DSP Homework 06

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Contents

Abstract

Videos

To sum up the videos we watched this week.

Sampling Method developed with Fourier Series

According to Fourier series, the sampling method using $\delta(t)$ will involve infinity energy which is not realizable. So, I try to develop a new method.

1 Summary and Thoughts

1.1 Bluetooth

In this video, we know about the technique about Bluetooth, I found something special about the Bluetooth as below.

Points Worth Noting

1. Working on 123 mm wave length EM wave
2. Using 120.7 mm represent 1, 124.9 mm represent 0
3. Sending a Million Bits per Second in the format of Packets
4. Having 79 Communication Channel and shifting among them continuously

In these points, I want to write more about 3. and 4.

Packets

The packets are a set of lots of $'0'$ and $'1'$.

First 72 bits are called Access Code, which are used to synchronize smartphone and earbuds to make sure that it's your specific earbuds that is that received the message.

Next 54 bits are called Header. These data are used to decide how much data should be transfer next.

Last $136 \sim 8186$ bits are called Payload. There will be control signals like pause and play or music data in this part. So, this part can be long and short which is decided by the Header part.

Frequency Hopping Spread Spectrum

My understanding is there is a list of the number of 79 Communication Channels. The phone and the earbuds should use different channel once a packet is transferred. So, the list decides in which order the Communication Channels are used.

Time slots

The Bluetooth protocol states that a packet should be transferred in 625 μs, and the earbuds and the phone alternately use a time slot for data transmission.

1.2 Learning to Learn

This is a lecture about why we should keep learning and how we can learn better.

Life-long Learning

The lecturer made a list of famous rich people, and try to make audience find out the similarity among them. And it turns out that the most precious ability is life-long learning. So, as the lecturer said, our learning ability decides our earning capacity. I don't quite know whether this is right, but keeping on learning new things on a regular basis won't be bad.

More OUTPUT rather than INPUT

Saying goes that, knowledge, use it or lose it. Here's some suggestions about learning from the lecturer.

1. Focus on what we learn, in other words, do single-tasking, or we will kill our motivation
2. Reflect what we learn, in other words, don't read and wait for forgetting
3. To implement is far more better than just remembering
4. Share our knowledge with others, which is good for ourselves

1.3 Further Thoughts

In the Frequency Hopping Spread Spectrum technology, I don't fully understand how to keep the sender and the receiver knowing which channel to use next. So, I searching for how it works and I find that the Bluetooth protocol states that there must be a PIN code if the two devices want to pair. Then, the number of channels will be generated by pseudo-random code which is decided by the PIN code, so if other devices don't know the PIN code, it's impossible to hijack.

2 My Sampling Method

2.1 Restatement

Use the Fourier series to develop a sampling method and compare it with the Shannon/Nyquist sampling method through examples.

2.2 Derivation

In class, we are considering the energy of the Fourier transform of the $s(t)$, we have

$$s(t) = \sum_{n=-\infty}^{\infty} c_n \, e^{j2\pi nfT} \tag{1}$$

and we have

$$\begin{aligned} E &= \int_{-\frac{T}{2}}^{\frac{T}{2}} s(t) \cdot s^*(t) \, \mathrm{d}t \\ &= \int_{-\frac{T}{2}}^{\frac{T}{2}} \sum_{n=-\infty}^{\infty} c_m \cdot c_n \; e^{j2\pi(n-m)fT} \, \mathrm{d}t \\ &= \sum_{n=-\infty}^{\infty} c_m \cdot c_n \int_{-\frac{T}{2}}^{\frac{T}{2}} e^{j2\pi(n-m)fT} \, \mathrm{d}t \end{aligned}$$

and the integral of $e^{j2\pi(n-m)fT}$ usually be 0 except that $n-m$, so

$$E = \sum_{n=-\infty}^{\infty} c_n^2 \cdot T$$

I will try to prove that if $E < \infty$, we must ensure that $n \to \infty, c_n \to 0$, which means

$$E < \infty \Rightarrow n \to \infty, c_n \to 0 \tag{2}$$

if that

$$n \to \infty, c_n \to C\,(C \neq 0)$$

we can always find a big number N which makes $c_n^2 > 0\,(n > N)$, and we can easily find that

$$\sum_{n=N}^{\infty} c_n^2 > (\infty - N) \cdot c_{n(min)}^2 \to \infty$$

So, $E \to \infty$ when $n \to \infty, c_n \to C\,(C \neq 0)$. Therefore, Equa.2 is proved. Next, we need to find a energy-limited signals to do the sampling. And we know that

$$c_n = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} s(t)\, e^{-j2\pi n f_s t}\, \mathrm{d}t$$

So, the same as I find out last week in my weekly report, we know that if we let the square impulse have the width of 2τ, we now have

$$\begin{aligned}
s(t) &= \sum_{n=-\infty}^{\infty} \mathrm{rect}(\frac{t-nT}{\tau}) \\
&= \sum_{n=-\infty}^{\infty} F_n e^{j2\pi n f_s t} \\
&= \sum_{n=-\infty}^{\infty} \left[\frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \mathrm{rect}(\frac{t}{\tau}) e^{-j2\pi n f_s t} \mathrm{d}t\right] e^{j2\pi n f_s t} \\
&= \sum_{n=-\infty}^{\infty} \left[f_s \int_{-\tau}^{\tau} e^{-j2\pi n f_s t} \mathrm{d}t\right] e^{j2\pi n f_s t} \\
&= \sum_{n=-\infty}^{\infty} 2 f_s \,\mathrm{sinc}\,(2 f_s n \tau)\, e^{j2\pi n f_s t}
\end{aligned} \tag{3}$$

Here,

$$\begin{aligned}
s(t) &= \textstyle\sum_{n=-\infty}^{\infty} \mathrm{rect}(\frac{t-nT}{\tau}) \\
c_n &= 2 f_s \,\mathrm{sinc}\,(2 f_s n \tau)
\end{aligned}$$

Therefore

$$\begin{aligned}
E &= T \cdot \sum_{n=-\infty}^{\infty} c_n^2 \\
&= T \cdot \sum_{n=-\infty}^{\infty} sinc^2(2 f_s n \tau)
\end{aligned} \tag{4}$$

In help of Wolfram Alpha [1], I know that when a is real, this series (Eq. 4) converges. The result is shown in Fig. 1.

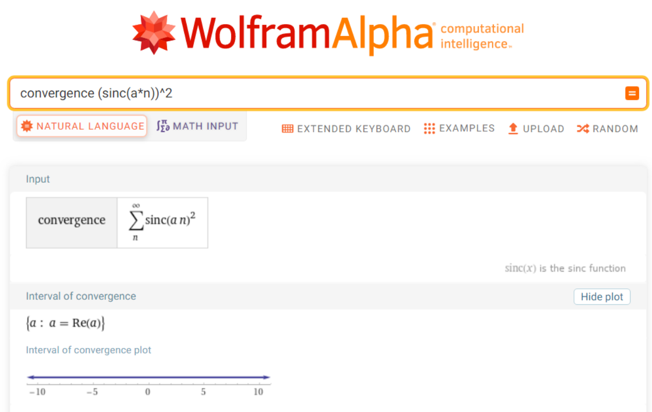

Figure 1: Convergence of $sinc^2(a \times n)$

Therefore

$$\sum_{n=-\infty}^{\infty} sinc^2(2f_s n\tau) < \infty$$

we can now say that the $s(t)$ mentioned in Equa.3 is energy-limited.

And back to Equa.4, we can control the order of the sum to improve this equation because the c_n decrease rapidly. Therefore

$$E = T \cdot \sum_{n=-N}^{N} sinc^2(2f_s n\tau)$$

So, let's use python to draw a picture to see more clearly, see Fig. 2.

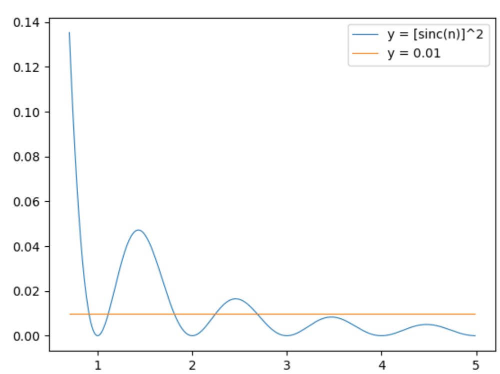

Figure 2: Orders of Sinc Function

It's easy to find that if we make n in this picture less than 5 or even 3 , little energy will be lost. This means we can rewrite the Equa. 4 like

$$E = T \cdot \sum_{n=-N}^{N} sinc^2(2f_s n\tau) \qquad N = [\frac{5}{2f_s\tau}] + 1 \tag{5}$$

If we look back on the $\delta(t)$ sampling, we have the energy E of $s(t)$:

$$E = \sum_{n=-\infty}^{\infty} T \to \infty \tag{6}$$

This (using $\sum \delta(t)$ to sampling) is not realizable. And, we can say that not the square function, but all energy-limited signals may be good for sampling.

3 Conclusion

Videos

After doing the summary of these videos, I further researched something about FHSS.

Sampling Method developed with Fourier Series

With the help of Fourier Series, I proved that it is more realizable to sample other signals with energy-limited signals like square function.

References

[1] https://www.wolframalpha.com/.

Appendix A Code Listing

```
import numpy as np
from sympy import sinc
from matplotlib import pyplot as plt

n = np.arange(0.7, 5, 0.01)
y = (np.sinc(n))**2
line = [0.01 for x in n]

fig = plt.figure()
plt.plot(n, y, linewidth=0.8, label="y = [sinc(n)]^2")
plt.plot(n, line, linewidth=0.8, label="y = 0.01")
plt.legend()
plt.show()
```