1. If $\boldsymbol{a} =< 1,0,2 >$ and $\boldsymbol{b} =< 3,2,1 >$, then $\boldsymbol{a} \times \boldsymbol{b} =$__________.
2. For a given $\boldsymbol{F} =< y^2,\ y^2 - x^2,\ 2z^2 >$, curl$\boldsymbol{F}$=__________.
3. If $\boldsymbol{F} =< \mathrm{e}^x \cos y,\ -\mathrm{e}^x \sin y + ay,\ 3z >$ and div$\boldsymbol{F}$=0, then a=__________.
4. A normal vector for surface $\boldsymbol{r}(u,v) =< u,\ v,\ 1 - u^2 - v^2 >$ is__________.
5. The type of the PDE $3u_{xx} + u_{xy} + u_{yy} = 0$ is__________.
6. If $\boldsymbol{F} =< x,\ 2y + 3 >$ and the curve C is $\boldsymbol{r}(t) =< t,\ t^2 - 1 >$, $0 \le t \le 1$, then the line integral $\int_C \boldsymbol{F} \cdot d\boldsymbol{r} =$__________.
7. Show that the line integral
$$\int_{(0,0,0)}^{(1,1,1)} (6xy^3 + 2z^2)dx + 9x^2y^2dy + (4xz + 1)dz$$
is independent of path and evaluate it.

8. Use Green's theorem to evaluate the line integral
$$\oint_C (x^3 + 2y)dx + (4x - 3y^2)dy\,,$$
Where C is the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

9. Find the general solution $u(x, y)$ of the PDE

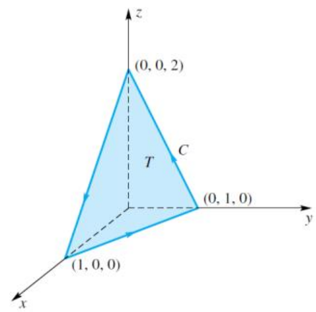
$$u_y + 2yu = 0.$$

10. Evaluate the surface integral $\iint_S \mathbf{F} \cdot \mathbf{n} dA$.

(a) $\boldsymbol{F} =< x^2,\ y^2,\ 1 >,\ \boldsymbol{r}(u, v) =< u,\ v,\ 2u - 3v >, 0 \leq u \leq 1, 0 \leq v \leq 2.$

(b) $\boldsymbol{F} =< yz,\ x + y,\ \mathrm{e}^x \mathrm{cos} y + z >$, S is the surface of $2 \leq x^2 + y^2 + z^2 \leq 4.$

11. Using Stokes's theorem to evaluate $\oint_C \mathbf{F}\cdot\mathbf{n}ds$, where $\boldsymbol{F} =< 2z,\ 8x-3y,\ 3x+y >$ and C is the triangular curve in the figure.



12. A period function of period $2L$ is defined by

$$f(x) = \begin{cases} x^2, -L \le x < 0 \\ L,\ \ 0 \le x < L \end{cases}.$$

(a) Sketch the graph of $f(x)$ in the range $-3L \le x \le 3L$.

(b) State the values the Fourier series will converge to at $x = 0, \frac{L}{2}, L, \frac{3L}{2}$.

(c) Find the Fourier series of $f(x)$ in $-L \le x < L$ and give the first three non-zero terms.

13. Given the PDE

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} + u = 0, 0 \le x < a, a > 0,$$

Where the function $u(x,t)$ satisfies the boundary conditions

$$u(0,t) = u(a,t) = 0, t \ge 0,$$

and the initial conditions

$$u(x,0) = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \sin\frac{n\pi x}{a}$$

and

$$u_t(x,0) = \sum_{n=1}^{\infty} \frac{\sqrt{n^2\pi^2 + a^2}}{\sqrt{n}\, a} \sin\frac{n\pi x}{a}.$$

(a) Using separation of variables with $u(x,t) = X(x)T(t)$, deduce that $X(x)$ and $T(t)$ satisfy the ordinary differential equations

$$X''(x) + \alpha_n^2 X(x) = 0$$

and

$$T''(t) + (\alpha_n^2 + 1)T(t) = 0,$$

where α_n is a constant.

(b) Solve the first ODE and show that $X(x) = A_n \sin\frac{n\pi}{a}x$, where A_n is a constant.

(c) Show that the solution is

$$u(x,t) = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \sin\frac{n\pi x}{a}\left[cos\frac{\sqrt{n^2\pi^2+a^2}}{a}t + sin\frac{\sqrt{n^2\pi^2+a^2}}{a}t\right].$$