Physics 303/573

Homework 3 due Wednesday, September 27.

1. A charged particle with mass μ and positive charge e moves in a uniform electric and magnetic field $\vec{E} = E\hat{x}$ and $\vec{B} = B\hat{x}$. The Lorentz force law tells us that $\vec{F} = e(\vec{E} + \vec{v} \times \vec{B})$.

a) Write the equations of motion in cartesian coordinates.

b) Solve the equations.

c) Sketch the particle's motion if it starts at the origin with $\vec{v}_0 = v_0\hat{y}$.

2. Two people, one with mass m_1 and the other with mass m_2, stand on a stationary sled with mass M on a frozen lake. Assume that the ice is frictionless.

a) The first person (with mass m_1) jumps off the sled with a speed u relative to the sled. What is his speed? Use conservation of momentum to find the speed of the sled (which is still carrying the second person).

b) Now the second person jumps with the same speed u relative to the sled. What is his speed? What is the the speed of the sled?

c) What is the change in the total kinetic energy of the two people and the sled?

d) If we assume that $m_1 + m_2 = m_{total}$ is held constant, we can write $m_1 = am_{total}$ and $m_2 = (1 - a)m_{total}$. What value of a with $0 \leq a \leq 1$ maximizes the speed of the sled?

3. Consider a force $\vec{F} = 2x^2\hat{x} - xy\hat{y}$.

a) How much work would this force do along a path that goes along the x-axis from the origin to the point $\hat{x}$ and then up parallel to the y-axis to the point $\hat{x} + \hat{y}$?

b) How much work would this force do along a path given by $y = x^2$, that is, along the path $x\hat{x} + x^2\hat{y}$, again from the origin to the point $\hat{x} + \hat{y}$? Is the force conservative?

c) For two constants a and b, how much work would this force do along a path $t^a\hat{x} + t^b\hat{y}$, again from the origin to the point $\hat{x} + \hat{y}$?

4. Consider a frictionless table with a hole in the center. A string passes through the hole. On the table, attached to the string, there is mass m moving in a circle with radius r_0 and angular velocity ω_0. The string has just enough tension to keep the mass moving around in a circle.

a) What is the angular momentum L_0 of the mass? What is its kinetic energy T_0?

The string is now gradually shortened.

b) What is the angular momentum $L(r)$ as a function of r? What is the angular velocity $\omega(r)$ as a function of r? What is the kinetic energy energy $T(r)$ as a function of r?

c) What is the tension in the string (the force on the mass) needed to keep the motion circular as a function of r? What is the work done by this force as the radius changes from r_0 to r? How does this compare to the change in the kinetic energy?

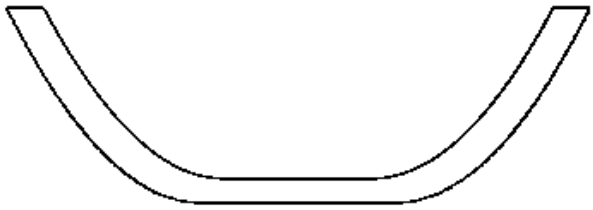
5. Calculate the center of mass for a cylindrically symmetric bowl made of clay with uniform density ρ and with a shape described by an inner and an outer curve. The outer curve goes from $0 \leq z \leq 1$ and is given by

$$R_{out} = \frac{1}{2} + \sqrt{z} \tag{0.1}$$

the inside of the bowl goes from $\frac{1}{8} \leq z \leq 1$ and is given by

$$R_{in} = \frac{3}{8} + \sqrt{z - \frac{1}{8}} \tag{0.2}$$

(see figure)



Hint: First calculate the total mass of the bowl. You will get some messy but elementary integrals to do.