a) Since $V=\infty$ for $x<0$, must have $\psi=0$ there.

Must also have $\psi(x=\infty)\to 0$ s.t. $|\psi|^2$ can be normalized.

b) $\psi = 2\alpha^{3/2} x e^{-\alpha x}$ ← try this

The TISE reads, for $x>0$

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\left(\cancel{2\alpha^{3/2}} x e^{-\alpha x}\right) - \frac{1}{4\pi\varepsilon_0}\frac{e^2}{4\cancel{x}}\left(\cancel{2\alpha^{3/2}}\cancel{x}e^{-\alpha x}\right)$$

$$= E\left(\cancel{2\alpha^{3/2}} x e^{-\alpha x}\right)$$

$$-\frac{\hbar^2}{2m}\frac{d}{dx}\left(e^{-\alpha x} - \alpha x e^{-\alpha x}\right) - \frac{e^2}{16\pi\varepsilon_0}e^{-\alpha x} = Exe^{-\alpha x}$$

$$-\frac{\hbar^2}{2m}\left(-\alpha \cancel{e^{-\alpha x}} - \alpha\left(\cancel{e^{-\alpha x}} - \alpha x \cancel{e^{-\alpha x}}\right)\right) - \frac{e^2}{16\pi\varepsilon_0}\cancel{e^{-\alpha x}} = Ex\cancel{e^{-\alpha x}}$$

$$\frac{\hbar^2}{2m}\left(2\alpha - \alpha^2 x\right) - \frac{e^2}{16\pi\varepsilon_0} = Ex$$

$$\left(E + \frac{\hbar^2\alpha^2}{2m}\right)x - \left(\frac{e^2}{16\pi\varepsilon_0} - \frac{\hbar^2\alpha}{m}\right) = 0$$

This eqn. can only be solved for all x if both terms in ( ) are zero.

$$\Rightarrow \boxed{\alpha = \frac{m}{\hbar^2}\frac{e^2}{16\pi\varepsilon_0} = \frac{1}{4a_0}}$$

where a_0 is the Bohr radius

$$E = -\frac{\hbar^2\alpha^2}{2m} = \frac{-1}{16}\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2\left(\frac{m}{\hbar^2}\right)^{\cancel{2}}\left(\frac{\cancel{\hbar^2}}{m}\right)\frac{1}{2}$$

$$= -\frac{1}{16}\underbrace{\frac{m}{2\hbar^2}\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2}_{}$$

$$Ry = 13.6\,eV$$

$$\boxed{E = \frac{-Ry}{16} = -0.85\,eV}$$

c) At $x=0$ $\quad \psi \propto 0\,e^0 = 0$

$x=\infty$ $\quad \psi \propto \infty e^{-\infty} \rightarrow 0$

Nothing wins against an exp.

d) From problem "Wavefcn pushups"

$$\langle x\rangle = \frac{3}{2}\frac{1}{\alpha} = \frac{3}{2}4a_0 = 6a_0$$

$$= 6\times 0.529\,\text{Å}$$

$$\boxed{\langle x\rangle = 3.174\,\text{Å}}$$