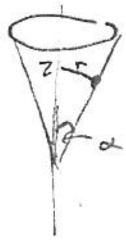
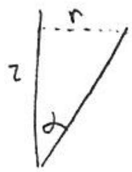
Particle ON A Cone

Particle of mass moves on inside surface of cone of half Angle α. It is subjected to a gravitational Force. Determine Lagrangian



What IS the constraints looking at the geometry

$Z =$



$\tan\alpha = \frac{O}{A}$

$\cot\alpha = \frac{A}{O}$

Derivative of equation of constraint

$\dot{z} =$

What IS The kinetic energy IN each coordinate system. Pick A coordinate System.

What is moving : particle on cylindrical
what coordinates change?

$T = \frac{1}{2} m v^2 = \frac{1}{2} m (\quad) \text{ OR } \frac{1}{2} m (\quad) \text{ or } \frac{1}{2}$

write cylindrical — write Cartesian — write spherical.

Use: $T =$

$=$

$=$

WHat does The Potential Energy (Circle one) : gravity ②
Coulomb.
Force Related.
Spring

$U = mgZ =$

<u>What IS The Lagrangian?</u>

$$\boxed{L = \frac{1}{2}m(\quad + \quad) - }$$

what Is lagrange equation : which coordinates Do you Do it for
What IS equation of motion?

<u>Θ</u>

$$\frac{\partial L}{\partial t}\frac{\partial L}{\partial \theta} = 0$$

$$\frac{\partial L}{\partial \dot{\theta}} = \qquad = \text{Constant.}$$

This expresses The conservative Angular momentum About The Axis of symmetry.

<u>R</u>

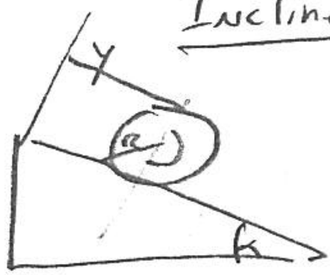
$$\frac{\partial L}{\partial r} = \frac{d}{dt}\frac{\partial L}{\partial \dot{r}}$$

<u>equation of motion?</u>

$$= 0$$

Inclined Plane with Constraint

2.5



Relation Between Coordinates.

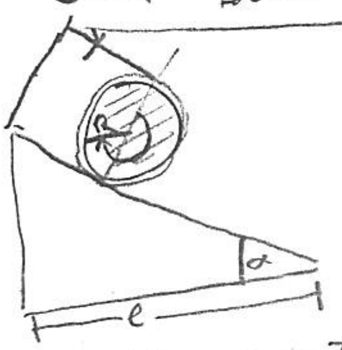
$$y =$$

Equation of constraint:

$$= 0$$

Lagrangians with constraints:

Disk down inclined plane:



what is moving? : Disk so you need translational + Rotational

what is T:

$T = \quad\quad = \frac{1}{2}m\dot{y}^2 + .$

what is I:? $I =$ moment of inertia.

what is U? $U = Mg(\quad ,)$

what is equation of constraint?

$y - R\theta = 0$

what is Lagrangian

$L = T - U = \frac{1}{2} \quad\quad + Mg(\quad\quad + \lambda(y - R\theta)$

$\boxed{\text{Options}}$

Equation of motion w/ constraint put in form

OR you can put constraint in Lagrangian and do it normal.

$$\frac{\partial L}{\partial y} - \frac{\partial}{\partial t}\frac{\partial L}{\partial \dot{y}} + \lambda\frac{\partial F}{\partial y} = 0$$

$$\frac{\partial L}{\partial \theta} - \frac{\partial}{\partial t}\frac{\partial L}{\partial \dot{\theta}} + \lambda\frac{\partial F}{\partial y}$$

equation of motion .

constraint equation $y = R\theta$

2nd Dirivative of constraint equation.

$$\ddot{\theta} = \frac{\ddot{y}}{R}$$

Solve for constraint λ:

$$\lambda = -\frac{\cdot}{2}$$

Find $\ddot{y}$:

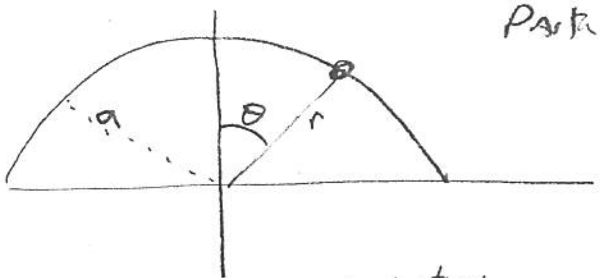
$$\ddot{y} = \frac{a}{3}$$

$$\lambda = - \frac{\cdot}{\cdot}$$

Final equations $\ddot{\theta} = \ldots$

Partik on Hemisphere:

⑥



Equaton of Constract? + Derivatives.

What is relaton ?

$r = a \longrightarrow \lambda(r-a)$ or $f(r,\theta) = \quad = 0$

$\dot{r} = 0 \quad \ddot{r} = 0$

T, U, L is ?

$T = \frac{m}{2}(\; , \quad)$

$U =$

$L = T - U = \frac{m}{2}(\quad) -$

Equaton of moton w Constraint

$$\frac{\partial L}{\partial r} - \frac{\partial}{\partial t}\frac{\partial L}{\partial \dot{r}} + \lambda \frac{\partial f}{\partial r} = 0$$

$$\frac{\partial L}{\partial t} - \frac{\partial}{\partial t}\frac{\partial L}{\partial \dot{\theta}} + \lambda \frac{\partial F}{\partial \theta} = 0$$

$$\frac{\partial F}{\partial r} = 1 \qquad \frac{\partial F}{\partial \theta} = 0$$

equations are:

$\quad - m \quad \cos\theta \quad \ddot{r} \quad 0$

$m g r \sin$

Simplify w/ Constraint Derivatives

$$m a \dot{\theta}^2 - m g \cos\theta + \lambda = 0$$

$$m g a \sin\theta - m a^2 \ddot{\theta} = 0$$

get $\ddot{\theta}, \lambda$

equations are

$$\ddot{\theta} = \ldots$$

Integrate:

$$\ddot{\theta} = \frac{d}{dt}\frac{d\theta}{dt} = \frac{d\dot{\theta}}{dt} = \frac{d\dot{\theta}}{d\theta}\frac{d\theta}{dt} = \dot{\theta}\frac{d\dot{\theta}}{d\theta}$$

$$\int \dot{\theta}\, d\dot{\theta} = \frac{g}{a}\int \sin\theta \, d\theta$$

$$\frac{\dot{\theta}^2}{2} = -\frac{g}{\ldots} \ldots + \ldots$$

Integration constant

$$\lambda = m \ldots$$

when does particle fall off. This is a condition when $\lambda = 0$ at angle θ_0

$$\lambda = 0 = mg(3\cos\theta_0 - 2)$$

$$\theta_0 = \cos^{-1}\left(\frac{2}{3}\right)$$