DSP Homework—week 5

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Abstract

This article mainly tells the summary and my thoughts of this week's videos, how to measure your maximum instantaneous running speed, the derivation of Shannon/Nyquist sampling theorem and matters needing attention when applying the actual sampling theorem. In this article, I reviewed the Shannon/Nyquist sampling theorem learned this week.

1 Problem 1 Thinking of videos

1.1 The summary of videos

1.1.1 Our Planet —— Coastal Seas

This video introduces the past, present situation and changes of marine ecosystem.

In the former marine ecosystem, a wide variety of marine organisms performed their duties: sharks, as the highest predators of coral reefs, maintained the health of coral reefs and the ecological balance of fish communities; sea otters and sheep-headed wrasse prey on sea urchins in kelp forests to prevent them from spreading like plague. However, in today's marine ecosystem, due to the rise of sea water temperature, a large number of symbiotic algae and plants in coral reefs have left, and corals cannot get nutrition and are gradually whitening and dying; excessive industrial fishing has led to a sharp decline in the number of fish in the world, and useless jellyfish are flooding instead of fish. Our ocean is facing a great ecological crisis.

However, the marine ecosystem has great resilience. We can promote the self-repair of the ocean by strengthening control, establishing marine ecological reserves and controlling the time of industrial fishing. As long as we are willing to do it, there is still time for the marine ecosystem to be repaired.

1.1.2 8 principles to achieve optimum mental health

This video tells about 8 principles of optimum mental health: new start, food diet, exercise, water, sunshine, temperance, rest and trust, and the most important three are exercise, rest and trust.

Exercise doesn't have to be done outdoors, you can also do it indoors if you don't have much time. Of course, it is best to exercise outdoors. You can bask in the sun and breathe fresh air at the same time. A good rest means that you need to have at least eight hours of night sleep. As for trust, you can often get together with your family to strengthen the connection and trust between you.

1.1.3 Why SpaceX is using a new fuel

This video introduces the change of rocket fuel from kerosene to liquid hydrogen to methane, and why SpaceX chose methane this new fuel.

Traditional kerosene is difficult to burn completely, and incomplete combustion of kerosene will produce a lot of soot, which will block the rocket engine, thus making it impossible to reuse the engine. Compared with kerosene, liquid hydrogen will not have these problems at all, but the density of liquid hydrogen is small, which requires a larger oil tank, and the boiling point of liquid hydrogen is low, so we need to use highly insulated materials to prevent its gasification and expansion. In addition, liquid hydrogen may degrade and weaken metal materials. The performance of methane is between the two fuels. And if we settle in Mars in the future, methane will be the best fuel available. On the whole, methane is an excellent fuel for SpaceX, since SpaceX's plan is for humans to settle on Mars.

1.2 Further thoughts

1.2.1 Our Planet —— Coastal Seas

The marine ecosystem is different from other ecosystems, and many creatures can't stay in the sea water for too long, so there are a lot of cooperative relationships in the marine ecosystem, such as the white-headed reef shark and the gray reef shark, the sea otter and the sheep-headed wrasse, corals and algae. These symbiotic or cooperative relationships are really amazing, nature not just have a scene where snipes and clams compete.

In this video, global warming not only affects the melting of glaciers, but also affects all ecosystems of the earth. We should not only control the killing of marine life by human beings, but also control our pollution to the earth's environment.

1.2.2 8 principles to achieve optimum mental health

In today's society, we are all striving for life. Most people don't pay attention to their physical health, let alone their mental health. However, optimum mental health can make us have higher efficiency in accomplishing all kinds of things, and it is conducive to longevity.

Although our life is very busy, we also need to remember: drink eight glasses of water every day, sleep for 8-9 hours, exercise for an hour, have less suspicion and more trust, and go out for fresh air.

1.2.3 Why SpaceX is using a new fuel

With the increasing consumption of traditional fuels, our planet can't afford the supply of common fuels. Scientists are gradually exploring various new fuels, of which methane is only a small part. However, if humans can really migrate to Mars in the future, then according to this video, methane is indeed the best fuel, because we can use reverse water gas conversion and water electrolysis to produce a lot of methane, and at the same time produce excess oxygen.

2 Problem 2 When applying the sampling theorem in practice, should we do approximation/modification about sampling signal ?

2.1 Yes or No

I don't think it's necessary to do approximation or modification.

2.2 What to do further or Why ?

Very good.

We only use impulse function $\delta(t)$ as a simple function to deduce the Shannon/Nyquist sampling theorem, but after we get the conclusion, we can directly apply this conclusion. In actual sampling, we don't need to use $\delta(t)$ as sampling signal, but only need to find an actual signal that satisfies the sampling theorem to achieve the perfect sampling.

In the previous class, we talked about two ways to solve problems, one is continuous experiment, and the other is direct application of existing theories. In my opinion, predecessors have deduced the Shannon/Nyquist sampling theorem, so we can apply it directly.

Just like 1+1 = 2, we are not limited to these two numbers in actual use, but extended to the addition of any two real numbers.

3 Problem 3 How to measure your maximum instantaneous running speed ?

3.1 Theoretical basis

The highest instantaneous running speed of human is 44.722 km/h (about 12.4228 m/s), which was created by Bolt, a Jamaican flyer. Generally, the maximum instantaneous running speed of girls is about 6 m/s. Need ref support.

Considering the method of using a timer to measure the instantaneous speed, we can measure the distance (*accurate to millimeter*) you run every second and my plan is to measure ten seconds in total. Then, we can get ten points in the x-t image. Using *excel*, we can directly generate a smooth x-t curve.

According to the instantaneous velocity formula

$$v=x/t$$

then, the slope of x-t image is the instantaneous running speed we need.

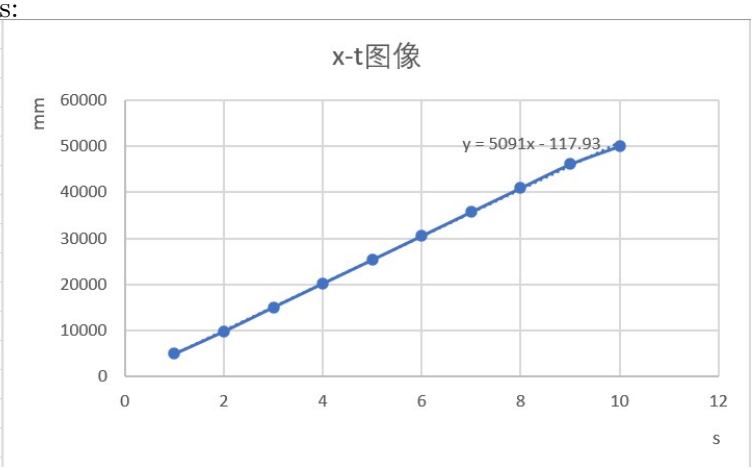
When the slope is maximum, the instantaneous running speed obtained is the maximum instantaneous running speed required.

Need details of your experiment

3.2 Result

Here is the details:

s	mm
1	4973
2	9837
3	15079
4	20233
5	25370
6	30557
7	35758
8	40932
9	46086
10	50000



the maximum instantaneous running speed of me is about 5091 mm/s.

4 Problem 4 Derive the result of the Shannon/Nyquist sampling theorem and the perfect reconstruction formula

4.1 Proof of the result of the Shannon/Nyquist sampling theorem

The content of Shannon/Nyquist sampling theorem is that the sampling frequency f_s must be greater than or equal to twice the highest frequency component of the original signal and the expression is

$f_s \geq 2f_m$.

Here is the proof:

Suppose the analog signal is x(t), sampling signal is $\delta_T(t) = \sum_{m=-\infty}^{\infty}\delta(t - mT)$, then the post-sampling signal is x_s(t).

$$x_s(t) = x(t)\delta_T(t) = \sum_{m=-\infty}^{\infty}x(mT)\delta(t - mT)$$

Considering the convolution property of Fourier transform,

$$\widetilde{x_s}(\omega) = \frac{1}{2\pi}\widetilde{x}(\omega) * \widetilde{\delta_T}(\omega)$$

Do Fourier expansion for $\delta_T(t)$, then

$$\delta_T(t) = \frac{1}{T}\sum_{n=-\infty}^{\infty}e^{jn\omega_s t}$$?

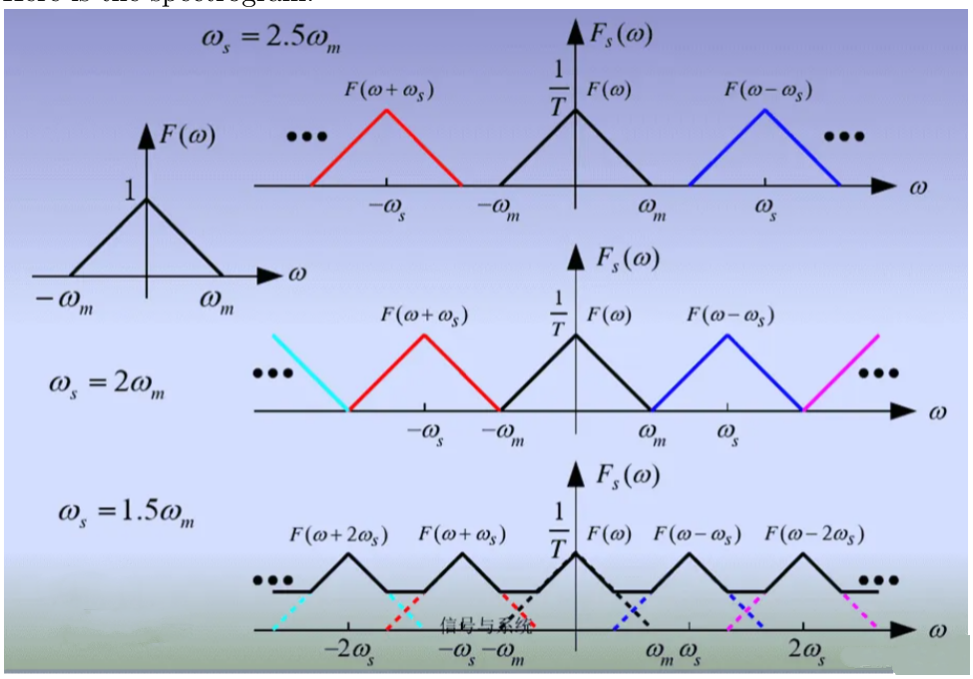
then,

$$\widetilde{\delta_T}(\omega) = \frac{2\pi}{T_s}\sum_{n=-\infty}^{\infty}\delta(\omega - n\omega)$$

then,

$$\widetilde{x_s}(\omega) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty}\widetilde{x}(\omega - n\omega)$$

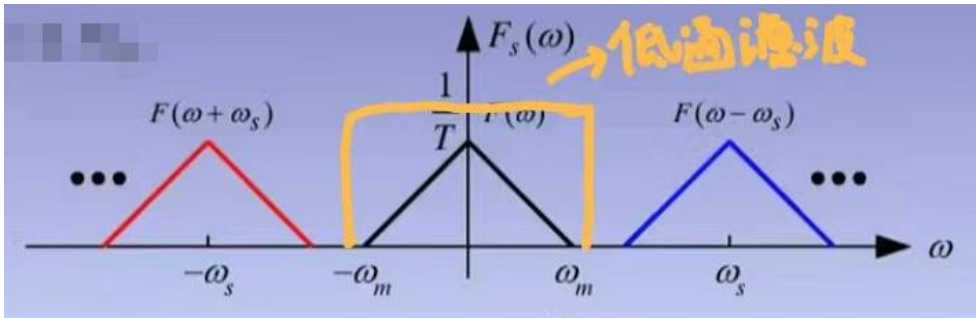
Here is the spectrogram:



Obviously, when $\omega_s \geq 2\omega_m(f_s \geq 2f_m)$, the spectrum will not overlap and get perfect sampling.

4.2 Proof of the perfect reconstruction formula

If we want to extract the original signal from the sampled signal, we can do low-pass filtering on the sampled signal to leave the original signal.



Assume that the spectrum of the low-pass filter is $\widetilde{h}(f)$, then the extracted signal is $\widetilde{y}(f) = \widetilde{x_s}(f)\widetilde{h}(f)$.

$$\widetilde{h}(f) = \begin{cases} T, |\omega| < \frac{\omega_s}{2} \\ 0, |\omega| \geq \frac{\omega_s}{2} \end{cases}$$

then, $h(t) = sinc(\frac{t}{T})$, $x_s(t) = \sum_{m=-\infty}^{\infty} x(mT)\delta(t - mT)$, y(t) $= x_s(t) * h(t)$.

then, $y(t) = \sum_{m=-\infty}^{\infty} x(mT)h(t - mT) = \sum_{m=-\infty}^{\infty} x(mT)sinc(\frac{t-mT}{T})$, y(t) is the perfect reconstruction formula.

5 Conclusions

In this article, I reviewed the Shannon/Nyquist sampling theorem I learned last week and fully understood the sampling and reconstruction of this sampling theorem. What's more, I known how fast I can run.