Description of Frauchiger-Renner example

Nuriya Nurgalieva, Simon Mathis, Lídia del Rio, and Renato Renner

Institute for Theoretical Physics, ETH Zürich, 8049 Zürich, Switzerland

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The aim of this document is to give the reader the idea and intuition behind the Frauchiger-Renner thought experiment [1], which can be used as an example of a possible protocol for software. The step-by-step explanation of the implementation of the protocol can be found in the Jupyter notebooks in this folder: one with the agents using what we call Copenhagen interpretation, and another in the view of the collapse theories. Here we describe the setting in terms of Copenhagen interpretation to pinpoint the arising paradox.

The thought experiment involves four agents, Alice, Bob, Ursula and Wigner, and two qubits R and S (Figure 1). The initial state of the R is $\sqrt{\frac{1}{3}}|0\rangle_R + \sqrt{\frac{2}{3}}|1\rangle_R$.

The agents proceed as follows:

$t = 1$ Alice measures system R in basis $\{|0\rangle_R, |1\rangle_R\}$. She records the result in her memory A, and prepares S accordingly: if she obtains outcome $a = 0$ she keeps her memory in state $|0\rangle_A$ and S in state $|0\rangle_S$; if she obtains outcome $a = 1$ she changes her memory to $|1\rangle_A$ and S to $\frac{1}{\sqrt{2}}(|0\rangle_S + |1\rangle_S)$. Finally, she gives system S to Bob.

$t = 2$ Bob measures system S in basis $\{|0\rangle_S, |1\rangle_S\}$ and records the outcome b in his memory B, similarly to Alice.

$t = 3$ Ursula measures Alice's lab (consisting of R and the memory A) in basis $\{|ok\rangle_{RA}, |fail\rangle_{RA}\}$, where

$$|ok\rangle_{RA} = \sqrt{\frac{1}{2}}(|0\rangle_R|0\rangle_A - |1\rangle_R|1\rangle_A)$$
$$|fail\rangle_{RA} = \sqrt{\frac{1}{2}}(|0\rangle_R|0\rangle_A + |1\rangle_R|1\rangle_A).$$

$t = 4$ Wigner measures Bob's lab (consisting of S and the memory B) in basis $\{|ok\rangle_{SB}, |fail\rangle_{SB}\}$, where

$$|ok\rangle_{SB} = \sqrt{\frac{1}{2}}(|0\rangle_S|0\rangle_B - |1\rangle_S|1\rangle_B)$$
$$|fail\rangle_{SB} = \sqrt{\frac{1}{2}}(|0\rangle_S|0\rangle_B + |1\rangle_S|1\rangle_B).$$

$t = 5$ Ursula and Wigner compare the outcomes of their measurements. If they were both "ok", they halt the experiment. Otherwise, they reset the timer and all systems to the initial conditions, and repeat the experiment.

In the original paper [1] it is shown that agents can come to a contradiction while reasoning about each others' outcomes in the case of Ursula and Wigner both getting outcomes "ok". For a pedagogical introduction look up [2].

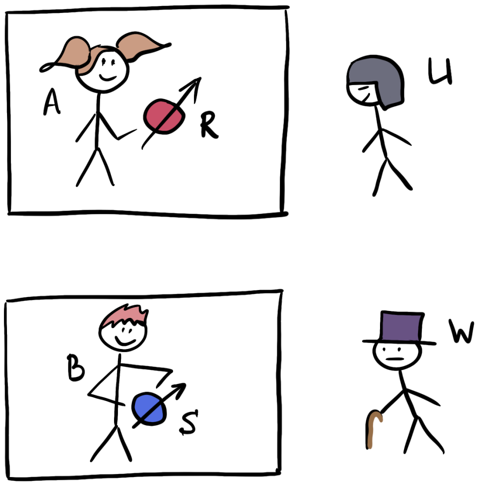

Figure 1: **FR thought experiment** [1]**.** The setting involves four agents: Alice, Bob, Ursula and Wigner, and two qubit systems R and S. Alice measures system R and prepares system S based on her outcome. The system S is then measured by Bob. Ursula and Wigner measure Alice's and Bob's labs respectively. The agents are allowed to make predictions with certainty about each other's outcomes.

Backward inference. FR is a good protocol example where one can examine the machinery of inference-making, both in backward and forward directions. The forward inference making is similar to the process described in the example of Alice and Bob sharing a Bell state. The backward inference making is, for example, Bob at $t = 2$, making a prediction about Alice's outcome at $t = 1$. To do so, Bob models Alice's lab's evolution, and projects the state to one of Alice's memory subspaces, corresponding to her getting outcome 0 or 1. The modelling is then carried out further for each of Alice's outcome options separately until Bob's measurement, where Bob also models his own memory as a qubit. By this, the inference table is constructed by correlating the contents of Alice's and Bob's memory qubits.

- Case $a = 0$:

  1. R is initialized in $\frac{1}{\sqrt{3}}|0\rangle_R + \sqrt{\frac{2}{3}}|1\rangle_R$; Alice measures R and writes the result down to her memory,

  $$\frac{1}{\sqrt{3}}|0\rangle_R|0\rangle_A + \sqrt{\frac{2}{3}}|1\rangle_R|1\rangle_A.$$

  2. Bob projects the state onto the subspace of Alice's memory corresponding to $a = 0$, $|0\rangle_R|0\rangle_A$.
  3. Alice prepares the system S and sends it to Bob, who measures it, writing down the result to his memory,

  $$|0\rangle_R|0\rangle_A|0\rangle_S|0\rangle_B.$$

  4. Now Bob can conclude that if he gets $b = 0$, Alice could have gotten $a = 0$.

- Case $a = 1$:

1. R is initialized in $\frac{1}{\sqrt{3}}|0\rangle_R + \sqrt{\frac{2}{3}}|1\rangle_R$; Alice measures R and writes the result down to her memory,

$$\frac{1}{\sqrt{3}}|0\rangle_R|0\rangle_A + \sqrt{\frac{2}{3}}|1\rangle_R|1\rangle_A.$$

2. Bob projects the state onto the subspace of Alice's memory corresponding to $a = 1$, $|1\rangle_R|1\rangle_A$.
3. Alice prepares the system S and sends it to Bob, who measures it, writing down the result to his memory,

$$\frac{1}{\sqrt{2}}|1\rangle_R|1\rangle_A|0\rangle_S|0\rangle_B + \frac{1}{\sqrt{2}}|1\rangle_R|1\rangle_A|1\rangle_S|1\rangle_B.$$

4. Now Bob can conclude that if he gets $b = 0$, Alice could have gotten $a = 1$; if he gets $b = 1$, Alice could have gotten $a = 1$ as well. Combining these conclusions with the analysis of the previous case, Bob comes to the conclusion that the only outcome Alice could have gotten in the case $b = 1$, is $a = 1$: $b = 1 \Rightarrow a = 1$.

When considering the thought experiment in the (neo-)Copehagen intepretation, the paradox is reproduced in the post-selected case of both Ursula and Wigner getting outcome "ok". In the collapse interpretation module, on the other hand, agents do not come to a contradiction due to the reasoning being in different "branches" of the collapse tree.

The exact assumptions under which this result is derived, can be found in the description file of the folder *Interpretations*.

The users can run the protocol where agents employ modal logic consistency rules, and Copenhagen interpretation in a corresponding Jupyter Notebook in the folder *FrauchigerRennerExample*.

References

[1] Frauchiger, D. & Renner, R. Quantum theory cannot consistently describe the use of itself. *Nature Communications* **9**, 3711 (2018).

[2] Nurgalieva, N. & del Rio, L. Inadequacy of modal logic in quantum settings. *EPCTS* **287**, 267–297 (2019). arXiv:1804.01106.