主要内容

n阶行列式的定义、性质及其计算.

重点内容 行列式的计算

行列式运算

矩阵运算

向量组运算

第一节 二阶与三阶行列式

主要内容

- 二阶行列式
- ●三阶行列式
- ●小结

行列式运算

二阶行列式的引入

第一章 行列式

用消元法解二元线性方程组
$$\begin{cases} a_{11}x_1+a_{12}x_2=b_1 \ a_{21}x_1+a_{22}x_2=b_2 \end{cases}$$
, 得

$$x_1 = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}} ,$$

提 示:

$$\begin{split} &[a_{11}x_1 + a_{12}x_2 = b_1] \times a_{22} \Rightarrow a_{11}a_{22}x_1 + a_{12}a_{22}x_2 = b_1a_{22} \,, \\ &[a_{21}x_1 + a_{22}x_2 = b_2] \times a_{12} \Rightarrow a_{12}a_{21}x_1 + a_{12}a_{22}x_2 = a_{12}b_2 \,, \\ &\Rightarrow (a_{11}a_{22} - a_{12}a_{21})x_1 = b_1a_{22} - a_{12}b_2 \,. \end{split}$$

■1.1 二阶与三阶行列式

◆ 第一章 行列式

用消元法解二元线性方程组
$$\begin{cases} a_{11}x_1+a_{12}x_2=b_1 \ a_{21}x_1+a_{22}x_2=b_2 \end{cases}$$
, 得

$$x_1 = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}, \quad x_2 = \frac{a_{11} b_2 - b_1 a_{21}}{a_{11} a_{22} - a_{12} a_{21}}.$$

提示:

$$\begin{split} &[a_{11}x_1 + a_{12}x_2 = b_1] \times a_{21} \Rightarrow a_{11}a_{21}x_1 + a_{12}a_{21}x_2 = b_1a_{21} \,, \\ &[a_{21}x_1 + a_{22}x_2 = b_2] \times a_{11} \Rightarrow a_{11}a_{21}x_1 + a_{11}a_{22}x_2 = a_{11}b_2 \,, \\ &\Rightarrow (a_{11}a_{22} - a_{12}a_{21}) x_2 = a_{11}b_2 - b_1a_{21} \,. \end{split}$$

■ 1.1 二阶与三阶行列式

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● 第一章 行列式

用消元法解二元线性方程组
$$\begin{cases} a_{11}x_1 + a_{12}x_2 = b_1 \\ a_{21}x_1 + a_{22}x_2 = b_2 \end{cases}$$
, 得
$$x_1 = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}} , \quad x_2 = \frac{a_{11} b_2 - b_1 a_{21}}{a_{11} a_{22} - a_{12} a_{21}} .$$
 我们用符号 $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$ 表示代数和 $a_{11}a_{22} - a_{12}a_{21}$ 这样就有
$$x_1 = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}} , \quad x_2 = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}} .$$

■1.1 二阶与三阶行列式

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定义 由四个数排成二行二列(横排称行、竖排 称列)的数表

$$a_{11} \ a_{12}$$
 $a_{21} \ a_{22}$ (4)

表达式 $a_{11}a_{22} - a_{12}a_{21}$ 称为数表(4)所确定的二阶

行列式,并记作
$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$
 (5)

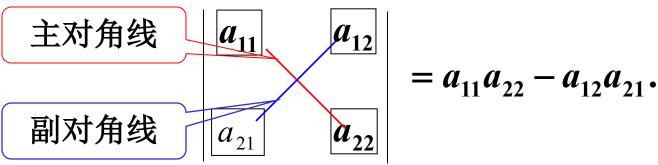
即
$$D = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$
. 行、列

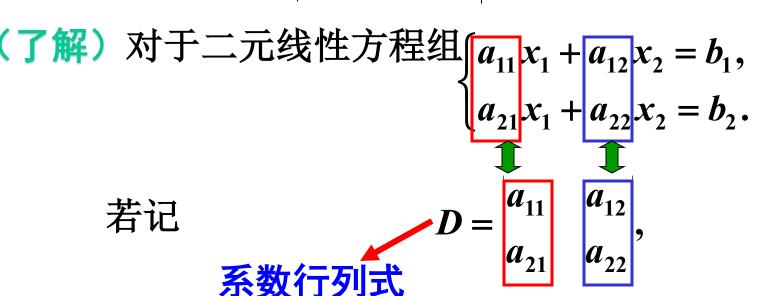
行列式运算

矩阵运算

向量组运算

二阶行列式的计算——对角线法则





行列式运算

矩阵运算

向量组运算

$$\begin{cases} a_{11}x_1 + a_{12}x_2 = b_1, \\ a_{21}x_1 + a_{22}x_2 = b_2, \\ D = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix},$$

$$\begin{cases} a_{11}x_1 + a_{12}x_2 = b_1, \\ a_{21}x_1 + a_{22}x_2 = b_2. \end{cases}$$

$$D_1 = \begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix} = b_1 a_{22} - a_{12} b_2,$$

$$\begin{cases} a_{11}x_1 + a_{12}x_2 = b_1, \\ a_{21}x_1 + a_{22}x_2 = b_2. \end{cases}$$



$$D = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix},$$

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$$\begin{cases} a_{11}x_1 + a_{12}x_2 = b_1, \\ a_{21}x_1 + a_{22}x_2 = b_2, \end{cases}$$

$$D_{1} = \begin{vmatrix} b_{1} & a_{12} \\ b_{2} & a_{22} \end{vmatrix} = b_{1}a_{22} - a_{12}b_{2},$$

$$\begin{cases} a_{11}x_1 + a_{12}x_2 = b_1, \\ a_{21}x_1 + a_{22}x_2 = b_2. \end{cases}$$

$$D_2 = \begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix} = a_{11}b_2 - b_1a_{21}.$$

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则二元线性方程组的解为

$$x_1 = rac{egin{array}{c|ccc} b_1 & a_{12} \ b_2 & a_{22} \ \hline a_{11} & a_{12} \ a_{21} & a_{22} \ \hline \end{array}}{a_{21} & a_{22}}, \qquad x_2 = rac{egin{array}{c|ccc} a_{11} & b_1 \ a_{21} & b_2 \ \hline a_{11} & a_{12} \ a_{21} & a_{22} \ \hline \end{array}}{a_{21} & a_{22} \ \end{array}.$$

注意 分母都为原方程组的系数行列式.

行列式运算

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例1 求解二元线性方程组

$$\begin{cases} 3x_1 - 2x_2 = 12, \\ 2x_1 + x_2 = 1. \end{cases}$$

解
$$D = \begin{vmatrix} 3 & -2 \\ 2 & 1 \end{vmatrix} = 3 - (-4) = 7 \neq 0,$$

$$D_1 = \begin{vmatrix} 12 & -2 \\ 1 & 1 \end{vmatrix} = 14, \quad D_2 = \begin{vmatrix} 3 & 12 \\ 2 & 1 \end{vmatrix} = -21,$$

$$\therefore x_1 = \frac{D_1}{D} = \frac{14}{7} = 2, \quad x_2 = \frac{D_2}{D} = \frac{-21}{7} = -3.$$

行列式运算

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二、三阶行列式

定义 设有9个数排成3行3列的数表

$$a_{11} \quad a_{12} \quad a_{13}$$

$$a_{21} \quad a_{22} \quad a_{23} \qquad (5)$$

$$a_{31} \quad a_{32} \quad a_{33}$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{21} & a_{22} & a_{23} \end{vmatrix} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \qquad (6)$$

$$-a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31},$$

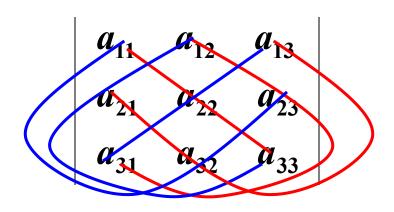
(6) 式称为数表(5) 所确定的三阶行列式.

行列式运算

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三阶行列式的计算 -- 对角线法则



$$= a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32}$$
$$-a_{13}a_{22}a_{31} - a_{12}a_{21}a_{33} - a_{11}a_{23}a_{32}.$$

注意 红线上三元素的乘积冠以正号,蓝线上三元素的乘积冠以负号.

说明 对角线法则只适用于二阶与三阶行列式.

行列式运算

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利用三阶行列式求解三元线性方程组(了解)

如果三元线性方程组
$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1, \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2, \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3; \end{cases}$$

的系数行列式
$$D = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \neq 0,$$

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$$D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \qquad D_1 = \begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix},$$

$$D_3 = \begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}$$
 $D_2 = \begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}$

则三元线性方程组的解为:

$$x_1 = \frac{D_1}{D}, \qquad x_2 = \frac{D_2}{D}, \qquad x_3 = \frac{D_3}{D}.$$

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三、小结

二阶与三阶行列式的计算 ——对角线法则

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}.$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}.$$

了解二阶与三阶行列式计算公式的特点.

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思考题

求一个二次多项式
$$f(x)$$
,使

$$f(1)=0$$
, $f(2)=3$, $f(-3)=28$.

思考题解答

解 设所求的二次多项式为

$$f(x) = ax^2 + bx + c,$$

由题意得 f(1)=a+b+c=0,

$$f(2)=4a+2b+c=3$$
, $f(-3)=9a-3b+c=28$,

得一个关于未知数 a,b,c 的线性方程组,

$$XD = -20 \neq 0$$
, $D_1 = -40$, $D_2 = 60$, $D_3 = -20$.

得
$$a = D_1/D = 2$$
, $b = D_2/D = -3$, $c = D_3/D = 1$

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故所求多项式为

$$f(x) = 2x^2 + -3x + 1.$$

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