ENGN2020 – HOMEWORK7

Problem 1

Part (a)-Euler Method

K21-1-1:

$$y' + 0.2y = 0, y(0) = 5, h = 0.2$$

Answer:

It can be solved that $y = ce^{-0.2x}$, since $y(0) = 5$, then $y = 5e^{-0.2x}$.

step	x_n	y'_n	y_n	exact values	error
0	0		5	5.0000	0.0000
1	0.2	-1	4.8	4.8039	0.0039
2	0.4	-0.96	4.608	4.6156	0.0076
3	0.6	-0.9216	4.4237	4.4346	0.0109
4	0.8	-0.8847	4.2467	4.2607	0.0140
5	1.0	-0.8493	4.0769	4.0937	0.0168
6	1.2	-0.8154	3.9138	3.9331	0.0193
7	1.4	-0.7828	3.7572	3.7789	0.0217
8	1.6	-0.7514	3.6069	3.6307	0.0238
9	1.8	-0.7214	3.4627	3.4884	0.0257
10	2.0	-0.6925	3.3242	3.3516	0.0274

K21-1-2:

$$y' = \frac{1}{2}\pi\sqrt{1 - y^2}, y(0) = 0, h = 0.1$$

Answer:

It can be solved that $y = \sin\left(c + \frac{1}{2}\pi x\right)$, since $y(0) = 0$, then $y = \sin\left(\frac{1}{2}\pi x\right)$.

step	x_n	y'_n	y_n	exact values	error
0	0		0.0000	0.0000	0.0000
1	0.1	1.5708	0.1571	0.1564	-0.0006
2	0.2	1.5513	0.3122	0.3090	-0.0032
3	0.3	1.4923	0.4614	0.4540	-0.0074
4	0.4	1.3936	0.6008	0.5878	-0.0130
5	0.5	1.2557	0.7264	0.7071	-0.0193
6	0.6	1.0796	0.8343	0.8090	-0.0253
7	0.7	0.8659	0.9209	0.8910	-0.0299
8	0.8	0.6122	0.9821	0.9511	-0.0311
9	0.9	0.2955	1.0117	0.9877	-0.0240
10	1	0.2409i	1.0117+0.0241i	1.0000	-0.0117-0.0241i

Part (b)- Improved Euler Method

K21-1-6:

$$y' = 2(1 + y^2), y(0) = 0, h = 0.05$$

Answer:

It can be solved that $y = \tan(c + 2x)$, since $y(0) = 0$, then $y = \tan(2x)$.

step	x_n	y_n	exact values	error
0	0	0	0	0
1	0.05	0.1005	0.1003	-0.0002
2	0.1	0.2030	0.2027	-0.0003
3	0.15	0.3098	0.3093	-0.0005
4	0.2	0.4234	0.4228	-0.0006
5	0.25	0.5470	0.5463	-0.0007
6	0.3	0.6849	0.6841	-0.0008
7	0.35	0.8429	0.8423	-0.0007
8	0.4	1.0299	1.0296	-0.0002
9	0.45	1.2593	1.2602	0.0009
10	0.5	1.5538	1.5574	0.0036

K21-1-7:

$$y' - xy^2 = 0, y(0) = 1, h = 0.1$$

Answer:

It can be solved that $y = \frac{-1}{0.5x^2 + c}$, since $y(0) = 1$, then $y = \frac{-1}{0.5x^2 - 1}$.

step	x_n	y_n	exact values	error
0	0	1.0000	1.0000	0.0000
1	0.1	1.0050	1.0050	0.0000
2	0.2	1.0204	1.0204	0.0001
3	0.3	1.0470	1.0471	0.0001
4	0.4	1.0868	1.0870	0.0002
5	0.5	1.1426	1.1429	0.0003
6	0.6	1.2190	1.2195	0.0005
7	0.7	1.3234	1.3245	0.0011
8	0.8	1.4684	1.4706	0.0022
9	0.9	1.6758	1.6807	0.0049
10	1	1.9881	2.0000	0.0119

Part (c)- Comparison

K21-1-9:

$$y' - xy^2 = 0, y(0) = 1, h = 0.1$$

Answer: By using Euler Method, the result is shown as below:

step	x_n	y_n	exact values	error
0	0	1.0000	1.0000	0.0000
1	0.1	1.0000	1.0050	0.0050
2	0.2	1.0100	1.0204	0.0104
3	0.3	1.0304	1.0471	0.0167
4	0.4	1.0623	1.0870	0.0247
5	0.5	1.1074	1.1429	0.0355
6	0.6	1.1687	1.2195	0.0508
7	0.7	1.2507	1.3245	0.0738

8	0.8	1.3601	1.4706	0.1104
9	0.9	1.5081	1.6807	0.1725
10	1	1.7129	2.0000	0.2871

The error is larger than the one obtained by Improved Euler Method

K21-1-10:

$$y' - xy^2 = 0, y(0) = 1, h = 0.05$$

Answer: By using Improved Euler Method with 20 steps, the result is shown as below:

step	x_n	y_n	exact values	error
0	0	1.0000	1.0000	0.0000
1	0.05	1.0013	1.0013	0.0000
2	0.1	1.0050	1.0050	0.0000
3	0.15	1.0114	1.0114	0.0000
4	0.2	1.0204	1.0204	0.0000
5	0.25	1.0322	1.0323	0.0000
6	0.3	1.0471	1.0471	0.0000
7	0.35	1.0652	1.0652	0.0000
8	0.4	1.0869	1.0870	0.0000
9	0.45	1.1126	1.1127	0.0000
10	0.5	1.1428	1.1429	0.0001
11	0.55	1.1781	1.1782	0.0001
12	0.6	1.2194	1.2195	0.0001
13	0.65	1.2677	1.2678	0.0002
14	0.7	1.3243	1.3245	0.0002
15	0.75	1.3910	1.3913	0.0003
16	0.8	1.4701	1.4706	0.0005
17	0.85	1.5648	1.5656	0.0008
18	0.9	1.6795	1.6807	0.0012
19	0.95	1.8204	1.8223	0.0019
20	1	1.9969	2.0000	0.0031

The error is much smaller than previous solution with 10 steps.