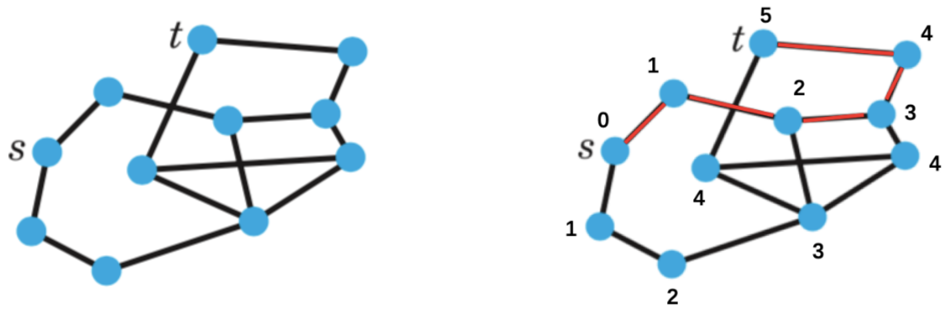
ENGN2020 – HOMEWORK6

Problem 1

(a) K23-2-1:

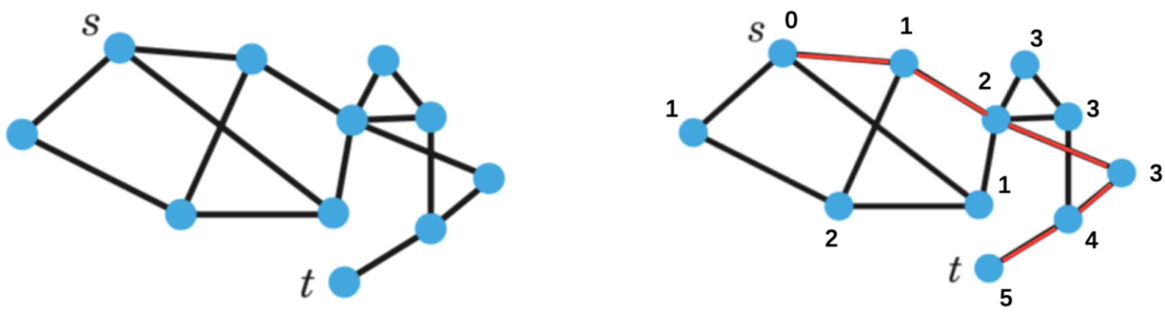
The given graph is shown as below:



(a) **(b)**

Fig 1. (a) The given graph. **(b)** The shortest path from ***s*** to ***t***

(b) K23-2-2:



(a) **(b)**

Fig 2. (a) The given graph. **(b)** The shortest path from ***s*** to ***t***

Problem 2

(a) K23-2-13:

The postman problem is shown as below:

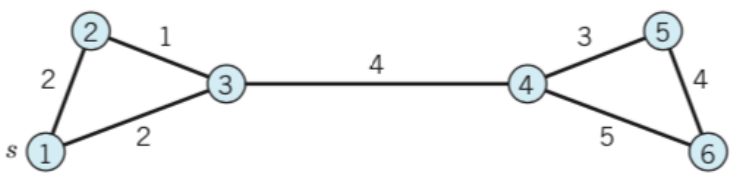


Fig 3. The given graph

The path should be: 1->2->3->4->5->6->4->3->1, the total length of the path is 26.

(b) K23-2-13 with modification:

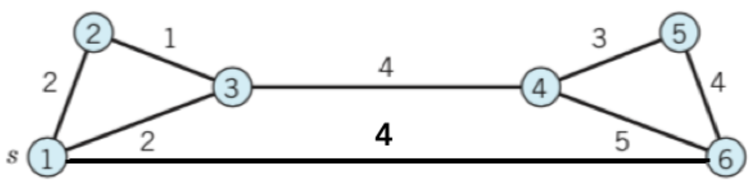


Fig 4. The given graph

The path should be :1->2->3->4->5->6->1, the total length of the path is 18.

Problem 3

(b) get_new_neighbors method:

```
def get_new_neighbors(self, neighbors):
    """Compares the vertex names in the list "neighbors"
    to the neighbors already contained in steptable.
    Returns a (shorter) list of names of neighbors that have
    not yet been discovered.
    """
    #loop all items in the neighbors
    for i in neighbors:
        #loop all vertices that in the step table
        for item in self.data:
            #if already exits in the step talbe, remove it
            if item[0].name == i:
                neighbors.remove(i)
                break
    return neighbors
```

(c) get_reverse_path method:

```
def get_reverse_path(self, vertex_name):
    """Starting at the vertex named "vertex_name", traces
    backwards through the step table to find the shortest
    distance to the origin. Note that this should only be
    called*after*vertex_name has been discovered.
    """
    #initial list with given vertex name
    result = [vertex_name]
    #get length of the recorded data
    length = len(self.data)
    #set the current vertex name
    current = vertex_name
    #loop from step table
    for i in range(length):
        for vertex in self.data:
            #find the current vertex in table
            if current in vertex[2]:
                result.insert(0,vertex[0].name)
                current = vertex[0].name

    return result
```

(d) Moore’ s algorithm

The function to get the shortest path from a given vertex to another given vertex is shown as below, please find the complete code in Appendix.

'''

```
* @name: findPath
* @description: use Moore`s algorithm to find the shortest path from start to end vertices
* @param start: the name of the start vertex
* @param end: the name of the end vertex
* @return: list, the shortest path from start to end vertices
'''
def findPath(start,end):
    #declare the queue
    bfs = Queue();
    #declare the step table
    record = StepTable();
    #create the start vertex
    start = Vertex(start);
    #append this start point into queue
    bfs.append(start,0)
    #save the this start point into step table
    record.append(start,0);

    #use Moore`s algorithm or so called bfs
    while(bfs.queue!=[]):
        #get the front the queue
        nextNode = bfs.next()
        #get the distance of this point
        currentDistance = nextNode["distance"]
        #get the neighbors of this point
        neighbors = nextNode["vertex"].get_neighbors();

        #loop all neighbors
        for item in neighbors:
            #use a bool value to see if the vertex has been visited
            visited = False
            #loop all records in the step table to see if visited
            for step in record.data:
                if step[0].name == item:
                    visited = True
                    break
            #if not visited
            if(not visited):
                #create the vertex
                temp = Vertex(item);
                #push it into queue
                bfs.append(temp,currentDistance+1);
                #record this step in step table
                record.append(temp,currentDistance+1);

    #record.print()
    #get the path from start to end
    path = record.get_reverse_path(end);
```

```
#print path
print(path)
```

The test result of this function to get the shortest path from “B” to “F” is:
['B', 'E', 'A', 'D', 'F']
The test result of this function to get the shortest path from “B” to “J” is:
['B', 'E', 'C', 'I', 'J']

(e) shortest distance between two websites

Please find the complete code in Appendix.

The test result of this function to get the shortest path from “'https://www.brown.edu/academics/engineering/” to “https://www.brown.edu/academics/engineering/about” is:
['https://www.brown.edu/academics/engineering/', 'https://www.brown.edu/academics/engineering/about']

The test result of this function to get the shortest path from “'https://www.brown.edu/academics/engineering/” to “https://www.brown.edu/academics/engineering/graduate-study/masters-and-phd-programs” is:
['https://www.brown.edu/academics/engineering/',
'https://www.brown.edu/academics/engineering/graduate-study'
'https://www.brown.edu/academics/engineering/graduate-study/masters-and-phd-programs']

Problem 4

(a) K23-3-4:

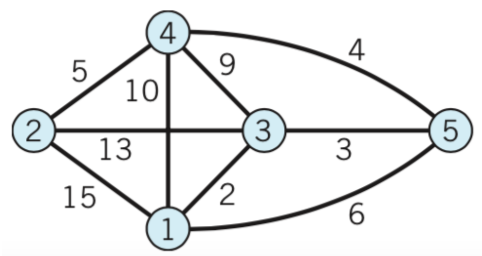


Fig 5. The given graph

Dijkstra’ s Algorithm result is shown as below:

Table 1. Dijkstra’ s Algorithm result

Step	1	2	3	4	5
0	0	“15”	“2”	“10”	“6”
1	0	“15”	2	“10”	“5”
2	0	“15”	2	“9”	5
3	0	“14”	2	9	5
4	0	14	2	9	5

(b) K23-3-5:

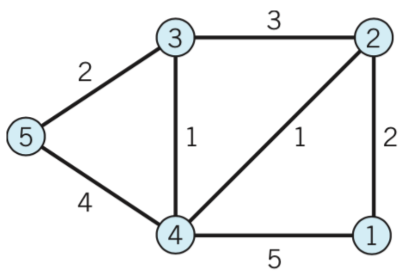


Fig 6. The given graph

Dijkstra’ s Algorithm result is shown as below:

Table 2. Dijkstra’ s Algorithm result

Step	1	2	3	4	5
0	0	“2”	“∞”	“5”	“∞”
1	0	2	“5”	“3”	“∞”
2	0	2	“4”	3	“7”
3	0	2	4	3	“6”
4	0	2	4	3	6

Problem 5

(a) K23-4-4

(a) **(b)**

Fig 7. (a) The given graph. **(b)** The shortest spanning tree by Kruskal’ s algorithm

(b) K23-4-5

(a) **(b)**

Fig 8. (a) The given graph. **(b)** The shortest spanning tree by Kruskal’ s algorithm

Problem 6

(a) Implementation of Prim Algorithm

```
import numpy as np
'''
* @name: Prim
```

```
* @description: the class to implement Prim`s algorithm for shortest spanning tree
* @param adjMatrix: the given graph stored in adjacency matrix
'''
class Prim:
    '''
    * @name: __init__
    * @description: constructor of the class
    * @param adjMatrix: the given graph stored in adjacency matrix
    '''
    def __init__(self,adjMatrix):
        #the input value is a adjacency matrix
        self.graph = adjMatrix

    '''
    * @name: nextVertex
    * @description: decide which is the next vertex to add to the tree
    * @param U: the vertices that already in the spanning tree
    * @param visited: list that save whether the vertex is used
    * @return: list, the new vertex added to the tree and the parent vertex of the new vertex
    '''
    def nextVertex(self,U,visited):
        #get the number of vertices
        vertexNum = self.graph.shape[0]

        #initial the values
        minValue = 10000
        minIndex = 0
        parent = 0

        #loop all unvisited vertices
        for i in range(vertexNum):
            if not visited[i]:
                #loop all nodes that already in the tree
                for j in U:
                    #find the nearest distance to the vertices in the tree
                    if self.graph[i][j]< minValue and self.graph[i][j]!=0:
                        minValue = self.graph[i][j]
                        minIndex = i
                        parent = j
        #return the list of the new vertex added to the tree and the parent vertex of the new vertex
        return [minIndex,parent]

    '''
    * @name: prim
    * @description: use Prim`s algorithm to create the shortest spanning tree
    * @param U: the vertices that already in the spanning tree
    * @param visited: list that save whether the vertex is used
    '''
```

```
def prim(self):
    #get the number of the vertices in the graph
    vertexNum = self.graph.shape[0]

    #set the all vertices by unvisited
    visited = [False]*vertexNum
    #start with vertex 0
    visited[0] = True
    U =[0]
    #loop all vertices
    for i in range(vertexNum):
        #call the member function to find next vertex
        nextStep = self.nextVertex(U,visited)
        #add it in U
        U.append(nextStep[0]);
        #set it as visited
        visited[nextStep[0]] = True;
        #print it
        if nextStep[0]!=nextStep[1]:
            print("Parent: "+str(nextStep[1]+1)+"      next: "+str(nextStep[0]+1))
```

(b) K23-5-6

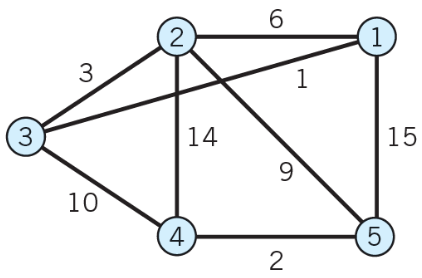


Fig 9. The given graph

The adjacency matrix of the given graph is:

$$\begin{bmatrix} 0 & 6 & 1 & 0 & 15 \\ 6 & 0 & 3 & 14 & 9 \\ 1 & 3 & 0 & 0 & 0 \\ 0 & 14 & 0 & 0 & 2 \\ 15 & 9 & 0 & 2 & 0 \end{bmatrix}$$

Use the following code to call the class defined in previous section:

```
A = np.array([[0, 6, 1, 0, 15],
              [6, 0, 3, 14, 9],
              [1, 3, 0, 10, 0],
              [0, 14, 0, 0, 2],
              [15, 9, 0, 2, 0]])
a = Prim(A);
a.prim();
```

The output is:

```
Parent: 1     next: 3
Parent: 3     next: 2
Parent: 2     next: 5
```

Parent: 5	next: 4

Based on the calculated result, the shortest spanning tree is show as below:

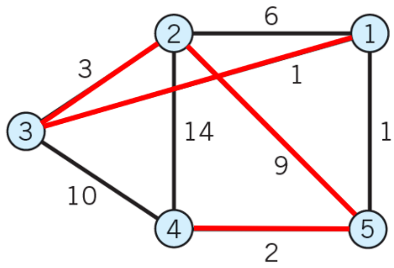


Fig 10. The shortest spanning tree by Prim`s algorithm

(c) K23-5-7

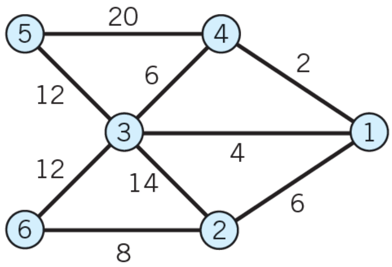


Fig 11. The given graph

The adjacency matrix of the given graph is:

$$\begin{bmatrix} 0 & 6 & 4 & 2 & 0 & 0 \\ 6 & 0 & 14 & 0 & 0 & 8 \\ 4 & 14 & 0 & 6 & 12 & 12 \\ 2 & 0 & 6 & 0 & 20 & 0 \\ 0 & 0 & 12 & 20 & 0 & 0 \\ 0 & 8 & 12 & 0 & 0 & 0 \end{bmatrix}$$

Use the following code to call the class defined in previous section:

```
A = np.array([[0, 6, 4, 2, 0, 0],
              [6, 0, 14, 0, 0, 8],
              [4, 14, 0, 6, 12,12],
              [2, 0, 6, 0, 20, 0],
              [0, 0, 12, 20, 0, 0],
              [0, 8, 12, 0, 0, 0]])
a = Prim(A);
a.prim();
```

The output is:

Parent: 1	next: 4

Parent: 1	next: 3

Parent: 1	next: 2

Parent: 2	next: 6

Parent: 3	next: 5

Based on the calculated result, the shortest spanning tree is show as below:

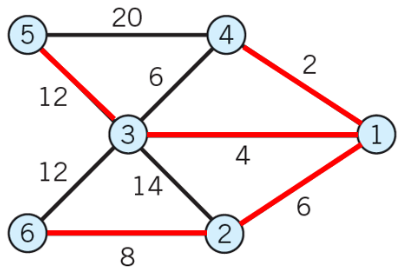


Fig 12. The shortest spanning tree by Prim`s algorithm

(d) K23-5-8

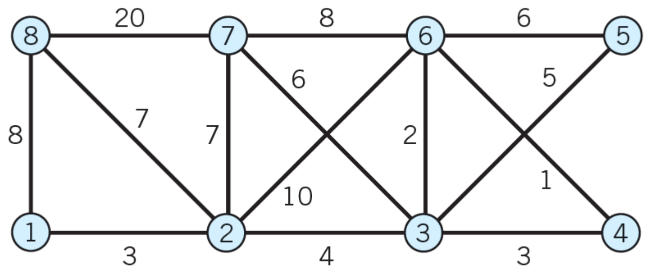


Fig 13. The given graph

The adjacency matrix of the given graph is:

$$\begin{bmatrix} 0 & 3 & 0 & 0 & 0 & 0 & 0 & 8 \\ 3 & 0 & 4 & 0 & 0 & 10 & 7 & 7 \\ 0 & 4 & 0 & 3 & 5 & 2 & 6 & 0 \\ 0 & 0 & 3 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 & 6 & 0 & 0 \\ 0 & 10 & 2 & 1 & 6 & 0 & 8 & 0 \\ 0 & 7 & 6 & 0 & 0 & 8 & 0 & 20 \\ 8 & 7 & 0 & 0 & 0 & 0 & 20 & 0 \end{bmatrix}$$

Use the following code to call the class defined in previous section:

```
A = np.array([[0, 3, 0, 0, 0, 0, 0, 8],
              [3, 0, 4, 0, 0, 10, 7, 7],
              [0, 4, 0, 3, 5, 2, 6, 0],
              [0, 0, 3, 0, 0, 1, 0, 0],
              [0, 0, 5, 0, 0, 6, 0, 0],
              [0, 10, 2, 1, 6, 0, 8, 0],
              [0, 7, 6, 0, 0, 8, 0, 20],
              [8, 7, 0, 0, 0, 0, 20, 0]])
a = Prim(A);
a.prim();
```

The output is:

```
Parent: 1    next: 2
Parent: 2    next: 3
Parent: 3    next: 6
Parent: 6    next: 4
Parent: 3    next: 5
Parent: 3    next: 7
Parent: 2    next: 8
```

Based on the calculated result, the shortest spanning tree is show as below:

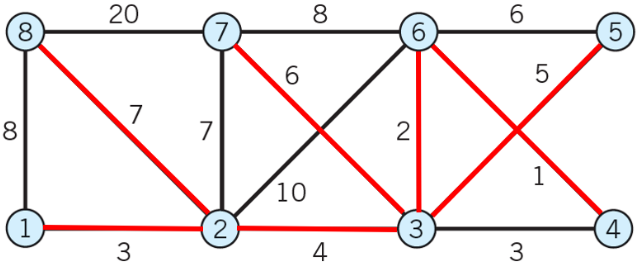


Fig 14. The shortest spanning tree by Prim`s algorithm

(e) K23-5-9

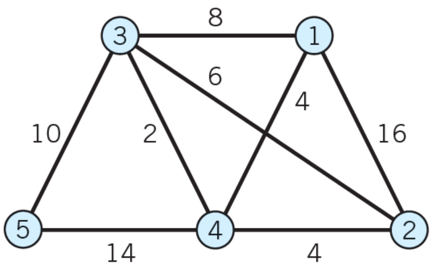


Fig 15. The given graph

The adjacency matrix of the given graph is:

$$\begin{bmatrix} 0 & 16 & 8 & 4 & 0 \\ 16 & 0 & 6 & 4 & 0 \\ 8 & 6 & 0 & 2 & 10 \\ 4 & 4 & 2 & 0 & 14 \\ 0 & 0 & 10 & 14 & 0 \end{bmatrix}$$

Use the following code to call the class defined in previous section:

```
A = np.array([[0, 3, 0, 0, 0, 0, 0, 8],
              [3, 0, 4, 0, 0, 10, 7, 7],
              [0, 4, 0, 3, 5, 2, 6, 0],
              [0, 0, 3, 0, 0, 1, 0, 0],
              [0, 0, 5, 0, 0, 6, 0, 0],
              [0, 10, 2, 1, 6, 0, 8, 0],
              [0, 7, 6, 0, 0, 8, 0, 20],
              [8, 7, 0, 0, 0, 0, 20, 0]])
a = Prim(A);
a.prim();
```

The output is:

Parent: 1 next: 4
Parent: 4 next: 3
Parent: 4 next: 2
Parent: 3 next: 5

Based on the calculated result, the shortest spanning tree is show as below:

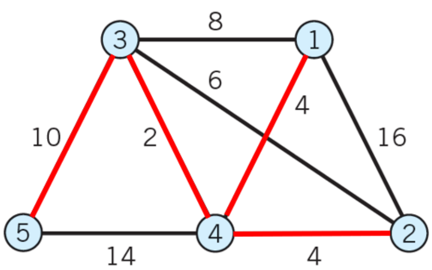


Fig 16. The shortest spanning tree by Prim`s algorithm

Appendix

1. Code of Problem 3(b)

```
class Vertex:
    def __init__(self, name):
        self.name = name
    def get_neighbors(self):
        return Vertex.links[self.name];
    #save the link in the vertex class
    links = {
    'A': ['B', 'D', 'G'],
    'B': ['E', 'G', 'H'],
    'C': ['A', 'H', 'I'],
    'D': ['F'],
    'E': ['H', 'A', 'C'],
    'F': ['G', 'I'],
    'G': ['C'],
    'H': ['A', 'E'],
    'I': ['C', 'J'],
    'J': []
    }

class Queue:
    def __init__(self):
        self.queue = []

    def next(self):
        if(len(self.queue)!=0):
            result =    self.queue[0]
            self.queue.remove(result);
            return result;

    def append(self,vertex,distance):
        result = {"distance":distance,"vertex":vertex}
        self.queue.append(result)

class StepTable:
    """Container to remember the steps taken by Moore's algorithm.
    "data" is a list of steps, where each step contains the vertex
    object, the distance from the origin, and the new neighbors
    encountered of that vertex. E.g.,

    data = [ (<vertex object>, 0, ['B', 'D']),
            (<vertex object>, 1, ['C']),
            (<vertex object>, 1, ['F']),
            ...]
    """
```

```
def __init__(self, data=None):
    if data is None:
        self.data = []
    else:
        self.data = data

def append(self, vertex, distance):
    """Adds the given vertex object to the step table.
    Distance is the distance from the origin."""
    neighbors = vertex.get_neighbors()
    new_neighbors = self.get_new_neighbors(neighbors)
    self.data.append((vertex, distance, new_neighbors))

def get_new_neighbors(self, neighbors):
    """Compares the vertex names in the list "neighbors"
    to the neighbors already contained in steptable.
    Returns a (shorter) list of names of neighbors that have
    not yet been discovered.
    """
    #loop all items in the neighbors
    for i in neighbors:
        #loop all vertices that in the step table
        for item in self.data:
            #if already exits in the step talbe, remove it
            if item[0].name == i:
                neighbors.remove(i)
                break
    return neighbors

def get_reverse_path(self, vertex_name):
    """Starting at the vertex named "vertex_name", traces
    backwards through the step table to find the shortest
    distance to the origin. Note that this should only be
    called*after*vertex_name has been discovered.
    """
    #initial list with given vertex name
    result = [vertex_name]

    #get length of the recorded data
    length = len(self.data)

    #set the current vertex name
    current = vertex_name

    #loop from step table reversely
    for i in range(length):
        for vertex in self.data:
            #find the current vertex in table
```

```
                    if current in vertex[2]:
                        result.insert(0,vertex[0].name)
                        current = vertex[0].name

            return result

        def print(self):
            """Attempts to pretty print the contents of the
            step table."""
            for row in self.data:
                print('{:10s} {:3d} {:s}'.format(row[0].name, row[1], str(row[2])))
'''
* @name: findPath
* @description: use Moore`s algorithm to find the shortest path from start to end vertices
* @param start: the name of the start vertex
* @param end: the name of the end vertex
* @return: list, the shortest path from start to end vertices
'''
def findPath(start,end):
    #declare the queue
    bfs = Queue();
    #declare the step table
    record = StepTable();
    #create the start vertex
    start = Vertex(start);
    #append this start point into queue
    bfs.append(start,0)
    #save the this start point into step table
    record.append(start,0);

    #use Moore`s algorithm or so called bfs
    while(bfs.queue!=[]):
        #get the front the queue
        nextNode = bfs.next()
        #get the distance of this point
        currentDistance = nextNode["distance"]
        #get the neighbors of this point
        neighbors = nextNode["vertex"].get_neighbors();

        #loop all neighbors
        for item in neighbors:
            #use a bool value to see if the vertex has been visited
            visited = False

            #loop all records in the step table to see if visited
            for step in record.data:
                if step[0].name == item:
                    visited = True
```

```
                break
            #if not visited
            if(not visited):
                #create the vertex
                temp = Vertex(item);
                #push it into queue
                bfs.append(temp,currentDistance+1);
                #record this step in step table
                record.append(temp,currentDistance+1);

    #record.print()
    #get the path from start to end
    path = record.get_reverse_path(end);
    #print path
    print(path)

    return path



findPath('B','F')
```

2. Code of Problem 3(e)

```
import requests
from lxml import html

class Vertex:
    def __init__(self, name):
        self.name = name

    def get_neighbors(self):
        #load the page
        page = requests.get(self.name)
        #get the tree structure of the urls
        tree = html.fromstring(page.content)
        #get all links
        links = tree.xpath('//a/@href')

        result = []
        #save all links related to engineering school
        for i in links:
            if i.startswith('https://www.brown.edu/academics/engineering/'):
                    result.append(str(i))
            if i.startswith('/academics/engineering'):
                result.append('https://www.brown.edu'+i)

        return result
```

```
class Queue:
    def __init__(self):
        self.queue = []

    def next(self):
        if(len(self.queue)!=0):
            result =   self.queue[0]
            self.queue.remove(result);
            return result;

    def append(self,vertex,distance):
        result = {"distance":distance,"vertex":vertex}
        self.queue.append(result)


class StepTable:
    """Container to remember the steps taken by Moore's algorithm.
    "data" is a list of steps, where each step contains the vertex
    object, the distance from the origin, and the new neighbors
    encountered of that vertex. E.g.,

    data = [ (<vertex object>, 0, ['B', 'D']),
            (<vertex object>, 1, ['C']),
            (<vertex object>, 1, ['F']),
            ...]
    """

    def __init__(self, data=None):
        if data is None:
            self.data = []
        else:
            self.data = data

    def append(self, vertex, distance):
        """Adds the given vertex object to the step table.
        Distance is the distance from the origin."""
        neighbors = list(set(vertex.get_neighbors()))
        new_neighbors = self.get_new_neighbors(neighbors)
        self.data.append((vertex, distance, new_neighbors))

    def get_new_neighbors(self, neighbors):
        """Compares the vertex names in the list "neighbors"
        to the neighbors already contained in steptable.
        Returns a (shorter) list of names of neighbors that have
        not yet been discovered.
        """
        #loop all items in the neighbors
        for i in neighbors:
```

```
        #loop all vertices that in the step table
        for item in self.data:
            #if already exits in the step talbe, remove it
            visited = False
            if item[0].name == i:
                neighbors.remove(i)
                visited = True
                break
            for neighbor in item[2]:
                if(i == neighbor):
                    neighbors.remove(i)
                    visited = True
                    break
            if(visited):
                break;

    return neighbors

def get_reverse_path(self, vertex_name):
    """Starting at the vertex named "vertex_name", traces
    backwards through the step table to find the shortest
    distance to the origin. Note that this should only be
    called*after*vertex_name has been discovered.
    """
    #initial list with given vertex name
    result = [vertex_name]

    #get length of the recorded data
    length = len(self.data)

    #set the current vertex name
    current = vertex_name

    start = self.data[0][0].name

    #loop from step table reversely
    for i in range(length):
        for vertex in self.data:
            #find the current vertex in table
            if start == current:
                return result

            if current in vertex[2]:
                result.insert(0,vertex[0].name)
                current = vertex[0].name

    return result
```

```
    def print(self):
        """Attempts to pretty print the contents of the
        step table."""
        for row in self.data:
            print('{:10s} {:3d} {:s}'.format(row[0].name, row[1], str(row[2])))



'''
* @name: findPath
* @description: use Moore`s algorithm to find the shortest path from start to end vertices
* @param start: the name of the start vertex
* @param end: the name of the end vertex
* @return: list, the shortest path from start to end vertices
'''
def findPath(start,end):
    #declare the queue
    bfs = Queue();
    #declare the step table
    record = StepTable();
    #create the start vertex
    start = Vertex(start);
    #append this start point into queue
    bfs.append(start,0)
    #save the this start point into step table
    record.append(start,0);

    #use Moore`s algorithm or so called bfs
    while(bfs.queue!=[]):
        #get the front the queue
        nextNode = bfs.next()

        currentName = nextNode["vertex"].name
        #if find the target then break the while loop
        if currentName==end:
            break
        #get the distance of this point
        currentDistance = nextNode["distance"]
        #get the neighbors of this point
        neighbors = list(set(nextNode["vertex"].get_neighbors()));

        #loop all neighbors
        for item in neighbors:
            #use a bool value to see if the vertex has been visited
            visited = False

            #loop all records in the step table to see if visited
            for step in record.data:
```

```
            if step[0].name == item:
                visited = True
                break
        #if not visited
        if(not visited):
            #create the vertex
            temp = Vertex(item);
            #push it into queue
            bfs.append(temp,currentDistance+1);
            #record this step in step table
            record.append(temp,currentDistance+1);

    #record.print()
    #get the path from start to end
    path = record.get_reverse_path(end);
    #print path
    print(path)

    return path

findPath('https://www.brown.edu/academics/engineering/',\
        'https://www.brown.edu/academics/engineering/graduate-study/masters-and-phd-programs')
```