - j0) = 24,000 \text{ W} + j0 \text{ var}$$
$$S_2 = VI_2^* = 1200\angle 0°(40 + j80) = 48,000 \text{ W} + j96,000 \text{ var}$$
$$S_3 = VI_3^* = 1200\angle 0°(20 - j20) = 24,000 \text{ W} - j24,000 \text{ var}$$

The total load complex power adds up to

$$S = S_1 + S_2 + S_3 = 96,000 \text{ W} + j72,000 \text{ var}$$

Alternatively, the sum of complex power delivered to the load can be obtained by first finding the total current.

$$\begin{aligned} I &= I_1 + I_2 + I_3 = (20 + j0) + (40 - j80) + (20 + j20) \\ &= 80 - j60 = 100\angle{-36.87°} \text{ A} \end{aligned}$$

and

$$\begin{aligned} S = VI^* &= (1200\angle 0°)(100\angle 36.87°) = 120,000\angle 36.87° \text{ VA} \\ &= 96,000 \text{ W} + j72,000 \text{ var} \end{aligned}$$

A final insight is contained in Figure 2.6, which shows the current phasor diagram and the complex power vector representation.

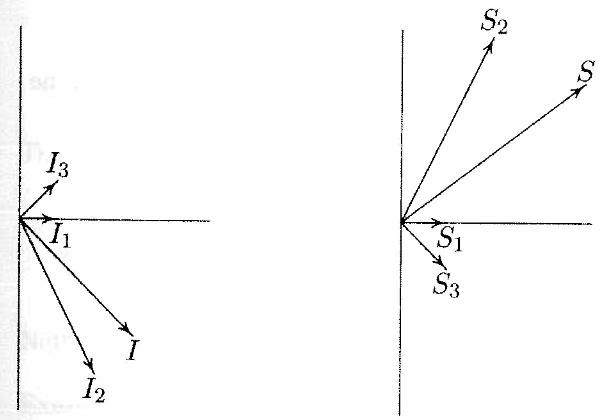

FIGURE 2.6
Current phasor diagram and power plane diagram.

The complex powers may also be obtained directly from (2.14)

$$S_1 = \frac{|V|^2}{Z_1^*} = \frac{(1200)^2}{60} = 24,000 \text{ W} + j\ 0$$
$$S_2 = \frac{|V|^2}{Z_2^*} = \frac{(1200)^2}{6 - j12} = 48,000 \text{ W} + j96,000 \text{ var}$$
$$S_3 = \frac{|V|^2}{Z_3^*} = \frac{(1200)^2}{30 + j30} = 24,000 \text{ W} - j24,000 \text{ var}$$

2.5 POWER FACTOR CORRECTION

It can be seen from (2.7) that the apparent power will be larger than P if the power factor is less than 1. Thus the current I that must be supplied will be larger for $PF < 1$ than it would be for $PF = 1$, even though the average power P supplied is the same in either case. A larger current cannot be supplied without additional cost to the utility company. Thus, it is in the power company's (and its customer's) best interest that major loads on the system have power factors as close to 1 as possible. In order to maintain the power factor close to unity, power companies install banks of capacitors throughout the network as needed. They also impose an additional charge to industrial consumers who operate at low power factors. Since industrial loads are inductive and have low lagging power factors, it is beneficial to install capacitors to improve the power factor. This consideration is not important for residential and small commercial customers because their power factors are close to unity.

Example 2.3 (chp2ex3)

Two loads $Z_1 = 100 + j0\ \Omega$ and $Z_2 = 10 + j20\ \Omega$ are connected across a 200-V rms, 60-Hz source as shown in Figure 2.7.

(a) Find the total real and reactive power, the power factor at the source, and the total current.

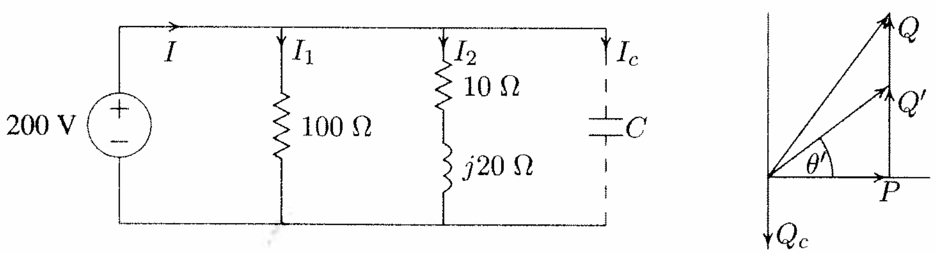

FIGURE 2.7
Circuit for Example 2.3 and the power triangle.

$$
\begin{aligned}
I_1 &= \frac{200\angle 0^\circ}{100} = 2\angle 0^\circ \text{ A} \\
I_2 &= \frac{200\angle 0^\circ}{10 + j20} = 4 - j8 \text{ A} \\
S_1 &= VI_1^* = 200\angle 0^\circ(2 - j0) = 400 \text{ W} + j0 \text{ var} \\
S_2 &= VI_2^* = 200\angle 0^\circ(4 + j8) = 800 \text{ W} + j1600 \text{ var}
\end{aligned}
$$

Total apparent power and current are

$$
\begin{aligned}
S &= P + jQ = 1200 + j1600 = 2000\angle 53.13^\circ \text{ VA} \\
I &= \frac{S^*}{V^*} = \frac{2000\angle -53.13^\circ}{200\angle 0^\circ} = 10\angle -53.13^\circ \text{ A}
\end{aligned}
$$

Power factor at the source is

$$PF = \cos(53.13) = 0.6 \text{ lagging}$$

(b) Find the capacitance of the capacitor connected across the loads to improve the overall power factor to 0.8 lagging.

Total real power $P = 1200$ W at the new power factor 0.8 lagging. Therefore

$$
\begin{aligned}
\theta' &= \cos^{-1}(0.8) = 36.87^\circ \\
Q' &= P\tan\theta' = 1200\tan(36.87^\circ) = 900 \text{ var} \\
Q_c &= 1600 - 900 = 700 \text{ var} \\
Z_c &= \frac{|V|^2}{S_c^*} = \frac{(200)^2}{j700} = -j57.14\ \Omega \\
C &= \frac{10^6}{2\pi(60)(57.14)} = 46.42\ \mu\text{F}
\end{aligned}
$$

The total power and the new current are

$$
\begin{aligned}
S' &= 1200 + j900 = 1500\angle 36.87^\circ \\
I' &= \frac{S'^*}{V^*} = \frac{1500\angle -36.87^\circ}{200\angle 0^\circ} = 7.5\angle -36.87^\circ
\end{aligned}
$$

Note the reduction in the supply current from 10 A to 7.5 A.

Example 2.4 (chp2ex4)

Three loads are connected in parallel across a 1400-V rms, 60-Hz single-phase supply as shown in Figure 2.8.

Load 1: Inductive load, 125 kVA at 0.28 power factor.

Load 2: Capacitive load, 10 kW and 40 kvar.

Load 3: Resistive load of 15 kW.

(a) Find the total kW, kvar, kVA, and the supply power factor.

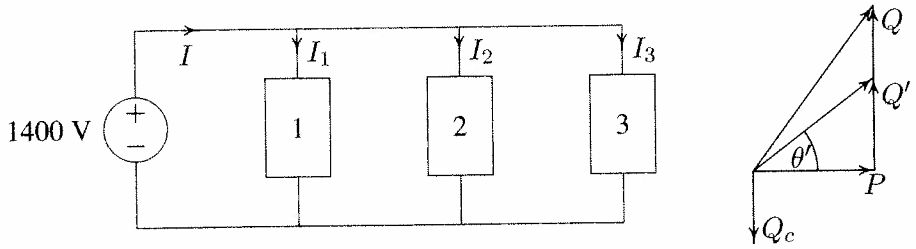

FIGURE 2.8
Circuit for Example 2.4.

An inductive load has a lagging power factor, the capacitive load has a leading power factor, and the resistive load has a unity power factor.

For Load 1:

$$\theta_1 = \cos^{-1}(0.28) = 73.74^\circ \text{ lagging}$$

The load complex powers are

$$\begin{aligned} S_1 &= 125\angle 73.74 \text{ kVA} = 35 \text{ kW} + j120 \text{ kvar} \\ S_2 &= 10 \text{ kW} - j40 \text{ kvar} \\ S_3 &= 15 \text{ kW} + j0 \text{ kvar} \end{aligned}$$

The total apparent power is

$$\begin{aligned} S = P + jQ &= S_1 + S_2 + S_3 \\ &= (35 + j120) + (10 - j40) + (15 + j0) \\ &= 60 \text{ kW} + j80 \text{ kvar} = 100\angle 53.13 \text{ kVA} \end{aligned}$$

The total current is

$$I = \frac{S^*}{V^*} = \frac{100,000\angle -53.13^\circ}{1400\angle 0^\circ} = 71.43\angle -53.13^\circ \text{ A}$$

The supply power factor is

$$PF = \cos(53.13) = 0.6 \text{ lagging}$$

(b) A capacitor of negligible resistance is connected in parallel with the above loads to improve the power factor to 0.8 lagging. Determine the kvar rating of this capacitor and the capacitance in μF.

Total real power $P = 60$ kW at the new power factor of 0.8 lagging results in the new reactive power Q'.

$$\begin{aligned} \theta' &= \cos^{-1}(0.8) = 36.87^\circ \\ Q' &= 60\tan(36.87^\circ) = 45 \text{ kvar} \end{aligned}$$

Therefore, the required capacitor kvar is

$$Q_c = 80 - 45 = 35 \text{ kvar}$$

and

$$\begin{aligned} X_c &= \frac{|V|^2}{S_c^*} = \frac{1400^2}{j35,000} = -j56\ \Omega \\ C &= \frac{10^6}{2\pi(60)(56)} = 47.37\ \mu\text{F} \end{aligned}$$

and the new current is

$$I' = \frac{S'^*}{V^*} = \frac{60,000 - j45,000}{1400\angle 0^\circ} = 53.57\angle -36.87^\circ \text{ A}$$

Note the reduction in the supply current from 71.43 A to 53.57 A.

2.6 COMPLEX POWER FLOW

Consider two ideal voltage sources connected by a line of impedance $Z = R + jX\ \Omega$ as shown in Figure 2.9.

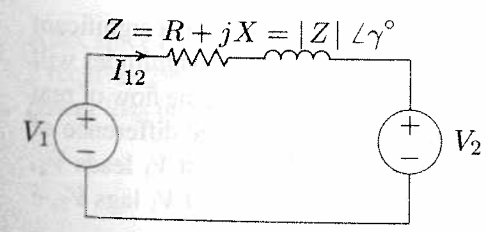

FIGURE 2.9
Two interconnected voltage sources.

Let the phasor voltage be $V_1 = |V_1|\angle\delta_1$ and $V_2 = |V_2|\angle\delta_2$. For the assumed direction of current

$$I_{12} = \frac{|V_1|\angle\delta_1 - |V_2|\angle\delta_2}{|Z|\angle\gamma} = \frac{|V_1|}{|Z|}\angle\delta_1 - \gamma - \frac{|V_2|}{|Z|}\angle\delta_2 - \gamma$$

The complex power S_{12} is given by

$$S_{12} = V_1 I_{12}^* = |V_1|\angle\delta_1 \left[\frac{|V_1|}{|Z|}\angle\gamma - \delta_1 - \frac{|V_2|}{|Z|}\angle\gamma - \delta_2\right]$$
$$= \frac{|V_1|^2}{|Z|}\angle\gamma - \frac{|V_1||V_2|}{|Z|}\angle\gamma + \delta_1 - \delta_2$$

Thus, the real and reactive power at the sending end are

$$P_{12} = \frac{|V_1|^2}{|Z|}\cos\gamma - \frac{|V_1||V_2|}{|Z|}\cos(\gamma + \delta_1 - \delta_2) \tag{2.16}$$
$$Q_{12} = \frac{|V_1|^2}{|Z|}\sin\gamma - \frac{|V_1||V_2|}{|Z|}\sin(\gamma + \delta_1 - \delta_2) \tag{2.17}$$

Power system transmission lines have small resistance compared to the reactance. Assuming $R = 0$ (i.e., $Z = X\angle 90°$), the above equations become

$$P_{12} = \frac{|V_1||V_2|}{X}\sin(\delta_1 - \delta_2) \tag{2.18}$$
$$Q_{12} = \frac{|V_1|}{X}[\,|V_1| - |V_2|\cos(\delta_1 - \delta_2)\,] \tag{2.19}$$

Since $R = 0$, there are no transmission line losses and the real power sent equals the real power received.

From the above results, for a typical power system with small R/X ratio, the following important observations are made :

1. Equation (2.18) shows that small changes in δ_1 or δ_2 will have a significant effect on the real power flow, while small changes in voltage magnitudes will not have appreciable effect on the real power flow. Therefore, the flow of real power on a transmission line is governed mainly by the angle difference of the terminal voltages (i.e., $P_{12} \propto \sin\delta$), where $\delta = \delta_1 - \delta_2$. If V_1 leads V_2, δ is positive and the real power flows from node 1 to node 2. If V_1 lags V_2, δ is negative and power flows from node 2 to node 1.

2. Assuming $R = 0$, the theoretical maximum power (static transmission capacity) occurs when $\delta = 90°$ and the maximum power transfer is given by

$$P_{max} = \frac{|V_1||V_2|}{X} \tag{2.20}$$

   In Chapter 3 we learn that increasing δ beyond the static transmission capacity will result in loss of synchronism between the two machines.

3. For maintaining transient stability, the power system is usually operated with small load angle δ. Also, from (2.19) the reactive power flow is determined by the magnitude difference of terminal voltages, (i.e., $Q \propto |V_1| - |V_2|$).

Example 2.5 (chp2ex5)

Two voltage sources $V_1 = 120\angle{-5}$ V and $V_2 = 100\angle 0$ V are connected by a short line of impedance $Z = 1 + j7\,\Omega$ as shown in Figure 2.9. Determine the real and reactive power supplied or received by each source and the power loss in the line.

$$I_{12} = \frac{120\angle{-5°} - 100\angle 0°}{1 + j7} = 3.135\angle{-110.02°}\ \text{A}$$
$$I_{21} = \frac{100\angle 0° - 120\angle - 5°}{1 + j7} = 3.135\angle 69.98°\ \text{A}$$
$$S_{12} = V_1 I_{12}^* = 376.2\angle 105.02° = -97.5\ \text{W} + j363.3\ \text{var}$$
$$S_{21} = V_2 I_{21}^* = 313.5\angle{-69.98°} = 107.3\ \text{W} - j294.5\ \text{var}$$

Line loss is given by

$$S_L = S_1 + S_2 = 9.8\ \ \text{W} + j68.8\ \text{var}$$

From the above results, since P_1 is negative and P_2 is positive, source 1 receives 97.5 W, and source 2 generates 107.3 W and the real power loss in the line is 9.8 W. The real power loss in the line can be checked by

$$P_L = R|I_{12}|^2 = (1)(3.135)^2 = 9.8\ \text{W}$$

Also, since Q_1 is positive and Q_2 is negative, source 1 delivers 363.3 var and source 2 receives 294.5 var, and the reactive power loss in the line is 68.6 var. The reactive power loss in the line can be checked by

$$Q_L = X|I_{12}|^2 = (7)(3.135)^2 = 68.8\ \text{var}$$

Example 2.6 (chp2ex6), (chp2ex6gui)

This example concerns the direction of power flow between two voltage sources. Write a *MATLAB* program for the system of Example 2.5 such that the phase angle of source 1 is changed from its initial value by $\pm 30°$ in steps of $5°$. Voltage magnitudes of the two sources and the voltage phase angle of source 2 is to be kept constant. Compute the complex power for each source and the line loss. Tabulate the real power and plot P_1 , P_2, and P_L versus voltage phase angle δ. The following commands

```
E1 = input('Source # 1 Voltage Mag. = ');
a1 = input('Source# 1 Phase Angle  = ');
E2 = input('Source # 2 Voltage Mag. = ');
a2 = input('Source # 2 Phase Angle  = ');
R  = input('Line Resistance = ');
X  = input('Line Reactance = ');
Z  =  R + j*X;                              % Line impedance
a1 = (-30+a1:5:30+a1)';        % Change a1 by +/- 30, col. array
a1r = a1*pi/180;                      % Convert degree to radian
k = length(a1);
a2 = ones(k,1)*a2;   % Create col. array of same length for a2
a2r = a2*pi/180;                      % Convert degree to radian
V1 = E1.*cos(a1r) + j*E1.*sin(a1r);
V2 = E2.*cos(a2r) + j*E2.*sin(a2r);
I12 = (V1 - V2)./Z;  I21=-I12;
S1 = V1.*conj(I12);  P1 = real(S1);  Q1 = imag(S1);
S2 = V2.*conj(I21);  P2 = real(S2);  Q2 = imag(S2);
SL = S1+S2;          PL = real(SL);  QL = imag(SL);
Result1 = [a1, P1,  P2, PL];
disp('   Delta 1     P-1        P-2        P-L ')
disp(Result1)
plot(a1, P1,  a1, P2,  a1,PL)
xlabel('Source #1 Voltage Phase Angle')
ylabel(' P, Watts'),
text(-26, -550, 'P1'), text(-26, 600,'P2'),
text(-26, 100, 'PL')
```

result in

```
Source # 1 Voltage Mag. = 120
Source # 1 Phase Angle  = -5
Source # 2 Voltage Mag. = 100
Source # 2 Phase Angle  = 0
Line Resistance = 1
Line Reactance =  7
```

Delta 1	P-1	P-2	P-L
-35.0000	-872.2049	967.0119	94.8070
-30.0000	-759.8461	832.1539	72.3078
-25.0000	-639.5125	692.4848	52.9723
-20.0000	-512.1201	549.0676	36.9475
-15.0000	-378.6382	402.9938	24.3556
-10.0000	-240.0828	255.3751	15.2923
-5.0000	-97.5084	107.3349	9.8265
0	48.0000	-40.0000	8.0000
5.0000	195.3349	-185.5084	9.8265
10.0000	343.3751	-328.0828	15.2923
15.0000	490.9938	-466.6382	24.3556
20.0000	637.0676	-600.1201	36.9475
25.0000	780.4848	-727.5125	52.9723

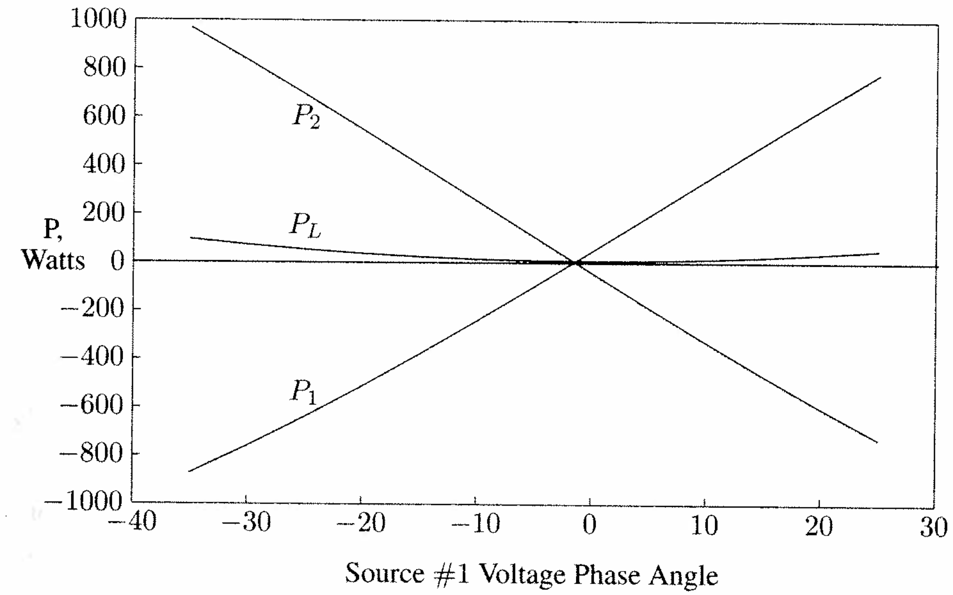

FIGURE 2.10
Real power versus voltage phase angle δ.

Examination of Figure 2.10 shows that the flow of real power along the interconnection is determined by the angle difference of the terminal voltages. Problem 2.9 requires the development of a similar program for demonstrating the dependency of reactive power on the magnitude difference of terminal voltages.

Run the new GUI program (**chp2ex6gui**). This allows you to see instantly the effect of sweeping voltage phase angle δ_1 on the direction of real power flow, and the effect of sweeping voltage magnitude V_1 on the direction of the reactive power flow.

2.7 BALANCED THREE-PHASE CIRCUITS

The generation, transmission and distribution of electric power is accomplished by means of three-phase circuits. At the generating station, three sinusoidal voltages are generated having the same amplitude but displaced in phase by 120°. This is called a *balanced source*. If the generated voltages reach their peak values in the sequential order ABC, the generator is said to have a *positive phase sequence*, shown in Figure 2.11(a). If the phase order is ACB, the generator is said to have a *negative phase sequence*, as shown in Figure 2.11(b).

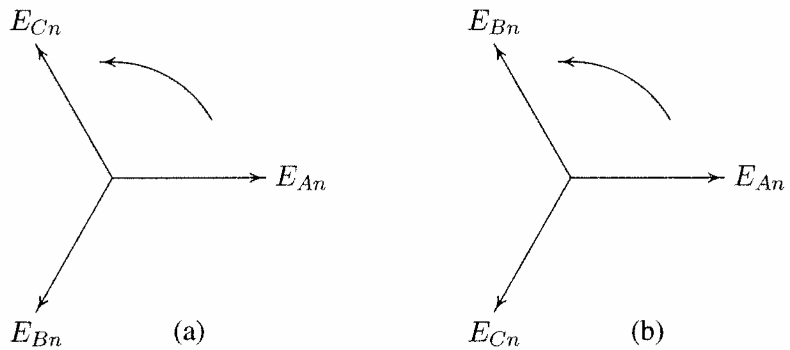

FIGURE 2.11
(a) Positive, or ABC, phase sequence. (b) Negative, or ACB, phase sequence.

In a three-phase system, the instantaneous power delivered to the external loads is constant rather than pulsating as it is in a single-phase circuit. Also, three-phase motors, having constant torque, start and run much better than single-phase motors. This feature of three-phase power, coupled with the inherent efficiency of its transmission compared to single-phase (less wire for the same delivered power), accounts for its universal use.

A power system has Y-connected generators and usually includes both Δ- and Y-connected loads. Generators are rarely Δ-connected, because if the voltages are not perfectly balanced, there will be a net voltage, and consequently a circulating current, around the Δ. Also, the phase voltages are lower in the Y-connected generator, and thus less insulation is required. Figure 2.12 shows a Y-connected generator supplying balanced Y-connected loads through a three-phase line. Assuming a positive phase sequence (phase order ABC) the generated voltages are:

$$\begin{aligned} E_{An} &= |E_p|\angle 0^\circ \\ E_{Bn} &= |E_p|\angle{-120^\circ} \\ E_{Cn} &= |E_p|\angle{-240^\circ} \end{aligned} \tag{2.21}$$

In power systems, great care is taken to ensure that the loads of transmission lines are balanced. For balanced loads, the terminal voltages of the generator V_{An}, V_{Bn} and V_{Cn} and the phase voltages V_{an}, V_{bn} and V_{cn} at the load terminals are balanced. For "phase A," these are given by

$$V_{An} = E_{An} - Z_G I_a \tag{2.22}$$

$$V_{an} = V_{An} - Z_L I_a \tag{2.23}$$

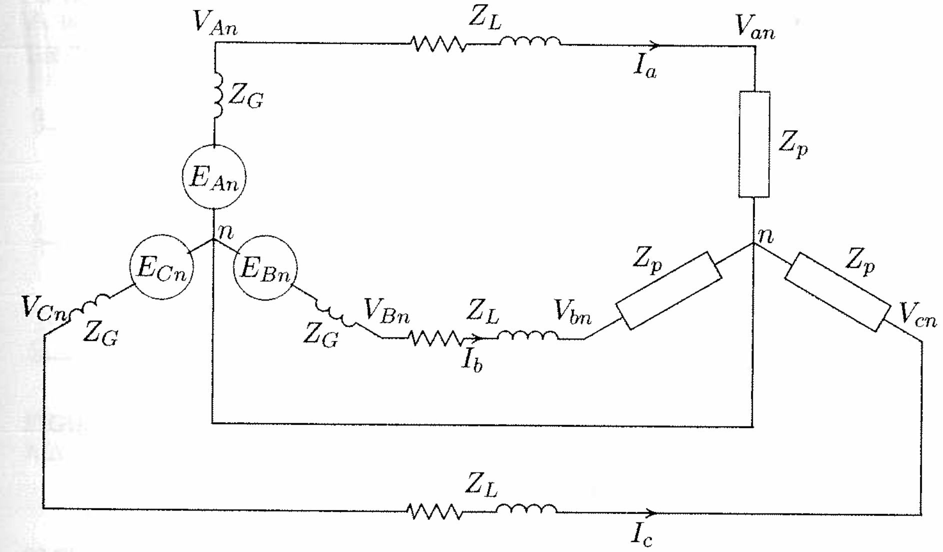

FIGURE 2.12
A Y-connected generator supplying a Y-connected load.

2.8 Y-CONNECTED LOADS

To find the relationship between the line voltages (line-to-line voltages) and the phase voltages (line-to-neutral voltages), we assume a positive, or ABC, sequence. We arbitrarily choose the line-to-neutral voltage of the a-phase as the reference, thus

$$\begin{aligned} V_{an} &= |V_p|\angle 0^\circ \\ V_{bn} &= |V_p|\angle{-120^\circ} \\ V_{cn} &= |V_p|\angle{-240^\circ} \end{aligned} \tag{2.24}$$

where $|V_p|$ represents the magnitude of the phase voltage (line-to-neutral voltage).

The line voltages at the load terminals in terms of the phase voltages are found by the application of Kirchhoff's voltage law

$$\begin{aligned} V_{ab} &= V_{an} - V_{bn} = |V_p|(1\angle 0^\circ - 1\angle{-120^\circ}) = \sqrt{3}|V_p|\angle 30^\circ \\ V_{bc} &= V_{bn} - V_{cn} = |V_p|(1\angle{-120^\circ} - 1\angle{-240^\circ}) = \sqrt{3}|V_p|\angle{-90^\circ} \\ V_{ca} &= V_{cn} - V_{an} = |V_p|(1\angle{-240^\circ} - 1\angle 0^\circ) = \sqrt{3}|V_p|\angle 150^\circ \end{aligned} \tag{2.25}$$

The voltage phasor diagram of the Y-connected loads of Figure 2.12 is shown in Figure 2.13. The relationship between the line voltages and phase voltages is demonstrated graphically.

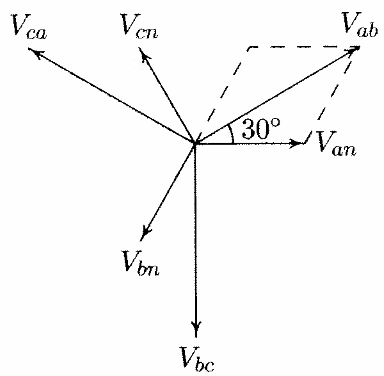

FIGURE 2.13
Phasor diagram showing phase and line voltages.

If the rms value of any of the line voltages is denoted by V_L, then one of the important characteristics of the Y-connected three-phase load may be expressed as

$$V_L = \sqrt{3}\,|V_p|\angle 30^\circ \tag{2.26}$$

Thus in the case of Y-connected loads, the magnitude of the line voltage is $\sqrt{3}$ times the magnitude of the phase voltage, and for a positive phase sequence, the set of line voltages leads the set of phase voltages by 30°.

The three-phase currents in Figure 2.12 also possess three-phase symmetry and are given by

$$\begin{aligned} I_a &= \frac{V_{an}}{Z_p} = |I_p|\angle -\theta \\ I_b &= \frac{V_{bn}}{Z_p} = |I_p|\angle -120^\circ - \theta \\ I_c &= \frac{V_{cn}}{Z_p} = |I_p|\angle -240^\circ - \theta \end{aligned} \tag{2.27}$$

where θ is the impedance phase angle.

The currents in lines are also the phase currents (the current carried by the phase impedances). Thus

$$I_L = I_p \tag{2.28}$$

2.9 Δ-CONNECTED LOADS

A balanced Δ-connected load (with equal phase impedances) is shown in Figure 2.14.

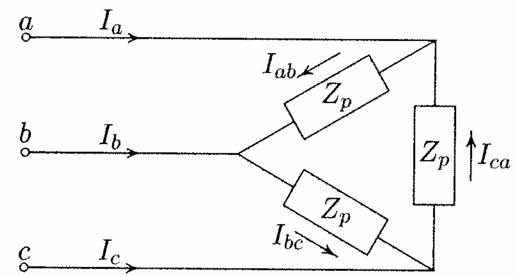

FIGURE 2.14
A Δ-connected load.

It is clear from the inspection of the circuit that the line voltages are the same as phase voltages.

$$V_L = V_p \tag{2.29}$$

Consider the phasor diagram shown in Figure 2.15, where the phase current I_{ab} is arbitrarily chosen as reference. we have

$$\begin{aligned} I_{ab} &= |I_p|\angle 0^\circ \\ I_{bc} &= |I_p|\angle -120^\circ \\ I_{ca} &= |I_p|\angle -240^\circ \end{aligned} \tag{2.30}$$

where $|I_p|$ represents the magnitude of the phase current.

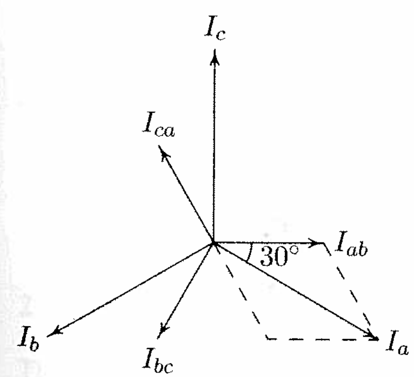

FIGURE 2.15
Phasor diagram showing phase and line currents.

The relationship between phase and line currents can be obtained by applying Kirchhoff's current law at the corners of Δ.

$$\begin{aligned} I_a &= I_{ab} - I_{ca} = |I_p|(1\angle 0^\circ - 1\angle{-240^\circ}) = \sqrt{3}|I_p|\angle{-30^\circ} \\ I_b &= I_{bc} - I_{ab} = |I_p|(1\angle{-120^\circ} - 1\angle 0^\circ) = \sqrt{3}|I_p|\angle{-150^\circ} \\ I_c &= I_{ca} - I_{bc} = |I_p|(1\angle{-240^\circ} - 1\angle{-120^\circ}) = \sqrt{3}|I_p|\angle 90^\circ \end{aligned} \tag{2.31}$$

The relationship between the line currents and phase currents is demonstrated graphically in Figure 2.15.

If the rms of any of the line currents is denoted by I_L, then one of the important characteristics of the Δ-connected three-phase load may be expressed as

$$I_L = \sqrt{3}|I_p| \angle{-30^\circ} \tag{2.32}$$

Thus in the case of Δ-connected loads, the magnitude of the line current is $\sqrt{3}$ times the magnitude of the phase current, and with positive phase sequence, the set of line currents lags the set of phase currents by 30°.

2.10 Δ-Y TRANSFORMATION

For analyzing network problems, it is convenient to replace the Δ-connected circuit with an equivalent Y-connected circuit. Consider the fictitious Y-connected circuit of Z_Y Ω/phase which is equivalent to a balanced Δ-connected circuit of Z_Δ Ω/phase, as shown in Figure 2.16.

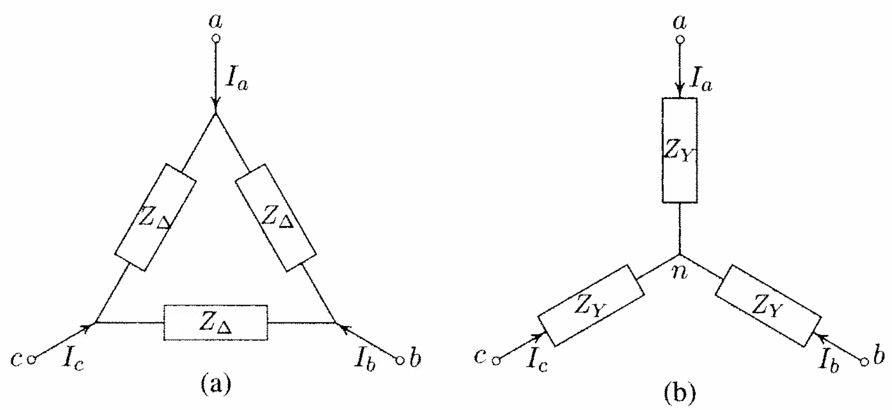

FIGURE 2.16
(a) Δ to (b) Y-connection.

For the Δ-connected circuit, the phase current I_a is given by

$$I_a = \frac{V_{ab}}{Z_\Delta} + \frac{V_{ac}}{Z_\Delta} = \frac{V_{ab} + V_{ac}}{Z_\Delta} \tag{2.33}$$

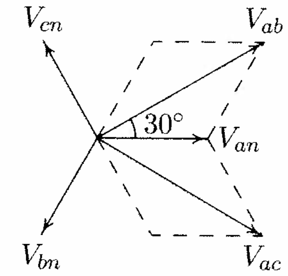

FIGURE 2.17
Phasor diagram showing phase and line voltages.

The phasor diagram in Figure 2.17 shows the relationship between balanced phase and line-to-line voltages. From this phasor diagram, we find

$$V_{ab} + V_{ac} = \sqrt{3}\,|V_{an}|\angle 30^\circ + \sqrt{3}\,|V_{an}|\angle{-30^\circ} \tag{2.34}$$

$$= 3V_{an} \tag{2.35}$$

Substituting in (2.33), we get

$$I_a = \frac{3V_{an}}{Z_\Delta}$$

or

$$V_{an} = \frac{Z_\Delta}{3} I_a \tag{2.36}$$

Now, for the Y-connected circuit, we have

$$V_{an} = Z_Y I_a \tag{2.37}$$

Thus, from (2.36) and (2.37), we find that

$$Z_Y = \frac{Z_\Delta}{3} \tag{2.38}$$

2.11 PER-PHASE ANALYSIS

The current in the neutral of the balanced Y-connected loads shown in Figure 2.12 is given by

$$I_n = I_a + I_b + I_c = 0 \tag{2.39}$$

Since the neutral carries no current, a neutral wire of any impedance may be replaced by any other impedance, including a short circuit and an open circuit. The return line may not actually exist, but regardless, a line of zero impedance is included between the two neutral points. The balanced power system problems are then solved on a "per-phase" basis. It is understood that the other two phases carry identical currents except for the phase shift.

We may then look at only one phase, say "phase A," consisting of the source V_{An} in series with Z_L and Z_p, as shown in Figure 2.18. The neutral is taken as datum and usually a single-subscript notation is used for phase voltages.

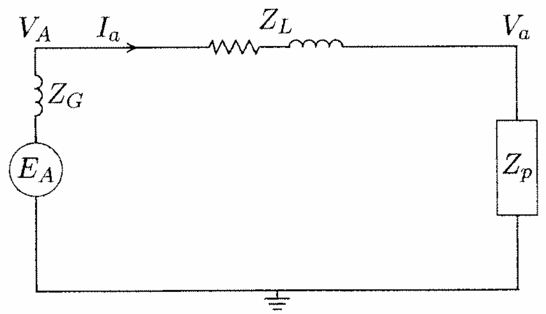

FIGURE 2.18
Single-phase circuit for per-phase analysis.

If the load in a three-phase circuit is connected in a Δ, it can be transformed into a Y by using the Δ-to-Y transformation. When the load is balanced, the impedance of each leg of the Y is one-third the impedance of each leg of the Δ, as given by (2.38), and the circuit is modeled by the single-phase equivalent circuit.

2.12 BALANCED THREE-PHASE POWER

Consider a balanced three-phase source supplying a balanced Y- or Δ- connected load with the following instantaneous voltages

$$\begin{aligned} v_{an} &= \sqrt{2}|V_p|\cos(\omega t + \theta_v) \\ v_{bn} &= \sqrt{2}|V_p|\cos(\omega t + \theta_v - 120^\circ) \\ v_{cn} &= \sqrt{2}|V_p|\cos(\omega t + \theta_v - 240^\circ) \end{aligned} \tag{2.40}$$

For a balanced load the phase currents are

$$\begin{aligned} i_a &= \sqrt{2}|I_p|\cos(\omega t + \theta_i) \\ i_b &= \sqrt{2}|I_p|\cos(\omega t + \theta_i - 120^\circ) \\ i_c &= \sqrt{2}|I_p|\cos(\omega t + \theta_i - 240^\circ) \end{aligned} \tag{2.41}$$

where $|V_p|$ and $|I_P|$ are the magnitudes of the rms phase voltage and current, respectively. The total instantaneous power is the sum of the instantaneous power of each phase, given by

$$p_{3\phi} = v_{an}i_a + v_{bn}i_b + v_{cn}i_c \tag{2.42}$$

Substituting for the instantaneous voltages and currents from (2.40) and (2.41) into (2.42)

$$\begin{aligned} p_{3\phi} = &\ 2|V_p||I_p|\cos(\omega t + \theta_v)\cos(\omega t + \theta_i) \\ &+2|V_p||I_p|\cos(\omega t + \theta_v - 120^\circ)\cos(\omega t + \theta_i - 120^\circ) \\ &+2|V_p||I_p|\cos(\omega t + \theta_v - 240^\circ)\cos(\omega t + \theta_i - 240^\circ) \end{aligned}$$

Using the trigonometric identity (2.4)

$$\begin{aligned} p_{3\phi} = &\ |V_p||I_p|[\cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i)] \\ &+|V_p||I_p|[\cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i - 240^\circ)] \\ &+|V_p||I_p|[\cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i - 480^\circ)] \end{aligned} \tag{2.43}$$

The three double frequency cosine terms in (2.43) are out of phase with each other by 120° and add up to zero, and the three-phase instantaneous power is

$$P_{3\phi} = 3|V_p||I_p|\cos\theta \tag{2.44}$$

$\theta = \theta_v - \theta_i$ is the angle between phase voltage and phase current or the impedance angle.

Note that although the power in each phase is pulsating, the total instantaneous power is constant and equal to three times the real power in each phase. Indeed, this constant power is the main advantage of the three-phase system over the single-phase system. Since the power in each phase is pulsating, the power, then, is made up of the real power and the reactive power. In order to obtain formula symmetry between real and reactive powers, the concept of complex or apparent power (S) is extended to three-phase systems by defining the three-phase reactive power as

$$Q_{3\phi} = 3|V_p||I_p|\sin\theta \tag{2.45}$$

Thus, the complex three-phase power is

$$S_{3\phi} = P_{3\phi} + jQ_{3\phi} \tag{2.46}$$

or

$$S_{3\phi} = 3V_pI_p^* \tag{2.47}$$

Equations (2.44) and (2.45) are sometimes expressed in terms of the rms magnitude of the line voltage and the rms magnitude of the line current. In a Y-connected load the phase voltage $|V_p| = |V_L|/\sqrt{3}$ and the phase current $I_p = I_L$.

In the Δ-connection $V_p = V_L$ and $|I_p| = |I_L|/\sqrt{3}$. Substituting for the phase voltage and phase currents in (2.44) and (2.45), the real and reactive powers for either connection are given by

$$P_{3\phi} = \sqrt{3}|V_L||I_L|\cos\theta \tag{2.48}$$

and

$$Q_{3\phi} = \sqrt{3}|V_L||I_L|\sin\theta \tag{2.49}$$

A comparison of the last two expressions with (2.44) and (2.45) shows that the equation for the power in a three-phase system is the same for either a Y or a Δ connection when the power is expressed in terms of line quantities.

When using (2.48) and (2.49) to calculate the total real and reactive power, remember that θ is the phase angle between the phase voltage and the phase current. As in the case of single-phase systems for the computation of power, it is best to use the complex power expression in terms of phase quantities given by (2.47). The rated power is customarily given for the three-phase and rated voltage is the line-to-line voltage. Thus, in using the per-phase equivalent circuit, care must be taken to use per-phase voltage by dividing the rated voltage by $\sqrt{3}$.

Example 2.7 (chp2ex7)

A three-phase line has an impedance of $2 + j4$ Ω as shown in Figure 2.19.

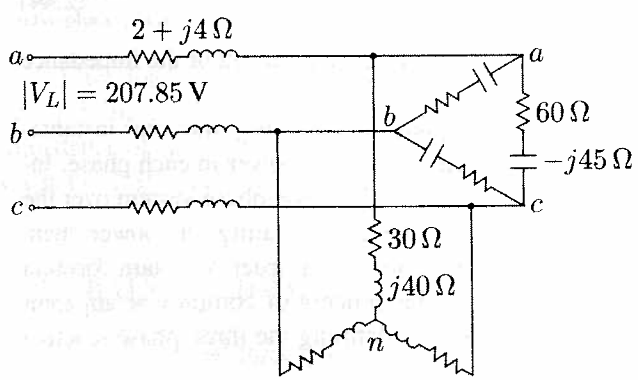

FIGURE 2.19
Three-phase circuit diagram for Example 2.7.

The line feeds two balanced three-phase loads that are connected in parallel. The first load is Y-connected and has an impedance of $30 + j40$ Ω per phase. The second load is Δ-connected and has an impedance of $60 - j45$ Ω. The line is energized at the sending end from a three-phase balanced supply of line voltage 207.85 V. Taking the phase voltage V_a as reference, determine:

(a) The current, real power, and reactive power drawn from the supply.
(b) The line voltage at the combined loads.
(c) The current per phase in each load.
(d) The total real and reactive powers in each load and the line.

(a) The Δ-connected load is transformed into an equivalent Y. The impedance per phase of the equivalent Y is

$$Z_2 = \frac{60 - j45}{3} = 20 - j15 \ \Omega$$

The phase voltage is

$$V_1 = \frac{207.85}{\sqrt{3}} = 120 \text{ V}$$

The single-phase equivalent circuit is shown in Figure 2.20.

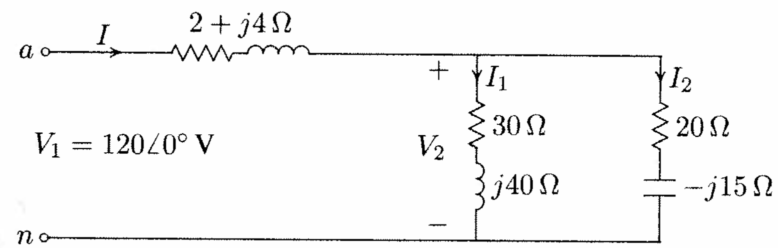

FIGURE 2.20
Single-phase equivalent circuit for Example 2.7.

The total impedance is

$$\begin{aligned} Z &= 2 + j4 + \frac{(30 + j40)(20 - j15)}{(30 + j40) + (20 - j15)} \\ &= 2 + j4 + 22 - j4 = 24 \ \Omega \end{aligned}$$

With the phase voltage V_{an} as reference, the current in phase a is

$$I = \frac{V_1}{Z} = \frac{120\angle 0^\circ}{24} = 5 \text{ A}$$

The three-phase power supplied is

$$S = 3V_1 I^* = 3(120\angle 0^\circ)(5\angle 0^\circ) = 1800 \text{ W}$$

(b) The phase voltage at the load terminal is

$$\begin{aligned} V_2 &= 120\angle 0^\circ - (2 + j4)(5\angle 0^\circ) = 110 - j20 \\ &= 111.8\angle -10.3^\circ \text{ V} \end{aligned}$$

The line voltage at the load terminal is

$$V_{2ab} = \sqrt{3}\angle 30^\circ\ V_2 = \sqrt{3}\,(111.8)\angle 19.7^\circ = 193.64\angle 19.7^\circ \text{ V}$$

(c) The current per phase in the Y-connected load and in the equivalent Y of the Δ load is

$$I_1 = \frac{V_2}{Z_1} = \frac{110 - j20}{30 + j40} = 1 - j2 = 2.236\angle -63.4^\circ \text{ A}$$
$$I_2 = \frac{V_2}{Z_2} = \frac{110 - j20}{20 - j15} = 4 + j2 = 4.472\angle 26.56^\circ \text{ A}$$

The phase current in the original Δ-connected load, i.e., I_{ab} is given by

$$I_{ab} = \frac{I_2}{\sqrt{3}\angle -30^\circ} = \frac{4.472\angle 26.56^\circ}{\sqrt{3}\angle -30^\circ} = 2.582\angle 56.56^\circ \text{ A}$$

(d) The three-phase power absorbed by each load is

$$S_1 = 3V_2I_1^* = 3(111.8\angle -10.3^\circ)(2.236\angle 63.4^\circ) = 450 \text{ W} + j600 \text{ var}$$
$$S_2 = 3V_2I_2^* = 3(111.8\angle -10.3^\circ)(4.472\angle -26.56^\circ) = 1200 \text{ W} - j900 \text{ var}$$

The three-phase power absorbed by the line is

$$S_L = 3(R_L + jX_L)|I|^2 = 3(2 + j4)(5)^2 = 150 \text{ W} + j300 \text{ var}$$

It is clear that the sum of load powers and line losses is equal to the power delivered from the supply, i.e.,

$$\begin{aligned} S_1 + S_2 + S_L &= (450 + j600) + (1200 - j900) + (150 + j300) \\ &= 1800 \text{ W} + j0 \text{ var} \end{aligned}$$

Example 2.8 (chp2ex8)

A three-phase line has an impedance of $0.4 + j2.7\ \Omega$ per phase. The line feeds two balanced three-phase loads that are connected in parallel. The first load is absorbing 560.1 kVA at 0.707 power factor lagging. The second load absorbs 132 kW at unity power factor. The line-to-line voltage at the load end of the line is 3810.5 V. Determine:

(a) The magnitude of the line voltage at the source end of the line.
(b) Total real and reactive power loss in the line.
(c) Real power and reactive power supplied at the sending end of the line.

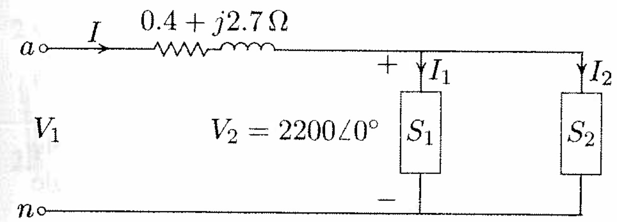

FIGURE 2.21
Single-phase equivalent diagram for Example 2.8.

(a) The phase voltage at the load terminals is

$$V_2 = \frac{3810.5}{\sqrt{3}} = 2200 \text{ V}$$

The single-phase equivalent circuit is shown in Figure 2.21.
The total complex power is

$$\begin{aligned} S_{R(3\phi)} &= 560.1(0.707 + j0.707) + 132 = 528 + j396 \\ &= 660\angle 36.87^\circ \text{ kVA} \end{aligned}$$

With the phase voltage V_2 as reference, the current in the line is

$$I = \frac{S^*_{R(3\phi)}}{3V_2^*} = \frac{660,000\angle -36.87^\circ}{3(2200\angle 0^\circ)} = 100\angle -36.87^\circ \text{ A}$$

The phase voltage at the sending end is

$$V_1 = 2200\angle 0^\circ + (0.4 + j2.7)100\angle -36.87^\circ = 2401.7\angle 4.58^\circ \text{ V}$$

The magnitude of the line voltage at the sending end of the line is

$$|V_{1L}| = \sqrt{3}|V_1| = \sqrt{3}(2401.7) = 4160 \text{ V}$$

(b) The three-phase power loss in the line is

$$\begin{aligned} S_{L(3\phi)} &= 3R|I|^2 + j3X|I|^2 = 3(0.4)(100)^2 + j3(2.7)(100)^2 \\ &= 12 \text{ kW} + j81 \text{ kvar} \end{aligned}$$

(c) The three-phase sending power is

$$S_{S(3\phi)} = 3V_1I^* = 3(2401.7\angle 4.58^\circ)(100\angle 36.87^\circ) = 540 \text{ kW} + j477 \text{ kvar}$$

It is clear that the sum of load powers and the line losses is equal to the power delivered from the supply, i.e.,

$$S_{S(3\phi)} = S_{R(3\phi)} + S_{L(3\phi)} = (528 + j396) + (12 + j81) = 540 \text{ kW} + j477 \text{ kvar}$$