Unit 6: Elements of Digital Electronics

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INTRODUCTION

Binary

To change a number to binary or to convert back to decimal, one can use following equation:

$$\begin{aligned} &(B_n B_{n-1} \cdots B_2 B_1 B_0)_2 \\ :=& \sum_{i=0}^{n} B_i 2^i = \sum_{i=0}^{k} D_i 10^i \\ =:&(D_k D_{k-1} \cdots D_2 D_1 D_0)_{10} \end{aligned} \tag{1}$$

On the other hand, for any base-n to base-m numberal system:

$$\begin{aligned} &(N_a N_{a-1} \cdots N_2 N_1 N_0)_2 \\ :=& \sum_{i=0}^{a} N_i n^i = \sum_{i=0}^{b} M_i m^i \\ =:&(M_b M_{b-1} \cdots M_2 M_1 M_0)_{10} \end{aligned} \tag{2}$$

For example, if one wants to convert $(137)_{10}$, using Equation 2, one can find following:

$$\begin{aligned} &(137)_{10} \\ &=1 \times 10^2 + 3 \times 10^1 + 7 \times 10^0 \\ &=1 \times 2^7 + 1 \times 2^3 + 1 \times 2^0 \\ &=(10001001)_2 \end{aligned}$$

On the other hand, if one wants to convert $(100101)_2$ to decimal, using Equation 2,one can obtain:

$$\begin{aligned} &(100101)_2 \\ &=1 \times 2^5 + 1 \times 2^2 + 1 \times 2^0 \\ &=3 \times 10^1 + 7 \times 10^0 \\ &=37 \end{aligned}$$

Logic and Truth Table

It is well-know that from logic equation to truth table, one can just plug in all possible input, and calculate the output. On the other hand, one may wants to convert a truth table into logic equation. To do so, one can chose each output 1 line from the truth table, and add (or) them up.

For example, a selector have n input, and $\log_2(n)$ select input. The select input select the input to be the output.

A	B	X	Y	O
1	0	0	1	1
1	1	0	1	1
0	0	0	1	0
0	1	0	1	0
0	1	1	0	1
1	1	1	0	1
0	0	1	0	0
1	0	0	0	0

TABLE I: Truth table of a 2 selector. A,B are input; X,Y are select input

For example, a 2 selector have truth table as Table I. We can chose the output 1 one and add them, Thus we have

$$O = A\bar{B}\bar{X}Y + AB\bar{X}Y + \bar{A}BX\bar{Y} + ABX\bar{Y}$$

Transistor-transistor Logic

In TTL system, the binary number can be present as high or low voltage. In ideal case, 0V is low and 5V is high. However, in real world, it is not possible to always obtain a 5V or 0V voltage. Thus, for TTL standard, any input higher than 2V is high, and any input lower than 0.8V is low. On the other hand, any output 3.5V is high and any ouput 0.5V is low.

Logic Gate

A logic gate accept some inputs and gives an output. The logic gate's truth table is same as mathematical logic. In the lab, we use NOT, AND, NAND, OR, NOR, XOR, and XNOR gate. The truth table is given in the appendix.

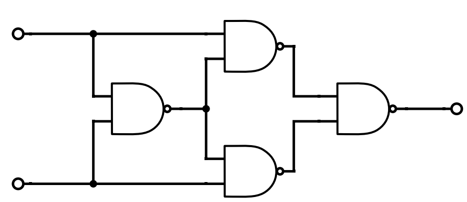


FIG. 1: Form a XOR gate using NAND gate

One can use NAND gate to form any other logic gate. For example, Figur 1 present how to form a XOR gate from NAND gate. The truth table is in the appendix.

Full-adder

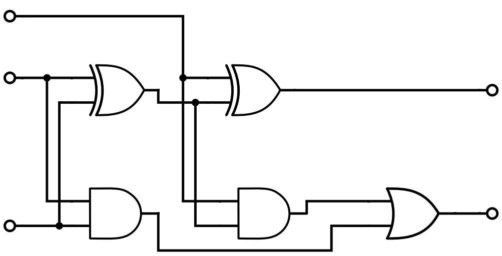


FIG. 2: A full-adder

One importent application of logic gate is full-adder, presented in Figure 2, where the left side is input and the right side is output. The output of a full-adder is the result of binary addition of the input.

Two Bit Half-adder

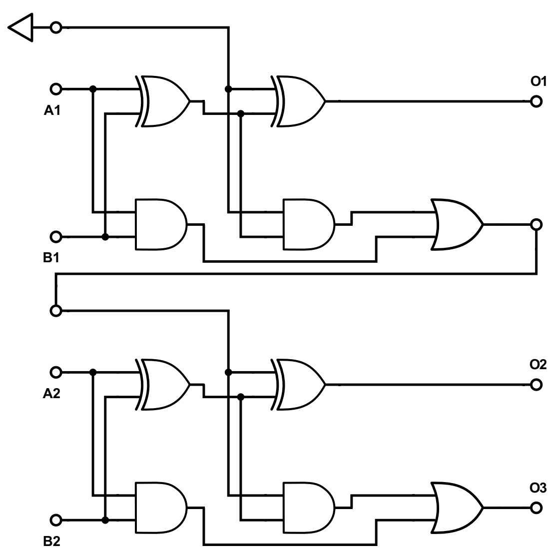


FIG. 3: A Two Bit Half-adder

Once obtain a full-adder, one can use full-adder to form a higher bit adder. For example, one can obtain a two bit half-adder as in Figure 3

Flip-flop

A Flip-flop, shown in Figure 4, can output one or zern when input are different and hold the output when two ouput are zero. There are a few application of flip-flop.

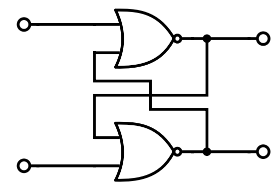
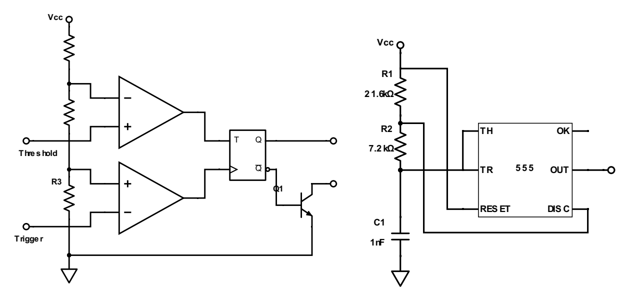


FIG. 4: A Flip-flop

Clocks



(a) 555 Chip (b) A Clock

FIG. 5: An application of flip-flop

A nice application of flip-flop is clock. The key of the clock is the 555 chip, which is an application of flip-flop. The circuit of a 555 chip is present in Figure 5a, where all resistors are 5kΩ. The 555 chip divide V_{CC} into 3 parts, when trigger voltage is larger than 1/3 V_{CC} and threshold voltage is smaller than 2/3 V_{CC}, the out put is hold, otherwise it the output is 1 or zero if trigger voltage is below 1/3 V_{CC} or threshold voltage is higher than 2/3 V_{CC}.

Than the clock is present in Figure 5b. This circuit can let the voltage periodically output zero or one, depending on the RC circuit. Thus, using by calculate the voltage in capastor, one can find the period of the circuit.

$$
\begin{aligned}
V_C &= V_{\mathrm{CC}}(1 - e^{-\frac{t}{RC}}) \\
\Rightarrow t_{\mathrm{high}} &= (R_1 + R_2)C\ln(2) \\
t_{\mathrm{low}} &= R_2 C\ln(2) \\
\Rightarrow T &= (R_1 + 2R_2)C\ln(2)
\end{aligned}
$$

The duty cycle is how much time the voltage ouput is high:

$$
\frac{t_{\mathrm{high}}}{T} = \frac{R_1 + R_2}{R_1 + 2R_2}
$$

For example, if one wants to have a 40kHz frequency clock with a 80% duty cycle, by above equation, one can use a 2.164kΩ and 7.213kΩ resistors and a 1nF capastor, as the value shown in Figure 5b.

Clock Edge

Another example of using flip-flop is D Clock Edge. The D flip-flop accepts a clock inputs.

DATA AND CALCULATION

ANALYSIS

CONCLUSION

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