a)

$$|\phi\rangle = a|f\rangle + b|g\rangle$$

We seek a normalized $N|\phi\rangle$ such that

$$|N|^2 \langle\phi|\phi\rangle = 1$$

$$|N|^2\left[\left(\langle f|a^* + \langle g|b^*\right)\left(a|f\rangle + b|g\rangle\right)\right] = 1$$

$$1 = |N|^2\left[|a|^2\underbrace{\langle f|f\rangle}_{1} + a^*b\,\cancel{\langle f|g\rangle}^{0} + b^*a\,\cancel{\langle g|f\rangle}^{0} + |b|^2\underbrace{\langle g|g\rangle}_{1}\right] \quad (1)$$

$$|N|^2\left[|a|^2 + |b|^2\right] = 1$$

$$\boxed{|N| = \frac{1}{\sqrt{|a|^2 + |b|^2}}}$$

Any complex number that satisfies this will work

b) Now equation (1) becomes

$$1 = |N|^2\left[|a|^2 + a^*bS + ab^*S^* + |b|^2\right]$$

$$\Rightarrow \boxed{|N| = \frac{1}{\sqrt{|a|^2 + |b|^2 + 2\mathrm{Re}(a^*bS)}}}$$

Any complex number that satisfies this will work.