3. 在结点 (x_i, t_k) 处考虑微分方程，有

$$\frac{\partial u}{\partial t}(x_i, t_k) = a\frac{\partial^2 u}{\partial x^2}(x_i, t_k) + b\frac{\partial u}{\partial x}(x_i, t_k) + c\,u(x_i, t_k).$$

由微分公式可建立如下显式差分格式

$$\frac{u_i^{k+1} - u_i^k}{\tau} = a\cdot\frac{u_{i+1}^k - 2u_i^k + u_{i-1}^k}{h^2} + b\cdot\frac{u_{i+1}^k - u_{i-1}^k}{2h} + c\,u_i^k.$$

截断误差

$$R_i^k = \frac{u(x_i, t_{k+1}) - u(x_i, t_k)}{\tau} - a\cdot\frac{u(x_{i+1}, t_k) - 2u(x_i, t_k) + u(x_{i-1}, t_k)}{h^2}$$

$$-b\cdot\frac{u(x_{i+1}, t_k) - u(x_{i-1}, t_k)}{2h} - c\,u(x_i, t_k)$$

$$= \frac{\partial u(x_i, t_k)}{\partial t} + \frac{\tau}{2}\frac{\partial^2 u(x_i, \eta_i^k)}{\partial t^2} - a\left[\frac{\partial^2 u(x_i, t_k)}{\partial x^2} + \frac{h^2}{12}\frac{\partial^4 u(\xi_i^k, t_k)}{\partial x^4}\right]$$

$$-b\left[\frac{\partial u(x_i, t_k)}{\partial x} + \frac{h^2}{6}\frac{\partial^3 u(\tilde{\xi}_i^k, t_k)}{\partial x^3}\right] - c\,u(x_i, t_k)$$

$$= \frac{\tau}{2}\frac{\partial^2 u(x_i, \eta_i^k)}{\partial t^2} - \frac{h^2}{12}\left[a\frac{\partial^4 u(\xi_i^k, t_k)}{\partial x^4} + 2b\frac{\partial^3 u(\tilde{\xi}_i^k, t_k)}{\partial x^3}\right],$$

其中 $\eta_i^k \in (t_k, t_{k+1})$，$\xi_i^k \in (x_{i-1}, x_{i+1})$，$\tilde{\xi}_i^k \in (x_{i-1}, x_{i+1})$.