Pauli Exclusion Principle

For part b), see MATLAB

a) show that wavefunction satisfies the TISE

set $L=1$

$$\hat{H} = \frac{-\hbar^2}{2m}\frac{d^2}{dx_1^2} - \frac{\hbar^2}{2m}\frac{d^2}{dx_2^2} + V(x_1,x_2)$$

$$V(x_1,x_2) = \begin{cases} 0, & 0<x_1<1 \text{ and } 0<x_2<1 \\ \infty, & \text{otherwise} \end{cases}$$

$$\psi_e = \left[\phi_1(x_1)\phi_2(x_2) - \phi_1(x_2)\phi_2(x_1)\right]\alpha(1)\alpha(2)$$

ϕ_1 and ϕ_2 are separately eigenstates of the Hamiltonian

$\hat{H}\phi_1 = E_1\phi_1$

e.g.

$$\frac{-\hbar^2}{2m}\frac{d^2}{dx_1^2}\phi_1 = \sqrt{2}\left(\frac{+\hbar^2}{2m}\right)\sin(\pi x_1)\pi^2$$

$$\frac{-\hbar^2}{2m}\frac{d^2}{dx_2^2}\phi_1(x_1) = 0$$

$$= \frac{\hbar^2\pi^2\overbrace{(1)^2}^{n^2}}{2m\underbrace{(1)^2}_{L^2}} = E_1\phi_1(x_1)$$

Write $\hat{H} = \hat{h}(x_1) + \hat{h}(x_2)$

$$\hat{h}(x_1) = \frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x_1^2}$$

$$\hat{h}(x_2) = \frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x_2^2}$$

$\hat{h}(x_1)\phi_1(x_1) = E_1\phi_1(x_1)$ $\quad\quad$ $\hat{h}(x_2)\phi_2(x_2) = E_2\phi_2(x_2)$

$\hat{h}(x_1)\phi_1(x_2) = 0$

$\hat{h}(x_2)\phi_1(x_2) = E_2\phi_2(x_2)$ $\quad$ etc...

$$\hat{H} = \hat{h}(x_1) + \hat{h}(x_2)$$

$$\hat{H}\psi_e = \left(\hat{h}(x_1) + \hat{h}_2(x_2)\right)\left(\phi_1(x_1)\phi_2(x_2) - \phi_1(x_2)\phi_2(x_1)\right)$$

$$= E_1\phi_1(x_1)\phi_2(x_2) - E_2\phi_1(x_2)\phi_2(x_1)$$
$$+ E_2\phi_1(x_1)\phi_2(x_2) - E_1\phi_1(x_2)\phi_2(x_1)$$

$$= (E_1 + E_2)\left(\phi_1(x_1)\phi_2(x_2) - \phi_1(x_2)\phi_2(x_1)\right)$$

$$= (E_1 + E_2)\psi_e$$