

习题 5.2

1. 选择题:

(1) 设 $F(x) = \int_a^x \cos(f(t))dt$, 则 $F'(x) =$ ()

- A.
- $\sin(f(x))$
- B.
- $\cos(f(x))$
- C.
- $\cos(f(x))f'(x)$
- D.
- $-\sin(f(x))f'(x)$
- .

(2) 设 $f(x) = \int_0^x (t-1)dt$, 则 $f(x)$ 有 () .

- A. 极小值
- $\frac{1}{2}$
- B. 极小值
- $-\frac{1}{2}$
- C. 极大值
- $\frac{1}{2}$
- D. 极大值
- $-\frac{1}{2}$

(3) 设 $F(x) = \int_a^x xf(t)dt$, 则 $F'(x) =$ ()

- A.
- $\int_a^x f(t)dt + xf(x)$
- B.
- $xf(x)$
- C.
- $(x-a)f(x)$
- D.
- $(x-a)[f(x)-f(a)]$

(4) 下列关系错误的是 ()

- A. $(\int_a^{-x} f(t)dt)' = -f(-x)$ B. $(\int_x^a f(t)dt)' = -f(x)$
- C. $(\int_x^{2x} f(t)dt)' = 2f(2x) - f(x)$ D. $(\int_a^b f(t)dt)' = f(b) - f(a)$

2. 填空题:

(1) 设 $f(x)$ 在 $[a, b]$ 上连续, 则 $d[\int_a^b f(x)dx] =$ _____; $\frac{d}{dx}[\int_{-x}^1 f(t)dt] =$ _____;

(2) $\frac{d}{du} \int_{\cos^2 u}^{\sin u} e^{t^2} dt =$ _____;

(3) 当 $a =$ _____ 时, $f(x) = \int_0^x (a \cos t + \cos 3t)dt$ 在 $x = \frac{\pi}{3}$ 处有极值;

(4) $\int_1^3 |(x-1)(x-3)| dx =$ _____;

(5) 设 $f(x) = \frac{1}{1+x^2} + \sqrt{1-x^2} \int_0^1 f(x)dx$, 则 $\int_0^1 f(x)dx =$ _____;

3. 求下列极限:

(1) $\lim_{x \rightarrow 0} \frac{\int_0^x \sin t^2 dt}{\int_0^x t(e^{2t}-1)dt}$;

(2) $\lim_{x \rightarrow 0} \frac{\int_0^{\sin x} \sin t^2 dt}{x^3 + x^4}$.

4. 计算下列积分:

(1) $\int_0^{\frac{\pi}{4}} \tan^2 x dx$;

(2) $\int_{\frac{\pi^2}{4}}^{\pi^2} \frac{\cos \sqrt{x}}{\sqrt{x}} dx$;

(3) $\int_{-3}^2 \min\{2, x^2\} dx$;

(4) $\int_a^b |x| dx \quad (a < b)$;

(5) $\int_0^{\pi} \sqrt{1 - \sin 2x} dx$;

(6) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{1 + \sin x} dx$.

5. 设 $f(x) = \begin{cases} 2x & 0 \leq x \leq 1 \\ \frac{1}{2} & 1 < x \leq 2 \end{cases}$, 求 $\varphi(x) = \int_0^x f(t) dt$ 在 $0 \leq x \leq 2$ 时的表达式.