Hamiltonian Notes:

express the lagrangian in general coordinates called general momentum

$$P \equiv \frac{\partial L}{\partial \dot{q}} \qquad \dot{P} = \frac{\partial L}{\partial q}$$

$$H(p,q,t) = \sum p\dot{q} - L(q,\dot{q},t)$$

momentum + position. position, velocity

$$\boxed{\text{Hamiltons equations of motion} \quad \dot{q} = \frac{\partial H}{\partial p} \quad -\dot{p} = \frac{\partial H}{\partial q}}$$

known as canonical equations of motion

If H does not contain time then Hamiltonian is a conserved quantity

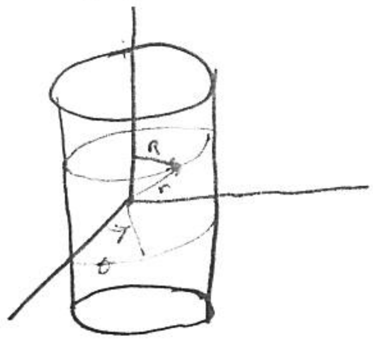
H = T + U = E If Rules Apply:

1. equations conveying rectangular and generalized coordinates must be independent of time.
2. Potential energy must be velocity independent.

In moving coordinate system Hamiltonian is not equal to total energy

Find the equations of motion of a particle of mass m moving on the surface of a cylinder Defined by $x^2 + y^2 = R^2$

The particle is subjected to a force directed toward the origin And proportional to the distance of the particle from origin $F = -kr$



Particle on surface of cylinder

Whats involved: types of Potential + kinetic energies: Is it moving? Is there Potential magic

Potential: $F = -kr \longrightarrow +\frac{kr^2}{2} = U =$

$F = -\frac{\partial U}{\partial r}$

Pick your coordinate system?

given: $x^2 + y^2 = R^2$

$U = \qquad \frac{1}{2} k (R^2 + z^2)$

Is it moving: kinetic energy

$T = \frac{1}{2} m (\qquad) \longrightarrow \frac{1}{2} m (\qquad)$

what is constant

What is Lagrangian?

$$L = T - U = 2$$

What are generalized coordinates + generalized momenta? get my p out

What is moving?

$$p_\theta = \frac{\partial L}{\partial \dot\theta} =$$

$$p_z = \frac{\partial L}{\partial \dot z} =$$

Is the system conservative If yes $H = T + U$

Hamiltonian is in term of position + momentum. (p, q)

Rewrite T and U with p

$$T = \frac{1}{2} m (R^2 \dot\theta^2 + \dot z^2) \rightarrow \frac{1}{2} m R^2 \dot\theta \dot\theta + \frac{1}{2} m \dot z \dot z =$$

$$U = \frac{1}{2} K (R^2 + Z^2) \rightarrow$$

Are there any constants. And If so they can be eliminated since not moving
You can keep them But they dont count
Its called Hamiltonian Dynamics
For a reason.

$$H = T + U = \frac{p_\theta^2}{2mR^2} + \quad +$$

Make Hamiltons equations $\dot p$ But equations

$$\dot p_\theta = -\frac{\partial H}{\partial \theta} = 0$$

$$-\dot p_z = \frac{\partial H}{\partial z} =$$

Now q dots or Things that move.

$$\dot{\Theta} = \frac{\partial H}{\partial p_\Theta} =$$

these are Duplicates From what we said earlier

$$\dot{z} = \frac{\partial H}{\partial p_z} =$$

$$P_\Theta = mR^2 \dot{\Theta} = \quad = \text{constant}$$

<u>get an equation of motion:</u>

$$p_z = m\dot{z} \xrightarrow{\text{Derivative}} \dot{p}_z = m\ddot{z} =$$

From Before $\dot{p}_z =$

$$m\ddot{z} + kz = 0$$

$$\omega_0^2 = \frac{k}{m}$$

$$\ddot{z} + \omega_0^2 z = 0$$

The motion in the z Direction is simply Harmonic

Rule

Recipe

Energies

Potential

Kinetic

Lagrangian. Form $q, \dot{q}$

Lagrangian equation.

Lagrangian momenta

Hamiltonian

Convert to Everything (P and q's) of moving parts.

Equations of motions $\dot{p}$ for each moving part.

Equation of motion $\dot{q}$ for each moving part.

WANT equation of motion take derivative of generalized momentum
Set it equal to Hamiltons $\dot{p}$

Pro's For Hamiltonian

Greater Freedom for choosing Variables since
Since q and p are independent
p are not mixed with q's
$\dot{p}$ Are mixed with q's

Provides Base to other Fields

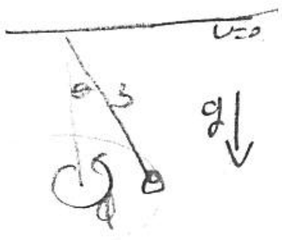
Pro's for Lagrangian

Easier to get equation of motion

Cyclic. Coordinates not appearing in T and U
Coordinate cyclic in L is also in H

If q doesn't apear in hamiltonian $\dot{p} = 0$
And Conjugate momentum p is Constant of motion

Equation of motion for spherical pendulum w/ Hamiltonian



$$T = \frac{1}{2} m \dot{r}^2 = \frac{1}{2} m (\qquad \qquad)$$

$$U =$$

Generalized momentum is?

what coordinates are moving $\boxed{\theta \mid \varphi}$

$$p_\theta = \frac{\partial L}{\partial \dot{\theta}} =$$

$$p_\varphi = \frac{\partial L}{\partial \dot{\varphi}} =$$

$H = T + U$ in terms of (p, q)

$$H = T + U = \frac{1}{2} m b^2 (\_\_\_ \qquad \_\_\_\_)$$

$$H(p_\theta, p_\varphi, \theta, \varphi) = \frac{p_\theta^2}{2 m b^2} + \frac{p_\varphi^2}{2 m b^2 \sin^2\theta}$$

Equations of motion Hamiltonian

Conjugate momentum dictates EOM. Equal it to hamiltonian to get Equation of motion

LAST

$$\dot{\theta} = \frac{\partial H}{\partial p_\theta} =$$

$$\dot{\varphi} = \frac{\partial H}{\partial p_\varphi} =$$

$$\dot{p}_\theta = -\frac{\partial H}{\partial \theta} =$$

$$\dot{p}_\varphi = -\frac{\partial H}{\partial \varphi} = 0$$

Φ is cyclic so momentum about axis is constant

Lagrangian Eq 2

Consider projectile motion in 2D. Find equation of motion in both Cartesian + Polar.

Energies: Is it moving? what magic? coordinates which directions?

$T =$

$U =$

Lagrangian:

$L = T - U =$

Lagrange equations:

$$\frac{\partial L}{\partial x} = \frac{\partial}{\partial t}\frac{\partial L}{\partial \dot{x}} \longrightarrow$$

$$\frac{\partial L}{\partial y} = \frac{\partial}{\partial t}\frac{\partial L}{\partial \dot{y}} \longrightarrow -mg = m\ddot{y} \longrightarrow$$

In Polar: $T = \frac{1}{2} m \dot{r}^2 + \frac{1}{2} m (r\dot{\theta})^2$

$$L = T - U = \frac{1}{2} m \dot{r}^2 + \frac{1}{2} m r^2 \dot{\theta}^2 - mgr\sin\theta$$

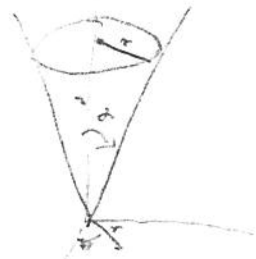
r : $$\frac{\partial L}{\partial r} = \frac{\partial}{\partial t}\frac{\partial L}{\partial \dot{r}} \longrightarrow m r \dot{\theta}^2 - mg\sin\theta - \frac{\partial}{\partial t}(m\dot{r}) = 0$$

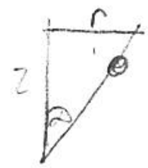
$$r\dot{\theta}^2 - g\sin\theta - \ddot{r} = 0$$

θ : $$\frac{\partial L}{\partial \theta} - \frac{\partial}{\partial t}\frac{\partial L}{\partial \dot{\theta}} = 0$$

$$-mgr\cos\theta - \frac{\partial}{\partial t}(m r^2 \dot{\theta}) = 0 \longrightarrow -gr\cos\theta - 2r\dot{r}\dot{\theta} - r^2\ddot{\theta} = 0$$

Particle Constraint to move on cone





$\tan\alpha = \quad \rightarrow \quad z = \quad = r\cot\alpha$

1. generalized coordinates (r, θ, z)

2. equation of constraint, variable

$z =$

$\dot{z} =$ $\quad r$

3

$$T = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2 + \dot{z}^2\right) = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2 + \dot{r}^2\cot^2\alpha\right)$$

$$= \frac{1}{2}m\left(\dot{r}^2\csc^2\alpha + r^2\dot{\theta}^2\right)$$

$U =$ $\quad r$

$L = \frac{1}{2}m(\quad) -$

$\frac{\partial L}{\partial\theta} = 0 \qquad \frac{\partial L}{\partial\dot{\theta}} = m\, r^2\dot{\theta} =$

Angular momentum

$m\, r^2 \omega = \text{constant.}$

Angular momentum $L = I \times \omega =$

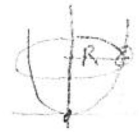
$\frac{d}{dt}\frac{\partial L}{\partial\dot{r}} =$

$\frac{\partial L}{\partial r} = \quad r$

$\cancel{m}\ddot{r}\csc^2\alpha = \cancel{m}r\dot{\theta}^2 - \cancel{m}g\cot\alpha$

$$\boxed{\ddot{r} - r\dot{\theta}^2\sin^2\alpha + g\sin\alpha\cos\alpha = 0.}$$

Bead Slides Along Smooth wire bent in shape
of Parabola $z = cr^2$



1)

1) Pick general coordinate: (r, θ, z)

$$T = \frac{m}{2}[\ .\ ]$$

2) Choose zero point of U.

$U =$

3). Chose equation of constraint.

$z = cr^2$ and $\theta = -$

and $\dot{z} = 2cr\dot{r}$ $\dot{\theta} = \omega$ $\dot{z}^2 = 4c^2r^2\dot{r}^2$

4) Lagrangian:

$$L = \frac{m}{2}(\ ) - r$$

5) equation of motion

$$\frac{\partial L}{\partial \dot{r}} = 1 \quad \ldots$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{r}} =$$

$$\ddot{r}(1 + 4c^2r^2) + \dot{r}^2(4c^2r) + r(2gc - \omega^2) = 0$$

if Bead stays $r = R =$ constant $\dot{r} = \ddot{r} = 0$ $\longrightarrow$ $R(2gc - \omega^2) = 0$