APMA 1650 Homework 6 Common Mistakes

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1. (a) No common mistakes

   (b) To find the marginal of Y, we have to consider two cases. This is because the bounds of integration in dx change for $0 \leq y < 1$ vs $1 \leq y \leq 2$. If you don't see this, draw the picture!

   (c) No common mistakes.

   (d) There are no cases based on the values of x here, but many students tried to solve the problem this way. To see this, draw the picture and notice that $x \leq y \leq 2 - x$ are the bounds on y everywhere.

   (e) The bounds for $f_{Y|X=x}$ are from $y = x$ to $y - 2 - x$, so the integral is over $\int_{0.6}^{1.1}$, not $\int_{0}^{1.1}$.

2. The most common mistake was getting the bounds of integration wrong. Always draw the region! Also, make sure you remember the formula for conditional probability: $P(A|B) = \frac{P(A \cap B)}{P(B)}$.

3. Remember that the integral gives the area under the curve, not above the curve. Thus, to find the area above the curve, we need to subtract it from the total area of the rectangle it's in, i.e.

$$\text{Area of the shaded region } = 2 - \int_{-1}^{1} x^2 dx.$$

   Students who tried to solve this problem using the joint density often had the wrong bounds. Again, draw the region!

4. No common mistakes.

5. A lot of people had trouble setting up the random variables needed to apply linearity of expectations. We want to count the number of runs, but the length of the runs can vary, so it's easiest to count where the runs start or end. A run after the first position starts with $0, 1$, so an appropriate random variable to record the start of the runs would be

$$X_i = \begin{cases} 1, & \text{if digit } i - 1 = 0 \text{ and digit } i = 1 \\ 0, & \text{otherwise} \end{cases}$$

for $i = 2, \ldots, n+m$. We have not yet considered the case when the sequence starts with a 1. For this, we define a new random variable Y by
$Y = \begin{cases} 1, & \text{if the first digit is 1} \\ 0, & \text{otherwise} \end{cases}$

Then, we have covered all cases of runs of 1's without double counting, so the total number of runs is $Y + \sum_{i=2}^{n+m} X_i$. A similar approach to count where the runs end is presented in the solutions.

Note that counting the number of times we have $0, 1$ **or** $1, 0$ is not a valid approach. Consider the sequence $0, 1, 0$. This method of counting would say we have 2 runs, when in fact we only have 1.

6. A number of students forgot to define their random variables. Tell the reader that X is the total number of people, Y is the number of men, and Z is the number of women who enter CVS, so that your proof is easy to follow.

   Most students lost points for not justifying why the joint pmf of the number of women and men conditioned on the total number of people is

$$P(Y = y, Z = z | Y + Z = y + z) = \binom{y+z}{y} p^y (1-p)^z.$$

   The other common mistake was forgetting to mention why Y and Z are independent.