# Tutorial 2 Path independence

1. (Page 425) Show that the form under the integral sign is exact in the plane or in space and evaluate the integral. Show the details of your work.

   (1) $\int_{(\pi/2,\pi)}^{(\pi,0)} \left( \frac{1}{2}\cos\frac{x}{2}\cos 2y dx - 2\sin\frac{x}{2}\sin 2y dy \right)$

   (2) $\int_{(4,0)}^{(6,1)} e^{4y}(2x dx + 4x^2 dy)$

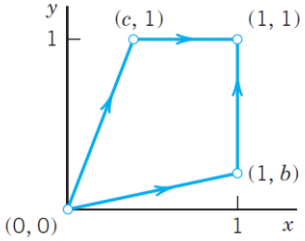
   (3) $\int_{(0,0,\pi)}^{(2,1/2,\pi 2)} e^{xy}(y\sin z dx + x\sin z dy + \cos z dz)$

   (4) $\int_{(0,0,0)}^{(1,1,0)} e^{x^2+y^2+z^2}(x dx + y dy + z dz)$

   (5) $\int_{(0,2,3)}^{(1,1,1)} (yz\sinh(xz)dx + \cosh(xz)dy + xy\sinh(xz)dz)$

2. (a) Show that $I = \int_C (x^2 y dx + 2xy^2 dy)$ is path dependent in the *xy*-plane.

   (b) Integrate from (0,0) along the straight-line segment to (1,*b*), $0 \le b \le 1$, and then vertically up to (1,1); see the figure below. For which *b* is *I* maximum? What is its maximum value?




   (c) Integrate I from (0,0) along the straight-line segment to (c,1), $0 \le c \le 1$, and then horizontally to (1,1). For c=1, do you get the same value as for b=1 in (b)? For which c is I maximum? What is its maximum value?

3. Check, and if independent, integrate from (0,0,0) to (*a*,*b*,*c*).

   (1) $2e^{x^2}(x\cos 2y dx - \sin 2y dy)$

   (2) $\sinh(xy)(z dx - x dz)$

   (3) $x^2 y dx - 4xy^2 dy + 8z^2 x dz$

   (4) $e^y dx + (xe^y - e^z)dy - ye^z dz$