半导体物理及固体物理基础

第四章：非平衡过剩载流子

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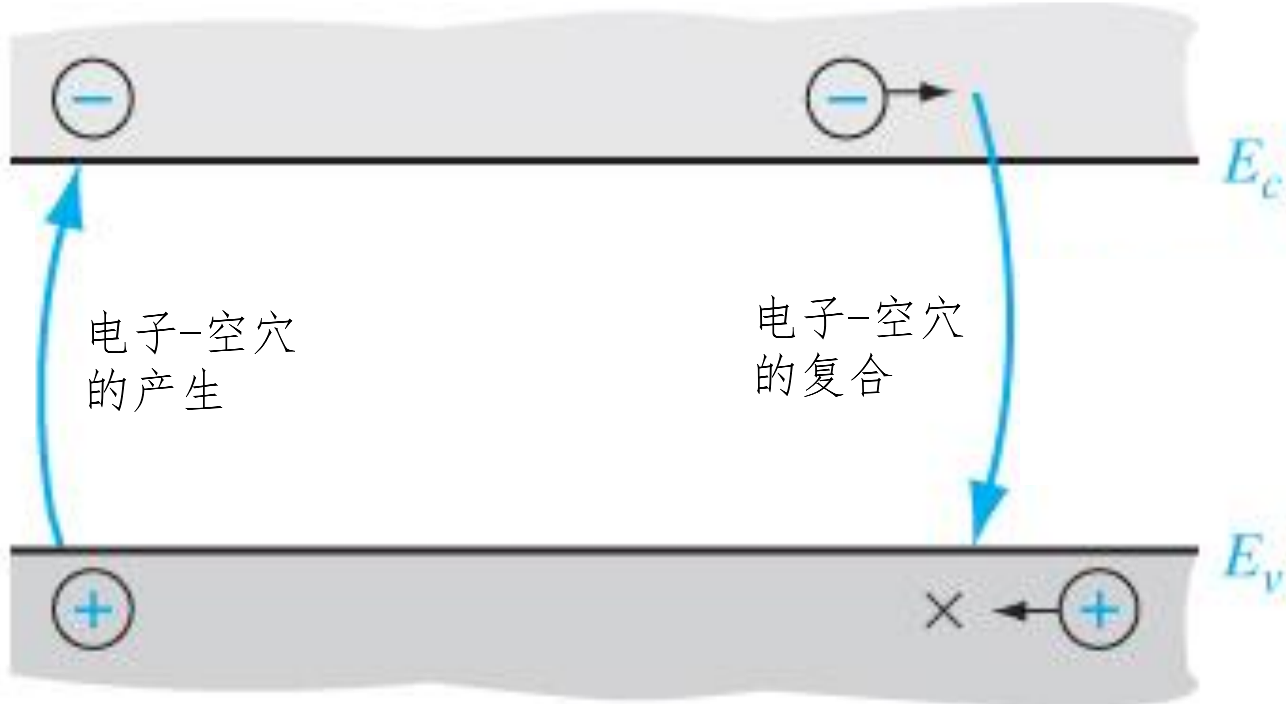
载流子的产生与复合

直接带间产生、直接带间复合

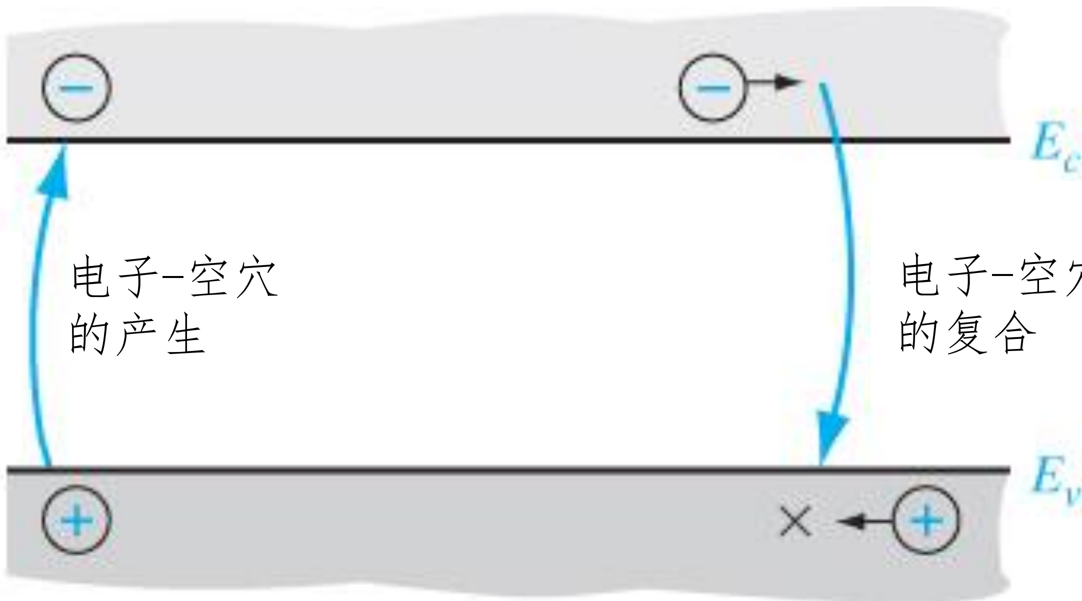


产生(Generation)：电子和空穴生成的过程

复合(Reconbination)：电子和空穴消失的过程

热平衡状态下的产生和复合

电子和空穴成对产生、成对复合



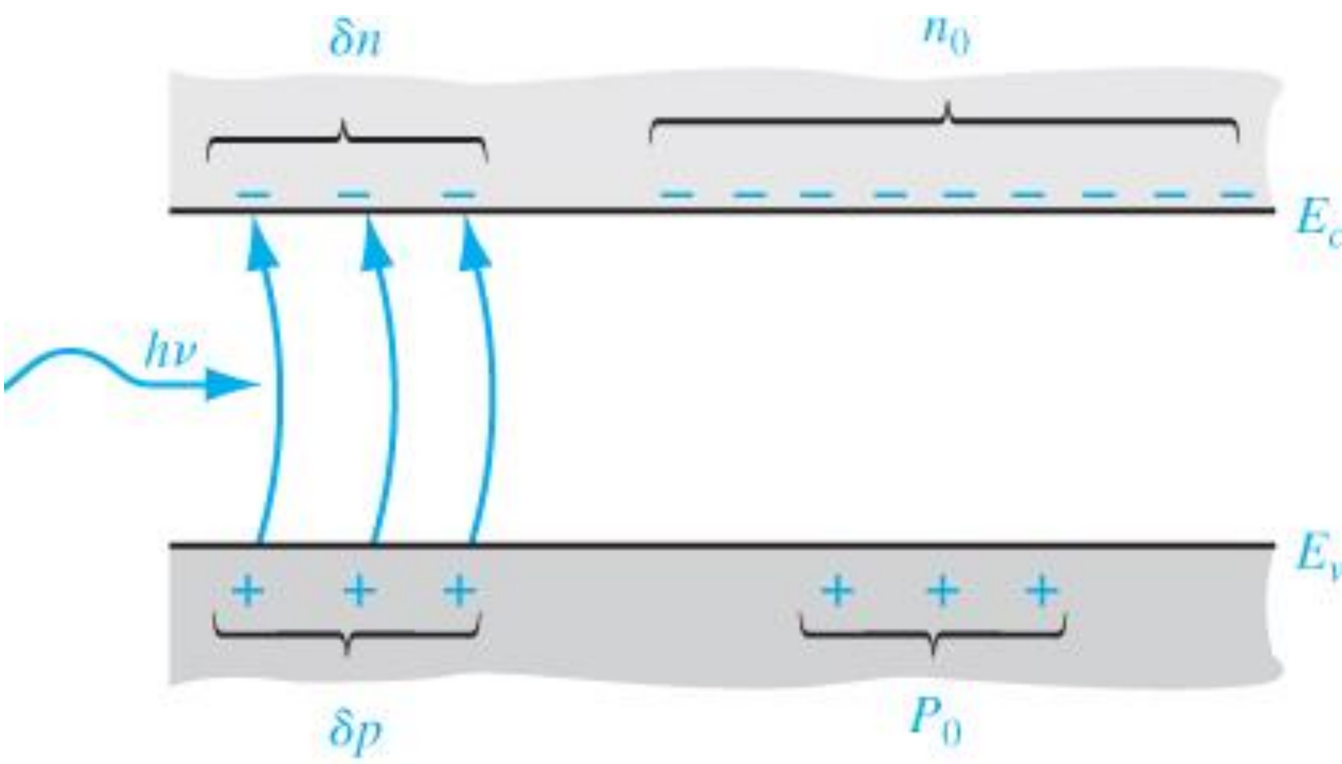
电子和空穴的产生率：$G_{n0} = G_{p0}$ #/cm^3-s

电子和空穴的复合率：$R_{n0} = R_{p0}$ #/cm^3-s

热平衡状态下：$G_{n0} = G_{p0} = R_{n0} = R_{p0}$

过剩载流子的产生

高能光子（能量大于禁带宽度）射入半导体，价带电子跃迁进入导带



$$n = n_0 + \boxed{\delta n}$$
$$p = p_0 + \boxed{\delta p}$$

过剩电子、空穴

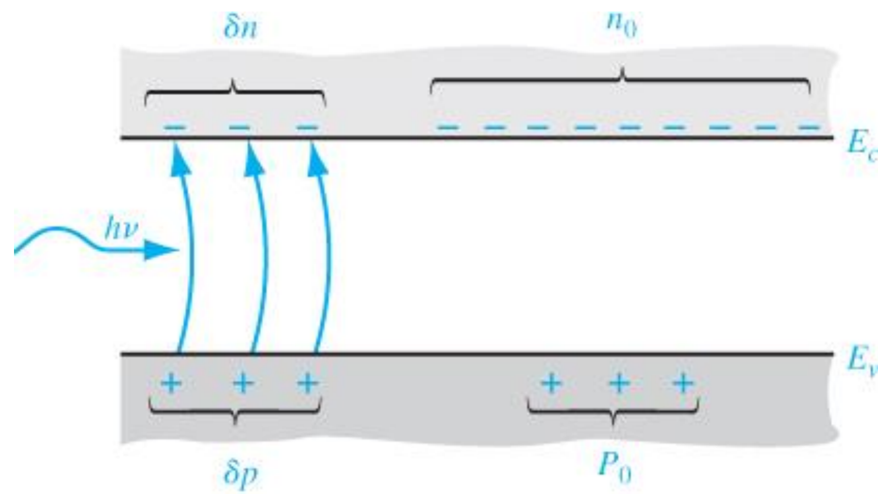
过剩载流子的产生

过剩载流子：

<u>在外界作用下</u>，导带或价带中产生的额外的载流子。

对于直接的带间产生来讲，
过剩电子和空穴也是成对出现的：

$$g_n' = g_p'$$



产生了过剩载流子之后：

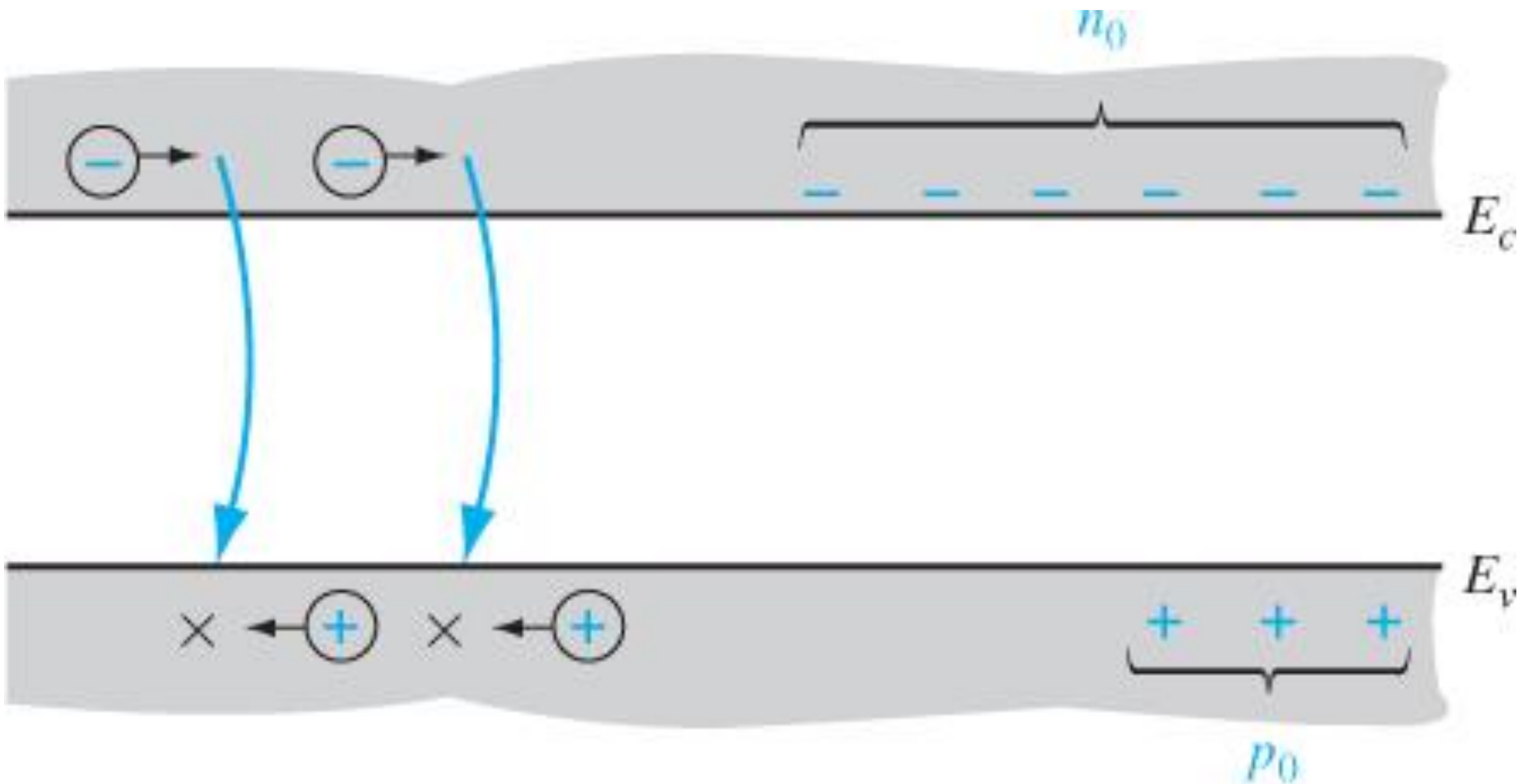
$$n = n_0 + \delta n$$

$$p = p_0 + \delta p$$

$$\boxed{np = (n_0 + \delta n)(p_0 + \delta p) \neq n_i^2}$$

过剩载流子的复合率

关闭光照后，过剩载流子复合，重建热平衡



过剩电子和空穴也是成对复合的：$R_n' = R_p'$

过剩流子的复合

复合率和电子与空穴的浓度成正比：

热平衡状态下：

$$G_{n0}=R_{n0}=a_r n_0 p_0$$

a_r是比例系数，与浓度无关

非平衡状态下：

$$R_n=a_r n(t)p(t)$$

$$n(t)=n_0+\delta n(t) \quad p(t)=p_0+\delta p(t)$$

过剩流子的复合

电子浓度的变化率：

$$\boxed{\begin{aligned} n(t) &= n_0 + \delta n(t) \\ p(t) &= p_0 + \delta p(t) \end{aligned}}$$

$$\frac{dn(t)}{dt} = a_r n_i^2 - a_r n(t) p(t)$$

$$= a_r \left[n_i^2 - \left(n_0 + \delta n(t)\right)\left(p_0 + \delta p(t)\right) \right]$$

$$= a_r \left[n_i^2 - n_0 p_0 - n_0 \delta p(t) - p_0 \delta n(t) - \delta n(t) \delta p(t) \right]$$

$$= -a_r \delta n(t) \left[\left(n_0 + p_0\right) + \delta n(t) \right]$$

$$\frac{d\delta n(t)}{dt} = -a_r \delta n(t)\left[\left(n_0 + p_0\right) + \delta n(t)\right]$$

$$= -a_r p_0 \delta n(t)$$

p型半导体;

小注入条件$\left(\delta n(t) \ll p_0\right)$

$$\delta n(t) = \delta n(0)\mathrm{e}^{-a_r p_0 t}$$

$$= \delta n(0)\mathrm{e}^{-t/\tau_{n0}}$$

$\tau_{n0} = \dfrac{1}{\alpha_r p_0}$，是一个常数，

τ_{n0}称为过剩少数载流子(电子)寿命

描述了少数载流子的衰减。

过剩流子的复合

$$\delta n(t)=\delta n(0)e^{-t/\tau_{n0}}$$

过剩少数载流子电子的复合率定义为一个正数：

$$R_n^{'}=-\frac{d\delta n(t)}{dt}=\frac{\delta n(t)}{\tau_{n0}}$$

$$R_n^{'}=R_p^{'}$$

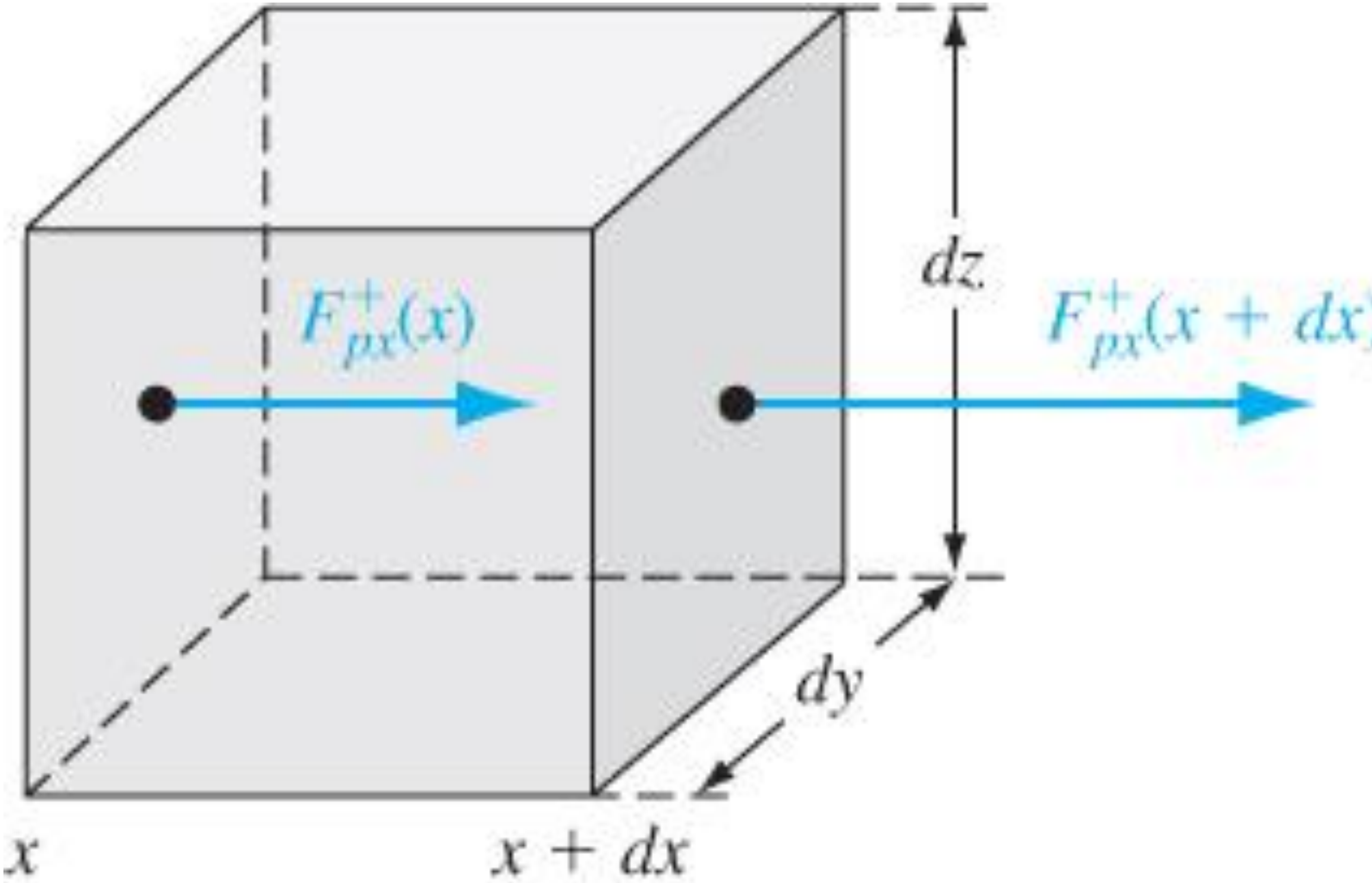
小注入条件下的n型半导体：

$$R_p^{'}=\frac{\delta p(t)}{\tau_{p0}}$$

$$\tau_{p0}=\frac{1}{\alpha_r n_0}$$

$$R_n^{'}=R_p^{'}$$

连续性方程



连续性方程

$$\frac{\partial p}{\partial t} = -\frac{\partial F_p^+}{\partial x} + g_p - \frac{p}{\tau_{pt}} \qquad \frac{\partial n}{\partial t} = -\frac{\partial F_n^-}{\partial x} + g_n - \frac{p}{\tau_{nt}}$$

$$\frac{\partial(\delta p)}{\partial t} = D_p \frac{\partial^2(\delta p)}{\partial x^2} - \mu_p \left(E \frac{\partial(\delta p)}{\partial x} + p \frac{\partial E}{\partial x} \right) + g_p - \frac{p}{\tau_{pt}}$$

$$\frac{\partial(\delta n)}{\partial t} = D_n \frac{\partial^2(\delta n)}{\partial x^2} + \mu_n \left(E \frac{\partial(\delta n)}{\partial x} + n \frac{\partial E}{\partial x} \right) + g_n - \frac{n}{\tau_{pt}}$$

$$\frac{\partial(\delta p)}{\partial t}=D_p\frac{\partial^2(\delta p)}{\partial x^2}-\mu_p\left(E\frac{\partial(\delta p)}{\partial x}+p\frac{\partial E}{\partial x}\right)+g_p-\frac{p}{\tau_{pt}}$$

$$\frac{\partial(\delta n)}{\partial t}=D_n\frac{\partial^2(\delta n)}{\partial x^2}+\mu_n\left(E\frac{\partial(\delta n)}{\partial x}+n\frac{\partial E}{\partial x}\right)+g_n-\frac{n}{\tau_{pt}}$$

$E=E_{app}+E_{int}$ 假设$|E_{int}|\ll|E_{app}|$，但是$\frac{\partial E}{\partial x}$不可忽略

$$\frac{\partial E}{\partial x}=\frac{\partial\left(E_{app}+E_{int}\right)}{\partial x}=\frac{\partial E_{int}}{\partial x}$$

$$\because \delta p = \delta n$$

$$\frac{\partial(\delta p)}{\partial t} = D_p \frac{\partial^2(\delta p)}{\partial x^2} - \mu_p \left(E \frac{\partial(\delta p)}{\partial x} + p \frac{\partial E}{\partial x} \right) + g_p - \frac{p}{\tau_{pt}}$$

$$\boxed{\begin{aligned} \frac{\partial(\delta n)}{\partial t} &= D_p \frac{\partial^2(\delta n)}{\partial x^2} - \mu_p \left(E \frac{\partial(\delta n)}{\partial x} + p \frac{\partial E}{\partial x} \right) + g_p - \frac{p}{\tau_{pt}} \\ \frac{\partial(\delta n)}{\partial t} &= D_n \frac{\partial^2(\delta n)}{\partial x^2} + \mu_n \left(E \frac{\partial(\delta n)}{\partial x} + n \frac{\partial E}{\partial x} \right) + g_n - \frac{n}{\tau_{pt}} \end{aligned}}$$

$$\frac{\partial(\delta n)}{\partial t}=D_p\frac{\partial^2(\delta n)}{\partial x^2}-\mu_p\left(E\frac{\partial(\delta n)}{\partial x}+\boxed{p\frac{\partial E}{\partial x}}\right)+g-R$$

$$\frac{\partial(\delta n)}{\partial t}=D_n\frac{\partial^2(\delta n)}{\partial x^2}+\mu_n\left(E\frac{\partial(\delta n)}{\partial x}+\boxed{n\frac{\partial E}{\partial x}}\right)+g-R$$

$$\Downarrow$$

$$\frac{\partial(\delta n)}{\partial t}=D'\frac{\partial^2(\delta n)}{\partial x^2}+\mu' E\frac{\partial(\delta n)}{\partial x}+g-R$$

$$\frac{\partial(\delta n)}{\partial t} = D'\frac{\partial^2(\delta n)}{\partial x^2} + \mu' E\frac{\partial(\delta n)}{\partial x} + g - R$$

$$D' = \frac{\mu_n n D_p + \mu_p p D_n}{\mu_n n + \mu_p p} \qquad \mu' = \frac{\mu_n \mu_p (p - n)}{\mu_n n + \mu_p p}$$

双极输运

双极扩散系数：

$$D' = \frac{\mu_n n D_p + \mu_p p D_n}{\mu_n n + \mu_p p}$$

$$\boxed{D' = \frac{D_n D_p (n+p)}{D_n n + D_p p}}$$

$$\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = \frac{kT}{e}$$

双极迁移率：

$$\boxed{\mu' = \frac{\mu_n \mu_p (p-n)}{\mu_n n + \mu_p p}}$$

$$D' = \frac{D_n D_p (n+p)}{D_n n + D_p p}$$

$$= \frac{D_n D_p \left[(n_0 + \delta n) + (p_0 + \delta p)\right]}{D_n (n_0 + \delta n) + D_p (p_0 + \delta p)}$$

$$\mu' = \frac{\mu_n \mu_p (p-n)}{\mu_n n + \mu_p p}$$

$$= \frac{\mu_n \mu_p \left[(p_0 + \delta p) - (n_0 + \delta n)\right]}{\mu_n (n_0 + \delta n) + \mu_p (p_0 + \delta p)}$$

$$D' = \frac{D_n D_p\left[(n_0+\delta n)+(p_0+\delta p)\right]}{D_n(n_0+\delta n)+D_p(p_0+\delta p)}$$

$$\mu' = \frac{\mu_n \mu_p\left[(p_0+\delta p)-(n_0+\delta n)\right]}{\mu_n(n_0+\delta n)+\mu_p(p_0+\delta p)}$$

对于p型半导体： $n_0 \ll p_0 \quad \delta n \ll p_0$

$$D' = D_n \qquad \mu' = \mu_n$$

对于n型半导体： $n_0 \gg p_0 \quad \delta n \gg p_0$

$$D' = D_p \qquad \mu' = -\mu_p$$

$$\frac{\partial(\delta n)}{\partial t}=D'\frac{\partial^2(\delta n)}{\partial x^2}+\mu' E\frac{\partial(\delta n)}{\partial x}+\boxed{g-R}$$

$$D'=\frac{\mu_n n D_p+\mu_p p D_n}{\mu_n n+\mu_p p} \qquad \mu'=\frac{\mu_n\mu_p(p-n)}{\mu_n n+\mu_p p}$$

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双极输运方程中电子的产生率和复合率：

$$
\begin{aligned}
g - R &= g_n - R_n \\
&= \left(G_{n0} + g_n^{'}\right) - \left(R_{n0} + R_n^{'}\right) \\
&= g_n^{'} - R_n^{'} \\
&= g_n^{'} - \frac{\delta n}{\tau_n}
\end{aligned}
$$

双极输运方程中的产生和复合

电子和空穴的复合率相等：

$$R_n = R_p = n/\tau_{nt} = p/\tau_{pt}$$

$1/\tau_{nt} = \alpha_r p \approx \alpha_r p_0$，单位时间内电子遇到空穴发生复合的概率

多数载流子空穴的浓度几乎不变，

$\tau_{nt} = \tau_{n0}$

小注入条件下的n型半导体：

$1/\tau_{pt} = \alpha_r n \approx \alpha_r n_0$，单位时间内空穴遇到电子发生复合的概率

多数载流子电子的浓度几乎不变，

$\tau_{pt} = \tau_{p0}$

双极输运方程中的产生和复合

双极输运方程中电子的产生率和复合率：

$$g - R = g_n - R_n$$

$$= \left(G_{n0} + g_n^{'}\right) - \left(R_{n0} + R_n^{'}\right)$$

$$= g_n^{'} - R_n^{'}$$

$$= g_n^{'} - \frac{\delta n}{\tau_n}$$

$$= g^{'} - \frac{\delta n}{\tau_{n0}} \qquad g_n^{'} = g_p^{'} = g^{'}$$

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双极输运方程中空穴的产生率和复合率：

$$g - R = g_p - R_p$$

$$= \left(G_{p0} + g_p'\right) - \left(R_{p0} + R_p'\right)$$

$$= g_p' - R_p'$$

$$= g_p' - \frac{\delta p}{\tau_p}$$

$$= g' - \frac{\delta p}{\tau_{p0}} \qquad g_n' = g_p' = g'$$

双极输运方程

双极输运方程就可以写成少子参数项的形式：

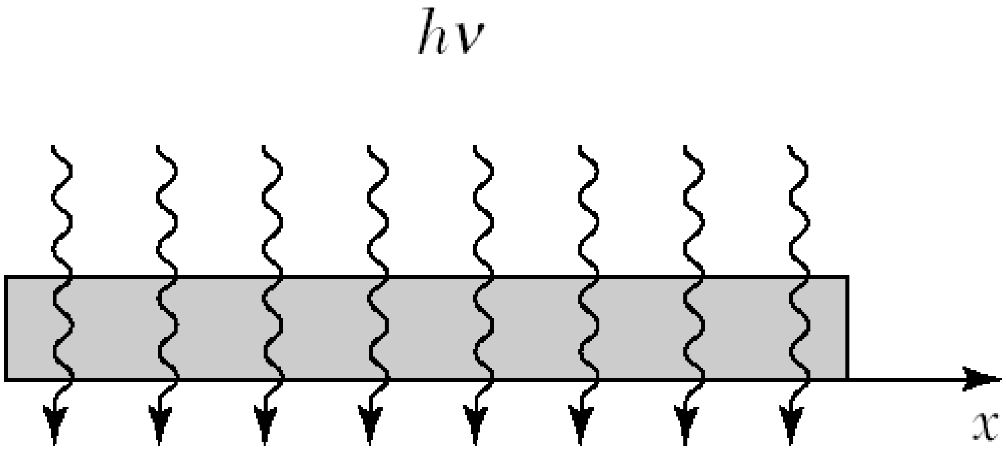
$$\frac{\partial(\delta n)}{\partial t}=D_n\frac{\partial^2(\delta n)}{\partial x^2}+\mu_n E\frac{\partial(\delta n)}{\partial x}+g'-\frac{\delta n}{\tau_{n0}}$$

$$\frac{\partial(\delta p)}{\partial t}=D_p\frac{\partial^2(\delta p)}{\partial x^2}-\mu_p E\frac{\partial(\delta p)}{\partial x}+g'-\frac{\delta p}{\tau_{p0}}$$

过剩少子的空间和时间函数就可以通过少子的漂移、扩散、产生和复合来描述。

过剩多子的状态就可以由少子的参数来决定。

双极输运方程的应用I



n型半导体

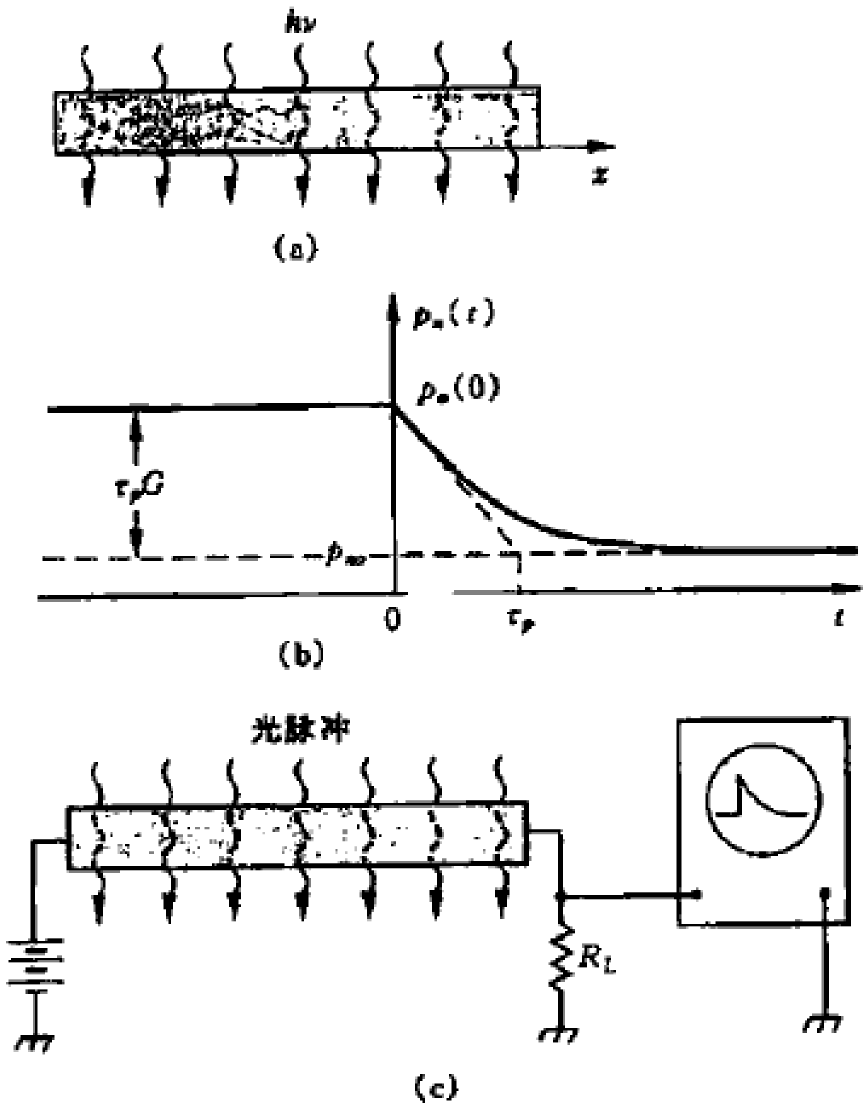
t ≥ 0时，均匀的产生率**g'**

满足小注入条件

$$\frac{\partial(\delta n)}{\partial t} = D_n \frac{\partial^2(\delta n)}{\partial x^2} + \mu_n E \frac{\partial(\delta n)}{\partial x} + g' - \frac{\delta n}{\tau_{n0}}$$

$$\frac{\partial(\delta p)}{\partial t} = D_p \frac{\partial^2(\delta p)}{\partial x^2} - \mu_p E \frac{\partial(\delta p)}{\partial x} + g' - \frac{\delta p}{\tau_{p0}}$$

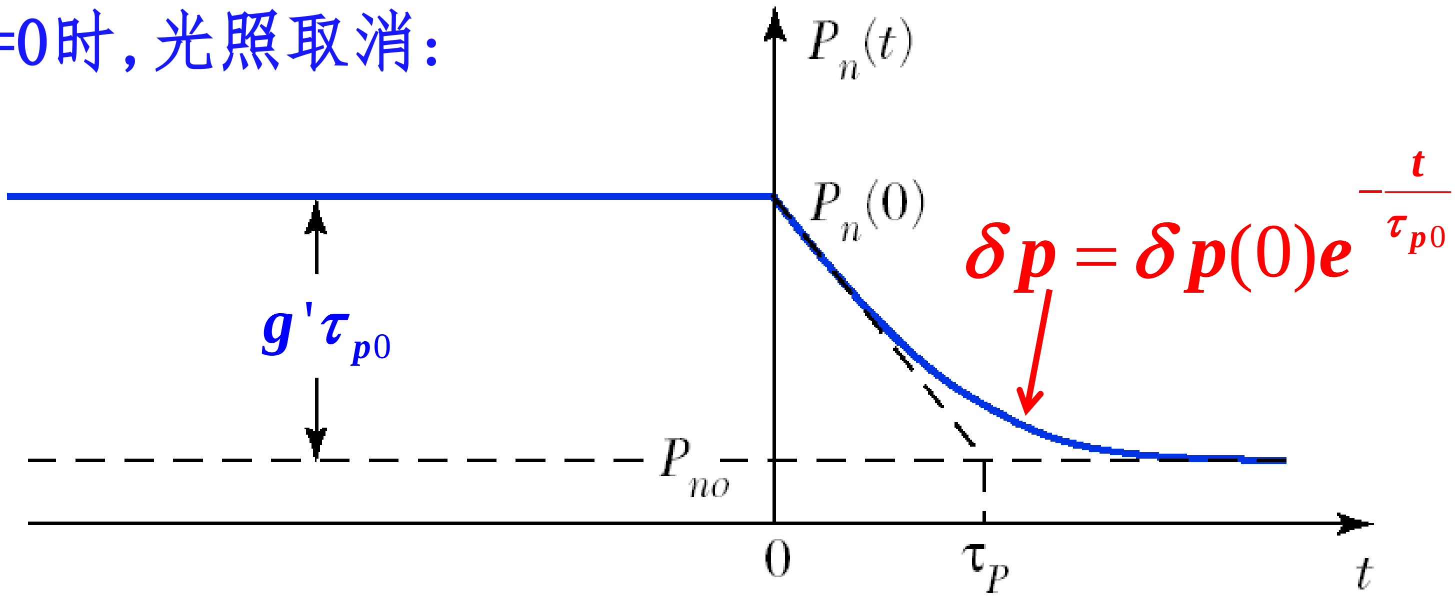
双极输运方程的应用II



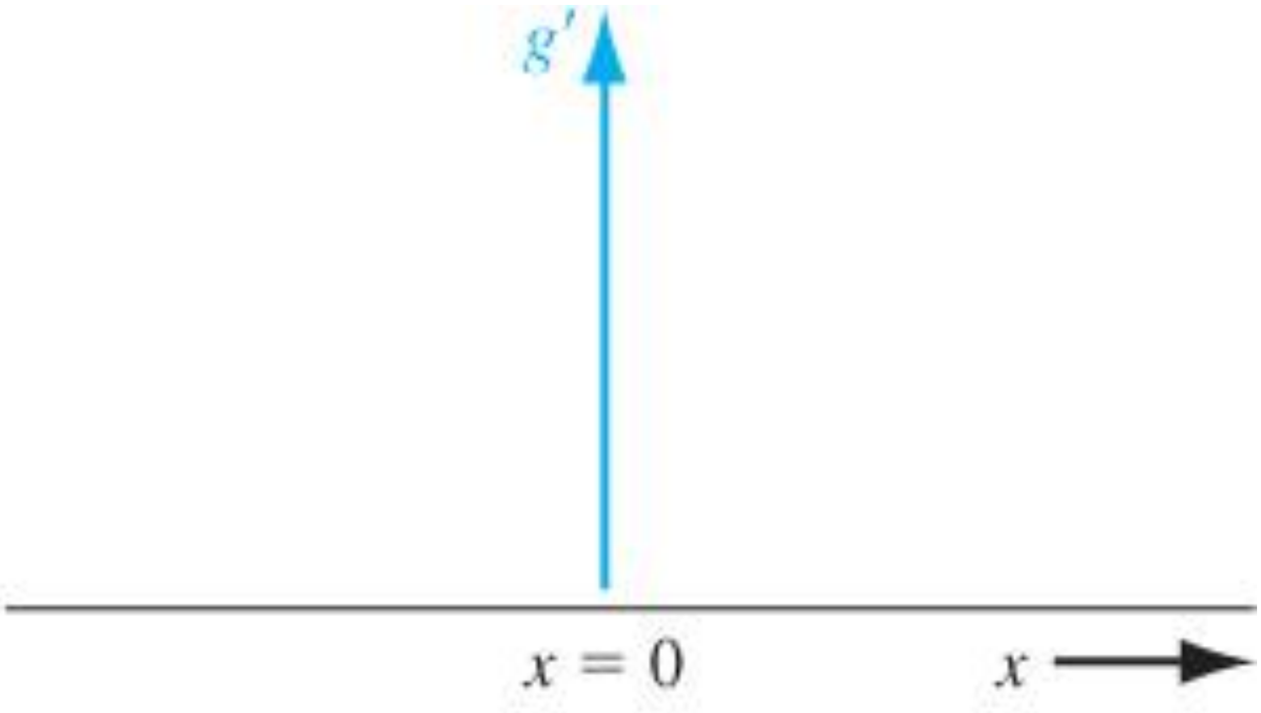
双极输运方程的应用II

$$\frac{\partial(\delta p)}{\partial t}=D_p\frac{\partial^2(\delta p)}{\partial x^2}-\mu_p E\frac{\partial(\delta p)}{\partial x}+g'-\frac{\delta p}{\tau_{p0}}$$

t=0时，光照取消：

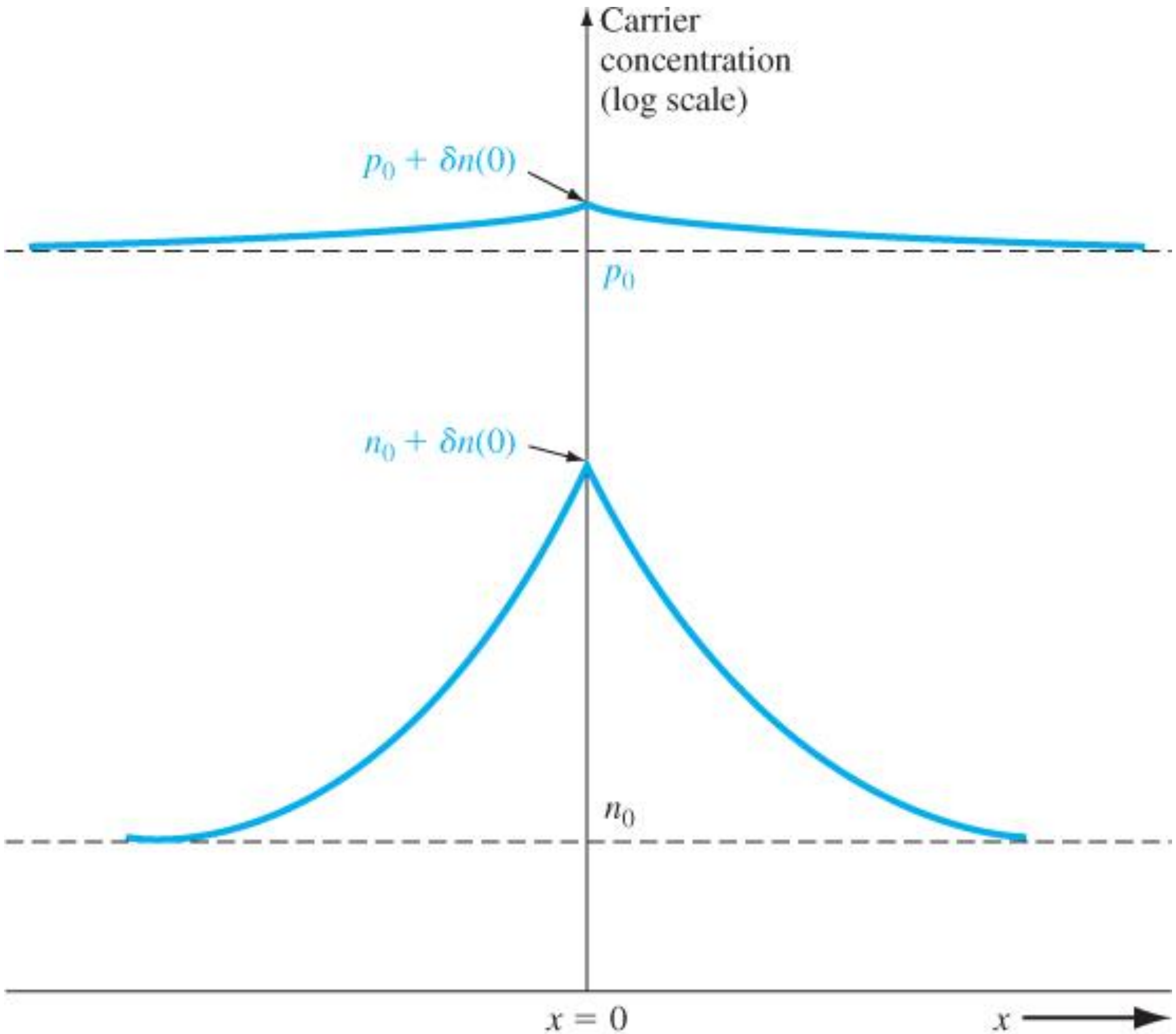


双极输运方程的应用III

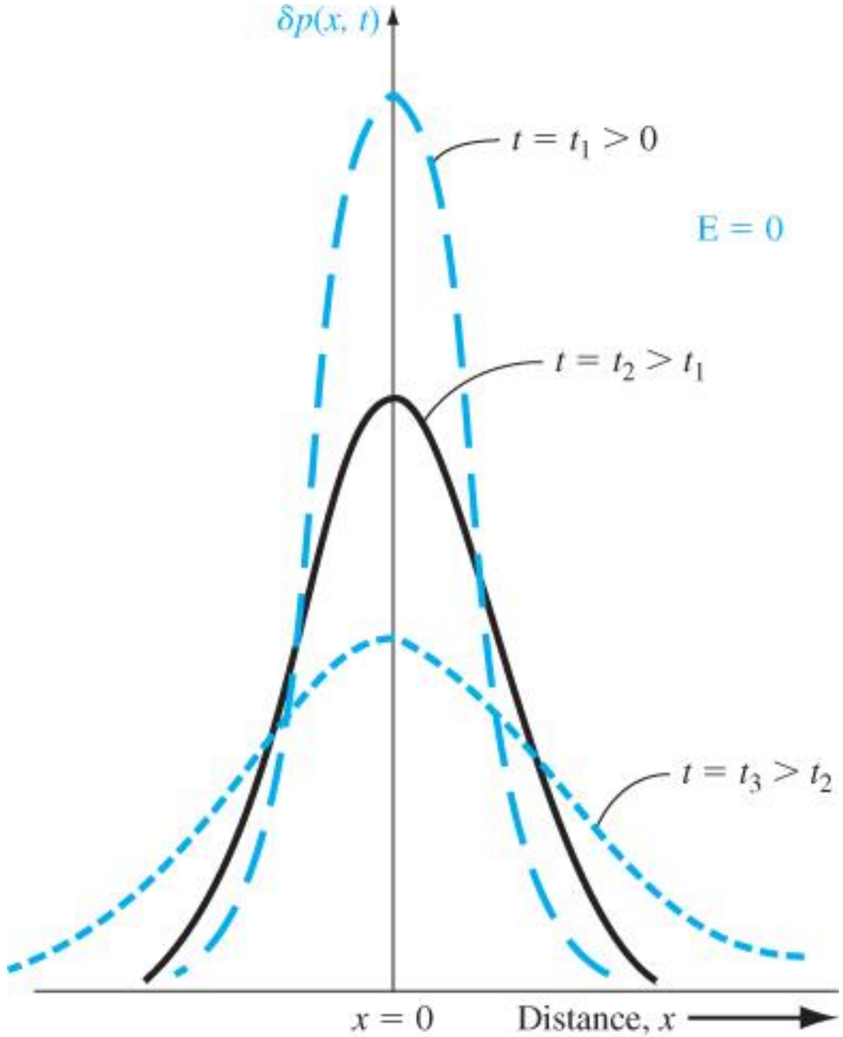


$$\frac{\partial(\delta n)}{\partial t} = D_n \frac{\partial^2(\delta n)}{\partial x^2} + \mu_n E \frac{\partial(\delta n)}{\partial x} + g' - \frac{\delta n}{\tau_{n0}}$$

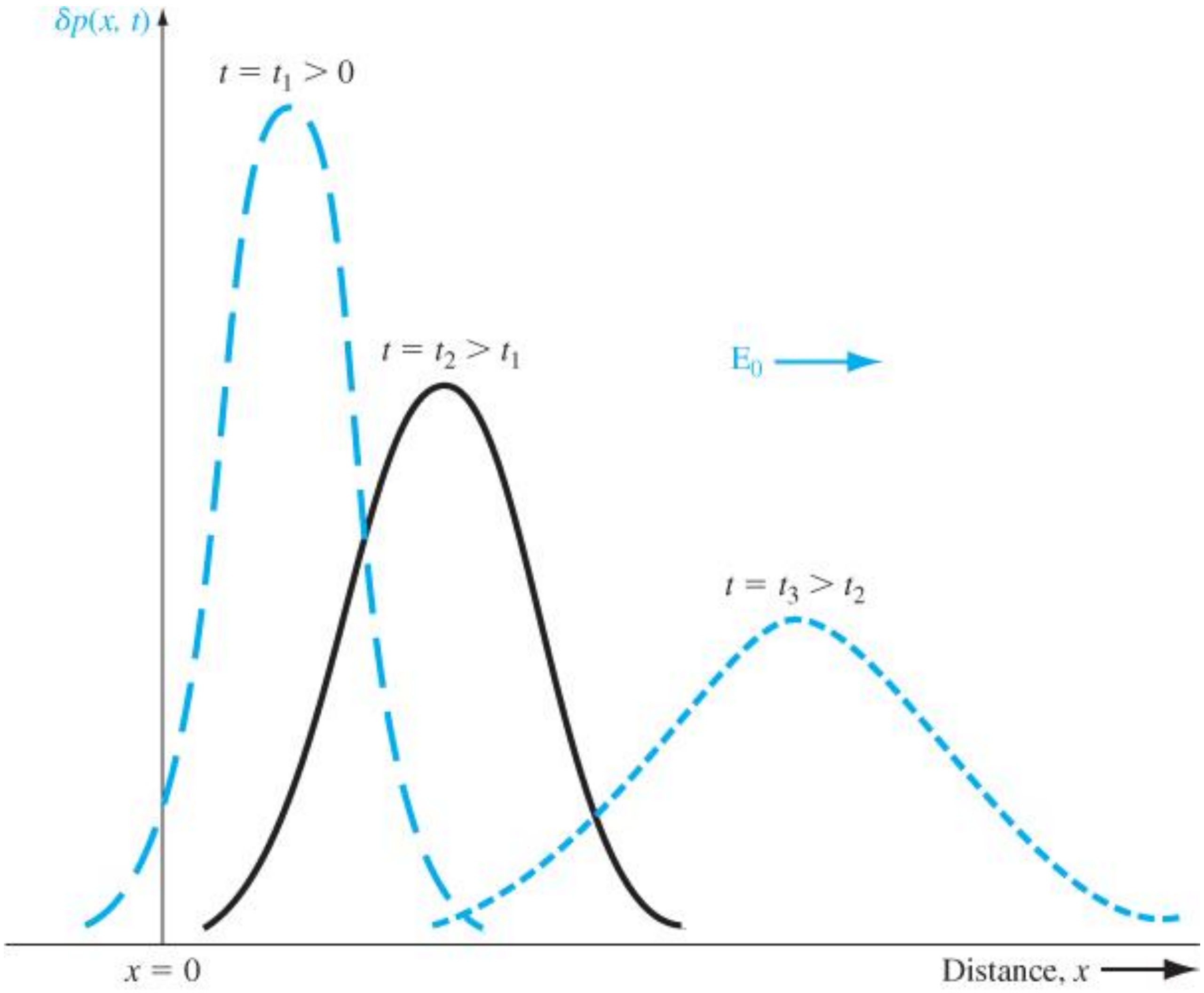
双极输运方程的应用III



双极输运方程的应用IV

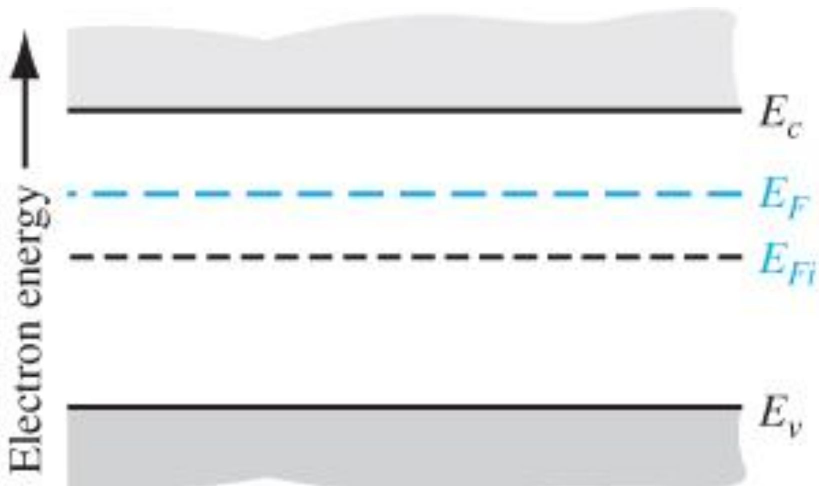


双极输运方程的应用IV



费米能级(热平衡状态)

$$n_0 = n_i \exp\left(\frac{E_F - E_{Fi}}{kT}\right) \qquad p_0 = n_i \exp\left(\frac{E_{Fi} - E_F}{kT}\right)$$

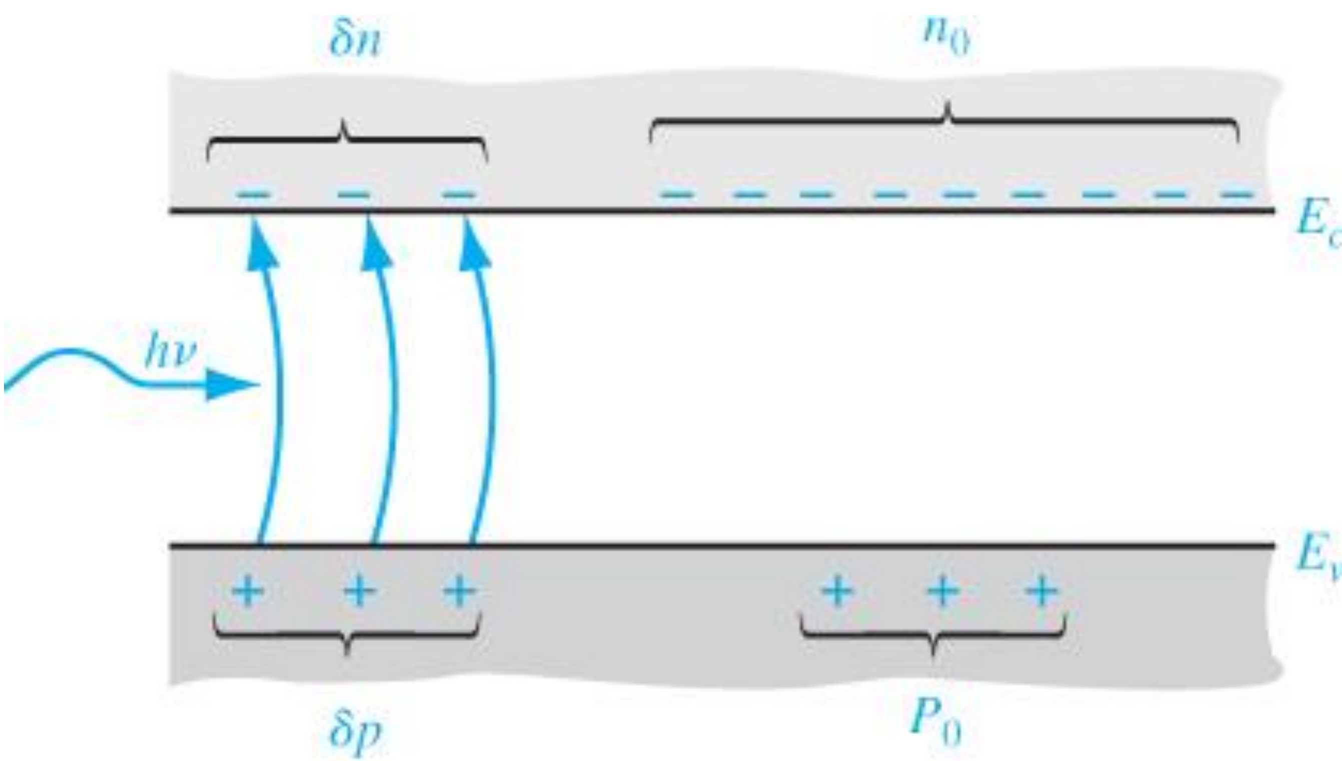


n型

p型

非平衡过剩载流子产生

高能光子（能量大于禁带宽度）射入半导体，价带电子跃迁进入导带



$$n = n_0 + \boxed{\delta n}$$
$$p = p_0 + \boxed{\delta p}$$

过剩电子、空穴

准费米能级 E_{Fn}, E_{Fp}

$$n = n_i \exp\left(\frac{E_{Fn} - E_{Fi}}{kT}\right)$$

$$p = n_i \exp\left(\frac{E_{Fi} - E_{Fp}}{kT}\right)$$

$$= n_i \exp\left(\frac{E_{Fn} - E_F + E_F - E_{Fi}}{kT}\right)$$

$$= n_i \exp\left(\frac{E_F - E_{Fi}}{kT}\right) \exp\left(\frac{E_{Fn} - E_F}{kT}\right)$$

$$= n_0 \exp\left(\frac{E_{Fn} - E_F}{kT}\right)$$

$$p = p_0 \exp\left(\frac{E_F - E_{Fp}}{kT}\right)$$

准费米能级 $E_{\mathrm{Fn}}, E_{\mathrm{Fp}}$

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$$n = n_0 \exp\left(\frac{E_{Fn} - E_F}{kT}\right) \qquad p = p_0 \exp\left(\frac{E_F - E_{Fp}}{kT}\right)$$

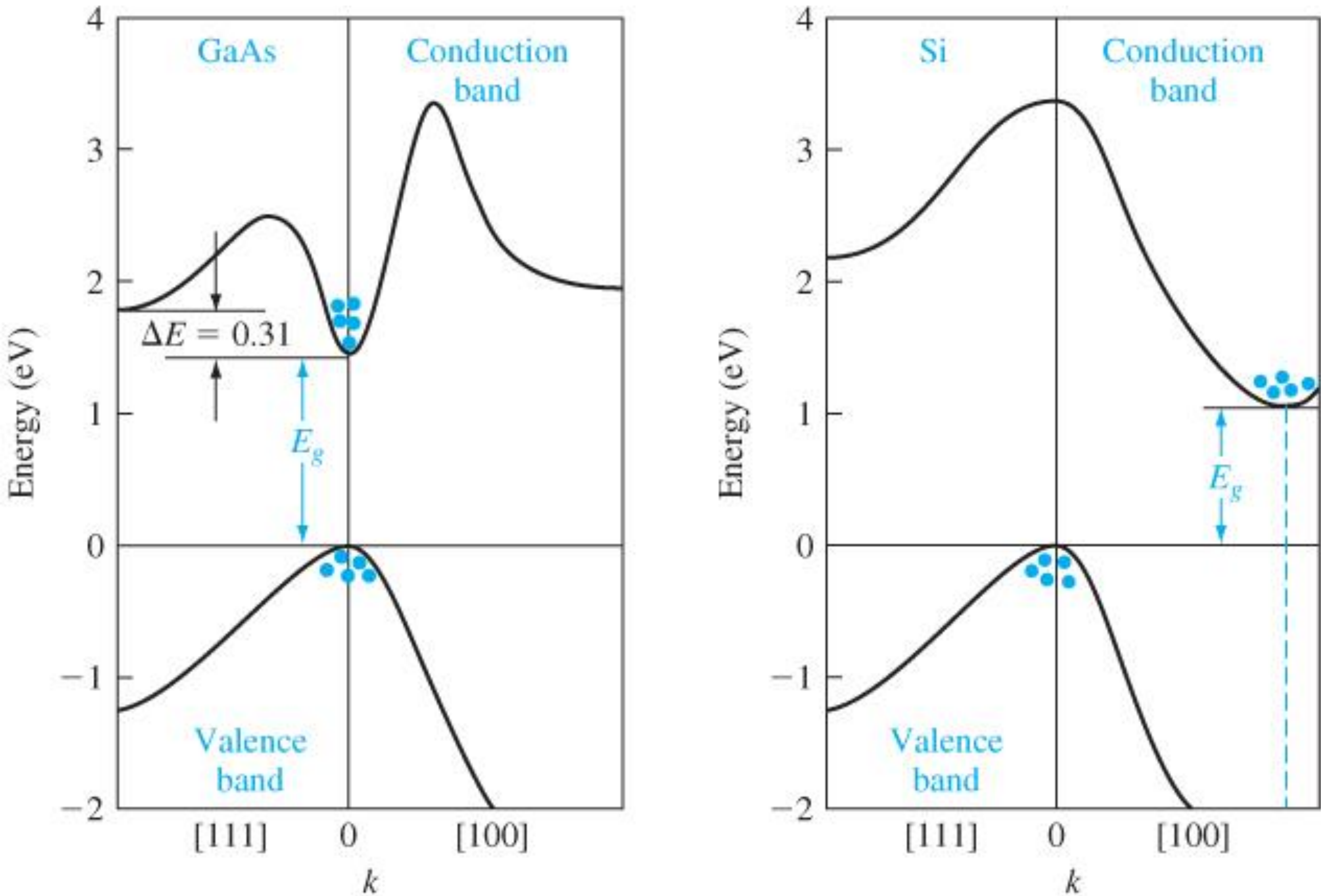
$$np = n_0 \exp\left(\frac{E_{Fn} - E_F}{kT}\right) \cdot p_0 \exp\left(\frac{E_F - E_{Fp}}{kT}\right)$$

$$= n_0 p_0 \exp\left(\frac{E_{Fn} - E_{Fp}}{kT}\right)$$

$$= n_i^2 \exp\left(\frac{E_{Fn} - E_{Fp}}{kT}\right)$$

载流子复合

直接带隙、间接带隙半导体

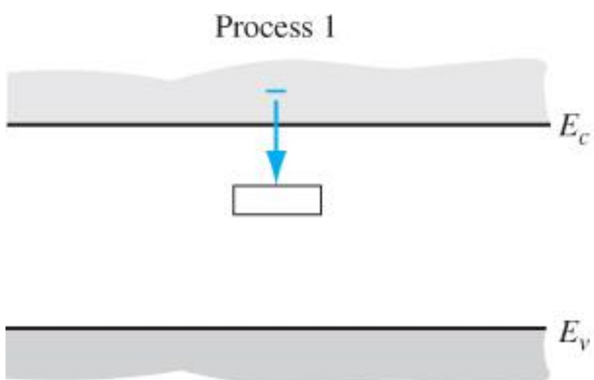


SRH复合理论

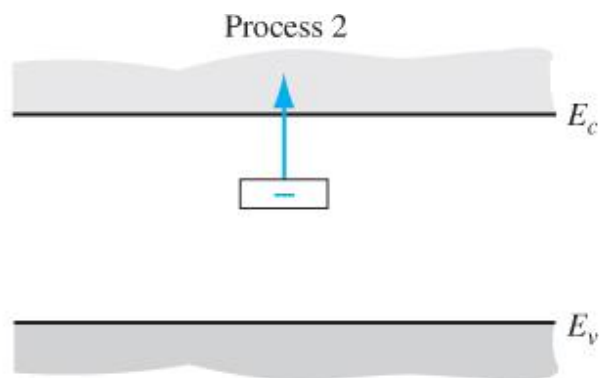
硅是间接带隙半导体，直接复合概率低，常通过复合中心间接复合。

复合中心:由于晶体中存在缺陷而在禁带中产生分立的电子能态。

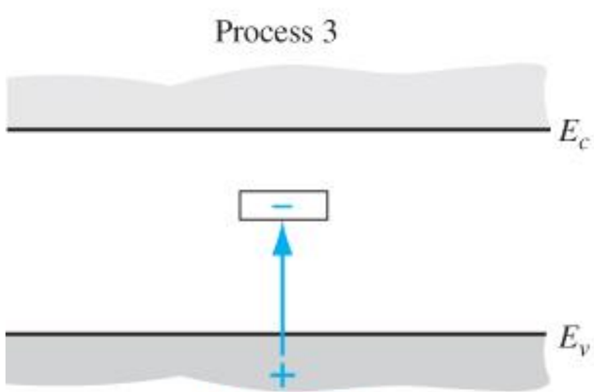
Shockley-Read-Hall (SRH) 复合理论



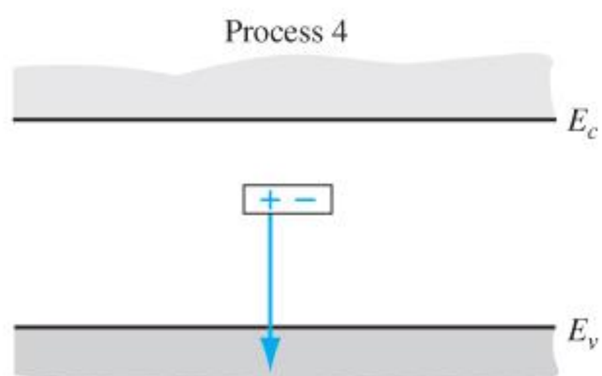
电子俘获



电子辐射

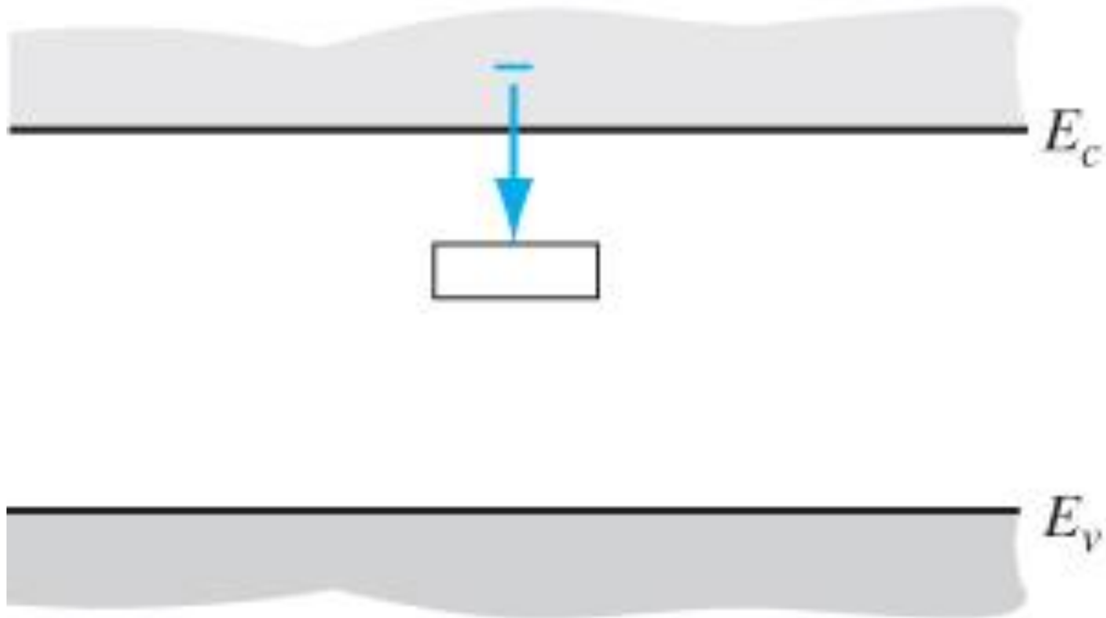


空穴俘获



空穴辐射

SRH复合理论

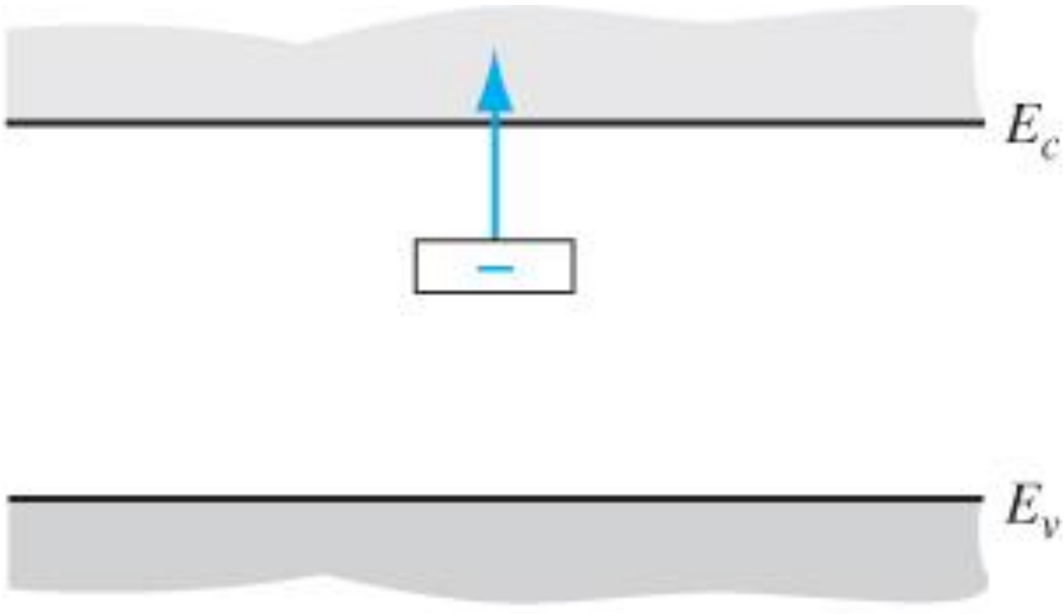


电子俘获

$$R_{cn} = C_n N_t \left[1 - f_F(E_t)\right] n$$

$$f_F(E_t) = \frac{1}{1 + \exp\left(\dfrac{E_t - E_F}{kT}\right)}$$

SRH复合理论

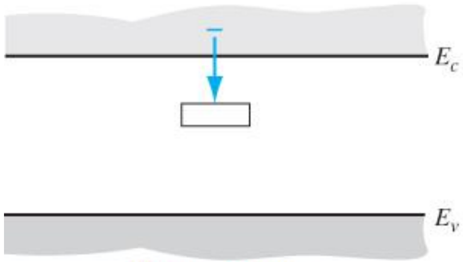


电子辐射

$$R_{en} = E_n N_t f_F(E_t)$$

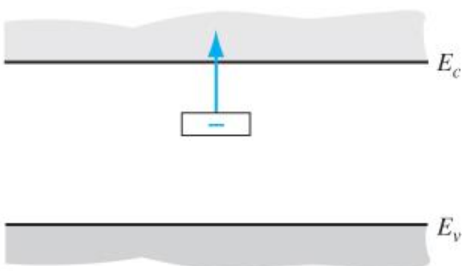
SRH复合理论

热平衡状态下：



电子俘获

$$R_{cn}=C_nN_t\left[1-f_{F0}(E_t)\right]n_0$$



电子辐射

$$R_{en}=E_nN_tf_{F0}(E_t)$$

$$C_nN_t\left[1-f_{F0}(E_t)\right]n=E_nN_tf_{F0}(E_t)$$

$$f_{F0}(E_t)=\frac{1}{1+\exp\left(\dfrac{E_t-E_F}{kT}\right)}=\exp\left(-\frac{E_t-E_F}{kT}\right)$$

热平衡状态下：

$$E_n N_t f_{F0}(E_t) = C_n N_t \left[1 - f_{F0}(E_t)\right] n_0$$

$$f_{F0}(E_t) = \frac{1}{1 + \exp\left(\frac{E_t - E_F}{kT}\right)} = \exp\left(-\frac{E_t - E_F}{kT}\right)$$

$$1 - f_{F0}(E_t) = \frac{1}{1 + \exp\left(-\frac{E_t - E_F}{kT}\right)} \approx 1$$

热平衡状态下：

$$\boxed{n_0 = N_C \exp\left(\frac{E_F - E_C}{kT}\right)}$$

$$E_n = C_n n_0 \exp\left(\frac{E_t - E_F}{kT}\right)$$

$$= C_n N_C \exp\left(\frac{E_t - E_C}{kT}\right)$$

$$= C_n n' \qquad n' = N_C \exp\left(\frac{E_t - E_C}{kT}\right)$$

SRH复合理论

非平衡状态下，导带电子被俘获的净概率：

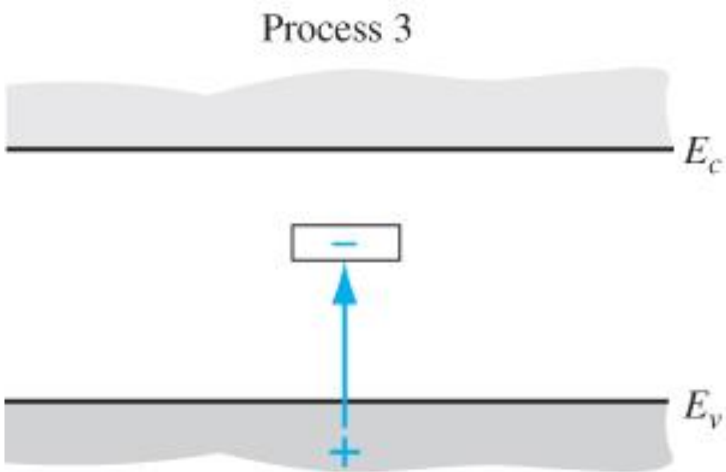
$$R_n = R_{cn} - R_{en}$$

$$= C_n N_t \left[1 - f_F(E_t)\right] n - E_n N_t f_F(E_t)$$

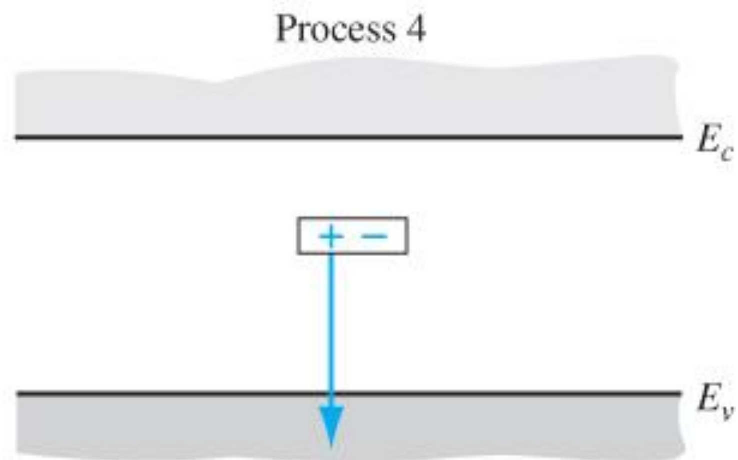
$$= C_n N_t \left[n\left(1 - f_F(E_t)\right) - n' f_F(E_t) \right]$$

$$n' = N_C \exp\left(\frac{E_t - E_C}{kT} \right)$$

SRH复合理论



空穴俘获



空穴辐射

非平衡状态下，价带空穴被俘获的净概率：

$$R_p = R_{cp} - R_{ep} \qquad p' = N_V \exp\left(\frac{E_V - E_t}{kT}\right)$$

$$= C_p N_t \left[f_F(E_t) p - p'\left(1 - f_F(E_t)\right) \right]$$

非平衡状态下，导带电子被俘获的净概率：

$$R_n = R_{cn} - R_{en}$$
$$= C_n N_t \left[n\left(1 - f_F(E_t)\right) - n' f_F(E_t) \right]$$

非平衡状态下，价带空穴被俘获的净概率：

$$R_p = R_{cp} - R_{ep}$$
$$= C_n N_t \left[f_F(E_t) p - p'\left(1 - f_F(E_t)\right) \right]$$

SRH复合理论

$$R_n = C_n N_t \left[n\left(1 - f_F(E_t)\right) - n' f_F(E_t) \right]$$

$$R_p = C_n N_t \left[f_F(E_t) p - p'\left(1 - f_F(E_t)\right) \right]$$

稳态情况下：

$$R_n = R_p$$

$$= \frac{C_n C_p N_t (np - n_i^2)}{C_n(n + n') + C_p(p + p')}$$

SRH复合理论

$$R_n = R_p = \frac{C_n C_p N_t (np - n_i^2)}{C_n(n+n') + C_p(p+p')}$$

$$= C_n N_t \frac{np - n_i^2}{n + n' + p + p'} \qquad (C_n = C_p)$$

$$= C_n N_t \frac{p - p_{n0}}{1 + \frac{(n' + p')}{n}} \qquad (n \gg p)$$

SRH复合理论

$$R=C_nN_t\frac{p-p_{n0}}{1+\frac{(n'+p')}{n}}$$

$$n'=N_C\exp\left(\frac{E_t-E_C}{kT}\right)$$

$$=N_C\exp\left(\frac{E_t-E_{Fi}+E_{Fi}-E_C}{kT}\right)$$

$$=N_C\exp\left(\frac{E_{Fi}-E_C}{kT}\right)\exp\left(\frac{E_t-E_{Fi}}{kT}\right)$$

$$=n_i\exp\left(\frac{E_t-E_{Fi}}{kT}\right)$$

SRH复合理论

$$R=C_nN_t\frac{p-p_{n0}}{1+\frac{(n'+p')}{n}}$$

$$p'=N_V\exp\left(\frac{E_V-E_t}{kT}\right)$$

$$=N_V\exp\left(\frac{E_V-E_{Fi}+E_{Fi}-E_t}{kT}\right)$$

$$=N_V\exp\left(\frac{E_V-E_{Fi}}{kT}\right)\exp\left(\frac{E_{Fi}-E_t}{kT}\right)$$

$$=n_i\exp\left(\frac{E_{Fi}-E_t}{kT}\right)$$

$$R = C_n N_t \frac{p - p_{n0}}{1 + \frac{(n' + p')}{n}}$$

$$p' = n_i \exp\left(\frac{E_{Fi} - E_t}{kT}\right)$$

$$n' = n_i \exp\left(\frac{E_t - E_{Fi}}{kT}\right)$$

$$= C_n N_t \frac{p - p_{n0}}{1 + \frac{n_i}{n}\left(\exp\left(\frac{E_{Fi} - E_t}{kT}\right) + \exp\left(\frac{E_t - E_{Fi}}{kT}\right)\right)}$$

$$= C_n N_t \frac{p - p_{n0}}{1 + 2\left(\frac{n_i}{n}\right)\cosh\left(\frac{E_{Fi} - E_t}{kT}\right)}$$

SRH复合理论

$$R = C_n N_t \frac{p - p_{n0}}{1 + 2\left(\frac{n_i}{n}\right)\cosh\left(\frac{E_{Fi} - E_t}{kT}\right)}$$

小注入下n型半导体：

$$R = C_p N_t \delta p$$

$$= \frac{\delta p}{1/C_p N_t} = \frac{\delta p}{\tau_{p0}}$$

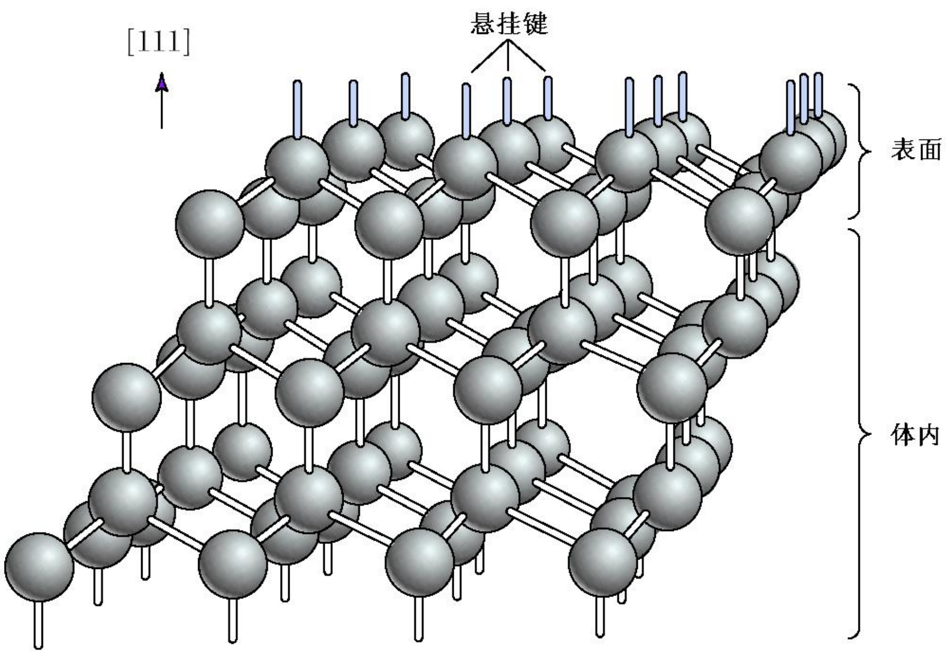
小注入下p型半导体：

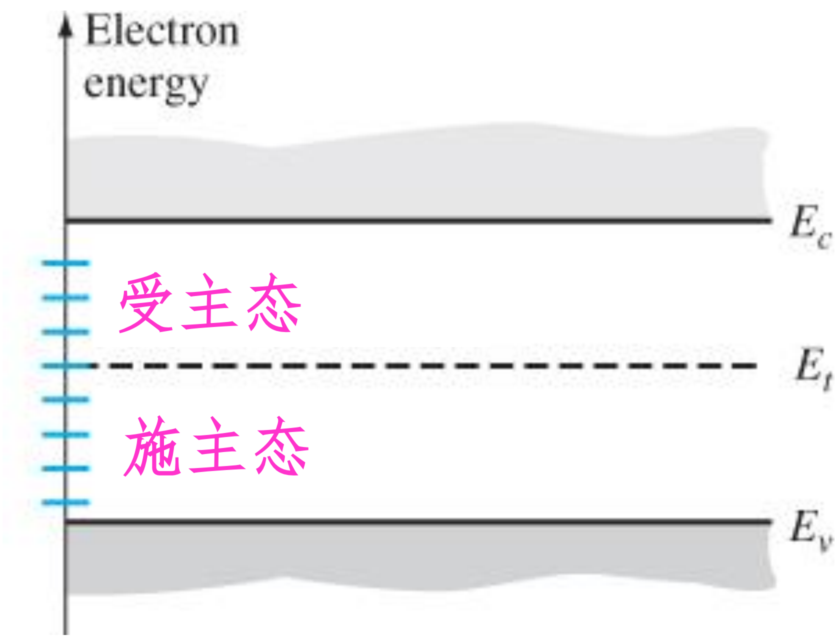
$$R = C_n N_t \delta n$$

$$= \frac{\delta n}{1/C_n N_t} = \frac{\delta n}{\tau_{n0}}$$

表面态:

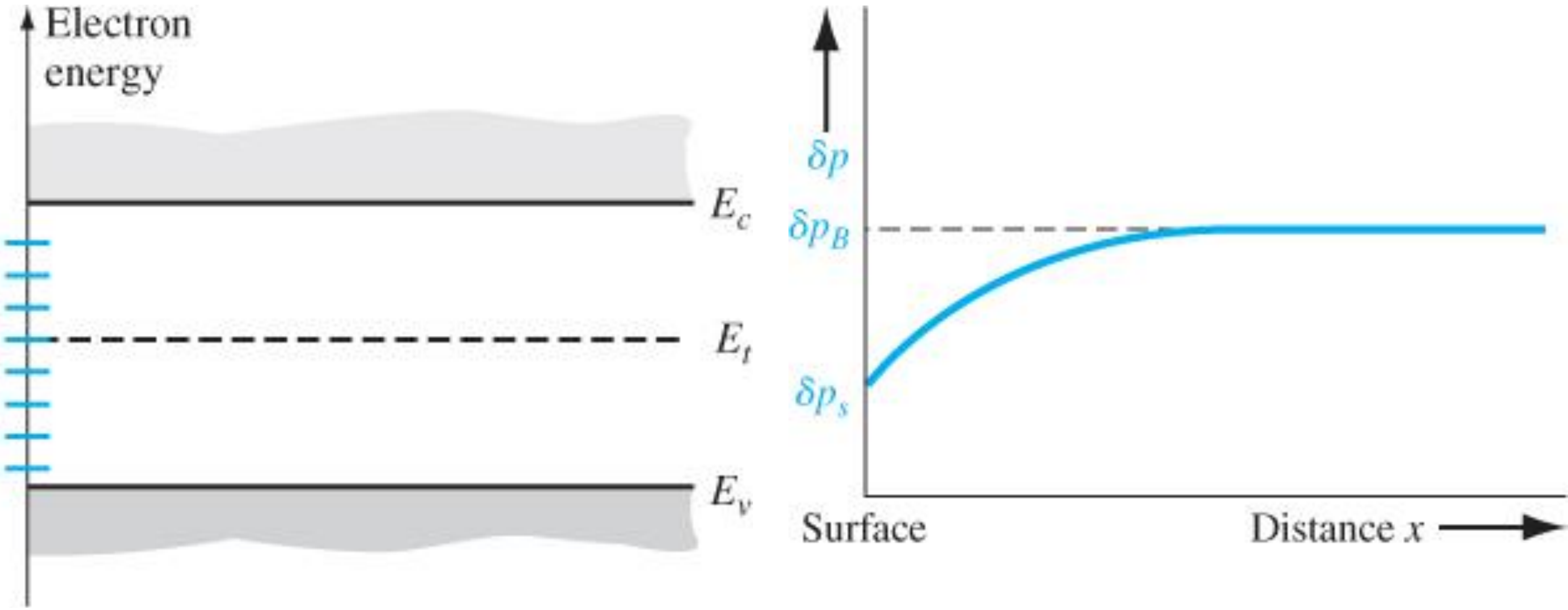
由于在晶体结构在表面中断,产生局部的能态(产生-复合中心).





表面　　　　体内

表面复合



Electron
energy
E_c
E_t
E_v
δp
δp_B
δp_s
Surface
Distance x

表面　　　　体内

非平衡载流子小结

1) 产生（Generation）、复合（Recombination）,直接、间接
2) 产生率、复合率
3) 非平衡态（光照、辐射、施加电压等）
4) 过剩载流子的产生与复合
   - 过剩载流子（δn, δp）
   - 载流子的总浓度（ $n = n_0 + \delta n$, $p = p_0 + \delta p$ ）
   - 小注入（过剩载流子浓度远小于热平衡多子浓度）
5) 连续性方程
6) 双极输运方程
7) 准费米能级（E_{Fn} , E_{Fp}）
8) Shockley-Read-Hall(SRH)复合理论
9) 表面复合