DSP Homework 03

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1.1 Our Planet

This video is basically about how beautiful our planet is and the truth that we are breaking its stability.

Like how our human social works, the nature has its our system and every single pieces of our beautiful planet are firmly connected. For example, the sea water evaporates at all times. The water vapor full of air will always condensates into drops, and fall from the air in the form of rain at one time. This is the natural water cycle, and the strong proof of the internal connection of nature.

What we are doing is making our planet warmer and warmer, and one of the results is the melting of glaciers. The fresh water from the glaciers will disturb the sea's current, change the salinity, and eventually affect the nature ecology.

The nature is more complex than what we thought. The rapid development of mankind from last century has been damaging the health of our only Mother Earth. Just as somebody says, 'We're the first generation to know what we're doing, and the last who have a chance to put things right'. There definitely are something which must be preserved if we are to ensure a future where humans and nature can thrive.

1.2 Wireless Earbuds

This video do the 4 things below:

1. Earbuds Disassembly and Introduction to Every Module in the Earbuds
2. Discuss the Audio Codec
3. Sampling Rate and Bit-depth
4. Music File Formats

For wired headphones, electricity flows from our smartphones through the wires to the headphones. In the wire, there's only analog signals travel between the smartphone and the headphone. However, for wireless earbuds, this doesn't work because it's difficult to send analog signals to such a tiny thing through the channel of air. The AirPods 2 relies on the technology of Bluetooth, DAC and the theory of the speaker to provide its service.

About the audio codec, codec stands for coding and decoding. In playing music, codec do the decoding, which means it converts the music data from digital values to analog waves. So, the coding means the reverse of that process.

We were talking about the sampling rate last week, and that's how we make continuous waves into a set of discrete pulses. The most common rate is 44.1 kHz, and the second popular one is 48 kHz.

However, we cannot save the pulses with irrational size as binary, so we must do the quantization – making all numbers in an interval fall into the center point (or other certain points) of this interval. The bit-depth is the number how many binary numbers (this also means how many levels do we have in the process of quantization) we use to represent one pulse.

Audio File Format can be divided into lossy format and lossless format according to whether part of the original audio data will be deleted. MP3 and AAC is a common lossy format, and their algorithm will find and delete the part of the original audio data that is hard for human ears to detect. ALAC and FLAC are common lossless formats. They only compress the original audio data in use of lossless compression algorithm and don't delete any original data.

The wireless earbuds is everywhere in our life. Maybe we all know that the sound is wave and the earbuds are just the processer of them, but few of us try to know about how pratically this tiny thing works. So, keep curiosity is crucial.

1.3 Benefits of Exercise

In this TED talks, we are suggested to do the proper amount of the exercise to improve our brain.

Our brains are get older while our bodies are getting older, especially Hippocampus, this talker mentioned. But we do have ways to slow that process – exercise.

She, the talker, experienced the change exercise gives her and her brain, so she shifted the goal of her lab and find out a new way to make or just keep people's minds brilliant.

Now, she's going to find out what the most proper amount of exercise of one specific person, and I think this means a lot for those who sit before their computers all day long.

Saying goes, life lies in sports, and we all know it. What's more, people today are clever enough to find out the most complex relationship between 'life' and 'sports'.

2 Problem 2

3 Problem 3

3.1 Problem Restatement

The Fourier transform pair can be defined as

$$\begin{aligned} \widetilde{x}(f) &= \int_{-\infty}^{\infty} x(t)\ e^{-j2\pi ft}\mathrm{d}t \\ x(f) &= \int_{-\infty}^{\infty} \widetilde{x}(f)\ e^{j2\pi ft}\mathrm{d}f \end{aligned}$$

Prove the following

$$\begin{array}{llll}
(1) & ax(t)+by(t) & \leftrightarrow\ a\widetilde{x}(f)+b\widetilde{y}(f) & \text{Linearity} \\
(2) & x(st) & \leftrightarrow\ \frac{1}{|s|}\widetilde{x}(\frac{f}{s}) & \text{Scaling} \\
(3) & x^*(t) & \leftrightarrow\ \widetilde{x}^*(-f) & \text{Conjugate} \\
(4) & \widetilde{x}(t) & \leftrightarrow\ x(-t) & \text{Duality} \\
(5) & x(t-t_0) & \leftrightarrow\ e^{-j2\pi t_0 f}\widetilde{x}(f) & \text{Timeshift} \\
(6) & e^{-j2\pi f_0 t}x(st) & \leftrightarrow\ \widetilde{x}(f-f_0) & \text{Frequencyshift} \\
(7) & x'(t) & \leftrightarrow\ j2\pi f\widetilde{x}(f) & \text{Differentiation} \\
(8) & \int x(\tau)y(t-\tau)\mathrm{d}\tau & \leftrightarrow\ \frac{1}{|s|}\widetilde{x}(\frac{f}{s}) & \text{Convolution}
\end{array}$$

3.2 Proof

(1)

$$\begin{aligned}\mathcal{F}[ax(t)+by(t)] &= \int_{-\infty}^{\infty}[ax(t)+by(t)]e^{-j2\pi ft}\mathrm{d}t \\ &= \int_{-\infty}^{\infty} ax(t)e^{-j2\pi ft}\mathrm{d}t + \int_{-\infty}^{\infty} by(t)e^{-j2\pi ft}\mathrm{d}t \\ &= a\widetilde{x}(t)+b\widetilde{y}(t)\end{aligned}$$

(2) if $s>0$,

$$\mathcal{F}[x(st)] = \int_{-\infty}^{\infty} x(st)\ e^{-j2\pi ft}\mathrm{d}t$$
$$\mathcal{F}[x(st)] = \frac{1}{s}\int_{-\infty}^{\infty} x(st)e^{-j2\pi(\frac{f}{s})(st)}\mathrm{d}(st)$$

let t' represent st,

$$\begin{aligned}\mathcal{F}[x(t')] &= \frac{1}{s}\int_{-\infty}^{\infty} x(t')e^{-j2\pi(\frac{f}{s})(t')}\mathrm{d}(t') \\ &= \frac{1}{s}\ \widetilde{x}(\frac{f}{s}) = \frac{1}{s}\ \widetilde{x}(\frac{f}{|s|})\end{aligned}$$

If $s\leq 0$,

$$\begin{aligned}\mathcal{F}[x(st)] &= \frac{1}{s}\int_{\infty}^{-\infty} x(st)e^{-j2\pi(\frac{f}{s})(st)}\mathrm{d}(st) \\ &= \frac{1}{-s}\int_{-\infty}^{\infty} x(st)e^{-j2\pi(\frac{f}{s})(st)}\mathrm{d}(st) \\ &= \frac{1}{-s}\ \widetilde{x}(\frac{f}{s}) = \frac{1}{|s|}\ \widetilde{x}(\frac{f}{s})\end{aligned}$$

(3)

$$\begin{aligned}\mathcal{F}[x^*(t)] &= \int_{-\infty}^{\infty} x^*(t)\ e^{-j2\pi ft}\mathrm{d}t \\ &= \left[\int_{-\infty}^{\infty}[x^*(t)\ e^{-j2\pi ft}]^*\mathrm{d}t\right]^* \\ &= \left[\int_{-\infty}^{\infty}[x(t)]\ e^{-j2\pi(-f)t}\mathrm{d}t\right]^* \\ &= [\widetilde{x}(-f)]^*\end{aligned}$$

(4)

$$\widetilde{x}(f) = \int_{-\infty}^{\infty} x(t)\ e^{-j2\pi ft}\mathrm{d}t$$

Exchange f and t,

$$\begin{aligned}
\widetilde{x}(t) &= \int_{-\infty}^{\infty} x(f)\ e^{-j2\pi ft}\mathrm{d}f \\
&= \int_{\infty}^{-\infty} x(-f)\ e^{j2\pi ft}\mathrm{d}(-f) \\
&= \int_{-\infty}^{\infty} x(-f)\ e^{j2\pi ft}\mathrm{d}f \\
&= \mathcal{F}[x(-f)]
\end{aligned}$$

(5)

$$\mathcal{F}[x(t-t_0)] = \int_{-\infty}^{\infty} x(t-t_0)e^{-j2\pi ft}\mathrm{d}t$$

let t be $t+t_0$,

$$\begin{aligned}
\mathcal{F}[x(t-t_0)] &= \int_{-\infty}^{\infty} x(t)e^{-j2\pi f(t+t_0)}\mathrm{d}(t+t_0) \\
&= e^{-j2\pi ft_0}\int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}\mathrm{d}(t) \\
&= e^{-j2\pi ft_0}\ \widetilde{x}(t)
\end{aligned}$$

(6)

$$\begin{aligned}
\mathcal{F}\left[e^{-j2\pi f_0 t}x(t)\right] &= \int_{-\infty}^{\infty} x(t)\ e^{-j2\pi(f-f_0)t}\mathrm{d}t \\
&= \widetilde{x}(f-f_0)
\end{aligned}$$

(7)

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}x(t) &= \frac{\mathrm{d}}{\mathrm{d}t}\mathcal{F}^{-1}[\widetilde{x}(f)] \\
&= \frac{\mathrm{d}}{\mathrm{d}t}\int_{-\infty}^{\infty} \widetilde{x}(f)\cdot e^{j2\pi ft}\cdot\mathrm{d}f \\
&= \int_{-\infty}^{\infty} \widetilde{x}(f)\cdot\frac{\mathrm{d}}{\mathrm{d}t}e^{j2\pi ft}\cdot\mathrm{d}t \\
&= \int_{-\infty}^{\infty} \widetilde{x}(f)\cdot j2\pi f\cdot e^{j2\pi ft}\cdot\mathrm{d}t \\
&= \int_{-\infty}^{\infty} [j2\pi f\cdot\widetilde{x}(f)]\cdot e^{j2\pi ft}\mathrm{d}t \\
&= \mathcal{F}[j2\pi f\cdot\widetilde{x}(f)]
\end{aligned}$$

(8)

$$\begin{aligned}
\mathcal{F}[x_1(t)*x_2(t)] &= \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} x_1(\tau)\ x_2(t-\tau)\mathrm{d}\tau\right]\ e^{-j2\pi ft}\mathrm{d}t \\
&= \int_{-\infty}^{\infty} x_1(\tau)e^{-j2\pi f\tau}\mathrm{d}\tau\int_{-\infty}^{\infty} x_2(t-\tau)e^{-j2\pi f(t-\tau)}\mathrm{d}(t-\tau) \\
&= \int_{-\infty}^{\infty} x_1(\tau)e^{-j2\pi f\tau}\mathrm{d}\tau\ \int_{-\infty}^{\infty} x_2(t)e^{-j2\pi ft}\mathrm{d}t \\
&= \widetilde{x}_1(f)\cdot\widetilde{x}_2(f)
\end{aligned}$$