The key to this problem is to write the Hamiltonian in the form

$$\hat{H} = \hat{a}^{+}\hat{a} + \hbar\omega/2 \qquad (1)$$

and use $[\hat{a},\hat{a}^{+}] = \hbar\omega$

$$[\hat{H},\hat{a}] = [\hat{a}^{+}\hat{a},\hat{a}] = \hat{a}^{+}\cancel{[\hat{a},\hat{a}]}^{0} + [\hat{a}^{+},\hat{a}]\hat{a}$$

$$= -\hbar\omega\hat{a}$$

similarly

$$[\hat{H},\hat{a}^{+}] = [\hat{a}^{+}a, a^{+}] = a^{+}[a,a^{+}] + \cancel{[a^{+},a^{+}]}^{0} a$$

$$= \hbar\omega a$$

so we have for the Heisenberg equations of motion

$$\frac{d\hat{a}}{dt} = \frac{i}{\hbar}(-\hbar\omega)\hat{a}$$

$$\frac{d\hat{a}}{dt} = -i\omega\hat{a} \Rightarrow \boxed{\hat{a}(t) = \hat{a}(t=0)\,e^{-i\omega t}} \qquad (2)$$

Similarly

$$\frac{d\hat{a}^{+}}{dt} = i\omega a^{+} \Rightarrow \boxed{a^{+}(t) = a^{+}(t=0)\,e^{i\omega t}} \qquad (3)$$