

CSCI 570 Summer 2016 Homework 2

On the first page of your homework, please write your name and USC ID # clearly. Please do something to indicate which is your last (family) name, such as underlining it. If your submission is multiple pages, you must *staple* them together. These are requirements of every homework assignment this semester.

This homework is due **July 27**. Note that this is a later due date than listed in the syllabus; this is so you can focus on this after quiz 1 if you prefer (greedy algorithms will be on quiz 2).

These two problems require a *greedy* algorithm. Please strive to describe your algorithm concisely; for example, for the scheduling with deadlines example from lecture, a sufficient description of the algorithm would be “sort in non-decreasing order of deadline.” You do not need to write pseudo-code or provide the running time (although both problems can be solved in $O(n \log n)$ time; please don’t make yours significantly worse).

In addition to providing the algorithm, prove that your algorithm is correct and optimal. Both of these can be proven with relatively short proofs; you shouldn’t need multiple pages.

1. Let X be a set of n intervals on the real line (each has a start coordinate s_i and an end coordinate f_i). A subset of intervals $Y \subseteq X$ is called a *tiling path* if the intervals in Y “cover” the intervals in X ; that is, any real value that is contained in some interval in X is also contained in some interval in Y . The size of a tiling cover is just the number of intervals; our goal is to find the smallest tiling cover.

Give an efficient **greedy algorithm** to solve this problem and prove it finds the optimal sized cover.

2. We’re asked to help the captain of the USC tennis team to arrange a series of matches against UCLA’s team. Both teams have n players; the tennis rating (a positive number, where a higher number can be interpreted to mean a better player) of the i th member of USC’s team is t_i and the tennis rating for the k th member of UCLA’s team is b_k . We would like to set up a competition in which each person plays one match against a player from the opposite school. Because we get to select who plays against whom, our goal is to make sure that in *as many matches as possible*, the USC player has a higher tennis rating than his or her opponent.

If two problems aren’t enough for you, and you would like additional practice, the following questions are suggested. Do not submit them for credit; these are for your own practice.

Chapter 10, G&T: problems A-10.1, A-10.4, A-10.6, A-10.7

Chapter 4, K&T: problems 3, 4, 5, 6, 7, 9, 10, 13, 14, 15, 16, 17, 29.