a) To constitute a vector space, the elements of a set must satisfy

$$\alpha f_1 + \beta f_2 = f_3 \in \text{vector space} \tag{1}$$

If we take any two polynomials $f_1(x)$ and $f_2(x)$ with degree less than N, e.g. of the form

$$f_1(x) = a_0 + a_1 x + \dots + a_N x^N$$
$$f_2(x) = b_0 + b_1 x + \dots + b_N x^N \tag{2}$$

Their sum (1) will be

$$\alpha f_1 + \beta f_2 = (\alpha a_0 + \beta b_0) + (\alpha a_1 + \beta b_1) x + \dots + (\alpha a_N + \beta b_N) x^N \tag{3}$$

which is a polynomial with degree $\leq N$. Also, addition is commutative, e.g.

$$f_1 + f_2 = f_2 + f_1 \tag{4}$$

So the polynomials form a vector space.

b) This is easily seen by evaluating the trace. For a vector space of dimension N

$$\text{trace}(I) = N \tag{5}$$

where I is the identity operator

But finite dimensional vector spaces also satisfy the trace rule

$$\mathrm{trace}(AB) = \mathrm{trace}(BA) \tag{6}$$

so if

$$[A, B] = I \tag{7}$$

we would have

$$\mathrm{trace}(AB - BA) = \mathrm{trace}(AB) - \mathrm{trace}(BA) = N$$

$$0 = N \tag{8}$$

which cannot be satisfied for finite N. The trace of an operator in an infinite dimensional vector/Hilbert space is more subtle, and there it is possible to have $[\hat{A}, \hat{B}] = \hat{I}$