Ejercicio 3.29

Mérida de León Luis Ernesto

Encontrar los coeficientes complejos de Fourier y dibujar los espectros de frecuencia para la función diente de sierra definida por:

$$f(t) = -\frac{1}{T}t + \frac{1}{2} \quad \text{para } 0<t<T \text{ y } f(t+T)=f(t) \longrightarrow w_0 = \frac{2\pi}{T}$$

Calculamos los coeficientes:

- $$a_0 = \frac{2}{T}\int_0^T f(t)dt = \frac{2}{T}\int_0^T \left(-\frac{t}{T}+\frac{1}{2}\right)dt = -\frac{2}{T^2}\int_0^T t\,dt + \frac{2}{T}\left(\frac{1}{2}\right)\int_0^T dt = -\frac{2}{T^2}\left[\frac{t^2}{2}\right]\Big|_0^T + \frac{1}{T}\left[t\right]\Big|_0^T$$

$$a_0 = -\frac{2}{T^2}\left[\frac{T^2}{2}-0\right] + \frac{1}{T}\left[T-0\right] = -1+1 = 0 \qquad \therefore a_0 = 0$$

- $$a_n = \frac{2}{T}\int_0^T f(t)\cos(nw_0t)dt = \frac{2}{T}\int_0^T\left(-\frac{t}{T}+\frac{1}{2}\right)\cos\left(n\frac{2\pi}{T}t\right)dt = -\frac{2}{T^2}\int_0^T \underbrace{t\cos\left(\frac{2n\pi t}{T}\right)dt}_{I} + \frac{1}{T}\int_0^T \underbrace{\cos\left(\frac{2n\pi t}{T}\right)dt}_{II}$$

Calculando I:

$$\int t\cos\left(\frac{2n\pi t}{T}\right)dt = t\left(\frac{T}{2\pi n}\cdot \operatorname{sen}\left(\frac{2\pi nt}{T}\right)\right) - \int \frac{T}{2\pi n}\cdot\operatorname{sen}\left(\frac{2\pi nt}{T}\right)dt$$

$u = t$
$du = dt$

$dv = \cos\left(\frac{2\pi nt}{T}\right)$

$v = \int\cos\left(\frac{2\pi nt}{T}\right)dt$

$w = \frac{2\pi nt}{T}$

$dw = \frac{2\pi n}{T}dt \longrightarrow dw\left(\frac{T}{2\pi n}\right) = dt$

$v = \int\cos(w)\cdot dw\left(\frac{T}{2\pi n}\right) = \frac{T}{2\pi n}\cdot\operatorname{sen}\left(\frac{2\pi nt}{T}\right)$

$$= \frac{tT}{2\pi n}\operatorname{sen}\left(\frac{2\pi nt}{T}\right) - \frac{T}{2\pi n}\underbrace{\int\operatorname{sen}\left(\frac{2\pi nt}{T}\right)dt} = \frac{tT}{2\pi n}\operatorname{sen}\left(\frac{2\pi nt}{T}\right) - \frac{T}{2\pi n}\left(-\frac{T}{2\pi n}\cos\left(\frac{2\pi nt}{T}\right)\right)$$

$w = \frac{2\pi nt}{T} \longrightarrow \int\operatorname{sen}(w)\cdot dw\left(\frac{T}{2\pi n}\right) = -\frac{T}{2\pi n}\cos\left(\frac{2\pi nt}{T}\right)$

$dw\left(\frac{T}{2\pi n}\right) = dt$

$$= \frac{tT}{2\pi n}\operatorname{sen}\left(\frac{2\pi nt}{T}\right) - \frac{T}{2\pi n}\left(-\frac{T}{2\pi n}\cos\left(\frac{2\pi nt}{T}\right)\right) = \frac{tT}{2\pi n}\operatorname{sen}\left(\frac{2\pi nt}{T}\right) + \frac{T^2}{(2\pi n)^2}\cos\left(\frac{2\pi nt}{T}\right)$$

Calculando II:

$$\int\cos\left(\frac{2\pi nt}{T}\right)dt \qquad \int\cos(w)\cdot dw\left(\frac{T}{2\pi n}\right) = \frac{T}{2\pi n}\cdot\operatorname{sen}\left(\frac{2\pi nt}{T}\right)$$

$w = \frac{2\pi nt}{T}$

$dw = \frac{2\pi n}{T}dt \longrightarrow dw\left(\frac{T}{2\pi n}\right) = dt$

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Sustituyendo a_n:

$$a_n = -\frac{2}{T^2}\left[\cancel{\frac{tT}{2\pi n}\,\text{sen}\left(\frac{2\pi n t}{T}\right)}^{\,0} + \frac{T^2}{(2\pi n)^2}\cos\left(\frac{2\pi n t}{T}\right)\right]\Bigg|_0^T + \frac{1}{T}\left[\cancel{\frac{T}{2\pi n}\cdot \text{sen}\left(\frac{2\pi n t}{T}\right)}^{\,0}\right]\Bigg|_0^T$$

$$a_n = -\frac{2}{T^2}\left[\frac{T^2}{(2n\pi)^2}\cos\left(\frac{2\pi n t}{T}\right)\right]\Bigg|_0^T = -\frac{2}{T^2}\left[\frac{T^2}{(2\pi n)^2}\cos\left(\frac{2\pi n T}{T}\right) - \frac{T^2}{(2\pi n)^2}\cos\left(\cancel{\frac{2\pi n(0)}{T}}^{\,0}\right)\right]$$

$$a_n = -\frac{2}{\cancel{T^2}}\left[\frac{\cancel{T^2}}{(2\pi n)^2}\cos(2\pi n) - \frac{\cancel{T^2}}{(2\pi n)^2}(1)\right]$$

$$a_n = -\frac{2}{(2\pi n)^2}\cos(2\pi n) + \frac{2}{(2\pi n)^2}$$

$$a_n = \frac{2}{(2\pi n)^2}\left(1 - \cos(2\pi n)\right)$$

$$a_n = \frac{2}{(2\pi n)^2}\left(\cancel{1-1}^{\,0}\right)$$

n	$\cos(2\pi n)$
1	$\cos(2\pi) = 1$
2	$\cos(4\pi) = 1$
3	$\cos(6\pi) = 1$
4	$\cos(8\pi) = 1$

$\cos(2\pi n) = 1$

$$\therefore\ a_n = 0$$

- $$b_n = \frac{2}{T}\int_0^T f(t)\,\text{sen}(n\omega_0 t)\,dt = \frac{2}{T}\int_0^T\left(-\frac{t}{T}+\frac{1}{2}\right)\text{sen}\left(n\frac{2\pi}{T}t\right)dt = -\frac{2}{T^2}\underbrace{\int_0^T t\,\text{sen}\left(\frac{2n\pi t}{T}\right)dt}_{\text{III}} + \frac{1}{T}\underbrace{\int_0^T \text{sen}\left(\frac{2n\pi t}{T}\right)dt}_{\text{IV}}$$

Calculando III:

$$\int t\,\text{sen}\left(\frac{2n\pi t}{T}\right)dt = t\left(-\frac{T}{2\pi n}\cos\left(\frac{2\pi n t}{T}\right)\right) - \frac{T}{2\pi n}\int\cos\left(\frac{2\pi n t}{T}\right)dt = -\frac{tT}{2\pi n}\cos\left(\frac{2\pi n t}{T}\right) - \frac{T}{2\pi n}\int\cos\left(\frac{2\pi n t}{T}\right)dt$$

$u = t$ $\quad dv = \text{sen}\left(\frac{2\pi n t}{T}\right)$

$du = dt$ $\quad v = \int \text{sen}\left(\frac{2\pi n t}{T}\right)dt$

$w = \frac{2\pi n t}{T}$

$dw = \frac{2\pi n}{T}dt \longrightarrow dw\left(\frac{T}{2\pi n}\right) = dt$

$v = \int \text{sen}(w)\cdot dw\left(\frac{T}{2\pi n}\right) = -\frac{T}{2\pi n}\cos\left(\frac{2\pi n t}{T}\right)$

$$= -\frac{tT}{2\pi n}\cos\left(\frac{2\pi n t}{T}\right) - \frac{T}{2\pi n}\int\cos\left(\frac{2\pi n t}{T}\right)dt = -\frac{tT}{2\pi n}\cos\left(\frac{2\pi n t}{T}\right) - \frac{T}{2\pi n}\left(\frac{T}{2\pi n}\,\text{sen}\left(\frac{2\pi n t}{T}\right)\right)$$

$w = \frac{2\pi n t}{T} \longrightarrow \int \cos(w)\cdot dw\left(\frac{T}{2\pi n}\right) = \frac{T}{2\pi n}\,\text{sen}\left(\frac{2\pi n t}{T}\right)$

$dw\left(\frac{T}{2\pi n}\right) = dt$

$$= -\frac{tT}{2\pi n}\cos\left(\frac{2\pi n t}{T}\right) - \frac{T^2}{(2\pi n)^2}\,\text{sen}\left(\frac{2\pi n t}{T}\right)$$

Calculando IV:

$$\int \text{sen}\left(\frac{2n\pi t}{T}\right)dt = \int \text{sen}(w)\cdot dw\left(\frac{T}{2\pi n}\right) = -\frac{T}{2\pi n}\cos\left(\frac{2\pi n t}{T}\right)$$

$w = \frac{2\pi n t}{T}$

$dw = \frac{2\pi n}{T}dt \longrightarrow dw\left(\frac{T}{2\pi n}\right) = dt$

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Sustituyendo III, IV en bn:

$$b_n = \frac{2}{T}\int_0^T f(t)\,sen(n\omega_0 t)\,dt = -\frac{2}{T^2}\underbrace{\int_0^T t\,sen\left(\frac{2n\pi t}{T}\right)dt}_{III} + \frac{1}{T}\underbrace{\int_0^T sen\left(\frac{2n\pi t}{T}\right)dt}_{IV}$$

$$= -\frac{2}{T^2}\left[-\frac{tT}{2\pi n}\cos\left(\frac{2\pi n t}{T}\right) - \frac{T^2}{(2\pi n)^2}\cancel{sen\left(\frac{2\pi n t}{T}\right)}^{0}\right]\Bigg|_0^T + \frac{1}{T}\left[-\frac{T}{2\pi n}\cos\left(\frac{2\pi n t}{T}\right)\right]\Bigg|_0^T$$

$$= -\frac{2}{T^2}\left[\frac{-tT}{2\pi n}\cos\left(\frac{2\pi n t}{T}\right)\right]\Bigg|_0^T + \frac{1}{T}\left[-\frac{T}{2\pi n}\cos\left(\frac{2\pi n t}{T}\right)\right]\Bigg|_0^T$$

$$= -\frac{\cancel{2}}{\cancel{T^2}}\left[\frac{-T(T)}{\cancel{2}\pi n}\cos\left(\frac{2\pi n \cancel{T}}{\cancel{T}}\right) - 0\right] + \frac{1}{T}\left[-\frac{T}{2\pi n}\cos\left(\frac{2\pi n \cancel{T}}{\cancel{T}}\right) + \frac{T}{2\pi n}\cancel{\cos(0)}^{1}\right]$$

$$= \frac{1}{\pi n}\cos(2\pi n) - \frac{1}{2\pi n}\cos(2\pi n) + \frac{1}{2\pi n}$$

$\boxed{\cos(2\pi n) = 1 \longleftarrow \text{En el calculo de } a_n \text{ se muestra el porqué es } 1}$

Lo aplicamos a bn:

$$b_n = \frac{1}{\pi n}(1) - \cancel{\frac{1}{2\pi n}(1)} + \cancel{\frac{1}{2\pi n}}$$

$$\therefore b_n = \frac{1}{\pi n}$$

Ahora, formulamos Cn (número complejo):

$$\boxed{C_n = \frac{a_n - i b_n}{2}} \quad ; \quad \boxed{C_0 = \frac{a_0}{2}}$$

Recordemos los valores: $a_0 = 0,\ a_n = 0,\ b_n = \frac{1}{\pi n}$

$$C_n = \frac{0 - i\left(\frac{1}{\pi n}\right)}{2} = \frac{\frac{-i}{\pi n}}{\frac{2}{1}} = \frac{-i}{2\pi n}\left(\frac{i}{i}\right) = \frac{-i^2}{2\pi n i} = \frac{-(-1)}{2\pi n i} = \frac{1}{2\pi n i}$$

$$\therefore C_n = \frac{1}{2\pi n i}$$

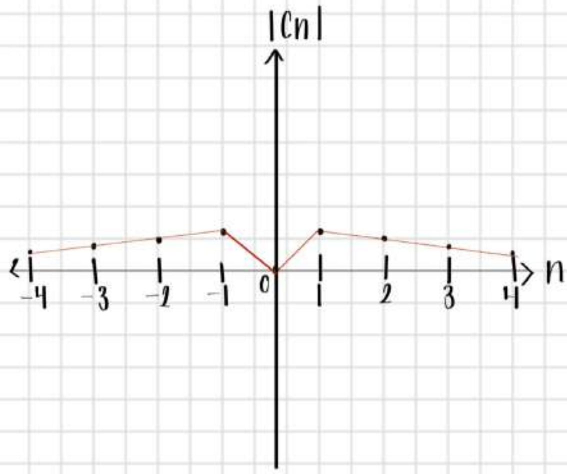
$$C_0 = \frac{0}{2} = 0$$

$$\therefore C_0 = 0$$

Graficamos C_0, $|C_n|$ vs $n\omega_0$

n	
1	$\frac{1}{2\pi(1)} = 0.1591$
2	$\frac{1}{2\pi(2)} = 0.07957$
3	$\frac{1}{2\pi(3)} = 0.05305$
4	$\frac{1}{2\pi(4)} = 0.03978$

n	
-1	$\frac{1}{2\pi(1)} = -0.1591$
-2	$\frac{1}{2\pi(2)} = -0.07957$
-3	$\frac{1}{2\pi(3)} = -0.05305$
-4	$\frac{1}{2\pi(4)} = -0.03978$



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Geogebra: