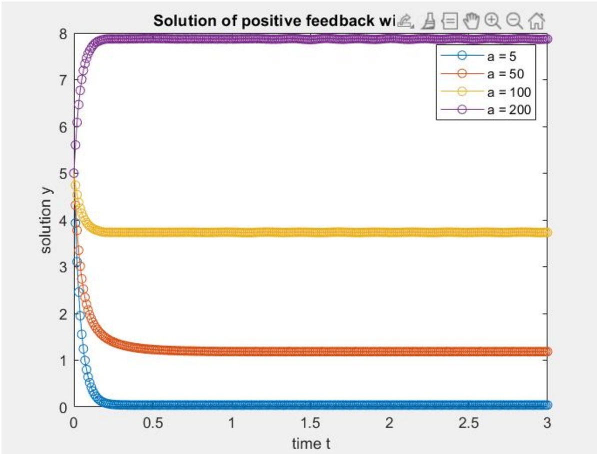
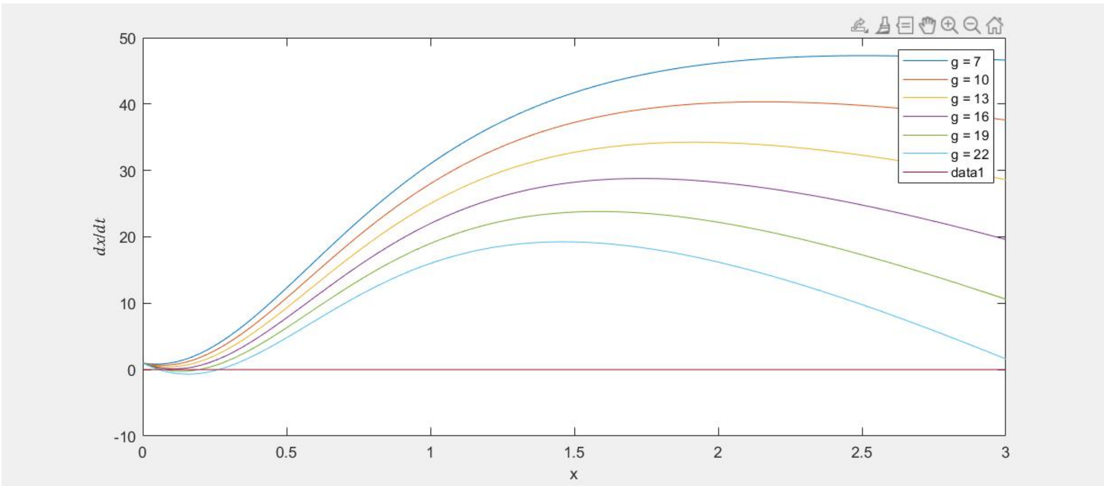
Question 2

(a) This equation represents a positive feedback circuit. Parameter “a” determines the strength of positive feedback. The larger “a”, the higher solution y we get.



(b)



(c)

For g=7 and a=75, fixed point is 10.6214

For g=10 and a=75, fixed point is 7.3661

For g=13 and a=75, fixed point is 5.5929

For g=16 and a=75, fixed point is 4.4668

For g=19 and a=75, fixed points are 3.6795, 0.1943, 0.0736

For g=22 and a=75, fixed points are 3.0903, 0.2629, 0.0560

```
>> roots([g_list(1) -a g_list(1) -1])

ans =

  10.6214 + 0.0000i
   0.0464 + 0.1063i
   0.0464 - 0.1063i
```

```
>> roots([g_list(2) -a g_list(2) -1])

ans =

   7.3661 + 0.0000i
   0.0670 + 0.0954i
   0.0670 - 0.0954i
```

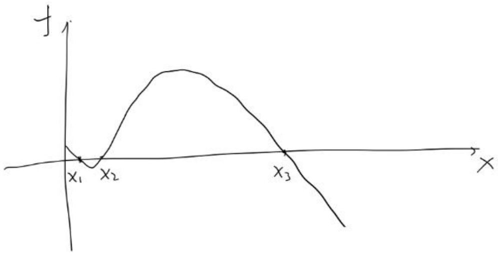
```
>> roots([g_list(3) -a g_list(3) -1])

ans =

   5.5929 + 0.0000i
   0.0882 + 0.0773i
   0.0882 - 0.0773i
```

```
>> roots([g_list(4) -a g_list(4) -1])

ans =

   4.4668 + 0.0000i
   0.1104 + 0.0425i
   0.1104 - 0.0425i
```

```
>> roots([g_list(5) -a g_list(5) -1])

ans =

    3.6795
    0.1943
    0.0736
```

```
>> roots([g_list(6) -a g_list(6) -1])

ans =

    3.0903
    0.2629
    0.0560
```

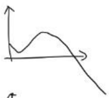
(d)

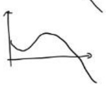


$f'(x_1) < 0$, stable

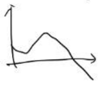
$f'(x_2) > 0$, unstable

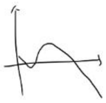
$f'(x_3) < 0$, stable

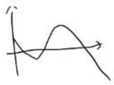
for $g = 7$, $a = 75$  $f'(10.6214) < 0$, this fix point is stable.

for $g = 10$, $a = 75$  $f'(7.3461) < 0$, this fix point is stable.

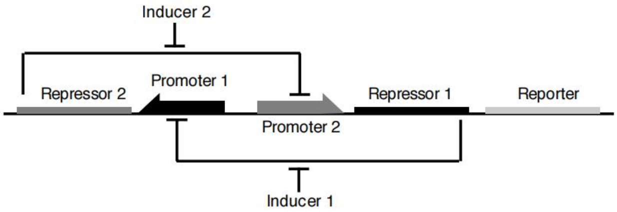
for $g = 13$, $a = 75$  $f'(5.5929) < 0$, this fix point is stable.

for $g = 16$, $a = 75$  $f'(4.4668) < 0$, this fix point is stable.

for $g = 19$, $a = 75$  $f'(3.6795) < 0$, $f'(0.1943) > 0$, $f'(0.0736) < 0$
unstable fix point: 0.1943. stable fix point: 3.6795 and 0.0736.

for $g = 22$, $a = 75$  $f'(3.0903) < 0$, $f'(0.2629) > 0$, $f'(0.0560) < 0$
unstable fix point: 0.2629. stable fix point: 3.0903 and 0.0560

Question 3



$$\frac{du}{dt} = \frac{\alpha_1}{1+v^{\beta}} - u \qquad \frac{dv}{dt} = \frac{\alpha_2}{1+u^{\gamma}} - v$$

(a)

First term: cooperative repression of constitutively transcribed promoters

Second term: degradation of the repressors (Degradation rate set as 1 here)

u: the concentration of repressor 1

v: the concentration of repressor 2

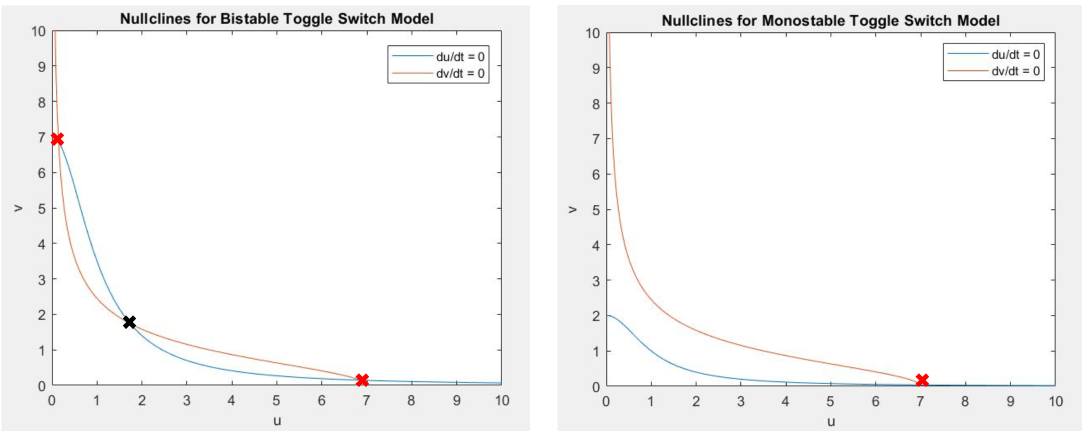
alpha1: the effective rate of synthesis of repressor 1

alpha2: the effective rate of synthesis of repressor 2

beta: Hill coefficient, the cooperativity of repression of promoter 2

gamma: Hill coefficient, the cooperativity of repression of promoter 1

(b)



✖ This black point is the unstable steady-state.

✖ This red point is the stable steady-state.

For bistable systems, there are two stable states and one unstable state.

For monostable systems, there is typically only one stable steady state.

(c)

First, the two alpha1 and alpha2 should be the same. Second, both beta and gamma should be the same and both greater than 2. The protein degradation rates should also be rough equal.

(d)

In the toggle switch system described above, there are two genes, A and B, that are mutually inhibitory. One gene creates a protein that prevents the other gene from being expressed when it is active. Due to the feedback loop this generates, each gene's activation or repression is reinforced, resulting in bistability.

The system can exist in one of two stable states, depending on the initial conditions and the parameters of the system. Gene B is active and gene A is repressed in one condition, while the opposite is true in the other state. By altering the input signal, such as by adding an inducer or deleting a repressor, the system can alternate between these states. Due to the system's bistable characteristic, once the system enters a new state, it stays there even if the input signal is turned off.

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