Exam. 1.

Q1 earth (x, y, z, t) ; spaceship (x', y', z', t') ; $x_A = x_A' = 0$; $t_A = t_A' = 0$

1). $x_B' = \gamma(x_B - V t_B) = \frac{5}{3}(L - \frac{4}{5}c \cdot 0) = \frac{5}{3}L = 5000\,m.$

$t_B' = \gamma(t_B - \frac{V x_B}{c^2}) = \frac{5}{3}(0 - \frac{\frac{4}{5}c \cdot L}{c^2}) = -\frac{4}{3}\frac{L}{c} = -1.33\times10^{-5}$ sec

2). $t_{AB} = \frac{L}{u_A + u_B} = \frac{3000}{6/5\, c} = 0.833\times10^{-5}$ sec

3). $t_{AB}' = \gamma(t_{AB} - \frac{V x_{AB}}{c^2}) = \frac{5}{3}(0.833\times10^{-5} - \frac{\frac{4}{5}c \cdot \frac{L}{2}}{c^2}) = 0.722\times10^{-5}$ sec

4). $u_x' = \frac{u_x - V}{1 - \frac{u_x V}{c^2}}$ $\therefore$ $u_A' = \frac{\frac{3}{5}c - \frac{4}{5}c}{1 - \frac{3}{5}\times\frac{4}{5}} = \frac{-\frac{1}{5}c}{\frac{13}{25}} = -\frac{5}{13}c$

$u_B' = \frac{-\frac{3}{5}c - \frac{4}{5}c}{1 + \frac{3}{5}\times\frac{4}{5}} = \frac{-\frac{7}{5}c}{\frac{37}{25}} = \frac{-35}{37}c$

2. set (x, y, z, t) on the electron source, (x', y', z', t') on the ground

$u_x = 0$; $u_y = 0.9c$; $V = -0.5c$

$u_x' = \frac{u_x - V}{1 - \frac{u_x V}{c^2}} = \frac{0.5c}{1} = 0.5c$

$u_y' = \frac{u_y}{\gamma(1 - \frac{u_x V}{c^2})} = 0.9c \times \sqrt{1 - 0.5^2} = 0.779c$

$u' = \sqrt{u_x'^2 + u_y'^2} = 0.926c$

$\phi = \tan^{-1}(\frac{u_y'}{u_x'}) = 57.3°$

3. $m_1 \rightarrow v_1$ $\quad \overset{v_2}{\leftarrow} m_2$ $\quad \Rightarrow \quad M_0 \rightarrow V$

$$\begin{cases} \gamma_1 m_1 c^2 + \gamma_2 m_2 c^2 = \gamma M_0 c^2 \\ \gamma_1 m_1 v_1 + \gamma_2 m_2 v_2 = \gamma M_0 V \end{cases}$$

$m_1 = 0.5\frac{MeV}{c^2}$; $v_1 = \frac{3}{5}c$; $\gamma_1 = \frac{5}{4}$

$m_2 = 1\frac{MeV}{c^2}$; $v_2 = -\frac{4}{5}c$; $\gamma_2 = \frac{5}{3}$

$$\begin{cases} \frac{5}{4}\cdot 0.5\frac{MeV}{c^2}\times c^2 + \frac{5}{3}1\times\frac{MeV}{c^2}\cdot c^2 = \gamma M_0 \cdot c^2 & \langle 1\rangle \\ \frac{5}{4}\, 0.5\frac{MeV}{c^2}\cdot\frac{3}{5}c - \frac{5}{3}1\frac{MeV}{c^2}\cdot\frac{4}{5}c = \gamma M_0 V & \langle 2\rangle \end{cases}$$

$\frac{\langle 2\rangle}{\langle 1\rangle} \Rightarrow \frac{V}{c^2} = \frac{\frac{5}{4}\times\frac{1}{2}\times\frac{3}{5}c - \frac{5}{3}\cdot 1\cdot\frac{4}{5}}{\frac{5}{4}\times\frac{1}{2}c^2 + \frac{5}{3}\cdot 1 c^2} \Rightarrow V = \frac{-23}{55}c$

$\gamma = \frac{1}{\sqrt{1 - (\frac{23}{55})^2}} = 1.1$ from $\langle 1\rangle$: $\frac{55}{24}$ MeV $= 1.1\, M_0 c^2$

$M_0 = 2.08\ MeV/c^2$