ENGN 2020:

Homework #8

Brown University School of Engineering

Assigned: April 15, 2019, Due: April 22, 2019

Problem 1

Part a. In biological reactors, the rate-determining step is often the transport of O_2 into the liquid media. Consider the vessel shown in Figure 1 which is designed to add O_2 to a continuous flow of water. Water flows in the top of the vessel and out the bottom; meanwhile O_2 bubbles out of a sparger at the bottom of the vessel and is collected at (and recycled from) the top. The differential equation that describes this system might be derived to be

$$0 = -v\frac{dC}{dz} + ka(C^{\text{sat}} - C)$$

where v [L/s] is the volumetric flow rate of water, C [mol/L] is the concentration of O_2 in the water, C^{sat} is the maximum concentration of O_2 in the water (a physical property), z [m] is the distance measured from the top, k is a mass transport coefficient, and a [m^{-1}] is the total bubble surface area per column volume. What is the initial condition for this physical problem? Nondimensionalize this equation, showing your work.

Paper Submission

(Turn in a PDF to Canvas.)
Points: 1

Part b. Consider a circuit whose differential equation can be written as

$$R\frac{dq}{dt} + \frac{1}{C}q = V$$

where q is the (time-dependent) charge on a capacitor and V is a constant. The initial condition is $q(t = 0) = 0$. Non-dimensionalize this equation.

Paper Submission

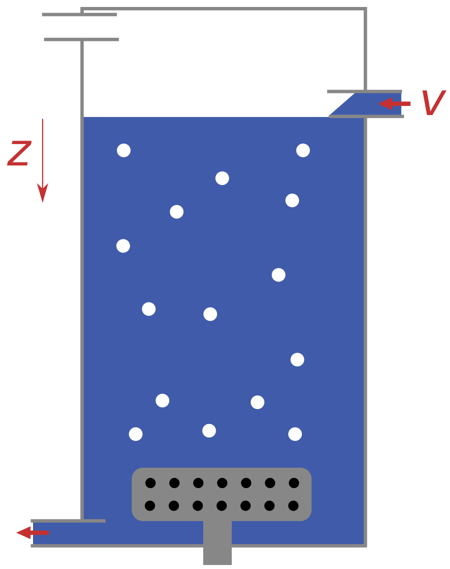


Figure 1: An oxygenator.

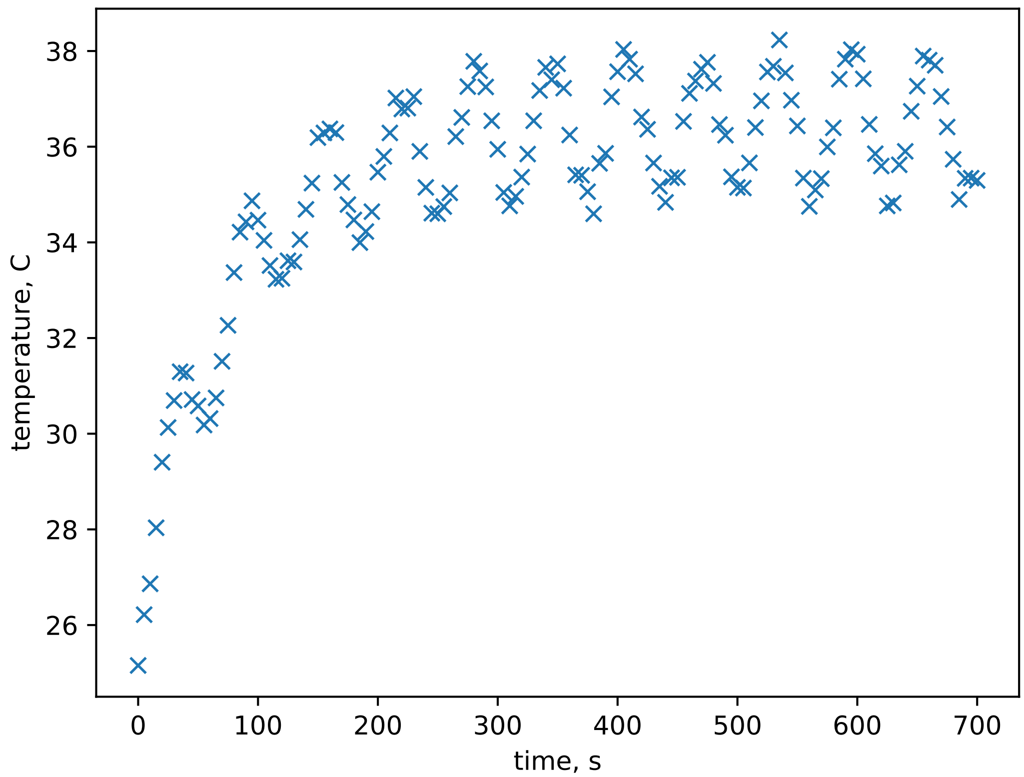
(Turn in a PDF to Canvas.)
Points: 1

Problem 2

Consider a 1-kg block of material whose thermodynamics are dictated by

$$mC_P\frac{dT}{dt} = -hA\left(T - T^\infty\right) + a\left(1 + \sin bt\right)$$

The final term is heat supplied by a heater that you control. You set a equal to 200 W and b equal to 0.1 s^{-1}, and observe the response below. The surface area A available for heat transfer to the surroundings is 200 cm^2 and the block starts at T^∞ = 25°C. What is the best-fit estimate of the material's heat capacity C_P [J/kg/K] and surface heat-transfer coefficient h [W/m^2/K]? Also turn in a plot like the one below containing your best-fit model solution in addition to the experimental points.



You can download[1] the data file from Canvas. You must solve this via numerical integration with a systematic approach to finding the best-fit parameters that minimize the sum of square residuals.

Paper Submission

(Turn in a PDF to Canvas.)
Points: 3

Problem 3

Consider a block of metal which undergoes one-dimensional conduction of heat out of single end at $x = 0$ (with $-k\,A\,\partial T/\partial x = -h\,A\,(T - T^{\infty})$) and is perfectly insulated ($-k\,A\,\partial T/\partial x = 0$) on the opposite end at $x = L$. The block is initially equilibrated with its surroundings at T_0, but at $t > 0$ the surrounding temperature is changed to T^{∞}, which is less than T_0. The governing equation for the internal of the block is

$$\rho C_P \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2}$$

[1] In numpy's .npz format which can be loaded with `numpy.load`.

Part a. First non-dimensionalize the governing equations, using appropriate dimensions from the problem and justifying your choices. When you finish, your governing equations should be

$$\frac{\partial \theta}{\partial \tau} = \frac{\partial^2 \theta}{\partial z^2}$$

$$\frac{\partial \theta}{\partial z} = \eta \theta \quad \text{at } z = 0$$

$$\frac{\partial \theta}{\partial z} = 0 \quad \text{at } z = 1$$

where θ, τ and z are non-dimensional variables and η is a non-dimensional parameter.

Paper Submission

(Turn in a PDF to Canvas.)
Points: 1

Part b. Convert this into a set of n ordinary differential equations by discretizing these equations over the spatial domain, with point $i = 0$ at $z = 0$ and point $i = n - 1$ at $z = 1$. Write a class that behaves as follows:

```
dydt = Derivatives(n=15, eta=50.)
yprimes = dydt(y, t)
```

That is, the `Derivatives` class makes a function `dydt` that can be fed to an ODE solver of your choice. *Hint:* You should define a `__call__` method in your class.

Paper Submission

(Turn in a PDF to Canvas.)
Points: 1

Part c. Solve your system using a suitable integration function from numpy/scipy. Create a series of solutions showing the effect of η on the system's behavior. For each value of η, your plot(s) should show the time evolution of the temperature profile inside your block.

Paper Submission

(Turn in your solution, including appropriate plots and code, as a PDF to Canvas.)
Points: 1