Physics 303/573

Homework 4 due Wednesday, October 4.

From Taylor: Problem 4.38 part (a) only.

1. Calculate the gradient in cartesian coordinates of the following functions:

a) $f(\vec{r}) = \alpha\,|r|^n \equiv \alpha\,(x^2 + y^2 + z^2)^{p/2}$ where p and α are constants.

b) $f(\vec{r}) = \beta\,x^2 y^3 z^4$ where β is a constant.

c) $f(\vec{r}) = \gamma(x^2 + y^2)^{p/2} + \alpha\,z^q$ where γ, α, p, q are constants.

2. a) Calculate the gradients in problem 1 in cylindrical coordinates.

b) Repeat the calculation in spherical polar coordinates.

Hint: Use the formulas in section 1.1 of the Lecture notes on multivariate calculus.

The following problem counts as two problems:

3. and 4. Recall (see equation (2.26) in the notes for lecture 2) that in spherical polar coordinates,

$$d\vec{r} = dr\,\hat{r} + r(d\theta\,\hat{\theta} + d\phi\sin\theta\,\hat{\phi})$$

a) Compute the $\hat{r}$ component of the curl of a vector $\vec{v}$ from the fundamental definition: write

$$\vec{v} = v_r\hat{r} + v_\theta\hat{\theta} + v_\phi\hat{\phi}$$

and compute the circulation of $\vec{v}$ (line integral) around a little square perpendicular to $\hat{r}$. Then divide by the area of the square and take the limit as the area vanishes to find

$$\hat{r}\cdot(\vec{\nabla}\times\vec{v})$$

b) Now compute the $\hat{\theta}$ and $\hat{\phi}$ components of the curl using the same approach.

c) Show that any vector of the form $\vec{v} = f(r)\hat{r}$ has vanishing curl.