Boolean Logic

Combinatoric logic in computing is based on Boolean algebra. The use follows from basic definitions and ten formal mathematical postulates and theorems.[1] The first definition is that a Boolean variable can take one of two possible values:

TRUE (or equvalent notation of "on" or "1")

FALSE (or equvalent notation of "off" or "0")

This is closely related to the first postulate below. The second definition is that Boolean algebra has two operations:

AND denoted as "$\cdot$" (or $\times$ or $\cap$ or $\wedge$)

OR denoted as "+" (or $\cup$ or $\vee$

Also, there is the usual concept of equality and substitution of equal expressions. The "0" and "1" and "$\cdot$" and "+" notations are used in Phy335, and when grouping parentheses are omitted "AND" has precedence over "OR". Furthermore, the "$\cdot$" is often omitted.

The ten postulates and theorems are given below. Of these, De Morgan's theorem is probably the most routinely used.

1. **Existance of 0 and 1:** If a is a boolean value then
   - $a \cdot 1 = a$,
   - $a + 0 = a$
2. **Commutivity:** For a and b representing boolean values
   - $a + b = b + a$,
   - $a \cdot b = b \cdot a$
3. **Associativity:**
   - $a + (b + c) = (a + b) + c$,
   - $a \cdot (b \cdot c) = (a \cdot b) \cdot c$,
4. **Distributivity:**
   - $a + (b \cdot c) = (a + b) \cdot (a + c)$,
   - $a \cdot (b + c) = (a \cdot b) + (a \cdot c)$,

[1] There are altenative equivalent formulations which have expanded lists of theorems, but these can be derived from the definition given here. Also, some authors use postulates where definitions are used here.

5. **Complement:** If $\overline{a}$ denotes the complement (or "not") of a, then it satisfies
   - $a \cdot \overline{a} = 0$
   - $a + \overline{a} = 1$
6. **Idempotency:**
   - $a + a = a$
   - $a \cdot a = a$
7. Uniqueness of values
   - $a + 1 = 1$
   - $a \cdot 0 = 0$
8. **Absorbtion**
   - $a + a \cdot b = a$
   - $a \cdot (a + b) = a$
9. **Elimination of redundant elements**
   - $a + \overline{a} \cdot b = a + b$
   - $a \cdot (\overline{a} + b) = a \cdot b$
10. **De Morgan's Theorem:**
    - $\overline{a \cdot b} = \overline{a} + \overline{b}$
    - $\overline{a + b} = \overline{a} \cdot \overline{b}$