Op Amp Feedback and Effective Gain

For the Op Amp circuit with no feedback, like that one shown in Fig. 1, the output voltage v_{out} is given by the expression

$$v_{out} = g^{OL}(v_+ - v_-). \tag{1}$$

Over a wide frequency range, the open loop gain is $g^{OL} = O(10^6)$. For typical rail voltages V_{CC+} and V_{CC-} of 15 V, the maximum usable input difference $(v_+ - v_-)$ is therefore limited to 15 V/$\mathrm{g^{OL}}$ = 15μV.

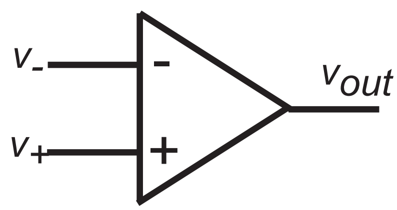

Figure 1: Op Amp circuit without feedback.

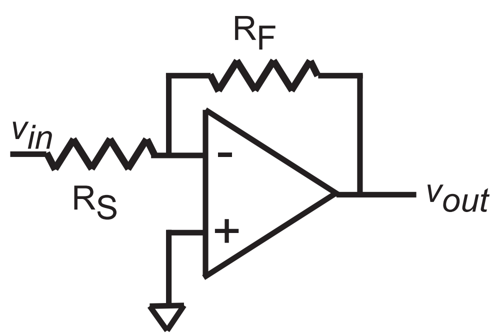

Figure 2: Op Amp circuit with feedback.

However, adding *negative* feedback makes the (closed loop) voltage gain $g \equiv v_{out}/v_{in}$ controllable by external components. For the circuit shown in Fig. 2, and using the high input impedance of an Op Amp so that $i_+ = i_- = 0$, one finds the equations governing the currents and voltages in the resistors to be

$$\begin{aligned} v_{out} &= g^{OL}(0 - v_-) \\ i_s &= \frac{v_{in} - v_-}{R_S} \\ i_f &= \frac{v_{out} - v_-}{R_F} \end{aligned}$$

where the first line comes from the Op Amp basic definition, the second and third are simply from Ohm's Law, and $v_+ = 0$ because it is connected to ground. Because $i_s = -i_f$ and $v_{out} = g^{OL}v_-$, v_- can be eliminated from the above equations:

$$\begin{aligned} i_s &= -i_f \\ \frac{1}{R_S}\Big(v_{in} + \frac{v_{out}}{g^{OL}}\Big) &= \frac{-1}{R_F}\Big(v_{out}g^{OL}\Big) \\ \frac{1}{R_S}v_{in} &= -\Big(\frac{1}{g^{OL}R_S} + \frac{1}{g^{OL}R_F} + \frac{1}{R_F}\Big)v_{out} \end{aligned}$$

(continued on back)

and since $g^{OL} R_S >> R_F$ and $g^{OL} R_F >> R_F$, the last line becomes

$$\frac{1}{R_S} v_{in} = -\frac{1}{R_F} v_{out}$$

which is trivially rearranged to become

$$g \equiv \frac{v_{out}}{v_{in}} = -\frac{R_F}{R_S}. \tag{2}$$

Back substitution then shows that $v_- = 0$ which gives rise to the second golden rule $v_+ = v_-$ (which we did not use, but derived).