Quantum or Classical

DOS

a) For particle on a ring

$$E = \frac{\hbar^2 k^2}{2m} \qquad k \cdot \cancel{2\pi} R = n \cancel{2\pi}$$

integer (n)

$$k = \frac{n}{R}$$

For moon, $\lambda \approx 10^{-59}$ m (see last problem)

$$R \approx 4 \times 10^{8} \text{ m}$$

$$\Rightarrow n = \frac{2\pi}{10^{-59}\,\cancel{m}} \times 4\times10^{8}\,\cancel{m}$$

$$= 2.5 \times 10^{68}$$

$$E = \frac{\hbar^2}{2m}\frac{n^2}{R^2}$$

$$\Delta E = \frac{\cancel{2}\hbar^2}{\cancel{2}mR^2}\Delta n\, n \qquad \Delta n = 1 \text{ for adjacent states}$$

$$\Delta E = \frac{\hbar^2 n}{MR^2}$$

$$= \frac{(1\times10^{-34}\text{ J-s})^2 \times 2.5\times10^{68}}{7.3\times10^{22}\text{ kg} \times (3.8\times10^{8}\text{ m})^2}$$

$$= 2.4\times10^{-40}\text{ J}$$

$$K_BT = 1.38\times10^{-23}\text{ J/K} \times 1700\text{ K}$$

$$= 2.34\times10^{-20}\text{ J}$$

$$\boxed{\frac{K_BT}{\Delta E} \sim 10^{20}}$$

Very Classical

b) $\Delta E = \left(-\frac{1}{2^2} - \frac{1}{1^2}\right) 13.6\,eV$

$= \frac{3}{4} 13.6\,eV = 10.2\,eV$

$k_B T = 0.025\,eV$ at Room Tem

$$\boxed{\frac{k_B T}{\Delta E} \approx \frac{0.025\,eV}{10.2\,eV} = 2 \times 10^{-3}}$$ quantum

Only $e^{-\Delta E/k_B T} = e^{-500}$ prob. of finding excited H atom at room temp.

c) From Ashcroft and Mermin, the DOS at the Fermi level is

$$\left.\frac{dN}{d\varepsilon}\right|_{\varepsilon_F} = \frac{3}{2}\frac{n}{\varepsilon_F}$$

For Al $n \approx 18 \times 10^{22}$ cm^3 $\varepsilon_F = 11.7\,eV$

$$\left.\frac{dN}{d\varepsilon}\right|_{\varepsilon_F} = \frac{3}{2}\frac{2 \times 10^{23}\,cm^{-3}}{11.7\,eV}$$

$$= 2.6 \times 10^{22}\ \text{states}/cm^3\text{-}eV$$

So for $1\,cm^3$ and $kT = 25\,meV$

we have $\sim 2.6 \times 10^{22} \frac{\text{states}}{\cancel{cm^3}\text{-}\cancel{eV}} \times 0.025\,\cancel{eV} \times \cancel{1\,cm^3}$

$= \boxed{7 \times 10^{20}\ \text{states}}$

$\Rightarrow$ Huge. $\Rightarrow$ Classical

From this perspective the electrons near the Fermi energy behave classically. Indeed many transport properties of metals can be deduced w/ reasonable accuracy treating the e^- classically $\rightarrow$ the "Drude model", but other properties, such as the heat capacity and bulk modulus, require the (quantum) Sommerfeld model.

d) Now we must consider a <u>relativistic</u> particle on a ring.

$$p = \frac{E}{c} = \hbar k$$

Boundary conditions

$$k(792\text{m}) = 2\pi n$$

$$\frac{2\pi n}{792\text{m}} = \frac{E}{\hbar c}$$

$$\Rightarrow n = \frac{3\text{ GeV} \cdot 792\text{m}}{2\pi \cdot 6.582\times10^{-16}\text{eV-s} \cdot 3\times10^{8}\text{m/s}}$$

$$n = 2\times10^{19}$$

for $\Delta n = 1$

$$\Delta E = \frac{2\pi\hbar c}{792\text{m}} = 10^{-9}\text{eV}$$

0.1% energy bandwidth is 3 MeV

$$\Rightarrow \frac{3\text{ MeV}}{\underset{\uparrow\ \Delta E}{10^{-9}\text{eV}}} = 3\times10^{15}\text{ states}$$

<u>Very</u> Classical

e) PIB $E = \frac{\hbar^2 n^2 \pi^2}{2ma^2}$

$$\Delta E = \frac{\hbar^2 \pi^2}{2ma^2} n \, \Delta n$$

take n, $\Delta n = 1$
diff. between
$n=1$
and $n=2$

$$\Delta E = \frac{\hbar^2 \pi^2}{2ma^2}$$

$$= \frac{(6 \times 10^{-34} \text{ J-s})^2 (\pi)^2}{(2 \times 1.7 \times 10^{-27} \text{ kg}) \times (7.5 \times 10^{-15} \text{ m})^2}$$

$$= 2 \times 10^{-11} \text{ J} \approx 100 \text{ MeV}$$

$$\boxed{\frac{K_B T}{100 \text{ MeV}} \approx 10^{-9} \rightarrow \text{Quantum}}$$

f) Harmonic Oscillator State spacing is $\hbar\omega$

$$\hbar\omega = 6.58 \times 10^{-16} \text{ eV-}\cancel{s} \times 80 \times 10^{12} \cancel{s}^{-1} \times 2\pi$$

$$= 0.33 \text{ eV}$$

$K_B T = 0.025 \text{ eV}$ at R.T.

$\Rightarrow$ Somewhat Quantum

$$\boxed{\frac{K_B T}{\hbar\omega} \sim 10}$$

Q/C w/ DOS

g) Same as f)

$$\boxed{\frac{k_B T}{\hbar\omega} \sim 10}$$

h) The natural frequency is still $\omega \sim 4.4$ rad/s

$$\hbar\omega = (4.4 \text{ rad/s})(6.58\times10^{-16} \text{ eV-s})$$

$$\approx 2\times10^{-15} \text{ eV}$$

$k_B T = 2.5\times10^{-2}$ eV at R.T.

$\Rightarrow \frac{k_B T}{\hbar\omega} \approx 10^{13} \rightarrow$ Very Classical!

i) Now $\hbar\omega = 2\pi \times 10^5 \text{ Hz} \times 6.582\times10^{-16} \text{ eV-s}$

$$= 4\times10^{-10} \text{ eV}$$

$\Rightarrow \frac{k_B T}{\hbar\omega} \approx 6\times10^{7}$ still Classical, although small

j) Box is 30 cm now

$M = 0.05$ kg

$$\Delta E = \frac{\hbar^2\pi^2}{2ma^2} = \frac{(6\times10^{-34} \text{ J-s})^2 \pi^2}{2\times 0.05\text{kg} \times (0.3\text{m})^2}$$

$= 4\times10^{-64}$ eV $\Rightarrow \boxed{10^{62} \text{ states}}$ Very Classical!

$k_B T \sim 10^{-2}$ eV