

习题 7.5

1. 填空题

(1) 微分方程 $\frac{d^3 y}{dx^3} = e^{ax} + x^b (a \neq 0, b > 0)$ 的通解为_____.

(2) 微分方程 $\frac{d^2 y}{dx^2} = x \sin x$ 的通解为_____.

(3) 微分方程 $(1+x)y'' = y'$ 的通解为_____.

2. 求下列微分方程的通解.

(1) $y'' = \frac{2x}{1+x^2} y'$;

(2) $y \frac{d^2 y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$

(3) $y'' = (y')^3 + y'$;

(4) $y^{(4)} - \frac{1}{x} y^{(3)} = 0$;

3. 求下列方程满足所给初值条件的特解:

(1) $y'' = -2xy'^2$, $y|_{x=0}=1$, $y'|_{x=0}=-\frac{1}{2}$; (2) $y'' = 2y(y')$, $y|_{x=0}=1$, $y'|_{x=0}=2$;

4. 求下列微分方程的通解

(1) $xy'' - y' \ln y' + y' \ln x = 0$

(2) $yy'' - y^2 = (y')^2$