DSP Homework 10

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3 Relationship Between FT and DTFT

We can first put the expression of perfect reconstruction here

$$x_s(t) = \sum_n x(nT)\,\delta(t - nT) = x_a(t) * s(t) = x_a(t) * \sum_n [\delta(t - nT)] \tag{1}$$

We put (1) into FT

$$\begin{aligned} \widetilde{x}_s(f) &= \int_{-\infty}^{\infty} \sum_n [x(nT)\,\delta(t - nT)]\ e^{-j2\pi ft}\ \mathrm{d}t \\ &= \sum_n x(nT)\, e^{-j2\pi fnT} \\ &= \sum_n x(n)\, e^{-j2\pi nf'} \end{aligned} \tag{2}$$

The value of T (also $\frac{1}{f_s}$) doesn't matter, so we can make

$$f' = \frac{f}{f_s}$$

The (2) can be changed into

$$\widetilde{x}_s(f) = \sum_n x(n)\, e^{-j2\pi nf'}$$

Therefore

$$\begin{aligned}
\widetilde{x}_s(f) &= \mathcal{F}\left[x_s(t)\right] \\
&= \mathcal{F}\left[x_a(t) * s(t)\right] \\
&= \widetilde{x}_a(f) \times \widetilde{s}(f) \\
&= \widetilde{x}_a(f) \times \left[f_s \sum_n \delta(f - nf_s)\right]
\end{aligned}$$

Therefore, the relationship can be described as

$$\widetilde{x}(f) = \widetilde{x}_s(f) = \widetilde{x}_a(f) \times \left[f_s \sum_n \delta(f - nf_s)\right] \tag{3}$$

4 From Analog Signal to DFT

Assuming there's an analog signal $x_a(t)$, computer cannot deal with analog values, so we can sample it use Shannon/Nyquist sampling method.

$$x(n) = x_s(t) = x_a(t) * \sum_n \left[\delta(t - nT)\right] = \sum_n \; x(nT)\, \delta(t - nT) \tag{4}$$

Then, we should analyze $x(n)$ in frequency domain, calculate the FT of $x(n)$ as below. Also, because computer cannot deal with analog values, so we should assume $n \in [0, N)$ to make $x_a(t)$ time-limited.

$$\begin{aligned}
\widetilde{x}_s(f) &= \int_{-\infty}^{\infty} \sum_{n=0}^{N-1} \; \left[x(nT)\, \delta(t - nT)\right] \; e^{-j2\pi f t} \; \mathrm{d}t \\
&= \int_{-\infty}^{\infty} \sum_{n=0}^{N-1} \; \left[x(nT)\, e^{-j2\pi f nT}\; \delta(t - nT)\right] \; \mathrm{d}t \\
&= \sum_{n=0}^{N-1} \; \left[x(nT)\, e^{-j2\pi f nT} \; \int_{-\infty}^{\infty} \; \delta(t - nT)\, \mathrm{d}t\right] \\
&= \sum_{n=0}^{N-1} \; x(nT)\, e^{-j2\pi nT\cdot f} \\
&= \sum_{n=0}^{N-1} \; x(n)\, e^{-j2\pi n\cdot f}
\end{aligned} \tag{5}$$

Because now $\widetilde{x}_s(f)$ is discrete and we don't care about the sampling period T, so we make $f = f \times T$ in (5).

However, $\widetilde{x}_s(f)$ is still not discrete. So, as what we did to $x_a(t)$, we should sample $\widetilde{x}_s(f)$ in frequency domain again. Rewrite the summation of (5) as below:

$$\widetilde{x}_s(f) = x(0) + x(1)e^{-j2\pi f} + x(2)e^{-j2\pi\cdot 2f} + \cdots + x(N-1)e^{-j2\pi\cdot(N-1)\,f}$$

The period is decided by $x(1)e^{-j2\pi f}$ and is 1 here. We should sample it in $[0, 1)$. Assuming $f = \frac{k}{N} \quad k \in [0, N)$, we have

$$\widetilde{x}_s(\frac{k}{N}) = \widetilde{x}_s(k) = \sum_{n=0}^{N-1} \; x(n)\, e^{-j2\pi n\cdot \frac{k}{N}}$$

$x_s(k)$ is also discrete now, so that it can be processed with the computer.

5 Analog Frequency in DFT

Variables in (4) are n and k.

First, here's my thoughts about n. Since $\widetilde{x}_s(k)$ is just a sampling of $\widetilde{x}_s(f)$, if we can find high analog frequency in $\widetilde{x}_s(f)$, the same method works for $\widetilde{x}_s(k)$. Go back and see (3), it tells us that $\widetilde{x}_s(f)$ is just many copies and frequency-shift of $\widetilde{x}_a(f)$. Therefore, the bigger n gets, the higher $\widetilde{x}_s(f)$ reach.

Then, k also matters. k has its source from f, when k (also f) gets bigger, no doubt the frequency gets bigger. However, n decides on frequency domain which period we are in. If n is fixed, we cannot reach other other frequency out of this specific interval.

6 Conclusion