Simple examples

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Here we describe few simple protocols which can be used to test the rules of consistency and the theory employed for an ability to yield simplest predictions.

Example I: Alice and Bob measure a Bell state.

Setting. Alice and Bob share a Bell pair $(|00\rangle_{RS} + |11\rangle_{RS})/\sqrt{2}$ (Figure 1a). The experiment proceeds as follows:

$t = 1$ Alice measures her qubit R in basis $\{|0\rangle_R, |1\rangle_R\}$, and records the result in her memory A.

$t = 2$ Alice makes a prediction about Bob's outcome.

$t = 3$ Bob measures his qubit S in basis $\{|0\rangle_S, |1\rangle_S\}$, and records the result in his memory B.

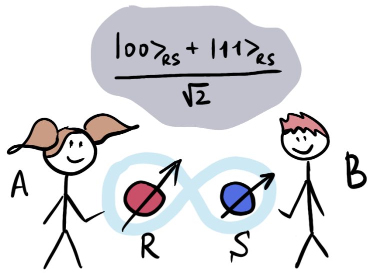
Expected result. In most interpretations of quantum theory we expect Alice to correctly guess that their outcomes are perfectly correlated.

Forward reasoning. Using her knowledge of quantum theory, Alice can run a simulation of the whole experiment what is alice simulating exactly? what registers? (before step t_1), and update her instruction registers to reflect the following: "if my measurement outcome is 0, I should predict that Bob will obtain 0; if my measurement outcome is 1, I should predict that Bob will obtain 1." This can be economically encoded by initializing her prediction qubit to $|0\rangle$ (the default prediction), setting her first instruction qubit to $|0\rangle$ ("if I see 0, I should not change my prediction") and her second instruction qubit to $|1\rangle$ ("if I see 1, transform the prediction"). When the experiment actually runs, one can simulate Alice's reasoning by running her circuit between steps t_1 and t_2, obtain her prediction, and correlate it with Bob's outcome at step t_3. The protocol is run until Alice's measurement, and each outcome is analyzed separately by projecting the state into the subspace corresponding to Alice getting the said outcome. For example, here the analysis would be carried out in the following way.

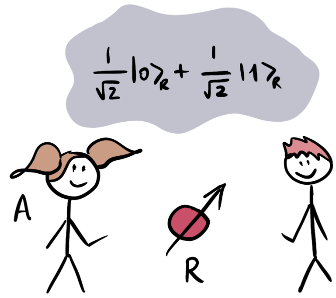
- Case $a = 0$:

  1. RS is initialized in the Bell state $\frac{1}{\sqrt{2}}|00\rangle_{RS} + \frac{1}{\sqrt{2}}|11\rangle_{RS}$; Alice measures R and writes the result down to her memory,
  $$\frac{1}{\sqrt{2}}|00\rangle_{RS}|0\rangle_A + \frac{1}{\sqrt{2}}|11\rangle_{RS}|1\rangle_A$$
  2. The state is projected onto the subspace of Alice's memory corresponding to $a = 0$, $|00\rangle_{RS}|0\rangle_A$. [1]

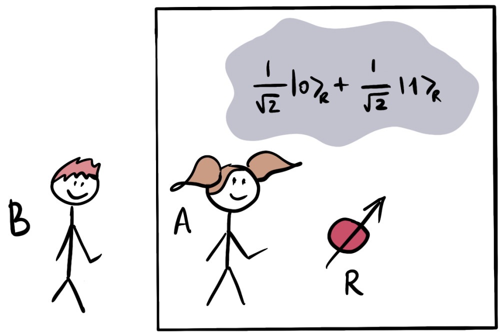
[1] Any prediction qubits associated with Alice's inference for $a = 0$ will also be captured in this subspace; here we don't add them explicitly to make the explanation simpler. For the full state, see the inference making function output in the jupyter notebooks.



(a) **Alice and Bob measure a Bell state.** Alice and Bob each have access to a half of a Bell state, and measure their own qubit in the computational basis. Alice should guess Bob's outcome.



(b) **Alice and Bob measure the same qubit.** Alice and Bob share a qubit in state $|+\rangle$. Alice measures it, and then tries to predict the outcome of Bob's subsequent measurement.



(c) **Bob measures Alice.** Alice measures a qubit (in state $|+\rangle$), and then tries to predict the outcome of Bob's future measurement. Bob reverses Alice's memory update and measures her memory.

Figure 1: **Simplest test protocols.** Here two agents measure a shared state and reason about each other's outcomes; there are no measurements of each other's memories involved. These "tutorial" examples act as simple tests of whether an interpretation was implemented correctly in QThought and makes the expected predictions in familiar settings.

3. Bob measures the state

$$|00\rangle_{RS}|0\rangle_A|0\rangle_B.$$

4. Now Alice can conclude that if she gets $a = 0$, Bob obtains $b = 0$.

- Case $a = 1$:

1. RS is initialized in the Bell state $\frac{1}{\sqrt{2}}|00\rangle_{RS} + \frac{1}{\sqrt{2}}|11\rangle_{RS}$; Alice measures R and writes the result down to her memory,

$$\frac{1}{\sqrt{2}}|00\rangle_{RS}|0\rangle_A + \frac{1}{\sqrt{2}}|11\rangle_{RS}|1\rangle_A$$

2. The state is projected onto the subspace of Alice's memory corresponding to $a = 1$, $|11\rangle_{RS}|1\rangle_A$.

3. Bob measures the state, and writes the result down to his memory,

$$|11\rangle_{RS}|1\rangle_A|1\rangle_B.$$

4. Now Alice can conclude that if she gets $a = 1$, Bob obtains $b = 1$.

The way this example is implemented, is explained at length in the corresponding Jupyter Notebook in the folder *simpleExamples*.

Example II: Alice and Bob make sequential measurements. This is an almost trivial experiment that is used as a quick test for new interpretations and logical axioms.

Setting. Alice and Bob have access to the same qubit R , initially in state $|+\rangle_R$ (Figure 1b). The experiment proceeds as follows:

$t = 1$ Alice measures system R in basis $\{|0\rangle_R, |1\rangle_R\}$, and records the result in her memory A.

$t = 2$ Bob measures system R in basis $\{|0\rangle_R, |1\rangle_R\}$, and records the result in his memory B.

$t = 3$ Alice and Bob reason about each other's outcomes.

Expected result. According to most standard quantum interpretations, we should expect their results to be the same, and them to be able to correctly guess each other's outcome; this is also our experience in the lab.

Applications. Suppose that the user wants to avoid contradictions in complex thought experiments like FR, and tries to restrict the logical axioms or to apply a bespoke interpretation (for example by disallowing any trust chains, or implementing an extreme version of many worlds). It is not sufficient to show that those settings avoid the paradox: they should also allow agents to make standard inferences in very simple, everyday setups like the present one. The user can then test their interpretation and logical axioms in this kind of scenarios — and indeed often those restrictions don't allow Alice and Bob to make the simple predictions of this example [1]. According to the laws of quantum theory, their results should be the same, and no contradiction should arise. This is implemented in *simple example II.*

Example III: Bob measures Alice.

Setting. Alice has access to a qubit R, and Bob has access to Alice's memory (Figure 1c). The initial state of R is $\frac{1}{\sqrt{2}}(|0\rangle_R + |1\rangle_R)$. The experiment proceeds as follows:

$t = 1$ Alice measures system R in basis $\{|0\rangle_R, |1\rangle_R\}$, and records the result in her memory qubit A.

$t = 2$ Alice makes a prediction about Bob's outcome at $t = 4$.

$t = 3$ Bob applies a CNOT gate on Alice's lab, which is controlled on the state of the qubit R.

$t = 4$ Bob measures Alice's memory A.

Expected result. According to the neo-Copenhagen interpretation, Alice's prediction about Bob's outcome will be that he gets 0, as Alice's memory qubit A starts out in the state $|0\rangle_A$. According to the collapse interpretation, Alice cannot make a deterministic prediction about Bob's outcome at a later step; however, Bob's measurement result corresponds to her state $|0\rangle_A$.

References

[1] Nurgalieva, N. & del Rio, L. Inadequacy of modal logic in quantum settings. *EPCTS* **287**, 267–297 (2019). arXiv:1804.01106.