Quantum or Classical
Wave Perspective

a) $R \approx 384 \times 10^3 \text{ km}$

$\approx 3.8 \times 10^8 \text{ m}$

$x = 2\pi R = 2.4 \times 10^9 \text{ m}$

What is momentum?

Orbit is ~ 27 days $= T$

$$V = \frac{2\pi R}{T} = \omega R$$

$$= \frac{2.4 \times 10^9 \text{ m}}{27 \cancel{\text{days}} \times 24 \cancel{\text{h}}/\cancel{\text{day}} \times 60 \cancel{\text{min}}/\cancel{\text{hour}} \times 60\text{s}/\cancel{\text{min}}}$$

$$= 1029 \text{ m/s}$$

$M_{moon} = 7.34 \times 10^{22} \text{ kg}$

$$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34} \text{ J-s}}{7.3 \times 10^{22} \text{ kg} \cdot 10^3 \text{ m/s}}$$

$$= 10^{-59} \text{ m}$$

$$\boxed{\frac{x}{\lambda} \approx 10^{69}}$$ Very classical

b)

$$\frac{1}{2}mv^2 = 13.6\,eV = \frac{p^2}{2m}$$

$$p = \sqrt{2mE}$$

$$p = \sqrt{2 \times 0.511 \times 10^6\, eV/c^2 \times 13.6\, eV}$$

$$= 3700\ eV/c^2$$

$$\lambda = \frac{h}{p} = \frac{2\pi \times 6.58 \times 10^{-16}\ \cancel{eV}\text{-}s}{3700\ \cancel{eV}/c}$$

$$= 1 \times 10^{-18}\ c\text{-}s$$

$$= 3 \times 10^{-10}\ m$$

$$\lambda = 3Å$$

$$X = 2\pi \times a_0 \approx 3Å \Rightarrow \boxed{\frac{X}{\lambda} \sim 1}$$

Definitely Quantum

c) The de Broglie wavelength is about the same as the hydrogen case above since KE is the same and nearly X is also close.

$$\boxed{\frac{X}{\lambda} \sim 1}$$ Quantum.

d) Now the particle is relativistic

$$p \approx \frac{E}{c} = 1\ \frac{GeV}{c}$$

$$\lambda = \frac{h}{p} = \frac{2\pi \times 6.58\times10^{-16}\ eV\text{-}s}{10^9\ eV/c}$$

$$= 4\times10^{-24}\ \cancel{s} \times 3\times10^8\ m/\cancel{s}$$

$$= 1.24\times10^{-15}\ m$$

$$\boxed{\frac{X}{\lambda} \approx 10^5} \rightarrow \text{classical}$$

e) $M_{neutron} = 940\ MeV/c^2$

$$p = \sqrt{2\times 940\ MeV/c^2 \times 4\ MeV}$$

$$= 87\ \frac{MeV}{c}$$

$$\lambda = \frac{2\pi \times 6.58\times10^{-16}\ eV\text{-}s}{87\times10^6\ eV/c}$$

$$= 14\ fm$$

$$\Rightarrow \boxed{\frac{X}{\lambda} \sim \frac{1}{2} \Rightarrow \text{Quantum.}}$$

f) $v \approx \sqrt{\frac{kT}{m}} \approx \sqrt{\frac{0.025eV}{938\ MeV/c^2}} = 5\times10^{-6}c$

Room Temp

$= 1500\ m/s$

$$p = mv = 1.7\times10^{-27}\,kg \times 1500\,m/s$$

$$= 2.6\times10^{-24}\ kg\text{-}m/s$$

$$\lambda = \frac{h}{p} = \frac{6\times10^{-34}\ J\text{-}s}{2.6\times10^{-24}\ kg\text{-}m/s} \approx 2\times10^{-10}\ m$$

$$= 2\,\mathring{A}$$

$\boxed{\frac{x}{\lambda} \sim 1}$ $\Rightarrow$ Quantum effects are important, although in practice classical molecular dynamics or semiclassical methods (e.g. path-integral MD) can approach this problem!

g) Harmonic oscillator $x = 0.1\,\mathring{A}\cos(2\pi\times100\,THz\times t)$

$$\dot{x}_{max} = (0.1\,\mathring{A})(2\pi\times100\ THz)$$

$$= 6300\ m/s$$

This is a similar speed as f) (and the particle will go slower sometimes). So we again expect $\boxed{\frac{x}{\lambda} \sim 1}$ Quantum

Note: mass of proton and mass of H-atom are almost the same since (e^-)'s are light!

h) Simple pendulum

$$\omega = \sqrt{\frac{g}{L}} = \sqrt{\frac{9.8\,m/s^2}{50\,cm}}$$

$$\omega = 4.43\ rad/s$$

$$V_{max} = \omega X_{max}$$

$$= 4.4\,rad/s \times 10^{-19}\,m = 4.4 \times 10^{-19}\ m/s$$

$$p = 40\,kg \times V_{max} = 1.8 \times 10^{-17}\ m/s \cdot kg$$

$$\lambda = \frac{h}{p} = \frac{6 \times 10^{-34}\ J\text{-}s}{1.8 \times 10^{-17}\ kg\text{-}m/s}$$

$$\boxed{\lambda \approx 3 \times 10^{-17}\,m} \qquad \frac{\overset{10^{-19}m}{x}}{\lambda} < 1$$

Interesting that this comes out less than one... The real resonant frequency could be higher due to radiation pressure. I think the motion of the test masses in LIGO is still considered to be classical. Certainly the (frequency integrated) excursions are much larger than 10^{-19} m.

Q/C waves

i) Agarn

$$v_{max} = \omega x_{max}$$

$$= 2\pi \times 10^{5}\,Hz \times 10^{-9}\,m$$

$$\approx 10^{-3}\,m/s$$

$$\lambda = \frac{h}{m v_{max}} = \frac{6\times10^{-34}\,J\text{-}s}{5\times10^{-12}\,kg \times 10^{-3}\,m/s}$$

$$= 10^{-19}\,m \ll 10\,nm$$

$$\frac{x}{\lambda} \approx 10^{11} \qquad \frac{x}{\lambda} \gg 1 \qquad \text{classical}$$

j) $M_{Homework} \approx 2\,oz. \approx 0.05\,kg$

$$\lambda = \frac{6\times10^{-34}\,J\text{-}s}{0.05\,kg \times 1\,m/s} \approx 1\times10^{-32}\,m$$

$$\boxed{\frac{x}{\lambda} \approx 3\times10^{31}}$$ <u>Very</u> classical