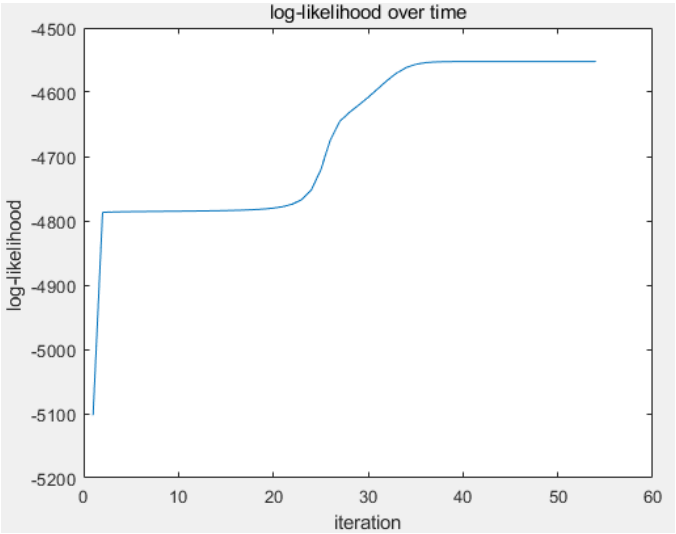
ENGN2520 Homework 5

Ming Xu (Banner ID: B01532164)

Problem1--A mixture of 2 Gaussians

(a) A plot of the log-likelihood over time for the best choice of initial parameters



(b) The resulting parameters of the mixture models.

$$\mu_1 = [9.8872 \quad 5.0366]$$

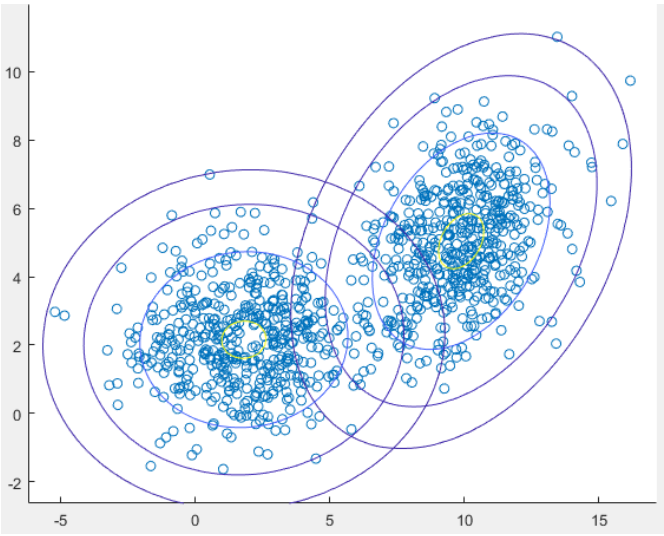
$$\mu_2 = [1.8013 \quad 2.1581]$$

$$\Sigma_1 = \begin{bmatrix} 3.1686 & 1.0605 \\ 1.0605 & 2.9009 \end{bmatrix}$$

$$\Sigma_2 = \begin{bmatrix} 4.4389 & 0.1106 \\ 0.1106 & 1.9539 \end{bmatrix}$$

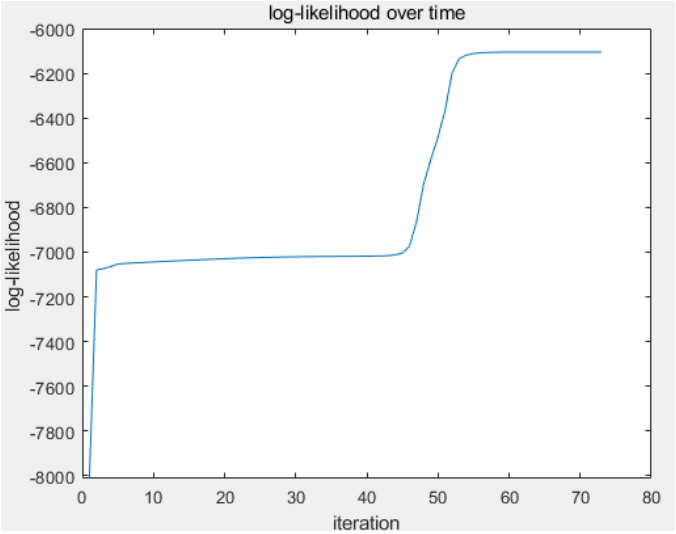
$$\pi = [0.5056 \quad 0.4944]$$

(c) A visualization of the estimated means and covariances over the datasets.



Problem2 --A mixture of 3 Gaussians

(a) A plot of the log-likelihood over time for the best choice of initial parameters For multiclass



(b) The resulting parameters of the mixture models.

$$\mu_1 = [2.9943 \quad 4.5667]$$
$$\mu_2 = [2.8963 \quad 0.0474]$$
$$\mu_3 = [3.0100 \quad 9.9408]$$

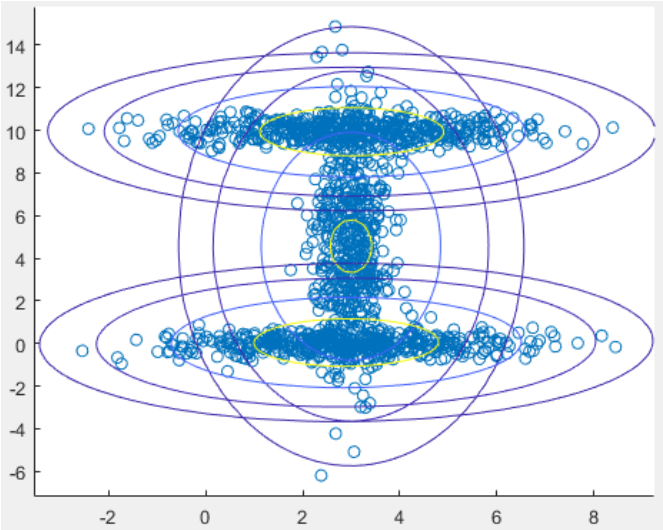
$$\Sigma_1 = \begin{bmatrix} 1.0000 & 0.0023 \\ 0.0023 & 8.4118 \end{bmatrix}$$

$$\Sigma_2 = \begin{bmatrix} 2.9204 & 0.0406 \\ 0.0406 & 1.0000 \end{bmatrix}$$

$$\Sigma_3 = \begin{bmatrix} 2.8622 & -0.0108 \\ -0.0108 & 1.0000 \end{bmatrix}$$

$$\pi = [0.2893 \quad 0.3480 \quad 0.3627]$$

(c) A visualization of the estimated means and covariances over the datasets.



Source Code

1.The matlab function to calculate probability:

```
function [prob] = probabilityCalculation(X,K,mu,sigma)
    [N,~] = size(X);
    prob = zeros(N, K);
    for j = 1:K
        prob(:, j) = mvnpdf(X, mu(j,:), squeeze(sigma(j,:,:)));
    end
end
```

2.The matlab function to implement ER algorithm:

```
function [maxMU,maxSigma,maxPI,likelyhood] = EM_Algorithm(data, K,
numIterations , tolForInteration ,thresholdForCovMatrix)
    %get dimension of input training data;
    [N,D] = size(data);
    %initialization
    maxLikelyhood = -inf;

    %iterate for certain times
    for i = 1:numIterations
       %set initial values for mu, pi, sigma of this iteration
       %initialize mu by randomly selecting K data points
       mu = data(randsample(N, K), :);
       %initialize sigma by using  the overall data covariance.
       sigma = reshape(cov(data),1,D,D);
       sigma = repmat(sigma, K, 1, 1);
       pi = ones(1,K) / K;
       %calculate the initial value of log-likelyhood
       lastLikelyhood = -inf;
       currentLikelyhood = sum(log(probabilityCalculation(data, K, mu , sigma) *
pi'));
       allLikelyhoods = currentLikelyhood;
       %loop until likelyhood becomes stable
       while currentLikelyhood - lastLikelyhood > tolForInteration
           lastLikelyhood = currentLikelyhood;
           prob = probabilityCalculation( data, K, mu , sigma).*pi;
           %calculate rj and Nj
           rj = prob./sum(prob,2);
           Nj = sum(rj , 1);
           %calculate mu
           mu = (rj'*data)./(Nj');
           %calculate sigma
           for j = 1:K
               X_mu = data-repmat(mu(j,:),N,1);
               sigmaJ = (rj(:,j).*X_mu)'*X_mu;
               sigmaJ = sigmaJ/Nj(j);
               for d = 1:D
                   sigmaJ(d,d) = max(sigmaJ(d,d), thresholdForCovMatrix);
               end
               sigma(j, :, :) = sigmaJ;
           end
           %caculate pi
           pi = Nj/N;
           %update likelyhood
           currentLikelyhood = sum(log(probabilityCalculation(data, K, mu, sigma)
* pi'));
           %save current likelyhood
           allLikelyhoods = [allLikelyhoods,currentLikelyhood];
       end

       %save the best likelyhood ever have
       if currentLikelyhood>maxLikelyhood && currentLikelyhood<-1000
```

```
            maxLikelyhood = currentLikelyhood ;
            maxMU = mu;
            maxSigma = sigma;
            maxPI = pi;
            likelyhood = allLikelyhoods;
        end
    end
end
```

3. The matlab function for visualization:

```
function [] = visualization(data,K,mu,sigma)
    figure
    hold on
    %plot the training data
    scatter(data(:,1), data(:,2))
    maxValue = max(data, [], 1) + 1;
    minValue = min(data, [], 1) - 1;
    x1 = minValue(1) :.2: maxValue(1);
    x2 = minValue(2) :.2: maxValue(2);
    [X1,X2] = meshgrid(x1,x2);
    X = [X1(:) X2(:)];
    for k = 1:K
        y = mvnpdf(X,   mu(k,:), squeeze(sigma(k,:,:)));
        y = reshape(y,length(x2),length(x1));
        contour(x1 ,x2 ,y ,[0.0001 0.001 0.01 0.05 0.15 0.25 0.35]);
    end
end
```

4.Main Function:

```
clc;clear;
numIterations = 20;
tolForInteration = 1e-7;
thresholdForCovMatrix = 1;
%% 2 Gaussian
load data2;
K = 2;
[mu, sigma,pi,likelyhood] = EM_Algorithm(data, K, numIterations,
tolForInteration, thresholdForCovMatrix);
pi
for i = 1:K
   mu(i,:)
   squeeze(sigma(i,:,:))
end
plot(likelyhood)
title('log-likelihood over time')
xlabel('iteration')
ylabel('log-likelihood ')
visualization(data, K, mu , sigma);

%% 3 Gaussian
load data3;
K = 3;
[mu, sigma,pi,likelyhood] = EM_Algorithm(data, K, numIterations,
tolForInteration, thresholdForCovMatrix);
pi
for i = 1:K
   mu(i,:)
   squeeze(sigma(i,:,:))
end
plot(likelyhood)
title('log-likelihood over time')
xlabel('iteration')
ylabel('log-likelihood ')
visualization(data, K, mu , sigma);
```