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1 Chem 30324, Spring 2018, Homework 1 Solution

1.1 Problem 1 : Discrete, Probably

1.1.1 How many different 5-card hands are there? (Remember, in poker the order in which the cards are received does not matter.)

The number of 5 card hands is $52C_5$ = 2,598,960.

1.1.2 What is the probability of being dealt four of a kind (a card of the same rank from each suit)?

No. of ways to choose the rank = $13C_1$ = 13
No. of ways to choose the 5th card = $48C_1$ = 48
Total no. of ways to get a four of a kind = 48*13 = 624
Probability of getting a four of a kind = 624/2598560 = 0.00024010

1.1.3 What is the probability of being dealt a flush (five cards of the same suit)?

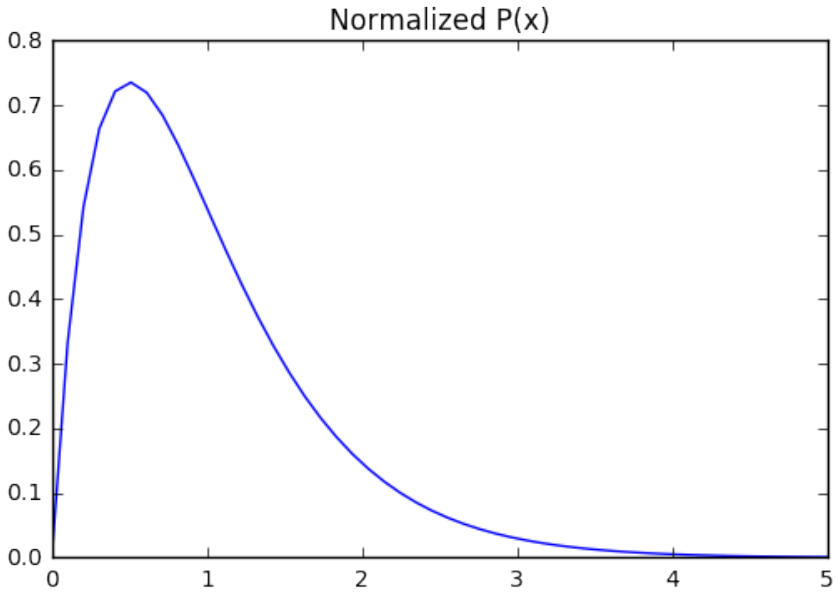
No. of ways to choose the suit = $4C_1$ = 4
No. of ways of choosing 5 cards = $13C_5$ = 1287
No. of ways of being dealt a flush = 1287*4 = 5148
Probability = 5148/2598960 = 0.00198079

1.2 Problem 2: Continuous, probably

1.2.1 Is P (x) normalized? If not, normalize it. Plot the normalized P (x) .

$$\int P(x)\,dx$$

from x=0 to infinity gives 1/4. So P(x) is not normalized
Hence the normalized P(x) = P(x)/(1/4) = $4xe^{-2x}$
The plot looks like



1.2.2 What is the most probable value of x ?

The most probable value is found by setting

$$\frac{dP(x)}{dx} = 0$$

The most probable value is x = 0.5.

1.2.3 What is the expectation value of x ?

The expectation value is found by evaluating

$$\int xP(x)dx$$

from x=0 to infinity.
The expectation value is 1.0.

1.2.4 What is the variance of x ?

The variance is found by evaluating

$$Variance = \int x^2 P(x)dx - (\int xP(x)dx)^2$$

The answer is 0.5.

1.3 Problem 3: One rough night

1.3.1 What is the furthest distance the person could travel after 20 steps?

The furthest distance is when 20 steps are in the same direction. 20 metres.

1.3.2 What is the probability that the person won't have traveled any net distance at all after 20 steps?

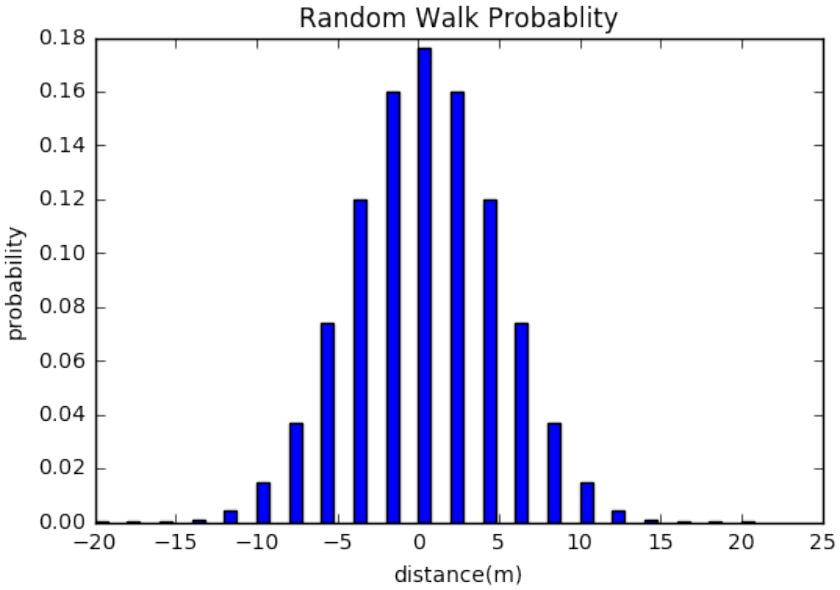
For no net distance, the person must take 10 steps north and 10 steps south.
Since any step of the person is either north or south
Number of ways for no net distance = Number of ways of choosing 10 steps from 20.
= $20C_{10}$ = 184756
Total number of ways the person can take steps = 2^20.
Probability of no net distance = 184756/2^20 = 0.1762.

1.3.3 What is the probability that the person has traveled half the maximum distance after 20 steps?

Half the maximum distance = 10m
There are two ways to get to 10m distance. 15 steps north, 5 steps south or vice versa.
Total ways = 2 * number fo ways to choose 15 steps from 20.
= 2* $20C_{15}$ = 31008.
Probability of half the maximum distance = 31008/2^20 = 0.0296.

1.3.4 Plot the probability of traveling a given distance vs distance. Does the probability distribution look familiar? You'll see it again when we talk about diffusion.

The probability distribution looks like :



This looks like a Guassian distribution.

1.4 Problem 4: Now this is what I call equilibrium

1.4.1 What is the expectation value of the velocity v of a particle?

The normalized probability as a function of velocity v is :

$$P(v) = \sqrt{\frac{m}{2pik_BT}} e^{\frac{mv^2}{2kT}}$$

The expectation value of the velocity is :

$$< v >= \int_0^{inf} vP(v)\, dv$$

The answer is 0.

1.4.2 What is the expectation value of the kinetic energy K of a particle? How does your answer depend on the particle mass? On temperature?

The expectation value of kinetic energy is calculated as

$$< K >= \int_0^{inf} \frac{mv^2}{2} P(v) dv$$

The answer is $0.5 k_B T$