

$$1/2 \times 1/2$$

1
+1
+1/2 +1/2
1
1 0
0 0
+1/2 -1/2
1/2 1/2
1
-1/2 +1/2
1/2 -1/2
-1
-1/2 -1/2
1

$$1 \times 1/2$$

3/2
+3/2
+1 +1/2
1
+1/2 +1/2
3/2 1/2
+1 -1/2
1/3 2/3
3/2 1/2
0 +1/2
2/3 -1/3
-1/2 -1/2
0 -1/2
2/3 1/3
3/2
-1 +1/2
1/3 -2/3
-3/2
-1 -1/2
1

$$2 \times 1$$

3
+3
+2 +1
1
+2 +2
3 2
+2 0
1/3 2/3
3 2 1
+1 +1
2/3 -1/3
+2 -1
1/15 1/3
3/5
+1 0
8/15 1/6
-3/10
0 +1
6/15 -1/2
1/10
3 2 1
0 0 0
+1 -1
1/5 1/2
3/10
0 0
-2/5
-1 +1
1/5 -1/2
3/10
3 2 1
-1 -1 -1

$$1 \times 1$$

2
+2
+1 +1
1
+1 +1
+1 0
1/2 1/2
2 1 0
0 +1
1/2 -1/2
0 0 0
+1 -1
1/6 1/2
1/3
0 0
2/3 0
-1/3
-1 +1
1/6 -1/2
1/3
2 1
-1 -1
0 -1
1/2 1/2
2
-1 0
1/2 -1/2
-2
-1 -1
1

$$3/2 \times 1$$

5/2
+5/2
+3/2 +1
1
+3/2 +3/2
+3/2 0
2/5 3/5
5/2 3/2
+1/2 +1/2
+1/2
+1
3/5 -2/5
+1/2 +1/2
+1/2
+3/2 -1
1/10 2/5
1/2
+1/2 0
3/5 1/15
-1/3
-1/2 +1
3/10 -8/15
1/6
5/2 3/2
1/2
-1/2 -1/2
-1/2

+1/2 -1
3/10 8/15
1/6
-1/2 0
3/5 -1/15
-1/3
-3/2 +1
1/10 -2/5
1/2
5/2 3/2
-3/2 -3/2
-1/2 -1
3/5 2/5
5/2
-3/2 0
2/5 -3/5
-5/2
-3/2 -1
1
0 -1
6/15 1/2
1/10
-1 0
8/15 -1/6
-3/10
-2 +1
1/15 -1/3
3/5
3 2
-2 -2
-1 -1
2/3 1/3
3
-2 0
1/3 -2/3
-3
-2 -1
1

$$2 \times 1/2$$

5/2
+5/2
+2 1/2
1
5/2 3/2
3/2 +3/2
+2 -1/2
1/5 4/5
5/2 3/2
+1 +1/2
4/5 -1/5
+1/2 +1/2
+1 -1/2
2/5 3/5
5/2 3/2
0 +1/2
3/5 -2/5
-1/2 -1/2
0 -1/2
3/5 2/5
5/2 3/2
-1 +1/2
2/5 -3/5
-3/2 -3/2
-1 -1/2
4/5 1/5
5/2
-2 +1/2
1/5 -4/5
-5/2
-2 -1/2
1

$$3/2 \times 1/2$$

2
+2
+3/2 +1/2
1
+1 +1
+3/2 -1/2
1/4 3/4
2 1
+1/2 +1/2
3/4 -1/4
0 0
+1/2 -1/2
1/2 1/2
2 1
-1/2 +1/2
1/2 -1/2
-1 -1
-1/2 -1/2
3/4 1/4
2
-3/2 +1/2
1/4 -3/4
-2
-3/2 -1/2
1

-1/2 -1/2
3/4 1/4
2
-3/2 +1/2
1/4 -3/4
-2
-3/2 -1/2
1
+1/2 -1
3/10 8/15
1/6
-1/2 0
3/5 -1/15
-1/3
-3/2 +1
1/10 -2/5
1/2
5/2 3/2
-3/2 -3/2
-1/2 -1
3/5 2/5
5/2
-3/2 0
2/5 -3/5
-5/2
-3/2 -1
1


$$\vec{J} = \vec{L} + \vec{S}$$

2p states of hydrogen:

$$l = 1$$

$$s = 1/2$$

$$|l = 1, s = 1/2; jm\rangle =$$

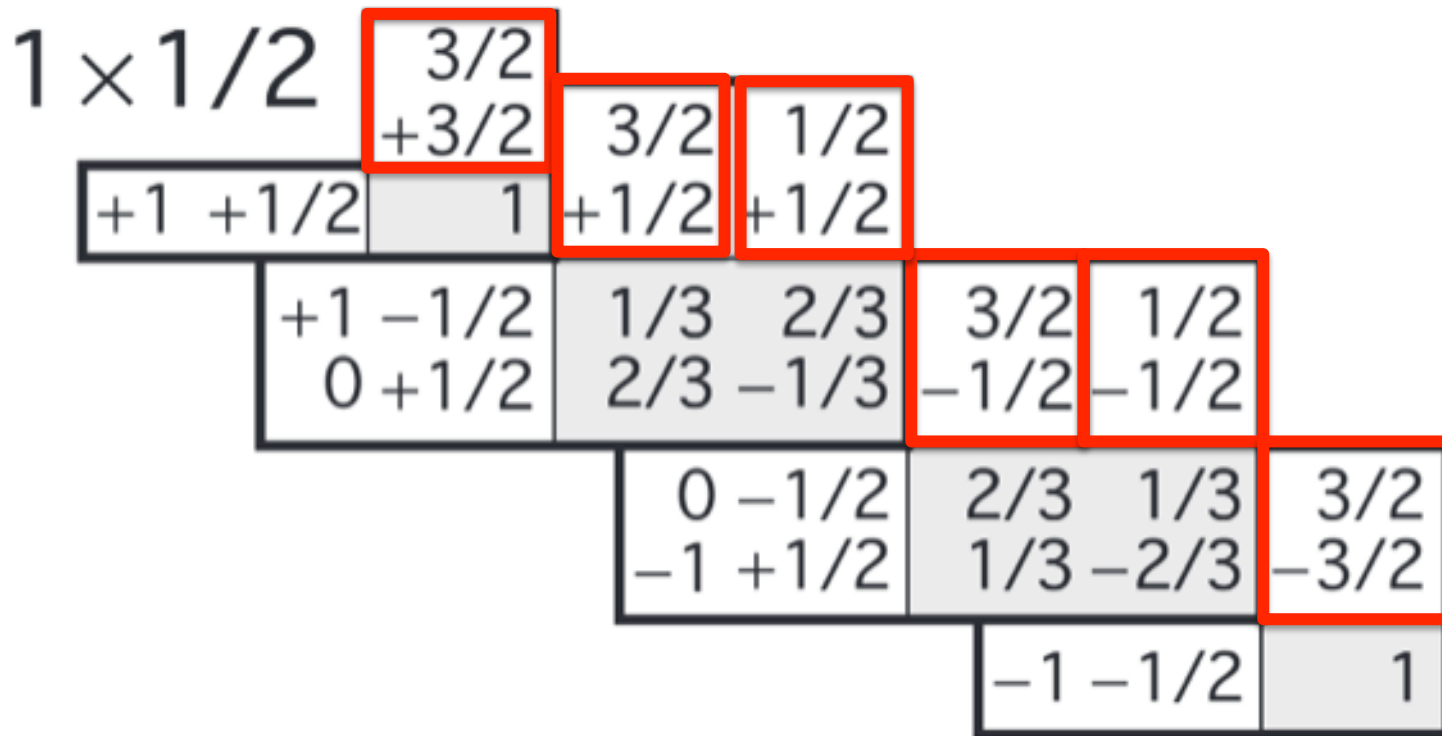
$$\sum_{m_1 m_2} \langle l = 1, s = 1/2; m_1 m_2 | l = 1, s = 1/2; jm \rangle |l = 1, s = 1/2; m_1 m_2\rangle$$


Clebsch-Gordan Coefficients!



$$-1 - 1/2$$

$$(2 \times 1 + 1) \left( 2 \times \frac{1}{2} + 1 \right) = 6$$



$$\left(2 \times \frac{3}{2} + 1\right) + \left(2 \times \frac{1}{2} + 1\right) = 6$$

Diagram illustrating a triangular arrangement of boxes, likely representing a Pascal's triangle of fractions or a similar combinatorial structure. The boxes are arranged in rows, with each row having one more box than the previous. The top row has one box with  $1 \times 1/2$ . The second row has two boxes:  $+3/2$  and  $3/2 \ 1/2$ . The third row has three boxes:  $+1 \ +1/2$ , a box with  $1$  (highlighted with a red border), and  $+1/2 \ +1/2$ . The fourth row has four boxes:  $+1 \ -1/2$ ,  $0 \ +1/2$ , a box with  $1/3 \ 2/3$  (shaded), and  $3/2 \ 1/2$ . The fifth row has five boxes:  $0 \ -1/2$ ,  $-1 \ +1/2$ , a box with  $2/3 \ 1/3$  (shaded),  $1/3 \ -2/3$  (shaded), and  $3/2$ . The sixth row has six boxes:  $-1 \ -1/2$ , a box with  $1$  (shaded), and  $3/2$ . The seventh row has seven boxes:  $-1 \ -1/2$ , a box with  $1$  (shaded), and  $3/2$ . The eighth row has eight boxes:  $-1 \ -1/2$ , a box with  $1$  (shaded), and  $3/2$ .

$$\langle m_l = +1, m_s = +1/2 | j = 3/2, m = +3/2 \rangle = \sqrt{1}$$

$$|j = 3/2, m = +3/2\rangle = \sqrt{1} \times |m_l = +1, m_s = +1/2\rangle$$

$1 \times 1/2$		$3/2$				
		$+3/2$	$3/2$	$1/2$		
$+1$	$+1/2$		$1$	$+1/2$	$+1/2$	
		$+1$	$-1/2$	$1/3$	$2/3$	$3/2$
		$0$	$+1/2$	$2/3$	$-1/3$	$1/2$
				$0$	$-1/2$	$2/3$
				$-1$	$+1/2$	$1/3$
						$3/2$
						$-3/2$
				$-1$	$-1/2$	$1$

$$|j = 3/2, m = +1/2\rangle =$$

$$\sqrt{\frac{1}{3}} |m_l = +1, m_s = -1/2\rangle + \sqrt{\frac{2}{3}} |m_l = 0, m_s = +1/2\rangle$$

$1 \times 1/2$		$\begin{array}{c} 3/2 \\ +3/2 \end{array}$		$\begin{array}{cc} 3/2 & 1/2 \end{array}$	
$+1$	$+1/2$	$1$	$+1/2$	$+1/2$	
$\begin{array}{cc} +1 & -1/2 \\ 0 & +1/2 \end{array}$		$\begin{array}{c} 1/3 \\ 2/3 \end{array}$	$\begin{array}{c} 2/3 \\ -1/3 \end{array}$	$\begin{array}{cc} 3/2 & 1/2 \\ -1/2 & -1/2 \end{array}$	
$\begin{array}{cc} 0 & -1/2 \\ -1 & +1/2 \end{array}$		$\begin{array}{cc} 2/3 & 1/3 \\ 1/3 & -2/3 \end{array}$	$\begin{array}{c} 3/2 \\ -3/2 \end{array}$		
		$-1$	$-1/2$	$1$	

$$|j = 1/2, m = +1/2\rangle =$$

$$\sqrt{\frac{2}{3}} |m_l = +1, m_s = -1/2\rangle - \sqrt{\frac{1}{3}} |m_l = 0, m_s = +1/2\rangle$$



Can also run it backwards!

$1 \times 1/2$		$3/2$			
		$+3/2$		$3/2$	$1/2$
$+1$	$+1/2$		$1$	$+1/2$	$+1/2$
		$+1$	$-1/2$	$1/3$	$2/3$
		$0$	$+1/2$	$2/3$	$-1/3$
		$0$	$-1/2$	$2/3$	$1/3$
		$-1$	$+1/2$	$1/3$	$-2/3$
				$-1$	$-1/2$
					$1$

$$|m_l = 1, m_s = -1/2\rangle =$$

$$\sqrt{\frac{1}{3}} |j = 3/2, m = +1/2\rangle + \sqrt{\frac{2}{3}} |j = 1/2, m = +1/2\rangle$$