CSCI 570, Summer 2016 Homework 1

On the first page of your homework, please write your name and USC ID # clearly. Please do something to indicate which is your last (family) name, such as underlining it. If your submission is multiple pages, you must *staple* them together. These are requirements of every homework assignment this semester.

Each of these questions requires a **dynamic programming** algorithm for the solution; for problems 2 and 3, (1) write out the base case and recurrence expressions; (2) explain (briefly) why their expressions are the correct things to compute; and (3) analyze the running time for the iterative version, explaining any important implementation details. You do not need to actually write out the iterative version for these two.

1. Suppose you owe someone v cents and must repay using only coins. You wish to use the fewest coins possible. Suppose you were in a country that mints coins with the following values: one cent, 10 cents, 30 cents, and 40 cents.

   (a) Give a dynamic programming algorithm that will determine the minimum *number* of coins necessary to make change for v cents. For this part, the recurrence expression is sufficient.

   (b) Write the iterative version of this algorithm and state its running time. $O(v)$ is possible.

   (c) Write the code that would follow after (b) if your goal were to output the actual coins. Don't worry about the formatting of the output.

2. When their respective sport is not in season, USC's student-athletes are very involved in their community, helping people and spreading goodwill for the school. Unfortunately, NCAA[1] regulations limit each student-athlete to at most one community service project per semester, so the athletic department is not always able to help every deserving charity. For the upcoming semester, we have S student-athletes who want to volunteer their time, and B busses to help get them between campus and the location of their volunteering[2]. There are F projects under consideration; project i requires s_i student-athletes and b_i busses to accomplish, and will generate $g_i > 0$ units of goodwill for the university.

   Use dynamic programming to produce an algorithm to determine which projects the athletic department should undertake to maximize goodwill generated. Note that each project must be undertaken entirely or not done at all – we cannot choose, for example, to do half of project i to get half of g_i goodwill. Give the running time of your algorithm. For full credit, your algorithm should have runtime $O(FBS)$.

3. Suppose you are organizing a company party. The corporation has a hierarchical ranking structure; that is, the CEO is the root node of the hierarchy tree, and the CEO's immediate subordinates are the children of the root node, and so on in this fashion. To keep the party fun for all involved, you will not invite any employee whose immediate superior is invited. Each employee j has a value v_j (a positive integer), representing how enjoyable their presence would be at the party. Produce an algorithm that will determine which employees should be invited, subject to the above constraints.

If you would like additional practice, there are many good questions in the textbooks we suggest for the course. In order to make the best use of your study time, the following questions are suggested. Do not submit them for credit; these are for your own practice.
Goodrich & Tamassia textbook, Chapter 12, exercises C-12.1, C-12.2, C-12.3, C-12.4, C-12.9, A-12.2, A-12.3
Kleinberg & Tardos textbook, Chapter 6, problems 1, 2, 3, 4, 5, 6, 11, 15, 20, 26, 27, and 28.

[1] Regulations mentioned in this problem are not necessarily accurate to reality and should be considered parody.

[2] The NCAA also won't permit student-athletes to use their own vehicles for these purposes. Furthermore, each bus can only be used for one project.