APMA 1650 Homework 09

Due: December 06, 2018

Due before class on Thursday, Dec 06, 2018. It can be dropped off in the APMA 1650 homework box on the first floor of the APMA department, 182 George St OR at class (before it starts).

Please attach the HW cover sheet to *the front of your HW* assignment. It can be found on Canvas. **Show all work and you MUST write up your own solutions.**

1. Let X be a random variable whose pdf is

$$f_X(x) = \begin{cases} \frac{2(\theta - x)}{\theta^2}, & 0 < x < \theta, \\ 0, & \text{elsewhere} \end{cases}.$$

   (a) Find the cdf of X.

   (b) Find the cdf of X/θ.

   (c) Use (b), find a 90% one-sided lower confidence limit for θ.

   (d) Use (b), find a 90% one-sided upper confidence limit for θ.

2. The administrators for a hospital wished to estimate the average number of days required for inpatient treatment of patients between the ages of 25 and 34. A random sample of size n hospital patients between these ages produced a mean and standard deviation equal to $\hat{\mu}_n$ and $\hat{\sigma}_n$ days, respectively.

   (a) Suppose $n = 500$ with $\hat{\mu}_{500} = 5.4$ and $\hat{\sigma}_{500} = 3.1$. Construct a 95% confidence interval for the mean length of stay for the population of patients from which the sample was drawn.

   (b) Due to the limited budget, the administrators can only afford a random sample of size $n = 21$. It seems that the average number of days follow a normal distribution with unknown μ and σ. It turns out $\hat{\mu}_{21} = 6.58$ and $\hat{\sigma}_{21} = 7.4$. Find a 95% confidence interval for the mean length of stay for the population of patients from which the sample was drawn.

3. The ages of a random sample of five Brown professors are 29, 44, 61, 72, and 59. It is assumed that the ages of Brown professors are normally distributed.

   (a) Find a 99% confidence interval for the population mean of the ages of all professors at Brown.

   (b) Find a 99% confidence interval for the population variance of the ages of all professors at Brown.

You may want to use one of the following facts:

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\begin{aligned}
Z \sim \mathcal{N}(0,1), &\qquad P(-2.58 < Z < 2.58) = 0.99,\\
Z \sim \mathcal{N}(0,1), &\qquad P(-1.96 < Z < 1.96) = 0.95,\\
T \sim \mathcal{T}_{20}, &\qquad P(-2.086 < T < 2.086) = 0.95,\\
T \sim \mathcal{T}_{20}, &\qquad P(-2.845 < T < 2.845) = 0.99,\\
T \sim \mathcal{T}_{4}, &\qquad P(-4.604 < T < 4.604) = 0.99,\\
V \sim \chi^2_4, &\qquad P(0.20699 < V < 14.8602) = 0.99,
\end{aligned}
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