Semiconductor Fundamentals – (III)

2.5 Boltzmann approximation & E_F, n, p

2.6 Carrier drift and diffusion

Material developed by Prof. C. Z. Zhao

Last lecture:

- Negative charges:
  - Conduction electrons (density = n) mobile
  - Ionized acceptor atoms (density = N_A^-) immobile
- Positive charges:
  - Holes (density = p) mobile
  - Ionized donor atoms (density = N_D^+) immobile

- The net charge density (C/cm^3) in a semiconductor is

$$\rho = q(p - n + N_D^+ - N_A^-)$$

- Law of Mass Action: $\boldsymbol{n \bullet p = n_i^2}$

质量作用定律

How to deduce the relationship between E_F and n/p?

2.5 Boltzmann approximation & E_F, n, p

- **Fermi function and Fermi level**
- Density of States
- Boltzmann Approximation
- Electron and hole Concentrations

Thermal Equilibrium

- No external forces are applied:
  - electric field = 0, magnetic field = 0
  - mechanical stress = 0
  - no light

- Dynamic situation in which every process is balanced by its inverse process
  - Electron-hole pair (EHP) generation rate = EHP recombination rate

- Thermal agitation → electrons and holes exchange energy with the crystal lattice and each other

  → Every energy state in the conduction band and valence band has a certain probability of being occupied by an electron

Statistical Thermodynamics: Fermi energy

- The Fermi energy, E_F, is the energy associated with a particle, which is in thermal equilibrium with the system of interest. The energy is strictly associated with the particle and does not consist even in part of heat or work. This same quantity is called the electro-chemical potential, m, in most thermodynamics texts.

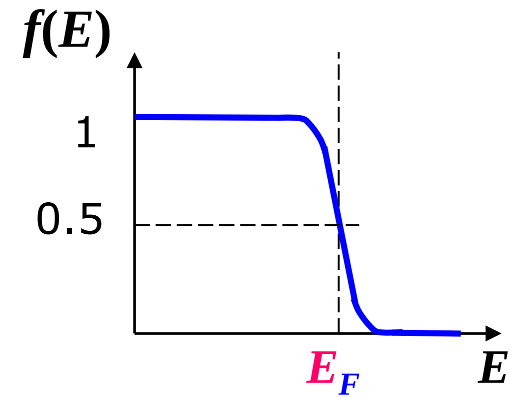
- http://hyperphysics.phy-astr.gsu.edu/Hbase/solids/fermi.html#c2
- http://hyperphysics.phy-astr.gsu.edu/Hbase/solids/fermi.html#c1

Fermi function and Fermi level

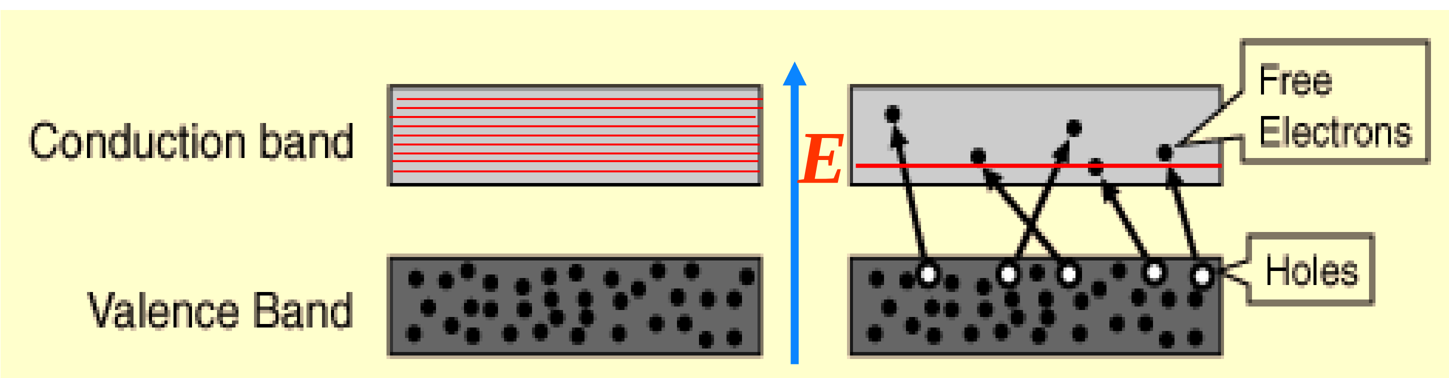
PL

- Probability **that** a **state** at energy level, *E*, is occupied by one electron is,

$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$



- $f(E)$: Fermi-Dirac function
- An increase in E will reduce $f(E)$
- **E_F --- Fermi-level**
  - **When $E = E_F$, $f(E=E_F) = 0.5$.**



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Fermi function and Fermi level

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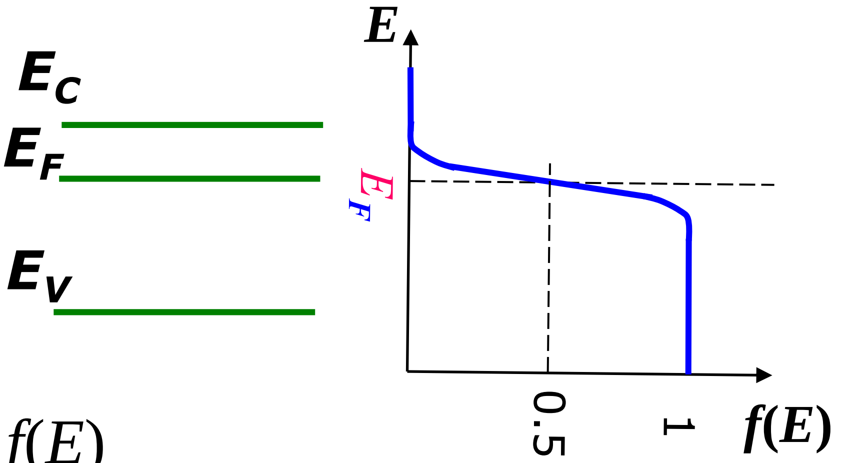

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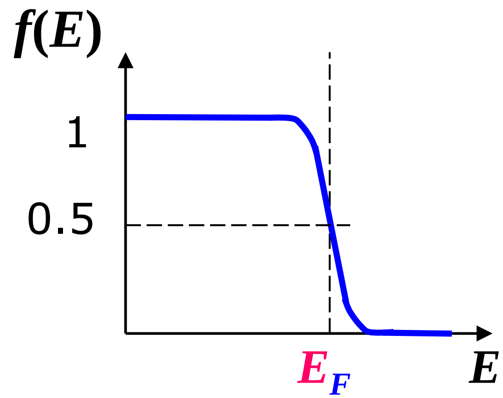
1. Need simplify the Fermi-Dirac function
2. What is the states' density?

Fermi function and Fermi level

PL

- Probability **that** a **state** at energy level, E, is occupied by one electron is,

$$f(E) = \frac{1}{1+\exp(\frac{E-E_F}{kT})}$$



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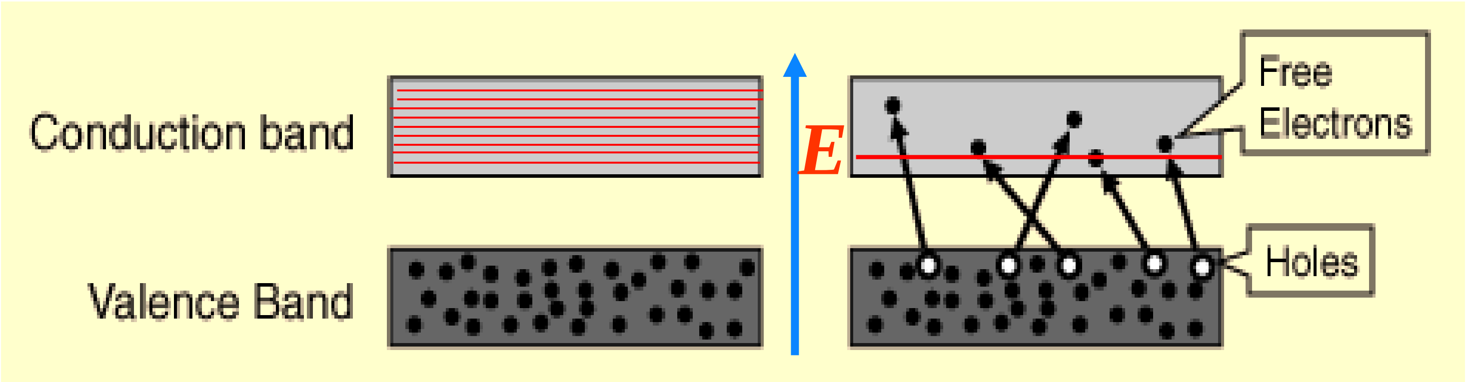

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2.5 Boltzmann approximation & E_F, n, p

- Fermi function and Fermi level
- **Density of States** 态密度
- Boltzmann Approximation
- Electron and hole Concentrations

Density of States

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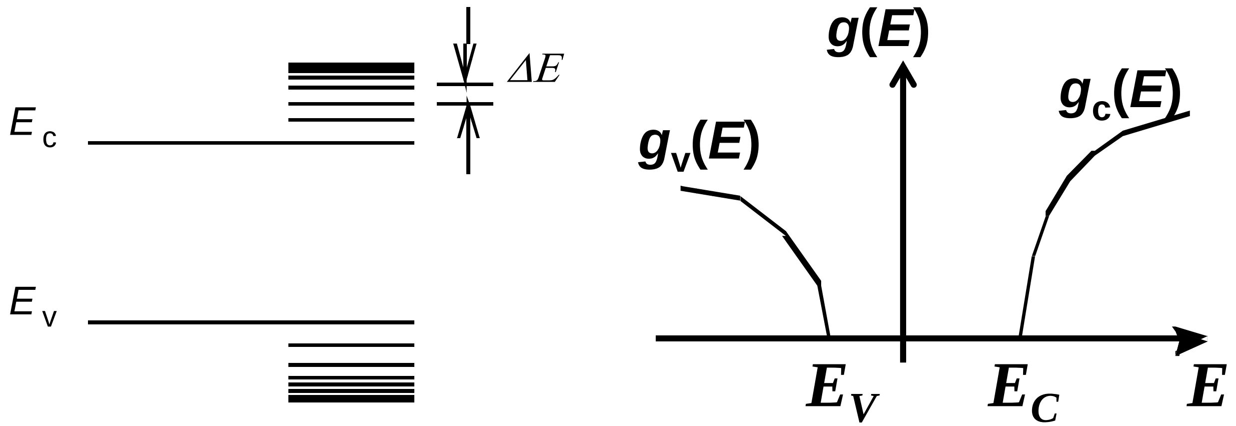


$g(E)\Delta E$ = number of states per cm^3 in the energy range between E and $E+\Delta E$

Near the band edges:

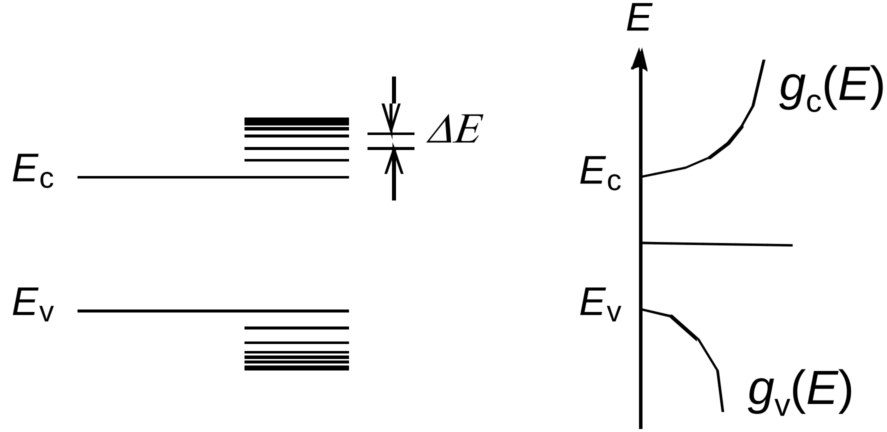
$$g_c(E) = \frac{m_n^* \sqrt{2m_n^*(E - E_c)}}{\pi^2 \hbar^3} \qquad E \geq E_c$$

density of states in the conduction band

$$g_v(E) = \frac{m_p^* \sqrt{2m_p^*(E_v - E)}}{\pi^2 \hbar^3} \qquad E \leq E_v$$

density of states in the valence band

Density of States



$g(E)dE$ = number of states per cm^3 in the energy range between E and $E+dE$

有效质量

Near the band edges:

$$g_c(E) = \frac{m_n^* \sqrt{2m_n^*(E-E_c)}}{\pi^2 \hbar^3} \qquad E \geq E_c$$

$$g_v(E) = \frac{m_p^* \sqrt{2m_p^*(E_v-E)}}{\pi^2 \hbar^3} \qquad E \leq E_v$$

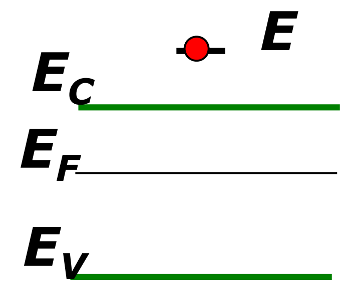
2.5 Boltzmann approximation & E_F, n, p

- Fermi function and Fermi level
- Density of States
- **Boltzmann Approximation**
- Electron and hole Concentrations

Boltzmann Approximation

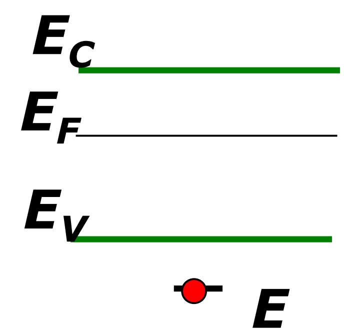
$$f(E) = \frac{1}{1+\exp(\frac{E-E_F}{kT})}$$

$$\text{If } E - E_F > 3kT,\ f(E) \cong e^{-(E-E_F)/kT}$$



because of exp[(E-E_F)/(kT)] >> 1

$$\text{If } E_F - E > 3kT,\ f(E) \cong 1 - e^{(E-E_F)/kT}$$



Probability that a state is **empty**:

$$\underline{1 - f(E)} \cong e^{(E-E_F)/kT} = e^{-(E_F-E)/kT}$$

Probability that a state is occupied by a hole

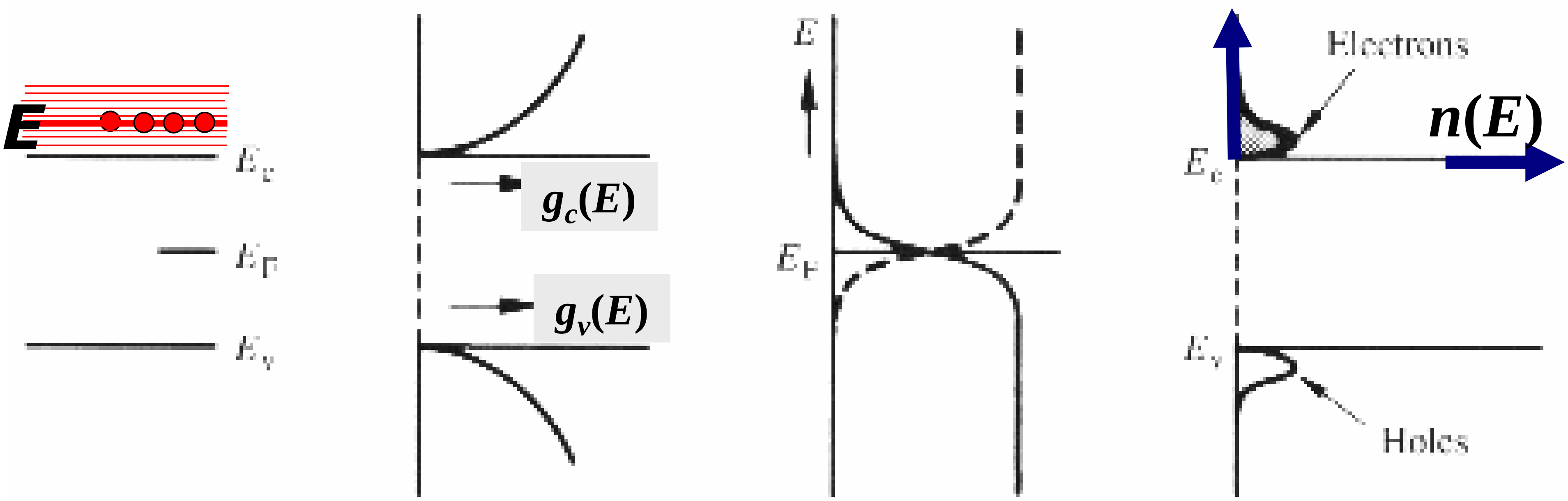
2.5 Boltzmann approximation & E_F, n, p

- Fermi function and Fermi level
- Density of States
- Boltzmann Approximation
- **Electron and hole Concentrations**

Equilibrium Distribution of Electrons

- Obtain $n(E)$ by multiplying $g_c(E)$ and $f(E)$

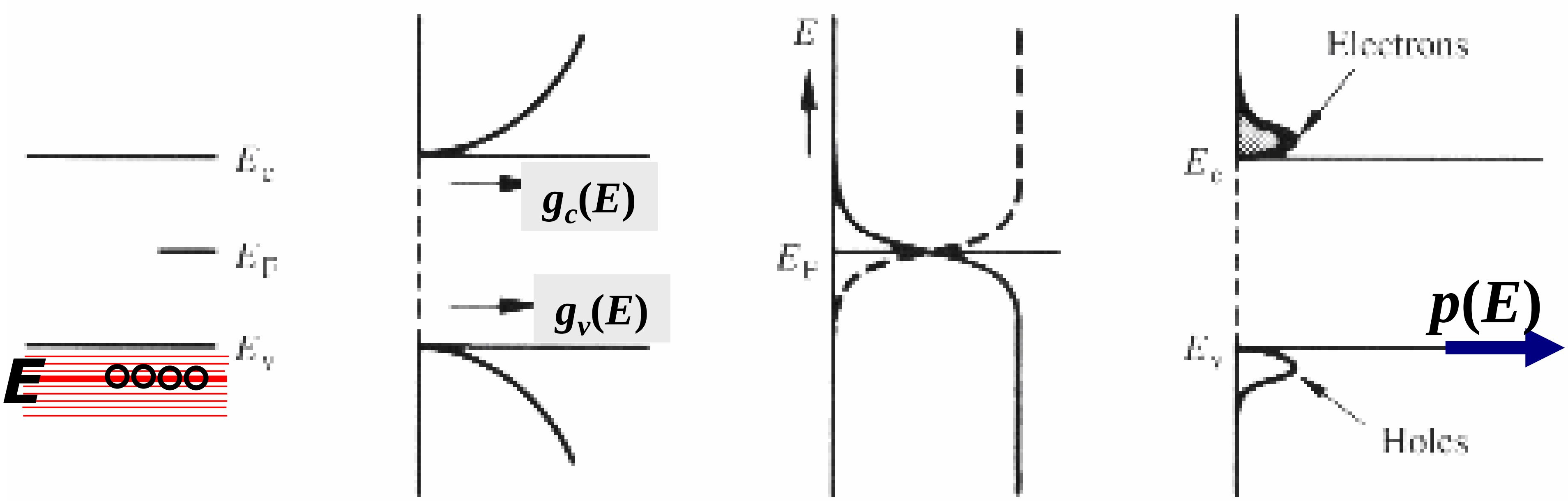
Energy band diagram	Density of States	Probability of occupancy	Carrier distribution



Equilibrium Distribution of Holes

- Obtain $p(E)$ by multiplying $g_v(E)$ and $1\text{-}f(E)$

Energy band diagram | **Density of States** | **Probability of occupancy** | **Carrier distribution**



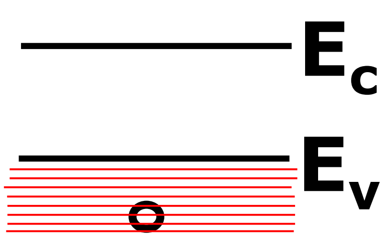
Equilibrium Hole Concentrations

- Integrate *p*(*E*) over all the energies in the valence band to obtain *p*:

$$p = \int_{-\infty\ \text{of valenceband}}^{E_v} g_v(E)\left[1 - f(E)\right]dE$$

- By using the Boltzmann approximation, and extending the integration limit to -∞, we obtain

$$p = N_v e^{-(E_F - E_v)/kT} \quad \text{where} \quad N_v = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{3/2}$$



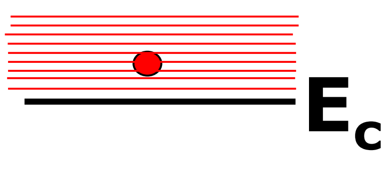
Equilibrium Electron Concentrations

- Integrate *n*(*E*) over all the energies in the conduction band to obtain *n*:

$$n = \int_{E_c}^{\text{top of conduction band}} g_c(E) f(E)\, dE$$

- By using the Boltzmann approximation, and extending the integration limit to ∞, we obtain

$$n = N_c e^{-(E_c - E_F)/kT} \quad \text{where } N_c = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2}$$



Intrinsic Carrier Concentration

$$np = \left(N_c e^{-(E_c - E_F)/kT}\right)\left(N_v e^{-(E_F - E_v)/kT}\right)$$

$$= N_c N_v e^{-(E_c - E_v)/kT} = N_c N_v e^{-E_g/kT}$$

$$= n_i^2$$

Law of Mass Action

$$n_i = \sqrt{N_c N_v} e^{-E_g/2kT}$$

Electron and hole concentrations

$$\boxed{\begin{aligned} n &= N_C \exp\left[\frac{-(E_C - E_F)}{kT}\right] \\ p &= N_V \exp\left[\frac{-(E_F - E_V)}{kT}\right] \\ n \cdot p &= n_i^2 \end{aligned}}$$

+

$$n_i = N_C \exp\left[\frac{-(E_C - E_i)}{kT}\right]$$

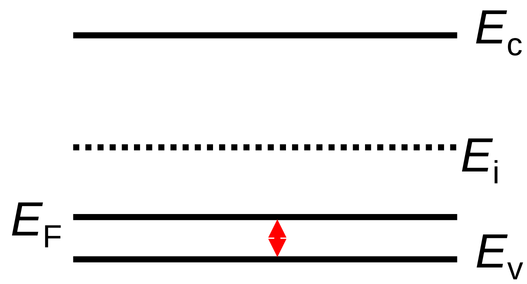
$$n_i = N_V \exp\left[\frac{-(E_i - E_V)}{kT}\right]$$

$$\boxed{\begin{aligned} n &= n_i \exp\left[\frac{(E_F - E_i)}{kT}\right] \\ p &= n_i \exp\left[\frac{-(E_F - E_i)}{kT}\right] \end{aligned}}$$

p.83,ref1, $E_{Fi} \approx E_i$

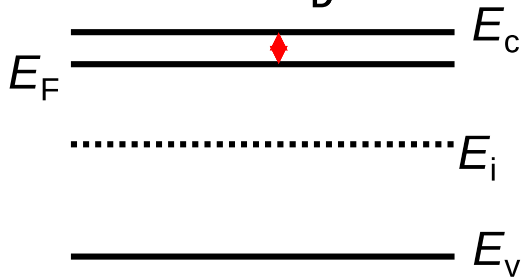
At RT

***p* = *N*$_A$**



At RT

***n* = *N*$_D$**



HW3: Energy-band diagram

Question: Where is E_F for $n = 10^{17}$ cm^{-3} ?

2.6 Carrier drift and diffusion

- **Carrier scattering** 载流子散射

- Carrier drift:
  - *Carrier mobility*
  - *Conductivity &Resistivity*
  - *Energy band model*

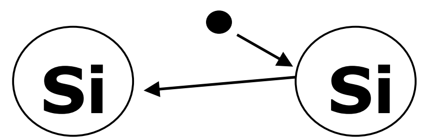
- Carrier diffusion

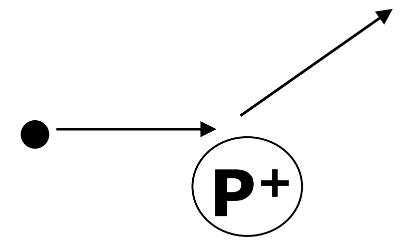
Reading: Chapter 2.6

Thermal Motion

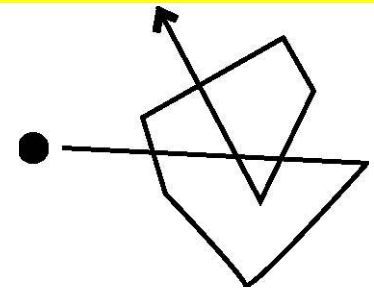
载流子

- In thermal equilibrium, **carriers** are not sitting still:
  - undergo collisions with vibrating Si atoms (Brownian motion)
  - electrostatically interact with charged dopants and with each other
- Characteristic time constant of thermal motion
  - **mean free time** between collisions: $\tau_c \equiv$ collision time [s]
  - In between collisions, carriers acquire high velocity: $\mathbf{v}_{th} \equiv$ **thermal velocity** [cm/s]
  - ...but get nowhere! (on aveerge)
- Characteristic length of thermal motion:
  - $\lambda \equiv$ **mean free path** [cm], $\lambda = \mathbf{v}_{th}\, \tau_c$





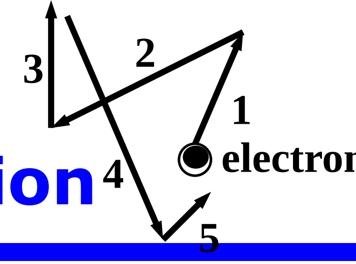
平均自由时间



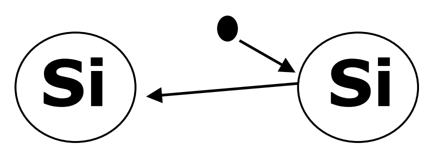
平均自由程

Carrier Scattering

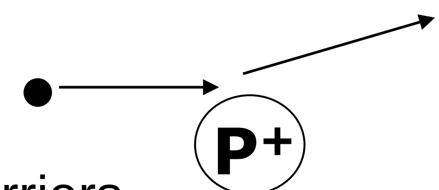
random motion



- Mobile electrons and atoms in the Si lattice are always in random thermal motion.
  - Average velocity of thermal motion for electrons in Si:
    ~**10^7 cm/s @ 300K**
  - Electrons make frequent "collisions" with the vibrating atoms 晶格散射或声子散射
    - "lattice scattering" or "**phonon scattering**"



  - Other scattering mechanisms:
    - deflection by ionized impurity atoms
    - deflection due to **Coulombic** force between carriers



库仑散射

- **The average current in any direction is zero, if no electric field is applied**.

Effective Mass

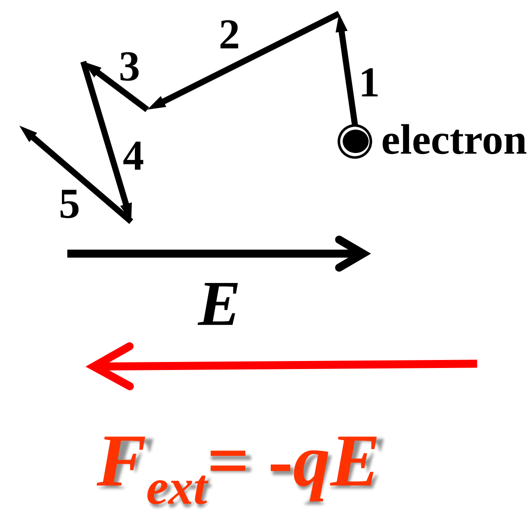
- Under an externally applied force, F_{ext}, the movement of electrons (or holes) is influenced by the positively charged protons and by negatively charged electron in the lattice. So, the movement in the crystal is different from that in vacuum.
- The total force F_{total}

$$F_{total} = F_{ext} + F_{\text{int}} = ma$$

where a is the acceleration, F_{int} is the internal force. We can write

$$F_{ext} = m^{*} a$$

where m^* is called **effective mass**.

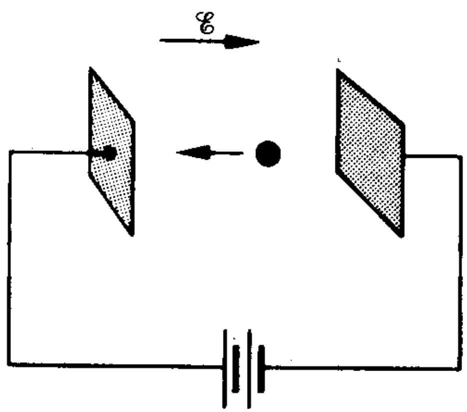


Notation: m_n^{*} for electrons, m_p^{*} for holes,

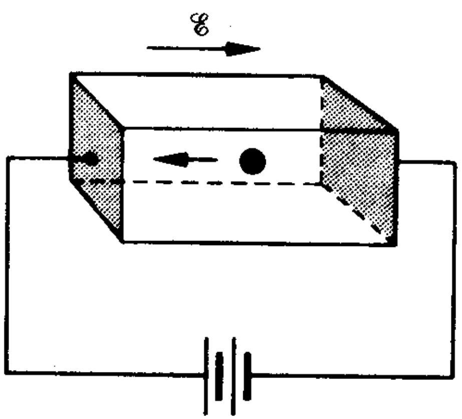
有效质量

Electrons as Moving Particles

In vacuum



In semiconductor



$F = (-q)\mathrm{E} = m_o a$

$F_{ext} = (-q)\mathrm{E} = m_n^* a$

where m_n^* is the electron effective mass.

If τ_{cn} is electron mean free time between collisions,

$$|a| = dv/dt \approx v_e/\tau_{cn}$$

$$|a| = q\mathrm{E}/m_n^*$$

$$\Rightarrow \quad v_e = \frac{q\tau_{cn}\mathrm{E}}{m_n^*}, \quad v_h = \frac{q\tau_{cp}\mathrm{E}}{m_p^*}$$

平均漂移速度：*average* ***drift velocity***

2.6 Carrier drift and diffusion

- Carrier scattering

- **Carrier drift:**

  载流子漂移

  - *Carrier mobility*

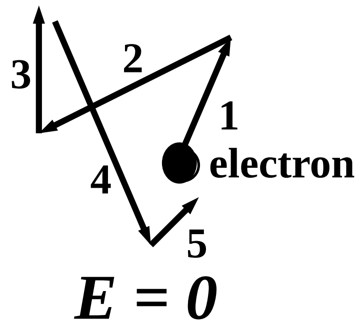
    载流子迁移率

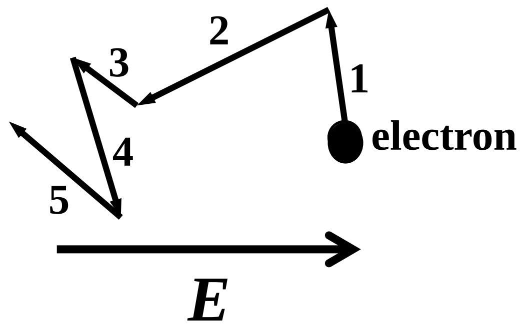
  - *Conductivity & Resistivity*
  - *Energy band model*

- Carrier diffusion

Carrier Drift

- **When an electric field** (*e.g.*, due to an externally applied voltage) **is applied to a semiconductor, mobile charge-carriers will be accelerated by the electrostatic force. This force superimposes on the random motion of electrons:**



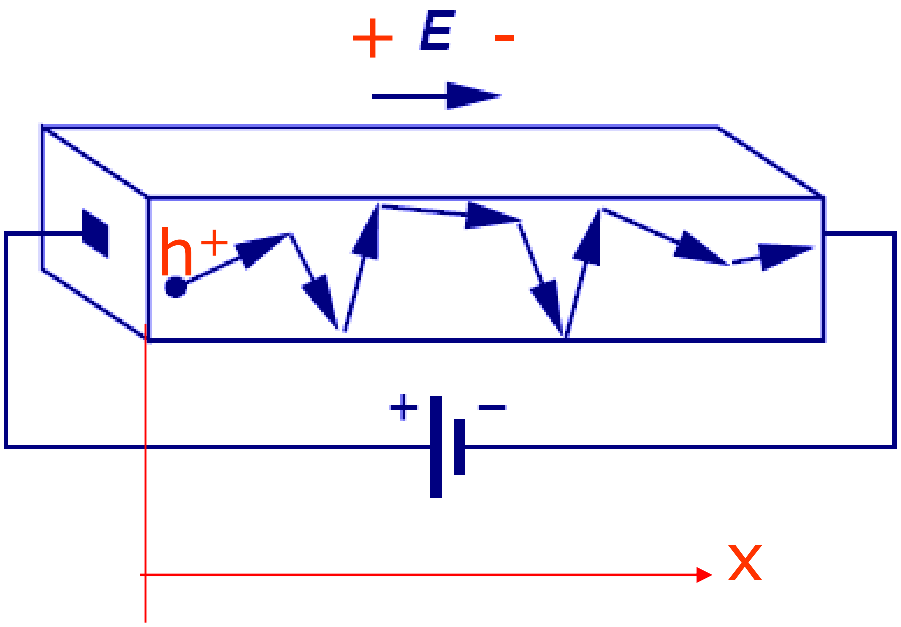


- **Electrons *drift* in the direction opposite to the *E*-field**
  → Current flows

❖ **Because of scattering, electrons in a semiconductor do not achieve constant acceleration. However, they can be viewed as classical particles moving at a constant average *drift velocity*.**

Carrier Drift

- The process in which charged particles move because of an electric field is called ***drift***. 漂移
- Charged particles within a semiconductor move with an average velocity proportional to the electric field.
  - The proportionality constant is the carrier ***mobility***. 迁移率



Hole velocity $\vec{v}_h = \mu_p \vec{E}$

Electron velocity $\vec{v}_e = -\mu_n \vec{E}$

Notation:

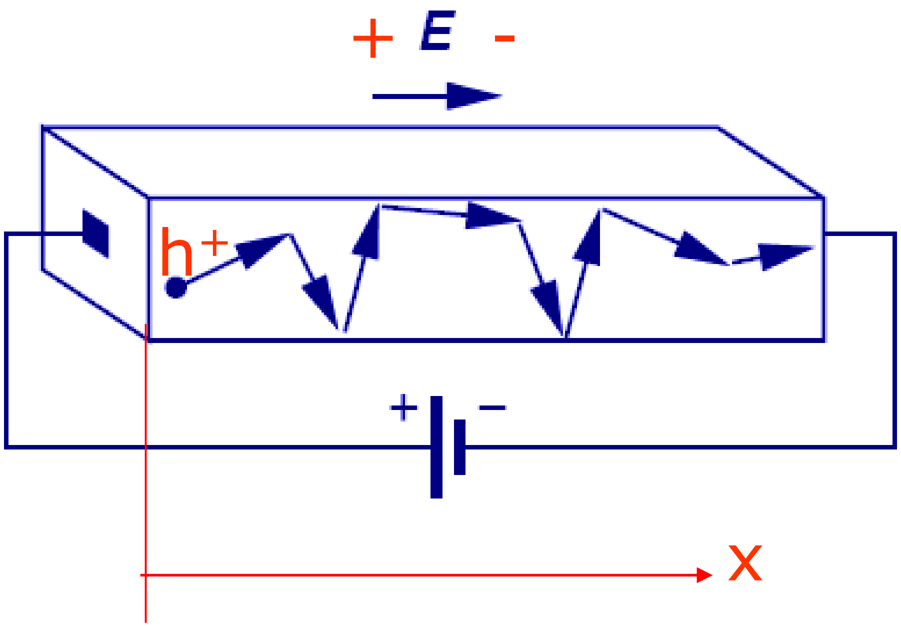
$\mu_p \equiv$ hole mobility (cm²/V·s)

$\mu_n \equiv$ electron mobility (cm²/V·s)

Carrier Drift

$$v_e = \frac{q\tau_{cn}\mathrm{E}}{m_n^*}, \qquad v_h = \frac{q\tau_{cp}\mathrm{E}}{m_p^*} \qquad \Rightarrow$$

$$\mu_n = \frac{q\tau_{cn}}{m_n^*}, \qquad \mu_p = \frac{q\tau_{cp}}{m_p^*}$$



Hole velocity $\vec{v}_h = \mu_p \vec{E}$

Electron velocity $\vec{v}_e = -\mu_n \vec{E}$

Notation:

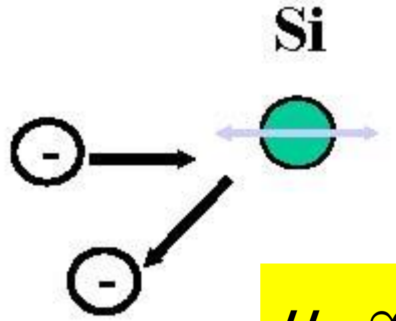
$\mu_p \equiv$ hole mobility ($cm^2/V \cdot s$)

$\mu_n \equiv$ electron mobility ($cm^2/V \cdot s$)

Carrier Mobility

$$1/\mu = 1/\mu_L + 1/\mu_I$$

- Mobile carriers are always in random thermal motion. If no electric field is applied, the average current in any direction is zero.



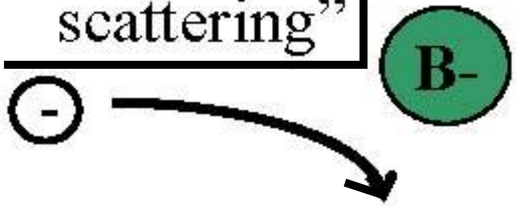
- **Mobility is reduced by**

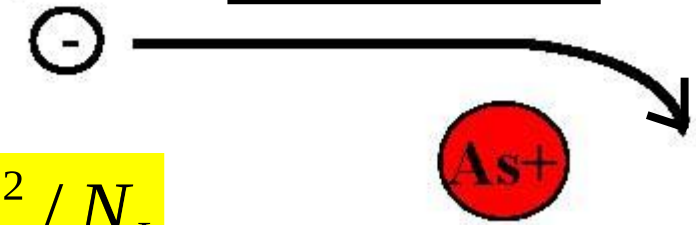
1) collisions with the vibrating atoms

"phonon scattering"

$$\mu_L \propto T^{-3/2}$$

2) deflection by ionized impurity atoms "Coulombic scattering"





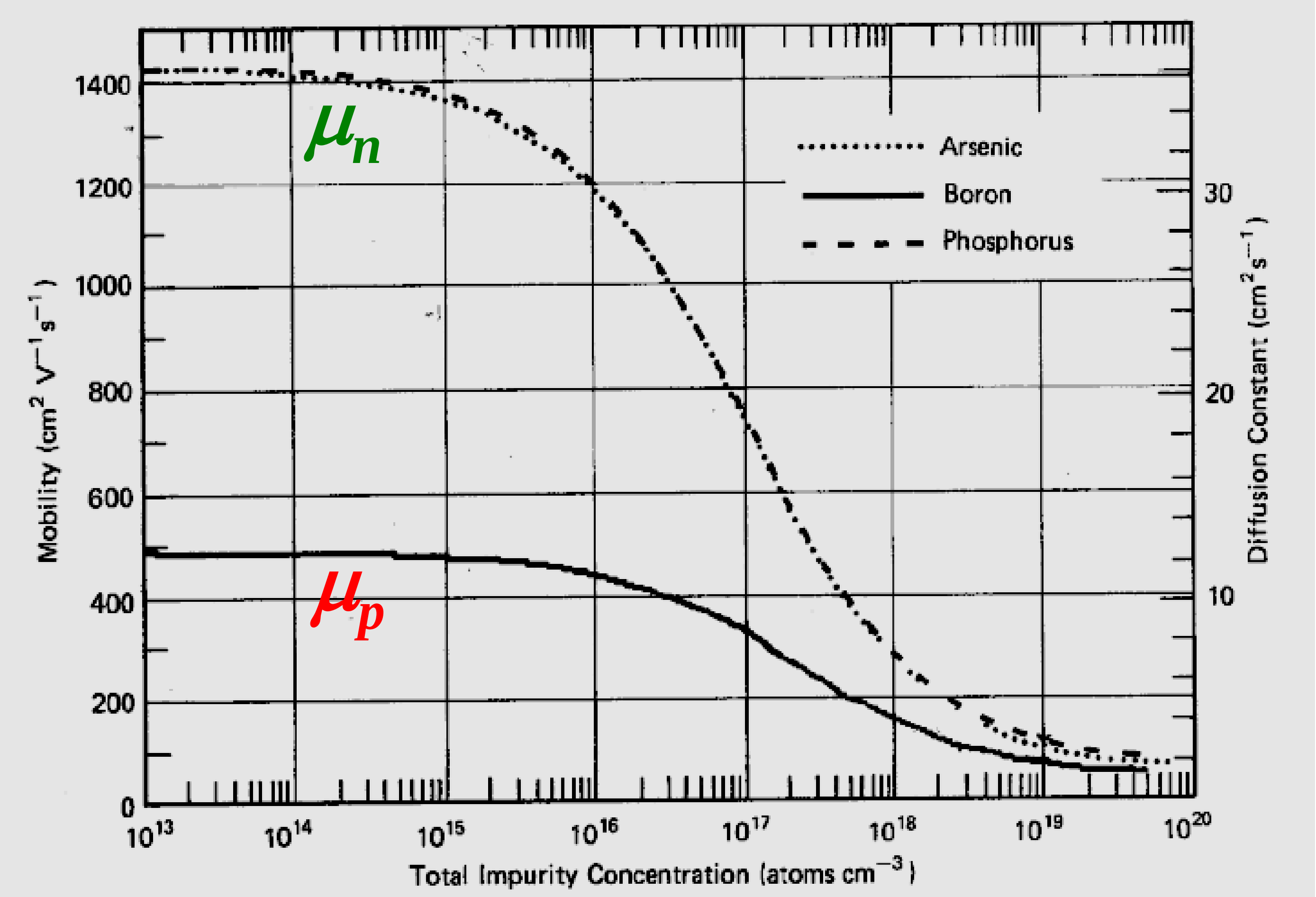
$$\mu_I \propto T^{+3/2} / N_I$$

Drift Velocity and Carrier Mobility

Mobile charge-carrier drift velocity is proportional to applied *E*-field:

$$\boxed{|v| = \mu E}$$

μ is the ***mobility*** **(Units: cm²/V•s)**

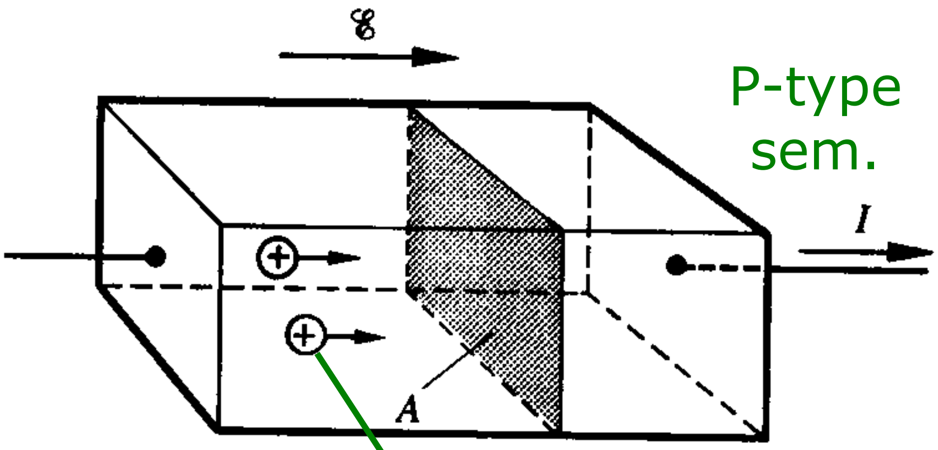


Note: Carrier mobility depends on *total* dopant concentration (N_D + N_A) !

Drift Current

- Drift current is proportional to the carrier velocity and carrier <u>concentration</u>:

1) p---hole density
2) q =1.6×10^{-19} C
--- One electron charge
3) Charges passing through 'A' per second
--- The definition of current.



$v_h\, t\, A$ = volume from which all holes cross plane in time t

$p\, v_h\, t$ A = # of holes crossing plane in time t

$q\, p\, v_h\, t$ A = charge crossing plane in time t

$q\, p\, v_h\, A$ = charge crossing plane per unit time = hole current

➔ **Hole current per unit area (*i.e.* current density) $J_{p,drift} = q\, p\, v_h$**

Electrical Conductivity σ

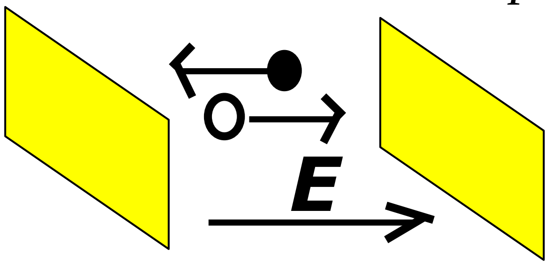
Negatively charged electron

Direction of electron drift

When an electric field is applied, current flows due to drift of mobile electrons and holes:

electron current density: $$J_n = (-q)nv_e = qn\mu_n E$$

hole current density: $$J_p = (+q)pv_h = qp\mu_p E$$



total current density: $$J = J_n + J_p = (qn\mu_n + qp\mu_p)E$$

$$J = \sigma E$$

电导率

conductivity $$\boxed{\sigma \equiv qn\mu_n + qp\mu_p}$$ **Units: $(\Omega\bullet\text{cm})^{-1}$**

Electrical Resistivity ρ

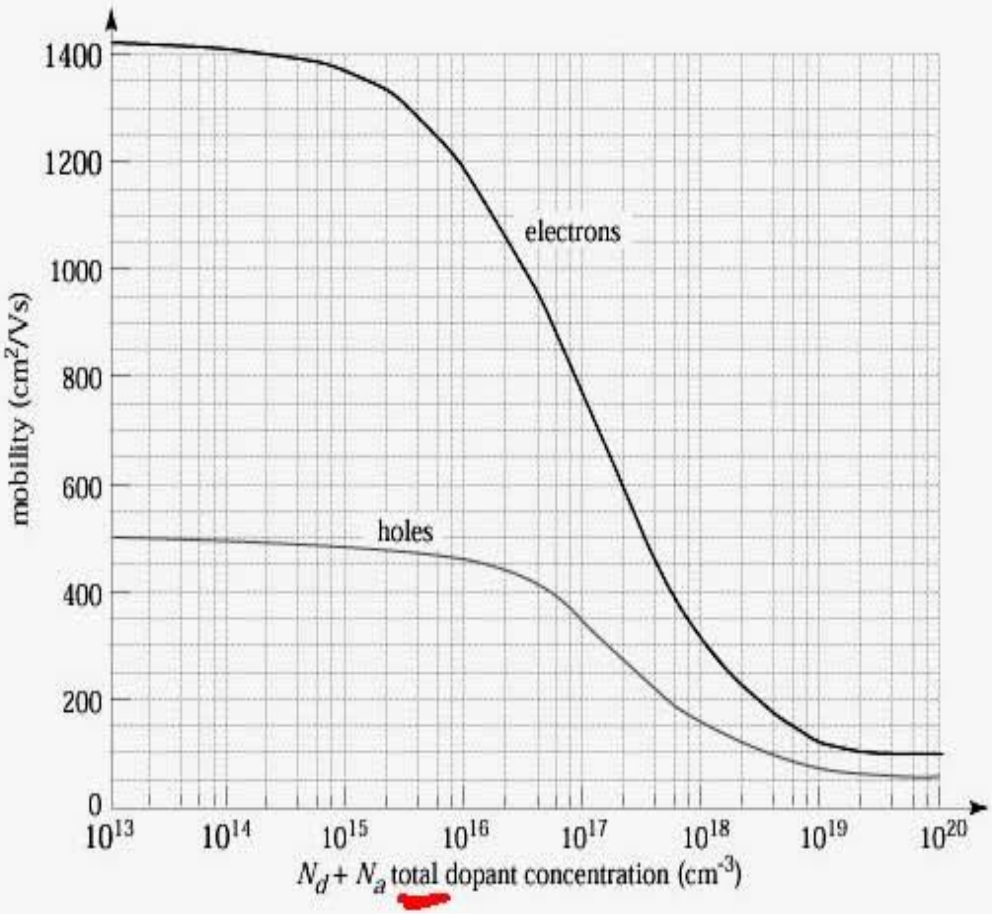
$$\boxed{\rho \equiv \frac{1}{\sigma} = \frac{1}{qn\mu_n + qp\mu_p}}$$

$$\rho \cong \frac{1}{qn\mu_n}$$ **for n-type mat'l**

$$\rho \cong \frac{1}{qp\mu_p}$$ **for p-type mat'l**

(Units: ohm•cm)

HW4

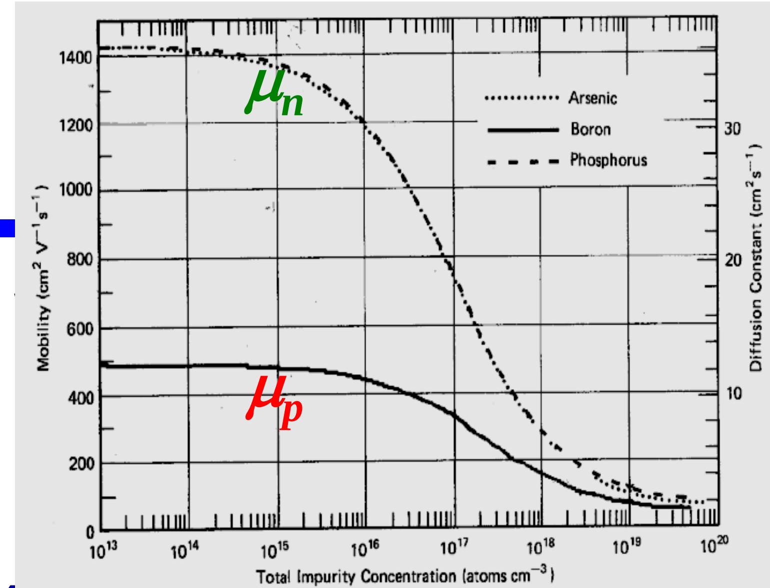


- Estimate the resistivity of a Si sample doped with phosphorus to a concentration of 10^{15} cm^{-3} and boron to a concentration of 10^{17} cm^{-3}.

$\boxed{\rho \simeq ?}$

- The electron mobility and hole mobility are 700 cm^2/Vs and 350 cm^2/Vs, respectively.

Example



Consider a Si sample doped
What is its resistivity?

Answer:

$N_A = 10^{16}/\text{cm}^3$, $N_D = 0$ $(N_A >> N_D \rightarrow$ p-type)

$\rightarrow p \approx 10^{16}/\text{cm}^3$ and $n \approx 10^4/\text{cm}^3$

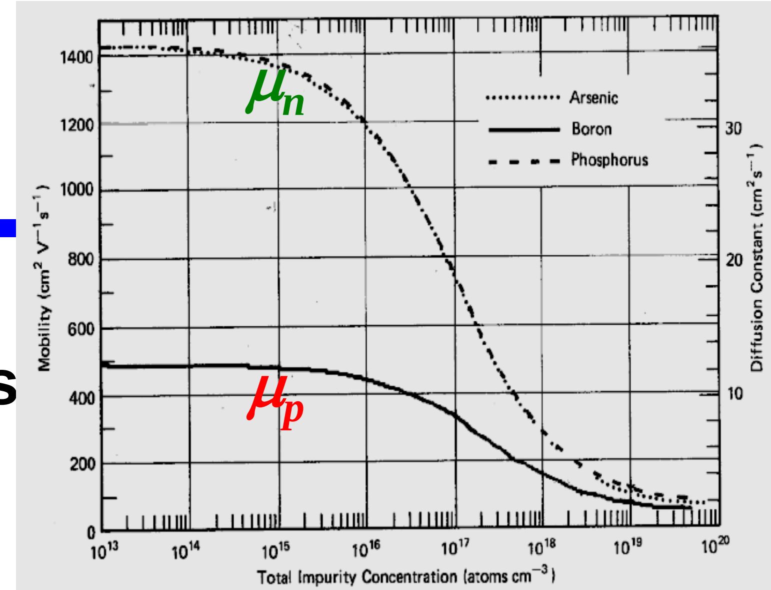
$$\rho = \frac{1}{qn\mu_n + qp\mu_p} \cong \frac{1}{qp\mu_p}$$

$$= \left[(1.6\times10^{-19})(10^{16})(450)\right]^{-1} = 1.4\,\Omega\cdot\text{cm}$$

From μ vs. ($N_A + N_D$) plot

Example (cont'd)

Consider the same Si sample, with 10^{17}/cm^3 Arsenic. What is



Answer:

$N_A = 10^{16}$/cm^3, $N_D = 10^{17}$/cm^3 **($N_D >> N_A \rightarrow$ n-type)**

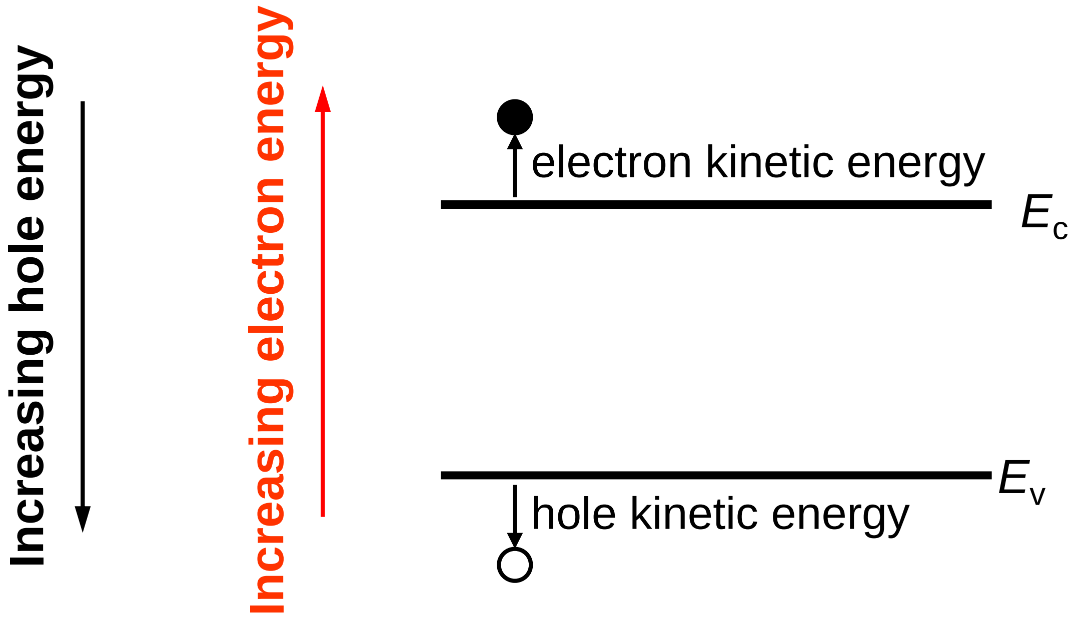
$\rightarrow n \approx 9\text{x}10^{16}$/cm^3 **and** $p \approx 1.1\text{x}10^{3}$/cm^3

$$\rho = \frac{1}{qn\mu_n + qp\mu_p} \cong \frac{1}{qn\mu_n}$$

$$= \left[(1.6\times10^{-19})(9\times10^{16})(700)\right]^{-1} = 0.10\,\Omega\cdot\text{cm}$$

The sample is converted to n-type material by adding more donors than acceptors, and is said to be "compensated".

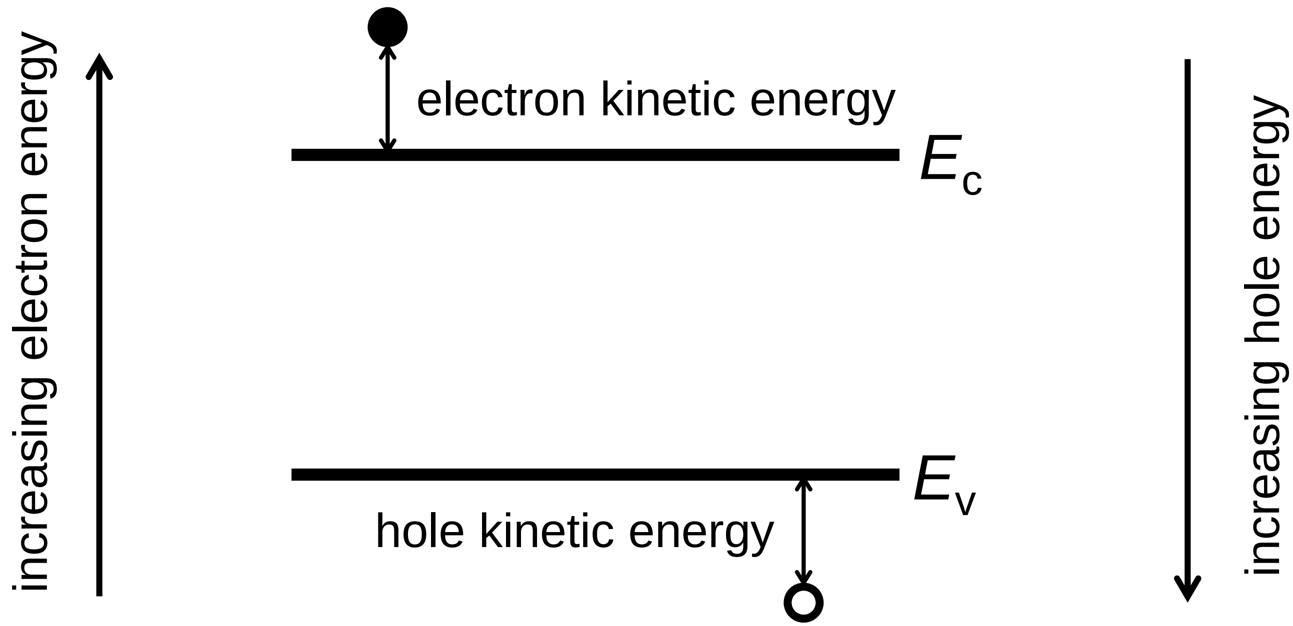
Electrons and Holes (Band Model)



- Electrons and holes tend to seek lowest-energy positions
  - Electrons tend to fall
  - Holes tend to float up (like bubbles in water)

Potential *vs.* Kinetic Energy

势能和动能

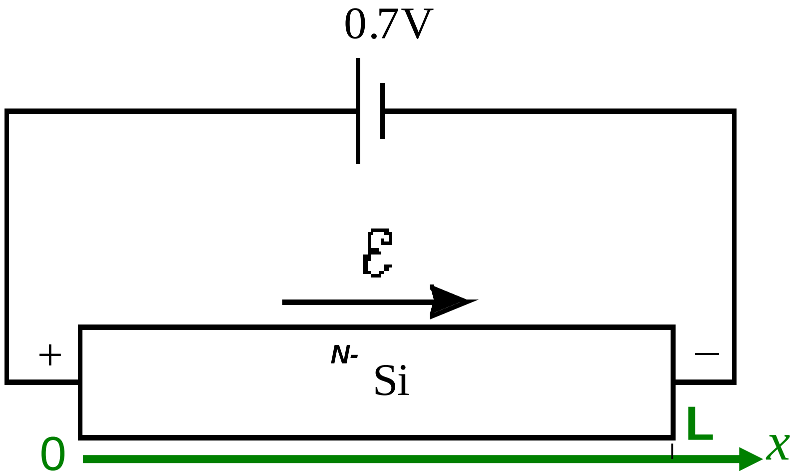


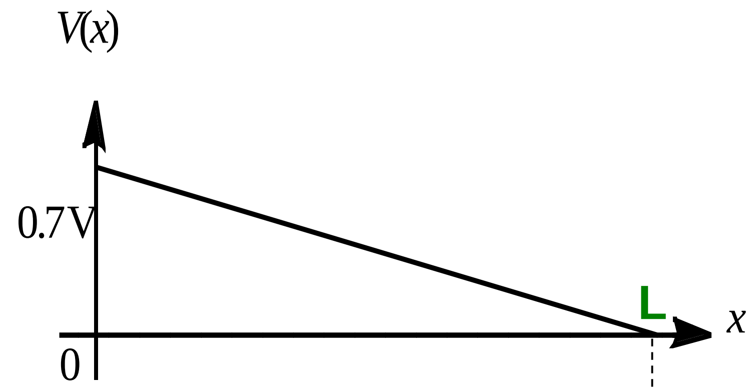
E_c represents the electron potential energy:

$$\text{P.E.} = E_c - E_{\text{reference}}$$

电子的势能

Electrostatic Potential, *V* 电势或电位



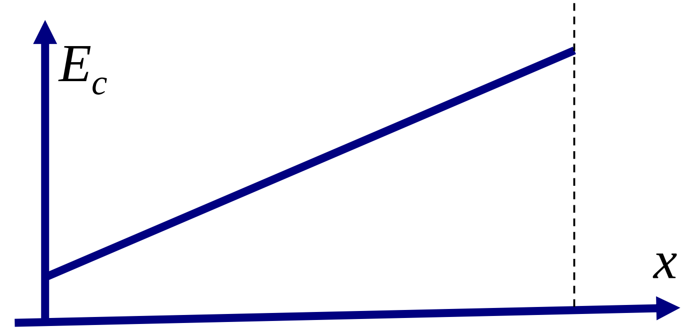


- The potential energy of a particle with charge -*q* is related to the electrostatic potential *V*(*x*):

$$\text{P.E.} = -qV$$

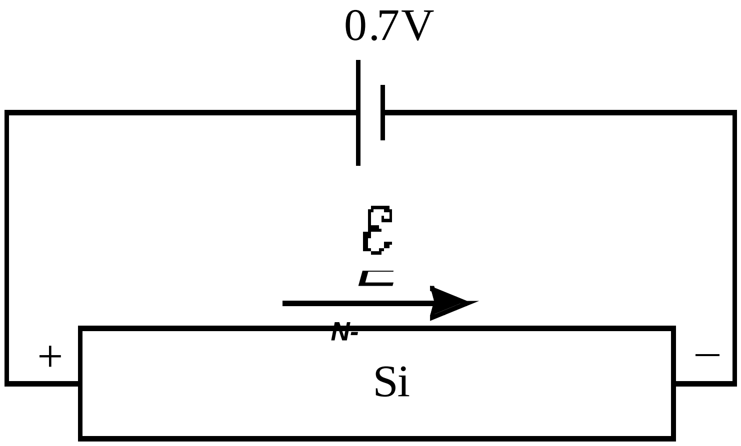
$$V = \frac{1}{q}(E_{\text{reference}} - E_c)$$

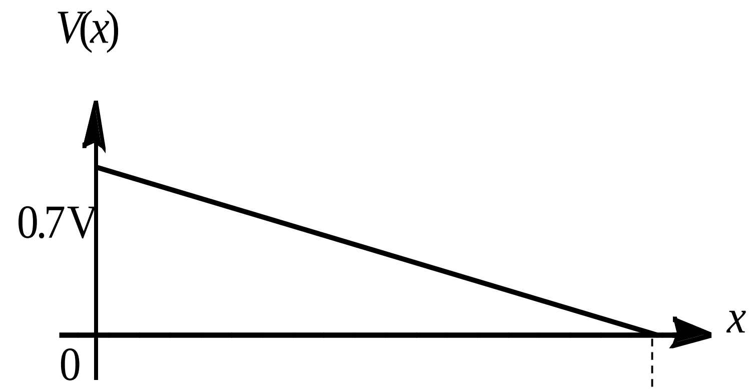
$E_{\text{reference}}$ is constant



Electric Field, ε

$$\mathcal{E} = -\frac{dV}{dx}$$

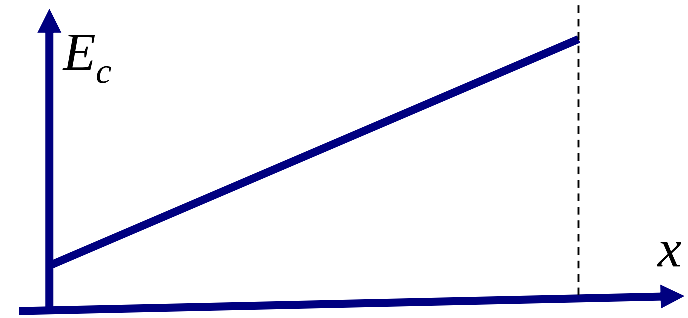




$$V = \frac{1}{q}(E_{\text{reference}} - E_c)$$

$$\Downarrow$$

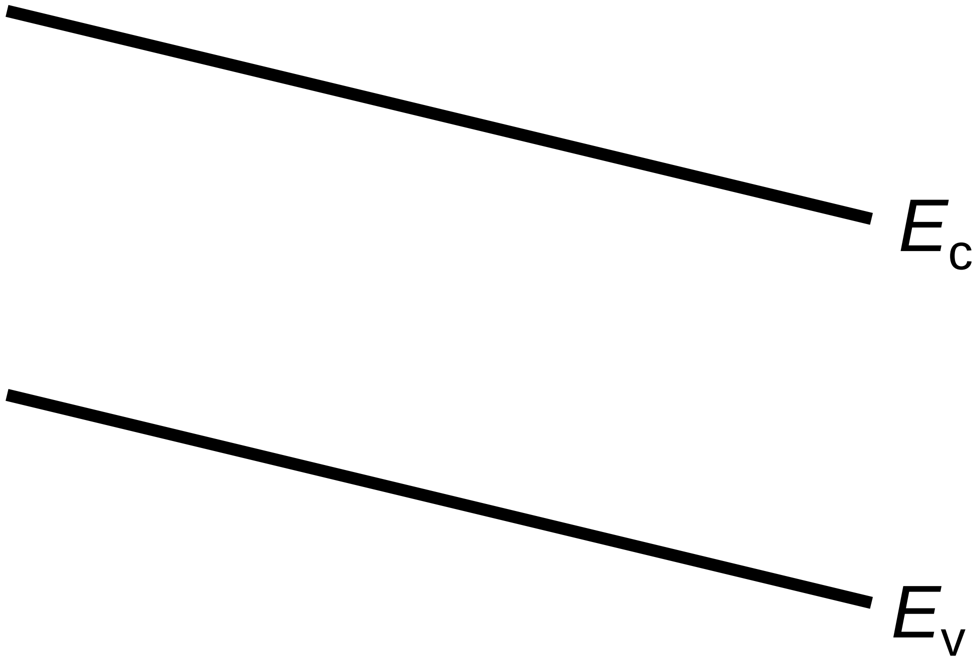
$$\boxed{\mathcal{E} = -\frac{dV}{dx} = \frac{1}{q}\frac{dE_c}{dx}}$$



- Variation of E_c with position is called ***"band bending."***

HW 5: Carrier Drift (Band Diagram Visualization)

$$\mathcal{E} = -\frac{dV}{dx} = \frac{1}{q}\frac{dE_c}{dx}$$



Q1: what is the direction of electric field?

Q2: what is the direction of carriers' drift?

2.6 Carrier drift and diffusion

- Carrier scattering

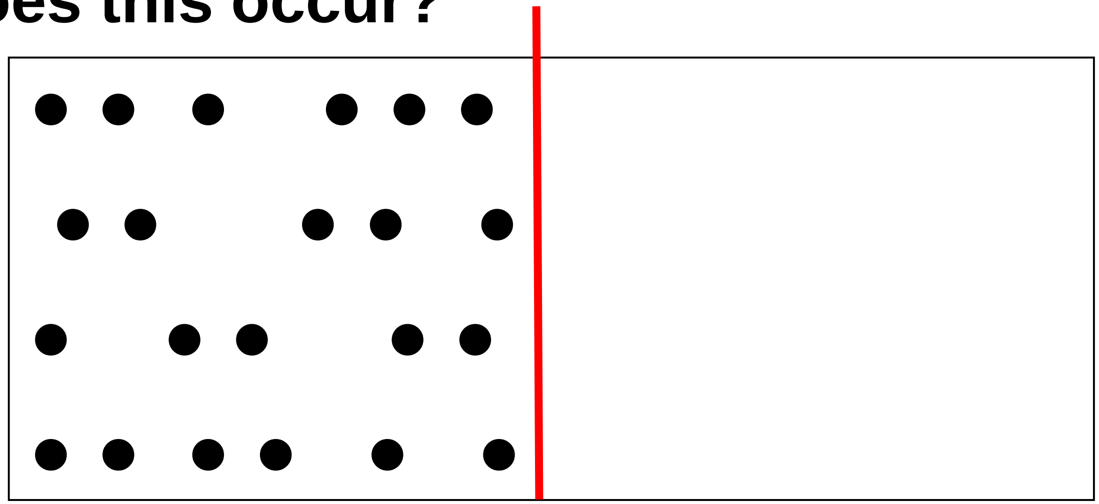
- **Carrier drift:**
  - *Carrier mobility*
  - *Conductivity & Resistivity*
  - *Energy band model*

载流子扩散

- **Carrier diffusion**

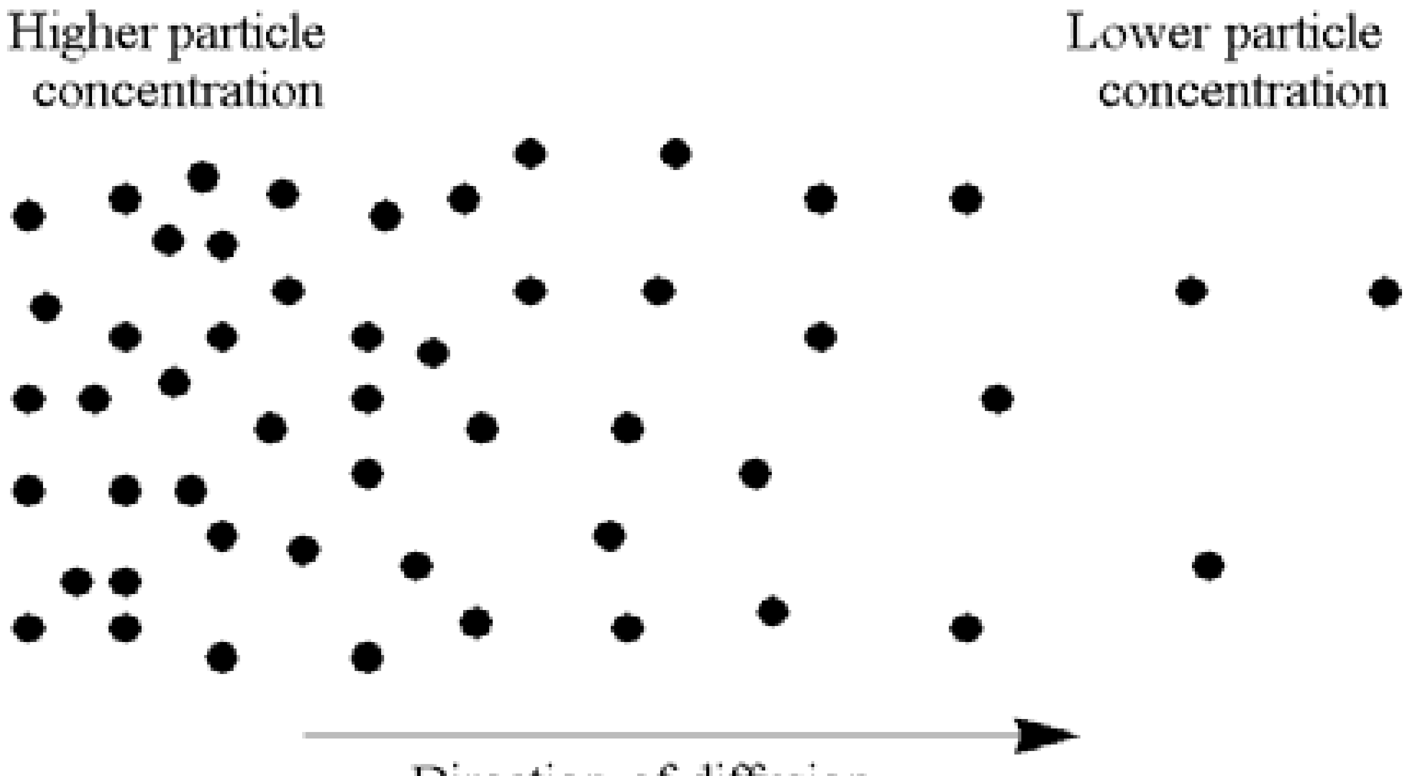
Diffusion

- Diffusion occurs when there exists a concentration gradient
- In the figure below, imagine that we fill the left chamber with a gas at temperate *T*
- If we suddenly remove the divider, what happens?
- The gas will fill the entire volume of the new chamber.
- How does this occur?



Diffusion

- Particles diffuse from higher concentration to lower concentration locations.

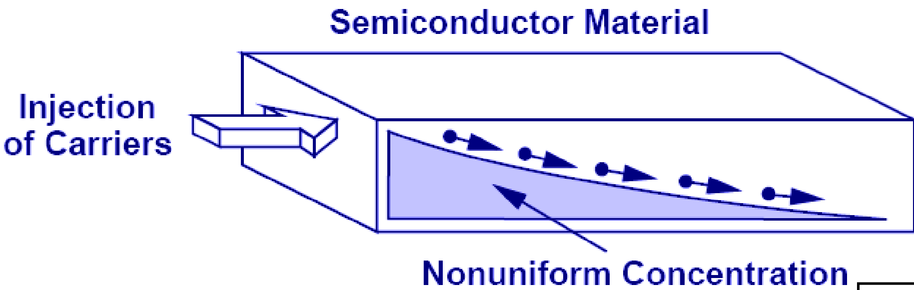


Carrier Diffusion

- Due to thermally induced random motion, mobile particles tend to move from a region of high concentration to a region of low concentration.
  - Analogy: ink droplet in water

浓度梯度

- Current flow due to mobile charge diffusion is proportional to the carrier **concentration gradient**.
  - The proportionality constant is the ***diffusion constant***.



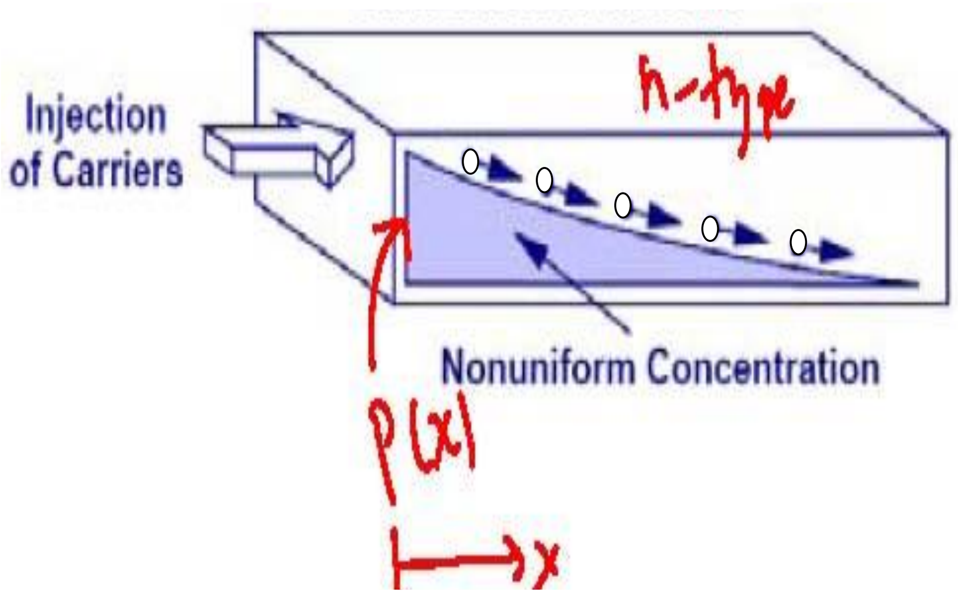
$$J_p = -qD_p \frac{dp}{dx}$$

Notation:

$D_p \equiv$ **hole diffusion constant (cm²/s)**

$D_n \equiv$ **electron diffusion constant (cm²/s)**

$$J_p = -qD_p \frac{dp}{dx}$$



Notation:

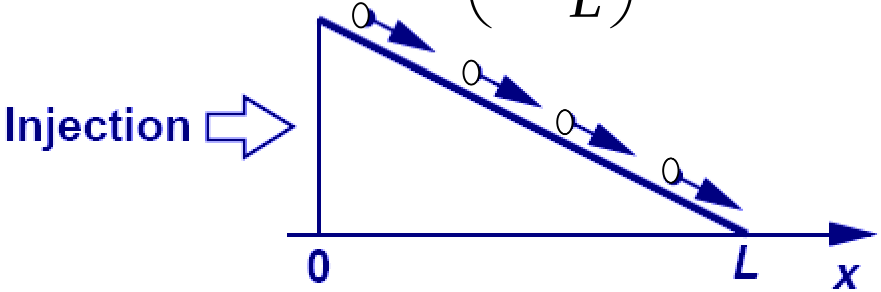
$D_p \equiv$ hole diffusion constant (cm^2/s)

$D_n \equiv$ electron diffusion constant (cm^2/s)

Diffusion Examples

- Linear concentration profile
  → constant diffusion current

$$p = N\left(1 - \frac{x}{L}\right)$$

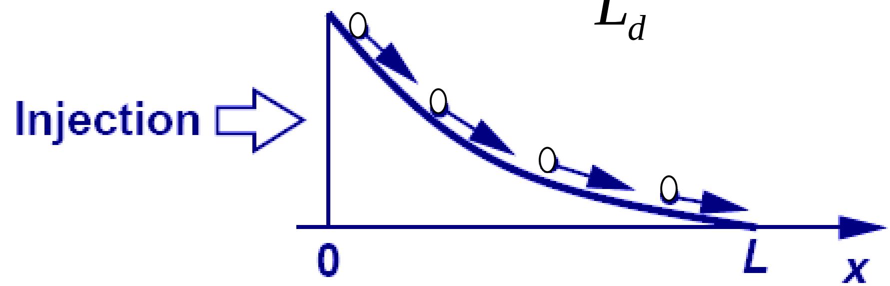


$$J_{p,diff} = -qD_p \frac{dp}{dx}$$

$$= qD_p \frac{N}{L}$$

- Non-linear concentration profile
  → varying diffusion current

$$p = N \exp \frac{-x}{L_d}$$



$$J_{p,diff} = -qD_p \frac{dp}{dx}$$

$$= \frac{qD_p N}{L_d} \exp \frac{-x}{L_d}$$

Total **Diffusion** Current

- Due to the non-uniform distribution of carriers

$$J_n = qD_n \frac{dn}{dx}$$

- D_n --- e Diffusion constant.
- Driving force: thermal energy, not electric field
- dn/dx--- density gradient
- **Total diffusion current**
  - $J = J_n + J_p$

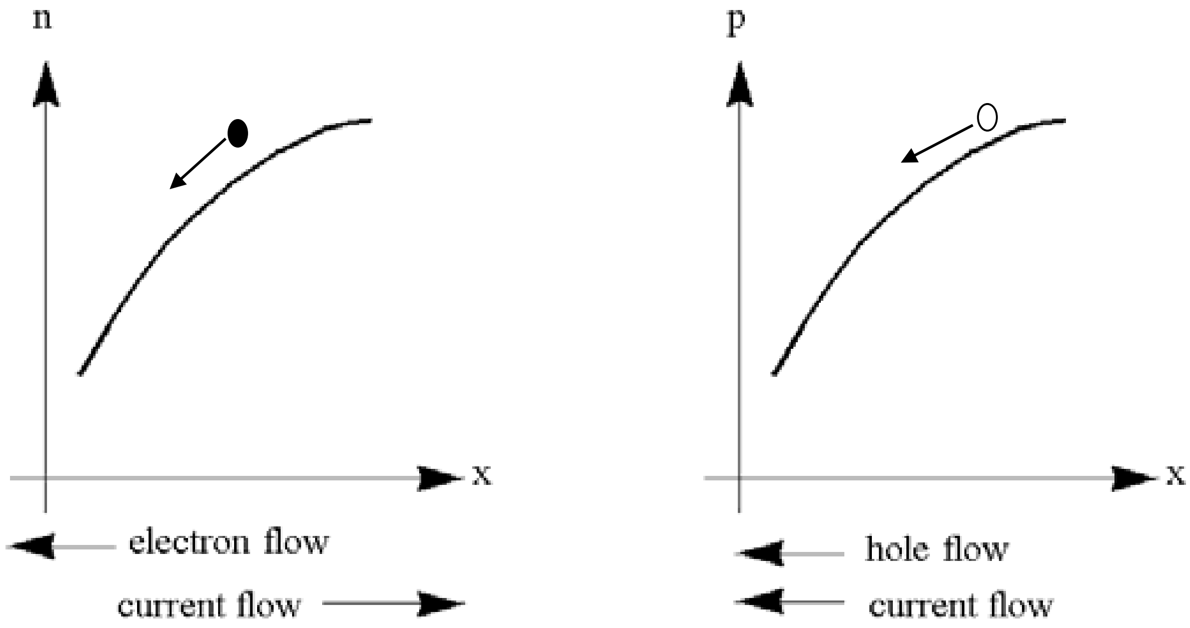
Concentration gradient

Total **Diffusion** Current

- Diffusion current within a semiconductor consists of hole and electron components:

$$J_{p,diff} = -qD_p \frac{dp}{dx} \qquad J_{n,diff} = qD_n \frac{dn}{dx}$$

$$J_{tot,diff} = q(D_n \frac{dn}{dx} - D_p \frac{dp}{dx})$$



The **total** current

- The total current flowing in a semiconductor is the sum of **drift current** and **diffusion current**:

$$\boxed{J_{tot} = J_{p,drift} + J_{n,drift} + J_{p,diff} + J_{n,diff}}$$

$$J_{p,drift} = qp\mu_p E, \qquad J_{n,drift} = qn\mu_n E$$

$$J_{p,diff} = -qD_p \frac{dp}{dx}, \qquad J_{n,diff} = qD_n \frac{dn}{dx}$$

The Einstein Relation

- The characteristic constants for drift and diffusion are related:

$$D_n = \frac{kT}{q}\mu_n$$

$$D_p = \frac{kT}{q}\mu_p$$

$$\boxed{\frac{D}{\mu} = \frac{kT}{q}} \simeq 26\ \text{mV at } T = 300\text{K}$$

- Note that $\dfrac{kT}{q} \cong 26\text{mV}$ at room temperature (300K)
  - This is often referred to as the **"thermal voltage"**.

Important Constants

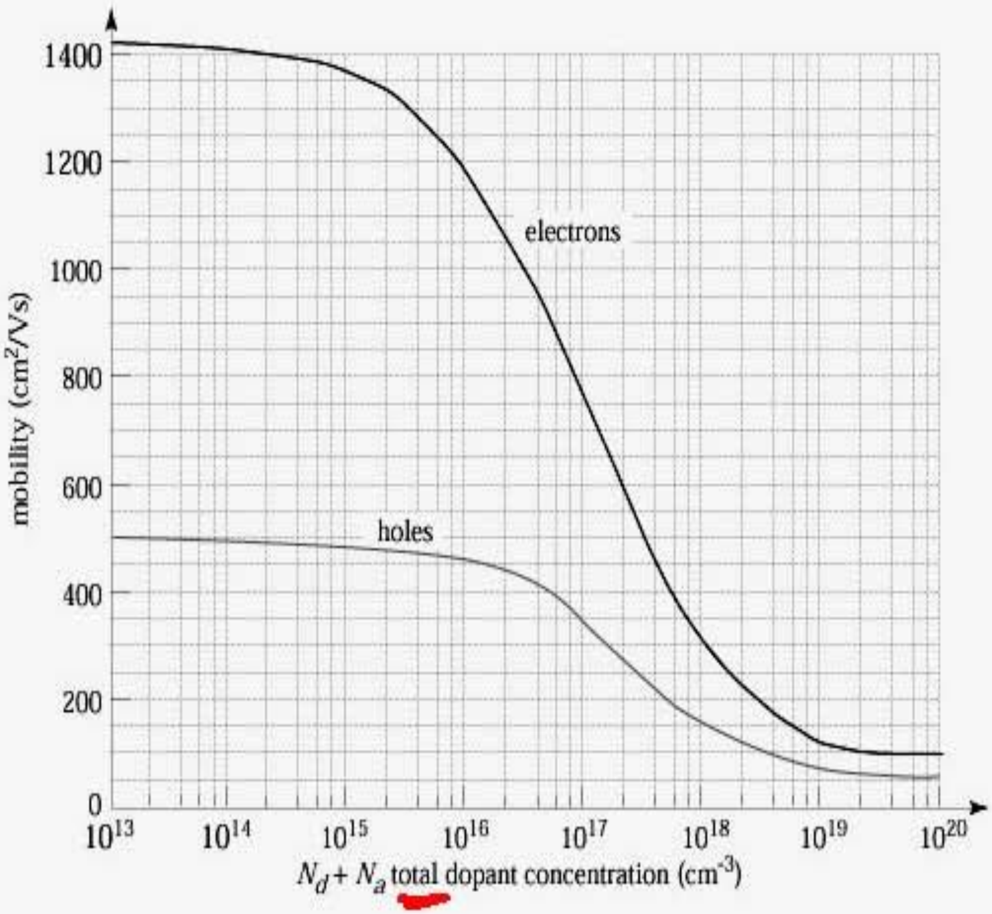
- Electronic charge, $q = 1.6\times10^{-19}$ C
- Permittivity of free space, $\varepsilon_o = 8.854\times10^{-14}$ F/cm
- Boltzmann constant, $k = 8.62\times10^{-5}$ eV/K
- Planck constant, $h = 4.14\times10^{-15}$ eV•s
- Free electron mass, $m_o = 9.1\times10^{-31}$ kg
- Thermal voltage $kT/q = 26$ mV, at T=300K

HW3: Energy-band diagram

Question: Where is E_F for $n = 10^{17}$ cm^{-3} ?

$$n = n_i \exp\left[\frac{(E_F - E_i)}{kT}\right]$$

HW4



$N_D = 10^{15}\ cm^{-3}$, $N_A = 10^{17}\ cm^{-3}$

$N_A - N_D \approx 10^{17}\ cm^{-3}$

$p = 10^{17}\ cm^{-3}$, $n \simeq 10^{3}\ cm^{-3}$

negligible

$\sigma = q\cancel{\mu_n n} + q\mu_p p$

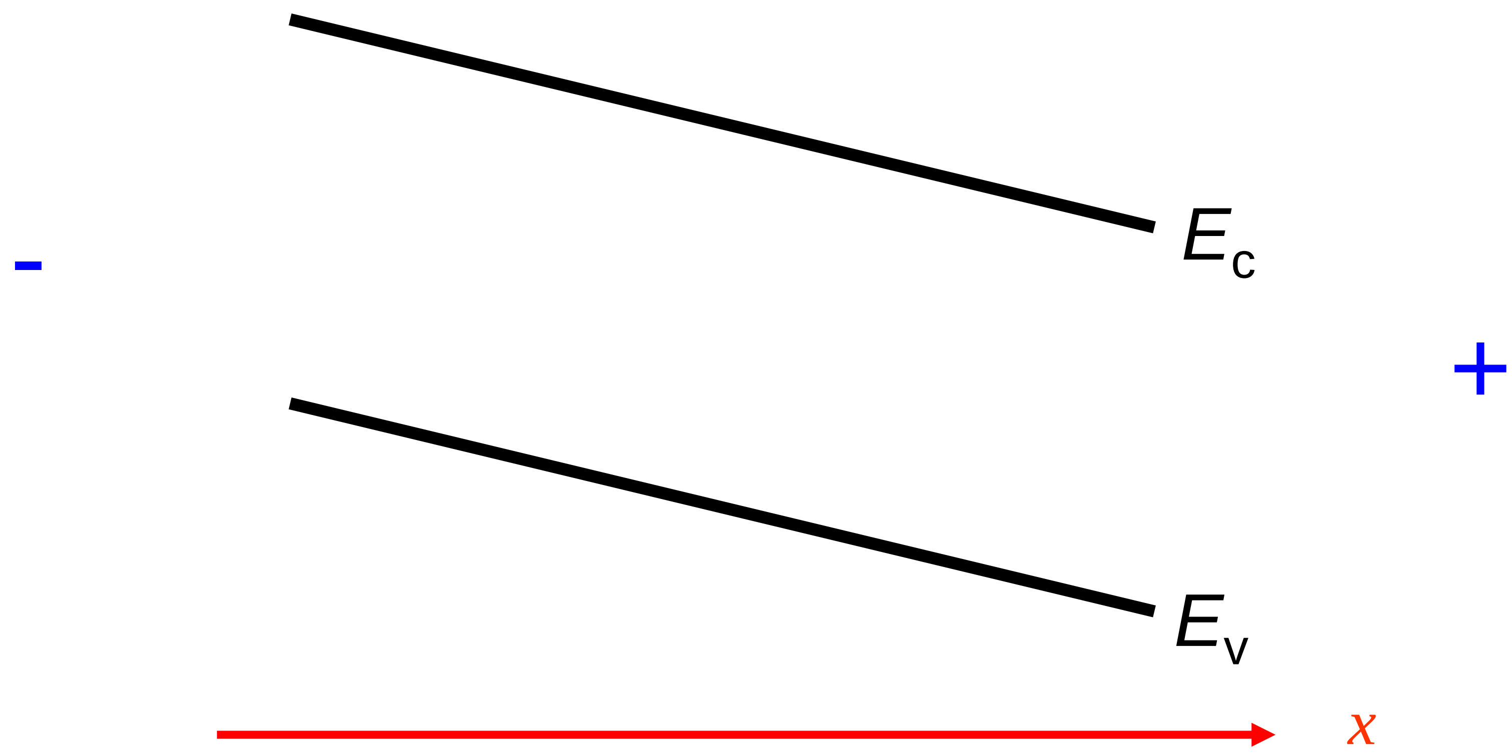
$\sigma = 1.6\times10^{-19}(350)(10^{17})$

$\approx 5\times10^{0}$

$\boxed{\rho \simeq 0.2\ \Omega\text{-cm}}$

- The electron mobility and hole mobility are 700 cm^2/Vs and 350 cm^2/Vs, respectively.

HW5: Carrier Drift (Band Diagram Visualization)



Q1: what is the direction of electric field?

Q2: what is the direction of carriers' drift?