

$$a) \quad \frac{d\sigma}{d\Omega} = \frac{\sigma}{4\pi} (1 + \beta P_2(\cos\theta))$$

collecting at all angles amounts to integrating over all angles

$$\int d\Omega \frac{d\sigma}{d\Omega} = \sigma + \frac{\beta\sigma}{4\pi} \int_0^{2\pi} d\phi \int_0^\pi d\theta \sin\theta P_2(\cos\theta) \quad (1)$$

$$P_2(\cos\theta) = \frac{1}{2}(3\cos^2\theta - 1)$$

Substituting $u = \cos\theta$, $du = -\sin\theta d\theta$

$$\begin{aligned} \int_0^{2\pi} d\phi \int_0^\pi \sin\theta d\theta &= -2\pi \int_1^{-1} du \frac{1}{2}(3u^2 - 1) \\ &= \pi \int_{-1}^1 du (3u^2 - 1) \\ &= \pi \left[u^3 - u \right]_{-1}^1 = 0 \end{aligned}$$

So the second term of (1) vanishes and you have

$$\boxed{\int \frac{d\sigma}{d\Omega} d\Omega = \sigma \quad \text{independent of } \beta}$$

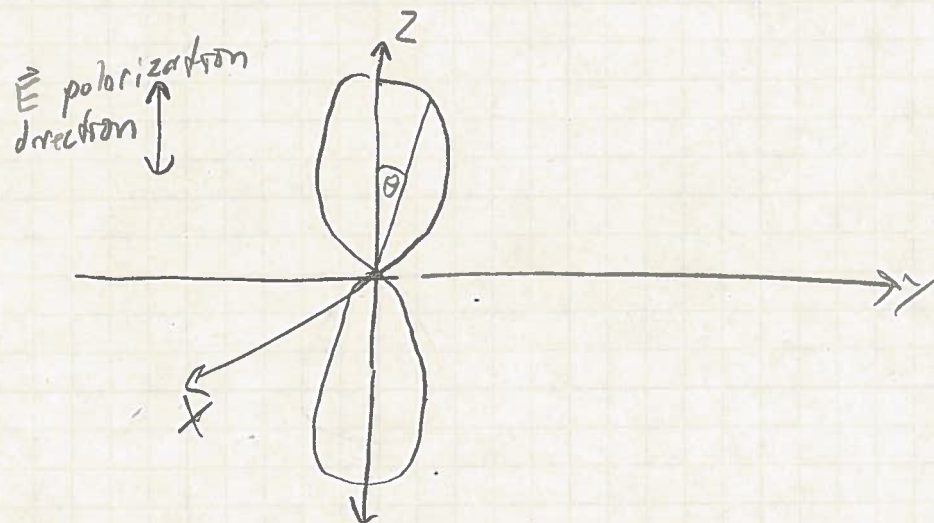
b) The second term vanishes when

$$P_2(\cos\theta) = 0 = \frac{1}{2}(3\cos^2\theta - 1)$$

$$\text{or } \cos^2\theta = \frac{1}{3} \quad \boxed{\theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) = 54.736^\circ}$$

The magic angle

c) For $\beta=2$, the intensity is maximal for $\theta=0$ (along the light polarization) and zero for $\theta=90^\circ$. The distribution looks like this on a radial plot



Assuming the dissociation is much faster than molecular rotation (and judging from the potential curves in the quoted paper, it definitely is), the molecular dipole must be \parallel to the molecular axis, so that the molecules that absorbed light and dissociated were the ones that were \parallel to \vec{E} .