

### 5.3 基本习题

#### 1. 选择题:

(1) 设  $\varphi(x) = \int_0^x f(t)dt$ , 则下列结论正确的是 ( ).

- A. 当  $f(x)$  为奇函数时,  $\varphi(x)$  为偶函数      B. 当  $f(x)$  为奇函数时,  $\varphi(x)$  为奇函数  
C. 当  $f(x)$  为偶函数时,  $\varphi(x)$  为偶函数      D. 以上都不对

#### 2. 填空题:

(1)  $\int_{-2}^2 \frac{x^4 \sin x}{x^2 + 1} dx =$  \_\_\_\_\_; (2)  $\int_{-1}^1 (x^2 + \sin x)|x|dx =$  \_\_\_\_\_;

(3)  $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 4 \cos^4 x dx =$  \_\_\_\_\_;

(4) 已知  $\int_{-a}^a (2x + \arctan x - 1)dx = -4$ , 则  $a =$  \_\_\_\_\_;

(5) 设  $f(x)$  为连续函数, 则  $\int_{\frac{1}{2}}^2 (1 - \frac{1}{x^2})f'(x + \frac{1}{x})dx =$  \_\_\_\_\_;

(6) 设  $M = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin x}{1+x^2} \cos^4 x dx$ ,  $N = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\sin^3 x + \cos^4 x)dx$ ,

$P = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (x^2 \sin^3 x - \cos^4 x)dx$ , 则从小到大的顺序为\_\_\_\_\_.

(7) 设  $f''(x)$  为连续函数, 且  $f'(b) = f'(a) = 0$ , 则  $\int_a^b xf''(x)dx =$  \_\_\_\_\_.

#### 3. 计算:

(1)  $\int_1^9 \frac{1}{1+\sqrt{x}} dx$  ;

(2)  $\int_0^2 \frac{1}{\sqrt{1+x} + \sqrt{(x+1)^3}} dx$ ;

(3)  $\int_1^2 \frac{1}{x^2 \sqrt{1+x^2}} dx$  ;

(4)  $\int_0^\pi x \sin^2 x dx$  ;

(5)  $\int_0^4 e^{\sqrt{2x+1}} dx$  ;

(6)  $\int_1^{16} \arctan \sqrt{\sqrt{x}-1} dx$  ;

(7)  $\int_1^e \sin(\ln x) dx$  ;

(8)  $\int_0^{\frac{\pi}{2}} e^{2x} \cos x dx$  .

4. 已知  $\int_a^{2\ln 2} \frac{dt}{\sqrt{e^t-1}} = \frac{\pi}{6}$ , 求  $a$  的值.