APMA 1650 Homework 02

Due: September 27, 2018

Due before class on Thursday, Sept 27, 2018. It can be dropped off in the APMA 1650 homework box on the first floor of the APMA department, 182 George St by 5pm OR at class (before it starts) on Thursday.

Please attach the HW cover sheet to *the front of your HW* assignment. It can be found on Canvas/Files.

Show all work and you MUST write up your own solutions.

1. Let A, B, and C be events such that $P(A) > P(B)$ and $P(C) > 0$.

   (a) Construct an example to demonstrate that it is possible that $P(A|C) < P(A)$.

   (b) Construct an example to demonstrate that it is possible that $P(A|C) > P(A)$.

   (c) Construct an example to demonstrate that it is possible that $P(A|C) < P(B|C)$.

2. Suppose 1 in 1000 people carry a disease. There is a diagnostic test for the disease and it has the following accuracies. If an individual carries the disease, the test correctly detects this 99% of the time. If an individual does not carry the disease, the test incorrectly reads positive with probability 5%. Suppose a person is given the test and the outcome is positive. What is the probability that he/she carries the disease? (*Does the answer surprise you?*)

3. Suppose we have identical biased two coins with the probability of Heads p and the probability of Tails $1 - p$. Let flip both. If the coins match, we record nothing and flip both coins again. If the coins do not match, we stop and record the outcome of the second. What is the probability of recording Heads?

4. Two people play a game. They take turns flipping a 'fair' coin, the first to flip H wins. A round constitutes just one flip (only one person gets to "go".)

   (a) What is the probability that the player who goes second wins?

   (b) What is the probability that the game lasts at least 4 rounds?

   (c) Given they have played 4 rounds, what is the probability that the game lasts at least 6 rounds?

   (d) Suppose that the coin is biased with the probability of Heads equal to p. Suppose the game lasts exactly 4 rounds. What value of p maximizes the chances of the game lasting exactly 4 rounds? For k rounds?

5. Suppose you only have one dollar, $1. One day, on the Thayer street, a guy suggests you play a game. The game is very simple. First, you pay $$x$ and toss a coin. If the coin shows a head, you got $$2x$. If the coin shows a tail, you got $$0.5x$. The probability of the coin showing a head is p and p is unknown to you. At each game, you have to pay all money you got.

(a) Suppose you play this game 3 times. Find the probability distribution for the total amount of money you have.

(b) What is the expected amount of money after the k-th game play?

6. Let X be a discrete random variable that assigns positive probabilities to only the positive integers, $\{1, 2, 3, \cdots\}$. Show that

$$E(X) = \sum_{k=1}^{\infty} P(X \geq k). \tag{1}$$