

a) Integer Spin $\Rightarrow$ boson

b) $I_{min} = |I_1 - I_2| = |1-1| = 0$

$I_{max} = |I_1 + I_2| = |1+1| = 2$

$I = 2, 1, 0$ are the allowed values.

Degeneracy is $2I+1$, so

$I=2$ is 5-fold degenerate

$I=1$ is 3-fold degenerate

$I=0$ is nondegenerate (1 state)

So there are 9 possible states. Does this make sense? Each individual spin has 3 possible states, and there are 2 spins.

$\boxed{3^2 = 9}$ states.

c)

$I=2 \Rightarrow m_I = 2, 1, 0, -1, -2$

$I=1 \Rightarrow m_I = 1, 0, -1$

$I=0 \Rightarrow m_I = 0$

d)

All even under interchange

$|2\,2\rangle = |1\,1\rangle|1\,1\rangle$

$|2\,1\rangle = \frac{1}{\sqrt{2}}|1\,1\rangle|1\,0\rangle + \frac{1}{\sqrt{2}}|1\,0\rangle|1\,1\rangle$

$I=2$ $|2\,0\rangle = \frac{1}{\sqrt{6}}|1\,1\rangle|1\,{-1}\rangle + \sqrt{\frac{2}{3}}|1\,0\rangle|1\,0\rangle + \frac{1}{\sqrt{6}}|1\,{-1}\rangle|1\,1\rangle$

$|2\,{-1}\rangle = \frac{1}{\sqrt{2}}|1\,0\rangle|1\,{-1}\rangle + \frac{1}{\sqrt{2}}|1\,{-1}\rangle|1\,0\rangle$

$|2\,{-2}\rangle = |1\,{-1}\rangle|1\,{-1}\rangle$

Nuc. Spin. Stats

$I=1 \rightarrow$ odd under interchange

$$|1\,1\rangle = \frac{1}{\sqrt{2}}|1\,1\rangle|1\,0\rangle - \frac{1}{\sqrt{2}}|1\,0\rangle|1\,1\rangle$$

$$|1\,0\rangle = \frac{1}{\sqrt{2}}|1\,{-1}\rangle|1\,1\rangle - \frac{1}{\sqrt{2}}|1\,1\rangle|1\,{-1}\rangle$$

$$|1\,{-1}\rangle = \frac{1}{\sqrt{2}}|1\,0\rangle|1\,{-1}\rangle - \frac{1}{\sqrt{2}}|1\,{-1}\rangle|1\,0\rangle$$

$I=0$ symmetric (even) under interchange

$$|0\,0\rangle = \frac{1}{\sqrt{3}}|1\,{+1}\rangle|1\,{-1}\rangle - \frac{1}{\sqrt{3}}|1\,0\rangle|1\,0\rangle + \frac{1}{\sqrt{2}}|1\,{-1}\rangle|1\,1\rangle$$

e) $I=2$ even
$I=1$ odd
$I=0$ even

f) Must have total wavefcn symmetric under C_2. $\psi_{\substack{nuc\\vib.}}\,\psi_{el}$ is <u>even</u>

must have

$$\psi^{(rot)}\,\psi^{(nuc.\ spin)} = \text{even}$$

$\Rightarrow$ even J must have $I=0$ or $I=2$

odd J must have $I=1$

g) Total degeneracy is a combination of m_J degeneracy and I, m_I degeneracy.

For even J, have $2J+1$ m_J states and each of these has $\underbrace{2\cdot 2+1}_{I=2} + \underbrace{2\cdot 0+1}_{I=0} = 6$

Nuclear spin states. So for even J, have

$$6(2J+1) \quad \text{even } J$$

For odd J, have only $I=1$, w 3 states, so degeneracy is

$$3(2J+1) \quad \text{odd } J.$$

h) since $e^{-\Delta E/k_BT} \to 1$, it is the ratio of the degeneracies that matters.

$$J=0 \Rightarrow 6\cdot(2\cdot 0+1) = 6 \text{ states}$$

$$J=1 \Rightarrow 3\cdot(2\cdot 1+1) = 9 \text{ states}$$

$$P(J=1) = \frac{9}{6+9} = \frac{9}{15} = \frac{3}{5} = 60\%$$

$$P(J=0) = \frac{6}{6+9} = \frac{6}{15} = \frac{2}{5} = 40\%$$

$\Rightarrow$ only 50% more likely.