COMP9334 Assignment

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Question 1.

According to the question, we could get:

$T = 90\,min = 5400s$

$C(0) = 676$, $C(CPU) = 1377$, $C(disk) = 1515$

$Busy(CPU) = 4729s$, $Busy(disk) = 2565s$

(a). We could get the system throughput $X(0) = \frac{C(0)}{T} = \frac{676}{5400}$ job/s.

The utilisation: $U(CPU) = \frac{Busy(CPU)}{T} = \frac{4729}{5400}$

$U(disk) = \frac{Busy(disk)}{T} = \frac{2565}{5400}$

According to service demand law, we get: $D(j) = \frac{U(j)}{X(0)}$

$\Rightarrow$ CPU's service demand $= \frac{U(CPU)}{X(0)} = \frac{4729}{5400} \div \frac{676}{5400} = 7.00s$.

disk's service demand $= \frac{U(disk)}{X(0)} = \frac{2565}{5400} \div \frac{676}{5400} = 3.79s$.

(b). According to utilisation law: $U(j) = S(j) \cdot X(j)$ and forced flow law: $V(j) = \frac{X(j)}{X(0)}$.

we could get: $X(j) = V(j) \cdot X(0)$

$\Rightarrow U(j) = S(j) \cdot V(j) \cdot X(0)$

As utilisation $U(j)$ must be less or equal to 1

$\Rightarrow U(j) = S(j) \cdot V(j) \cdot X(0) \le 1$

$\Rightarrow X(0) \le \frac{1}{S(j) \cdot V(j)}$

$\Rightarrow X(0) \le \frac{1}{\max[S(j) \cdot V(j)]}$

In this question, $S(cpu) = \frac{Busy(CPU)}{C(CPU)} = \frac{4729}{1377}$ s/job, $V(CPU) = \frac{C(CPU)}{C(0)} = \frac{1377}{676}$

$S(disk) = \frac{Busy(disk)}{C(disk)} = \frac{2565}{1515}$ s/job, $V(disk) = \frac{C(disk)}{C(0)} = \frac{1515}{676}$

$\Rightarrow S(CPU) V(CPU) = \frac{4729}{1377} \times \frac{1377}{676} = 7.00s$

$S(disk) \cdot V(disk) = \frac{2565}{1515} \times \frac{1515}{676} = 3.79s$

$\Rightarrow X(0) \le \frac{1}{7.00} = 0.143$ job/s

Hence, it's possible to determine the bottleneck of the system without calculating the service demand, which is 0.143 job/s.

(c). According to the question, the number of interactive users job $N=30$, thinking time $Z=31s$,

According to the bottleneck analysis: $X(0) \le \min\left[\frac{1}{\max D_i}, \frac{N}{Z+\sum_{i=1}^{K} D_i}\right]$

Bound 1: $\frac{1}{\max D_i} = \frac{1}{7.00} = 0.143$ job/s.

Bound 2: $\frac{N}{Z+\sum_{i=1}^{K} D_i} = \frac{30}{31+7.00+3.79} = 0.72$ job/s

$\Rightarrow X(0) \le \min(0.143, 0.72) = 0.143$ job/s

$\Rightarrow$ The asymptotic bound of the system throughput is 0.143 job/s

(d). According to Little's law, we get: $N = X(0) \cdot$ (response time + thinking time Z)

$\Rightarrow$ response time + thinking time $Z = \frac{N}{X(0)} \ge \frac{30}{0.143} = 209.79$

$\Rightarrow$ response time $+ 31 \ge 209.79$

$\Rightarrow$ response time $\ge 178.79s$

$\Rightarrow$ The minimum possible response time is $178.79s$.

Question 2:

(a). Let's set an observational period of T.

During this period, the number of arriving requests to the dispatcher would be: λT.

The the number of requests to system 1 would be: $\lambda T P$,

the number of requests to system 2 would be: $\lambda T(1-P)$

$\Rightarrow$ Busy time for system 1: $t_1 = \frac{\lambda T P}{\mu_1}$,

Busy time for system 2: $t_2 = \frac{\lambda T(1-P)}{\mu_2}$.

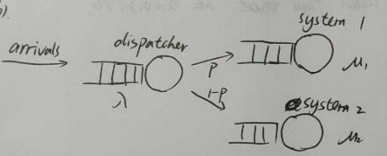
As system 1 and system 2 have the same utilisation then $t_1 = t_2$.

$\Rightarrow \frac{\lambda T P}{\mu_1} = \frac{\lambda T(1-P)}{\mu_2}$

$\Rightarrow \frac{P}{10} = \frac{1-P}{15}$

$\Rightarrow P = 0.4$

(b).



Let's set an observation period of T.

During this period, the number of requests to system 1 would be $\lambda T P$,

and the number of requests to system 2 would be $\lambda T(1-P)$.

As the arrivals to the dispatcher is possion distribution and the service time distributions of both systems are both exponentially distribution.

Then, this two systems could be considered as two "M/M/1" structures.

And for system 1: $\lambda_1 = \frac{P \cdot \lambda T}{T} = P\lambda$, for system 2: $\lambda_2 = \frac{\lambda T(1-P)}{T} = (1-P)\lambda$.

According to the lecture slide: "the response time of an M/M/1 queue $= \frac{1}{\mu - \lambda}$".

$\Rightarrow$ The response time t_1 of the system $1 = \frac{1}{\mu_1 - \lambda_1} = \frac{1}{10 - 0.4 \times 20} = \frac{1}{2}s$,

the response time t_2 of the system $2 = \frac{1}{\mu_2 - \lambda_2} = \frac{1}{15 - (1-0.4) \times 20} = \frac{1}{3}s$.

$\Rightarrow$ The mean response time of the server farm

$= P \times t_1 + (1-P) \times t_2 = 0.4 \times \frac{1}{2} + 0.6 \times \frac{1}{3} = 0.4s$.

(c). According to the way of (b),

for system 1: $\lambda_1 = p\lambda$, system 2: $\lambda_2 = (1-p)\lambda$.

$\Rightarrow \begin{cases} \text{response time } t_1 \text{ for system 1: } \frac{1}{\mu_1 - \lambda_1} \\ \text{response time } t_2 \text{ for system 2: } \frac{1}{\mu_2 - \lambda_2} \end{cases}$

The mean response time for the server farm $t = p \times t_1 + (1-p) \times t_2$

$\Rightarrow \lambda_1 = 20p,\ \lambda_2 = 20(1-p)$

$t_1 = \frac{1}{10-20p}$, $t_2 = \frac{1}{15-20(1-p)} = \frac{1}{20p-5}$

And we need $\begin{cases} 10-20p > 0 \\ 20p-5 > 0 \end{cases} \Rightarrow \frac{1}{4} < p < \frac{1}{2}$

$\Rightarrow t = p \times t_1 + (1-p) \times t_2 = \frac{p}{10-20p} + \frac{1-p}{20p-5}$ $(\frac{1}{4} < p < \frac{1}{2})$

set $f(x) = \frac{x}{10-20x} + \frac{1-x}{20x-5}$ $(\frac{1}{4} < x < \frac{1}{2})$

We could draw the graph for $f(x)$, and we could find that at $x = 0.3876$, $f(x)$ gets the minimum value which is 0.3949.

$\Rightarrow p = 0.3876$

Question 3.

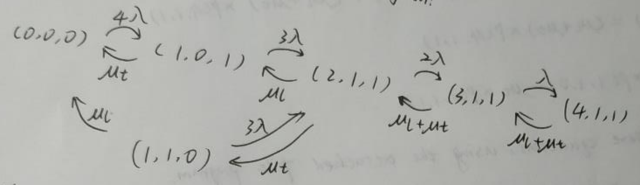
(a) According to the question, we can get:

rate of failure for one machine $= \lambda = \frac{1}{600}$

leader's repairing rate $= \mu_l = \frac{1}{60}$

trainee's repairing rate $= \mu_t = \frac{1}{90}$

And here is the state transitive diagram:



When there are M working machine, the failure rate is $M\lambda$.

At state (1,0,1), when 1 machine fails, trainee would work on repairing the failed machine, then the repairing rate is μ_t.

At state (2,1,1), when it goes to state (1,0,1), as trainee is working on another machine and 1 machine is repaired by 1 staff, then the leader would repair this machine, then the repairing rate is μ_l.

At state (3,1,1) or (4,1,1), both leader and trainee could repair the machine, then the repairing rate is $(\mu_l + \mu_t)$

Besides, there could have another state (1,1,0), which comes from state (2,1,1), which means trainee has finished the repairing work, then the repairing rate is μ_t, and it could goes to state (0,0,0), and the leader would do the repairing, then the repairing rate is μ_l.

And there is no other states as 1 staff could only repair 1 machine at a time.

(b) ① $P(0,0,0) + P(1,0,1) + P(1,1,0) + P(2,1,1) + P(3,1,1) + P(4,1,1) = 1$

② $4\lambda \times P(0,0,0) = \mu t \times P(1,0,1) + \mu c \times P(1,1,0)$

③ $(\mu t + 3\lambda) \times P(1,0,1) = 4\lambda \times P(0,0,0) + \mu c \times P(2,1,1)$

④ $(\mu c + \mu t + 2\lambda) \times P(2,1,1) = 3\lambda \times P(1,0,1) + 3\lambda \times P(1,1,0) + (\mu c + \mu t) \times P(3,1,1)$

⑤ $(\mu c + \mu t + \lambda) \times P(3,1,1) = 2\lambda \times P(2,1,1) + (\mu c + \mu t) \times P(4,1,1)$

⑥ $\lambda \times P(3,1,1) = (\mu c + \mu t) \times P(4,1,1)$

⑦ $(\mu c + 3\lambda) \times P(1,1,0) = \mu t \times P(2,1,1)$

(c). According to these equations, using the attached file program, we could get: $P(0,0,0) = 0.5918$

$P(1,0,1) = 0.3081$

$P(1,1,0) = 0.0004$

$P(2,1,1) = 0.0313$

$P(3,1,1) = 0.0611$

$P(4,1,1) = 0.0073$

(d). Prob(at least 3 machines are avilable) $= P(0,0,0) + P(1,0,1) + P(1,1,0)$

$= 0.5918 + 0.3081 + 0.0004$

$= 0.9003$

(e) Mean number of failed machines

$= 0 \times P(0,0,0) + 1 \times P(1,0,1) + 1 \times P(1,1,0) + 2 \times P(2,1,1) + 3 \times P(3,1,1) + 4 \times P(4,1,1)$

$= 0 \times 0.5918 + 1 \times 0.3081 + 1 \times 0.0004 + 2 \times 0.0313 + 3 \times 0.0611 + 4 \times 0.0073$

$= 0.5836$

(f) MTTR = Mean time to repair = Queuing time for repair + actual repairing time

Using Little's law, we get: