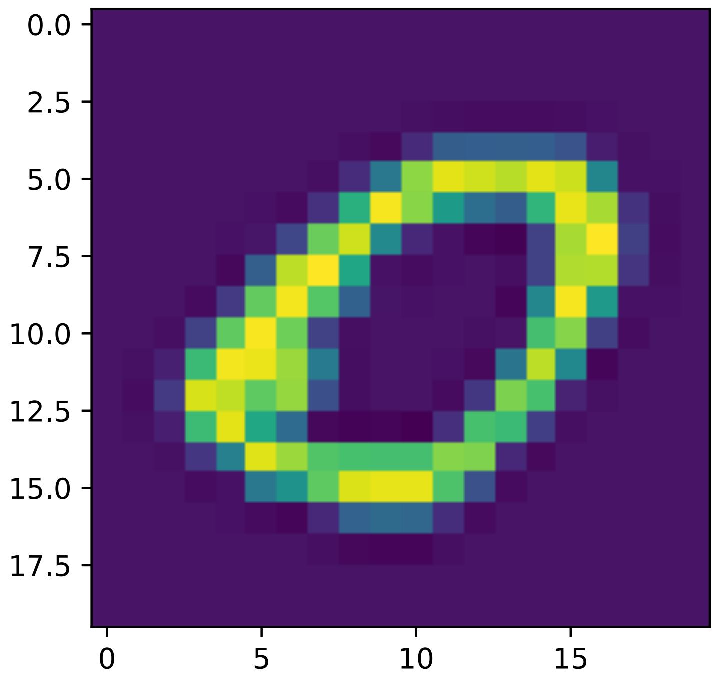
Q1. One-vs-All Logistic Regression for Handwritten Digits

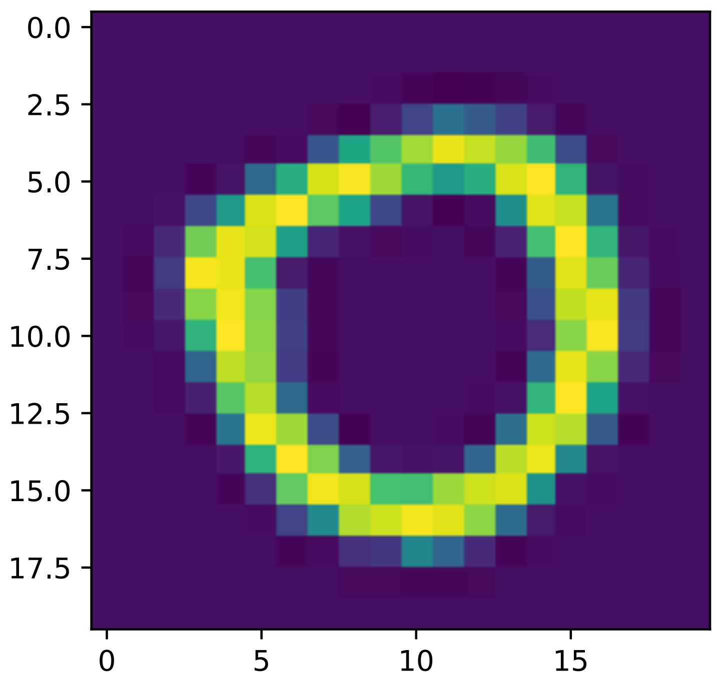
(a)

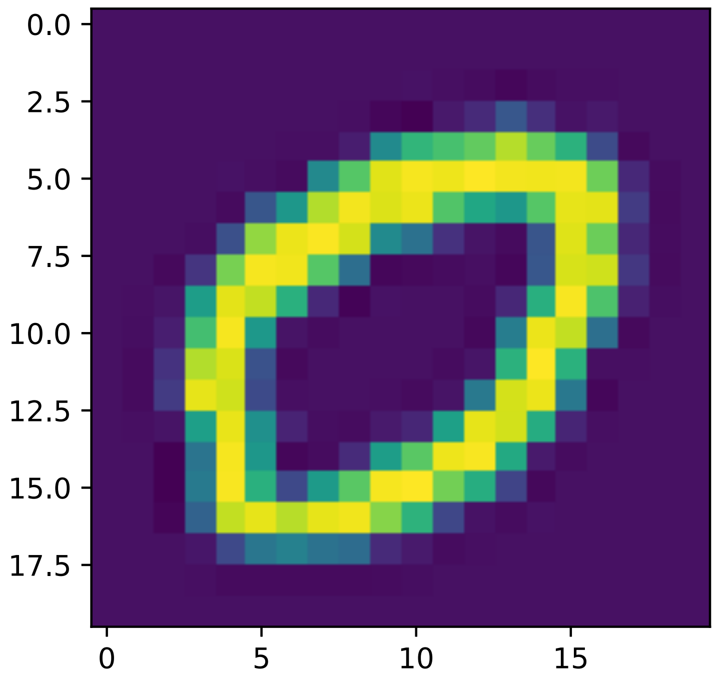
In [2]:
```
#load dataset
import numpy as np
import scipy.io as sio
import matplotlib.pyplot as plt
data = sio.loadmat("digits.mat")
x = data['X']
y = np.squeeze(data['y'])
np.place(y,y==10,0)
numExamples = x.shape[0]
numFeatures = x.shape[1]
numLabels = 10
```

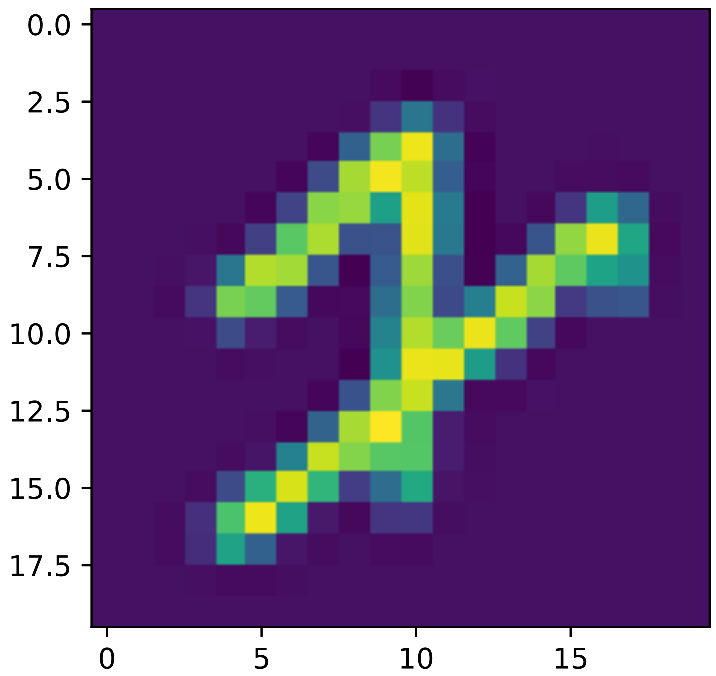
In [17]:
```
x1 = x[0,:].reshape(20,20)
x23 = x[22,:].reshape(20,20)
x233 = x[232,:].reshape(20,20)
x2333 = x[2332,:].reshape(20,20)
plt.figure(1)
plt.imshow(x1)
plt.figure(2)
plt.imshow(x23)
plt.figure(3)
plt.imshow(x233)
plt.figure(4)
plt.imshow(x2333)
```

Out[17]: <matplotlib.image.AxesImage at 0x7fb4008f3850>









(b)

In [27]:
```
def sigmoid(z):
    return 1 / (1 + np.exp(-z))
def cost(theta, X, y):
    predictions = sigmoid(X @ theta)
    predictions[predictions == 1] = 0.999 #log(1)=0 causes error in division
    error = -y * np.log(predictions) - (1 - y) * np.log(1 - predictions)
    return sum(error) / len(y)
def costGradient(theta, X, y):
    predictions = sigmoid(X @ theta)
    return X.transpose() @ (predictions - y) / len(y)
```

In [120…
```
#split data
def split_data(mat):
    x_train = np.empty((0,mat.shape[1]))
    x_test = np.empty((0,mat.shape[1]))
    for i in range(10):
        train = mat[i*500:i*500+400,:]
        test = mat[i*500+400:i*500+500,:]
        x_train = np.vstack((x_train,train))
        x_test = np.vstack((x_test,test))
    return x_train,x_test
y_train,y_test = split_data(y.reshape(-1,1))
x_train,x_test = split_data(x)
```

In [118…

```
#learding w
def w_learn(theta,X,y,max_iter,tol,learning_rate):
    J = []
    iter = 0
    while iter<max_iter:
        m = cost(theta, X, y)
        J.append(m)
        G = costGradient(theta, X, y).reshape(-1,1)
        last_theta = theta
        theta = theta-learning_rate*G
        iter = iter +1
        if abs(cost(theta, X, y)-cost(last_theta, X, y)) < tol:
            break
    return theta,iter,J
```

In [137…

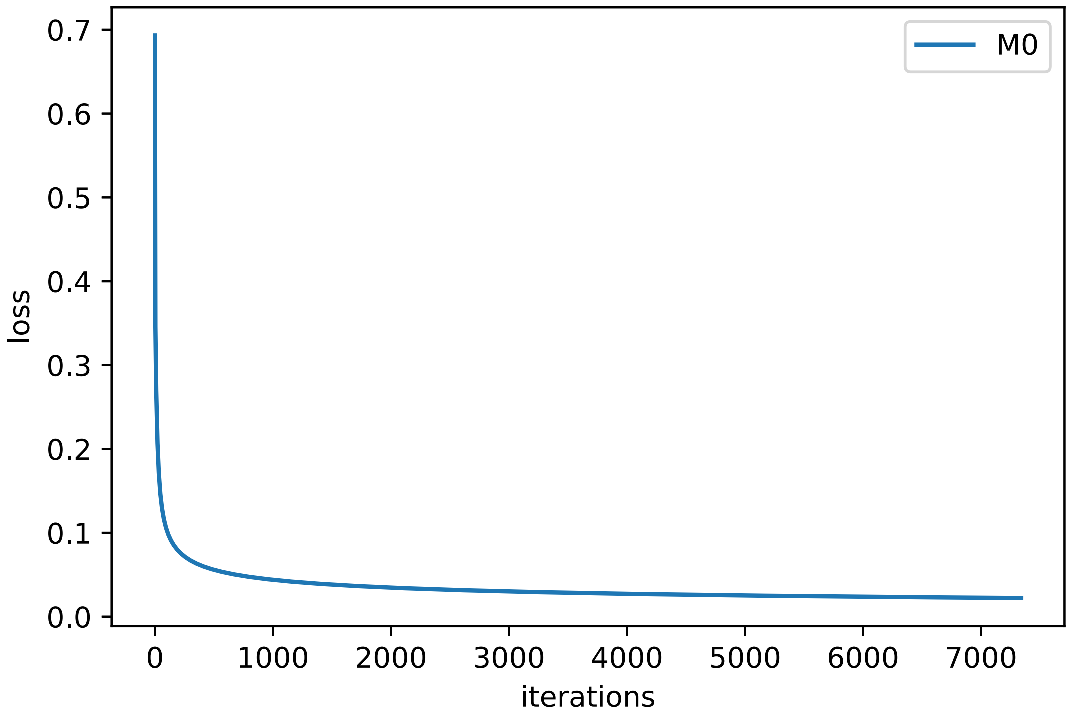
```
one = np.ones((len(x_train))).reshape(-1,1)
X_train = np.hstack((one, x_train))
coef = []
iters = []
loss = []
for i in range(10):
    Y = np.zeros((y_train.shape))
    Y[np.argwhere(y_train == i)] = 1
    Y[np.argwhere(y_train != i)] = 0
    theta = np.zeros((401,1))
    theta,iter,J = w_learn(theta,X_train,Y,1e6,1e-6,0.1)
    coef.append(theta)
    iters.append(iter)
    loss.append(J)
```

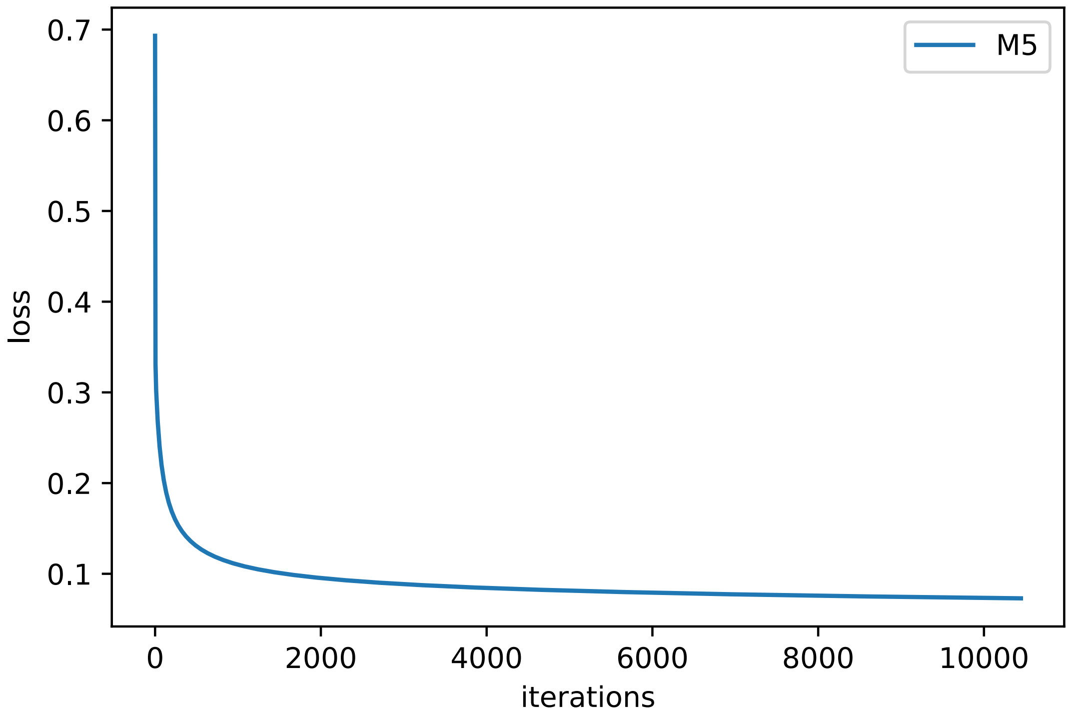
In [138…

```
def loss_figure(iter_times, loss,n):
    iter_times = np.arange(0,iter_times,1)
    plt.figure(n)
    plt.plot(iter_times,loss[n],label=f"M{n}")
    plt.xlabel("iterations")
    plt.ylabel("loss")
    plt.legend()
```

In [139…

```
loss_figure(iters[0], loss,0)
loss_figure(iters[5], loss,5)
```





(c)

In [140…

```
# predict
one_1 = np.ones((len(x))).reshape(-1,1)
X = np.hstack((one_1, x))
x_prob = np.empty((5000,0))
for i in range(10):
    prob = sigmoid(X@coef[i])
    x_prob = np.hstack((x_prob,prob))
label = np.argmax(x_prob, axis=1)
print("1st label",label[0])
print("23th label",label[22])
print("233th label",label[232])
print("2333th label",label[2332])
```

```
1st label 0
23th label 0
233th label 0
2333th label 4
```

In [141…

```
label_train,label_test = split_data(label.reshape(-1,1))
train_accuracy = sum(label_train == y_train)/len(y_train)
test_accuracy = sum(label_test == y_test)/len(y_test)
print("train accuracy",train_accuracy)
print("test accuracy",test_accuracy)
```

```
train accuracy [0.932]
test accuracy [0.901]
```

In [205...

```
from sklearn import linear_model
clf = linear_model.LogisticRegression(solver = "liblinear",
                    tol = 1e-6, max_iter = int(1e6))
X_train,X_test = split_data(X)
prob_train = np.empty((len(X_train),0))
prob_test = np.empty((len(X_test),0))
for i in range(10):
    Y = np.zeros((y_train.shape))
    Y[np.argwhere(y_train == i)] = 1
    Y[np.argwhere(y_train != i)] = 0
    clf.fit(X_train,np.ravel(Y))
    pred_train= clf.predict_proba(X_train)[:,1]
    pred_test= clf.predict_proba(X_test)[:,1]
    prob_train = np.hstack((prob_train,pred_train.reshape(-1,1)))
    prob_test = np.hstack((prob_test,pred_test.reshape(-1,1)))
train_label_sklearn = np.argmax(prob_train, axis=1).reshape(-1,1)
test_label_sklearn = np.argmax(prob_test, axis=1).reshape(-1,1)
train_accuracy_sklearn = sum(train_label_sklearn == y_train)/len(y_train)
test_accuracy_sklearn = sum(test_label_sklearn == y_test)/len(y_test)
print("train accuracy(sklearn)",train_accuracy_sklearn)
print("test accuracy(sklearn)",test_accuracy_sklearn)
```

```
train accuracy(sklearn) [0.94725]
test accuracy(sklearn) [0.908]
```

Q2. Data Normalization and Error

(a)

In [33]:

```
# Transform points to 10D space
def map_feature(x1,x2,m):
    A = np.empty(shape=(len(x1),0))
    for i in range(m+1):
        for j in range(i+1):
            x = (x1**j)*(x2**(i-j))
            A = np.hstack((A,x))
    return A
```

In [4]:

```
def add_one(x):
    one = np.ones((len(x))).reshape(-1,1)
    x_one = np.hstack((one,x))
    return x_one
```

In [4]:

```
Test = np.loadtxt('test.txt')
Test_x = Test[:,0:2]
Test_y = Test[:,2]
Train = np.loadtxt('train.txt')
Train_x = Train[:,0:2]
Train_y = Train[:,2]
```

In [177...

```
#A_train = add_one(Train_x)
A_train = map_feature(Train_x[:,0].reshape(-1,1),Train_x[:,1].reshape(-1,1),3)
eigenvalue,featurevector = np.linalg.eig(A_train.T@A_train)
print("eigenvalue :",eigenvalue)
ratio = np.max(eigenvalue)/np.min(eigenvalue)
print("ratio of max/min :",ratio)
```

```
eigenvalue : [0.00000000e+00 3.69838267e+27 2.55647952e+20 1.84299854e+17
 2.39035796e+13 7.29695886e+09 1.18666015e+07 2.25891684e+06
 4.11216311e+04 4.45716497e+02]
ratio of max/min : inf
<ipython-input-177-4bc81f37e6cc>:5: RuntimeWarning: divide by zero encountered in double_scalars
  ratio = np.max(eigenvalue)/np.min(eigenvalue)
```

(b)

In [188...

```
w = np.linalg.inv(A_train.T@A_train)@A_train.T@Train_y
A_test = map_feature(Test_x[:,0].reshape(-1,1),Test_x[:,1].reshape(-1,1),3)
#A_test = add_one(Test_x)
pred = A_test@w
pred
```

Out[188...

```
array([  45.84719945,   50.55510689,    9.47781831,   23.62176656,
        -20.1923248 ,   49.60034083,  119.99144615,   17.39498871,
        124.87883493,   87.62907354,  120.21445624,  116.57534952,
         53.99790826,   -2.59935654,   61.82501526,   59.98939604,
        -23.86384411,   45.76169792,   41.46209501,   94.03043285,
         72.35192554,   69.09409652,   61.37915413,    5.68083591,
         60.15787888,   66.68030402,   13.10122091,  -19.6281454 ,
         12.21877151,   59.93585183,  118.84117673,   23.84811   ,
```

```
 40.529519  ,   43.79447987,   63.30032366,   18.62917888,
-10.69191046,   68.98636344,    5.6559924 ,   54.6624325 ,
 10.83823224,   26.31228683,   -7.72989691,   30.71087427,
 19.33590502,   38.49654738,  111.24757195,  -14.12426599,
100.44747044,   24.77493104,   89.61506808,   -5.1774381 ,
-12.7518068 ,   41.69450743,    2.92541621,   -1.56793046,
 30.83157549,  105.84753946,   87.54482528,  -38.46377955,
 69.72541612,  102.58021786,   12.38501048,  -31.92980151,
-23.60325316, -106.8369771 ,   -4.53013082,   20.54344649,
  0.21794713,   23.264502  ,   24.50419764,   -2.76266286,
123.9616875 ,   -0.90564187,  114.95527158,    8.77520576,
 37.44088991,  109.19894142,   76.41143613,   18.92618484,
  0.6521195 ,   28.33741192,   23.21946589,   88.77625688,
 38.70173164,    8.97923098,   38.00142374,   19.79791099,
 17.48482775,   56.41886518,    9.88641784,   58.5966141 ,
104.16148059,  -18.81158429,   21.61085591,  -14.92045705,
 11.23970307,  -25.13723881,   98.76730949,   34.52078404,
-17.06268795,   82.02615526,   97.93854328,  125.89045848,
  4.70518452,   74.98877834,   34.12648312,   36.31705167,
 38.97991586,   85.265674  ,   22.01692031,   30.15902276,
 33.44295296,   91.70267135,   15.60887699,   83.46498596,
 30.59183167,   35.50046247,    3.58693482,   46.8640332 ,
-23.58665937,  -19.89117816,   33.80896736,  -26.70594535,
 23.25656133,  105.18103057,   57.41622361,    4.84094701,
 50.02434108,   26.07534808,   -4.27450524,   59.23833729,
 27.15117683,   -1.91285058,   43.67632731,  110.51017995,
 55.35505948,  115.55837163,  102.58309103,   69.39861212,
 62.08402952,   25.71849885,   63.17897703,   61.53646866,
  9.90841573,   96.4314573 ,   53.64282578,   26.21314097,
 41.23026777,   22.13235994,   30.9015558 ,   34.59167327,
-26.70921698,  127.32174185,  -11.33733514,   57.95023841,
 81.07661399,   53.38553009,   61.8193463 ,  105.60575384,
106.39076358,  114.95238433,  -26.59955522,  108.21382881,
 32.40913232,  -17.28312218,  -16.80033629,   45.01609841,
112.70561423,  -21.96971447,   53.74508352,  -63.817655  ,
  3.44161053,   22.64169078,  -12.83113638,    4.05281095,
 28.70391716,  -20.22087877,    0.56885243,   70.37811885,
 58.53328525,   12.04110591,   -8.40096157,   -2.44109869,
 53.20097985,  -13.25989399,   67.73501044,   16.84904323,
 38.30805797,   77.20374153,   25.60393729,    6.91706033,
 18.28417519,  105.78280534,    7.05966951,   -6.06145613,
-27.85074166,  131.91971461,   19.82451483,  -13.35315285,
 63.55221799,  119.69819697,   31.14378442,   15.95058721,
 22.61518895,   62.00015509,   17.45351329,  108.21383136,
114.16179527,  128.64449268,   66.53398827,   14.9190679 ,
 76.85001073,   -8.85810722,   47.70217234,   11.37769593,
  2.82854146,   96.82778131,   31.76175301,   13.74884296,
 62.13626211,    0.6165078 ,    7.40057761,   27.50958081,
-22.82735084,   26.67423277,  -11.66880358,   58.73772315,
 84.79487823,   93.76143204,   -3.62451617,   78.05759625,
 57.65065134,    3.57872855,    7.36016302,   78.61673026,
-23.82170742,   96.36591777,   37.16815631,   75.09218377,
 94.64978057,  -25.18688068,   96.43303962,  120.68208952,
 10.39530142,  108.47563673,   53.69561511,   85.37749551,
102.98283108,   70.93507978,   69.6071153 ,   27.24749855,
 13.3840817 ,   72.72995342,   39.89423893,   -0.72333558,
-11.88463648,   85.81868734,  -20.67242941,   58.09305319,
 25.014048  ,    5.89400453,   23.26740468,  -21.60293779,
 93.49974198,   73.92253401,    0.62155967,  130.76480049,
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 70.09177361,    5.70568004,   62.45766063,  -39.82958466,
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 29.57134858,   53.72933722,   29.10332811,   34.88909728,
-15.64210035,  113.11281981,   82.82906507,  -26.27422204,
 70.94161794,   23.21513323,   27.4731183 ,   18.5328889 ,
 69.68201617,    3.87241341,   86.66227773,   75.36107654,
 19.16329408,  -22.67575065,   36.4606322 ,   17.39743364,
 66.91334149,   31.24329091,   52.80866248,   11.89350422,
 89.08575411,  123.28288371,  131.77173516,  109.38929142,
 97.38408043,   31.32657812,  109.78816572,   -7.44793644,
 94.52230034,   81.17634181,   23.00477654,   25.12214872,
 24.50488895,   21.54378903,  -26.3300953 ,   90.03287244,
 58.5088923 ,   90.71388178,   82.72536248,   30.87284896,
102.9191352 ,  107.71841698,   10.53066649,    3.42438667,
124.9577909 ,   -8.2286174 ,   42.54626137,   10.10228128,
 81.27326107,  -27.12492802,   56.65161851,   -0.37412808,
 40.002072  ,   -2.38177783,  109.30211901,  -24.20605127,
 59.3378655 ,   64.85503028,  -21.80196913,   74.44527871,
 -1.30705544,   -7.79530859,  124.8218441 ,   39.01038592,
-10.56603351,   26.75076889,   19.48467877,   12.37977069,
 19.48408193,   19.9805553 ,   19.14245913,   32.43090445,
  8.06118494,  -16.23273911,   87.16334378,  121.47471096,
 36.83186485,   62.96035639,   74.56813836,   -9.55935134,
 18.73653098,   -1.00870361,   26.35349858,   26.71130407,
-18.84693708,   16.45330756,  112.5737098 ,  -19.47110249,
 38.32520302,   11.75286336,   11.71171598,   70.15142171,
-18.44651255,   11.19157414,  123.17089072,  100.58227336,
 60.05744541,   -7.42110195,  121.45972405,  -10.00387321,
```

```
          0.75218487,   12.89308626,   50.83214302,   33.21880669,
        129.78792857,   47.67000286,  119.52117291,   35.88700312,
         52.43071184,   70.35433126,   89.01025421,  114.82425535,
        -13.95469729,   -5.91429895,   24.86414788,   27.93571126,
         -9.75123556,   30.15874682,    2.73886233,    4.63170185,
         21.56799101,   61.76735093,   40.37330532,   90.43931298,
        132.15385709,   93.92077209,   15.38928198,  105.10230111,
        118.03188554,   88.29626168,   25.37571255,   -9.33801836,
        111.69638989,   -5.84118801,   -1.25063452,   59.81183713,
         28.50728425,   47.37696793,  -15.91015286,  -18.93726823,
         85.2703658 ,   99.45772824,   69.94083088,  -12.75010745,
         60.71003861,   56.04855094,    1.65423979,   13.47232806,
         11.96580082,    8.97909076,  -25.62380928,   46.96138385,
         26.31569139,  -37.99114448,   11.48792308,   38.09199026,
         77.54131479,  -19.948314  ,   41.23651202,   -9.22737095,
         -1.37563022,    4.83836547,   -7.48966005,    7.53710309,
         -7.70881307,    9.21456784,    8.70912283,   67.34845968,
        108.5696863 ,   68.52615911,  120.51945385,  -13.60256171,
         55.45089055,   51.52404538,  -15.13575511,   70.5790007 ,
        127.74472427,  117.14908124,   78.22085312,   69.03452278,
         10.47828985,  -36.51595163,   53.08333185,  -17.55183797,
         33.5007922 ,   29.6771821 ,   -5.42980288,  106.55344265,
         27.60208044,  102.37372418,    9.49539146,   75.98953312,
        104.10224396,   -4.94302814,    2.80768985,  126.17291344,
         26.31826867,   54.21986643,  102.69946261,  114.20625834,
          7.59201996,   30.71605375,   67.00025064,   30.95319875,
         15.59815516,  114.01471589,  100.92694207,   29.8757142 ])
```

(c)

In [200…

```python
error = np.sqrt(np.mean((Test_y.reshape(-1,1)-pred.reshape(-1,1))**2,axis = 0))
print("error :",error)
```

```
error : [18.91742004]
```

(d)

In [220…

```python
mu_x = np.mean(Train_x,axis = 0)
#mu_x[0] = 0
std_x = np.std(Train_x,axis = 0)
#std_x[0] = 1
x_new = (Train_x - mu_x)/std_x
A_train_new =map_feature(x_new[:,0].reshape(-1,1),x_new[:,1].reshape(-1,1),3)
#A_x_new = add_one(x_new)
eigenvalue_new,featurevector = np.linalg.eig(A_train_new.T@A_train_new)
print("eigenvalue of standardization:",eigenvalue_new)
ratio_new = np.max(eigenvalue_new)/np.min(eigenvalue_new)
print("ratio of max/min (standardization):",ratio_new)
```

```
eigenvalue of standardization: [6278.01972693 5438.37641409 3424.40744854 1080.47322434  210.31827492
  112.52071394  100.41022042  839.68902588  662.31826487  705.42569236]
ratio of max/min (standardization): 62.52371223172851
```

The ratio can represent condition number of the square matrix. If the ratio is infinity, the matrix is said to be ill-conditioned and almost singular. This is because when the feature values across different columns differ by orders of magnitude. Therefore, after standardization, the ratio goes from infinity to 62.52.

(e)

In [222…

```python
w_std = np.linalg.inv(A_train_new.T@A_train_new)@A_train_new.T@Train_y
x_new_test = (Test_x - mu_x)/std_x
A_test_new = map_feature(x_new_test[:,0].reshape(-1,1),x_new_test[:,1].reshape(-1,1),3)
#A_x_new_test = add_one(x_new_test)
pred_std = A_test_new@w_std
pred_std
```

Out[222…

```
array([  45.84719945,   50.55510691,    9.47781833,   23.62176654,
        -20.19232475,   49.60034084,  119.99144611,   17.3949887 ,
        124.87883488,   87.62907344,  120.21445619,  116.57534948,
         53.99790827,   -2.5993565 ,   61.82501524,   59.98939605,
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```

(f)

In [209…

```python
error_std = np.sqrt(np.mean((Test_y.reshape(-1,1)-pred_std.reshape(-1,1))**2,axis = 0))
print("standardization error :",error_std)
```

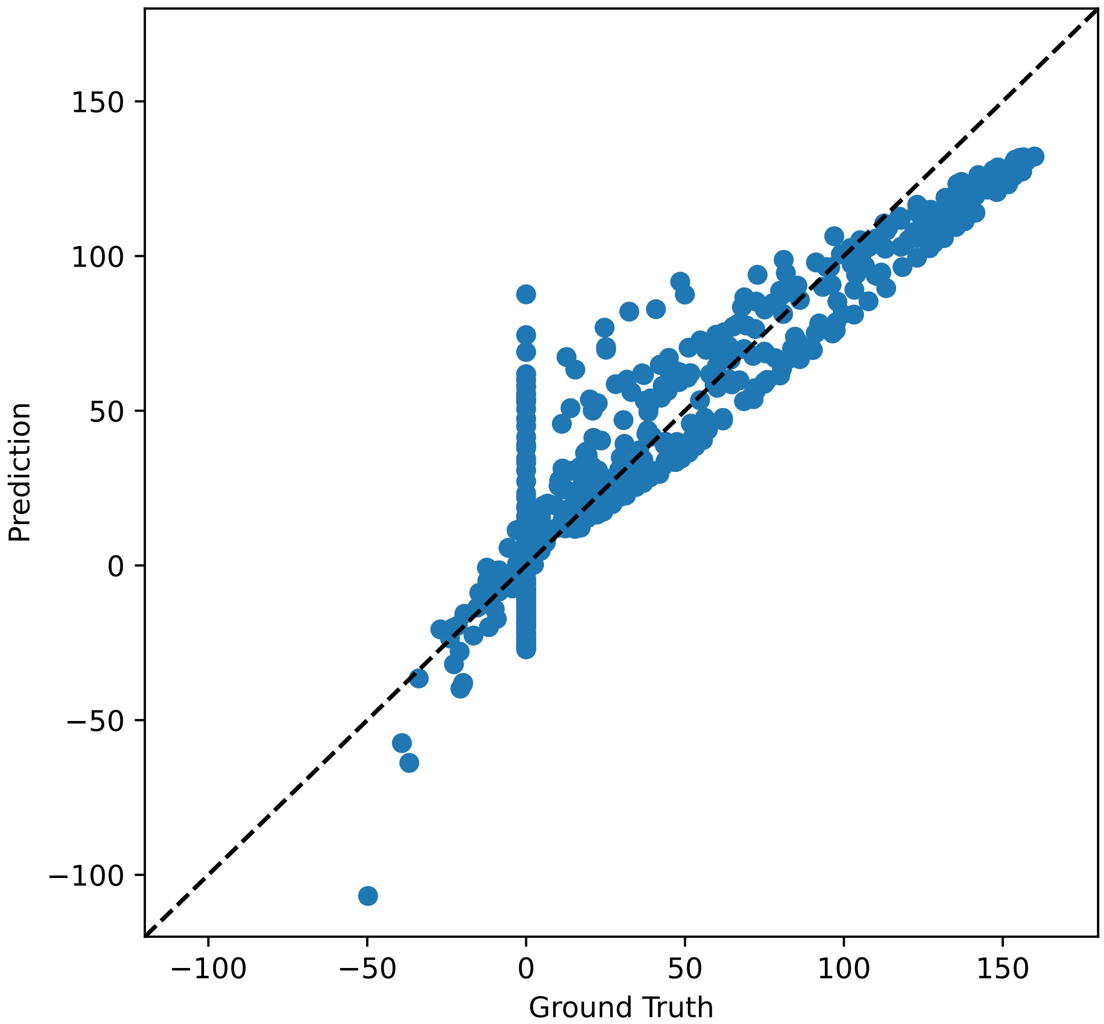
```
standardization error : [18.91742004]
```

(g)

In [218…

```python
f, ax = plt.subplots(figsize=(6, 6))
plt.scatter(Test_y,pred_std)
plt.ylim(-120,180)
plt.xlim(-120,180)
ax.plot([0, 1], [0, 1], transform=ax.transAxes,ls='--',c='k')
plt.xlabel("Ground Truth")
plt.ylabel("Prediction")
R2 = 1 - np.sum((Test_y - pred_std)**2) / np.sum((Test_y - np.mean(Test_y))**2)
#plt.text("R-square = ",R2)
n = len(Test_y)
p = 9
adj_R2 = 1-(1-R2)*(n-1)/(n-p-1)
print("adjusted R-squareed value",adj_R2)
```

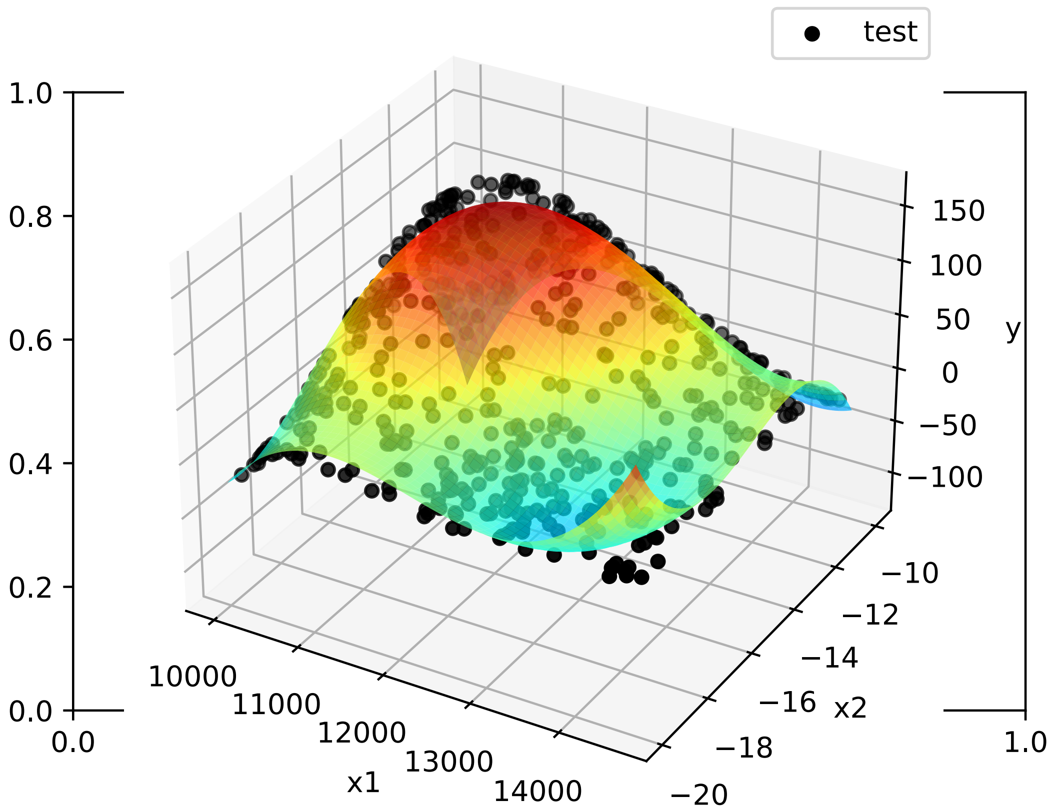
```
adjusted R-squareed value 0.8431209052235497
```



(h)

In [214…
```
from mpl_toolkits.mplot3d import Axes3D
x1 = np.linspace(np.min(Test_x[:,0]),np.max(Test_x[:,0]),100)
x2 = np.linspace(np.min(Test_x[:,1]),np.max(Test_x[:,1]),100)
x1_new, x2_new = np.meshgrid(x1,x2)
std_x1 = ((x1_new.ravel()).reshape(-1,1)-mu_x[0])/std_x[0]
std_x2 = ((x2_new.ravel()).reshape(-1,1)-mu_x[1])/std_x[1]
X_new = map_feature(std_x1,std_x2,3)
result_new = X_new@w_std
fig, ax = plt.subplots(1)
ax = Axes3D(fig)
ax.scatter(Test_x[:,0].reshape(-1,1),Test_x[:,1].reshape(-1,1),Test_y.reshape(-1,1),c='black',label="test")
ax.plot_surface(x1_new,x2_new,result_new.reshape(x1_new.shape),cmap='jet',alpha = 0.7)
ax.set(xlabel="x1",ylabel="x2",zlabel="y")
ax.legend()
```

Out[214… <matplotlib.legend.Legend at 0x7fe109710d90>



Q3. K-Nearest Neighbors

(a)

In [5]:
```
def standardize(X):
    mean = np.mean(X,axis = 0)
    std = np.std(X,axis = 0)
    std_x = (X - mean)/std
    return std_x,mean,std
Train_x_std,mean,std = standardize(Train_x)
Test_x_std = (Test_x - mean)/std
```

In [6]:
```
def Euclidian_dis(x_try,X):
    dis = np.empty((len(X),0))
    for i in range(len(x_try)):
        m = ((np.sum((x_try[i,:].reshape(1,-1) - X)**2,axis = 1))**0.5).reshape(-1,1)
        dis = np.hstack((dis,m))
    idx = np.argsort(dis,axis = 0)
    sorted_dis = np.sort(dis,axis = 0)

    return idx,sorted_dis
```

In [15]:
```
def KNN_regre(K,test,X,Y):
    idx,dis = Euclidian_dis(test,X)
    neighbors = dis[0:K,:]
    index = idx[0:K,:]
    y_mat = np.empty((K,0))
    for j in range(len(test)):
        b = Y[index[:,j],:]
        y_mat = np.hstack((y_mat,b))
    weight = (neighbors**(0))*(1/K)
    pred = np.sum(weight*y_mat,axis = 0)/np.sum(weight,axis = 0)
    return pred
```

In [16]:
```
# K = 1
KNN_regre(1,Test_x_std,Train_x_std,Train_y.reshape(-1,1)).reshape(-1,1)
```

Out[16]:
```
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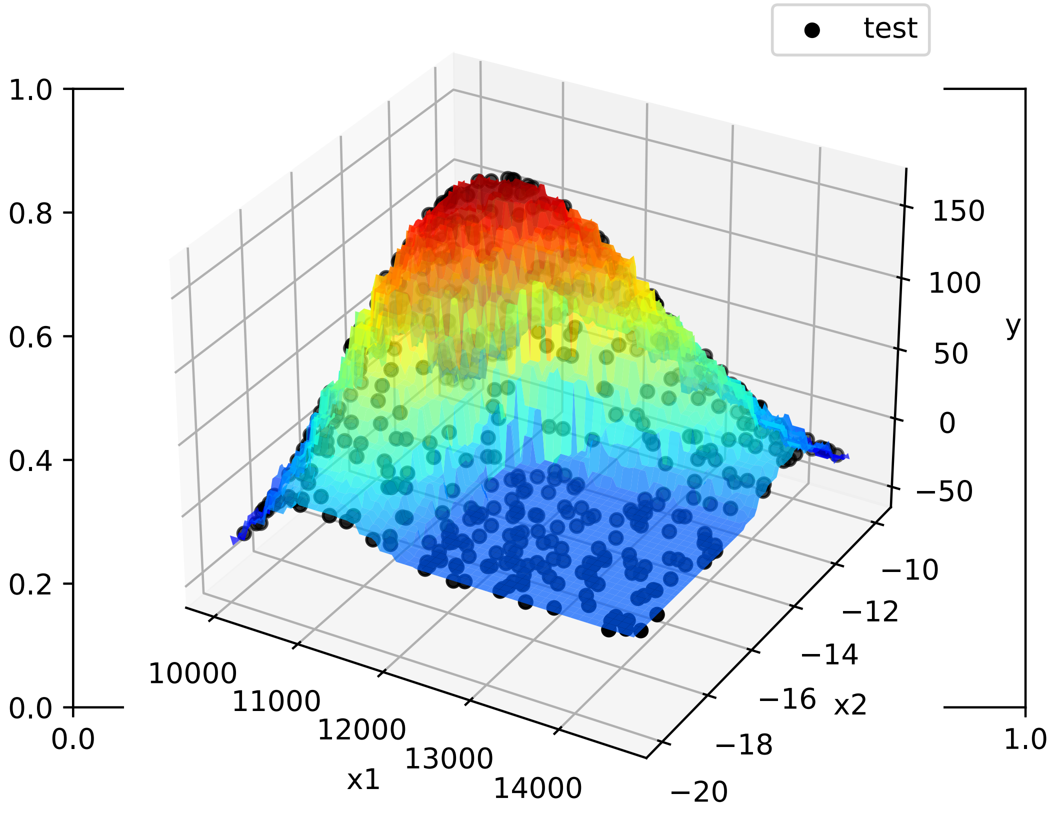
```

(b)

In [18]:

```
from mpl_toolkits.mplot3d import Axes3D
x1 = np.linspace(np.min(Test_x[:,0]),np.max(Test_x[:,0]),100)
x2 = np.linspace(np.min(Test_x[:,1]),np.max(Test_x[:,1]),100)
x1_new, x2_new = np.meshgrid(x1,x2)
X_New = np.hstack((x1_new.ravel().reshape(-1,1), x2_new.ravel().reshape(-1,1)))
X_new_std = (X_New-mean)/std
result_new = KNN_regre(1,X_new_std,Train_x_std,Train_y.reshape(-1,1))
fig, ax2 = plt.subplots(1)
ax2 = Axes3D(fig)
ax2.scatter(Test_x[:,0].reshape(-1,1),Test_x[:,1].reshape(-1,1),Test_y.reshape(-1,1),c='black',label="test")
ax2.plot_surface(x1_new,x2_new,result_new.reshape(x1_new.shape),cmap='jet',alpha = 0.7)
ax2.set(xlabel="x1",ylabel="x2",zlabel="y")
ax2.legend()
```

Out[18]: <matplotlib.legend.Legend at 0x7fe9b136d340>



(c)

In [17]:
```
# K = 4
KNN_regre(4,Test_x_std,Train_x_std,Train_y.reshape(-1,1)).reshape(-1,1)
```

Out[17]:
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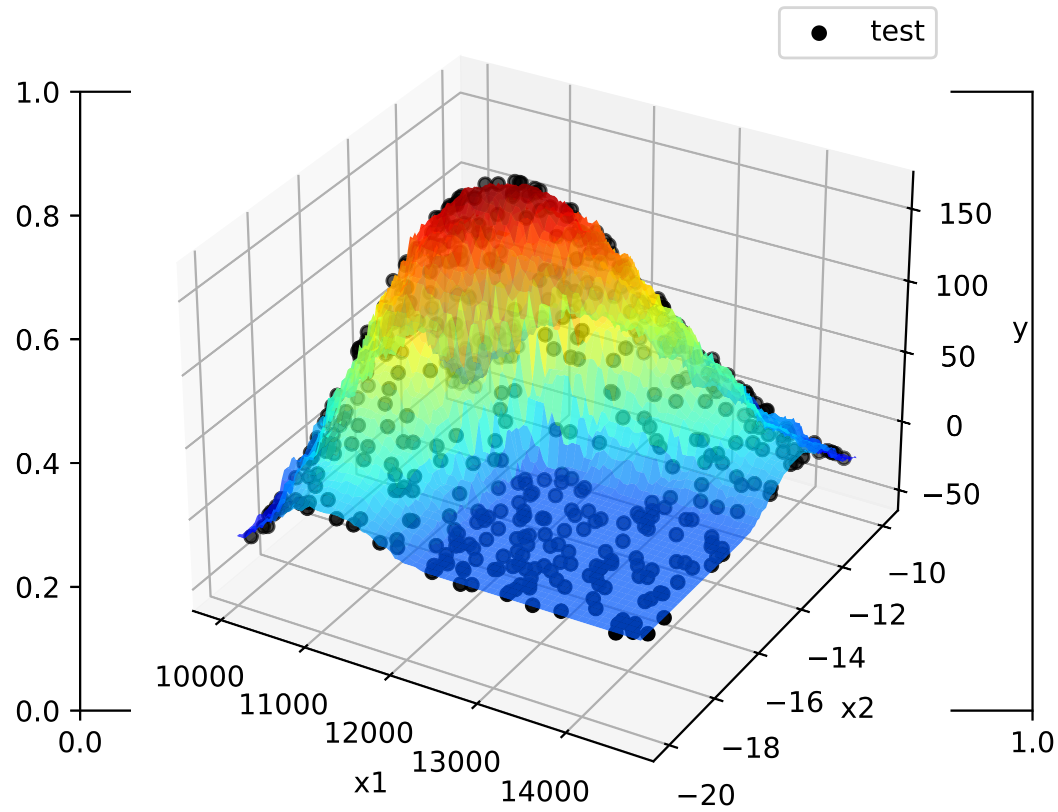
```

(d)

In [19]:

```
from mpl_toolkits.mplot3d import Axes3D
x1 = np.linspace(np.min(Test_x[:,0]),np.max(Test_x[:,0]),100)
x2 = np.linspace(np.min(Test_x[:,1]),np.max(Test_x[:,1]),100)
x1_new, x2_new = np.meshgrid(x1,x2)
X_New = np.hstack((x1_new.ravel().reshape(-1,1), x2_new.ravel().reshape(-1,1)))
result_new = KNN_regre(4,X_new_std,Train_x_std,Train_y.reshape(-1,1))
fig, ax3 = plt.subplots(1)
ax3 = Axes3D(fig)
ax3.scatter(Test_x[:,0].reshape(-1,1),Test_x[:,1].reshape(-1,1),Test_y.reshape(-1,1),c='black',label="test")
ax3.plot_surface(x1_new,x2_new,result_new.reshape(x1_new.shape),cmap='jet',alpha = 0.7)
ax3.set(xlabel="x1",ylabel="x2",zlabel="y")
ax3.legend()
```

Out[19]: <matplotlib.legend.Legend at 0x7fe9c05a1070>



(e)

From these two figures, I can see Figure (K = 4) is more smooth than the figure (K = 1), which means when K = 1, the model might be overfitting. But it's not said it's better when K increases, when K is too large, the model is underfitting. Therefore, we should choose the optimal K value.