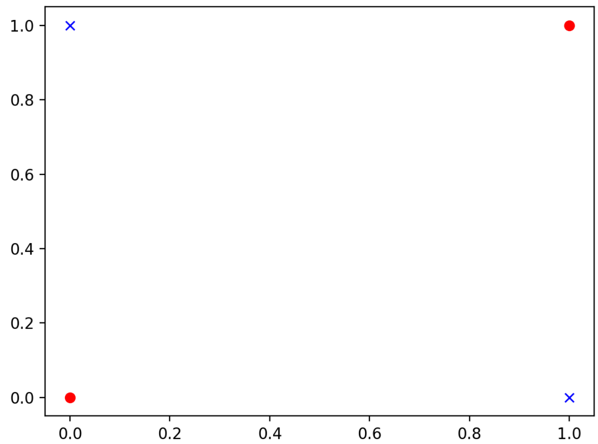
Homework 2

Problem1:

(a) **Conjunction:**
w = $\{$1,1, ..., 1,1$\}$
$b = -d + 1$
$h_w(x) = sign(< w, x > +b)$
w is all 1 so that $< w, x >$ is d if and only if all values of x is 1. In this case, $h_w(x)$ is 1. In all others cases, $h_w(x) = 0or - 1$.

(b) **Majority:**
w = $\{$1,1, ..., 1,1$\}$
$b = -d/2$
$h_w(x) = sign(< w, x > +b)$
w is all 1. When more than half of d values of x are 1, $< w, x >$ is greater than d/2. In this case, $h_w(x)$ is 1. Otherwise, $< w, x > +b$ is less or equal to 0, which leads to $h_w(x) = 0or - 1$

Problem2: The plot of different scenarios of the given function.



The red dots are points where $h_{equiv} = 1$, while the blue crosses are points where $h_{equiv} = 0$. There is no way to draw to line to separate those points. Therefore, h_{equiv} can not be represented with a halfspace.

Problem3: We already know that $\boldsymbol{w}$ is a vector perpendicular to the decision boundary, where $< w, x >= 0$.

For a point x that is out of the decision boundary, the vector $\boldsymbol{x}$ can be written as the sum of the projection vector on decision boundary $\boldsymbol{x_{proj}}$ and the vector that perpendicular to the decision boundary $\boldsymbol{x_{per}}$

$$\boldsymbol{x} = \boldsymbol{x_{proj}} + \boldsymbol{x_{per}}$$

Besides, $\boldsymbol{x_{per}}$ can be written as $d\frac{\boldsymbol{w}}{||\boldsymbol{w}||_2}$ where d is the distance of x to the decision boundary and $\frac{\boldsymbol{w}}{||\boldsymbol{w}||_2}$ is a unit vector that perpendicular to the decision boundary. So,

$$\boldsymbol{x} = \boldsymbol{x_{proj}} + d\frac{\boldsymbol{w}}{||\boldsymbol{w}||_2}$$

Multiply the equation with $\boldsymbol{w^T}$:

$$\boldsymbol{w^T x} = \boldsymbol{w^T x_{proj}} + d\frac{\boldsymbol{w^T w}}{||\boldsymbol{w}||_2}$$

Then,

$$| < w, x > | = | < w, x_{proj} > | + d * ||w||_2$$

Because x_{proj} is on the decision boundary, $| < w, x_{proj} > | = 0$. Therefore,

$$d = \frac{| < w, x > |}{||w||_2}$$