Complex Absorbing Potentials

a) From the TDSE

$$\frac{\partial}{\partial t}|\Psi(x,t)|^2 = \Psi^* \frac{\partial \Psi}{\partial t} + \Psi \frac{\partial \Psi^*}{\partial t}$$

$$= \Psi^*\left(\frac{1}{i\hbar}\left(\frac{\hat{p}^2}{2m}\Psi + V\Psi\right)\right) + \Psi\left(\frac{-1}{i\hbar}\left(\frac{\hat{p}^2}{2m}\Psi^* + V^*\Psi^*\right)\right)$$

Note $\hat{p}^2$ is Hermitian so $(\hat{p}^2)^\dagger = \hat{p}^2$

$$= \frac{i}{\hbar}\left(\Psi \hat{p}^2 \Psi^* - \Psi^* \hat{p}^2 \Psi\right) + \frac{i}{\hbar}|\Psi|^2(V^* - V)$$

This term will integrate to zero b/c $\hat{p}^2$ is Hermitian, as shown in Griffiths

$$V^* - V = 2i\Gamma$$

so $$\frac{\partial}{\partial t}|\Psi|^2 = -\frac{2\Gamma}{\hbar}|\Psi|^2$$

$$\Rightarrow \boxed{\frac{dP}{dt} = -\frac{2\Gamma}{\hbar}P}$$

b) $\boxed{P(t) = P_0 e^{-t/\tau}}$ w/ $\boxed{\tau = \frac{\hbar}{2\Gamma}}$

$1/e$ lifetime