Xi'an Jiaotong-Liverpool University
西交利物浦大學

EEE204 Continuous and Discrete Time Signals and Systems II

2018–2019 Semester 2

Electrical and Electronic Engineering

Xi'an Jiaotong-Liverpool University

Week 5

Causality, memory, stability and LTI?

$$y[k] = ax[k] + b$$

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Causality, memory, stability and LTI?

$y[k] = \sum_{m=-\infty}^{k} x[m]$

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Causality, memory, stability and LTI?

$y[k] = \sum_{m=-\infty}^{k} x[m]$

Discrete-time(DT) LTI Systems

DT LTI Systems

We know

$$
\begin{aligned}
y[n] &= T\{x[n]\}, \\
&= T\left\{\sum_{k=-\infty}^{+\infty} x[k]\delta[n-k]\right\}, \text{Slide 28 of Week 1} \\
&= \sum_{k=-\infty}^{+\infty} x[k] \cdot T\left\{\delta[n-k]\right\}, \\
&= \sum_{k=-\infty}^{+\infty} x[k]h[n-k], \text{Assume } T\{\cdot\} \text{ is LTI} \\
&= x[n] * h[n]. \; h[n] \text{ is called the impulse response}
\end{aligned}
$$

- $$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k]$$
  is referred to as the convolution sum.
- Given $h[n]$, it is possible to calculate the output $y[n]$ due to any input $x[n]$ using the convolution sum
- An LTI system is completely characterised by its impulse response $h[n]$.

Convolution Sum

$y[n] = x[n] * h[n]$

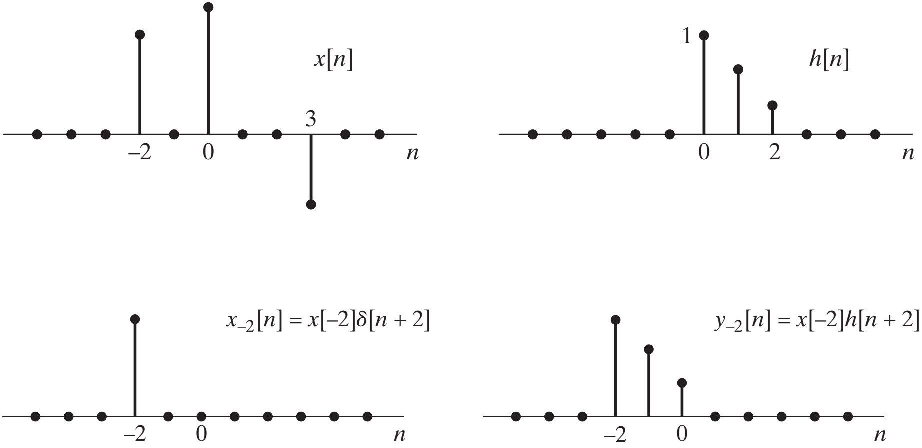
What is the expression for $y[n - n_0]$?

$y[n] = x[n] * h[n]$

- $y[n] = x[n] * h[n]$ is shorthand notation for $\sum_{k=-\infty}^{+\infty} x[k]h[n-k]$ and any use of the shorthand form should be referred back to the full expression of convolution sum.
- Blindly trying the substitution or any other transformation may lead to wrong answers.

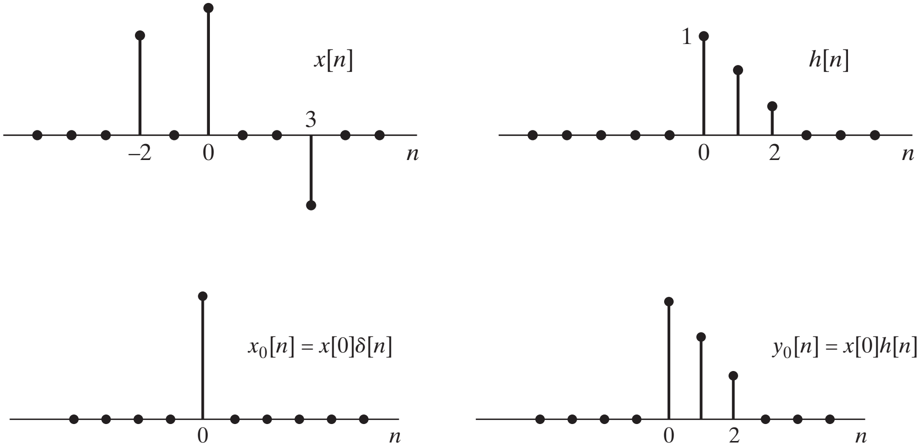
Example

Graphical Approach I



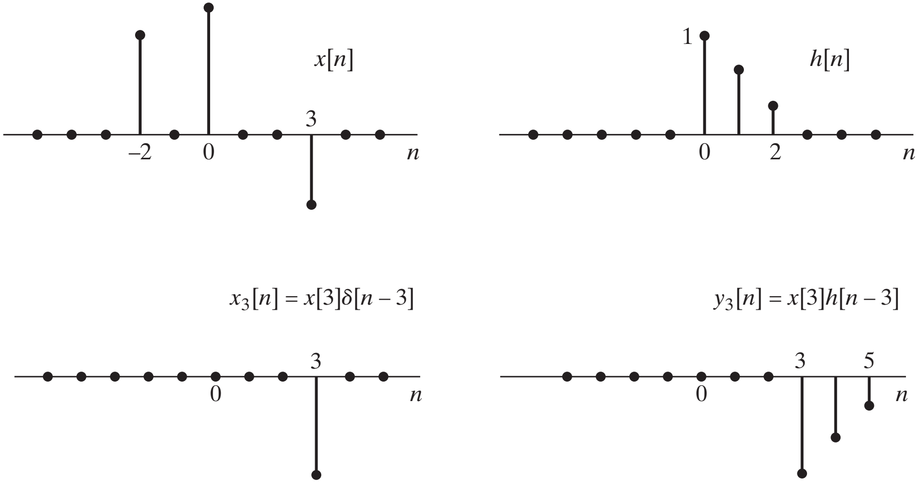
Example

Graphical Approach I



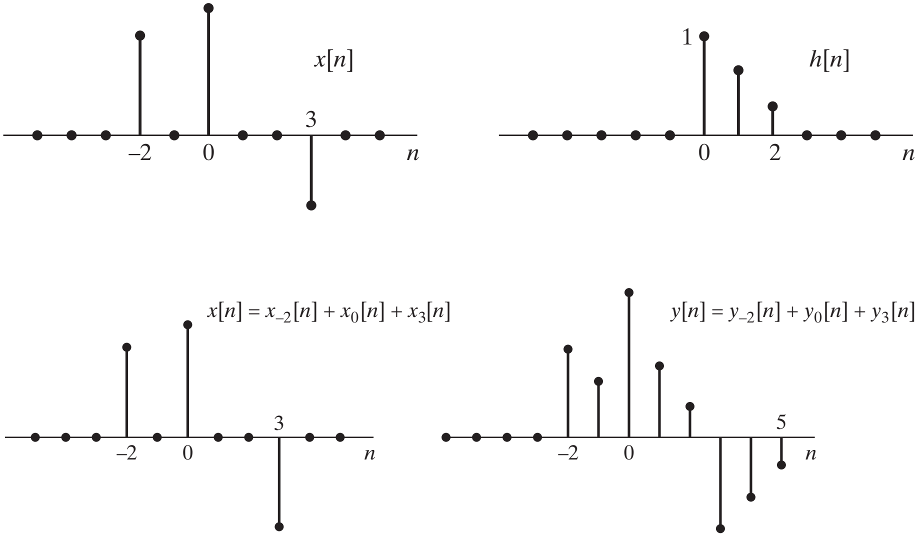
Example

Graphical Approach I



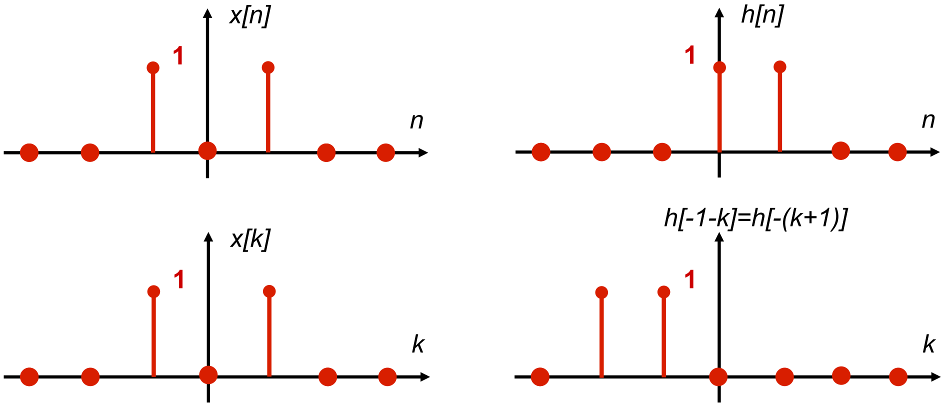
Example

Graphical Approach I



Example

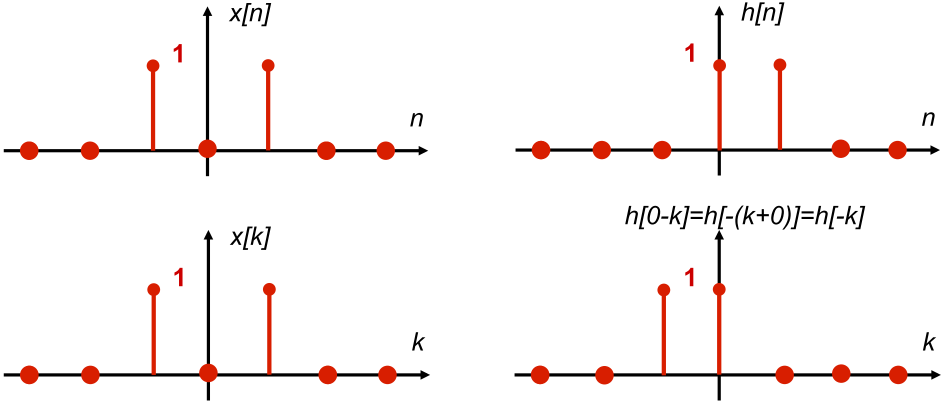
Graphical Approach II



$$y[-1] = 1$$

Example

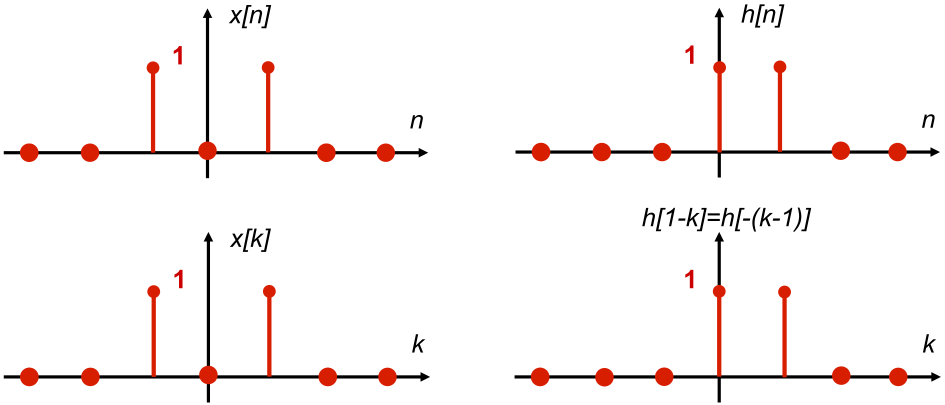
Graphical Approach II



$$y[0] = 1$$

Example

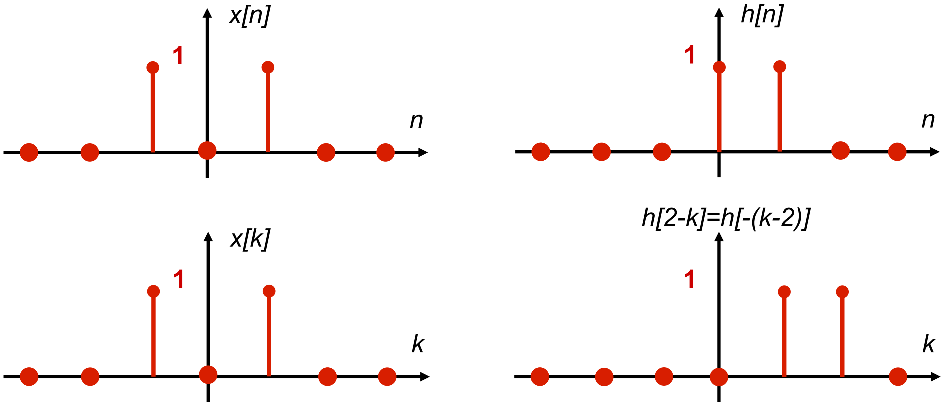
Graphical Approach II



$$y[1] = 1$$

Example

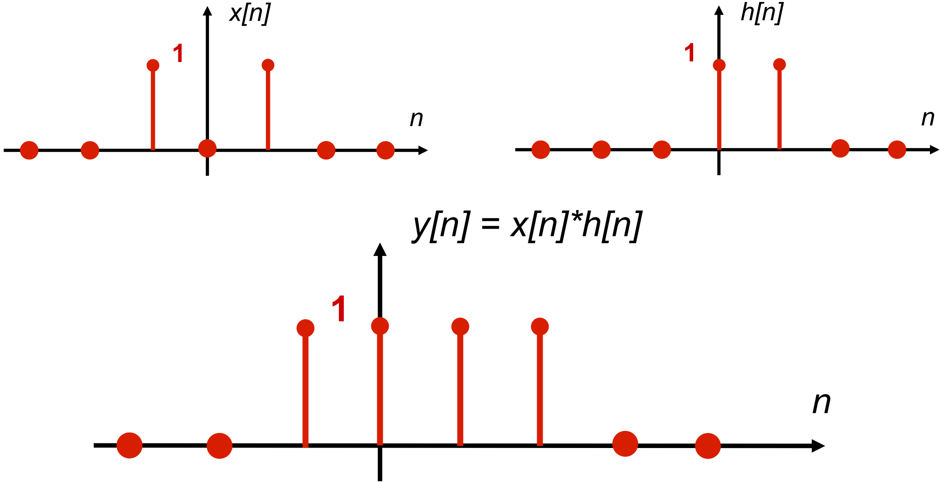
Graphical Approach II



$$y[2] = 1$$

Example

Graphical Approach II



$$y[n] = u[n+1] - u[n-3]$$

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Analytical Approach

Consider a system with impulse response

$$h[n] = u[n] - u[n-N].$$

The input is

$$x[n] = a^n u[n].$$

Find the output $y[n]$ at a particular index n.

Example

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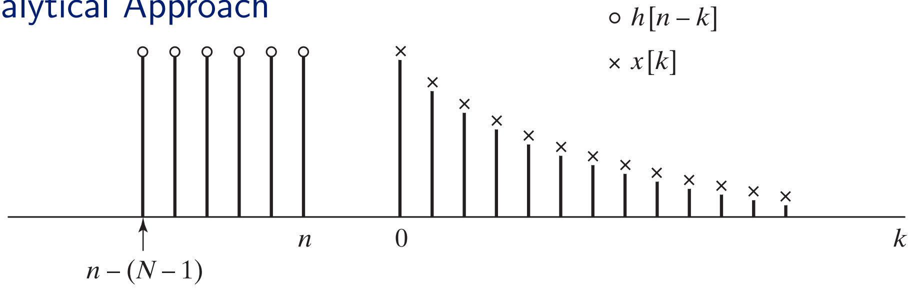
Analytical Approach

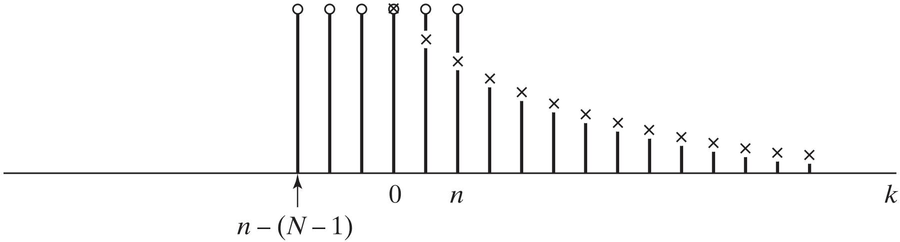


All negative values of n give a similar picture; i.e., the nonzero portions of the sequences $x[k]$ and $h[n-k]$ do not overlap, so

$$y[n] = 0, \text{ for } n < 0.$$

Example

Analytical Approach

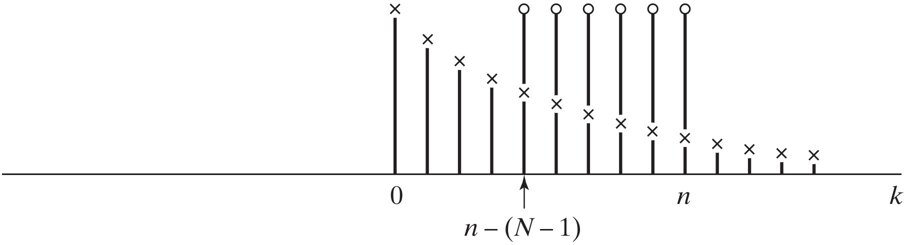


$$y[n] = \sum_{k=0}^{n} x[k]h[n-k],$$

$$= \sum_{k=0}^{n} a^k = \frac{1-a^{n+1}}{1-a}, \text{ for } 0 \leqslant n \leqslant N-1.$$

Example

Analytical Approach



$$y[n] = \sum_{k=n-N+1}^{n} x[k]h[n-k],$$

$$= \sum_{k=n-N+1}^{n} a^k = \frac{a^{n-N+1}(1-a^N)}{1-a}, \text{ for } n > N-1.$$

Example

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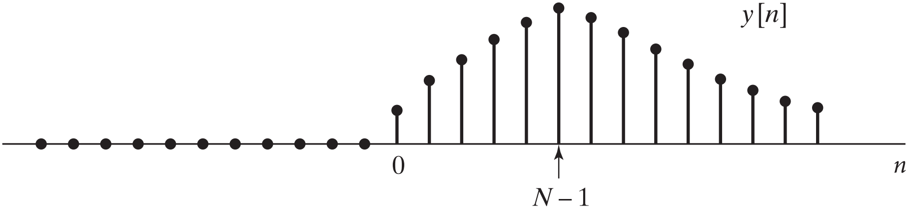
Analytical Approach



$$y[n] = \begin{cases} 0, & n < 0, \\ \dfrac{1 - a^{n+1}}{1 - a}, & 0 \leqslant n \leqslant N - 1, \\ \dfrac{a^{n-N+1}(1 - a^N)}{1 - a}, & n > N - 1. \end{cases}$$

Homework

- Page 74–90, 103–116 read section 2.0–2.1, 2.3;
- Page 137, Q2.1: (a)–(c);
- Page 138, Q2.2;
- Page 138, Q2.3;
- Page 138, Q2.4;
- Page 138, Q2.5;
- Page 138, Q2.6;
- Page 138–139, Q2.7: (a)–(d).

Thank you for your attention.