EEE225 Advanced Electrical Circuits and Electromagnetics

Lecture 10 Electromagnetic fields

(Static and Time-varying fields)

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Content

- Faraday's Law
  - Faraday's experiments
  - Lenz's law
  - Faraday's law
- Displacement current
  - Modified Ampere's law
- Complete Maxwell's equations
- Boundary conditions
- Time-harmonic fields (sinusoidal fields)

Static Electric and Magnetic Fields

Fundamental Relations	Electrostatic Model	Magnetostatic Model
Governing equations	$\nabla \times \mathbf{E} = 0$ $\nabla \cdot \mathbf{D} = \rho$	$\nabla \cdot \mathbf{B} = 0$ $\nabla \times \mathbf{H} = \mathbf{J}$
Constitutive relations (linear and isotropic media)	$\mathbf{D} = \epsilon \mathbf{E}$	$\mathbf{H} = \frac{1}{\mu}\mathbf{B}$

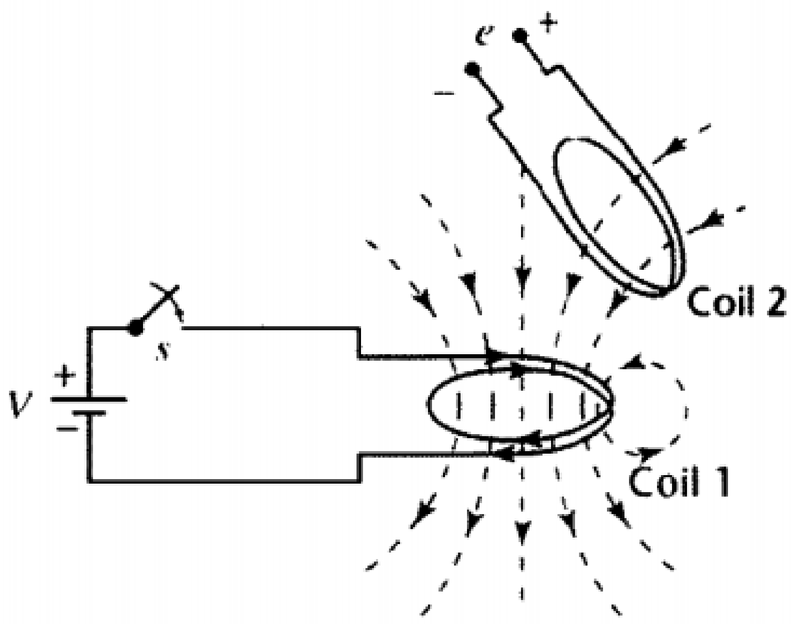
- The electric fields produced by stationary charges.
- The magnetic fields produced by moving charges (currents).

- Imposing an electric field on a conductor gives rise to a current which in turn generates a magnetic field.
  - Oersted’s experiment
  - Faraday’s modified experiment
  - Ampere’s Law

- Whether or not an electric field could be produced by a magnetic field?

Faraday's Experiments

Experiment 1



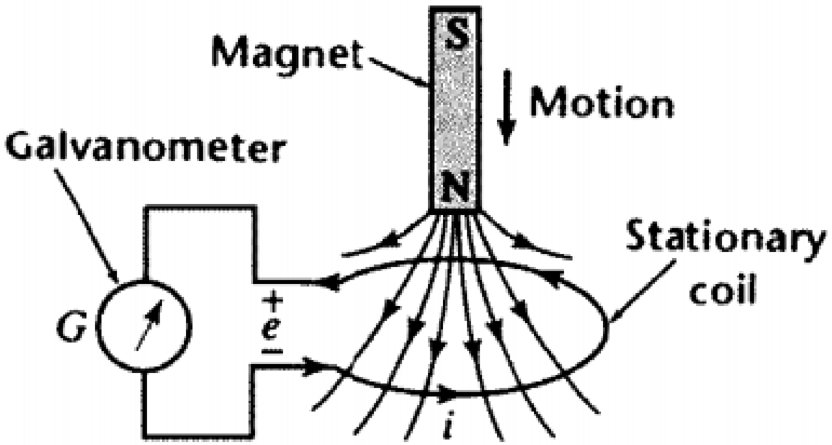
Induced voltage can be detected in coil 2 at the time of opening or closing the switch *s*.

- Experiment setup:

Video 1

Faraday's Experiments

Experiment 2



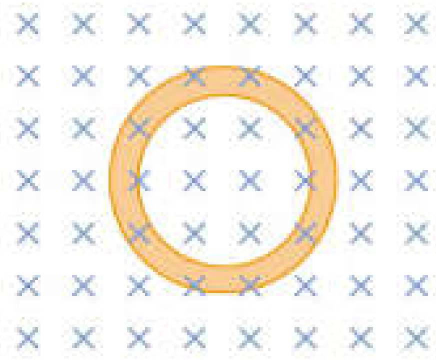
Induced voltage and current can be detected in the coil when moving the magnet towards or away from the coil.

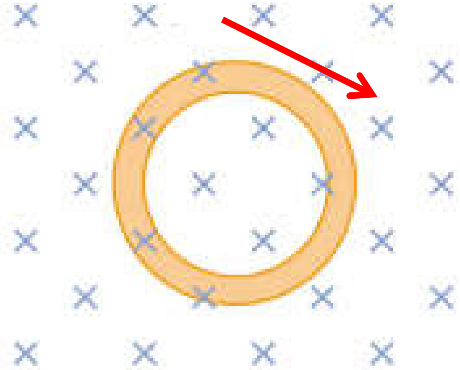
- Lenz's Law - The induced current produces magnetic fields which tend to oppose the change in magnetic flux that induces such currents.
  - Most "human" law;
  - Determine the "direction of induced current".

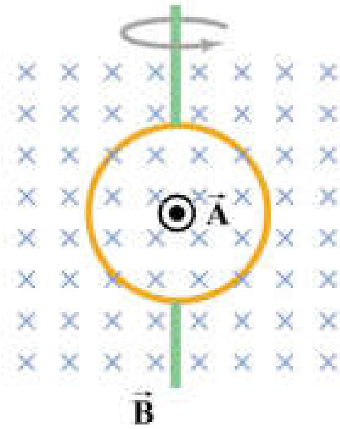
Video 2

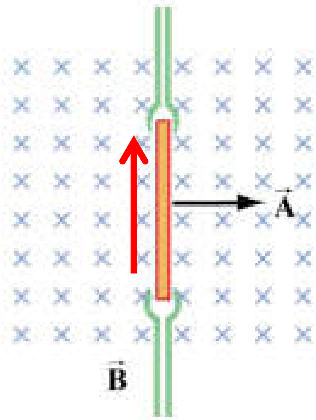
Lenz's Law

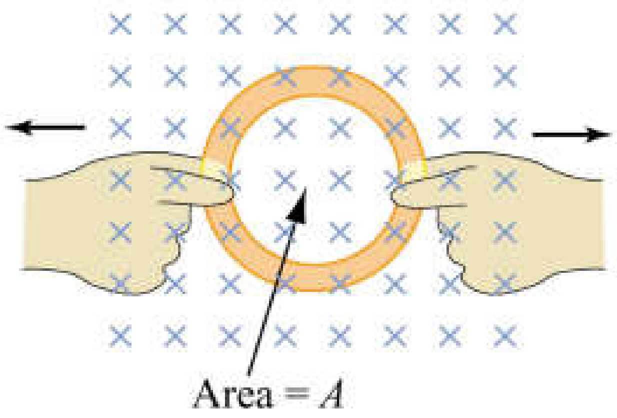
- Examples

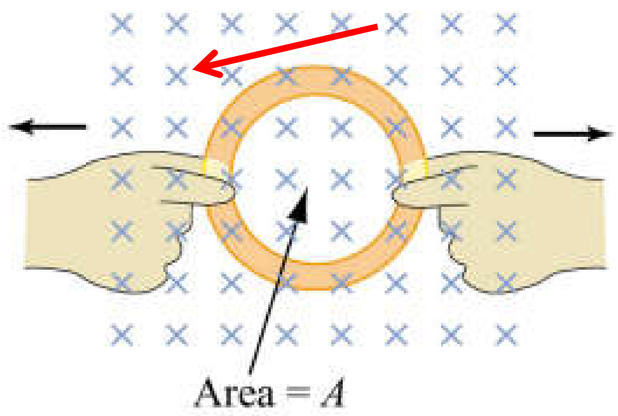












Faraday’s Experiments

- The process of inducing a voltage in a coil (also called a loop) by placing it in a time-varying magnetic field is now commonly referred to as an ***electromagnetic induction***.
  - The induced current in a closed conducting path is a consequence of the ***induced voltage*** in the loop.
  - Experiment:

Faraday's Experiments

- Recall the Faraday's experiment setup:

Video 3

Video 4

- The flux Φ_B is confined inside the solenoid, which is irrelevant to loop 2;
  - So the induced current is irrelevant to the size and shape of loop 2.
  - But the induced current is proportional to the turns of loop 2.

Faraday's Law

- Faraday's Law - a time-varying magnetic field produces an **electromotive force (*emf*)** that may establish a current in a suitable closed circuit.
  - An electromotive force is merely a voltage that arises from conductors moving in a magnetic field or changing magnetic fields

$$emf = -\frac{d\Phi}{dt} \ (V)$$

  - The minus sign is again, from Lenz's Law.
- Lenz's Law – an indication that the *emf* is in such a direction as to produce a current whose flux, if added to the original flux, would reduce the magnitude of the *emf*.

Faraday's Law

- Define the induced *emf* in a conductor in terms of the induced electric field intensity inside the conductor as:

$$emf = \oint_C \vec{E} \cdot \overrightarrow{dl}$$

- The total flux enclosed by contour *c* is

$$\Phi = \iint_S \vec{B} \cdot \overrightarrow{ds}$$

- Therefore, the Faraday's law can be written as:

$$\oint_C \vec{E} \cdot \overrightarrow{dl} = -\frac{d}{dt} \iint_S \vec{B} \cdot \overrightarrow{ds}$$

The direction of the surface ***ds*** is defined by the direction of contour *c* and the right-hand rule.

$$= -\iint_S \frac{\partial \vec{B}}{\partial t} \cdot \overrightarrow{ds}$$

$\rightarrow$ Integral form of Faraday's Law

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Faraday's Law

$$\boxed{emf = \oint_C \vec{E} \cdot \overrightarrow{dl} = -\iint_S \frac{\partial \vec{B}}{\partial t} \cdot \overrightarrow{ds}}$$

- Using Stokes' theorem:

$$\oint_C \vec{E} \cdot \overrightarrow{dl} = \iint_S (\nabla \times \vec{E}) \cdot \overrightarrow{ds} = -\iint_S \frac{\partial \vec{B}}{\partial t} \cdot \overrightarrow{ds}$$

- The integrand should be equal on both sides, so

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \longrightarrow \boxed{\text{Differential form of Faraday's Law}}$$

- The electric field intensity in a region of time-varying magnetic flux density is ***nonconservative***.

Faraday's Law – *Example of stationary circuit*

- Example 1: A stationary circuit in a time-varying magnetic field. A circular loop of N turns of conducting wire lies in the xy-plane with its center at the origin of a magnetic field specified by $\mathbf{B} = \mathbf{a_z} B_0 \cos(\pi/2b)\sin(\omega t)$, where b is the radius of the loop and ω is the angular frequency. Find the emf induced in the loop.
- Solution:
  - The magnetic flux linking each turn of the circular loop is

$$\Phi = \int_S \mathbf{B} \cdot d\mathbf{s} = \int_0^b \left[\mathbf{a}_z B_0 \cos \frac{\pi}{2b} \sin \omega t \right] \cdot (\mathbf{a}_z 2\pi r \, dr)$$

  - Since there are N turns, the total flux is $N\Phi$, so

$$emf = -N \frac{d\Phi}{dt}$$

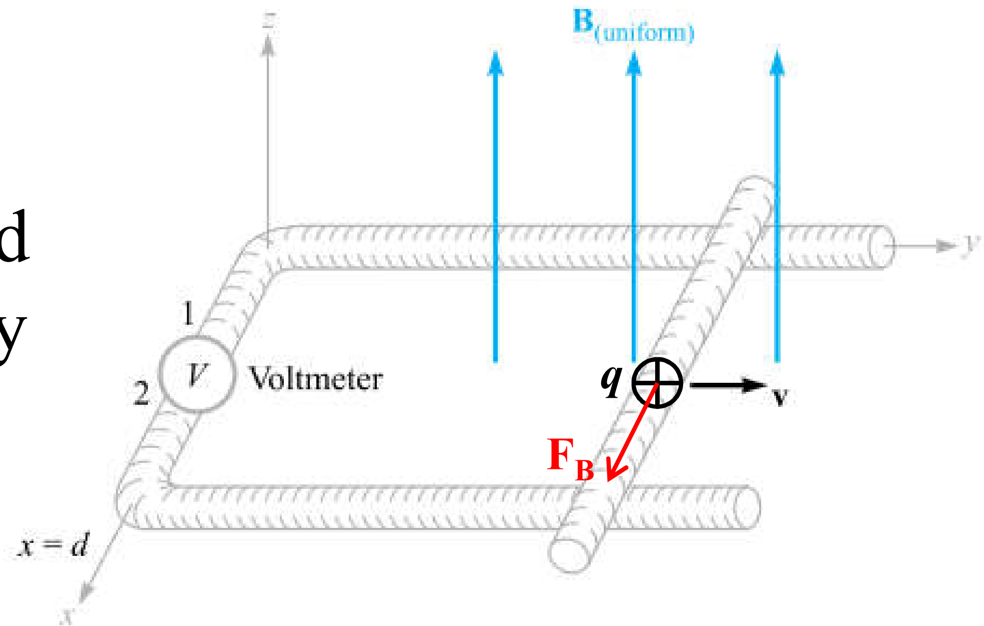
Faraday's Law

- In fact, the electromagnetic induction will take place as long as one of the following conditions holds.
  - 1. A time-changing magnetic flux intensity **B** in a stationary loop;
  - 2. Relative motion between a steady flux and a closed path (loop)
    - The coil continuously changes its shape, position or orientation;

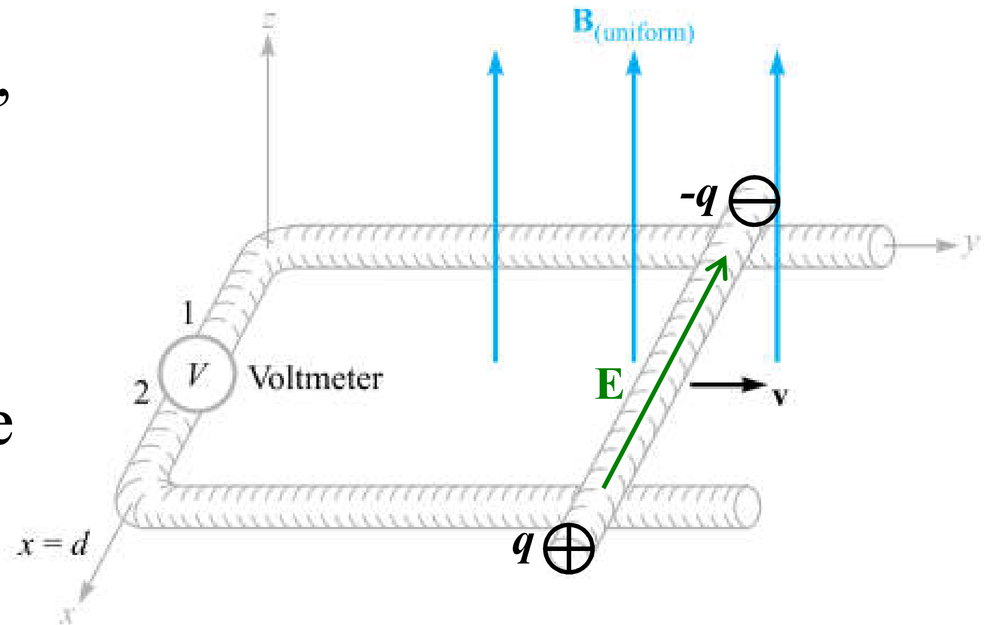
Video 5

Faraday's Law – *Example of Motional EMF*

- In the constant magnetic field **B**, the shorting bar moves to the right with a velocity *v*, and the circuit is completed through the two rails and an extremely small high-resistance voltmeter is used to read the emf.

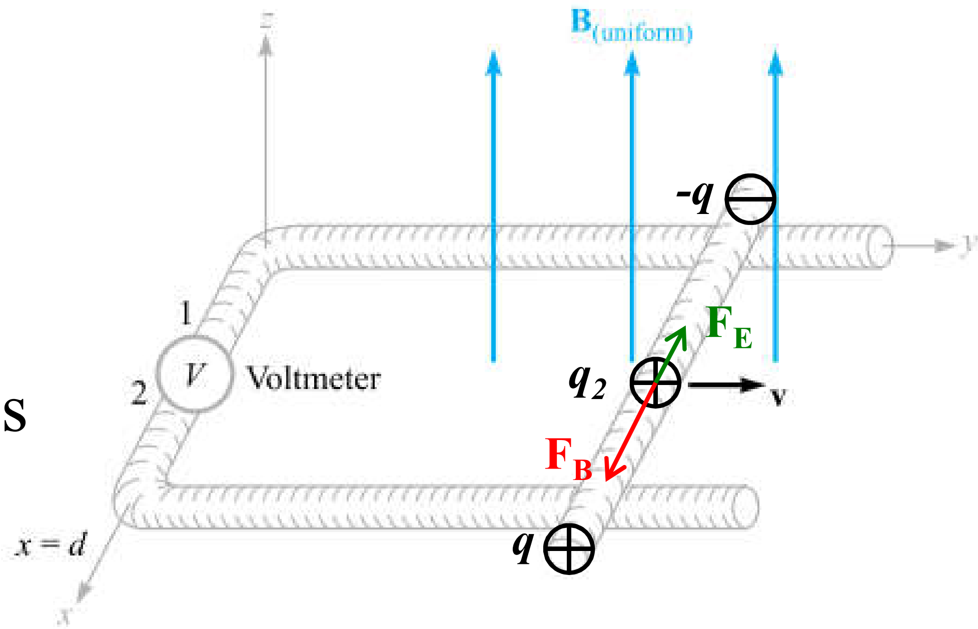


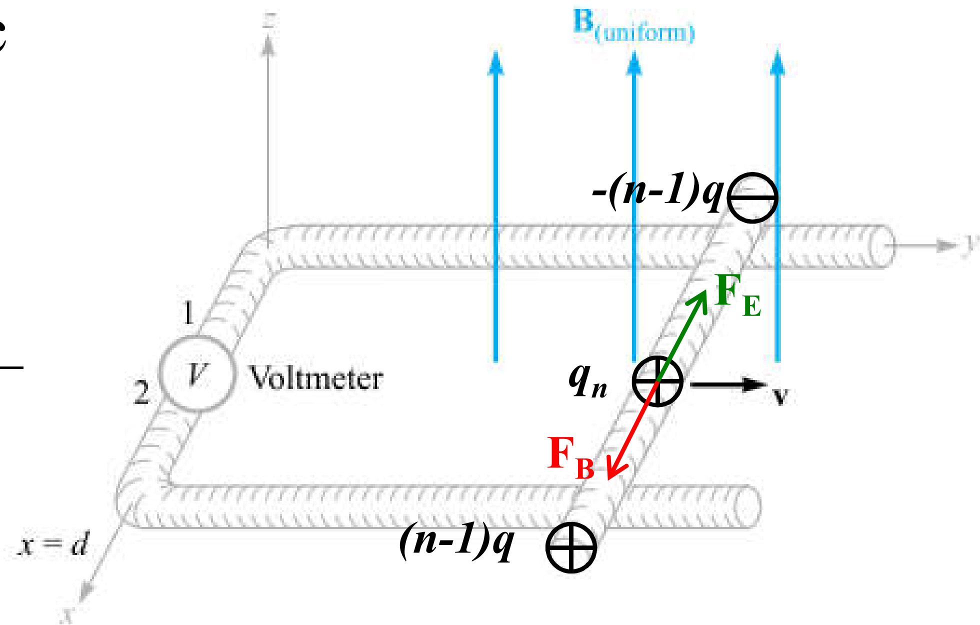
- Analyses:
  - Consider a charge *q* on the conductor, which experiences a Lorentz's force $\mathbf{F_B}$, make it drifted to the lower end (+*x* direction) of the conducting bar.
  - The whole bar is neutral, so a positive and negative charge pair built an internal **E** field inside the bar.



Faraday's Law – Example of Motional EMF

- Analyses (continued):
  - Consider a new charge q_2, which experiences two forces, the Lorentz's force due to the field **B** and the electric force due to the field **E**. In this case, $\mathbf{F_B} > \mathbf{F_E}$, so q_2 drifts to $+x$ direction and contributes to field **E**.
  - After a while (very short), the electric field **E** increases to a value large enough to generate the force $\mathbf{F_E} = \mathbf{F_B}$. Now the charges can move in y direction without x direction drifting – equilibrium state.





Faraday's Law – *Example of Motional EMF*

- The force per unit charge is called the motional electric field intensity $\mathbf{E_m}$:

$$\boldsymbol{E_m} = \frac{\boldsymbol{F}}{Q} = \boldsymbol{v} \times \boldsymbol{B}$$

- The voltage produced by the induced motional electric field intensity $\mathbf{E_m}$ is then:

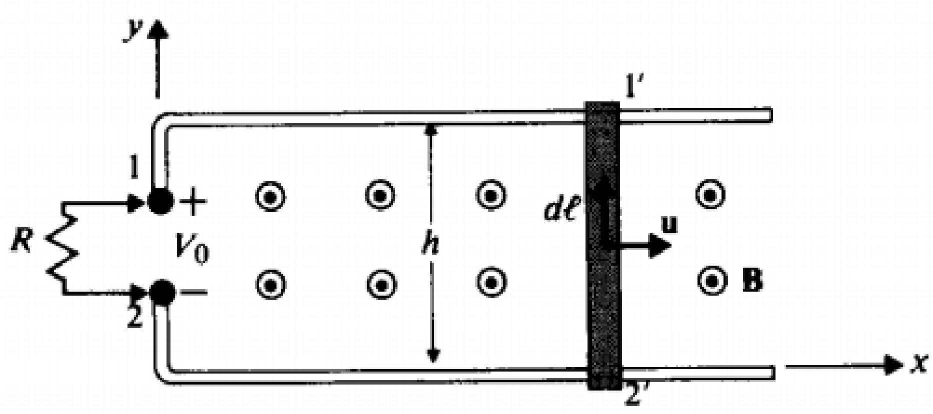
$$V_{12} = \oint \boldsymbol{E_m} \cdot dL = \oint (\boldsymbol{v} \times \boldsymbol{B}) \cdot d\boldsymbol{L} = \int_{\mathrm{d}}^{0} vBdx = -Bvd$$

  – where *d* is the length of the conducting bar.

- This is referred to as a ***flux cutting emf*** or a ***motional emf***.
  – Obviously, only the part of the circuit that moves in a direction not parallel to the magnetic flux will contribute to V.

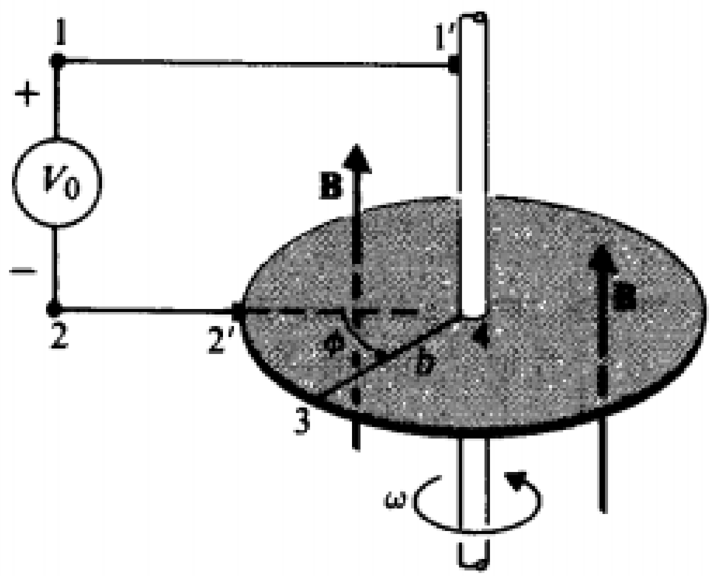
Faraday's Law – Example of Motional EMF

- Example 2: A metal bar slides over a pair of conducting rails in a uniform magnetic field $\mathbf{B} = \mathbf{a}_z B_0$ with a constant velocity $\mathbf{u}$, as shown in the figure.
  - a) Determine the open-circuit voltage V_0 that appears across terminals 1 and 2;
  - b) Assuming that a resistance R is connected between the terminals, find the electric power dissipated in R;
  - c) Show that this electric power is equal to the mechanical power required to move the sliding bar with a velocity u. Neglect the electric resistance of the metal bar and the conducting rails.



Faraday's Law – Example of Motional EMF

- Example 3: The Faraday disk generator consists of a circular metal disk rotating with a constant angular velocity ω in a uniform and constant magnetic filed of flux density $\boldsymbol{B} = \mathbf{a}_z B_0$ that is parallel to the axis of rotation. Brush contacts are provided at the axis and on the rim of the disk, as shown by the figure on the right.
- Determine the open-circuit voltage of the generator if the radius of the disk is b.



A Moving Circuit in a Time-varying Magnetic Field

- Recall the ***Lorentz's force equation***: for a charge q moving in a region where both **E** and **B** fields exist, the EM force **F** on q is measured by:

$$\boldsymbol{F} = q(\boldsymbol{E} + \boldsymbol{u} \times \boldsymbol{B}) = q\boldsymbol{E}'$$

- Therefore, when a conducting circuit with contour C and surface S moves with a velocity $\boldsymbol{u}$ in a field (**E**, **B**), the total emf is:

$$emf = \oint_C \boldsymbol{E}' \cdot d\boldsymbol{l} = \boxed{-\iint_S \frac{\partial \boldsymbol{B}}{\partial t} \cdot d\boldsymbol{s}} + \boxed{\oint_C (\boldsymbol{u} \times \boldsymbol{B}) \cdot d\boldsymbol{l}} \quad (V)$$

  - This is the general form of ***Faraday's Law***.

Transformer emf – due to the time variation of **B**

Motional emf – due to the motion of the circuit

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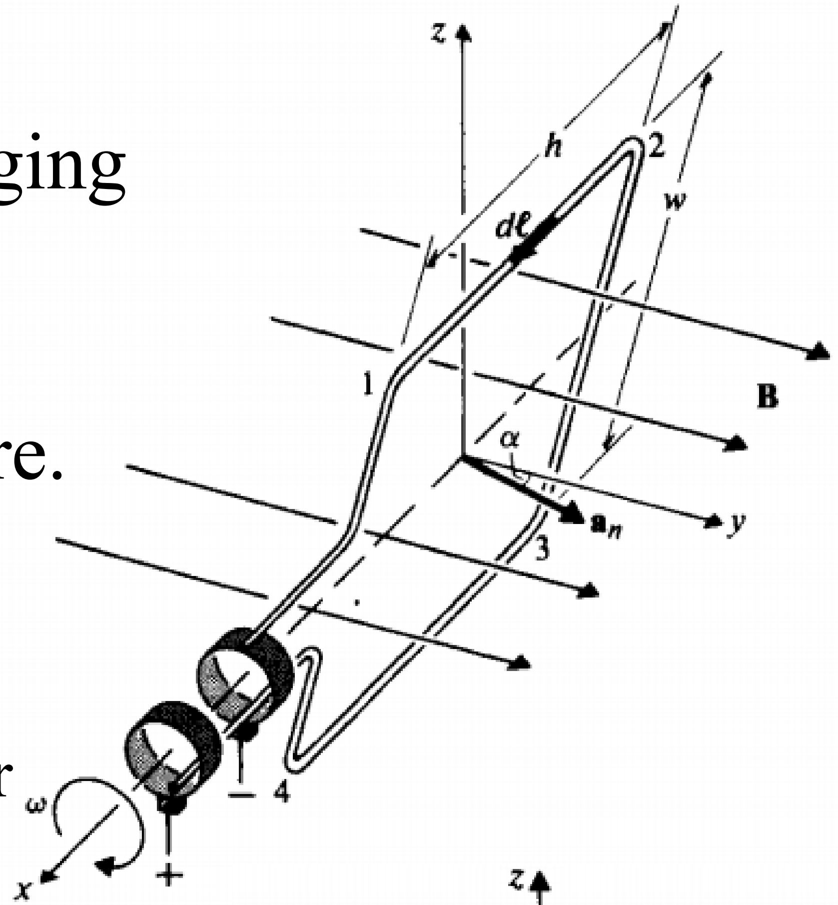
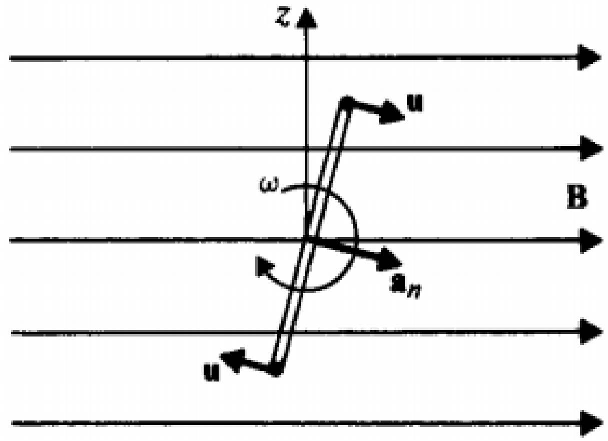
Faraday's Law – Example of a generator

- Example 4: An h by w rectangular conducting loop is situated in a changing magnetic field $\mathbf{B} = \mathbf{a}_y B_0 \sin(\omega t)$. The normal of the loop initially makes an angle α with $\mathbf{a}_y$, as shown in the figure.
- Find the induced emf in the loop:
  - a) when the loop is at rest;
  - b) when the loop rotates with an angular velocity ω about the x-axis.

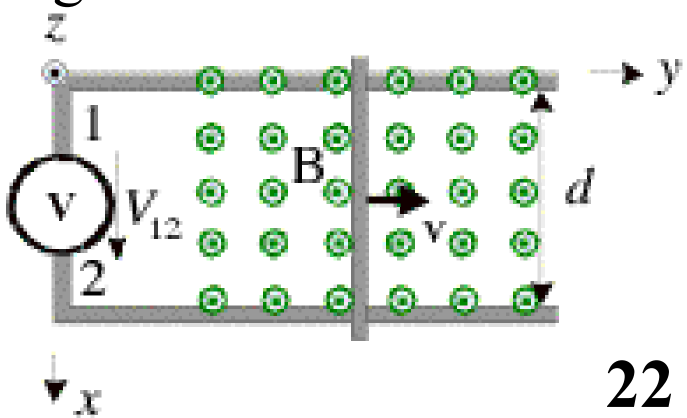




Quiz

- 1. A square loop of area A = 100 cm^2 and N = 200 turns rotates in an external steady magnetic field **B** = 1.2 $\mathbf{a}_x$ T. The axis of rotation is $\mathbf{a}_z$. The loop rotates at a rate of 6000 RPM (revolutions per minute). What is the magnitude of the voltage em induced at the terminals of the loop (at open circuit)?
  - (a) em = 1508 V; (b) em = 1120 V;
  - (c) em = 965 V; (d) em = 557 V.

- 2. An external force is applied to a conducting bar supported by conducting rails, with which the bar is in perfect electrical contact (see figure). The bar moves with a constant velocity $\boldsymbol{v}$ = 40 $\mathbf{a}_y$ m/s. A steady magnetic field is present: **B** = 0.5 $\mathbf{a}_z$ T. The width between the supporting rails is d = 0.5 m. Find the voltage V_{12} measured by the ideal voltmeter.
  - (a) V_{12} = 20 V; (b) V_{12} = -20 V;
  - (c) V_{12} = 10 V; (d) V_{12} = -10 V.



Maxwell's Equations (So far)

- So far, we have the equations:

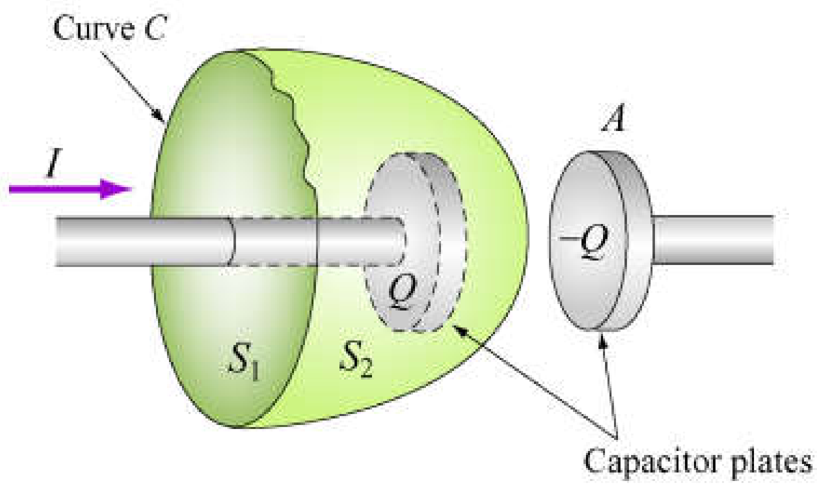
Law	Integral	Differential	Physical meaning
Gauss's law for **E**	$\oiint_S \boldsymbol{D}\cdot d\boldsymbol{s} = Q$	$\nabla\cdot\boldsymbol{D} = \rho$	Electric flux through a closed surface is proportional to the charged enclosed
Faraday's law	$\oint_C \boldsymbol{E}\cdot d\boldsymbol{l} = -\frac{d\Phi_B}{dt}$	$\nabla\times\boldsymbol{E} = -\frac{\partial\boldsymbol{B}}{\partial t}$	Changing magnetic flux produces an electric field
Gauss's law for **B**	$\oiint_S \boldsymbol{B}\cdot d\boldsymbol{s} = 0$	$\nabla\cdot\boldsymbol{B} = 0$	The total magnetic flux through a closed surface is zero
Ampere's law	$\oint_C \boldsymbol{H}\cdot d\boldsymbol{l} = I$	$\nabla\times\boldsymbol{H} = \boldsymbol{J}$	Electric current produces a magnetic field

- Changing magnetic field produces electric field.

- Can changing electric field produces magnetic field?

Displacement Current

- Consider a capacitor which is being charged by a DC current *I*.
  - If the surface bounded by the path is the flat surface S_1, then the enclosed current is $I_{enc} = I$.
  - If we choose S_2 to be the surface bounded by the curve, then $I_{enc} = 0$ since no current passes through S_2.



Surfaces S_1 and S_2 bound by C

- Solution: adding an extra term

$$I_d = \varepsilon_0 \frac{d\Phi_E}{dt} \longrightarrow \boxed{\text{Displacement current}}$$

  - This term involves a change in electric flux.

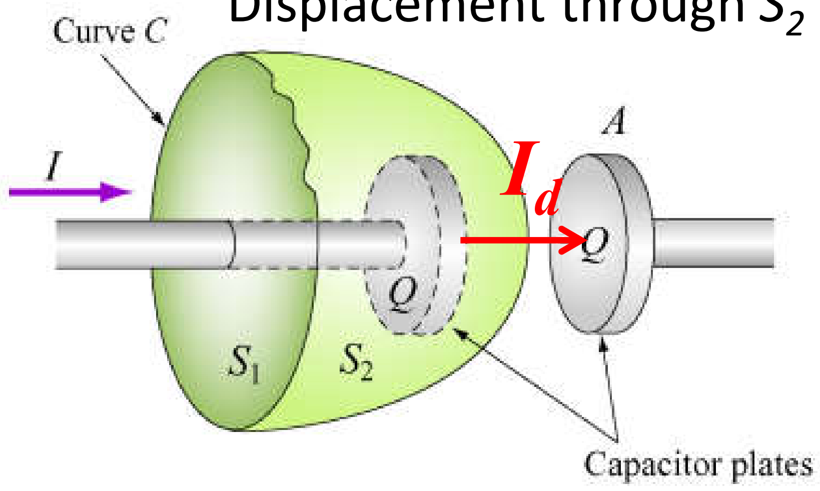
Displacement Current

- The electric flux passes through S_2 is given by:

$$\Phi_E = \iint_S \boldsymbol{E} \cdot d\boldsymbol{s} = EA = \frac{Q}{\varepsilon_0}$$

  - Since $I_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \frac{dQ}{dt}$, which is the rate of increase of charge on the plate, with $\frac{dQ}{dt} = I$, so $I_d = I$.
  - So, no matter how to choose the surface, there always exists $I_{enc} = I$.

Displacement through S_2



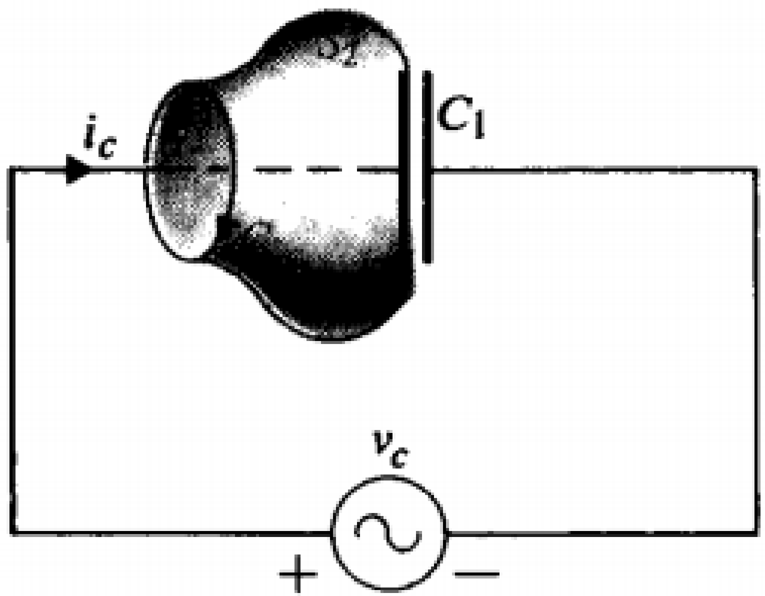
- So the generalized Ampere's (or the Ampere-Maxwell) law:

$$\oint_C \boldsymbol{H} \cdot d\boldsymbol{l} = I + \varepsilon_0 \frac{d\Phi_E}{dt} \longrightarrow \boxed{\text{Integral form}}$$

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t} \longrightarrow \boxed{\text{Differential form}}$$

Displacement Current – Example

- Example 5: An AC voltage source of amplitude V_0 and angular frequency ω, $v_c = V_0\sin(\omega t)$, is connected across a parallel-plate capacitor C_1, as shown in the figure.
  - a) Verify that the displacement current in the capacitor is the same as the conduction current in the wires.
  - b) Determine the magnetic field intensity at a distance r from the wire.



Maxwell's Equations

- Finally, the Maxwell's equations are:

Law	Integral	Differential	Physical meaning
Gauss's law for **E**	$\oiint_S \boldsymbol{D} \cdot d\boldsymbol{s} = Q$	$\nabla \cdot \boldsymbol{D} = \rho$	Electric flux through a closed surface is proportional to the charged enclosed
Faraday's law	$\oint_C \boldsymbol{E} \cdot d\boldsymbol{l} = -\dfrac{d\Phi_B}{dt}$	$\nabla \times \boldsymbol{E} = -\dfrac{\partial \boldsymbol{B}}{\partial t}$	Changing magnetic flux produces an electric field
Gauss's law for **B**	$\oiint_S \boldsymbol{B} \cdot d\boldsymbol{s} = 0$	$\nabla \cdot \boldsymbol{B} = 0$	The total magnetic flux through a closed surface is zero
Generalized Ampere's law	$\oint_C \boldsymbol{H} \cdot d\boldsymbol{l} = I + \varepsilon_0 \dfrac{d\Phi_E}{dt}$	$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \dfrac{\partial \boldsymbol{D}}{\partial t}$	Electric current and changing electric flux produces a magnetic field

Electromagnetic Boundary Conditions

- The boundary conditions for time-varying fields are exactly the same as those for static fields.

Scalar form	Vector form
$E_{t1} = E_{t2}$	$\vec{\mathbf{a}}_n \times (\vec{\mathbf{E}}_1 - \vec{\mathbf{E}}_2) = 0$
$H_{t1} - H_{t2} = J_s$	$\vec{\mathbf{a}}_n \times (\vec{\mathbf{H}}_1 - \vec{\mathbf{H}}_2) = \vec{\mathbf{J}}_s$
$B_{n1} = B_{n2}$	$\vec{\mathbf{a}}_n \cdot (\vec{\mathbf{B}}_1 - \vec{\mathbf{B}}_2) = 0$
$D_{n1} - D_{n2} = \rho_s$	$\vec{\mathbf{a}}_n \cdot (\vec{\mathbf{D}}_1 - \vec{\mathbf{D}}_2) = \rho_s$

We can make the following general statements about the EM BCs:

1. The tangential component of an **E** field is continuous across an interface;
2. The tangential component of an **H** field is discontinuous across an interface where a surface current exists, the amount of discontinuity being determined by the $\boldsymbol{J_s}$;
3. The normal component of a **D** field is discontinuous across an interface where a surface charge exists, the amount of discontinuity being determined by ρ_s;
4. The normal component of a **B** field is continuous across an interface.

EM Boundary Conditions –

Interface between two lossless linear media

- A lossless linear medium can be specified by a permittivity ε and a permeability μ, with $\sigma = 0$.
- There are usually no free charges and no surface currents at the interface between two lossless media => $\rho_s = 0$ and $\boldsymbol{J}_s = 0$.
- Therefore, the BCs are:

$$E_{1t} = E_{2t} \rightarrow \frac{D_{1t}}{D_{2t}} = \frac{\epsilon_1}{\epsilon_2}$$

$$H_{1t} = H_{2t} \rightarrow \frac{B_{1t}}{B_{2t}} = \frac{\mu_1}{\mu_2}$$

$$D_{1n} = D_{2n} \rightarrow \epsilon_1 E_{1n} = \epsilon_2 E_{2n}$$

$$B_{1n} = B_{2n} \rightarrow \mu_1 H_{1n} = \mu_2 H_{2n}$$

EM Boundary Conditions –

Interface between a dielectric and a perfect conductor

- In the interior of a perfect conductor, the electric fields (**E**, **D**) are zero, and any charges the conductor will have will reside on the surface only.
- For a time-varying EM field, the (**E**, **D**) is zero ensures that (**B**, **H**) are also zero in the interior of a conductor.
- Therefore, the BCs are:

On the Side of Medium 1	On the Side of Medium 2
$E_{1t} = 0$	$E_{2t} = 0$
$\mathbf{a}_{n2} \times \mathbf{H}_1 = \mathbf{J}_s$	$H_{2t} = 0$
$\mathbf{a}_{n2} \cdot \mathbf{D}_1 = \rho_s$	$D_{2n} = 0$
$B_{1n} = 0$	$B_{2n} = 0$
dielectric	conductor

Boundary Conditions – Example

- Example 6: An electric field intensity in a source-free dielectric medium is given by $\boldsymbol{E} = C\cos(\omega t - \beta z)\,\boldsymbol{a_x}$ V/m, where C is the amplitude of the field, ω is the frequency, and β is a constant quantity.
- Under what condition can this field exist? What are the other field quantities?

Poynting Theorem

- With some derivation of the Maxwell's equations, we get:

$$\nabla \cdot \boxed{(\vec{\mathbf{E}} \times \vec{\mathbf{H}})} + \vec{\mathbf{J}} \cdot \vec{\mathbf{E}} + \vec{\mathbf{H}} \cdot \frac{\partial \vec{\mathbf{B}}}{\partial t} + \vec{\mathbf{E}} \cdot \frac{\partial \vec{\mathbf{D}}}{\partial t} = 0$$

Differential form of Poynting's theorem

Poynting vector: with the unit of power density, W/m^2, is the instantaneous flow of power per unit area.
Defined as: **S** = **E x H**, where **S** is the Poynting vector, normal to the plane containing **E** and **H**.

- With some more modifications, get:

$$\oint_s \vec{\mathbf{S}} \cdot \vec{ds} + \int_v \vec{\mathbf{J}} \cdot \vec{\mathbf{E}}\, dv + \frac{d}{dt}\int_v w_m\, dv - \frac{d}{dt}\int_v w_e\, dv = 0$$

Integral form of Poynting's theorem

  – where $w_m = \frac{1}{2}\vec{\mathbf{B}} \cdot \vec{\mathbf{H}} = \frac{1}{2}\mu H^2$

$$w_e = \frac{1}{2}\vec{\mathbf{D}} \cdot \vec{\mathbf{E}} = \frac{1}{2}\epsilon E^2$$

Poynting Theorem

$$\oint_s \vec{\mathbf{S}} \cdot \overrightarrow{ds} + \int_v \vec{\mathbf{J}} \cdot \vec{\mathbf{E}}\, dv + \frac{d}{dt} \int_v w_m\, dv - \frac{d}{dt} \int_v w_e\, dv = 0$$

- The first term represents the power crossing the closed surface *S* bounding the volume *v*.
- The second integral represents the power supplied to the charged particles by the field.
- The third term represents the rate of change of stored magnetic energy.
- The final term represents the rate of change of stored energy in the electric field.

$$-\oint_s \vec{\mathbf{S}} \cdot \overrightarrow{ds} = \int_v \vec{\mathbf{J}} \cdot \vec{\mathbf{E}}\, dv + \frac{d}{dt} \int_v (w_m + w_e)\, dv$$

- The negative sign on the left indicates that the net power must flow into volume v in order to account for (a) the power dissipation in the region as heat and (b) the increase in the energy stored in electric and magnetic fields.

Poynting Theorem – Example

- Example 7: The electric field intensity in a dielectric medium is given as $\mathbf{E} = E_0 \cos(\omega t - kz)\, \mathbf{a_x}$ V/m, where E_0 is its peak value, and k is a constant quantity.
- Determine:
  - (a) the magnetic field intensity in the region;
  - (b) the direction of power flow;
  - (c) the average power density.

Time-harmonic Fields

- Time-harmonic (sinusoidal) field: the excitation source varies sinusoidally in time with a single frequency. In a linear system, a sinusoidally varying source generates fields that also vary sinusoidally in time at all points in the system.
- For example, if the **E** field is given as:

$$\vec{\mathbf{E}}(x,y,z,t) = E_x(x,y,z,t)\vec{\mathbf{a}}_x + E_y(x,y,z,t)\vec{\mathbf{a}}_y + E_z(x,y,z,t)\vec{\mathbf{a}}_z$$

  – Where

$$E_x(x,y,z,t) = E_x(r,t) = E_{x0}(r)\cos[\omega t + \alpha(r)] = \mathrm{Re}[E_{x0}(r)e^{j\alpha(r)}e^{j\omega t}] = \mathrm{Re}[\tilde{E}_x(r)e^{j\omega t}]$$
$$E_y(x,y,z,t) = E_y(r,t) = E_{y0}(r)\cos[\omega t + \beta(r)] = \mathrm{Re}[E_{y0}(r)e^{j\beta(r)}e^{j\omega t}] = \mathrm{Re}[\tilde{E}_y(r)e^{j\omega t}]$$
$$E_z(x,y,z,t) = E_z(r,t) = E_{z0}(r)\cos[\omega t + \gamma(r)] = \mathrm{Re}[E_{z0}(r)e^{j\gamma(r)}e^{j\omega t}] = \mathrm{Re}[\tilde{E}_z(r)e^{j\omega t}]$$

  – So

$$\vec{\mathbf{E}}(r,t) = \mathrm{Re}\{[\tilde{E}_x(r)\vec{\mathbf{a}}_x + \tilde{E}_y(r)\vec{\mathbf{a}}_y + \tilde{E}_z(r)\vec{\mathbf{a}}_z]e^{j\omega t}\}$$
$$= \mathrm{Re}[\tilde{\vec{\mathbf{E}}}(r)e^{j\omega t}]$$

  – where $\tilde{\vec{\mathbf{E}}}(r) = \tilde{E}_x(r)\vec{\mathbf{a}}_x + \tilde{E}_y(r)\vec{\mathbf{a}}_y + \tilde{E}_z(r)\vec{\mathbf{a}}_z \longrightarrow$ Phasor form

Time-harmonic Fields in Phasor Form

- For the field in its phasor form

$$\tilde{\vec{\mathbf{E}}}(r) = \tilde{E}_x(r)\vec{\mathbf{a}}_x + \tilde{E}_y(r)\vec{\mathbf{a}}_y + \tilde{E}_z(r)\vec{\mathbf{a}}_z$$

- The time rate of change of the **E** fields is

$$\frac{\partial \vec{\mathbf{E}}(r, t)}{\partial t} = \mathrm{Re}[j\omega \tilde{\vec{\mathbf{E}}}(r) e^{j\omega t}]$$

- Example 8: The E field in a source-free dielectric region is given as $\mathbf{E} = C \sin\alpha x \cos(\omega t - kz)\, \mathbf{a}_y$ V/m. Using phasor form, determine:
  - (a) the magnetic field intensity
  - (b) the time-average power flow per unit area.

Time-harmonic Fields – Maxwell's equations

Law	Integral	Differential
Gauss's law for **E**	$\oiint_S \boldsymbol{D} \cdot d\boldsymbol{s} = Q$	$\nabla \cdot \boldsymbol{D} = \rho$
Faraday's law	$\oint_C \boldsymbol{E} \cdot d\boldsymbol{l} = -j\omega \iint_S \boldsymbol{B} \cdot d\boldsymbol{s}$	$\nabla \times \boldsymbol{E} = -j\omega\boldsymbol{B}$
Gauss's law for **B**	$\oiint_S \boldsymbol{B} \cdot d\boldsymbol{s} = 0$	$\nabla \cdot \boldsymbol{B} = 0$
Generalized Ampere's law	$\oint_C \boldsymbol{H} \cdot d\boldsymbol{l} = I + j\omega \iint_S \boldsymbol{D} \cdot d\boldsymbol{s}$	$\nabla \times \boldsymbol{H} = \boldsymbol{J} + j\omega\boldsymbol{D}$

Next Two Lectures

- Maxwell's prediction: WAVES!
  - Plane waves
    - Uniform plane wave in free space
  - Guided waves
    - Waves in transmission lines

Extra

Video 6

Conservative field	Non-conservative field
Kirchhoff's voltage law (KVL)	Faraday's law
$\oint_{loop} \vec{E} \cdot d\vec{l} = 0$	$\oint_{loop} \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$
Circuit:	Circuit:
Results: V2 = 0.9 (V) V1 = 0.9 (V)	Results: V2 = 0.9 (V) V1 = -0.1 (V)