

# DSP Homework 05

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## Abstract

## 1 Summary and Thoughts

### 1.1 Mental Health

This lecture want to tell us what is important to improve our mental health. There are 8 principles the lecturer mentioned.

#### **nutrition**

The research shows that those whose norm was to achieve 100 years or older had the basis of their diet plant-based.

#### **exercise**

Exercise help our body produce good happy chemicals.

#### **water**

70 percent of our body is water. Water helps to travel nutrients to our brain which are the raw materials of the chemicals mentioned above, and then helps you to become happy.

#### **sunshine**

The sunshine make our sleep sounder, and decrease the pain someone has.

#### **temperance**

Don't use too much cigarettes, alcohol and coffee.

**air**

Help clarify our blood.

**rest**

This world is almost outcome driven, we all want to do much better. But, in the process of pushing ourselves, we often underestimate the effect of rest and overestimate the effect of the time we spent.

**trust**

The lecturer quote the words from Ellen White:

The greatest want of the world is the want of . . . men whose conscience is as true to duty as the needle to the pole; men who will stand for the right though the heavens fall.

Nowadays, many people have lost in the competition with others and have given up the basic trust between people around them. What we should treasure is the pure thoughts we had once we were a baby. In this video, such as, love and trust.

## 1.2 Our Planet

This video shows us the beautiful scenes of shallow seas in earth and the danger of the animals and plants in those areas face.

We human's activities have been effecting the balance of the shallow seas which are the habitat of the living things.

One is the emitting of  $CO_2$ , which is one of the results of global warming. In this video, the little change will let the coral get white and die.

The other is fishing. Over fishing break the food chain in the ecological system, which will lead to, in this video, sea urchins falling the kelp forests.

Thankfully, we human have taken action to prevent the bad things from happening. The seas around Misool, mentioned in the last part of this video, has been set as a protected area. In a decade, the vivid scene recovered, and this gives us confidence to make other areas vivid again.

## 1.3 New Fuel

First, this video makes a comparison between the hydrogen, the kerosene and methane.

The hydrogen has good performance as fuel. It can give more impulse with less mass. But it is hard to store because the small density of itself.

The kerosene is easier to store than hydrogen, but the problem is that it cannot be fully burn (into carbon dioxide and water). This makes the residue which is solid have the change of getting stuck in the engine.

The methane achieve the balance of the two fuels. But also that means methane doesn't have the best effect in every area.

The goodness of the methane is that it can be synthesized from  $CO_2$ . Also, the raw material of this chemical reaction which is carbon dioxide is easy to get on Mars. And if we can get  $H_2O$  on Mars, this reaction can even provide excess of oxygen for human breathing. This extra bonus helps us make human settlement in the future.

The scientists are now developing a new technology which can better solve the energy problem on Mars. Moreover, carbon capture on Earth is more to look forward to. If we can improve the efficiency of the reaction which change the carbon dioxide to methane, a clean renewable way of consuming the energy can be found.

## 1.4 Thoughts

In the video of mental health, the lecturer told us that sunshine is good for our health. But he mentioned little about why the sunshine is good for us. So I searched on the Internet and found the points below.

### Improves your sleep

Your body creates a hormone called melatonin that is critical to helping you sleep. Sunshine regulates your circadian rhythm by telling your body when to increase and decrease your melatonin levels. So, the more daylight exposure you can get, the better your body will produce melatonin when it's time to go to sleep.

## Maintains strong bones

One of the best (and easiest) ways to get vitamin D is by being outside. Our bodies produce vitamin D when exposed to sunlight—about 15 minutes in the sun a day is adequate if you're fair skinned. And since Vitamin D helps your body maintain calcium and prevents brittle, thin, or misshapen bones, soaking in sun may be just what the doctor ordered.

## Fights off depression

It's not just in your head; there's a scientific reason being in the sunshine improves your mood. Sunshine boosts your body's level of serotonin, which is a chemical that improves your mood and helps you stay calm and focused. Increased exposure to natural light may help ease the symptoms of seasonal affective disorder—a change in mood that typically occurs in the fall and winter months when there are fewer hours of daylight.

# 2 Problem 2

## 2.1 Restatement

In the development of the Shannon/Nyquist sampling theorem, the impulse function  $\delta(t)$  is used. But  $\delta(t)$  is not a practical signal. Does that mean that when applying the sampling theorem in practice, we need some approximation/modification? If yes, what is it that needs to be further done? If no, why?

## 2.2 Improvement

Though the  $\delta(t)$  function isn't a practical signal, but it works well in our thought experiment. We should try to use some other similar signals to do the sampling, such as the square wave. Square function has finite values, but has infinite derivative values. It is also a function that hard to generate, but it is realizable. To explore whether this method is good, we should try to find out the expression of  $\tilde{x}_s(t)$  first.

If we let the square wave last the length of  $2\tau$ , we now have

$$\begin{aligned} s(t) &= \sum_{n=-\infty}^{\infty} \text{rect}\left(\frac{t-nT}{\tau}\right) \\ &= \sum_{n=-\infty}^{\infty} F_n e^{j2\pi n f_s t} \\ &= \sum_{n=-\infty}^{\infty} \left[ \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \text{rect}\left(\frac{t}{\tau}\right) e^{-j2\pi n f_s t} dt \right] e^{j2\pi n f_s t} \\ &= \sum_{n=-\infty}^{\infty} \left[ f_s \int_{-\tau}^{\tau} e^{-j2\pi n f_s t} dt \right] e^{j2\pi n f_s t} \\ &= \sum_{n=-\infty}^{\infty} 2f_s \text{sinc}(2f_s n\tau) e^{j2\pi n f_s t} \end{aligned} \tag{1}$$

Therefore

$$\tilde{s}(f) = \sum_{n=-\infty}^{\infty} 2f_s \text{sinc}(2f_s n\tau) \delta(f - nf_s) \tag{2}$$

And considering the expression below

$$\begin{aligned} \tilde{x}_s(f) &= \frac{1}{f_c} \cdot \tilde{x}(f) * \tilde{s}(f) \\ &= \frac{1}{f_c} \cdot \tilde{x}(f) * \sum_{n=-\infty}^{\infty} 2f_s \text{sinc}(2f_s n\tau) \delta(f - nf_s) \\ &= \frac{1}{f_c} \cdot \sum_{n=-\infty}^{\infty} 2f_s \text{sinc}(2f_s n\tau) [\tilde{x}(f) * \delta(f - nf_s)] \\ &= \sum_{n=-\infty}^{\infty} \frac{2f_s}{f_c} \text{sinc}(2f_s n\tau) \tilde{x}(f - nf_s) \end{aligned} \tag{3}$$

If we find a LPF  $H_a(f)$  whose transfer function is 1 over  $[2f_s T \text{sinc}(2f_s \tau)]$  and bandwidth  $f_c$  is between  $B$  and  $f_s - B$ . Then we have

$$\tilde{x}_s(f) \cdot H_a(f) = \tilde{x}(f) \quad (4)$$

Now I has proved the perfect square wave can be used in the sampling. If we find a way to make the rise time of a ladder-shaped function far less than the  $\tau$ , this function can be used as the square wave, which makes this kind of sampling reasonable.

### 3 Problem 3

### 4 Problem 4

#### 4.1 Restatement

Derive the result of the Shannon/Nyquist sampling theorem and the perfect reconstruction formula which is as below.

##### Nyquist sampling theorem

if no actual information is lost in the sampling process, there must be

$$f_s \geq 2B \quad (5)$$

in which B is the bandwidth of the original signal.

##### Perfect Reconstruction Formula

$$x(t) = 2f_s T \sum_{n=-\infty}^{\infty} x(nT) \text{sinc}[2f_s(t - nT)] \quad (6)$$

#### 4.2 Proof

##### Nyquist sampling theorem

When  $x(t)$  is a function with the Fourier transform  $\tilde{x}(f)$  we have the sampling functions below:

$$\begin{aligned} s(t) &= \begin{cases} \delta(t - nT) & t = nT \\ 0 & \text{otherwise} \end{cases} \\ &= \sum_{n=-\infty}^{\infty} \delta(t - nT) \end{aligned} \quad (7)$$

Assuming that  $\tilde{x}(f)$  is the , it is easy to write the sampled  $x_s(t)$  as:

$$\begin{aligned} \because x_s(t) &= x(t) \times s(t) \\ &= \sum_{n=-\infty}^{\infty} x(nT) \delta(t - nT) \end{aligned} \quad (8)$$

$$\therefore \tilde{x}_s(f) = \tilde{x}(f) * \tilde{s}(f) \quad (9)$$

To calculate  $\tilde{s}(f)$ , we should find out the Fourier series of  $s(t)$  first.

Assuming that

$$\begin{aligned} f_s &= \frac{1}{T} \\ s(t) &= \sum_{n=-\infty}^{\infty} F_n e^{j2\pi n f_s t} \end{aligned}$$

Easy to find that

$$\begin{aligned}
F_n &= \frac{1}{T} \int_{-\frac{1}{T}}^{\frac{1}{T}} \sum_{n=-\infty}^{\infty} \delta(t - nT) e^{-j2\pi n f_s t} dt \\
&= \frac{1}{T} \int_{-\frac{1}{T}}^{\frac{1}{T}} \delta(t) dt \\
&= \frac{1}{T} \\
&= f_s
\end{aligned}$$

Therefore

$$s(t) = f_s \sum_{n=-\infty}^{\infty} e^{j2\pi n f_s t}$$

Therefore

$$\begin{aligned}
\tilde{s}(f) &= f_s \sum_{n=-\infty}^{\infty} \mathcal{F} [e^{j2\pi n f_s t}] \\
&= f_s \sum_{n=-\infty}^{\infty} \delta(f - n f_s)
\end{aligned} \tag{10}$$

Put Equa.10 with Equa.9, we have

$$\begin{aligned}
\tilde{x}_s(t) &= f_s \sum_{n=-\infty}^{\infty} \tilde{x}(f) * \delta(f - n f_s) \\
&= f_s \sum_{n=-\infty}^{\infty} \tilde{x}(f - n f_s)
\end{aligned} \tag{11}$$

In Fig. we can see there are lots of shifted  $\tilde{x}(f)$  in the spectrum of  $\tilde{x}_s(t)$ . The center frequencies of  $\tilde{x}(f)$  are  $f_s$  away from each other, so if we don't want the spectrum of them to mix up, we should make

$$f_s \leq 2B$$

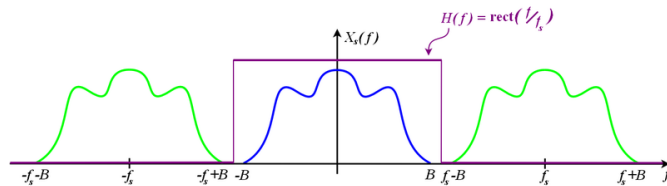


Figure 1: Spectrum of  $\tilde{x}_s$

### Perfect Reconstruction Formula

To prove this equation, we need to calculate  $x(t)$

$$\begin{aligned}
x(t) &= \frac{1}{f_s} \mathcal{F}^{-1} [\tilde{x}(f)] \\
&= \frac{1}{f_s} [x_s(t) * h(t)]
\end{aligned} \tag{12}$$

The  $h(t)$  is the transfer function of the LPF (like the one in Fig.1).

$$\begin{aligned}
\because \tilde{h}(f) &= u(f + f_c) - u(f - f_c) \\
\therefore h(t) &= \int_{-f_c}^{f_c} e^{j2\pi ft} \, df \\
&= \left[ \frac{1}{j2\pi t} e^{j2\pi ft} \right]_{-f_c}^{f_c} \\
&= \frac{1}{\pi t} \frac{e^{j2\pi f_c t} - e^{-j2\pi f_c t}}{2j} \\
&= 2f_c \frac{\sin 2\pi f_c t}{2f_c \pi t} \\
&= 2f_c \text{sinc} 2f_c t
\end{aligned} \tag{13}$$

Put Equa.13 in Equa.12, and we have Equa.8, therefore

$$\begin{aligned}
x(t) &= \frac{1}{f_s} \left\{ \sum_{n=-\infty}^{\infty} x(nT) [\delta(t - nT) * 2f_c \text{sinc} 2f_c t] \right\} \\
&= 2\frac{f_c}{f_s} \sum_{n=-\infty}^{\infty} x\left(\frac{n}{f_s}\right) \text{sinc} \left[ 2f_c \left(t - \frac{n}{f_s}\right) \right]
\end{aligned} \tag{14}$$

## Appendix A Code Listing