

# Image Understanding - Assignment #5

## Optical Flow

**Due: Friday, December 14**

For this programming assignment you will estimate dense optical flow using one of the methods we discussed in class. The approach combines the optical flow constraint at each pixel equation and a smoothness constraint across the image.

You should turn in your source code and a PDF writeup. Your writeup should include example results computed using your implementation. You should include the results of your method on the two image pairs available on the class website. The results will depend on a smoothing parameter  $\lambda$  (see below). You should experiment with different values of  $\lambda$  and comment on how they affect the results. What do you think is a good value for  $\lambda$ ?

Please turn in a ZIP file via email with the subject “ENGN1610 HOMEWORK 5”.

Let

$$I_x = \frac{\partial I}{\partial x}, \quad I_y = \frac{\partial I}{\partial y}, \quad I_t = \frac{\partial I}{\partial t}.$$

Let  $u(p)$  and  $v(p)$  denote the horizontal and vertical components of the optical flow for each pixel  $p$ . You will estimate  $u$  and  $v$  by minimizing the objective,

$$F(u, v) = \sum_{\text{pixels } p} \|I_x(p)u(p) + I_y(p)v(p) + I_t(p)\|^2 + \lambda \sum_{p, q \text{ neighbors}} (u(p) - u(q))^2 + (v(p) - v(q))^2$$

(a) Compute  $\frac{\partial F}{\partial u(p)}$ .

(b) Compute  $\frac{\partial F}{\partial v(p)}$ .

(c) Write a matlab function for computing the optical flow between two images using the system of linear equations obtained by setting the quantities in (a) and (b) equal to zero.

You should use finite differences to approximate  $I_x$ ,  $I_y$  and  $I_t$ . When computing  $I_x$  and  $I_y$  you should smooth the image first by convolving it with a gaussian with  $\sigma = 1$ .

The resulting linear system is sparse and can be solved efficiently with the Jacobi method as discussed in class. In this case we repeatedly go over the pixels and update the value for  $u(p)$  and  $v(p)$  in turn, using the linear equations  $\frac{\partial F}{\partial u(p)} = 0$  and  $\frac{\partial F}{\partial v(p)} = 0$ .

(d) Make a visualization of the resulting optical flow fields. You can display the flow at each pixel using two images, one for visualizing  $u$  and one for visualizing  $v$ . You can also visualize the flow magnitude at each pixel with a single image to combine the results.