Environments

Announcements

Environments for Higher-Order Functions

Environments Enable Higher-Order Functions

Functions are first-class: Functions are values in our programming language

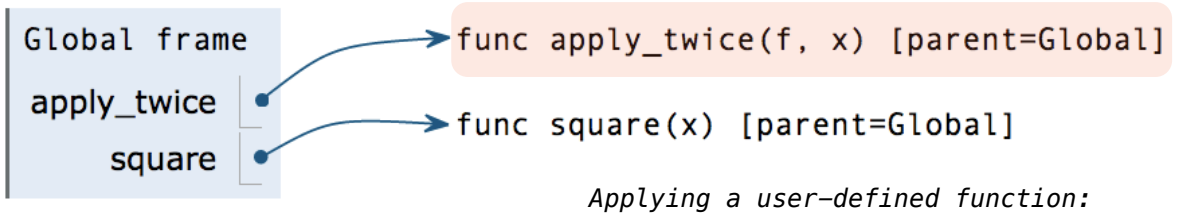
Higher-order function: A function that takes a function as an argument value **or**
A function that returns a function as a return value

Environment diagrams describe how higher-order functions work!

(Demo)

Names can be Bound to Functional Arguments

```
1  def apply_twice(f, x):
2      return f(f(x))
3
4  def square(x):
5      return x * x
6
7  result = apply_twice(square, 2)
```



Applying a user-defined function:

- Create a new frame
- Bind formal parameters (f & x) to arguments
- Execute the body: return f(f(x))

```
1  def apply_twice(f, x):
2      return f(f(x))
3
4  def square(x):
5      return x * x
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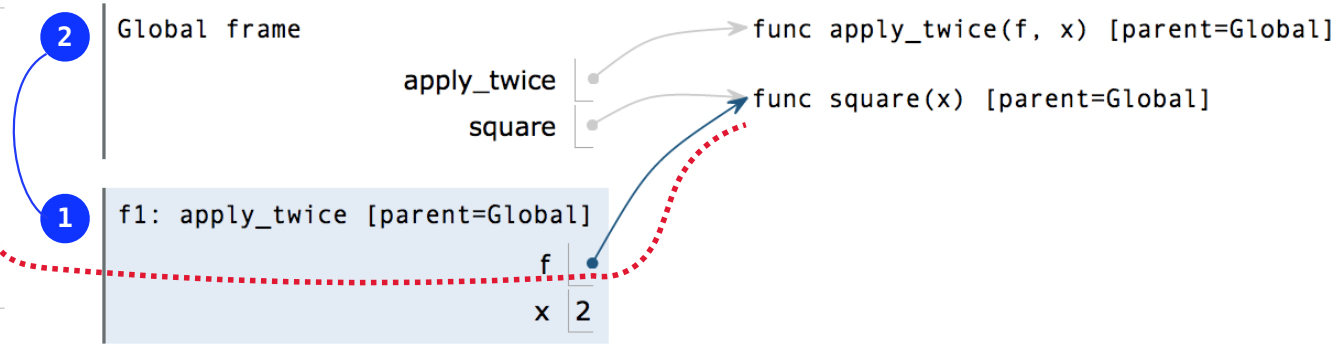

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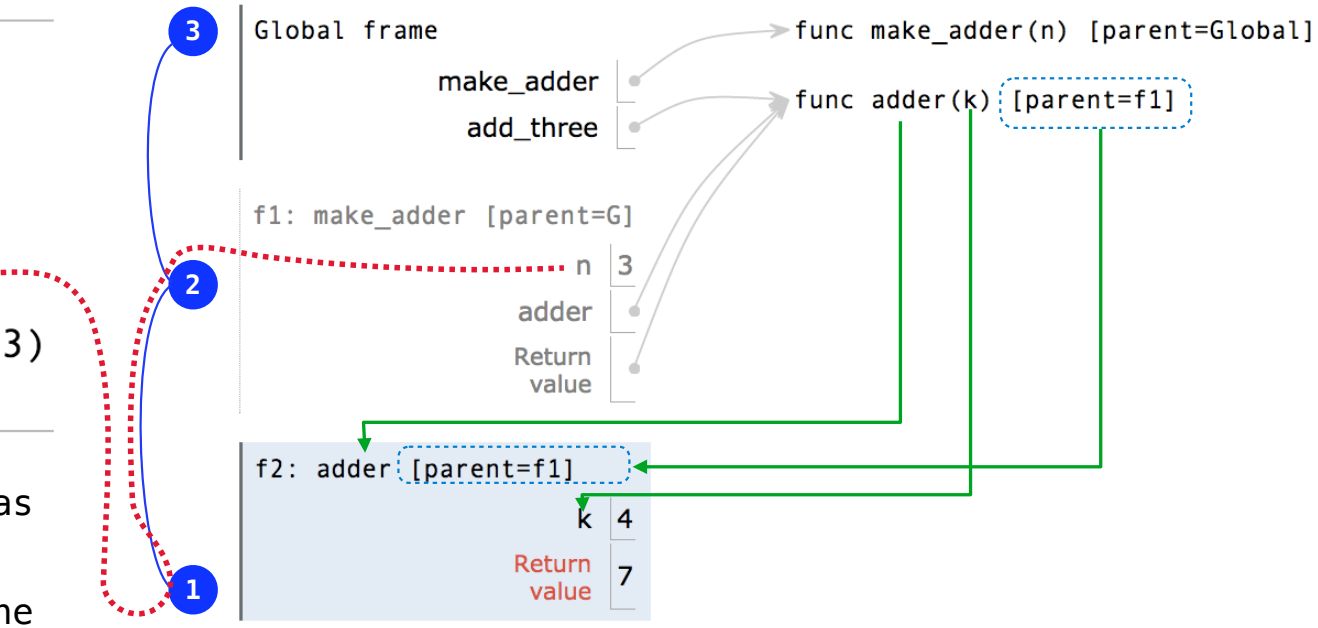
Environments for Nested Definitions

(Demo)

Environment Diagrams for Nested Def Statements

Nested def

```
1  def make_adder(n):
2      def adder(k):
3          return k + n
4      return adder
5
6  add_three = make_adder(3)
7  add_three(4)
```



- Every user-defined function has a parent frame (often global)
- The parent of a function is the frame in which it was defined
- Every local frame has a parent frame (often global)
- The parent of a frame is the parent of the function called

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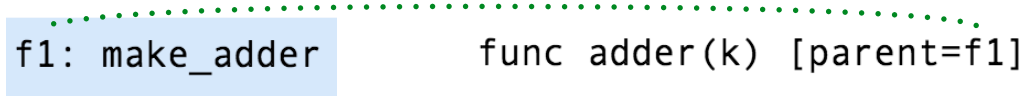
How to Draw an Environment Diagram

When a function is defined:

Create a function value: func <name>(<formal parameters>) [parent=<label>]

Its parent is the current frame.



Bind <name> to the function value in the current frame

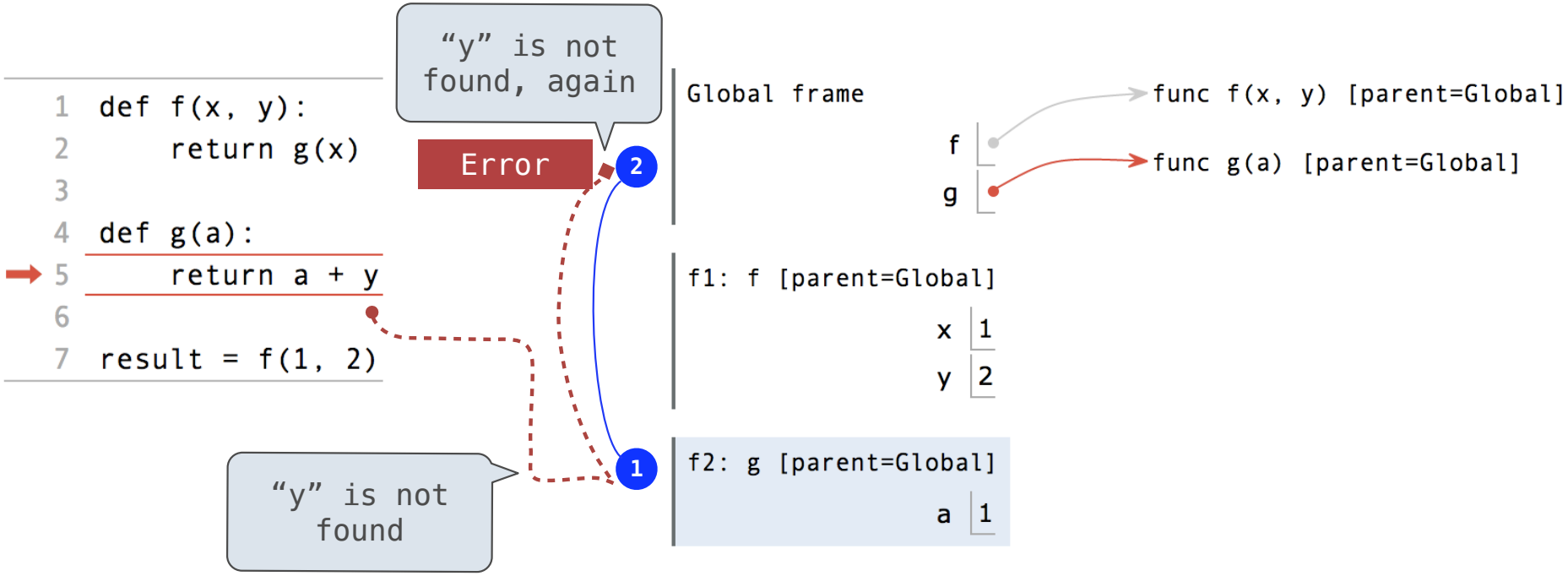
When a function is called:

1. Add a local frame, titled with the <name> of the function being called.
2. Copy the parent of the function to the local frame: [parent=<label>]
3. Bind the <formal parameters> to the arguments in the local frame.
4. Execute the body of the function in the environment that starts with the local frame.

Local Names

(Demo)

Local Names are not Visible to Other (Non-Nested) Functions



- An environment is a sequence of frames.
- The environment created by calling a top-level function (no def within def) consists of one local frame, followed by the global frame.

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%20y%29%3A%0A%20%20%20%20return%20g%28x%29%0A%0Adef%20g%28a%29%3A%0A%20%20%20%20return%20a%20%2B%20y%0A%20%20%20%20%0Aresult%20%3D%20f%281,%202%29&cumulative=true&curInstr=0&mode=display&origin=composingprograms.js&py=3&rawInputLstJSON=%5B%5D

Lambda Expressions

(Demo)

Lambda Expressions

```
>>> x = 10
>>> square = x * x
>>> square = lambda x: x * x
>>> square(4)
16
```

An expression: this one evaluates to a number

Also an expression: evaluates to a function

Important: No "return" keyword!

A function with formal parameter x that returns the value of "x * x"

Must be a single expression

Lambda expressions are not common in Python, but important in general

Lambda expressions in Python cannot contain statements at all!

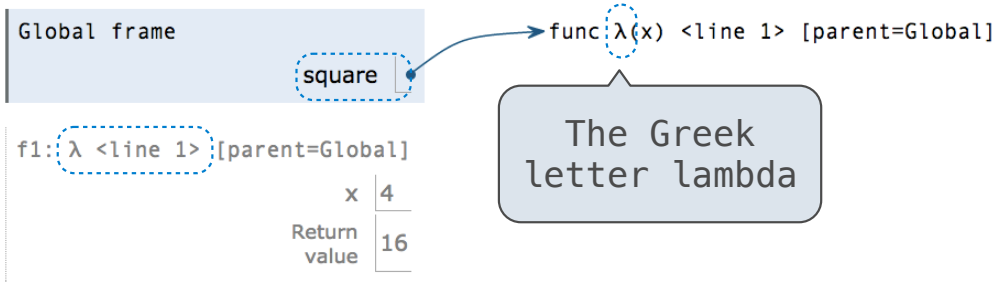
Lambda Expressions Versus Def Statements

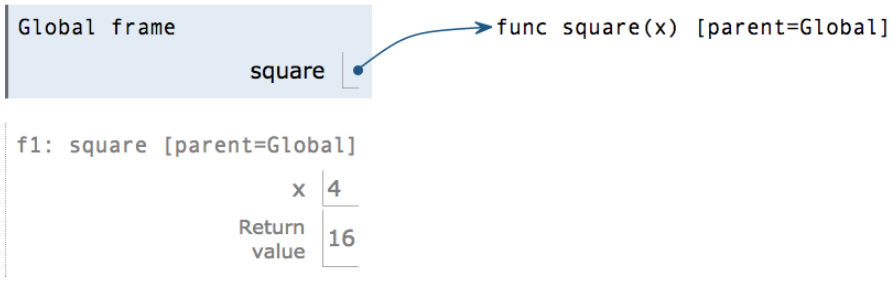
```
square = lambda x: x * x
```

VS

```
def square(x):
    return x * x
```

- Both create a function with the same domain, range, and behavior.
- Both bind that function to the name square.
- Only the def statement gives the function an intrinsic name, which shows up in environment diagrams but doesn't affect execution (unless the function is printed).





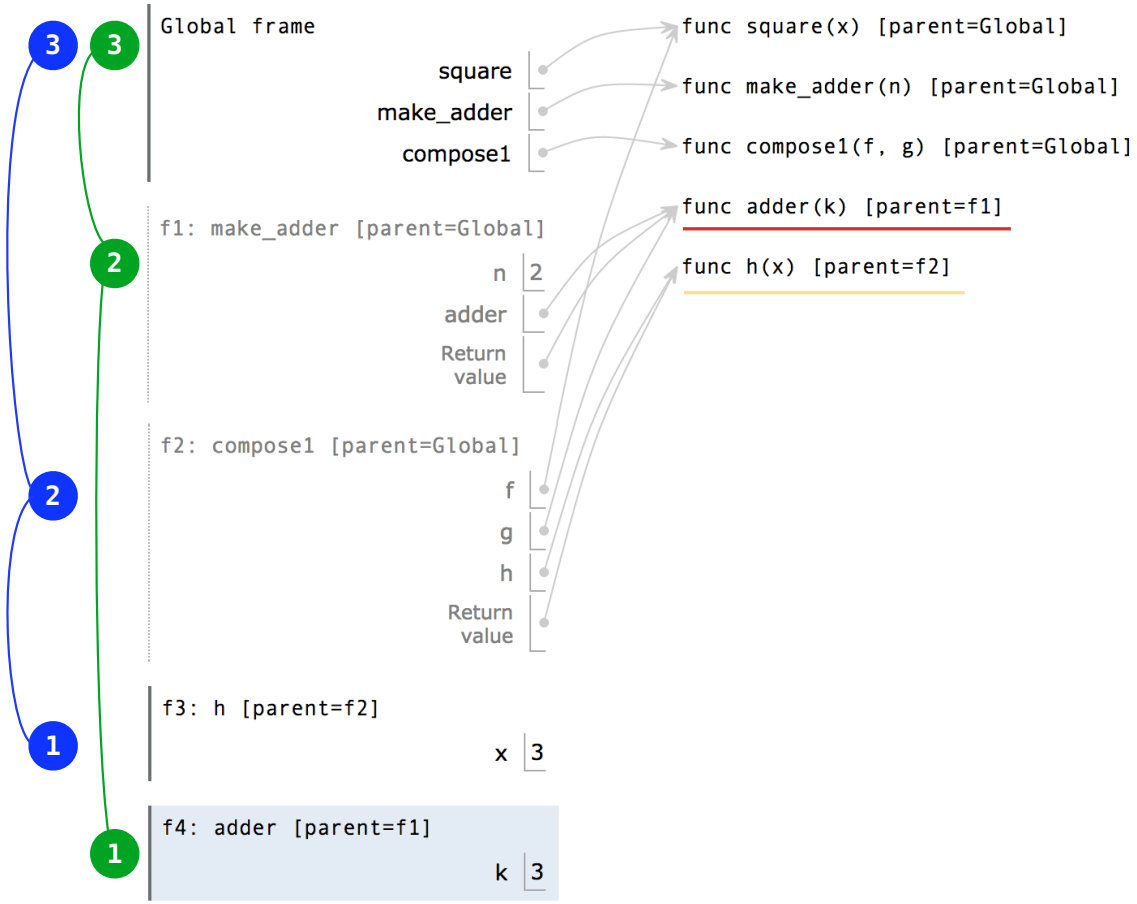
Function Composition

(Demo)

The Environment Diagram for Function Composition

```
1  def square(x):
2      return x * x
3
4  def make_adder(n):
5      def adder(k):
6          return k + n
7      return adder
8
9  def compose1(f, g):
10     def h(x):
11         return f(g(x))
12     return h
13
14 compose1(square, make_adder(2))(3)
```

Return value of make_adder is an argument to compose1

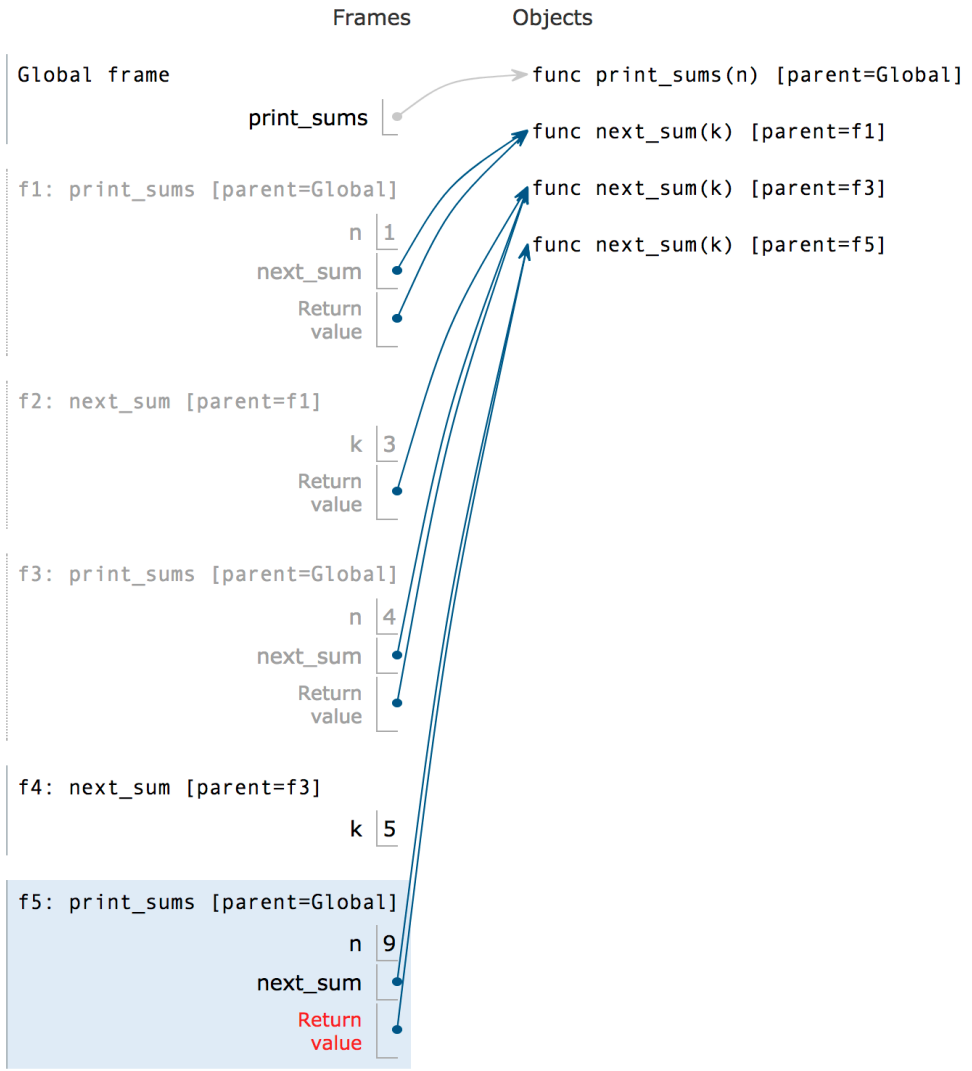


Self-Reference

(Demo)

Returning a Function Using Its Own Name

```
1  def print_sums(n):
2      print(n)
3      def next_sum(k):
4          return print_sums(n+k)
5      return next_sum
6
7  print_sums(1)(3)(5)
```



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http://pythontutor.com/composingprograms.html#code=def%20print_sums%28n%29%3A%0A%20%20%20%20print%28n%29%0A%20%20%20%20def%20next_sum%28k%29%3A%0A%20%20%20%20%20%20%20%20return%20print_sums%28n%2Bk%29%0A%20%20%20%20return%20next_sum%0A%0Aprint_sums%281%29%283%29%285%29&cumulative=true&curInstr=0&mode=display&origin=composingprograms.js&py=3&rawInputLstJSON=%5B%5D

Currying

Function Currying

```
def make_adder(n):
    return lambda k: n + k
```

```
>>> make_adder(2)(3)
5
>>> add(2, 3)
5
```

There's a general relationship between these functions

(Demo)

Curry: Transform a multi-argument function into a single-argument, higher-order function