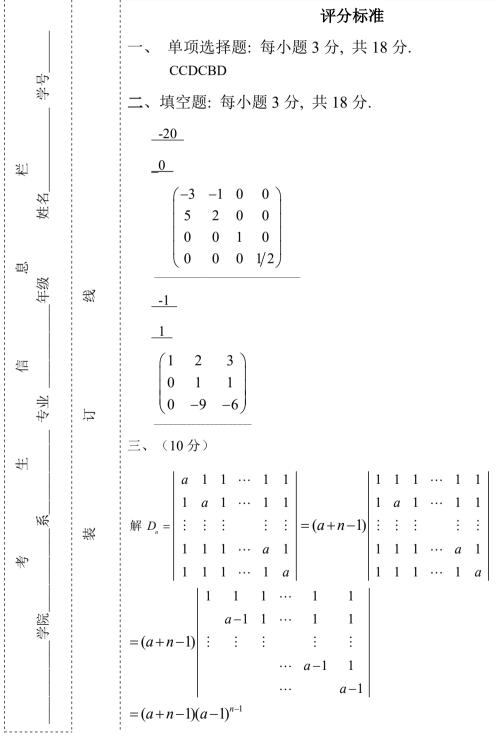
## 福建师范大学 (公共课) 数统 学院

## <u>2022</u>—<u>2023</u>学年第 2 学期 期中考 试卷



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四、(15分)

解:

(1) 
$$AB = (E + \alpha \beta^T)(E - \alpha \beta^T) = E;$$

(2) 因为A,B是方阵且AB = E, 所以A 可逆.

$$\alpha \beta^{T} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix},$$

$$B = E - \alpha \beta^{T} = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}, |B| = 1, ||B| A^{-1}| = ||B| B| = 1.$$

$$(3) A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}, |A| = 1, A^* = |A|A^{-1} = |A|B = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}.$$

五、(12分)

解:

$$|A| = -1$$

$$ABA^* = BA^{-1} - E \Longrightarrow -AB = B - A \Longrightarrow (A + E)B = A.$$

$$(A+E,A) = \begin{pmatrix} 2 & 2 & 3 & 1 & 2 & 3 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ 1 & 1 & 3 & 1 & 1 & 2 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 3 & 1 & 1 & 2 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ 2 & 2 & 3 & 1 & 2 & 3 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 3 & 1 & 1 & 2 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & -3 & -1 & 0 & -1 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix}
1 & 1 & 0 & 0 & 1 & 1 \\
0 & 1 & 0 & 0 & \frac{1}{2} & 0 \\
0 & 0 & -3 & -1 & 0 & -1
\end{pmatrix}
\rightarrow
\begin{pmatrix}
1 & 0 & 0 & 0 & \frac{1}{2} & 1 \\
0 & 1 & 0 & 0 & \frac{1}{2} & 0 \\
0 & 0 & 1 & \frac{1}{3} & 0 & \frac{1}{3}
\end{pmatrix}$$

$$B = \begin{pmatrix} 0 & \frac{1}{2} & 1 \\ 0 & \frac{1}{2} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} \end{pmatrix}$$

六、(12分)

解:

$$(A,b) \to \cdots \to \begin{pmatrix} 1 & 0 & -3 & 0 & \frac{3}{2} \\ 0 & 1 & 5 & 0 & -2 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

R(A) = R(A,b) = 3 < 4,有无穷多解取 $x_3 = C$ 为自由未知量

通解:

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \mathbf{c} \begin{pmatrix} 3 \\ -5 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} \frac{3}{2} \\ -2 \\ 0 \\ 0 \end{pmatrix} (\mathbf{c} \in \mathbf{R})$$

七、(15分)

解:

(1)  

$$A = P\Lambda P^{-1}$$
  
 $f(A) = A^6 + 2E = P\Lambda^6 P^{-1} + 2E = 3E$ 

(2)