APMA 1650 Homework 2 Common Mistakes

Rebecca Santorella

1. Most students came up with good examples, but some people lost points for not defining their sample space or only using descriptions of examples without numbers. In part (c), many students forgot the requirement that $P(A) > P(B)$.

2. This problem was fine overall.

3. A lot of students did not take into account that the flips continued until the coins did not match. It is not sufficient to consider only the cases HH, HT, TH, TT since the flips continue until the coins do not match.

4. In part (b), a lot of students assumed that the game ended on round 4 and found $P(rounds = 4)$ rather than $P(rounds \geq 4)$. In part (d), a common mistake was to write that $P(rounds = k) = p(1-p)^k$, but it should be $P(rounds = k) = p(1-p)^{k-1}$ since the coin must be tails $k-1$ times to be heads on the kth round.

5. (a) Some students used $p = \frac{1}{2}$ even though the probability was not given.

   (b) A common mistake was forgetting the binomial coefficient $\binom{k}{x}$. Some students also calculated the probability but not the expected value.

6. Almost every student had the right concept for the proof. A point was taken off for solutions with no words. It is fine to use mathematical notation in proofs, but there must be words justifying why certain steps were taken. In this proof, it is sufficient to say that we can go from $\sum_{k=1}^{\infty} \sum_{j=1}^{k} P(X = k)$ to $\sum_{j=1}^{\infty} \sum_{k=j}^{\infty} P(X = k)$ because we can switch the order of summation for a positive series.

   A few students tried to use induction to prove this, but there is no base case or previous cases to induct on. Even if you restated the problem as $\sum_{k=1}^{\infty} kP(X = k) = \sum_{k=1}^{\infty} P(X \geq k)$, induction only applies to finite cases, and then you would need to take a limit to go from $\sum_{k=1}^{N}$ to $\sum_{k=1}^{\infty}$.