CSE341T/CSE549T **August 25, 2014**

Master Method Cheat Sheet

1 Master Method - Formal Version

The Master method applies to many recurrences of the form

$$T(n) = aT\left(\frac{n}{b}\right) + f(n),$$

with constants $a \geq 1$ and $b > 1$. We assume that there exists some constant n_0 such that for all $n < n_0$, $T(n) = \Theta(1)$.

Case 1: If $f(n) = O(n^{\log_b a - \varepsilon})$ for some $\varepsilon > 0$, then $T(n) = \Theta(n^{\log_b a})$.

Case 2: If $f(n) = \Theta(n^{\log_b a} \lg^k n)$ for some constant $k \geq 0$, then $T(n) = \Theta(n^{\log_b a} \lg^{k+1} n)$.

Case 3: If $f(n) = \Omega(n^{\log_b a + \varepsilon})$ for some $\varepsilon > 0$, and $af(n/b) \leq cf(n)$ for some constant $c < 1$ and sufficiently large n, then $T(n) = \Theta(f(n))$.

2 Master Method - Informal Version

Given $T(n) = aT(n/b) + f(n)$, take the following steps:

- Compute $x = \log_b a$.
- If you can, put $f(n)$ in the form $\Theta(n^y \log^k n)$, for some constant $k \geq 0$. If $f(n) = 2n$, $y = 1, k = 0$. If $f(n) = \log n$, we have $y = 0, k = 1$. If $f(n) = \Theta(1)$, we have $y = 0, k = 0$. They are all valid values of y and k, and you can apply the below intuition.
- Compare x and y. The bigger one gets to be in the answer.

  Case 1: If $x > y$, then the solution is $T(n) = \Theta(n^x)$.

  Case 2: If $x = y$, then the solution is $T(n) = \Theta(n^x \log^{k+1} n) = \Theta(n^y \log^{k+1} n) = \Theta(f(n) \log n)$.

  Case 3: If $y > x$, then the solution is $T(n) = \Theta(f(n))$. (All functions of the form $n^y \log^k n$ satisfy the regularity condition $af(n/b) \leq cf(n)$.)

What if $f(n)$ doesn't fit the form $n^y \log^k n$, for $k \geq 0$? For example if $f(n) = n/\log n$ or $f(n) = n \log \log n$. In essence, the above intuition still works for cases 1 and 3 for any

$$f(n) = n^y.\text{any small (smaller than polynomial) function,}$$

since they have O and Ω in their definitions and these are "inequality functions". Therefore, if you have such functions, compare x and y and if $x > y$ or $y > x$, then you can continue to use the above intuition. Check the regularity condition for case 3, though.

If $x = y$, however, then you *can not* use the above intuition. Case 2 requires a Θ, and therefore requires that $f(n) = n^y \log^k n$. If you happen to fall into case 2 with a different function like $f(n)$ like $n/\log n$ or $n \log \log n$, then Master Theorem doesn't apply definitively. You can still use the Master Theorem to guess your solution, but you have to prove it using induction.