§ 3. 3 条件分布

一、离散型随机变量的条件分布

设 $P{X = x_i, Y = y_i} = p_{ii}$, 对于固定的 i, 称

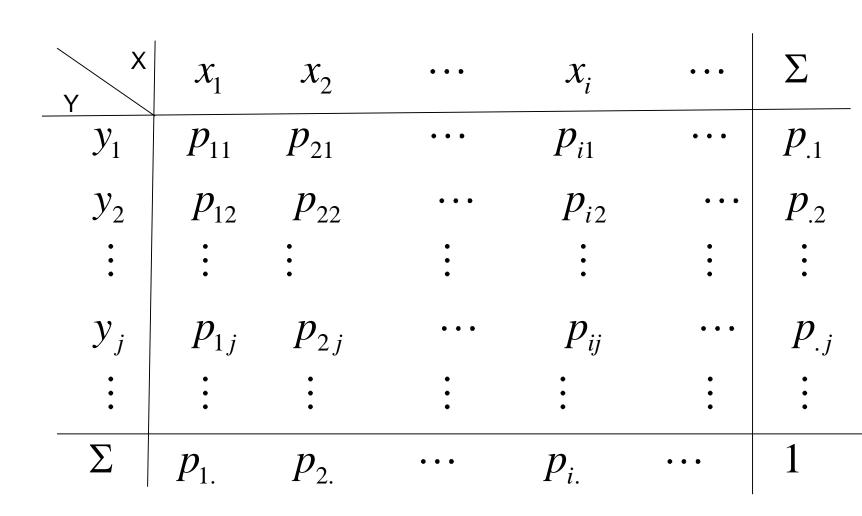
$$P\{Y = y_{j} | X = x_{i}\} = \frac{P\{X = x_{i}, Y = y_{j}\}}{P\{X = x_{i}\}} = \frac{p_{ij}}{p_{i}}$$

为在已知 $X = x_i$ 的条件下,Y的条件(概率)分布

类似地,称

$$P\{X = x_i | Y = y_j\} = \frac{P\{X = x_i, Y = y_j\}}{P\{Y = y_j\}} = \frac{p_{ij}}{p_{ij}}$$

为在已知 $Y = y_i$ 的条件下,X的条件(概率)分布



从表上可见,边缘分布,条件分布就是什么?

因为
$$\frac{p_{ij}}{p_{i\bullet}} \ge 0$$
且 $\sum_{j} \frac{p_{ij}}{p_{i\bullet}} = 1$

条件概率也是概率。可见,条件概率 分布也是概率分布。 例2 一射手进行射击,击中目标的概率为p(0 ,射击直至击中目标两次为止。设以X表示首次击中目标所进行的射击次数,以Y表示总共进行的射击次数,试求X和Y的联合分布律及条件分布律。

解: 当 $n \ge m$ 时, $P\{X = n, Y = m\} = 0$ 当n < m时, $P\{X = n, Y = m\}$ = $P\{$ 第n次和第m次击中,其它各次均未击中} = $p^2(1-p)^{m-2}$, $1 \le n < m < +\infty$

所以联合分布为

$$P\{X = n, Y = m\} = \begin{cases} p^2 (1-p)^{m-2}, & 1 \le n < m \\ 0, & n \ge m \ge 1 \end{cases}$$

边缘分布为

$$P\{X = n\} = \sum_{m=n+1}^{\infty} P\{X = n, Y = m\} = \sum_{m=n+1}^{\infty} p^{2} q^{m-2} = pq^{n-1}, \qquad n = 1, 2, \dots$$

$$P\{Y = m\} = \sum_{m=n+1}^{\infty} P\{X = n, Y = m\}$$

$$=\sum_{m=1}^{m-1}p^2q^{m-2}=(m-1)p^2q^{m-2}, \qquad m=2,3,\cdots$$

条件分布为:

$$P\{Y = m \mid X = n\} = \frac{P\{X = n, Y = m\}}{P\{X = n\}}$$

$$= \frac{p^{2}(1-p)^{m-2}}{pq^{n-1}} = p(1-p)^{m-n-1}, \quad m = n+1, n+2, n+3, \dots$$

$$P{X = n \mid Y = m} = \frac{P{X = n, Y = m}}{P{Y = m}}$$

$$= \frac{p^2 (1-p)^{m-2}}{(m-1)p^2 q^{m-2}} = \frac{1}{(m-1)}, \qquad n = 1, 2, ..., m-1$$

补例:投掷一枚骰子直至出现两次"6点"为止。设首次"6点"出现在第X次,以Y表示总共投掷次数,试求X和Y的联合分布律及条件分布律。

解:
$$P\{X = n, Y = m\} = \begin{cases} \frac{5^{m-2}}{6^m}, & 1 \le n < m \\ 0 & n \ge m \ge 1 \end{cases}$$

$$P\{X = n\} = \frac{5^{n-1}}{6^n}, \quad n = 1, 2, \dots$$

$$P\{Y = m\} = \frac{5^{m-2}}{6^m}, \quad m = 2, 3, \dots$$

条件分布为:

$$P{Y = m \mid X = n} = \frac{5^{m-n-1}}{6^{m-n}}, \qquad m = n+1, n+2, n+3, \dots$$

$$P{X = n \mid Y = m} = \frac{1}{(m-1)}, \qquad n = 1, 2, ..., m-1$$

二、连续型(不讲)

作业: 10, 12