

# Hierarchical Task Allocation and Planning for Multi-Robots under Hierarchical Temporal Logic Specifications

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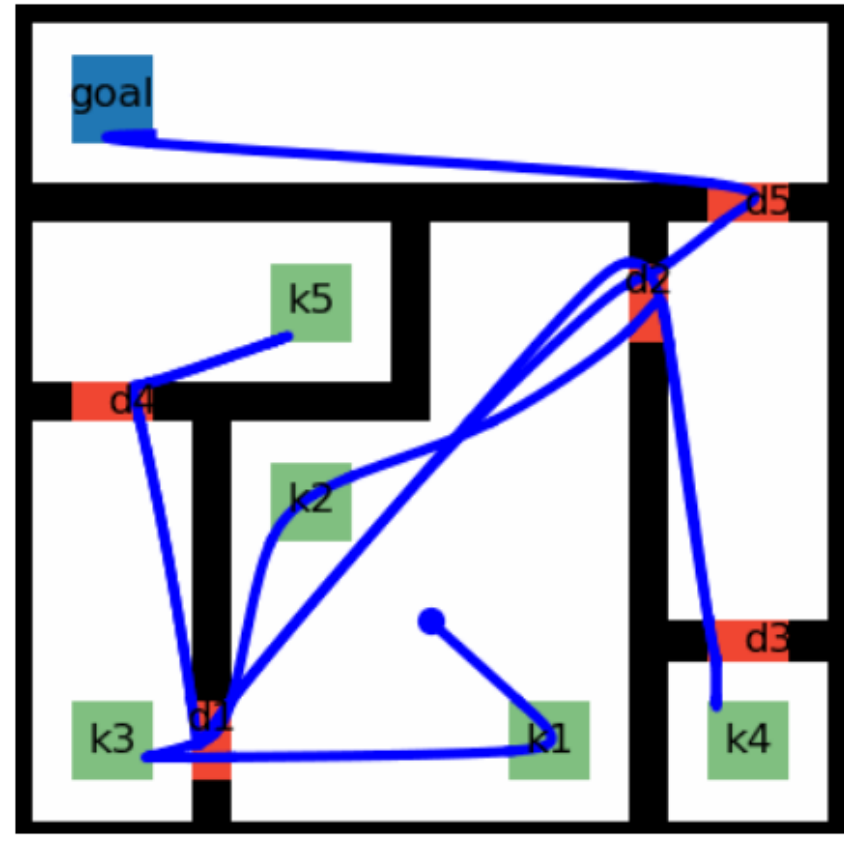
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## Introduction

### Motivation:

- Linear temporal logic (LTL) is expressive for long-term tasks but struggles with computational scalability.



**Task:** Pick up five keys in order to pass through corresponding doors before eventually reaching a goal.

⚠ Conversion from LTL to automaton required 32 minutes  
(Intel i7 CPU and 32-GB RAM)

## Hierarchical sc-LTL

Luo et al. Arxiv 2024

- Hierarchical syntactically co-safe LTL (sc-LTL) improves computational efficiency by structuring the specifications in a hierarchical manner.
- There are currently no planning algorithms for hierarchical sc-LTL that explicitly account for inter-robot interactions.

**Contribution:** To the best of knowledge, our work is **the first one** that can tackle multi-robot collaboration under hierarchical LTL specifications.

## Problem Formulation

### Hierarchical sc-LTL for multi-robot pickup and delivery:

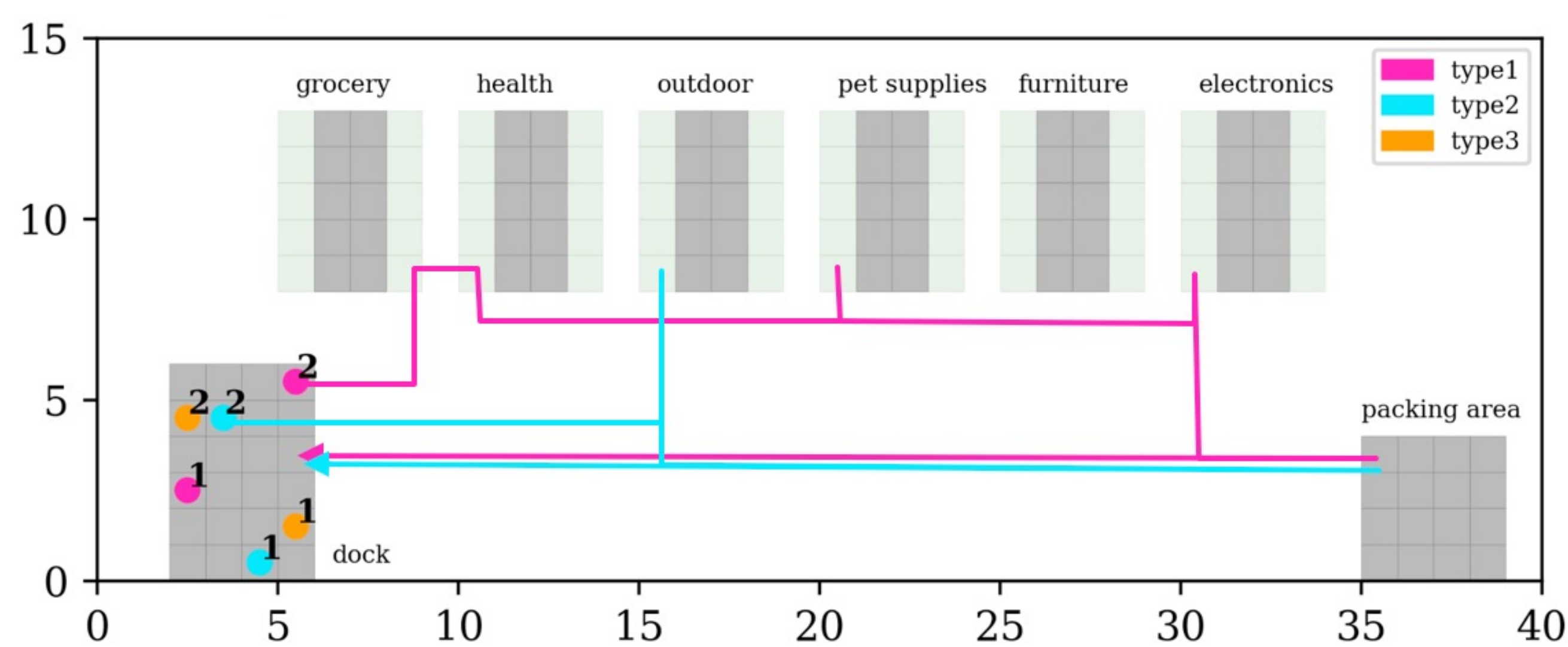


Figure 1: Topological map of the supermarket.

- A type 1 robot gathers items from the health, grocery, electronics and pet sections **in any order**;
- Either** a type 2 robot **or** a type 3 robot gathers items from the outdoor section;
- After** the items are delivered to the packing area, all robots **eventually** return to the dock.

### Hierarchical sc-LTL:

$$\begin{aligned} \text{high level} \\ \phi(1,1) &= \Diamond (\phi(2,1) \wedge \Diamond \phi(2,2)) \\ &\quad \wedge \Diamond (\phi(2,3) \vee \phi(2,4)) \end{aligned}$$

$$\begin{aligned} \text{low level} \\ \phi(2,1) &= \Diamond \pi_{pet}^1 \wedge \Diamond \pi_{electronics}^1 \wedge \Diamond \pi_{health}^1 \wedge \Diamond \pi_{furniture}^1 \\ \phi(2,2) &= \Diamond (\pi_{pack}^1 \wedge \Diamond \pi_{dock}^1) \\ \phi(2,3) &= \Diamond (\pi_{outdoor}^2 \wedge \Diamond (\pi_{pack}^2 \wedge \Diamond \pi_{dock}^2)) \\ \phi(2,4) &= \Diamond (\pi_{outdoor}^3 \wedge \Diamond (\pi_{pack}^3 \wedge \Diamond \pi_{dock}^3)) \end{aligned}$$

**Problem formulation:** Given domain of multi-robot navigation or manipulation and a task represented by hierarchical sc-LTL specifications, the goal is to generate a satisfying plan.

## Decomposition-based Planning

### Overview:

#### Task allocation

What robot to do which subtask at what time step  
(e.g., robot 1 of type 1 reaches pet section at time 10)

Mixed Integer Linear Programming (MILP)



#### Motion planning

How to carry out the subtask while abiding by the temporal relations.

### Task decomposition:

- Extract the temporal relations between sub-tasks for each specification;
- Deduce the temporal relations between sub-tasks across different specifications to construct the task network;



Figure 2: Sub-tasks along with temporal relations in specification  $\phi(2,2)$ .

### Task network:

- Sub-tasks inherit the temporal relations of their respective composite propositions at the upper level.

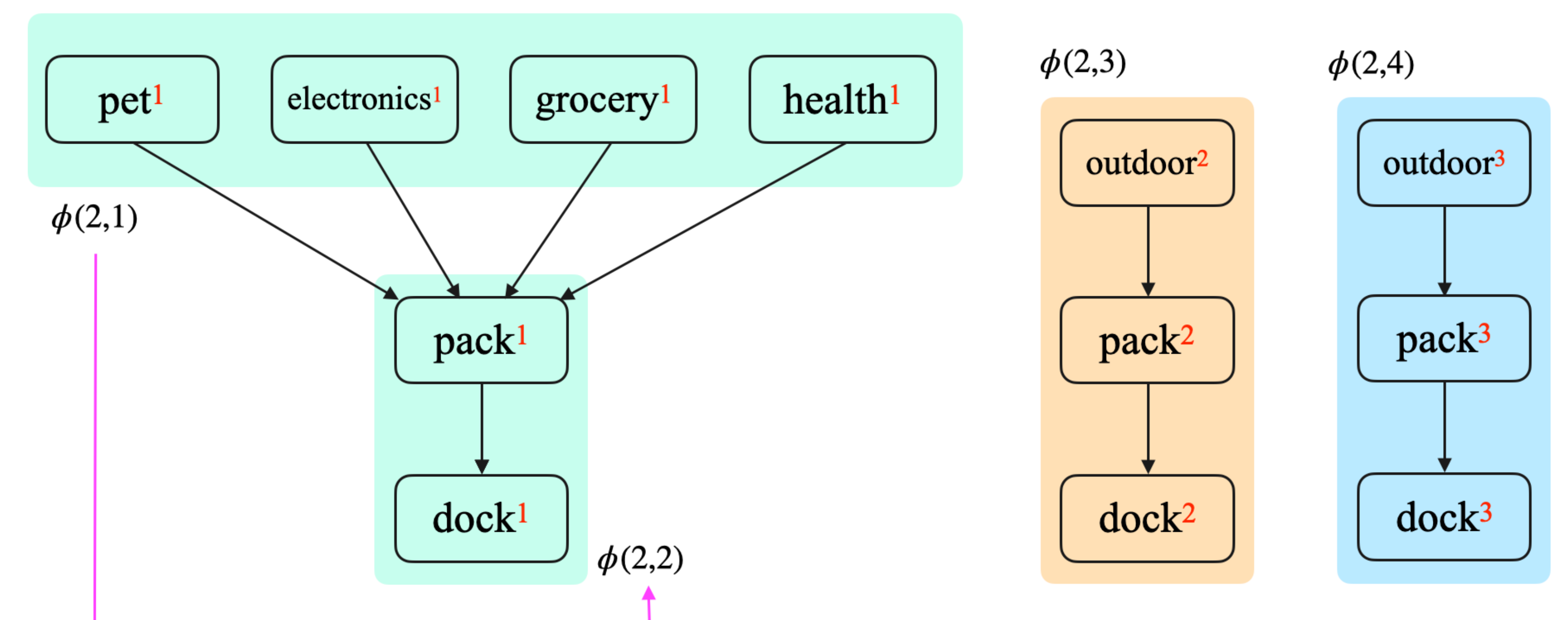


Figure 3: Task network consisting of all sub-tasks at the lowest level.

## Simulation Results

### Navigation:

task	$l_{std}$	$l_{hier}$	$\mathcal{A}_{std}$	$\mathcal{A}_{hier}$	$t_{std}$	$t_{hier}$	$c_{std}$	$c_{hier}$
1	51	35	(387, 13862)	(33, 45)	126.1±2.7	18.4±0.8	289.3±3.0	237.5±3.3
2	45	19	(12, 63)	(20, 28)	30.8±0.5	24.8±0.4	115.6±1.9	114.8±1.6
3	35	27	(113, 1891)	(35, 101)	25.8±1.8	21.4±0.5	148.1±2.1	147.7±2.1

Table 1: Comparative results.  $l_{std}$ ,  $l_{hier}$ : length of standard and hierarchical sc-LTL formulas,  $\mathcal{A}$ : size of automata (nodes, edges),  $t$ : runtimes (seconds),  $c$ : plan horizons.

### Manipulation:

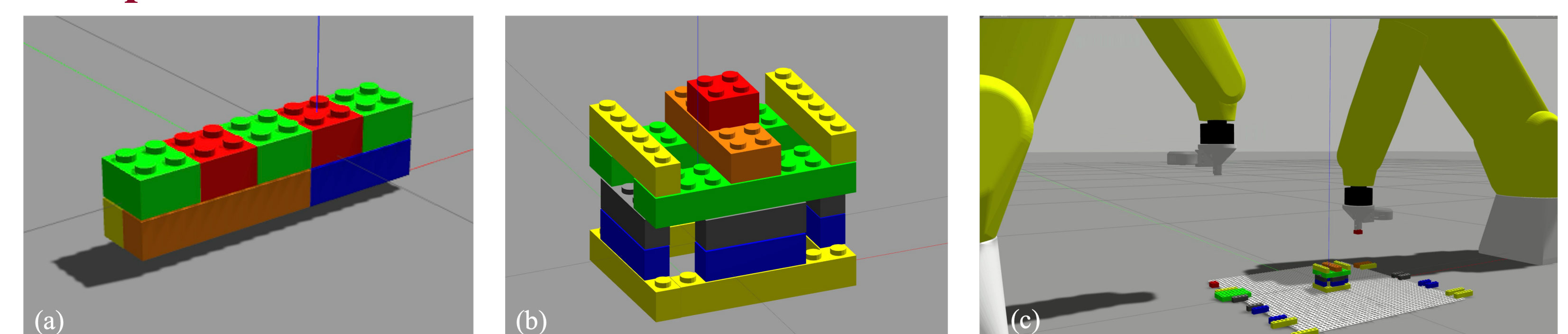


Figure 4: Collaborative construction of LEGO models. Time: (a) 4.9s (b) 4.8s