Xi'an Jiaotong-Liverpool University
西交利物浦大学

EEE220 Instrumentation and Control System

2018-19 Semester 2

Dr. Qing Liu

Email: qing.liu@xjtlu.edu.cn

Office: EE516

Department of Electrical and Electronic Engineering

11 April, 2019

Lecture 14

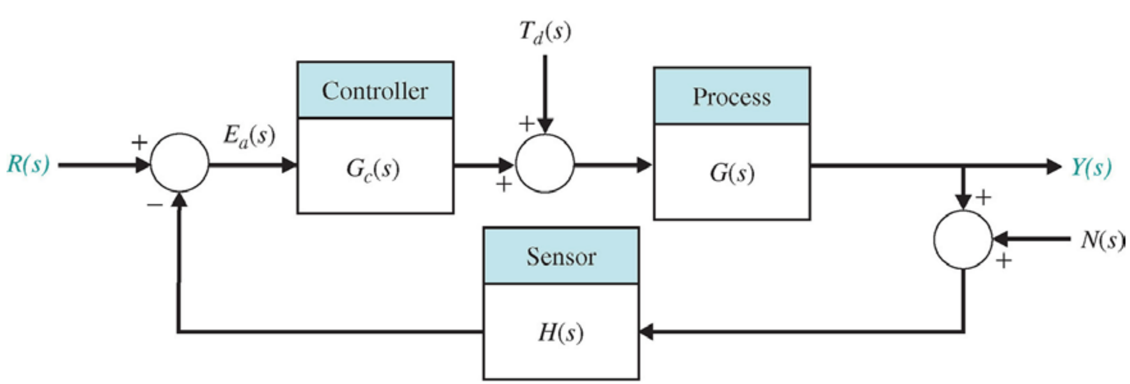
Outline

The Time-Domain Performance of Feedback Systems

- ❑ **Test Input Signals**
- ❑ **Performance of Second-Order System**
- ❑ **Effects of a Third Pole and a Zero on the Second-Order System Response**
- ❑ The s-Plane Root Location and the Transient Response
- ❑ The Steady-State Error of Feedback Control Systems
- ❑ System Simulation Using Matlab

Overview

- Easy control and adjustment of the transient and steady-state response of a control system is a distinct advantage of feedback control systems;
- To analyze and design a control system, we must define and measure its performance, the controller parameters may be adjusted to provide the desired response which is often described by **design specifications**.
- Control systems are inherently dynamic, their performance is usually specified in terms of both the transient response the steady-state response.
  - **Transient response** is the response that disappears with time;
  - **Steady-state response** is the response that exists for a long time following an input signal initiation.



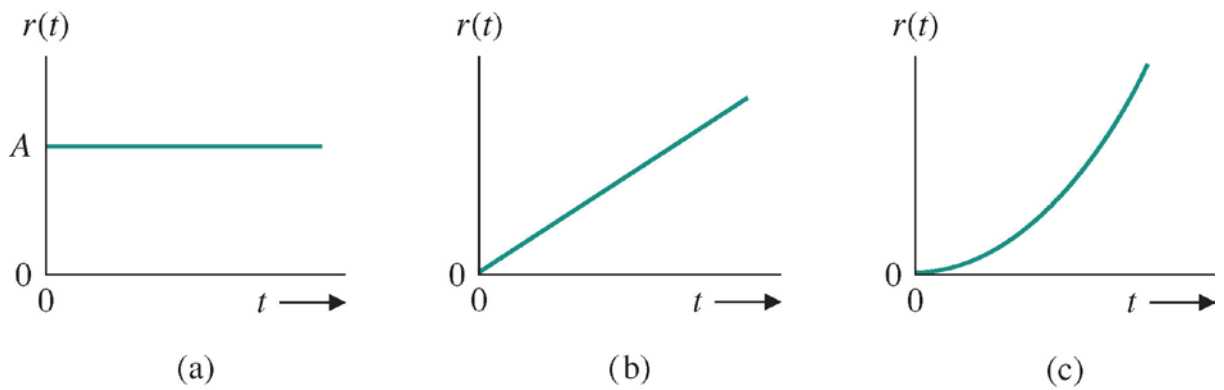
Closed-loop System

$$T(s) = \frac{Y(s)}{R(s)} = \frac{G_cG}{1 + G_cGH}$$

Test Input Signals

Control systems are inherently time-domain systems, so the system transient or time performance is the response of prime interest for control systems.

- Is the system stable? (will be discussed in the following lectures)
- If stable, how to measure and compare the performance of several competing designs?
  - Provide several measures of performance (response time, percent overshoot etc.)
  - Test the system by standard test input signals.



Three Standard Test Input Signals

Test Input Signal in Time and s-Domain

- There is a reasonable correlation between the response of a system to a standard test input and the system's ability to perform under normal operating conditions.
- Actually, many control systems experience input signals that are very similar to the standard test signals.

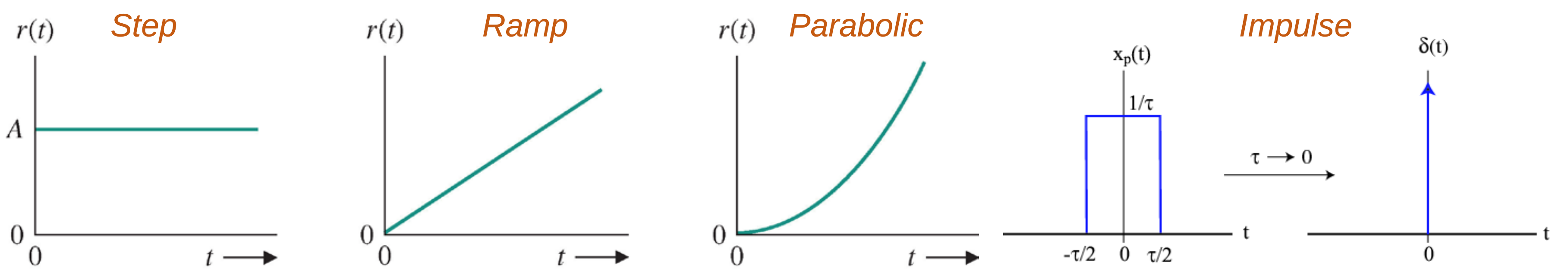


Table 5.1 Test Signal Inputs		
Test Signal	$r(t)$	$R(s)$
Step	$r(t) = A, t > 0$ $= 0, t < 0$	$R(s) = A/s$
Ramp	$r(t) = At, t > 0$ $= 0, t < 0$	$R(s) = A/s^2$
Parabolic	$r(t) = At^2, t > 0$ $= 0, t < 0$	$R(s) = 2A/s^3$

$$\boldsymbol{r(t) = t^n}$$

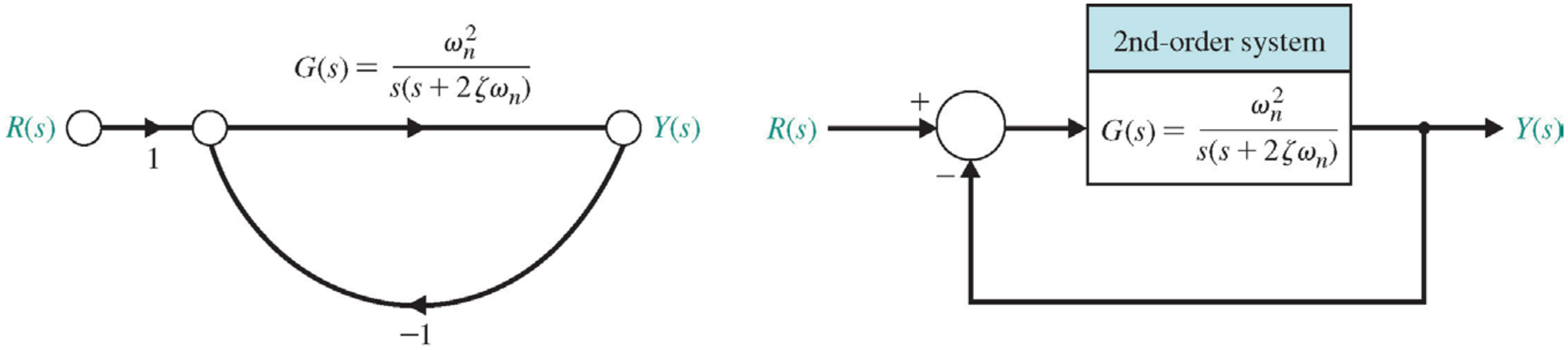
$$\boldsymbol{R(s) = \frac{n!}{s^{n+1}}}$$

except for impulse input:

$$r(t) = \delta(t), R(s) = 1$$

Second-Order System

$$Y(s) = \frac{G(s)}{1 + G(s)} R(s)$$



$$\boxed{Y(s) = \frac{{\omega_n}^2}{s^2 + 2\zeta\omega_n s + {\omega_n}^2} R(s)}$$

ω_n: Natural Frequency;

ζ : Damping Ratio.

Time Response to Impulse Input

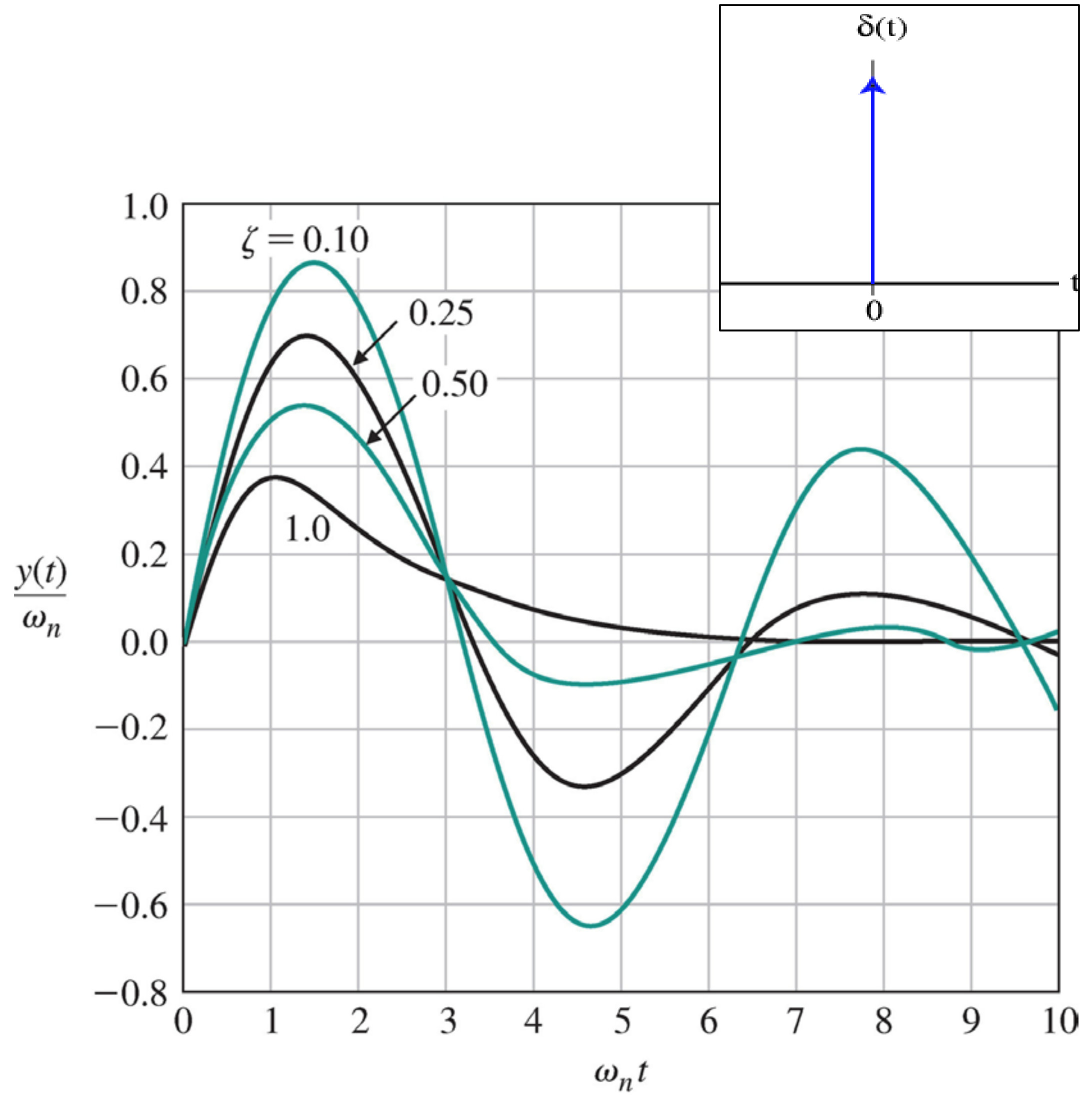
$$R(s) = 1$$

$$Y(s) = \frac{{\omega_n}^2}{s^2 + 2\zeta\omega_n s + {\omega_n}^2}$$

$$\boxed{\boldsymbol{y(t) = \frac{\omega_n}{\beta} e^{-\zeta\omega_n t} \sin(\omega_n \beta t)}}$$

where $\beta = \sqrt{1 - \zeta^2}$,

$0 < \zeta < 1.$



Time Response to Step Input

$$R(s) = \frac{1}{s}$$

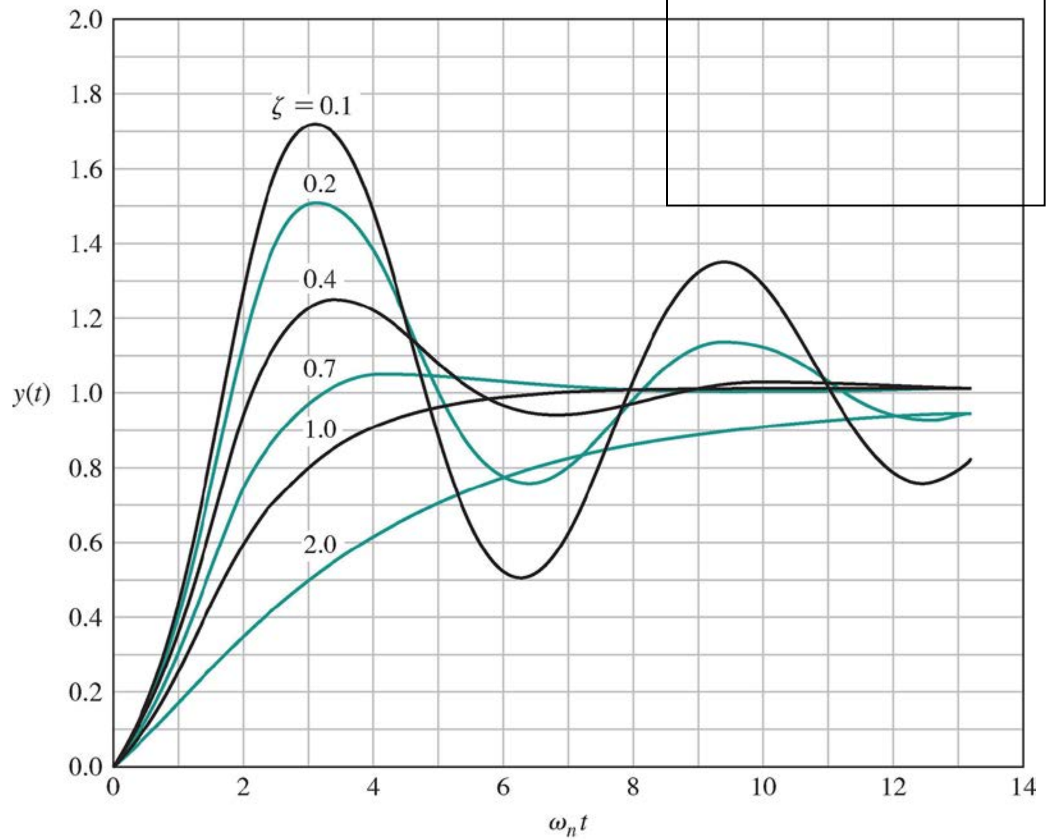
$$Y(s) = \frac{{\omega_n}^2}{s^2 + 2\zeta\omega_n s + {\omega_n}^2}\frac{1}{s}$$

$$\boxed{\boldsymbol{y(t) = 1 - \frac{1}{\beta} e^{-\zeta\omega_n t} \sin(\omega_n \beta t + \theta)}}$$

where $\beta = \sqrt{1-\zeta^2}$,

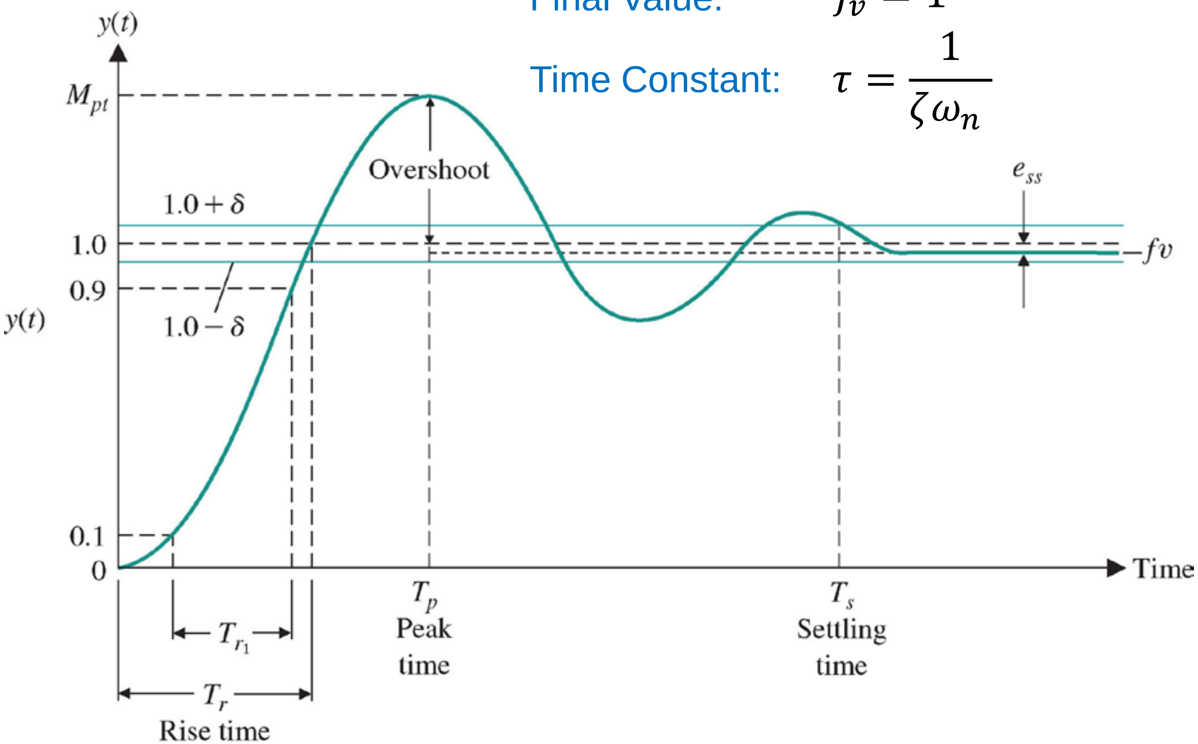
$\theta = \cos^{-1}\zeta$

$0 < \zeta < 1.$



Standard Performance Measures

Standard performance measures are often defined in terms of the **unit step response** of the closed-loop system.



Peak Value: M_{pt}

Final Value: $f_v = 1$

Time Constant: $\tau = \dfrac{1}{\zeta\omega_n}$

Peak Time:

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$

Rise Time:

$$T_{r1} = \frac{2.16\zeta + 0.60}{\omega_n} \quad (0.3 < \zeta < 0.8)$$

2% Settling Time:

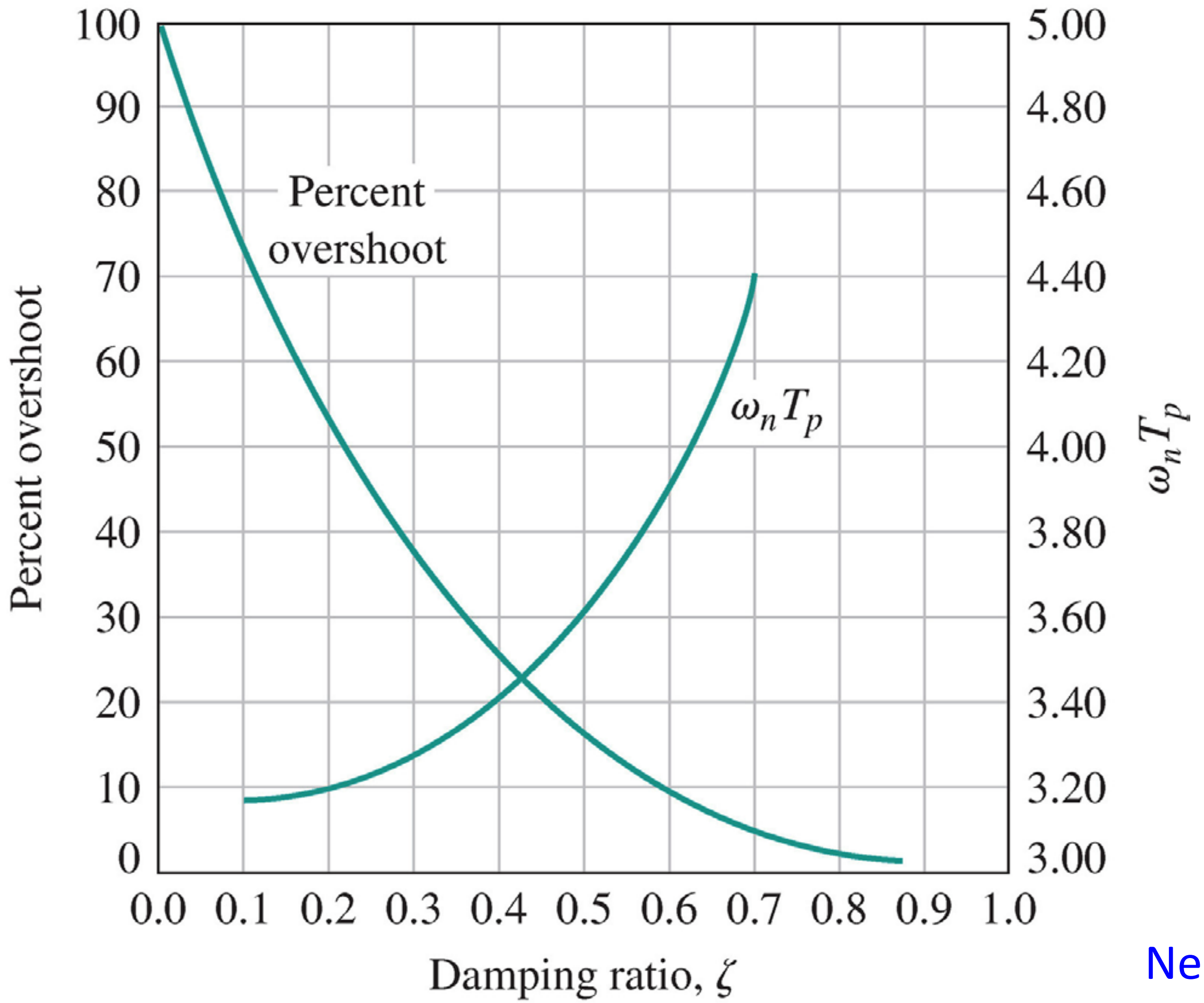
$$\boxed{T_s \cong \frac{4}{\zeta\omega_n}}$$

Percent Overshoot:

$$\boxed{P.O. = 100e^{-\zeta\pi/\sqrt{1-\zeta^2}}}$$

$$P.O. = \frac{M_{pt-fv}}{fv} \times 100\%$$

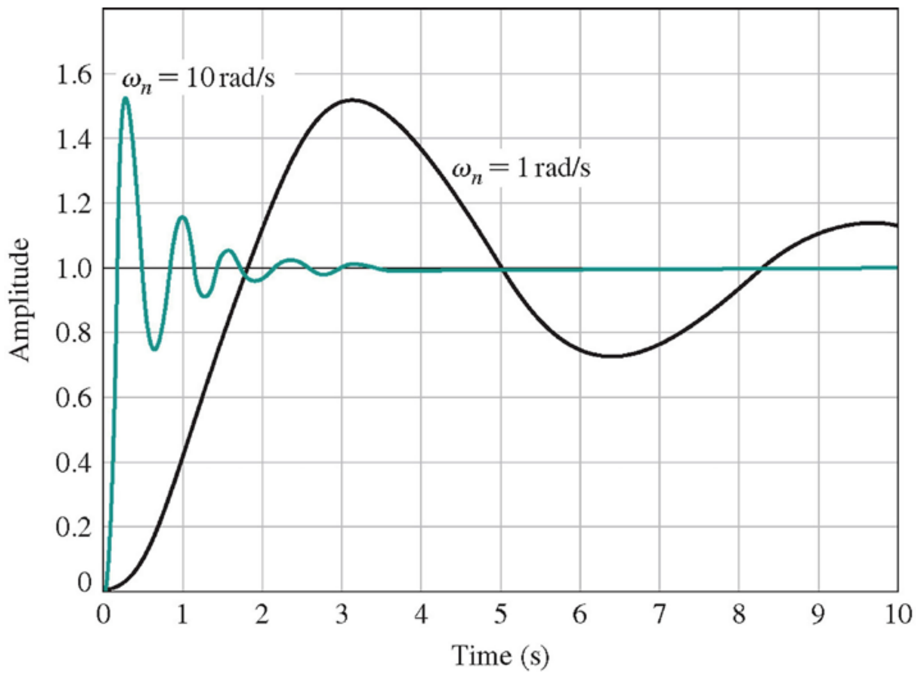
P.O. and Normalized Peak Time vs. ζ



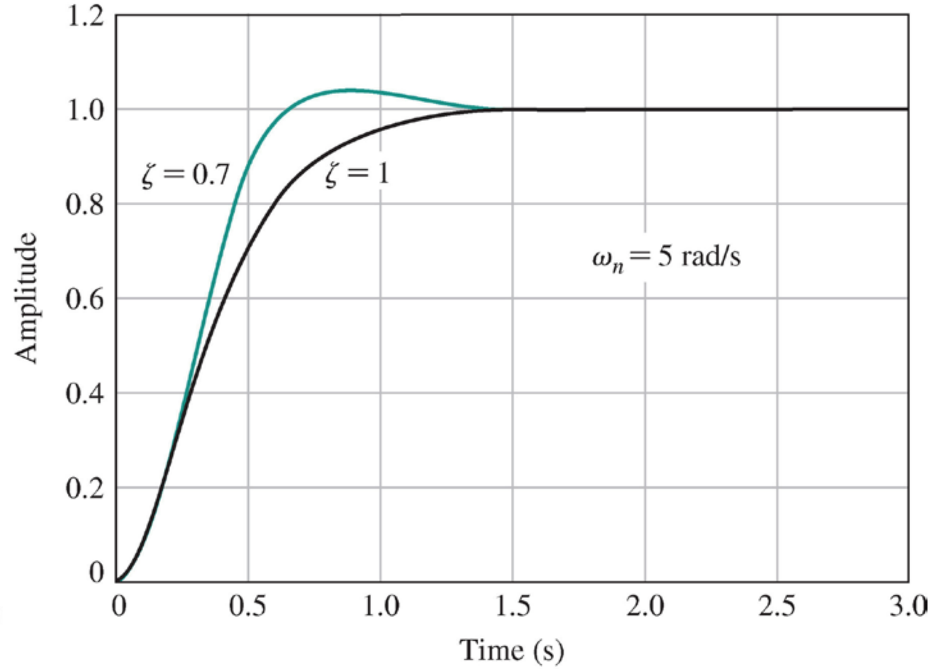
Need Compromise!

Effects of ω_n and ζ on The Step Response

with ζ =0.2,
different ω_n



with $\omega_n = 5$,
different ζ



Peak Time:

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$

Rise Time:

$$T_{r1} = \frac{2.16\zeta + 0.60}{\omega_n} \quad (0.3 < \zeta < 0.8)$$

2% Settling Time:

$$\boxed{T_s = \frac{4}{\zeta\omega_n}}$$

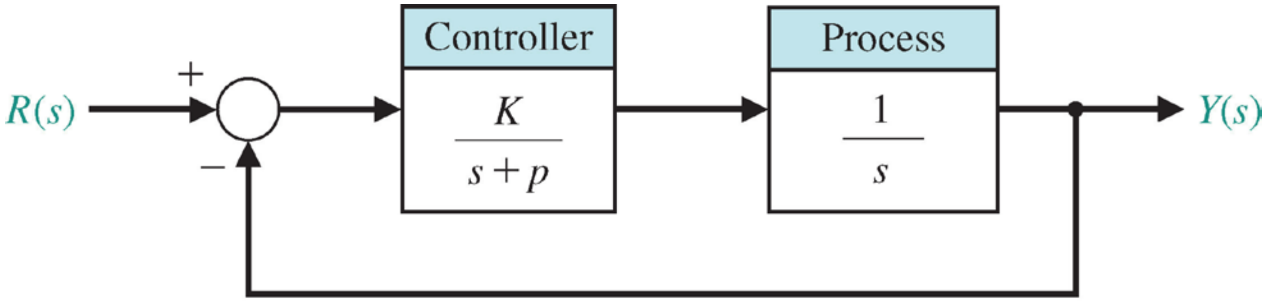
Percent Overshoot:

$$\boxed{P.O. = 100e^{-\zeta\pi/\sqrt{1-\zeta^2}}}$$

Example 14.1

Consider the following system, select gain K and the parameter p so that the time-domain specifications to a unit step input are satisfied.

- ***Specifications**: 2% settling time $T_s \le 4$ s; and percent overshoot $P.O. \le 5\%$.*



Step 1. Transfer function:

$$T(s) = \frac{K}{s^2 + ps + K} \qquad \left(= \frac{{\omega_n}^2}{s^2 + 2\zeta\omega_n s + {\omega_n}^2}\right)$$

$$2\zeta\omega_n = p, \qquad {\omega_n}^2 = K$$

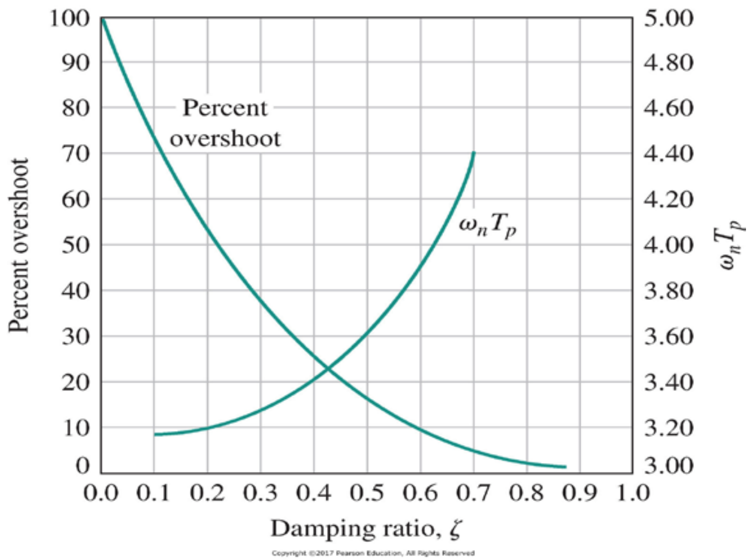
Step 2. To satisfy settling time requirement:

$$\frac{4}{\zeta\omega_n} \le 4 \longrightarrow \zeta\omega_n \ge 1$$

2% Settling Time:

$$\boxed{T_s = \frac{4}{\zeta\omega_n}}$$

Step 3. To satisfy the P.O. requirement:



Percent Overshoot:

$$\boxed{P.O. = 100e^{-\zeta\pi/\sqrt{1-\zeta^2}}}$$

$$P.O. \le 5\% \longrightarrow \zeta \ge 0.69$$

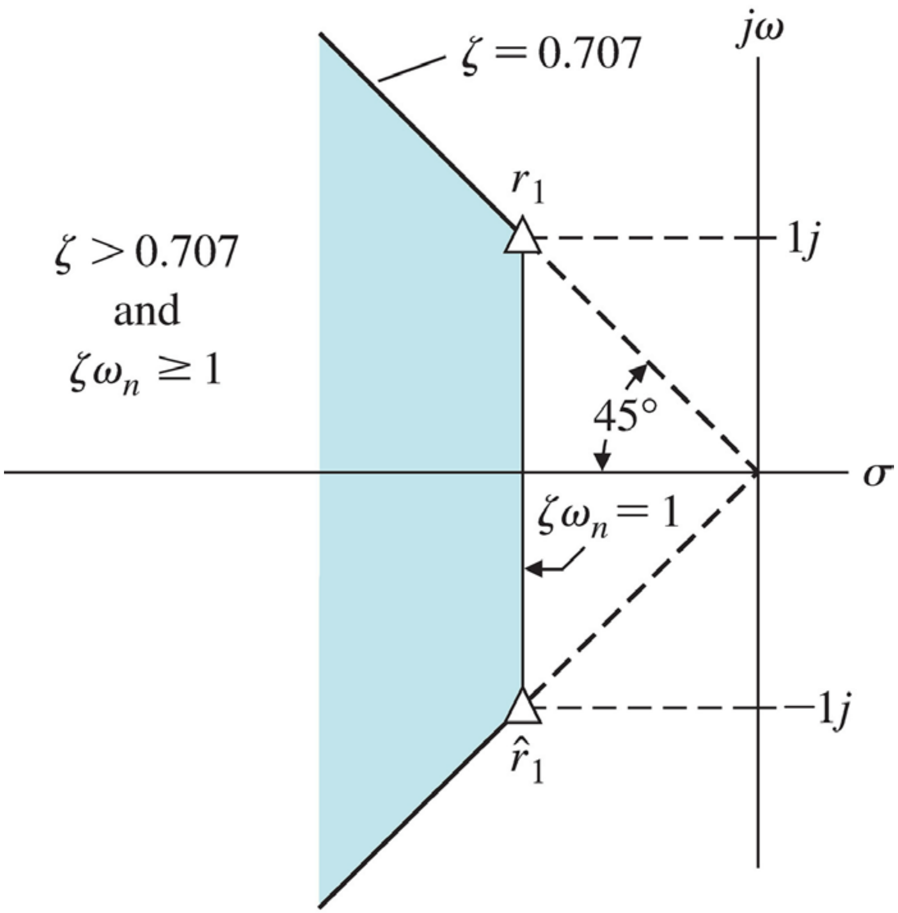
Step 4. Choose suitable values:

Can choose $\zeta\omega_n = 1$

$$\zeta = 0.707 = \frac{1}{\sqrt{2}}$$

$$\longrightarrow \omega_n = \sqrt{2} \longrightarrow \begin{matrix} p = 2 \\ K = 2 \end{matrix}$$

Specifications and Root Locations



$$T(s) = \frac{K}{s^2 + 2s + 2}$$

Poles: $-1 \pm j1$

$$T(s) = \frac{{\omega_n}^2}{s^2 + 2\zeta\omega_n s + {\omega_n}^2}$$

Poles:

$$p_{1,2} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$

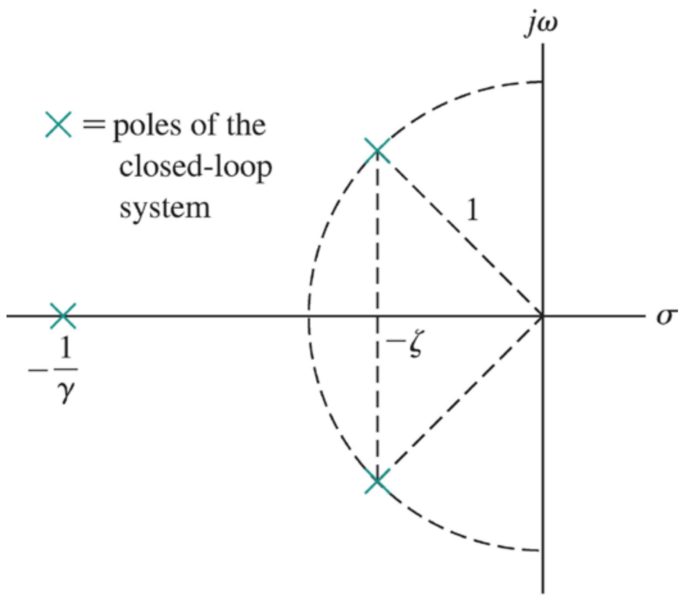
Xi'an Jiaotong-Liverpool University 西交利物浦大学

Effects of A Third Pole

Assume $\omega_n = 1$, Consider a system with two complex poles and an additional pole

$$T(s) = \frac{1}{(s^2 + 2\zeta\omega_n s + 1)(\gamma s + 1)}$$

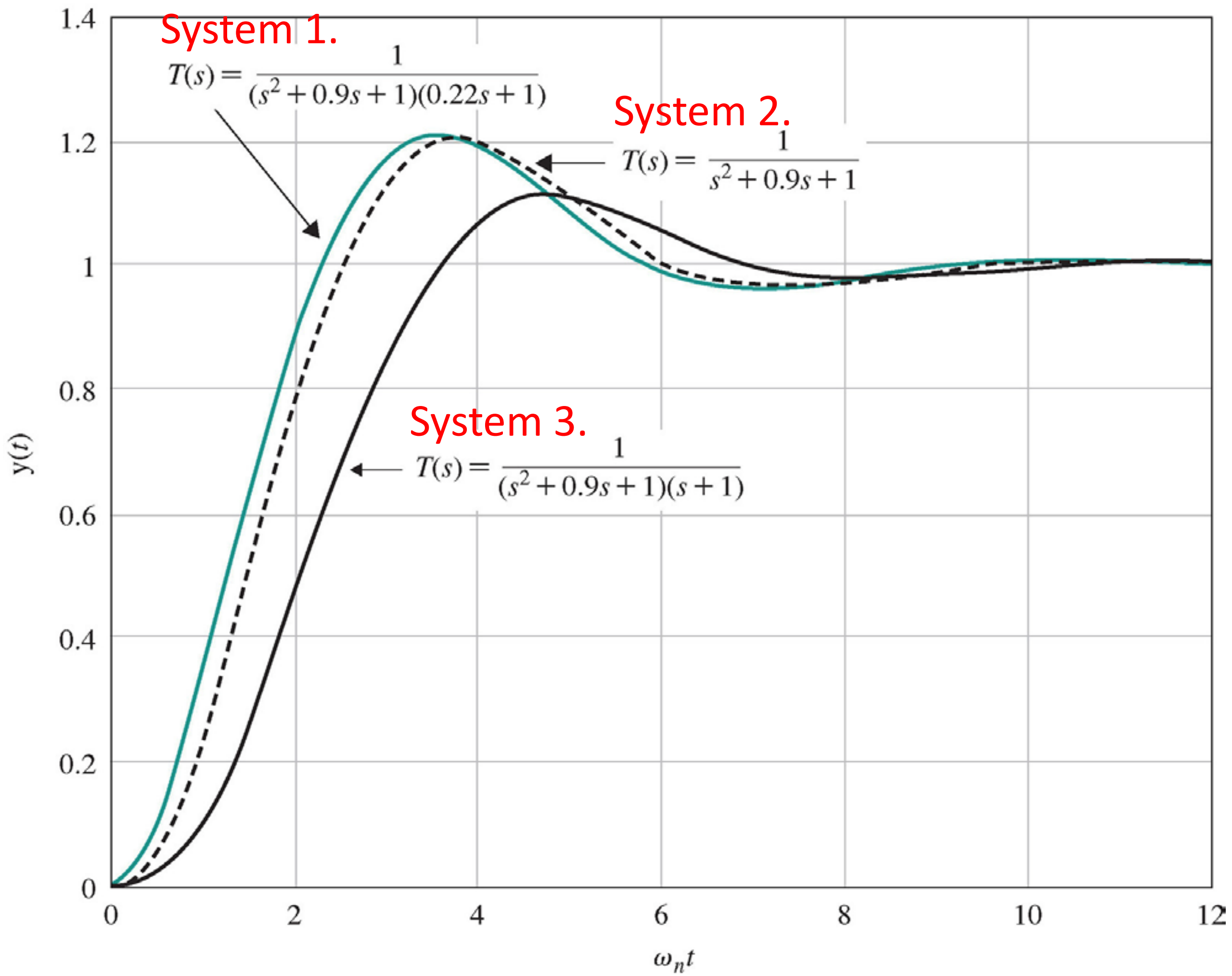
The time response of a third-order system can be approximated by the **dominant roots** of the second-order system as long as the real part of the dominant roots is less than one tenth of the real part of the third pole.



$$\left|{}^{1}/_{\gamma}\right| \geq 10|\zeta\omega_n|$$

NOTE: DC gain $T(0)$ should be kept the same after approximation.

Example 14.2



System 1 can be approximated by system 2, while system 3 can NOT!

Effects of A Finite Zero

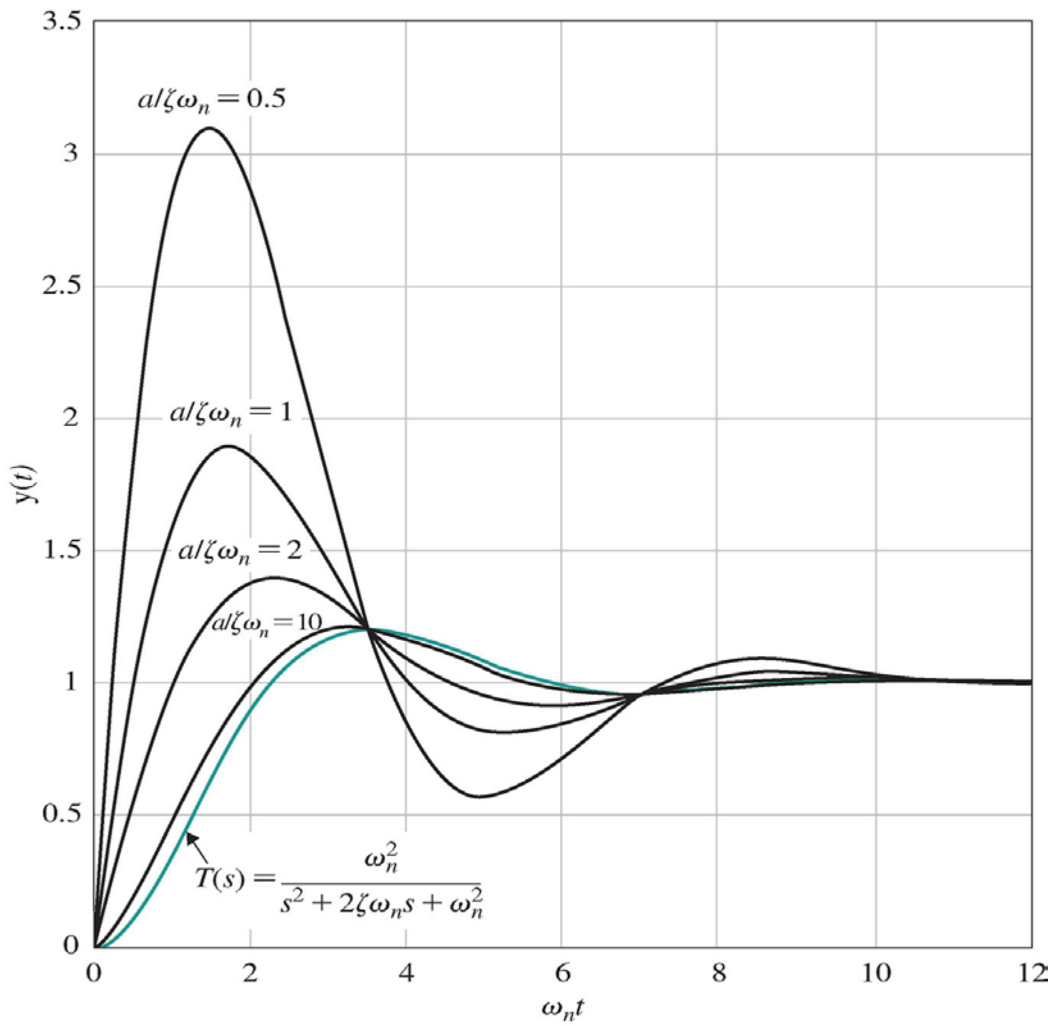
Consider a system:

$$T(s) = \frac{\frac{{\omega_n}^2}{a}(s+a)}{s^2 + 2\zeta\omega_n s + {\omega_n}^2}$$

If $a \gg \zeta\omega_n$:

the system can be simplified as:

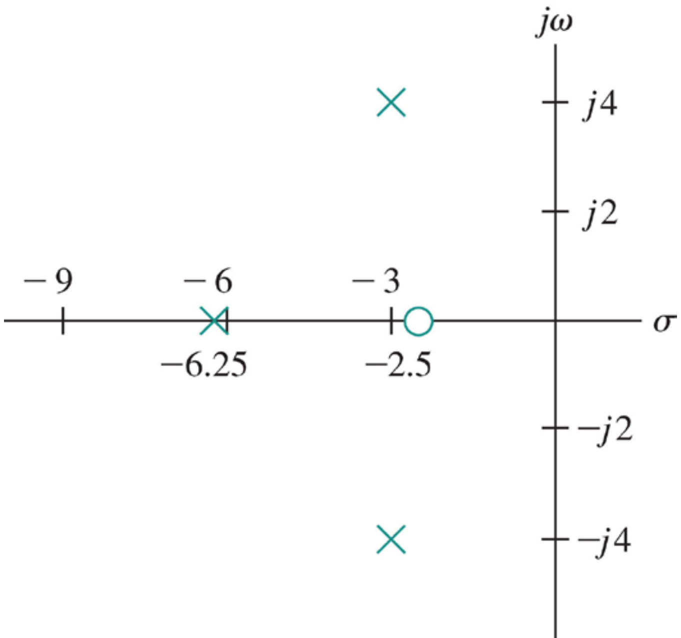
$$T(s) \approx \frac{{\omega_n}^2}{s^2 + 2\zeta\omega_n s + {\omega_n}^2}$$



Example 14.3

$$T(s) = \frac{1.6(s+2.5)}{(s^2+6s+25)(0.16s+1)}$$

$$T(s) = \boxed{\frac{\frac{\omega_n{}^2}{a}(s+a)}{(s^2+2\zeta\omega_n s+\omega_n{}^2)(1+\tau s)}}$$



$$\zeta\omega_n = 3, a = 2.5, \tau = 0.16$$

For this system, zero and third pole can **NOT** be neglected!

For the actual third-order system:

Ts=1.6s, P.O.=38%

For the second-order system: $T(s) = \frac{25}{s^2+6s+25}$

Ts=1.33s, P.O.=9.5%

Quiz 14.1

Consider the following system, can we neglect the effects of third pole? If yes, obtain approximated transfer function and estimate P.O.

$$T(s) = \frac{Y(s)}{R(s)} = \frac{2500}{(s + 20)(s^2 + 10s + 125)}$$

Quiz 14.2

For a second order system, determine the root locations in s-plane which satisfies:

1. 10% < P.O. < 20%
2. Settling time < 0.6.

Thank You！