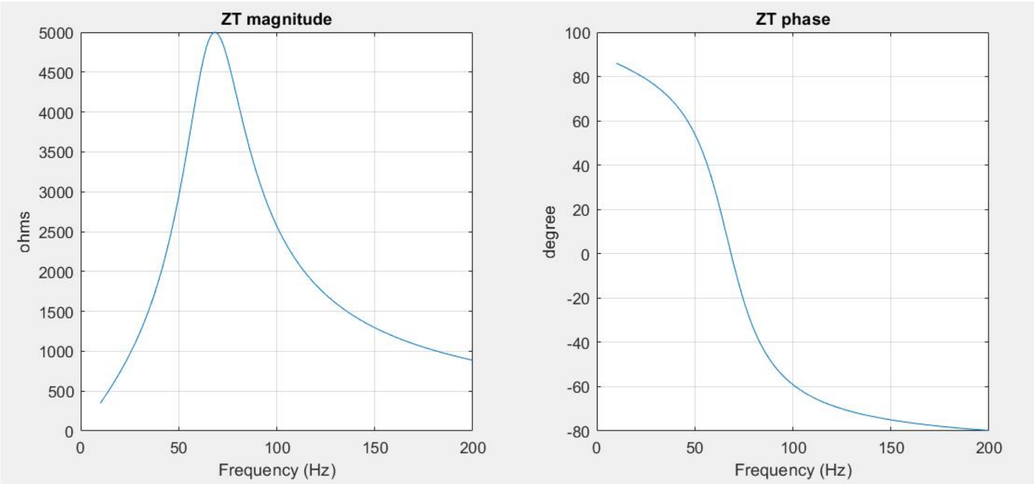
1. A parallel LRC circuit has R = 5000 ohms, C=1 μF and L = 5.4 henrys.

(a) Find the impedance, magnitude (ohms) and phase (degrees) and plot them versus frequency. (You should reproduce the graph below.)



$$Z_R = R = 5000\ ohms$$
$$Z_L = i\omega L = 5.4 i\omega\ ohms$$
$$Z_C = \frac{1}{i\omega C} = \frac{1000000}{i\omega}\ ohms$$

$$Z_T = \frac{1}{\frac{1}{Z_R}+\frac{1}{Z_L}+\frac{1}{Z_C}} = \frac{1}{\frac{1}{R}+\frac{1}{i\omega L}+i\omega C} = \frac{R}{1+\frac{R}{i\omega L}+i\omega CR} = \frac{R}{1+\frac{iRC}{\omega}(\omega^2-\omega_0^2)} \qquad (\omega_0 = \frac{1}{\sqrt{LC}})$$

$$\text{magnitude } |Z_T| = \frac{\mathrm{R}}{[\sqrt{(\frac{RC}{\omega}(\omega^2-\omega_0^2))^2+1}]} \mathrm{e}^{\mathrm{i}\varphi}$$

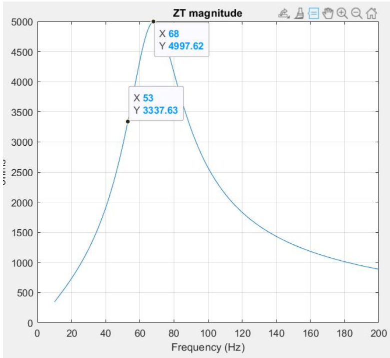
$$\text{phase } \varphi \ = \ -\arctan(\frac{RC}{\omega}(\omega^2-\omega_0^2))$$

(b) Confirm that the analytic value for Q (from class notes) is consistent with what you measure off the graph.

$$\mathrm{Q} = \mathrm{R}\sqrt{\frac{\mathrm{C}}{\mathrm{L}}} = 5000\sqrt{\frac{1\mathrm{e}-6}{5.4}} \approx 2.1517$$

$$\mathrm{Q} = \omega_0 \mathrm{RC}\ (\omega_0 = 2\pi \mathrm{f})$$
$$\mathrm{Q} = 2\pi * 68 * 5000 \ * \ 1\mathrm{e}\text{-}6 \approx 2.1352$$

Also, from this graph we can get f0 and f2.



At $f_0(68)$, $|Z_T| = 4997.62$

At $f_2(53)$, $|Z_T| = 3337.63$ $(3337.62 \ * \ \sqrt{2} \ = 4720)$

$$\omega_0 = 2\pi f = 2\pi * 68 = 136\pi$$

$$\omega_2 = 2\pi * 53 = 108\pi$$

$$Q \equiv \frac{\omega_0}{2|\omega_2 - \omega_0|} = 2.26$$

The analytic value for Q is consistent with what we measure off the graph, within a reasonable margin of error.

2. a. Find the impedance, magnitude and phase, of this circuit in terms of L, C, R and ω (ω=2πf, where f is the driving frequency in Hz.)

$$Z_C = \frac{1}{i\omega C}$$

$$Z_L = i\omega L$$

$$Z_R = R$$

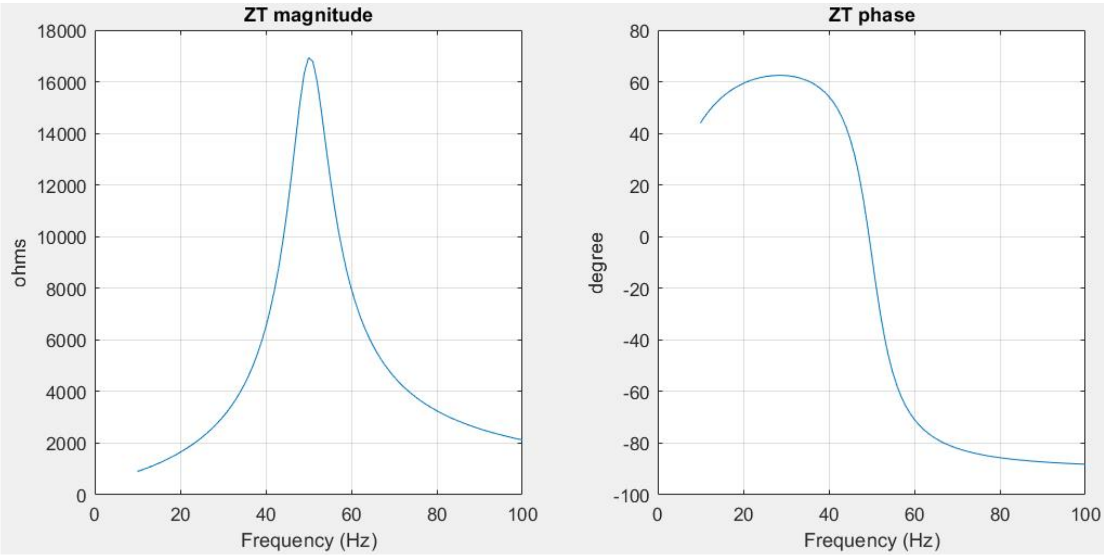
$$Z_T = \frac{1}{\frac{1}{Z_R + Z_L} + \frac{1}{Z_C}} = \frac{1}{\frac{1}{R + i\omega L} + i\omega C} = \frac{R + i\omega L}{1 + (R + i\omega L) i\omega C} = \frac{R + i\omega L}{1 + i\omega CR - \omega^2 LC}$$

$$= \frac{R^2 + \omega^2 L^2}{R + i\omega(CR^2 - L + \omega^2 L^2 C)}$$

$$\text{magnitude } |Z_T| = \frac{R^2 + \omega^2 L^2}{[\sqrt{(\omega(CR^2 - L + \omega^2 L^2 C))^2 + \mathrm{R}^2}]}$$

$$\text{phase } \varphi = -\arctan(\omega(CR^2 - L + \omega^2 L^2 C)/R)$$

b. Plot Z (mag and phase (in degrees)) for R=600 ohms, C= 1μF and L = 10 henrys. Plot the result for frequency from 10 to 100 Hz.



c. Show analytically that the Q of this circuit ≅ (1/R)√(L/C). (See lecture notes for hints.)

$$|Z_T| = \frac{R^2 + \omega^2 L^2}{\sqrt{(\omega(CR^2 - L + \omega^2 L^2 C))^2 + \mathrm{R}^2}}$$

$$\text{when } \omega = \omega_0, |Z_T| \text{max occurs.}$$

$$|Z_T|\text{max} = \frac{R^2 + \omega^2 L^2}{\sqrt{(\omega_0(CR^2 - L + \omega_0{}^2 L^2 C))^2 + \mathrm{R}^2}}$$

$$\text{plug in } \omega_0 = \frac{1}{\sqrt{\mathrm{LC}}}$$

$$|Z_T|\text{max} = \frac{R^2 + \frac{L}{C}}{\mathrm{R}\sqrt{\frac{CR^2 + \mathrm{L}}{\mathrm{L}}}} = \frac{\sqrt{\mathrm{C}LR^2 + L^2}}{\mathrm{CR}}$$

when $\omega = \omega_2$, $|Z_T| = |Z_T|\max/\sqrt{2}$.

$$\frac{R^2 + \omega_2{}^2L^2}{\sqrt{(\omega_2(CR^2 - L + \omega_2{}^2L^2C))^2 + \mathrm{R}^2}} = \frac{\sqrt{CLR^2 + L^2}}{\sqrt{2}\mathrm{CR}}$$

$$\omega_2 = \frac{2\sqrt{\mathrm{LC}} \pm \mathrm{RC}}{2\mathrm{LC}}$$

$$|\omega_2 - \omega_0| = \frac{\mathrm{R}}{2\mathrm{L}}$$

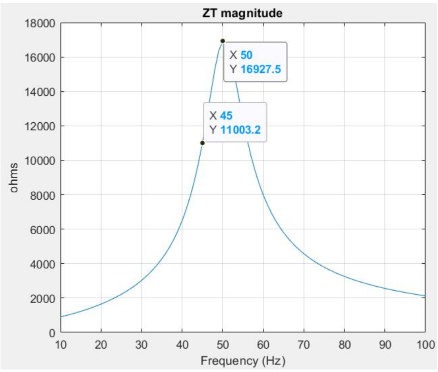
$$\mathrm{Q} \equiv \frac{\omega_0}{2|\omega_2 - \omega_0|} = \frac{1}{\mathrm{R}}\sqrt{\frac{\mathrm{L}}{\mathrm{C}}}$$

$$\text{therefore, } \mathrm{Q} \cong \frac{1}{\mathrm{R}}\sqrt{\frac{\mathrm{L}}{\mathrm{C}}}$$

d. Show that this makes sense based on measuring Q from your plots in (b).

$$\mathrm{Q} = \frac{1}{\mathrm{R}}\sqrt{\frac{\mathrm{L}}{\mathrm{C}}} = \frac{1}{600}\sqrt{\frac{10}{1\mathrm{e}-6}} \approx 5.27$$

From this graph, we can get f0 and f2.



At $\mathrm{f}_0(50)$, $|Z_T| = 16927.5$

At $\mathrm{f}_2(45)$, $|Z_T| = 11003.2$ $(11003.2 \ * \ \sqrt{2} \ = 15560.9)$

$$\omega_0 = 2\pi\mathrm{f} = 2\pi * 50 = 100\pi$$

$$\omega_2 = 2\pi * 45 = 90\pi$$

$$\mathrm{Q} \equiv \frac{\omega_0}{2|\omega_2 - \omega_0|} = 5$$