Spin Precession

a) $\hat{H} = -\vec{\mu} \cdot \vec{B}$

$$\boxed{\hat{H} = -\gamma B_0 \hat{I}_z}$$

b) $E_\uparrow = \langle \uparrow | \hat{H} | \uparrow \rangle = -\frac{\gamma B_0 \hbar}{2}$

$E_\downarrow = \langle \downarrow | \hat{H} | \downarrow \rangle = +\frac{\gamma B_0 \hbar}{2}$

$\boxed{\Delta E = \hbar \gamma B_0}$ $\boxed{\omega \equiv \gamma B_0}$

$$\boxed{|\Psi(t)\rangle = \frac{1}{\sqrt{2}}|\uparrow\rangle + \frac{1}{\sqrt{2}} e^{-i\Delta E t/\hbar} |\downarrow\rangle}$$

Note, we do not need to keep track of any overall phase factor b/c it has no impact on observables.

c) Easiest to work w/ Pauli matrices

$$I_x = \frac{\hbar}{2}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\langle \hat{I}_x \rangle = \frac{\hbar}{2}\left(\frac{1}{\sqrt{2}}, \frac{e^{i\omega t}}{\sqrt{2}}\right)\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{pmatrix} 1/\sqrt{2} \\ e^{-i\omega t}/\sqrt{2} \end{pmatrix}$$

$$= \frac{\hbar}{2}\left(\frac{1}{\sqrt{2}}, \frac{e^{i\omega t}}{\sqrt{2}}\right)\begin{pmatrix} e^{-i\omega t}/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$$

$$\langle I_x \rangle = \frac{\hbar}{2}\left(\frac{e^{-i\omega t}}{2} + \frac{e^{i\omega t}}{2}\right)$$

$$= \frac{\hbar}{2}\cos(\omega t)$$

$$\langle \hat{I}_y \rangle = \frac{\hbar}{2}\left(\frac{1}{\sqrt{2}}, \frac{e^{i\omega t}}{\sqrt{2}}\right)\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}\begin{pmatrix} 1/\sqrt{2} \\ e^{-i\omega t}/\sqrt{2} \end{pmatrix}$$

$$= \frac{\hbar}{2}\left(\frac{1}{\sqrt{2}}, \frac{e^{i\omega t}}{\sqrt{2}}\right)\begin{bmatrix} -i\frac{e^{-i\omega t}}{\sqrt{2}} \\ i/\sqrt{2} \end{bmatrix}$$

$$= -\frac{\hbar}{2}\,\frac{e^{i\omega t} - e^{-i\omega t}}{2i}$$

$$\boxed{\langle \hat{I}_y \rangle = -\frac{\hbar}{2}\sin(\omega t)}$$

$$\langle I_z \rangle = \frac{\hbar}{2}\left(\frac{1}{\sqrt{2}}, \frac{e^{i\omega t}}{\sqrt{2}}\right)\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{pmatrix} 1/\sqrt{2} \\ \frac{e^{-i\omega t}}{\sqrt{2}} \end{pmatrix}$$

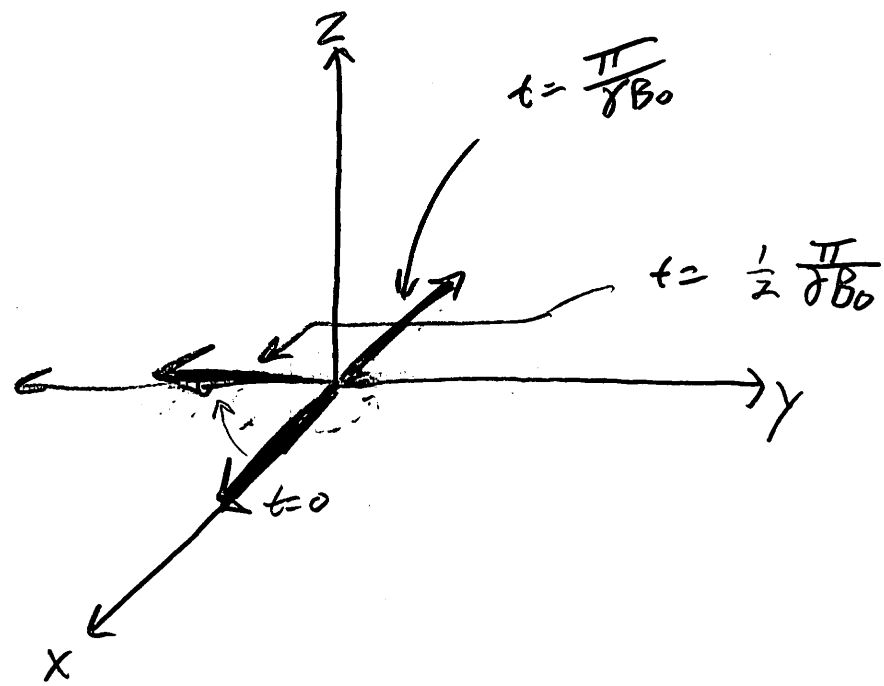
$$= \frac{\hbar}{2}\left(\frac{1}{2} - \frac{1}{2}\right) = \boxed{0 = \langle I_z \rangle}$$

Spin Precession

$\omega t = 0 \quad \langle \vec{I} \rangle = \frac{\hbar}{2} \hat{e}_x$

$\omega t = \frac{\pi}{2} \quad \langle \vec{I} \rangle = \frac{\hbar}{2} \hat{e}_y$

$\omega t = \pi \quad \langle \vec{I} \rangle = -\frac{\hbar}{2} \hat{e}_x$



spin precesses around magnetic field in x, y plane.