a) Take $\lambda = 550\,nm$ for visible light

$$\phi \approx \frac{2\pi}{\lambda} a_0 = \frac{2\pi}{5500\,Å} \cdot 0.5\,Å = 0.6\,mrad$$ (1)

Yes neglecting this phase is customary. This is called the electric dipole approximation.

b) For a plane wave

$$I = \frac{1}{2} c \varepsilon_0 E_0^2$$

$$E_0 = \sqrt{\frac{2I}{\varepsilon_0 c}} = \sqrt{\frac{2 \times 1000\,W/m^2}{8.85\times10^{-12}\,C^2/Nm^2 \times 3\times10^8\,m/s}}$$

$$\boxed{E_0 = 868\,V/m}$$ (2)

The atomic unit of electric field is

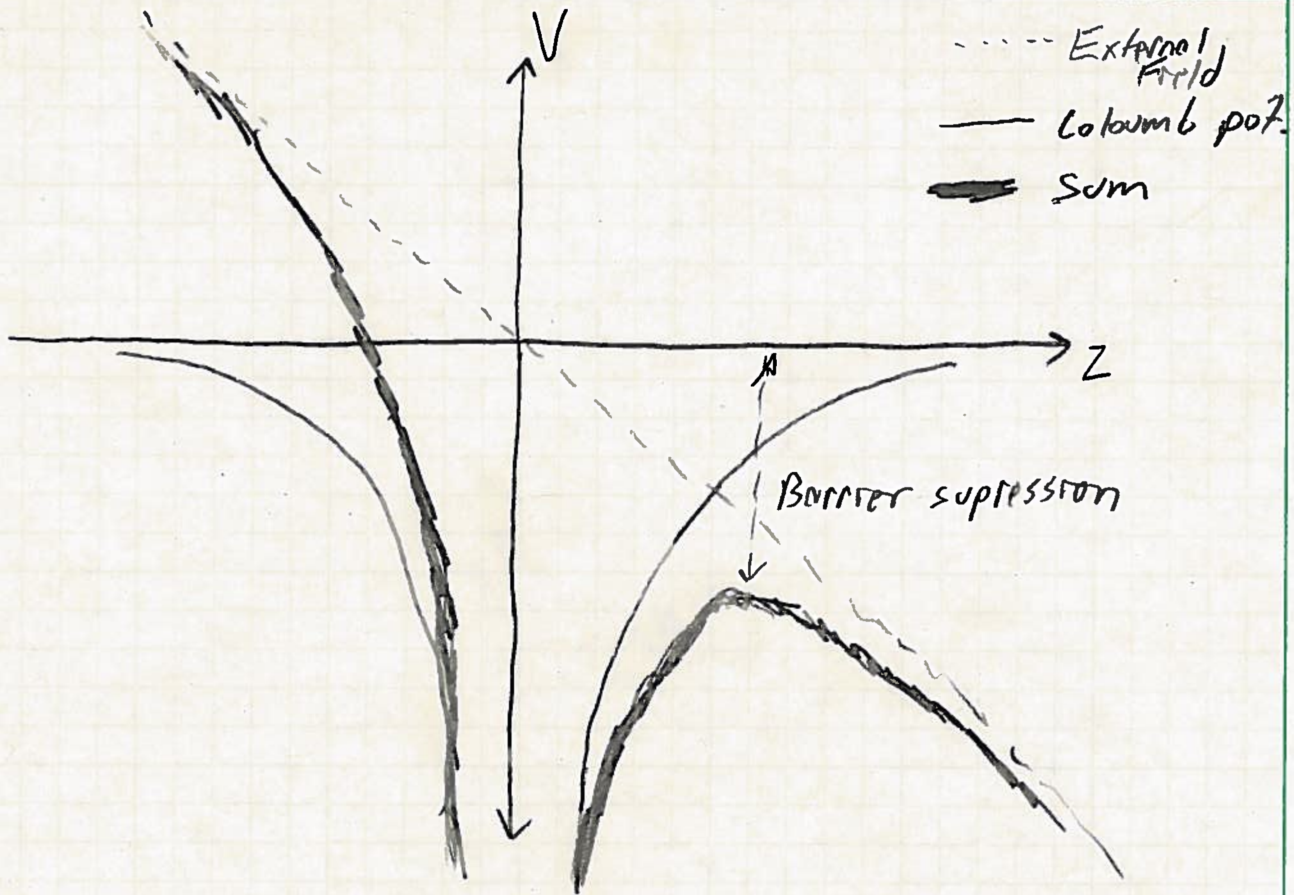
$$E_a = \frac{1\ \text{Hartree}}{e a_0} = \frac{e}{4\pi\varepsilon_0 a_0^2} = 5.14\times10^{11}\,V/m = 51.4\,V/Å$$

So the field of the solar light wave is 1.7×10^{-9} a.u.

c) In atomic units, the potential is

$$V = -\frac{1}{r} - zE_0$$

This is sketched on the next page for a slice along the z-direction



(3)

To find the saddle point, we can look at $\frac{\partial V}{\partial z} = 0$ w/ $x=0, y=0$ (z-axis)

$$V(0,0,z) = \frac{-1}{z} - zE_0$$

$$\frac{\partial V}{\partial z} = +\frac{1}{z^2} - E_0 = 0 \Rightarrow z_{max} = \frac{1}{\sqrt{E_0}}$$

so we want the barrier height $= -1\,Ry = -\frac{1}{2}$ a.u.

$$V(z_{max}) = -\sqrt{E_0} - \sqrt{E_0} = -\frac{1}{2} \text{ a.u.}$$

$$= -2\sqrt{E_0} = -\frac{1}{2} \text{ a.u.}$$

$$\Rightarrow \boxed{E_0 = \frac{1}{16} \text{ a.u.} = 3.25\ V/Å = 3.25\times10^{10}\ V/m}$$ (4)

The corresponding intensity is

$$I = \frac{1}{2} c \varepsilon_0 E_0^2$$

$$= \frac{1}{2}(3\times10^8 \, m/s)(8.85\times10^{-12} \, C^2/Nm^2)(3.25\times10^{10} \, V/m)^2$$

$$\boxed{I = 1.4\times10^{18} \, W/m^2} \quad (5)$$

This sounds like a lot, but consider a 1mJ, 100fs laser pulse focused to an area of $(10\mu m)^2$

$$I \approx \frac{1\times10^{-3} \, J}{1\times10^{-13} s \times (10^{-5} m)^2} = 10^{20} \, W/m^2 \quad (6)$$

Well in excess of the requirement.