Homework 1

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PROBLEM 2.16

A golfer hits his ball with speed v_0 at an angle θ above the hoizotal ground. Assuming that the angle θ is fixed and that air resistance can be neglected, what is the minimul speed v_0 for wich the ball will clear a wall of hight h, a distance d away? Your solution should get into trouble if the angle θ is such taht $\tan\theta < \frac{h}{d}$. Explain. What is the minimum speed if $\theta = 25°$, $d = 50$m, and $h = 2$m.

Solution

Since air resistance can be neglected, we could have $\vec{F} = m\ddot{\vec{r}} = -mg\hat{y}$. We can see that $v_x(0) = \cos\theta v$ and $v_y(0) = \sin\theta v$. Thus we have following equations:

$$\frac{\mathrm{d}r_x}{\mathrm{d}t} = v_x(0)$$
$$\frac{\mathrm{d}^2 r_y}{\mathrm{d}t^2} = -g$$

Easy to find

$$\begin{aligned} r_x &= \int v_x(0)\mathrm{d}t = v_x(0)t + r_x(0) \\ \frac{\mathrm{d}r_y}{\mathrm{d}t} &= \int -g\mathrm{d}t = -gt + v_y(0) \\ r_y &= \int -gt\mathrm{d}t + \int v_y(0)\mathrm{d}t \\ &= -g\int t\mathrm{d}t + v_y(0)t + r_y(0) \\ &= -g\frac{t^2}{2} + v_y(0)t + r_y(0) \end{aligned}$$

Let $r(0) = (0,0)$, we could have:

$$\begin{aligned} r_x &= v_x(0)t \\ r_y &= -g\frac{t^2}{2} + v_y(0)t \end{aligned}$$

Now we can plug back v and θ, we have:

$$\begin{aligned} r_x &= \cos\theta v t \\ r_y &= -g\frac{t^2}{2} + \sin\theta v t \end{aligned}$$

From the first euqation we can find the time t that the golfer reach the ground when $r_x = d$. And this give us $t = \frac{d}{\cos\theta v}$.

Now plug it in to 2nd equation:

$$\begin{aligned} r_y &= -g\frac{t^2}{2} + \sin\theta v\frac{d}{\cos\theta v} \\ r_y &= -g\frac{d^2}{2\cos^2\theta v^2} + \tan\theta d \end{aligned}$$

In our case, we wish $r_y > h$. Let look at the case where $\theta = \frac{h}{d}$. We have;

$$\begin{aligned} r_y &= -g\frac{d^2}{2\cos^2\theta v^2} + \frac{h}{d}d \\ h &< -g\frac{d^2}{2\cos^2\theta v^2} + h \end{aligned}$$

Notice that $g\frac{d^2}{2\cos^2\theta v^2}$ always larger than zero. Thus for any t it is not possable to have $h < h - a$ where $a > 0$. Thus this means it is not possable to clear such a wall.

When $\tan\theta < \frac{h}{d}$. $\tan\theta d < \frac{h}{d}d$, we will have:

$$h < -a + h < -a + \tan\theta d$$

where $a > 0$. This means right side is always smaller than h. It is not possable to clear such a wall.

Now plug in the numbers in to equation, we have:

$$2 < -10\frac{50^2}{2\cos^2 25° v^2} + 50\tan 25°$$

Solve the equation we have the min $v = 26.72$m/s.

PROBLEM 2.36

Consider the following quote from Galileo's *Dialogues Concerning Two New Sciences*:

Aristotle says that "an iron ball of 100 pounds falling from a height of one hundred cubits reches the ground before a one-pound ball has fallen a single cubit." I say taht they arrive at the same time. You find, on making the experiment, that the larger outstrips the smaller by two finger-breadths, that is, when the larger has reached the ground, the other is short of it by two finger-breadths.

We know that the statement attributed to Aristotle is totally worong, but just how close is Galileo's claim that the difference is just "two finger breadths"?

1. Given that the density of iron is about $8\,\mathrm{g\,cm^{-3}}$, find the terminal speed of the two iron balls.

2. Given that a cubit is about 2 feet, use Equation (2.58) to find the time for the heavier ball to land and then the position of the lighter ball at that time. How far apart are they.

$$y = \frac{v_{\text{ter}}^2}{g}\ln\left[\cosh\frac{gt}{v_{\text{ter}}}\right]$$

Use quadratic drag and $x = \dot{x} = 0$.

Solution

1. 100 Pounds is 45kg. We can calculate $V_{100} = 0.0056\text{m}^3$, giving $r = 0.11$m.Plug the value into equation (2.50) of the book we have $v_{\text{ter}} = \sqrt{\frac{mg}{\gamma D^2}} = \sqrt{\frac{45\times 10}{0.25\times(2\times 0.11)^2}} = 192.847\text{m/s}$.

   1 Pounds is 0.45kg. Thus we have $V_1 = 0.00005626\text{m}^3$, giving $r = 0.024$m. Thus we have $v_{\text{ter}} = \sqrt{\frac{mg}{\gamma D^2}} = \sqrt{\frac{0.45\times 10}{0.25\times(2\times 0.024)^2}} = 88.39\text{m/s}$.

2. Using the equation we have for 100 pounds ball:

$$61 = \frac{192.847^2}{10}\ln[\cosh(\frac{10t}{192.847})]$$

   Solve the equation we have t = 3.50s. Now plug it into the euqation for the smaller ball:

$$y = \frac{88.39^2}{10}\ln[\cosh(\frac{10\times 3.50}{88.39})] = 59.7\text{m}$$

   And we can find the difference is 1.3m.

PROBLEM 1

Consider a body with a cross-sectional area A moving with speed $\|v\|$ through a fluid with density ρ. We showed using dimensional analysis that this gave rise to a drag force proportional to $\rho A\|v\|^2$.

1. At what rate does the body encounter the fluid, that is how much mass per second does it hit?

2. To accelerate all this fluid to the bodys speed $\|v\|$, what force needs to be exerted?

   In reality, the body doesn't accelerate all the fluid it encounters to its own speed, but this gives a rough measure of the drag force. For a sphere, it turns out that the force is about $\frac{\rho A\|v\|^2}{4}$. In air room temperature and presure (STP), $\rho \approx 1.3\text{kg}\,\text{m}^{-3}$

3. Find the constant c in equation (3.35) of the notes in terms of the diameter D of the sphere. How does this result compare to (2.6) in the book?

$$\ddot{\vec{r}} = -g\hat{z} - \frac{c}{m}v\vec{v}$$

$$\gamma = 0.25\text{N}\,\text{s}^2\,\text{m}^{-4}$$

Solution

1. When the body encounter the fluid. The volume of the fluid that is been hit is $\int A\|v\|\,\text{d}t$. Giveing the rate is $A\|v\|$ And to find the mass, knowing the dencity, we can find the rate is $A\|v\|\,\rho$.

2. The mass that it accelerate is $A\|v\|\,\rho$ pre sec. Thus we have $F = A\|v\|\,\rho a$ per sec. We wish to accelerate the fluid to $\|v\|$ from 0. Thus we have $a = v$. THus we have $F = F = A\|v\|^2\,\rho$.

3. Knowing the force, we could set up the euqation:

$$\frac{\rho A(t)v^2}{4}m = -g\hat{z} - \frac{c}{m}v\vec{v}$$

   where $A = \pi(\sin(\arccos\frac{R-tv}{R})R)^2 = \pi(1 - \frac{R-tv}{R})R^2$. Thus we have:

$$\frac{\rho\pi(1-\frac{R-tv}{R})R^2\pi(1-\frac{R-tv}{R})R^2v^2}{4}m = -g\hat{z} - \frac{c}{m}v\vec{v}$$

PROBLEM 2

For one-dimensional motion along the z-axis, if $F_z = f(z)$ for an arbitrary function f, write the general solution relating v_z and z in terms of an integral (hintmultiply the equation through by $v_z \equiv \dot{z}$).

Solution

$$\begin{aligned}
F_z = f(z) &= m\ddot{r}_z \\
\frac{\text{d}^2 z}{\text{d}t^2} &= \frac{f(z)}{m} \\
v_z = \frac{\text{d}z}{\text{d}t} &= \frac{1}{m}\int f(z)\text{d}t \\
&= \frac{1}{m}\int_0^d f(z)\text{d}d + v_z(0)
\end{aligned}$$

PROBLEM 3

For one-dimensional motion along the z-axis, if $F_z = f(v_z)$ for an arbitrary function f, write the general solution relating t and v_z in terms of an integral.

Solution

$$F_z = f(v_z) = m\ddot{r}_z$$
$$\frac{\mathrm{d}v_z}{\mathrm{d}t} = \frac{f(v_z)}{m}$$
$$\mathrm{d}v_z = \frac{f(v_z)}{m}\mathrm{d}t$$
$$\frac{\mathrm{d}v_z}{f(v_z)} = \frac{1}{m}\mathrm{d}t$$
$$\int \frac{\mathrm{d}v_z}{f(v_z)} = \frac{1}{m}t + C$$

PROBLEM 4

For quadratic drag, suppose $\dot{z}(0) = 0$

1. What is the initial slope w_0 in this case? What is the integration constant A in equation (3.44) of the notes?

2. The mass falls; after some time, $v_x = -v_z$. Use (3.38) of the note, the definition of the slope w, and (3.43) to find what the velocity is. Notice how this simplifies when the drag $c = 0$; what is the physical reason for this?

3. More generally, use (3.38), the definition of the slop w, (3,43), and (3,44) to write down the exact relation between v_x and v_z for arbitrary $\dot{x}_0$ and $\dot{z}_0$ - this is a complicated equation relating v_z, v_z, $\dot{x}_0$, $\dot{z}_0$.

Solution

1. Since $w(0) = \frac{v_z}{v_x} = 0$. We have $A = 0 + \ln(0 + \sqrt{1+0}) + \frac{mg}{c\dot{x}_0^2} = \frac{mg}{c\dot{x}_0^2}$.

2. We could have from (3.38)
$$\dot{w} = -\frac{g}{v_x}$$
and plug it in to (3.43) to have
$$\frac{g^2}{v_x^2} = -\frac{cg}{m}(w\sqrt{1+w^2} + \ln(w + \sqrt{q+w^2}) - \frac{mg}{c\dot{x}_0^2})$$
Knowing that $v_x = -v_z$, we could have $w = -1$. Thus we have
$$\frac{g}{v_x^2} = -\frac{c}{m}(-\sqrt{2} + \ln(-1+\sqrt{2}) - \frac{mg}{c\dot{x}_0^2})$$
Easy to solve for v_x
$$v_x = \sqrt{\frac{mg}{\sqrt{2}c - c\ln(\sqrt{2}-1) + \frac{mg}{\dot{x}_0^2}}}$$
And $v_z = -v_x$.
When $c = 0$, we can find that $v_x = \sqrt{\frac{mg}{(\frac{mg}{\dot{x}_0^2})}} = x_0$.
Physicaly, that is how we solve the problem when air resistance can be neglected, i.e., the velocity in x direction preserve.

3. We would have:
$$-\frac{g}{v_x} = -\frac{cg}{m}(\frac{v_z}{v_x}\sqrt{1+\frac{v_z^2}{v_x^2}} + \ln[\frac{v_z}{v_x} + \sqrt{1+\frac{v_z^2}{v_x^2}}] - A)$$
where
$$A = \frac{\dot{z}_0}{\dot{x}_0}\sqrt{1+\frac{\dot{z}_0^2}{\dot{x}_0^2}} + \ln(\frac{\dot{z}_0}{\dot{x}_0} + \sqrt{1+\frac{\dot{z}_0^2}{\dot{x}_0^2}}) + \frac{mg}{c\dot{x}_0^2}$$