

Homework 3

Problem1:

- (a) From the given conditions, we know that:

$$f'(0) = 10$$

$$f'(1) = 20$$

$$f'(-1) = -20$$

.

Therefore, one example that satisfies these conditions is $f'(x) = 10x^2 + 20x - 10$.

$$f(x) = \frac{10}{3}x^3 + 10x^2 - 10x$$

- (b) We can assume the $f'(x)$ between 0 and 1 is extremely small. In this case, the function can be $f(x) = 0.000001x^2$
- (c) In each step, record the value of $f(x)$. If the current $f(x)$ is smaller than previous value of $f(x)$, double the step size as the new step size. Otherwise, take the half of current step size as new step size. For a convex function, the algorithm will finally find the minimum.
- (d) Use Taylor series to expand $f(x)$. Then:

$$f(x_{min}) = f(x) + f'(x) * (x - x_{min}) + \frac{f''(x)(x - x_{min})^2}{2} + \dots$$

Since $|x - x_{min}| < \epsilon$ and ϵ is a very small number, then

$$f(x_{min}) = f(x) + f'(x) * (x - x_{min})$$

Therefore:

$$|f(x_{min}) - f(x)| \leq f'(x) * \epsilon \leq r * \epsilon$$