DSP Homework 06

Xu, Minhuan

October 9, 2022

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Abstract

1 Summary and Thoughts

1.1 Bluetooth

In this video, we know about the technique about Bluetooth, I found something special about the Bluetooth as below.

Points Worth Noting

1. Working on 123 mm wave length EM wave
2. Using 120.7 mm represent 1, 124.9 mm represent 0
3. Sending a Million Bits per Second in the format of Packets
4. Having 79 Communication Channel and shifting among them continuously

In these points, I want to write more about 3. and 4.

Packets

The packets are a set of lots of $'0'$ and $'1'$.

First 72 bits are called Access Code, which are used to synchronize smartphone and earbuds to make sure that it's your specific earbuds that is that received the message.

Next 54 bits are called Header. These data are used to decide how much data should be transfer next.

Last $136 \sim 8186$ bits are called Payload. There will be control signals like pause and play or music data in this part. So, this part can be long and short which is decided by the Header part.

Frequency Hopping Spread Spectrum

My understanding is there is a list of the number of 79 Communication Channels. The phone and the earbuds should use different channel once a packet is transferred. So, the list decides in which order the Communication Channels are used.

Time slots

The Bluetooth protocol states that a packet should be transferred in 625 μs, and the earbuds and the phone alternately use a time slot for data transmission.

1.2 2

2 My Sampling Method

2.1 Restatement

Use the Fourier series to develop a sampling method and compare it with the Shannon/Nyquist sampling method through examples.

2.2 Derivation

In class, we are considering the energy of the Fourier transform of the $s(t)$, we have

$$s(t) = \sum_{n=-\infty}^{\infty} c_n \, e^{j2\pi nfT} \tag{1}$$

and we have

$$\begin{aligned} E &= \int_{-\frac{T}{2}}^{\frac{T}{2}} s(t) \cdot s^*(t) \, \mathrm{d}t \\ &= \int_{-\frac{T}{2}}^{\frac{T}{2}} \sum_{n=-\infty}^{\infty} c_m \cdot c_n \; e^{j2\pi(n-m)fT} \, \mathrm{d}t \\ &= \sum_{n=-\infty}^{\infty} c_m \cdot c_n \; \int_{-\frac{T}{2}}^{\frac{T}{2}} \; e^{j2\pi(n-m)fT} \, \mathrm{d}t \end{aligned}$$

and the integral of $e^{j2\pi(n-m)fT}$ usually be 0 except that $n-m$, so

$$E = \sum_{n=-\infty}^{\infty} c_n^2 \cdot T$$

I will try to prove that if $E < \infty$, we must ensure that $n \to \infty, c_n \to 0$, which means

$$E < \infty \Rightarrow n \to \infty, c_n \to 0 \tag{2}$$

if that

$$n \to \infty, c_n \to C \, (C \neq 0)$$

we can always find a big number N which makes $c_n^2 > 0 \, (n > N)$, and we can easily find that

$$\sum_{n=N}^{\infty} c_n^2 > (\infty - N) \cdot c_{n(min)}^2 \to \infty$$

So, $E \to \infty$ when $n \to \infty, c_n \to C \, (C \neq 0)$. Therefore, Equa.2 is proved. And we know that

$$c_n = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} s(t) \, e^{-j2\pi nf_s t} \, \mathrm{d}t$$

So, as I find out last week in my weekly report, if we let the square wave last the length of 2τ, we now have

$$\begin{aligned}
s(t) &= \sum_{n=-\infty}^{\infty} \mathrm{rect}(\frac{t-nT}{\tau}) \\
&= \sum_{n=-\infty}^{\infty} F_n e^{j2\pi n f_s t} \\
&= \sum_{n=-\infty}^{\infty} \left[\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \mathrm{rect}(\frac{t}{\tau}) e^{-j2\pi n f_s t} \mathrm{d}t\right] e^{j2\pi n f_s t} \\
&= \sum_{n=-\infty}^{\infty} \left[f_s \int_{-\tau}^{\tau} e^{-j2\pi n f_s t} \mathrm{d}t\right] e^{j2\pi n f_s t} \\
&= \sum_{n=-\infty}^{\infty} 2f_s \,\mathrm{sinc}\,(2f_s n\tau)\, e^{j2\pi n f_s t}
\end{aligned} \tag{3}$$

Here,

$$\begin{aligned}
s(t) &= \sum_{n=-\infty}^{\infty} \mathrm{rect}(\tfrac{t-nT}{\tau}) \\
c_n &= 2f_s \,\mathrm{sinc}\,(2f_s n\tau)
\end{aligned}$$

Therefore

$$\begin{aligned}
E &= T \cdot \sum_{n=-\infty}^{\infty} c_n^2 \\
&= T \cdot \sum_{n=-\infty}^{\infty} sinc^2(2f_s n\tau)
\end{aligned} \tag{4}$$

In help of Wolfram Alpha [1], I know that when a is real, this series (Eq. 4) converges. The result is shown in Fig. 1.

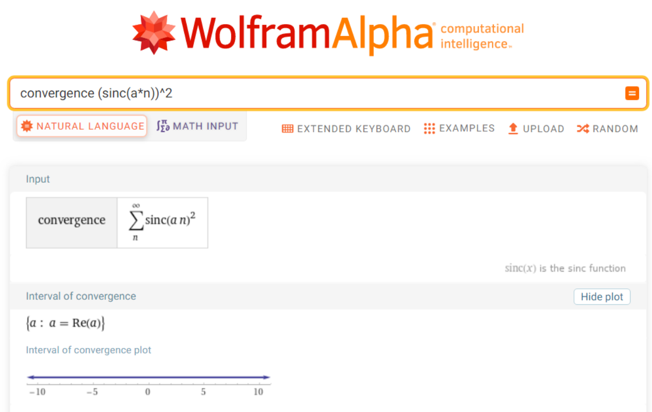

Figure 1: Convergence of $sinc^2(a \times n)$

Therefore

$$\sum_{n=-\infty}^{\infty} sinc^2(2f_s n\tau) < \infty$$

we can now say that the $s(t)$ mentioned in Equa.3 is energy-limited.

And back to Equa.4, we can control the order of the sum to improve this equation because the c_n decrease rapidly. Therefore

$$E = T \cdot \sum_{n=-N}^{N} sinc^2(2f_s n\tau)$$

So, let's use python to draw a picture to see more clearly, see Fig. 2.

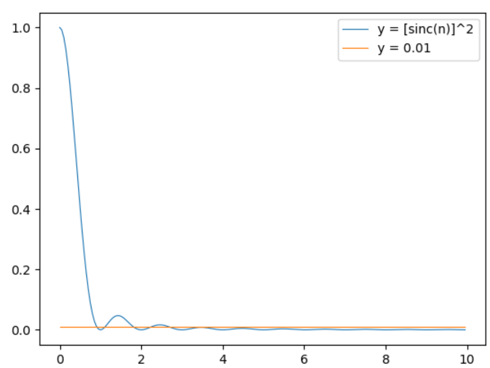

Figure 2: Orders of Sinc Function

It's easy to find that if we make n in this picture less than 5 or even 2 , little energy will be lost. This means we can rewrite the Equa. 4 like

$$E = T \cdot \sum_{n=-N}^{N} sinc^2(2f_s n\tau) \qquad N = [\frac{5}{2f_s\tau}] + 1 \tag{5}$$

If we look back on the $\delta(t)$ sampling, we have the energy E of $s(t)$:

$$E = \sum_{n=-\infty}^{\infty} T \to \infty \tag{6}$$

This (using $\sum \delta(t)$ to sampling) is not realizable.

3 Conclusion

References

[1] https://www.wolframalpha.com/.

Appendix A Code Listing

```
import numpy as np
from matplotlib import pyplot as plt

n = np.arange(0, 10, 0.05)
y = (np.sinc(n))**2
line = [0.01 for x in n]

fig = plt.figure()
plt.plot(n, y, linewidth=0.8, label="y = [sinc(n)]^2")
plt.plot(n, line, linewidth=0.8, label="y = 0.01")
plt.legend()
plt.show()
```