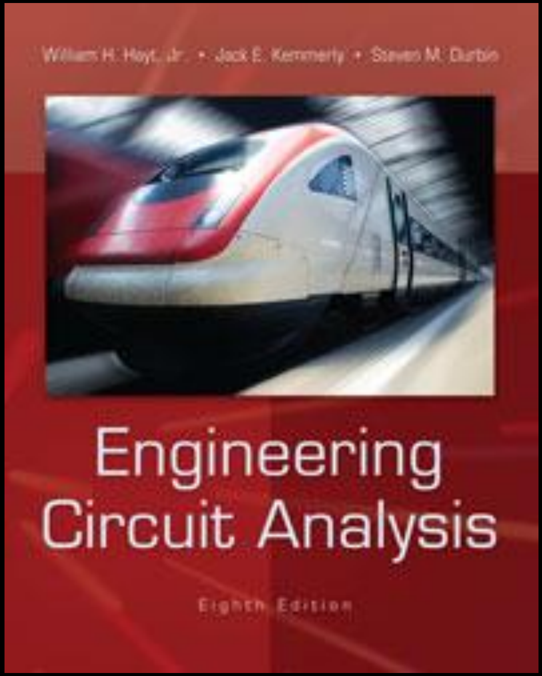
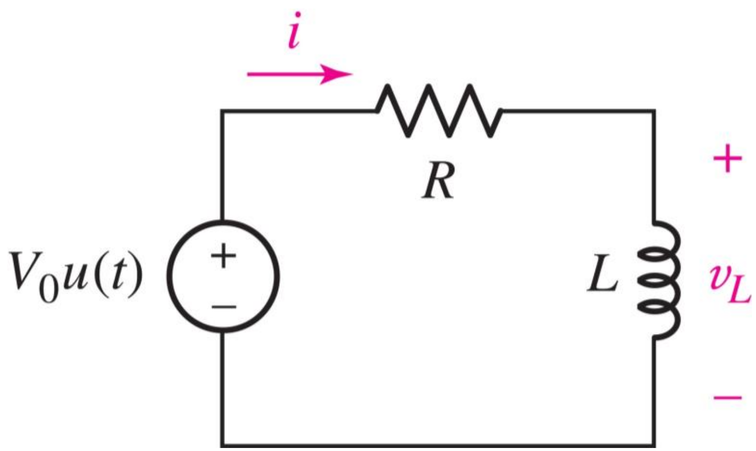
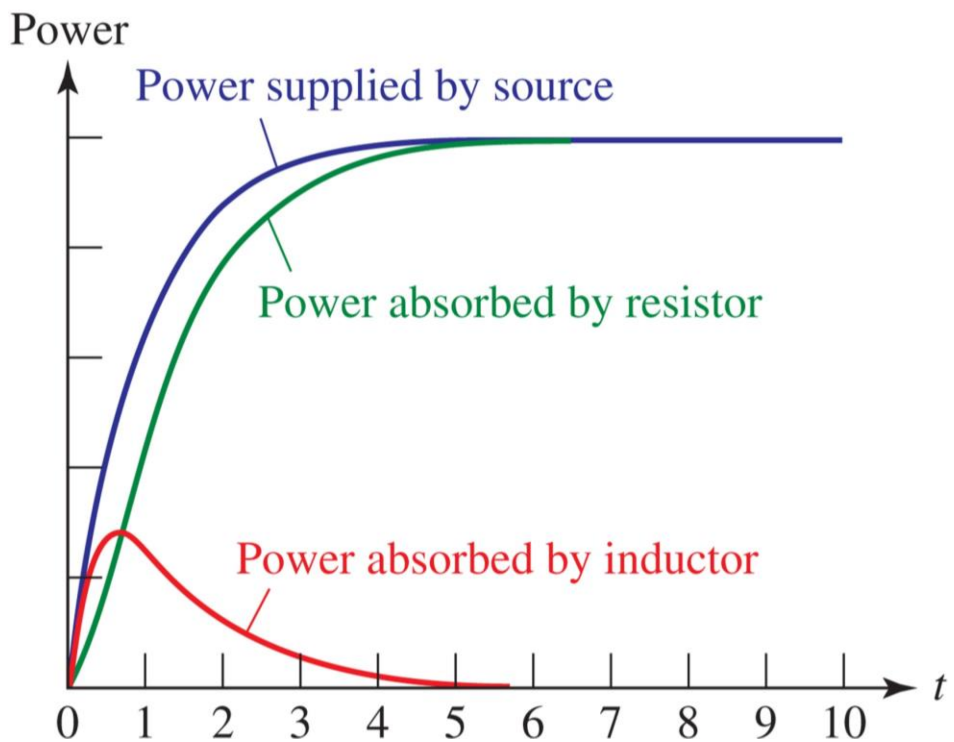
Chapter 11 AC Circuit Power Analysis



Instantaneous Power



The instantaneous power is $p(t)=v(t)i(t)$.



At all times t,

power supplied = power absorbed

Power from Sinusoidal Source

If in the same RL circuit, the source is $V_m cos(\omega t)$, then

$$i(t) = I_m \cos(\omega t + \phi)$$

$$I_m = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \qquad \text{and} \qquad \phi = -\tan^{-1}\frac{\omega L}{R}$$

and so the power will be

$$p(t) = v(t)i(t) = V_m I_m \cos(\omega t + \phi)\cos\omega t$$

$$= \frac{V_m I_m}{2}[\cos(2\omega t + \phi) + \cos\phi]$$

Constant Term

Double Frequency Term

$$= \frac{V_m I_m}{2}\cos\phi + \frac{V_m I_m}{2}\cos(2\omega t + \phi)$$

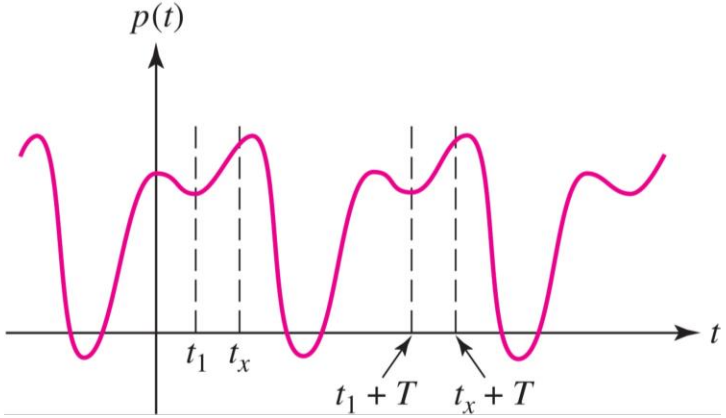
Average Power

The average power over an arbitrary interval from t_1 to t_2 is

$$P = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} p(t)\, dt$$

When the power is periodic with period T, the average power is calculated over *any* one period:

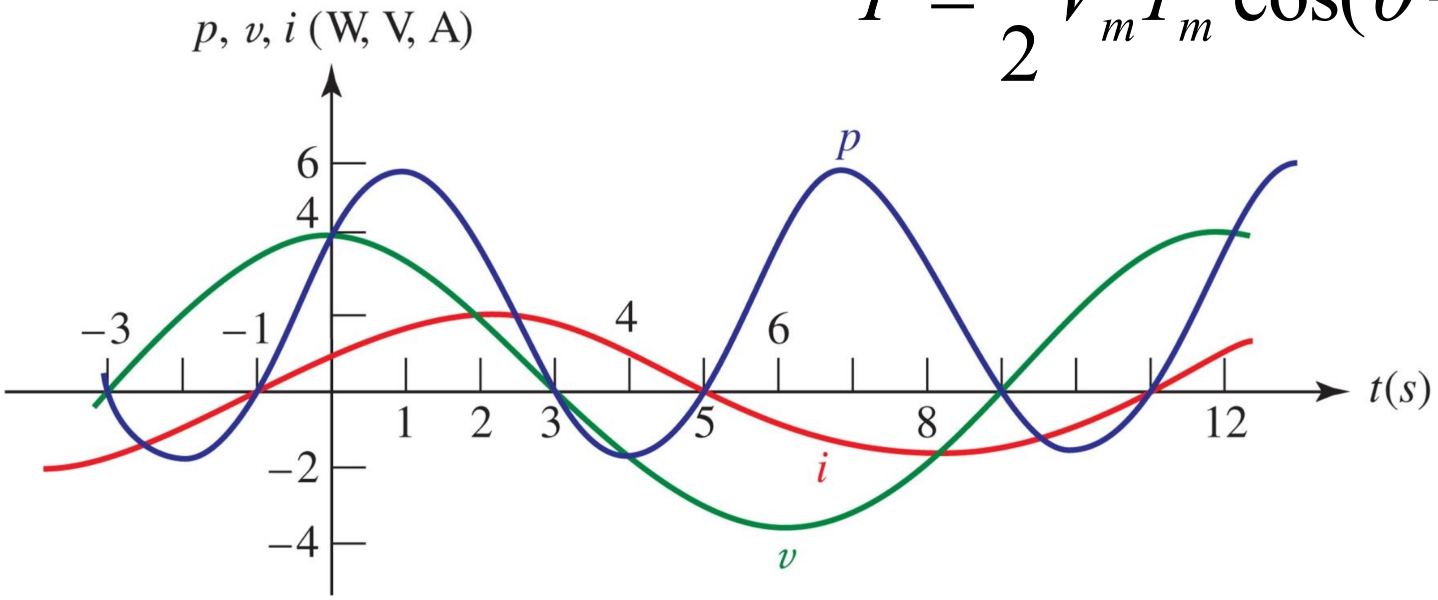
$$P = \frac{1}{T} \int_{t_x}^{t_x + T} p(t)\, dt$$



Average Power: Sinusoidal Steady State

If $v(t)=V_m cos(\omega t+\theta)$ and $i(t)=I_m cos(\omega t+\phi)$, then

$$P=\frac{1}{2}V_m I_m \cos(\theta-\phi)$$



Average Power for Elements

- The average power absorbed by a resistor R is

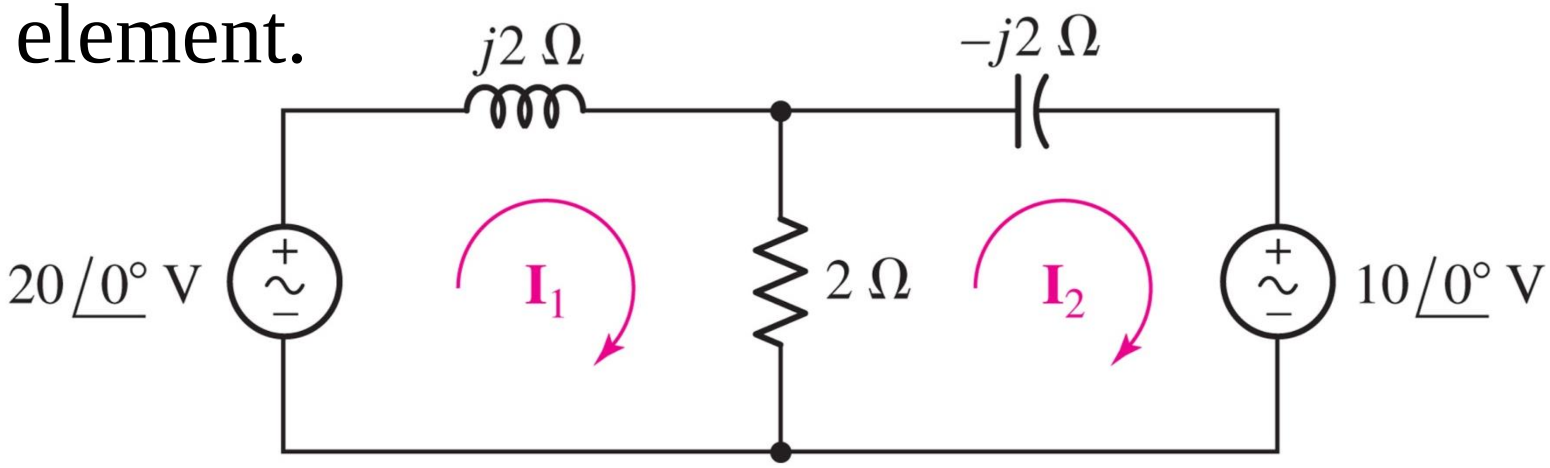
$$P_R = \frac{1}{2}\frac{V_m^2}{R}$$

- The average power absorbed by a purely reactive element(s) is zero, since the current and voltage are 90 degrees out of phase:

$$P_X = 0$$

Example: Average Power

Find the average power absorbed by each element.



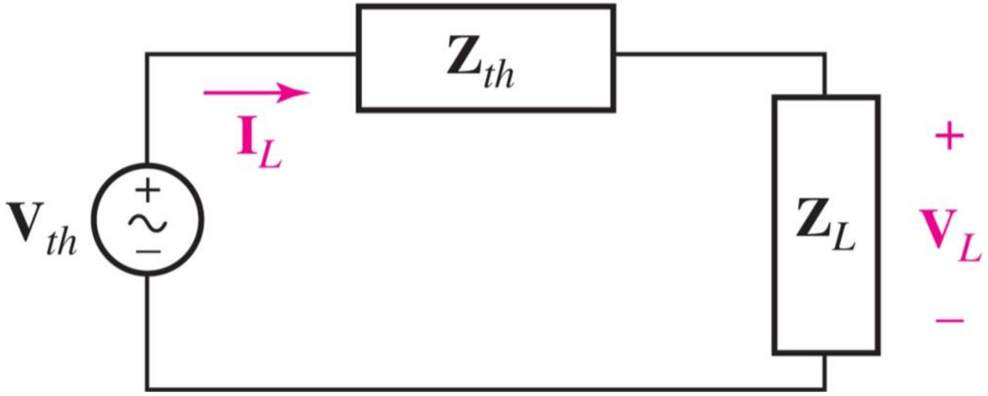
Answer: $P_L = 0\ W$ $P_C = 0\ W,$ $P_R = 25\ W$

$P_{left} = -50\ W$ $P_{right} = 25\ W$

Maximum Power Transfer

An independent voltage source in *series with an impedance* $\mathbf{Z}_{th}$ delivers a maximum average power to that load impedance $\mathbf{Z}_L$ which is the conjugate of $\mathbf{Z}_{th}$:

$$\mathbf{Z}_L = \mathbf{Z}_{th}^*$$



Maximum Power Transfer Derivation

First, solve for the load power:

$$P = \frac{\frac{1}{2}|\mathbf{V}_{th}|^2\sqrt{R_L^2 + X_L^2}}{(R_{th} + R_L)^2 + (X_{th} + X_L)^2}\cos\left(\tan^{-1}\left(\frac{X_L}{R_L}\right)\right)$$

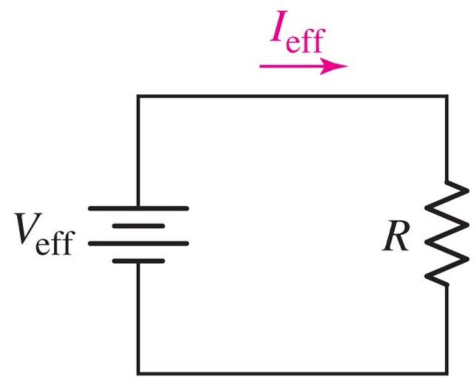
$$= \frac{\frac{1}{2}|V_{th}|^2 R_L}{(R_{th} + R_L)^2 + (X_{th} + X_L)^2}$$

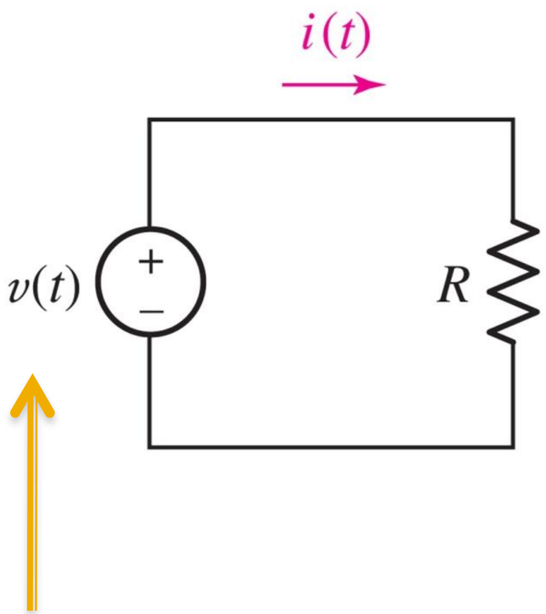
Clearly, P is largest when $X_L + X_{th} = 0$

Solving $dP/dR_L = 0$ will show that $R_L = R_{th}$

Effective Values of Current and Voltage

The same power is delivered to the resistor in the circuits shown.





periodic, period T

$$I_{\text{eff}} = \sqrt{\frac{1}{T}\int_0^T i^2\,dt}$$

Effective (RMS) for Sine Wave

- The effective value is often referred to as the root-mean-square or RMS value.

$$\boxed{I_{\text{eff}} = \sqrt{\frac{1}{T}\int_0^T i^2\,dt}}$$

- For sine waves: $V_{eff} = \frac{1}{\sqrt{2}} V_m \cong 0.707 V_m$

- Power is now $P = I_{eff}^2 R$

Apparent Power & Power Factor

- If $v(t)=V_m cos(\omega t+\theta)$ and $i(t)=I_m cos(\omega t+\phi)$, then

$$P=\frac{1}{2}V_m I_m \cos(\theta-\phi)=V_{eff} I_{eff} \cos(\theta-\phi)$$

- the apparent power is defined as $V_{eff} I_{eff}$ and is given the units volt-ampere V•A

Example: Average Power

Find the average power being delivered to an impedance $Z_L = 8 - j11\ \Omega$ by a current $I = 5e^{j20^\circ}$ A.

Only the 8-Ω resistance enters the average-power calculation, since the *j11*-Ω component will not absorb any *average power.*

Thus,

$$P = (1/2)(5^2)8 = 100\ W$$

Apparent Power & Power Factor

Power factor is defined as

$$PF = \frac{average\ power}{apparent\ power} = \frac{P}{V_{eff} I_{eff}}$$

- for a resistive load, $PF=1$
- for a purely reactive load, $PF=0$
- generally, $0 \le PF \le 1$

Power Factor: Lagging & Leading

- Since the power factor for sine waves is

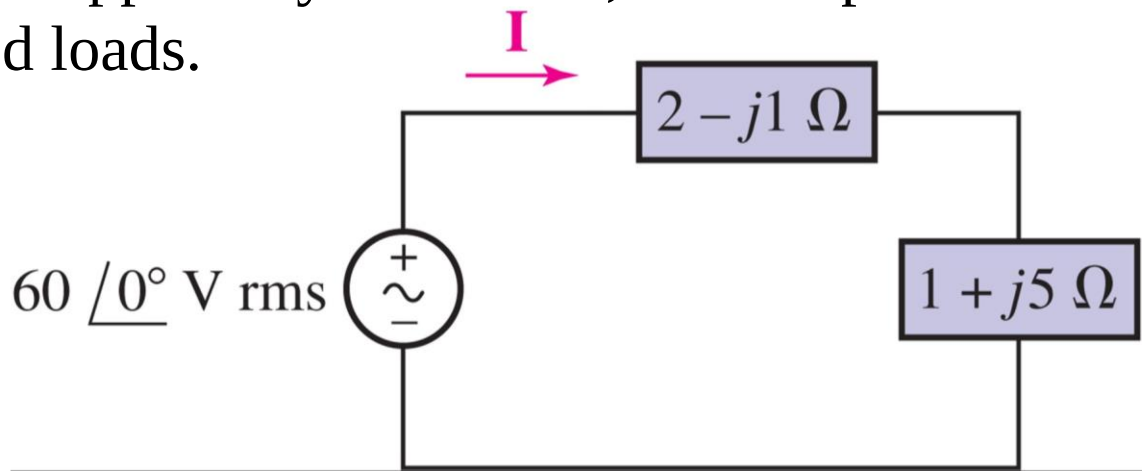
$$PF = \cos(\theta - \phi)$$

the information as to whether current leads or lags voltage is lost, so we add the adjective to the power factor term.

- An inductive load has a *lagging* PF.
- A capacitive load has a *leading* PF.

Example: Power Factor

Find the average power delivered to each of the two loads, the apparent power supplied by the source, and the power factor of the combined loads.

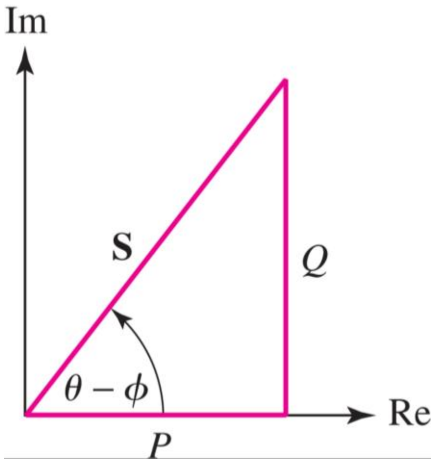


Answer: 288 W, 144 W, 720 VA, PF=0.6 (lagging)

Complex Power

Define the complex power **S** as

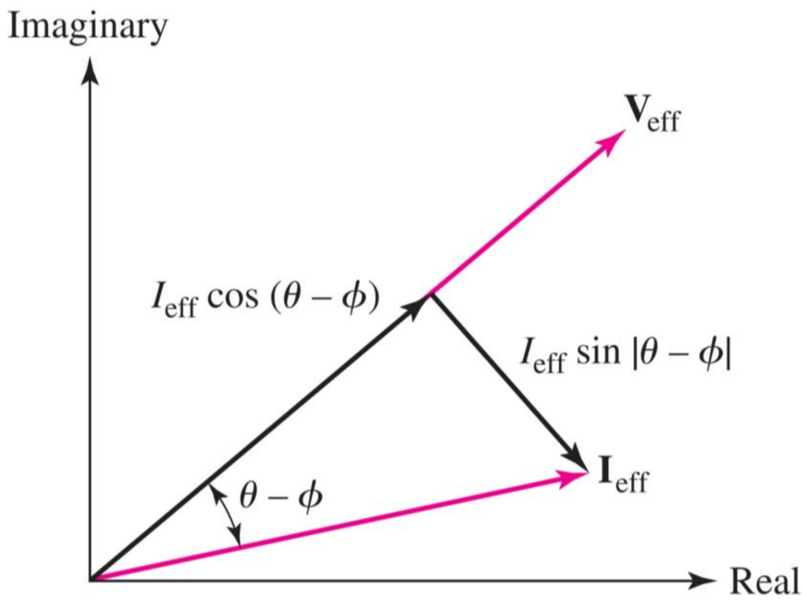
$$\mathbf{S} = \mathbf{V}_{eff}\mathbf{I}^*_{eff} = V_{eff} I_{eff} e^{j(\theta-\phi)} = P + jQ$$



- the real part of **S** is P, the average power
- the imaginary part of **S** is Q, the reactive power, which represents the flow of energy back and forth from the source (utility company) to the inductors and capacitors of the load (customer)

Complex Power

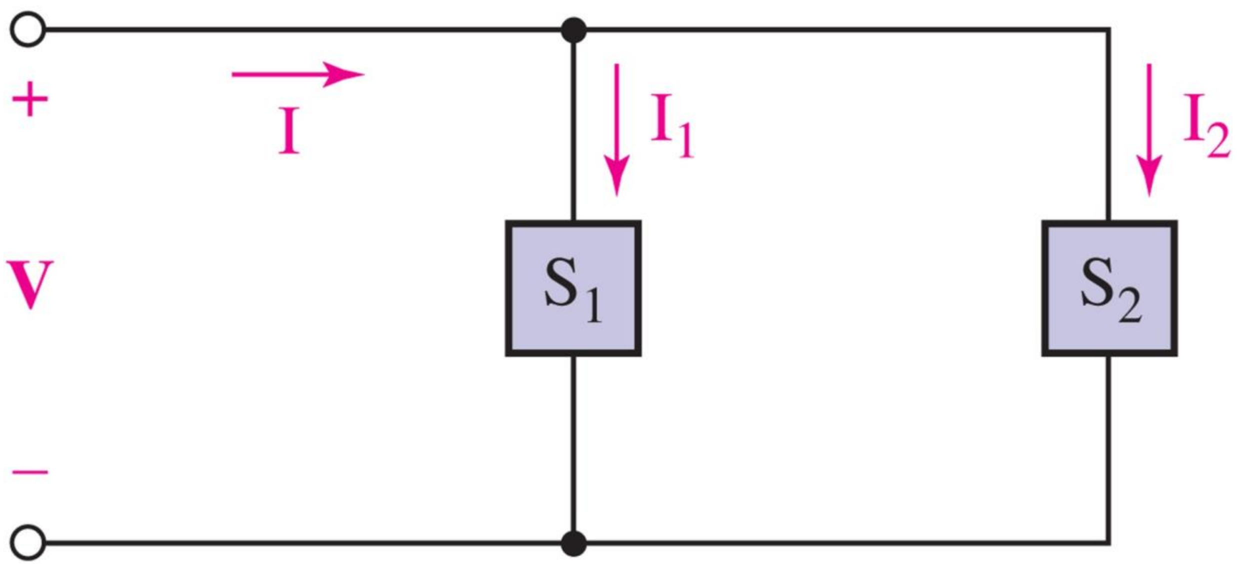
Splitting the current phasor $\mathbf{I}_{\text{eff}}$ into in-phase and out-of-phase components is another way of visualizing the complex power.



Complex Power

Complex powers to loads *add*:

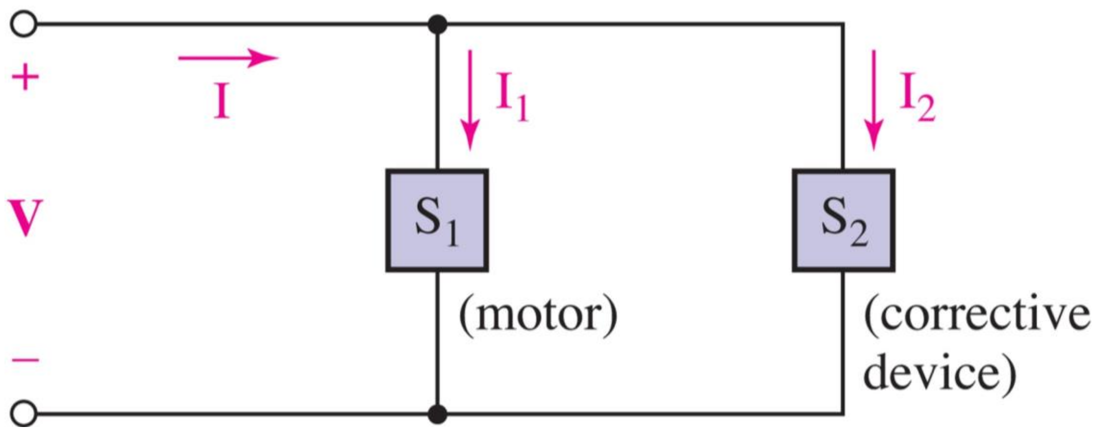
$$\boldsymbol{S} = \mathbf{V}\mathbf{I}^* = \mathbf{V}(\mathbf{I}_1 + \mathbf{I}_2)^* = \mathbf{V}(\mathbf{I}_1^* + \mathbf{I}_2^*) = \boldsymbol{S}_1 + \boldsymbol{S}_2$$



Example: Power Factor Correction

An industrial consumer is operating a 50 kW induction motor at a lagging PF of 0.8. The source voltage is 230 V rms. In order to obtain lower electrical rates, the customer wishes to raise the PF to 0.95 lagging.

Specify a suitable solution.



Answer: deploy a capacitor in parallel with the motor, as shown above.

At 60 Hz, C=1.056 mF