Unit 5: Negative Feedback and Operational Amplifiers

Xueqi Li*
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INTRODUCTION

An operational amplifier (opamp) can amplifie the input voltage by a large factor. Since it have a large gain, it might not stable in many case and the output is very uncontrollable, we usualy build up a circuits with it to achieve a much stable circuit.

An opamp, other than power-supply pins (V_{CC} and V_{EE}), have two input and one output, where the input are known as non-inverting input (+) and inverting input (−). The ouput is given as:

$$v_{\mathrm{out}} = A_0(v_+ - v_-) \tag{1}$$

where A_0 is a very large number. Usually it is higher than 10^5. The impedance of the opamp can be think as ideal.

Negative Feedback

A negative feedback is such that the output of the opamp connect to the inverting input. That is to say, $v_- = v_{\mathrm{out}}$. This leads to following equation:

$$v_+ = v_- \tag{2a}$$
$$i_+ = i_- = 0 \tag{2b}$$

Follower

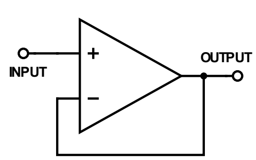

FIG. 1: A follower

The simplest negative feedback circuit is a follower, so as in Figure 1. From Equation 2a, we see that

$$V_{\mathrm{out}} = V_{\mathrm{in}} \tag{3}$$

Inverting Amplifier

An operational amplifier have to be able to be an amplifier! An Inverting Amplifier is presented as Figure 2.

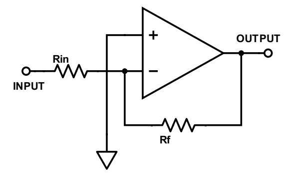

FIG. 2: An Inverting Amplifier

Since the non-inverting input is grounded, we must have $v_+ = v_- = 0\mathrm{V}$. Thus, we can find the current in the input side are given as

$$I_{\mathrm{in}} = \frac{V_{\mathrm{in}}}{R_{\mathrm{in}}}$$

Follow the Equation 2b, this current have to be also go through R_f, where the voltage drop on R_f just so happened to be output voltage ($v_- - V_{\mathrm{out}}$):

$$V_{\mathrm{out}} = -IR_f = -V_{\mathrm{in}}\frac{R_f}{R_{\mathrm{in}}} \tag{4}$$

where we see that the gain is just given as:

$$\mathrm{Gain} = -\frac{R_f}{R_{\mathrm{in}}}$$

Notice that for the inverting amplifier, the input impedance is give as:

$$Z_{\mathrm{in}} = \frac{\mathrm{d}V_{\mathrm{in}}}{\mathrm{d}I_{\mathrm{in}}} = R_{\mathrm{in}}$$

which is not a very high input impedance. In the other hand, easy to see the output impedance is ideal.

Non-inverting Amplifier

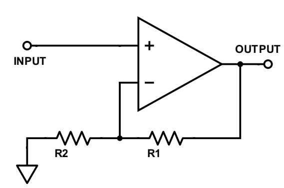

FIG. 3: An non-inverting Amplifier

It is nice to have a non-inverting amplifier. A non-inverting amplifier is shown as Figure 3. Since the input is directly connect to the non-inverting input, we have $V_{\text{input}} = v_+ = v_-$. In the end of R_2, we must have 0V sicne it is grounded. Thuse we have:

$$I_2 = \frac{V_{\text{in}}}{R_2}$$

From Equation 2b, this current have to go through R_1:

$$V_1 = IR_1 = V_{\text{in}}\frac{R_1}{R_2}$$

where the other end of R_2 just so happened to be the output:

$$V_{\text{output}} = V_{\text{in}} + V_1 = V_{\text{in}}(1 + \frac{R_1}{R_2}) \tag{5}$$

which give as a gain as:

$$\text{Gain} = (1 + \frac{R_1}{R_2})$$

Integrator

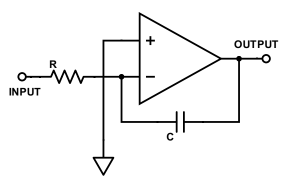

FIG. 4: An Integrator

The circuit presented in Figure 4 is an integrator. Since the non-inverting input is grounded, we have $v_- = v_+ =$ 0V. Thus we can find the current in the capacitor is the same as the current go through the resistor:

$$I = -C\frac{\mathrm{d}V_{\text{out}}}{\mathrm{d}t} = \frac{V_{\text{in}}}{R}$$

solve the equation, we have:

$$V_{\text{out}} = -\frac{1}{RC}\int V_{\text{in}}\mathrm{d}t \tag{6}$$

However, there is a problem for the integrator. Usually, the input might have an offset. That is to say, there is a constent term in the input voltage. This will give us a growing or falling output after integration. To solve it, one can parallel a resistor with the capacitor.

T Network

To achieve a large parallel resistor, one can use a T network, so as in Figure 5. Ignore the capacitor, we can

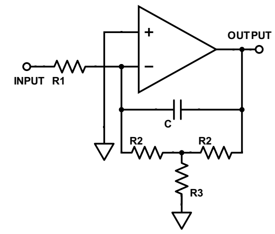

FIG. 5: An Integrator with T network

just calculate the effect of the T network. First, we still have the current on the R_1 will go through the left R_2:

$$I_1 = \frac{V_{\text{in}}}{R_1} = I_{2\text{L}} = I_{2\text{R}} + I_3$$

On the other hand, the voltage between R_1 and $R_{2\text{L}}$ is 0V. Thus we can find the voltage between two R_2:

$$V_2 = 0 - I_1R_2 = -V_{\text{in}}\frac{R_2}{R_1}$$

And this voltage will dorp to zero once it go through R_3:

$$I_3 = \frac{V_2}{R_3} = -V_{\text{in}}\frac{R_2}{R_1R_3}$$

In the output end, we can write the current go through R_2 as following:

$$I_2 = \frac{V_2 - V_{\text{out}}}{R_2} = -\frac{V_{\text{in}}}{R_1} - \frac{V_{\text{out}}}{R_2}$$

This relate the output voltage to the current. Now we have $I_1 = I_2 + I_3$:

$$\frac{V_{\text{in}}}{R_1} = -V_{\text{in}}\frac{R_2}{R_1R_3} - \frac{V_{\text{in}}}{R_1} - \frac{V_{\text{out}}}{R_2}$$

Solve the equation, we find

$$V_{\text{out}} = -V_{\text{in}}\frac{R_2}{R_1}(2 + \frac{R_2}{R_3})$$

This give us the impedance of the T network:

$$Z_f = R_2(2 + \frac{R_2}{R_3})$$

Thus, we can achieve a high impedance if we just chose two two small resistor with a large ratio.

Differentiator

By interchange the resistor and capacitor of integrator, one can achieve a differentiator, as in Figure 6. Since the non-inverting input is grounded, we have to have 0V on v_-. This means the current go through the capacitor and the resistor are the same:

$$V_{\text{out}} = -IR = -RC\frac{\mathrm{d}V\text{in}}{\mathrm{d}t} \tag{7}$$

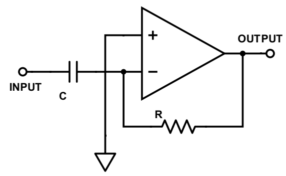

FIG. 6: An Differentiator

Limit of the Opamp

A realistic opamp are not ideal. There are a few importent things we have to consider when use it:

1. The voltage gain (A_0) will drop linearly to the frequency in logarithm scale;
2. There might be a phase shift in the output;
3. For the feedback loop, Equation 2b is approximate, i.e., i_+, $i_- \approx 0$;
4. For the feedback loop, Equation 2a is approximate, i.e., $v_+ \approx v_-$;
5. There is a small delay to the output. When input voltage change suddenly, the output voltage will learly increase and reach the theoretical value. This is called slew rate.

DATA AND CALCULATION

ANALYSIS

CONCLUSION

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* Partner: Tianming Hai