

# Renewable Energy and Distributed Generation

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## Doubly Fed Induction Generator WECS

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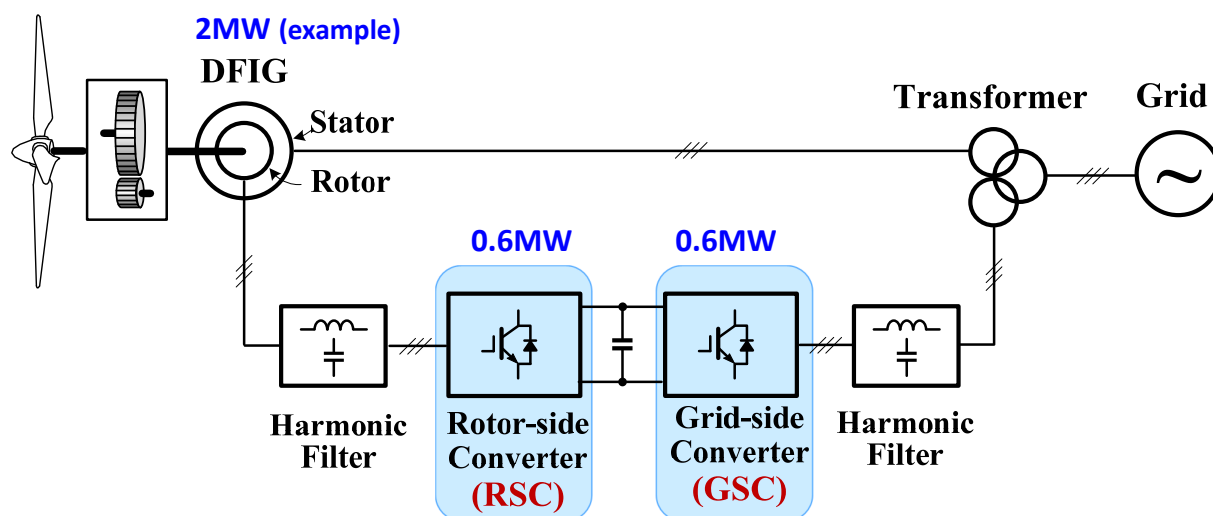
## DFIG WECS

### Main Topics

1. Introduction
2. Super- and Sub-synchronous Modes of Operation
3. DFIG Steady State Model
4. Steady State Analysis
5. Stator Voltage Oriented Control of DFIG WECS
6. DFIG WECS Start-up and Experiments

# Introduction

## System Configuration



Block diagram of DFIG WECS

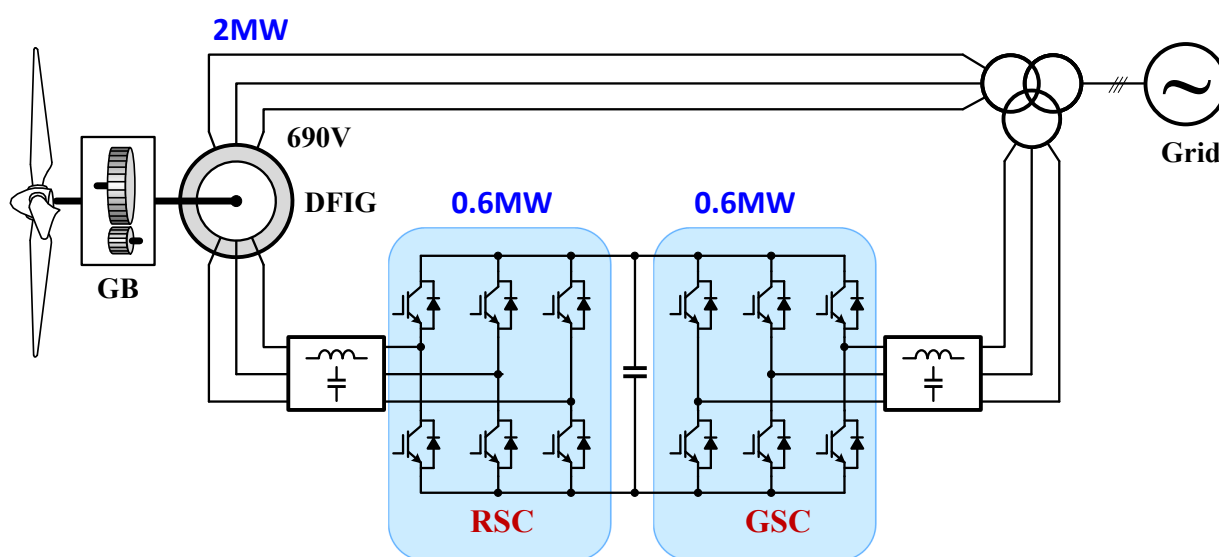
### Main Feature:

The converter power rating is only around 30% of the generator rated power

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# Introduction

## System Configuration



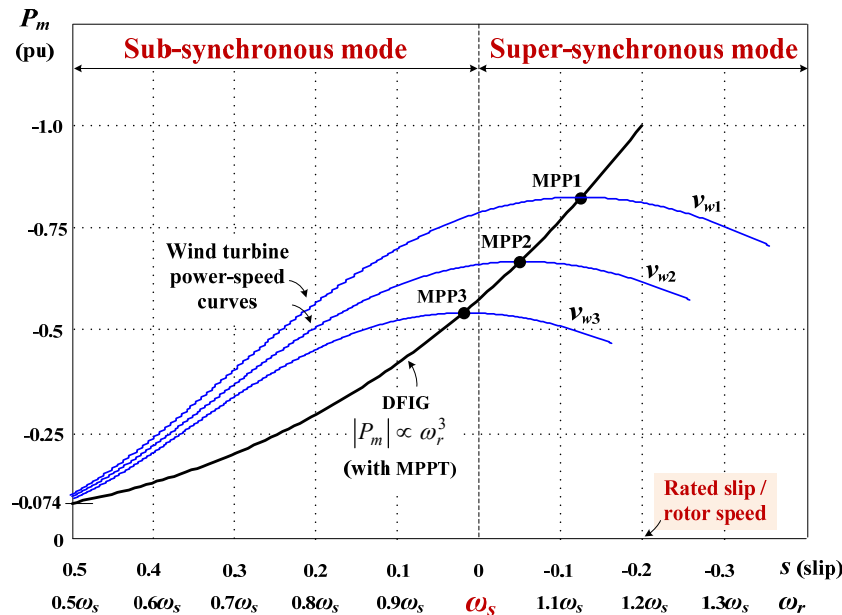
DFIG WECS with 2-level VSC

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# Super- and Sub-synchronous Operation

## Power-speed Characteristics

- Rotor speed range:  $(0.7 \sim 1.3) \omega_s$
- $\omega_s$  is the synchronous speed of the generator



Sub-synchronous speed operation:

$$\omega_r < \omega_s$$

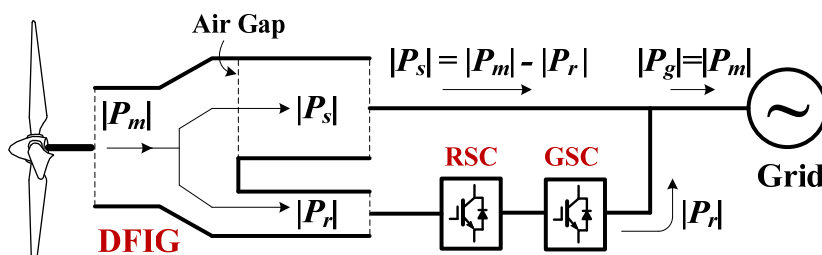
Super-synchronous speed operation:

$$\omega_r > \omega_s$$

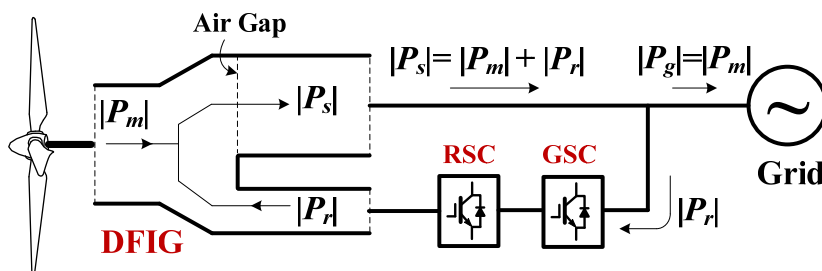
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# Super- and Sub-synchronous Operation

## Power Flow in DFIG WECS



(a) Super-synchronous mode



(b) Sub-synchronous mode

Assumptions:  
Power losses in DFIG and converters are ignored.

$P_m$  – mechanical power  
 $P_s$  – stator power  
 $P_r$  – rotor power  
 $P_g$  – grid power

$$Q: |P_s| = |P_m| + |P_r|$$

Is it too much for the stator?

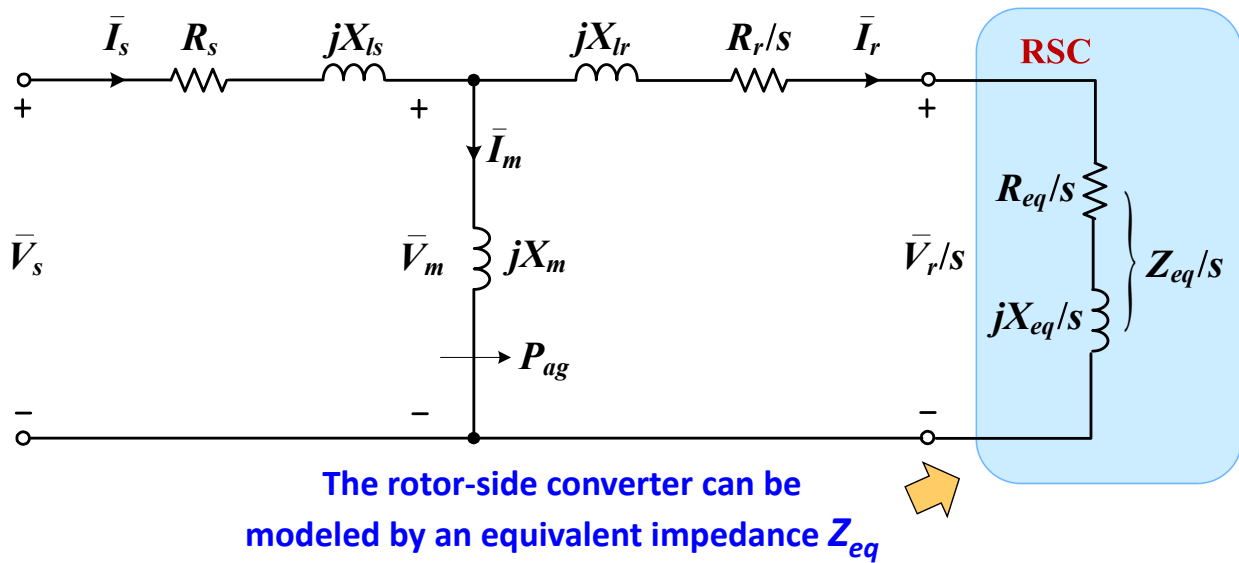
Note: Power flow in the rotor and converters are bidirectional

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# DFIG Steady State Model

## Equivalent Impedance of RSC

DFIG Steady-state Equivalent Circuit



Q: How to find  $R_{eq}$  and  $X_{eq}$ ?

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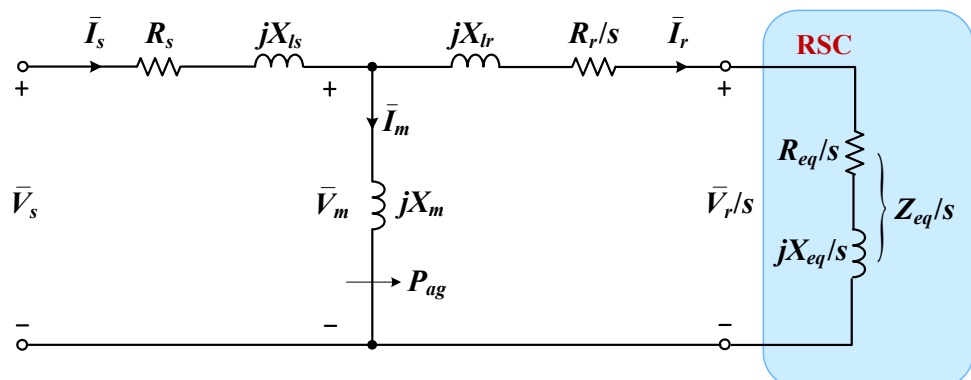
# DFIG Steady State Model

## Equivalent Impedance of RSC

Q: How to find  $R_{eq}$  and  $X_{eq}$ ?

**Note:**

The stator is connected to the grid.



**Step 1** – Find  $I_s$  given  $V_s$ ,  $\omega_s$  and  $T_m$  (or  $P_m$ )

**Step 2** – Find  $V_m$  and  $I_m$

**Step 3** – Find  $I_r$  and  $V_r$

**Step 4** – Find  $R_{eq}$  and  $X_{eq}$

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# DFIG Steady State Model

## Equivalent Impedance of RSC

The equivalent impedance of the converter is defined by

$$\bar{Z}_{eq} = R_{eq} + jX_{eq} = R_{eq} + j\omega_{sl}L_{eq}$$

Dividing by the slip yields:

$$\bar{Z}_{eq} / s = R_{eq} / s + j\omega_{sl}L_{eq} / s = R_{eq} / s + j\omega_s L_{eq}$$

where  $\omega_{sl} = s\omega_s$

### Step 1 – Find the stator current $I_s$

Assuming that the stator operates at a **unity power factor**, the air-gap power is given by

$$P_{ag} = 3V_m I_s = 3(V_s - I_s R_s) I_s \quad (\text{from the equivalent circuit})$$

The air-gap power can also be defined by  $P_{ag} = \frac{\omega_s T_m}{P}$  (from the machine theory)

Combining both:  $\frac{\omega_s T_m}{P} = 3(V_s - I_s R_s) I_s \Rightarrow R_s I_s^2 - V_s I_s + \frac{\omega_s T_m}{3P} = 0$

# DFIG Steady State Model

## Equivalent Impedance of RSC

Solving for  $I_s$ :

$$R_s I_s^2 - V_s I_s + \frac{\omega_s T_m}{3P} = 0 \Rightarrow I_s = \frac{V_s \pm \sqrt{V_s^2 - \frac{4R_s \omega_s T_m}{3P}}}{2R_s}$$

### Step 2 – Find $V_m$ and $I_m$

The voltage across the magnetizing branch is

$$\bar{V}_m = \bar{V}_s - \bar{I}_s (R_s + j\omega_s L_s)$$

where  $\bar{V}_s = V_s \angle 0^\circ$  and  $\bar{I}_s = I_s \angle 180^\circ$  **Note:** The stator current angle of  $180^\circ$  is due to the generating mode and unity power factor operation.

The magnetizing current is

$$\bar{I}_m = \frac{\bar{V}_m}{j\omega_s L_m}$$

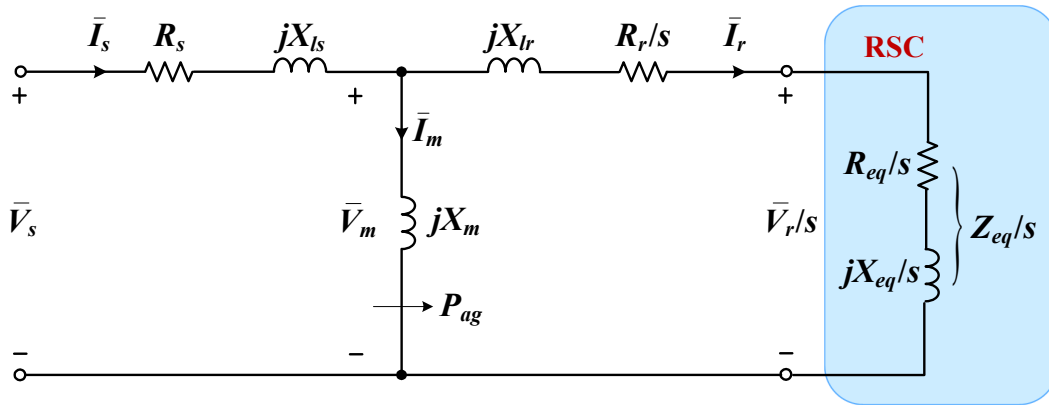
# DFIG Steady State Model

## Equivalent Impedance of RSC

Step 3 – Find  $\bar{I}_r$  and  $\bar{V}_r$

$$\bar{I}_r = \bar{I}_s - \bar{I}_m$$

$$\bar{V}_r / s = \bar{V}_m - \bar{I}_r \left( \frac{R_r}{s} + j\omega_s L_{lr} \right) \Rightarrow \bar{V}_r = s \bar{V}_m - \bar{I}_r (R_r + js\omega_s L_{lr})$$



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# DFIG Steady State Model

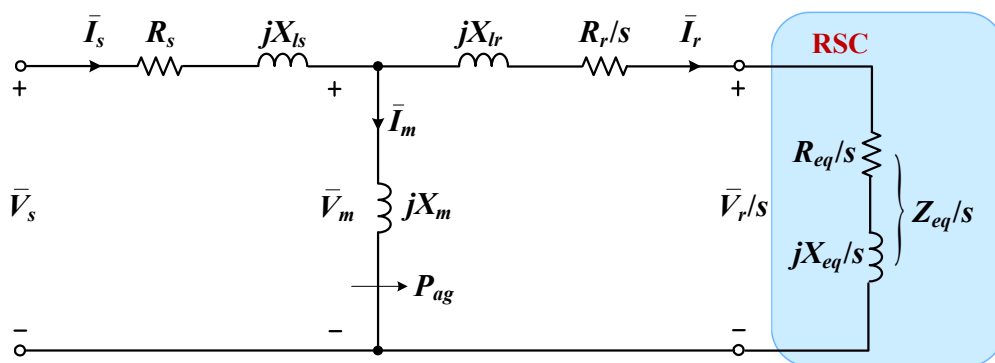
## Equivalent Impedance of RSC

Step 4 – Find  $R_{eq}$  and  $X_{eq}$

$$\frac{\bar{V}_r / s}{\bar{I}_r} = R_{eq} / s + jX_{eq} / s$$

from which

$$R_{eq} + jX_{eq} = \frac{\bar{V}_r}{\bar{I}_r}$$



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# DFIG Steady State Model

## Case Study 1 – Equivalent Impedance of RSC

Consider:

- DFIG: 1.5MW/690V/1068A/50Hz/1750rpm
- Parameters given in Table B-5 of Appendix B
- Generator operates with an MPPT scheme
- Mechanical torque  $T_m$  is proportional to the square of the rotor speed
- Unity stator power factor operation

Q: Number of pole pairs?

A:  $P = 2$

**Investigate** relationship between the rotor voltage, rotor current, and the equivalent impedance of the rotor-side converter when the DFIG operates at super-synchronous speed of 1750rpm

# DFIG Steady State Model

## Case Study 1 – Equivalent Impedance of RSC

DFIG operating at super-synchronous speed (1750 rpm, rated)

Stator current

Q: Synchronous speed?

A: 1500rpm.

$$I_s = \frac{V_s \pm \sqrt{V_s^2 - \frac{4R_s T_m \omega_s}{3P}}}{2R_s} = -1068.2 \text{ A} \quad (\text{the other solution } I_s = 1.514 \times 10^5 \text{ A omitted})$$

Rated torque due to the rated rotor speed of 1750rpm

where

$$V_s = 690 / \sqrt{3} \text{ V}, \quad T_m = -8185.1 \text{ N.m}, \quad \omega_s = 2\pi \times 50 \text{ rad/sec}, \quad R_s = 6.25 \text{ m}\Omega \text{ and } P = 2.$$

Voltage across the magnetizing branch

$$\begin{aligned} \bar{V}_m &= \bar{V}_s - \bar{I}_s (R_s + j\omega_s L_{ls}) \\ &= 690 / \sqrt{3} \angle 0^\circ - 1068.2 \angle 180^\circ \times (0.00625 + j100\pi \times 0.1687 \times 10^{-3}) \\ &= 401.2 + j56.6 = 405.2 \angle 8^\circ \text{ V} \end{aligned}$$

# DFIG Steady State Model

## Case Study 1 – Equivalent Impedance of RSC

**Magnetizing current**

$$\bar{I}_m = \frac{\bar{V}_m}{j\omega_s L_m} = 32.92 - j233.26 = 235.6 \angle -82.0^\circ \text{ A}$$

**The rotor current**

$$\bar{I}_r = \bar{I}_s - \bar{I}_m = -1101.1 + j233.26 = 1125.6 \angle 168.0^\circ \text{ A}$$

**The rotor voltage**

$$\bar{V}_r = s\bar{V}_m - \bar{I}_r(R_r + js\omega_s L_{lr}) = 67.97 \angle -164.9^\circ$$

**where the slip is**  $s = (\omega_s - \omega_r) / \omega_s = -0.1667$

**Finally, the equivalent impedance can be calculated by**

$$\bar{Z}_{eq} = \bar{V}_r / \bar{I}_r = 0.05375 \Omega + j0.2751 \Omega \Rightarrow \begin{cases} R_{eq} = 0.05375 \Omega \\ X_{eq} = 0.02751 \Omega \end{cases}$$

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# DFIG Steady State Model

## Case Study 1 – Equivalent Impedance of RSC

**Question:** When the DFIG operates at the synchronous speed (1500 rpm), how to calculate the mechanical torque?

Rated  
mechanical  
speed
Rated  
mechanical  
torque

↓
↓

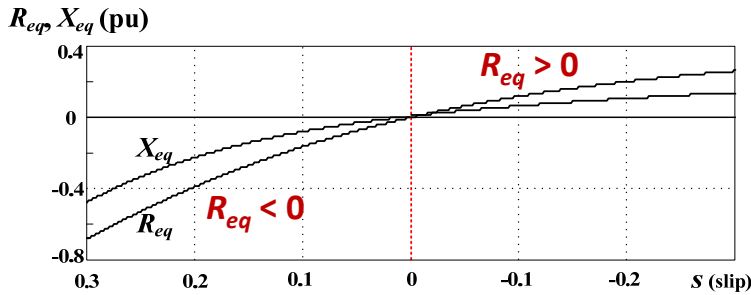
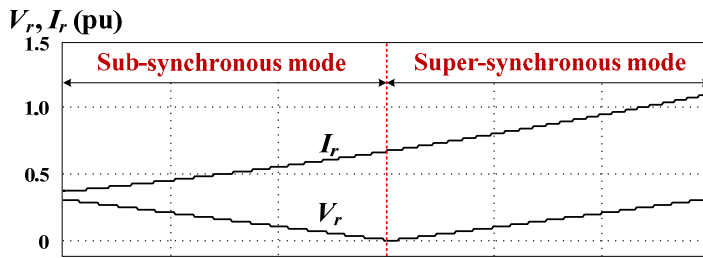
$$T_m = -(1500/1750)^2 \times 8.1851 = -6.0135 \text{ kN.m} \quad \text{Why?}$$

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# DFIG Steady State Model

## Case Study 1 – Equivalent Impedance of RSC



**Q:** Physical meaning of positive and negative  $R_{eq}$  from the power flow point of view ?

**A:**

$R_{eq} > 0 \rightarrow$  rotor delivers power to the grid, super-synch mode;

$R_{eq} < 0 \rightarrow$  rotor absorbs power from the grid, sub-synch mode;

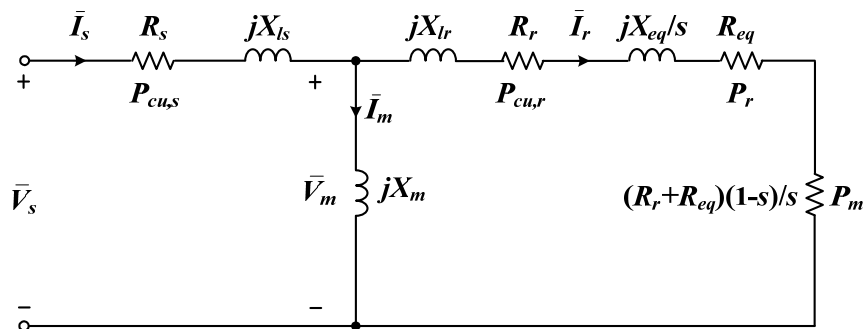
See Fig. 8.3-6 on Slide 70

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## Steady State Analysis

### Steady-state analysis

Consider the steady-state equivalent circuit



Neglecting rotational losses, the transferred and dissipated powers are

$$\begin{cases} P_m = 3I_r^2(R_r + R_{eq})(1-s)/s \\ P_r = 3I_r^2 R_{eq} \\ P_{cu,r} = 3I_r^2 R_r \\ P_{cu,s} = 3I_s^2 R_s \\ P_s = 3V_s I_s \cos \varphi_s \end{cases} \quad \Rightarrow \quad |P_g| = \begin{cases} |P_s| + |P_r| & \text{for super-synchronous mode} \\ |P_s| - |P_r| & \text{for sub-synchronous mode} \end{cases}$$

**Power transferred to the grid**

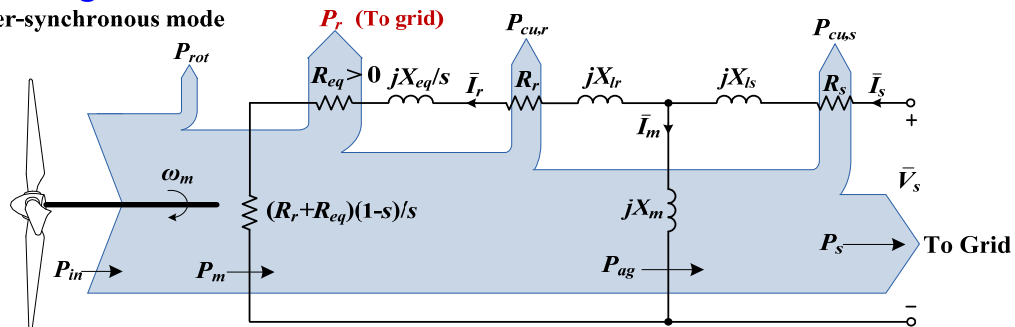
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# Steady State Analysis

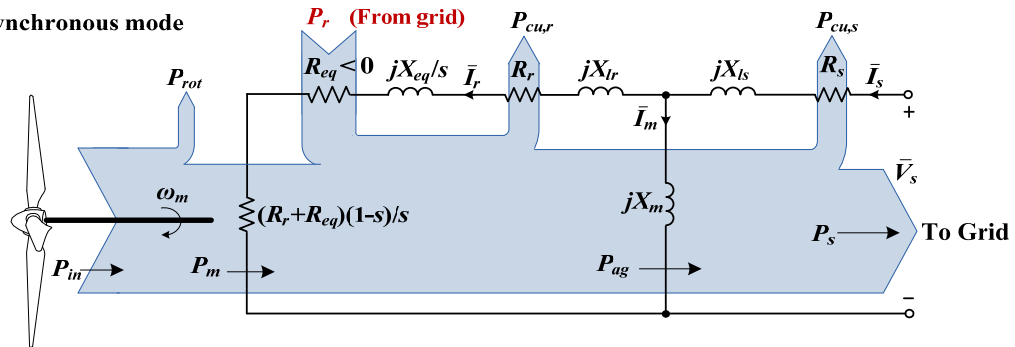
## Steady-state analysis

### Power flow diagrams

(a) Super-synchronous mode



(b) Sub-synchronous mode



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# Steady State Analysis

## Case Study 2 – Steady-state Analysis with $PF_s = 1$

Consider the 1.5MW/690V DFIG operating under the same conditions as those in Case Study 1. Calculate the stator and rotor winding losses and the efficiency of the generator when it operates at the rotor speed of 1750rpm (super-synchronous mode).

With the **rotor current** calculated in Case Study 1, the mechanical power of the DFIG can be calculated by

$$\begin{aligned}
 P_m &= 3I_r^2(R_{eq} + R_r)(1-s)/s \quad \text{(from the equivalent circuit)} \\
 &= 3 \times 1125.6^2(0.05375 + 0.00263)(1 + 0.1667)/(-0.1667) \\
 &= -1500 \text{ kW}
 \end{aligned}$$

Alternatively,

$$P_m = T_m \omega_m = -8185.1 \times 1750 \times 2\pi / 60 = -1500 \text{ kW} \quad \text{(from the machine theory)}$$

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# Steady State Analysis

## Case Study 2 – Steady-state Analysis with $PF_s = 1$

The rotor power

$$P_r = 3(I_r)^2 R_{eq} = 3 \times 1125.6 \times 0.05375 = 204.29 \text{ kW}$$

The rotor and stator winding losses

$$P_{cu,r} = 3(I_r)^2 R_r = 10.0 \text{ kW}$$

$$P_{cu,s} = 3(I_s)^2 R_s = 9.07 \text{ kW}$$

Generating mode  
with unity PF

The stator active power

$$P_s = 3V_s I_s \cos \varphi_s = 690 / \sqrt{3} \times 1068.2 \times \cos(180^\circ) = -1276.64 \text{ kW}$$

Total power delivered to the grid

$$|P_g| = |P_s| + |P_r| = 1480.93 \text{ kW} \quad \Rightarrow$$

Efficiency:

$$\eta = P_g / P_m = 1480.93 / 1500 = 98.7\%$$

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# Steady State Analysis

## Case Study 2 – Steady-state Analysis with $PF_s = 1$

Simplified calculations

In large wind generators, the stator resistance is small ( $< 0.01\text{pu}$ ).

Assumption:  $R_s \approx 0$

The air-gap power

$$P_{ag} \approx P_s = 3 V_s I_s \quad (PF_s = 1)$$

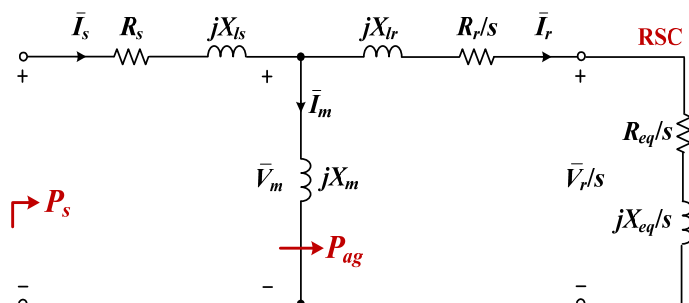
From electric machine theory:

$$P_{ag} = T_m \omega_s / P$$

from which the stator current can be calculated by

$$I_s = \frac{T_m \omega_s / P}{3 V_s}$$

With  $I_s$  calculated, all other analysis can be performed the same way as shown in Case Study 1.



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# Steady State Analysis

## Case Study 2 – Steady-state Analysis

Calculations with leading and lagging power factor

With leading or lagging PF, the stator current can be calculated by

$$I_s = \frac{T_m \omega_s / P}{3V_s \cos \varphi_s} \quad \leftarrow \text{Leading or lagging PF}$$

Following the same procedures given in Case Study 1, the calculated results with leading and lagging PF are as follows:

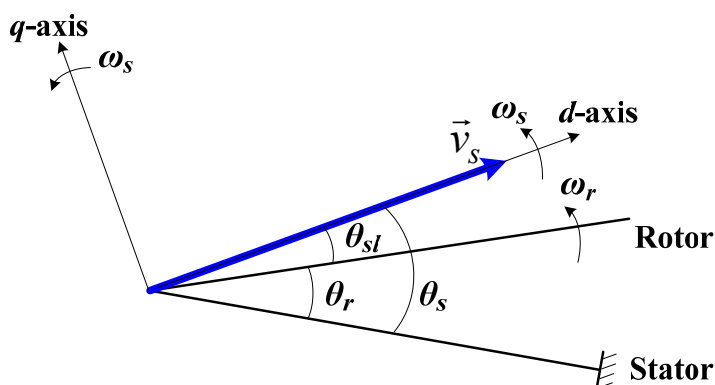
Stator PF	$\omega_m$ (rpm)	1200
	$s$ (Slip)	0.2
Leading PF (0.95) $\varphi_s = -161.8^\circ$	$\bar{V}_r$ [V]	$86.89 \angle 5.7^\circ$
	$\bar{I}_r$ [A]	$659.3 \angle 142.2^\circ$
	$R_{eq}$ [ $\Omega$ ]	-0.0957
	$X_{eq}$ [ $\Omega$ ]	-0.0906
Lagging PF (0.95) $\varphi_s = 161.8^\circ$	$\bar{V}_r$ [V]	$80.65 \angle 6.9^\circ$
	$\bar{I}_r$ [A]	$525.2 \angle 173.3^\circ$
	$R_{eq}$ [ $\Omega$ ]	-0.1493
	$X_{eq}$ [ $\Omega$ ]	-0.0360

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## Stator Voltage Oriented Control

### Principles of SVOC

- The stator of the DFIG is directly connected to the grid.
- Since stator voltage is fixed by the grid, it is convenient to use voltage oriented control (VOC) instead of field oriented control (FOC)
- Space vector diagram



Stator voltage vector aligned with the d-axis of the synch frame:

$$v_{qs} = 0$$

$$v_{ds} = v_s$$

Speed of the rotating frame:

$$\omega_s = 2\pi f_s$$

Slip angle:

$$\theta_{sl} = \theta_s - \theta_r$$

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# Stator Voltage Oriented Control

## Principles of SVOC

By definition, the active and reactive power are

$$\begin{cases} P_s = \frac{3}{2}(v_{ds}i_{ds} + v_{qs}i_{qs}) \\ Q_s = \frac{3}{2}(v_{qs}i_{ds} - v_{ds}i_{qs}) \end{cases}$$

With  $v_{qs} = 0$  for SVOC, 
$$\begin{cases} P_s = \frac{3}{2}v_{ds}i_{ds} \\ Q_s = -\frac{3}{2}v_{ds}i_{qs} \end{cases} \quad (8.64)$$

Substituting (8.55) into (8.64) yields

$$\begin{cases} P_s = \frac{3}{2}v_{ds}\left(\frac{\lambda_{ds} - L_m i_{dr}}{L_s}\right) \\ Q_s = -\frac{3}{2}v_{ds}\left(\frac{\lambda_{qs} - L_m i_{qr}}{L_s}\right) \end{cases} \quad (8.65)$$

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# Stator Voltage Oriented Control

## Principles of SVOC

From (8.65)

$$\begin{cases} i_{dr} = -\frac{2L_s}{3v_{ds}L_m}P_s + \frac{1}{L_m}\lambda_{ds} \\ i_{qr} = \frac{2L_s}{3v_{ds}L_m}Q_s + \frac{1}{L_m}\lambda_{qs} \end{cases} \quad (8.66)$$

From (8.66)

$$i_{qr} = \frac{2L_s}{3v_{ds}L_m}Q_s - \frac{v_{ds}}{\omega_s L_m} \quad (b) \quad (8.68)$$

**Note:** The stator active and reactive power can be controlled through rotor currents and voltages.

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# Stator Voltage Oriented Control

## Case Study 3 – Steady-state analysis of SVOC

With the same parameters of Case Study 1 and at 0.7pu wind speed

The mechanical torque of the generator is

$$T_m = -(\omega_{m,pu})^2 \times T_{m,R} = (0.7)^2 \times 8185 = -4010.7 \text{ N.m}$$

The stator current can be calculated by

$$I_s = \frac{V_s \pm \sqrt{V_s^2 - \frac{4R_s T_m \omega_s}{3P}}}{2R_s} = -525.3 \text{ A} \quad (I_s = 1.509 \times 10^5 \text{ A omitted})$$

where

$$V_s = 690 / \sqrt{3} \text{ V}, \quad \omega_s = 2\pi \times 50 \text{ rad/sec}, \quad R_s = 6.25 \text{ m}\Omega \text{ and } P = 2.$$

The voltage of the magnetizing branch is

$$\begin{aligned} \bar{V}_m &= \bar{V}_s - \bar{I}_s (R_s + j\omega_s L_{ls}) \\ &= 690 / \sqrt{3} \angle 0^\circ - 525.3 \angle 180^\circ \times (0.00625 + j100\pi \times 0.1687 \times 10^{-3}) \\ &= 399.8 + j27.8 = 400.7 \angle 4.0^\circ \text{ V} \end{aligned}$$

# Stator Voltage Oriented Control

## Case Study 3 – Steady-state analysis of SVOC

The magnetizing current is calculated by

$$\bar{I}_m = \frac{\bar{V}_m}{j\omega_s L_m} = 16.19 - j232.42 = 233.0 \angle -86.0^\circ$$

The rotor current is

$$\bar{I}_r = \bar{I}_s - \bar{I}_m = -541.50 + j232.42 = 589.27 \angle 156.8^\circ$$

The rotor voltage is determined by

$$\bar{V}_r = s\bar{V}_m - \bar{I}_r (R_r + js\omega_s L_{lr}) = 76.99 \angle 6.5^\circ$$

where  $s = (\omega_s - \omega_r) / \omega_s = 0.1833$

Then the slip frequency at the 0.7 pu rotor speed is

$$\omega_{sl} = s\omega_s = 0.1833 \times 2\pi \times 50 = 57.58 \text{ rad/sec} \quad (9.165 \text{ Hz})$$



# Stator Voltage Oriented Control

## Case Study 3 – Steady-state analysis of SVOC

The peak values of the rotor voltage and rotor current are

$$\begin{cases} i_r = \sqrt{2} I_r = \sqrt{2} \times 589.27 = 833.25 \text{ A} & (0.664 \text{ pu}) \\ v_r = \sqrt{2} V_r = \sqrt{2} \times 76.99 = 108.9 \text{ V} & (0.273 \text{ pu}) \end{cases}$$

The rotor power factor angle is

$$\varphi_r = 6.5^\circ - 156.8^\circ = -150.3^\circ$$

The equivalent impedance for the rotor-side converter is

$$\bar{Z}_{eq} = \bar{V}_r / \bar{I}_r = -0.11351 \Omega + j - 0.06472 \Omega$$

from which

$$\begin{cases} R_{eq} = -0.11351 \Omega \\ X_{eq} = -0.06472 \Omega \end{cases}$$

# Stator Voltage Oriented Control

## Case Study 3 – Steady-state analysis of SVOC

The mechanical power of the generator is

$$\begin{aligned} P_m &= 3 I_r^2 (R_{eq} + R_r) (1-s) / s \\ &= 3 \times 589.27^2 (-0.11351 + 0.00263) (1 + 0.1833) / (-0.1833) = -514.50 \text{ kW} \end{aligned}$$

Alternatively,

$$P_m = T_m \omega_m = -4010.7 \times 1750 \times 0.7 \times 2\pi / 60 = -514.50 \text{ kW}$$

The rotor power is

$$P_r = 3 (I_r)^2 R_{eq} = 3 \times 589.27^2 \times (-0.11351) = -118.24 \text{ kW}$$

The rotor and stator winding losses are

$$P_{cu,r} = 3 (I_r)^2 R_r = 2.74 \text{ kW}, \quad P_{cu,s} = 3 (I_r)^2 R_s = 2.19 \text{ kW}$$

The stator active power is

$$P_s = 3 V_s I_s \cos \varphi_s = 690 / \sqrt{3} \times 589.27 \times \cos(180^\circ) = -627.81 \text{ kW}$$

**Note:** the DFIG operates as a generator with unity PF

# Stator Voltage Oriented Control

## Case Study 3 – Steady-state analysis of SVOC

The total power delivered to the grid is

$$|P_g| = |P_s| - |P_r| = 627.81 - 118.24 = 509.57 \text{ kW}$$

The efficiency of the DFIG is then

$$\eta = |P_g| / |P_m| = 509.57 / 514.50 = 99.0\%$$

The grid current is calculated by

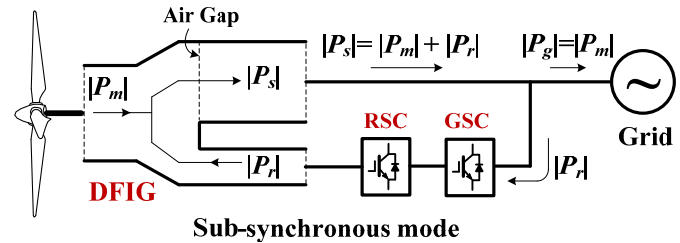
$$|I_g| = \frac{|P_g|}{3V_g} = \frac{509.6 \times 10^3}{3 \times 398.4} = 426.37 \text{ A}$$

The peak grid current is

$$i_g = \sqrt{2} |I_g| = \sqrt{2} \times 426.37 = 603.0 \text{ A} \quad (0.48 \text{ pu})$$

The peak stator current is

$$i_s = \sqrt{2} |I_s| = \sqrt{2} \times 525.3 = 742.9 \text{ A} \quad (0.59 \text{ pu})$$



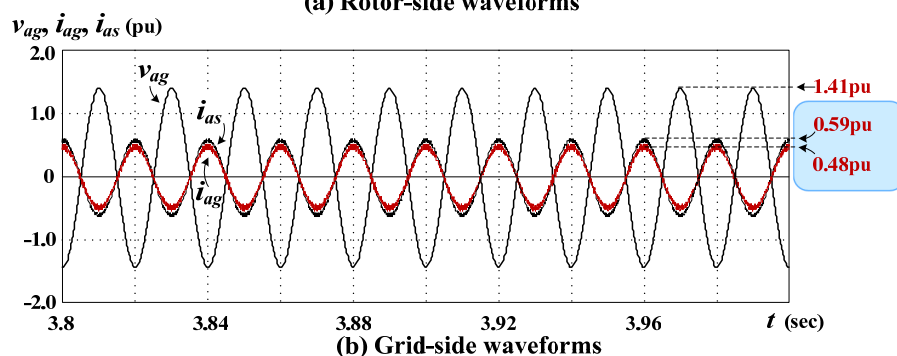
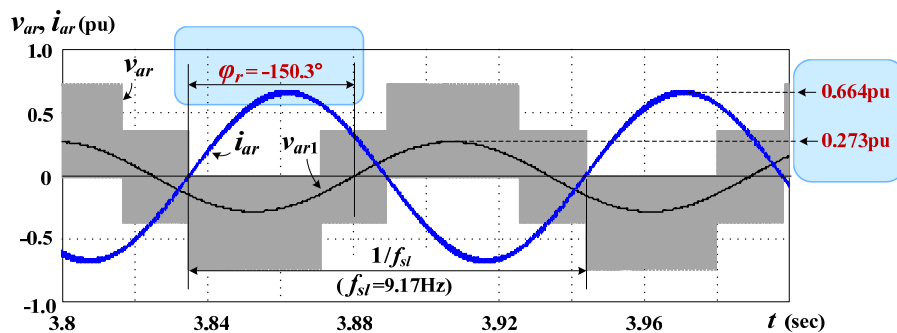
**Q:** Why is the grid current lower than stator current?

**A:** Because the grid power is equal to the stator power minus the rotor power in the sub-synch mode

# Stator Voltage Oriented Control

## Case Study 3 – Steady-state analysis of SVOC

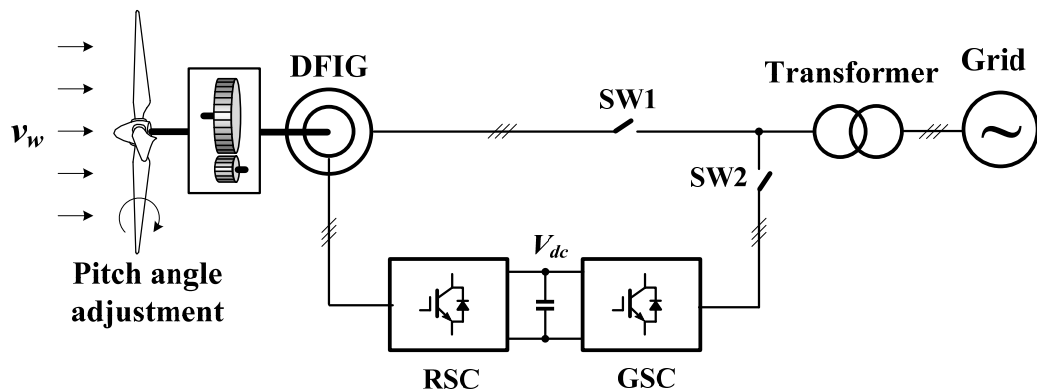
Verifying steady-state values by simulation



# DFIG Startup and Experiments

## Startup Process

System block diagram



Initial parking state

- Switches SW1 and SW2 are open
- Turbine blades are pitched out of the wind
- Turbine does not rotate

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# DFIG Startup and Experiments

## Startup Process

Step 1 – Stator voltage generation

- Mechanic safety brake is released
- Turbine blades are slightly pitched to produce starting torque
- Turbine and generator start to rotate
- SW2 is closed
- dc-link voltage is built by the GSC
- RSC provides excitation to the rotor and a rotating magnetic field is produced
- A 3-phase voltage is generated in the stator
- Torque reference is set to zero → no power is generated

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# DFIG Startup and Experiments

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## Startup Process

### Step 2 – Synchronization

- The generated stator voltage/frequency are monitored and adjusted by RSC to match grid voltage/frequency for synchronization
- SW1 is closed when the synchronization is achieved
- The DFIG WECS is now connected to the grid

# DFIG Startup and Experiments

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## Startup Process

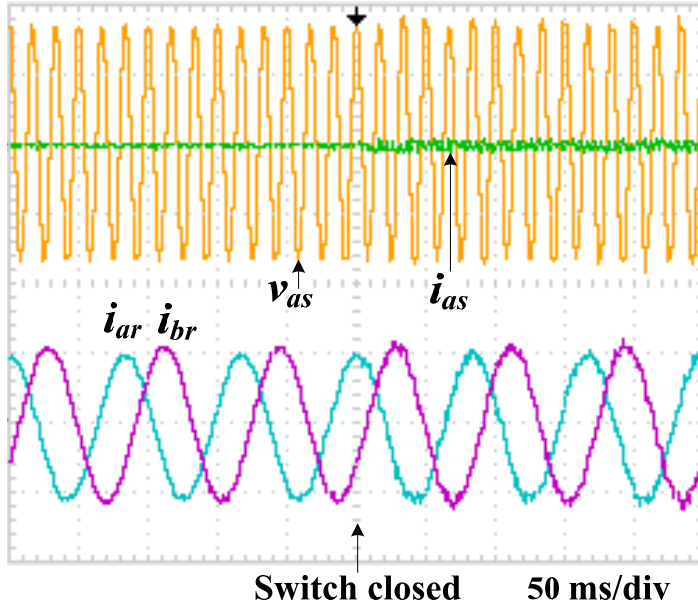
### Step 3 – Power generation

- The torque reference is increased from zero to a value by MPPT
- The blade pitch angle is adjusted to its optimal value
- The DFIG produces power to the grid
- The start-up process is completed

# DFIG Startup and Experiments

## Startup Process – Experimental Results

- Startup process with rotor speed at 0.8 pu (Steps 1 and 2 achieved)
- Results show Step 3 at which SW1 is closed

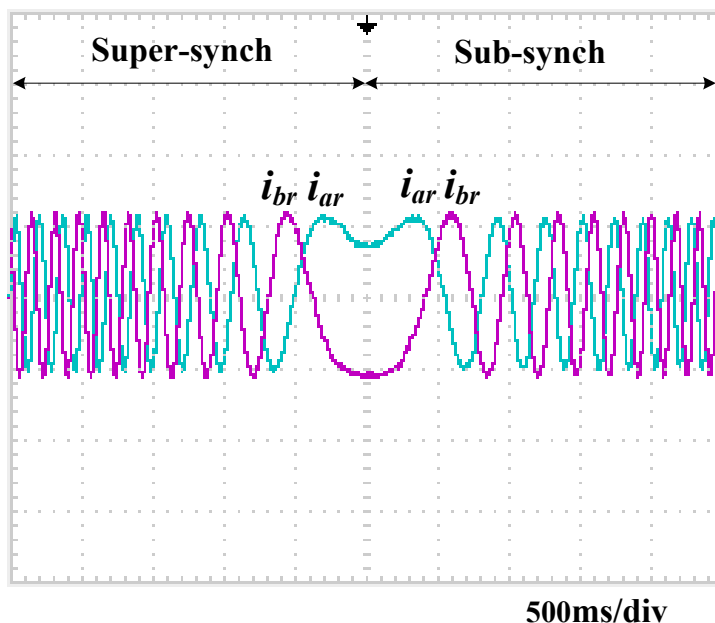


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# DFIG Startup and Experiments

## Startup Process – Experimental Results

Rotor currents during transition from super- to sub-synchronous speed



The rotor currents are DC when crossing synchronous speed

The Phase-*a* rotor current goes from lagging to leading the phase-*b* rotor current like seen in Case Study 4

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# Summary

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## 1. Super- and Sub-synchronous Modes of Operation

Bidirectional power flow in the rotor circuit:

Super-synch operation: power delivered from the rotor to the grid

Sub-synch operation: power delivered from the grid to the rotor

Power flow in the stator circuit: always from the stator to the grid

## 2. DFIG Steady State Model

Derivation of  $R_{eq}$  and  $X_{eq}$  to represent the operation of the rotor-side converter

## 3. Steady State Analysis

Calculation of mechanical power, rotor power, losses, power factor, power delivered to the grid, and efficiency of the DFIG

## 4. Stator Voltage Oriented Control

SVOC principle, equations, steady state analysis, calculations

## 5. DFIG WECS Start-up Process

Discussed the start procedure of a DFIG WECS

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Thanks