DSP Homework 08

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Abstract

1 Comparison Between Two Sampling Methods

1.1 Understanding of Wan Sampling Method

The Taylor Theorem is expressed as below.

$$f(x) = \sum_{i=0}^{n} \frac{f^{(i)}(x_0)}{i!}\ (x - x_0)^i + R_n(x)$$

Applying to our signal $x(t)$, we can make $t_0 = nT, n = 2$ here and get

$$x(t) = x(nT) + x'(nT)(x - nT) + \frac{x''(t_0)}{2}(x - t_0)^2 + R_2(t) \tag{1}$$

What exactly we get after sampling is only the values of $x(nT)$, but It is easy to do derivation. Therefore, using simple circuit, we know the values of $x'(nT)$, $x''(nT)$, $x'''(nT)$, $\cdots$

So, the reconstructed signal $\hat{x}(t)$ can be expressed as

$$\hat{x}(t) = x(nT) + x'(nT)(x - nT) + R_1(t) \tag{2}$$

but also as

$$\hat{x}(t) = x(nT) + R_0(t) \tag{3}$$

The difference of the above two lies on the error of the reconstructed signal. It time to do the quantitative analysis, we assume that

$$\begin{aligned} |\hat{x}(t) - x(t)| &< \epsilon \\ |x^{(n)}(t)| \qquad &< \eta_n \end{aligned} \tag{4}$$

So, to ensure the accuracy, the need to make the sampling period T satisfy the equations below.

$$T < \begin{cases} \frac{2\epsilon}{\eta_1} & n = 1 \\ 2\sqrt{\frac{2\epsilon}{\eta_2}} & n = 2 \end{cases} \tag{5}$$

2 Conclusion