ENGN2020 – HOMEWORK2

Problem 3

(1) Answer:

Let matrix **A** = $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{bmatrix}$, vector b = $\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}$

a) $\boldsymbol{E_1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$\boldsymbol{E_1} \times \boldsymbol{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{31} & a_{32} & a_{33} \\ a_{21} & a_{22} & a_{23} \\ a_{41} & a_{42} & a_{43} \end{bmatrix}$, which is **A** after changing row 2 and row 3,

$\boldsymbol{E_1} \times b = \begin{bmatrix} a_1 \\ a_3 \\ a_2 \\ a_4 \end{bmatrix}$, which is b after changing row 2 and row 3.

b) $\boldsymbol{E_2} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -5 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$\boldsymbol{E_2} \times \boldsymbol{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} - 5a_{11} & a_{32} - 5a_{12} & a_{33} - 5a_{13} \\ a_{41} & a_{42} & a_{43} \end{bmatrix}$, which is A after the third row subtracting 5 times the first row.

$\boldsymbol{E_2} \times b = \begin{bmatrix} a_1 \\ a_2 \\ a_3 - 5a_1 \\ a_4 \end{bmatrix}$, which is b after the third row subtracting 5 times the first row.

c) $\boldsymbol{E_3} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 8 \end{bmatrix}$

$\boldsymbol{E_3} \times \boldsymbol{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ 8a_{41} & 8a_{42} & 8a_{43} \end{bmatrix}$, which is A after multiplying the fourth row by 8.

$\boldsymbol{E_3} \times b = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ 8a_4 \end{bmatrix}$, which is b after multiplying the fourth row by 8.

Let **A** be a 4 X 2 matrix $\begin{bmatrix} a_{11} & a_{21} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{bmatrix}$,

then **B** = $\boldsymbol{E_3 E_2 E_1 A} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 8 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -5 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} a_{11} & a_{21} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -5 & 1 & 0 & 0 \\ 0 & 0 & 0 & 8 \end{bmatrix} \times \begin{bmatrix} a_{11} & a_{21} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{bmatrix}$

$= \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{31} & a_{32} & a_{33} \\ a_{21} - 5a_{11} & a_{22} - 5a_{12} & a_{23} - 5a_{13} \\ 8a_{41} & 8a_{42} & 8a_{43} \end{bmatrix}$

then C $=\boldsymbol{E_1 E_2 E_3 A} = \begin{bmatrix}1&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&1\end{bmatrix} \times \begin{bmatrix}1&0&0&0\\0&1&0&0\\-5&0&1&0\\0&0&0&1\end{bmatrix} \times \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&8\end{bmatrix} \times \begin{bmatrix}a_{11}&a_{21}\\a_{21}&a_{22}\\a_{31}&a_{32}\\a_{41}&a_{42}\end{bmatrix} = \begin{bmatrix}1&0&0&0\\-5&0&1&0\\0&1&0&0\\0&0&0&8\end{bmatrix} \times \begin{bmatrix}a_{11}&a_{21}\\a_{21}&a_{22}\\a_{31}&a_{32}\\a_{41}&a_{42}\end{bmatrix}$

$= \begin{bmatrix}a_{11}&a_{12}&a_{13}\\a_{31}-5a_{11}&a_{32}-5a_{12}&a_{33}-5a_{13}\\a_{21}&a_{22}&a_{23}\\8a_{41}&8a_{42}&8a_{43}\end{bmatrix}$

Therefore, **B** doesn`t equal to **C**

(2) Answer:

Proof: Since $\mathbf{A} = \mathbf{I} \times \mathbf{A}$, do the same row operation on both sides, left side still equals to the right side.
M is obtained from **A** by an elementary row operation while right side equals to **E**×**A**,
therefore: **M** = **E**×**A**

Problem 4

(1) Chapter7-section4-14:

If A is not square, either the row vectors or the column vectors of A are linearly dependent.

Proof: Since **A** is not square, let **A** is a $m \times n$ matrix, $m \neq n$.

THEOREM 4: Consider p vectors each having n components. If n<p, **then these vectors** *are linearly dependent.*

If $m>n$, there are n column vectors, each vector has m components, according to Theorem 4, column vectors are dependent.

If $m<n$, there are m row vectors, each vector has n components, those row vectors are dependent.

Therefore, either the row vectors or the column vectors are linearly dependent.

(2) Chapter7-section4-15:

If the row vectors of a square matrix are linearly independent, so are the column vectors, and conversely.

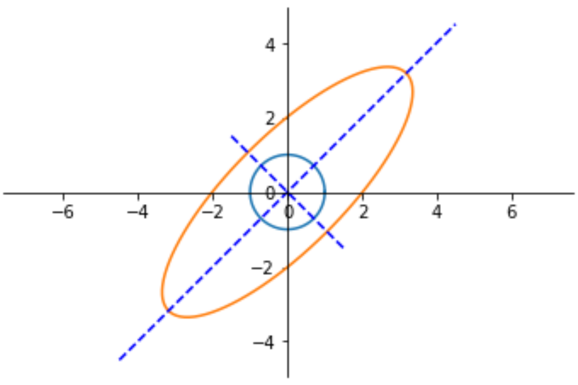
Proof: Let matrix **A** be a square $n \times n$ matrix. If the row vectors are linearly independent, as the definition of the rank of a matrix, rank **A** = n.

Assume: column vectors are not linearly independent, then the rank of matrix **A** has to be smaller than n, which is controversial with rank **A**= n, therefore the column vectors are linearly independent.

Problem 5

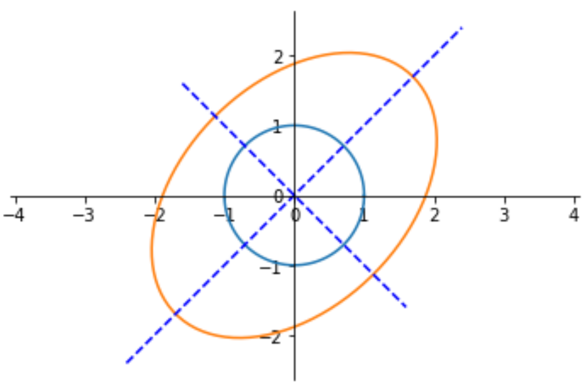
(1) Chapter8-section2-1:

A = $\begin{bmatrix}3.0&1.5\\1.5&3.0\end{bmatrix}$



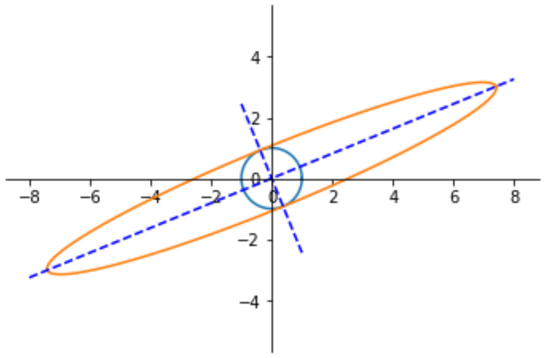
(2) Chapter8-section2-2:

A = $\begin{bmatrix}2.0&0.4\\0.4&2.0\end{bmatrix}$



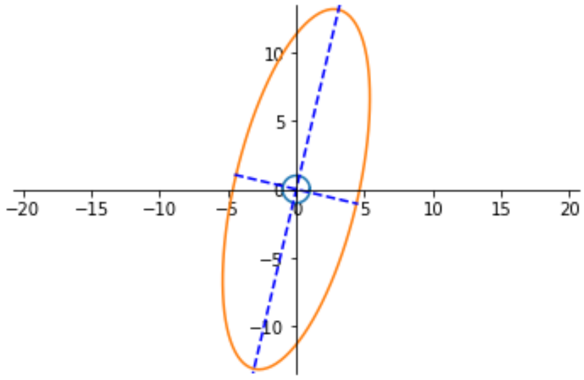
(3) Chapter8-section2-3:

A = $\begin{bmatrix} 7.0 & \sqrt{6} \\ \sqrt{6} & 2.0 \end{bmatrix}$



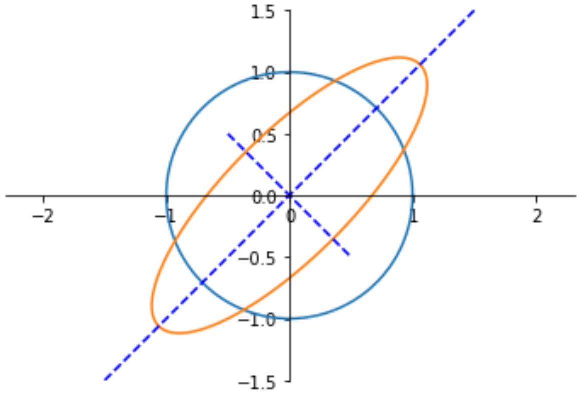
(4) Chapter8-section2-4:

A = $\begin{bmatrix} 5 & 2 \\ 2 & 13 \end{bmatrix}$



(5) Chapter8-section2-5:

A = $\begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}$



(6) Chapter8-section2-6:

A = $\begin{bmatrix} 1.25 & 0.75 \\ 0.75 & 1.25 \end{bmatrix}$