Introduction to Systems

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System ….

- **System:**
  - A physical entity that operates on a set of primary signals (the inputs) to produce a corresponding set of signals (the outputs).
    - The operations, or processing, may take several forms: decomposition, filtering, extraction of parameters, combination, etc.
  - A system may contain many subsystems with their own inputs / outputs

Systems : some examples …

- **Biometric systems :**
  - Enables the identification, verification or authentication of an individual based on physiological, behavioral and molecular characteristics. Biometric techniques include recognizing faces, hands, voices, signatures, irises, fingerprints, DNA patterns, etc.

Input signal

Is it an authorized person?

Biometric system S{.}

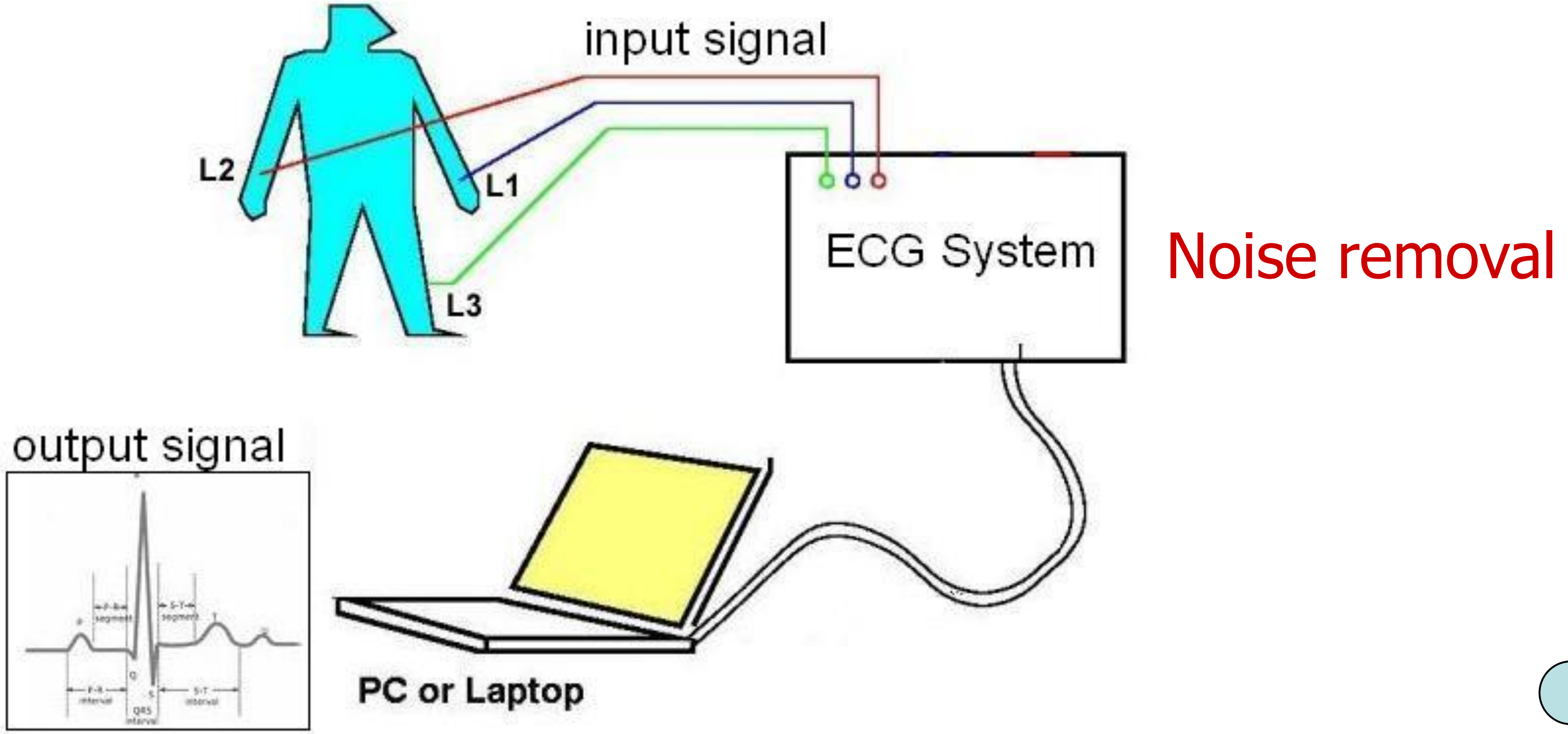
-Yes !

-NO !

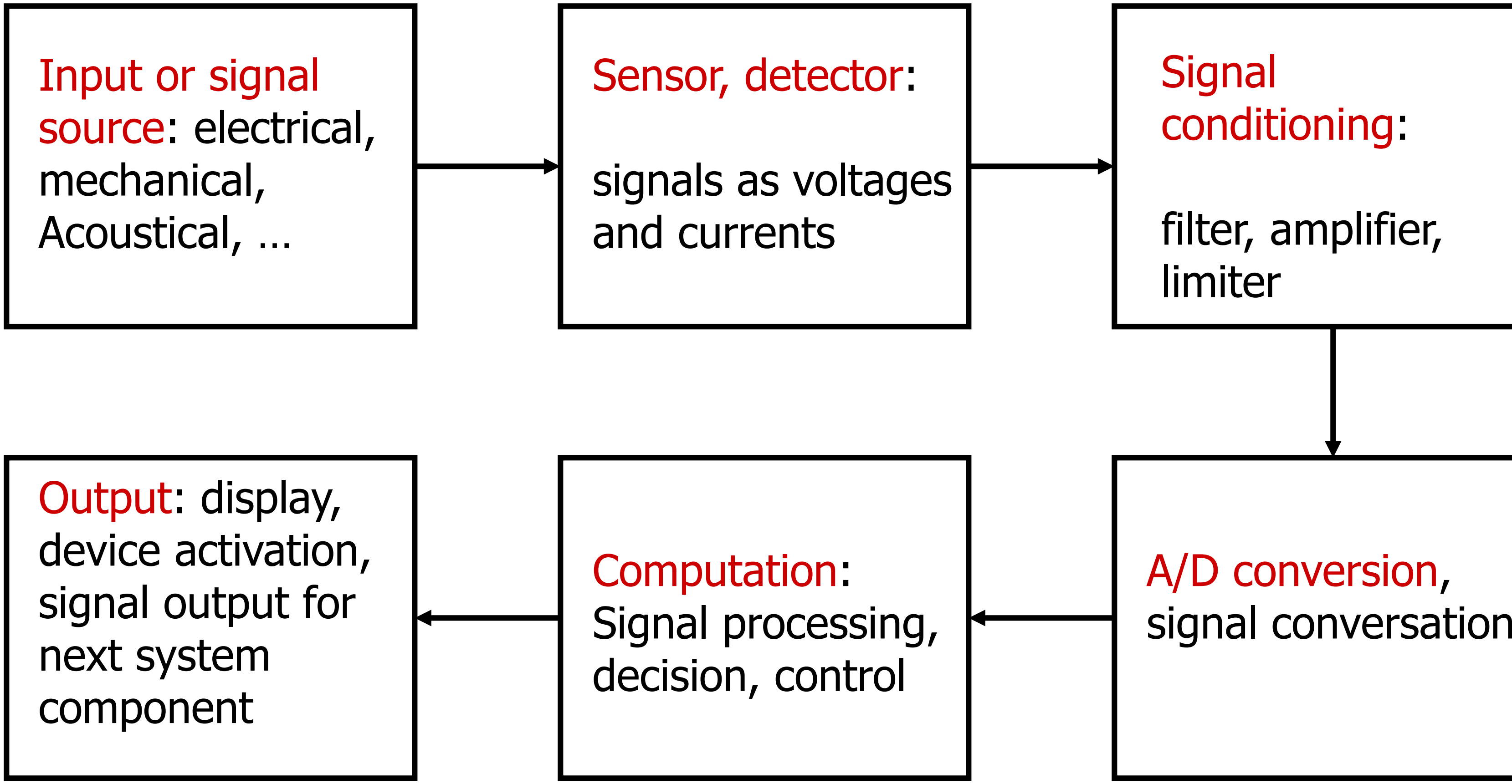
Output signal

Systems : some examples …

- **Electrocardiogram (ECG) :**
  - An electrocardiogram (ECG) is a noninvasive graphic approach that records the electrical activity of the heart over time.

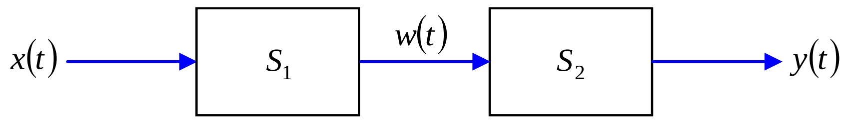


A Generic System Diagram

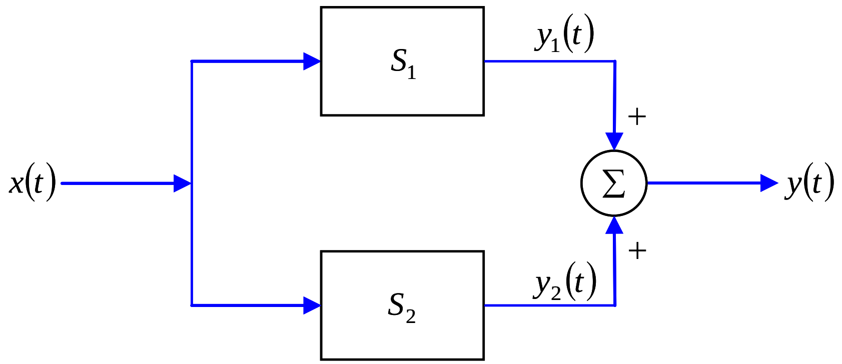


Interconnection of systems

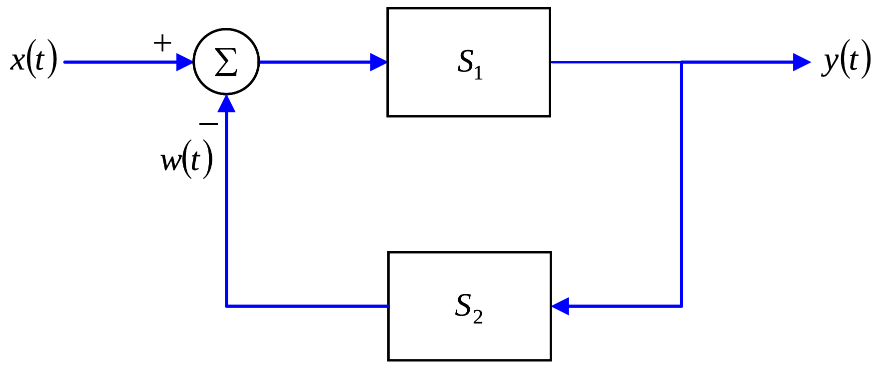
Cascaded configuration



Parallel configuration



Feedback configuration



Discrete-Time (DT) systems can be represented in different ways to more easily address different types of issues.

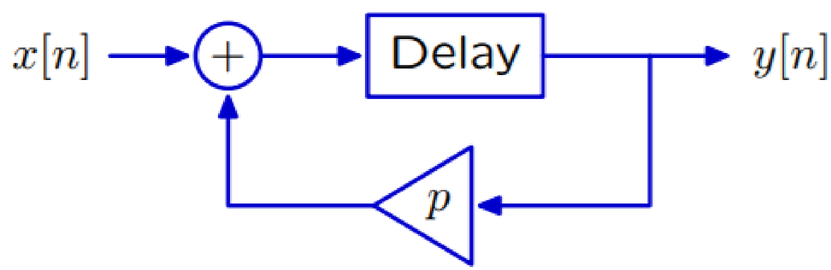
Verbal descriptions: preserve the rationale.

> \Next year, your account will contain p times your balance from this year plus the money that you added this year."

Difference equations: mathematically compact.

$$y[n+1] = x[n] + py[n]$$

Block diagrams: illustrate signal flow paths.



Operator representations: analyze systems as polynomials.

$$(1 - p\mathcal{R})\,Y = \mathcal{R}X$$

Similar representations for Continuous-Time (CT) systems.

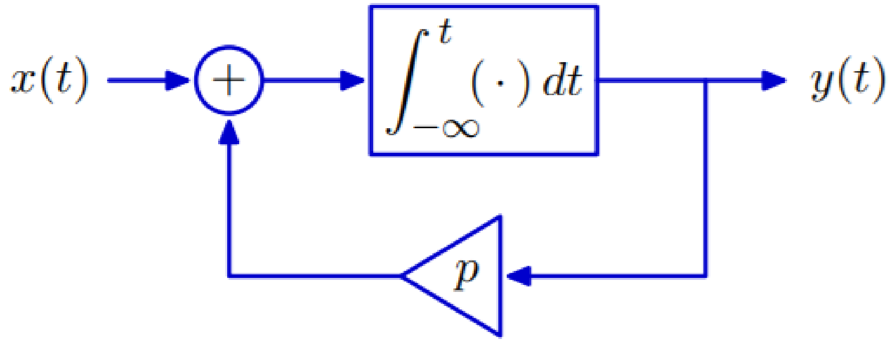
Verbal descriptions: preserve the rationale.

\Your account will grow in proportion to your balance plus the rate at which you deposit."

Differential equations: mathematically compact.

$$\frac{dy(t)}{dt} = x(t) + py(t)$$

Block diagrams: illustrate signal flow paths.



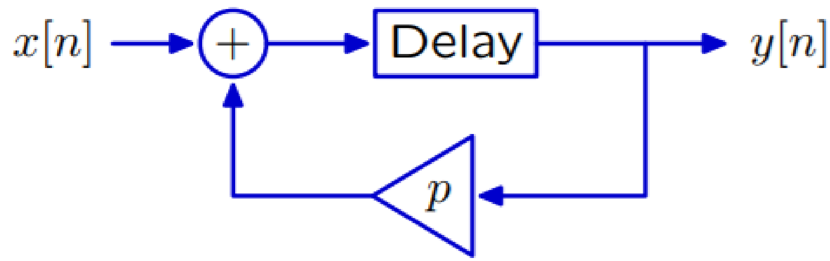
Operator representations: analyze systems as polynomials.

$$(1 - p\mathcal{A})Y = \mathcal{A}X$$

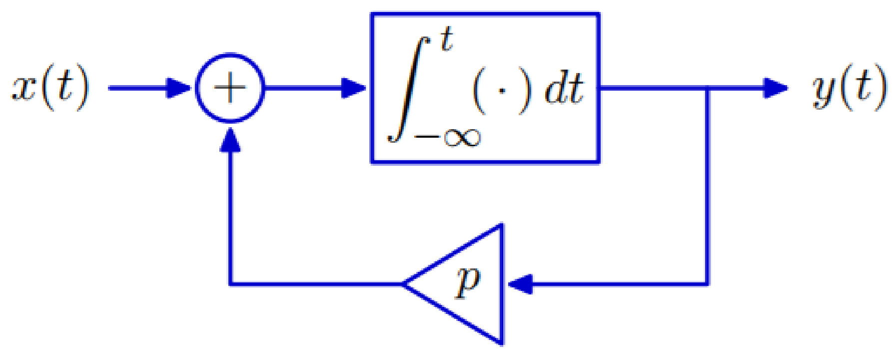
Block Diagrams

Block diagrams illustrate signal flow paths.

DT: adders, scalers, and delays { represent systems described by linear difference equations with constant coeff cents.



CT: adders, scalers, and integrators { represent systems described by a linear differential equations with constant coefficients.



Delays in DT are replaced by integrators in CT.

Operator Representation

CT Block diagrams are concisely represented with the $\mathcal{A}$ **operator**.

Applying $\mathcal{A}$ to a CT signal generates a new signal that is equal to the integral of the first signal at all points in time.

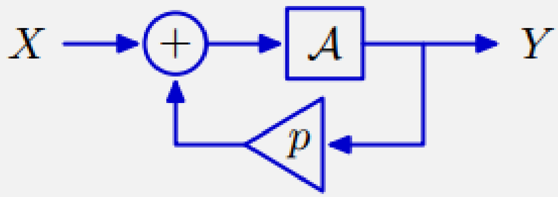
$$Y = \mathcal{A}X$$

is equivalent to

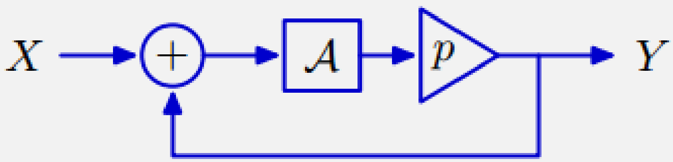
$$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$$

for **all** time t.

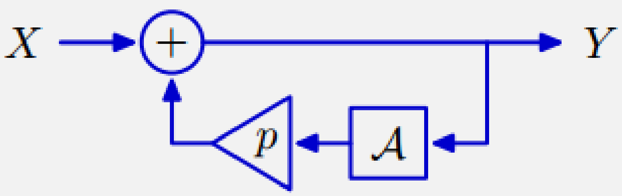
Test yourself



$$\dot{y}(t) = \dot{x}(t) + py(t)$$

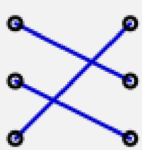
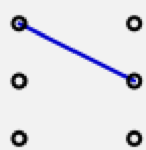
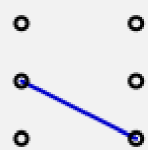
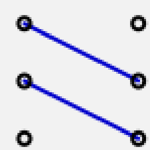


$$\dot{y}(t) = x(t) + py(t)$$



$$\dot{y}(t) = px(t) + py(t)$$

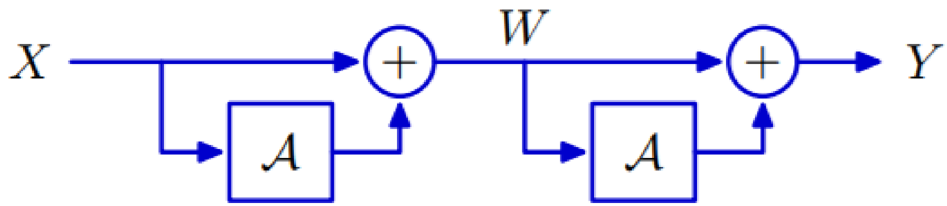
Which block diagrams correspond to which equations?

1. 
2. 
3. 
4. 
5. none

Evaluating Operator Expressions

As with $\mathcal{R}$, $\mathcal{A}$ expressions can be manipulated as polynomials.

Example:



$$w(t) = x(t) + \int_{-\infty}^{t} x(\tau)d\tau$$

$$y(t) = w(t) + \int_{-\infty}^{t} w(\tau)d\tau$$

$$y(t) = x(t) + \int_{-\infty}^{t} x(\tau)d\tau + \int_{-\infty}^{t} x(\tau)d\tau + \int_{-\infty}^{t} \left(\int_{-\infty}^{\tau_2} x(\tau_1)d\tau_1 \right) d\tau_2$$

$$W = (1 + \mathcal{A})\, X$$

$$Y = (1 + \mathcal{A})\, W = (1 + \mathcal{A})(1 + \mathcal{A})\, X = (1 + 2\mathcal{A} + \mathcal{A}^2)\, X$$

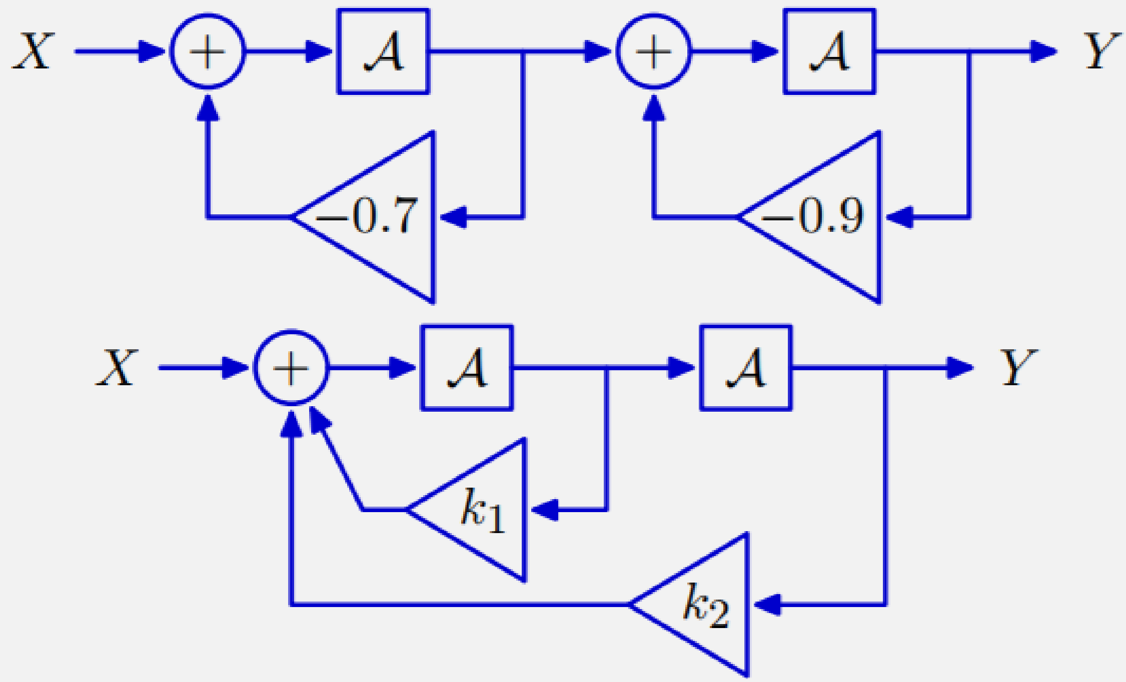
Evaluating Operator Expressions

Expressions in $\mathcal{A}$ can be manipulated using rules for polynomials.

- Commutativity: $\mathcal{A}(1-\mathcal{A})X = (1-\mathcal{A})\mathcal{A}X$
- Distributivity: $\mathcal{A}(1-\mathcal{A})X = (\mathcal{A}-\mathcal{A}^2)X$
- Associativity: $\Big((1-\mathcal{A})\mathcal{A}\Big)(2-\mathcal{A})X = (1-\mathcal{A})\Big(\mathcal{A}(2-\mathcal{A})\Big)X$

Test yourself

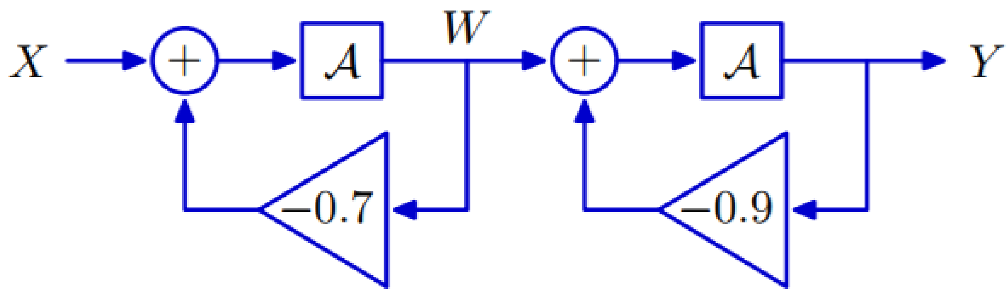
Determine k_1 so that these systems are "equivalent."



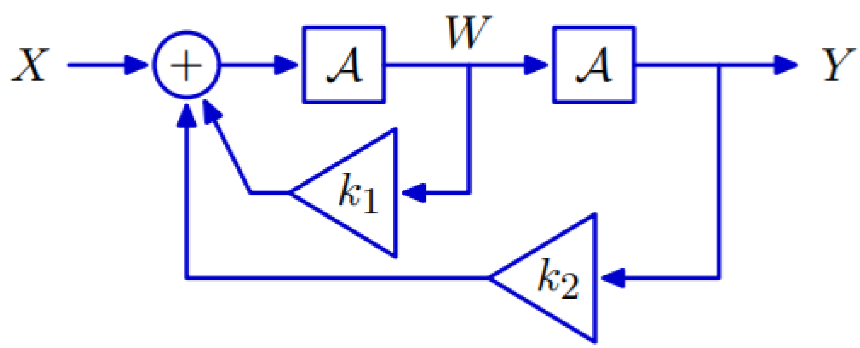
1. 0.7
2. 0.9
3. 1.6
4. 0.63
5. none of these

Test yourself

Write operator expressions for each system.



$$\begin{array}{l} W = \mathcal{A}(X - 0.7W) \\ Y = \mathcal{A}(W - 0.9Y) \end{array} \rightarrow \begin{array}{l} (1+0.7\mathcal{A})W = \mathcal{A}X \\ (1+0.9\mathcal{A})Y = \mathcal{A}W \end{array} \rightarrow \begin{array}{l} (1+0.7\mathcal{A})(1+0.9\mathcal{A})Y = \mathcal{A}^2X \\ (1+1.6\mathcal{A}+0.63\mathcal{A}^2)Y = \mathcal{A}^2X \end{array}$$



$$\begin{array}{c} W = \mathcal{A}(X + k_1W + k_2Y) \\ Y = \mathcal{A}W \end{array} \rightarrow \begin{array}{c} Y = \mathcal{A}^2X + k_1\mathcal{A}Y + k_2\mathcal{A}^2Y \\ (1 - k_1\mathcal{A} - k_2\mathcal{A}^2)Y = \mathcal{A}^2X \end{array}$$

$k_1 = -1.6$

Unit-Impulse and Unit-Step Signals

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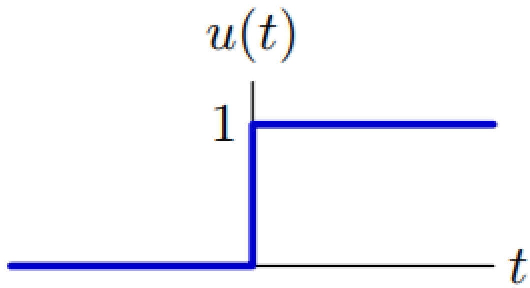
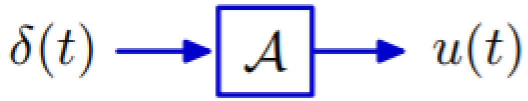
The indefinite integral of the unit-impulse is the unit-step.

$$u(t) = \int_{-\infty}^{t} \delta(\lambda)\, d\lambda = \begin{cases} 1; & t > 0 \\ 0; & \text{otherwise} \end{cases}$$

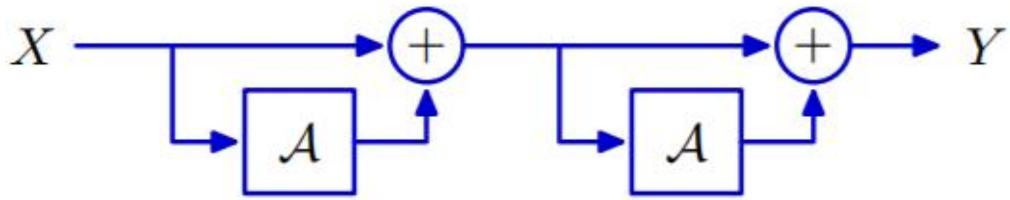


Equivalently



Impulse Response of Acyclic CT System

If the block diagram of a CT system has no feedback (i.e., no cycles), then the corresponding operator expression is "imperative."



$$Y = (1 + \mathcal{A})(1 + \mathcal{A})\,X = (1 + 2\mathcal{A} + \mathcal{A}^2)\,X$$

If $x(t) = \delta(t)$ then

$$y(t) = (1 + 2\mathcal{A} + \mathcal{A}^2)\,\delta(t) = \delta(t) + 2u(t) + tu(t)$$

CT Feedback

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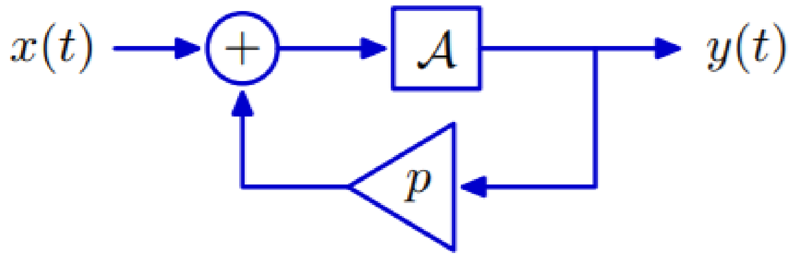
Find the impulse response of this CT system with feedback.



Method 1: find differential equation and solve it.

$$\dot{y}(t) = x(t) + py(t)$$

Linear, first-order difference equation with constant coefficients.

Try $y(t) = Ce^{\alpha t}u(t)$.

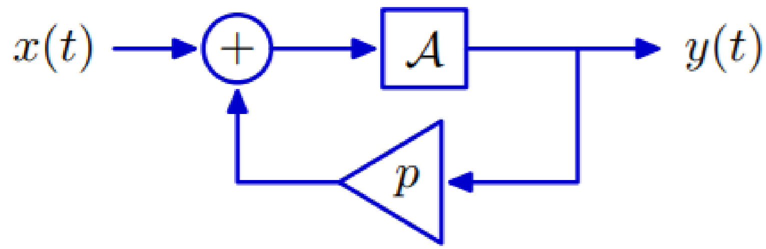
Then $\dot{y}(t) = \alpha Ce^{\alpha t}u(t) + Ce^{\alpha t}\delta(t) = \alpha Ce^{\alpha t}u(t) + C\delta(t)$.

Substituting, we find that $\alpha Ce^{\alpha t}u(t) + C\delta(t) = \delta(t) + pCe^{\alpha t}u(t)$.

Therefore $\alpha = p$ and $C = 1 \quad \rightarrow \quad y(t) = e^{pt}u(t)$.

CT Feedback

Find the impulse response of this CT system with feedback.



Method 2: use operators.

$$Y = \mathcal{A}\,(X + pY)$$
$$\frac{Y}{X} = \frac{\mathcal{A}}{1 - p\mathcal{A}}$$

Now expand in ascending series in $\mathcal{A}$:

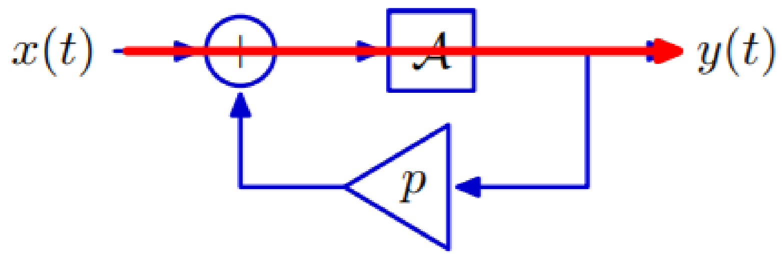
$$\frac{Y}{X} = \mathcal{A}(1 + p\mathcal{A} + p^2\mathcal{A}^2 + p^3\mathcal{A}^3 + \cdots)$$

If $x(t) = \delta(t)$ then

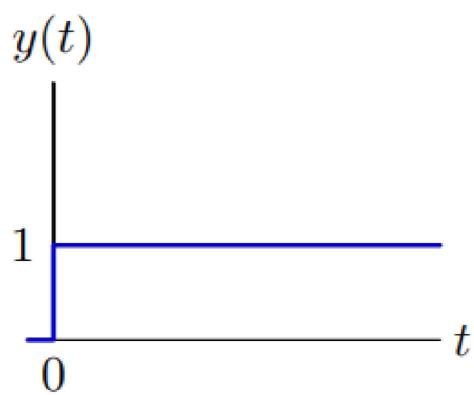
$$y(t) = \mathcal{A}(1 + p\mathcal{A} + p^2\mathcal{A}^2 + p^3\mathcal{A}^3 + \cdots)\,\delta(t)$$
$$= (1 + pt + \frac{1}{2}p^2t^2 + \frac{1}{6}p^3t^3 + \cdots)\,u(t) = e^{pt}u(t)\,.$$

CT Feedback

We can visualize the feedback by tracing each cycle through the cyclic signal path.



$$
\begin{aligned}
y(t) &= \left(\mathcal{A} + p\mathcal{A}^2 + p^2\mathcal{A}^3 + p^3\mathcal{A}^4 + \cdots\right)\delta(t) \\
&= \left(1 + pt + \frac{1}{2}p^2t^2 + \frac{1}{6}p^3t^3 + \cdots\right)u(t)
\end{aligned}
$$



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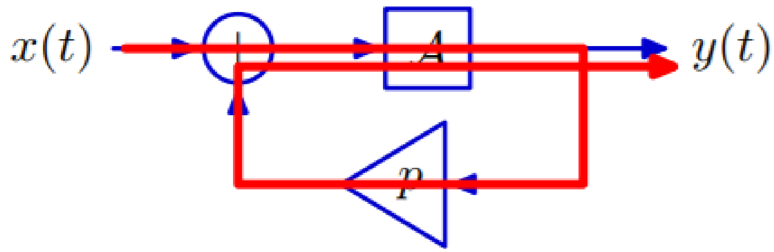

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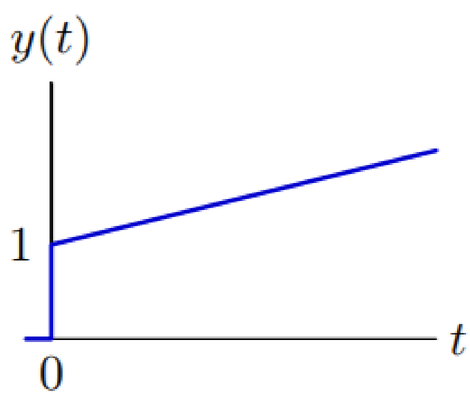

CT Feedback

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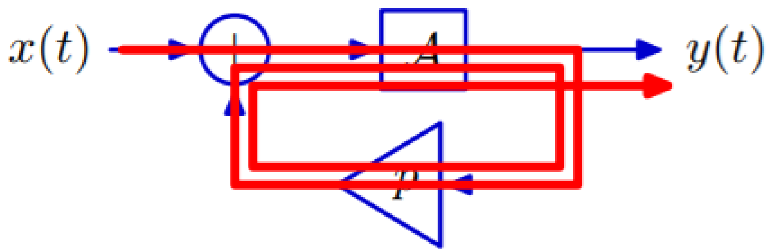

$$y(t) = (\mathcal{A} + p\mathcal{A}^2 + p^2\mathcal{A}^3 + p^3\mathcal{A}^4 + \cdots)\,\delta(t)$$
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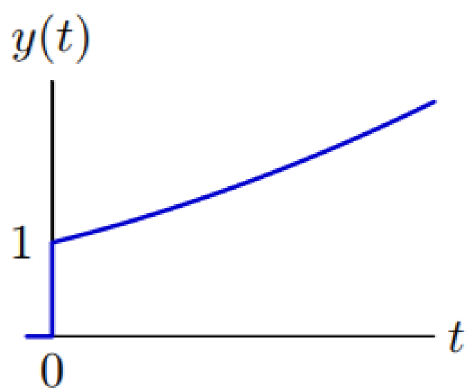

CT Feedback

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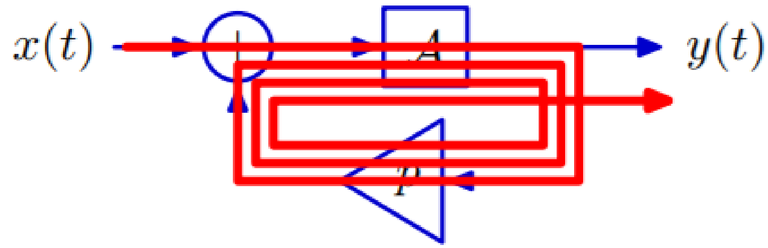
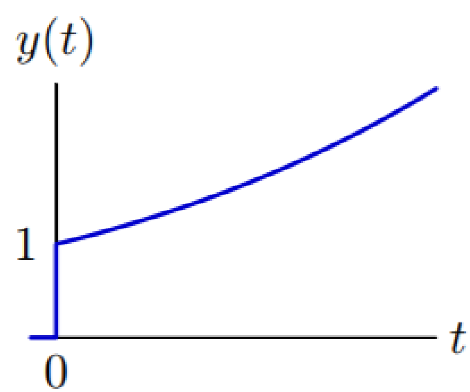
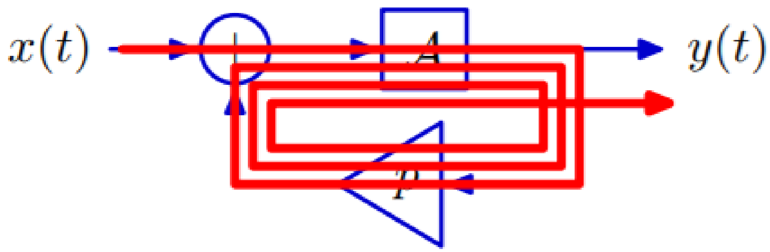


$$
\begin{aligned}
y(t) &= (\mathcal{A} + p\mathcal{A}^2 + p^2\mathcal{A}^3 + p^3\mathcal{A}^4 + \cdots)\,\delta(t) \\
&= (1 + pt + \frac{1}{2}p^2t^2 + \frac{1}{6}p^3t^3 + \cdots)\,u(t)
\end{aligned}
$$

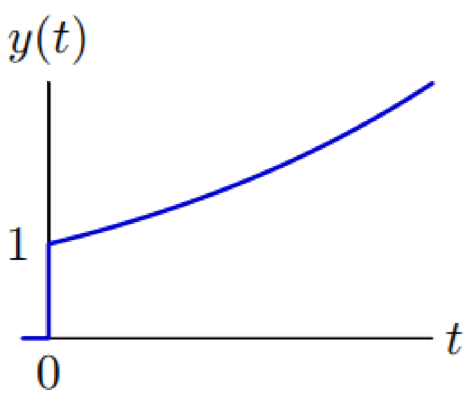


CT Feedback

We can visualize the feedback by tracing each cycle through the cyclic signal path.

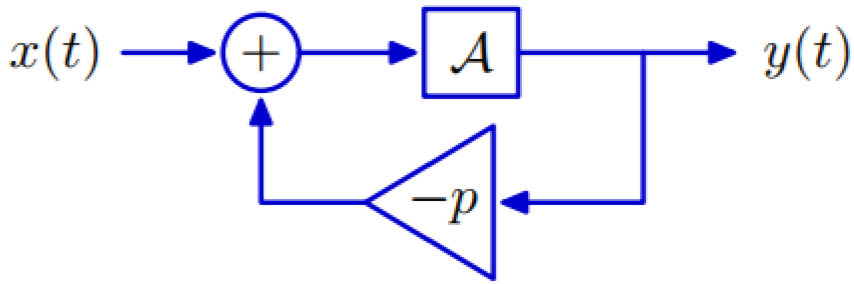


$$y(t) = (\mathcal{A} + p\mathcal{A}^2 + p^2\mathcal{A}^3 + p^3\mathcal{A}^4 + \cdots)\,\delta(t)$$
$$= (1 + pt + \frac{1}{2}p^2t^2 + \frac{1}{6}p^3t^3 + \cdots)\,u(t) = e^{pt}u(t)$$



CT Feedback

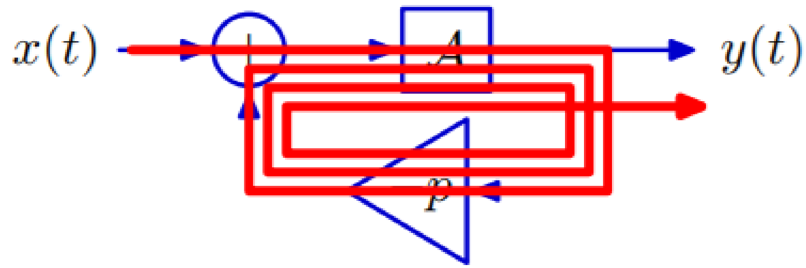
Making p negative makes the output converge (instead of diverge).



$$
\begin{aligned}
y(t) &= \left(\mathcal{A} - p\mathcal{A}^2 + p^2\mathcal{A}^3 - p^3\mathcal{A}^4 + \cdots\right)\delta(t) \\
&= \left(1 - pt + \frac{1}{2}p^2t^2 - \frac{1}{6}p^3t^3 + \cdots\right)u(t)
\end{aligned}
$$

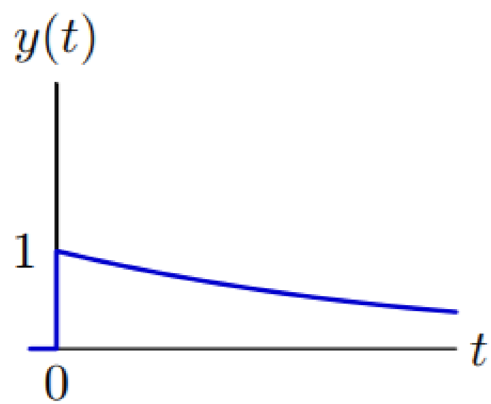
CT Feedback

Making p negative makes the output converge (instead of diverge).



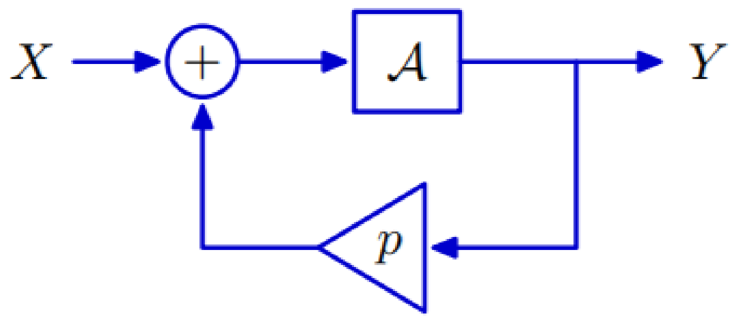
$$y(t) = (\mathcal{A} - p\mathcal{A}^2 + p^2\mathcal{A}^3 - p^3\mathcal{A}^4 + \cdots)\,\delta(t)$$

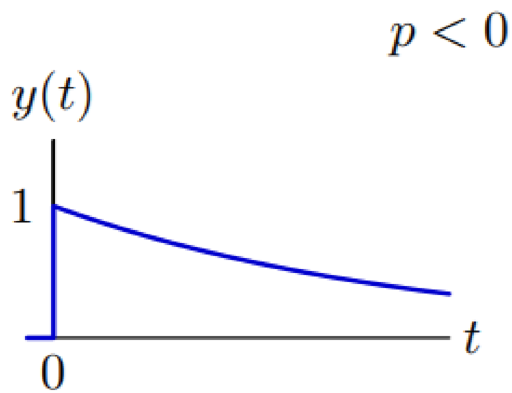
$$= (1 - pt + \frac{1}{2}p^2t^2 - \frac{1}{6}p^3t^3 + \cdots)\,u(t) = e^{-pt}u(t)$$

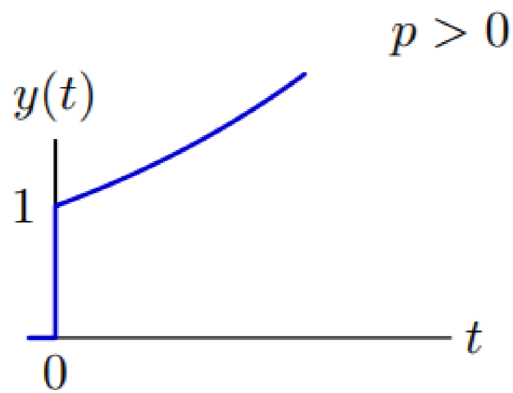


Convergent and Divergent Poles

The fundamental mode associated with p converges if $p < 0$ and diverges if $p > 0$.







System classification

System classification

- **Non-linear/linear system**
- **Time variant/invariant systems**
- **Invertible/non-invertible systems**
- **Systems with/without memory**
- **Causal and non-causal system**

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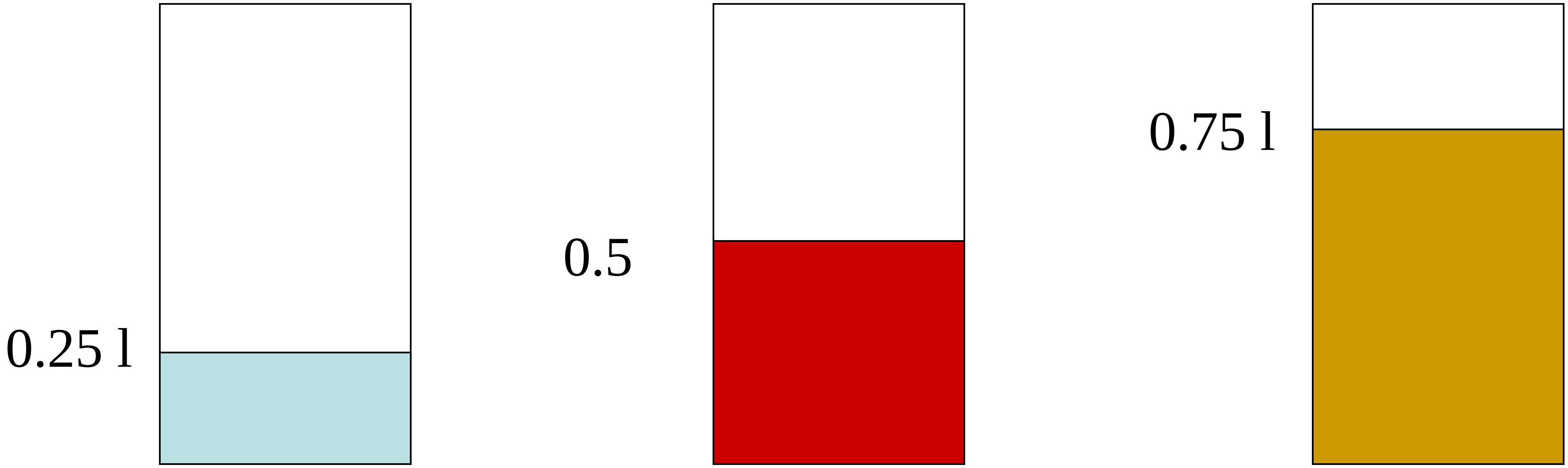
Linear systems

Linear systems

- **Superposition principle :**
  - In physics and systems theory, the superposition principle (superposition property), states that, the net response at a given place and time caused by two or more stimuli is the sum of the responses which would have been caused by each stimulus individually.

Volume in a cup as function of the quanity of liquids



Linear systems

If an input is the sum of two signals, then the output will be the sum of the outputs obtained from the two signals separately.

- **Linear systems :**

$x_1(t)$

$x_2(t)$

$a\,x_1(t) + b\,x_2(t)$

Linear systems $S\{.\}$

$y_1(t) = S\{x_1(t)\}$

$y_2(t)=S\{x_2(t)\}$

$a\,y_1(t) + b\,y_2(t)$

Principle of Superposition

Additive property

$$x_1(t) + x_2(t) \rightarrow y_1(t) + y_2(t)$$

Homogeneity property

$$ax_1 \rightarrow ay_1(t)$$

'Zero-input, zero-output property.'

Test yourself

Determine whether the CT systems are linear.

$$(1)\ \ y(t) = t\,x(t);$$
$$(2)\ y(t) = e^{x(t)};$$
$$(3)\ y(t) = 7x(t);$$
$$(4)\ \ y(t) = 7x(t) + 7;$$

Test yourself

Determine whether the CT systems are linear.

(1) $y(t) = t\,x(t)$;
(2) $y(t) = e^{x(t)}$;
(3) $y(t) = 7x(t)$;
(4) $y(t) = 7x(t) + 7$;

Answer:
(1) Linear;
(2) Non-linear;
(3) Linear
(4) Non-linear

Linear systems

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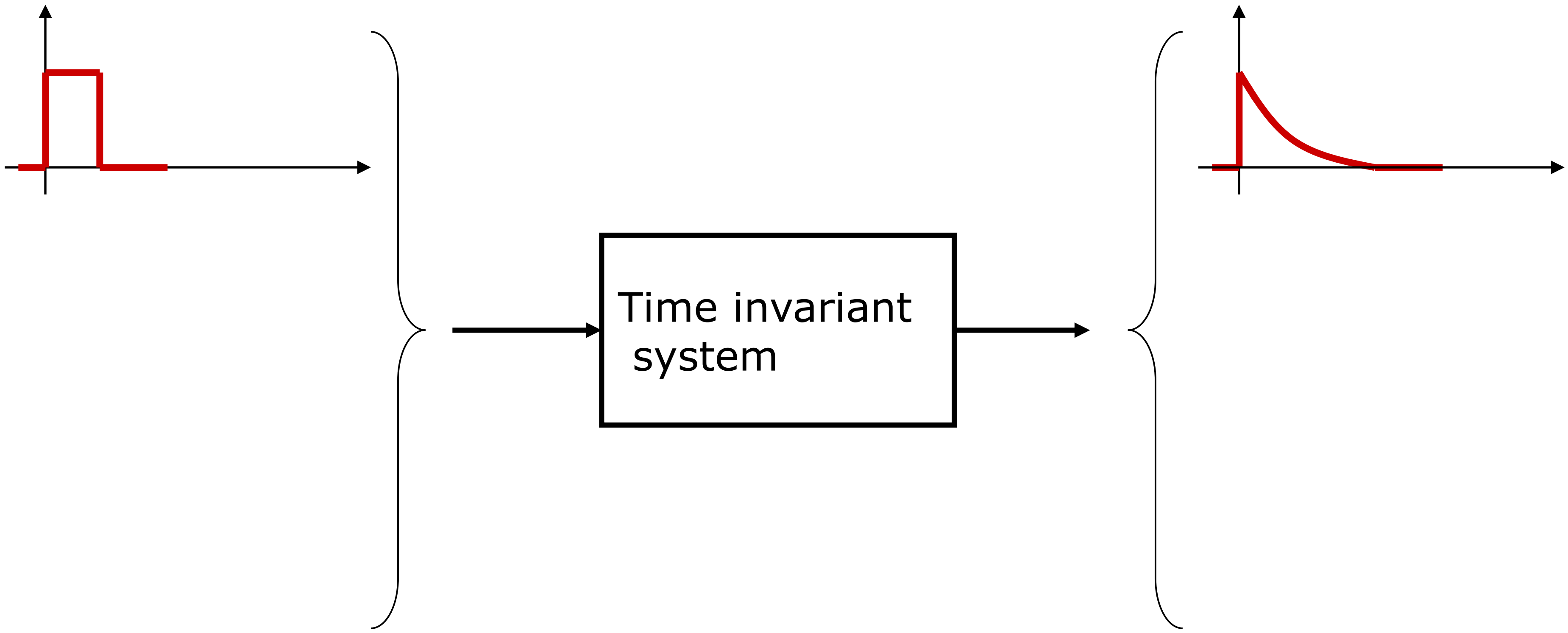
- **System:**
  - Linear systems are relatively simple, nonlinear systems are not.
  - The output of a linear system is predictable:
    - where the input is the sum of two signals, the output will be the sum of the outputs obtained for the separate inputs ➔ this is not usually true for nonlinear systems.

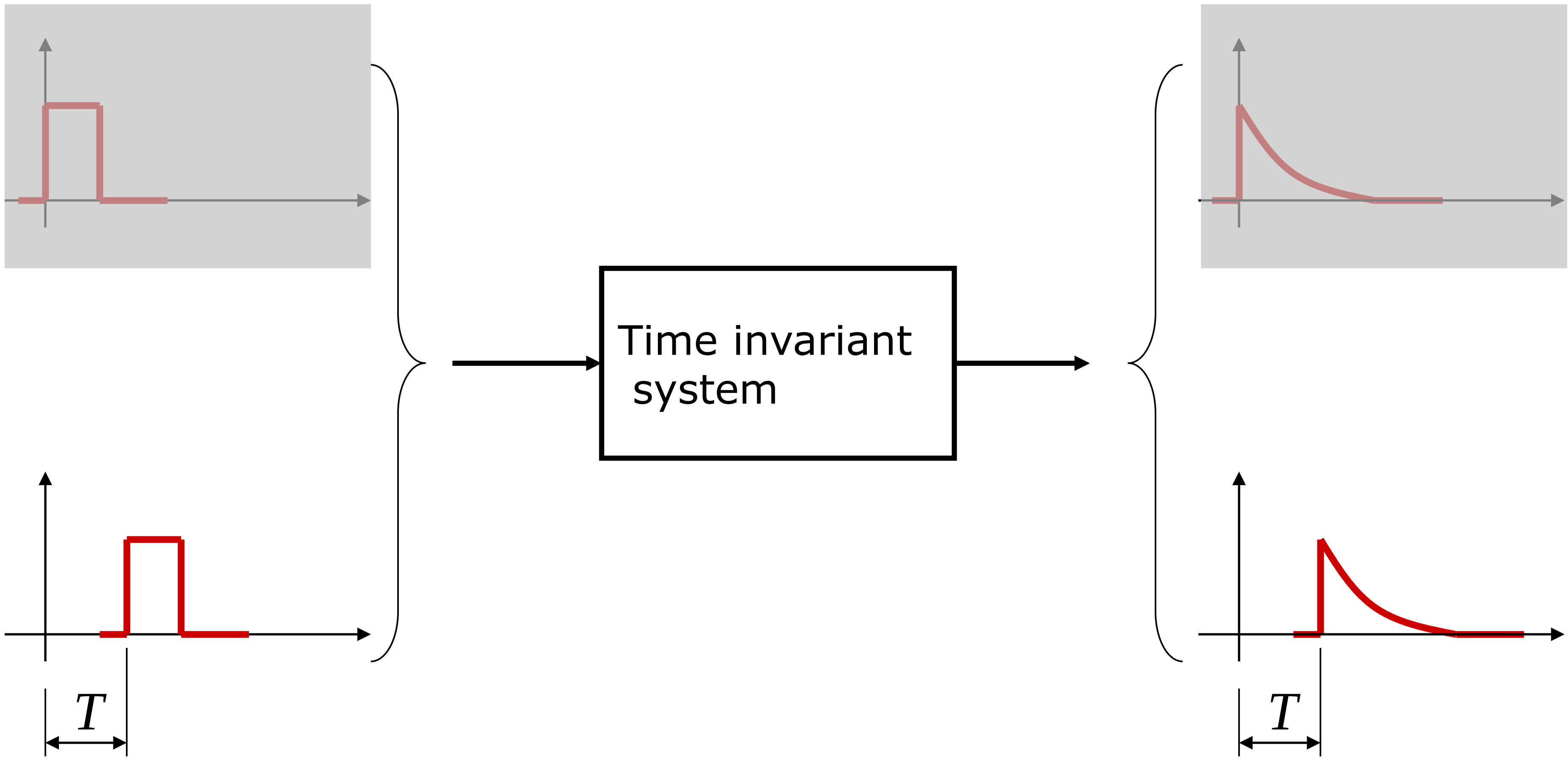
  - In practice, all real systems have some nonlinear properties, but we can usually treat them as linear as long as we apply "small" signals (although what counts as "small" will differ from system to system).

Time invariant systems

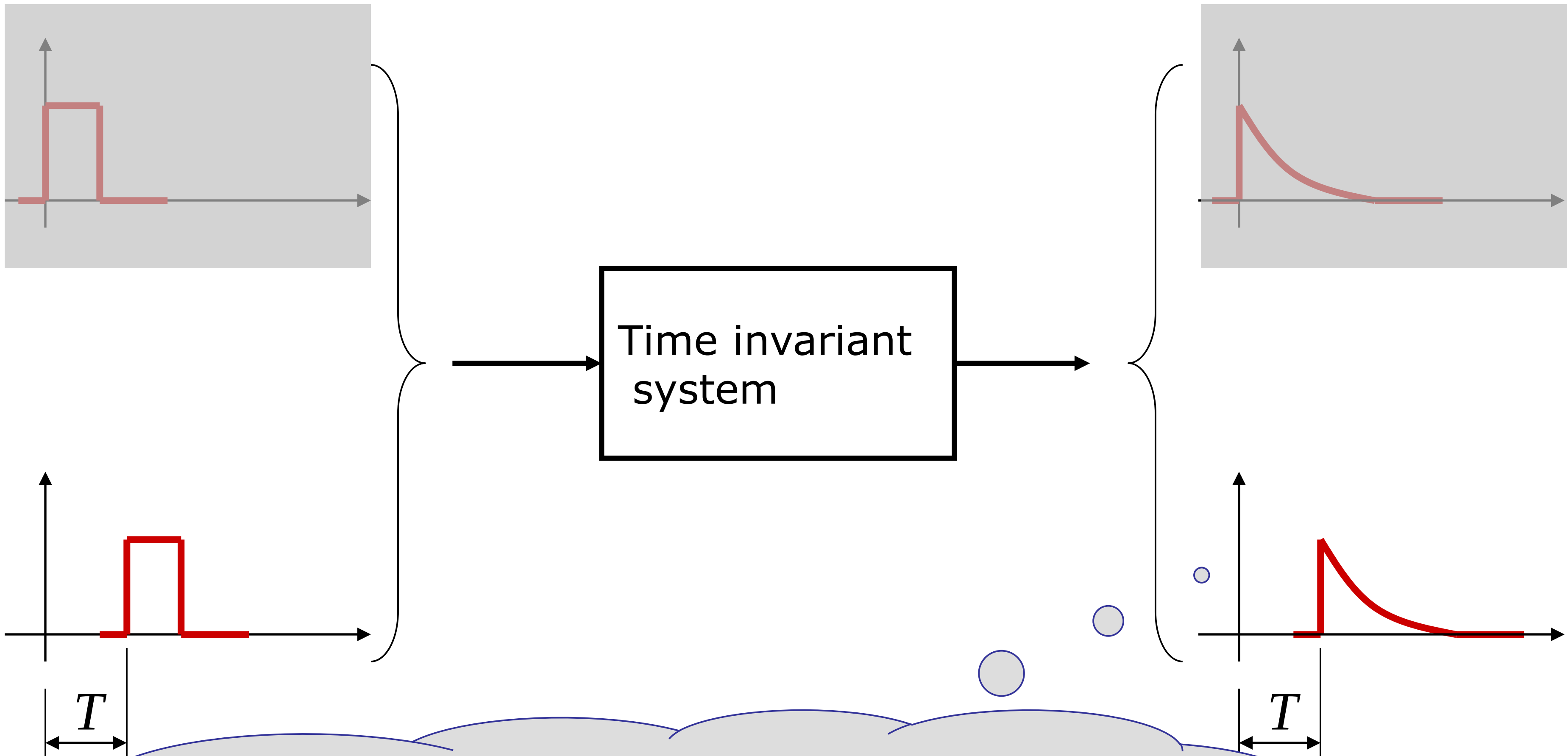
Time invariant systems …



Time invariant systems …



Time invariant systems …



Except for a time shift in the output, the system responds exactly the same way no matter when the input signal is applied.

Time invariant systems …

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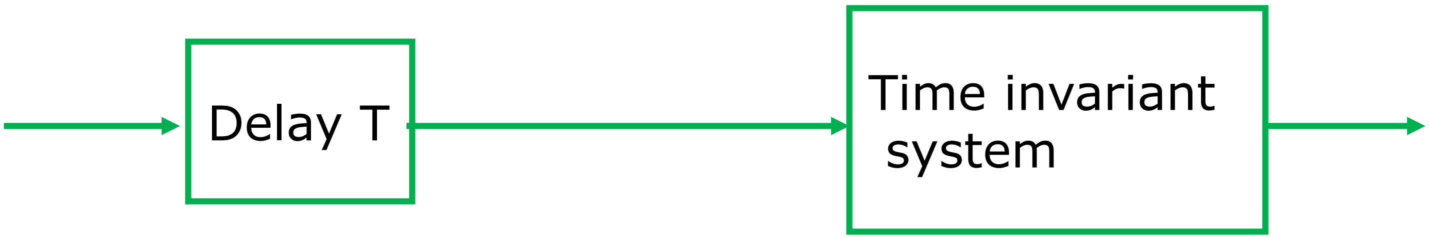

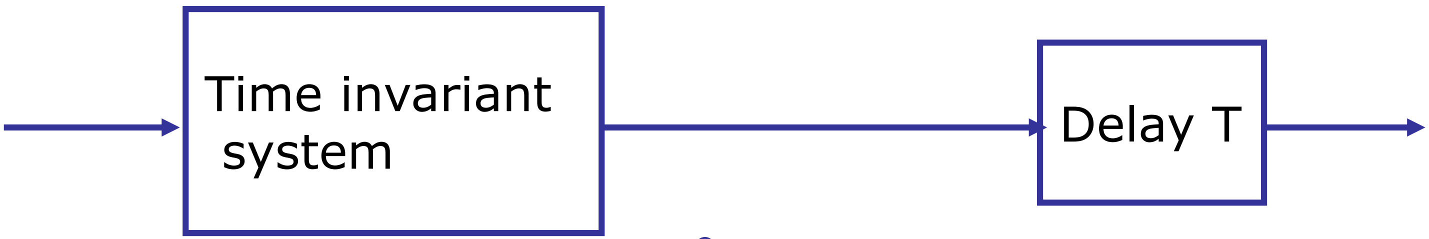

In a time invariant system delaying the input or the output leads to the same result.

Time invariant systems …

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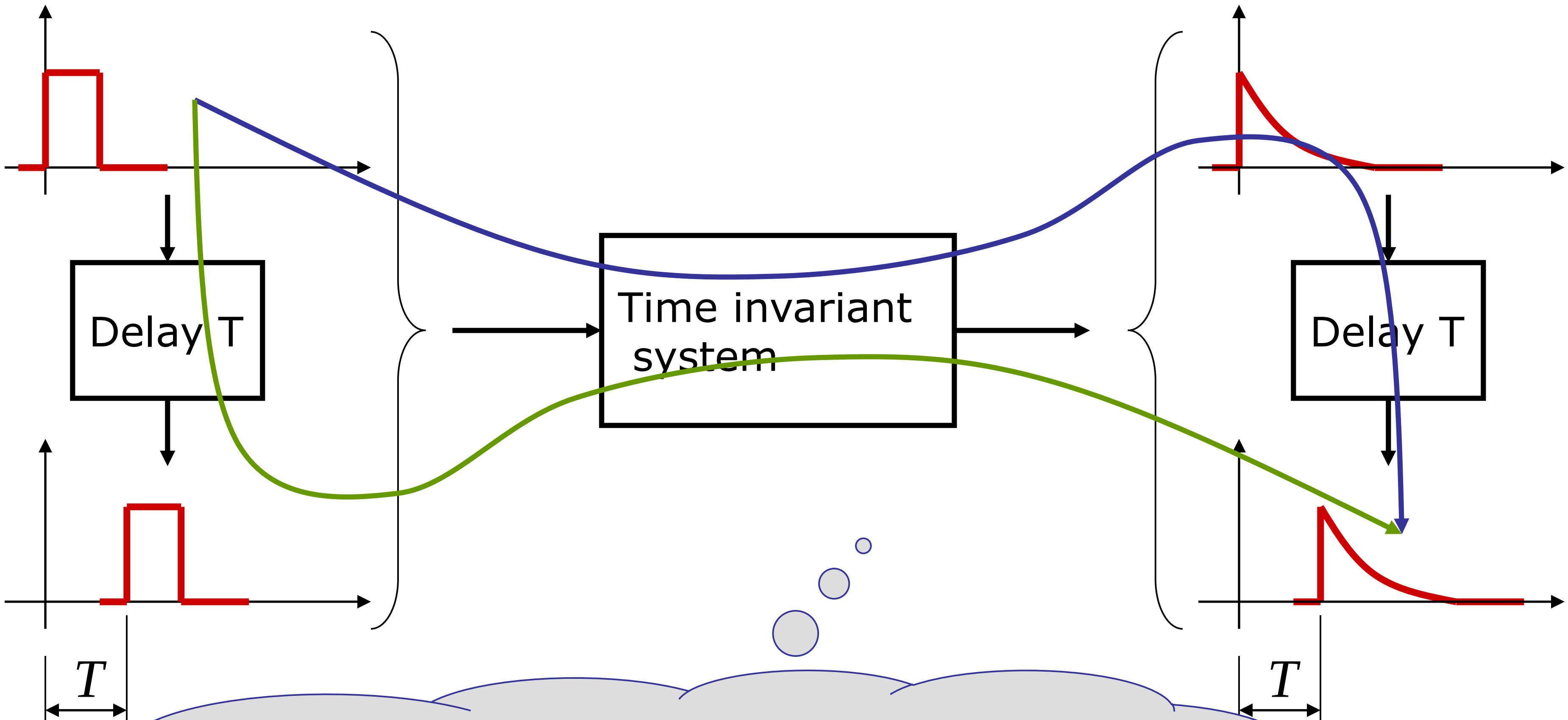

In a time invariant system delaying the input or the output leads to the same result.

Time invariant systems …

- **Time invariant systems:**

$$x(t) \rightarrow y(t) \quad \longrightarrow \quad x(t-\tau) \rightarrow y(t-\tau)$$

  - Time invariant systems are good models to describe systems with time-non-varying parameters.
  - Time invariant systems do not change with time → they are rare, since most things change with time, but as long as the time scale for the variations is longer than the time scale over which the calculation is performed (or the system operates), then time invariance is also a reasonably good approximation.

$y(t) = t \sin(x(t))$; is it time-invariant system ?

Invertible/non-invertible systems

Invertible/non-invertible systems

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A CT system is invertible if the input signal x(t) can be uniquely determined from the output y(t) produced in response to x(t) for all time $t \in (-\infty, \infty)$

For example:

$$y(t) = 3x(t) + 5 \ggg x(t) = \frac{1}{3}[y(t) - 5]$$

Therefore, the system is invertible.

Another example:

$$y(t) = \cos[x(t)]$$

$$x(t) = cos^{-1}[y(t)] + 2\pi m$$

Systems with/without memory

- **Systems without memory**
  - A system is memoryless if its output at any time depends only on the value of the input at that same time → *simpler*.
- **Systems with memory (*dynamic* system)**
  - If the response of the system depends on the previous input → *it has more capabilities in terms of signal processing.*

$$y(t) = ax(t)$$

$$y(t) = x(t/2)$$

$$y(t) = |x(t)|$$

$$y(t) = \int_{-\infty}^{t} x(\tau)d\tau$$

Is it memoryless or with memory system ?

Causal and non-causal system

Causal and non-causal system

- **Causal systems**
  - The output *y(t)* only depends on the current and previous values (time/space) of the input *x(t)*.
    - Therefore these kinds of systems have outputs and internal states that depend only on the current and previous input values.
  - Causality is based on the observation that an effect comes after the cause.
    - Classically, nature or physical reality has been considered to be a causal system.
  - A system that has some dependence on input values from the following values is termed a non-causal.

Causal and non-causal system

All memoryless systems are Causal systems

Is it Causal or non-causal system ?

$$y(t) = x(t-5)$$
$$y(t) = x(2t)$$
$$y(t) = x(\frac{t}{2})$$
$$y(t) = x(-t)$$
$$y(t) = x(t-2) + x(t+2)$$

Some examples

Continuous-time	
Memoryless systems	Systems with memory
$y(t) = 3x(t) + 5$	$y(t) = x(t - 5)$
$y(t) = \sin\{x(t)\} + 5$	$y(t) = x(t + 2)$
$y(t) = e^{x(t)}$	$y(t) = x(2t)$
$y(t) = x^2(t)$	$y(t) = x(t/2)$

CT systems	
Causal	Non-causal
$y(t) = x(t - 5)$	$y(t) = x(t + 2)$
$y(t) = \sin\{x(t - 4)\} + 3$	$y(t) = \sin\{x(t + 4)\} + 3$
$y(t) = e^{x(t-2)}$	$y(t) = x(2t)$
$y(t) = x^2(t - 2)$	$y(t) = x(t/2)$
$y(t) = x(t - 2) + x(t - 5)$	$y(t) = x(t - 2) + x(t + 2)$

For the next lecture

Read Chapter 2.

2.0

2.2

2.3