DSP Homework 06

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Abstract

In this article, I write two summaries about the video we watched, first one is about how blue tooth works, it send waves that have different wavelength to distinguish '0' and '1'. The second is a speech about how we can learn fast, the speaker tells us, we should pay more attention to the 'output' part of things we learn, he prove it himself that it is useful for us. Then I write my thoughts after watching this two video, the first video eliminate my worries about using gadgets based on blue tooth technology will bring radiation to human. Then second video provide a new learn method for me to learn. What's more, I derive the sampling method using Fourier series, and compare it with Shannon/Nyquist sampling method, and I think the Fourier series sampling method is better.

1 About videos

1.1 Summary of two videos

1.1.1 How our wireless earbuds communicate with our phone using blue tooth

At first, the video introduce how traffic lights works by using different wave length to show different color so that it convert different information to our brain. Our smart phone use 121mm wave to send '1' and 124mm to send '0'. It can send about 1000000 bits/second to our earbud. And then it shows four different animation demonstration of different vision to show electromagnetic wave. After showing this, it says that blue tooth antenna can only receive the a specific range frequency wave, it divide the frequency band into 79 channels, when phone communicate with our earbuds, they take a channel, but when there are a lot of blue tooth using the same frequency band, it may lead trouble. So the information our phone send is consist of three part, the first part is access code to make sure our earbuds have received the information; the second part is header provide details about information phone send; the last part is payload, namely the information our phone send. And in fact when our phone communicate with our earbuds, they don't stay at a specific channel fixedly, they jump among different channels, when they jump once they send a information package we mentioned. These is named frequency hopping. Also the wave our blue tooth use is similar to the wave our microwave oven, but it is safe due to they have a low power. The blue tooth microchip can achieve packets sends, frequency hopping, error detection and noise filtering to avoid errors. The '0', '1'codes need to be send in form of electromagnetic wave. So it introduce the frequency modulation: frequency-shift keying and phase-shifted on the heels of it.

1.1.2 How to learn things fast

At the beginning of speech, the speaker tell us that we can train our brain to remember better, learn faster and to master anything we want in our life. And then he demonstrate how we can do with train our brain taking himself as an example. He says he is a forgetful guy before, when his mom let him go to shop to buy something, he taking a circle to shop, but when he leave the shop he doesn't remember that he ride to the shop until two days later. In the video, he invite 5 people to write 6 random numbers on the broad each person. After they all done, he let the last person read these numbers to him. After it he says he had remembered these numbers and speaks them out to the audiences. He says he is a grandmaster of memory and break a lot of records compared to a forgetful guy that he used to be. Then he tells us his way of how we can learn or master anything in a fun, easy way. He said that people like Maske and Bill Gates, have committed to life learning. The most important thing comes he says that: your learning ability decides your earning ability, if we learn faster, we can stand out from everyone else. then he says that if we want to learn deep and well we should pay attention to output rather than input. The input is the things we learn while the output is reflect, implement and share. And if the time we spend on output is twice more compared to the time we spend on the input section, we can be a master at achieving everything we want. At the end of the video, speaker recite the numbers that participants write on the board on reverse direction to prove his method is useful.

1.2 Further thoughts about two videos

1.2.1 How our wireless earbuds communicate with our phone using blue tooth

In this video, it shows us how blue tooth works, and it mentioned that the wave blue tooth use is like our microwave oven use, but due to its low power, it does no harm to our human. So after watching this video, I have eliminate the fear of blue tooth technology. In the past, I usually fear of the radiation that blue tooth produce, so the blue tooth on my phone is usually off. Also to avoid radiation, I don't use blue tooth earbuds, I think that the radiation it produced will influence my brain. After watching this video these worries all gone, I can live more relaxed.

1.2.2 How to learn things fast

In the video, the speaker says that the 'output' is more important than 'input'. The output section is consisted of reflect, implement and share. If we want to learn things fast and well, we should pay more attention to the output section. It give me a new learn method, and it is efficient because the speaker had already verify it for us. Also there are traditional Chinese saying goes: 'Learning without thoughts is Labour lost; thinking without learning is perilous', 'Practice makes perfect'. Both in Chinese culture and Western culture people also find this learning method, so it is not different for us to infer that this learning method is useful. Thus, I want to try this method in my subsequent learning time. At first I learn the knowledge, and then I reflect on it usually to find if there is something I still don't know about this knowledge. Next I will apply what I learned into practice. Last, if someonee ask me about this knowledge I will share what I know with him, so I review the knowledge again.

2 Sampling method using Fourier series and its difference compared with the Shannon/Nyquist sampling method

2.1 Derivation of sampling method using Fourier series

For a analog signal x(t), suppose that $t \in [-T/2, T/2]$, we can expand x(t) into Fourier series as the Equation 1 shows.

$$\begin{aligned} x(t) &= \sum_n C_n e^{j2\pi nft} = \sum_n C_n e^{j2\pi nt/T} \\ where \quad C_n &= \frac{1}{T}\int_{-T/2}^{T/2} x(t) e^{j2\pi nt/T} dt \end{aligned} \tag{1}$$

The energy of signal x(t) can be represented by E, where E can be showed as the Equation 2

$$\begin{aligned} E &= \int_{-T/2}^{T/2} |x(t)|^2 dt \\ &= \int_{-T/2}^{T/2} \big(\sum_n C_n e^{j2\pi nt/T}\big)\big(\sum_m C_m^* e^{-j2\pi mt/T}\big) dt \\ &= \sum_n C_n \sum_m C_m^* \int_{-T/2}^{T/2} e^{j2\pi(n-m)t/T} \\ &= T \sum_n C_n \sum_m C_m^* \delta(n-m) \\ &= T \sum_n |C_n|^2 \end{aligned} \tag{2}$$

$\delta(n)$ is defined as the Equation 3

$$\delta(n) = \begin{cases} 1 & n = 0 \\ 0 & else \end{cases} \tag{3}$$

Where all know that the energy of any signal exist in nature is limited. So we let

$$\lim_{n\to\infty} E = E_0 \tag{4}$$

then, we let $s(n) = T\sum_n |C_n|^2$, so we can know that

$$\begin{aligned} T|C_n|^2 &= T\sum_n |C_n|^2 - T\sum_n |C_{n-1}|^2 \\ &= s(n) - s(n-1) \end{aligned} \tag{5}$$

because $\lim_{n\to\infty} s(n) = E_0$,

$$\begin{aligned} \lim_{n\to\infty} T|C_n|^2 &= \lim_{n\to\infty} s(n) - \lim_{n\to\infty} s(n-1) \\ &= E_0 - E_0 \\ &= 0 \end{aligned} \tag{6}$$

so that $\lim_{n\to\infty} |C_n| = 0$.

Thus, we can choose finite sum of Fourier series to represent x(t), as the Equation 7 shows(N is very big).

$$x(t) \approx \sum_{-N}^{N} C_n e^{j2\pi nt/T} \tag{7}$$

So, after sampling, x(t) can be reconstructed showed as the equation 8

$$\begin{aligned} x(t) &= \sum_{-N}^{N} C_n e^{j2\pi nt/T} \\ where \qquad C_n &= \frac{1}{T}\int_{-T/2}^{T/2} x(t) e^{j2\pi nt/T} dt \end{aligned} \tag{8}$$

2.2 Difference between Fourier series sampling method and Shannon/Nyquist sampling method

We know that when we use Shannon/Nyquist sampling method, x(t) can be reconstructed as Equation 9 shows,

$$x(t) = 2f_cT\sum_n x(nT)sinc[2f_c(t-nT)] \tag{9}$$

while using the Fourier series sampling method, x(t) can be reconstructed as the Equation 8 shows.

In Shannon/Nyquist sampling method, the sampling signal we use is $s(t) = \sum_{n=-\infty}^{\infty}\delta(t-nT)$, in fact $\delta(t)$ is not truly exist, so when we use this method in real sampling process, we need use periodic narrow pulse. Suppose that our sampling signal in one period $t \in [-T/2, T/2]$ can be shown as the Equation 10

$$s_0(t) = \begin{cases} A & -\tau/2 < t < \tau/2 \\ 0 & else \end{cases} \tag{10}$$

So, the real sampling signal can be represent by$s(t) = s_0(t) * \sum_n \delta(t-nT)$, while we have already know that the $\sum_n \delta(t-nT) \leftrightarrow \frac{1}{T}\sum_n \delta(f-nf_s)(f_s = 1/T)$, also

$$\begin{aligned} \tilde{s_0}(f) &= \int_{-\infty}^{\infty} s_0(t)e^{-j2\pi ft}dt \\ &= \int_{-\tau/2}^{\tau/2} Ae^{-j2\pi ft}dt \\ &= A\tau sinc(f\tau) \end{aligned} \tag{11}$$

thus,

$$\begin{aligned} \tilde{s}(f) &= \tilde{s_0}(f)\sum_n \delta(f-nf_s) \\ &= \frac{A\tau}{T}sinc(f\tau)\sum_n \delta(f-nf_s) \\ &= \frac{A\tau}{T}\sum_n sinc(nf_s\tau)\delta(f-nf_s) \end{aligned} \tag{12}$$

Now let's go back to our analog signal x(t), suppose that $x(t) \leftrightarrow \tilde{x}(f)$, so after sampling we get $x_s(t) = x(t)s(t)$, then

$$\begin{aligned} \tilde{x_s}(f) &= \tilde{x}(f) * \frac{A\tau}{T}\sum_n sinc(nf_s\tau)\delta(f-nf_s) \\ &= \frac{A\tau}{T}\sum_n sinc(nf_s\tau)\tilde{x}(f-nf_s) \end{aligned} \tag{13}$$

here we can find that we still use a low pass filter to pass x(t) out, and it is similar to ideal sampling(because in frequency base band, $\tilde{x_s}(f)$ is in direct proportion to $\tilde{x}(f)$, only coefficient difference). $x(t) = 2f_cT/A\tau\sum_n x(nT)sinc[2\pi f_c(t-nT)]$. While A, τ, f_c, we all know, we can use switch circuit to get x(nT), and as for $\sum_n x(nT)sinc[2\pi f_c(t-nT)]$, we need to get a N that is very big to make a similarity, it is obvious that when N is larger, the effect produced by the trailing phenomenon of sinc(t) is less. We need to send x(nT) at the transmit end. So using these method, we just need some add circuit and multiplying unit we can reconstruct x(t), and it can be totally done in receive end. Good realization.

In fact, we don't need to consider $\delta(t)$ when reconstruct x(t) *(I just understand reconstruct formula on Monday, thing above this paragraph is my thinking process, so I don't delete it)*, because from reconstruct formula Equation 9, there is no $\delta(t)$, we just use x(nT) to reconstruct x(t). But is it seems easy? No, if the signal x(t) is not a periodic function, its sampling frequency is difficlut to determine.

If we use Fourier series sampling method, x(t) can be reconstructed as the Equation 8 shows. It seems that, we just need to get C_n and use multiplying and adder unit, we can get x(t). But from the Equation 8, we can find that to get C_n, we need to integral $x(t)e^{j2\pi nt/T}$ to t and then multiply by 1/T. so to achieve this, we need integrator, multiplier. So just look at the variety of unit two method use, Fourier series sampling method needs more basic circuit. What is more important is that, when we calculate C_n, we need x(t), we're using x(t) before we reconstruct x(t), it is impossible for receive end to get C_n, so C_n need to be provide in transmit end. It need us to just send C_n from transmit end. Also, when derive the reconstruction formula using this method, we suppose that x(t) only has values at $t \in [-T/2, T/2]$, in real life, when x(t) isn't a time-limited signal, it is very difficult to do integral.

Compare two method, when using Shannon/Nyquist sampling method, we all operate with real function when rebuild x(t), also the equipment we need is less complex, but sampling frequency is difficult to determine; when using Fourier series sampling method, we operate with complex functions, we need to get the real part of the final result.(**We know that Fourier series both has exponential form and triangular form, to compare two method in a simple way, later we use triangular form for comparison. There are same in principle**). And in Fourier series way, the circuit we use is more complex. I think error rate of the former is less, because we can see things in our daily life, the more complex one thing is, the more error it will have. A car has more problem than a bike, and a bike has more problem than a chair. From these aspect it is hard for us to distinguish better or worse. Is it really that Shannon/Nyquist sampling method better than Fourier series sampling method?

We just consider the circuit two way use, but we don't consider effect produce by N. Both two ways we can't get a infinity sum, so we need to choose a finite N for reconstruction. Here I define a function f(t) as the Equation 14 shows. And I choose different N to reconstruct f(t). But before we do reconstruction, we need to decide the sampling frequency f_s. I take the fft f(t) in positive frequency axis, and as the Figure 1 shows. So we can suppose that f_m of f(t) is 7, so we can choose $f_s = 20$ then the cut-off frequency f_c of LPF is 7.

$$f(t) = \begin{cases} 1+t & -1<t<0 \\ 1-t & 0<t<1 \\ 0 & else \end{cases} \tag{14}$$

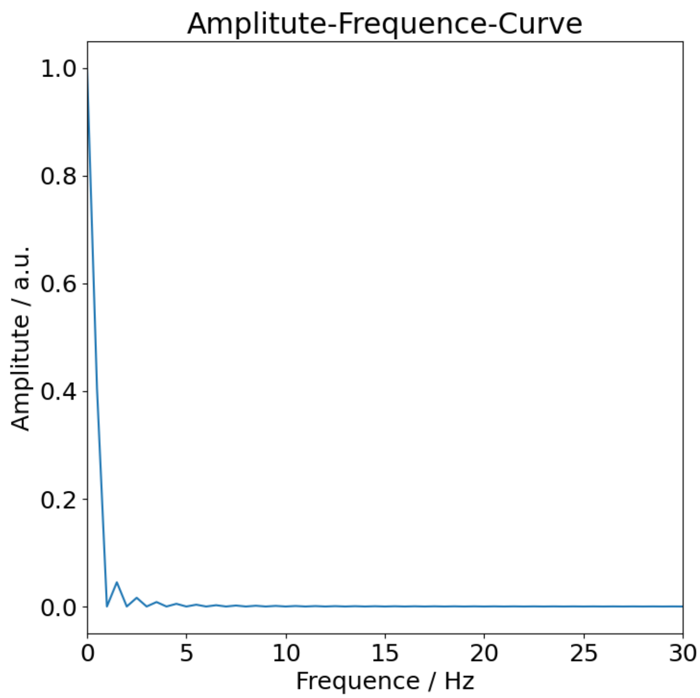


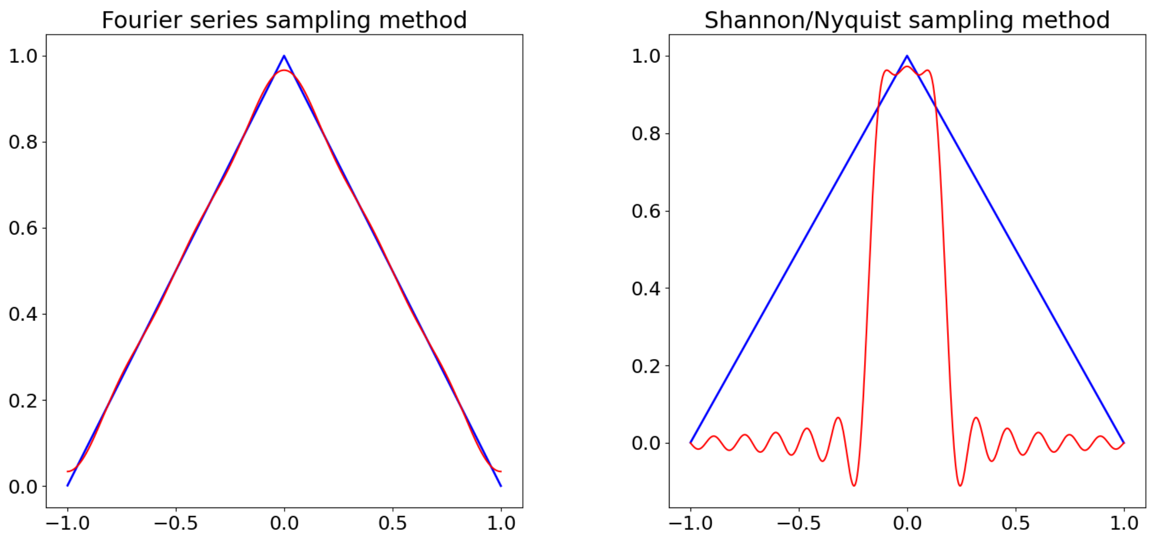
Figure 1: FFT of f(t)

So after determining these, we choose N= 6, 30, 60 to see the reconstruction result. As the Figure 2 shows. The red line is the reconstruction signal, and the blue line is f(t). From the figures, we can see that when N is same, reconstructed signal use Fourier series sampling method is more similar to f(t). And a small N can make signal reconstructed by Fourier series sampling method more close to real signal f(t). What does this mean? It meas that we only need a little samples we can reconstruct the signal very well. So all in all, I think Fourier series sampling method is better.

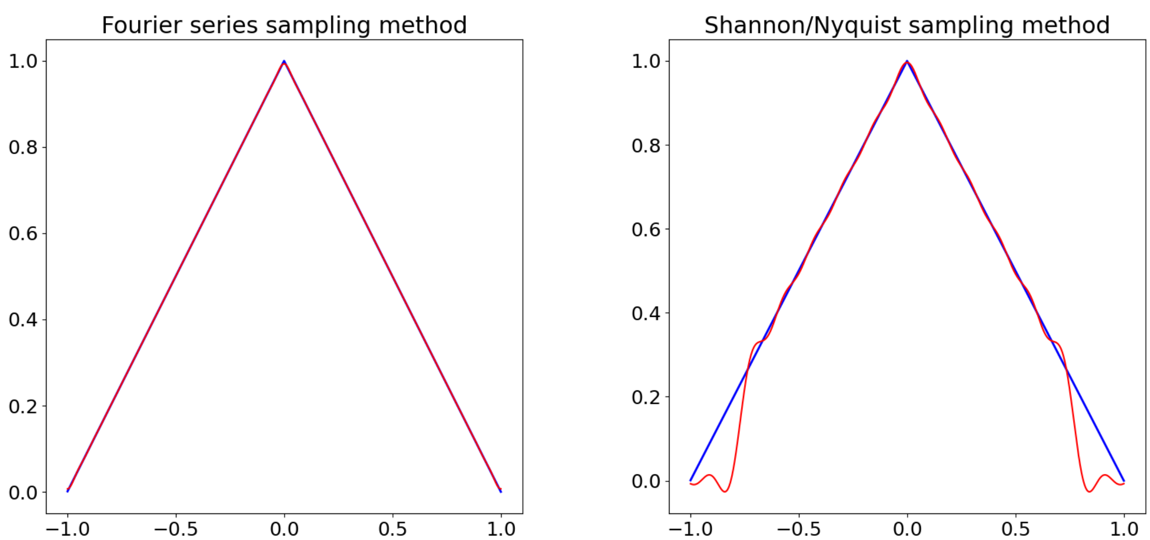
3 Conclusions

(1) Fourier series sampling method is better for reconstruction than Shannon/Nyquist sampling method.

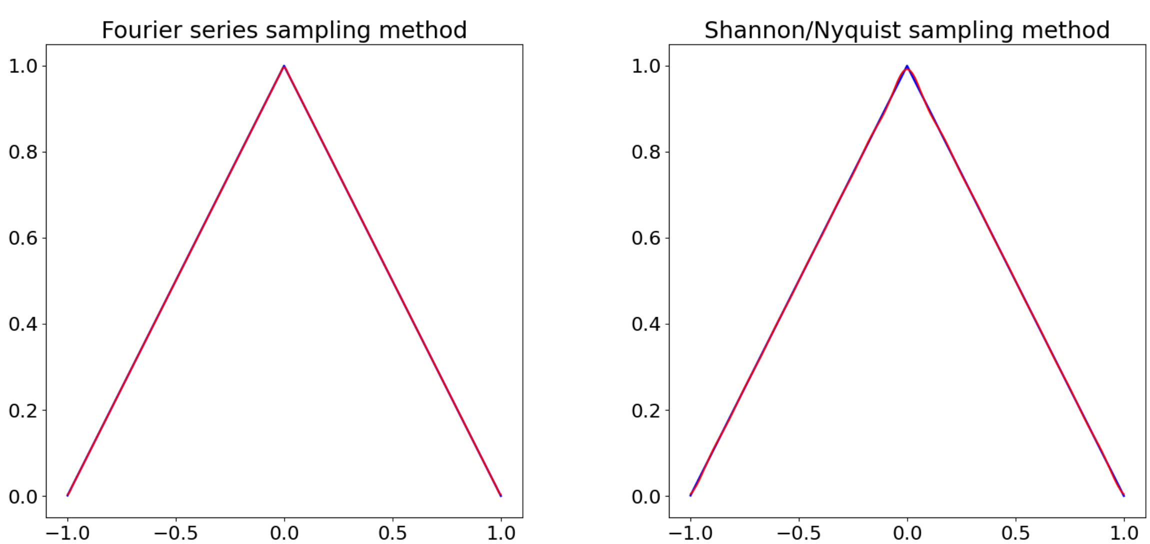
(2) A small number of summation terms can reconstruct a very good signal using Fourier series sampling method.



(a) N=6



(b) N=30



(c) N=60

Appendix A Code listings

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.fft import fft

plt.rcParams['figure.figsize'] = [8, 8]
plt.rcParams.update({'font.size': 18})

#lineNumber = 60

#lineNumber = 6

lineNumber = 30 # N

# Define domain
dx = 0.001
L = 1
x = L * np.arange(-1+dx,1+dx,dx)

def func(t):                # a function we use as x(t)
    if t<-1:
        return 0
    elif t<0:
        return 1+t
    elif t<1:
        return 1-t
    else:
        return 0
f=np.array([func(t) for t in x])

def myfft(x,t):             # fourier transform to determine f_s
    fft_x = fft(x)                                   #  fft computer
    amp_x = abs(fft_x)/len(x)*2
    label_x = np.linspace(0,int(len(x)/2)-1,int(len(x)/2)) # generate frequency range
    amp = amp_x[0:int(len(x)/2)]                     # only compute first half
    # amp[0] = 0                                     # choose whether to remove the direct traffic signal
    fs =1/( t[2]-t[1])                               # Calculate sampling frequency
    fre = label_x/len(x)*fs
    pha = np.unwrap(np.angle(fft_x))                 # The phase Angle was calculated and the 2pi jump was removed
    return amp,fre,pha

amp,fre,pha = myfft(f, x)

plt.figure(1)
plt.plot(fre,amp)
plt.xlim(0, 30)
plt.title('Amplitute-Frequence-Curve')
plt.ylabel('Amplitute / a.u.')
plt.xlabel('Frequence / Hz')


# Compute Fourier series triangle form
A0 = np.sum(f * np.ones_like(x)) * dx
fFS = A0/2

A = np.zeros(lineNumber)
B = np.zeros(lineNumber)
for k in range(lineNumber):
    A[k] = np.sum(f * np.cos(np.pi*(k+1)*x/L)) * dx # retangle integral
    B[k] = np.sum(f * np.sin(np.pi*(k+1)*x/L)) * dx
    fFS = fFS + A[k]*np.cos((k+1)*np.pi*x/L) + B[k]*np.sin((k+1)*np.pi*x/L) # fFs is reconstruce signal use
        fourier series
plt.figure(2)
plt.title('Fourier series sampling method')
plt.plot(x,f,'-',color='b',linewidth=2)
```

```python
    plt.plot(x,fFS,':', color='r')

    fc = 7 # from Fourier trans get

    Ts = 0.05 # from Fourier trans get

    n_rt = np.zeros_like(x)
    def recon(t, fc, k, Ts):
        return np.sinc(2 * fc * (t - k * Ts))

    for k in np.arange(-lineNumber/2,lineNumber/2 + 1):
        n_rt = func(k*Ts) * np.array([recon(t, fc, k, Ts) for t in x]) + n_rt

    n_rt =n_rt * fc * Ts * 2 # reconstruct use Nyquist way


    plt.figure(3)
    plt.title('Shannon/Nyquist sampling method')
    plt.plot(x,f,'-',color='b',linewidth=2)
    plt.plot(x,n_rt, ':', color='r')
    plt.show()
```