书本

1.1 (1) 不是，$\vec{B}$与$\vec{C}$有可能垂直，$\vec{A}\cdot\vec{B}=\vec{A}\cdot\vec{C}=\vec{0}$

(2) ~~$\vec{A}\cdot(\vec{B}\times\vec{C}) = |\vec{A}|\,|\vec{B}|\,|\vec{C}|\sin\langle\vec{B},\vec{C}\rangle \cos$~~

~~假设$\langle\vec{A},\vec{B}\rangle=\alpha$，$\langle\vec{B},\vec{C}\rangle=\beta$，$\langle\vec{A},\vec{C}\rangle=\gamma$~~

假设 $\vec{A}=(a_x,a_y,a_z)$，$\vec{B}=(b_x,b_y,b_z)$，$\vec{C}=(c_x,c_y,c_z)$

$\vec{B}\times\vec{C}=(b_yc_z-b_zc_y,\ b_zc_x-b_xc_z,\ b_xc_y-b_yc_x)$

$\vec{C}\times\vec{A}=(c_ya_z-c_za_y,\ c_za_x-c_xa_z,\ c_xa_y-c_ya_x)$

$\vec{A}\times\vec{B}=(a_yb_z-a_zb_y,\ a_zb_x-a_xb_z,\ a_xb_y-a_yb_x)$

$\vec{A}\cdot(\vec{B}\times\vec{C})=a_x(b_yc_z-b_zc_y)+a_y(b_zc_x-b_xc_z)+a_z(b_xc_y-b_yc_x)$

$\vec{B}\cdot(\vec{C}\times\vec{A})=b_x(c_ya_z-c_za_y)+b_y(c_za_x-c_xa_z)+b_z(c_xa_y-c_ya_x)$

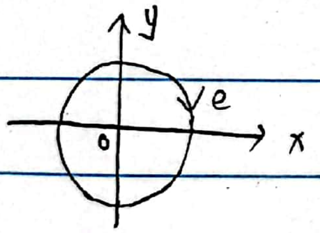
$\vec{C}\cdot(\vec{A}\times\vec{B})=c_x(a_yb_z-a_zb_y)+c_y(a_zb_x-a_xb_z)+c_z(a_xb_y-a_yb_x)$

上述三式最终结果均为 $a_xb_yc_z+a_yb_zc_x+a_zb_xc_y-a_xb_zc_y-a_yb_xc_z-a_zb_yc_x$

1.3 矢量线方程 $-\frac{y}{dy}=\frac{x}{dx}$，两边同时积分得 $x^2+y^2=C^2$

将矢量场表达式转变为圆柱坐标系，得 $\vec{F}(r)=\hat{a}_x(-y)+\hat{a}_y(x)=\hat{a}_e e$

故此矢量场的矢量线为一组圆心在原点的同心圆，方向为e的方向.



1.4 $P_1(-3,1,4)$，$P_2(2,-2,3)$

(1) $\vec{r}_1=-3\hat{a}_x+\hat{a}_y+4\hat{a}_z$，$\vec{r}_2=2\hat{a}_x-2\hat{a}_y+3\hat{a}_z$

(2) $\vec{r}=\vec{r}_2-\vec{r}_1=5\hat{a}_x-3\hat{a}_y-\hat{a}_z$，~~$|\vec{r}|=\sqrt{5^2+3^2+(-1)^2}$~~

方向由P_1指向P_2，$|\vec{r}|=\sqrt{5^2+3^2+1^2}=\sqrt{35}$

(3) $|\vec{r}_1|\cdot\cos\theta=\frac{\vec{r}_1\cdot\vec{r}_2}{|\vec{r}_2|}=\frac{4}{\sqrt{17}}$

1.7 (1) $r=\sqrt{x^2+y^2+z^2}=5\sqrt{2}$

$|\vec{E}| = \frac{\vec{E}}{\hat{a}_r}\Big|_{r=5\sqrt{2}} = 0.5$

$z=-5=r\cdot\cos\theta$, $\cos\theta=-\frac{\sqrt{2}}{2}$

$E_z = |\vec{E}|\cos\theta = -\frac{\sqrt{2}}{4}$

(2) $\vec{B} = \hat{a}_r\sqrt{2^2+2^2+1^2} = 3\hat{a}_r$, $\vec{E}=0.5\hat{a}_r$

所以夹角 $\alpha = \arccos\frac{\vec{E}\cdot\vec{B}}{|\vec{E}||\vec{B}|} = \arccos(-\frac{19}{15\sqrt{2}})$

指导书:

1.1 $P_1(1,1,3)$, $P_2(0,-2,1)$

(1) $\vec{r}_1=\hat{a}_x+\hat{a}_y+3\hat{a}_z$, $\vec{r}_2=-2\hat{a}_y+\hat{a}_z$

(2) $\vec{r}=\vec{r}_2-\vec{r}_1=-\hat{a}_x-3\hat{a}_y-2\hat{a}_z$

(3) $|\vec{r}_1|\cos\theta = \frac{\vec{r}_1\cdot\vec{r}_2}{|\vec{r}_2|} = \frac{1}{\sqrt{5}}$

1.3 标量场 $\psi=x^2+y^2+e^z$

(1) $x^2+y^2+e^z = 1^2+2^2+e^3 = 5+e^3$

(2) $\text{grad}\,\psi = \nabla\psi = 2x\,\hat{a}_x + 2y\,\hat{a}_y + e^z\,\hat{a}_z$

(3) $\frac{\partial\psi}{\partial l}\Big|_{(1,1,0)} = \nabla\psi\cdot\hat{a}_l\Big|_{(1,1,0)} = (2\hat{a}_x+2\hat{a}_y+\hat{a}_z)\cdot\frac{2\hat{a}_x-2\hat{a}_y+\hat{a}_z}{\sqrt{2^2+2^2+1^2}} = \frac{1}{3}$

1.6 $\oint_c\vec{A}\cdot d\vec{l} = \int_0^1\vec{A}\cdot\hat{a}_x\,dx\big|_{y=0} + \int_0^1\vec{A}\cdot\hat{a}_y\,dy\big|_{x=1} + \int_0^1\vec{A}\cdot(-\hat{a}_x)\,dy\big|_{y=1}$

$+\int_0^1\vec{A}(-\hat{a}_y)\,dy\big|_{x=0} = \frac{1}{2}$

$\int_s\nabla\times\vec{A}\,d\vec{S} = \int_0^1\int_0^1(y\hat{a}_e)\cdot\hat{a}_z\,dx\,dy = \frac{1}{2}$

$\therefore \oint_c\vec{A}\cdot d\vec{l} = \int_s\nabla\times\vec{A}\,d\vec{S}$，即斯托克斯定理成立.

1.7 (1) $u = xy^2 + x$

$\mathrm{grad}\, u = \hat{a}_x (y^2+1) + \hat{a}_y\, 2xy$

(2) $u = x^2yz + y^2z$

$\mathrm{grad}\, u = \hat{a}_x\, 2xyz + \hat{a}_y (x^2z + 2yz) + \hat{a}_z (x^2y + y^2)$

1.8 (1) $\vec{A} = \hat{a}_x x + \hat{a}_y y^2 + \hat{a}_z (3z - x)$ 在点 $P(1,2,-1)$

$\mathrm{div}\, \vec{A}\big|_{(1,2,-1)} = (1 + 2y + 3)\big|_{(1,2,-1)} = 1 + 2\times 2 + 3 = 8$

(2) $\vec{A} = \hat{a}_x x^2y + \hat{a}_y yz$ 在点 $P(0,1,2)$

$\mathrm{div}\, \vec{A}\big|_{(0,1,2)} = (2xy + z)\big|_{(0,1,2)} = 2$

1.9 (1) $\vec{A} = \hat{a}_x (x^2+1) + \hat{a}_z\, 3z^2$

$\mathrm{rot}\vec{A} = \nabla \times \vec{A} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2+1 & 0 & 3z^2 \end{vmatrix} = 0$，故 $\vec{A}$ 旋度为0，标量源分布 $\rho = 2x+6$

无矢量源

(2) $\vec{A} = \hat{a}_x\, xyz$

$\mathrm{rot}\vec{A} = \nabla \times \vec{A} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xyz & 0 & 0 \end{vmatrix} = \hat{a}_y\, xy - \hat{a}_z\, xz$

$\rho = \nabla \cdot \vec{A} = yz$

故 $\vec{A}$ 旋度为 $\hat{a}_y\, xy - \hat{a}_z\, xz$，标量源分布为 yz，矢量源分布为

$\hat{a}_y xy - \hat{a}_z xz$