DSP Homework 05 9

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October 02,2022

Contents

Abstract

The assignment is divided into three modules: in the last week we watched the Marine ecological environment, respectively, aviation fuel power problem and how to maintain a healthy state of mind of video, I felt after fulfilling the video to watch the beauty of biodiversity, energy that play an important role in modern science and technology, and positive life attitude on the important effects of mental health; Question 2 needs to be considered together with question 4. First, we need to clarify the process of Nyquist sampling theory derivation and how to achieve the recovery of the signal, and then through the analysis of this process to judge whether δ(t) has an impact on the recovery of the signal; Finally, consider question 3. The difficulty of this question lies in how to get the fastest speed point as far as possible and how to approach the distance within one second. The answer to this question is also based on solving these two problems.

1 Write a summary of this week's video(s) and your further thoughts on the content.

1.1 What the videos are talking?

This week I watched three videos about Marine life, jet fuel and mental health. In the first video explaining the sharks, whales, clown fish, seabirds, such as biological life environment, in the place where we can't see still has a mutual restriction between biological, some fish feed on sea anemones, larger fish feed on other fish, birds can even capture, coral provides a place for avoiding capture small fish. The ecosystem is a harmonious whole, and observing the life of the creatures in nature is a great way to relieve stress as time wears on; In the second video, we learned about the fuel power of the aerospace system. In the video, we found that C, H and O are relatively important elements in aerospace power. For example, CH4 can be used as an important fuel to burn in oxygen and release huge energy. Since fuel power will account for a large proportion of the total weight of space equipment in launch, reducing the weight of fuel will be an important direction of future research. In the third video, it is mentioned that our life often needs a new start. We should exercise in the sunshine more, breathe the fresh air outside, pay attention to rest, trust people more, drink less coffee and other foods that make us nervous, protect our mental health, and live happily every day.

1.2 Further thoughts

Among the three videos, I am interested in the third one. With the rapid development of society today, the communication between people has become less and less, the young people's life pressure is becoming more and more heavy, mental health problems have developed to the same important position as physical health. During the learning process of the video, I came up with the idea that the wearable device can detect the outdoor exercise and rest, and input the food intake that can easily stimulate human tension, such as coffee, as indicators to detect the mental health status of the human body, and make timely adjustments to the living habits through indicators.

2 In the development of the Shannon/Nyquist sampling theorem, the impulse function δ(t) is used. But δ(t) is not a practical signal. Does that mean that when applying the sampling theorem in practice, we need some approximation/modification?

2.1 Whether it right or not?

I think that isn't right. ?

2.2 How I prove my opinion.

Consider the difference between square wave sampling in a real-world application and impulse sampling in an ideal situation,take continuous functions x(t). Through transform characteristics of the periodic function which repeat by T_0,

$$\mathcal{F}(f(t)) = \sum_{n=-\infty}^{\infty} F_n \delta(f - nf_0), f_0 = \frac{1}{T0}$$

$$F_n = \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} f(t) e^{-jn2\pi f_0 t} dt$$

we could get the Periodic square wave pulse p(t) transform way:

$$\widetilde{p}(f) = \sum_{n=-\infty}^{\infty} P_n \delta(f - nf_0)$$

$$P_n = \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} p(t) e^{-jn2\pi f_0 t} dt$$

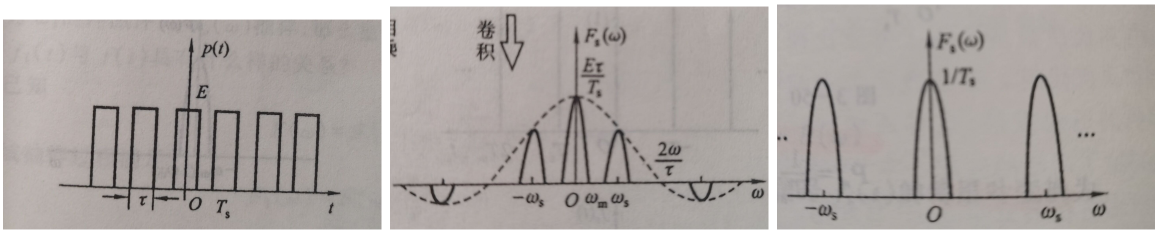
By the convolution characteristics,we can know that,

$$\begin{aligned} X_s(f) &= \widetilde{x}(f) * \widetilde{p}(f) \\ &= \sum_{n=-\infty}^{\infty} P_n \widetilde{x}(f - nf_0) \end{aligned}$$

$$
\begin{aligned}
P_n &= \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} p(t)e^{-jn2\pi f_0 t}dt \\
&= \frac{1}{T_0}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} Ee^{-jn2\pi f_0 t}dt \\
&= \frac{Er}{T_0}Sa(n\pi f_0 t)
\end{aligned}
$$

$$
\widetilde{x_s}(f) = \frac{Er}{T_0}\sum_{n=-\infty}^{\infty} Sa(n\pi f_0 t)\widetilde{x}(f-nf_0)
$$

compare with the result of Q4:$\widetilde{x_s}(t) = \widetilde{x}(f-nf_0)$,we can get that The repeating structure of the function is the same only if the amplitude changes,which like the pictures,and there's nothing wrong about $f_0 > 2W$.



(a) Square wave pulse sampling (b) Square wave pulse p(t) (c) Impulse pulse sampling

Figure 1: the pictures we need

3 Find a way to measure your maximum instantaneous running speed, accurate to 1mm/s (millimeter per second).

3.1 The problems need to be solved.

Thoughts on this issue mainly focus on two aspects:

1. How can the measured data approximate the distance in a second as closely as possible?
2. At which stage in the process of movement can the measurement measure the fastest speed?

3.2 How do I carry it out?

In the process of designing this experiment, I was inspired by the measurement method (as shown in the picture below) of the motion speed of the high school physics car and made the following design:

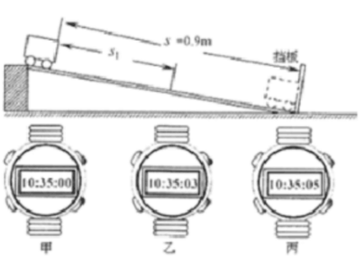


Figure 2: Design inspiration

1. Video recording is used to determine the time required for position movement.
2. Mark the ground position distance, select the site as shown below for data collection, and measure the width of each grid as 30cm.
3. Since the speed cannot reach the maximum at the beginning of running and stopping, it is considered to intercept the data in the middle 1/3 of the video for testing.

4.The movement time (about 2S as far as possible) was determined by the duration of the middle part shown in the mobile video, and the movement distance was determined by counting the number of slabs crossed, and then the maximum movement speed was found by calculation.

5.Test several times to find the maximum speed.


(a) measure the length (b) running test

Figure 3: carry it out

3.3 My answer

Analysis?

By testing,I find the maximum speed is 2700mm/s.

4 Derive the result of the Shannon/Nyquist sampling theorem and the perfect reconstruction formula.

4.1 How I prove it?

Introduce the continuous signals x(t) likes the following figure and the sampling signals s(t) which meet following conditions,and let $\mathcal{F}(x(t)) = \widetilde{x}(t)$.

$$s(t) = \sum_{n=-\infty}^{\infty} \Delta(t - nT_0)$$

Then we can get the function after sampling $x_s(t)$ which is,

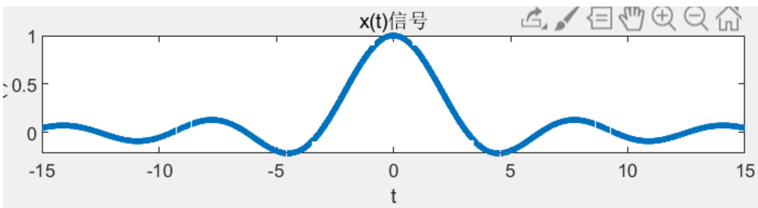


Figure 4: signal x(t)

$$x_s(t) = x(t)s(t)$$

From the convolution characteristics we can get that,

$$\begin{aligned}\mathcal{F}[x_s(t)] &= \mathcal{F}[x(t)s(t)] \\ &= \widetilde{x}(f) * \widetilde{s}(f)\end{aligned}$$

Think the Fourier series expansion of s(t),

$$
\begin{aligned}
s(t) &= \sum_{n=-\infty}^{\infty} \Delta(t-nT_0) \\
&= \sum_{k=-\infty}^{\infty} a_k e^{j2\pi k f_0 t}, f_0 = \frac{1}{T_0}
\end{aligned}
$$

$$
\begin{aligned}
a_k &= \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} s(t)e^{jkt}dt \\
&= \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \sum_{n=-\infty}^{\infty} \Delta(t-nT_0)e^{jkt}dt, when \qquad k=n, a_k \neq 0 \\
&= \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \Delta(t)e^{jnt}dt \\
&= \frac{1}{T_0}, whenk = n.
\end{aligned}
$$

Then

$$
\begin{aligned}
\widetilde{s}(t) &= \mathcal{F}[\frac{1}{T0}\sum_{n=-\infty}^{\infty} e^{j2\pi n f_0 t}] \\
&= \frac{1}{T0}\Delta(f-nf_0)
\end{aligned}
$$

And we can know that,

$$
\begin{aligned}
\widetilde{x}(f) * \widetilde{s}(f) &= \widetilde{x}(f) * \frac{1}{T0}\Delta(f-nf_0) \\
&= \frac{1}{T0}\widetilde{x}(f-nf_0)
\end{aligned}
$$

which was that $\mathcal{F}(x_s(t)) = \widetilde{x}(f-nf_0)$,like the following picture.

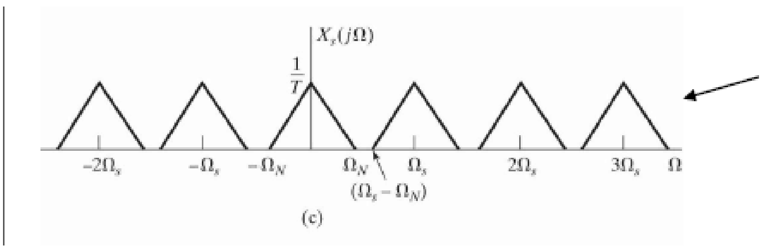


Figure 5: transform magic

In the picture, we can find that if the lower limit of the latter in adjacent waveforms is less than the upper limit of the former, it will cause the single waveform after Fu's transformation to be seriously distorted from the original waveform, so it will be distortion.Then $\Omega > 2W$.

4.2 Reconstruction it.

Use the low-pass filter pair to get $\widetilde{x_s}(f)$,which meet the h(f).

$$
\widetilde{h}(f) = \begin{cases} 1, if \qquad \Omega_N - < f < \Omega_N + \\ 0, otherwise \end{cases}
$$

so $\widetilde{x}(f) = T_0\widetilde{x_s}(f)h(f)$,according to the convolution characteristics we can get that,

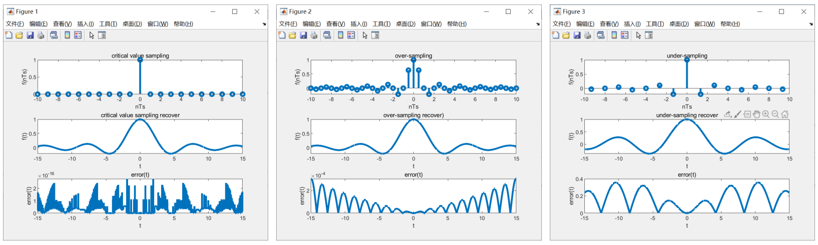
$$
x(t) = T_0 x_s(t) * h(t)
$$

consider that $\mathcal{F}(2f_cSa(2f_ct)) = \widetilde{h}(f)$,so

$$\begin{aligned} x(t) &= T_0 x_s(t) * h(t) \\ &= T_0 \sum_{n=-\infty}^{\infty} x(t)\Delta(t-nT_0) * h(t) \\ &= T_0 \sum_{n=-\infty}^{\infty} x(nT)\Delta(t-nT_0) * h(t) \\ &= T_0 \sum_{n=-\infty}^{\infty} x(nT)h(t-nT_0) \\ &= T_0 2f_c \sum_{n=-\infty}^{\infty} x(nT)Sa(2f_c(t-nT_0)) \end{aligned} \tag{1}$$

4.3 Realize the process in matlab

In order to understand Nyquist sampling theorem more intuitively, I use Matlab to draw the sampling results of Sa(t) function under three frequencies of critical value sampling, over-sampling and under-sampling, image restoration, and the error between the actual image. The effect diagram is as follows,and we can find that at the under-sampling,the recover signal isn't well.



(a) critical value sampling (b) over-samplings (c) under-samplings

Figure 6: Jackson Yee

Reference

Appendix A Code Listings

```
%% critical value samplings
figure(1);
wm = 1;
ws = 2 * wm;
wc = 0.5 * ws;
Ts = 2*pi/ws;
N = 10;
n = -N:N;
nTs = n * Ts;
fs = sinc(nTs/pi);
subplot(311);
stem(nTs/pi,fs,'LineWidth',3);
xlabel("nTs");
ylabel("f(nTs)");
title("critical value sampling");
```

```
% recover
Dt = 0.005;
t = -15:Dt:15;
fa = Ts*wc/pi * fs * sinc((wc/pi)*(ones(length(nTs),1)*t-nTs'*ones(1,length(t))));
subplot(312);
plot(t,fa,'LineWidth',3);
xlabel("t");
ylabel("f(t)");
title("critical value sampling recover");
% error
error = abs(fa-sinc(t/pi));
subplot(313);
plot(t,error,'LineWidth',3);
xlabel("t");
ylabel("error(t)");
title("error(t)");
%%over_sampling
figure(2);
wm = 1;
ws = 4 * wm;
wc = 0.5 * ws;
Ts = 2*pi/ws;
N = 20;
n = -N:N;
nTs = n * Ts;
fs = sinc(nTs/pi);
subplot(311);
stem(nTs/pi,fs,'LineWidth',3);
xlabel("nTs");
ylabel("f(nTs)");
title("over-sampling");
% recover
Dt = 0.005;
t = -15:Dt:15;
fa = fs*Ts*wc/pi * sinc((wc/pi)*(ones(length(nTs),1)*t-nTs'*ones(1,length(t))));
subplot(312);
plot(t,fa,'LineWidth',3);
xlabel("t");
ylabel("f(t)");
title("over-sampling recover)");
% error
error = abs(fa-sinc(t/pi));
subplot(313);
plot(t,error,'LineWidth',3);
xlabel("t");
ylabel("error(t)");
title("error(t)");
%% under_sampling
figure(3);
wm = 1;
ws = 1.5 * wm;
wc = 0.5 * ws;
Ts = 2*pi/ws;
N = 7;
n = -N:N;
nTs = n * Ts;
```

```
fs = sinc(nTs/pi);
subplot(311);
stem(nTs/pi, fs, 'LineWidth', 3);
xlabel("nTs");
ylabel("f(nTs)");
title("under-sampling");
% recover
Dt = 0.005;
t = -15:Dt:15;
fa = fs*Ts*wc/pi * sinc((wc/pi)*(ones(length(nTs),1)*t-nTs'*ones(1,length(t))));
subplot(312);
plot(t, fa, 'LineWidth', 3);
xlabel("t");
ylabel("f(t)");
title("under-sampling recover");
%error
error = abs(fa-sinc(t/pi));
subplot(313);
plot(t, error, 'LineWidth', 3);
xlabel("t");
ylabel("error(t)");
title("error(t)");
```