

习题 4.2

1. 选择题

(1) 函数 $2(e^{2x} - e^{-2x})$ 的原函数有 ()

- A. $e^{2x} + e^{-2x}$ B. $e^{2x} - e^{-2x}$ C. $-e^{2x} + e^{-2x}$ D. $-e^{2x} - e^{-2x}$

(2) 设 $\int f(x)dx = \ln x^2 + C$, 则下面式子正确的是 ()

- A. $\int e^x f(e^x)dx = e^{2x} + C$ B. $\int e^x f(e^x)dx = 2x + C$
C. $\int e^{-x} f(e^{-x})dx = -2x + C$ D. $\int e^{-x} f(e^{-x})dx = e^{-2x} + C$

2. 填空题

(1) 若 $\int f(x)dx = F(x) + C$, 则 $\int xf(1-x^2)dx =$ _____.

(2) 已知 $f'(2x) = \sec^2 x$, 则 $f(x) =$ _____.

(3) 若 $f'(\tan 2x) = \sec^2 x$, 则 $\int \sec^2(2x)f''(\tan 2x)dx =$ _____.

(4) 已知 $\sin x$ 是 $f(x)$ 的原函数, 则 $\int \frac{f'(\ln x)}{x}dx =$ _____.

(5) $\int \tan x \sec^2 x dx =$ _____, $\int \frac{e^x}{x^2}dx =$ _____.

3. 利用第一类换元计算下列不定积分

(1) $\int \frac{2^{\arccos x}}{\sqrt{1-x^2}}dx$

(2) $\int \frac{x}{\sqrt{x^2-1}}dx$

(3) $\int \frac{dx}{x(1+x^n)} (n \in \mathbb{N})$

(4) $\int \frac{\ln 2x}{x \ln 4x}dx$

4. 利用第二类换元计算下列不定积分

(1) $\int \frac{x^2}{\sqrt{1-x^2}} dx$

(2) $\int \frac{dx}{x^4 \sqrt{x^2-1}}.$

(3) $\int \frac{1}{(\sqrt{1+x^2})^3} dx$

5. 计算下列不定积分

(1) $\int \frac{1}{\sqrt{-2+3x-x^2}} dx$

(2) $\int \frac{1}{\sqrt{2-3x+x^2}} dx$