APMA 1650 Homework 03

Due: October 4, 2018

Due before class on Thursday, Oct. 4, 2018. It can be dropped off in the APMA 1650 homework box on the first floor of the APMA department, 182 George St by **5pm** OR at class (before it starts).

Please attach the HW cover sheet to *the front of your HW* assignment. It can be found on Canvas. **Show all work and you MUST write up your own solutions.**

1. (a) Construct a discrete random variable whose expectation is infinite.

   (b) Let X be a random variable and α and β be constants. Prove that

   $$\text{Var}[\alpha X + \beta] = \alpha^2 \text{Var}[X].$$

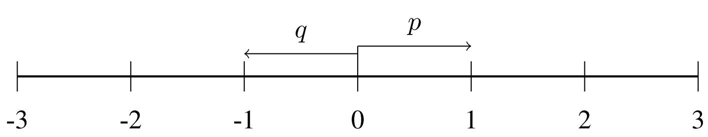
   (c) Let X be a random variable. Let define a new random variable Y such that

   $$Y = \frac{X - E[X]}{\sqrt{\text{Var}[X]}}.$$

   Compute $E[Y]$ and $\text{Var}[Y]$.

2. The following gambling game is known as the wheel of fortune. A player bets on one of the numbers 1 through 6. Three dice are then rolled. If the number bet by the player appears j times, $j = 1, 2, 3$, then the player wins j units. If the number bet by the player does not appear on any of the dice, then the player loses 1 unit. In order to determine whether or not this is a fair game for the player, compute the expectation of the player's winnings in the game. Based on the expectation, is this game fair to the player?

3. 1D Random Walk: Let consider the following game associated with the coin toss. In the integer number line, $\mathbb{Z}$, we have a stone at 0. We toss a coin and move the stone to +1 if a head shows up and to -1 if a tail shows up. Let X_n be the position of the stone after n tosses.



   (a) What is the probability distribution of X_3? Let $Y_3 \sim B(3, p)$. By comparing distributions of Y_3 and X_3, find a relation between Y_3 and X_3.

   (b) By generalizing (a), find a relation between $Y_n \sim B(n, p)$ and X_n. Based on this relation, compute $E[X_n]$ and $\text{Var}[X_n]$.

4. There are 64 teams who play single elimination tournaments, hence 6 rounds, and you have to predict all the winners in all 63 games. Your score is then computed as follows: 32 points for correctly predicting the final winner, 16 points for each correct finalist, and so on, down to 1 point of every correctly predicted winner for the first round. (The maximum number of points you can get is thus 192.) Knowing nothing about any team, you flip fair coins to decide every one of your 63 bets. Compute the expected number of points.

5. An instructor who taught two sections of mathematical statistics last term, the first with 20 students and the second with 30, decided to assign a term project. After all projects had been turned in, the instructor randomly ordered them before grading. Consider the first 15 graded projects.

   (a) What is the probability that at least 10 of these are from the second section?

   (b) What is the probability that at least 10 of these are from the same section?

   (c) What are the mean value and standard deviation of the number of projects among these 15 that are from the second section?

   (d) What are the mean value and standard deviation of the number of projects not among these 15 that are from the second section?

6. Let $X \sim Geo(p)$. Find the probability of X being an even number, $P(X \text{is an even number})$.