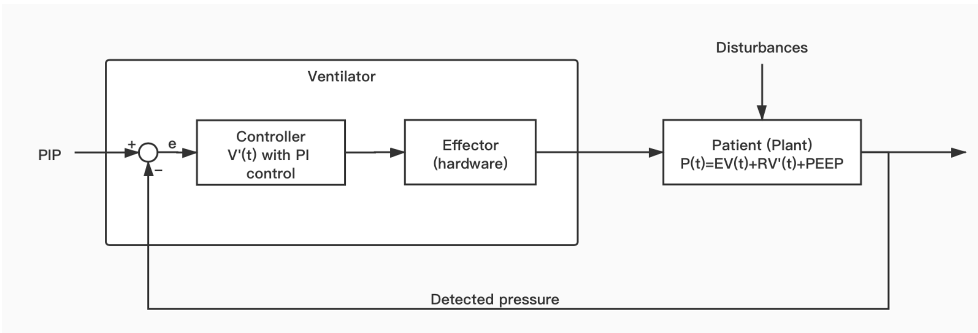
1 Inhalation

1.1 Control Circuit



The control circuit for inhalation. The input PIP (Peak Inspiratory Pressure) is the expected level of pressure to reach. Error e is the difference between actual pressure $P(t)$ measured by the sensor and PIP. $V(t)$ is the volume of air in patient's alveoli and $V'(t)$ would represent the flow speed of the injected gas. PI control is used in the circuit. The formula for the controller is :

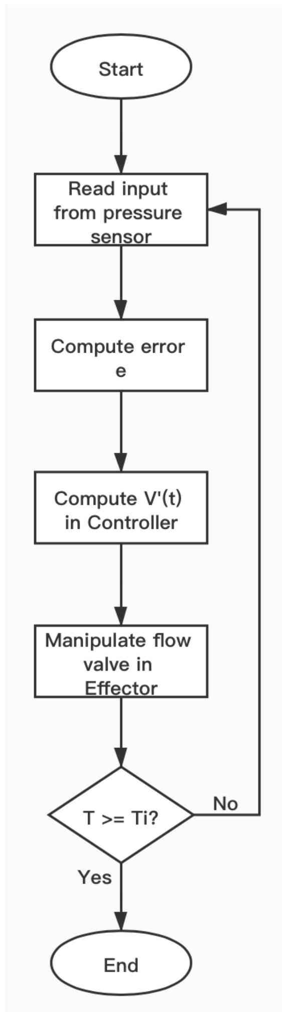
$$V'(t) = K_p e + K_i \int_0^t e dt$$

For the plant, the formula

$$P(t) = EV(t) + RV'(t) + PEEP$$

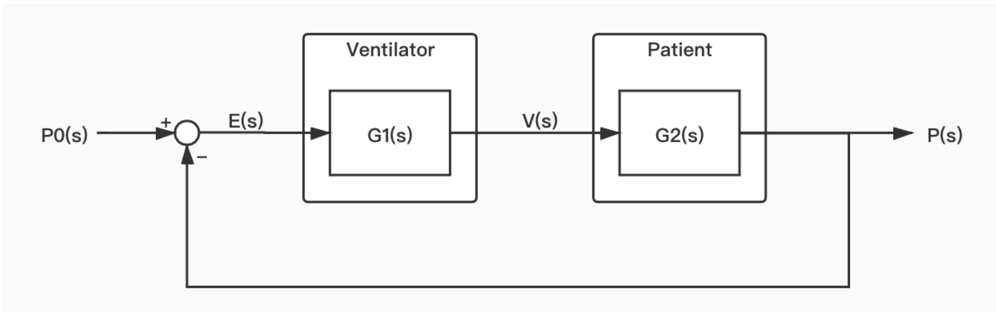
is the equation of motion, which shows a mathematical relation between the physical properties in the patient's lung. E represents the elastance (inverse of the compliance) of the patient's lung and R represents the airway resistance. $PEEP$ (Positive End Expiratory Pressure) is the remaining positive pressure in the patient's lung after exhalation. It is usually set as positive manually to prevent atelectasis.

1.2 Flow Diagram



The flow diagram for a single inhalation. It can be triggered either by the machine or by patient. When the time T has exceed the preset inspiratory time Ti, the inhalation would end and the exhalation would begin.

1.3 Transfer Function



$E(s)$, $V(s)$, $P(s)$ is the Laplace transforms of the error $e(t)$, volume $v(t)$ and the pressure $p(t)$. The input $P_0(s)$ is the Laplace transform of p_0, while p_0 is $PIP - PEEP$. $PEEP$ is subtracted from PIP so that the zero initial condition of the system can be satisfied ($p(0) = 0$). $G_1(s)$ is the transfer function of the ventilator and $G_2(s)$ is the transfer function of the patient.

For ventilator:

$$v'(t) = K_p e + K_i \int_0^t e dt$$

Do Laplace transform on both sides:

$$sV(s) = K_p E(s) + \frac{K_i}{s} E(s)$$

$$G_1(s) = \frac{V(s)}{E(s)} = \frac{K_p}{s} + \frac{K_i}{s^2}$$

For patient:

$$p(t) = Ev(t) + Rv'(t)$$

Do Laplace transform on both sides:

$$P(s) = EV(s) + RsV(s)$$

$$G_2(s) = \frac{P(s)}{V(s)} = E + Rs$$

$$G(s) = \frac{P(s)}{E(s)} = G_1(s) \cdot G_2(s) = (\frac{K_p}{s} + \frac{K_i}{s^2})(E + Rs)$$

$$P(s) = G(s) \cdot E(s) = G(s) \cdot (P_0(s) - P(s))$$

$$\frac{P(s)}{P_0(s)} = \frac{G(s)}{1 + G(s)}$$

Therefore, the transfer function of the system is:

$$\frac{P(s)}{P_0(s)} = \frac{G(s)}{1 + G(s)} = \frac{K_p R s^2 + (K_p E + K_i R)s + K_i E}{(K_p R + 1)s^2 + (K_p E + K_i R)s + K_i E}$$

2 Exhalation

Constructing