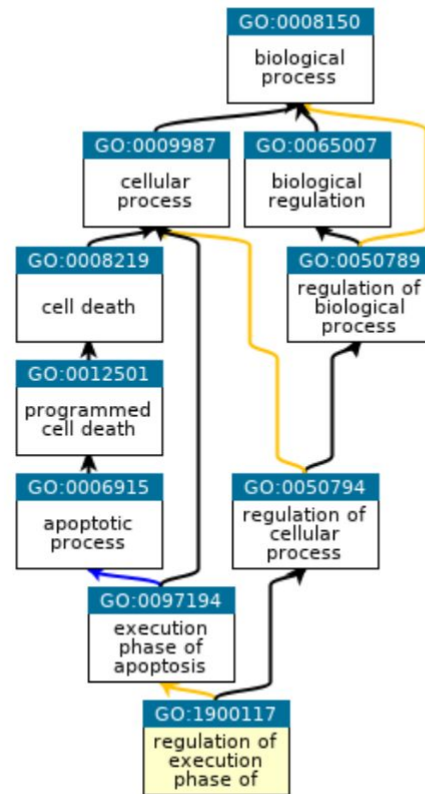


Graph embeddings

The 'Graph' part

Treat the ontology as a graph

Vertices and edges

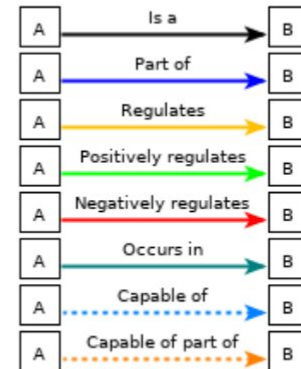
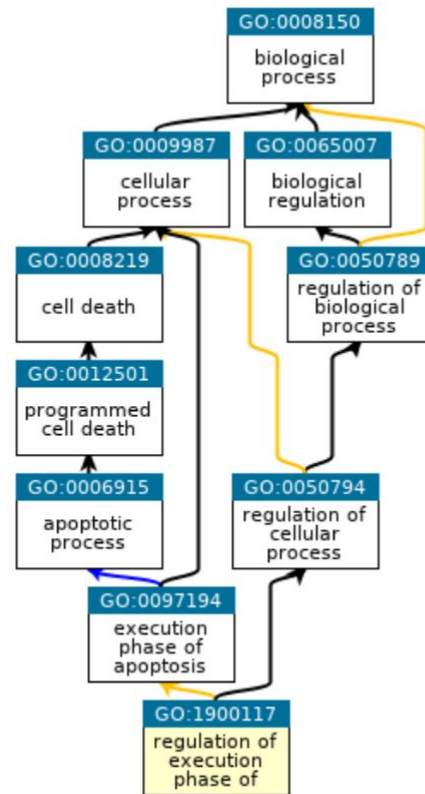


The 'Graph' part

Different graphs

For example:

- Using different relations

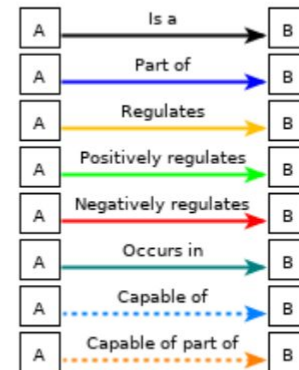
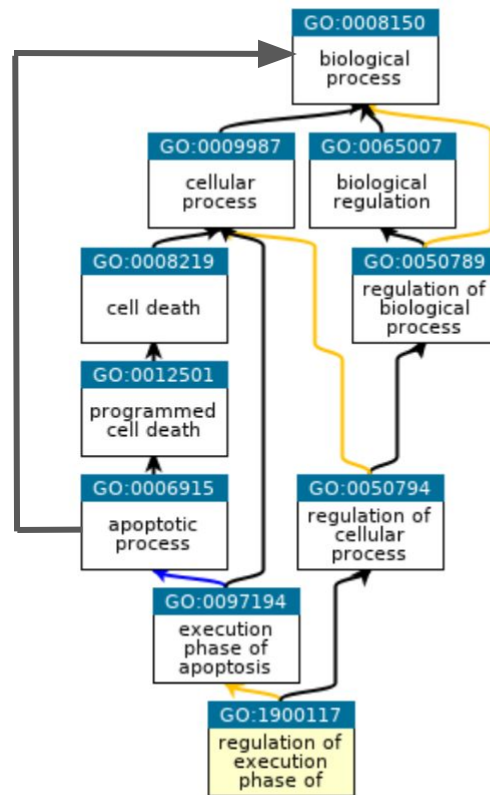


The 'Graph' part

Different graphs

For example:

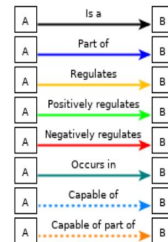
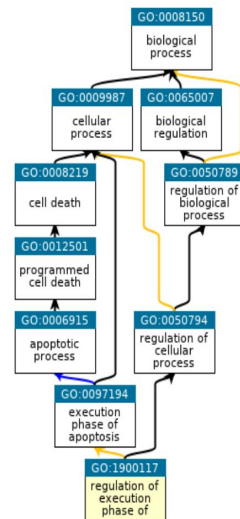
- Using different relations
- Reasoning (Deductive closure)



The ‘embeddings’ part

Definition

Let $KG = (V, E, L; \vdash)$ be an ontology graph with a set of vertices V , a set of edges $E \subseteq V \times V$, a label function $L : V \cup E \mapsto Lab$ that assigns labels from a set of labels Lab to vertices and edges, and an inference relation \vdash . An ontology graph embedding is a function $f_\eta : L(V) \cup L(E) \mapsto \mathbf{R}^n$.

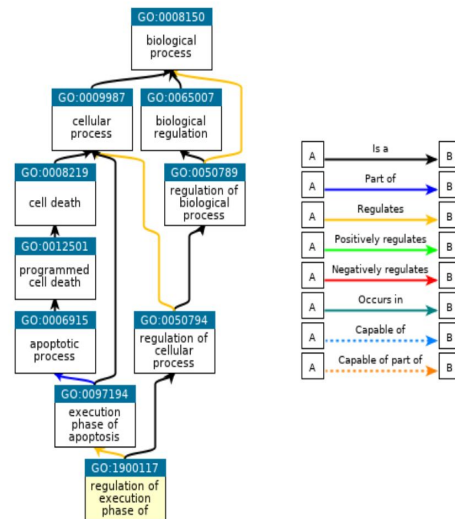


What is an embedding?

Definition

Let $KG = (V, E, L; \vdash)$ be an ontology graph with a set of vertices V , a set of edges $E \subseteq V \times V$, a label function $L : V \cup E \mapsto Lab$ that assigns labels from a set of labels Lab to vertices and edges, and an inference relation \vdash . An ontology graph embedding is a function $f_\eta : L(V) \cup L(E) \mapsto \mathbf{R}^n$.

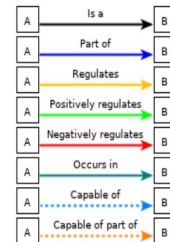
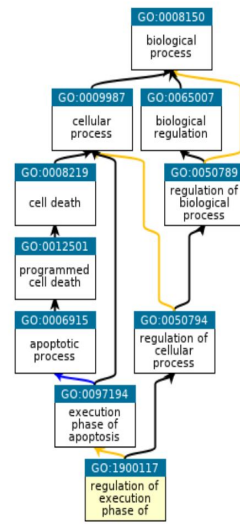
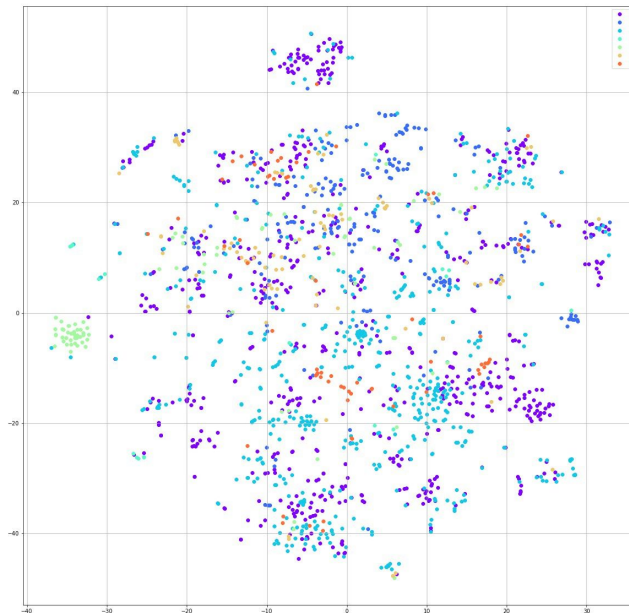
- ★ Preserve some structure of the graph in \mathbf{R}^n
- ★ This structure is preserved under operations in \mathbf{R}^n
- ★ These operations represent operations between the entities (connectedness, similarity, and others)



What is an embedding?

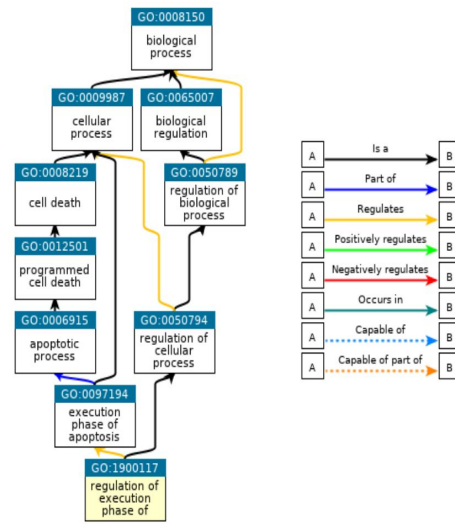
You can think of this as:

A mapping of entities to some numerical representation in some R^n



Why embeddings?

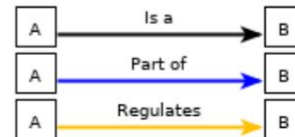
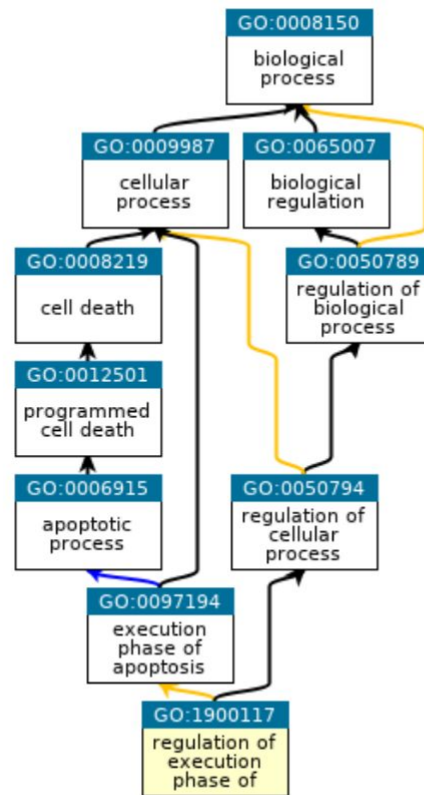
- Represent entities in a compact dimension
- Visualize entities and their relations
- Cluster entities
- Compute semantic similarity



Graph Embeddings

We'd like to capture:

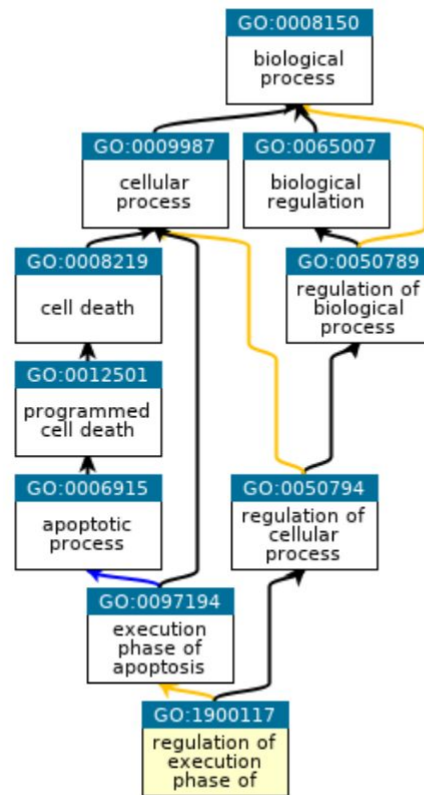
- Graph structure
- Adjacency
- Hub-nodes and communities



How do we capture the graph?

To capture the graph:

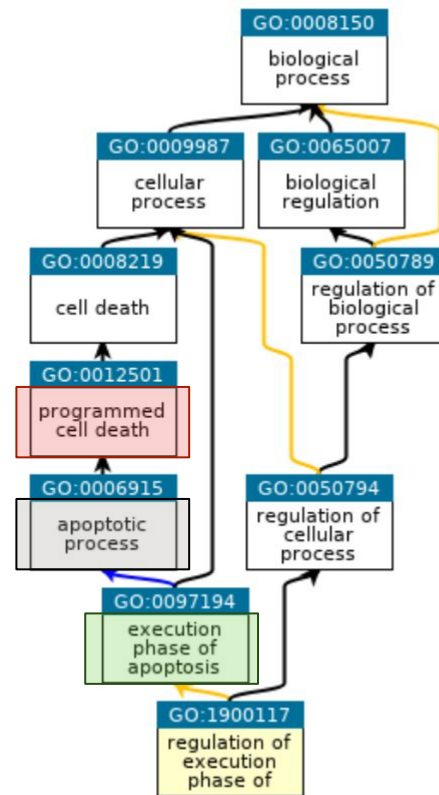
Traverse it using *walks*



How do we capture the graph?

To capture the graph:

Traverse it using *walks*

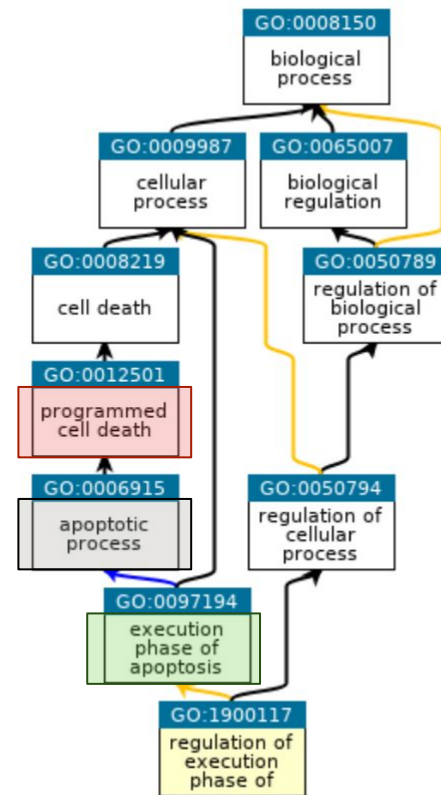


GO:1900117 **regulates** GO:0097194 **Part of** GO:0006915 **is a** GO:0012501

Why walks?

Exploring!

For a given node, walks will:



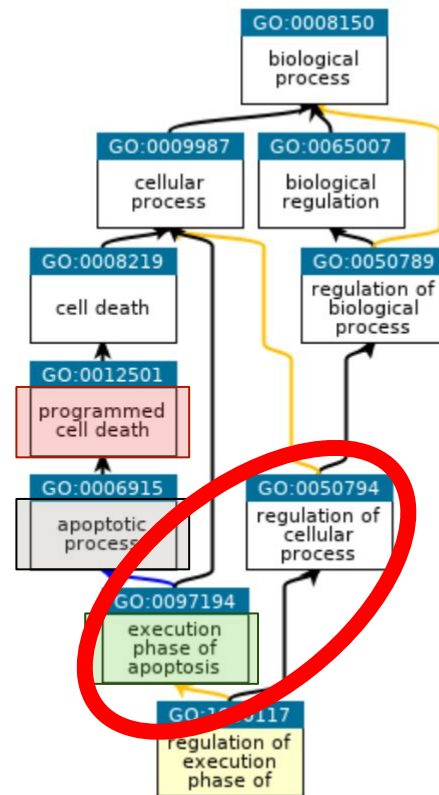
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Why walks?

Exploring!

For a given node, walks will:

Capture directly adjacent nodes very well



GO:1900117 **regulates** GO:0097194 **Part of** GO:0006915 **is a** GO:0012501

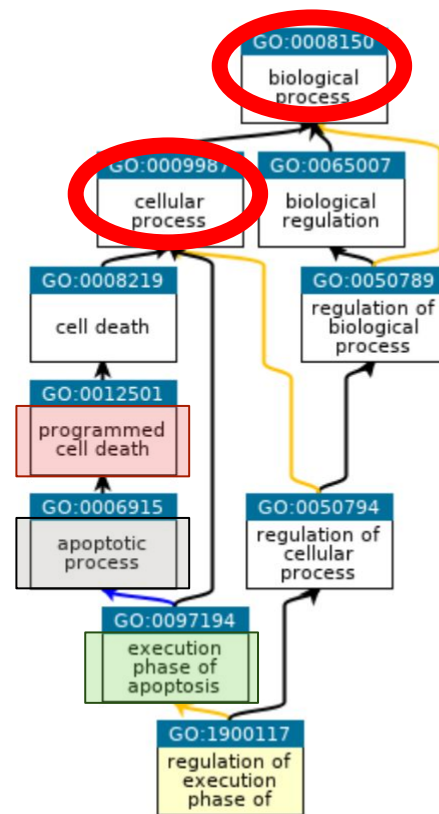
Why walks?

Exploring!

For a given node, walks will:

Capture directly adjacent nodes very well

Capture nodes with multiple paths better



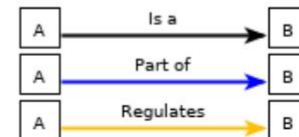
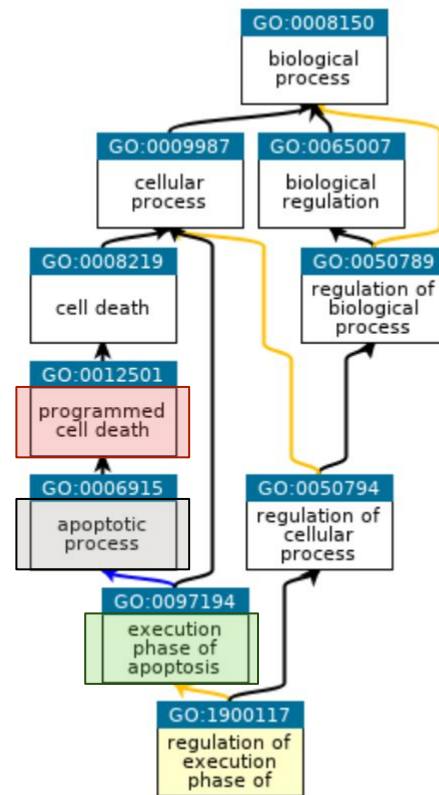
GO:1900117 **regulates** GO:0097194 **Part of** GO:0006915 **is a** GO:0012501

Walks

Variables to consider:

Walk length

- How long should you go?
- How many nodes and edges?



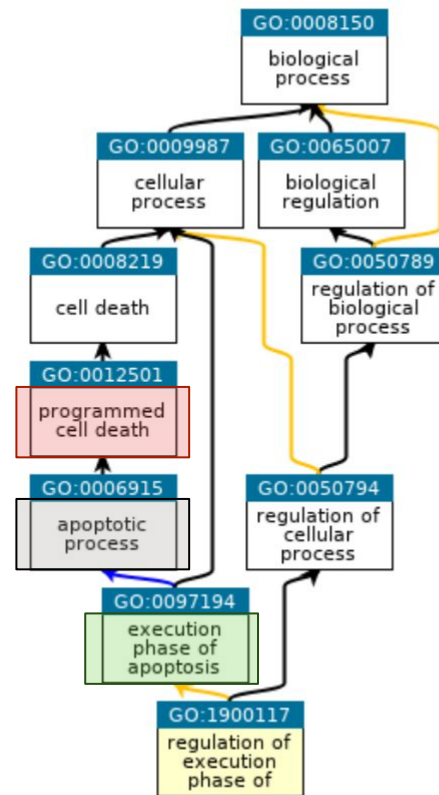
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Walks

Variables to consider:

Walk direction

- Randomly (Random walk)
- With a bias (Node2Vec)



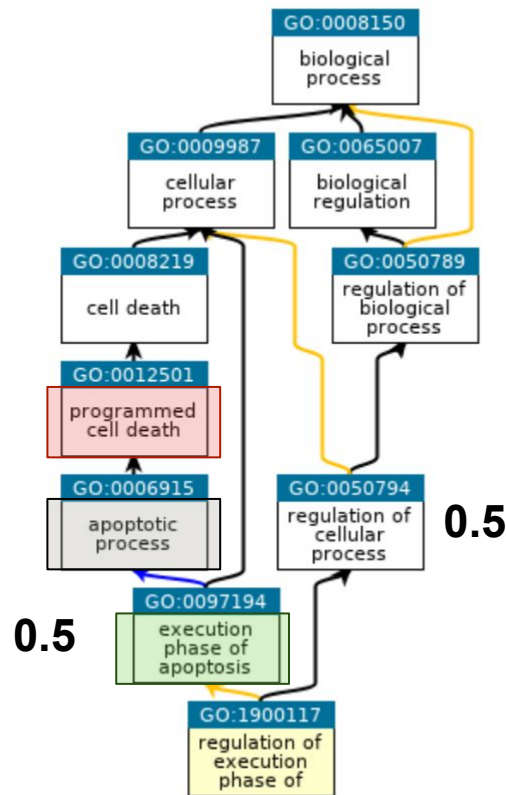
GO:1900117 **regulates** GO:0097194 **Part of** GO:0006915 **is a** GO:0012501

Walks

Variables to consider:

Walk direction

- Randomly (Random walk)
 - Candidate nodes have equal probabilities of getting chosen



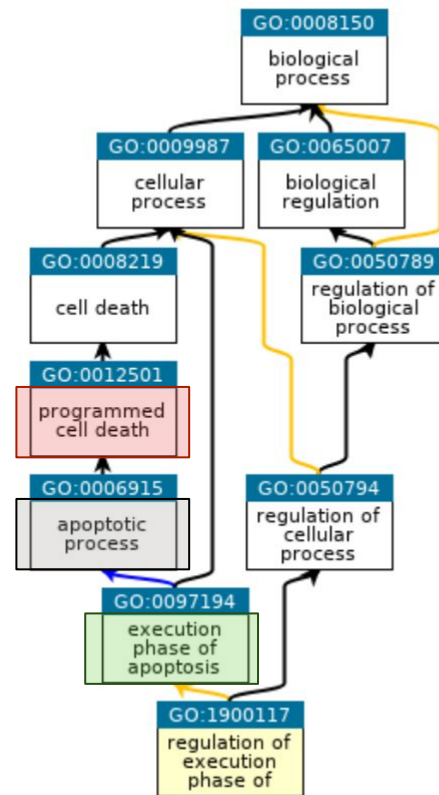
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Walks

Variables to consider:

Walk direction

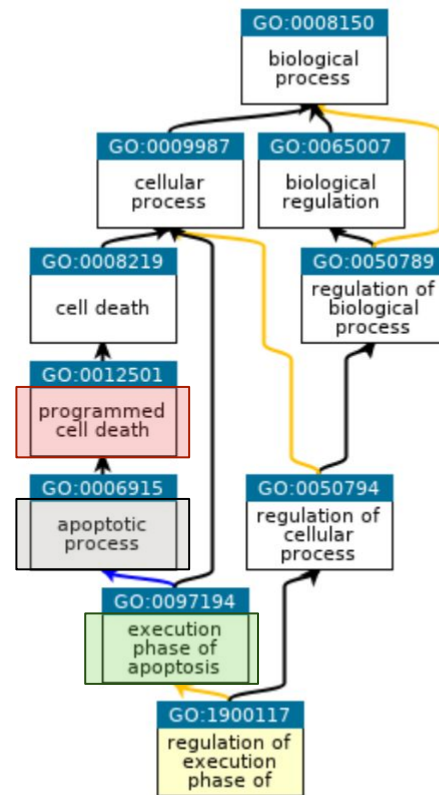
- With a bias (Node2Vec)
 - P probability to go back to previous node
 - Q probability to explore further nodes



GO:1900117 **regulates** GO:0097194 **Part of** GO:0006915 **is a** GO:0012501

Walks → embeddings

How do we go from walks to embeddings?



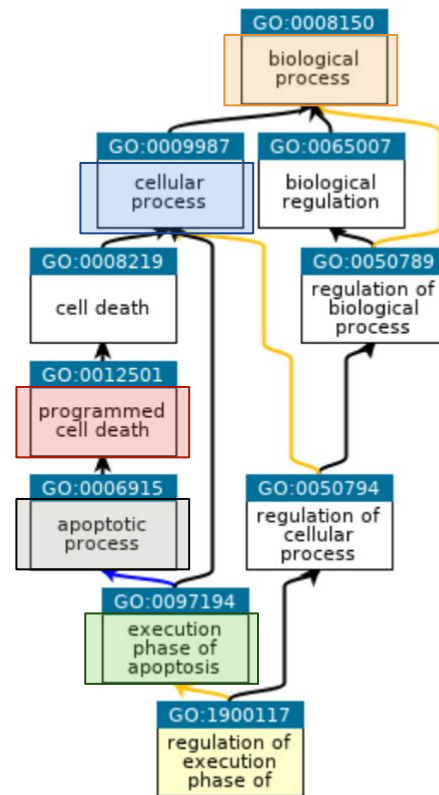
GO:1900117 **regulates** GO:0097194 **Part of** GO:0006915 **is a** GO:0012501

Walks → embeddings

How do we go from walks to embeddings?

- Generate many such walks

(Many walks will give us an idea about the graph adjacency and structure)



GO:1900117 **regulates** GO:0097194 **Part of** GO:0006915 **is a** GO:0012501

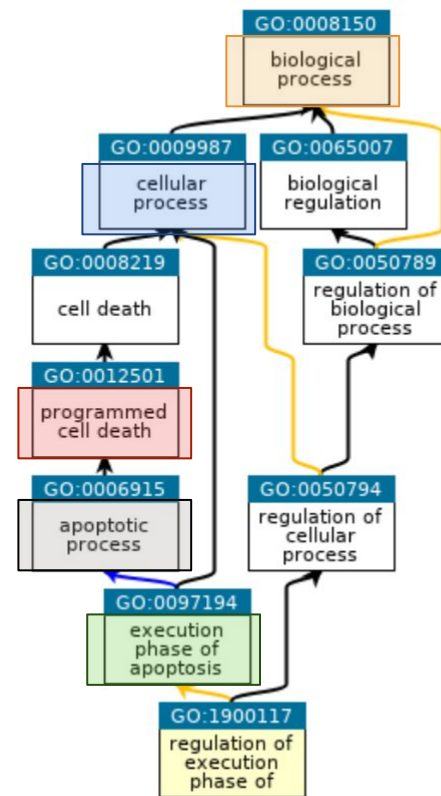
...

GO:1900117 **regulates** GO:0097194 **is a** GO:0009987 **is a** GO:0008150

Walks → embeddings

How do we go from walks to embeddings?

- Generate many such walks
- Each walk is a sentence
- Sentences form a corpus



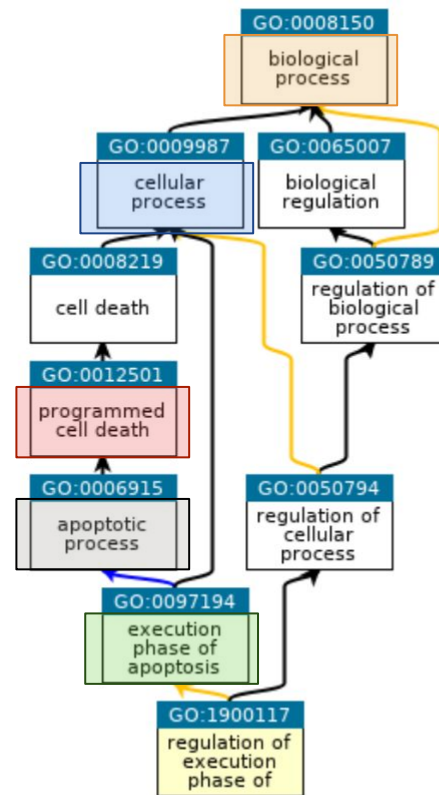
GO:1900117 **regulates** GO:0097194 **Part of** GO:0006915 **is a** GO:0012501

...

GO:1900117 **regulates** GO:0097194 **is a** GO:0009987 **is a** GO:0008150

Word2Vec

- Well-known method
- Generates embeddings that capture co-occurrences based on a corpus
- Embeddings are in the form of n -dimensional vectors



GO:1900117 **regulates** GO:0097194 **Part of** GO:0006915 **is a** GO:0012501

...

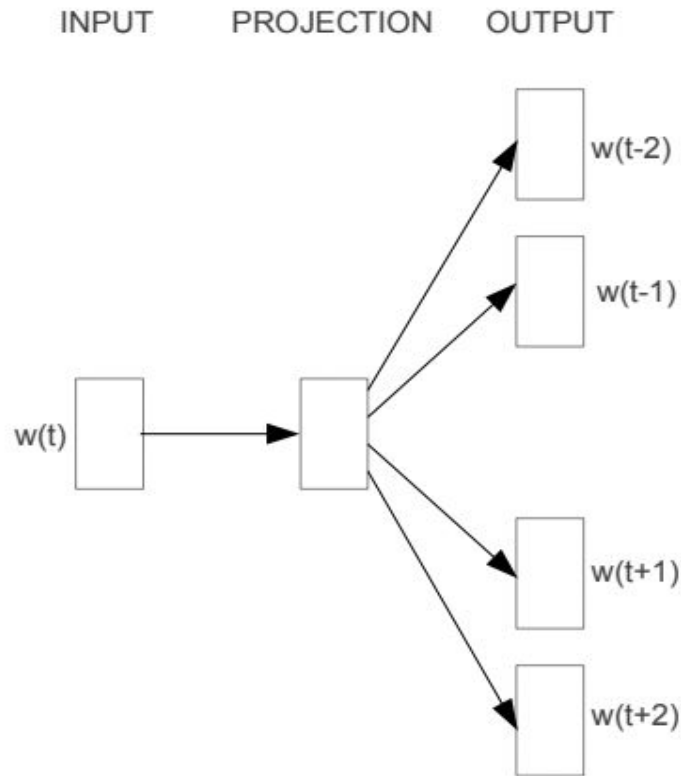
GO:1900117 **regulates** GO:0097194 **is a** GO:0009987 **is a** GO:0008150

Word2Vec

Word2Vec captures co-occurrences

Given a node:

- Capture the nodes it frequently co-occurred with in the given walks



Skip-gram

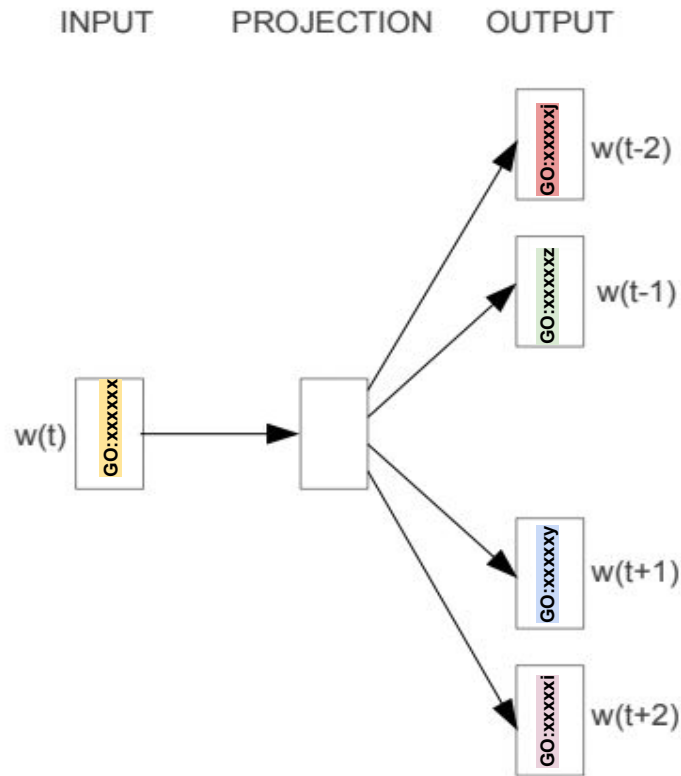
Figure from the original paper: Efficient Estimation of Word Representations in Vector Space, Mikolov et al.

Word2Vec

Word2Vec captures co-occurrences

Given a node:

- Capture the nodes it frequently co-occurred with in the given walks
- Minimize the cross-entropy loss



Skip-gram

Figure from the original paper: Efficient Estimation of Word Representations in Vector Space, Mikolov et al.

Word2Vec

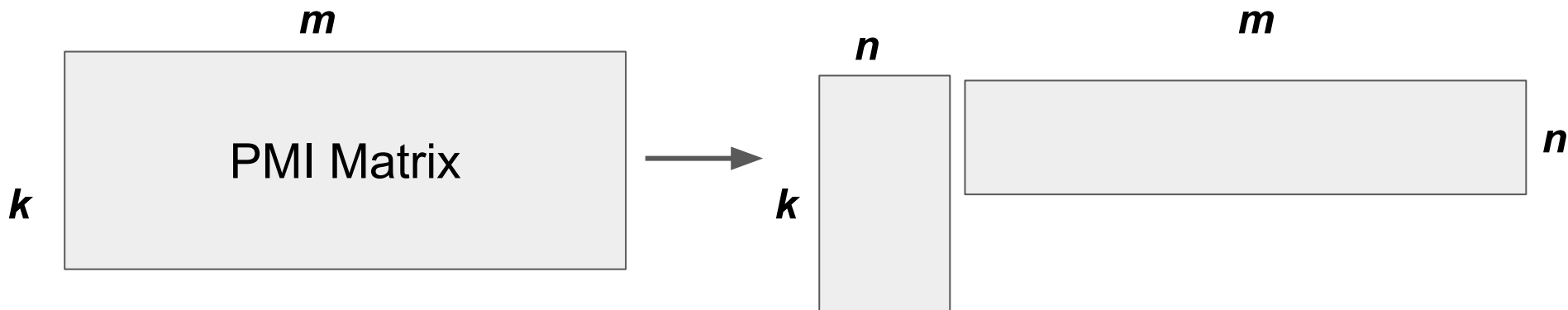
- You can think of it as a factorization of a Pointwise Mutual Information (PMI) matrix

	GO:xxxxxx	GO:xxxxxj	GO:xxxxxz	GO:xxxxxy	GO:xxxxxi
GO:xxxxxx	0	2	1	1	4
GO:xxxxxj	2	0	1	1	2
GO:xxxxxz	1	1	0	2	2
GO:xxxxxy	1	1	2	0	3
GO:xxxxxi	4	2	2	3	0

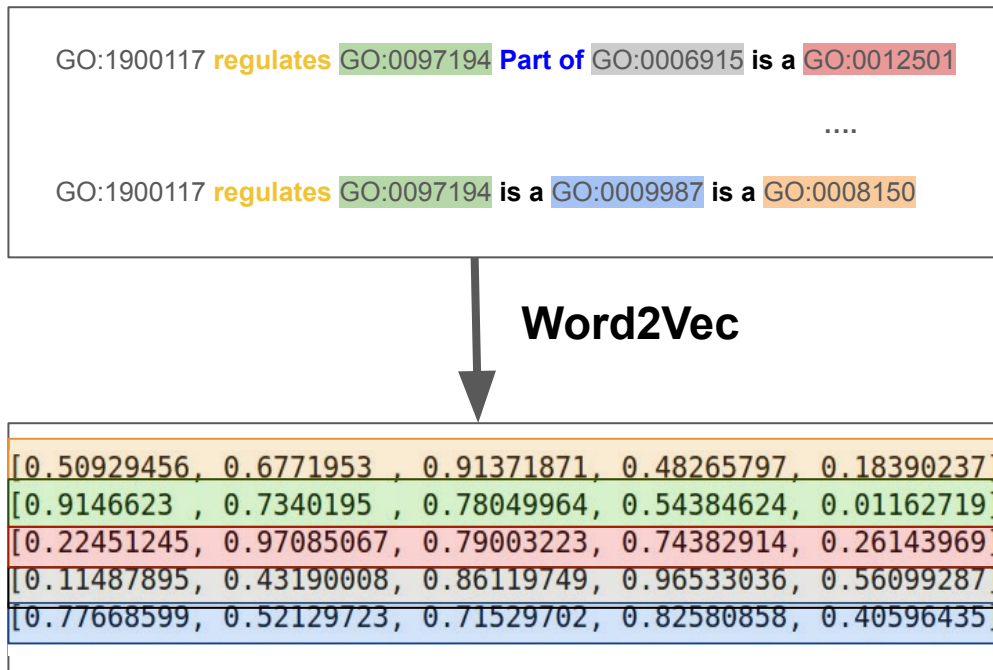
$$\text{pmi}(x; y) \equiv \log \frac{p(x, y)}{p(x)p(y)}$$

Word2Vec

You can think of it as an efficient factorization of a Pointwise Mutual Information (PMI) matrix



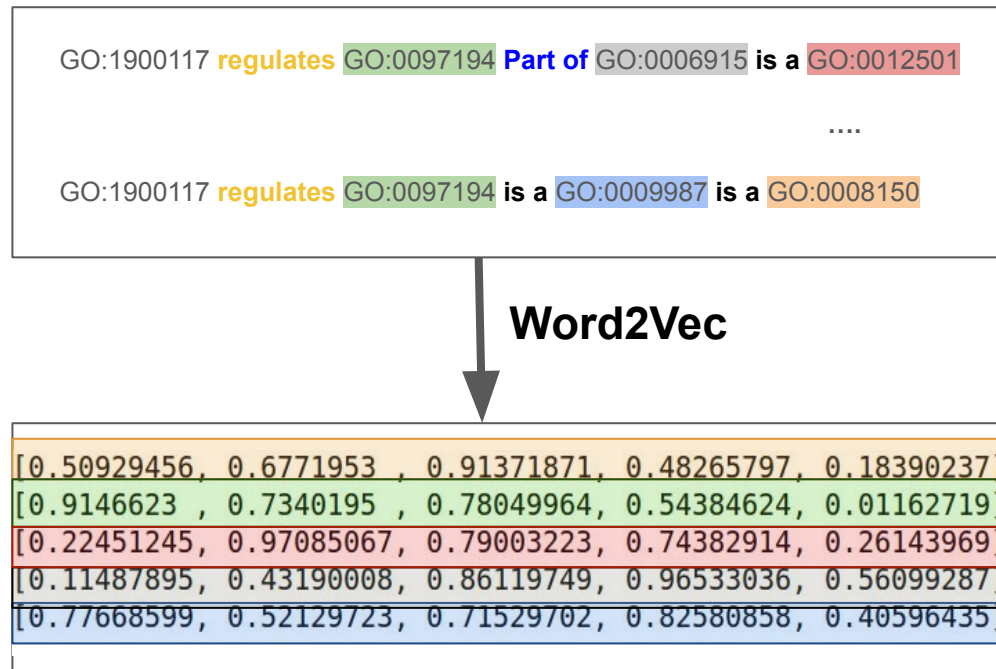
Walks → embeddings



What do these vectors capture?

Remember: Co-occurrence

- The graph structure
- Adjacency
- Hub-nodes and communities



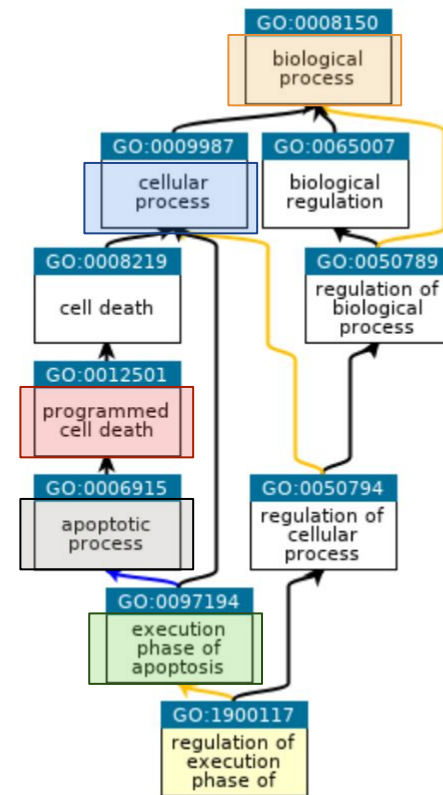
More parameters

Again we are faced with more parameters:

- How many walks per node?
- Restart probability

Recall:

- Length of walk



GO:1900117 **regulates** GO:0097194 **Part of** GO:0006915 **is a** GO:0012501

...

GO:1900117 **regulates** GO:0097194 **is a** GO:0009987 **is a** GO:0008150

Ontology graph embedding methods

Many methods

- OWL2Vec
 - Recall

Axiom of condition 1	Axiom or triple(s) of condition 2	Projected triple(s)
$A \sqsubseteq \Box r.D$ or $\Box r.D \sqsubseteq A$	$D \equiv B \mid B_1 \sqcup \dots \sqcup B_n \mid B_1 \sqcap \dots \sqcap B_n$	$\langle A, r, B \rangle$ or
$\exists r.T \sqsubseteq A$ (domain) $A \sqsubseteq \exists r.\{b\}$	$T \sqsubseteq \forall r.B$ (range) $B(b)$	$\langle A, r, B_i \rangle$ for $i \in 1, \dots, n$
$r \sqsubseteq r'$ $r' \equiv r^-$	$\langle A, r', B \rangle$ has been projected $\langle B, r', A \rangle$ has been projected	
$s_1 \circ \dots \circ s_n \sqsubseteq r$ $B \sqsubseteq A$	$\langle A, s_1, C_1 \rangle \dots \langle C_n, s_n, B \rangle$ have been projected –	$\langle B, rdfs:subClassOf, A \rangle$ $\langle A, rdfs:subClassOf^-, B \rangle$
$A(a)$	–	$\langle a, rdf:type, A \rangle$ $\langle A, rdf:type^-, a \rangle$
$r(a, b)$	–	$\langle a, r, b \rangle$

Ontology graph embedding methods

Many methods

- DL2Vec

Condition 1	Condition 2	Triple(s)
$A \sqsubseteq QR_0$ $\dots QR_m D$ A $\equiv QR_0$ $\dots QR_m D$	$D :$ $= B_1 \sqcup$ $\dots \sqcup B_n$ $ $ $B_1 \sqcap$ $\dots \sqcap B_n$	$\langle A,$ $(R_0$ $\dots R_m),$ $B_i \rangle$ for $i \in 1$ $\dots n$
$A \sqsubseteq B$		$\langle A,$ $SubClassOf,$ $B \rangle$
$A \equiv B$		$\langle A,$ $EquivalentTo,$ $B \rangle$

Ontology graph embedding methods

