Continuously Maintaining Order Statistics over Data Streams (Extended Abstract)

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1 Introduction

A rank query is essentially to find a data element with a given rank against a monotonic order specified on data elements. It has several equivalent variations [8, 17, 30]. Rank queries over data streams have been investigated in the form of *quantile computation*. A *ϕ-quantile* ($\phi \in (0,1]$) of a collection of N data elements is the element with rank $\lceil \phi N \rceil$ against a monotonic order specified on data elements. Rank and quantile queries have many applications [1, 3, 6, 7, 10, 14–16, 26, 27], including monitoring high speed networks, trends and fleeting opportunities detection in the stock market, sensor data analysis, Web ranking aggregation and log mining, etc. In these applications, they not only play very important roles in the decision making but also have been used in summarizing data distributions of data streams. The following example shows a popular tool to compare the distributions of two data sets (data streams).

Example 1 *An information provider may provide various real-time statistics of the stock market to its clients, through the Internet or telecommunication, for trends' analysis. One of the most popular charts is the* quantile-quantile *(Q-Q) plot [28] for comparing two data distributions. In a stock market, price and volume distributions are two key indexes to monitor. Figure 1 illustrates such a Q-Q plot by using two real datasets AOL and Technique_section. In AOL,* $1.3M$ *(millions) "tick-tick" transactions during the period Dec/2000 - July/2001 sorted increasingly against the volume of each transaction (deal) are collected from NYSE (New York Stock Exchange) for the stock AOL. In Technique_section,* $27M$ *tick-tick transactions are collected in the same period for the stocks csco, ibm, dell, sun, ca and also sorted on volumes. The figure demonstrates that clients can view the global chart (with very coarse information due to physical limits of display) for a general comparison, and can also click on such a chart graph to zoom in a particular range of quantiles interactively for more accurate information. Such Q-Q plots combining with other statistic display tools greatly facilitate clients detection of trade trends and thus make good trade decisions.*

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In such an application, clients may require on-line updates (processing) of Q-Q plots to monitor trading trends and detect fleeting opportunities in real time.

It has been shown in [20] that an exact computation of rank queries requires memory size linearly proportional to the size of a dataset by any one-scan technique; this may be impractical in on-line data stream computation where streams are massive in size and fast in arrival speed. Approximately computing rank queries over data streams, thus, has received a great deal of attention recently. The main paradigm is to continuously maintain a small space data structure, called summary or sketch, to summarize *order statistics*. In this talk, we will introduce the space- and time-efficient one-scan techniques of continuously maintaining order statistics for supporting approximate rank (quantile) queries. These include deterministic and randomized approximate techniques. We also discuss open issues in the area.

2 ϵ-approximation

In the problem setting, an element x may be augmented to (x, v) where $v = f(x)$ (called "value") is to rank elements according to a monotonic order of v, and f is a pre-defined function. Without loss of generality, we assume $v > 0$ and a monotonic order is always an increasing order. We study the following rank queries over a data stream S.

Rank Query (RQ) : Given a rank r, find the rank r element in S.

Suppose that r is the given rank in a RQ query, and r' is the rank of an approximate solution. We could use the constant-based absolute error metric; that is, enforce $|r' - r| \leq \epsilon$ for a given ϵ. It is immediate that such an absolute error precision guarantee leads to the space requirement $\Omega(N)$ even for an off-line computation where $N = |S|$. Therefore, two error metrics have been used.

Uniform Error. $\frac{|r'-r|}{N} \leq \epsilon$.

Relative Error. $\frac{|r'-r|}{r} \leq \epsilon$.

An answer to a RQ regarding r is *uniform ϵ-approximate* if its rank r' has the precision $|r' - r| \leq \epsilon N$; it is *relative ϵ-approximate* if its rank r' has the precision $|r' - r| \leq \epsilon r$. In this talk, we present the techniques of continuously maintaining a sketch (consisting of several sub-sketches) over a data stream S such that at any time, the sketch can be used to return a (relative or uniform) ϵ-approximate answer to a RQ. The focus is to minimize the *maximum memory space required in such a continuous computation* of sketch/summary.

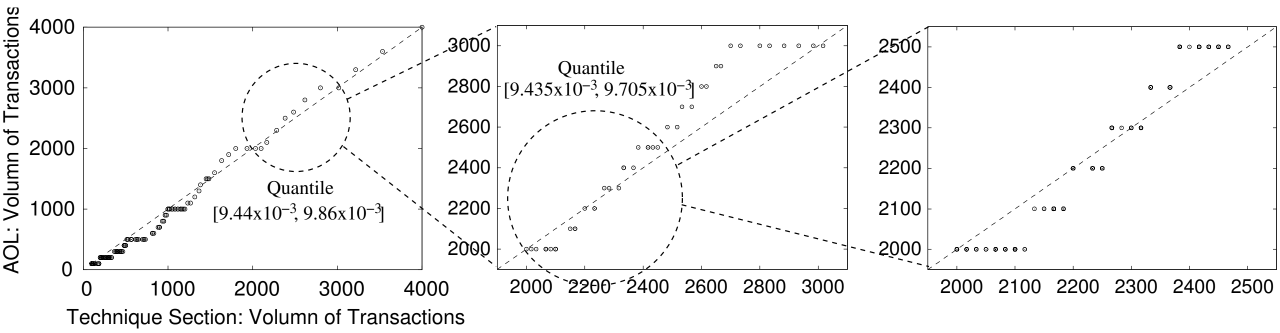

Figure 1: Q-Q Plot

3 Uniform Error Techniques

It has been shown in [15,25] that a space-efficient, uniform ϵ-approximate quantile summary can be maintained so that, for a quantile ϕ, it is always possible to find an element at rank r' with the precision guarantee $|\lceil \phi N \rceil - r'| \leq \epsilon N$. Greenwald and Khanna [15] developed a one-scan technique with $O(\frac{1}{\epsilon}\log(\epsilon N))$ space bound and the deterministic error guarantee $|r - r'| \leq \epsilon N$.

Manku *et al* [24] provided a space efficient randomized algorithm, based on an adaptive sampling technique, to achieve the uniform precision guarantee ϵN with confidence (probability) at least $1 - \delta$ and space $O(\frac{1}{\epsilon}\log^2 \frac{1}{\epsilon\delta})$. It has been shown that if the GK-algorithm [15] is applied, the space bound can be reduced to $O(\frac{1}{\epsilon}\log\frac{1}{\epsilon\delta})$.

Gilbert *et al* [13] proposed space-efficient randomized techniques for processing rank queries (quantile queries) when elements already seen may be updated; the uniform precision guarantee ϵN is also used. Cormode and Muthukrishnan [5] showed that an application of their *count-min sketch* technique can reduce the space bound in [13] from $O(\frac{1}{\epsilon^2}\log^2|U|\log\frac{\log|U|}{\delta})$ to $O(\frac{1}{\epsilon}\log^2|U|\log\frac{\log|U|}{\delta})$ where U is the value domain.

In [27], Shrivastava *et al* investigated the problem of minimizing uniform errors, when a space bound is pre-given, with the applications in sensor networks.

While quantile (order statistic) summaries over whole data streams have their applications, such summaries do not have the concept of *aging*, that is, quantiles are computed for *all* N data elements seen so far, including those seen long time ago. There are a broad spectrum of applications where data elements seen early could be outdated and quantile summaries for the most recently seen data elements are more important. For example, the top ranked Web pages among most recently accessed N pages should produce more accurate web page access prediction than the top ranked pages among all pages accessed so far as users' interests are changing. In financial market, investors are often interested in the price quantile of the most recent N bids. Motivated by this, in [21] we developed space- and time- efficient *sliding window* techniques to continuously maintaining order statistics over fixed-length sliding windows and variable sliding window techniques, respectively. Our techniques are based on a combination of GK-algorithm [15] and the *exponential histogram* techniques in [11]; they provide uniform ϵ-approximation. Arasu and Manku [2] replace the exponential histogram based data structure by an interval-tree like data structure to improved the space bound in our paper [21].

4 Relative Error Techniques

In many applications, it is desirable to investigate relative errors (or *biased* errors); that is, enforce error precision ϵr (relative ϵ-approximation) instead of ϵN. As pointed out in [7], finer error guarantees at higher ranks are often desired in network management.[1] This is because IP traffic data often exhibits skew towards the tail and it is exactly in the most skewed region where one wants finer rank error guarantees, to get more precise information about changes in values.

The problem of finding approximate quantiles with relative error guarantees was first studied by Gupta and Zane [17], who developed a one-scan randomized technique with $O(\frac{1}{\epsilon^3}\log^2 N)$ space requirement for approximately counting *inversions*, by maintaining an order sketch with the relative rank error guarantee ϵ. However, the technique requires advance knowledge of (an upper bound on) N to do one-scan sampling. This potentially limits its applications. Cormode *et al.* [7] studied the related problem - computing *biased quantiles*, that is, the set of quantiles $\Phi = \{\phi_i = \phi_0^i : 1 \leq i \leq k\}$, for a fixed k and some ϕ_0, which are estimated with precision $\epsilon\phi_i N$. [7] gives an algorithm to approximate such biased quantiles with deterministic error guarantees which performs very well against many real data sets. As shown in [30], to enforce ϵ-approximation of rank queries it requires a linear space $\Omega(N)$ in the worst case.

In [30], we developed a novel, space- and time- efficient multi-layer sampling technique. It guarantees relative ϵ-approximation with high confidence $1 - \delta$ ($\delta > 0$) and requires space $O(\frac{1}{\epsilon^2}\log\frac{1}{\delta}\log\epsilon^2 N)$ in the worst case and $O(\frac{1}{\epsilon}\log(\frac{1}{\epsilon}\log\frac{1}{\delta})\frac{\log^{2+\alpha}\epsilon N}{1-1/2^\alpha})$ (for $\alpha > 0$) on average.

Restricted to a fixed value domain, Cormode *et al* [8] recently developed a novel deterministic algorithm, by significantly extending the technique in [27], to ensure relative ϵ-approximation with space bound $O(\frac{\log|U|}{\epsilon}\log\epsilon N)$.

The problem of sliding windows is not well solved though there are some discussions in [7].

5 Duplicate-insensitive

In many real applications, duplicates may occur when data elements are observed and recorded multiple times at different data sites. For instance, as pointed out in [7,9] the same packet may be seen at many tap points within an IP network depending on how the

[1] Note that the form of our relative error metric is biased towards the *head* (i.e., finer error guarantees towards lower ranks). Clearly, finer error guarantees towards the *tail* may be obtained if the data elements are ordered in reverse.

packet is routed; thus it is important to discount those duplicates while summarizing data distributions by rank queries (quantiles). Moreover, to deal with possible communication loss TCP retransmits lost packets and leads to the same packet being seen even at a given monitor more than once. Furthermore, duplicates may often occur due to the projection on a subspace if elements have multiple attributes.

In such applications, there may be many *duplicated elements* in a data stream S. To discount the duplicates in S, rank queries have to be issued against D_S instead of S where D_S denote the set of distinct data elements in S. Note that in D_S, there are no duplicates but many different elements may happen to have the same values. Consequently, all the challenges in approximately computing rank queries remain the same. The unique challenge is to discount duplicates without keeping all elements in a summary/sketch.

With the recent data-intensive applications in sensor/P2P networks, duplicate-insensitive techniques are also highly desirable to achieve high communication fault-tolerance. In [23], Manjhi, Nath, and Gibbons propose an effective adaption paradigm for in-network aggregates computation over stream data with the aim to minimize communication costs and to achieve high fault-tolerance. As indicated, a duplicate-insensitive technique for approximately computing quantiles may be immediately obtained by a combination of their tree-based approximation technique and the existing *distinct counting* technique in [4]. It can be immediately applied to a single site, where a data stream has duplicated elements, with the uniform precision guarantee $|r' - r| \leq \epsilon n$ by confidence $1 - \delta$ and space $O(1/\epsilon^3 \log 1/\delta \log m)$ where $n = |D_S|$.

In [9], Cormode and Muthukkrishnan present a *DISTINCT RANGE SUMS* technique by applying the FM [12] technique on the top of the *count-min* [5]. The technique can be immediately used to approximately processing RQ with the uniform precision guarantee $|r' - r| \leq \epsilon n$, confidence $1 - \delta$, and space $O(\frac{1}{\epsilon^3} \log \frac{1}{\delta} \log^2 m)$. Independently, Hadjieleftheriou, Byers, and Kollios [18] also developed two novel duplicate-insensitive techniques to approximately compute quantiles in a distributed environment. Applying their techniques to a single site immediately leads the uniform precision guarantee $|r' - r| \leq \epsilon n$ by confidence $1 - \delta$ and space $O(\frac{1}{\epsilon^3} \log \frac{1}{\delta} \log m)$.

Very recently, we developed the first space- and time- efficient, duplicate-insensitive algorithms [31] to continuously maintain a sketch of order statistics over data stream to enforce relative ϵ-approximation. They have been developed based on the probabilistic counting techniques in [4, 12]. They not only improve the existing precision guarantee (from uniform ϵ-approximation to relative ϵ-approximation) but also reduce the space from $O(\frac{1}{\epsilon^3} \log \frac{1}{\delta} \log m)$ to $O(\frac{1}{\epsilon^2} \log \frac{1}{\delta} \log m)$ where m is the element domain size.

The sliding window problem remains open.

6 Miscellaneous

Continuous queries are issued once and run continuously to update query results along with updates of the underlying datasets. In [6], Cormode *et al* provide a novel algorithm to continuously processing a rank query in a sensor/network environment with the aim to minimize the communication costs. We [22] recently developed novel techniques to efficiently processing a massive set of rank queries. The objective is to share the computation as much as possible.

Quantiles computation against multi-dimensional datasets has been recently investigated in [19,29]. Yiu *et al* [29] developed an efficient R-tree based algorithm to provide exact solutions regarding an off-line computing environment, while Hershberger *et al* [19] presented an effective one-scan approximation technique to maintain a small space sketch with the uniform precision guarantee ϵN. The problems of relative error guarantee and duplicate-insensitiveness over data streams remain open.

7 Future Studies

While a number of theoretical problems still remain open, the following two new applications in continuously maintaining order statistics may be worth some exploration.

Uncertainty. In many applications, we may need to deal with data sets with uncertainty; that is, the value of a data element is not fixed. In such applications, ranks of data elements have to be specified probabilistically. The challenge is to effectively build a small space summary by one-scan techniques while the distribution of each data element is continuously sampled.

Graphs. Graphs are very common to model many real applications; for instance, IP network and communication network. To manage and explore such networks, summarizing distributions of various node degrees information is an important issue. The challenge is to maintain a small space summary by one-scan while the underlying graph structure is continuously "disclosed".

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