Problem 1: Support Vector Machines

Instructions:

- 1. Please use this q1.ipynb file to complete SVM using cvxopt
- 2. You may create new cells for discussions or visualizations

```
In [3]: # Import modules
import numpy as np
import matplotlib.pyplot as plt
from cvxopt import matrix, solvers
```

a): Linearly Separable Dataset

```
data = np.loadtxt('clean_lin.txt', delimiter='\t')
In [11]:
          x = data[:, 0:2]
          y = data[:, 2].reshape(-1,1)
          n = x.shape[0]
          Q = matrix(np.eye((3)),tc = "d")
          Q[2,2] = 0
          p = matrix(np.zeros((3,1)),tc = "d")
          h = matrix(-np.ones((n,1)),tc = "d")
          ones = np.ones((n)).reshape(-1,1)
          G1 = (-y) * np.hstack((x,ones))
          G = matrix(G1, tc = "d")
          solvers.options['show_progress'] = False
          sol = solvers.qp(Q, p, G, h)
          print(sol['x'])
          z = sol['x']
          # Plot points
          x point = np.linspace(0,1.4,100)
          y_{point} = (-z[2]-z[0]* x_{point}) / z[1]
          y_{point1} = (1-z[2]-z[0]* x_{point}) / z[1]
          y_{point2} = (-1-z[2]-z[0]* x_{point}) / z[1]
          plt.figure(1)
          scatter = plt.scatter(x[:,0],x[:,1],c = y)
          plt.xlabel("x1")
          plt.ylabel("x2")
          legend1 = plt.legend(*scatter.legend_elements(),loc="upper right", title="Classes")
          plt.plot(x_point,y_point,c = "b")
          plt.plot(x_point,y_point1,c = "green",linestyle="-.")
          plt.plot(x_point,y_point2,c = "green",linestyle="-.")
          plt.ylim(-0.6,1.1)
         [ 6.83e+00]
         [ 6.38e+00]
         [-5.43e+00]
```

Out[11]: (-0.6, 1.1)

1.0 Classes -1 8.0 1 0.6 0.4 0.0 -0.2-0.4-0.60.2 0.4 0.6 0.0 8.0 1.0 1.2 x1

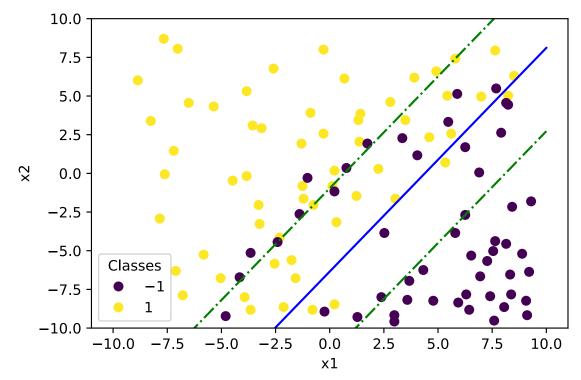
```
In [14]: print("Q matrix:",Q)
    print("p matrix:",p)
    print("G matrix:",G)
    print("h matrix:",h)
```

```
Q matrix: [ 1.00e+00 0.00e+00 0.00e+00]
[ 0.00e+00 1.00e+00 0.00e+00]
[ 0.00e+00 0.00e+00 0.00e+00]
p matrix: [ 0.00e+00]
[ 0.00e+00]
[ 0.00e+00]
G matrix: [-4.78e-02 -9.56e-01 -1.00e+00]
[-1.42e+00 3.96e-01 -1.00e+00]
[-2.51e-01 -8.97e-01 -1.00e+00]
[-2.55e-01 -7.99e-01 -1.00e+00]
[-3.38e-01 -7.25e-01 -1.00e+00]
[-5.35e-01 -5.45e-01 -1.00e+00]
[-7.32e-01 -5.37e-01 -1.00e+00]
[-7.77e-01 -7.09e-01 -1.00e+00]
[-6.59e-01 -8.03e-01 -1.00e+00]
[-9.81e-01 -7.66e-01 -1.00e+00]
[-8.77e-01 -4.39e-01 -1.00e+00]
[-8.32e-01 -1.73e-01 -1.00e+00]
[-6.52e-01 -3.08e-01 -1.00e+00]
[-1.42e+00 -9.21e-01 -1.00e+00]
[-1.28e+00 -6.51e-01 -1.00e+00]
[-1.30e+00 -3.78e-01 -1.00e+00]
[-1.19e+00 -1.90e-01 -1.00e+00]
[-1.09e+00 1.37e-01 -1.00e+00]
[-1.44e+00 -1.12e-01 -1.00e+00]
[ 9.59e-02 3.37e-01 1.00e+00]
[ 7.52e-02 1.57e-01 1.00e+00]
[ 1.79e-01 2.10e-01 1.00e+00]
[ 2.55e-01 3.37e-01 1.00e+00]
[ 3.24e-01 2.51e-01 1.00e+00]
[ 4.93e-01 1.65e-01 1.00e+00]
[ 5.97e-01 3.43e-02 1.00e+00]
[ 6.01e-01 -1.17e-01 1.00e+00]
[ 7.18e-01 -2.35e-01 1.00e+00]
[ 5.45e-01 -3.01e-01 1.00e+00]
[ 2.27e-01 -3.09e-01 1.00e+00]
[ 7.80e-01 -3.42e-01 1.00e+00]
[ 9.57e-01 -3.38e-01 1.00e+00]
[ 1.03e-01 -3.09e-01 1.00e+00]
[ 5.79e-02 -1.88e-02 1.00e+00]
[ 1.93e-01 5.47e-02 1.00e+00]
[ 3.86e-01 -3.09e-01 1.00e+00]
h matrix: [-1.00e+00]
```

b): Linearly Non-separable Dataset

```
# Load the data set that is not linearly separable
In [15]:
          data = np.loadtxt('dirty_nonlin.txt', delimiter='\t')
          x = data[:, 0:2]
          y = data[:, 2].reshape(-1,1)
          n = x.shape[0]
          Q = np.zeros((n+3,n+3))
          Q[0,0] = 1
          Q[1,1] = 1
          Q = matrix(Q,tc = "d")
          p = np.ones((n+3,1))
          p[0:3,:] = 0
          p = matrix(0.05*p,tc = "d")
          h = -np.ones((2*n,1))
          h[n:2*n,:] = 0
          h = matrix(h,tc = "d")
          ones = np.ones((n)).reshape(-1,1)
          G1 = (-y) * (np.hstack((x,ones)))
          G2 = -np \cdot eye((n))
          G3 = np.zeros((n,3))
          Ga = np.hstack((G1,G2))
          Gb = np.hstack((G3,G2))
          G = np.vstack((Ga,Gb))
          G = matrix(G,tc = "d")
          sol = solvers.qp(Q, p, G, h)
          #print(sol['x'])
          z = sol['x']
          # Plot points
          x_point = np.linspace(-10,10,200)
          y_{point} = (-z[2]-z[0]* x_{point}) / z[1]
          y_point1 = (1-z[2]-z[0]* x_point) / z[1]
          y_{point2} = (-1-z[2]-z[0]* x_{point}) / z[1]
          plt.figure(1)
          scatter = plt.scatter(x[:,0],x[:,1],c = y)
          plt.xlabel("x1")
          plt.ylabel("x2")
          legend1 = plt.legend(*scatter.legend_elements(),loc="lower left", title="Classes")
          plt.plot(x_point,y_point,c = "b")
          plt.plot(x_point,y_point1,c = "green",linestyle="-.")
          plt.plot(x_point,y_point2,c = "green",linestyle="-.")
          plt.ylim(-10,10)
```

Out[15]: (-10.0, 10.0)



```
print("Q matrix:",Q)
In [16]:
         print("p matrix:",p)
         print("G matrix:",G)
         print("h matrix:",h)
        Q matrix: [ 1.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 ... ]
                                                                   0.00e+00 ... ]
                             0.00e+00 0.00e+00 0.00e+00 0.00e+00
         [ 0.00e+00 1.00e+00
        [ 0.00e+00  0.00e+00  0.00e+00  0.00e+00  0.00e+00  0.00e+00
                                                                   0.00e+00 ... ]
        [ 0.00e+00  0.00e+00  0.00e+00  0.00e+00  0.00e+00  0.00e+00  0.00e+00  ... ]
        [ 0.00e+00 0.00e+00 0.00e+00 0.00e+00
                                                0.00e+00 0.00e+00 0.00e+00 ...]
        [ 0.00e+00 0.00e+00 0.00e+00 0.00e+00
                                                0.00e+00 0.00e+00
                                                                   0.00e+00 ... ]
        [ 0.00e+00 0.00e+00 0.00e+00 0.00e+00
                                                0.00e+00 0.00e+00
                                                                   0.00e+00 ... ]
        [ 0.00e+00 0.00e+00 0.00e+00 0.00e+00
                                                0.00e+00 0.00e+00
                                                                   0.00e+00 ... ]
        [ 0.00e+00 0.00e+00 0.00e+00 0.00e+00
                                                0.00e+00 0.00e+00
                                                                   0.00e+00 ... ]
        [ 0.00e+00 0.00e+00 0.00e+00 0.00e+00
                                                0.00e+00 0.00e+00
                                                                   0.00e+00 ... ]
        [ 0.00e+00 0.00e+00 0.00e+00 0.00e+00
                                                0.00e+00 0.00e+00
                                                                   0.00e+00 ... ]
        [ 0.00e+00 0.00e+00 0.00e+00 0.00e+00
                                                0.00e+00 0.00e+00
                                                                   0.00e+00 ... ]
        [ 0.00e+00  0.00e+00  0.00e+00  0.00e+00
                                                0.00e+00 0.00e+00
                                                                   0.00e+00 ... ]
```

[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	1
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	· · · ĺ
	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00		
							•••]
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	•••]
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00]
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00]
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00		i
							-
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00		•••]
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00]
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00]
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00		· · · ĺ
							-
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00		•••]
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00]
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00]
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	1
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00		• • •]
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00]
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00]
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00		· · ·
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00		• • •]
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	• • •]
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00]
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	į
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00]
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	• • •]
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00]
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	į
	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00		
]
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00		•••]
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00]
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	į
	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00		
]
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00		•••]
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00]
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00]
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	1
-							-
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	•••]
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	•••]
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00]
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00]
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0 00 .00	· · ·
•						0 00 .00	
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00]
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	• • •]
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00]
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00]
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0 00 .00	
-						0 00 .00]
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00		• • •]
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00]
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00]
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	أ
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0 00 .00	
-							•••]
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00]
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00]
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00]
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0 00 .00	
-							•••]
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	•••]
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00]
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00]
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0 00 .00	i
-				0.00e+00	0.00e+00	0 00 .00	
[0.00e+00	0.00e+00	0.00e+00	0.00e+00]
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00		•••]
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00]
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00]
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0 00 .00	· · · ĺ
[0.00e+00	0.00e+00				0.000100		
-			0.000 ± 00	0.000+00		0.000 ± 00	
[0.00e+00		0.00e+00	0.00e+00	0.00e+00	0.00e+00]
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00 0.00e+00	0.00e+00]
	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00]
[0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00]
-	0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00]
[0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00	···] ···] ···]
[0.00e+00 [0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00	···] ···] ···] ···]
[0.00e+00 [0.00e+00 [0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	···] ···] ···] ···] ···]
[0.00e+00 [0.00e+00 [0.00e+00 [0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	···] ···] ···] ···]
[0.00e+00 [0.00e+00 [0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	···] ···] ···] ···] ···]
[0.00e+00 [0.00e+00 [0.00e+00 [0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	···] ···] ···] ···] ···]
[0.00e+00 [0.00e+00 [0.00e+00 [0.00e+00 [0.00e+00 [0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00]]]]]]]]
[0.00e+00 [0.00e+00 [0.00e+00 [0.00e+00 [0.00e+00 [0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00]]]]]]]]
[0.00e+00 [0.00e+00 [0.00e+00 [0.00e+00 [0.00e+00 [0.00e+00 [0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00]]]]]]]]
[0.00e+00 [0.00e+00 [0.00e+00 [0.00e+00 [0.00e+00 [0.00e+00 [0.00e+00 [0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00]]]]]]]]
[0.00e+00 [0.00e+00 [0.00e+00 [0.00e+00 [0.00e+00 [0.00e+00 [0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00]]]]]]]]
[0.00e+00 [0.00e+00 [0.00e+00 [0.00e+00 [0.00e+00 [0.00e+00 [0.00e+00 [0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00]]]]]]]]]
[0.00e+00 [0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00]]]]]]]]]]]
[0.00e+00 [0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00]]]]]]]]]]]
[0.00e+00 [0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00]]]]]]]]]]]
[0.00e+00 [0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00]]]]]]]]]]]
[0.00e+00 [0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00]]]]]]]]]]]
[0.00e+00 [0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00]]]]]]]]]]]]
[0.00e+00 [0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00]]]]]]]]]]]]]
[0.00e+00 [0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00	
[0.00e+00 [0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00	
[0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00	
[0.00e+00 [0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00	
[0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00	
[0.00e+00 [0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00	
[0.00e+00 [0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00	

```
[ 0.00e+00  0.00e+00  0.00e+00  0.00e+00  0.00e+00  0.00e+00  0.00e+00  ... ]
[ 0.00e+00  0.00e+00  0.00e+00  0.00e+00  0.00e+00  0.00e+00  0.00e+00  ... ]
[ 0.00e+00  0.00e+00  0.00e+00  0.00e+00  0.00e+00  0.00e+00  0.00e+00  ... ]
[ 0.00e+00  0.00e+00  0.00e+00  0.00e+00  0.00e+00  0.00e+00  0.00e+00  ... ]
[ 0.00e+00  0.00e+00  0.00e+00  0.00e+00  0.00e+00  0.00e+00  0.00e+00  ... ]
[ 0.00e+00  0.00e+00  0.00e+00  0.00e+00  0.00e+00  0.00e+00  0.00e+00  ... ]
[ 0.00e+00  0.00e+00  0.00e+00  0.00e+00  0.00e+00  0.00e+00  0.00e+00  ... ]
[ 0.00e+00  0.00e+00  0.00e+00  0.00e+00  0.00e+00  0.00e+00  0.00e+00  ... ]
[ 0.00e+00  0.00e+00  0.00e+00  0.00e+00  0.00e+00  0.00e+00  0.00e+00  ... ]
[ 0.00e+00  0.00e+00  0.00e+00  0.00e+00  0.00e+00  0.00e+00  0.00e+00  ... ]
[ 0.00e+00  0.00e+00  0.00e+00  0.00e+00  0.00e+00  0.00e+00  0.00e+00  ... ]
p matrix: [ 0.00e+00]
[ 0.00e+00]
[ 0.00e+00]
[ 5.00e-02]
```

[5.00e-02]

```
[ 5.00e-02]
G matrix: [-8.51e+00 -6.31e+00 -1.00e+00 -1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...]
[-8.24e+00 -5.02e+00 -1.00e+00 -0.00e+00 -1.00e+00 -0.00e+00 -0.00e+00 ...]
[-6.99e+00 -4.96e+00 -1.00e+00 -0.00e+00 -0.00e+00 -1.00e+00 -0.00e+00 ...]
[-5.61e+00 -2.57e+00 -1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -1.00e+00 ...]
[-4.60e+00 -2.34e+00 -1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...]
[-3.49e+00 -3.45e+00 -1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...]
[-1.32e+00 -3.45e+00 -1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...]
[1.30e+00 -1.93e+00 -1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...]
[ 3.15e+00 -2.92e+00 -1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ... ]
[ 3.84e+00 1.75e-01 -1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ... ]
[ 1.26e+00  8.18e-01 -1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ... ]
[-2.19e-01 -1.75e-01 -1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...]
[-1.37e+00 -2.04e+00 -1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...]
[ 4.48e+00     4.67e-01     -1.00e+00     -0.00e+00     -0.00e+00     -0.00e+00     -0.00e+00     ... ]
[ 7.62e+00 5.84e-02 -1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ... ]
[ 7.85e+00 2.92e+00 -1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ... ]
[ 7.11e+00 6.31e+00 -1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ... ]
[ 6.79e+00 7.88e+00 -1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ... ]
[ 5.03e+00 6.77e+00 -1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ... ]
[ 2.55e+00 5.84e+00 -1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ... ]
[-3.11e-01 \quad 3.15e+00 \quad -1.00e+00 \quad -0.00e+00 \quad -0.00e+00 \quad -0.00e+00 \quad -0.00e+00 \quad ... ]
[-2.25e+00 -2.92e-01 -1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...]
[-5.33e+00 -7.01e-01 -1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...]
[-3.91e+00 -6.19e+00 -1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...]
[ 2.88e-01 -8.00e+00 -1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ... ]
[ 2.59e+00 -6.77e+00 -1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ... ]
[ 7.02e+00 -8.06e+00 -1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ... ]
[ 7.66e+00 -8.70e+00 -1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ... ]
[ 8.86e+00 -6.01e+00 -1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ... ]
[8.26e+00 -3.39e+00 -1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...]
[ 5.36e+00 -4.32e+00 -1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ... ]
[7.20e+00 -1.46e+00 -1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...]
[ 6.51e+00 -4.55e+00 -1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ... ]
[ 3.84e+00 -5.31e+00 -1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ... ]
[ 8.87e-01 -3.91e+00 -1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ... ]
[-1.42e+00 -3.85e+00 -1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...]
[-2.80e+00 -4.61e+00 -1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...]
[-4.92e+00 -6.60e+00 -1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...]
[-5.79e+00 -7.42e+00 -1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...]
```

[-7.64e+00 -7.94e+00 -1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...][-5.43e+00 -5.02e+00 -1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] $[-1.27e-01 \quad 8.18e-01 \quad -1.00e+00 \quad -0.00e+00 \quad -0.00e+00 \quad -0.00e+00 \quad -0.00e+00 \quad ... \]$ $[-1.23e+00 \quad 1.46e+00 \quad -1.00e+00 \quad -0.00e+00 \quad -0.00e+00 \quad -0.00e+00 \quad -0.00e+00 \quad ...]$ $[-3.03e+00 \quad 1.64e+00 \quad -1.00e+00 \quad -0.00e+00 \quad -0.00e+00 \quad -0.00e+00 \quad -0.00e+00 \quad ...]$ [7.49e-01 2.04e+00 -1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...][2.32e+00 4.15e+00 -1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] [1.76e+00 5.61e+00 -1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] [1.58e+00 6.77e+00 -1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...]

[7.95e-01 8.82e+00 -1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] $[-2.19e-01 \quad 8.47e+00 \quad -1.00e+00 \quad -0.00e+00 \quad -0.00e+00 \quad -0.00e+00 \quad -0.00e+00 \quad ...]$ [3.93e+00 8.00e+00 -1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] [5.82e+00 5.26e+00 -1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] [3.56e+00 -3.09e+00 -1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] [-6.80e-01 -6.13e+00 -1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] [2.88e-01 -2.57e+00 -1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] [-4.80e+00 -9.23e+00 1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] [-4.16e+00 -6.72e+00 1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] [-3.65e+00 -5.14e+00 1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] [-2.41e+00 -4.44e+00 1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] [-1.39e+00 -2.63e+00 1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] [2.19e-01 -1.17e+00 1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] [7.72e-01 3.50e-01 1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] [3.35e+00 2.28e+00 1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] [5.47e+00 3.33e+00 1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] [4.04e+00 1.17e+00 1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] [6.30e+00 -7.82e+00 1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] $[7.41e+00 -7.94e+00 \ 1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 \dots]$ [7.55e+00 -5.02e+00 1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] [8.33e+00 -6.54e+00 1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] [8.38e+00 -7.82e+00 1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] [9.21e+00 -6.37e+00 1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] [9.07e+00 -8.23e+00 1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] $\begin{bmatrix} 6.44 \\ e+00 \\ -8.82 \\ e+00 \end{bmatrix}$ $\begin{bmatrix} 1.00 \\ e+00 \\ -0.00 \\ e+00 \\ -0.00 \\ e+00 \end{bmatrix}$ $\begin{bmatrix} -0.00 \\ e+00 \\ -0.00 \\ e+00 \end{bmatrix}$ $\begin{bmatrix} -0.00 \\ e+00 \\ -0.00 \\ e+00 \end{bmatrix}$ [2.98e+00 -9.58e+00 1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] [2.38e+00 -8.00e+00 1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] [3.68e+00 -6.95e+00 1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] [6.26e+00 -2.69e+00 1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] [6.90e+00 5.84e-02 1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] [7.91e+00 2.63e+00 1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] [8.15e+00 4.55e+00 1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] [7.68e+00 5.49e+00 1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] [5.89e+00 5.14e+00 1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] $[-1.03e+00 -2.92e-01 \ 1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 \dots]$ [2.52e+00 -3.85e+00 1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] [4.32e+00 -6.25e+00 1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] [6.95e+00 -6.66e+00 1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] [7.64e+00 -4.38e+00 1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] [8.15e+00 -4.55e+00 1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] [8.42e+00 -2.16e+00 1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] [8.88e+00 -5.20e+00 1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] [7.59e+00 -9.52e+00 1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] [2.98e+00 -9.17e+00 1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] [1.28e+00 -9.28e+00 1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] $[-2.42e-01 -8.93e+00 \ 1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 \dots]$ [3.58e+00 -8.18e+00 1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] [4.78e+00 -8.23e+00 1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] [5.93e+00 -8.35e+00 1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] $\begin{bmatrix} 8.05e+00 & -8.64e+00 & 1.00e+00 & -0.00e+00 & -0.00e+00 & -0.00e+00 & -0.00e+00 & ... \end{bmatrix}$ [9.11e+00 -9.17e+00 1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] [9.30e+00 -1.81e+00 1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] [5.79e+00 -3.85e+00 1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] [6.53e+00 -5.31e+00 1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] $[7.27e+00 -5.66e+00 \ 1.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 \dots]$ $[\ 0.00e+00 \ \ 0.00e+00 \ \ 0.00e+00 \ \ -1.00e+00 \ \ -0.00e+00 \ \ -0.00e+00 \ \ -0.00e+00 \ \ ... \]$ $[\ 0.00e+00 \ \ 0.00e+00 \ \ 0.00e+00 \ \ -0.00e+00 \ \ -1.00e+00 \ \ -0.00e+00 \ \ -0.00e+00 \ \ ... \]$ $[\ 0.00e+00 \ \ 0.00e+00 \ \ 0.00e+00 \ \ -0.00e+00 \ \ -0.00e+00 \ \ -1.00e+00 \ \ -0.00e+00 \ \dots \]$ $[\ 0.00e+00 \ \ 0.00e+00 \ \ 0.00e+00 \ \ -0.00e+00 \ \ -0.00e+00 \ \ -0.00e+00 \ \ -1.00e+00 \ \dots \]$ $[\ 0.00e+00 \ \ 0.00e+00 \ \ 0.00e+00 \ \ -0.00e+00 \ \ -0.00e+00 \ \ -0.00e+00 \ \ -0.00e+00 \ \ ... \]$ $[\ 0.00e+00 \ \ 0.00e+00 \ \ 0.00e+00 \ \ -0.00e+00 \ \ -0.00e+00 \ \ -0.00e+00 \ \ -0.00e+00 \ \ ... \]$ $[\ 0.00e+00 \ \ 0.00e+00 \ \ 0.00e+00 \ \ -0.00e+00 \ \ -0.00e+00 \ \ -0.00e+00 \ \ -0.00e+00 \ \ ... \]$ $[\ 0.00e+00 \ \ 0.00e+00 \ \ 0.00e+00 \ \ -0.00e+00 \ \ -0.00e+00 \ \ -0.00e+00 \ \ -0.00e+00 \ \ ... \]$ $[\ 0.00e+00 \ \ 0.00e+00 \ \ 0.00e+00 \ \ -0.00e+00 \ \ -0.00e+00 \ \ -0.00e+00 \ \ -0.00e+00 \ \ ... \]$ $[\ 0.00e+00 \ \ 0.00e+00 \ \ 0.00e+00 \ \ -0.00e+00 \ \ -0.00e+00 \ \ -0.00e+00 \ \ -0.00e+00 \ \ ... \]$ [0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] [0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] [0.00e+00 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] $[\ 0.00e+00 \ \ 0.00e+00 \ \ 0.00e+00 \ \ -0.00e+00 \ \ -0.00e+00 \ \ -0.00e+00 \ \ -0.00e+00 \ \ ... \]$ $[\ 0.00e+00 \ \ 0.00e+00 \ \ 0.00e+00 \ \ -0.00e+00 \ \ -0.00e+00 \ \ -0.00e+00 \ \ -0.00e+00 \ \ ... \]$ $[\ 0.00e+00 \ \ 0.00e+00 \ \ 0.00e+00 \ \ -0.00e+00 \ \ -0.00e+00 \ \ -0.00e+00 \ \ -0.00e+00 \ \ ... \]$ 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] [0.00e+00 [0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] [0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] [0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] [0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] [0.00e+00 $[\ 0.00e+00 \ \ 0.00e+00 \ \ 0.00e+00 \ \ -0.00e+00 \ \ -0.00e+00 \ \ -0.00e+00 \ \ -0.00e+00 \ \ ... \]$

0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] 0.00e+00 0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] [0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] [0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] [0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] [0.00e+00 0.00e+00 0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 -0.00e+00 ...] $[0.00e+00 \quad 0.00e+00 \quad 0.00e+00 \quad -0.00e+00 \quad -0.00e+00 \quad -0.00e+00 \quad -0.00e+00 \quad ...$ $[\ 0.00e+00 \ \ 0.00e+00 \ \ 0.00e+00 \ \ -0.00e+00 \ \ -0.00e+00 \ \ -0.00e+00 \ \ -0.00e+00 \ \ ... \]$ $[\ 0.00e+00 \ \ 0.00e+00 \ \ 0.00e+00 \ \ -0.00e+00 \ \ -0.00e+00 \ \ -0.00e+00 \ \ -0.00e+00 \ \ ... \]$ h matrix: [-1.00e+00] [-1.00e+00] [-1.00e+00][-1.00e+00]

[-1.00e+00]

[-1.00e+00]

[-1.00e+00]

[-1.00e+00][-1.00e+00]

[-1.00e+00]

- [-1.00e+00]
- [-1.00e+00]
- [-1.00e+00]
- [-1.00e+00]
- [-1.00e+00]
- [-1.00e+00]
- [-1.00e+00]
- [-1.00e+00]
- [-1.00e+00]
- [-1.00e+00]
- [-1.00e+00]
- [-1.00e+00]
- [-1.00e+00]
- [-1.00e+00]
- [-1.00e+00]
- [-1.00e+00]
- [-1.00e+00]
- [-1.00e+00]
- [-1.00e+00]
- [-1.00e+00]
- [-1.00e+00]
- [-1.00e+00]
- [-1.00e+00]
- [-1.00e+00]
- [-1.00e+00]
- [-1.00e+00]
- [-1.00e+00]
- [-1.00e+00]
- [-1.00e+00]
- [-1.00e+00]
- [-1.00e+00]
- [-1.00e+00]
- [-1.00e+00]
- [-1.00e+00]
- [-1.00e+00]
- [-1.00e+00]
- [-1.00e+00]
- [-1.00e+00]
- [-1.00e+00][-1.00e+00]
- [-1.00e+00]
- [-1.00e+00][-1.00e+00]
- [-1.00e+00]
- [-1.00e+00]
- [-1.00e+00]
- [-1.00e+00][-1.00e+00]
- [-1.00e+00]
- [-1.00e+00]
- [-1.00e+00]
- [-1.00e+00][-1.00e+00]
- [-1.00e+00]
- [-1.00e+00]
- [-1.00e+00][-1.00e+00]
- [-1.00e+00]
- [-1.00e+00][-1.00e+00]
- [-1.00e+00]
- [-1.00e+00]
- [-1.00e+00][-1.00e+00]
- [-1.00e+00]
- [-1.00e+00]
- [-1.00e+00][-1.00e+00]
- [-1.00e+00]
- [-1.00e+00]
- [-1.00e+00]
- [-1.00e+00]
- [-1.00e+00][-1.00e+00]
- [-1.00e+00]
- [-1.00e+00][-1.00e+00]
- [-1.00e+00]
- [-1.00e+00]
- [-1.00e+00]
- [-1.00e+00][-1.00e+00]
- [-1.00e+00]
- [-1.00e+00]
- [-1.00e+00][-1.00e+00]
- [-1.00e+00]
- [-1.00e+00]
- [-1.00e+00]
- [-1.00e+00]

- [-1.00e+00] [-1.00e+00]
- [0.00e+00]
- [0.00e+00] [0.00e+00]
- [0.00e+00] [0.00e+00]
- [0.00e+00] [0.00e+00]
- [0.00e+00]
- [0.00e+00]
- [0.00e+00] [0.00e+00]
- [0.00e+00]
- [0.00e+00]
- [0.00e+00]
- [0.00e+00] [0.00e+00]
- [0.00e+00]
- [0.00e+00]
- [0.00e+00] [0.00e+00]
- [0.00e+00]
- [0.00e+00]
- [0.00e+00]
- [0.00e+00] [0.00e+00]
- [0.00e+00] [0.00e+00]
- [0.00e+00]
- [0.00e+00]
- [0.00e+00] [0.00e+00]
- [0.00e+00]
- [0.00e+00]
- [0.00e+00] [0.00e+00]
- [0.00e+00] [0.00e+00]
- [0.00e+00]
- [0.00e+00]
- [0.00e+00]
- [0.00e+00]

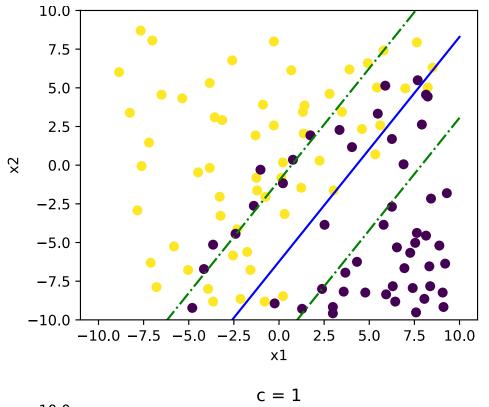
[0.00e+00] [0.00e+00]

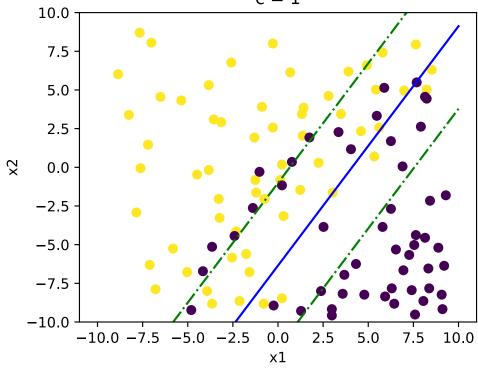
c): Output 4 plots & Explain your observations here:

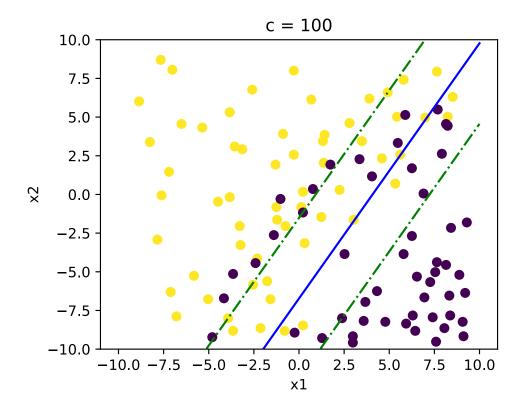
```
def svm(data,C):
In [263...
              x = data[:, 0:2]
              y = data[:, 2].reshape(-1,1)
              n = x.shape[0]
              Q = np.zeros((n+3,n+3))
              Q[0,0] = 1
              Q[1,1] = 1
              Q = matrix(Q, tc = "d")
              p = np.ones((n+3,1))
              p[0:3,:] = 0
              p = matrix(C*p,tc = "d")
              h = -np.ones((2*n,1))
              h[n:2*n,:] = 0
              h = matrix(h,tc = "d")
              ones = np.ones((n)).reshape(-1,1)
              G1 = (-y) * (np.hstack((x,ones)))
              G2 = -np \cdot eye((n))
              G3 = np.zeros((n,3))
              Ga = np.hstack((G1,G2))
              Gb = np.hstack((G3,G2))
              G = np.vstack((Ga,Gb))
              G = matrix(G,tc = "d")
              sol = solvers.qp(Q, p, G, h)
              #print(sol['x'])
              z = sol['x']
              # Plot points
              x_point = np.linspace(-10,10,200)
              y_{point} = (-z[2]-z[0]* x_{point}) / z[1]
              y_{point1} = (1-z[2]-z[0]* x_{point}) / z[1]
              y point2 = (-1-z[2]-z[0]* x point) / z[1]
              f, ax = plt.subplots(figsize=(5,4))
              ax.scatter(x[:,0],x[:,1],c = y)
              ax.set_xlabel("x1")
              ax.set_ylabel("x2")
              #legend1 = plt.legend(*scatter.legend_elements(),loc="upper right", title="Classes")
              ax.plot(x_point,y_point,c = "b")
              ax.plot(x_point,y_point1,c = "green",linestyle="-.")
              ax.plot(x_point,y_point2,c = "green",linestyle="-.")
              ax.set_ylim(-10,10)
              ax.set_title(f"c = {C}")
              return f
In [264...
          fig1 = svm(data, 0.1)
```

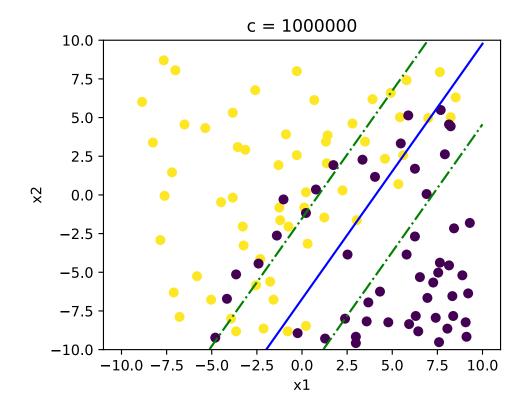
c = 0.1

fig2 = svm(data,1)
fig3 = svm(data,100)
fig4 = svm(data,1000000)







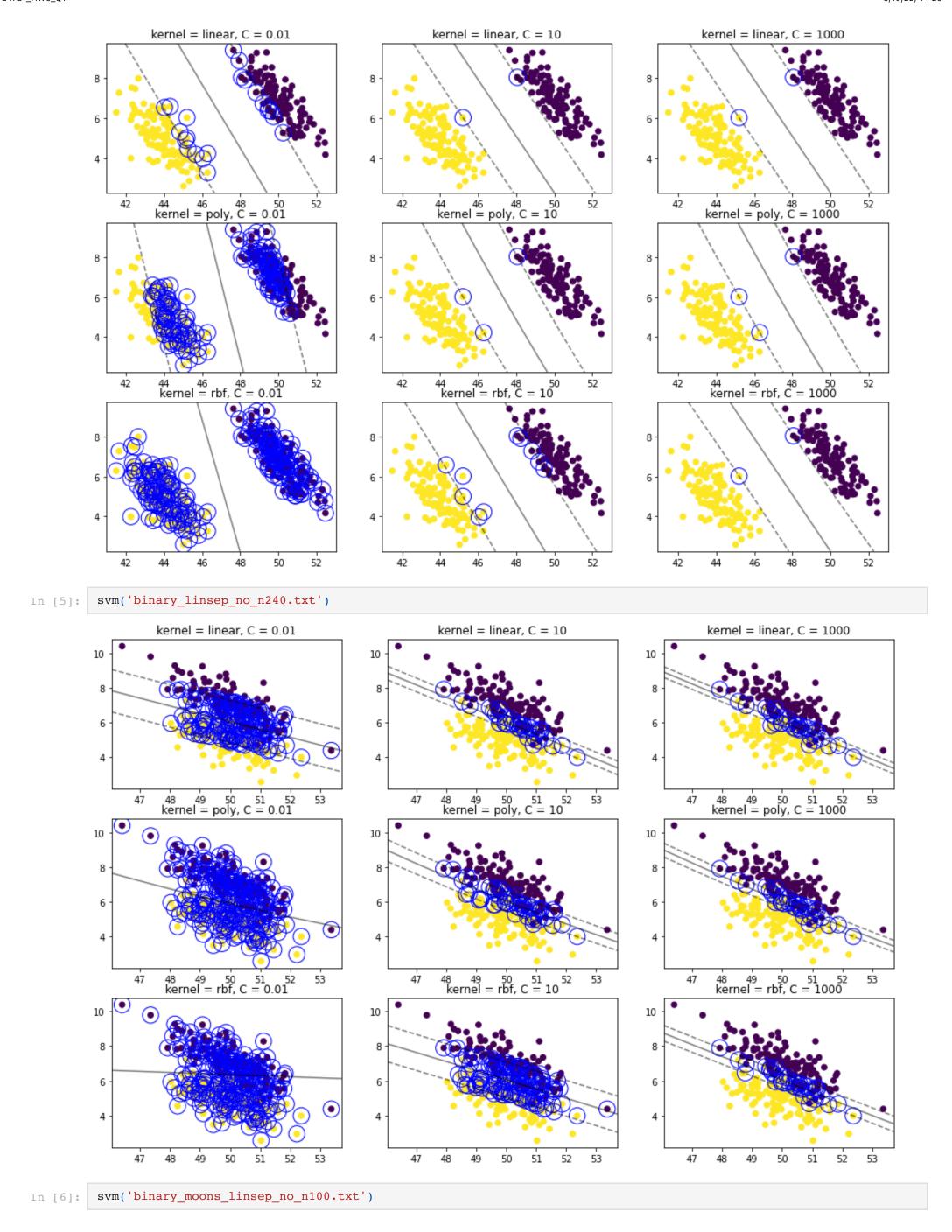


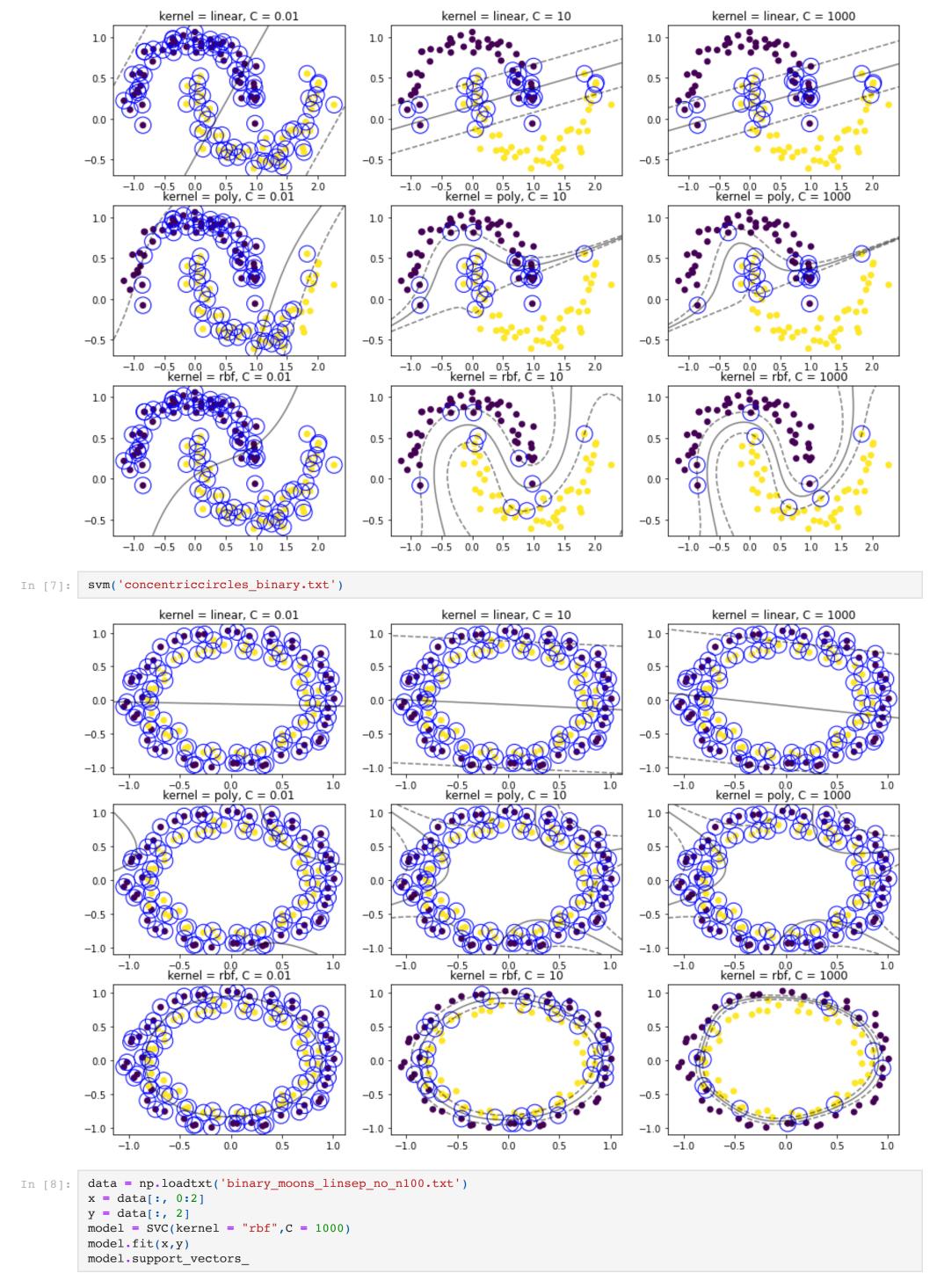
I think there are not huge differences between these four figures. Generally, with the increase of C, the constraints are harder to satisfy, the width between margins will become more narrow. However, this is a linearly non-separable case, no matter how hard the constraints are, these classes cannot be linearly separate, which means there aren't big differences between different C values.

Problem 2: Support Vector Machine Experiments with sklearn

(a)

```
from sklearn.svm import SVC
In [1]:
         from plot_svc_decision_function import plot_svc_decision_function
         def svm(data):
In [3]:
             data = np.loadtxt(data)
             x = data[:, 0:2]
             y = data[:, 2]
             fig, axs = plt.subplots(3,3,figsize = (15,10))
             kernel = ["linear", "poly", "rbf"]
             C = [0.01, 10, 1000]
             for i in range(3):
                 for j in range(3):
                     model = SVC(kernel = kernel[i],C = C[j])
                     model.fit(x,y)
                     axs[i,j].scatter(x[:,0],x[:,1],c = y)
                     axs[i,j].set_title(f"kernel = {kernel[i]}, C = {C[j]}")
                     plot_svc_decision_function(model,axs[i,j])
         svm('binary linsep yes n240.txt')
In [4]:
```





(b)

Kernel:

for the first datasets, linear kernel is better because it takes less time.

for the second datasets, three types of kernel can't linearly separate the datasets. Therefore, I think the simple one is better.

for the third and fourth datasets, RBF did better.

Conclusion: I think we should choose different kernel according to different datasets.

C value:

The larger the C, the greater punishment SVM receives when it commits a misclassification. As a result, the decision boundary will rely on fewer support vectors as the margin becomes narrower.

Moons dataset

I think the model with 5 support vectors is better. Because these two models have identical classification accurary, more support vectors means it's more expensive to train model. Therefore, less support vectors model is better.

Problem 3: Support Vector Regression

(a)

```
In [246...
          from sklearn.svm import SVR
          data = np.loadtxt('train.txt')
          test = np.loadtxt("test.txt")
          x = data[:, 0:2]
          y = data[:, 2]
          test_x = test[:,0:2]
          test_y = test[:,2]
          mu_x = np.mean(x,axis = 0)
          std_x = np.std(x,axis = 0)
          x_new = (x - mu_x)/std_x
          test new = (\text{test } x - \text{mu } x)/\text{std } x
          model = SVR(kernel = "rbf", C = 1, epsilon = 0.0001)
          model.fit(x_new,y)
          y pred = model.predict(test new)
          rmse = np.sqrt(np.mean((y pred.reshape(-1,1))-test y.reshape(-1,1))**2,axis = 0))
```

Out[246... array([24.03386007])

(b)

```
In [236...

def svr(C,epsilon):
    model = SVR(kernel = "rbf",C = C, epsilon = epsilon)
    model.fit(x_new,y)
    y_pred = model.predict(test_new)
    rmse = np.sqrt(np.mean((y_pred.reshape(-1,1)-test_y.reshape(-1,1))**2,axis = 0))
    return rmse
```

```
print("C = 0.01, epsilon = 0.0001, RMSE = ", svr(0.01, 0.0001))
In [242...
          print("C = 0.01, epsilon = 0.01, RMSE = ",svr(0.01,0.01))
          print("C = 0.01, epsilon = 1, RMSE = ", svr(0.01,1))
          print("C = 10, epsilon = 0.0001, RMSE = ", svr(10, 0.0001))
          print("C = 10, epsilon = 0.01, RMSE = ",svr(10,0.01))
          print("C = 10, epsilon = 1, RMSE = ", svr(10,1))
          print("C = 100, epsilon = 0.0001, RMSE = ", svr(100, 0.0001))
          print("C = 100, epsilon = 0.01, RMSE = ",svr(100,0.01))
          print("C = 100, epsilon = 1, RMSE = ",svr(100,1))
         C = 0.01, epsilon = 0.0001, RMSE = [49.86230822]
         C = 0.01, epsilon = 0.01, RMSE = [49.86230822]
         C = 0.01, epsilon = 1, RMSE = [49.71919847]
         C = 10, epsilon = 0.0001, RMSE = [7.2066139]
         C = 10, epsilon = 0.01, RMSE = [7.20308351]
         C = 10, epsilon = 1, RMSE = [7.19590857]
         C = 100, epsilon = 0.0001, RMSE = [4.35615181]
         C = 100, epsilon = 0.01, RMSE = [4.35497666]
         C = 100, epsilon = 1, RMSE = [4.18971781]
```

C value

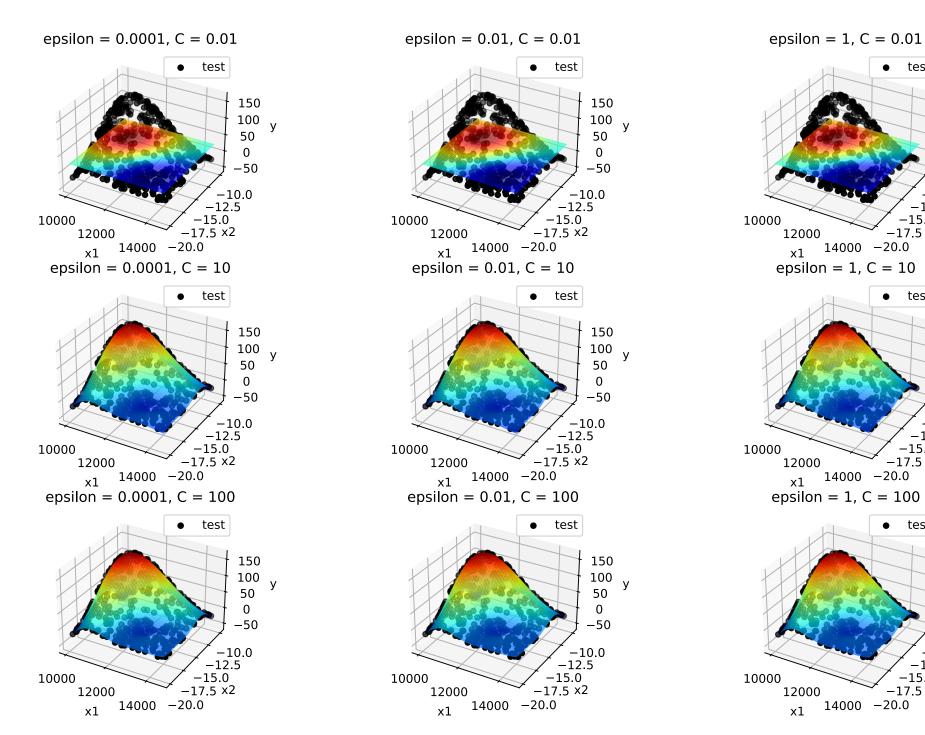
From these number, I can figure out when C becomes bigger, RMSE will decrease. But this doesn't mean that bigger C is better. When C is too big, the model might be overfitted.

Epsilon

The value of epsilon defines a margin of tolerance where no penalty is given to errors. From numbers above, I can see there is no huge difference between the models which have the same C value.

(c)

```
In [256...
         #from mpl_toolkits.mplot3d import Axes3D
          x1 = np.linspace(np.min(test_x[:,0]),np.max(test_x[:,0]),100)
          x2 = np.linspace(np.min(test x[:,1]),np.max(test x[:,1]),100)
          x1_{new}, x2_{new} = np.meshgrid(x1,x2)
          std_x1 = ((x1_new.ravel()).reshape(-1,1)-mu_x[0])/std_x[0]
          std_x2 = ((x2\_new.ravel()).reshape(-1,1)-mu_x[1])/std_x[1]
          X_new = np.hstack((std_x1,std_x2))
         fig = plt.figure(figsize=(15,10))
In [255...
          epsilon = [0.0001, 0.01, 1]
          C = [0.01, 10, 100]
          for i in range(3):
              for j in range(3):
                  model = SVR(kernel = "rbf", C = C[i], epsilon = epsilon[j])
                  model.fit(x_new,y)
                  y_pred = model.predict(X_new)
                  \#axs[i,j] = Axes3D(fig)
                  ax = fig.add_subplot(3,3,3*i+j+1, projection='3d')
                  ax.scatter(test x[:,0].reshape(-1,1),test x[:,1].reshape(-1,1),test y.reshape(-1,1),c='black',label="test"
                  ax.plot_surface(x1_new,x2_new,y_pred.reshape(x1_new.shape),cmap='jet',alpha = 0.7)
                  ax.set(xlabel="x1",ylabel="x2",zlabel="y")
                  ax.legend()
                  ax.set_title(f"epsilon = {epsilon[j]}, C = {C[i]}")
```



test

150

50

0

150

50

0

-50

150

0

-50

-10.0

-12.5

-15.0

-17.5 x2

100 50

-10.0

-12.5

-15.0 -17.5 x2

test

100 y

-10.0

-15.0

-17.5 x2

test

100 y