Physics 303/573

Homework 1 due Wednesday, September 13.

September 6, 2017

Please go through Example 1.1 and Example 1.2 in the book carefully.

Consider a vector

$$\vec{r}(t) = \sin(\pi t)\hat{x} + \cos(\pi t)\hat{y} - \sqrt{7}\hat{z}$$

(given with respect to a fixed Cartesian basis). Here t means time.

(1) a) What is the length of $\vec{r}(t)$?

b) What is the scalar product $\vec{r} \cdot \vec{w}$ with a fixed vector

$$\vec{w} = \hat{x} - 2\hat{y} + \sqrt{7}\hat{z}$$

c) What is the cross product $\vec{r} \times \vec{w}$?

(2) a) What is the first derivative

$$\vec{v}(t) = \frac{d\vec{r}}{dt}$$

b) What is the second derivative?

(3) Sketch the trajectory of the vector $\vec{r}(t)$. What kind of a shape does it sweep out?

(4) Suppose that $\vec{r}(t)$ describes the motion of a body with mass m.

a) What is the momentum $\vec{p} = m\vec{v}$ of this body?

b) What is the force $\vec{F} = m\vec{a}$ needed to make it move along the path $\vec{r}(t)$?

(5) a) Write $\vec{r}(t)$ and $\vec{v}(t)$ in a cylindrical coordinate basis (in lecture and in my notes, I use coordinates R, ϕ, z and basis vectors $\hat{R}, \hat{\phi}, \hat{z}$ where the book uses coordinates ρ, ϕ, z for cylindrical coordinates).

b) What is the time derivative

$$\frac{d\hat{R}}{dt}$$

of the basis vector $\hat{R}$ for this motion?

c) Compute the first derivative of $\vec{r}(t)$ directly in a cylindrical coordinate basis, and show that it agrees with the expression that you found for $\vec{v}(t)$ in question (5) a).

(6) a) Compute the length of $\vec{p}(t)$.

b) Compute

$$\hat{z} \cdot (\vec{r} \times \vec{p})$$

How do both of these depend on time? Do you know why?