

7. $h=0.2$, $x_i = ih$, $0 \leq i \leq 5$, $\tau = sh = 0.2$, $t_k = k\tau$, $0 \leq k \leq 3$.

$$u_i^0 = \sin \pi x_i, \quad 0 \leq i \leq 5;$$

$$u_0^k = 0, \quad u_5^k = 0, \quad 1 \leq k \leq 3$$

$$u_i^1 = u_i^0 + \tau \frac{\partial u(x_i, 0)}{\partial t} = u_i^0 + \tau x_i(1-x_i), \quad 1 \leq i \leq 4.$$

(I) 显格式

$$\frac{1}{\tau^2} (u_i^{k+1} - 2u_i^k + u_i^{k-1}) - \frac{1}{h^2} (u_{i+1}^k - 2u_i^k + u_{i-1}^k) = 0, \quad 1 \leq i \leq 4, \quad 1 \leq k \leq 2.$$

计算得

$\begin{matrix} u_i^k \\ k \backslash i \end{matrix}$	1	2	3	4
0	0.5877853	0.5037611	0.2235402	-0.1397310
1	0.9510565	0.8113255	0.3640301	-0.2237552
2	0.9510565	0.813255	0.3640301	-0.2237552
3	0.5877853	0.5037611	0.2235402	-0.1397310

(II) 隐格式

$$\frac{1}{\tau^2} (u_i^{k+1} - 2u_i^k + u_i^{k-1}) - \frac{1}{2h^2} (u_{i+1}^{k+1} - 2u_i^{k+1} + u_{i-1}^{k+1} + u_{i+1}^{k-1} - 2u_i^{k-1} + u_{i-1}^{k-1}) = 0,$$

$1 \leq i \leq 4, \quad 1 \leq k \leq 2.$

改写为

$$\begin{pmatrix} 2 & -\frac{1}{2} & 0 & 0 \\ -\frac{1}{2} & 2 & -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & 2 & -\frac{1}{2} \\ 0 & 0 & -\frac{1}{2} & 2 \end{pmatrix} \begin{pmatrix} u_1^{k+1} \\ u_2^{k+1} \\ u_3^{k+1} \\ u_4^{k+1} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} u_1^k \\ u_2^k \\ u_3^k \\ u_4^k \end{pmatrix} + \begin{pmatrix} -2 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & -2 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & -2 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & -2 \end{pmatrix} \begin{pmatrix} u_1^{k-1} \\ u_2^{k-1} \\ u_3^{k-1} \\ u_4^{k-1} \end{pmatrix}, \quad 1 \leq k \leq 2$$

计算得

$$u_1^2 = 0.25681, \quad u_2^2 = 0.41223, \quad u_3^2 = 0.41223, \quad u_4^2 = 0.25681;$$

$$u_1^3 = -0.0737, \quad u_2^3 = -0.11833, \quad u_3^3 = -0.11833, \quad u_4^3 = -0.0737.$$