

第四章总复习题

1. 选择题

(1) $\int |x| dx = (\quad)$.

A. $|x| + C$

B. $x|x| + C$

C. $\frac{1}{2}x^2 + C$

D. $\frac{1}{2}x|x| + C$

(2) 若 $f'(x) = \sin(x^2)$, 则 $\int x f''(x^2) dx = (\quad)$

A. $-\frac{1}{2}\cos(x^4) + c$

B. $-\frac{1}{2}\cos(x^4)$

C. $-\frac{1}{2}\sin(x^4) + c$

D. $\frac{1}{2}\sin(x^4) + c$

(3) 已知 $\sin x$ 是 $f(x)$ 的原函数, 则 $\int \frac{f'(\ln x)}{x} dx = (\quad)$

A. $-\sin(\ln x) + C$

B. $\sin(\ln x) + C$

C. $\cos(\ln x) + C$

D. $-\cos(\ln x) + C$

(4) 设 $I_m = \int x^\alpha \ln^m x dx$, m 为正整数, $\alpha > 0$, 则 $I_m = (\quad)$

A. $\frac{1}{\alpha+1} x^{\alpha+1} \ln^m x - \frac{1}{\alpha+1} I_{m-1}$

B. $x^\alpha I_{m-1} + C$

C. $(\alpha+1)x^{\alpha+1} \ln^m x - (\alpha+1)m I_{m-1} + C$

D. $\frac{1}{\alpha+1} x^{\alpha+1} \ln^m x - \frac{m}{\alpha+1} I_{m-1}$

2. 求下列不定积分

(1) $\int \frac{\cot x}{1 + \sin x} dx;$

(2) $\int \frac{dx}{x\sqrt{1+x^2}};$

(3) $\int e^x \frac{1 + \sin x + \cos x}{(1 + \cos x)^2} dx;$

(4) $\int \frac{1 + \sin x}{\sin x (1 + \cos x)} dx;$

(5) $\int \frac{\arctan e^x}{e^{2x}} dx;$

(6) $\int \frac{x}{\sqrt{3x+1} + \sqrt{2x+1}} dx;$

(7) $\int \frac{\arctan x}{x^2(1+x^2)} dx;$

(8) $\int \frac{dx}{\sqrt{x(4-x)}};$

(9) $\int x^2 \sqrt{1-x^2} dx;$

(10) $\int \frac{dx}{(2x^2+1)\sqrt{x^2+1}};$

3. 设 $f(x) = \begin{cases} \sin 2x, & x < 0 \\ 0, & x = 0 \\ \ln(2x+1), & x > 0 \end{cases}$ 求 $\int f(x)dx$.

4. 设 $f(\ln x) = \frac{\ln(1+x)}{x}$ 计算不定积分 $\int f(x)dx$.

5、设 $f(x)$ 是一可导函数，且 $f(x)$ 的反函数 $f^{-1}(x)$ 存在，则

$$\int f^{-1}(x)dx = xf^{-1}(x) - F(f^{-1}(x)) + C, \text{ 其中 } F(x) \text{ 为 } f(x) \text{ 的一个原函数.}$$