APMA 1650 Homework 06

Due: November 01, 2018

Due before class on Thursday, Nov 01, 2018. It can be dropped off in the APMA 1650 homework box on the first floor of the APMA department, 182 George St OR at class (before it starts).

Please attach the HW cover sheet to *the front of your HW* assignment. It can be found on Canvas. **Show all work and you MUST write up your own solutions.**

1. Suppose X and Y have joint probability density function $f(x, y)$, given by

$$f(x,y) = \begin{cases} cx^2y & \text{if } 0 \leq x \leq y, x + y \leq 2 \\ 0 & \text{elsewhere} \end{cases}$$

   (a) Find the value of c that makes $f(x, y)$ a probability density function.
   (b) Find the marginal density functions for X and Y.
   (c) Are X and Y independent?
   (d) Find the conditional density of Y given $X = x$.
   (e) Find $P(Y < 1.1|X = 0.6)$.

2. Let X and Y be independent exponentially distributed random variable with mean $\mu = 1$. Find $P(X > Y|X < 2Y)$.

3. Consider the following quadratic equation:

$$x^2 + 2Bx + C = 0.$$

   Suppose the coefficients B, C are independent random variables such that $B \sim \mathcal{U}(-1, 1)$ and $C \sim \mathcal{U}(-1, 1)$. Find the probability that the equation does not have real solutions.

4. A building has n floors numbered 1,2,$\cdots$, n. At floor 0, m people get on the elevator together. Each gets off at one of the n floors, uniformly at random (and independently of everybody else). What is the expected number of floors the elevator stops at?

5. Suppose that a sequence of n 1's and m 0's is randomly permuted so that each of the possible arrangements is equally likely. Any consecutive string of 1's is said to constitute a run of 1's. For example, if $n = 6, m = 4$, and the ordering is

$$1, 1, 1, 0, 1, 1, 0, 0, 1, 0$$

   then, there are 3 runs of 1's. Find the expected number of runs of 1.

6. Suppose that the number of people who enter the CVS on Thayer street on a given day follows a Poisson distribution with λ. Also suppose each person who enters the CVS is a male with probability p and a female with probability $q = 1 - p$. Show that the number of males and females entering the CVS are independent Poisson random variables with parameters λp and λq, respectively.