APMA 1650 Homework 7 Common Mistakes

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1. A number of students said that $\text{Var}(X) = E[X^2]$ and $E[X^2] = E[X]^2$. Neither is true. Remember that

$$\text{Var}(X) = E[(X - E[X])^2] = E[X^2] - E[X]^2.$$

2. On part (d), some students found $E[X]$ and $\text{Var}(X)$ rather than $E[\bar{X}]$ and $\text{Var}(\bar{X})$. Make sure you read the questions carefully.

3. (a) Students did way more work than was necessary for this problem. There should be no integration needed. Since $X_1 = Z$ and $X_2 = Z^2$ and we know Z has mean 0 and variance 1, we can instantly find that $E[X_1] = E[Z] = 0$. To find $E[X_2] = E[Z^2]$, we just need to solve the variance equation $\text{Var}(Z) = E[Z^2] - E[Z]^2$ for $E[Z^2]$ to get $E[Z^2] = \text{Var}(Z) + E[Z]^2 = 1 + 0 = 1$.

   (b) No common mistakes

   (c) No common mistakes

   (d) $\text{Cov}(X_1, X_2) = 0$ does **not** imply that X_1 and X_2 are independent. Also, $E[X_1]E[X_2] = E[X_1X_2]$ does **not** imply that X_1 and X_2 are independent. To check independence, it is enough to note the relation $X_2 = X_1^2$, so X_2 depends on X_1.

   If you still don't see why this is true, go back to one of our many equivalent definitions of independence. The easiest one to check here is that X_1 and X_2 are independent if for any events I and J,

$$P(X_1 \in I, X_2 \in J) = P(X_1 \in I)P(X_2 \in J). \tag{1}$$

   If we take the intervals $I = (1, 2)$ and $J = (10, \infty)$, then the LHS of (1) is

$$P(X_1 \in I, X_2 \in J) = P(1 < Z < 2, Z^2 > 10) = 0.$$

   For RHS of (1), we know

$$P(X_1 \in I)P(X_2 \in J) = P(1 < Z < 2)P(Z^2 > 10) > 0,$$

   so these two quantities cannot be equal, and thus, X_1 and X_2 are dependent.

4. (a) No common mistakes

   (b) Many student omitted explanations for why this distribution is multinomial or omitted the bounds for the distribution.

   (c) Covariance is not linear. If you approach this problem using indicator functions, you need to account for the cross terms in the covariance and justify why they're zero.

5. Almost everyone got this problem correct, but a number of students did not justify their answers. There are enough quantities in this problem that it is not obvious what the probabilities involved are, and there should be some explanation.

6. No common mistakes