Physics 303/573

Homework 6 due Wednesday, October 18.

Consider a mass m that hangs in the air suspended by four identical springs under a table. The other end of the springs are attached to the corners of the table; the table is square and its sides have length d.

1.

a) Sketch this arrangement.

b) If the springs have spring constant k and **the springs are ideal (the unstretched length is zero: $\ell = 0$)**, what is the potential energy?

c) What is the equilibrium point?

d) What are effective spring constants k_x, k_y, k_z?

2. Find the solution for the system in problem 1. with initial conditions

$$x(0) = d/2 \quad , \quad y(0) = z(0) = 0 \quad , \quad v_x(0) = 0 = v_z(0) = 0 \quad , \quad v_y(0) = A\frac{d}{2}\sqrt{k/m}$$

where A is a constant. For what value of A is the motion a circle.

3. Redo problem 1. for the case when the unstretched length of each spring is some finite positive value $\ell > 0$.

4.

a) Find the fourier series corresponding to a square wave with amplitude F/m and period T_F.

b) Find the steady state solution for a one-dimensional damped harmonic oscillator subject to this driving force.

5. A rubber band that is stretched more than a tiny bit is not a very good approximation to a harmonic oscillator. Suppose its potential is

$$U(x) = \frac{1}{2}k(x-\ell)^2 + \frac{1}{3}a(x-\ell)^3 + \frac{1}{4}b(x-\ell)^4$$

where a, b are both small positive constants.

a) Sketch the potential for $k = 1$, $a = 1/2$, $b = 1/8$.

b) Show that the only stable equilibrium point is still at $x = \ell$.

c) Find the frequency of small oscillations about this point.

d) Find the corrections to frequency and the apparent equilibrium point related to nonlinear effects when the oscillations are not so small.

NOTE: Part d) is a hard and advanced problem!