

Math 407: Interesting Discovery

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Introduction

I want to present an interesting finding by comparing a set of two different solutions to one homework question. This interesting finding might help people better understand the meaning of shadow prices and my concerns about the meaning behind shadow prices.

1 Problem Background Revisit

I am referring to problem 1.6 from Chvatal. I will brief all background information here. A meat packing plant produces 480 hams, 400 pork bellies, and 230 picnic hams every day; each of these products can be sold either fresh or smoked. The total number of hams, bellies, and picnics that can be smoked during a normal day is 420; in addition, up to 250 products can be smoked on overtimes at a higher cost.

The prices for fresh ham, smoked on regular time ham and smoked on overtime ham are \$8, \$14 and \$11. The prices for fresh bellies, smoked on regular time bellies and smoked on overtime bellies are \$4, \$12 and \$7.

The prices for fresh picnics, smoked on regular time picnics and smoked on overtime picnics are \$4, \$13 and \$9.

1.1 First Approach

Define x_1 is the number of hams being smoked, x_2 is the number of hams smoked overnight, x_3 is the number of bellies smoked, x_4 is the number of bellies smoked overnight, x_5 is the number of picnic smoked, x_6 is the number of picnics smoked overnight.

Then we have the following LP:

Max: $6x_1 + 3x_2 + 8x_3 + 3x_4 + 9x_5 + 5x_6$:

such that:

$$x_1 + x_2 \leq 480$$

$$x_3 + x_4 \leq 400$$

$$x_5 + x_6 \leq 230$$

$$x_1 + x_3 + x_5 \leq 420$$

$$x_2 + x_4 + x_6 \leq 250$$

$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$$

As a result, we have the following values: $x_1 = 0$, $x_2 = 40$, $x_3 = 400$, $x_4 = 0$, $x_5 = 20$, and $x_6 = 210$. The corresponding shadow prices are $y_1 = 0$, $y_2 = 1$, $y_3 = 2$, $y_4 = 7$ and $y_5 = 3$.

1.2 Second Approach

Define x_1 is the number of fresh hams, x_2 is the number of hams being smoked, x_3 is the number of hams smoked overnight, x_4 is the number of fresh bellies, x_5 is the number of bellies smoked, x_6 is the number of bellies smoked overnight, x_7 is the number of fresh picnics, x_8 is the number of picnic smoked, x_9 is the number of picnics smoked overnight.

Then we have the following LP:

Max: $8x_1 + 14x_2 + 11x_3 + 4x_4 + 12x_5 + 7x_6 + 4x_7 + 13x_8 + 9x_9$:
such that:

$$x_1 + x_2 + x_3 \leq 480$$

$$x_4 + x_5 + x_6 \leq 400$$

$$x_7 + x_8 + x_9 \leq 230$$

$$x_2 + x_5 + x_8 \leq 420$$

$$x_3 + x_6 + x_9 \leq 250$$

$$x_1, x_2, x_3, x_5, x_6, x_7, x_8, x_9 \geq 0$$

As a result, we have the following values: $x_1 = 440$, $x_2 = 0$, $x_3 = 40$, $x_4 = 0$, $x_5 = 400$, $x_6 = 0$, $x_7 = 0$, $x_8 = 20$ and $x_9 = 210$. The corresponding shadow prices are $y_1^* = 8$, $y_2^* = 5$, $y_3^* = 6$, $y_4^* = 7$ and $y_5^* = 3$.

2 Discussion

With little surprise, both methods present the same solution to the exact same solution concerning on how to maximize the profit. However, I want to point out that the difference between y and y^* is 8, 4, 4, 0, 0 where differences on first three elements present exactly the price for the fresh hams, fresh bellies and fresh picnics.

This result is not surprising at all. We had extra variables taking all the costs into considerations. However, my concerns about the definitions about shadow prices have been arisen.

I quote from the book in Chvatal, y_i is often called the marginal value of the i th resource, the adjective marginal referring to the difference between the trading price and the actual worth of the resource.

Based on the two alternative solutions for the problem, I find that there are some contradictions in a clear manner. Specifically, if we put no limitations on how to construct an LP at the beginning, we cannot be sure about the meanings of dual solutions. Based on the example I presented, the dual solutions can either become the shadow prices or the shadow prices plus the cost of the resources. This is also a very practical and critical point that we should consider because the result will influence pricing and many decisions if we are trying to interpret the solutions from simplex methods.

It might be fun to prove my thoughts rigorously because the vagueness of the definition of shadow prices made me confused for a couple of days. However, this example serves as an interesting piece of the meaning of shadow prices, but also makes me wonder whether Chvatal works on examples like I presented above.