

# ENGN 2520 Midterm exam

**NAME:**

**BANNER ID:**

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problem	grade	memo
1		
2		
3		
4		
total		

## Problem 1

We want to classify fish into two types: bass and salmon. We have 2 real valued features: length and weight.

Suppose we assume the length and weight of a fish are independent conditional on the fish type, and each is distributed according to a gaussian distribution with mean and variance that depends on the fish type.

We estimate the parameters of

- $p(\text{length}|\text{type} = \text{bass})$ ,
- $p(\text{weight}|\text{type} = \text{bass})$ ,
- $p(\text{length}|\text{type} = \text{salmon})$ ,
- $p(\text{weight}|\text{type} = \text{salmon})$ ,
- $p(\text{type} = \text{salmon})$ ,
- $p(\text{type} = \text{bass})$ ,

using maximum likelihood estimation with a training set  $T$ .

(a) What is the classification rule defined by the Bayes optimal classifier in terms of the six distributions specified above. You should use a 0/1 loss to derive classifier.

(b) Suppose the resulting classifier has poor performance on a test set. Give 3 different reasons why this might happen in practice.

## Problem 2

Given short answers for the questions below.

- (a) Suppose we want to use regression to fit a polynomial function to some training data  $T$ . How can we pick the degree of the polynomial to use in the regression?
- (b) What is an advantage of a multilayer neural network over a linear classifier?
- (c) What is an advantage of a linear classifier over a multilayer neural network?
- (d) Consider a binary classification problem with a set of training examples  $T$ . Let  $H_1$  and  $H_2$  be two sets of classifiers and suppose the VC dimension of  $H_1$  is smaller than the VC dimension of  $H_2$ . Suppose we find classifiers  $c_1 \in H_1$  and  $c_2 \in H_2$  that both correctly classify all examples in  $T$ . How should we pick between  $c_1$  and  $c_2$ ? Give a brief justification.

### Problem 3

Consider a binary classification problem where the input is a point in  $\mathbb{R}$ .

Suppose we are interested in learning classifiers that are defined in terms of intervals  $[a, b]$  with  $a < b$ . Examples inside the interval are classified as positive, and examples outside the interval are classified as negative.

Define a feature map  $\phi : \mathbb{R} \rightarrow \mathbb{R}^D$  such that every interval classifier can be represented by a linear classifier in the feature space defined by  $\phi$ . Justify your answer.

## Problem 4

Suppose you have an instrument that measures the speed of a running animal. You install the instrument in a habitat that contains two kinds of animals: tigers and cheetahs. The speed of a tiger is a random variable with probability density  $p_1(x)$ . The speed of a cheetah is a random variable with probability density  $p_2(x)$ . Suppose  $p_1$  and  $p_2$  are known. These densities are different but they have significant overlap, so it is hard to decide if an animal is a tiger or a cheetah based on a single observation of its speed.

(a) Let  $x$  be the speed of a random animal in the habitat. Let  $w_1$  be the fraction of animals in the habitat that are tigers, and  $w_2$  be the fraction of animals in the habitat that are cheetahs (since there are no other animals in this habitat we have  $w_1 + w_2 = 1$ ). What is the probability density of  $x$ ?

(b) Suppose we record the speed of  $n$  animals  $x_1, \dots, x_n$ . Assume each measurement comes from a random animal in the habitat. How can we estimate  $w_1$  and  $w_2$  from these measurements? Give an algorithm for estimating  $w_1$  and  $w_2$ . Justify your answer.