CSCI 570, Summer 2016 Homework 5

For each question, to prove it is $\mathcal{NP}$-complete, you must prove it is in $\mathcal{NP}$, provide an appropriate reduction, and prove that the reduction is correct.

1. In a graph G, a Dominating Set in a subset $V' \subseteq V$ of the vertices such that every vertex either is in V' or is adjacent to one that is. Prove that the problem of determining if a dominating set of size k is present in a general graph is $\mathcal{NP}$-complete.

2. Sometimes you can know people for years and never really understand them. Take your friends Raj and Alanis, for example. Neither of them is a morning person, but now they're getting up at 6AM every day to visit local farmers' markets, gathering fresh fruits and vegetables for the new health-food restaurant they've opened.

   In the course of trying to save money on ingredients, they've come across the following problem. There's a large set of n possible raw ingredients they could buy $I_1, I_2, \ldots I_n$. Ingredient I_j must be purchased in units of size s_j grams, costs c_j dollars per unit, and is safe to use for t_j days from the date of purchase.

   Over the next k days, they want to make a set of k different daily specials, one each day. The order in which they schedule the specials is up to them. The ith daily special uses a subset $S_i \subseteq \{I_1, I_2, \ldots, I_n\}$ of the raw ingredients. Specifically, it requires $a_{i,j}$ grams of ingredeient I_j. Furthermore, the ingredients are partitioned into two subsets: those that must be purchased on the very day the special is offered, and those that can be used until they expire.

   This is where the opportunity to save money on ingredients comes up. Often, when they buy a certain ingredient I_j, they don't need the whole thing for the special they're making that day. Thus, if they can follow up quickly with another special that uses I_j, but doesn't require it to be fresh that day, they can save money by not having to purchase I_j again. Of course, scheduling the basil recipes so close together makes it harder to schedule the goat cheese recipes close together, and so forth – that's where the complexity comes in.

   So we define the Daily Special Scheduling problem as follows: given data on ingredients and recipes as above, and a budget x, is there a way to schedule the k daily specials so that the total money spent on ingredients over the course of all k days is at most x?

   Prove that Daily Special Scheduling is $\mathcal{NP}$-complete.

Additional Study: The following are good additional study questions:
G & T: A-17.1, A-17.2, A-17.3, A-17.4, A-17.5, A-17.6, A-17.7
K & T: 8.3, 8.4, 8.5, 8.6, 8.12, 8.15, 8.16, 8.19, 8.26, 8.27, 8.28, 8.31, 8.37
Do not submit your solutions to these for credit, although you may still discuss them with course staff.