ENGN2520 Homework 4

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Problem1

(a)

Suppose $w_y^T > w_{\hat{y}}^T + 1$:

$$L\big((w_1, \dots, w_k), (x, y)\big) = 0$$

Therefore:

$$\frac{\partial L\big((w_1, \dots, w_k), (x, y)\big)}{\partial w_{j,l}} = 0$$

(b)

Suppose $w_y^T < w_{\hat{y}}^T + 1$ and $j = y$:

$$L\big((w_1, \dots, w_k), (x, y)\big) = w_{\hat{y}}^T x + 1 - w_y^T x$$

Therefore:

$$\frac{\partial L\big((w_1, \dots, w_k), (x, y)\big)}{\partial w_{j,l}} = -x_{j,l}$$

(c)

Suppose $w_y^T < w_{\hat{y}}^T + 1$ and $j = \hat{y}$:

$$L\big((w_1, \dots, w_k), (x, y)\big) = w_{\hat{y}}^T x + 1 - w_y^T x$$

Therefore:

$$\frac{\partial L\big((w_1, \dots, w_k), (x, y)\big)}{\partial w_{j,l}} = x_{j,l}$$

(d)

Suppose $w_y^T < w_{\hat{y}}^T + 1$ and $j \neq y$ and $j \neq \hat{y}$:

$$L\big((w_1, \dots, w_k), (x, y)\big) = w_{\hat{y}}^T x + 1 - w_y^T x$$

Therefore:

$$\frac{\partial L\big((w_1, \dots, w_k), (x, y)\big)}{\partial w_{j,l}} = 0$$

Problem2

(a)

For multiclass SVM, the classifer $\boldsymbol{y}$ is defined as below:

$$y = \operatorname*{argmax}_j w_j^T x$$

When there are 2 class, y can be written as：

$$y = \begin{cases} 1 & w_1^T x \geq w_2^T x \\ 2 & w_1^T x < w_2^T x \end{cases}$$

Equivalently, y can be written as:

$$y = \begin{cases} 1 & w_1^T x - w_2^T x \geq 0 \\ 2 & w_1^T x - w_2^T x < 0 \end{cases}$$

Let $w = w_1 - w_2$, then y becomes:

$y = \begin{cases} 1 & w^T x \geq 0 \\ 2 & w^T x < 0 \end{cases}$, which is a linear perceptron classifier.

(b)

For multiclass SVM, the classifer $\boldsymbol{y}$ is defined as below:

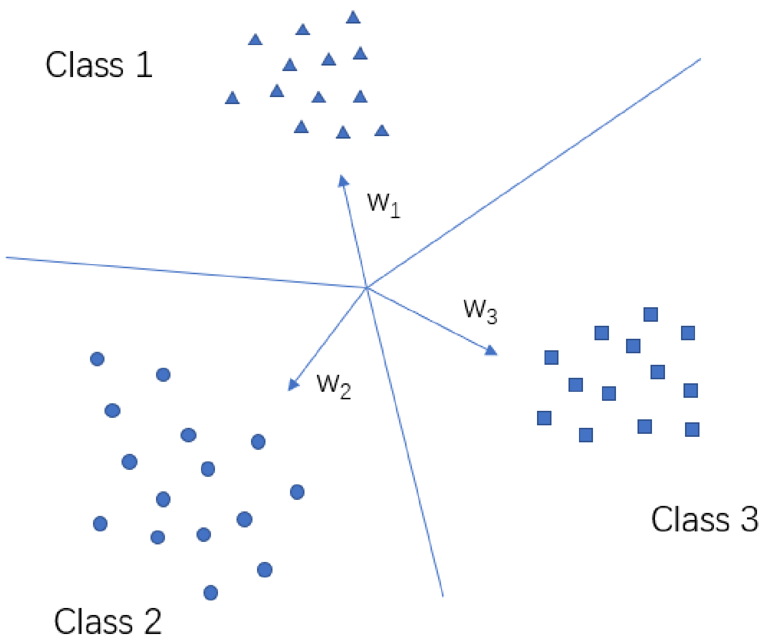
$$y = \underset{j}{\operatorname{argmax}}\, w_j^T x$$

For K classes, the classifier can be written as below:

If $w_k^T x - w_j^T x \geq 0\ for\ \forall j \neq k$, then $y = k$.

Therefore, the boundaries between each decision regions are like linear perceptions.

Below is an example of 3 class problem in the plane(D=2) where a multiclass SVM is used. The boundary between decision regions are lines determined by $(w_1 - w_2)$, $(w_2 - w_3)$ and $(w_1 - w_3)$.



Problem3

The matlab function to calculate $E(x)$ and $\nabla E(x)$:

```
function [Ew,gradientEw] = calculateEw(w,x,y,C)
    %k classes
    [~,k] = size(w);
    %n training datas
    [n,~] = size(x);
    %initialization
    Ew = 0;
    gradientEw = w;

    %loop all training datas
    for index = 1:n
```

```
        temp = x(index,:)*w;
        y_train = y(index,1);

        %find yhat, make sure yhat = argmax(temp) where yhat!=y
        temp(y_train) = -Inf;
        [~,y_hat] = max(temp);

        %calculate w_y_x and w_y_hat_t
        w_y_x = x(index,:)*w(:,y_train);
        w_y_hat_x = x(index,:)*w(:,y_hat);

        if w_y_x<w_y_hat_x+1
            %update E
            Ew = Ew+C*(w_y_hat_x+1-w_y_x);
            %update gradient E
            gradientEw(:,y_hat) = gradientEw(:,y_hat) + C*x(index,:)';
            gradientEw(:,y_train) = gradientEw(:,y_train)-C*x(index,:)';
        end
    end

    for i = 1 : k
        Ew = Ew + w(i,:)*w(i,:)'/2;
    end
end
```

Problem4

(a)

The matlab function for gradient descent algorithm:

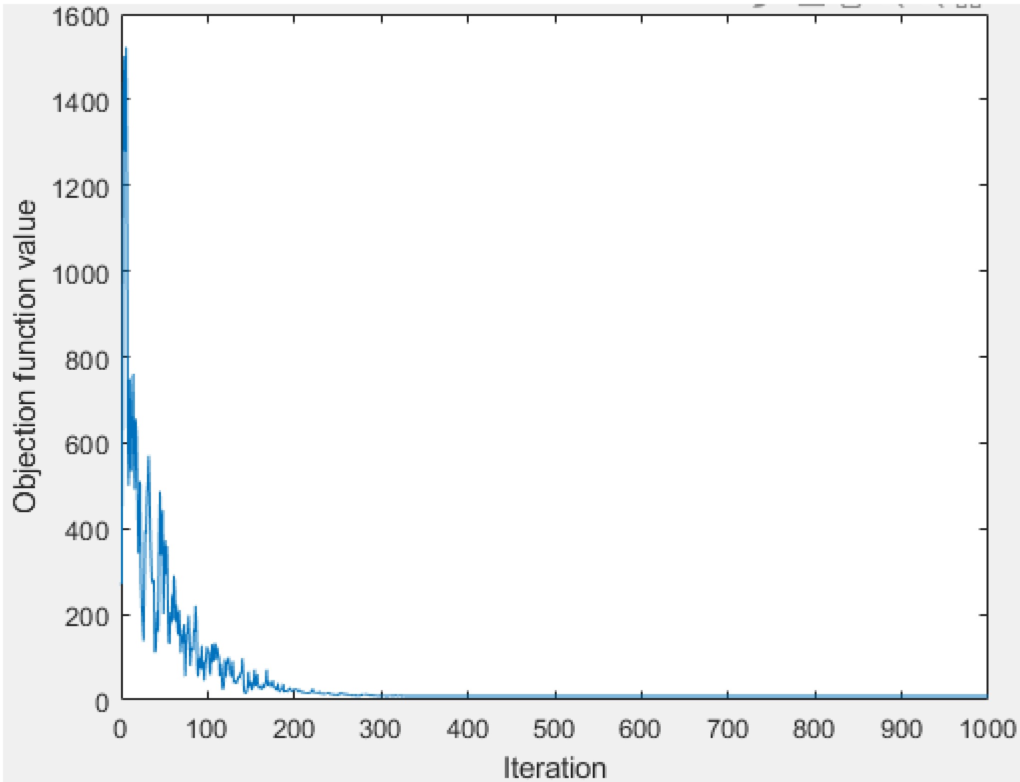
```
function [w,Loss] = gradientDescent(x,y,C,r,T)
    %generate a random w matrix
    w = rand(785,10,'double');
    Loss = [];
    d = 1.01;

    %iteration for T times
    for i = 1:T
        %calculate Loss and gradient of the loss
        [E, G] = calculateEw(w,x,y,C);
        %save loss for further visulization
        Loss = [Loss, E];
        %update w
```

```
        w = w - r * G;
        %decrease stepsize
        r = r / d;
    end
end
```

(b)

Below is a figure of objection function value over time, when C = 0.01, r = 0.01, and T = 1000:



(c)

The different values of C and its corresponding correct fraction is shown in the table below. In this experiment, r = 0.01 and T=1000.

Index	C	Correct fraction
1	0.0001	74.66%
2	0.001	84.40%
3	0.01	87.32%
4	0.1	85.16%
5	1	84.26%
6	10	83.90%
7	100	84.02%

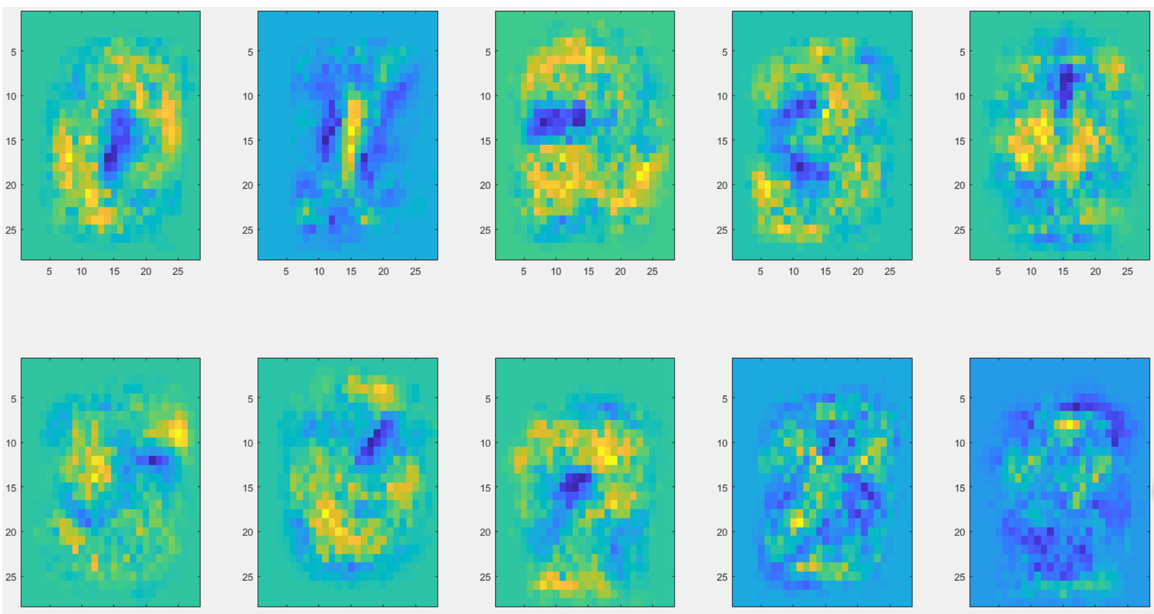
From the table, we can find when C=0.01, the model has minimum number of erros.

The confusion matrix, when C = 0.01, is show as below:

	0	1	2	3	4	5	6	7	8	9
0	464	0	13	2	0	7	8	0	4	2
1	0	478	6	6	1	3	4	0	2	0
2	5	11	410	17	5	2	8	20	17	5
3	4	0	13	406	0	35	3	15	16	8
4	2	0	0	4	440	1	9	7	11	26
5	12	2	11	27	16	379	9	7	30	7
6	8	3	13	0	17	18	434	5	1	1
7	0	5	22	17	7	0	0	417	0	32
8	5	5	21	31	12	18	2	11	384	11
9	3	2	4	12	43	4	1	31	11	389

(d)

When C = 0.01, the visualization of each digit is shown as below:



(e) Source code

1. Code to prepare training set:

```
clear;clc;
load('digits');
x = [];
y = [];
for i = 1 : 10
    x = [x; ones(500,1), eval(['train' num2str(i-1)])];
    y = [y; ones(500,1) * i];
end
```

2. Code to plot figure of objection function value vs iteration time

```
%% Objective function value vs iteration time
r = 0.1;
T = 1000;
C = 0.01;
[w,Loss] = gradientDescent(x,y,C,r,T);
plot(Loss);
xlabel('Iteration');
ylabel('Objection function value');
```

3. Function to do experiments on different values of C:

```
%% Do experiments of different values of C and find minimum
classification erros
figure();
hold on;
acc_max = 0;
C = [0.0001,0.001,0.01, 0.1, 1, 10, 100];
r = 0.1;
T = 1000;
for i = 1 : size(C, 2)
    [w,Loss] = gradientDescent(x,y,C(i),r,T);

    [acc, classification_map] = Test(w, 10, 500);
    fprintf('C = %f, accuracy = %f \n', C(i), acc);
    if acc > acc_max
        acc_max = acc;
        c_best = C(i);
        W_best = w;
        result = classification_map;
    end
end
```

4. Function to visualize digits of w:

```
figure();
for i = 1 : 10
    subplot(2,5,i);
    w_i = normalize(W_best(2 : end, i), 'range');
    imagesc(reshape(w_i,28,28)');
end
```