WMGCAMB

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1   The flat special WMG Model

The Friedmann equation in the flat special WMG (eqn. (2.33) in [1]) is

$$H^2 = \frac{\mu}{3}\rho_{\mathrm{M}} + 2\left(M^2 H_0^2 - \frac{\mathcal{A}_1 m_4^2}{6}\right) + M H_0 \sqrt{\frac{\mu}{3}\rho_{\mathrm{M}} + N H_0^2}, \tag{1}$$

where the subscript 'M' denotes 'Matter' (including pressureless matter, radiation, neutrinos, etc.), with

$$M = \pm\frac{\mu}{\kappa H_0}, \quad N = \left(\frac{\mu^2}{\kappa^2 H_0^2} - \frac{\mathcal{A}_1 m_4^2}{6 H_0^2}\right)^2 \left(\frac{\mu}{\kappa H_0}\right)^{-2} - \frac{\Lambda_{\mathrm{eff}}}{6 H_0^2}, \quad \mathcal{C} = 0.$$

After taking the time derivative of (1), we get

$$\dot{H} = -\frac{\mu}{2}\left(\rho_{\mathrm{M}} + P_{\mathrm{M}}\right)\left(1 + \frac{M}{\sqrt{\frac{\mu}{3H_0^2}\rho_{\mathrm{M}} + N}}\right), \tag{2}$$

where $\dot{\rho_{\mathrm{M}}} = -3H\left(\rho_{\mathrm{M}} + P_{\mathrm{M}}\right)$ has been used.

2   Modifications in `CAMB`

When performing the perturbation calculations, `CAMB` determines the Hubble parameter H and its time derivative $\dot{H}$ using the relations:

$$H^2 = \frac{\mu}{3}\rho_{\mathrm{total}}, \tag{3}$$

$$\dot{H} = -\frac{\mu}{2}\left(\rho_{\mathrm{total}} + P_{\mathrm{total}}\right). \tag{4}$$

To implement the WMG model in this framework, it is useful to derive the "effective" ρ_{total} and P_{total} that satisfy these equations. Combining (2) and (4), we obtain the expression for the effective total pressure:

$$P_{\mathrm{total}} = \left(\rho_{\mathrm{M}} + P_{\mathrm{M}}\right)\left(1 + \frac{M}{\sqrt{\Omega_{\mathrm{M}}\frac{H^2}{H_0^2} + N}}\right) - \rho_{\mathrm{total}}, \tag{5}$$

or equally,

$$P_{\mathrm{total}} = P_{\mathrm{M}} + \left(\rho_{\mathrm{total}} - \rho_{\mathrm{M}}\right)\left(\frac{M\left(\rho_{\mathrm{M}} + P_{\mathrm{M}}\right)}{\left(\sqrt{\Omega_{\mathrm{M}}\frac{H^2}{H_0^2} + N}\right)\left(\rho_{\mathrm{total}} - \rho_{\mathrm{M}}\right)} - 1\right). \tag{6}$$

Specifically, the modification of ρ_{total} can be found in the Fortran file `DarkEnergyInterface.f90`'s subroutine 'BackgroundDensityAndPressure', where the dark enery density has been modified, contributing to the total energy density. The modification of P_{total} can be found in the Fortran file `Equation.f90`'s subroutines 'dervis' and 'dervisv' when the `gpres` is calculated.

All key modifications related to the WMG model can be identified by searching for the identifier '`!!!!WMG!!!!`' within these two `f90` source files.

References

[1] S. Garcia-Saenz, Y. Wei and X. Zhou, *Cosmology in warped massive gravity*, [2509.09270].