

DSP Homework 06

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Abstract

1 Summary and Thoughts

2 My Sampling Method

2.1 Restatement

Use the Fourier series to develop a sampling method and compare it with the Shannon/Nyquist sampling method through examples.

2.2 Improvement

In class, we are considering the energy of the Fourier transform of the $s(t)$, we have

$$s(t) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f T} \quad (1)$$

and we have

$$\begin{aligned} E &= \int_{-\frac{T}{2}}^{\frac{T}{2}} s(t) \cdot s^*(t) dt \\ &= \int_{-\frac{T}{2}}^{\frac{T}{2}} \sum_{n=-\infty}^{\infty} c_n e^{-j2\pi n f T} \cdot \sum_{m=-\infty}^{\infty} c_m e^{j2\pi m f T} dt \\ &= \int_{-\frac{T}{2}}^{\frac{T}{2}} \sum_{n=-\infty}^{\infty} c_m \cdot c_n e^{j2\pi(n-m)fT} dt \\ &= \sum_{n=-\infty}^{\infty} c_m \cdot c_n \int_{-\frac{T}{2}}^{\frac{T}{2}} e^{j2\pi(n-m)fT} dt \end{aligned}$$

and the integral of $e^{j2\pi(n-m)fT}$ usually be 0 except that $n = m$, so

$$E = \sum_{n=-\infty}^{\infty} c_n^2 \cdot T$$

I will try to prove that if $E < \infty$, we must ensure that $n \rightarrow \infty, c_n \rightarrow 0$, which means

$$E < \infty \Leftrightarrow n \rightarrow \infty, c_n \rightarrow 0 \quad (2)$$

if that

$$n \rightarrow \infty, c_n \rightarrow C \ (C \neq 0)$$

we can always find a big number N which makes $c_n^2 > 0 \ (n > N)$, and we can easily find that

$$\sum_{n=N}^{\infty} c_n^2 > (\infty - N) \cdot c_{n(min)}^2 \rightarrow \infty$$

So, $E \rightarrow \infty$ when $n \rightarrow \infty, c_n \rightarrow C \ (C \neq 0)$. Therefore, Equa.2 is proved. And we know that

$$c_n = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} s(t) e^{-j2\pi n f_s t} dt \quad (3)$$

So, as I find out last week in my weekly report, if we let the square wave last the length of 2τ , we now have

$$\begin{aligned} s(t) &= \sum_{n=-\infty}^{\infty} \text{rect}\left(\frac{t-nT}{\tau}\right) \\ &= \sum_{n=-\infty}^{\infty} F_n e^{j2\pi n f_s t} \\ &= \sum_{n=-\infty}^{\infty} \left[\frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \text{rect}\left(\frac{t}{\tau}\right) e^{-j2\pi n f_s t} dt \right] e^{j2\pi n f_s t} \\ &= \sum_{n=-\infty}^{\infty} \left[f_s \int_{-\tau}^{\tau} e^{-j2\pi n f_s t} dt \right] e^{j2\pi n f_s t} \\ &= \sum_{n=-\infty}^{\infty} 2f_s \text{sinc}(2f_s n\tau) e^{j2\pi n f_s t} \end{aligned} \quad (4)$$

Here,

$$\begin{aligned} s(t) &= \sum_{n=-\infty}^{\infty} \text{rect}\left(\frac{t-nT}{\tau}\right) \\ c_n &= 2f_s \text{sinc}(2f_s n\tau) \end{aligned}$$

Therefore

$$\begin{aligned} E &= T \cdot \sum_{n=-\infty}^{\infty} c_n^2 \\ &= T \cdot \sum_{n=-\infty}^{\infty} \text{sinc}^2(2f_s n\tau) \end{aligned}$$

In help of wolfram, I know that when a is real:

$$\sum_{n=-\infty}^{\infty} \text{sinc}^2(a \times n) < \infty$$

Therefore

$$\sum_{n=-\infty}^{\infty} \text{sinc}^2(2f_s n\tau) < \infty \quad (5)$$

we can now say that the $s(t)$ mentioned in Equa.4 is energy-limited.

If we look back on the $\delta(t)$ sampling, we have the energy E of $s(t)$:

$$E = \sum_{n=-\infty}^{\infty} T \rightarrow \infty \quad (6)$$

which is not realizable.

3 Conclusion

Appendix A Code Listing

```
import numpy as np
from sympy import symbols, integrate, sinc

x, a = symbols('x, a')

print(integrate((sinc(a*x))**2,(x,-np.inf,np.inf)))
```