COMPRESSED IMAGE RECOVERY

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Abstract. Sparsity is a common phenomenon in signal processing. This report explores compression and signal recovery in image data. We first used Discrete Cosine Transformation (DST) with different percentile of coefficient testing for quality of compressed image. Then image was recovered with different amount of observed pixels. Finally an unknown image was used to test the algorithm.

1. Introduction and Overview

The goal of this project is to explore the signal compression and recovery using a resized version of the Son of Man image. After the DCT, we keep different percentile of coefficients and set the rest to zero and apply inverse DCT to inspect the quality of compressed image. Image recovery was done by using a small amount of observed pixels and apply Lasso regression to take care of sparsity nature of image data. With the preparation of the steps above, an unknown image is tested and the unknown image is identified as Nyan Cat.

2. Theoretical Background

Given a discrete signal $\mathbf{f} \in \mathbb{R}^K$, we define DCT(f) as

$$F_k = \sqrt{\frac{1}{K}}[f_0 cos(\frac{\pi k}{2K}) + \sqrt{2}\sum_{j=1}^{K-1} f_j cos(\frac{\pi k(2j+1)}{2K})]$$

and define inverse DCT as

$$f_k = \sqrt{\frac{1}{K}}[F_0 cos(\frac{\pi k}{2K}) + \sqrt{2}\sum_{j=1}^{K-1} F_j cos(\frac{\pi k(2j+1)}{2K})]$$

Let $f \in \mathbb{R}^N$ be the flatten version of the image. DCT(f) can be calculated by Df where D is DCT forward matrix constructed using the equation above. The original signal can get back by applying $D^{-1}Df$.

According to [Brunton and Kutz, 2019], in lasso regression, $\hat{\beta}$ is defined as

$$\hat{\beta} = \underset{\beta}{\operatorname{argmin}} \frac{1}{2\sigma^2}\|A\beta - Y\|^2 + \lambda\|\beta\|_1$$

Since we only have a small proportion of data, the problem is underdetermined. However, if we assume β is S-sparse,then we can get a specific solution, and this is the advantage of using L1 norm as the penalty instead of L2. The rigorous proof can be found in relevant research papers.

The question above is equivalent to

$$\min_{\beta}\|\beta\|_1 \ \textit{such that}\ A\beta - Y = 0$$

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as an optimization problem.

Let $B \in \mathbb{R}^{M \times N}$ by randomly selecting M rows from I(n). Let $y = Bf$ be the measurement of original signal f, where $y \in \mathbb{R}^M$. Define $A = BD^{-1} \in \mathbb{R}^{M \times N}$ and solve for x^* using the formation above for optimization problem. The recovered image can be inspected by reshaping the original signal.

3. Algorithm Implementation and Development

The main package used in this project is **Numpy** and **cvxpy**.

The following algorithm illustrates how to compress the image.

Algorithm 1 Image Compression

$DCTf \leftarrow DCT(f)$
$threshold \leftarrow percentile(DCTF, p)$
$DCTf[i] = DCTF[i]\ if\ DCTF[i] > threshold\ else\ 0\ for\ i\ in\ len(DCTf)$
$im \leftarrow D^{-1}DCTF$ reshape to 2D
repeat for different values of p

The following algorithm illustrates how image is recovered.

Algorithm 2 Image Recovery

$B = perm(I(n))[0 : M]$
$y = B * flatten(f)$
$A = BD^{-1}$
solve x defined in theoretical backgroud
reshape $D^{-1}x$ to 2D
repeat with different M

4. Computational Results

5. Summary and Conclusions

Wrap up your report with a brief summary of what you did and what you discovered. Finish with some conclusions and possibly future directions if any.

Acknowledgements

We first present a bar graph of the coefficient of flattened image f after DCT. As we could see in Figure 1, most of the coefficients are zero, which strongly indicating sparsity.

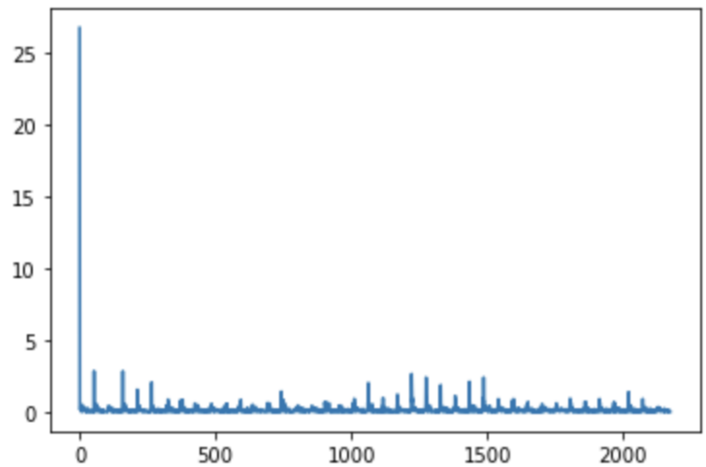

FIGURE 1. absolute value of DCT coefficient

Figure 2 shows compressed image using top 5,10,20,40 percent of DCT coefficients. It is obvious that as we incorporating more coefficients we get clearer compressed image.

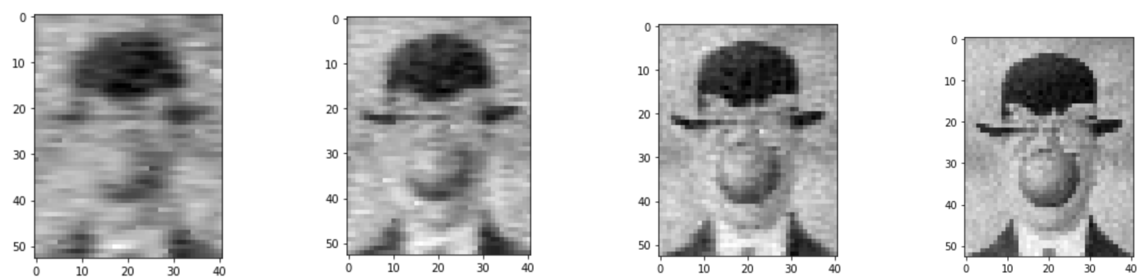

FIGURE 2. compressed image with top 5,10,20,40 DCT coefficients

Figure 3 shows the recovered image using 20 percent of observations.

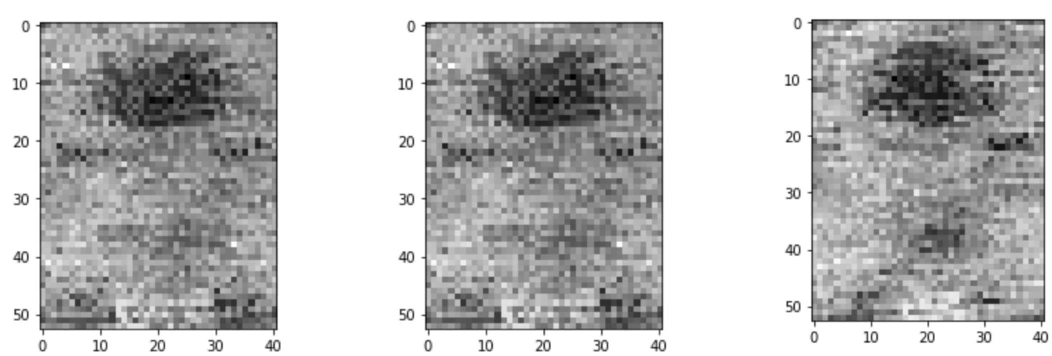

FIGURE 3. Recovered Image using 20 percent of observations

Figure 4 shows the recovered image using 40 percent of observations.

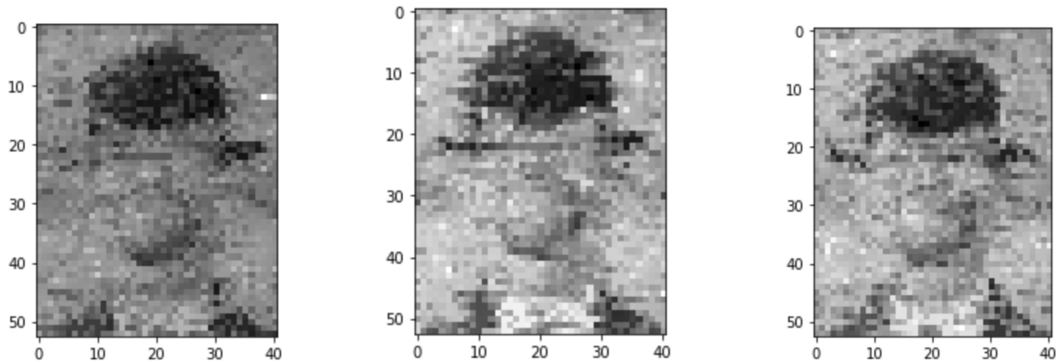

FIGURE 4. Recovered Image using 40 percent of observations

Figure 5 shows the recovered image using 60 percent of observations.

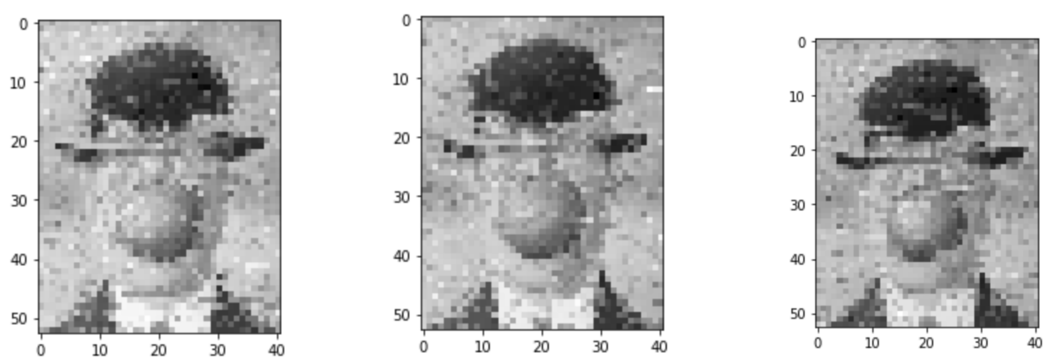

FIGURE 5. Recovered Image using 60 percent of observations

We get slightly different result each time even using the same amount of observations, which is caused by using different subsets of observations to recover the original image. In addition, as we using more observations, we get better recovery of images.

Finally, by applying the technique discussed above to the unknown image, figure 6 shows what it looks like and it appears to be an Nyan cat.

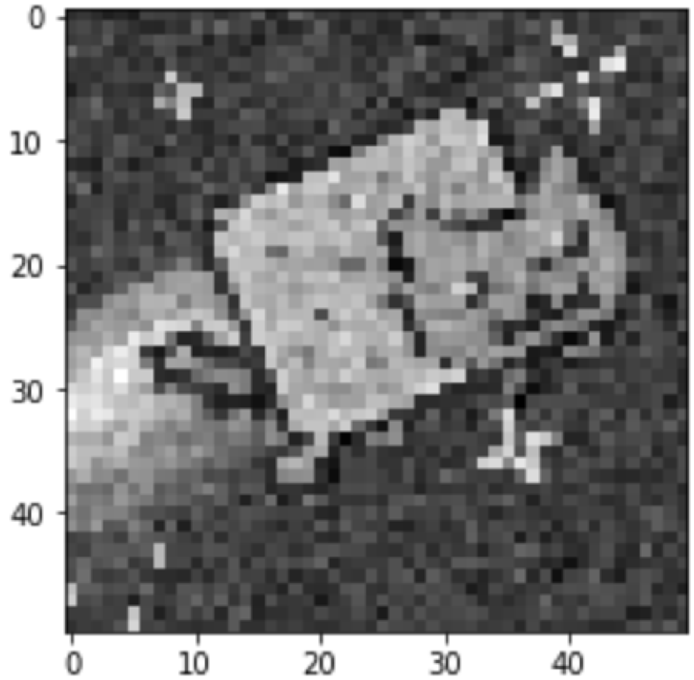

Figure 6. unknown image

6. Summary and Conclusions

In summary, image compression and recovery are explored on resized the son of man image. We see L1 norm is a powerful tool to solving underdetermined sparse optimization problem.One potential drawback is that recovering the signal using more observations makes computation expensive.Future work can be explored on how to speed up the optimization process.

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References

[Brunton and Kutz, 2019] Brunton, S. and Kutz, J. (2019). *Data-Driven Science and Engineering: Machine Learning, Dynamical Systems, and Control*. Cambridge University Press.