Week 4 Project

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Question 1

- Calculate and compare the expected value and standard deviation of price at time t (P_t) , given each of the 3 types of price returns, assuming $r_t \sim \; N(0, \sigma^2)$. Simulate each return equation using $r_t \sim \; N(0, \sigma^2)$ and show the mean and standard deviation match your expectations.

Solution

- 1. Brownian Motion

$$E[P_t|P_{t-1}] \ = \ P_{t-1}, \qquad Std[P_t|P_{t-1}] = \sigma$$

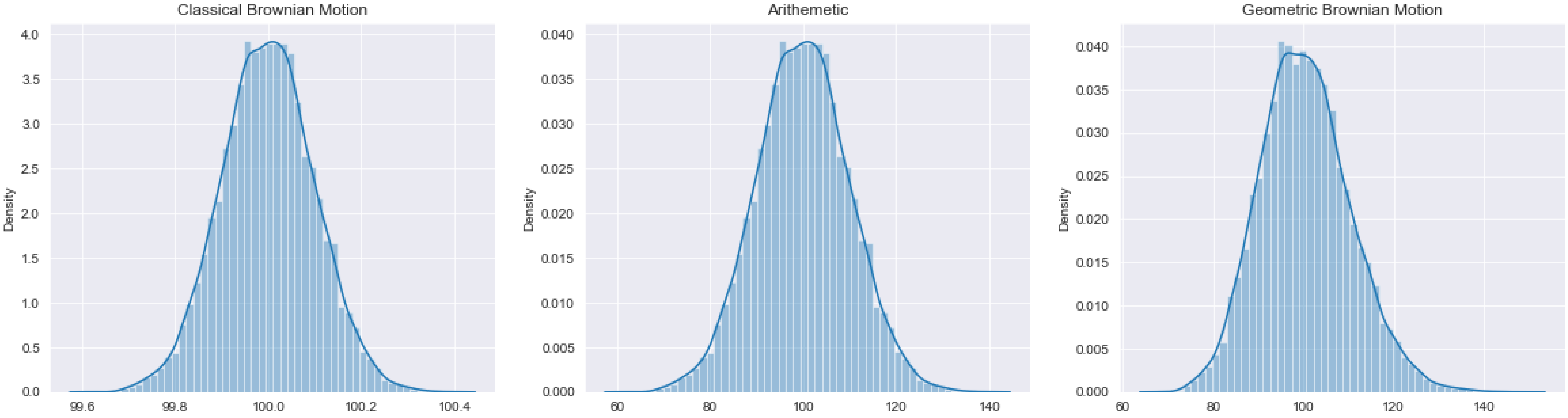
- 2. Arithmetic

$$E[P_t|P_{t-1}] \ = \ P_{t-1}, \qquad Std[P_t|P_{t-1}] = P_{t-1}\sigma$$

- 3. Geometric Brownian Motion

$$E[\ln(P_t)|P_{t-1}] \ = \ \ln(P_{t-1}), \qquad Std[\ln(P_t)|P_{t-1}] = \sigma$$

Distributions



Question 2

- Implement a function similar to the “return_calculate()” in this week’s code. Allow the user to specify the method of return calculation.

- Use DailyPrices.csv. Calculate the arithmetic returns for INTC. Remove the mean from the series so that the mean(INTC)=0. Calculate VaR
  1. Using a normal distribution.
  2. Using a normal distribution with an Exponentially Weighted variance ($\lambda = 0.94$)
  3. Using a MLE fitted T distribution.
  4. Using a Historic Simulation.

- Compare the 4 values. Look at the empirical distribution of returns, in sample.

Download from Yahoo! Finance the prices since the end of the data in the CSV file (about 2 weeks). Look the empirical distribution of returns, out of sample.

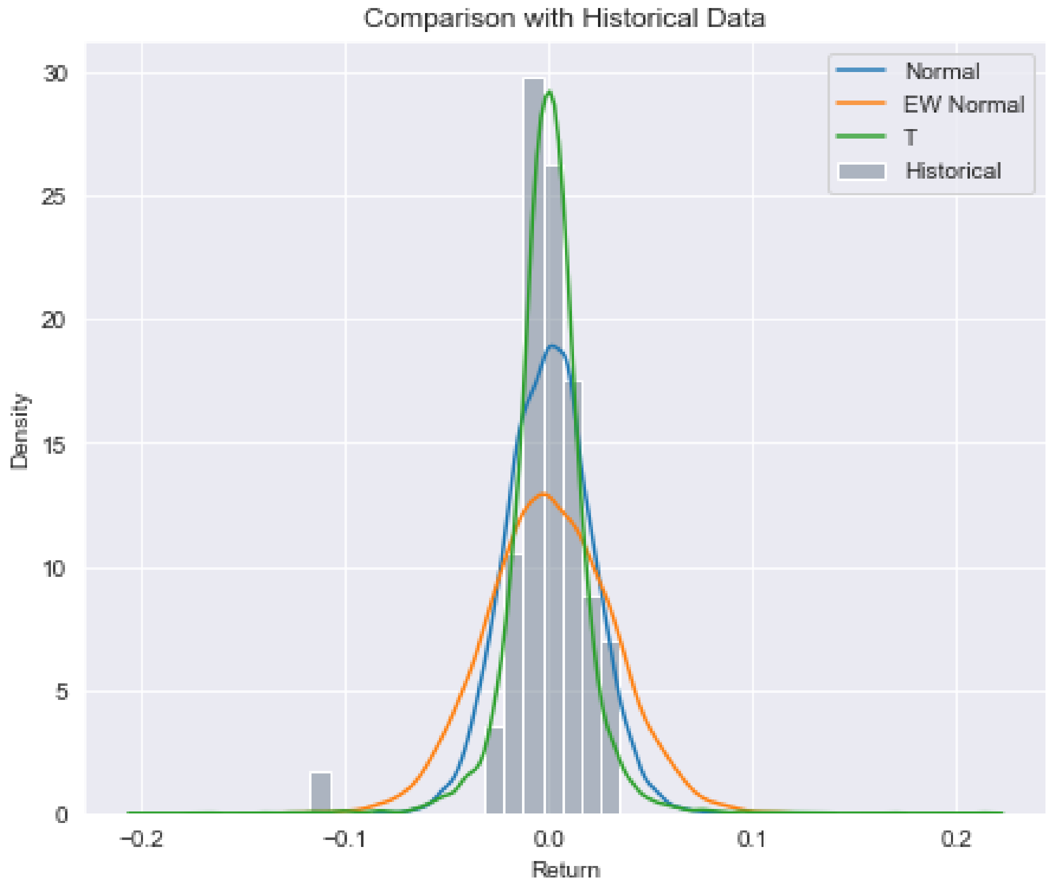
Discuss the ability of these models to describe the risk in this stock.

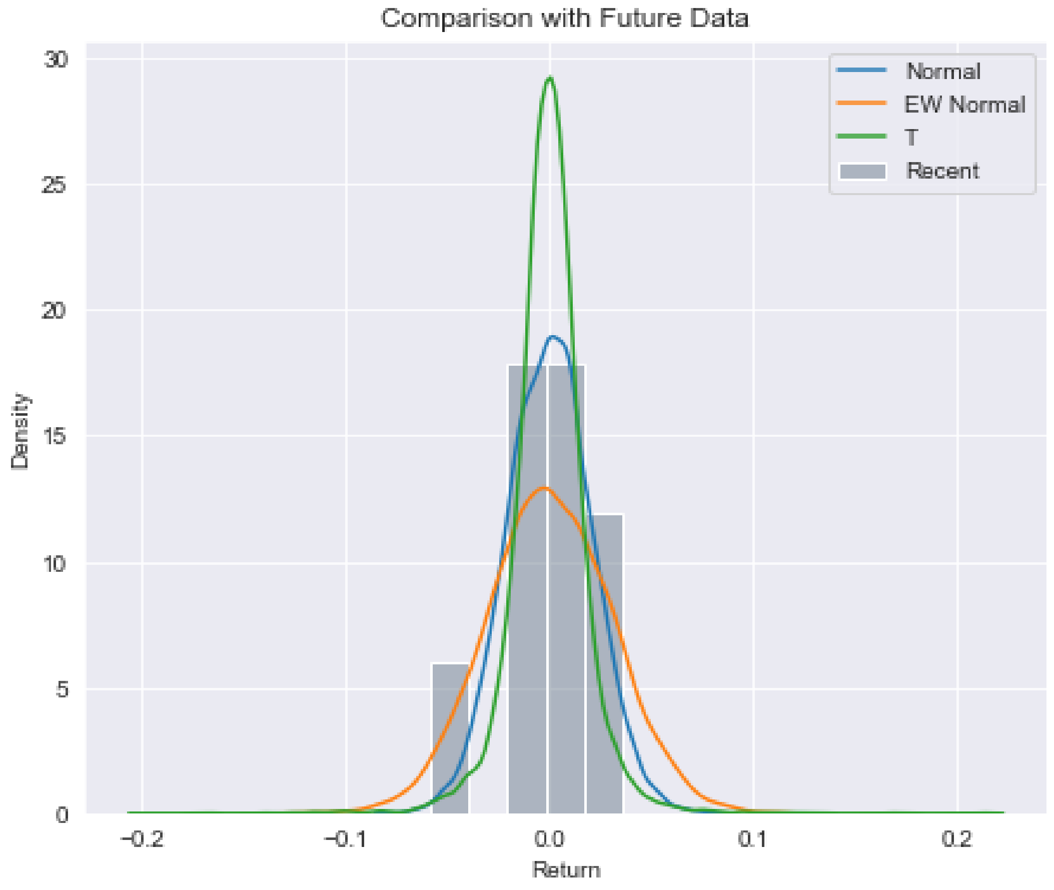
Solution

- VaR values:
  - Historical = 2.07%
  - Normal = 3.39%
  - EW Normal = 5.02%
  - T = 2.74%

- The T simulation is the best to fit the history, but the worst to predict the future.
  The EW Normal simulation is neither good at explaining the past nor predicting the future.
  The Normal simulation is just good enough for both objectives.

How well the simulations fit the data?





Question 3

- Using Portfolio.csv and DailyPrices.csv. Assume the expected return on all stocks is 0.
- This file contains the stock holdings of 3 portfolios.
- You own each of these portfolios. Calculate the VaR of each portfolio as well as your total VaR (VaR of the total holdings).
- Discuss your methods, why you chose those methods, and your results.

Are returns normally distributed?

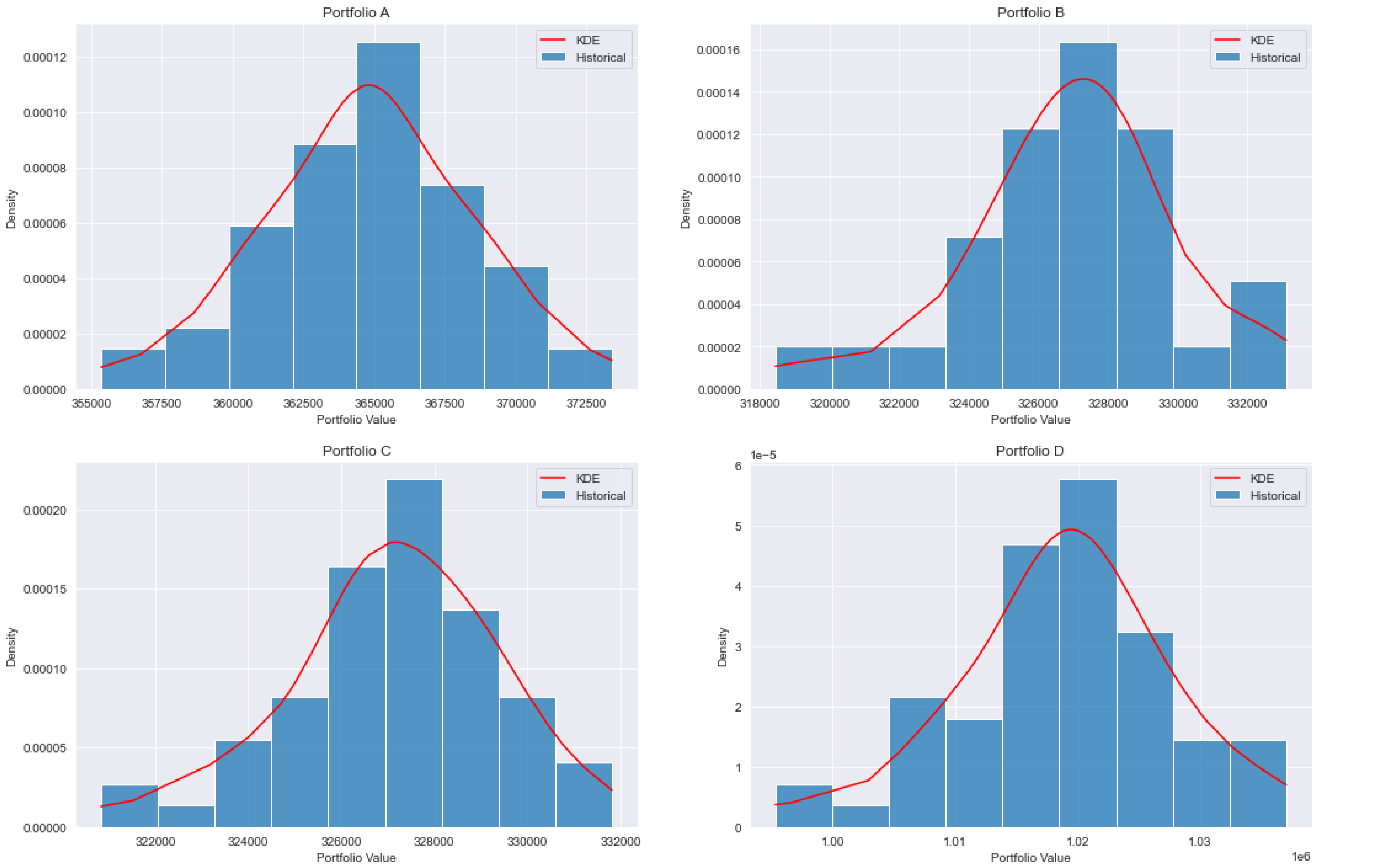
- Apply the Shapiro-Wilk test to each asset's returns. If the return of one asset is not normally distributed, than the returns of the whole portfolio is not likely to follow a multivariate normal distribution.

- Another way is to apply Henze-Zirkler multivariate normality test on the whole return matrix.

- Both methods show the returns are not normally distributed

Using Historical Data and KDE

- use the historical data to calculate the VaR
- Because of the short history, use KDE to create a continuous distribution.
- VaR:
  - Portfolio A: 6364.90
  - Portfolio B: 5730.95
  - Portfolio C: 3986.19
  - Total Holdings: 15427.42



simulative portfolio value distribution and the KDE-smoothed distribution