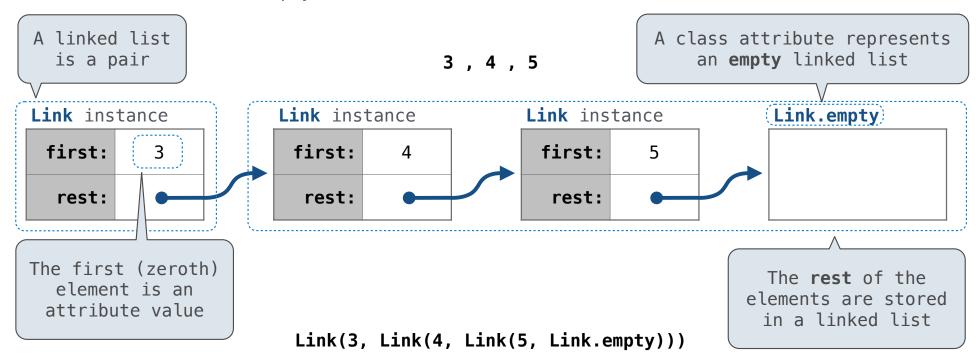


Linked List Structure

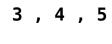
A linked list is either empty **or** a first value and the rest of the linked list

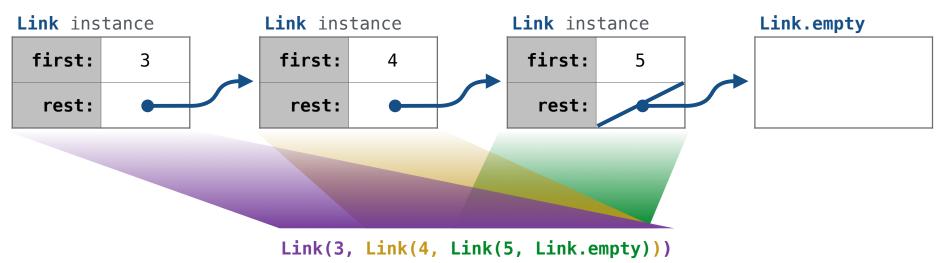


4

Linked List Structure

A linked list is either empty \mathbf{or} a first value and the rest of the linked list

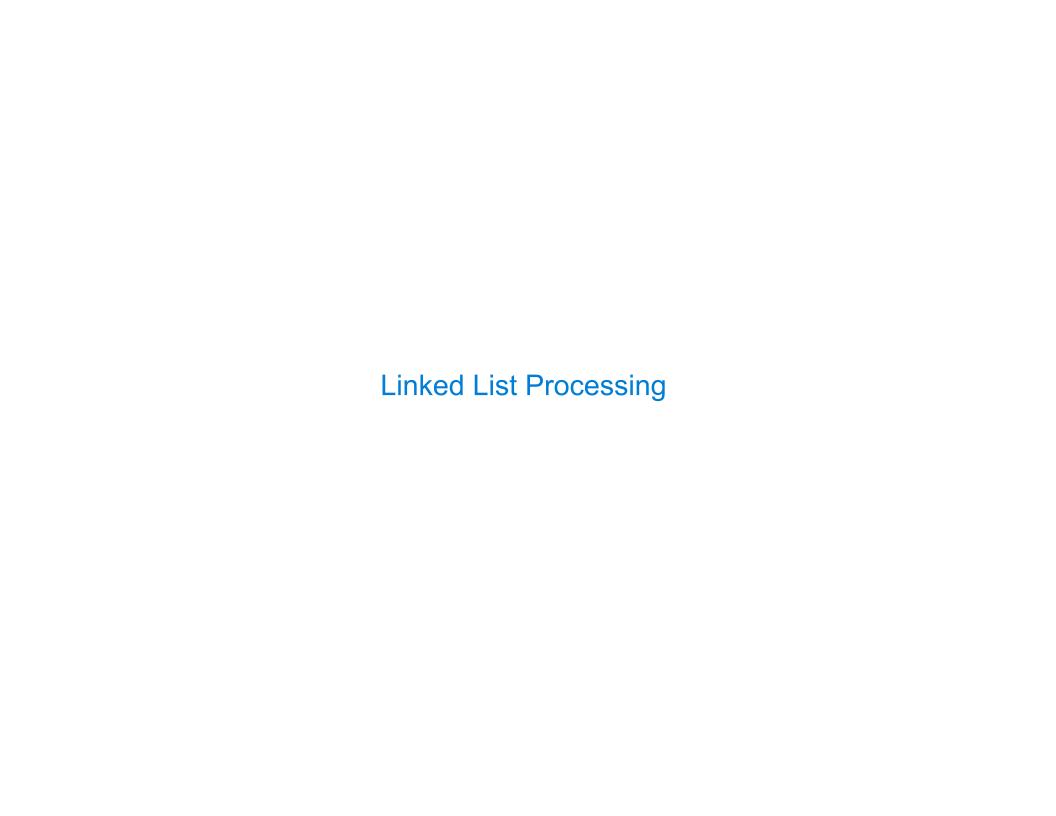




Linked List Class

```
Linked list class: attributes are passed to __init__
  class Link:
                    Some zero-length sequence
      empty = ()
      def __init__(self, first, rest=empty):
          assert rest is Link.empty or isinstance(rest, Link)
          self.first = first
          self.rest = rest
                                        Returns whether
                                         rest is a Link
help(isinstance): Return whether an object is an instance of a class or of a subclass thereof.
                          Link(3, Link(4, Link(5)
                                                           )))
                                          (Demo)
```

6



Example: Range, Map, and Filter for Linked Lists

```
square, odd = lambda x: x * x, lambda x: x % 2 == 1
list(map(square, filter(odd, range(1, 6))))
                                                   # [1, 9, 25]
map_link(square, filter_link(odd, range_link(1, 6))) # Link(1, Link(9, Link(25)))
def range link(start, end):
    """Return a Link containing consecutive integers from start to end.
    >>> range link(3, 6)
    Link(3, Link(4, Link(5)))
def map link(f, s):
    """Return a Link that contains f(x) for each x in Link s.
    >>> map link(square, range link(3, 6))
    Link(9, Link(16, Link(25)))
def filter link(f, s):
    """Return a Link that contains only the elements x of Link s for which f(x)
    is a true value.
    >>> filter link(odd, range link(3, 6))
    Link(3, Link(5))
    \mathbf{H}^{-}\mathbf{H}^{-}\mathbf{H}
```

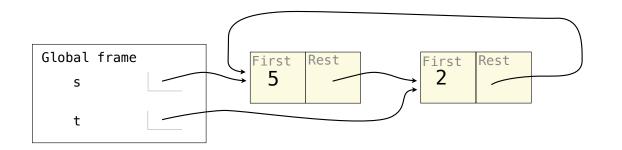


Linked Lists Can Change

Attribute assignment statements can change first and rest attributes of a Link

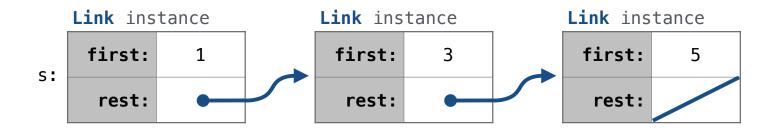
The rest of a linked list can contain the linked list as a sub-list

```
>>> s = Link(1, Link(2, Link(3)))
>>> s.first = 5
>>> t = s.rest
                         Global frame
                                               First
                                                     Rest
                                                                      Rest
                                                                 First
                                                                                        Rest
                                                                                  First
>>> t.rest = s
                                                                                   3
                                                                  2
                                                 1
                             S
>>> s.first
>>> s.rest.rest.rest.rest.rest.first
```

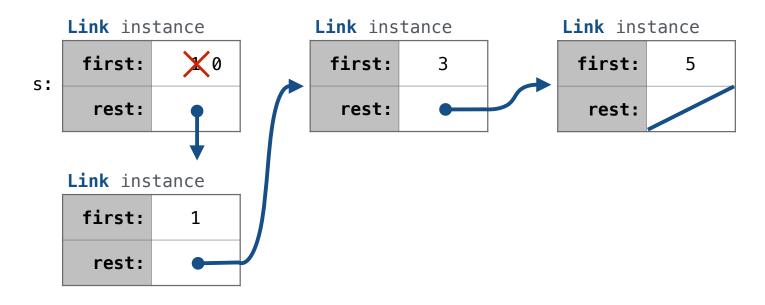


Note: The actual environment diagram is much more complicated.

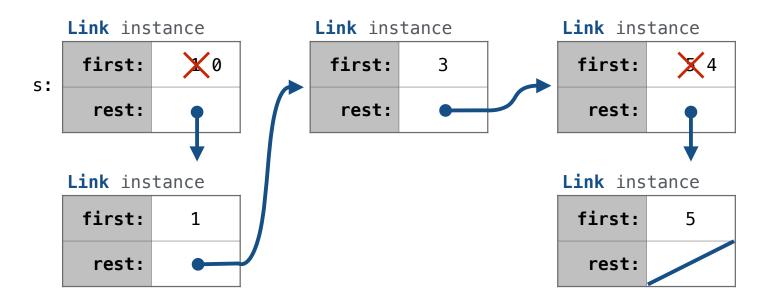




```
def add(s, v):
    """Add v to an ordered list s with no repeats, returning modified s."""
    (Note: If v is already in s, then don't modify s, but still return it.)
    add(s, 0)
```

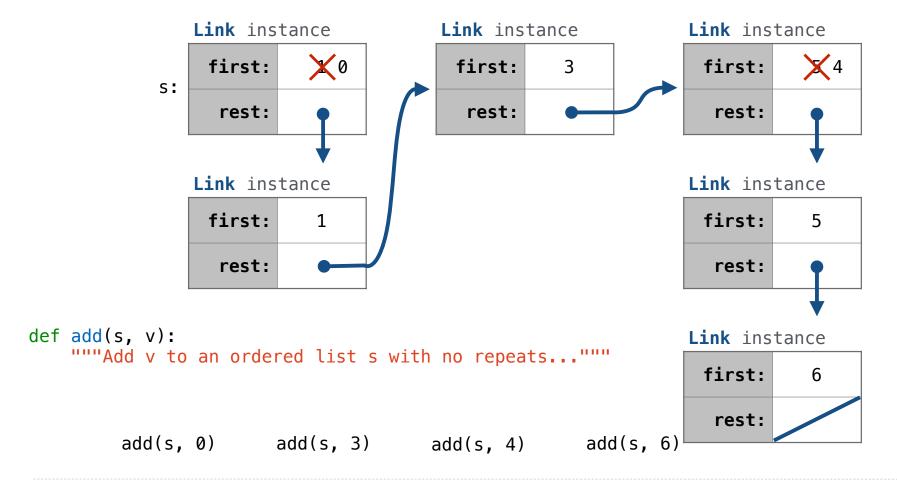


```
def add(s, v):
    """Add v to an ordered list s with no repeats, returning modified s."""
    (Note: If v is already in s, then don't modify s, but still return it.)
    add(s, 0) add(s, 3) add(s, 4)
```



```
def add(s, v):
    """Add v to an ordered list s with no repeats..."""
```

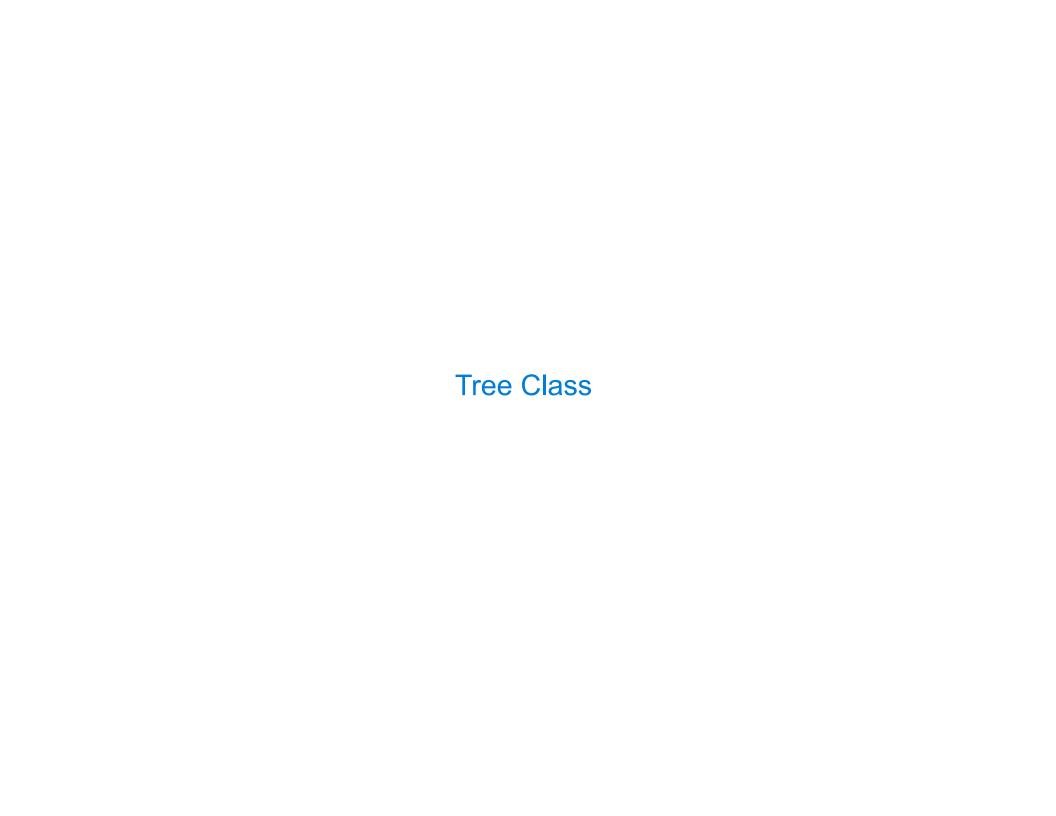
add(s, 0) add(s, 3) add(s, 4) add(s, 6)



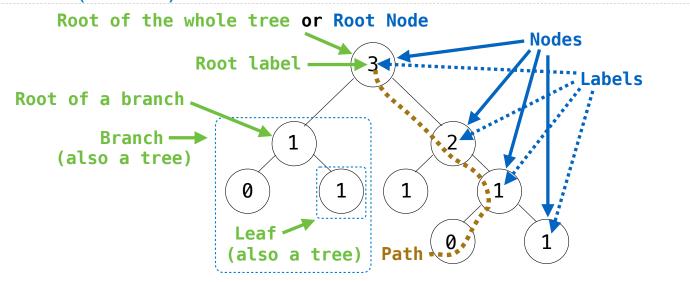
Adding to a Set Represented as an Ordered List

```
def add(s, v):
                                                            Link instance
                                                                             Link instance
                                                                                             Link instance
    """Add v to s, returning modified s."""
                                                                    X0
                                                             first:
                                                                              first:
                                                                                              first:
                                                         s:
    >>> s = Link(1, Link(3, Link(5)))
                                                                              rest:
                                                              rest:
                                                                                              rest:
    >>> add(s, 0)
    Link(0, Link(1, Link(3, Link(5))))
                                                            Link instance
                                                                                             Link instance
    >>> add(s, 3)
                                                              first:
                                                                    1
                                                                                              first:
    Link(0, Link(1, Link(3, Link(5))))
                                                              rest:
                                                                                              rest:
    >>> add(s, 4)
    Link(0, Link(1, Link(3, Link(4, Link(5)))))
                                                                                             Link instance
    >>> add(s, 6)
                                                                                              first:
    Link(0, Link(1, Link(3, Link(4, Link(5, Link(6))
                                                                                              rest:
    assert s is not List.empty
    if s.first > v:
                                                                    Link(s.first, s.rest)
         s.first, s.rest =
    elif s.first < v and empty(s.rest):</pre>
                                                       Link(v)
         s.rest =
    elif s.first < v:
                                                    add(s.rest, v)
    return s
```

16



Tree Abstraction (Review)



Recursive description (wooden trees):

A tree has a root label and a list of branches
Each branch is a tree
A tree with zero branches is called a leaf
A tree starts at the root

Relative description (family trees):

Each location in a tree is called a **node**Each **node** has a **label** that can be any value

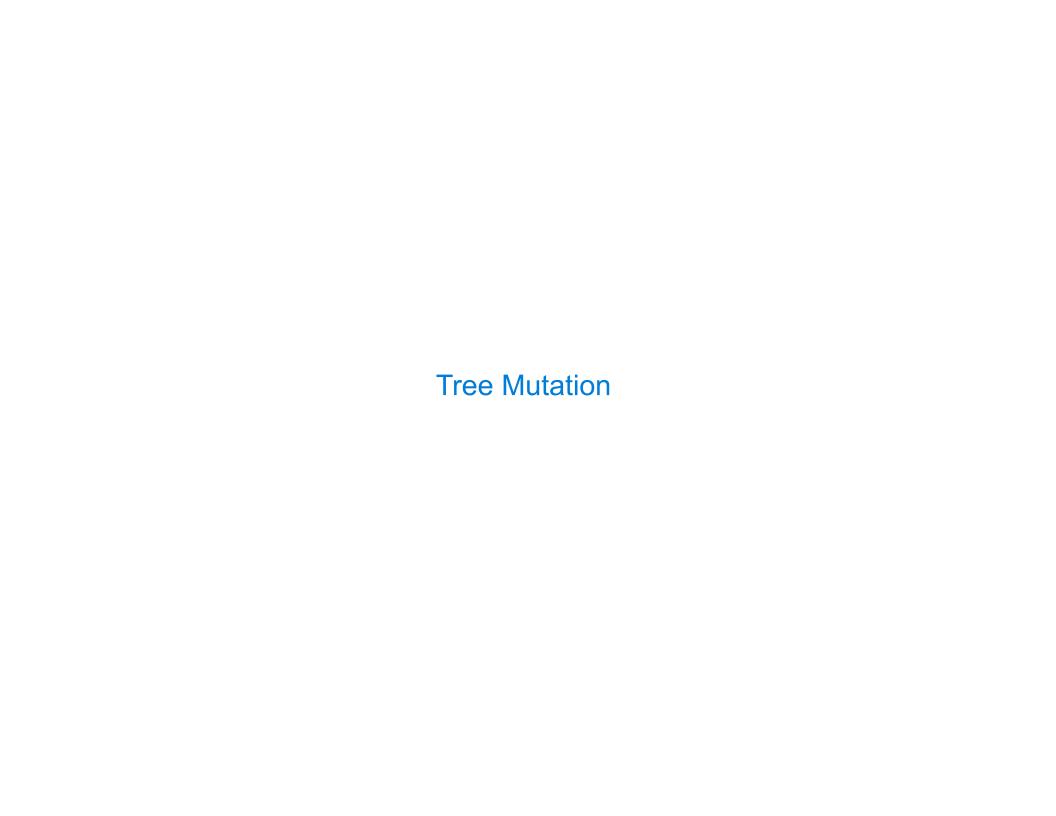
One node can be the **parent/child** of another

The top node is the **root node**

People often refer to labels by their locations: "each parent is the sum of its children"

Tree Class

```
A Tree has a label and a list of branches; each branch is a Tree
class Tree:
                                                    def tree(label, branches=[]):
    def __init__(self, label, branches=[]):
                                                        for branch in branches:
        self.label = label
                                                             assert is tree(branch)
        for branch in branches:
                                                         return [label] + list(branches)
            assert isinstance(branch, Tree)
                                                    def label(tree):
        self.branches = list(branches)
                                                         return tree[0]
                                                    def branches(tree):
                                                         return tree[1:]
def fib_tree(n):
                                                    def fib_tree(n):
    if n == 0 or n == 1:
                                                        if n == 0 or n == 1:
        return Tree(n)
                                                             return tree(n)
    else:
                                                        else:
        left = fib tree(n-2)
                                                             left = fib tree(n-2)
        right = fib tree(n-1)
                                                             right = fib tree(n-1)
        fib n = left.label + right.label
                                                             fib n = label(left) + label(right)
        return Tree(fib_n, [left, right])
                                                             return tree(fib n, [left, right])
                                           (Demo)
```



Example: Pruning Trees

Removing subtrees from a tree is called *pruning*

Prune branches before recursive processing

