EEE336 Signal Processing and Digital Filtering

Lecture 9 Discrete Fourier Transform

9_1 Introduction to DFT

Zhao Wang

Zhao.wang@xjtlu.edu.cn

Room EE322

From DTFT to DFT

- DTFT is an important tool in digital signal processing, as it provides the spectral content of a discrete time signal.
  - However, the computed spectrum, $X(\omega)$ is a continuous function of ω, and therefore cannot be computed using a computer
  - We need a method to compute the spectral content of a discrete time signal and have a spectrum – actually a discrete function
- A straightforward solution: Simply sample the frequency variable ω of the DTFT in frequency domain in the $[0, 2\pi]$ interval.
  - If we want N points in the frequency domain, then we divide ω in the $[0, 2\pi]$ interval into N equal intervals.
  - Then the discrete values of ω are $0, 2\pi/N, 2*2\pi/N, 3*2\pi/N, \ldots, (N-1)*2\pi/N$

Discrete Fourier Transform (DFT)

- The simplest relation between a length-N sequence x[n], defined for 0≤n≤N-1, and its DTFT X(ω) is obtained by uniformly sampling X(ω) on the ω-axis 0≤ω≤2π with

$$\omega_k = \frac{2\pi k}{N}, \quad k = 0, 1, \dots, N-1$$

- From the definition of DTFT, we thus have

$$X[k] = X\left(e^{j\omega}\right)\Big|_{\omega_k = 2\pi k/N} = X\left(e^{j\frac{2\pi k}{N}}\right) = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi kn}{N}}, \quad k = 0, 1, \dots, N-1$$

- Using the notation $W_N = e^{-j2\pi/N}$, the DFT is usually expressed as

$$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{kn}, \quad k = 0, 1, \dots, N-1$$

Twiddle factor

Twiddle Factor W_N

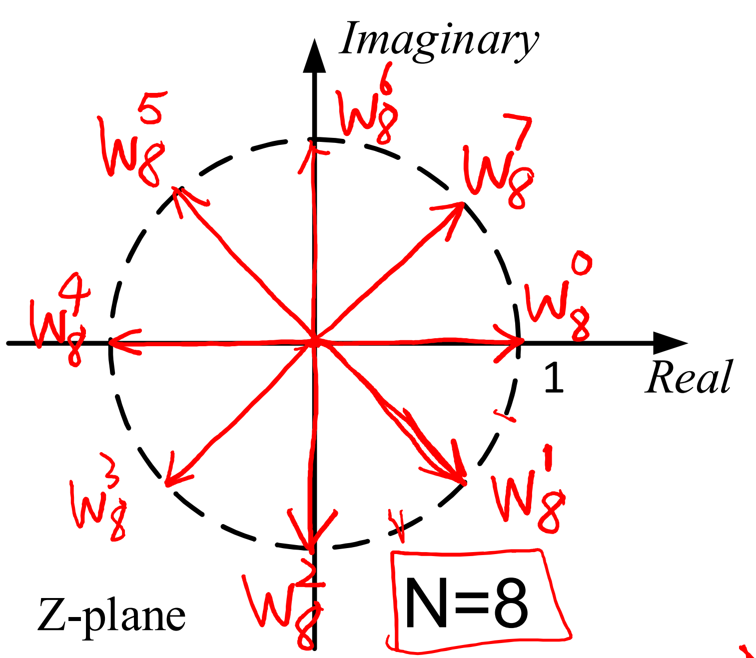
$e^{j\omega n}$ z^n $\{W_N^{kn}\}$

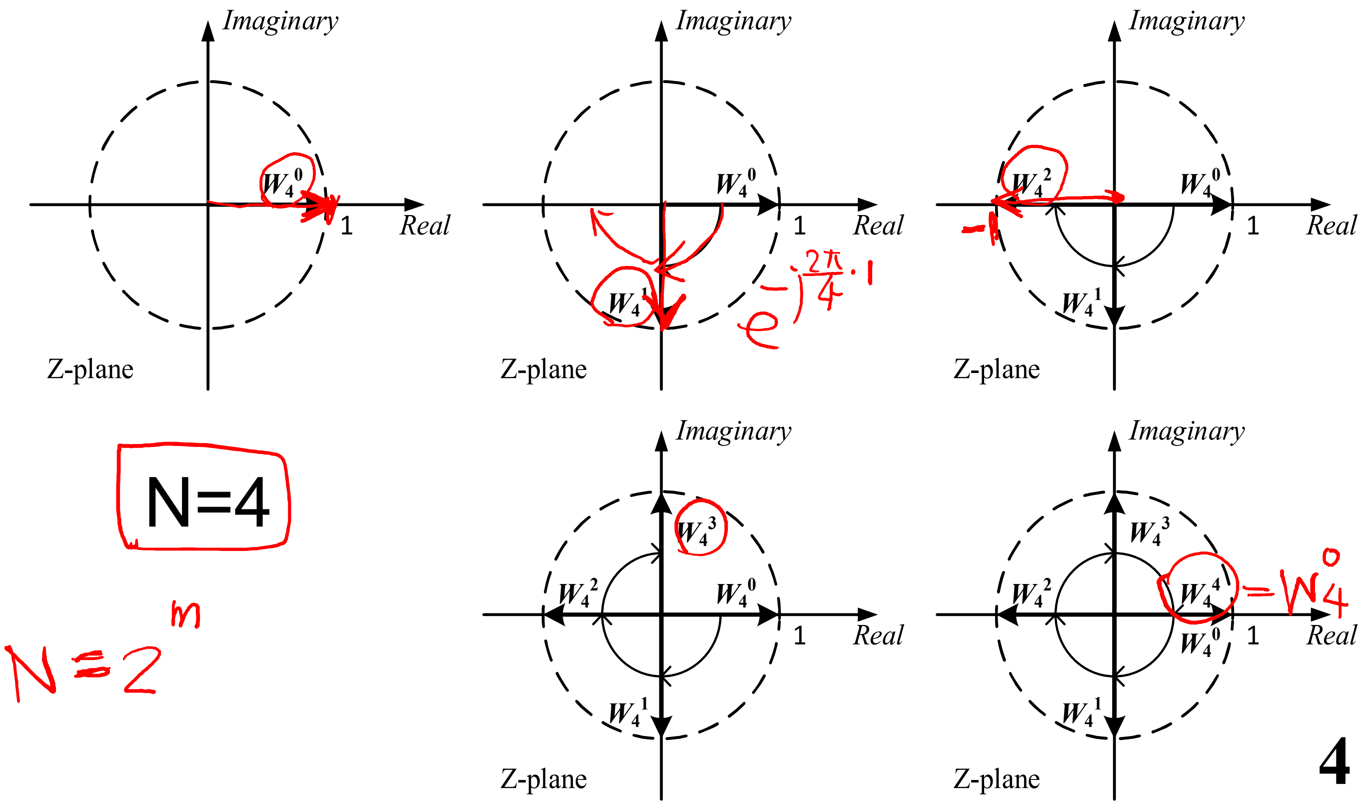
- The twiddle factor W_N is important in DFT

$$W_N = e^{-j\frac{2\pi}{N}} \qquad X[k] = \sum_{n=0}^{N-1} x[n] W_N^{kn}, 0 \le k \le N-1$$

  – Complex exponential wheel

$W_N = W_4^{kn}$





$N = 2^m$

Inverse Discrete Fourier Transform (IDFT)

- The IDFT is defined as

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]e^{j\frac{2\pi kn}{N}}, \quad k = 0, 1, \ldots, N-1$$

- Using the notation $W_N = e^{-j2\pi/N}$, the IDFT is usually expressed as

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]W_N^{-kn}, \quad k = 0, 1, \ldots, N-1$$

- To verify the above expression, we multiply both sides of IDFT equation by $e^{-j2\pi kn/N}$ and sum the result from n=0 to N-1.

DFT Pair

- The analysis equation

$$\boxed{X[k] = \sum_{n=0}^{N-1} x[n]\, W_N^{kn}, \quad k = 0, 1, \ldots, N-1}$$

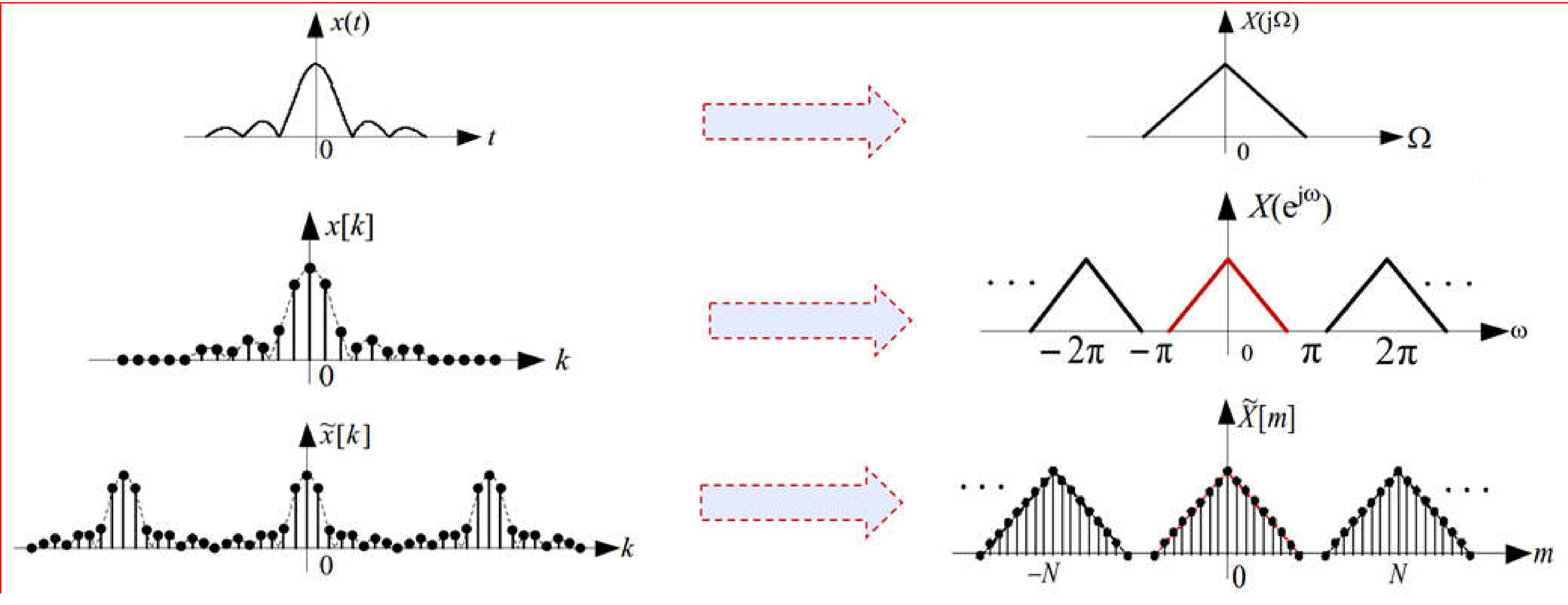
- The synthesis equation

$$\boxed{x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] W_N^{-kn}, \quad k = 0, 1, \ldots, N-1}$$

- The DFT pair is denoted as x[n] $\leftrightarrow$ X[k]

CTFT → DTFT → DFT

Time Domain	$x(t)$	$x[k]$	$\tilde{x}[k]$
	CTFT	DTFT	DFT
Frequency Domain	$X(j\Omega)$	$X(e^{j\omega})$	$X[m]$



DTFT from DFT

- DTFT is a continuous transform. Sampling the DTFT at regularly spaced intervals around the unit circle gives the DFT.
- Just like we can reconstruct a continuous time signal from its samples, DTFT can also be ***interpolated*** from its DFT samples, as long as the number of points N at which DFT is computed is equal to or larger than the samples of the original signal.
  - That is, given the N-point DFT X[k] of a length-N sequence x[n], its DTFT X(ω) can be uniquely determined from X[k]
- ***How?***

DTFT from DFT by Interpolation

$$X(e^{j\omega}) = \sum_{n=0}^{N-1} x[n]e^{-j\omega n}$$

$$= \sum_{n=0}^{N-1} \left\{ \frac{1}{N} \sum_{k=0}^{N-1} X[k] W_N^{-kn} \right\} e^{-j\omega n}$$

$$= \sum_{k=0}^{N-1} X[k] \underbrace{\left\{ \frac{1}{N} \sum_{n=0}^{N-1} e^{-j\left(\omega - \frac{2\pi k}{N}\right)n} \right\}} = \sum_{k=0}^{N-1} X[k] \cdot \Phi_k(\omega)$$

$$\Phi_k(\omega) = \frac{1}{N} \frac{1 - e^{-j\left(\omega - \frac{2\pi k}{N}\right)N}}{1 - e^{-j\left(\omega - \frac{2\pi k}{N}\right)}} = \frac{1}{N} \frac{\sin\left(\dfrac{\omega N - 2\pi k}{2}\right)}{\sin\left(\dfrac{\omega N - 2\pi k}{2N}\right)} e^{-j\left(\frac{N}{2} - \frac{1}{2}\right)\left(\omega - \frac{2\pi k}{N}\right)}$$

9_1 Wrap up

- Discrete Fourier Transform
  - Analyses equation
  - Syntheses equation

- Relationship between DFT and DTFT
  - From DTFT to DFT
    - Frequency domain sampling ←→ Time domain periodic
  - From DFT to DTFT
    - Time domain windowing ←→ Frequency domain interpolation

EEE336 Signal Processing and Digital Filtering

Lecture 9 Discrete Fourier Transform

9_2 Computation of DFT

Zhao Wang

Zhao.wang@xjtlu.edu.cn

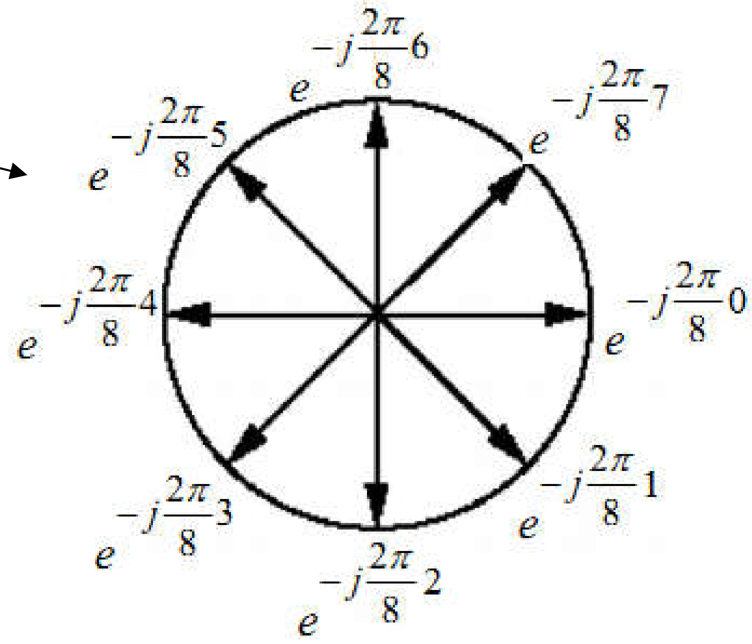
Room EE322

Computing DFT

- For any given k, the DFT is computed by multiplying each x[n] with each of the complex exponentials $W_N^{nk} = e^{-j2\pi nk/N}$ and then adding up all these components
- If, for example, we wish to compute an 8-point DFT, the complex exponentials are 8 unit vectors placed at equal distances from each other on the unit circle

Complex exponential wheel

$$\boxed{X[k] = \sum_{n=0}^{N-1} x[n]\, W_N^{kn}, \quad k = 0, 1, \ldots, N-1}$$



Example 1

- Find the DFT of a 4-point sequence $x_4[k]=\{1,1,1,1;\ k=0,1,2,3\}$

Example 2

- Find the 8-point DFT of the zero-padded sequence $x_8[k]=\{1,1,1,1,0,0,0,0;\ k=0,1,2,3,4,5,6,7\}$

Compare Example 1 and 2

- Find the DTFT of $x_4[k]$ and $x_8[k]$

$$X(\omega) = e^{-j\frac{3}{2}\omega}\left[2cos\left(\frac{\omega}{2}\right) + 2\cos\left(\frac{3\omega}{2}\right)\right]$$

- $x_4[k] = \{\mathbf{1}, 1,1,1\}$

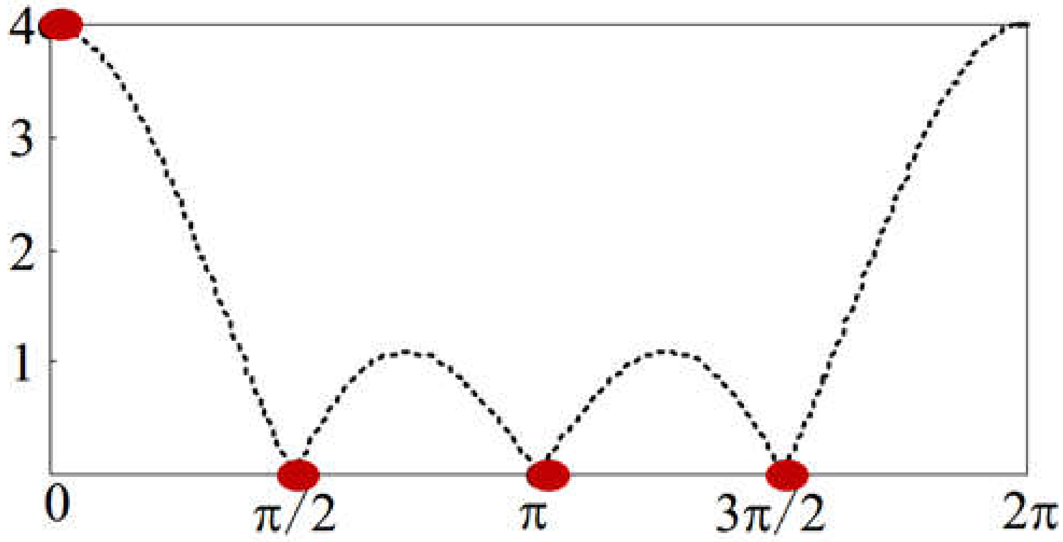
$$\Longrightarrow X_4[m] = X(\omega)\Big|_{\omega=\frac{2\pi m}{4}}, \quad \omega = 0, \quad \frac{\pi}{2}, \quad \pi, \quad \frac{3\pi}{2}$$

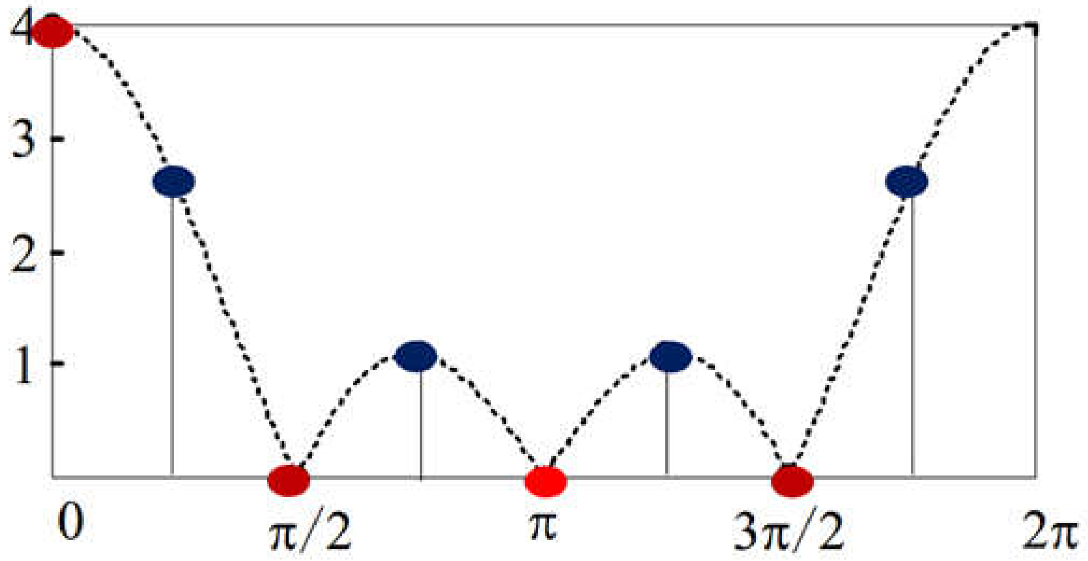
- $x_8[k] = \{\mathbf{1}, 1,1,1,0,0,0,0\}$

$$\Longrightarrow X_8[m] = X(\omega)\Big|_{\omega=\frac{2\pi m}{8}}, \quad \omega = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}$$

Compare Example 1 and 2

$$X_4[m] = X(\omega)\Big|_{\omega=\frac{2\pi m}{4}} \qquad X_8[m] = X(\omega)\Big|_{\omega=\frac{2\pi m}{8}}$$





- Zero-padded sequence provides more sampling points in frequency domain, i.e. more details in the spectrum.

Matrix computation of DFT

- The DFT samples defined by

$$\boxed{X[k]=\sum_{n=0}^{N-1} x[n]W_N^{kn}, \quad 0 \le k \le N-1}$$

can be expressed in a matrix form $\boxed{\mathbf{X}=\mathbf{D}_N\mathbf{x}}$

  - where X=[X[0] X[1] … X[N-1]]^T is the vectorized DFT sequence,
  - x=[x[0] x[1] … x[N-1]]^T is the vectorized time domain sequence,
  - And D_N is the NxN DFT matrix given by

$$\mathbf{D}_N = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & W_N^1 & W_N^2 & \cdots & W_N^{(N-1)} \\ 1 & W_N^2 & W_N^4 & \cdots & W_N^{2(N-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & W_N^{(N-1)} & W_N^{2(N-1)} & \cdots & W_N^{(N-1)^2} \end{bmatrix}$$

Matrix computation of DFT (cont.)

- Likewise, the IDFT relation given by

$$\boxed{x[n] = \sum_{k=0}^{N-1} X[k] W_N^{-kn}, \ 0 \le n \le N-1}$$

can be expressed in a matrix form $\boxed{\mathbf{x} = \mathbf{D}_N^{-1}\mathbf{X}}$

  – where D_N^{-1} is the NxN IDFT matrix

$$\mathbf{D}_N^{-1} = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & W_N^{-1} & W_N^{-2} & \cdots & W_N^{-(N-1)} \\ 1 & W_N^{-2} & W_N^{-4} & \cdots & W_N^{-2(N-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & W_N^{-(N-1)} & W_N^{-2(N-1)} & \cdots & W_N^{-(N-1)^2} \end{bmatrix}$$

- Note that DFT and IDFT matrices are related to each other $\boxed{\mathbf{D}_N^{-1} = \frac{1}{N}\mathbf{D}_N^{*}}$

9_2 Wrap up

- Computing DFT manually
  - Twiddle factor W_N
  - Calculating DFT using twiddle factor rotation in z-plane
  - DFT of zero-padded sequence

- Computing DFT using computer: matrix method
  - And IDFT using matrix method

EEE336   Signal Processing and Digital Filtering

Lecture 9 Discrete Fourier Transform

9_3 DFT Properties

Zhao Wang

Zhao.wang@xjtlu.edu.cn

Room EE322

DFT Properties

- Like the DTFT, the DFT also satisfies a number of properties which are useful in signal processing
- Some of these properties are essentially identical to those of the DTFT, while some others are somewhat different

Type of Property	Length-N Sequence	N-point DFT
	$g[n]$ $h[n]$	$G[k]$ $H[k]$
Linearity	$\alpha g[n] + \beta h[n]$	$\alpha G[k] + \beta H[k]$
Circular time-shifting	$g[\langle n - n_o \rangle_N]$	$W_N^{kn_o} G[k]$
Circular frequency-shifting	$W_N^{-k_o n} g[n]$	$G[\langle k - k_o \rangle_N]$
Duality	$G[n]$	$N g[\langle -k \rangle_N]$
N-point circular convolution	$\sum_{m=0}^{N-1} g[m] h[\langle n - m \rangle_N]$	$G[k] H[k]$
Modulation	$g[n] h[n]$	$\frac{1}{N} \sum_{m=0}^{N-1} G[m] H[\langle k - m \rangle_N]$

Orthogonality of twiddle factors

- Orthogonality of discrete exponentials
  - For integers *N*, *n*, *l* and *r*

$$\boxed{\sum_{n=0}^{N-1} e^{j\frac{2\pi}{N}nl} = \begin{cases} N, & l = rN \\ 0, & otherwise \end{cases}} = \boxed{\sum_{n=0}^{N-1} W_N^{-nl} = \begin{cases} N, & l = rN \\ 0, & otherwise \end{cases}}$$

  - In words, summation of harmonically related discrete complex exponentials of $2\pi l/N$ is N, if *l* is an integer multiple of N; zero, otherwise.
  - Proof: geometric series expansion:

$$\sum_{n=0}^{N-1} e^{j\frac{2\pi}{N}nl} = \sum_{n=0}^{N-1}\left(e^{j\frac{2\pi}{N}l}\right)^n = \frac{1-\left(e^{j\frac{2\pi}{N}l}\right)^N}{1-e^{j\frac{2\pi}{N}l}}$$

Always 1 for any integer *l*

Equals to 1 only when $l=rN$

Xi'an Jiaotong-Liverpool University
西交利物浦大学

Example

- Find the DFT of $g[n] = \cos\left(\frac{2\pi r}{N} n\right), 0 \le n \le N-1$
- g[n] can be expressed as:

$$g[n] = \frac{1}{2}\left(e^{j\frac{2\pi rn}{N}} + e^{-j\frac{2\pi rn}{N}}\right) = \frac{1}{2}(W_N^{-rn} + W_N^{rn})$$

$$\Big\downarrow$$

$$G[k] = \sum_{n=0}^{N-1} g[n] W_N^{kn} = \frac{1}{2}\left(\sum_{n=0}^{N-1} W_N^{-(r-k)n} + \sum_{n=0}^{N-1} W_N^{(r+k)n}\right)$$

- According to the modified orthogonality:

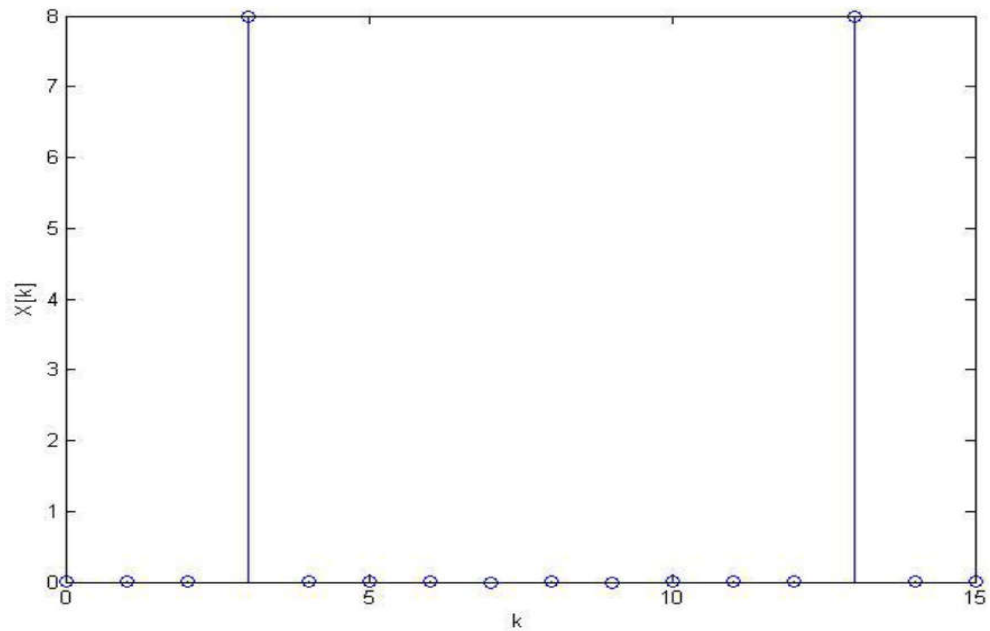
$$\boxed{\sum_{n=0}^{N-1} W_N^{-(k-l)n} = \begin{cases} N, & for\ k-l = mN \\ 0, & otherwise \end{cases}}$$

Example (cont.)

- The DFT is obtained:

$$G[k] = \begin{cases} N/2, & for\ k = r \\ N/2, & for\ k = N - r \\ 0, & otherwise \end{cases}$$

  - Eg: when N = 16, r = 3: $G[k], 0 \le k \le 15$

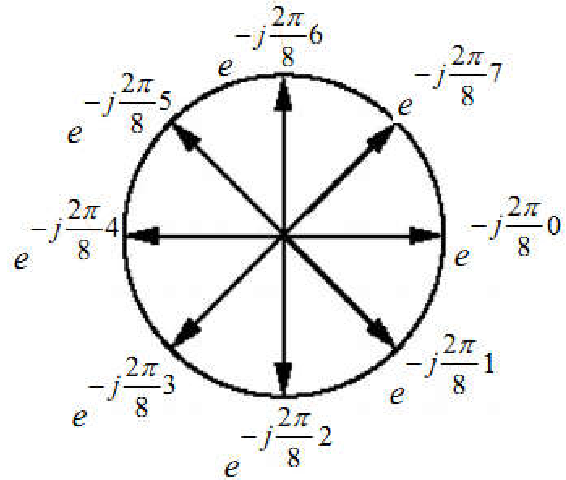


Periodicity in DFT

- DFT is periodic in both *time* and *frequency* domains
  - Even though the original time domain sequence to be transformed is not periodic!
- Periodicity can be explained by
  - Mathematically: both the analysis and synthesis equations are periodic by N.

$$x[n+N]=\frac{1}{N}\sum_{k=0}^{N-1}X[k]e^{j\frac{2\pi k(n+N)}{N}}=\frac{1}{N}\sum_{k=0}^{N-1}X[k]e^{j\frac{2\pi kn}{N}}=x[n]$$

$$X[k+N]=\sum_{k=0}^{N-1}x[n]e^{-j\frac{2\pi(k+N)n}{N}}=\sum_{k=0}^{N-1}x[n]e^{-j\frac{2\pi kn}{N}}=X[k]$$



  - From the complex exponential wheel: there are only N vectors around a unit circle, the transform will repeat itself every N points.

Parseval's relation

- Similar to DTFT, DFT also holds the Parseval's relation:

$$\sum_{n=0}^{N-1} |x[n]|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |X[k]|^2$$

  – The energy conservation in time and frequency domain is valid for DFT too.

Symmetry relations

- There are some very complex symmetry relations

Length-N Sequence	N-point DFT
$x[n]$	$X[k]$
$x^*[n]$	$X^*[\langle -k\rangle_N]$
$x^*[\langle -n\rangle_N]$	$X^*[k]$
$\mathrm{Re}\{x[n]\}$	$X_{\text{pcs}}[k] = \frac{1}{2}\{X[\langle k\rangle_N] + X^*[\langle -k\rangle_N]\}$
$j\,\mathrm{Im}\{x[n]\}$	$X_{\text{pca}}[k] = \frac{1}{2}\{X[\langle k\rangle_N] - X^*[\langle -k\rangle_N]\}$
$x_{\text{pcs}}[n]$	$\mathrm{Re}\{X[k]\}$
$x_{\text{pca}}[n]$	$j\,\mathrm{Im}\{X[k]\}$

Note: $x_{\text{pcs}}[n]$ and $x_{\text{pca}}[n]$ are the periodic conjugate-symmetric and periodic conjugate-antisymmetric parts of $x[n]$, respectively. Likewise, $X_{\text{pcs}}[k]$ and $X_{\text{pca}}[k]$ are the periodic conjugate-symmetric and periodic conjugate-antisymmetric parts of $X[k]$, respectively.

Length-N Sequence	N-point DFT
$x[n]$	$X[k] = \mathrm{Re}\{X[k]\} + j\,\mathrm{Im}\{X[k]\}$
$x_{\text{pe}}[n]$	$\mathrm{Re}\{X[k]\}$
$x_{\text{po}}[n]$	$j\,\mathrm{Im}\{X[k]\}$
Symmetry relations	$X[k] = X^*[\langle -k\rangle_N]$ $\mathrm{Re}\,X[k] = \mathrm{Re}\,X[\langle -k\rangle_N]$ $\mathrm{Im}\,X[k] = -\mathrm{Im}\,X[\langle -k\rangle_N]$ $\lvert X[k]\rvert = \lvert X[\langle -k\rangle_N]\rvert$ $\arg X[k] = -\arg X[\langle -k\rangle_N]$

Note: $x_{\text{pe}}[n]$ and $x_{\text{po}}[n]$ are the periodic even and periodic odd parts of $x[n]$, respectively.

9_3 Wrap up

- DFT properties
  - Linearity
  - Circular shifting, circular reversal, circular convolution

  - Orthogonality
  - Periodicity
  - Parseval's relation
  - Symmetry relation

EEE336 Signal Processing and Digital Filtering

Lecture 9 Discrete Fourier Transform

9_4 Circular Convolution and DFT

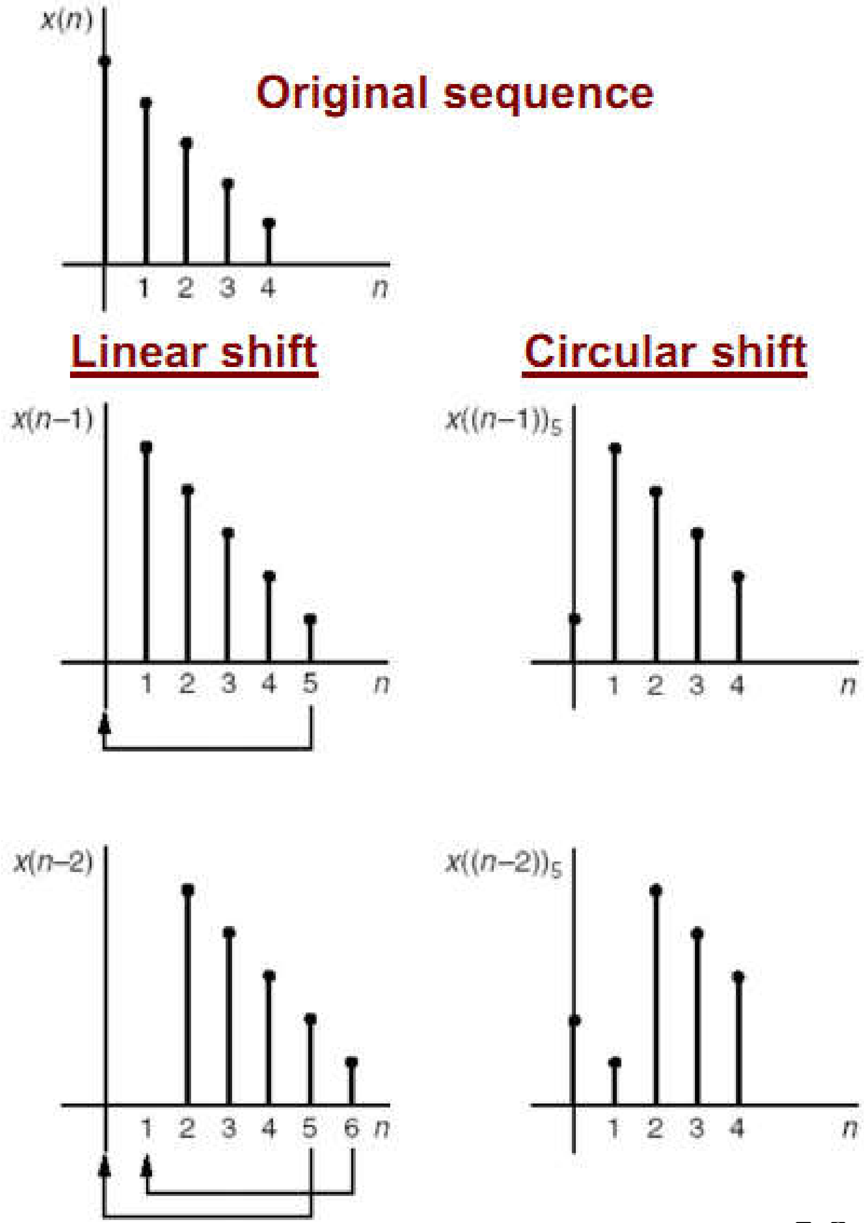
Zhao Wang

Zhao.wang@xjtlu.edu.cn

Room EE322

Circular shift

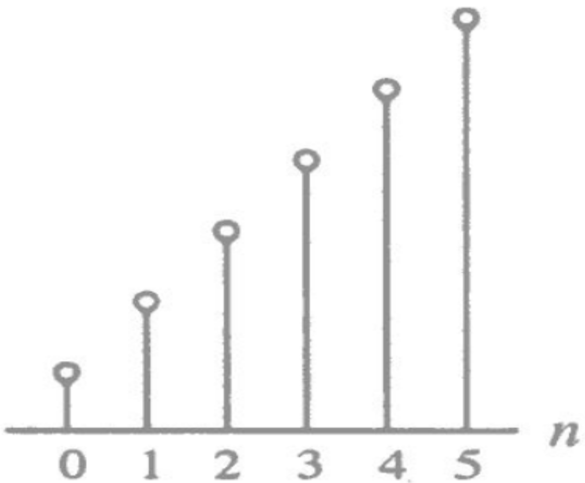
- A circularly shifted sequence is denoted by $x[\langle n-L\rangle_N]$
  - $\langle\cdot\rangle_N$ modulo operation;
  - L is the amount of shift;
  - N is the length of the previously determined base interval
- To obtain a circularly shifted sequence:
  - first linearly shift the sequence by L
  - then rotate the sequence in such a manner that the shifted sequence remain in the same interval originally defined by N.



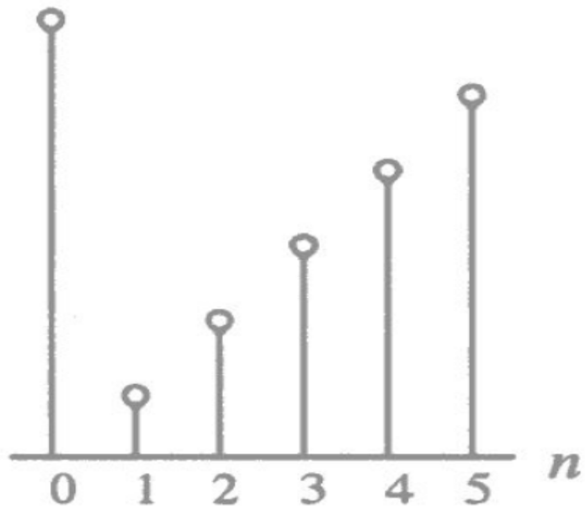
Circular shift

- To obtain a circularly shifted sequence:

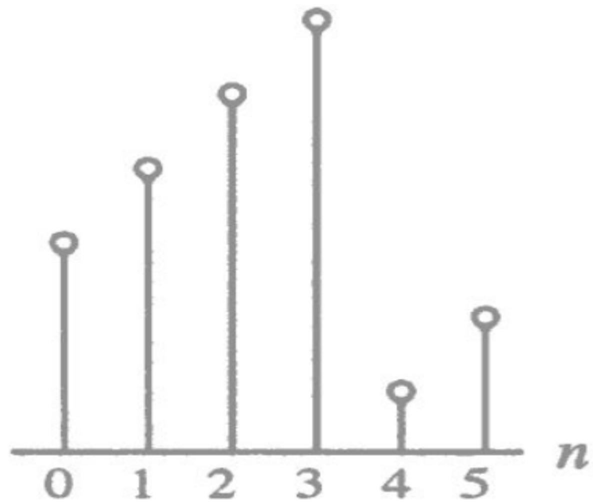
$$x[\langle n-L\rangle_N] = \begin{cases} x[n-L], & for\ L \le n \le N-1 \\ x[n-L+N], & for\ 0 \le n \le L \end{cases}$$



$x[n]$



$x[\langle n-1\rangle_N]$
$= x[\langle n+5\rangle_N]$

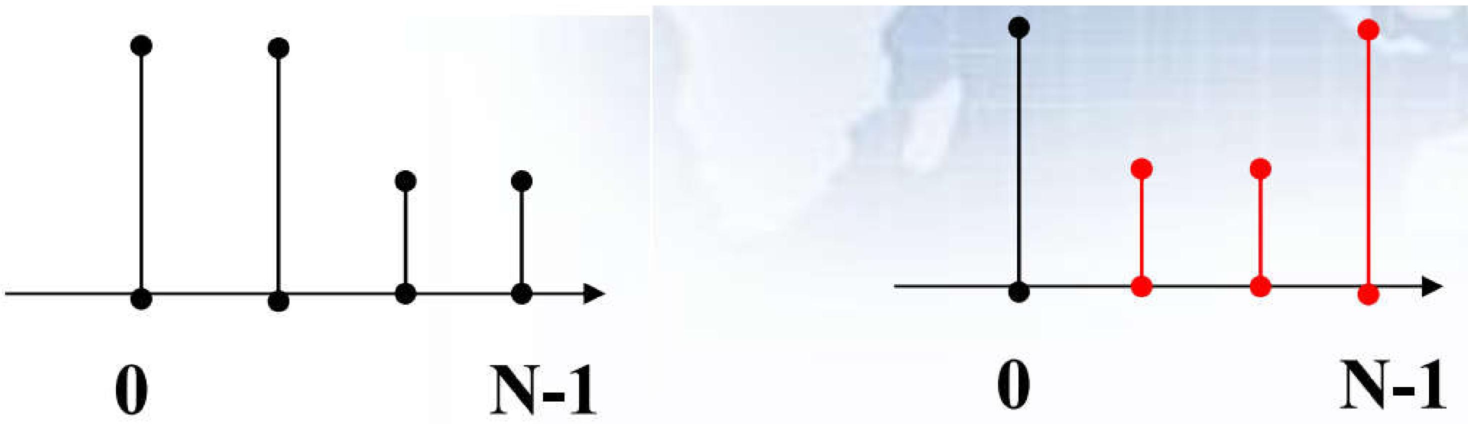


$x[\langle n-4\rangle_N]$
$= x[\langle n+2\rangle_N]$

Circular reversal

- Circular reversal of length-N sequence x[n]:

$$x[\langle -n\rangle_N] = \begin{cases} x[0], & for\ n = 0 \\ x[N-n], & for\ 1 \le n \le N-1 \end{cases}$$



$x[n]$ $x[\langle -n\rangle_4]$

Xi'an Jiaotong-Liverpool University
西交利物浦大學

Circular convolution

- Since the convolution operation involves shifting, we need to redefine the convolution for circularly shifted sequences:

$$y[k] = x_1[k] \circledast_N x_2[k] = \sum_{n=0}^{N-1} x_1[\langle n \rangle_N] x_2[\langle k-n \rangle_N]$$

- Expressed in matrix form, take N=4 as an example:

$$\begin{bmatrix} y[0] \\ y[1] \\ y[2] \\ y[3] \end{bmatrix} = \begin{bmatrix} x_2[0] & x_2[3] & x_2[2] & x_2[1] \\ x_2[1] & x_2[0] & x_2[3] & x_2[2] \\ x_2[2] & x_2[1] & x_2[0] & x_2[3] \\ x_2[3] & x_2[2] & x_2[1] & x_2[0] \end{bmatrix} \begin{bmatrix} x_1[0] \\ x_1[1] \\ x_1[2] \\ x_1[3] \end{bmatrix}$$

Circular convolution

- Example: Compute circular convolution of the following sequences x[n]=[**1** 2 3 4], h[n]=[**5** 6 7]

  - Circular convolution: $x[n] \circledast_N h[n]$:
  - Linear convolution:

    $x[n] * h[n] = \{5, 16, 34, 52, 45, 28\}$

4-point

$$y[k] = \begin{bmatrix} 1 & 4 & 3 & 2 \\ 2 & 1 & 4 & 3 \\ 3 & 2 & 1 & 4 \\ 4 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 6 \\ 7 \\ 0 \end{bmatrix} = \begin{bmatrix} 50 \\ 44 \\ 34 \\ 52 \end{bmatrix}$$

6-point

$$y[k] = \begin{bmatrix} 1 & 0 & 0 & 4 & 3 & 2 \\ 2 & 1 & 0 & 0 & 4 & 3 \\ 3 & 2 & 1 & 0 & 0 & 4 \\ 4 & 3 & 2 & 1 & 0 & 0 \\ 0 & 4 & 3 & 2 & 1 & 0 \\ 0 & 0 & 4 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 6 \\ 7 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 16 \\ 34 \\ 52 \\ 45 \\ 28 \end{bmatrix}$$

5-point

$$y[k] = \begin{bmatrix} 1 & 0 & 4 & 3 & 2 \\ 2 & 1 & 0 & 4 & 3 \\ 3 & 2 & 1 & 0 & 4 \\ 4 & 3 & 2 & 1 & 0 \\ 0 & 4 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 6 \\ 7 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 33 \\ 16 \\ 34 \\ 52 \\ 45 \end{bmatrix}$$

Linear VS circular convolution

- All LTI systems are based on the principle of **linear convolution**, as the output of an LTI system is the linear convolution of the system impulse response and the input to the system, which is equivalent to the product of the respective DTFTs in the frequency domain.
- However, if we use DFT instead of DTFT (so that we can compute it using a computer), then the result appear to be invalid:
  - DTFT is based on linear convolution, and DFT is based on circular convolution, and they are not the same!
  - For starters, they are not even of equal length: For two sequences of length N and M, the linear convolution is of length N+M-1, whereas circular convolution of the same two sequences is of length max(N,M), where the shorter sequence is zero padded to make it the same length as the longer one.

Linear VS circular convolution (cont.)

- Is there any relationship between the linear and circular convolutions? Can one be obtained from the other?
- YES!
  - FACT: If we zero pad both sequences x[n] and h[n], so that they are both of length N1+N2-1, then linear convolution and circular convolution result in identical sequences
  - Furthermore: If the respective DFTs of the zero padded sequences are X[k] and H[k], then the inverse DFT of X[k]·H[k] is equal to the linear convolution of x[n] and h[n]
  - Note that, normally, the inverse DFT of X[k].H[k] is the circular convolution of x[n] and h[n]. If they are zero padded, then the inverse DFT is also the linear convolution of the two.

Linear VS circular convolution (cont.)

- Compute circular convolution of x[n] $= [\mathbf{1}, 2, 3, 4]$, h[n]=[**5**, 6, 7], by appropriately zero padding the two:
  - 1. Zero pad signals with length L=N1+N2-1=6:
    - $x_L[n] = [\mathbf{1}, 2, 3, 4, 0, 0]$
    - $h_L[n] = [\mathbf{5}, 6, 7, 0, 0, 0]$
  - 2. Performing circular convolution using formula:

$$y_C[n] = x_L \circledast h_L = \sum_{m=0}^{L} x_L[m] h_L[\langle n - m \rangle_L]$$

  - 3. The solution

$$y_C[n] = y[n] = x[n] * h[n] = \sum_{m=-\infty}^{\infty} x[m] h\,[n - m]$$

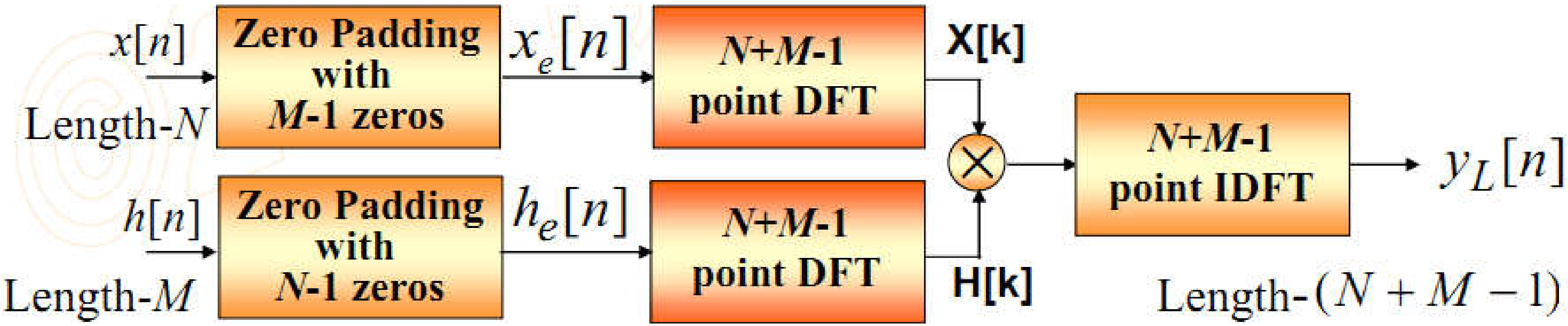
Xi'an Jiaotong-Liverpool University 西交利物浦大學

Computing convolution using DFT

- We can compute the output signal using no time domain operation: the inverse DFT of X[k]·H[k] is equal to the linear convolution of x[n] and h[n].

$$y[n] = x[n] \circledast_N h[n] = \left(x[n] * h[n]\right)_L = IDFT\{X[k] \cdot H[k]\}$$

- To ensure that one gets linear convolution, both sequences in the time domain must be zero padded to appropriate length L=N+M-1 as follows



9_4 Wrap up

- Facts about circular convolution:
  - Circular convolution ≠ Linear convolution
  - Linear convolution => sequence processed by system

$$y[n] = x[n] * h[n]$$

  - DFT multiplication (in FD) = circular convolution (in TD)

$$X[k] \cdot H[k] = x[n] \circledast h[n]$$

- How to use DFT multiplication to get sequence processed by system?
  - Zero-padding ➔ DFT ➔ multiplication ➔ IDFT

Chapter 9 Summary

- CTFT – DTFT – DFT
  - Time domain
  - Frequency domain
- Computation of DFT
  - Twiddle factor = complex exponential (wheel)
  - Matrix method
- Properties
- Circular convolution
  - Relationship to linear convolution
  - Frequency domain meaning
  - How to find the system output y[n] indirectly (in FD)