DSP Homework 09

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Abstract

1 1

2 Digital Number Representations

2.1 Exact Meanings of Common Representations

- byte:
- short integer:
- integer:
- float:
- double:
- quadruple types:
- fixed-point:
- floating-point:

3 3

4 Optimal Quantization Strategy When PDF Has Uniform Distribution

We have the quantization error of J which can be described as below:

$$J = \sum_{i=1}^{M} \int_{b_{i-1}}^{b_i} \left[Qi(x) - x \right]^2 p(x) \, \mathrm{d}x \tag{1}$$

In the special case of uniform distribution, we Have

$$J = \sum_{i=1}^{M} \int_{b_{i-1}}^{b_i} [q_i - x]^2 \, \mathrm{d}x \tag{2}$$

To find the best q_i, take the partial derivative of q_i

$$\begin{aligned} \frac{\partial J}{\partial q_i} &= \sum_{i=1}^{M} \int_{b_{i-1}}^{b_i} \frac{\partial}{\partial q_i}(q_i^2 - 2q_i x + x^2) \, \mathrm{d}x \\ &= \sum_{i=1}^{M} \int_{b_{i-1}}^{b_i} (2q_i - 2x) \, \mathrm{d}x \end{aligned}$$

We want $\frac{\partial J}{\partial q_i} = 0$. Therefore

$$\begin{aligned} q_i \int_{b_{i-1}}^{b_i} \mathrm{d}x &= \int_{b_{i-1}}^{b_i} x \, \mathrm{d}x \\ q_i(b_i - b_{i-1}) &= \frac{1}{2} \, (b_i^2 - b_{i-1}^2) \end{aligned}$$

Therefore

$$q_i = \frac{b_i + b_{i-1}}{2} \tag{3}$$

And, the same as the q_i, take the partial of b_i, and make $\frac{\partial J}{\partial b_i} = 0$

$$\begin{aligned} \frac{\partial J}{\partial b_i} &= \frac{\partial}{\partial b_i} \sum_{i=1}^{M} \int_{b_{i-1}}^{b_i} (q_i - x)^2 \, \mathrm{d}x \\ &= \frac{\partial}{\partial b_i} [\int_{b_{i-1}}^{b_i} (q_i - x)^2 \, \mathrm{d}x + \int_{b_i}^{b_{i+1}} (q_{i+1} - x)^2 \, \mathrm{d}x] \\ &= 0 \end{aligned}$$

We learned (4) in out freshman year that

$$\begin{aligned} \frac{d}{dx} \int_{a}^{x} f(t) \, \mathrm{d}t &= f(x) \\ \frac{d}{dx} \int_{x}^{a} f(t) \, \mathrm{d}t &= -f(x) \end{aligned} \tag{4}$$

Rewrite the (4)

$$\begin{aligned} (q_i - b_i)^2 &= (q_{i+1} - b_i)^2 \\ b_i - q_i &= q_{i+1} - b_i \end{aligned}$$

We can have the other result

$$b_i = \frac{q_{i+1} + q_i}{2} \tag{5}$$

If we combine (3) with (5). First, we can know that

$$\begin{aligned} q_i &= \frac{b_i + b_{i-1}}{2} \\ q_{i+1} &= \frac{b_{i+1} + b_i}{2} \end{aligned}$$

Put them in (5)

$$\begin{aligned} b_i &= \frac{q_i + q_{i+1}}{2} \\ &= \frac{1}{2} \, (b_i + \frac{b_{i-1} + b_{i+1}}{2}) \end{aligned}$$

Therefore

$$b_i = \frac{b_{i-1} + b_{i+1}}{2}$$

We can easily know that b_i is an arithmetic sequence, and because (3), q_i is an arithmetic sequence too.

In conclusion, if $p(x)$ is in the special case of uniform distribution, the range of $[0, 1]$ should be equally divided into M parts, and q_i should be the mean of b_i and b_{i+1}.

5 Conclusion