Elliptical Slice Sampling

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Slice sampling

Slice sampling was introduced by R. M. Neal in 2003 in the homonym paper published in the Annals of Statistics. Neal's idea was to present a new approach to sample from a probability density function, that tried to overcome some drawbacks of the two major Monte Carlo Markov Chain methods of sampling: the Gibbs sampler and the Metropolis Hastings method.

The aim of these three methods is to sample from a probability density function from which we are not able to sample directly (very often, from posterior distributions in Bayesian models). Their basic idea is to construct a Markov chain that have as stationary distribution the target one we care about, and samples from this chain will eventually come from the desired distribution. Gibbs sampler exploits the possibility of sampling from the full conditional density functions of each parameter given all the others (that are usually simpler to deal with than the original joint distribution), since the complete vector of parameters will eventually be sampled from the joint distribution of these. No tuning of parameters of the chain is required, but we must be able to sample from these conditional distributions in order to implement the method, and sometimes this could not be the case. Nevertheless, some algorithms to overcome this problem have been proposed, in particular ARS (adaptive rejection sampling) and MARS (adaptive rejection Metropolis sampling). Metropolis Hasting, instead, initializes the chain with a value sampled from a proposal distribution from which we are able to sample from and that attains some nice links with the target distribution. This sample can be accepted or rejected according to a probability that depends on the likelihood of the new candidate point and the previous one. This method presents some scale parameters for which there does not exist a certain rule of decision; moreover, the proposal distribution does not always come from a straightforward reasoning.

Slice sampling tries to overcome these flaws in the previous methods. The basic idea lies in the fact that if we want to sample x from a distribution $p(x)$ that is proportional to a certain function $f(x)$, it would be sufficient just to sample uniformly from the area below $f(x)$. By defining an auxiliary random variable y, we exploit Gibbs sampler to sample from the two full conditional distributions. In particular, $y|x$ is distributed as an $Uniform(0, f(x))$ and $x|y$ is distributed as an uniform on the so called slice $S = \{x : y < f(x)\}$. The joint density of x and y is, then

$$p(x, y) = \begin{cases} 1/Z & \text{for } 0 < y < f(x) \\ 0 & \text{otherwise} \end{cases}$$

where $Z = \int f(x)dx$. From this it can easily be seen that

$$p(x) = \int_0^{f(x)} p(x, y)dy = \frac{1}{Z} f(x)$$

as desired. Sampling on the slice can be difficult, and it is sometimes substituted with some update for x which leaves invariant the uniform distribution.

Elliptical Slice Sampling

Elliptical Slice Sampling is a particular case of Monte Carlo markov chain method that avoids the tuning of parameters, simpler and often faster than other methods to sample from the posterior distribution of models with multivariate Gaussian prior.

Let $\mathbf{f}$ be the vector of latent variables and a Gaussian distribution with zero vector mean and covariance matrix Σ:

$$\mathbf{f} \sim \mathcal{N}(\mathbf{f}; \mathbf{0}, Sigma) = |2\pi\Sigma|^{-1/2}\mathrm{exp}\left(-\frac{1}{2}\mathbf{f}^T\Sigma^{-1}\mathbf{f}\right)$$

Let

$$L(\mathbf{f}) = p(\mathrm{data}|\mathbf{f})$$

be the likelihood function. Our target distribution is the posterior of this model:

$$p^*(\mathbf{f}) \propto \mathcal{N}(\mathbf{f}; \mathbf{0}, Sigma)L(\mathbf{f})$$

Note the various macros within the `vignette` section of the metadata block above. These are required in order to instruct R how to build the vignette. Note that you should change the `title` field and the `\VignetteIndexEntry` to match the title of your vignette.

Styles

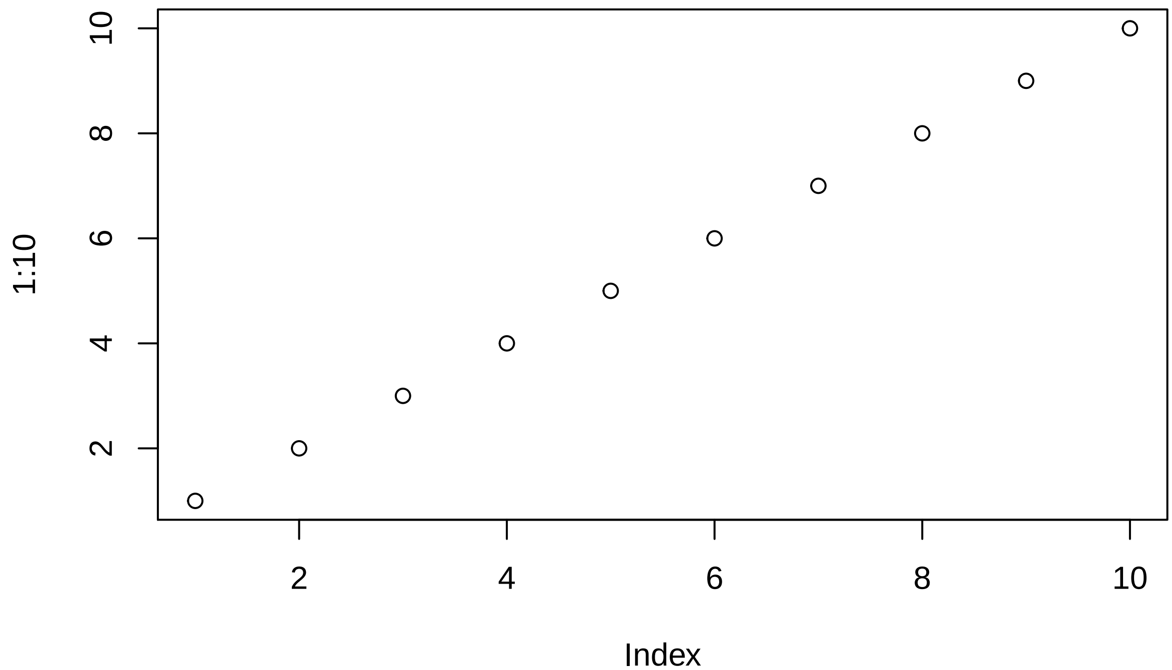
The `html_vignette` template includes a basic CSS theme. To override this theme you can specify your own CSS in the document metadata as follows:

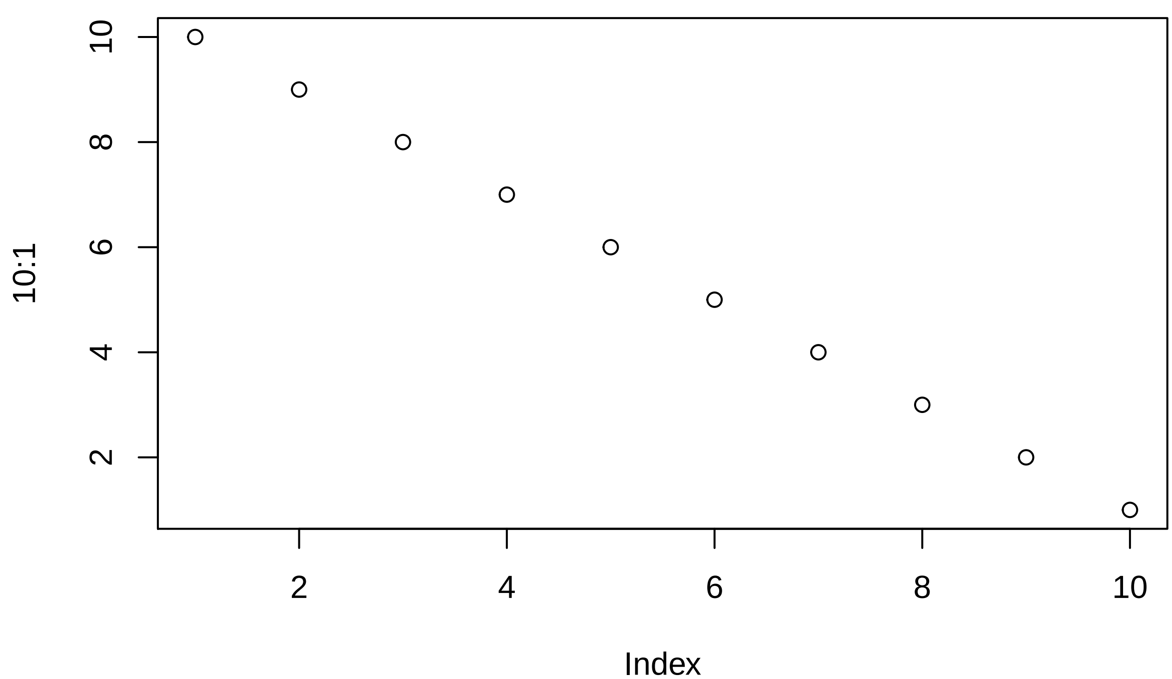
```
output:
  rmarkdown::html_vignette:
    css: mystyles.css
```

Figures

The figure sizes have been customised so that you can easily put two images side-by-side.

```
plot(1:10)
plot(10:1)
```


1:10
2
4
6
8
10
Index


10:1
2
4
6
8
10
Index

You can enable figure captions by `fig_caption: yes` in YAML:

```
output:
  rmarkdown::html_vignette:
    fig_caption: yes
```

Then you can use the chunk option `fig.cap = "Your figure caption."` in **knitr**.

More Examples

You can write math expressions, e.g. $Y = X\beta + \epsilon$, footnotes[1], and tables, e.g. using `knitr::kable()`.

	mpg	cyl	disp	hp	drat	wt	qsec	vs	am	gear	carb
Mazda RX4	21.0	6	160.0	110	3.90	2.620	16.46	0	1	4	4
Mazda RX4 Wag	21.0	6	160.0	110	3.90	2.875	17.02	0	1	4	4
Datsun 710	22.8	4	108.0	93	3.85	2.320	18.61	1	1	4	1
Hornet 4 Drive	21.4	6	258.0	110	3.08	3.215	19.44	1	0	3	1
Hornet Sportabout	18.7	8	360.0	175	3.15	3.440	17.02	0	0	3	2
Valiant	18.1	6	225.0	105	2.76	3.460	20.22	1	0	3	1
Duster 360	14.3	8	360.0	245	3.21	3.570	15.84	0	0	3	4
Merc 240D	24.4	4	146.7	62	3.69	3.190	20.00	1	0	4	2
Merc 230	22.8	4	140.8	95	3.92	3.150	22.90	1	0	4	2
Merc 280	19.2	6	167.6	123	3.92	3.440	18.30	1	0	4	4

Also a quote using `>`:

> "He who gives up [code] safety for [code] speed deserves neither." (via)

[1] A footnote here.