DSP Homework 14

1. Write a summary of this week's video(s) and your further thoughts on the content.

2. Common linear digital filters can be represented by

$$y(n) + b_1 y(n-1) + \cdots + b_Q y(n-Q) = a_0 x(n) + \cdots + a_P x(n-P). \tag{1}$$

   The total number of parameters is $M = P + Q + 1$.

   (a) Prove that when $Q = 0$, the system is FIR; when $Q > 0$, the system is IIR.

   (b) IIR filters have the stability problem. Set up the following polynomial

   $$H(z) = z^Q + b_1 z^{Q-1} + \cdots + b_Q. \tag{2}$$

       i. Give a reasonable definition of the digital filter stability.

       ii. Prove that FIR filters are always stable.

       iii. Prove that IIR filters are stable if and only if the roots of $H(z)$ are all within the unit circle (radius $= 1$).

       iv. In class it is proposed that when $Q = 2$, $P = 0$, a sufficient condition for the IIR system to be stable is

   $$|b_1| + |b_2| < 1 \tag{3}$$

       Prove or disprove this conjecture.

3. Through spectrum analysis or some other way, find out why some people's voice seem "nicer" than others'.