CSCI 570 Practice Quiz 2 Summer 2016

Last (family) Name	
Rest of Name	
ID	

DO NOT OPEN EXAM UNTIL INSTRUCTED TO DO SO

SILENCE MOBILE PHONE AND OTHER DEVICES

PLEASE READ THE COVER CAREFULLY

- You are allowed to have one normal-sized page of hand-written notes.

- You may not have any other materials available to you other than those used for writing.

- You will have 60 minutes, from **2:30 PM** until **3:30 PM**, to work on the real exam.

- When answering any given question, use only the page on which the question is printed.

- If you leave your seat without permission of a proctor, you may be required to submit your exam without any further changes.

- If you give multiple answers to a free-response question and do not clearly indicate which one you wish to be graded, we reserve the right to select which one to grade.

- The grader may elect to read only a portion of unnecessary long answers.

- Please keep this cover page and the question pages intact.
  (You may remove the scratch paper from the end if you would like)

Question	Points	Possible
1		10
2		10
Total		20

1. You are given two lists of positive integers, A and B, each of size n. You may rearrange the elements of each list however you like. Afterward, we will compute $\Pi_i A_i^{B_i}$: that is, the product of A_i to the B_i over all i. Our goal is to have this product as large as possible.

   (a) Give an efficient **greedy** algorithm to determine this. Your answer may be at most 20 words.

   (b) Prove that your algorithm optimizes the value obtained.
       Hint: Choose an order of B and use it to compare alternate permutations of A.

2. Lately, we have been getting a large number (n to be exact) of prank calls at Algorithmic Pizza HQ, all from blocked numbers. This prevents us from finding out the identity of any of the callers, and our phone service provider will not release the names or numbers to us. Instead, they will allow us to specify a pair of phone calls to determine if the callers were the same person. We would like to use this information to answer the following question: is it the case that *strictly more* than $\frac{n}{2}$ of the n calls we have received are from the same person? Because there is a charge for each use of this service, we want to ask this about as few of the pairs of calls as possible.

Give an efficient **divide and conquer** algorithm that solves this problem with asymptotically fewer pairs being checked than the brute force would do. State how many pairs of calls your algorithm will inspect, both as a recurrence relation and in closed form (O or Θ-notation is fine for the latter).

For full credit, you should only inspect $O(n)$ pairs of calls. A correct algorithm with $O(n \log n)$ pairs of calls inspected is worth 7/10

Extra page. You may use this for scratch paper, but nothing on this page will be graded.