

# APMA 1650 Homework 01

Due: September 20, 2018

Due before class on Thursday, Sept 20, 2018. It can be dropped off in the APMA 1650 homework box on the first floor of the APMA department, 182 George St by 5pm OR at class (before it starts) on Thursday.

**Please attach the HW cover sheet** to the front of your HW assignment. It can be found on Canvas/Files.

**Show all work and you MUST write up your own solutions.**

1. (a) Show *distributive laws* without drawing Venn diagrams.

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C), \quad A \cup (B \cap C) = (A \cup B) \cap (A \cup C).$$

- (b) Assuming the above distributive laws, prove that

$$A \cap \bigcup_{i=1}^k B_i = \bigcup_{i=1}^k A \cap B_i.$$

Hint: Apply mathematical induction.

2. We have a die such that the probability of the die showing a number is proportional to the number itself. For example, showing a six is two times as likely as showing a three.
- (a) When the die is tossed once, what is the probability we pick a odd number?
- (b) When the die is tossed two times, what is the probability that the sum of the two is more than 9?
- (c) When the die is tossed two times, what is the probability that the sum of the two is a prime number? (Note that 2, 3, 5, 7, 11 are prime numbers).
3. (a) Let toss a fair coin 20 times. What is the probability of flipping the first Head after 9th toss?
- (b) Let toss a fair coin 20 times. What is the probability that you do not flip two Heads nor two Tails in a row?
- (c) Let toss a fair coin 5 times. What is the probability of flipping two or more Heads in a row?
4. Assume there are 365 distinct possible birthdays. Suppose we ask the birthday of  $n$  students. Assume the probability of having any given birthday is equally likely.
- (a) What is the probability that there are no two students who have the same birth date?

- (b) What is the probability that at least two students share a birthday?
5. A bag contains 100 balls of 75 white balls and 25 black balls. At each time, we randomly pick a ball from the bag and check the color and throw the ball away. Let say we repeat this  $n$  times.
- (a) What is the probability that the number of observed black balls among  $n = 10$  trials is  $k = 4$ ?
- (b) What is the probability that the number of observed black balls among  $n$  trials is  $k$ ?
6. The probabilities are 0.4, 0.2, 0.3, and 0.1, respectively, that a delegate to a certain convention arrived by air, bus, automobile, or train. What is the probability that among 9 delegates randomly selected at this convention, 3 arrived by air, 3 arrived by bus, 1 arrived by automobile, and 2 arrived by train?