

习题 3.2

(一) 基本习题

1. 判断题

$$(1) \lim_{x \rightarrow 2} \frac{2x}{2x-1} = \lim_{x \rightarrow 2} \frac{(2x)'}{(2x-1)'} \quad (\quad)$$

$$(2) \lim_{x \rightarrow 0} \frac{e^{2x} - 1}{\sin x} = \lim_{x \rightarrow 0} \left(\frac{e^{2x} - 1}{\sin x} \right)' \quad (\quad)$$

2. 下列各式中正确运用洛必达法则求极限的是 ()

$$A. \lim_{x \rightarrow 0} \frac{\sin x}{e^x - 1} = \lim_{x \rightarrow 0} \frac{\cos x}{e^x} = \lim_{x \rightarrow 0} \frac{-\sin x}{e^x}$$

$$B. \lim_{x \rightarrow \infty} \frac{x + \sin x}{x} = \lim_{x \rightarrow \infty} (1 + \cos x) \text{ 不存在}$$

$$C. \lim_{x \rightarrow 0} \frac{1}{x} \left(\frac{1}{x} - \cot x \right) = \lim_{x \rightarrow 0} \frac{\sin x - x \cos x}{x^2 \sin x} = \lim_{x \rightarrow 0} \frac{\sin x - x \cos x}{x^3} = \lim_{x \rightarrow 0} \frac{x \sin x}{3x^2} = \frac{1}{3}$$

$$D. \lim_{x \rightarrow 0} \frac{e^{2x} - 1}{\sin x} = \lim_{x \rightarrow 0} \frac{e^{2x}}{\cos x} = 1$$

3. 填空题

$$(1) \lim_{x \rightarrow 0} \frac{e^x - e^{-x} - 2x}{x - \sin x} = \underline{\hspace{2cm}}.$$

$$(2) \lim_{x \rightarrow 0} \frac{e^x - e^{-x}}{\sin x} = \underline{\hspace{2cm}}.$$

$$(3) \lim_{x \rightarrow 0} \frac{x^2 \sin \frac{1}{x}}{\sin x} = \underline{\hspace{2cm}}.$$

$$(4) \lim_{x \rightarrow +\infty} \frac{\ln(1 + e^x)}{\sqrt{1 + x^2}} = \underline{\hspace{2cm}}.$$

$$(5) \lim_{x \rightarrow 0^+} \sqrt[3]{x^2} \ln x = \underline{\hspace{2cm}}.$$

$$(6) \lim_{x \rightarrow 0} \frac{\ln(1 + \sin 2x)}{\arcsin(x + x^2)} = \underline{\hspace{2cm}}.$$

4. 求下列极限

$$(1) \lim_{x \rightarrow 0} \frac{e^x - \cos x}{\sin x}$$

$$(2) \lim_{x \rightarrow 0} \frac{x - \sin x}{\sqrt{1 + x^3} - 1}$$

(3) $\lim_{x \rightarrow 0} \frac{\arctan x - x}{\ln(1 + 2x^3)}$

(4) $\lim_{x \rightarrow 0} \sin x \cdot \ln x$

(5) $\lim_{x \rightarrow 0} \left(\frac{1}{x^2} - \frac{1}{x \tan x} \right)$

(6) $\lim_{x \rightarrow +\infty} (1 + x)^{\frac{1}{x}}$

(7) $\lim_{x \rightarrow 0} \frac{[\sin x - \sin(\sin x)] \sin x}{x^4}$

(8) $\lim_{x \rightarrow +\infty} \frac{x + \ln(x + e^x)}{\sqrt{1 + x^2}}$