西交利物浦大学 Xi'an Jiaotong-Liverpool University

**EEE108 Electromagnetism and Electromechanics**

# Lecture 9

## Biot-Savart Law
## Ampere's Law
## Gauss's Law for Magnetism


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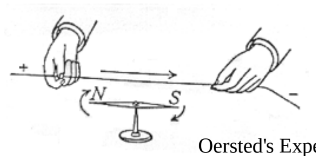
## The Big Step Forward

In year 1820 Oersted, a Danish physicist, realised that current flowing in a wire made the needle of a compass swing. The direction depends on the direction of the current:




Oersted's Experiment

Hans Christian Oersted (1777–1851). Besides his work in electricity and magnetism, Oersted was the first to prepare pure metallic aluminum (1825).

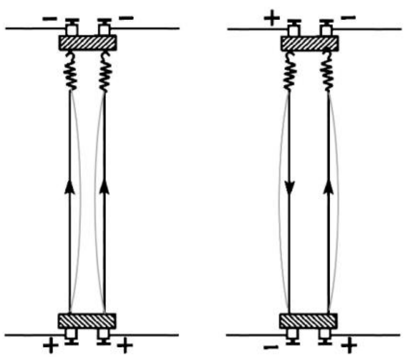
BIG discovery: proves that electricity and magnetism are related!!



## Ampere's Parallel Wire Experiments

Soon after, Ampere's experiment with parallel wires carrying current

- If currents are parallel, wires attract
- If anti-parallel, wires repel





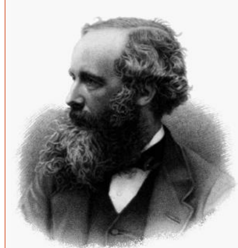

## A Treatise on Electricity and Magnetism

In 1873, A book "Treatise on Electricity and Magnetism", written by James Clerk Maxwell was published.

In the book, Maxwell unites the discoveries of Gauss, Coulomb, Oersted, Ampere, Faraday, and others into four elegantly constructed mathematical equations, now known as **Maxwell's Equations**, that relate the electric and magnetic fields to their sources, charge density and current density.

Gauss's law
Gauss's law for magnetism
Ampère's law
Faraday's law

**James Clerk Maxwell**



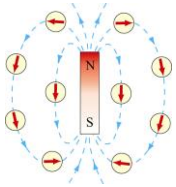
James Clerk Maxwell (1831–1879)
Scottish theoretical physicist and mathematician

## Sources of Magnetic Fields

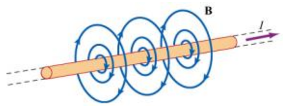
How are magnetic fields created?

Two sources

1. Permanent magnets




2. Electric current




From Oersted's observations, it was deduced that an electric current produces a magnetic field as it flows through a wire.

$I$ ⇔ ? **B**



## Today

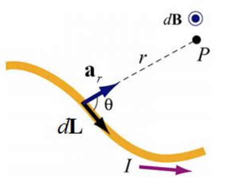
- Biot-Savart Law
- Ampere's Law
- Gauss's Law for Magnetism



## Biot-Savart Law

French scientists Jean Biot and Felix Savart arrived at an expression that results the magnetic flux density **B** at a point in space to the current $I$ that generates **B,** known as the Biot-Savart Law .

**B** ⇔ $I$




The Biot - Savart Law states that the differential magnetic flux density $d\mathbf{B}$ generated by a steady current $I$ flowing through a different length $d\mathbf{L}$ is given by

$$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{Id\mathbf{L}\times\mathbf{a}_r}{r^2}$$

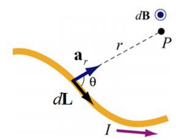
Adding up these contributions to find the magnetic field at the point $P$ requires integrating over the current source :

$$\boxed{\mathbf{B} = \int_l d\mathbf{B} = \frac{\mu_0 I}{4\pi}\int_l \frac{d\mathbf{L}\times\mathbf{a}_r}{r^2}}$$



## Biot-Savart Law

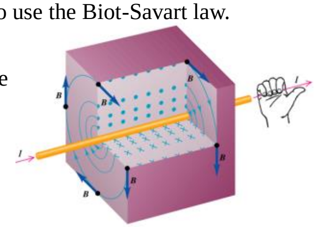
$$\mathbf{B} = \frac{\mu_0 I}{4\pi}\int_l \frac{d\mathbf{L}\times\mathbf{a}_r}{r^2}$$




The integral is a vector integral, which means that the expression for **B** is really three integrals, one for each component of **B**.

The vector nature of this integral appears in the cross product. Understanding how to evaluate this cross product and then perform the integral will be the key to learning how to use the Biot-Savart law.

Right-hand rule for the magnetic field due to a current element:
Point the thumb of the right hand in the direction of the current. The four fingers curl around the current element in the direction of the magnetic field lines.



$$\boxed{\mathbf{B} = \frac{\mu_0 I}{4\pi}\int_l \frac{d\mathbf{L}\times\mathbf{a}_r}{r^2}}$$

## B and H

| **B** | **H** (M) |
| --- | --- |
| • Magnetic flux density (vector) | • Magnetic field intensity (vector) |
| • Magnetic field | • H - field |
| • B - field | |

$\mathbf{B} = \mu\mathbf{H}$ or $\mathbf{B} = \mu_0\mathbf{H}$

$\mu = \mu_r\mu_0$, $\mu$ : magnetic permeability, $\mu_r$ : relative permeability

$\mu_0$ : permeability of free space, $= 4\pi\times10^{-7}$ H/m

$\Updownarrow$

$\mathbf{D} = \varepsilon\mathbf{E}$ or $\mathbf{D} = \varepsilon_0\mathbf{E}$

$\varepsilon = \varepsilon_r\varepsilon_0$, $\varepsilon$ : electric permittivity, $\varepsilon_r$ : relative permittivity

$\varepsilon_0$ : permittivity of free space, $= 8.85\times10^{-12}$ $\mathrm{C^2/N\cdot m^2} \cong \frac{1}{36\pi}\times10^{-9}$ F/m



## Biot-Savart Law

$$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{Id\mathbf{L}\times\mathbf{a}_r}{r^2} \quad\Rightarrow\quad \mathbf{B} = \int_l d\mathbf{B} = \frac{\mu_0 I}{4\pi}\int_l \frac{d\mathbf{L}\times\mathbf{a}_r}{r^2}$$

The Biot - Savart Law can also be written by using the magnetic field intensity **H** :

$$d\mathbf{H} = \frac{1}{4\pi}\frac{Id\mathbf{L}\times\mathbf{a}_r}{r^2} \quad \text{or} \quad \mathbf{H} = \int_l d\mathbf{H} = \frac{I}{4\pi}\int_l \frac{d\mathbf{L}\times\mathbf{a}_r}{r^2}$$

with $\mathbf{B} = \mu\mathbf{H}$ in free space $\boxed{\mathbf{B} = \frac{\mu_0 I}{4\pi}\int_l \frac{d\mathbf{L}\times\mathbf{a}_r}{r^2}}$



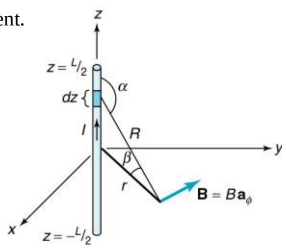
## Biot-Savart Law

### Magnetic Field of a Straight Wire

Determining the magnetic field due to a line current.

Consider the magnetic field only at distances *r* from the midpoint of and perpendicular to the current in order to simplify the integral. The differential contribution to the field due to a current element is:

$$\boxed{d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{Id\mathbf{L}\times\mathbf{a}_r}{r^2}}$$




(a)

Setting up the problem to take advantage of symmetry

$$d\mathbf{B} = \mathbf{a}_\phi \frac{\mu_0 I}{4\pi}\frac{dz}{R^2}\sin\alpha$$

where $R = \sqrt{r^2+z^2}$ and

$\sin\alpha = \sin(\beta+\pi/2) = \cos\beta = r/R$

The total field from all current elements is : $\mathbf{B} = \int_{-L/2}^{L/2}\frac{\mu_0 I}{4\pi}\frac{r\,dz}{(r^2+z^2)^{3/2}}\mathbf{a}_\phi$



## Biot-Savart Law

### Magnetic Field of a Straight Wire

By $\int \frac{dz}{(r^2\pm z^2)^{3/2}} = \frac{z}{r^2\sqrt{r^2\pm z^2}}$ we have

$$\mathbf{B} = \int_{-L/2}^{L/2}\frac{\mu_0 I}{4\pi}\frac{r\,dz}{(r^2+z^2)^{3/2}}\mathbf{a}_\phi = \mathbf{a}_\phi\frac{\mu_0 Ir}{4\pi}\left[\frac{z}{r^2\sqrt{r^2+z^2}}\right]_{-L/2}^{L/2}$$

$$= \frac{\mu_0 I}{2\pi r}\frac{L/2}{\sqrt{r^2+(L/2)^2}}\mathbf{a}_\phi$$

When $L\to\infty$, we have :

$$\mathbf{B} = \frac{\mu_0 I}{2\pi r}\mathbf{a}_\phi$$

This result is a very useful expression. It states that in the neighborhood of a linear conductor carrying a current *I*, the induced magnetic field forms concentric circles around the wire, and its intensity is directly proportional to *I* and inversely proportional to the distance *r*.

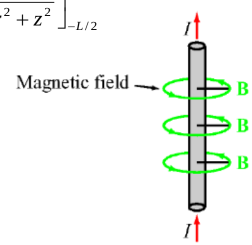



Illustration of the fact that the magnetic field about an infinite line current is circumferentially directed.

## Biot-Savart Law

### Magnetic Field of a Circular Loop

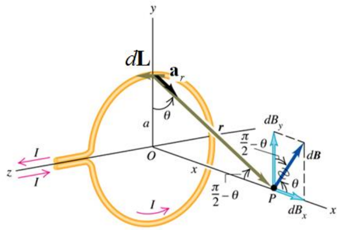
A circular loop of radius $a$ in the $y$-$z$ plane carries a steady current $I$. What is the magnetic field at a point $P$ on the axis of the loop, at a distance $x$ from the center?

$d\mathbf{L}$ and $\mathbf{a}_r$ are perpendicular and the direction of the field $d\mathbf{B}$ caused by this particular element $d\mathbf{L}$ lies in the $x$-$y$ plane.

As $r^2 = x^2 + a^2$

The magnitude $dB$ of the field due to element $d\mathbf{L}$ is:

$$dB = \frac{\mu_0 I}{4\pi}\frac{dL}{x^2 + a^2}$$




$$\boxed{d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{Id\mathbf{L}\times\mathbf{a}_r}{r^2}}$$



## Biot-Savart Law

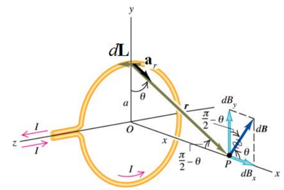
### Magnetic Field of a Circular Loop

From $dB = \frac{\mu_0 I}{4\pi}\frac{dL}{x^2 + a^2}$

The component of the vector $d\mathbf{B}$ are:

$$dB_x = dB\cos\theta = \frac{\mu_0 I}{4\pi}\frac{dL}{x^2 + a^2}\frac{a}{\sqrt{x^2 + a^2}}$$

$$dB_y = dB\sin\theta = \frac{\mu_0 I}{4\pi}\frac{dL}{x^2 + a^2}\frac{x}{\sqrt{x^2 + a^2}}$$



The situation has rotational symmetry about $x$-axis, so there cannot be a component of the total field perpendicular to $x$-axis and only the $x$-components survive:

$$B_x = \int_0^{2\pi} dB_x = \frac{\mu_0 Ia}{4\pi\left(x^2 + a^2\right)^{3/2}}\int_0^{2\pi} dL = \frac{\mu_0 Ia}{4\pi\left(x^2 + a^2\right)^{3/2}}2\pi a$$

$$B_x = \frac{\mu_0 Ia^2}{2\left(x^2 + a^2\right)^{3/2}}$$

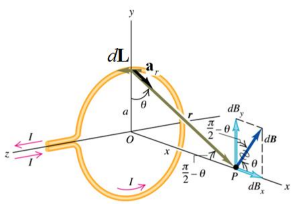


## Biot-Savart Law

### Magnetic Field of a Circular Loop

The direction of the magnetic field on the axis of a current-carrying loop is given by right-hand rule:

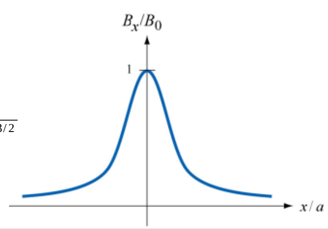
Curl the fingers of the right hand around the loop in the direction of the current, the right thumb points in the direction of the field.



$$\boxed{B_x = \frac{\mu_0 Ia^2}{2\left(x^2 + a^2\right)^{3/2}}}$$

If we let $B_0 = \frac{\mu_0 I}{2a}$, then $\frac{B_x}{B_0} = \frac{1}{\left[\left(\frac{x}{a}\right)^2 + 1\right]^{3/2}}$

$B_0$ is the magnetic flux density at $x = 0$.






## Biot-Savart Law

### Magnetic Field of a Circular Loop

$$\boxed{B_x = \frac{\mu_0 Ia^2}{2\left(x^2 + a^2\right)^{3/2}}}$$

Suppose that instead of the single loop, there is a coil consisting of $N$ loops, all with the same radius. The total field is $N$ times the field of a single loop:

$$B_x = \frac{\mu_0 NIa^2}{2(x^2 + a^2)^{3/2}}$$

The maximum value of the field is at $x = 0$, the centre of the loop or coil:

$$B_x = \frac{\mu_0 NI}{2a}$$

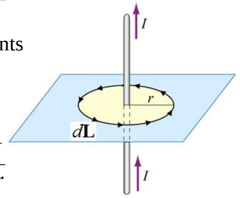
## Ampere's Law

Moving charges or currents are the source of magnetism.
What is the relationship between moving charges or currents and magnetism?

Consider again the magnetic field caused by a long, straight conductor carrying a current $I$ in free space.

The field at a distance $r$ from the conductor is : $B = \frac{\mu_0 I}{2\pi r}$

and the magnetic field lines are circles centered on the conductor.




Take the line integral of $\mathbf{B}$ around one such circle with radius $r$

$$\oint \mathbf{B} \bullet d\mathbf{L} = B \oint dL = \frac{\mu_0 I}{2\pi r}(2\pi r) = \mu_0 I$$

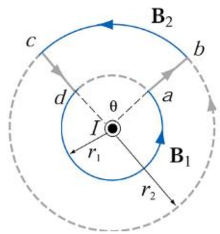
The line integral is independent of the radius of the circle.
The result only depends on $\mu_0$ and the current passing through the area bounded by the circle, or called ***Amperian loop***.

discovered by André-Marie Ampère in 1826



## Ampere's Law

Consider a more complicated closed path *abcda*.
The line integral of the magnetic flux density around the contour *abcda* is :




$$\oint_{abcda} \mathbf{B} \bullet d\mathbf{L} = \oint_{ab} \mathbf{B} \bullet d\mathbf{L} + \oint_{bc} \mathbf{B} \bullet d\mathbf{L} + \oint_{cd} \mathbf{B} \bullet d\mathbf{L} + \oint_{da} \mathbf{B} \bullet d\mathbf{L}$$
$$= 0 + B_2(r_2\theta) + 0 + B_1[r_1(2\pi - \theta)]$$

where the length of arc $bc = r_2\theta$,
and arc $da = r_1(2\pi - \theta)$

$B_1 = \frac{\mu_0 I}{2\pi r_1}$ and $B_2 = \frac{\mu_0 I}{2\pi r_2}$, the expression becomes

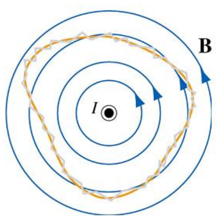
$$\oint_{abcda} \mathbf{B} \bullet d\mathbf{L} = \frac{\mu_0 I}{2\pi r_2}(r_2\theta) + \frac{\mu_0 I}{2\pi r_1}[r_1(2\pi - \theta)] = \frac{\mu_0 I}{2\pi}\theta + \frac{\mu_0 I}{2\pi}(2\pi - \theta) = \mu_0 I$$



## Ampere's Law

In cylindrical coordinates $(r, \varphi, z)$ with current flowing in the $+z$-axis, the magnetic flux density is given by :

$$\mathbf{B} = \frac{\mu_0 I}{2\pi r}\mathbf{a}_\varphi$$




An arbitrary length element in the cylindrical coordinates :

$$d\mathbf{L} = dr\mathbf{a}_r + rd\varphi\mathbf{a}_\varphi + dz\mathbf{a}_z$$

Then

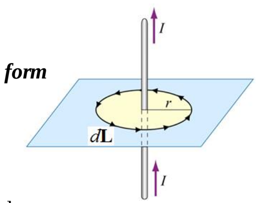
$$\oint_{\substack{\text{closed}\\ \text{path}}} \mathbf{B} \bullet d\mathbf{L} = \oint_{\substack{\text{closed}\\ \text{path}}} \frac{\mu_0 I}{2\pi r} r d\varphi = \frac{\mu_0 I}{2\pi} \oint_{\substack{\text{closed}\\ \text{path}}} d\varphi = \frac{\mu_0 I}{2\pi}(2\pi) = \mu_0 I$$



## Ampere's Law

The line integral of $\oint \mathbf{B} \bullet d\mathbf{L}$ around any closed Amperian loop is proportional to $I_{enc}$, the current encircled by the loop.

$$\oint \mathbf{B} \bullet d\mathbf{L} = \mu_0 I_{enc} \quad \Leftarrow \quad \textit{Ampere's law integral form}$$




***Differential form***

By the Kelvin-Stokes theorem, this equation can also be written in a differential form :

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} \quad \Leftarrow \quad \textit{Ampere's law differential form}$$

where $\mathbf{J}$ is the current density through the surface enclosed by the Amperian loop.

## Applications of Ampere's Law

Ampere's law in magnetism is analogous to Gauss's law in electrostatics.

In order to apply them, the system must possess certain symmetry.

| | | |
|---|---|---|
| Biot-Savart Law | $\boxed{\mathbf{B} = \frac{\mu_0 I}{4\pi}\int_l \frac{d\mathbf{L}\times\mathbf{a}_r}{r^2}}$ | General current source<br>Ex: finite wire |
| Ampere's Law | $\oint \mathbf{B}\bullet d\mathbf{L} = \mu_0 I_{enc}$ | Current source has certain symmetry<br>Ex: infinite wire |

***Ampere's law is applicable to the following current configurations:***

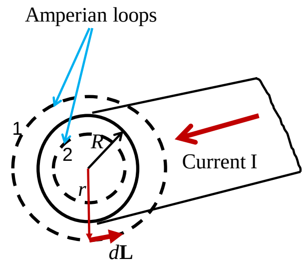
1. Infinitely long straight wires carrying a steady current $I$
2. Infinitely large sheet of thickness $b$ with a current density $J$
3. Infinite solenoid
4. Toroid



## Applications of Ampere's Law
### Field Inside and Outside a Current-Carrying Wire

Consider a long straight wire of radius $R$ carrying a current $I$ of uniform current density.

Find the magnetic field everywhere.




(i) ***Outside the wire*** where $r \geq R$

The Amperian loop (circle 1) completely encircles the current.

Applying Ampere's law :

$$\oint \mathbf{B}\bullet d\mathbf{L} = B\oint dl = B(2\pi r) = \mu_0 I$$

So $B = \frac{\mu_0 I}{2\pi r}$



## Applications of Ampere's Law
### Field Inside and Outside a Current-Carrying Wire

(ii) ***Inside the wire*** where $r < R$

The amount of current encircled by the Amperian loop (circle 2) :

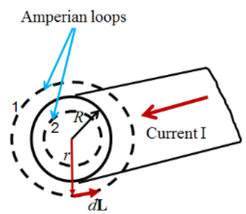
$$I_{enc} = \left(\frac{\pi r^2}{\pi R^2}\right) I$$

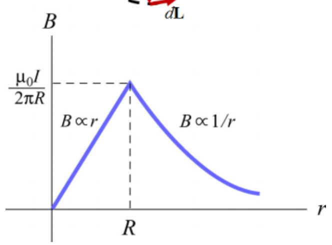
Then, applying Ampere's law :

$$\oint \mathbf{B}\bullet d\mathbf{L} = B(2\pi r) = \mu_0 I\left(\frac{\pi r^2}{\pi R^2}\right)$$

So $B = \frac{\mu_0 I r}{2\pi R^2}$ $\quad r < R$

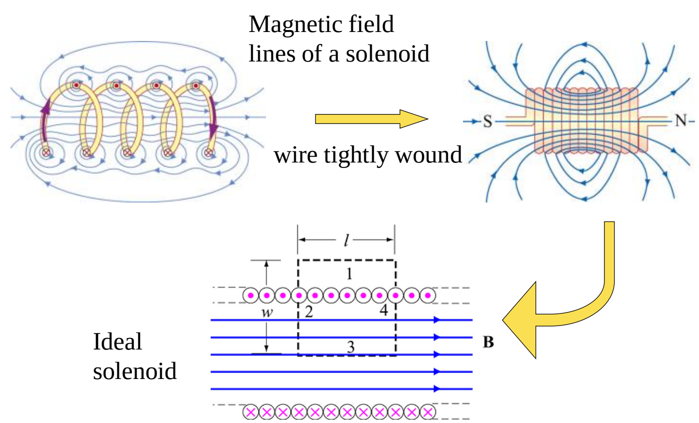
$$\boxed{B = \frac{\mu_0 I}{2\pi r} \quad r \geq R}$$






## Applications of Ampere's Law
### Solenoid



## Applications of Ampere's Law

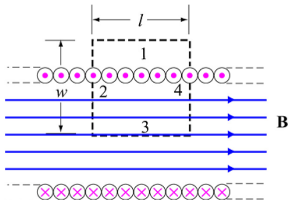
### Solenoid

The Amperian loop : 1234

The line integral of $\mathbf{B}$ along the loop :

$$\oint \mathbf{B}\bullet d\mathbf{L} = \int_1 \mathbf{B}\bullet d\mathbf{L} + \int_2 \mathbf{B}\bullet d\mathbf{L} + \int_3 \mathbf{B}\bullet d\mathbf{L} + \int_4 \mathbf{B}\bullet d\mathbf{L}$$

$$= 0 \qquad +0 \qquad +Bl \qquad +0$$




ApplyingAmpers's law :

$$\oint \mathbf{B}\bullet d\mathbf{L} = Bl = \mu_0 NI$$

$$B = \frac{\mu_0 NI}{l} = \mu_0 nI \quad \text{or} \quad B = \mu_0 K$$

where $n = N/l$ -- the number of turns per unit length

$k = nI$ -- current per unit length, or the surface current



## Applications of Ampere's Law

### Solenoid

Take a cross section of tightly packed loops at $x'$ with a thickness $dx'$.

The amount of current flowing through :

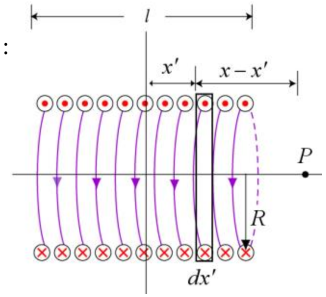
$dI = I(ndx') = I(N/l)dx'$

where $n = N/l$

The contribution to the magnetic field at $P$ due to this subset of loop :

$$dB_x = \frac{\mu_0 R^2}{2\left((x-x')^2+R^2\right)^{3/2}} dI$$

$$= \frac{\mu_0 R^2}{2\left((x-x')^2+R^2\right)^{3/2}} (nIdx')$$




Finite Solenoid

$$\boxed{B_x = \frac{\mu_0 NIa^2}{2(x^2+a^2)^{3/2}}}$$



## Applications of Ampere's Law

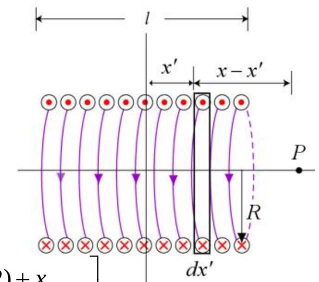
### Solenoid

Integrating over the entire length of the solenoid :

$$B_x = \frac{\mu_0 nIR^2}{2}\int_{-l/2}^{l/2} \frac{dx'}{\left((x-x')^2+R^2\right)^{3/2}}$$

$$= \frac{\mu_0 nIR^2}{2} \left. \frac{x'-x}{R^2\sqrt{(x-x')^2+R^2}} \right|_{-l/2}^{l/2}$$

$$= \frac{\mu_0 nI}{2}\left[\frac{(l/2)-x}{\sqrt{(x-l/2)^2+R^2}} + \frac{(l/2)+x}{\sqrt{(x+l/2)^2+R^2}}\right]$$




Ideal solenoid $\boxed{B = \frac{\mu_0 NI}{l} = \mu_0 nI}$

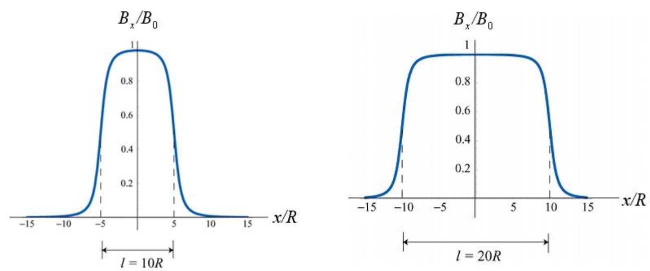


## Applications of Ampere's Law

### Solenoid

Let $B_0 = \mu_0 nI$ --the magnetic field of an infinite solenoid,

the $B_x/B_0$ is a function of $x/R$




The value of the magnetic field in the region $|x|<l/2$ is nearly uniform and approximately equal to $B_0$

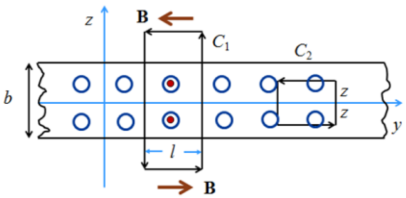
## Applications of Ampere’s Law
### Magnetic Field Due to an Infinite Current Sheet

Consider an infinitely large sheet of thickness $b$ lying in the $xy$ plane with a uniform current density $\mathbf{J} = J_0 \mathbf{a}_x$

Find the magnetic field everywhere.

**Solution**

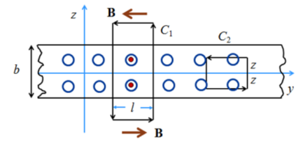
Take it as a set of parallel wires carrying currents in the $+x$-direction. The $z$-component vanishes after adding up the contributions from all wires.






## Applications of Ampere’s Law
### Magnetic Field Due to an Infinite Current Sheet

Applying Ampere’s law to find the magnetic field due to the current sheet:




For the field outside, the amount of current enclosed by $C_1$ is

$$I_{enc} = \iint \mathbf{J} \cdot d\,\mathbf{s} = J_0(b\ell)$$

Applying Ampere’s law

$$\oint \mathbf{B} \cdot d\mathbf{L} = B(2\ell) = \mu_0 I_{enc} = \mu_0 (J_0 b \ell)$$

or $\quad B = \mu_0 J_0 b / 2$

Note that the magnetic field outside the sheet is constant, independent of the distance from the sheet.

The magnetic field inside the sheet. The amount of current enclosed by path $C_2$ is: $I_{enc} = \iint \mathbf{J} \cdot d\,\mathbf{s} = J_0(2\,|\,z\,|\,\ell)$

$$\oint \mathbf{B} \cdot d\mathbf{L} = B(2\ell) = \mu_0 I_{enc} = \mu_0 J_0 (2\,|\,z\,|\,\ell)$$

or $\quad B = \mu_0 J_0\,|\,z\,|$

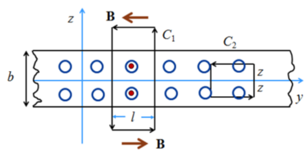
At $z = 0$, the magnetic field vanishes, as required by symmetry.



## Applications of Ampere’s Law
### Magnetic Field Due to an Infinite Current Sheet

The results can be summarized using the unit-vector notation as

$$\mathbf{B} = \begin{cases} -\dfrac{\mu_0 J_0 b}{2}\,\mathbf{a}_y, & z > b/2 \\ -\mu_0 J_0 z\,\mathbf{a}_y, & -b/2 < z < b/2 \\ \dfrac{\mu_0 J_0 b}{2}\,\mathbf{a}_y, & z < -b/2 \end{cases}$$



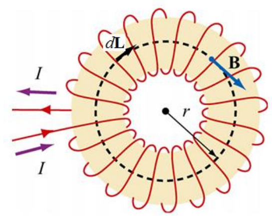



## Applications of Ampere’s Law
### Toroid

Consider a toroid which consists of $N$ turns.

Find the magnetic field everywhere.

Take a toroid as a solenoid wrapped around with its ends connected. Thus, the magnetic field is completely confined inside the toroid and the field points in the azimuthal direction (clockwise due to the way the current flows).




Using Ampere's law :

$$\oint \mathbf{B} \bullet d\mathbf{L} = B \oint dL = B(2\pi r) = \mu_0 N I$$

$$B = \frac{\mu_0 N I}{2\pi r}$$

Unlike the magnetic field of a solenoid, the magnetic field inside the toroid is non-uniform and decreases as $1/r$.

## Applying Ampere's Law

Procedure: 1. Identify regions in which to calculate $\mathbf{B}$ field

Get $\mathbf{B}$ direction by right hand rule

2. Choose Amperian loop : Symmetry

$\mathbf{B}$ is 0 or constant on the loop!

3. Calculate $\oint \mathbf{B} \bullet d\mathbf{L}$

4. Calculate current enclosed by the loop

5. Apply Ampere's law to solve for $\mathbf{B}$

$$\oint \mathbf{B} \bullet d\mathbf{L} = \mu_0 I_{enc}$$

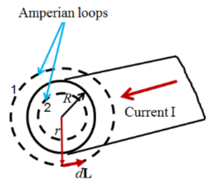
Apply Ampere's law to solve for $\mathbf{H}$

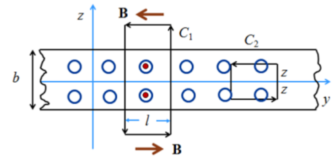
$$\oint \mathbf{H} \bullet d\mathbf{L} = I_{enc}$$



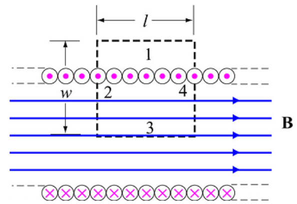
## Ampere's Law: $\oint \mathbf{B} \bullet d\mathbf{L} = \mu_0 I_{enc}$ $\oint \mathbf{H} \bullet d\mathbf{L} = I_{enc}$

Long Circular Symmetry

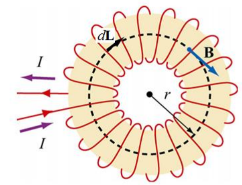



(Infinite) Current Sheet




Solenoid




Toroid

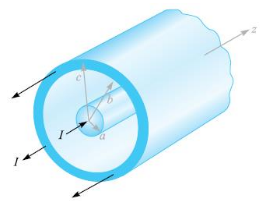


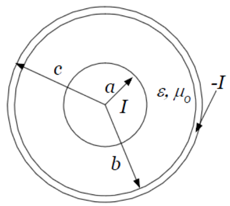
## Ampere's Law

### Example

The central conductor (of radius $a$) of a coaxial cable carries a current $I$. This current is returned in the outer cylindrical sheath (of inner and outer radii $b$ and $c$, respectively). The dielectric filling the space between the two concentric conductors has a permittivity $\varepsilon$ and a permeability of $\mu_0$.

Assuming a uniform current density carried by the cable, determine the magnetic field intensity as a function of the radius $r$ (for all $r > 0$).





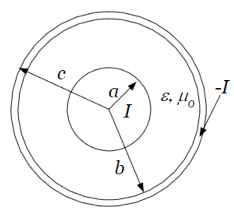

## Ampere's Law

### Example

From symmetry consideration, the magnetic field $H$ has only $H_\varphi$ component, and it depends on only the radial distance $r$.

From the Ampere's Law, we have

$$\oint_C \mathbf{H} \bullet d\mathbf{L} = 2\pi r H_\phi = I_{\text{enclosed}} \Rightarrow H_\phi = \frac{1}{2\pi r} I_{\text{enclosed}}$$



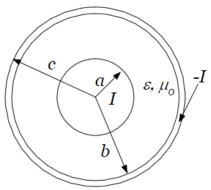

Assuming a uniform current through the cylinder of radius $a$ and also the cylindrical shell of inner and outer radii of $b$ and $c$, we will have the enclosed current in different regions as follows:

## Ampere's Law

### Example

$$H_\phi = \frac{1}{2\pi r} I_{\text{enclosed}}$$

$$= \frac{1}{2\pi r} \begin{cases} \dfrac{I}{\pi a^2}\left(\pi r^2\right) = I\left(\dfrac{r}{a}\right)^2 & 0 \le r \le a \\ I & a \le r \le b \\ I - \dfrac{I}{\pi c^2 - \pi b^2}\left(\pi r^2 - \pi b^2\right) = I\dfrac{\pi c^2 - \pi r^2}{\pi c^2 - \pi b^2} & b \le r \le c \\ I - I = 0 & r > c \end{cases}$$



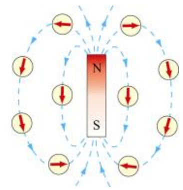

The dielectric filled between the inner conducting core and the outer conducting shell has no effect on the magnetic field intensity, and the magnetic field depends only on the current $I$ or its distribution.

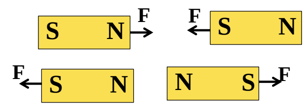
$\varepsilon$ does not need to be taken into account at all.



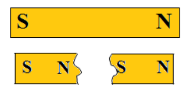
## Magnetic Field of Bar Magnet

- A bar magnet is a source of a magnetic field.
- The bar magnet consists of two poles, which are designated as the north (N) and the south (S).
- Magnetic fields are strongest at the poles.
- The magnetic field lines leave from the north pole and enter the south pole.
- The like poles repel each other while the opposite poles attract




### Bar Magnets Are Dipoles!




Two magnetic poles always come in a pair, or

Magnetic "monopoles" do not exist in isolation.



## Gauss's Law for Magnetism

**Recap**

$$\Phi_E = \oiint_S \mathbf{E} \bullet d\mathbf{s} = \frac{Q_{in}}{\varepsilon_0}$$

Electric flux is proportional to charge inside the volume enclosed by S

If the closed surface encloses an electric dipole, the total electric flux is zero as the total charge is zero.

Since the basic entity for magnetism is the magnetic dipole or magnetic monopoles do not exist, we can conclude:

The total magnetic flux through a closed surface is always zero.

$$\boxed{\oiint_S \mathbf{B} \bullet d\mathbf{s} = 0}$$

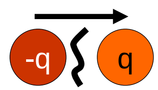
Gauss's law for magnetism in integral form



## Gauss's Law for Magnetism

### Magnetic Monopoles?

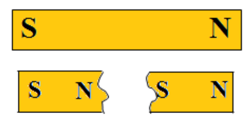
Electric Dipole




When cut:

2 monopoles (charges)

Magnetic Dipole




When cut: 2 dipoles

Magnetic monopoles do not exist in isolation

$$\oiint_S \mathbf{E} \bullet d\mathbf{s} = \frac{Q_{in}}{\varepsilon_0}$$

Gauss's Law

$$\oiint_S \mathbf{B} \bullet d\mathbf{s} = 0$$

Gauss's Law for Magnetism

## Gauss's Law for Magnetism

•Gauss's law for magnetism is one of Maxwell's equations.

•It states that the magnetic flux density **B** has divergence equal to zero. It is equivalent to the statement that magnetic monopoles do not exist.

•Rather than "magnetic charges", the basic entity for magnetism is the magnetic dipole. (Of course, if monopoles were ever found, the law would have to be modified.)

•Gauss's law for magnetism can be written in two forms, a differential form and an integral form. These forms are equivalent due to the divergence theorem.



## Gauss's Law for Magnetism

### Differential Form

The differential form for Gauss's law for magnetism is :

$$\boxed{\nabla \bullet \mathbf{B} = 0}$$

where **B** is the magnetic flux density, Wb/m$^2$.

**Recap**

If **A** is a vector

$$\nabla \bullet \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

Cartesian

$$\nabla \bullet \mathbf{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

Cylindrical

**A** is a vector,

$\nabla \bullet \mathbf{A}$(divergence of **A**) is a scalar



## Today

- Biot-Savart Law

$$d\mathbf{B} = \frac{\mu}{4\pi}\frac{Id\mathbf{L}\times\mathbf{a}_r}{r^2} \Rightarrow \mathbf{B} = \int_l d\mathbf{B} = \frac{\mu I}{4\pi}\int_l \frac{d\mathbf{L}\times\mathbf{a}_r}{r^2}$$

- Ampere's Law

$$\oint \mathbf{B}\bullet d\mathbf{L} = \mu I_{enc} \qquad \nabla\times\mathbf{B} = \mu\mathbf{J}$$

*Applicable to the following current configurations:*

1. Infinitely long straight wires carrying a steady current $I$
2. Infinitely large sheet of thickness $b$ with a current density $J$
3. Infinite solenoid
4. Toroid

- Gauss's Law for Magnetism — Magnetic monopoles do not exist.

$$\oiint_S \mathbf{B}\bullet d\mathbf{s} = 0 \qquad \nabla\bullet\mathbf{B} = 0$$



## Next

- Magnetic Flux and Magnetic Flux Density
- Magnetic Force and Magnetic Torque

Thanks for your attendance